

We begin with basic notation followed by the multiplication rule. We strongly suggest that you focus on the *intuitive* multiplication rule, because it is based on understanding instead of blind use of a formula.

#### Notation

$P(A \text{ and } B) = P(\text{event } A \text{ occurs in a first trial and event } B \text{ occurs in a second trial})$

$P(B|A)$  represents the probability of event  $B$  occurring after it is assumed that event  $A$  has already occurred. (Interpret  $B|A$  as “event  $B$  occurring after event  $A$  has already occurred.”)

#### Formal Multiplication Rule

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

#### Intuitive Multiplication Rule

To find the probability that event  $A$  occurs in one trial and event  $B$  occurs in another trial, multiply the probability of event  $A$  by the probability of event  $B$ , but *be sure that the probability of event  $B$  takes into account the previous occurrence of event  $A$ .*

When applying the multiplication rule and considering whether event  $B$  must be adjusted to account for the previous occurrence of event  $A$ , we are focusing on whether events  $A$  and  $B$  are *independent*.

**DEFINITIONS** Two events  $A$  and  $B$  are **independent** if the occurrence of one does not affect the *probability* of the occurrence of the other. (Several events are similarly independent if the occurrence of any does not affect the probabilities of the occurrence of the others.) If  $A$  and  $B$  are not independent, they are said to be **dependent**.

**CAUTION** Don't think that *dependence* of two events means that one is the direct *cause* of the other. Having a working TV in your room and having working lights in your room are dependent events (because they have the same power source), even though neither has a direct effect on the other.

In the wonderful world of statistics, sampling methods are critically important, and the following relationships hold:

- Sampling *with replacement*: Selections are *independent* events.
- Sampling *without replacement*: Selections are *dependent* events.

#### Exception: Treating Dependent Events as Independent

Some cumbersome calculations can be greatly simplified by using the common practice of treating events as independent when *small samples* are drawn from *large populations*. In such cases, it is rare to select the same item twice.

Here is a common guideline routinely used with applications such as analyses of survey results: