Uniformly Accelerated Motion

Sanjin Zhao

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Learning Outcome

I highly recommend you to finish this checklist to determine whether you've achieved the learning objectives.

- ☐ Determine displacement from the **area** under a *velocity–time graph*.
- ☐ Determine velocity using the **gradient** of a *displacement–time graph*.
- ☐ Determine acceleration using the **gradient** of a *velocity–time graph*.
- ☐ Derive, from the definitions of velocity and acceleration, equations that represent uniformly accelerated motion in a straight line.
- ☐ Solve problems using equations of uniformly accelerated motion. including free fall
- ☐ Draw *d*-*t*, *v*-*t* and *a*-*t* for stationary, uniform motion, uniformly acclerated motion, free fall or thrown up.
- ☐ Describe an experiment to determine the acceleration of free fall using a falling object
- ☐ Describe and explain motion due to a uniform velocity in one direction and a uniform acceleration in a perpendicular direction

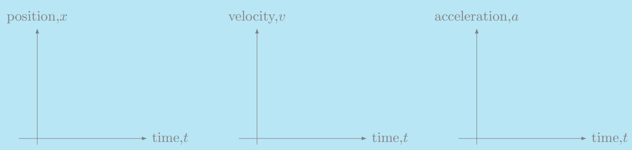
Leadin

Check the last section in Describe Motion, Calculus has been introduced in analysing the kinematics. Let's dig into how this can be applied.

Uniform Motion

A motion is said to be uniform when the velocity remains constant, this means not only the direction but also the magnitude of velocity is constant.

Task



draw the *d*-*t*, *v*-*t*, and *a*-*t* graph of an object experiencing uniform motion.

What will the graph look like if the velocity is said to be *'negative'*

Uniformly Accelerated Motion

However, uniform motion is not the most important topic today. We are talking about the Uniformaly Accelerated Motion[1].

[1] def:

a-t graph

Draw the *a*-*t* graph of a unifrom accelerated motion,using the space below:

due to the acceleration is constant, the $a-t$ graph is also a ____________ straight line. Has anyone draw a line which is negative? Does that make any sense?

v-t graph

The most import graph of a uniform accelerated motion must be *v*-*t* graph. Please draw the *v*-*t* graph of a unifrom accelerated motion,using the space below:

Recall how we can deduce the acceleration from *v*-*t* graph.

Summary

Acceleration is the gradient of the *v*-*t* graph.

For the initial velocity, u is utilized to resemble it. and v is used to symbolize the final velocity

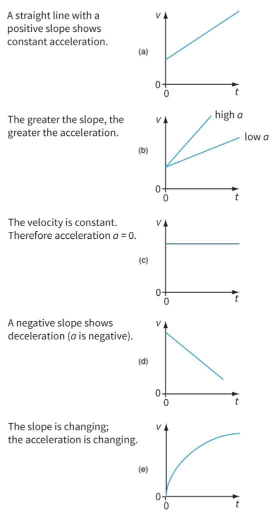


Figure 1: different *v*-*t* graphs

Derivation of displacement

The most important aspect of *integration* when Newton applied to analyze kinematics is find displacement when given velocities.

dispalcement when speed is constant

check the v-t graph of a uniform motion, it is not hard to deduce that the if the velocity is constant throughout the whole period of motion, then the displacement would be calculated as:

$$s = v \cdot t$$

Because v is the height of the v-axis, and t is the width in the time axis. thus the product of the two would be the *AREA* of the rectangle enclosed by the following:

1. v-t graph
2. t-axis
3. v-axis
4. another vertical line defining the end time of the motion

Task

In figure 1 (c), color the area which represent the displacement of the uniform motion.

displacement when velocity is changing constantly

Apply the process, but in a much more microscopic perpective. If I divide

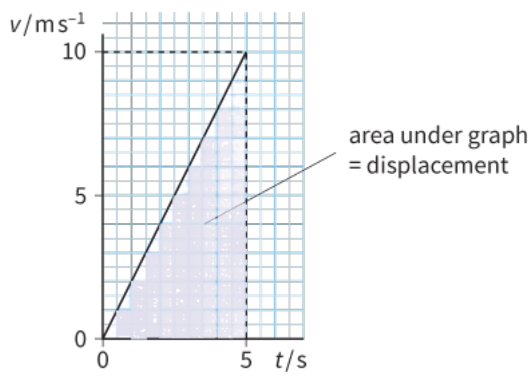


Figure 2: the v-t graph of a uniformly accelerated motion

the motion process into 9 segment, and for each segment, treat it like the object is keeping the motion using the initial velocity. Then each slim rectangle represent the 'displacement' of each segment, adding them together will lead us to an *approximation* of the total displacement. But that's not what we want, we need exact displacement not just the approximation. The process can move on to an **INIFINITELY SMALL** time interval, we denote it as dt, all the 'slim' rectangles will look like a slim straight line which will have **INIFINITELY SMALL** area which can be denoted as dx. The relationship between the dx and dt is now quite clear.

