Projectile Motion

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Learning Outcome

I highly recommend you to finish this checklist to determine whether you've achieved the learning objectives.

- ☐ Recognize the kinematic characteristics of *horizontal projectile motion* or *projectile motion*
- ☐ Grasp the way to decompose vectors and analyze the *Two Dimensional Motion*
- ☐ Using equation of free fall to solve problems related to horizontal projectile motion or projectile motion

Leadin

Figure 1: The multiflash photo of a motor racer

Life is so hard, how could every motion in the real world always be straight or linear? The investigate should be furthered, at least into two dimensional or *planar motion*.

Horizontal Projectile

One of the most frequently studied two dimensional motion is *projectile*[1], in which the object has an initial **velocity** and is subject only to **gravity**. The moving path is shown below:

[1] def:

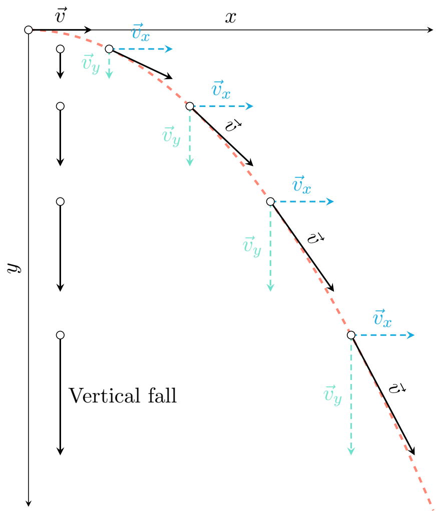


Figure 2: The path of an object's *horizontal projectile motion*

You can click on this Phet simulation to understand projectile better.

Decompose vectors

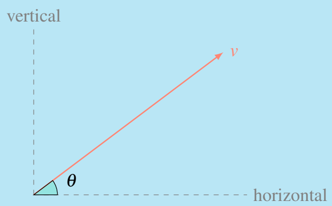
Recall that the displacement and vectors are vectors. Which means it can be viewed as a combination of two perpendicular components.

Task

In figure **??**, label the displacement of the final position, and decompose it into horizontal displacement $\vec{x}$ and vertical displacement$\vec{y}$

Just as the displacement, the velocity at any position can be decomposed into horizontal and vertical components $\vec{v}_x$ and v_y. In figure **??**. The blue arrow represent the ________ and the green arrow represent the ________. The black arrow is the resultant instantaneous and the true velocity of such motion.

Task



Try to decompose the velocity vector above, and using trigonometrical ratio to express the magnitude of horizontal and vertical component.

Final Conclusion

Check the Phet simulation, and turn on the components visualizaiton? What do you find?

Summary

The horizontal velocity
The vertical velocity ________________________________.
Thus:
If viewed only in the vertical direction, the object experiencing horizontal projectile will be exactly the same as another object which experience free fall. If viewed only in the horizontal direction, the horizontal projectile motion will experience uniform motion

Check this video to validate our conclusions.

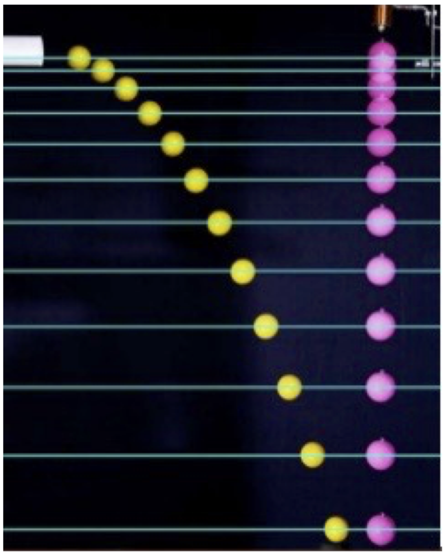


Figure 3: Two balls will have exactly same *veritcal displacement*

Quantitative Equations

Horizontal Projectile Motion will be better investigated through the view of decomposition and component. Again, acceleration, velocity, and displacement will be studied.

