Work and Energy

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Learning Outcome

I highly recommend you to finish this checklist to determine whether you've achieved the learning objectives.

- ☐ Use the concept of work and energy
- ☐ Derive and use the formulae for *kinetic energy*(k.e.) and *gravitational potential energy*(g.p.e.).
- ☐ Recall and apply the *principle of conservation of energy*[1]

[1] conservation is a quite important law in physics

Leadin

You parents might tell you to work hard in order not to be a manual hard-worker[2]. But in physics, you brain's activity does not do work, labor work such as moving a box is actually the 'work' in physics. And also, speaking of energy, it is the catalyst for two industrial revolutions, it is also the pearl that human might even start a war to capture.

[2] no offense, every hard-work.e.r derseves respect

Work Done

If you Push a Box without moving a distance, you are pretending to do work.

Work Done by Constant Force

Since work done by a **constant** force is defined as

Definition 0.1 *work done is the product of*

If expressed in formula

$$W = \vec{F} \cdot \vec{s} = F \cdot s \cdot \cos\theta_{(F,s)} \tag{1}$$

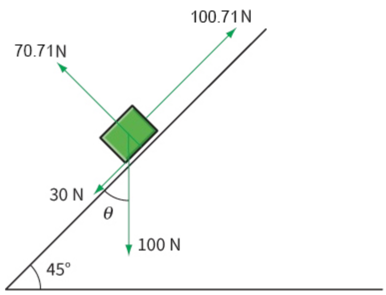


Figure 2: Multiple forces acting on the box

Task

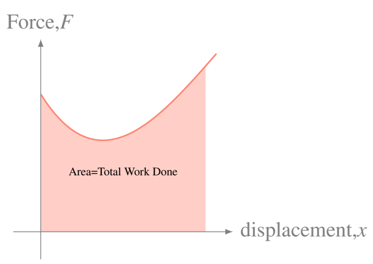
Figure 2 shows the forces acting on a box that is being pushed up a slope. Calculate the work done by each force if the box moves 0.50 m up the slope. Calculate the resultant force and determine the

work done by the resultant force.

Work Done by Varying forces

As I mentioned, life is so hard. sometimes, the force might be changing in magnitude, for example you are lifting a barbell against the weight. How could I solve the work done by the pulling force. The concept of of *Integration* has again come to our mind. Think about the fomula for $s = vt$ and $s = \int v dt$. How would you apply the integration in determine the work done by a changing force[3].

[3] This will be explained later



Summary

there are several important concepts or formulae to be emphasized.

- the formula for deciding the work done by a constant force is $W =$
- the unit for work is ________, the SI base unit is
- if the force is changing, while the F-s graph is provided, then the
- work is a <u>*scalar quantity*</u>, it can be positive or negative. The sign means such amount of work tends to ________________ of the object

Energy

It may be not a suprising thing that the label on food use the same unit kJ as the work done.

The two concept are really closely related. But one thing to k.e.ep in mind is that, the work done is a so-called ‘process quantity’, while Energy is ‘state quantity’[4]. But let’s now look at two forms of physics energy.

营养成分表-宏望

项目	每100g	NRV%
能量	1533kJ	18%
蛋白质	13.4g	22%
脂肪	0g	0%
碳水化合物	76.8g	26%
钠	85mg	4%

Figure 4: The nutrition in a buckwheat noodles

[4] it deserves further study

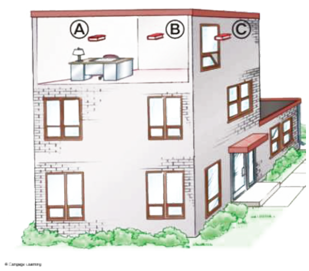


Figure 5: three books with difference reference

Gravitaional Potential Energy

Gravitaional Potential Energy(g.p.e.) is the energy used to quantify the energy related to gravity of the object. with the rise of height, the object tends to have more g.p.e.. Thus the formula of g.p.e. is defined as following:

$$E_p = mg\Delta h \tag{2}$$

The only thing need to pay attention is the meaning of Δh. Please articulate your understanding of Δh in figure 5. As for the derivation of

Kinetic Energy

Kinetic Energy(k.e.), just as its name suggest, is a type of energy related to an object with their motions.

$$E_k = {}^1/_2 mv^2 \tag{3}$$

Here, it is quite intuitious that kinetic energy is related to the <u>mass</u> and <u>velocity</u> of the object. But why square and why the coefficient of $^1/_2$ exist. Let’s go digger further in the work and energy relationship section.

Task

Car A is twice the mass of another car B, if the kinetic energy is the same for the both cars. How many times larger should the car B be than that of car A?

