Problem Set 7

Problem 1. Consider the following first-order systems of linear, homogeneous ODE:

(a)

$$\dot{\vec{\mathbf{x}}} = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix} \vec{\mathbf{x}}$$

(b)

$$\dot{\vec{\mathbf{x}}} = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix} \vec{\mathbf{x}}$$

(c)

$$\dot{\vec{\mathbf{x}}} = \begin{bmatrix} -3 & 2 \\ 2 & -3 \end{bmatrix} \vec{\mathbf{x}}$$

(d)

$$\dot{\vec{\mathbf{x}}} = \begin{bmatrix} 2 & 8 \\ -1 & -2 \end{bmatrix} \vec{\mathbf{x}}$$

For each of these systems:

(i) Compute the eigenvalues and eigenvectors of the system.

(ii) Use the eigenvalues to classify the equilibrium type of the origin.

(iii) Use the eigenvectors as guides to plot a phase portrait of the system.

(iv) Present a general solution to the system of ODE.

(v) Find the particular solution to this system of ODE if

$$\vec{\mathbf{x}}(0) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}.$$

Identify which curve in the phase portrait corresponds to this solution.

(vi) If we define

$$\vec{\mathbf{x}}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix},$$

plot on their own set of axes the solutions $x(t)$ and $y(t)$ corresponding to your particular solution in the previous part.