U(1) Quantum Lattice Gauge

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Abstract

• U(1) gauge theory is a model of the electromagnetic force that incorporates relativistic and quantum effects. Simulations are performed using U(1) gauge theory to study the photon field and the interactions between photons and charged particles. These simulations can be achieved through a discretization of space and time on a lattice. The propagation of the photon field through the lattice relies on Metropolis updates that select field states from a distribution given by the photon field Lagrangian. Measurements can be made on the simulated lattice to derive relationships between charged particles, such as the Coulomb potential, and to find the critical "temperature" at which charged particles become decoupled within the simulation.

Maxwell's Equations

• Classically, the electric and magnetic fields are solved using Maxwell's equations:

$$\begin{aligned}
\boldsymbol{\nabla}\times\boldsymbol{E}+\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} &= 0\\
\boldsymbol{\nabla}\times\boldsymbol{E}+\frac{1}{c}\frac{\partial(\boldsymbol{\nabla}\times\boldsymbol{A})}{\partial t} &= 0\\
\boldsymbol{\nabla}\times\boldsymbol{E}+\boldsymbol{\nabla}\times\frac{1}{c}\frac{\partial \boldsymbol{A}}{\partial t} &= 0\\
\boldsymbol{\nabla}\times\left(\boldsymbol{E}+\frac{1}{c}\frac{\partial \boldsymbol{A}}{\partial t}\right) &= 0
\end{aligned}$$

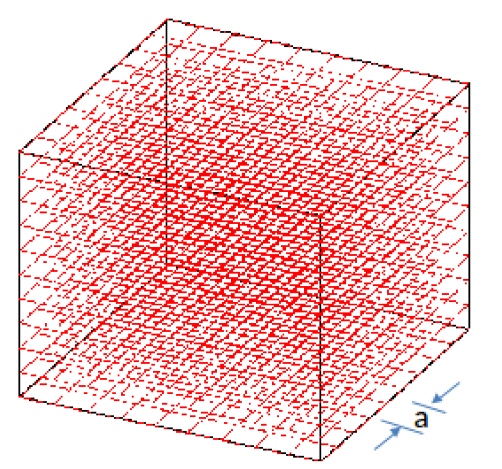
• The equations above imply that the electric and magnetic fields can be expressed using scalar and vector potentials:

$$\begin{aligned}
\boldsymbol{E} &= -\boldsymbol{\nabla}V-\frac{\partial \boldsymbol{A}}{\partial t}\\
\boldsymbol{B} &= \boldsymbol{\nabla}\times\boldsymbol{A}
\end{aligned}$$

• *Relativistically*, the components of the electric and magnetic fields are conveniently expressed using a field strength tensor:

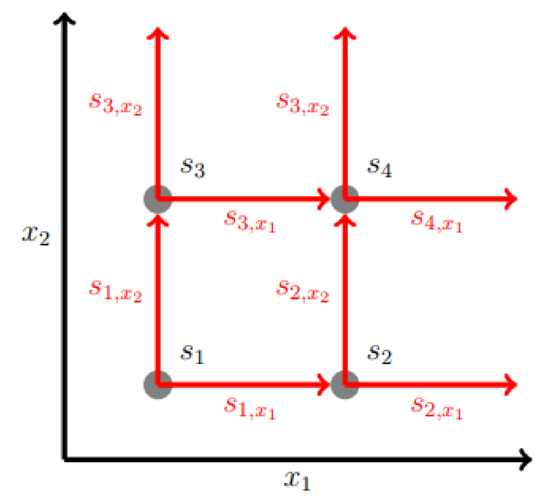
$$F^{\mu\nu}=\begin{pmatrix} 0 & -E_x & -E_y & -E_z\\ E_x & 0 & -B_z & B_y\\ E_y & B_z & 0 & -B_x\\ E_z & -B_y & B_x & 0\end{pmatrix}$$

