Panel data estimation and forecasting

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Forms of panel data

To define the problems of panel data management, consider a dataset in which each variable contains information on N panel units, each with T time-series observations. The second dimension of panel data need not be calendar time, but many estimation techniques assume that it can be treated as such, so that operations such as first differencing make sense.

These data may be commonly stored in either the *long form* or the *wide form*, in Stata parlance. In the long form, each observation has both an i and t subscript.

Long form data:

```
. list, noobs sepby(state)
```

state	year	pop
CT	1990	3291967
CT	1995	3324144
CT	2000	3411750
MA	1990	6022639
MA	1995	6141445
MA	2000	6362076
RI	1990	1005995
RI	1995	1017002
RI	2000	1050664

You often encounter data in the wide form, in which different variables (or columns of the data matrix) refer to different time periods.

Wide form data:

```
. list, noobs
```

state	pop1990	pop1995	pop2000
CT	3291967	3324144	3411750
MA	6022639	6141445	6362076
RI	1005995	1017002	1050664

In a variant on this theme, the wide form data could also index the observations by the time period, and have the same measurement for different units stored in different variables: essentially the transpose of this array.

The former kind of wide-form data, where time periods are arrayed across the columns, is often found in spreadsheets or on-line data sources.

These examples illustrate a *balanced panel*, where each unit is represented in each time period. That is often not available, as different units may enter and leave the sample in different periods (new countries may appear, companies may start operations or liquidate, household members may die, etc.) In those cases, we must deal with *unbalanced panels*. Stata's data transformation commands are uniquely handy in that context.

Estimation for panel data

We first consider estimation of models that satisfy the zero conditional mean assumption for OLS regression: that is, the conditional mean of the error process, conditioned on the regressors, is zero. This does not rule out non-*i.i.d.* errors, but it does rule out endogeneity of the regressors and, generally, the presence of lagged dependent variables. We will deal with these exceptions later.

The most commonly employed model for panel data, the *fixed effects* estimator, addresses the issue that no matter how many individual-specific factors you may include in the regressor list, there may be *unobserved heterogeneity* in a pooled OLS model. This will generally cause OLS estimates to be biased and inconsistent.

Given longitudinal data $\{y\ X\}$, each element of which has two subscripts: the unit identifier i and the time identifier t, we may define a number of models that arise from the most general linear representation:

$$y_{it} = \sum_{k=1}^{K} X_{kit}\beta_{kit} + \epsilon_{it},\ i = 1, N,\ t = 1, T \tag{1}$$

Consider a balanced panel of $N \times T$ observations. Since this model contains $K \times N \times T$ regression coefficients, it cannot be estimated from the data. We could ignore the nature of the panel data and apply pooled ordinary least squares, which would assume that $\beta_{kit} = \beta_k \; \forall \; k, i, t$, but that model might be viewed as overly restrictive and is likely to have a very complicated error process (e.g., heteroskedasticity across panel units, serial correlation within panel units, and so forth). Thus the pooled OLS solution is not often considered to be practical.

One set of panel data estimators allow for heterogeneity across panel units (and possibly across time), but confine that heterogeneity to the intercept terms of the relationship. These techniques, the *fixed effects* and *random effects* models, we consider below. They impose restrictions on the model above of $\beta_{kit} = \beta_k \; \forall i, t, \; k > 1$, assuming that β_1 refers to the constant term in the relationship.

The fixed effects estimator

The general structure above may be restricted to allow for heterogeneity across units without the full generality (and infeasibility) that this equation implies. In particular, we might restrict the slope coefficients to be constant over both units and time, and allow for an intercept coefficient that varies by unit or by time. For a given observation, an intercept varying over units results in the structure:

$$y_{it} = \sum_{k=2}^{K} X_{kit}\beta_k + u_i + \epsilon_{it} \tag{2}$$

There are two interpretations of u_i in this context: as a parameter to be estimated in the model (a so-called *fixed effect*) or alternatively, as a component of the disturbance process, giving rise to a composite error term $[u_i + \epsilon_{it}]$: a so-called *random effect*. Under either interpretation, u_i is taken as a random variable.

If we treat it as a fixed effect, we assume that the u_i may be correlated with some of the regressors in the model. The fixed-effects estimator removes the fixed-effects parameters from the estimator to cope with this incidental parameter problem, which implies that all inference is conditional on the fixed effects in the sample.

Use of the random effects model implies additional orthogonality conditions—that the u_i are not correlated with the regressors—and yields inference about the underlying population that is not conditional on the fixed effects in our sample.

We could treat a time-varying intercept term similarly: as either a fixed effect (giving rise to an additional coefficient) or as a component of a composite error term. We concentrate here on so-called *one-way fixed (random) effects* models in which only the individual effect is considered in the "large *N*, small *T*" context most commonly found in economic and financial research.

Stata's set of `xt` commands include those which extend these panel data models in a variety of ways. For more information, see `help xt`.

One-way fixed effects: the within estimator

Rewrite the equation to express the individual effect u_i as

$$y_{it} = X^*_{it}\beta^* + Z_i\alpha + \epsilon_{it} \tag{3}$$

In this context, the X^* matrix does not contain a units vector. The heterogeneity or individual effect is captured by Z, which contains a constant term and possibly a number of other individual-specific factors. Likewise, β^* contains $\beta_2 \ldots \beta_K$ from the equation above, constrained to be equal over i and t. If Z contains only a units vector, then pooled OLS is a consistent and efficient estimator of $[\beta^* \ \alpha]$.

However, it will often be the case that there are additional factors specific to the individual unit that must be taken into account, and omitting those variables from Z will cause the equation to be misspecified.

The *fixed effects* model deals with this problem by relaxing the assumption that the regression function is constant over time and space in a very modest way. A one-way fixed effects model permits each cross-sectional unit to have its own constant term while the slope estimates (β^*) are constrained across units, as is the σ^2_ϵ.

This estimator is often termed the *LSDV* (least-squares dummy variable) model, since it is equivalent to including ($N - 1$) dummy variables in the OLS regression of y on X (including a units vector). The *LSDV* model may be written in matrix form as:

$$y = X\beta + D\alpha + \epsilon \tag{4}$$

where D is a $NT \times N$ matrix of dummy variables d_i (assuming a balanced panel of $N \times T$ observations).

The model has $(K-1)+N$ parameters (recalling that the β^* coefficients are all slopes) and when this number is too large to permit estimation, we rewrite the least squares solution as

$$b = (X'M_D X)^{-1}(X'M_D y) \tag{5}$$

where

$$M_D = I - D(D'D)^{-1}D' \tag{6}$$

is an idempotent matrix which is block–diagonal in $M_0 = I_T - T^{-1}\iota\iota'$ (ι a T–element units vector).

Premultiplying any data vector by M_0 performs the demeaning transformation: if we have a T–vector Z_i, $M_0 Z_i = Z_i - \bar{Z}_i\iota$. The regression above estimates the slopes by the projection of demeaned y on demeaned X without a constant term.

The estimates a_i may be recovered from $a_i = \bar{y}_i - b'\bar{X}_i$, since for each unit, the regression surface passes through that *unit's* multivariate point of means. This is a generalization of the OLS result that in a model with a constant term the regression surface passes through the *entire sample's* multivariate point of means.

The large-sample VCE of b is $s^2[X'M_DX]^{-1}$, with s^2 based on the least squares residuals, but taking the proper degrees of freedom into account: $NT - N - (K - 1)$.

This model will have explanatory power *if and only if* the variation of the individual's *y* above or below the individual's mean is significantly correlated with the variation of the individual's *X* values above or below the individual's vector of mean *X* values. For that reason, it is termed the *within estimator*, since it depends on the variation *within* the unit.

It does not matter if some individuals have, e.g., very high *y* values and very high *X* values, since it is only the within variation that will show up as explanatory power. This is the panel analogue to the notion that OLS on a cross-section does not seek to explain the mean of *y*, but only the variation around that mean.

This has the clear implication that any characteristic which does not vary over time for each *unit* cannot be included in the model: for instance, an individual's gender, or a firm's three-digit SIC (industry) code, or the nature of a country as landlocked.

The unit-specific intercept term absorbs all heterogeneity in y and X that is a function of the identity of the unit, and any variable constant over time for each unit will be perfectly collinear with the unit's indicator variable.

The one-way individual fixed effects model may be estimated by the Stata command `xtreg` using the `fe` (fixed effects) option. The command has a syntax similar to `regress`:

```
xtreg depvar indepvars, fe [options]
```

As with standard regression, options include `robust` and `cluster()`. The command output displays estimates of σ_u^2 (labeled `sigma_u`), σ_ϵ^2 (labeled `sigma_e`), and what Stata terms `rho`: the fraction of variance due to u_i. Stata estimates a model in which the u_i are taken as deviations from a single constant term, displayed as `_cons`; therefore testing that all u_i are zero is equivalent in our notation to testing that all α_i are identical. The empirical correlation between u_i and the regressors in X^* is also displayed as `corr(u_i, Xb)`.

