Training a Neural Network

In this homework, our objective is to implement a simple neural network from scratch, in particular, error backpropagation and the gradient descent optimization procedure. We first import some useful libraries.

```
import numpy
import matplotlib
%matplotlib inline
from matplotlib import pyplot as plt
na = numpy.newaxis
numpy.random.seed(0)
```

We consider a two-dimensional moon dataset on which to train the network. We also create a grid dataset which we will use to visualize the decision function in two dimensions. We denote our two inputs as x_1 and x_2 and use the suffix `d` and `g` to designate the actual dataset and the grid dataset.

```
# Create a moon dataset on which to train the neural network
import sklearn,sklearn.datasets
Xd,Td = sklearn.datasets.make_moons(n_samples=100)
Xd = Xd*2-1
Td = Td * 2 - 1
X1d = Xd[:,0]
X2d = Xd[:,1]

# Creates a grid dataset on which to inspect the decision function
l = numpy.linspace(-4,4,100)
X1g,X2g = numpy.meshgrid(l,l)
```

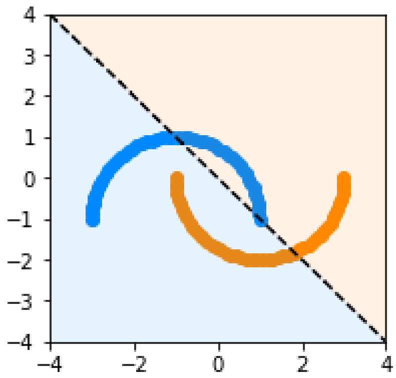
```
print(X1g.shape, X2g.shape)
```

```
(100, 100) (100, 100)
```

The moon dataset is plotted below along with some dummy decision function $x_1+x_2=0$.

```
def plot(Yg,title=None):
    plt.figure(figsize=(3,3))
    plt.scatter(*Xd[Td==-1].T,color='#0088FF')
    plt.scatter(*Xd[Td==1].T,color='#FF8800')
    plt.contour(X1g,X2g,Yg,levels=[0],colors='black',linestyles='dashed')
    plt.contourf(X1g,X2g,Yg,levels=[-100,0,100],colors=
['#0088FF','#FF8800'],alpha=0.1)
    if title is not None: plt.title(title)
    plt.show()

plot(X1g+X2g) # plot the dummy decision function
```



Part 1: Implementing Error Backpropagation (30 P)

We would like to implement the neural network with the equations:
\begin{align}
\forall{j=1}^{25}:~ z_j &= x_1 w{1j} + x_2 w{2j} + b_j\
\forall{j=1}^{25}:~a_j &= \max(0,z_j)\
y &= \sum_{j=1}^{25} a_j v_j
\end{align}
where x_1,x_2 are the two input variables and y is the output of the network. The parameters of the neural network are initialized randomly using the normal distributions $w{ij} \sim \mathcal{N}(\mu=0,\sigma^2=1/2)$, $b{j} \sim \mathcal{N}(\mu=0,\sigma^2=1)$, $v_{j} \sim \mathcal{N}(\mu=0,\sigma^2=1/25)$. The following code initializes the parameters of the network and implements the forward pass defined above. The neural network is composed of 50 neurons.

```
import numpy

NH = 50

W = numpy.random.normal(0,1/2.0**.5,[2,NH])
B = numpy.random.normal(0,1,[NH])
V = numpy.random.normal(0,1/NH**.5,[NH])

def forward(X1,X2):
    X = numpy.array([X1.flatten(),X2.flatten()]).T # Convert meshgrid into 
dataset
    Z = X.dot(W)+B
    A = numpy.maximum(0,Z)
    Y = A.dot(V)
    return Y.reshape(X1.shape) # Reshape output into meshgrid
```

```
print(W.shape, B.shape, V.shape)
```

```
(2, 50) (50,) (50,)
```

We now consider the task of training the neural network to classify the data. For this, we define the error function:

$$\mathcal{E}(\theta) = \sum_{k=1}^{N} \max(0, -y^{(k)} t^{(k)})$$

where N is the number of data points, y is the output of the network and t is the label.

Task:

- **Complete the function below so that it returns the gradient of the error w.r.t. the parameters of the model.**

```
import numpy as np

def backprop(X1,X2,T):
    X = numpy.array([X1.flatten(),X2.flatten()]).T

    # Compute activations
    Z = X.dot(W)+B
    A = numpy.maximum(0,Z)
    Y = A.dot(V)

    # Compute backward pass
    DY = (-Y*T>0)*(-T)
    DZ = numpy.outer(DY,V)*(Z>0)

    # Compute parameter gradients (averaged over the whole dataset)

    DV = DY.T @ A
    DB = DZ.T @ np.ones(100, )
    DW = X.T @ DZ

    return DW,DB,DV
```

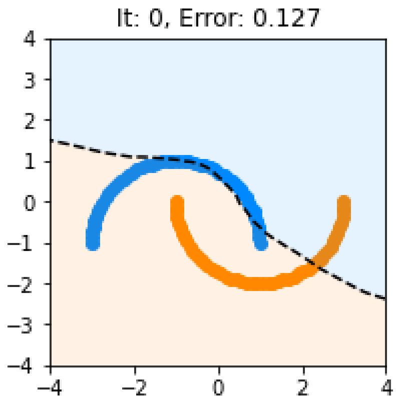
Exercise 2: Training with Gradient Descent (20 P)

