Poisson Distribution as an Approximation to the Binomial Distribution

The Poisson distribution is sometimes used to approximate the binomial distribution when n is large and p is small. One rule of thumb is to use such an approximation when the following two requirements are both satisfied.

Requirements for Using the Poisson Distribution as an Approximation to the Binomial

1. $n \geq 100$
2. $np \leq 10$

If both requirements are satisfied and we want to use the Poisson distribution as an approximation to the binomial distribution, we need a value for μ. That value can be calculated by using Formula 5-6 (first presented in Section 5-4):

Formula 5-6 Mean for Poisson Distribution as an Approximation to the Binomial

$$\mu = np$$

Example 2 Maine Pick 4

In the Maine Pick 4 game, you pay 50¢ to select a sequence of four digits, such as 2449. If you play this game once every day, find the probability of winning at least once in a year with 365 days.

Solution

Because the time interval is 365 days, $n = 365$. Because there is one winning set of numbers among the 10,000 that are possible (from 0000 to 9999), $p = 1/10{,}000$. With $n = 365$ and $p = 1/10{,}000$, the conditions $n \geq 100$ and $np \leq 10$ are both satisfied, so we can use the Poisson distribution as an approximation to the binomial distribution. We first need the value of μ, which is found as follows:

$$\mu = np = 365 \cdot \frac{1}{10{,}000} = 0.0365$$

Having found the value of μ, we can proceed to find the probability for specific values of x. Because we want the probability that x is "at least 1," we will use the clever strategy of first finding $P(0)$, the probability of no wins in 365 days. The probability of at least one win can then be found by subtracting that result from 1. We find $P(0)$ by using $x = 0$, $\mu = 0.0365$, and $e = 2.71828$, as shown here:

$$P(0) = \frac{\mu^x \cdot e^{-\mu}}{x!} = \frac{0.0365^0 \cdot 2.71828^{-0.0365}}{0!} = \frac{1 \cdot 0.9642}{1} = 0.9642$$

Using the Poisson distribution as an approximation to the binomial distribution, we find that there is a 0.9642 probability of no wins, so the probability of at least one win is $1 - 0.9642 = 0.0358$. If we use the binomial distribution, we get a probability of 0.0358, so the Poisson approximation is quite good here.