Homework 6 (Due Wednesday, July 2)

If the first test did not go too well, redo the problems (given below). After each problem provide a short paragraph with the analysis of what happened (what went wrong, what you learned, how you will avoid such mistakes in the future).

1. Find an equation of the tangent plane. Validate your solutions with Matlab plots.
   (a) $z = 4\,x^2 - y^2 + 2\,y$ at $(-1, 2, 4)$.
   (b) $y\,z = \ln(x + z)$ at $(0, 0, 1)$.
2. Find the critical points of $f(x, y) = 3\,x - x^3 - 2\,y^2 + y^4$. Produce a contour plot showing all critical points and revealing their nature.
3. Postal regulations specify that the combined length and girth of a parcel sent by parcel post may not exceed 130 inches. Find the dimensions of the rectangular package that would have the greatest possible volume under these regulations. *Provide thorough verbal exposition.*
4. Find the shortest distance from the point $(2, 1, -1)$ to the plane $x + y - z = 1$. Explain your work.

Review Taylor polynomials. Then try the following exercises.

1. Find the Taylor polynomial of order two for the following functions. Produce Matlab plots showing both the function and the Taylor quadratic.
   (a) $y = e^x$ centered at $x_0 = 1$.
   (b) $y = e^{-x^2}\cos(x)$ centered at $x_0 = 0$
   (c) $e\,e^{-y} = x - 1$ centered at $x_0 = 0$
2. Let T_2 be the Taylor quadratic for $y = f(x)$ centered at x_0. Derive the formula for T_2 from the following equations:

$$\begin{aligned} T_2(x_0) &= f(x_0) \\ T_2'(x_0) &= f'(x_0) \\ T_2''(x_0) &= f''(x_0) \end{aligned}$$

3. In the previous problem you derived the formula for T_2 using the principle: at x_0 the value of T_2 and the values of its first two derivatives agree with those of the function. Generalize this principle to functions of two variables and derive an explicit formula for the Taylor quadratic of $z = f(x, y)$ centered at (x_0, y_0).

4. Provided that you did the preceding two exercises, compute T_2 at each critical point of $f(x, y) = 3\,x - x^3 - 2\,y^2 + y^4$. Contour each T_2 on a small patch centered at the critical point.