Momentum

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Learning Outcome

I highly recommend you to finish this checklist to determine whether you've achieved the learning objectives.

- ☐ state and apply the principle of conservation of momentum to collisions in one and two dimensions
- ☐ apply the principle of conservation of momentum to solve simple problems, including elastic and inelastic interactions between objects in both one and two dimensions[1]
- ☐ solve the velocity after elastic and total inelastic collision
- ☐ recall that, for a perfectly elastic collision, the relative speed of approach is equal to the relative speed of separation
- ☐ discuss energy changes in perfectly elastic and inelastic collisions

[1] knowledge of the concept of *coefficient of restitution* is not required. But Sanjin Zhao would be much happier if you dig into that.

Leadin

In the story of "The Three Body", Dong Yang and lots of physicists committed suicide because of the result from the Large Hardon Colider does not obey the *principle of conservation of momentum*, which means the whole physics may not exist.

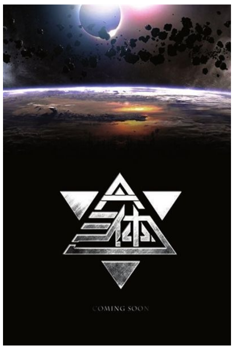


Figure 1: The cover page of "The Three Body" by Cixin Liu

Principle of Conservation of Momentum

So, the principle states that:

Theorem 0.1 *For a closed system, the total momentum before an interaction (e.g., collision) is equal to the total momentum after the interaction.*

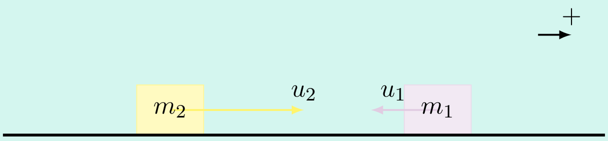
For the term of a closed system, it means there is *no external* resultant force acting on the system.

And don't worry, only in the imaginary world, could this conservation fail. It is an unbreakable rule governing everything in the physical world, from giant galaxies to microscopic particles.

Example

For a single object, if no external force exists, the resultant force is 0, according to the Newton ____________________ Law, the velocity of the object will remain the same. Thus, the velocity is always the same, even the direction keeps the same, so do momentum.

For a closed system, for example, two objects moving in a horzitonal **smooth** track. and make on a **head-on collision**.



For single objects, it is definitely not conserved. Because for the object, the forces from the other box is external force. Besides, the initial and final velocity are not the same(it seems nonsense, but the rationale of judging whether a object or system's momentum is conserved are not is critical).
Now let's look at the system's momentum. the way is by checking the impulse of single object.

$$\Delta p = F \cdot \Delta t$$

Suppose the time the collision takes is Δt, assuming the force is constant through the process (It is not possible, but let's suppose

that). Making the direction to the right is positive,
The impulse that m_1 exerted on m_2 is ______________________________;
The impulse that m_2 exerted on m_1 is ______________________________;
Thus the total impulse of the whole is system is _. Thus, the change of total momentum is **ZERO**. The total momentum is conserved.
Let's prove the principle through experiments later. But this can be used through Phet Simulation

Collision

With the principle, the motional states after collison can be investigated.

Figure 2: Who player snooker well will grasp collision well

Task

List several possible scenarios of collision:

- two cars
- hockey
- comet or asteroid
- hadrons in LHC
- galaxies

Perfect Elastic Collision

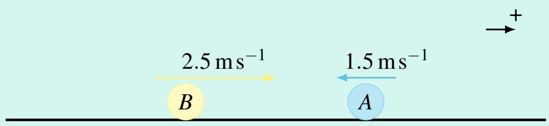
It is the ideal situation of collision, in which

Summary

- The momentum is conserved
- The kinetic energy is conserved

Example

Two identical balls A and B about to make a head-on perfect elastic collision. The mass of each ball is 4.0 kg. Determine the velocity of each ball, state the direction of the velocity as well



Before collision: The momentum and k.e. of A before collision is:

$p_{Ai} = m_A \cdot u_A =$ $\qquad KE_{Ai} =$

The momentum k.e. of B before collision is:

$p_{Bi} = m_B \cdot u_B =$ $\qquad KE_{Bi} =$

The total momentum and total k.e. of the **closed system** is

$\sum p =$ $\qquad \sum KE =$

After Collision:

Inelastic(Sticky) Collision

If two objects with equal mass and velocity but the direction are opposite, they are experiencing head-on collision. After the collision, the two objects will stick together. Let's investigate the momentum and k.e. of such collision before and after. Please finish the table below