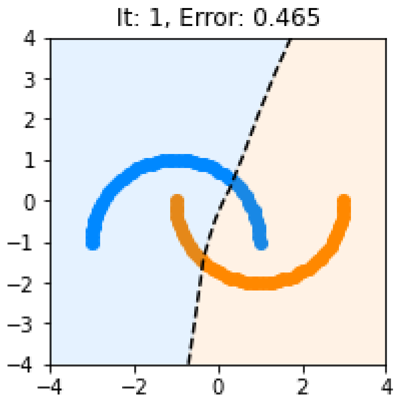
We would like to use error backpropagation to optimize the parameters of the neural network. The code below optimizes the network for 128 iterations and at some chosen iterations plots the decision function along with the current error.

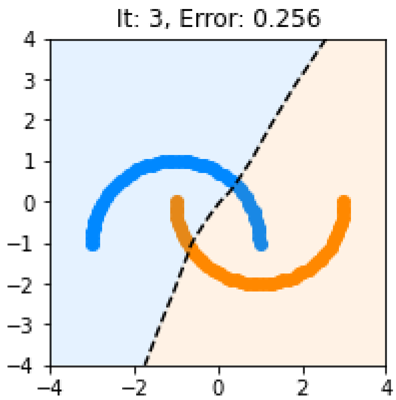
Task:

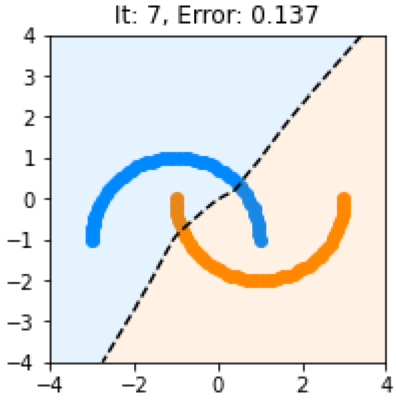
- **Complete the procedure above to perform at each iteration a step along the gradient in the parameter space. A good choice of learning rate is $\eta=0.1$.**

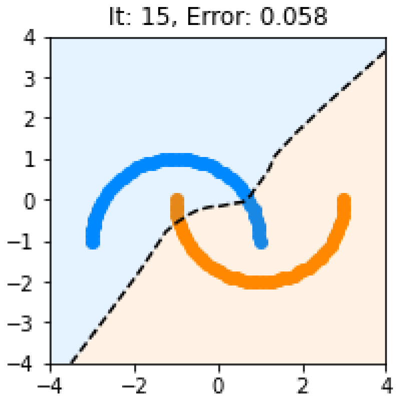
```
lr = 0.1
for i in range(128):
    if i in [0,1,3,7,15,31,63,127]:
        Yg = forward(X1g,X2g)
        Yd = forward(X1d,X2d)
        Ed = numpy.maximum(0,-Yd*Td).mean()
        plot(Yg,title="It: %d, Error: %.3f"%(i,Ed))
        DW,DB,DV = backprop(X1d, X2d, Td)
        W -= lr * DW
        B -= lr * DB
```

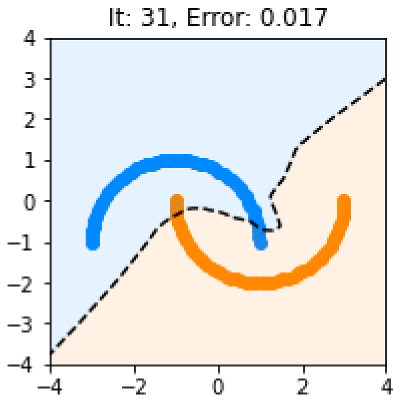

It: 0, Error: 0.127

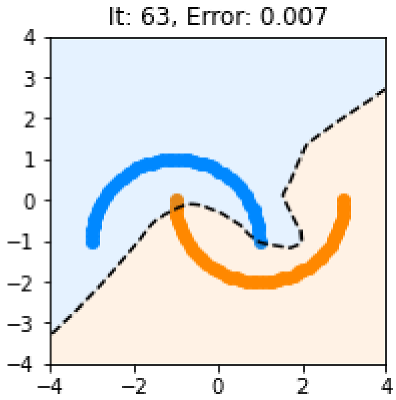

It: 1, Error: 0.465

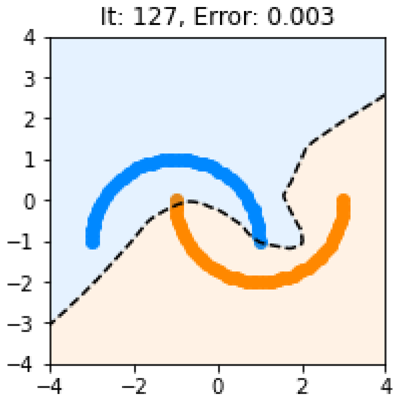

It: 3, Error: 0.256


It: 7, Error: 0.137


It: 15, Error: 0.058


It: 31, Error: 0.017


It: 63, Error: 0.007


It: 127, Error: 0.003
4
3
2
1
0
−1
−2
−3
−4
−4
−2
0
2
4