The fixed effects estimator does not require a balanced panel. As long as there are at least two observations per unit, it may be applied. However, since the individual fixed effect is in essence estimated from the observations of each unit, the precision of that effect (and the resulting slope estimates) will depend on N_i.

We wish to test whether the individual-specific heterogeneity of α_i is necessary: are there distinguishable intercept terms across units? `xtreg,fe` provides an *F*-test of the null hypothesis that the constant terms are equal across units. If this null is rejected, pooled OLS would represent a misspecified model.

The one-way fixed effects model also assumes that the errors are not contemporaneously correlated across units of the panel. This hypothesis can be tested (provided $T > N$) by the Lagrange multiplier test of Breusch and Pagan, available as the author's `xttest2` routine (`findit xttest2`).

In this example, using the `traffic` dataset, we have 1982–1988 state-level data for 48 U.S. states on traffic fatality rates (deaths per 100,000). We model the highway fatality rates as a function of several common factors: `beertax`, the tax on a case of beer, `spircons`, a measure of spirits consumption and two economic factors: the state unemployment rate (`unrate`) and state per capita personal income, $000 (`perincK`). We present descriptive statistics for these variables of the `traffic.dta` dataset.

Try it out:

```
. bcuse traffic, clear
. summarize fatal beertax spircons unrate perincK
```

Variable	Obs	Mean	Std. Dev.	Min	Max
fatal	336	2.040444	.5701938	.82121	4.21784
beertax	336	.513256	.4778442	.0433109	2.720764
spircons	336	1.75369	.6835745	.79	4.9
unrate	336	7.346726	2.533405	2.4	18
perincK	336	13.88018	2.253046	9.513762	22.19345

Try it out:

```
. xtreg fatal beertax spircons unrate perincK, fe
Fixed-effects (within) regression               Number of obs      =       336
Group variable (i): state                       Number of groups   =        48

R-sq:  within  = 0.3526                         Obs per group: min =         7
       between = 0.1146                                        avg =       7.0
       overall = 0.0863                                        max =         7

                                                F(4,284)           =     38.68
corr(u_i, Xb)  = -0.8804                        Prob > F           =    0.0000
```

fatal	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
beertax	-.4840728	.1625106	-2.98	0.003	-.8039508	-.1641948
spircons	.8169652	.0792118	10.31	0.000	.6610484	.9728819
unrate	-.0290499	.0090274	-3.22	0.001	-.0468191	-.0112808
perincK	.1047103	.0205986	5.08	0.000	.064165	.1452555
_cons	-.383783	.4201781	-0.91	0.362	-1.210841	.4432754
sigma_u	1.1181913					
sigma_e	.15678965					
rho	.98071823	(fraction of variance due to u_i)				

```
F test that all u_i=0:     F(47, 284) =    59.77                 Prob > F = 0.0000
```

All explanatory factors are highly significant, with the unemployment rate having a negative effect on the fatality rate (perhaps since those who are unemployed are income-constrained and drive fewer miles), and income a positive effect (as expected because driving is a normal good).

Note the empirical correlation labeled `corr(u_i, Xb)` of -0.8804. This correlation indicates that the unobserved heterogeneity term, proxied by the estimated fixed effect, is strongly correlated with a linear combination of the included regressors. That is not a problem for the fixed effects model, but as we shall see it is an important magnitude.

We have considered one-way fixed effects models, where the effect is attached to the individual. We may also define a two-way fixed effect model, where effects are attached to each unit and time period. Stata lacks a command to estimate two-way fixed effects models. If the number of time periods is reasonably small, you may estimate a two-way FE model by creating a set of time indicator variables and including all but one in the regression.

In Stata 11 onward, that is very easy to do using factor variables by specifying `i.year` in the regressor list. The joint significance of those variables may be assessed with `testparm`, as we illustrate below.

The joint test that all of the coefficients on those indicator variables are zero will be a test of the significance of time fixed effects. Just as the individual fixed effects (LSDV) model requires regressors' variation over time within each *unit*, a time fixed effect (implemented with a time indicator variable) requires regressors' variation over units within each *time period*.

If we are estimating an equation from individual or firm microdata, this implies that we cannot include a "macro factor" such as the rate of GDP growth or price inflation in a model with time fixed effects, since those factors do not vary across individuals.

We consider the two-way fixed effects model by adding time effects to the model of the previous example. **Try it out!**

```
. xtreg fatal beertax spircons unrate perincK i.year, fe
Fixed-effects (within) regression               Number of obs      =       336
Group variable: state                           Number of groups   =        48

R-sq:  within  = 0.4528                         Obs per group: min =         7
       between = 0.1090                                        avg =       7.0
       overall = 0.0770                                        max =         7

                                                F(10,278)          =     23.00
corr(u_i, Xb)  = -0.8728                        Prob > F           =    0.0000
```

fatal	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
beertax	-.4347195	.1539564	-2.82	0.005	-.7377878	-.1316511
spircons	.805857	.1126425	7.15	0.000	.5841163	1.027598
unrate	-.0549084	.0103418	-5.31	0.000	-.0752666	-.0345502
perincK	.0882636	.0199988	4.41	0.000	.0488953	.1276319
year						
1983	-.0533713	.030209	-1.77	0.078	-.1128387	.0060962
1984	-.1649828	.037482	-4.40	0.000	-.2387674	-.0911983
1985	-.1997376	.0415808	-4.80	0.000	-.2815908	-.1178845
1986	-.0508034	.0515416	-0.99	0.325	-.1522647	.050658
1987	-.1000728	.05906	-1.69	0.091	-.2163345	.0161889
1988	-.134057	.0677696	-1.98	0.049	-.2674638	-.0006503
_cons	.1290568	.4310663	0.30	0.765	-.7195118	.9776253
sigma_u	1.0987683					
sigma_e	.14570531					
rho	.98271904	(fraction of variance due to u_i)				

```
F test that all u_i=0:     F(47, 278) =    64.52                 Prob > F = 0.0000
```

```
. testparm i.year
( 1)  1983.year = 0
( 2)  1984.year = 0
( 3)  1985.year = 0
( 4)  1986.year = 0
( 5)  1987.year = 0
( 6)  1988.year = 0
      F(  6,   278) =    8.48
           Prob > F =    0.0000
```

The four quantitative factors included in the one-way fixed effects model retain their sign and significance in the two-way fixed effects model. The time effects are jointly significant, suggesting that they should be included in a properly specified model. Otherwise, the model is qualitatively similar to the earlier model, with a sizable amount of variation explained by the individual (state) fixed effect.

The between estimator

Another estimator that may be defined for a panel data set is the *between estimator*, in which the group means of y are regressed on the group means of X in a regression of N observations. This estimator *ignores* all of the individual-specific variation in y and X that is considered by the within estimator, replacing each observation for an individual with their mean behavior.

This estimator is not widely used, but has sometimes been applied in cross-country studies where the time series data for each individual are thought to be somewhat inaccurate, or when they are assumed to contain random deviations from long-run means. If you assume that the inaccuracy has mean zero over time, a solution to this measurement error problem can be found by averaging the data over time and retaining only one observation per unit.

This could be done explicitly with Stata's `collapse` command. However, you need not form that data set to employ the between estimator, as the command `xtreg` with the `be` (between) option will invoke it. Use of the between estimator requires that $N > K$. Any macro factor that is constant over *individuals* cannot be included in the between estimator, since its average will not differ by individual.

We can show that the pooled OLS estimator is a matrix weighted average of the within and between estimators, with the weights defined by the relative precision of the two estimators. We might ask, in the context of panel data: where are the interesting sources of variation? In individuals' variation around their means, or in those means themselves? The within estimator takes account of only the former, whereas the between estimator considers only the latter.

We illustrate with the traffic fatality dataset. **Try it out!**

```
. xtreg fatal beertax spircons unrate perincK, be
Between regression (regression on group means)  Number of obs      =       336
Group variable (i): state                       Number of groups   =        48
R-sq:  within  = 0.0479                         Obs per group: min =         7
       between = 0.4565                                        avg =       7.0
       overall = 0.2583                                        max =         7
                                                F(4,43)            =      9.03
sd(u_i + avg(e_i.))=  .4209489                  Prob > F           =    0.0000
```

fatal	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
beertax	.0740362	.1456333	0.51	0.614	-.2196614	.3677338
spircons	.2997517	.1128135	2.66	0.011	.0722417	.5272618
unrate	.0322333	.038005	0.85	0.401	-.0444111	.1088776
perincK	-.1841747	.0422241	-4.36	0.000	-.2693277	-.0990218
_cons	3.796343	.7502025	5.06	0.000	2.283415	5.309271

Note that cross-sectional (interstate) variation in `beertax` and `unrate` has no explanatory power in this specification, whereas they are highly significant when the within estimator is employed.

