



## 06 Authentication and Key Management

2020 Spring

Information Security

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March 26, 2020

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## **Before We Start**

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What we have learned:

- Symmetric Key Encryption.
- Asymmetric Key Encryption.
- Hash/MAC.

Where is the key??

## **TTP: Trusted Third Party**

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In the past, the sender and the receiver share the same codebook.

- Or use the newspaper.

## Quiz

Given  $n$  people, if one member wants to talk with others securely, how many keys does the member need?

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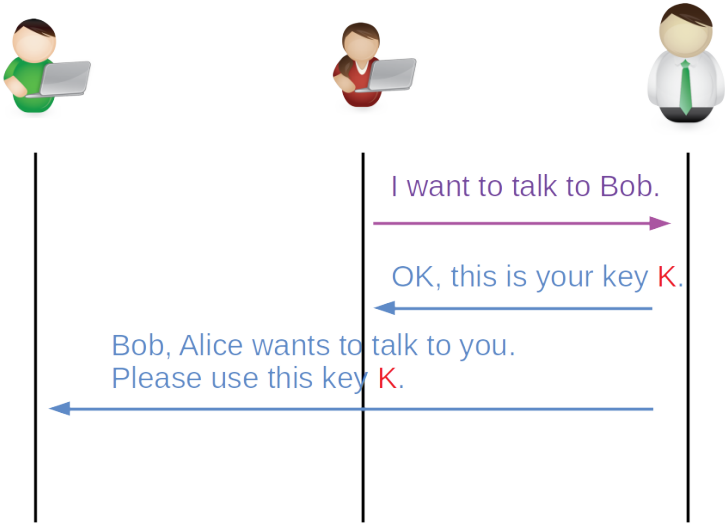
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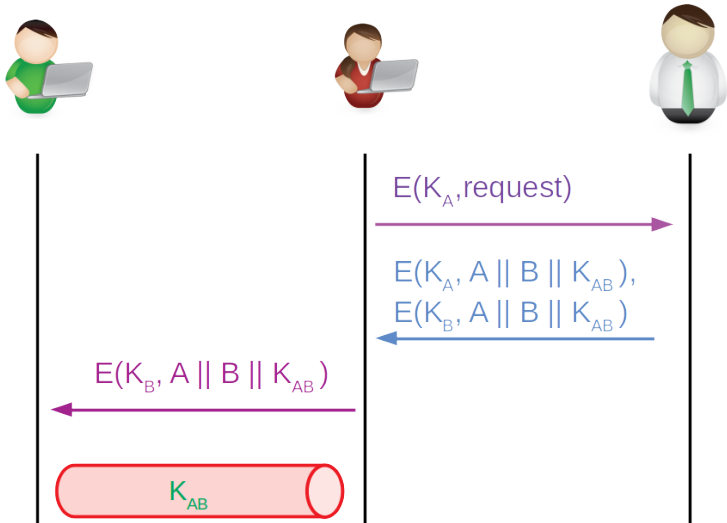
Key management is really a big issue.



## A Simple Solution: TTP



## A Simple Solution: TTP



How to attack??

How to attack??

Replay Attack.

How to attack??

Replay Attack.

How to defense?

How to attack??

Replay Attack.

How to defense?

Add timestamp.

# Is it Possible to Exchange Key without TTP

- TTP is a very simple solution.
- However, in some cases, we do not like TTPs.
  - TTP may not be always there.
  - Because there is no TTP in the real world.
    - Example: **Prism Project, Huawei.**
- How can we do this?
  - Remember, we always assume that the attacker can see your communication.

# Some Solutions

- **Merkle Puzzles.**
- **Diffie Hellman.**
- **Public Key Solution.**
  - Since the key is public, of course we can release to the attacker.
  - Yes, but how to release your public key?