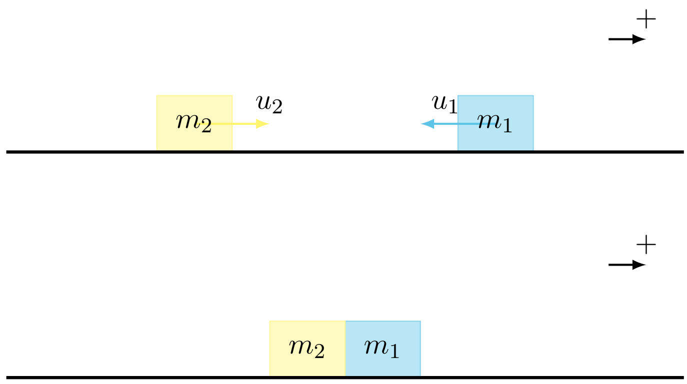


Figure 3: two objects stick together after collision

Before Collision

Object	Mass	Velocity	Momentum	k.e.
A				
B				

After Collision

A				
B				

Summary

In an inelastic collision,

- The total momentum is
- The total k.e. is ______________________________

You can use the use the Phet Simulation, and adjusting the *coefficient of elasticity*[2]. Validate the head-on elastic collision

[2] AKA, coefficient of restitution

Task

In the game of bowls, a player rolls a large ball towards a smaller, stationary ball. A large ball of mass 5.0 kg moving at $10.0\,\mathrm{m\,s^{-1}}$ strikes a stationary ball of mass 1.0 kg. The smaller ball flies off at $10.0\,\mathrm{m\,s^{-1}}$.

1. Determine the final velocity of the large ball after the impact
2. Calculate the kinetic energy before and after the impact.
3. Stete whether the collision is elastic or inelastic

Two Dimension Collision

Have the word "two dimension" remind you something in projectile motion? What is the rationale how we analyze the motion in two dimension? Now what am I going to elicit?

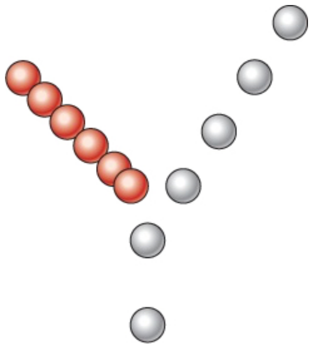


Figure 4: Such collision happens in two dimension

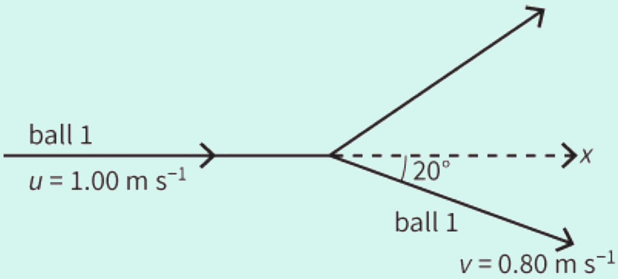
Summary

Momentum is a vector, it can be resolved. And depsite in two dimension collision, the total momentum is also conserved, which means the components in two directions must remain the same as well.

Look at the example:

Example

A snooker ball collides with a second identical ball as shown below



1. Determine the components of the velocity of the first ball in the x- and y-directions.
2. Hence, determine the components of the velocity of the second ball in the x- and y-directions.
3. Hence, determine the velocity (magnitude and direction) of the second ball.

1.

$$v_{1_x} = v \cdot \cos 20° = 0.8 \times \cos 20° = \underline{\hspace{8cm}}$$

$$v_{1_y} = v \cdot \sin 20° = 0.8 \times \sin 20° = \underline{\hspace{8cm}}$$

2. suppose the mass of the two balls are identical, denoted as m. the inital momentum in x-direction is

$$p_{x_i} = m \cdot u + m \cdot 0 = m \cdot 1.0 =$$

the final mometum in y-direction is

$$p_{x_f} = m \cdot v_{1_x} + m \cdot v_{2_x} =$$

By due to the conservation of linear momemtum (component as well)$p_{x_i} = p_{x_f}$

$$m \cdot 1.0 = m \cdot 0.75 + m \cdot v_{2_x}$$

$v_{2_x} =$ ________________________________

Repeat the same process

3. Since the

$$v_{2_x} = \underline{\hspace{4cm}}$$

$$v_{2_y} = \underline{\hspace{4cm}}$$

. The velocity of the second ball after collision is:

$$v_2 = \sqrt{\hspace{8cm}} = \underline{\hspace{2.5cm}}$$

The angle it makes with the horizontal is ______________ and above/downward

Figure 5: Is momentum in fireworks conserved?

Explosion and more cases

During a explosion, a whole object will split into two or more pieces, each pieces will have different direction, thus having different momentum. However, what we are sure is that the total momentum must be zero after the explosion since the momentum before explosion is zero.

Task

Suppose the candle sent from the firework has mass m, and the velocity is v after the ignition, the direction is slanted upward with an angle θ. The mass of the remaining part of the fireworks is M, Determine the velocity of the remaining part.

Figure 6: Roman Candle is a easier firework to investigate on the conservation of momentum