The random effects estimator

As an alternative to considering the individual-specific intercept as a "fixed effect" of that unit, we might consider that the individual effect may be viewed as a random draw from a distribution:

$$y_{it} = X^*_{it}\beta^* + [u_i + \epsilon_{it}] \tag{7}$$

where the bracketed expression is a composite error term, with the u_i being a single draw per unit. This model could be consistently estimated by OLS or by the between estimator, but that would be inefficient in not taking the nature of the composite disturbance process into account.

A crucial assumption of this model is that u_i is independent of X^*: individual i receives a random draw that gives her a higher wage. That u_i must be independent of individual i's measurable characteristics included among the regressors X^*. If this assumption is not sustained, the random effects estimator will yield inconsistent estimates since the regressors will be correlated with the composite disturbance term.

If the individual effects can be considered to be strictly independent of the regressors, then we can model the individual-specific constant terms (reflecting the unmodeled heterogeneity across units) as draws from an independent distribution. This greatly reduces the number of parameters to be estimated, and conditional on that independence, allows for inference to be made to the population from which the survey was constructed.

In a large survey, with thousands of individuals, a random effects model will estimate K parameters, whereas a fixed effects model will estimate $(K-1)+N$ parameters, with the sizable loss of $(N-1)$ degrees of freedom.

In contrast to fixed effects, the random effects estimator can identify the parameters on time-invariant regressors such as race or gender at the individual level.

Therefore, where its use can be warranted, the random effects model is more efficient and allows a broader range of statistical inference. The assumption of the individual effects' independence is testable using `hausman`, and should always be tested.

In actual empirical work, it is extremely unusual to find that the key assumption underlying the random effects model is satisfied. Beyond textbook examples, it is difficult to find instances where the unobserved random effect can plausibly be uncorrelated with all observable attributes of the unit.

For instance, if you applied the estimator to country-level data on GDP growth, you would attribute the country-specific random component of the error term to a draw from nature that is uncorrelated with all observable characteristics of the country's performance. Thus, we will not discuss this estimator in any greater detail.

The first difference estimator

The within transformation used by fixed effects models removes unobserved heterogeneity at the unit level. The same can be achieved by first differencing the original equation (which removes the constant term). In fact, if $T = 2$, the fixed effects and first difference estimates are identical. For $T > 2$, the effects will not be identical, but they are both consistent estimators of the original model.

Stata's `xtreg` does not provide the first difference estimator, but Mark Schaffer's `xtivreg2` from SSC provides this option as the `fd` model.

We illustrate the first difference estimator with the traffic data set.
Try it out!

```
. xtivreg2 fatal beertax spircons unrate perincK, fd nocons small
FIRST DIFFERENCES ESTIMATION
------------------------------

Number of groups =        48                  Obs per group: min =         6
                                                             avg =       6.0
                                                             max =         6

OLS estimation
--------------

Estimates efficient for homoskedasticity only
Statistics consistent for homoskedasticity only

                                                 Number of obs =       288
                                                 F(  4,   284) =      6.29
                                                 Prob > F      =    0.0001
Total (centered) SS     =  11.21286023           Centered R2   =    0.0812
Total (uncentered) SS   =  11.21590589           Uncentered R2 =    0.0814
Residual SS             =  10.30276586           Root MSE      =     .1905

------------------------------------------------------------------------------
     D.fatal |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     beertax |
         D1. |   .1187701   .2728036     0.44   0.664    -.4182035    .6557438
    spircons |
         D1. |    .523584   .1408249     3.72   0.000     .2463911     .800777
      unrate |
         D1. |    .003399   .0117009     0.29   0.772    -.0196325    .0264304
     perincK |
         D1. |   .1417981   .0372814     3.80   0.000     .0684152     .215181
------------------------------------------------------------------------------
Included instruments: D.beertax D.spircons D.unrate D.perincK
------------------------------------------------------------------------------
```

We may note that, as in the between estimation results, the `beertax` and `unrate` variables have lost their significance. The larger Root MSE for the `fd` equation, compared to that for `fe`, illustrates the relative inefficiency of the first difference estimator when there are more than two time periods.

The seemingly unrelated regression estimator

An alternative technique which may be applied to "small *N*, large *T*" panels is the method of *seemingly unrelated regressions* or SURE. The "small *N*, large *T*" setting refers to the notion that we have a relatively small number of panel units, each with a lengthy time series: for instance, financial variables of the ten largest U.S. manufacturing firms, observed over the last 40 calendar quarters, or annual data on the G7 countries for the last 30 years.

The SURE technique (implemented in Stata as `sureg`) requires that the number of time periods exceeds the number of cross-sectional units.

The concept of 'seemingly unrelated' regressions is that we have several panel units, for which we could separately estimate proper OLS equations: that is, there is no simultaneity linking the units' equations. The units might be firms operating in the same industry, or industries in a particular economy, or countries in the same region.

We might be interested in estimating these equations jointly in order to take account of the likely correlation, across equations, of their error terms. These correlations represent common shocks. Incorporating those correlations in the estimation can provide gains in efficiency.

The SURE model is considerably more flexible than the fixed-effect model for panel data, as it allows for coefficients that may differ across units (but may be tested, or constrained to be identical) as well as separate estimates of the error variance for each equation. In fact, the regressor list for each equation may differ: for a particular country, for example, the price of an important export commodity might appear, but only in that country's equation. To use `sureg`, your data must be stored in the 'wide' format: the same variable for different units must be named for that unit.

Its limitation, as mentioned above, is that it cannot be applied to models in which $N > T$, as that will imply that the residual covariance matrix is singular. SURE is a generalized least squares (GLS) technique which makes use of the inverse of that covariance matrix.

A limitation of official Stata's `sureg` command is that it can only deal with balanced panels. This may be problematic in the case of firm-level or country-level data where firms are formed, or merged, or liquidated during the sample period, or when new countries emerge, as in Eastern Europe.

I wrote an extended version of `sureg`, named `suregub`, which will handle SURE in the case of unbalanced panels as long as the degree of imbalance is not too severe: that is, there must be some time periods in common across panel units. It is available in the SSC package `itsp_ado`.

One special case of note: if the equations contain exactly the same regressors (that is, numerically identical), SURE results will exactly reproduce equation-by-equation OLS results. This situation is likely to arise when you are working with a set of demand equations (for goods or factors) or a set of portfolio shares, wherein the explanatory variables should be the same for each equation.

Although SURE will provide no efficiency gain in this setting, you may still want to employ the technique on such a set of equations, as by estimating them as a system you gain the ability to perform hypothesis tests across equations, or estimate them subject to a set of linear constraints. The `sureg` command supports linear constraints, defined in the same manner as single-equation `cnsreg`.

We illustrate `sureg` with a macro example using the Penn World Tables (v9.0) dataset, `pwt90`. For simplicity, we choose three countries from that dataset: Spain, Italy, and Greece for 1960–2007. Our model considers the consumption share of real GDP per capita (`csh_c`) as a function of its lagged value and the shares of investment and government spending (`csh_i, csh_g`).

```
. bcuse pwt90, nodesc clear
. keep if tin(1960,)
(1,820 observations deleted)
. keep csh_c csh_i csh_g countrycode year
. keep if inlist(countrycode, "ITA", "ESP", "GRC")
(9,845 observations deleted)
. levelsof countrycode, local(ctylist)
`"ESP"´ `"GRC"´ `"ITA"´
. reshape wide csh_c csh_g csh_i, i(year) j(countrycode) string
(note: j = ESP GRC ITA)
Data                               long   ->   wide
-----------------------------------------------------------------------------
Number of obs.                      165   ->      55
Number of variables                   5   ->      10
j variable (3 values)       countrycode   ->   (dropped)
xij variables:
                                  csh_c   ->   csh_cESP csh_cGRC csh_cITA
                                  csh_g   ->   csh_gESP csh_gGRC csh_gITA
                                  csh_i   ->   csh_iESP csh_iGRC csh_iITA
-----------------------------------------------------------------------------
. tsset year, yearly
        time variable:  year, 1960 to 2014
                delta:  1 year
```

We build up a list of equations for `sureg` using the list of country codes created by `levelsof`:

```
. loc eqns
. foreach c of local ctylist {
  2.         loc eqns "`eqns´ (csh_c`c´ L.csh_c`c´ csh_i`c´ csh_g`c´)"
  3. }
. display "`eqns´"
 (csh_cESP L.csh_cESP csh_iESP csh_gESP) (csh_cGRC L.csh_cGRC csh_iGRC csh_gGRC) (csh_cITA L.csh_cITA csh_
> iITA csh_gITA)
```

```
. sureg "`eqns´", corr
Seemingly unrelated regression
```