# Merkle Puzzle

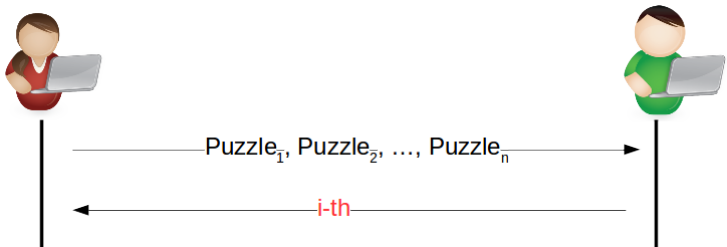
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## Puzzle-based Solution

1. Alice prepares  $n$  puzzles and sends all of them to Bob.
2. Bob randomly selects one puzzle and solves it.
3. After Bob solves it, Bob told Alice that I have solved  $i$ -th puzzle, we can communicate with the puzzle answer as a key.

How about the attacker??

## Puzzle-based Solution



- What is a puzzle?
  - A symmetric key encryption where key is  $n$ -bits.
  - Message: "This is message  $X$ . This is the symmetrical key  $Y$ ".
  - Example:  $n = 32$
- For Alice:  $O(n)$ .
- For Bob:  $O(n)$ .
- For Attacker:  $O(n^2)$ .

## **Diffie Hellman Key Exchange (Important)**

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## Before We Start

We need to start with some assumptions.

# What does Assumption Mean?

- In the crypto field, lots of mechanism are based on assumptions.
- If someone can crack our crypto system, it means (s)he can solve a hard problem.
  - The problem is assumed to be hard since we have no idea how to solve it.

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  - But the reason may be that we are not smart enough.

# Some Assumptions

## Discrete Logarithm Assumption

Given  $g, a \in \mathbb{G}$ , it is hard to find an integer  $x$  that  $g^x = a$ .

## Computational Diffie-Hellman Assumption

Given  $g^x, g^y \in \mathbb{G}$ , it is hard to compute  $g^{xy}$ .

## Decisional Diffie-Hellman Assumption

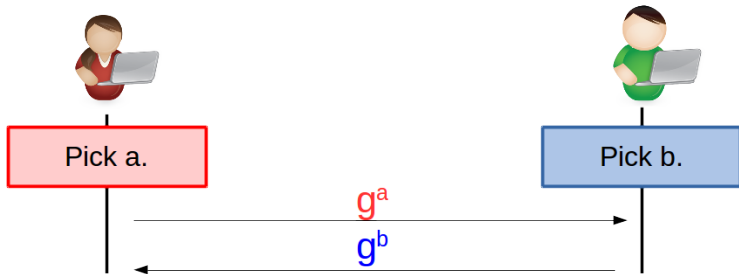
Given  $g^x, g^y, t \in \mathbb{G}$ , it is hard to verify if  $t = g^{xy}$ .



Please do not tell me that given 2 and 8, the answer is 3.

Which one is the hardest among three?

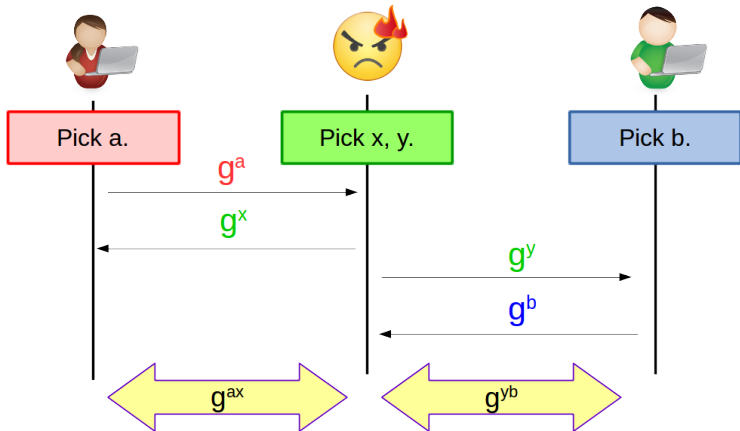
# Diffie-Hellman Protocol



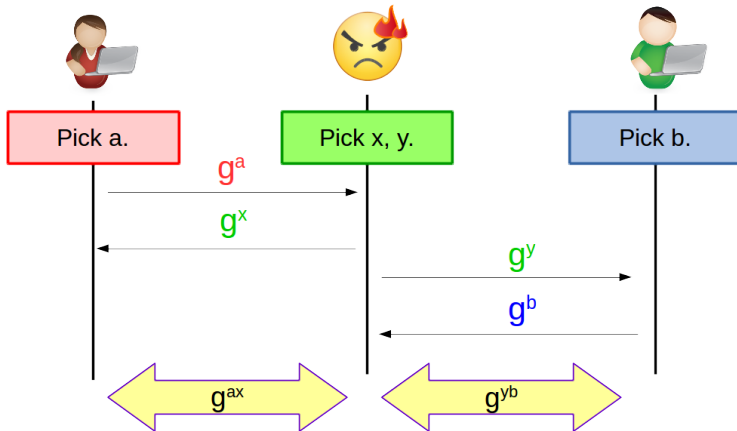
The key will be  $g^{ab}$ .

