When obtaining critical values of χ^2 from Table A-4, the numbers of degrees of freedom are consecutive integers from 1 to 30, followed by 40, 50, 60, 70, 80, 90, and 100. If a number of degrees of freedom (such as 52) is not found in the table, you can be conservative by using the next lower number of degrees of freedom, or you can use the closest critical value in the table, or you can get an approximate result with interpolation. For numbers of degrees of freedom greater than 100, use the equation given in Exercise 23, or use a more extensive table, or use technology.

Although s^2 is the best point estimate of σ^2, there is no indication of how good it actually is. To compensate for that deficiency, we develop an interval estimate (or confidence interval) that gives us a range of values associated with a confidence level.

Confidence Interval for Estimating a Population Standard Deviation or Variance

Objective

Construct a confidence interval used to estimate a population standard deviation or variance.

Notation

σ = population standard deviation

s = sample standard deviation

n = number of sample values

χ^2_L = left-tailed critical value of χ^2

σ^2 = population variance

s^2 = sample variance

E = margin of error

χ^2_R = right-tailed critical value of χ^2

Requirements

1. The sample is a simple random sample.
2. The population must have normally distributed values (even if the sample is large). The requirement of a normal distribution is much stricter here than in earlier sections, so departures from normal distributions can result in large errors.

Confidence Interval for the Population Variance σ^2

$$\frac{(n-1)s^2}{\chi^2_R} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_L}$$

Confidence Interval for the Population Standard Deviation σ

$$\sqrt{\frac{(n-1)s^2}{\chi^2_R}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_L}}$$

Round-Off Rule

1. When using the *original set of data* values to construct a confidence interval, round the confidence interval limits to one more decimal place than is used for the original set of data.
2. When using the *summary statistics* (n, s) for constructing a confidence interval, round the confidence interval limits to the same number of decimal places used for the sample standard deviation.

Confidence intervals can be easily constructed with technology or they can be constructed by using Table A-4 with the following procedure.