Question 1:

Lab 1

- Written questions

#1. a) $\frac{1}{4} + \frac{1}{3} = \frac{7}{12}$

b) $\frac{1}{3}$

c) $Var(x) = E[x^2] - (E[x])^2$

$= [\cancel{0^2 \times (\frac{1}{4} + \frac{1}{6})} + 1^2 (\frac{1}{4} + \frac{1}{3})]$

$- [\cancel{0 \times (\frac{1}{4} + \frac{1}{6})} + 1 (\frac{1}{4} + \frac{1}{3})]^2$

$= \frac{35}{144}$

d) $Var(X|Y=1) = (1^2 \times \frac{1}{3}) - (1 \times \frac{1}{3})^2 = \frac{2}{9}$

e) $E[X^3 + X^2 + 3Y^7 | Y=1] = (\frac{1}{3})^3 + (\frac{1}{3})^2 + 3 \times 1 = \frac{85}{27}$

Question 2:

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In [6]: # Written Problem 2
        V = np.array([[0,1],[1,0],[1,0]]) # matrix of basis vectors by column
        P1 = [3, 1, 0]
        P2 = [3, 2, 0]
        P3 = [3, 3, 1]
        point_mat = np.array([P1, P2, P3])
        projection = np.matmul(np.matmul(np.matmul(V, np.linalg.inv(np.matmul(V.T, V))), V.T), point_mat)
        projection

Out[6]: array([[3. , 1. , 0. ],
               [3. , 2.5, 0.5],
               [3. , 2.5, 0.5]])
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Question 3:

Using CLT,

Let $X = \begin{cases} 1 & \text{wp } \frac{2}{3} \\ 0 & \text{wp } \frac{1}{3} \end{cases}$

since $X_i$ are iid and Binomially distributed

$\mu = E[X_i] = np = 1\left(\frac{2}{3}\right) = \frac{2}{3}$

$\sigma^2 = var(X_i) = np(1-p) = 1 \cdot \frac{2}{3} \cdot \frac{1}{3} = \frac{2}{9}$

$S_{100} = \sum_{i=1}^{100} X_i$

$P[S_{100} \leq 50] = P\left[\frac{S_{100} - 100\mu}{\sqrt{100\sigma^2}} \leq \frac{50 - 66.\overline{6}}{10\sqrt{\frac{2}{9}}}\right]$

$= P\left[\frac{S_{100} - 100\mu}{\sqrt{100\sigma^2}} \leq -3.53\right]$

$= 1 - \Phi(-3.53)$

$= 1 - .49979 = .50021$

$\boxed{\approx 50\%}$