How to Attack??

# Man in the Middle Attack

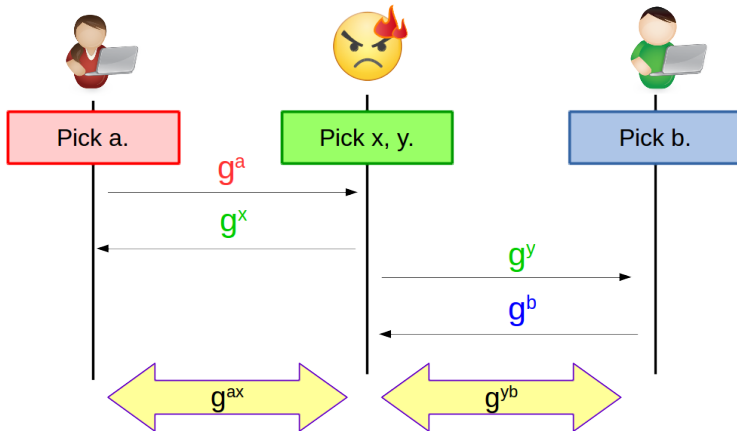


# Man in the Middle Attack



Note that this attack does not solve the hard problem.

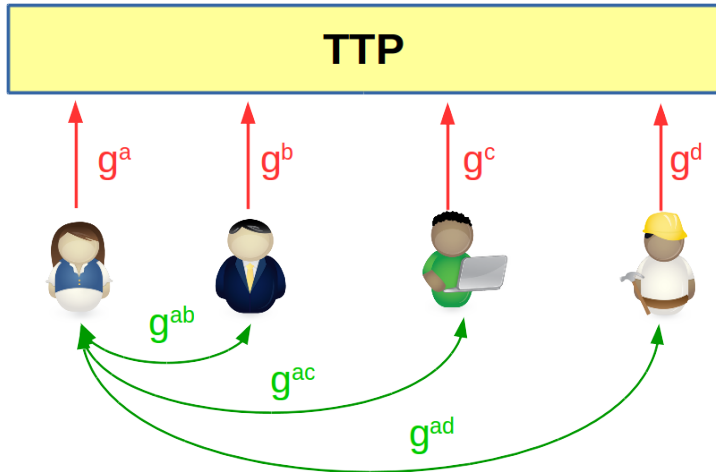
# Man in the Middle Attack



Note that this attack does not solve the hard problem.

How to defense?

## Another Usage





- How to establish a key between Alice, Bob and Charlie?

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  - **Bilinear Map Group.**

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- How to establish a key between Alice, Bob, Charlie, Dennis?

# Questions

- How to establish a key between Alice, Bob and Charlie?
  - **Bilinear Map Group.**
- How to establish a key between Alice, Bob, Charlie, Dennis?
  - **Multi-linear Map Group.**
  - **Open Question.**

# ElGamal Encryption

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- Wait a moment! I think we have already completed the public key encryption topic.
- Yes, but there is something we skip ... **the security proof**.
- Now we have some assumptions, we can prove the encryption system.

# ElGamal Encryption Scheme

- **KeyGen** $(1^\lambda) \rightarrow \{PK, SK\}$ :
  - A cyclic group  $\mathbb{G}$  of order  $p$  with generator  $g$ .
  - $x \xleftarrow{R} \mathbb{Z}_p^*$ .
  - $h \leftarrow g^x$ .
  - $PK: \{\mathbb{G}, p, g, h\}$ .
  - $SK: \{x\}$ .
- **Enc** $(PK, m) \rightarrow C$ :
  - $y \xleftarrow{R} \mathbb{Z}_p^*$ .
  - $C = (c_1, c_2) = (g^y, m \cdot h^y)$ .
- **Dec** $(SK, C) \rightarrow m$ :
  - $s = c_1^x = g^{xy}$ .
  - $c_2 \cdot s^{-1} = m \cdot h^y \cdot s^{-1} = m \cdot g^{xy} \cdot g^{-xy} = m$ .

