EE 460J Lab 2

Team:

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In [3]:
```
import pandas as pd
import numpy as np
import seaborn as sns
import matplotlib.pyplot as plt
from sklearn import datasets, linear_model
```

Problem 1a

When given a data matrix, an easy way to tell if any two columns are correlated is tolook at a scatter plot of each column against each other column. For a warm up, do this: Look at the data in DF1 in Lab2Data.zip. Which columns are (pairwise) correlated?

Figure out how to do this with Pandas, and also how to do this with Seaborn.

In [5]:
```
data_file1 = "./DF1"
data_file2 = "./DF2"
```

In [23]:
```
df1 = pd.read_csv(data_file1, header=None)
df1_mean = df1.mean()
df1.fillna(df1_mean, inplace=True)

display(df1)
print("Correlation Matrix:")
display(df1.corr())

print("Pandas pairplot (scatter matrix):")
pd_pairplot = pd.plotting.scatter_matrix(df1,figsize=(15, 15), alpha=.8)

sns_pairplot = sns.pairplot(df1)
#sns_pairplot.fig.suptitle("Seaborn pairplot:") # title doesn't appear how I wan
```

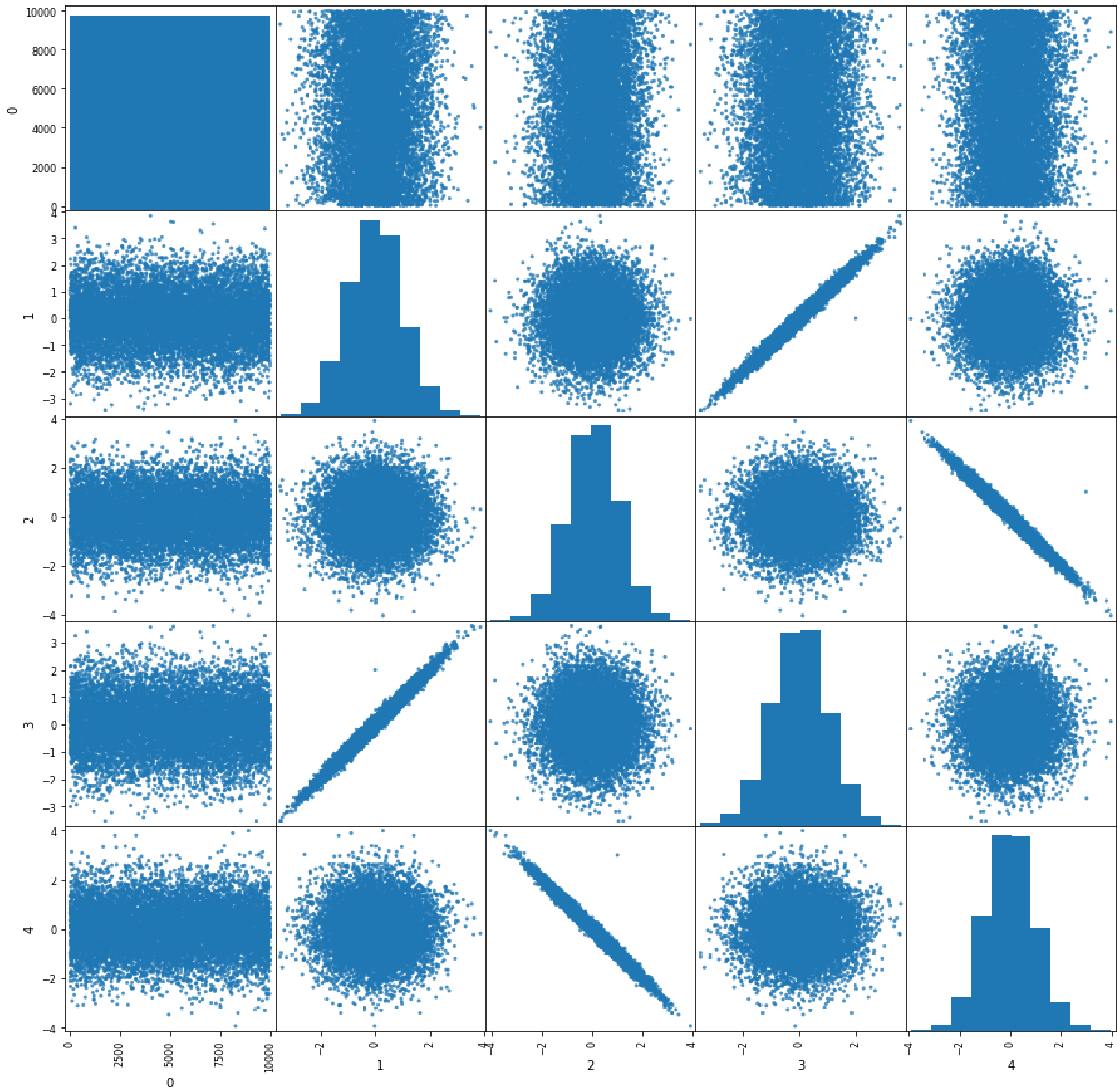
	0	1	2	3	4
0	4999.5	0.000000	1.000000	2.000000	3.000000
1	0.0	1.038502	0.899865	0.835053	-0.971528
2	1.0	0.320455	-0.647459	0.149079	0.352593
3	2.0	0.055480	2.234771	0.271672	-2.108739
4	3.0	-0.007260	-0.524299	-0.126550	0.670827
...	...	...	...	...	...
9996	9995.0	-0.632309	-0.145873	-0.797517	0.436184
9997	9996.0	0.679417	-0.530216	0.526470	0.439397
9998	9997.0	0.890697	-2.210855	1.072751	2.285372
9999	9998.0	0.475293	0.490971	0.536909	-0.195772
10000	9999.0	1.207406	0.819239	1.230797	-0.752397

