Problem Set 1

Problem 1. For each of the following differential equations, identify the independent and dependent variables and classify it based on its

- Linearity
- Order
- Type (be it ordinary or partial)
- Homogeneity (if linear)
- Coefficient class (be it constant or variable).

(a) $t^2\ddot{y} + t\dot{y} + 2y = \sin t$

(b) $\dot{y} + ty^2 = 0$

(c) $(1 + y^2)\frac{d^2y}{dx^2} + t\frac{dy}{dx} + y = e^t$

(d) $\frac{d^4}{dx^4} + \frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 1$

(e) $u_{xx} + u_{yy} + u_{zz} = -f(x, y, z)$ (Poisson's equation)

(f) $u_t + uu_x = 0$ (inviscid Burgers' equation)

Problem 2. Solutions to ODE should be the same regardless of which technique we use to solve them. Consider the following ODE:

$$\frac{dy}{dt} = f'(t)(A + By),$$

where $f(t)$ is some smooth function and A and $B \neq 0$ are both constant.

(a) Show that this ODE has the same solution if you use: (i) separation of variables or (ii) an integrating factor.

(b) If $\lim_{t\to\infty} f(t)$ does not exist, what is $\lim_{t\to\infty} y(t)$? Explain your response.

Problem 3. Consider *Newton's Law of Cooling*:

> The temperature $u(t)$ at time t of an object changes at a rate directly proportional to the difference $T - u(t)$ of its temperature and the temperature T of its ambient environment.

(a) Explain why the initial value problem

$$\begin{cases} \frac{du}{dt} = k(T - u), \quad k > 0 \\ u(0) = T_0 \end{cases}$$

is a good preliminary model for this scenario.

(b) A cup of coffee is poured from a pot kept at 95° C and left to cool in a room kept at a constant 20° C (room temperature). We check the temperature of the coffee after 5 minutes and notice that it is only 90° C, which is still too hot to drink.

If we can only drink the coffee safely after its temperature drops below 50° C, how much longer will we need to wait to drink it?

How long do we have to drink the coffee if it becomes cold and undrinkable after its temperature falls below 25° C?