Question 1:

Lab 1

- Written questions

#1. a) $\frac{1}{4} + \frac{1}{3} = \frac{7}{12}$

b) $\frac{1}{3}$

c) $$Var(X) = E[X^2] - (E[X])^2$$
$$= \left[\cancel{0^2 \times (1/4 + 1/6)} + 1^2 (1/4 + 1/3)\right] - \left[\cancel{0 \times (1/4 + 1/6)} + 1(1/4 + 1/3)\right]^2$$
$$= \frac{35}{144}$$

d) $Var(X \mid Y=1) = (1^2 \times \frac{1}{3}) - (1 \times \frac{1}{3})^2 = \frac{2}{9}$

e) $E[X^3 + X^2 + 3Y^7 \mid Y=1] = (\frac{1}{3})^3 + (\frac{1}{3})^2 + 3 \times 1 = \frac{85}{27}$

Question 2:

$V_1 = [1,1,1]$

$V_2 = [1,0,0]$

$P_1 = [3,3,3]$

$P_2 = [1,2,3]$

$P_3 = [0,0,1]$

P_1 Projected on Subspace $\mathbb{R}^3$ $= \frac{\langle P_1, V_1 \rangle}{\langle V_1, V_1 \rangle} V_1 + \frac{\langle P_1, V_2 \rangle}{\langle V_2, V_2 \rangle} V_2$

$= \frac{9}{3}[1,1,1] + \frac{3}{1}[1,0,0]$

$= [6,3,3]$

P_2 Projected on Subspace $\mathbb{R}^3$ $= \frac{\langle P_2, V_1 \rangle}{\langle V_1, V_1 \rangle} V_1 + \frac{\langle P_2, V_2 \rangle}{\langle V_2, V_2 \rangle} V_2$

$= \frac{6}{3}[1,1,1] + [1,0,0]$

$= [3,2,2]$

P_3 Projected on Subspace $\mathbb{R}^3$ $= \frac{\langle P_3, V_1 \rangle}{\langle V_1, V_1 \rangle} V_1 + \frac{\langle P_3, V_2 \rangle}{\langle V_2, V_2 \rangle} V_2$

$= \frac{1}{3}[1,1,1] = [1/3, 1/3, 1/3]$

Coordinates:

$(6,3,3), (3,2,2), (1/3, 1/3, 1/3)$

Question 3:

Using CLT,

Let $X = \begin{cases} 1 & \text{wp } \frac{2}{3} \\ 0 & \text{wp } \frac{1}{3} \end{cases}$

since X_i are iid and Binomially distributed

$$\mu = E[X_i] = np = 1\left(\tfrac{2}{3}\right) = \tfrac{2}{3}$$

$$\sigma^2 = var(X_i) = np(1-p) = 1 \cdot \tfrac{2}{3} \cdot \tfrac{1}{3} = \tfrac{2}{9}$$

$$S_{100} = \Sigma_{i=1}^{100} X_i$$

$$P[S_{100} \leq 50] = P\left[\frac{S_{100} - 100\mu}{\sqrt{100\sigma^2}} \leq \frac{50 - 66.\bar{6}}{10\sqrt{\frac{2}{9}}}\right]$$

$$= P\left[\frac{S_{100} - 100\mu}{\sqrt{100\sigma^2}} \leq -3.53\right]$$

$$= 1 - \Phi(-3.53)$$

$$= 1 - .49979 = .50021$$

$\boxed{\approx 50\%}$