Series and Parallel

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2nd Oct, 2022

Learning Outcome

I highly recommend you to finish this checklist to determine whether you've achieved the learning objectives.

- ☐ use Kirchhoff's laws to derive the formulae for the combined resistance of two or more resistors *in series and in parallel*
- ☐ recognise that ammeters are connected in series within a circuit and therefore should have low resistance
- ☐ recognise that voltmeters are connected in parallel across a component, or components, and therefore should have high resistance.

Leadin

In mechanics, if two springs with spring constant k_1 and k_2 are connected in parallel, the equivalent spring constant $k_{eq} = k_1 + k_2$. If connected in series, the equivalent spring consant can be determined by this formula:

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Now we are going to discuss the connection of two resistors.

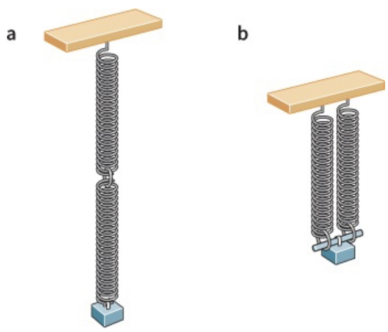

Figure 1: Specify the connection types

Series Connection

Two resistors are connected in series if one resistor is connected one after another, as shown in the Fig.2.

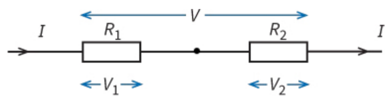

Figure 2: Two resistors in series

The current flows out of R_1 and then flows into R_2, since there is just one junction between R_1 and R_2, based on KCL, the conclusion in series circuits are:

currents are equal in all resistors in series

If one equivalent resistor are used to replace the combination of the two resistors, the requirements should be the following:

- p.d. across the equivalent resistor should be equal to the TOTAL p.d.s of the two
- the current flowing in the equivalent resistor should be equal to that of two resistors.

Example

According to the requirement and Fig.2, deduce the R_{eq} in terms of R_1 and R_2

$V = I \cdot R_1 + I \cdot R_2$, and in the equivalent resistor, $V = I \cdot R_{eq}$, thus

$$R_{eq} = \frac{V}{I} = \frac{I \cdot R_1 + I \cdot R_2}{I} = \underline{\hspace{4cm}}$$

And this rule can be applied further, if more than two resistors are connected in the circuits[1], such process can be repeated.

[1] Three laws are applied in determining this, state which laws they are.

Parallel Connection

Two resistors are connected in parallel if the wire seems parallel, or they are connected to the same junctions in the circuits, as shown in the Fig.3.

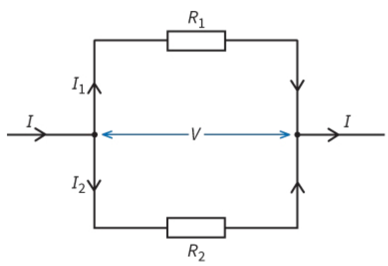

Figure 3: Two resistors in parallel

The currents will seprate into two individual ways, one flows into R_1,denoting it as I_1, and one flows into R_2, denoting it as I_2. since the two resistors are using common junctions, the potential difference no matter viewed from either resistor.

p.d.s are equal in all resistors in parallel

Again, if one equivalent resistor are used to replace the combination of the two resistors, the requirements should be the following:

- p.d. across the equivalent resistor should be equal to the TOTAL p.d.s of the two
- the current flowing in the equivalent resistor should be equal to that of two resistors.

Example

According to the requirement and Fig.3, deduce the R_{eq} in terms of R_1 and R_2

$V = I_1 \cdot R_1 = I_2 \cdot R_2$, and in the equivalent resistor, $V = I \cdot R_{eq}$, thus

$$R_{eq} = \frac{V}{I} = \frac{V}{I_1 + I_2} = \underline{\hspace{4cm}}$$

Summary

Total(equivalent) resistance in series connection is given by the equation:

$$R_{eq} = R_1 + R_2 + R_3 + \dots$$

Total(equivalent) resistance in parallel connection is given by the equation:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

Task

Compare that to the connection types of spring, think why similarities exsit.

Summary

In a more complex circuits, resistors are connected in various types, the most important thing is the find the smallest unit of connection.

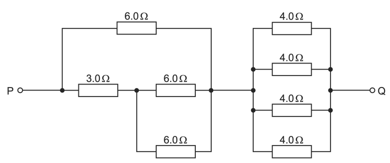

Figure 4: a more complicated circuits

Example

Without using Kirchhoff's Law, Find the total resistance between P and Q.

The smallest unit of connection is _______________, and the equivalent resistance is 3 Ω, then the circuit diagram can be simplified into the following Then, repeat the procedure.