How to Prove its Security?

How to Prove its Security?  
Which assumption should you use?

## Theorem

If the decisional Diffie-Hellman assumption (DDH) holds in  $\mathbb{G}$ , then ElGamal achieves semantic security.

Proof:

If I can break ElGamal, given  $g^x, g^y, t$ , I know how to check if  $t = g^{xy}$ .

# Security Proof



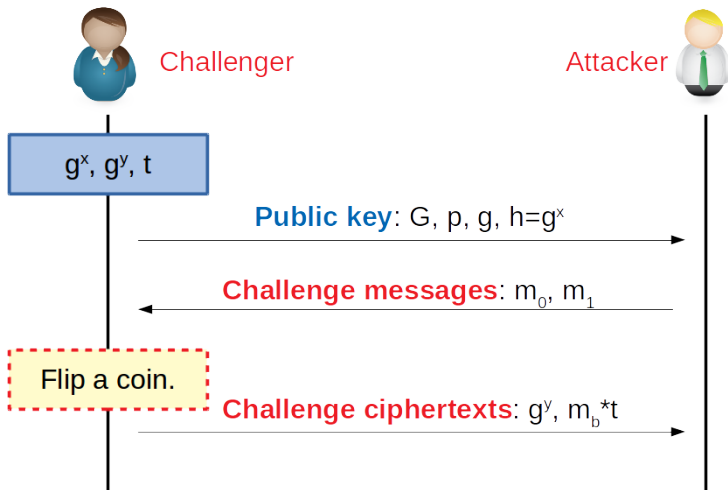
I give you  $g^x$ ,  $g^y$ ,  $t$ , would you please answer me if  $t = g^{xy}$  ?



I do not know.

(Wait ... I remember that Bob knows how to break ElGamal. Maybe I can ask him for help.)

# Security Proof



- If Bob cannot break the game:
  - The probability that Bob wins the game is  $\frac{1}{2}$ .
- If Bob can break the game:
  - The probability that Bob wins the game is  $\frac{1}{2} + \delta$ .

So if  $\delta$  is not negligible, that is, the challenge ciphertext is a **valid Elgamal ciphertext**, which implies  $t = g^{xy}$ .



# DSA: Digital Signature Algorithm

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- **Setup:**

- A large prime  $p$ .
- A prime  $q$  that divides  $p - 1$ .
- A hash function  $H$ , like SHA-256.
- $g = h^{\frac{p-1}{q}}$ .
  - $h$  is a random number  $\in \{2, \dots, p-2\}$ . Commonly  $h = 2$  is used.
  - $g^q \equiv ??$  .

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- **KeyGen:**

- Private key:  $x, x < q$ .
- Public key:  $g^x$ .

- **Sign:** given a message  $m$  and a private key  $x$ .
  - $k \xleftarrow{R} \mathbb{Z}_q$ .
  - $r = (g^k \bmod p) \bmod q$ .
  - $s = (H(m) + xr) \cdot k^{-1} \bmod q$ .
  - The signature is  $(r, s)$ .
- **Verify:** ??

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  - The signature is  $(r, s)$ .
- **Verify:** message  $m$ , signature  $(r, s)$ , public key  $g^x$ :
  - $a = H(m) \cdot s^{-1} \bmod q$ .
  - $b = r \cdot s^{-1} \bmod q$ .
  - $v = (g^a (g^x)^b \bmod p) \bmod q$ .
  - Check if  $v = r$ .

$$\begin{aligned}v &= (g^a(g^x)^b \bmod p) \bmod q \\&= (g^{H(m) \cdot s^{-1} \bmod q} (g^x)^{r \cdot s^{-1} \bmod q} \bmod p) \bmod q \\&= (g^{(H(m)+xr) \cdot s^{-1} \bmod q} \bmod p) \bmod q \\&= (g^k \bmod p) \bmod q\end{aligned}$$

How to prove DSA is secure?



