Preparations for A-Level Physics

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Learning Outcome

I highly recommend you to finish this checklist to determine whether you've achieved the leanring objectives.

- ☐ Understand the difference between *scalar and vector* quantities[1]
- ☐ **Add** and **subtract** coplanar vectors
- ☐ Represent a vector as **two perpendicular components**
- ☐ Using coordinate expression to calculate the scalar product

[1] def:

Scalar and Vector

For physic quantities, an important properties of which is the vector nature or scalar nature.[2].

To distinguish whether a quantities is vector or scalar, the most important way is evaluating whether you need to specify the ______________.

[2] Despite work and torque shares same dimension, they are completely different two physical quantities due to their natures

Task

Determine whether the following common physical quantities are vectors or scalars

- velocity
- speed
- acceleration
- time
- current [a]]this is tricky
- magnetic field
- distance travelled

[a] [

Operation Rules

Vectors can be added or subtracted, it must follows some specified laws or rules. there are basically two rules.

Triangle Rules

The first principles is **Triangle Rule**, which states the following steps:

You can demonstrate triangle rule using figure 1.

Parallelogram Rule

Second way to add vectors are **Parallelogram Rule**, which states the following steps:

You can demonstrate parallelogram rule using figure 2.

Summary

Triangle Rules and Parallogram Rules will always leads to the same resultant vectors, but what are the differences between them?

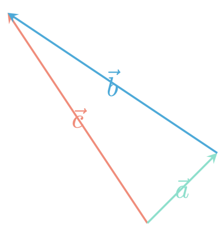

Figure 1: triangle rules addition

Substraction

The first step to carry out the substraction of vectors is to understand the opposite vector. if $\vec{b}$ is a vector shown below, try to label $-\vec{b}$ starting from the same origin.

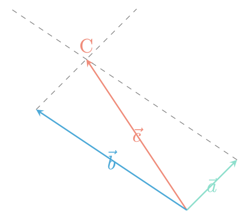

Figure 2: parallelogram rules addition

If you can undertand the equivalence between negative vector and opposite vector. then the subtraction $\vec{a}-\vec{b}$ can be effectively changed into:

$$\vec{a}-\vec{b}=\vec{a}+(-\vec{b})$$

Then, the subtraction is easily changed into addition.

Task

Show the resultant of $\vec{a}-\vec{b}$ in the figure 3.

Numerical Multiplication

What does $3\vec{a}$ means in terms of $\vec{a}$. if a vector is multiplied by a scalar(magnitude), the resultant has the same direction as the vector, but the magntidue is a multiple of the origin one.

$$|n\cdot\vec{a}| = n|\vec{a}| \quad (1)$$ [3]

Task

Show the resultant of $3\vec{a}$,$2\vec{b}$ and $3\vec{a}+2\vec{b}$ in the previous diagram.

Scalar Product

A vector can be multiplied by another vector, but the resultant would be quite differnt based on the way it is multiplied. There are two ways: *scalar product* [4] and *vector product*. And scalar product is discussed in this subsidary section. Usually the scalar product is connected by a $\cdot$, just like the following:

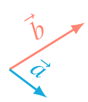

Figure 3: Two vectors starting from same point

[3] $|\vec{a}|$ means the magnitude of vector $\vec{a}$

$$\vec{a}\cdot\vec{b}$$

Because the final product would be a scalar, that's why such multiplication is called scalar product. The rules of scalar multiplication is defined as following:

$$\vec{a}\cdot\vec{b}=|\vec{a}|\times|\vec{b}|\times\cos(\theta_{<\vec{a},\vec{b}>}) \quad (2)$$

This is key to understanding **work done by a force** And we will discuss about the scalar product in another way.

Vector Product

In mathematics world. $\cdot$ and $\times$ are equivalent operations for numbers, but this is completely different for vectors. If $\vec{a} \times \vec{b}$ that means vector product[5] of the two vectors. And the result is completely different. The first thing to notice is that, the result would be a **vector**, which means apart from the magnitude, you shall also descibe the direction, which is to be determined by the **Right Hand Rule** [6]. The magnitude is calcualated by the following:

[4] sometimes it is also referred to as dot product

Figure 4: work done by a constant force

[5] sometimes it is also referred to as cross multiplication

$$|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \cdot \sin(\theta_{<\vec{a},\vec{b}>}) \quad (3)$$

[6] state the rule here:

Decomposition of vectors

The ultimate way to study vectors is using **coordinate geometry**. Setting up a coordinate system, stating two **unit vectors**[7], $\vec{i}$ for positve x direction and $\vec{j}$ for positve y direction. Then any vectors in that coordinate plane can be expressed by the **linear combination** of $\vec{i}$ and $\vec{j}$. The graph shows the principle behind it.

As seen in the picture, the vector $\vec{a}$ can be viewed as the sum of two **components**[8] $\vec{a}_x$ and $\vec{a}_y$ using triangle rule. If you treat a_x and a_y as magnitude, then the vector is decomposed into:

$$\vec{a} = a_x\vec{i} + a_y\vec{j}$$

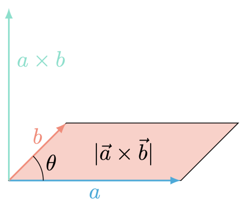

Figure 5: cross product of two vectors

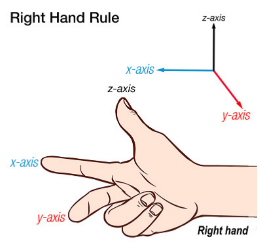

Figure 6: right hand rule

[7] def:

Summary

How to find the value of a_x and a_y given the magnitude of $\vec{a}$ and the angle θ that $\vec{a}$ makes with the horizontal axis?
How to determine the magnitude of $\vec{a}$ and direction vise and versa?

Thus a much more simplied notation of vector if the underlying coordinates has already been setup. it is denoted as:

$$\vec{a} = a_x\mathbf{i} + a_y\mathbf{j} \quad \text{or} \quad \vec{a} = \binom{a_x}{a_y}$$

Multiplication Using Coordinates

With the aid of coordinate system, one way to determine the scalar product is quite easy and simple in math. You will never be bothered to measure the magnitude and the angle. Before reaching the conclusion, determine the following scalar product:

$$\vec{i} \cdot \vec{i} = |\vec{i}| \cdot |\vec{i}| \cos 0^\circ = 1$$
$$\vec{i} \cdot \vec{j} = |\vec{i}| \cdot |\vec{j}| \cos \square^\circ = \_\_\_\_\_\_\_$$
$$\vec{j} \cdot \vec{j} = |\vec{i}| \cdot |\vec{j}| \cos \square^\circ = \_\_\_\_\_\_\_$$
$$\vec{j} \cdot \vec{i} = |\vec{i}| \cdot |\vec{j}| \cos \square^\circ = \_\_\_\_\_\_\_$$

so what happens when two vectors are being dot multiplied. Try to expand the following dot product

$$(a_x\mathbf{i} + a_y\mathbf{j}) \cdot (b_x\mathbf{i} + b_y\mathbf{j}) = \tag{4}$$