Modeling the Interior of Exoplanets

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Abstract

• In many cases, astronomers can measure the radius of a planet, but not its mass. We need a computational method for estimating the mass of different planets. We can get a theoretical estimation for a planet's mass by modeling the planet as a sphere that contains two layers, each made of a different material. Starting from the planet's center, we integrate two equations of planetary structure throughout the planet until we reach the surface. There are three variables that we do not know. Two of them are independent variables that we make a guess for: the pressure at the center of the planet and the radius of the planet's core. The third variable is dependent: the mass of the planet. With a guess for the planet's central pressure and the core radius, we integrate starting at the planet's center and continue until the pressure reaches 0 GPa at the planet's surface. At the same time, we compute incremental additions to the overall mass of the planet as we integrate outwards. At the end of the simulation, we get a result for the planet's mass, the radius, and the core mass fraction (CMF, the mass of the planet's core divided by the total mass of the planet). We use an optimization scheme to get the best guesses for the central pressure and the core radius.

• In many cases, we do not know the age of a planet. Older planets have solid cores while younger ones have liquid cores. The density of the core, and therefore the overall mass of the planet, depends on whether the core is solid or liquid. For planets with an Earth-like composition, our computational model should be able to tell us the error in our estimate for the mass of the planet based on the measured radius.

Some Equations and Integration Profile

• Equation of state:

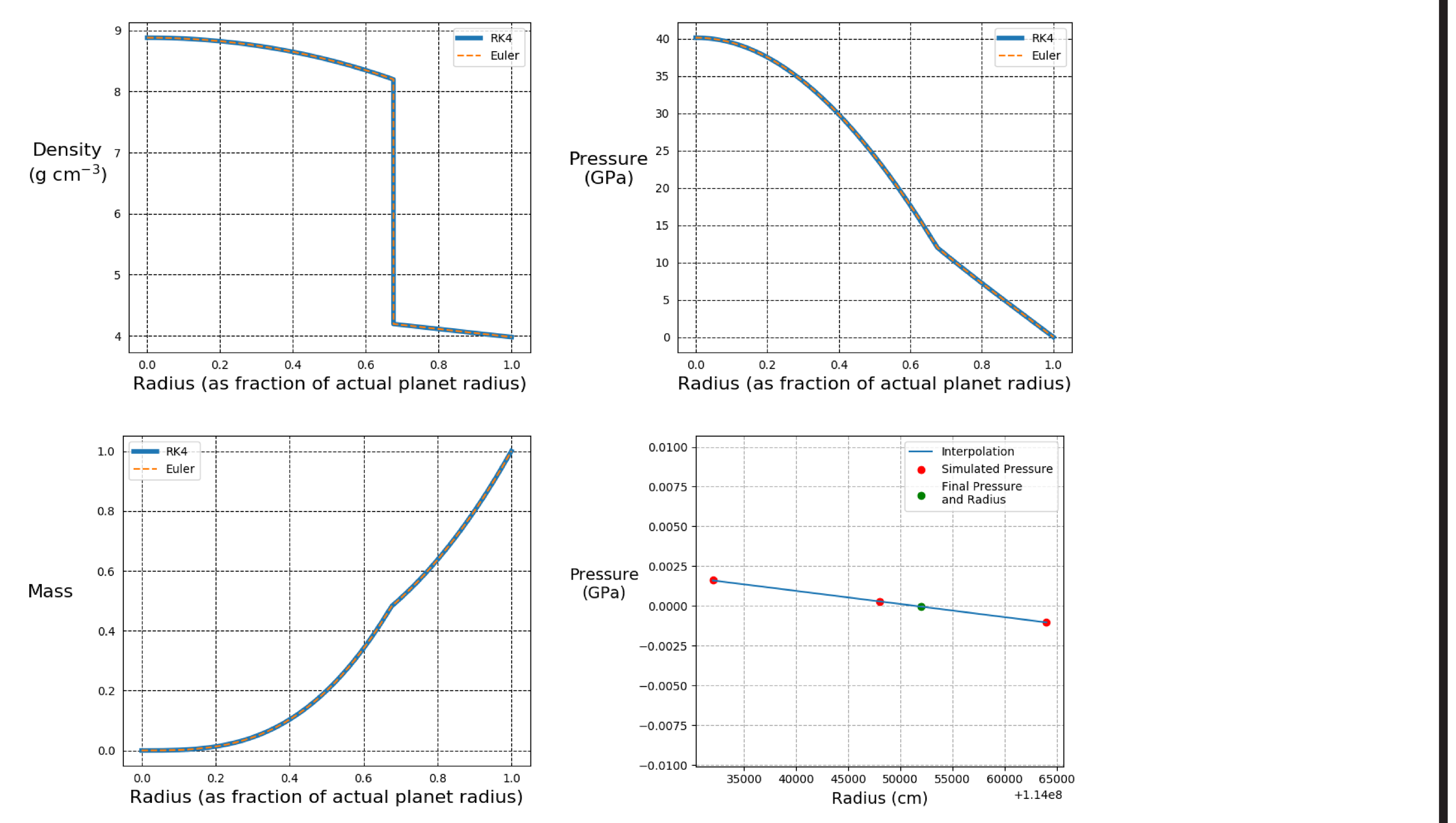
$$P = \frac{3K_0}{2}\left(\left(\frac{\rho(r)}{\rho_0}\right)^{\frac{7}{3}} - \left(\frac{\rho(r)}{\rho_0}\right)^{\frac{5}{3}}\right) \tag{1}$$