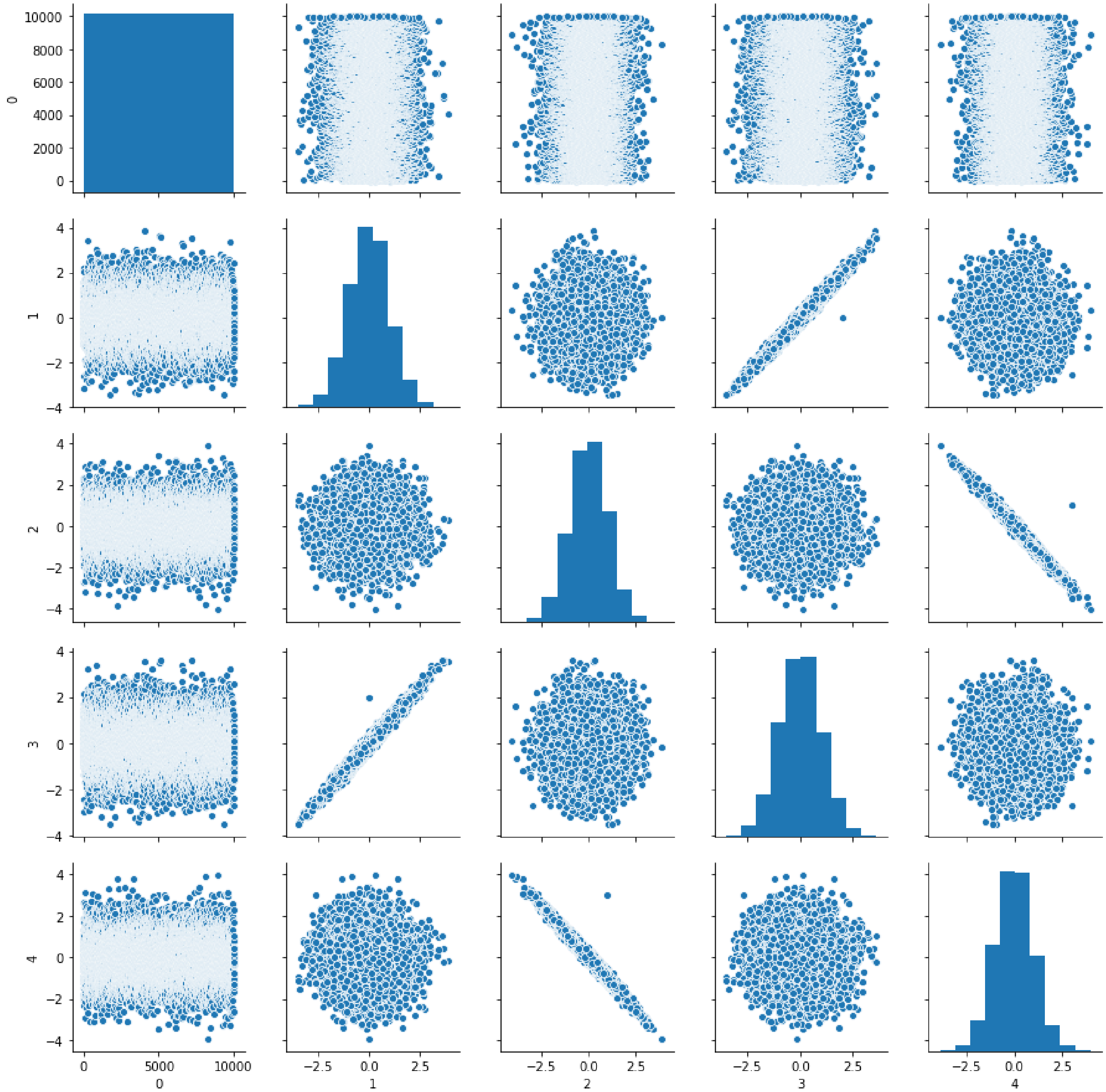
10001 rows × 5 columns

Correlation Matrix:

	0	1	2	3	4
0	1.000000	-0.003991	0.008788	-0.004043	-0.007083
1	-0.003991	1.000000	-0.003998	0.989869	0.004107
2	0.008788	-0.003998	1.000000	-0.003887	-0.989445
3	-0.004043	0.989869	-0.003887	1.000000	0.004662
4	-0.007083	0.004107	-0.989445	0.004662	1.000000

Pandas pairplot (scatter matrix):





From the graphs above, it appears that the graphs 1 and 3 are correlated (45 degree line)

And graphs 2 and 4 are correlated (-45 degree line)

Problem 1b

Compute the covariance matrix of the data. Write the explicit expression for what this is, and then use any command you like (e.g., np.cov) to compute the 4×4 matrix.

Explain why the numbers that you get fit with the plots you got.

$$Covariance(X,Y) = \sum_{i=1}^{n} \frac{(x_i - \bar{x})(y_i - \bar{y})}{N}$$

$$C_{i,j} = \sigma(x_i, x_j)$$

The covariance matrix corresponds to the covariance between variables for every element (i,j).

Two elements are strongly related if their covariance is close to 1.

As you can see below, element (1,3) (and thus also (3,1)) is 0.991523 ~ 1, corresponding to the positive 45 degree angle, showing their strong relationship.

Likewise, (2,4) (and thus also (4,2)) shows -0.995059 ~ -1, corresponding to the negative 45 degree angle, showing their strong relationship.

In [27]:
```
df1.cov()
```

Out[27]:

	0	1	2	3	4
0	8.333333e+06	-11.529682	25.437628	-11.681482	-20.508118
1	-1.152968e+01	1.001458	-0.004012	0.991523	0.004122
2	2.543763e+01	-0.004012	1.005376	-0.003901	-0.995059
3	-1.168148e+01	0.991523	-0.003901	1.001885	0.004680
4	-2.050812e+01	0.004122	-0.995059	0.004680	1.005973

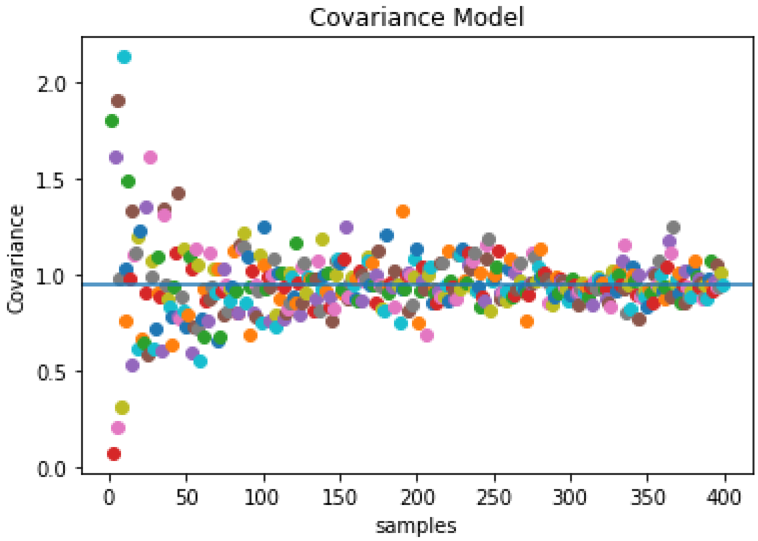
Problem 1c

The above problem in reverse. Generate a zero-mean multivariate Gaussian random variable in 3 dimensions, Z = (X1,X2,X3) so that (X1,X2) and (X1,X3) are uncorrelated, but (X2,X3) are correlated.

Specifically: choose a covariance matrix that has the above correlations structure, and write this down. Then find a way to generate samples from this Gaussian. Choose one of the non-zero covariance terms (Cij, if C denotes your covariance matrix) and plot it vs the estimated covariance term, as the number ofsamples you use scales. The goal is to get a visual representation of how the empirical covariance converges to the true (or family) covariance.

In [57]:
```
import warnings
warnings.filterwarnings("ignore")

covariance_structure = lambda mean,cov,samples,i,j: np.cov(np.random.multivariat

mean = [0,0,0]
cov = [[1, 0, 0], [0,1,0.95], [0,0.95, 1]] # correlation between X2 and X3 is de

fig, ax = plt.subplots(1,1)
for n in range(0,400):
    ignore = ax.plot(n, covariance_structure(mean, cov, n, 1, 2), marker='o')


ax.set_title("Covariance Model")
ax.axhline(0.95) # Set covariacnce - convergence point
ax.set_ylabel("Covariance")
ax.set_xlabel("samples")
```

Out[57]: Text(0.5, 0, 'samples')



Problem 2

Consider the two-dimensional data in DF2 in Lab2Data.zip. Look at a scatterplot of the data. It contains two points that look like potential outliers. Which one is “more” outlying? Propose a transformation of the data that makes it clear that the point at (−1,1) is more outlying than the point at (5.5,5), even though the latter point is “farther away” from the nearest points. Plot the data again after performing this transformation. Provide discussion as appropriate to justify your choice of transformation.