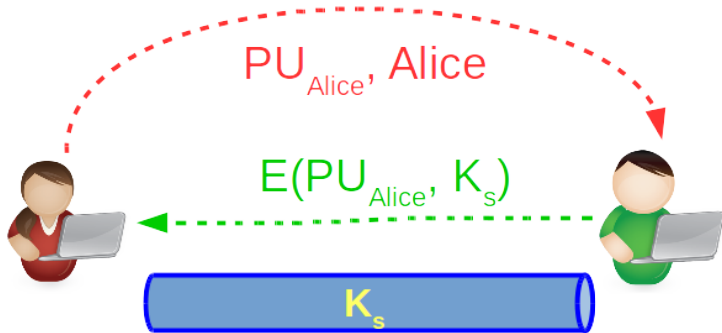
How to prove DSA is secure?

No one knows...

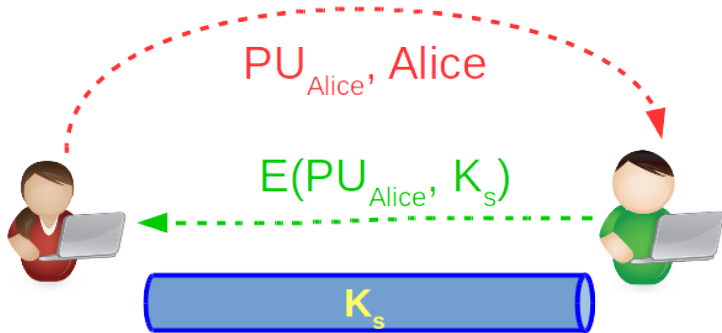
# Key Distribution through Public Key Encryption

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## A Trivial Way

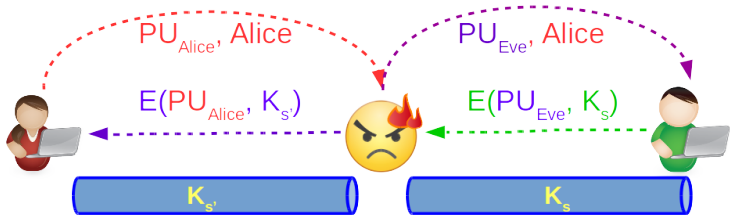


## A Trivial Way



**Quiz:** Is there any problem in this scheme?

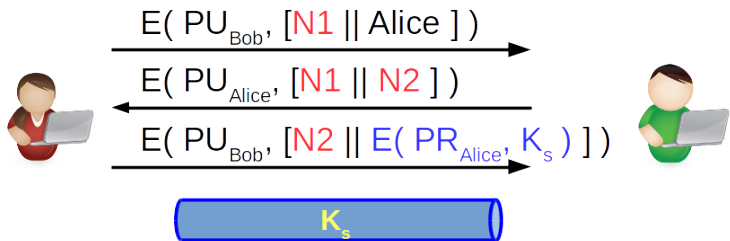
# Man in the Middle



We do not know the sender really has the private key.

Why do not we use public key encryption always?

## Key Distribution with Authentication



# One Important Problem

- Wait a moment! This scheme assumes that the public keys are well-known.
- How to distribute your public key?



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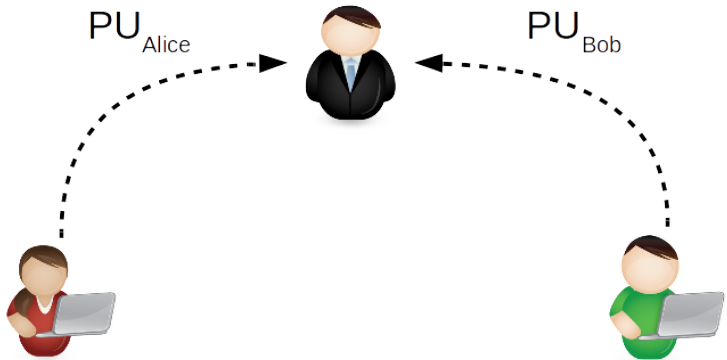
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BTW, is there any other problem about this scheme? **Replay attack.**