Equation	Obs	Parms	RMSE	"R-sq"	chi2	P
csh_cESP	54	3	.0082906	0.9644	1594.02	0.0000
csh_cGRC	54	3	.0176729	0.9160	606.11	0.0000
csh_cITA	54	3	.0093013	0.5868	87.74	0.0000

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
csh_cESP						
csh_cESP						
L1.	.7429981	.0803418	9.25	0.000	.5855311	.9004651
csh_iESP	-.1297547	.0366487	-3.54	0.000	-.2015848	-.0579245
csh_gESP	-.2479771	.1023932	-2.42	0.015	-.4486642	-.0472901
_cons	.2298568	.0693135	3.32	0.001	.0940048	.3657088
csh_cGRC						
csh_cGRC						
L1.	.6566215	.0779777	8.42	0.000	.5037881	.8094549
csh_iGRC	-.1672854	.0702747	-2.38	0.017	-.3050213	-.0295495
csh_gGRC	.2234098	.1396253	1.60	0.110	-.0502508	.4970704
_cons	.2305564	.0647088	3.56	0.000	.1037294	.3573833

(continued)

csh_cITA						
csh_cITA						
L1.	.6442806	.1024233	6.29	0.000	.4435346	.8450267
csh_iITA	-.1245503	.0658817	-1.89	0.059	-.253676	.0045753
csh_gITA	.0637873	.1391879	0.46	0.647	-.209016	.3365906
_cons	.2338423	.0622189	3.76	0.000	.1118955	.355789

Correlation matrix of residuals:

	csh_cESP	csh_cGRC	csh_cITA
csh_cESP	1.0000		
csh_cGRC	0.2292	1.0000	
csh_cITA	0.3216	0.1134	1.0000

Breusch-Pagan test of independence: chi2(3) = 9.115, Pr = 0.0278

Note from the displayed correlation matrix of residuals and the Breusch–Pagan test of independence that there is evidence of cross-equation correlation of the residuals.

Given our systems estimates, we may test hypotheses on coefficients in different equations: for instance, that the coefficients on `csh_g` are equal across equations. Note that in the `test` command we must specify in which equation each coefficient appears.

```
. test [csh_cESP]csh_gESP = [csh_cGRC]csh_gGRC = [csh_cITA]csh_gITA
 ( 1)  [csh_cESP]csh_gESP - [csh_cGRC]csh_gGRC = 0
 ( 2)  [csh_cESP]csh_gESP - [csh_cITA]csh_gITA = 0
           chi2(  2) =    8.12
         Prob > chi2 =    0.0173
```

We can produce *ex post* or *ex ante* forecasts from `sureg` with `predict`, specifying a different variable name for each equation's predictions:

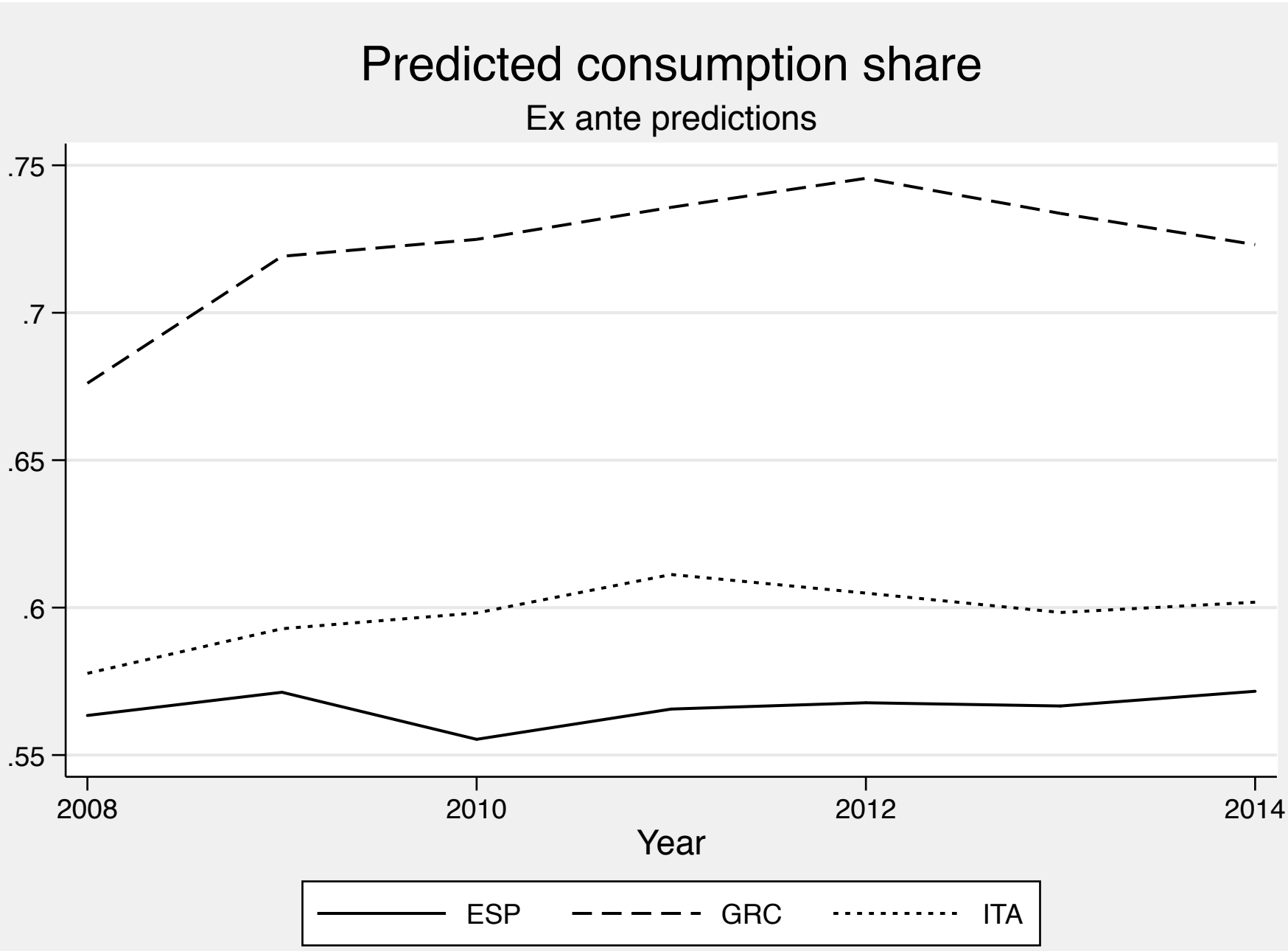
```
. sureg "`eqns´" if year<=2007, notable
Seemingly unrelated regression
```

Equation	Obs	Parms	RMSE	"R-sq"	chi2	P
csh_cESP	47	3	.0080839	0.9518	978.96	0.0000
csh_cGRC	47	3	.0163868	0.9053	453.21	0.0000
csh_cITA	47	3	.0089431	0.6135	81.72	0.0000

```
. foreach c of local ctylist {
  2.          qui predict double `c´hat if year>2007, xb equation(csh_c`c´)
  3.          label var `c´hat "`c´"
  4. }
. su *hat
```

Variable	Obs	Mean	Std. Dev.	Min	Max
ESPhat	7	.5659447	.0055302	.5553351	.5716213
GRChat	7	.7225971	.0223778	.6760783	.7455726
ITAhat	7	.5978607	.0106106	.5777059	.6112091

```
. tsline *hat if year>2007, legend(rows(1)) ti("Predicted consumption share") ///
> t2("Ex ante predictions") ylab(,angle(0) labs(small)) xlab(,labs(small)) ///
> legend(size(small)) scheme(s2mono)
```


Predicted consumption share
Ex ante predictions
.75
.7
.65
.6
.55
2008
2010
2012
2014
Year
ESP
GRC
ITA

Instrumental variables estimators for panel data

Linear instrumental variables (IV) models for panel data may be estimated with Stata's `xtivreg`, a panel-capable analog to `ivregress`. This command only fits standard two-stage least squares models, and does not support IV-GMM nor LIML. By specifying options, you may choose among the random effects (`re`), fixed effects (`fe`), between effects (`be`) and first-differenced (`fd`) estimators.

If you want to use IV-GMM or LIML in a panel setting, you may use Mark Schaffer's `xtivreg2` routine, which is a 'wrapper' for Baum–Schaffer–Stillman's `ivreg2`, providing all of its capabilities in a panel setting. However, `xtivreg2` only implements the fixed-effects and first-difference estimators.