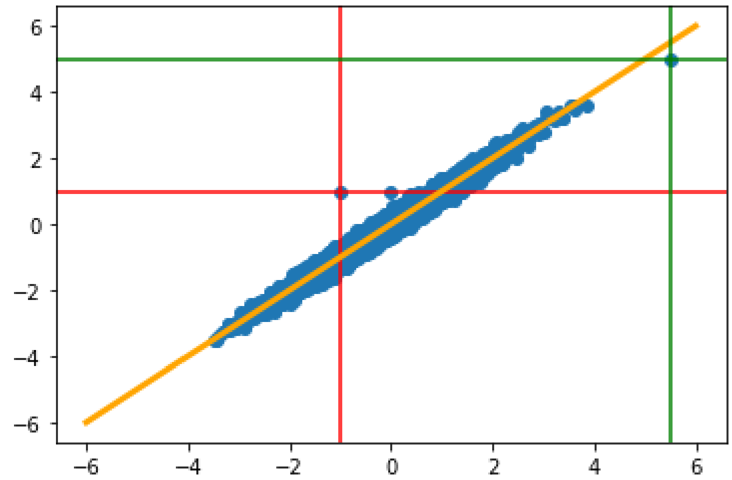
In [59]:

```
df2 = pd.read_csv(data_file2, header=None)
```

In [78]:

```
plt.scatter(df2[1], df2[2])
plt.axvline(-1, color='r')
plt.axhline(1, color='r')
plot = plt.plot([-6,6],[-6,6], color='orange', linewidth=3)
plt.axvline(5.5, color='g')
plt.axhline(5, color='g')
```

Out[78]: <matplotlib.lines.Line2D at 0x1ab30da690>



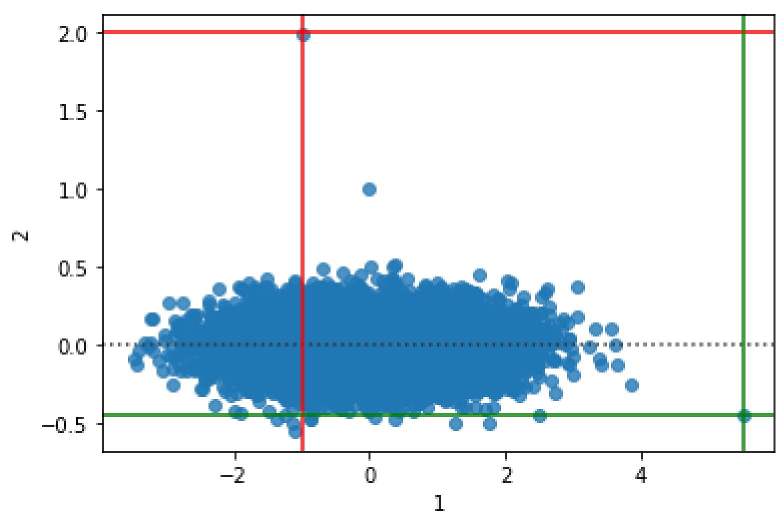
As you can see from the graph above, the point (-1,1) seems to be further away from the projection line. How can we tell for sure?

Let's plot a residual graph and witness the difference from the 45 degree line

In [84]:

```
plot = sns.residplot(df2[1], df2[2])

## the plot point (-1,1)
plt.axvline(-1, color='r')
plt.axhline(2, color='r')

## the plot point (5.5, 5)
plt.axvline(5.5, color='g')
plt.axhline(-.45, color='g')
```

Out[84]: <matplotlib.lines.Line2D at 0x1ab5fe7dd0>



As you can seefrom the plot above, the point (-1,1) corresponds to the residual point (-1, 2) - meaning that the diference between our prediction and the data is 2. Meanwhile, (5.5, 5) corresponds to the residual point (5.5, -.45) - meaning that the difference between our prediction and the data is 0.45

because 2 > 0.45, we know that the point (-1,1) is the greater outlier

Problem 3a

Generate n= 150 data points as follows:

xi~N(0,1)

ei~N(0,1)

yi=β∗0+xiβ∗1+ei, where β0=−3 and β∗1= 0. Note that since β∗1= 0,this means that yand x are unrelated!

Use either the closed form expression for β1(hat) or a linear regression package of your choice, to obtain the least-squares estimate for β1 (hat). Is it equal to β∗1? Either way, explain.

In [127…

```
mean = 0
std = 1
n= 150
x_i = np.random.normal(mean, std, n)

e_i = np.random.normal(mean, std, n)

beta_0 = -3
beta_1 = 0

y_i = beta_0 + beta_1*x_i + e_i
x_i = x_i[:, None]
reg = linear_model.LinearRegression().fit(x_i, y_i)
print(reg.coef_)
print(reg.intercept_)
print("The regression is not able to find a 0 slope, but the value appears to be
```

```
[0.03851592]
-3.0244479294235913
The regression is not able to find a 0 slope, but the value appears to be very c
lose to 0
```

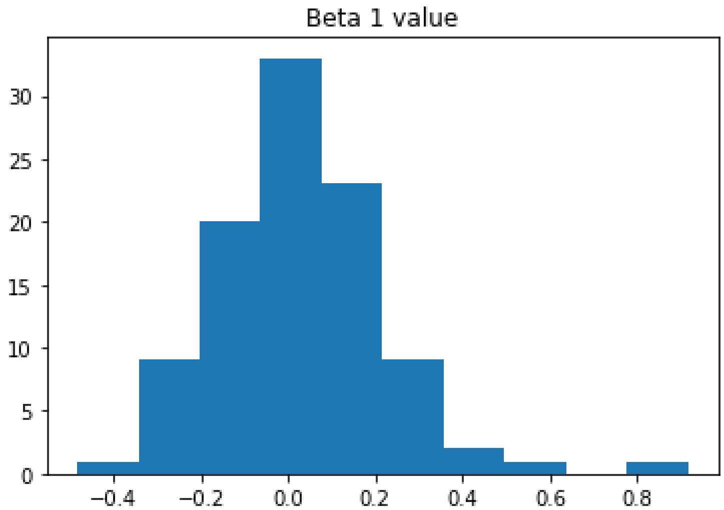
Problem 3b

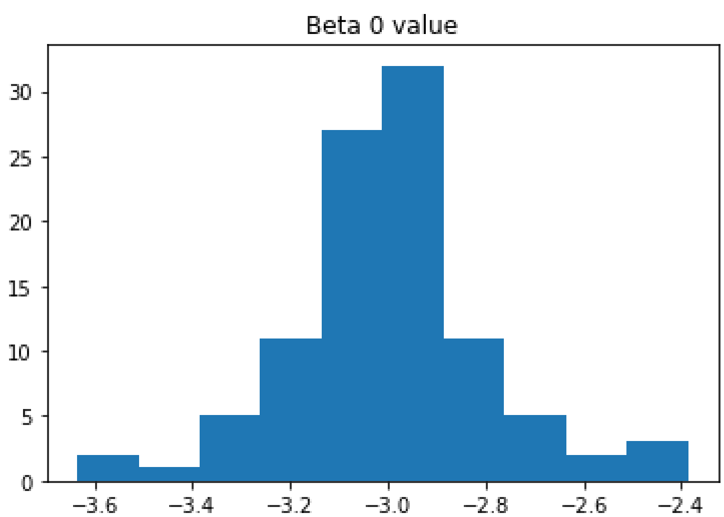
Now do the following M= 99 more times. Create fresh data exactly as you did above, but for the same β∗0 and β∗1. This is like going out and collecting more data from the same source. (For example, like I'm collecting statistics on sleep vs test grades for EE460J students, and I repeat the experiment each semester, each semester getting fresh data,i.e., new students). Once you are done with this, you will therefore have 100 different calculated values for β1(hat). Plot these in a histogram of sufficiently small bins to make it look nice.•

