Homework 6 (Due Wednesday, July 2)

1. Find and classify the critical points of the function $f = 4+x^3+y^3-3\,x\,y$. Illustrate your findings with a contour plot.

2. Repeat the previous exercise with $z = x\,y + \frac{1}{x} + \frac{1}{y}$.

3. A mechanical system consists of a mass m suspended on two identical springs with stiffness k and natural length L. The springs are anchored at the points $(-a, 0, 0)$ and $(a, b, 0)$. Let (x, y, z) be the coordinates of the mass.

   (a) Find the potential energy $U(x, y, z)$ of the system.

   (b) Using the expression for U find the stable equilibria of the system.

   (c) Illustrate your findings with plots for three different choices of parameters.