We spoke in an earlier lecture about the use of *cluster-robust standard errors*: a specification of the error term's VCE in which we allow for arbitrary correlation within *M* clusters of observations. Most Stata commands, including `regress, ivregress` and `xtreg`, support the option of `vce(cluster` *varname)* to produce the cluster-robust VCE.

In fact, if you use `xtreg, fe` or `xtivreg, fe` with the `robust` option, the VCE estimates are generated as cluster-robust, as Stock and Watson demonstrated (*Econometrica*, 2008) that it is necessary to allow for clustering to generate a consistent robust VCE when $T > 2$.

You can use `xtivreg2` to estimate fixed-effects or first-difference IV models with cluster-robust standard errors. In a panel context, you may also want to consider *two-way clustering*: the notion that dependence between observations' errors may not only appear within the time series observations of a given panel unit, but could also appear across units at each point in time.

The extension of cluster-robust VCE estimates to two- and multi-way clustering is an area of active econometric research.

Computation of the two-way cluster-robust VCE is straightforward, as Thompson (SSRN WP, 2006) illustrates. The VCE may be calculated from

$$VCE(\hat{\beta}) = VCE_1(\hat{\beta}) + VCE_2(\hat{\beta}) - VCE_{12}(\hat{\beta})$$

where the three VCE estimates are derived from one-way clustering on the first dimension, the second dimension and their intersection, respectively. As these one-way cluster-robust VCE estimates are available from most Stata estimation commands, computing the two-way cluster-robust VCE involves only a few matrix manipulations.

One concern that arises with two-way (and multi-way) clustering is the number of clusters in each dimension. With one-way clustering, we should be concerned if the number of clusters G is too small to produce unbiased estimates.

The theory underlying two-way clustering relies on asymptotics in the smaller number of clusters: that is, the dimension containing fewer clusters. The two-way clustering approach is thus most sensible if there are a sizable number of clusters in each dimension.

We illustrate with a fixed-effect IV model of `kc` from the Penn World Tables (v6.3) data set, in which regressors are specified as `openc` and `cgnp`, each instrumented with two lags. The model is estimated for an unbalanced panel of 99 countries for 38–46 years per country. We fit the model with classical standard errors (IID), cluster-robust by country (clCty) and cluster-robust by country and year (clCtyYr).

Table: Panel IV estimates of kc, 1960-2007

	(1) IID	(2) clCty	(3) clCtyYr
openc	-0.036***	-0.036*	-0.036*
	(0.007)	(0.018)	(0.018)
cgnp	0.800***	0.800***	0.800***
	(0.033)	(0.146)	(0.146)
N	4508	4508	4508

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The two-way cluster-robust standard errors are very similar to those produced by the one-way cluster-robust VCE. Both sets are considerably larger than those produced by the *i.i.d.* error assumption, suggesting that classical standard errors are severely biased in this setting.

Dynamic panel data estimators

The ability of first differencing to remove unobserved heterogeneity also underlies the family of estimators that have been developed for dynamic panel data (DPD) models. These models contain one or more lagged dependent variables, allowing for the modeling of a partial adjustment mechanism.

Nickell bias

A serious difficulty arises with the one-way fixed effects model in the context of a *dynamic panel data* (DPD) model particularly in the “small T, large N" context. As Nickell (*Econometrica*, 1981) shows, this arises because the demeaning process which subtracts the individual’s mean value of y and each X from the respective variable creates a correlation between regressor and error.

The mean of the lagged dependent variable contains observations 0 through $(T-1)$ on y, and the mean error—which is being conceptually subtracted from each ϵ_{it}—contains contemporaneous values of ϵ for $t = 1 \ldots T$. The resulting correlation creates a bias in the estimate of the coefficient of the lagged dependent variable which is not mitigated by increasing N, the number of individual units.

The demeaning operation creates a regressor which *cannot* be distributed independently of the error term. Nickell demonstrates that the inconsistency of $\hat{\rho}$ as $N \rightarrow \infty$ is of order $1/T$, which may be quite sizable in a "small T" context. If $\rho > 0$, the bias is invariably negative, so that the persistence of y will be underestimated.

For reasonably large values of T, the limit of $(\hat{\rho} - \rho)$ as $N \rightarrow \infty$ will be approximately $-(1 + \rho)/(T - 1)$: a sizable value, even if $T = 10$. With $\rho = 0.5$, the bias will be -0.167, or about 1/3 of the true value. The inclusion of additional regressors does not remove this bias. Indeed, if the regressors are correlated with the lagged dependent variable to some degree, their coefficients may be seriously biased as well.

Note also that this bias is not caused by an autocorrelated error process ϵ. The bias arises even if the error process is *i.i.d*. If the error process is autocorrelated, the problem is even more severe given the difficulty of deriving a consistent estimate of the *AR* parameters in that context.

The same problem affects the one-way random effects model. The u_i error component enters every value of y_{it} by assumption, so that the lagged dependent variable *cannot* be independent of the composite error process.

One solution to this problem involves taking first differences of the original model. Consider a model containing a lagged dependent variable and a single regressor X:

$$y_{it} = \beta_1 + \rho y_{i,t-1} + X_{it}\beta_2 + u_i + \epsilon_{it} \tag{8}$$

The first difference transformation removes both the constant term and the individual effect:

$$\Delta y_{it} = \rho \Delta y_{i,t-1} + \Delta X_{it}\beta_2 + \Delta \epsilon_{it} \tag{9}$$

There is still correlation between the differenced lagged dependent variable and the disturbance process (which is now a first-order moving average process, or $MA(1)$): the former contains $y_{i,t-1}$ and the latter contains $\epsilon_{i,t-1}$.

But with the individual fixed effects swept out, a straightforward instrumental variables estimator is available. We may construct instruments for the lagged dependent variable from the second and third lags of y, either in the form of differences or lagged levels. If ϵ is $i.i.d.$, those lags of y will be highly correlated with the lagged dependent variable (and its difference) but uncorrelated with the composite error process.

Even if we had reason to believe that ϵ might be following an $AR(1)$ process, we could still follow this strategy, "backing off" one period and using the third and fourth lags of y (presuming that the timeseries for each unit is long enough to do so).

The Arellano–Bond approach

The *DPD* (Dynamic Panel Data) approach of Arellano and Bond (1991) is based on the notion that the instrumental variables approach noted above does not exploit all of the information available in the sample. By doing so in a Generalized Method of Moments (GMM) context, we may construct more efficient estimates of the dynamic panel data model. The Arellano–Bond estimator can be thought of as an extension of the Anderson–Hsiao estimator implemented by `xtivreg, fd`.

Although DPD is usually associated with Arellano and Bond, the methodology was actually presented in Holtz-Eakin, Newey and Rosen, *Econometrica* 1988. The Arellano and Bond approach gained popularity because those authors freely distributed their software.

Arellano and Bond argue that the Anderson–Hsiao estimator, while consistent, fails to take all of the potential orthogonality conditions into account. Consider the equations

$$\begin{aligned} y_{it} &= X_{it}\beta_1 + W_{it}\beta_2 + v_{it} \\ v_{it} &= u_i + \epsilon_{it} \end{aligned} \tag{10}$$

where X_{it} includes strictly exogenous regressors, W_{it} are predetermined regressors (which may include lags of y) and endogenous regressors, all of which may be correlated with u_i, the unobserved individual effect. First-differencing the equation removes the u_i and its associated omitted-variable bias.

The AB approach, and its extension to the 'System GMM' context, is an estimator designed for situations with:

- 'small T, large N' panels: few time periods and many individual units
- a linear functional relationship
- one left-hand variable that is dynamic, depending on its own past realizations
- right-hand variables that are not strictly exogenous: correlated with past and possibly current realizations of the error
- fixed individual effects, implying unobserved heterogeneity
- heteroskedasticity and autocorrelation within individual units' errors, but not across them

The Arellano–Bond estimator sets up a generalized method of moments (*GMM*) problem in which the model is specified as a system of equations, one per time period, where the instruments applicable to each equation differ (for instance, in later time periods, additional lagged values of the instruments are available).

This estimator is available in Stata as `xtabond`. A more general version, allowing for autocorrelated errors, is available as `xtdpd`. An excellent alternative to Stata's built-in commands is David Roodman's `xtabond2`, available from SSC (`findit xtabond2`). It is very well documented in his paper "How to to do xtabond2." The `xtabond2` routine provides several additional features—such as the orthogonal deviations transformation discussed below—not available in official Stata's commands.

Constructing the instrument matrix

In standard 2SLS, including the Anderson–Hsiao approach, the twice-lagged level appears in the instrument matrix as

$$\mathbf{Z}_i = \begin{pmatrix} . \\ y_{i,1} \\ \vdots \\ y_{i,T-2} \end{pmatrix}$$

where the first row corresponds to $t = 2$, given that the first observation is lost in applying the FD transformation. The missing value in the instrument for $t = 2$ causes that observation for each panel unit to be removed from the estimation.