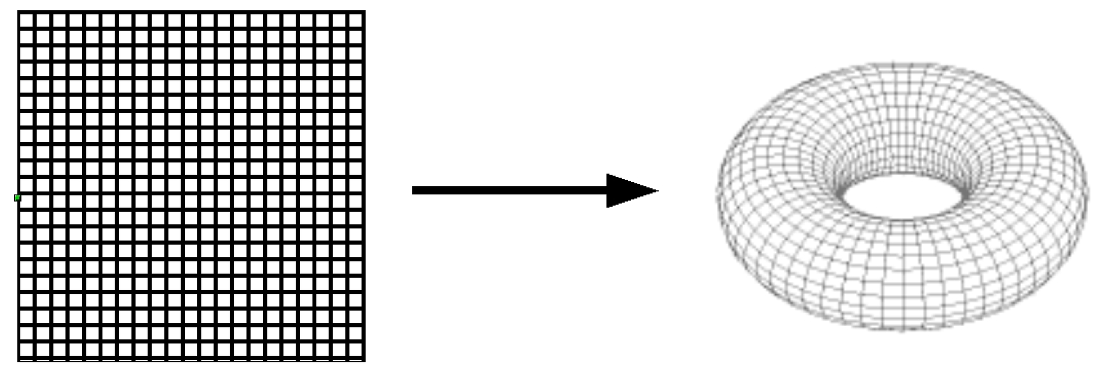
The Lattice



• A lattice is used to discretize spacetime. The cube on the left shows three of the four spacetime dimensions.

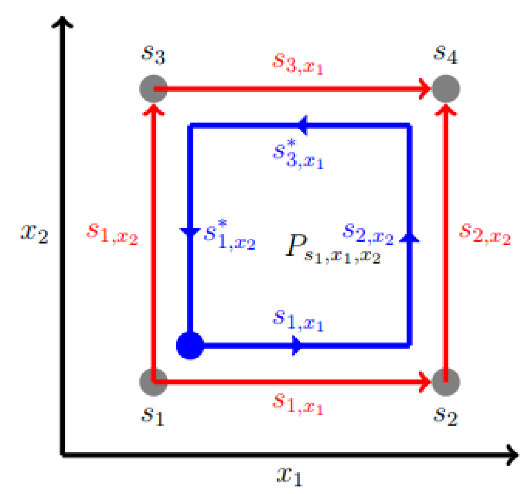
• The lattice is made of sites and links that connect the sites. Each link contains information about the photon field.





• Periodic boundary conditions are used on the lattice. This turns the lattice into a 4-dimensional torus.

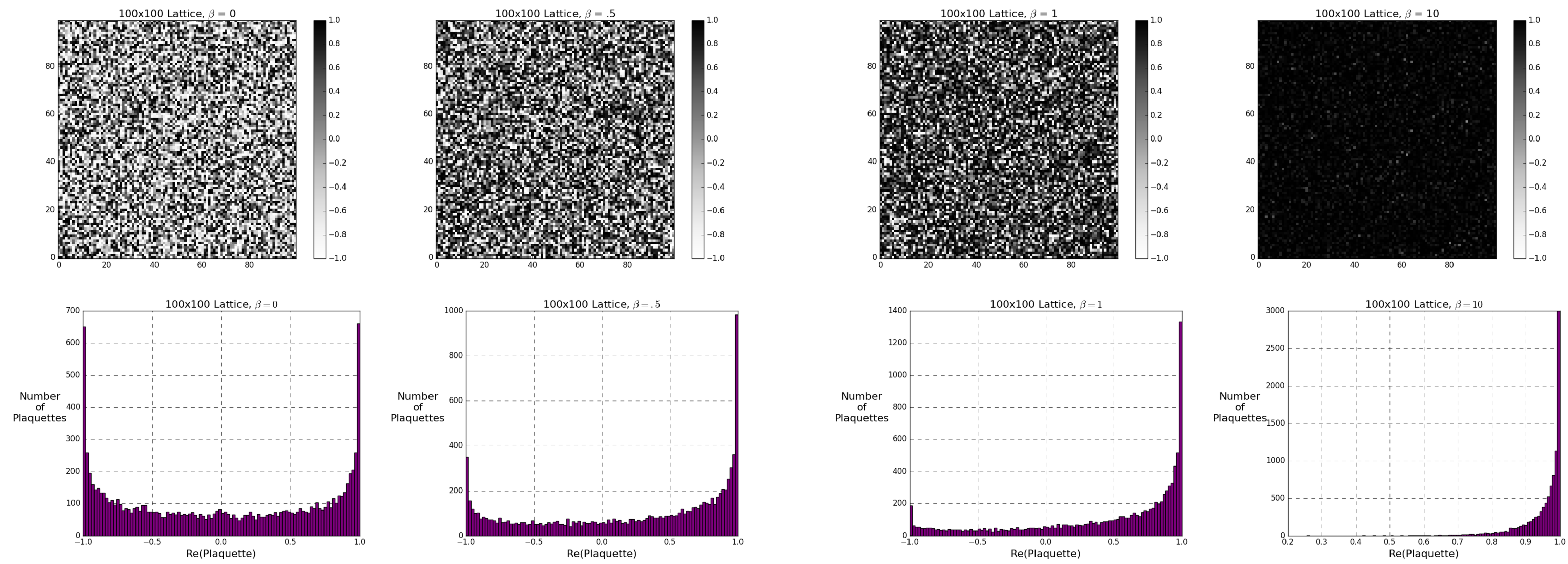
• Plaquettes are one of the measurements made on the lattice and are used in the update of the lattice.



