Momentum

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17th Sep, 2022

Learning Outcome

I highly recommend you to finish this checklist to determine whether you've achieved the learning objectives.

- ☐ Define and use *linear momentum*
- ☐ Define and use *impluse*[1]
- ☐ Recall and use that the **area** under F-t graph equals to the **change of momentum**
- ☐ Relate force to the rate of change of momentum and state Newton's second law of motion

[1] not required by CAIE but by me

Leadin

'People who succeed have momentum. The more they succeed, the more they want to succeed, and the more they find a way to succeed.'

Tony Robbins

In this quotes, momentum is defined as the inner state of mind. Like human, moving object also have momentum. So I change that quote into: 'Objects which move has momenta, the more speed they have, the more momenta they get, and the more they will remain their momenta'

Sanjin Zhao

Momentum

As mentioned before, momentum is a physical quantity related with a moving object. It is defined as the ______________________________.

Summary

If expressed in formula, the momentum $\vec{p}$ is :

$$\vec{p} = m \cdot \vec{v}$$

There are several things to notice:

1. momentum is a _____________ quantity, the direction of which is defined by the direction of the _____________ of the object
2. momentum is a <u>state/process</u> quantity.
3. the unit for momentum is __________________________, there is no other derived unit for momentum

Task

If a truck with a total mass of 2000 kg is moving to east at a speed of 80 km h^{-1}, what is the momentum of the truck?

In order for a man with mass of 60 kg to have the same momentum as the truck in previous question, what should the velocity of the man be? state both magnitude and direction of velocity.

k.e. and momentum

Using the mass of the object and the velocity(speed), both k.e. and momentum can be calculated.

Example

Say a object with mass m are having a momentum p. What is the velocity expressed in terms of m and p, and hence derive a formula to determine the k.e. of the object, using m and p and necessary scientific constant only.

Try to compare k.e. and momentum in the following aspect.

- vector or scalar nature
- process or state quantity
- similarities between the two quantities from the defining formulae
- difference between the two quantities

Impulse

This section is not required by the CAIE, but if it would be better off to learn this

Impulse by a Constant Force

Summary

Impulse is defined as the product of force and the time the force has been applied. Expressed in formula, that would be

$$\text{Impulse} = \vec{F} \times \Delta t$$

Several things to notice are that:

1. impulse is a ______________ quantity, the direction of which is defined by the direction of the
2. impulse is a state/process quantity.
3. the unit for impulse is ____________________________, there is no other derived unit for impulse. The base SI unit for N s is ______________.

Impulse by a Varying Force

Recall the process of how we determine the displacement from a varying velocity-time graph. *v*-*t* graph.
If velocity is constant, the displacement is $s = v \times \Delta t$
If velocity is changing, the displacent is $s = \int v(t)\,\mathrm{d}t$, this formula means the area under the *v*-*t* graph

Now let's apply that *integration* principle[2] to the impluse.
If the force is constant, the impluse is $\vec{I} = \vec{F} \times \Delta t$
If the force is changing with respect to time, the impulse that varying force exert to the object is $\vec{I} = \int \vec{F}(t)\,\mathrm{d}t$

Using a F-t graph would make this process more clear.

[2] which can be applied to every quantities having product relationship

Relationship Between Impulse and Mometum

As mentioned before, Impulse and Momentum have same SI base unit, and are both vectors, but impulse is a process quantity, while mometum is a state quantity. Do the special things reminds you the relationship between work and energy? Again, same process has been used, Let's look at the example of a UAM.

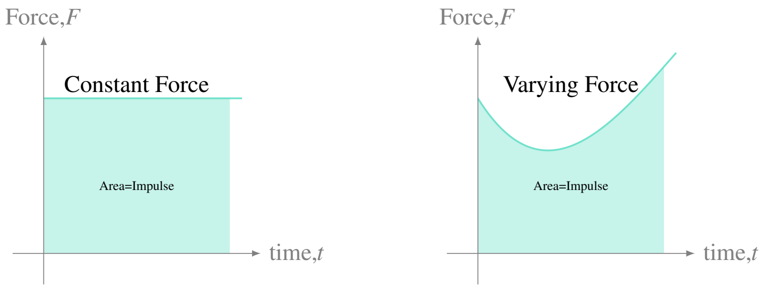


Figure 1: Area under F-t graph is impulse

Example

Suppose an object with mass m is moving with a initial velocity u, and now a constant resultant force F is acting on the object, the force has been applied on the object for a time period of t, the direction of the force is the same as the initial velocity. Calculate the impulse provided by the force F;
Determine the final velocity of the object, and the **change of momentum**;