If we also included the thrice-lagged level y_{t-3} as a second instrument in the Anderson–Hsiao approach, we would lose another observation per panel:

$$\mathbf{Z}_i = \begin{pmatrix} . & . \\ y_{i,1} & . \\ y_{i,2} & y_{i,1} \\ \vdots & \vdots \\ y_{i,T-2} & y_{i,T-3} \end{pmatrix}$$

so that the first observation available for the regression is that dated $t = 4$.

To avoid this loss of degrees of freedom, Holtz-Eakin et al. construct a set of instruments from the second lag of y, one instrument pertaining to each time period:

$$\mathbf{Z}_i = \begin{pmatrix} 0 & 0 & \dots & 0 \\ y_{i,1} & 0 & \dots & 0 \\ 0 & y_{i,2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & y_{i,T-2} \end{pmatrix}$$

The inclusion of zeros in place of missing values prevents the loss of additional degrees of freedom, in that all observations dated $t = 2$ and later can now be included in the regression. Although the inclusion of zeros might seem arbitrary, the columns of the resulting instrument matrix will be orthogonal to the transformed errors. The resulting moment conditions correspond to an expectation we believe should hold: $E(y_{i,t-2}\epsilon^*_{it}) = 0$, where ϵ^* refers to the FD-transformed errors.

It would also be valid to 'collapse' the columns of this **Z** matrix into a single column, which embodies the same expectation, but conveys less information as it will only produce a single moment condition. In this context, the collapsed instrument set will be the same implied by standard IV, with a zero replacing the missing value in the first usable observation:

$$\mathbf{Z}_i = \begin{pmatrix} 0 \\ y_{i,1} \\ \vdots \\ y_{i,T-2} \end{pmatrix}$$

This is specified in Roodman's `xtabond2` software by giving the `collapse` option.

Given this solution to the tradeoff between lag length and sample length, we can now adopt Holtz-Eakin et al.'s suggestion and include *all* available lags of the untransformed variables as instruments. For endogenous variables, lags 2 and higher are available. For predetermined variables that are not strictly exogenous, lag 1 is also valid, as its value is only correlated with errors dated $t-2$ or earlier.

Using all available instruments gives rise to an instrument matrix such as

$$\mathbf{Z}_i = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ y_{i,1} & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & y_{i,2} & y_{i,1} & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & y_{i,3} & y_{i,2} & y_{i,1} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

In this setup, we have different numbers of instruments available for each time period: one for $t = 2$, two for $t = 3$, and so on. As we move to the later time periods in each panel's timeseries, additional orthogonality conditions become available, and taking these additional conditions into account improves the efficiency of the AB estimator.

One disadvantage of this strategy should be apparent. The number of instruments produced will be quadratic in T, the length of the timeseries available. If $T < 10$, that may be a manageable number, but for a longer timeseries, it may be necessary to restrict the number of past lags used.

A useful feature of `xtabond2` is the ability to specify, for GMM-style instruments, the limits on how many lags are to be included. If T is fairly large (more than 7–8) an unrestricted set of lags will introduce a huge number of instruments, with a possible loss of efficiency. By using the lag limits options, you may specify, for instance, that only lags 2–5 are to be used in constructing the GMM instruments.

The System GMM estimator

A potential weakness in the Arellano–Bond *DPD* estimator was revealed in later work by Arellano and Bover (1995) and Blundell and Bond (1998). The lagged levels are often rather poor instruments for first differenced variables, especially if the variables are close to a random walk. Their modification of the estimator includes lagged levels as well as lagged differences.

The original estimator is often entitled *difference GMM*, while the expanded estimator is commonly termed *System GMM*. The cost of the System GMM estimator involves a set of additional restrictions on the initial conditions of the process generating y. This estimator is available in Stata as `xtdpdsys`.

Diagnostic tests

As the DPD estimators are instrumental variables methods, it is particularly important to evaluate the Sargan–Hansen test results when they are applied. Roodman's `xtabond2` provides *C* tests (as discussed in re `ivreg2`) for groups of instruments. In his routine, instruments can be either "GMM-style" or "IV-style". The former are constructed per the Arellano–Bond logic, making use of multiple lags; the latter are included as is in the instrument matrix. For the system GMM estimator (the default in `xtabond2`) instruments may be specified as applying to the differenced equations, the level equations or both.

Another important diagnostic in DPD estimation is the *AR* test for autocorrelation of the residuals. By construction, the residuals of the differenced equation should possess serial correlation, but if the assumption of serial independence in the original errors is warranted, the differenced residuals should not exhibit significant *AR*(2) behavior. These statistics are produced in the `xtabond` and `xtabond2` output. If a significant *AR*(2) statistic is encountered, the second lags of endogenous variables will not be appropriate instruments for their current values.

An empirical exercise

To illustrate the performance of the several estimators, we make use of the original AB dataset, available within Stata with `webuse abdata`. This is an unbalanced panel of annual data from 140 UK firms for 1976–1984. In their original paper, they modeled firms' employment `n` using a partial adjustment model to reflect the costs of hiring and firing, with two lags of employment.

Other variables included were the current and lagged wage level `w`, the current, once- and twice-lagged capital stock (`k`) and the current, once- and twice-lagged output in the firm's sector (`ys`). All variables are expressed as logarithms. A set of time dummies is also included to capture business cycle effects.

If we were to estimate this model ignoring its dynamic panel nature, we could merely apply `regress` with panel-clustered standard errors:

Try it out:

```
regress n nL1 nL2 w wL1 k kL1 kL2 ys ysL1 ysL2 yr*, cluster(id)
```

One obvious difficulty with this approach is the likely importance of firm-level unobserved heterogeneity. We have accounted for potential correlation between firms' errors over time with the cluster-robust VCE, but this does not address the potential impact of unobserved heterogeneity on the conditional mean.

We can apply the within transformation to take account of this aspect of the data: **Try it out:**

```
xtreg n nL1 nL2 w wL1 k kL1 kL2 ys ysL1 ysL2 yr*, fe cluster(id)
```

The fixed effects estimates will suffer from Nickell bias, which may be severe given the short timeseries available.

	OLS		FE	
nL1	1.045***	(20.17)	0.733***	(12.28)
nL2	-0.0765	(-1.57)	-0.139	(-1.78)
w	-0.524**	(-3.01)	-0.560***	(-3.51)
k	0.343***	(7.06)	0.388***	(6.82)
ys	0.433*	(2.42)	0.469**	(2.74)
N	751		751	

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

In the original OLS regression, the lagged dependent variable was positively correlated with the error, biasing its coefficient upward. In the fixed effects regression, its coefficient is biased downward due to the negative sign on ν_{t-1} in the transformed error. The OLS estimate of the first lag of *n* is 1.045; the fixed effects estimate is 0.733.

Given the opposite directions of bias present in these estimates, consistent estimates should lie between these values, which may be a useful check. As the coefficient on the second lag of *n* cannot be distinguished from zero, the first lag coefficient should be below unity for dynamic stability.

To deal with these two aspects of the estimation problem, we might use the Anderson–Hsiao estimator to the first-differenced equation, instrumenting the lagged dependent variable with the twice-lagged level: **Try it out:**

```
 ivregress 2sls D.n (D.nL1 = nL2) D.(nL2 w wL1 k kL1 kL2 ///
ys ysL1 ysL2 yr1979 yr1980 yr1981 yr1982 yr1983 )
```

	A-H	
D.nL1	2.308	(1.17)
D.nL2	-0.224	(-1.25)
D.w	-0.810**	(-3.10)
D.k	0.253	(1.75)
D.ys	0.991*	(2.14)
N	611	

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Although these results should be consistent, they are quite disappointing. The coefficient on lagged *n* is outside the bounds of its OLS and FE counterparts, and much larger than unity, a value consistent with dynamic stability. It is also very imprecisely estimated.

The difference GMM approach deals with this inherent endogeneity by transforming the data to remove the fixed effects. The standard approach applies the first difference (FD) transformation, which as discussed earlier removes the fixed effect at the cost of introducing a correlation between $\Delta y_{i,t-1}$ and $\Delta \nu_{it}$, both of which have a term dated $(t-1)$. This is preferable to the application of the within transformation, as that transformation makes every observation in the transformed data endogenous to every other for a given individual.

The one disadvantage of the first difference transformation is that it magnifies gaps in unbalanced panels. If some value of y_{it} is missing, then both Δy_{it} and $\Delta y_{i,t-1}$ will be missing in the transformed data. This motivates an alternative transformation: the forward orthogonal deviations (FOD) transformation, proposed by Arellano and Bover (*J. Econometrics*, 1995).