In [158…

```
mean = 0
std = 1
n= 150

beta_0 = -3
beta_1 = 0

reg_coefficients = []
reg_intercepts = []
```

```
for i in range(1, 100):
    x_i = np.random.normal(mean, std, i)
    e_i = np.random.normal(mean, std, i)

    y_i = beta_0 + beta_1*x_i + e_i
    x_i = x_i[:, None]
    reg_coefficients.append((linear_model.LinearRegression().fit(x_i, y_i)).coef
    reg_intercepts.append((linear_model.LinearRegression().fit(x_i, y_i)).interc

reg_coefficients = [i[0] for i in reg_coefficients]
plot1 = plt.hist(reg_coefficients, bins=10)
plt.title("Beta 1 value")
plt.show()
plot2 = plt.hist(reg_intercepts, bins=10)
plt.title("Beta 0 value")
plt.show()
```





Problem 3c

Compute the standard deviation of the above 100 points.

In [159…

```
np_coeff = np.array(reg_coefficients)
np_int = np.array(reg_intercepts)
```

```
print("B1 std deviation:" + str(np_coeff.std()))
print("B0 std deviation: " + str(np_int.std()))
```

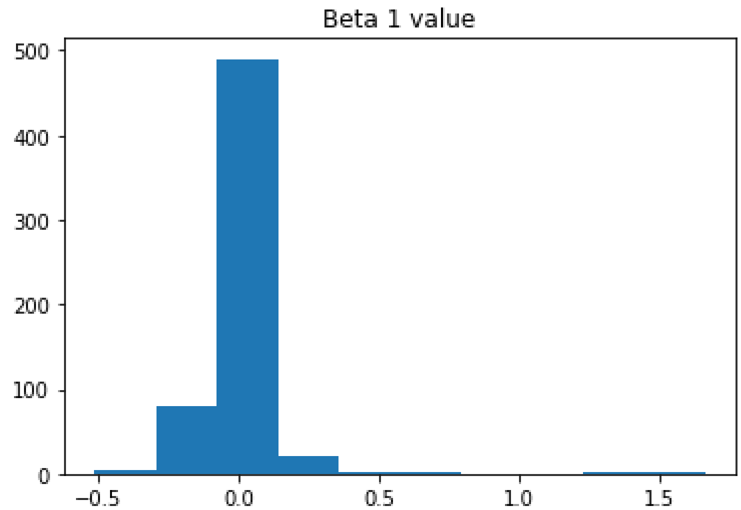
```
B1 std deviation:0.19694195048760887
B0 std deviation: 0.20407852509955823
```

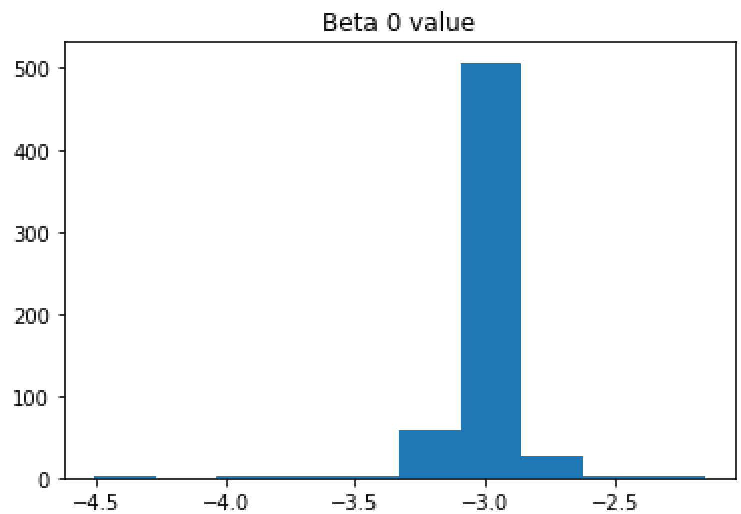
Problem 3d

Now repeat the experiment for a larger n, say n= 600. That is, each batch of data you use to compute β1(hat), will now have more data. Even before you started this class, you all knew that more data means more accurate. So we expect that the values of β1(hat) we compute, should be better estimates of β_*1. What exactly does "more accurate" mean? Now we are finding this out: it means less variance. You now have M= 100 values of β1(hat). Again, plot these, and compute the standard deviation. The $1/\sqrt{n}$ law tells us that we should expect the standard deviation to be about half of what it was before, since you now used 4× the data.

In [160…

```
reg_coefficients = []
reg_intercepts = []
for i in range(1, 600):
    x_i = np.random.normal(mean, std, i)
    e_i = np.random.normal(mean, std, i)

    y_i = beta_0 + beta_1*x_i + e_i
    x_i = x_i[:, None]
    reg_coefficients.append((linear_model.LinearRegression().fit(x_i, y_i)).coef
    reg_intercepts.append((linear_model.LinearRegression().fit(x_i, y_i)).interc

reg_coefficients = [i[0] for i in reg_coefficients]
plot1 = plt.hist(reg_coefficients, bins=10)
plt.title("Beta 1 value")
plt.show()
plot2 = plt.hist(reg_intercepts, bins=10)

plt.title("Beta 0 value")
plt.show()

np_coeff = np.array(reg_coefficients)
np_int = np.array(reg_intercepts)

print("B1 std deviation:" + str(np_coeff.std()))
print("B0 std deviation: " + str(np_int.std()))
```






```
B1 std deviation:0.1307916924854775
B0 std deviation: 0.12708097843105054
```

Problem 3e

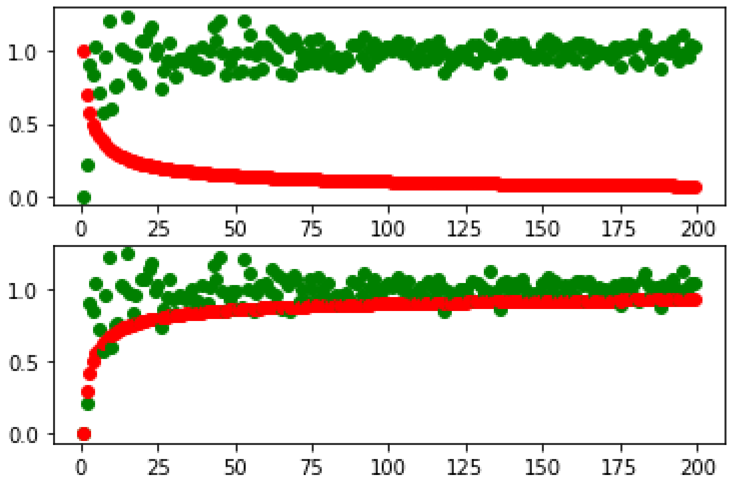
Finally, we will repeat this several more times, to try to empirically observe this 1/√n law. Thus: repeat the above experiment for different values of n. Plot these values, and on the same plot, try to fit c/√n for some constant c. You can choose which and how many values of n you choose. Think of yourselves as empirical statisticians, trying to learn some hidden law. So choose enough values of n so that you can “see” the right answer (which we know should scale like 1/√n).

