2.20

(a)

Solution.

$$\int_{-\infty}^{\infty} u_0(t)\cos(t)\,\mathrm{d}t = \cos 0 = 1.$$

(b)

Solution.

$$\int_{0}^{5} \sin(2\pi t)\delta(t+3)\,\mathrm{d}t = 0.$$

(c)

Solution.

$$\int_{-5}^{5} u_1(1-\tau)\cos(2\pi\tau)\,\mathrm{d}\tau = \int_{-5}^{1} \cos(2\pi\tau)\,\mathrm{d}\tau = 0.$$

2.21

(a)

Solution.

$$y[n] = \sum_{k=-\infty}^{\infty} x[n]h[n] = \sum_{k=0}^{n} \alpha^k\beta^{n-k} = \frac{\alpha^{n+1}-\beta^{n+1}}{\alpha-\beta}.$$

2.22

(c)

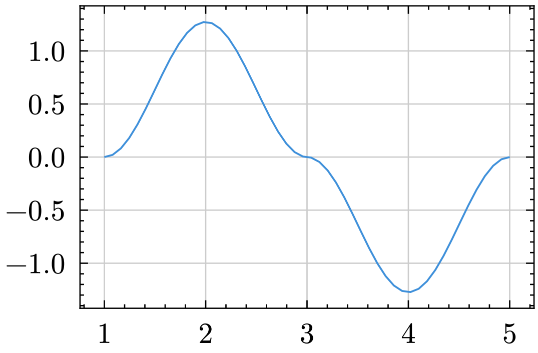
Solution. The desired convolution is

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,\mathrm{d}\tau = \int_{0}^{2} \sin(\pi\tau)h(t-\tau)\,\mathrm{d}\tau.$$

This gives us

$$y(t) = \begin{cases} 0, & t<1 \\ 2/\pi(1-\cos(\pi(t-1))), & 1<t<3 \\ 2/\pi(\cos(\pi(t-3))-1), & 3<t<5 \\ 0, & t>5 \end{cases}$$

as shown in the figure below.



2.28

(c)

Solution. Not causal because $h[n] = 2^n > 0$ for $n < 0$. Unstable because

$$\sum_{n=-\infty}^{\infty} h[n] = \sum_{n=0}^{\infty} 2^n = \infty.$$

2.29

(g)

Solution. Causal because $h(t) = 0$ for $t < 0$. Unstable because

$$\int_{-\infty}^{\infty} |h(\tau)| \,\mathrm{d}\tau \geq \int_{100}^{\infty} \left(e^{\frac{x}{100}-1} - 1\right) \mathrm{d}\tau = \infty.$$

2.33

Solution. We may solve this ODE first. Multiplying both sides by e^{2t}, we get

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(e^{2t} y(t)\right) = e^{2t} x(t).$$

Integrating both sides from t_0 to t now gives us

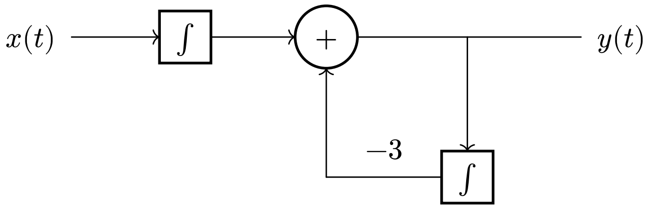
$$e^{2t} y(t) - e^{2t_0} y(t_0) = \int_{t_0}^{t} e^{2\tau} x(\tau) \,\mathrm{d}\tau.$$

For (a), let $t_0 = 0$. Then the linearity of the system follows from the linearity of the integral. For (b), since $\mathrm{d}t = \mathrm{d}(t - T)$, we can safely replace t with $t - T$ in the above equation without breaking its properties.

2.39

(b)

Solution. The block diagram is as shown in the figure below.



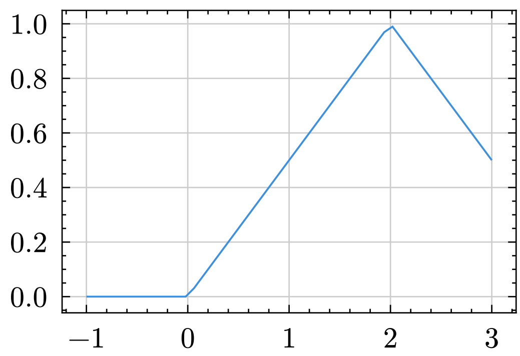
2.47

(b)

Solution. We have

$$y(t) = x(t) * h(t) = x_0(t) * h_0(t) - x_0(t-2) * h_0(t) = y_0(t) - y_0(t-2),$$

as shown in the figure below.



(d)

Solution. Not enough information.

(f)

Solution. We have

$$y(t) = \int_{-\infty}^{\infty} \frac{\partial x}{\partial t}(\tau) \frac{\partial h}{\partial t}(t-\tau)\,\mathrm{d}\tau = \frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(\int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,\mathrm{d}\tau\right) = \frac{\mathrm{d}^2}{\mathrm{d}t^2} y(t).$$

as shown in the figure below.