Error and Uncertainty

Sanjin Zhao

1th Sep 2022

Learning Outcome

I highly recommend you to finish this checklist to determine whether you've achieved the learning objectives.

- ☐ Distinguish **true value** and readings.
- ☐ Know the difference between *accuracy* and *precision*
- ☐ Distinguish between *Random Error* and *Systematic Error*[1]
- ☐ State methods to minimize such errors
- ☐ Express uncertainty in both *absolute* and *relative* forms
- ☐ Grasp the operation rules for uncertainty

[1] def:

Leadin

Why so serious? About sciences.

Despite we want exact value of measurement, but life is so hard. You can not claim your observation or measurement is the *true value*[2] of the object being measured. So we need to discuss errors, presions, and uncertainty during measurement[3].

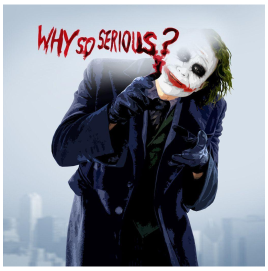


Figure 1: Why so serious about science?

[2] def:

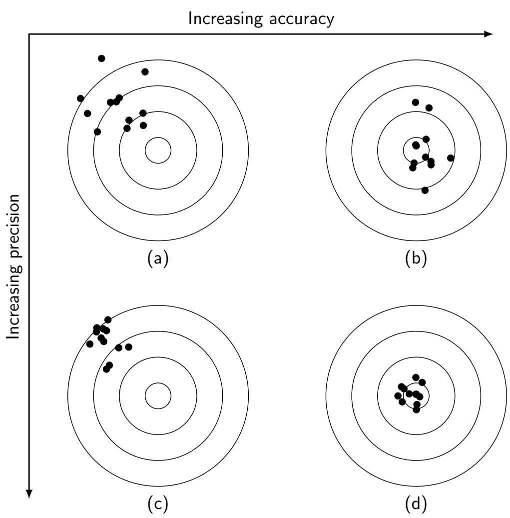
[3] You can refer to this error and uncertainty linkage to find more information.

[4] def:The closeness of the measured value to a standard or true value

[5] def:The closeness of two or more measurements to each other is known as the precision

Accuracy vs Precision

In physics and chemistry related subjects, Two quality is taken into consideration, Accuracy(Accurate)[4] and Precision(Precise)[5].



Summary

Try to compare accuracy and precision according to the figure

Errors

If we know the exact value, the difference between your reading and the exact value can be explained by *errors*.

Random Error

- Random errors cause **unpredictable** fluctuations in an instrument's readings as a result of uncontrollable factors, such as environmental conditions
- This affects the **precision of the measurements** taken, causing a wider spread of results about the mean value

In figure , all of them are showing random error.

Systematic Error

- Systematic errors arise from the use of faulty instruments used or from flaws in the experimental method.
- Systematic errors are biased[6]
- This type of error is repeated every time the instrument is used or the method is followed, which affects the **accuracy of all readings** obtained

[6] which means it always increased or decreased the reading value

In figure , (a) and (c) are showing systematic error.

Zero Error

Zero error is a classic systematic error, in which an instrument gives a reading when the true reading at that time is zero. This can be mitigated by **recalibrating** the instrument[7].

[7] ususally press the zero button

> **Summary**
>
> List the methods that random error and systematic error can be reduced or eliminated

Uncertainty

In order to solve the problem of reading, uncertainty is introduced. Just in the marginfigure, percentage or absolute uncertainty is introduced in labelling the mass or the volume of foods

It is usually be expressed by:

$$\text{reading value} \pm \text{uncertainty}$$

质量或体积定量包装商品标注净含量（Q_n）g或ml	允许短缺量（T） g或ml	
	Q_n的百分比	g或ml
0～50	9	——
50～100	——	4.5
100～200	4.5	——
200～300	——	9
300～500	3	——
500～1000	——	15
1000～10 000	1.5	——
10 000～15 000	——	150
15 000～50 000	1	——

Figure 2: shortage are allowed in foods labelling

For example, if you measure the mass of an apple, with uncertainty of 1 g. The result would be:

$$200 \pm 1\ \text{g}$$

This means the **true value** of the mass of the apple is some value within the range of 199 g to 201 g.

Absolute uncertainty

The above is an expression for **absolute uncertainty**. And usually the absolutely uncertainty is defined as half of the precision of the instrument. For example the precision of the micrometer is 0.01 mm, thus the uncertainty is 0.005 mm

Fractional uncertainty

Another form is **fractional uncertainty**, which gives the perception of the uncertainty relative to the reading. For example, The reading of the apple could also be written as:

$$200\,\mathrm{g}\ \pm 0.5\%$$

Summary

How the absolute uncertainty 1 g can be changed into percentage uncertainty

Operation Rules

The last thing is to calculate the combined uncertainty when dealing with multiple readings. The following are some rules to be utilized.

Addition or Subtraction

Serveral thing have to be kept in mind to deal with the addition or subtraction of uncertainty.

1. absolute uncertainty should be used, that means if percentage uncertainty is used, you must convert it before calculation, the fomula for converting is:

$$\text{percentage uncertainty} = \frac{\qquad\qquad}{\qquad\qquad} \times 100\%$$

2. no matter addition or subtraction, always **ADD** the uncertainties to determine the absolute uncertainty of the final result

Example

The length and width of an rectangle, with uncertainty, are measured to be 4.0 ± 0.4 m and 3.0 ± 0.3 m. What is the perimeter with uncertainty

Multiplication or Division

To determine the uncertainty of products or quotients, **Percentage Uncertainty** should be used

1. No matter multiplication or division, always **ADD** the uncertainties to determine the absolute uncertainty of the final result.

2. if not specified, the final result can be expressed in percentage. However, if absolute uncertainty is required, the formula for converting is:

$$\text{absolute uncertainty} = \underline{\hspace{6cm}}$$

If any physical quantites is multiplied or divided by an exact value or scientific constant. the final result would share same percentage uncertainty as orignial.

Example

The length and width of an rectangle, with uncertainty, are measured to be $4.0 \pm 0.4\,\text{m}$ and $3.0 \pm 0.3\,\text{m}$. What is the area with uncertainty

---

The radius of a circle is measured to be $10.0 \pm 0.5\,\text{m}$. What is the circumference of the cirle with uncertainty?

Powers or Roots

Since powers can be viewd as advanced multiplication, it follows the rule of multiplication. For example, the reading is x and the absolute uncertainty is Δx, if $r = x^n$, then

$$\frac{\Delta r}{r} = n \times \frac{\Delta x}{x}$$ [8]

Example

The radius of a circle is measured to be $10.0 \pm 0.5\,\text{m}$. What is the area of the cirle with uncertainty?

And because of the law of exponent[9]. $\sqrt[n]{x} = x^{\frac{1}{n}}$. If $r = \sqrt[n]{x}$, then:

$$\frac{\Delta}{r} = \frac{1}{n} \cdot \frac{\Delta x}{x}$$

[8] That is because $r = \underbrace{x \cdot x \cdot \ldots \cdot x}_{n}$, the percentage of each term should be added

Example

The area of a square is measured to be $25.0\,\text{m}^2$ with 10% of uncertainty. What is the side length of the square and its corresponding uncertainty?

The side length of a square is measured to be 5.0 m with 10% of uncertainty. What is the area of the square and its corresponding uncertainty?