Homework 8 (Due Thursday, July 17)

1. Let
$$I = \int_0^2 f(x)\,dx$$
where $f(x) = e^{-x}$. Denote by L_N and M_N approximations to I obtained using the left endpoint and midpoint rules with N subdivisions. Plot $\ln|L_N - I|$ and $\ln|M_N - I|$ as functions of N on the same plot. Submit the code, the plot, and your interpretation of the plot.

2. Repeat the previous exercise with
$$f(x) = \begin{cases} x^2, & x \neq 1, \\ 0, & x = 1. \end{cases}$$
What is the value of the integral? What affect does the discontinuity of the function have on it?

3. Find the following double integrals. Confirm symbolic answers using `Matlab`.

   (a) $\displaystyle\iint_R (x^3 + y^3)\,dA,\ R = \{(x, y) \mid 0 \leq x, y \leq 1\}$

   (b) $\displaystyle\iint_R \ln(x + y)\,dA,\ R = \{(x, y) \mid 1 \leq x, y \leq 2\}$

   (c) $\displaystyle\iint_R \sin(x)\,\sin(y)\,dA,\ R = \{(x, y) \mid 0 \leq x, y \leq \pi\}$

4. In each case below, conjecture a formula for the integral. Then validate your conjectures using special cases:

   (a) $\displaystyle\iint_R f(x) + g(y)\,dA,\ R = \{(x, y) \mid a \leq x \leq b,\ c \leq y \leq d\}$

   (b) $\displaystyle\iint_R f(x)\,g(y)\,dA,\ R = \{(x, y) \mid a \leq x \leq b,\ c \leq y \leq d\}$

   (c) $\displaystyle\iint_R \sum_{n=1}^{N} f_n(x)\,g_n(y)\,dA,\ R = \{(x, y) \mid a \leq x \leq b,\ c \leq y \leq d\}$

5. Approximate the value of the integral correct to four decimal places:

$$I = \iint_R J_0(\sqrt{x^2 + y^2})\, dA, \quad R = \{(x, y) \mid 0 - 1 \leq x, y \leq 1\}.$$

Here J_0 is the Bessel J-function of order zero (`besselj` in `Matlab`).

6. Let

$$I = \iint_R (x^2 + y^2)\, dA.$$

Write a program that approximates I for the case when R is a unit disk centered at the origin. If that proves difficult, explain what the difficulty is.

7. Repeat the previous exercise with R being a triangle. Any triangle.

8. Let R be some domain in the plane. What can you say about the integral $\iint_R 1\, dA$?