

Specific Alternative Value of $p$	$\beta$	Power of Test ( $1 - \beta$ )
0.6	0.820	0.180
0.7	0.564	0.436
0.8	0.227	0.773
0.9	0.012	0.988

### Interpretation

Based on the list of power values above, we see that this hypothesis test has power of 0.180 (or 18.0%) of rejecting  $H_0: p = 0.5$  when the population proportion  $p$  is actually 0.6. That is, if the true population proportion is actually equal to 0.6, there is an 18.0% chance of making the correct conclusion of rejecting the false null hypothesis that  $p = 0.5$ . That low power of 18.0% is not good.

There is a 0.564 probability of rejecting  $p = 0.5$  when the true value of  $p$  is actually 0.7. It makes sense that this test is more effective in rejecting the claim of  $p = 0.5$  when the population proportion is actually 0.7 than when the population proportion is actually 0.6. (When identifying animals assumed to be horses, there's a better chance of rejecting an elephant as a horse—because of the greater difference—than rejecting a mule as a horse.) In general, increasing the difference between the assumed parameter value and the actual parameter value results in an increase in power, as shown in the table above.

Because the calculations of power are quite complicated, the use of technology is strongly recommended. (In this section, only Exercises 35 and 36 involve power.)

## Power and the Design of Experiments

Just as 0.05 is a common choice for a significance level, a power of at least 0.80 is a common requirement for determining that a hypothesis test is effective. (Some statisticians argue that the power should be higher, such as 0.85 or 0.90.) When designing an experiment, we might consider how much of a difference between the claimed value of a parameter and its true value is an important amount of difference. If testing the effectiveness of the XSORT gender-selection method, a change in the proportion of girls from 0.5 to 0.501 is not very important. A change in the proportion of girls from 0.5 to 0.9 would be very important. Such magnitudes of differences affect power. When designing an experiment, a goal of having a power value of at least 0.80 can often be used to determine the minimum required sample size, as in the following example.

### Example 4 Finding the Sample Size Required to Achieve 80% Power

Here is a statement similar to one in an article from the *Journal of the American Medical Association*: “The trial design assumed that with a 0.05 significance level, 153 randomly selected subjects would be needed to achieve 80% power to detect a reduction in the coronary heart disease rate from 0.5 to 0.4.” Before conducting the experiment, the researchers selected a significance level of 0.05 and a power of at least 0.80. They also decided that a reduction in the proportion of coronary heart disease from 0.5 to 0.4 is an important difference that they wanted to detect (by correctly rejecting the false null hypothesis). Using a significance level of 0.05,