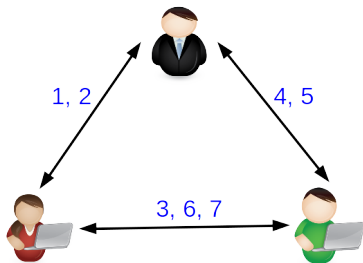
## X509 Certificate

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## Again, TTP is Always a Solution



## TTP-based Solution

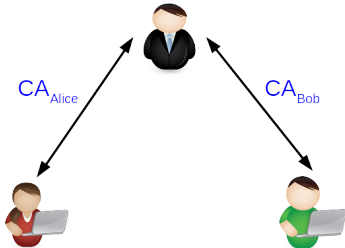


1.  $\text{Req} || T_1$ .
2.  $E(PR_{\text{TTP}}, PU_{\text{Bob}} || \text{Req} || T_1)$ .
3.  $E(PU_{\text{Bob}}, ID_{\text{Bob}} || N_1)$ .
4.  $\text{Req} || T_2$ .
5.  $E(PR_{\text{TTP}}, PU_{\text{Alice}} || \text{Req} || T_2)$ .
6.  $E(PU_{\text{Alice}}, N_1 || N_2)$ .
7.  $E(PU_{\text{Bob}}, N_2)$ .

## Some Drawbacks

- Everytime when Alice and Bob need to talk, TTP should always be there.
- TTP may be the bottleneck.

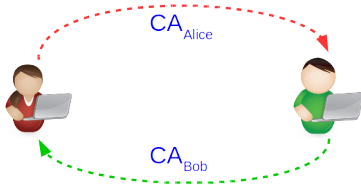
# Certificate-based Solution



- $E(PR_{\text{TTP}}, T || ID || PU_{\text{ID}})$ .

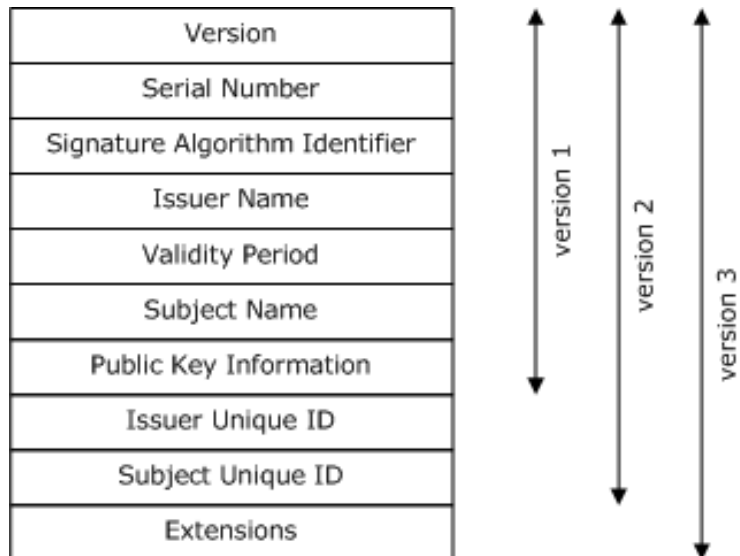


# Certificate-based Solution



- $E(PR_{TTP}, T || ID || PU_{ID})$ .

## X509 Certificate



## Certificate Example

Let's check some certificate.

- `http://w1.csie.ntnu.edu.tw/`
- `https://www.google.com/`
- `https://ftp.rsa.com/`

## Who can Issue a Certificate?

- Actually, everyone can issue a certificate.
  - `http://www.study-area.org/tips/certs/certs.html`
- However, is there anyone who dares to accept certificates issued by you?

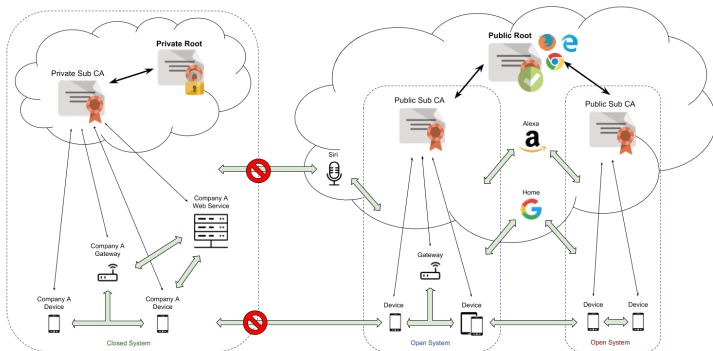
If you want to have a CA, maybe you can try **Let's Encrypt**.