In [178…

```
fig, ax = plt.subplots(2,1)

for i in range(1, 200):
    x_i = np.random.normal(mean, std, i)
    e_i = np.random.normal(mean, std, i)

    y_i = beta_0 + beta_1*x_i + e_i
    x_i = x_i[:, None]

   # sample = (linear_model.LinearRegression().fit(x_i, y_i))
    #sample_raw = [s[0] for s in sample]
    ax[0].plot(i, y_i.std(), marker='o', color='g')
```

```
    ax[0].plot(i, 1/np.sqrt(i), marker='o', color='r')

    ax[1].plot(i, y_i.std(), marker='o', color='g')
    ax[1].plot(i, -1/np.sqrt(i)+1, marker='o', color='r')



print("It looks as though the samples are reflected across ~0.5 from 1/sqrt(n)")
print("From the second plot shown, it appears that maybe we can reflect 1/sqrt(n
```

```
It looks as though the samples are reflected across ~0.5 from 1/sqrt(n)
From the second plot shown, it appears that maybe we can reflect 1/sqrt(n) and a
dd 1 to fit the graph
```



Problem 4a

Download the data in the auto-mpg.data-originaldata set from http://archive.ics.uci.edu/ml/machine-learning-databases/auto-mpg/.

Display all pairwise relationships of the data. There are a number of packages that will do this for you. You can use, for example, Pandas, or you can use Seaborn, or something else of your choice.

In [238…

```
auto_mpg_file = 'auto-mpg.data'
auto_mpg_data = open(auto_mpg_file, 'r').readlines()

auto_mpg_dict = {
    'mpg': [],
    'cylinders': [],
    'displacement': [],
    'horsepower': [],
    'weight': [],
    'acceleration': [],
    'model_year': [],
    'origin': [],
    'car name': []
}


for i in auto_mpg_data:
    split_data = (i.strip().split('\t'))
    auto_mpg_dict['car name'].append(split_data[1])
```

```
    other_vals = split_data[0].split()
    auto_mpg_dict['mpg'].append(other_vals[0])
    auto_mpg_dict['cylinders'].append(other_vals[1])
    auto_mpg_dict['displacement'].append(other_vals[2])
    auto_mpg_dict['horsepower'].append(other_vals[3])
    auto_mpg_dict['weight'].append(other_vals[4])
    auto_mpg_dict['acceleration'].append(other_vals[5])
    auto_mpg_dict['model_year'].append(other_vals[6])
    auto_mpg_dict['origin'].append(other_vals[7])

auto_mpg = pd.DataFrame(auto_mpg_dict)
auto_mpg = auto_mpg.replace('?', 0.0)
auto_mpg.mpg = auto_mpg.mpg.astype(float)
auto_mpg.cylinders = auto_mpg.cylinders.astype(float)
auto_mpg.displacement = auto_mpg.displacement.astype(float)
auto_mpg.horsepower = auto_mpg.horsepower.astype(float)
auto_mpg.weight = auto_mpg.weight.astype(float)
auto_mpg.acceleration = auto_mpg.acceleration.astype(float)
auto_mpg.model_year = auto_mpg.model_year.astype(float)
auto_mpg.origin = auto_mpg.origin.astype(float)


plot = sns.pairplot(auto_mpg_limited)
plot.set(xticklabels=[])
plot.set(yticklabels=[])
```

Out[238…

```
<seaborn.axisgrid.PairGrid at 0x1ad1579c90>
```

Problem 4b

Now you will explore the relationship between mass and fuel efficiency. First, try to fit a straight line (not necessarily through the origin) to the data, using squared loss as your loss function. That is, try to find the best-fit relationship of the form: MPG =β0+β1WEIGHT.

In [234…

```
weight = auto_mpg['weight'][:, None]
reg = linear_model.LinearRegression().fit(weight, auto_mpg['mpg'])

print("Beta 0 (intercept): " + str(reg.intercept_))
print("Beta 1 (slope): " + str(reg.coef_[0]))
```

```
Beta 0 (intercept): 46.31736442026565
Beta 1 (slope): -0.0076766100639264 7
```

Problem 4c

Now fit a quadratic, i.e., a relationship of the form: MPG =β0+β1WEIGHT +β2WEIGHT2. Do this in two different ways: (A) Use the numpy.polynomial package and ask it to fit a polynomial of degree 2. Plot the curve you get against the points.(B) Now repeat this, by explicitly adding a column that is the square of the weight, and then using linear (but multiple) regression.

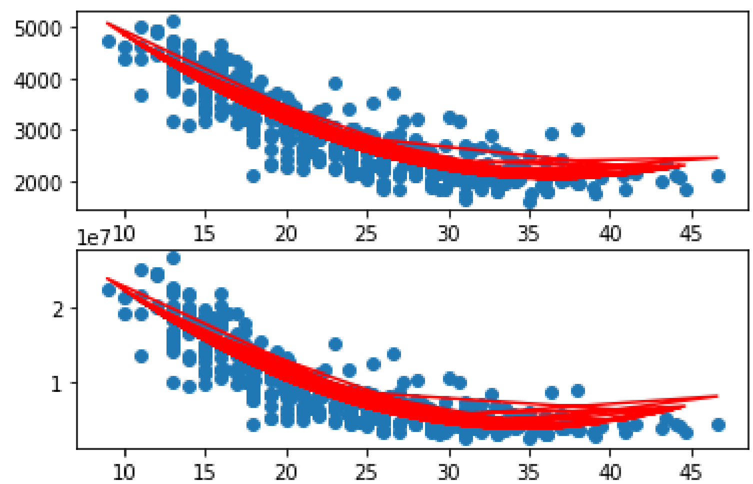
In [291…

```
fig, ax = plt.subplots(2,1)
auto_mpg['weight_2'] = auto_mpg['weight']**2

ax[0].scatter(auto_mpg['mpg'], auto_mpg['weight'])
ax[1].scatter(auto_mpg['mpg'], auto_mpg['weight_2'])

np_equation = np.polynomial.polynomial.Polynomial.fit(auto_mpg['mpg'], auto_mpg[
print(np_equation)

ax[0].plot(auto_mpg['mpg'], np_equation(auto_mpg['mpg']), color='r')


np_equation_2 = np.polyfit(auto_mpg['mpg'], auto_mpg['weight_2'], 2)
p = np.poly1d(np_equation_2)
ax[1].plot(auto_mpg['mpg'], p(auto_mpg['mpg']), color='r')
```

```
poly([ 2352.62000771 -1316.31449561  1416.20689285])
```