Acceleraiton in HP

The horizontal acceleration is $0\,\mathrm{m\,s^{-2}}$. or we'll say the horizontal motion is uniform motion The vertical acceleration is $9.81\,\mathrm{m\,s^{-2}}$, that's why the two balls will have same motion states. Because they are both subject to the **WEIGHT**

Velocity in HP

If the initial velocity is v then, the horizontal component is always:

$$v_x = v \tag{1}$$

While, the vertical can be analyzed as a free fall motion, the vertical component can be calculated as [2]:

[2] assuming downward is positive direction

$$v_y = 0 + gt = gt \tag{2}$$

If the total velcoity are required, using the vector addtion rule, but don't forget to specify the direction, which is proxied by the angle relative to the horizontal. The formula is:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{v^2 + (gt)^2} \tag{3}$$

$$\theta = \arctan\left(\frac{v_y}{v_x}\right) \tag{4}$$

Displacement in HP

Again, calculating the components will be much simpler. The horizontal displacement is:

$$x = v_x \cdot t = vt \tag{5}$$

The vertical displacement is:

$$y = {}^1\!/\!_2 \cdot gt^2 \tag{6}$$

If the total displacement[3] are required, using the vector addition rule, but don't forget to specify the direction, which is proxied by the angle relative to the horizontal. The formula is:

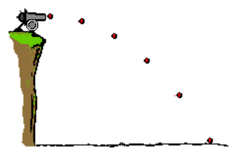


Figure 4: a canon being fired from a cliff

[3] The path is a curve, thus the distance travelled is a little bit complicated. But with the invention of Calculus, it has become nothing but a mathematical computation problem. If you are interested in this, please contact me

$$s = \sqrt{x^2 + y^2} = \sqrt{(vt)^2 + ({}^1\!/\!_2 gt^2)^2} \tag{7}$$

$$\alpha = \arctan\left(\frac{y}{x}\right) \tag{8}$$

With this, you are ready to solve anything in horizontal projectiles. Let's check one example.

Example

A canon in figure **??** from the top of a cliff, which is $176.58\,\mathrm{m}$ high, fires a projectile horizontally with an initial velocity of $75\,\mathrm{m\,s^{-1}}$.
Find: a) The time for which the canon travelled before hitting the ground
b) The Horizontal distance travelled.

a) $y = {}^1\!/\!_2 gt^2$, $t = \sqrt{{}^{2h}\!/\!_g} = \sqrt{{}^{2\times176.58}\!/\!_{9.81}} = 6\,\mathrm{s}$
b) $x = vt = 75 \times 6 = 450\,\mathrm{m}$

Projectile

Think about the figure **??**, the racer and his motor are actually launching with an initial velocity which is **NOT** horizontal. That seems make things a little bit confusing. But what's important here is to apply the process being utlized to another similar motion process.

Decompose the Initial Velocity

Suppose the racer has an initial velocity of u, and it makes an angle of θ with horizontal. Such initial velocity can be decomposed into two components.

$$u_x = u \cdot \cos\theta \tag{9}$$

$$u_y = u \cdot \sin\theta \tag{10}$$

Analyze the Process

In the horizontal direction, the racer will still move in uniform motion with a velocity of u_x.

In the vertical direction, the racer will resemble a thrown up motion, which decelerates to maximum height, and then accelerates downward. The initial vertical velocity is u_y. the acceleration[4] is $-g$.

[4] Why using negative sign here? What is the difference with horizontal projectile

Quantitative Equations

Acceleraiton in Projectile

The horizontal acceleration is $0\,\mathrm{m\,s^{-2}}$. or we'll say the horizontal motion is uniform motion The vertical acceleration is $-9.81\,\mathrm{m\,s^{-2}}$

Velocity in HP

If the initial velocity is u, the angle it makes with the horizontal is θ then, the horizontal component is **always**:

$$v_x = u_x = u\cos\theta \tag{11}$$

While, the vertical can be analyzed as a thrown up motion, the vertical component can be calculated as [5]:

[5] assuming upward is postive direction

$$v_y = u_y + (-g)t = u\sin\theta - gt \tag{12}$$

If the total velcoity are required, using the vector addtion rule, but don't forget to specify the direction, which is proxied by the angle relative to the horizontal. The formula is:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2} \tag{13}$$

$$\theta = \arctan\left(\frac{v_y}{v_x}\right) \tag{14}$$

Displacement in HP

Again, calculating the components will be much simpler. The horizontal displacement is:

$$x = v_x \cdot t = (u\cos\theta) \cdot t \tag{15}$$

The vertical displacement is:

$$y = u_y t + {}^1/_2 \cdot (-g)t^2 = (u \sin\theta) - {}^1/_2 \cdot gt^2 \tag{16}$$

The *displacement* is calculated as

$$s = \sqrt{x^2 + y^2} \tag{17}$$

$$\alpha = \arctan\left(\frac{y}{x}\right) \tag{18}$$