The impulse provided is

$$I = Ft$$

the direction of the impulse is the same as the resultant force.
The acceleartion is

$$a = {}^{F}/_{m}$$

thus, the final velocity is

$$v = u + at = u + {}^{F}/_{m} \cdot t$$

the direction is the same as the initial velocity
The final momentum

$$p_f = m \cdot (u + {}^{F}/_{m} \cdot t)$$

the initial momentum is

$$p_i = mu$$

thus the change of momentum is

$$\Delta p = \underline{\hspace{6cm}}$$

The direction of the change in momentum is the same as the initial velocity

From the example, it not hard to deduce the following relationship[3]

[3] Depsite we use simple UAM, Calculus can prove the same conclusion even if the force is changing

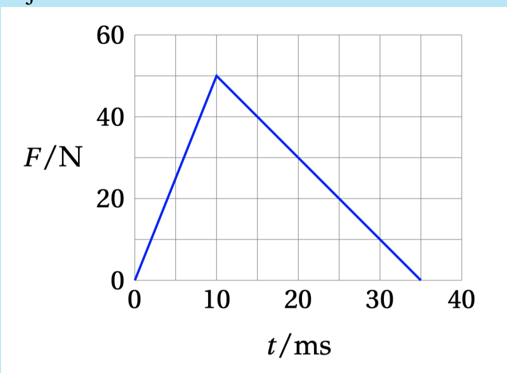
Summary

change in an object's momentum is equal to the area under the F-t graph (impulse)

$$\Delta\vec{p} = \vec{F} \cdot \Delta t$$

Task

An object of mass 70 g is initially at rest. A force that varies with time is exerted on the object. The graph shows the how the force varies during the time of impact. What is the final velocity of the object?

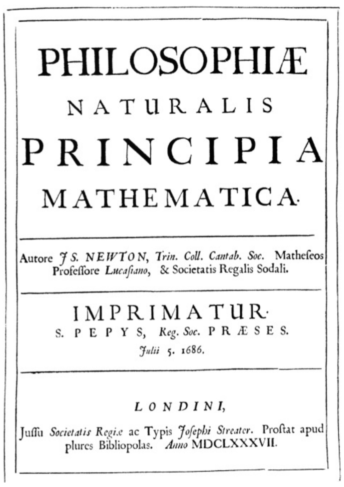


Refine Force and NSL

In the previous handout, force is defined as an interaction between objects, as a strict future physicist, this definition is not a good one. It is not quantitative enough. But now, with the introduction of momentum. $\Delta p = F \cdot \Delta t$, Reformulate the relationship, $\vec{F} = \Delta\vec{p}/\Delta t$ or expressed in differential form $\vec{F} = \mathrm{d}\vec{p}/\mathrm{d}t$. Finally, we arrive at a much more quantitative definition form of foce.

Theorem 0.1 *Force is defined as the rate of change in momentum.* $\vec{F} = \Delta\vec{p}/\Delta t$

And actually this is first introduced as Netwon's Second Law in his 'Philosophiae Naturalis Principia Mathematica'


PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore J. S. NEWTON, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

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S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

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Figure 2: the title page of this GIANT book

Summary

The resultant force acting on an object is directly proportional to the rate of change of the linear momentum of that object. The resultant force and the change in momentum are in the same direction.

This is the second form of NSL, which is also the original form proposed by Newton.

Average vs Instantaneous Force

According to NSL, average force equals to the total change in momentum divided by the total time. Using the *differentiation* concept, what is the instantaneous force if a *p*-*t* diagram is provided?

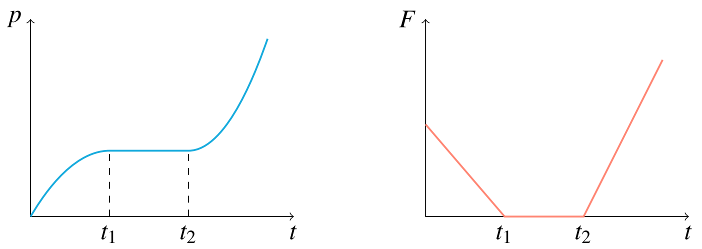

Figure 3: How to infer F from p-t graph?

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Figure 4: the air max unit can reduce the average force when touching the ground

Task

Explain the principle of Nike vapormax shoes, or the air cushion to save life.