In contrast to the within transformation, which subtracts the average of all observations' values from the current value, and the FD transformation, that subtracts the previous value from the current value, the FOD transformation subtracts the average of all available *future* observations from the current value. While the FD transformation drops the first observation on each individual in the panel, the FOD transformation drops the last observation for each individual. It is computable for all periods except the last period, even in the presence of gaps in the panel.

The FOD transformation is not available in any of official Stata's DPD commands, but it is available in David Roodman's `xtabond2` implementation of the DPD estimator, available from SSC.

To illustrate the use of the AB estimator, we may reestimate the model with `xtabond2`, assuming that the only endogeneity present is that involving the lagged dependent variable. **Try it out:**

```
 xtabond2 n L(1/2).n L(0/1).w L(0/2).(k ys) yr*, gmm(L.n) ///
iv(L(0/1).w L(0/2).(k ys) yr*) nolevel robust small
```

Note that in `xtabond2` syntax, every right-hand variable generally appears twice in the command, as instruments must be explicitly specified when they are instrumenting themselves. In this example, all explanatory variables except the lagged dependent variable are taken as "IV-style" instruments, entering the **Z** matrix as a single column. The lagged dependent variable is specified as a "GMM-style" instrument, where all available lags will be used as separate instruments. The `noleveleq` option is needed to specify the AB estimator.

	A-B	
L.n	0.686***	(4.67)
L2.n	-0.0854	(-1.50)
w	-0.608**	(-3.36)
k	0.357***	(5.95)
ys	0.609***	(3.47)
N	611	

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

In these results, 41 instruments have been created, with 17 corresponding to the "IV-style" regressors and the rest computed from lagged values of *n*. Note that the coefficient on the lagged dependent variable now lies within the range for dynamic stability. In contrast to that produced by the Anderson–Hsiao estimator, the coefficient is quite precisely estimated.

There are 25 overidentifying restrictions in this instance, as shown in the first column below. The `hansen_df` represents the degrees of freedom for the Hansen *J* test of overidentifying restrictions. The *p*-value of that test is shown as `hansenp`.

	All lags		lags 2-5		lags 2-4	
L.n	0.686***	(4.67)	0.835*	(2.59)	1.107***	(3.94)
L2.n	-0.0854	(-1.50)	0.262	(1.56)	0.231	(1.32)
w	-0.608**	(-3.36)	-0.671**	(-3.18)	-0.709**	(-3.26)
k	0.357***	(5.95)	0.325***	(4.95)	0.309***	(4.55)
ys	0.609***	(3.47)	0.640**	(3.07)	0.698***	(3.45)
hansen_df	25		16		13	
hansenp	0.177		0.676		0.714	

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

In this table, we can examine the sensitivity of the results to the choice of "GMM-style" lag specification. In the first column, all available lags of the level of *n* are used. In the second column, the `lag(2 5)` option is used to restrict the maximum lag to 5 periods, while in the third column, the maximum lag is set to 4 periods. Fewer instruments are used in those instances, as shown by the smaller values of `sar_df`.

The *p*-value of Hansen's *J* is also considerably larger for the restricted-lag cases. On the other hand, the estimate of the lagged dependent variable's coefficient appears to be quite sensitive to the choice of lag length.

We illustrate estimating this equation with both the FD transformation and the forward orthogonal deviations (FOD) transformation:

	First diff		FOD	
L.n	0.686***	(4.67)	0.737***	(5.14)
L2.n	-0.0854	(-1.50)	-0.0960	(-1.38)
w	-0.608**	(-3.36)	-0.563***	(-3.47)
k	0.357***	(5.95)	0.384***	(6.85)
ys	0.609***	(3.47)	0.469**	(2.72)
hansen_df	25		25	
hansenp	0.177		0.170	

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The results appear reasonably robust to the choice of transformation, with slightly more precise estimates for most coefficients when the FOD transformation is employed.

We might reasonably consider, as did Blundell and Bond (*J. Econometrics*, 1998), that wages and the capital stock should not be taken as strictly exogenous in this context, as we have in the above models.

Reestimate the equation producing "GMM-style" instruments for all three variables, with both one-step and two-step VCE:

Try it out:

```
 xtabond2 n L(1/2).n L(0/1).w L(0/2).(k ys) yr*, gmm(L.(n w k)) ///
iv(L(0/2).ys yr*) nolevel robust small
```

	One-step		Two-step	
L.n	0.818***	(9.51)	0.824***	(8.51)
L2.n	-0.112*	(-2.23)	-0.101	(-1.90)
w	-0.682***	(-4.78)	-0.711***	(-4.67)
k	0.353**	(2.89)	0.377**	(2.79)
ys	0.651***	(3.43)	0.662***	(3.89)
hansen_df	74		74	
hansenp	0.487		0.487	

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The results from both one-step and two-step estimation appear reasonable. Interestingly, only the coefficient on *ys* appears to be more precisely estimated by the two-step VCE. With no restrictions on the instrument set, 74 overidentifying restrictions are defined, with 90 instruments in total.

To illustrate system GMM, we follow Blundell and Bond, who used the same `abdata` dataset on a somewhat simpler model, dropping the second lags and removing sectoral demand. We consider wages and capital as potentially endogenous, with GMM-style instruments.

Estimate the one-step BB model.

Try it out:

```
 xtabond2 n L.n L(0/1).(w k) yr*, gmm(L.(n w k)) iv(yr*, equation(level)) ///
robust small
```

We indicate here with the `equation(level)` suboption that the year dummies are only to be considered instruments in the level equation. As the default for `xtabond2` is the BB estimator, we omit the `noleveleq` option that has called for the AB estimator in earlier examples.

	n	
L.n	0.936***	(35.21)
w	-0.631***	(-5.29)
k	0.484***	(8.89)
hansen_df	100	
hansenp	0.218	

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

We find that the α coefficient is much higher than in the AB estimates, although it may be distinguished from unity. 113 instruments are created, with 100 degrees of freedom in the test of overidentifying restrictions.

A second empirical exercise

We also illustrate DPD estimation using the Penn World Table cross-country panel. We specify a model for `csh_c` (the consumption share of real GDP per capita) depending on its own lag, the government share `csh_g`, and a set of time fixed effects.

We first estimate the two-step 'difference GMM' form of the model with (cluster-)robust VCE, using data for 1991–2014. We could use `testparm i.year` after estimation to evaluate the joint significance of time effects (listing of which has been suppressed).

Try it out:

```
. bcuse pwt90, nodesc
. keep if tin(1989,)
(7,098 observations deleted)
. xtabond2 csh_c L.csh_c csh_g i.year if tin(1991,), gmm(L.csh_c csh_g, ///
> lag(2 5)) iv(i.year) twostep robust noleveleq nodiffsargan
Favoring space over speed. To switch, type or click on mata: mata set matafavor speed, perm.
Warning: Number of instruments may be large relative to number of observations.
Warning: Two-step estimated covariance matrix of moments is singular.
  Using a generalized inverse to calculate optimal weighting matrix for two-step estimation.
Dynamic panel-data estimation, two-step difference GMM
```

Group variable: cty				Number of obs	=	4313
Time variable : year				Number of groups	=	182
Number of instruments = 200				Obs per group: min	=	8
Wald chi2(28) = 1670.07				avg	=	23.70
Prob > chi2 = 0.000				max	=	24

csh_c	Coef.	Corrected Std. Err.	z	P>|z|	[95% Conf.	Interval]
csh_c L1.	-.041	.0060394	-6.79	0.000	-.052837	-.0291629
csh_g	3.014836	.405835	7.43	0.000	2.219414	3.810258
year						

(*output omitted*)

(continued)

```
Instruments for first differences equation
  Standard
    D.(1989b.year 1990.year 1991.year 1992.year 1993.year 1994.year 1995.year
    1996.year 1997.year 1998.year 1999.year 2000.year 2001.year 2002.year
    2003.year 2004.year 2005.year 2006.year 2007.year 2008.year 2009.year
    2010.year 2011.year 2012.year 2013.year 2014.year)
  GMM-type (missing=0, separate instruments for each period unless collapsed)
    L(2/5).(L.csh_c csh_g)
------------------------------------------------------------------------------
Arellano-Bond test for AR(1) in first differences: z =  -1.60  Pr > z =  0.110
Arellano-Bond test for AR(2) in first differences: z =   1.15  Pr > z =  0.250
------------------------------------------------------------------------------
Sargan test of overid. restrictions: chi2(172)  =3419.89  Prob > chi2 =  0.000
  (Not robust, but not weakened by many instruments.)
Hansen test of overid. restrictions: chi2(172)  = 177.59  Prob > chi2 =  0.369
  (Robust, but weakened by many instruments.)
```

Given the relatively large number of time periods available, I have specified that the GMM instruments only be constructed for lags 2–5 to keep the number of instruments manageable. The autoregressive coefficient is -0.041, implying mean reversion, and the `csh_g` coefficient is positive and highly significant. Although not shown, the test for joint significance of the time effects has a p-value close to zero.