References

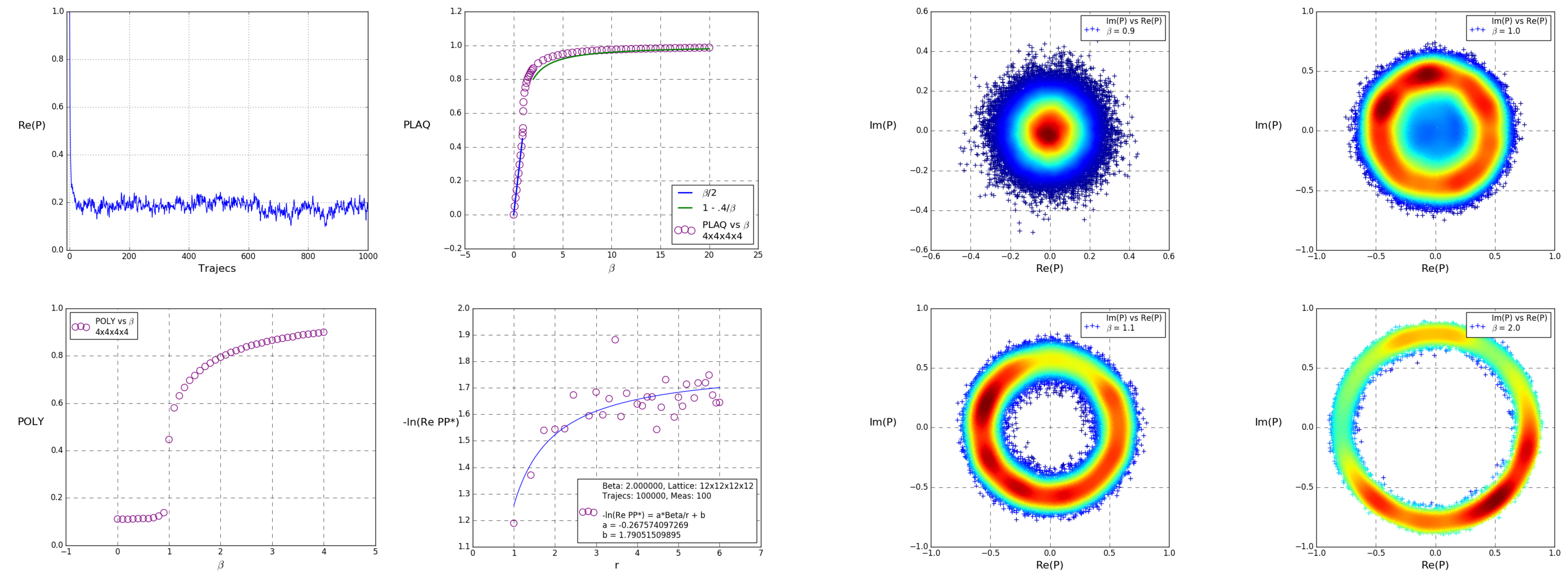
[1] D. Griffiths. Introduction to Elementary Particles. *Second, Revised Edition.*
[2] H. J. Rothe. Lattice Gauge Theories: An Introduction. *Second Edition.*
[3] H. Reinhardt. Effective Approaches to QCD.
[4] J. Hetrick. An Introduction to Lattice Gauge Theory. *Colloquium Lecture.*

Updates on the Lattice



• The photon field is updated by changing the value of the links at each site. Updates are made on the lattice using a Metropolis algorithm. To carry out the updates, we iterate over every link on the lattice and decide whether to accept or reject a new link value according to a distribution of Plaquettes. The top row of figures show the Real(Plaquette) values across a 100x100 lattice for different values of β. The β value is a parameter in the simulation that determines the distribution of Plaquette values that the Metropolis updates draw upon. The distributions for different β values can be seen in the bottom row of figures. As β increases, the distribution becomes more heavily weighted towards Real(Plaquette) = 1.

Measurements on the Lattice



• A plot of the average Plaquette versus β shows that the β_{crit} value is equal to 1. Another measurement that can be made on the lattice is the average Polyakov loop. The logarithm of the product of two Polyakov loops at different locations on the lattice is related the the Coulomb potential between two charged particles. Plots of Imaginary(Polyakov) versus Real(Polyakov) can be made for different β values. As β increases, the magnitude of the average Polyakov loop also increases.