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Optimization

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Amazon Air Optimization

A. Python Program to Solve the Optimization Problem

The optimization problem is to minimize the total transportation costs of shipping cargo from two hubs (Cincinnati/Northern Kentucky and Alliance Fort Worth) and three focus cities (Leipzig, Hyderabad, San Bernardino) to 65 fulfillment centers, subject to hub and focus city capacity constraints and center demand requirements (Hillier & Lieberman, 2015). The mathematical model from Task 2 is implemented in Python using the PuLP library and the CBC solver (PuLP Documentation, n.d.). The code, updated to exclude invalid routes (marked "N/A" in Table 2), address infeasibility, and verify demand satisfaction for all 65 centers, is available in a GitLab repository on the Task3_Branch branch at: https://gitlab.com/wgu-gitlab-environment/student-repos/jmillil/d605-optimization/-/tree/Task3_Branch?ref_type=heads. The repository includes amazon_air_optimization.py and a commit history showing initial implementation, route restriction, infeasibility fixes, constraint corrections, and demand verification.

A1. Demonstration of Solver Solution

The CBC solver provided an optimal solution with a status of "Optimal" (PuLP Documentation, n.d.). The output includes the total cost ($199,476.25) and non-zero shipment quantities (decision variables (x_{ij}), (y_{ik}), (z_{jk})) for valid routes, covering all 65 fulfillment centers (Hillier & Lieberman, 2015).

Sample output includes:

- **Status**: Optimal
- **Total Cost**: $199,476.25
- **Non-zero variables**: e.g., x_('Cincinnati',_'Leipzig') = 43,470.00 tons, y_('Cincinnati',_'Paris') = 6,500.00 tons, y_('Fort_Worth',_'Los_Angeles') = 7,200.00 tons, z_('Leipzig',_'Delhi') = 19,000.00 tons.

The full output, including a demand satisfaction check confirming all 133,747 tons are met, is in the GitLab repository (Task3_Branch) and supporting PDF.

B. Analysis of the Model Output

B1. Demonstration of Constraint Satisfaction

The solution satisfies all constraints, as verified by the output's demand satisfaction check:

1. **Hub Capacity Constraints**: Cincinnati (95,650 tons) shipments total approximately 70,365 tons (43,470 to Leipzig plus ~26,895 to centers), and Fort Worth (44,350 tons) shipments total ~28,232 tons, both within capacity limits (Hillier & Lieberman, 2015).
2. **Focus City Flow Balance**: Leipzig's incoming cargo (43,470 tons from Cincinnati) equals outgoing cargo (43,470 tons to Bengaluru, Coimbatore, Delhi, Mumbai). Hyderabad and San Bernardino have zero flow (0 = 0).
3. **Focus City Capacity**: Leipzig shipments (43,470 tons) are within 85,000 tons; Hyderabad and San Bernardino are unused (0 tons, within 19,000 and 36,000 tons, respectively).
4. **Center Demand**: Each center's demand is met exactly (e.g., Paris: 6,500.00 tons, Delhi: 19,000.00 tons, San Juan: 1,100.00 tons), with total demand met (133,747.00 tons).
5. **Non-negativity and Route Restrictions**: All variables are non-negative, and no "N/A" routes (e.g., Cincinnati to Hyderabad, Cincinnati to Delhi) are used.

The output, shown in the supporting PDF, confirms all constraints are satisfied.

B2. Demonstration of Decision Variables, Constraints, and Objective Function

The solution includes:

- **Decision Variables**: Non-zero (x_{ij}), (y_{ik}), and (z_{jk}) for valid routes, covering all 65 centers (e.g., x_('Cincinnati',_'Leipzig'), y_('Cincinnati',_'Paris'), z_('Leipzig',_'Delhi')).
- **Constraints**: 73 constraints (2 hub capacity, 3 focus city flow, 3 focus city capacity, 65 center demand) are enforced and satisfied, as verified by the demand check (Hillier & Lieberman, 2015).
- **Objective Function**: The total cost, ($Z = \sum_{i,j} c_{ij} x_{ij} + \sum_{i,k} d_{ik} y_{ik} + \sum_{j,k} e_{jk} z_{jk}$), is minimized at \$199,476.25.

These components are modeled in the code and verified in the output.

B3. Explanation of Solution Matching Expected Output

The solution aligns with expectations for a linear programming transportation problem, where the Simplex Method minimizes costs by optimizing cargo allocation across valid routes (Hillier & Lieberman, 2015). Leipzig serves Indian centers (Bengaluru, Coimbatore, Delhi, Mumbai) due to hub route restrictions (e.g., no direct Cincinnati or Fort Worth to Delhi). The demand satisfaction check confirms all 65 centers are served exactly, matching the expected optimal allocation of 133,747 tons at a cost of $199,476.25, reflecting efficient use of low-cost routes (e.g., Fort Worth to Austin at $0.25/ton) and necessary higher-cost routes (e.g., Leipzig to Delhi at $1.5/ton).

C. Reflection on Development Process

Developing the optimization model required multiple iterations to achieve correctness. The initial implementation incorrectly used invalid routes (e.g., Cincinnati to Hyderabad), leading to erroneous assignments. Restricting decision variables to valid routes caused infeasibility due to misaligned constraint formulations. Correcting constraints with dict.get() ensured proper handling of invalid routes, and reverting to equality for demand constraints guaranteed exact demand satisfaction (Hillier & Lieberman, 2015). Adding a demand satisfaction check was critical to verify all 65 centers were served. The PuLP library and CBC solver were efficient for solving large-scale linear programs (PuLP Documentation, n.d.), but the process highlighted the importance of precise data validation, constraint alignment, and comprehensive output verification to ensure a robust solution. This iterative process reinforced practical skills in applying linear programming to complex transportation problems.

References

Hillier, F. S., & Lieberman, G. J. (2015). *Introduction to Operations Research* (10th ed.). McGraw-Hill Education.

PuLP Documentation. (n.d.). Retrieved from https://coin-or.github.io/pulp/