Out[291…

```
[<matplotlib.lines.Line2D at 0x1ad65d3890>]
```



Problem 5

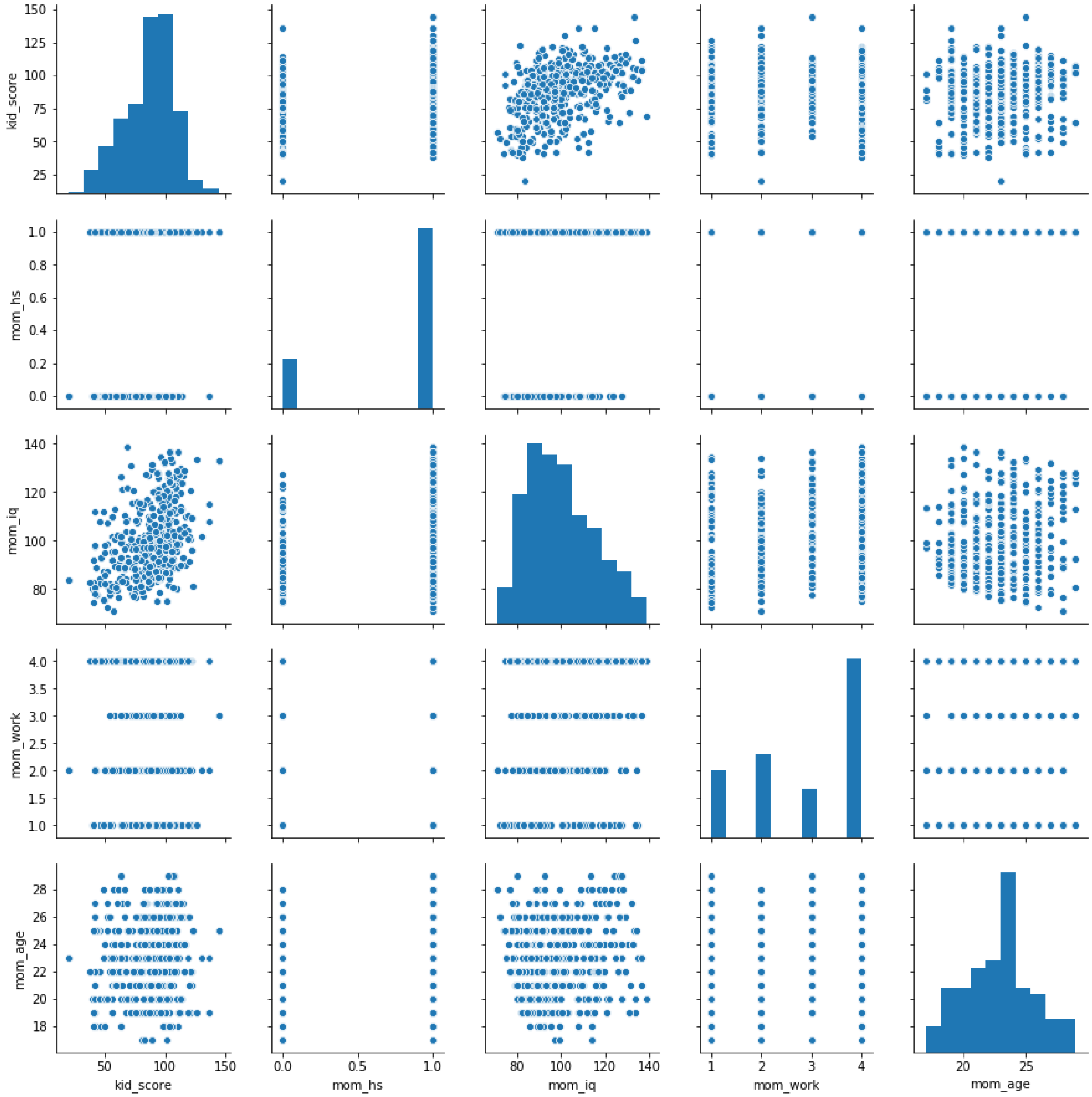
Run through the Jupyter Notebook from class on Child IQ and Mom IQ, from the textbook of Gelman and Hill. You will have to download the data set yourselves. Then do the exercise of adding an interaction term, as left for you at the end of the Jupyter notebook. Explain what you see, and how it relates to the graph you obtain before adding the interaction term. That is, use plots / visualization, to argue convincingly that the interaction term should or shouldn't be there, and then tell us what this means.

In [293…

```
kidiq_file = 'kidiq.dta'
childiq_file = 'child.iq.dta'

kidiq = pd.read_stata(kidiq_file)
sns.pairplot(kidiq)
```

```
sns.despine()
print("Tightest correlation looks to be between moms_iq and kid_score (some amou
```

```
Tightest correlation looks to be between moms_iq and kid_score (some amount of r
elational qualities)
```

