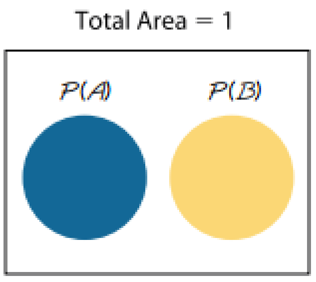

Figure 4-5 Venn Diagram for Disjoint Events

notation $P(A \cap B)$ is sometimes used in place of $P(A \text{ and } B)$, so the formal addition rule can be expressed as

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Whenever A and B are disjoint, $P(A \text{ and } B)$ becomes zero in the addition rule. Figure 4-5 illustrates that when A and B are disjoint, we have $P(A \text{ or } B) = P(A) + P(B)$.

Here is a summary of the key points of this section:

1. To find $P(A \text{ or } B)$, begin by associating use of the word *or* with addition.
2. Consider whether events A and B are disjoint; that is, can they happen at the same time? If they are not disjoint (that is, they can happen at the same time), be sure to avoid (or compensate for) double counting when adding the relevant probabilities. If you understand the importance of not double counting when you find $P(A \text{ or } B)$, you don't necessarily have to calculate the value of $P(A) + P(B) - P(A \text{ and } B)$.

CAUTION Errors made when applying the addition rule often involve double counting; that is, events that are not disjoint are treated as if they were. One indication of such an error is a total probability that exceeds 1; however, errors involving the addition rule do not always cause the total probability to exceed 1.

Complementary Events and the Addition Rule

In Section 4-2 we used $\overline{A}$ to indicate that A does not occur. Common sense dictates this principle: We are certain (with probability 1) that either an event A occurs *or* it does not occur, so it follows that $P(A \text{ or } \overline{A}) = 1$. Because events A and $\overline{A}$ must be disjoint, we can use the addition rule to express that commonsense principle as follows:

$$P(A \text{ or } \overline{A}) = P(A) + P(\overline{A}) = 1$$

This result of the addition rule leads to the following three expressions that are equivalent in the sense that they are different forms of the same principle.

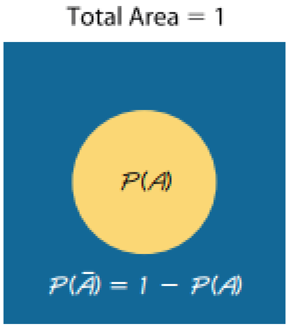

Figure 4-6 Venn Diagram for the Complement of Event *A*

Rule of Complementary Events

$$P(A) + P(\overline{A}) = 1 \qquad P(\overline{A}) = 1 - P(A) \qquad P(A) = 1 - P(\overline{A})$$

Figure 4-6 visually displays this relationship between $P(A)$ and $P(\overline{A})$.

Example 3 Devilish Belief

Based on data from a Harris Interactive poll, the probability of randomly selecting someone who believes in the devil is 0.6, so $P(\text{believes in the devil}) = 0.6$. If a person is randomly selected, find the probability of getting someone who does *not* believe in the devil.

Solution

Using the rule of complementary events, we get

$$P(\text{does } \textit{not} \text{ believe in the devil}) = 1 - P(\text{believes in the devil}) = 1 - 0.6 = 0.4$$

The probability of randomly selecting someone who does not believe in the devil is 0.4.

A major advantage of the *rule of complementary events* is that it simplifies certain probability problems, as we illustrate in Section 4-5.