We could also fit this model with the 'system GMM' estimator, which will be able to utilize one more observation per country in the level equation, and estimate a constant term in the relationship.

```
. xtabond2 csh_c L.csh_c csh_g i.year if tin(1991,), gmm(L.csh_c csh_g, ///
>  lag(2 4)) iv(i.year) twostep robust nodiffsargan
Favoring space over speed. To switch, type or click on mata: mata set matafavor speed, perm.
Warning: Number of instruments may be large relative to number of observations.
Warning: Two-step estimated covariance matrix of moments is singular.
  Using a generalized inverse to calculate optimal weighting matrix for two-step estimation.
Dynamic panel-data estimation, two-step system GMM
------------------------------------------------------------------------------
Group variable: cty                             Number of obs      =      4338
Time variable : year                            Number of groups   =       182
Number of instruments = 207                     Obs per group: min =         9
Wald chi2(28) =    705.96                                      avg =     23.84
Prob > chi2   =     0.000                                      max =        24
------------------------------------------------------------------------------
             |              Corrected
       csh_c |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       csh_c |
         L1. |  -.0402861   .0176262    -2.29   0.022    -.0748327   -.0057394
             |
       csh_g |   2.735404    .512415     5.34   0.000     1.731089    3.739719
             |
        year |
(output omitted)
```

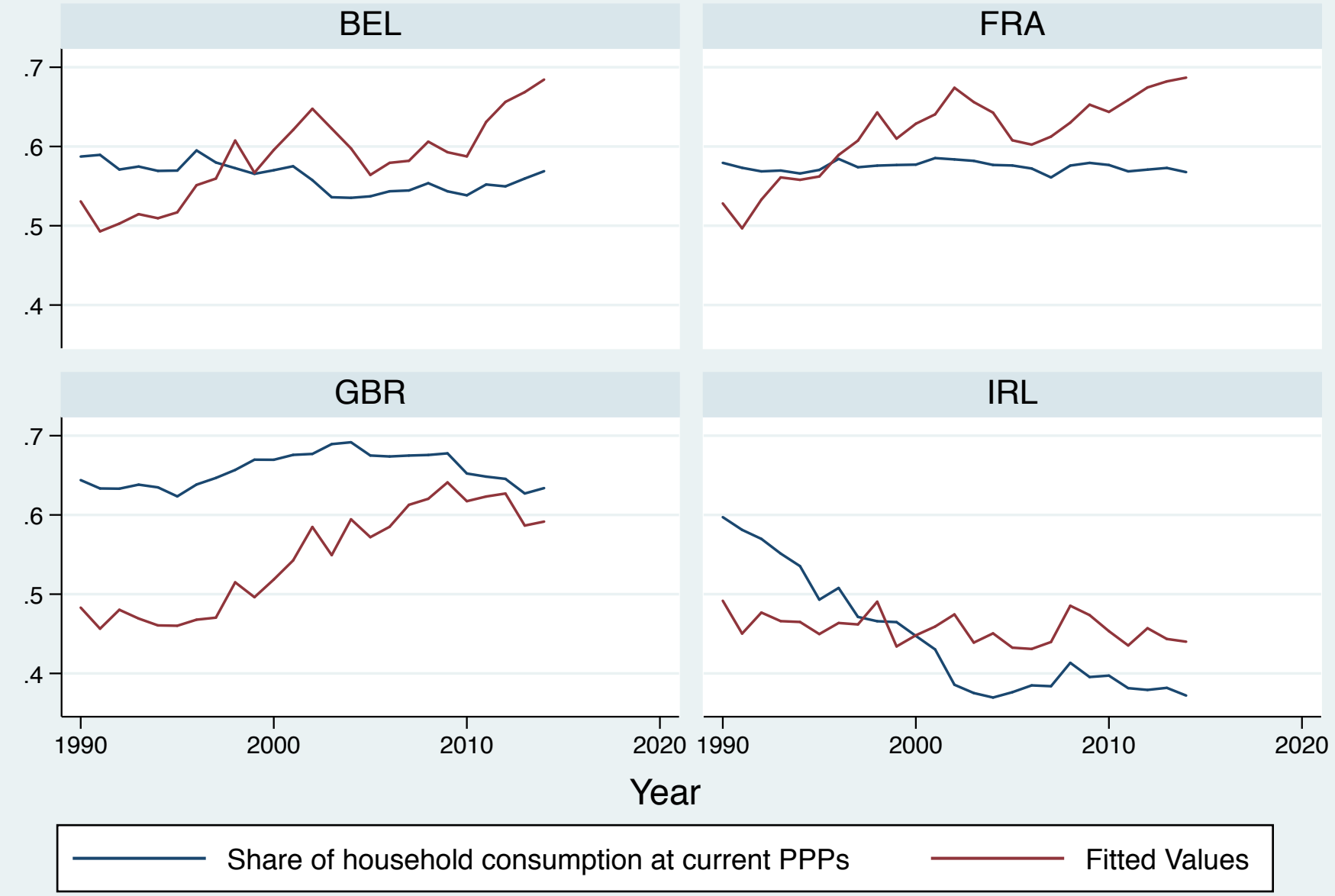
(continued)

```
Instruments for first differences equation
  Standard
    D.(1989b.year 1990.year 1991.year 1992.year 1993.year 1994.year 1995.year
    1996.year 1997.year 1998.year 1999.year 2000.year 2001.year 2002.year
    2003.year 2004.year 2005.year 2006.year 2007.year 2008.year 2009.year
    2010.year 2011.year 2012.year 2013.year 2014.year)
  GMM-type (missing=0, separate instruments for each period unless collapsed)
    L(2/4).(L.csh_c csh_g)
Instruments for levels equation
  Standard
    1989b.year 1990.year 1991.year 1992.year 1993.year 1994.year 1995.year
    1996.year 1997.year 1998.year 1999.year 2000.year 2001.year 2002.year
    2003.year 2004.year 2005.year 2006.year 2007.year 2008.year 2009.year
    2010.year 2011.year 2012.year 2013.year 2014.year
    _cons
  GMM-type (missing=0, separate instruments for each period unless collapsed)
    DL.(L.csh_c csh_g)
------------------------------------------------------------------------------
Arellano-Bond test for AR(1) in first differences: z =  -1.26  Pr > z =  0.209
Arellano-Bond test for AR(2) in first differences: z =   0.78  Pr > z =  0.433
------------------------------------------------------------------------------
Sargan test of overid. restrictions: chi2(178)  =6589.09  Prob > chi2 =  0.000
  (Not robust, but not weakened by many instruments.)
Hansen test of overid. restrictions: chi2(178)  = 175.74  Prob > chi2 =  0.534
  (Robust, but weakened by many instruments.)
```

I have constrained the number of lags used to 2–4 to keep the number manageable. The results are largely similar to those of the Diff-GMM specification.

After any DPD estimation command, we can save predicted values or residuals and graph them against the actual values:

```
. predict double csh_c_hat if inlist(countrycode,"BEL","GBR","FRA","IRL")
(option xb assumed; fitted values)
(4,632 missing values generated)
. xtline csh_c csh_c_hat if !mi(csh_c_hat), ylab(,angle(0) labs(small)) ///
>  xlab(,labs(small)) legend(size(small))
. graph export `sjl´pwt3.pdf, replace
(file ~/dropbox/baum/timberlake2013-2016/slides/pwt3.pdf written in PDF format)
```


BEL
FRA
GBR
IRL
.7
.6
.5
.4
1990
2000
2010
2020
Year
Share of household consumption at current PPPs
Fitted Values
Graphs by 3-letter ISO country code

Although the DPD estimators are linear estimators, they are highly sensitive to the particular specification of the model and its instruments: more so in my experience than any other regression-based estimation approach.

There is no substitute for experimentation with the various parameters of the specification to ensure that your results are reasonably robust to variations in the instrument set and lags used. A very useful reference for DPD modeling is David Roodman's paper "How to do `xtabond2`", available from `http://ideas.repec.org` or the *Stata Journal* website.

Panel unit root tests

Stata supports a variety of panel unit root tests via the `xtunitroot` command. These include:

- Levin–Lin–Chu test
- Harris–Tzavalis test
- Breitung test
- Im–Pesaran–Shin test
- Fisher-type (combining p-values) tests
- Hadri LM stationarity test

All but the Hadri test have H0: each unit of the panel is $I(1)$. The Hadri test, like the KPSS univarate test, has H(0): each unit of the panel is (trend) stationary.