a. If you bet \$5 on the number 7 in roulette, the probability of losing \$5 is 37/38 and the probability of making a net gain of \$175 is 1/38. (The prize is \$180, including your \$5 bet, so the net gain is \$175.) Find your expected value if you bet \$5 on the number 7 in roulette.

b. If you bet \$5 on the pass line in the dice game of craps, the probability of losing \$5 is 251/495 and the probability of making a net gain of \$5 is 244/495. (If you bet \$5 on the pass line and win, you are given \$10 that includes your bet, so the net gain is \$5.) Find your expected value if you bet \$5 on the pass line.

Which of the preceding two bets is better? Why?

Solution

a. Roulette The probabilities and payoffs for betting \$5 on the number 7 in roulette are summarized in Table 5-5. Table 5-5 also shows that the expected value is $\Sigma[x \cdot P(x)] = -26¢$. That is, for every \$5 bet on the number 7, you can expect to *lose* an average of 26¢.

Table 5-5 Roulette

Event	x	$P(x)$	$x \cdot P(x)$
Lose	−\$5	37/38	−\$4.868421
Win (net gain)	\$175	1/38	\$4.605263
Total			−\$0.26 (rounded) (or −26¢)

b. Dice The probabilities and payoffs for betting \$5 on the pass line in craps are summarized in Table 5-6. Table 5-6 also shows that the expected value is $\Sigma[x \cdot P(x)] = -7¢$. That is, for every \$5 bet on the pass line, you can expect to lose an average of 7¢.

Table 5-6 Dice

Event	x	$P(x)$	$x \cdot P(x)$
Lose	−\$5	251/495	−\$2.535353
Win (net gain)	\$5	244/495	\$2.464646
Total			−\$0.07 (rounded) (or −7¢)

Interpretation

The \$5 bet in roulette results in an expected value of −26¢ and the \$5 bet in craps results in an expected value of −7¢. Because you are better off losing 7¢ instead of losing 26¢, the craps game is better in the long run, even though the roulette game provides an opportunity for a larger payoff.

Rationale for Formulas 5-1 through 5-4

Instead of blindly accepting and using formulas, it is much better to have some understanding of why they work. When computing the mean from a frequency distribution, f represents class frequency and N represents population size. In the expression