• Equations of planetary structure:

$$\Delta M = 4\pi\rho(r)r^2\Delta r \tag{2}$$

$$\Delta P = -\frac{\rho(r)GM}{r^2}\Delta r \tag{3}$$

P: pressure, K_0: bulk modulus, ρ: density, ρ_0: density at zero pressure, M: mass, Δr: time-step, r: radial distant from the center of the planet, G: gravitational constant.

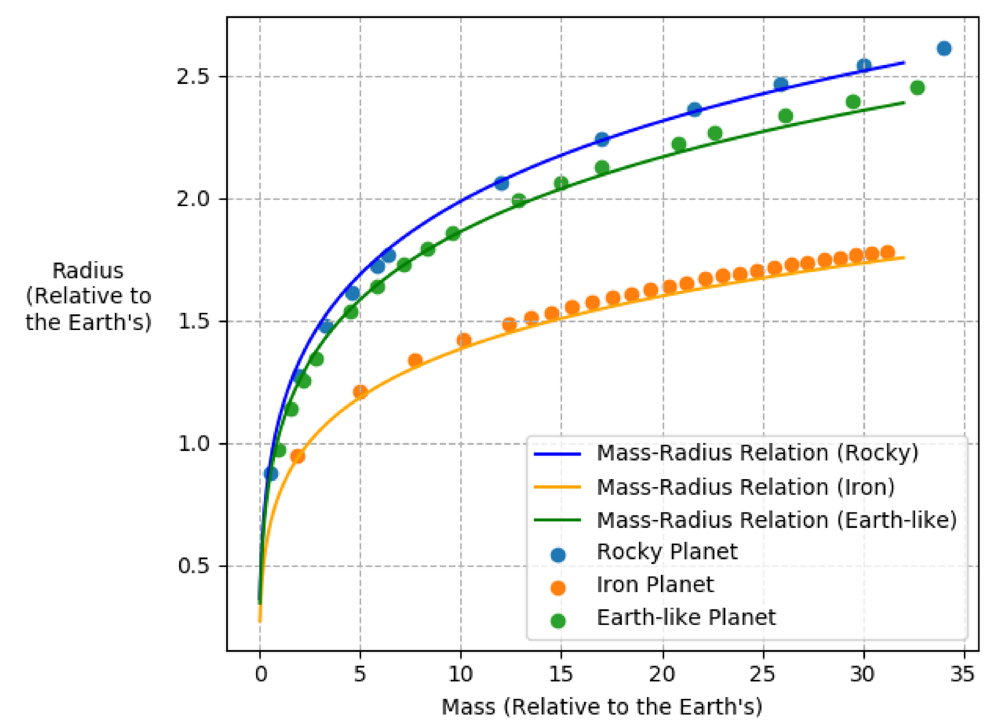


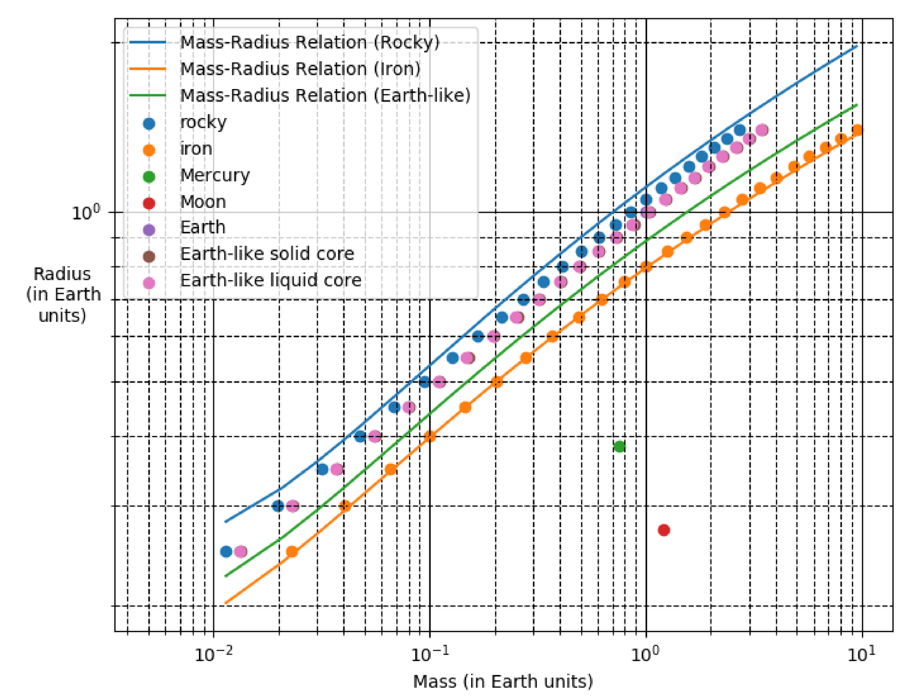
• The figures on above show the integration profile for a two-layer model of Mercury, with a solid iron core and a rocky mantle. The simulated density, pressure, and mass are plotted against the distance away from the center of the planet divided by the radius of Mercury. The simulation switches from solid iron to rocky mantle just before a radius fraction of .7. The results from using the 4th order Runge-Kutta (RK4) algorithm and the Euler algorithm are shown. The results from using both methods give similar results.

Optimization Scheme

• In order to get a good estimate for the mass of a planet, we must optimize our guesses for the the planet's central pressure and the radius of the planet's core. For a given guess for the central pressure and core radius, the simulation will produce certain values for the mass, radius, and CMF for the planet. If the simulated planetary radius and/or CMF are off from the desired (expected) values, we use linear interpolation or extrapolation to find better guesses for the independent variables. We continue this process until we get results for the dependent variables that are within a desired error tolerance. This gives us the "correct" guess for the planet's central pressure and core radius.