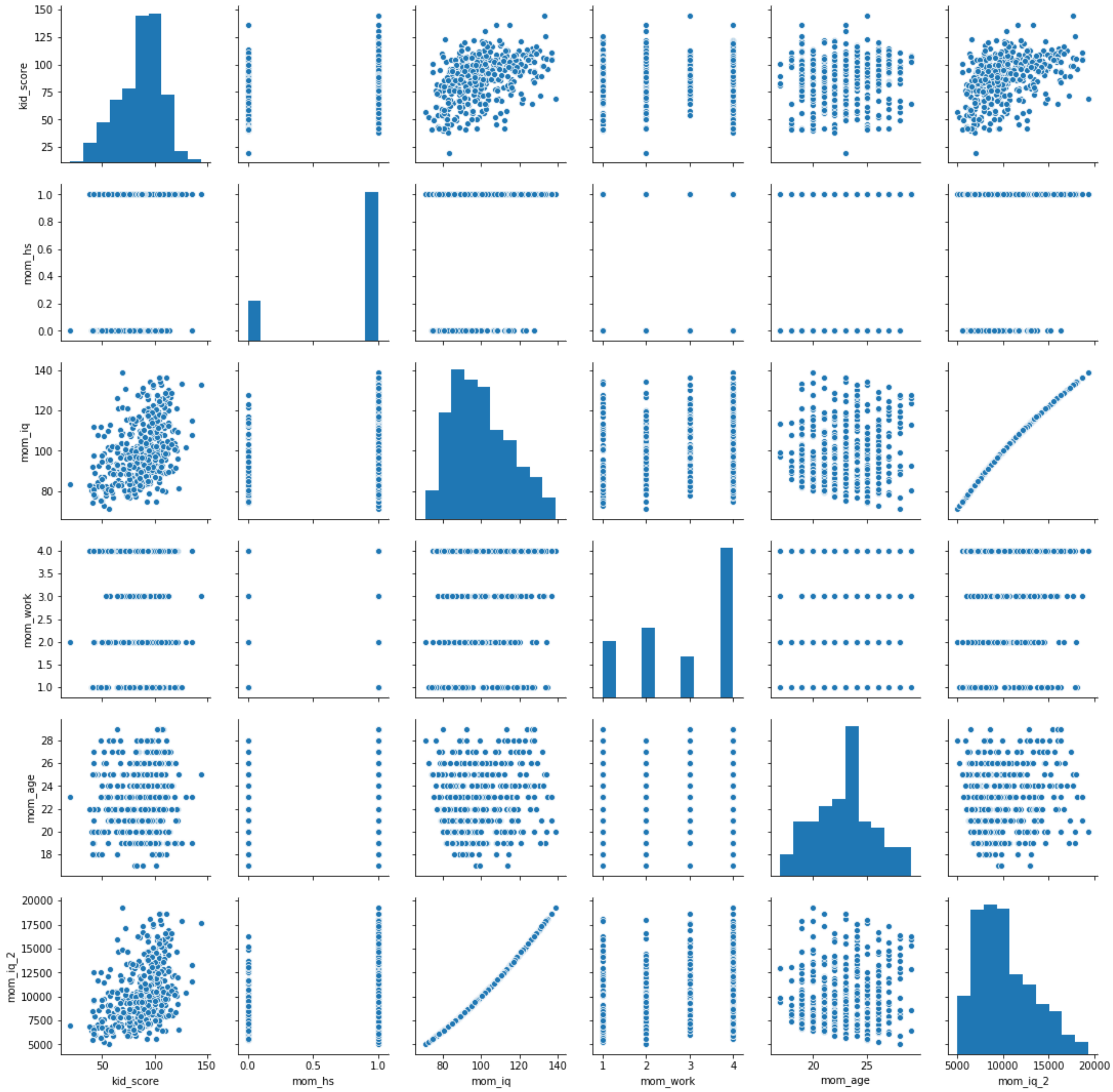


In [294…

```
kidiq['mom_iq_2'] = kidiq['mom_iq']**2
sns.pairplot(kidiq)
```

Out[294…

```
<seaborn.axisgrid.PairGrid at 0x1ad59de210>
```



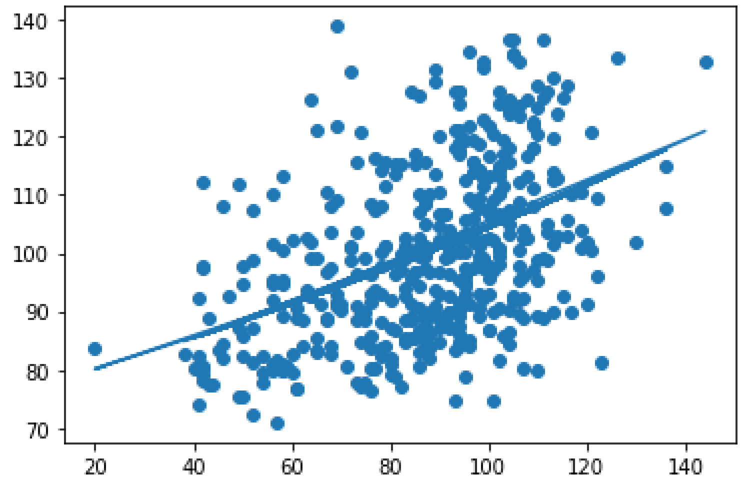
In [301…

```
mom_iq = kidiq['mom_iq']
reg = np.polyfit(kidiq['kid_score'], mom_iq, 2)
print(reg)
p = np.poly1d(reg)
plt.scatter(kidiq['kid_score'], kidiq['mom_iq'])
plt.plot(kidiq['kid_score'], p(kidiq['kid_score']))
```

```
[6.25336351e-04 2.26758742e-01 7.53469217e+01]
```

Out[301…

```
[<matplotlib.lines.Line2D at 0x1acc6b1810>]
```



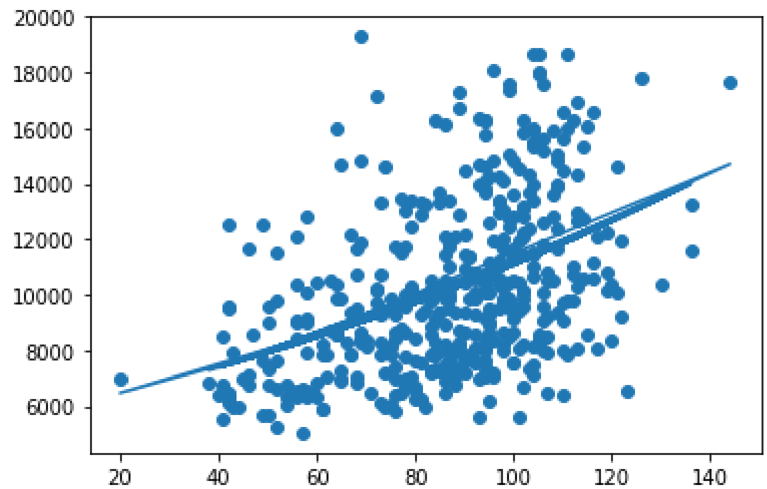
In [302…

```
mom_iq_2 = kidiq['mom_iq_2']
reg = np.polyfit(kidiq['kid_score'], mom_iq_2, 2)
print(reg)
p2 = np.poly1d(reg)
plt.scatter(kidiq['kid_score'], kidiq['mom_iq_2'])
plt.plot(kidiq['kid_score'], p2(kidiq['kid_score']))
```

```
[2.02381477e-01 3.32205856e+01 5.73221602e+03]
```

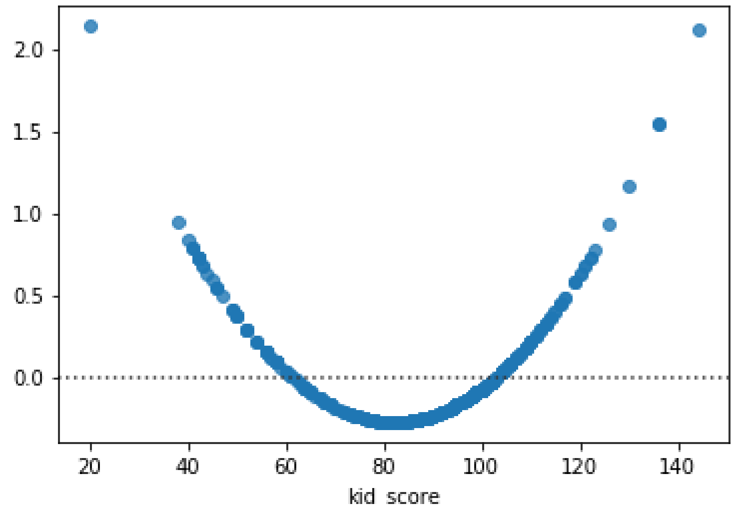
Out[302…

```
[<matplotlib.lines.Line2D at 0x1acc646210>]
```