- **Free.**
- Since it is free, lots of people will use this CA, including bad guys.

# **PKI: Public Key Infrastructure**

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A public key infrastructure (PKI) is a system for the **creation**, **storage**, and **distribution** of **digital certificates** which are used to verify that a particular public key belongs to a certain entity.





More details can be found at [https://www.e-reading.club/bookreader.php/142115/Choudhury\\_-\\_Public\\_Key\\_Infrastructure\\_implementation\\_and\\_design.pdf](https://www.e-reading.club/bookreader.php/142115/Choudhury_-_Public_Key_Infrastructure_implementation_and_design.pdf).

# Kerberos

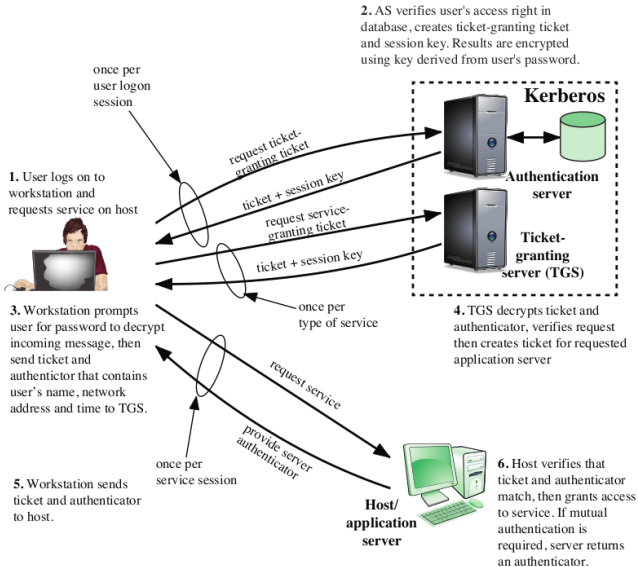
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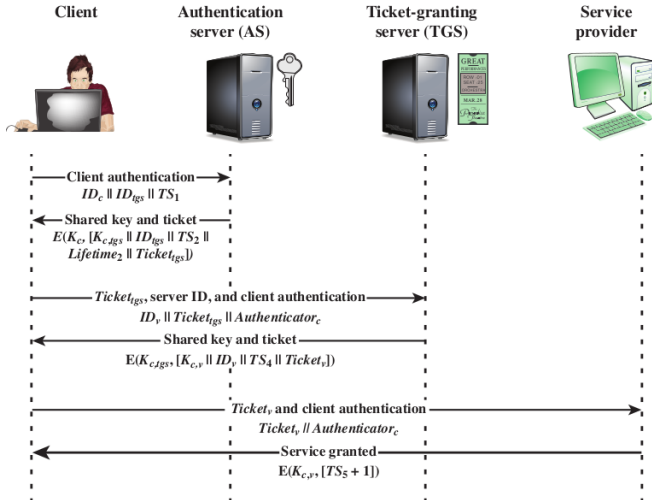


- Designed by MIT.
- Scenario:  
Assume an open **distributed** environment in which users at workstations wish to **access services** on servers distributed throughout the network.

# Kerberos



# Kerberos



# The End of Crypto

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What you should know:

1. Symmetric key encryption: DES, 3DES, AES.
2. Asymmetric key encryption: RSA.
3. Hash function.
4. Key Exchange: Diffie-Hellman.
5. How to read security protocols.
6. How to define the security of crypto.



## So Now I Know What Crypto is

Unfortunately, you are not even in the crypto gateway. I skip a lot of things:

1. Prime.
2. Number theory.
3. Other public key schemes, like ECC.

## More Crypto Topics

1. Zero knowledge commitment.
2. Identity-based encryption.
3. Attribute-based encryption.
4. Homomorphic encryption.
5. Post-quantum encryption.
6. Blockchain.