Conservation of Energy

By just giving you the fomulae of k.e. and g.p.e., I am behavioring like a knowledge bullier. Learning process is never memorizing the tedious formulae. Now, let's discover how the physics giants derive such units. But we have start from the relationship between work and energy.

Transform and Transfer

When warmer object are in contact with a colder object, the colder one will gain some temperature and we know that energy is **transfered** from the warmer one to the colder one. When an object falls from a height, the g.p.e. will decrease, but the k.e. will increase, that pheonomenon is called **transform**. Beside, the weight seems to do postive work on the object.

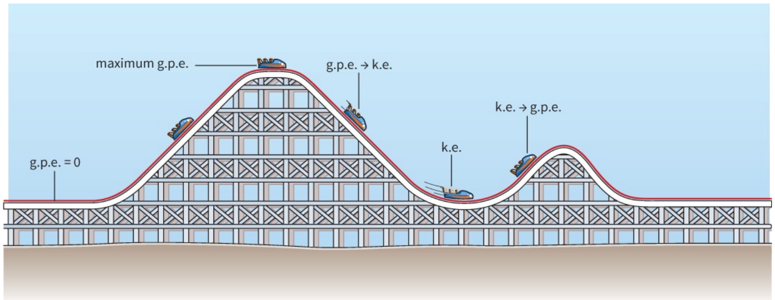


Figure 6: a roller coaster actually does not rely motor to move

Task

Use the figure 6 to explain the transformation of energy.

Relationships between Work and Energy

Due to people's or being's 'intelligence', each forms of energy can be transformed or transferred through **doing work**. Thus you are now supposed to understand why both work and energy are expressed in jouels. Because,

Summary

Work done = Energy transferred

Based on this principle, we can follow Coriolis's path to derive the g.p.e. and k.e. formula

If an object with mass m are lifted upward to a height of Δh with a *constant* velocity[5] from the ground. According to NFL, the force that applied to the object is equivalent to the weight, $F = mg$, Thus the work done by the force is $W = F \cdot \Delta h = mg\Delta h$. It is postive, which means that the g.p.e. has increased by that amount. Sincee on the ground, g.p.e. can be viewed as 0. Thus the g.p.e. at height Δh is equal to $0 + mg\Delta h = mg\Delta h$[6].

To determine k.e., assuming there is a resultant force F acting on an object with mass m, which is initally at rest[7],The force causes the object to accelerate with an accleration of $a = \frac{F}{m}$. The final velocity is v after moving a displacement of s. So, accroding to UAM equations. $s = v^2/2a$, the work

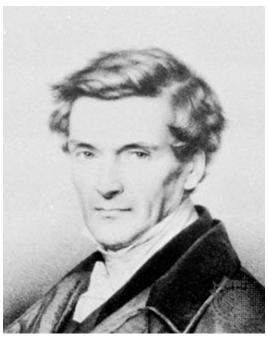


Figure 7: Gaspard-Gustave de Coriolis
1792-1843

[5] why require constant velocity
[6] why so stictly add 0?
[7] For convenience calculation

done by the force is $W = F \cdot s = F \cdot {}^{v^2}/_{2a} = ma \cdot {}^{v^2}/_{2a} = {}^{1}/_{2} \cdot mv^2$. Because the object is initially at rest, the k.e. can be assummed to be 0. Thus at the final state, the k.e. is $0 + {}^{1}/_{2} \cdot mv^2 = {}^{1}/_{2} \cdot mv^2$.

Have such process made you feel satisfied with the rigidity and precision of physics?

Conservation

Like in the roller coaster example, g.p.e. decreases when the coaster fall down through the track, while the k.e. increases. If all resistance can be ignored, it is not hard to imagine the relationship between the two changes in a quantitative way, which is

Summary

decrease in g.p.e = increase in k.e.

It seems that the total energy is fixed, one type increases, the other types decreases. That's the rudimentary glimpse of *conservation*. Watch this video to further understand the conservation of energy.

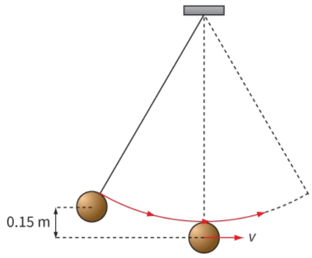


Figure 8: a pendulum

Example

A pendulum consists of a brass sphere of mass 5.0 kg hanging from a long string (see Figure 8).The sphere is pulled to the side so that it is 0.15 m above its lowest position. It is then released. How fast will it be moving when it passes through the lowest point along its path?

You might find it difficult to determine the velocity through the traditional kinematics perspective. But using energy transformation and conservation could be quite a boost. Because only two things needed is the initial and final states quantities.

Step1: determine the loss in g.p.e.

Step2: determine the final velocity from the equivalence between gain in k.e. and loss in g.p.e