$$\mathrm{d}s = v(t)\mathrm{d}t$$

The last step is to adding infinitely many small displacement together, to determine the total displacement, in Leibniz's notation system, it is written as:

$$s = \int(\mathrm{d}s) = \int v(t)\mathrm{d}t$$

However, you are not required to grasp this at this stage[2]. Back to the v-t graph, we find that the small straight line(with infnitely small area) will form a *trapezoid*[3] or a triangle. No matter which type, it is always the **Area Under the v-t graph**. That's the only thing to be used in deriving the displacement from the v-t graph is:

[2] Watch 3B1B essence of calculus.

[3] def

Summary

Area Under the v-t graph represent the ________ of a motion. no matter the velocity is changing or not changing.

Task

Determine the displacement of the motion show in figure 2.

displacement when velocity is changing but not constantly

In the figure 1 (e), the velocity is not changing with a constant acceleration, but the principle of integration can still be used. Thus, label the area which represent the displacement of such motion.[4]

[4] In ALevel, if $v(t)$ is not provided, students will not be required to give exact value of displacement in such scenario

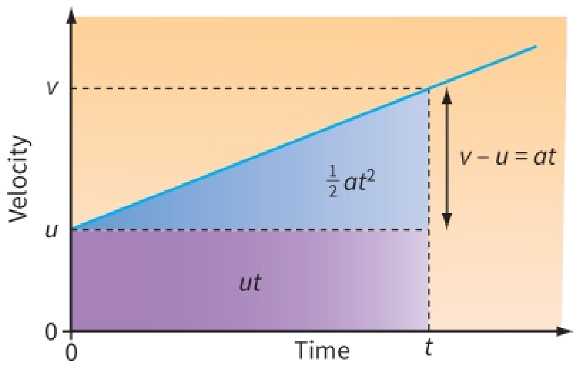
Summary

Equation of Motion in UAM

There is a set of equations that allows us to calculate the kinematics quantities involved when an object is moving with a *constant acceleration*. Those quantities include:

- u:
- v:
- a:
- t:
- s:

Everything strats from the v-t graph of a uniformly accelerated motion: Several things have to be kept in mind in order for using equations:



1. the path of the motion should be a straight
2. the object's acceleration must be
3. all vector quantities's directions are shown as $+$ or $-$, and such sign can not be ignored when using the equation.

Acceleration

Since the acceleration is the **Gradient**:

$$a = \frac{v-u}{t} \tag{1}$$

Velocity

Reformulate the acceleration formula, the initial velocity or final velocity or average velocity of the motion.

$$v = u + at \tag{2}$$

$$u = v - at \tag{3}$$

$$\bar{v} = \frac{u+v}{2} \tag{4}$$

Time

The moving time can also be derived if you change the side of the formula:

$$t = \frac{v-u}{t} \tag{5}$$

Displacement

It all starts with the conclusion from integration, displacement is the area of the v-t graph. Therefore, the easiest equation to determine displacement is:

$$s = \frac{v+u}{2} \cdot t \tag{6}$$

Also, we can substitue any quantities, therefore, more equations can be deduced.

$$s = \frac{v^2 - u^2}{2a} \tag{7}$$

$$s = u^2 + \frac{1}{2}at^2 \tag{8}$$

$$s = v^2 - \frac{1}{2}at^2 \tag{9}$$

Summary

It seems there exist so many equations derived from the UAM, only four of them are of the greatest value,(or we can call them the mother formulae), they are:

1. $a =$ ________________
2. $s = \frac{u+v}{\rule{2cm}{0.4pt}} \cdot$ ____________
3. $s = u \cdot$ ________ $+ \frac{1}{2} \cdot$ _____
4. $v^2 =$ ___________________

Task

A cyclist is travelling at $15\,\mathrm{m\,s^{-1}}$. The distance between her and the wall is $18\,\mathrm{m}$. If she brakes now so that she wouldn't collide with wall, what should the deceleration be?