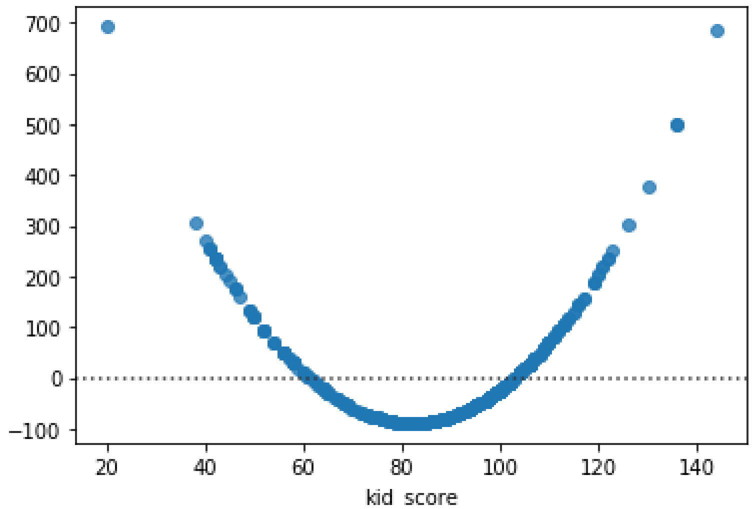
In [304…

```
plot = sns.residplot(kidiq['kid_score'], p(kidiq['kid_score']))
```



In [305…

```
plot = sns.residplot(kidiq['kid_score'], p2(kidiq['kid_score']))
```



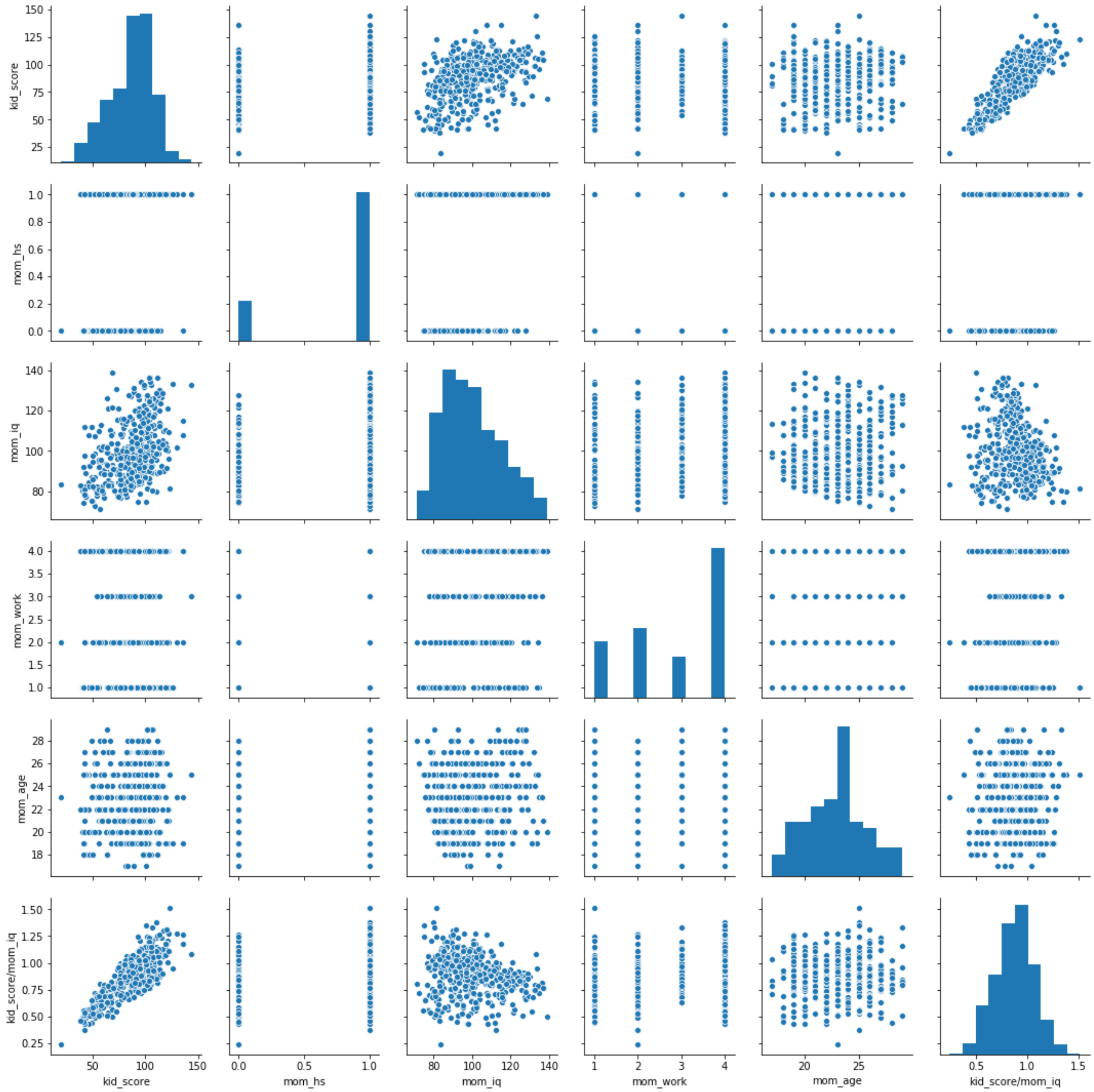
This does not appear to be very helpful, in part because the relationship between kid score and mom iq is not a simple quadratic. Because of this, we'll look into another potential interaction term

In [306…

```
kidiq['kid_score/mom_iq'] = kidiq['kid_score']/kidiq['mom_iq']
del kidiq['mom_iq_2']
sns.pairplot(kidiq)
```

Out[306…

```
<seaborn.axisgrid.PairGrid at 0x1acd61f710>
```



There are a few things to appreciate about this new interaction term. For one, the histogram closely resembles the normal curve, which is both pretty and a good sign that we can extract some value from the feature.

Another potential signature of this interactive term is the increased "tightness" of the plots in the columns mom_hs and mom_work in the bottom row. From this, we are able to uniquely

identify outliers in these relationships that other plots did not directly show

In [ ]:

```

```

Written question:

1) $\min : \frac{1}{n} \sum_{i=1}^{n} (x_i \beta - y_i)^2$

$$\frac{d}{d\beta_1}(\text{Loss}) = 0$$

$$\frac{1}{n} \sum_i 2(-x_i)(x_i \beta - y_i) = 0$$

$$\cancel{\frac{-2}{n}} \sum_i (\beta_1 x_i^2 - x_i y_i) = 0$$

$$\sum_i \beta_1 x_i^2 = \sum_i y_i x_i = \beta_1 \sum_j x_j^2$$

$$\hat{\beta}_1 = \frac{\sum_i y_i x_i}{\sum_j x_j^2}$$

-

$$\hat{\beta} = \frac{\Sigma_i x_i y_i}{\Sigma_j x_j^2}$$

$$y_i = x_i \cdot \beta^* + e_i$$

$$\hat{\beta} = \beta^* + Ze$$

$$\hat{\beta} = \frac{\Sigma_i \; x_i (x_i \cdot \beta^* + e_i)}{\Sigma_j \; x_j^2} = \frac{\Sigma_i \, x_i^2 \cdot \beta^* + e_i x_i}{\Sigma_j \, x_j^2}$$

$$\hat{\beta} = \frac{\beta^* \cdot \cancel{\Sigma x_i^2}}{\cancel{\Sigma_j x_j^2}} + \frac{\Sigma_i \; e_i x_i}{\Sigma_j x_j^2} = \beta^* + eZ$$

$$e = \Sigma e_i$$

$$Z = \frac{\Sigma_i x_i}{\Sigma_j x_j^2}$$