Step 5: Because the claim involves the proportion p, the statistic relevant to this test is the sample proportion $\hat{p}$ and the sampling distribution of sample proportions can be approximated by the normal distribution.

Step 6: The test statistic $z = 6.09$ is calculated as follows:

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{\frac{40}{41} - 0.50}{\sqrt{\frac{(0.5)(0.5)}{41}}} = 6.09$$

Refer to Figure 8-4 for the procedure for finding the P-value. Figure 8-4 shows that for this right-tailed test, the P-value is the area to the right of the test statistic. Using Table A-2, we see that $z = 6.09$ is off the chart, so the area to the right of $z = 6.09$ is 0.0001. (Technology provides a more accurate P-value of 0.000000000564.)

Step 7: Because the P-value is less than or equal to the significance level of 0.05, we reject the null hypothesis.

Interpretation

Because we reject the null hypothesis, we support the alternative hypothesis. We therefore conclude that there is sufficient evidence to support the Republican claim that the county clerk used a method that favored Democrats. The county clerk lost his job.

Critical Value Method

If we were to repeat Example 3 using the critical value method of testing hypotheses, we would see that in Step 6 the critical value is $z = 1.645$. In Step 7 we would reject the null hypothesis because the test statistic of $z = 6.09$ would fall within the critical region bounded by $z = 1.645$. We would then reach the same conclusion given in Example 3.

Confidence Interval method

If we were to repeat Example 3 using the confidence interval method, we would use a 90% confidence level because we have a right-tailed test. (See Table 8-1.) We get this 90% confidence interval: $0.936 < p < 1.015$. Because the confidence interval limits do not contain the value of 0.5, it is very unlikely that the success rate is actually 50%, so there is sufficient evidence to reject the 50% rate. In this case, the P-value method, critical value method, and confidence interval method all lead to the same conclusion.

Part 2: Exact Methods for Testing Claims About a Population Proportion p

Instead of using the normal distribution as an *approximation* to the binomial distribution, we can get *exact* results by using the binomial probability distribution itself. Binomial probabilities are a nuisance to calculate manually, but technology makes this approach quite simple. Also, this exact approach does not require that $np \geq 5$ and $nq \geq 5$, so we have a method that applies when that requirement is not satisfied. To test hypotheses using the exact binomial distribution, use the binomial probability distribution with the P-value method, use the value of p assumed in the null hypothesis, and find P-values as follows:

Lefties Die Sooner?

A study by psychologists Diane Halpern and Stanley Coren received considerable media attention and generated considerable interest when it concluded that left-handed people don't live as long as right-handed people. Based on their study, it appeared that left-handed people live an average of nine years less than righties. The Halpern/Coren study has been criticized for using flawed data. They used second-hand data by surveying relatives about people who had recently died. The myth of lefties dying younger became folklore that has survived many years. However, more recent studies show that left-handed people do *not* have shorter lives than those who are right-handed.