

## 10. SEPARATION OF VARIABLES EXERCISE

Work on this problem as preparation for the upcoming test. In particular, work on the exposition—try to make it clear and concise.

- (1) Let  $L$  be a positive number greater than one. Use separation of variables to solve the following Neumann problem for the Heat Equation:

$$\begin{aligned}\frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2}, & 0 < x < L, & \quad t > 0 \\ \frac{\partial u}{\partial x} \Big|_{x=0} &= \frac{\partial u}{\partial x} \Big|_{x=L} = 0, \\ u(x, 0) &= \begin{cases} 1, & \text{if } 0 < x < 1 \\ 0, & \text{otherwise.} \end{cases}\end{aligned}$$

- (2) Validate your solution numerically for  $L = 5, 10, 20$  and  $t = .1$ .
- (3) What happens to the solution as  $L \rightarrow \infty$ ? Write a paragraph summarizing your thoughts.