Solution

We have 49 different numbers and we must select 6 without replacement (because the selected numbers must be different). Because order does not count, we need to find the number of different possible *combinations*. With $n = 49$ numbers available and $r = 6$ numbers selected, the number of combinations is as shown below:

$$_{n}C_{r} = \frac{n!}{(n-r)!r!} = \frac{49!}{(49-6)!6!} = \frac{49!}{43! \cdot 6!} = 13{,}983{,}816$$

Interpretation

If you select one 6-number combination, your probability of winning is 1/13,983,816. Typical lotteries rely on the fact that people rarely know the value of this probability and have no realistic sense for how small that probability is. This is why the lottery is sometimes called a "tax on people who are bad at math."

Because choosing between permutations and combinations can often be tricky, we provide the following example that emphasizes the difference between them.

Example 6 Permutations and Combinations: Corporate Officials and Committees

The Teknomite Corporation must appoint a president, chief executive officer (CEO), and chief operating officer (COO). It must also appoint a Planning Committee with three different members. There are eight qualified candidates, and officers can also serve on the committee.

a. How many different ways can the officers be appointed?

b. How many different ways can the committee be appointed?

Solution

Note that in part (a), order is important because the officers have very different functions. However, in part (b), the order of selection is irrelevant because the committee members serve the same function.

a. Because order does count, we want the number of *permutations* of $r = 3$ people selected from the $n = 8$ available people. We get

$$_{n}P_{r} = \frac{n!}{(n-r)!} = \frac{8!}{(8-3)!} = 336$$

b. Because order does *not* count, we want the number of *combinations* of $r = 3$ people selected from the $n = 8$ available people. We get

$$_{n}C_{r} = \frac{n!}{(n-r)!r!} = \frac{8!}{(8-3)!3!} = 56$$

With order taken into account, there are 336 different ways that the officers can be appointed, but without order taken into account, there are 56 different possible committees.

This section presented five different counting rules summarized near the beginning of the section. Not all counting problems can be solved with these five rules, but

Bar Codes

In 1974, the first bar code was scanned on a pack of Juicy Fruit gum that cost 67¢. Now, bar codes or "Universal Product Codes" are scanned about 10 billion times each day. When used for numbers, the bar code consists of black lines that represent a sequence of 12 digits, so the total number of different bar code sequences can be found by applying the fundamental counting rule. The number of difference bar code sequences is $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^{12} = 1{,}000{,}000{,}000{,}000$. The effectiveness of bar codes depends on the large number of different possible products that can be identified with unique numbers.

When a bar code is scanned, the detected number is not price; it is a number that identifies the particular product. The scanner uses that identifying number to look up the price in a central computer. Shown below is the bar code representing the author's name, so that letters are used instead of digits. There will be no price corresponding to the bar code below, because this person is priceless—at least according to most members of his immediate family.

M a r i o T r i o l a

Go Figure

10^{80} is the number of particles in the observable universe. The probability of a monkey randomly hitting keys and typing Shakespeare's *Hamlet* is $10^{-216,159}$.