Homework 2 (Due Wednesday, June 18)

If your score on the Diagnostic Test is less than 80, redo it following the guidelines below. You may not be able to do everything in one day. You almost certainly will not redo it perfectly the first time around. Do the best you can. If necessary, I will ask you to revise your work later.

Exposition and problem solving guidelines

First, let me impress upon you that proper mathematical exposition is *the* most valuable skill that you can learn in calculus. Seriously. It is not so important whether you can (or cannot) produce some answer: answers are not very interesting by themselves. What is important is that you can explain your logic. And explain it well.

Learning mathematical exposition takes a long time. Just like any other type of writing. Think of this homework is a start of a long process. I will give you a short list of guidelines below. More guidelines will follow as we discuss things in class.

1. Good writing requires good reading. Read the problem several times and make sure that you understand it. Before you start writing, ask yourself: What do I need to find? What does the answer look like? Is it a number, a function, a True or False statement, or something else? What data am I given? Is it sufficient? Is anything superfluous? How can I validate the answer? Is there a way to estimate the answer quickly?

2. Choose symbolic notation. Use last letters of the Latin alphabet x, y, z, ... for unknowns and first letters a, b, c, ... for parameters that are treated as known quantities. Do not use acronyms of any sort! E.g., use s or S for 'surface area' instead of SA. Do not simply use the first letter of a noun to denote the corresponding variable. E.g., it is better to label the length, width, and height of a box using x, y, and z rather than l, w, and h.

3. Introduce symbolic notation using full sentences. E.g., Let x, y, and z denote the length, width, and the height of the box, respectively... Simply putting labels next to symbols is insufficient and bad form.

4. Think of the strategy. There is often a choice of several. Contemplate them all and choose what seems to be the best.

5. Work out the solution on scratch paper. Note the important steps and transitions.

6. Explain the logic of the solution: that is, tell the reader what you are about to do.

7. Present the solution. Show only major steps and explain, verbally, *how* and *why* you do things, not *what* you do. Telling the reader "Now I plug in blah into blah and do algebra" is unhelpful.

8. Number the important equations. Then you can refer them in your solution as Equation (1), Equation (2), and so forth.

9. Suppress the routine computational details. You do not need to show how you simplify some fraction. Just say, "Simplifying Equation (1), we get:" and then show the result.

10. Validate your answers! Say you found a formula for x. If x is speed of a car, is it positive? Is it less than the speed of light? Does it have the units of speed? Does it depend on parameters in appropriate ways?

11. Conclude with some introspection. Write a sentence where you summarize what you learned, what was successful, which dead ends you ran into, and so forth. Do it for extra credit. And do it for yourself!

12. If the problem is hard, explain why. Do not simply leave a blank space where the solution should be.

13. If the problem is too hard, simplify it, and work out a simpler analog. Then try to return to the hard problem and see if it has gotten simpler.

14. Formulate good questions. And ask them in class. A lot. Remember that it is better to ask a "dumb" question and have it resolved than to do something truly dumb on the final exam.

Below are the problems from the Diagnostic Test. Try to redo them following the guidelines.

1. A particle is moving with constant acceleration a along the x-axis. If the particle started its motion from the point x_0 with initial velocity v_0, what is the position of the particle at time t?

   (a) If you could not do this problem correctly on the Diagnostic Test, explain why or what went wrong.

   (b) Regardless of whether you received points for this problem or not, redo it *with verbal exposition*. Set up notation, explain the logic of the solution, and use full sentences.

2. Find the coordinates of all points where the graph of $y = \ln(1 + x^2)$ changes concavity. Illustrate your answer with an accurate sketch.

   (a) If you could not do this problem correctly on the Diagnostic Test, explain why or what went wrong.

   (b) Review curve sketching. Then sketch the curve by hand.

   (c) Produce a `Matlab` plot of the curve and compare it with your sketch.

   (d) Regardless of whether you received points for this problem or not, redo it *with verbal exposition*.

3. The equation $x^3 - 3\,x\,y + y^3 = 0$ defines a plane curve known as the Folium of Descartes. Find the slope of this curve at the point $(3/2, 3/2)$.

   (a) If you could not do this problem correctly on the Diagnostic Test, explain why or what went wrong.

   (b) Try to produce a `Matlab` plot of this curve (look up the `contour` command.)

   (c) Regardless of whether you received points for this problem or not, redo it *with verbal exposition*.

4. Compute the following limit or explain why it does not exist:

$$\lim_{\epsilon \to 0} \frac{1}{2\,\epsilon} \left(\frac{1}{x + \epsilon} - \frac{1}{x - \epsilon} \right).$$

   (a) If you could not do this problem correctly on the Diagnostic Test, explain why or what went wrong.

(b) Use `Matlab` to estimate the limit.

(c) Regardless of whether you received points for this problem or not, redo it *with verbal exposition*. Consider all possible approaches.

5. Suppose that $f(1) = -1$ and $\frac{df}{dx}(1) = 3$. Use the tangent line approximation to estimate $f(2)$.

   (a) If you could not do this problem correctly on the Diagnostic Test, explain why or what went wrong.

   (b) Regardless of whether you received points for this problem or not, redo it *with verbal exposition*.

   (c) Choose three different functions that satisfy the given conditions. Compute the estimates for each case and compare them with actual values. Try to explain the discrepancies.

6. A spherical balloon is being inflated at the rate of x m^3/sec. Find the rate at which the diameter of the balloon is changing when its volume is V m^3.

   (a) If you could not do this problem correctly on the Diagnostic Test, explain why or what went wrong.

   (b) Regardless of whether you received points for this problem or not, redo it *with verbal exposition*. Do not use the letter d to denote diameter to avoid monstrosities like $\frac{dd}{dt}$. Use y instead.

7. A rectangular aquarium is to be designed subject to the following constraints:

   (a) The aquarium must hold V cubic meters of water.

   (b) The walls are to be made from glass.

   (c) The bottom is to be made from slate.

   If the price of slate is k times the price of glass (per square meter), what dimensions of the aquarium will minimize the total cost?

   (a) This problem actually is impossible at this point. Nevertheless, provide verbal exposition where you set it up and explain the impossibility.

8. Consider the following statement:

   > The rate of population growth is proportional to the size of the population.

   Let $P(t)$ denote population size at time t. Translate the above statement into an equation satisfied by $P(t)$.

   (a) If you could not do this problem correctly on the Diagnostic Test, explain why or what went wrong.

   (b) Translate the following terms into math: (i) rate of growth, (ii) proportional. Now translate the full sentence.

9. Let $g(x) = \int_0^x e^{-t^2}\,dt$. Find $\frac{dg}{dx}(1)$

   (a) If you could not do this problem correctly on the Diagnostic Test, explain why or what went wrong.

   (b) Regardless of whether you received points for this problem or not, redo it *with verbal exposition.*

10. Let

$$f(x) = \begin{cases} x^2, & x \neq 1, \\ 0, & x = 1. \end{cases}$$

    Find the integral $\int_0^2 f(x)\,dx$ or explain why the integral does not exist.

    (a) If you could not do this problem correctly on the Diagnostic Test, explain why or what went wrong.

    (b) Use `Matlab` to estimate the integral.

    (c) Regardless of whether you received points for this problem or not, redo it *with verbal exposition.*

11. Write a paragraph where you (i) summarize the analysis of your performance on the Diagnostic Test and (ii) formulate a strategy for real tests.