Problem Set 3

Problem 1. *Using the second-order existence and uniqueness theorem*

(a) Determine whether or not a solution exists for the initial value problem

$$\begin{cases} (\sec t)\ y'' - t\ y' + (\ln t)\ y = \frac{t}{t^2-9} \\ y(2) = 1, y'(2) = -5 \end{cases}$$

If a solution exists, find the largest interval of t over which it is guaranteed to exist.

(b) How does your answer above change if we use the initial conditions $y(5) = 2$ and $y'(5) = -2$ instead?

(c) What can be said about the existence or uniqueness of any solutions if we now use $y(3) = 0$ and $y'(3) = 4$ as the initial conditions?

Problem 2. *Euler's method*

Consider the IVP

$$\begin{cases} \frac{dy}{dt} = \frac{-3x^2+4x-2}{2(y-1)} \\ y(0) = 2 \end{cases},$$

whose implicit solution can be expressed as the equation

$$(y-1)^2 = -x^3 + 2x^2 - 2x + 1.$$

(a) Use Euler's method to approximate the solution from $0 \leq t \leq 1$ with a time step of $h = \frac{1}{4} = 0.25$. Find the absolute error $|y_{approx}(t) - y_{actual}(t)|$ between your approximation and the actual solution at the appropriate values of t.

(b) How can we refine our approximation ion part (a)?

(c) Use a computer to plot the exact solution. Explain why it might not be such a great idea to try to approximate the solution for values $t > 1$.

(d) Could you have predicted your observations in part (b) from our Existence and Uniqueness Theorem? Why or why not?

Problem 3. Compute the general solution to each of the following ODE:

(a) $y'' - 6y' + 9y = 0$

(b) $y'' + 4y = 0$

(c) $y'' + 5y' + 6y = 0$

(d) $y'' + y' + y = 0$