Mass-Radius Relation





$$R = (0.0912\text{rmf} + 0.1603)(\log M)^2 + (0.3330\text{rmf} + 0.7387)\log M + (0.4639\text{rmf} + 1.1193)$$

• The equation above gives the mass-radius relation, where R is the radius of the planet, M is the mass of the planet, and rmf is the rock mass fraction of the planet. This equation is provided by Zeng et al.

• The figures show the radius of simulated planets plotted against their mass. The planets shown are made entirely out of rock or solid iron or have an Earth-like composition. Assuming an earth-like composition, the estimated mass could be off by 1.5% depending on the radius if the state of the metallic core is known (being solid or liquid).

References

[1] Li Zeng, Dimitar Sasselov, and Stein Jacobsen. *Mass-Radius Relation for Rocky Planets based on PREM.* The Astrophysical Journal. 2016, 819, p127; doi: 10.3847/0004-637X/819/2/127.

[2] J. J. Fortney and M. S. Marley and J. W. Barnes. *Planetary Radii Across Five Orders of Magnitude in Mass and Stellar Insolation: Application to Transits.* The Astrophysical Journal. 2007, 659, p1661-1672; doi: 10.1086/512120.

[3] S. Seager, M. Kuchner, C. A. Hier-Majumder, and B. Militzer. *Mass-Radius Relationships for Solid Exoplanets.* The Astrophysical Journal. 2007, 669, p1279-1297; doi: 10.1086/521346.