With a probability of 0.187661 for the first cell, we expect that among 253 subjects, there are $253 \cdot 0.187661 = 47.478$ subjects in the first cell. If we generalize these calculations, we get the following:

$$\text{Expected frequency } E = (\text{grand total}) \cdot \frac{(\text{row total})}{(\text{grand total})} \cdot \frac{(\text{column total})}{(\text{grand total})}$$

This expression can be simplified to

$$E = \frac{(\text{row total})(\text{column total})}{(\text{grand total})}$$

We now proceed to conduct a hypothesis test of independence, as in Example 3.

Example 3 Does the Choice of Treatment Affect Success?

If we analyze the data in Table 11-6, it appears that the choice of treatment does affect success. However, we must determine whether those differences are *significant,* and that is the purpose of the test of independence. Use a 0.05 significance level to test the claim that success is independent of the treatment group. What does the result indicate about the increasing trend to use surgery?

Solution

Requirement check (1) Based on the description of the study, we will treat the subjects as being randomly selected and randomly assigned to the different treatment groups. (2) The results are expressed as frequency counts in Table 11-6. (3) The expected frequencies are all at least 5. (The lowest expected frequency is 6.174.) The requirements are satisfied. ✓

The null hypothesis and alternative hypothesis are as follows:

H_0: Success is independent of the treatment.

H_1: Success and the treatment are dependent.

The significance level is $\alpha = 0.05$.

Because the data are in the form of a contingency table, we use the χ^2 distribution with this test statistic:

$$\chi^2 = \sum \frac{(O - E)^2}{E} = \frac{(54 - 47.478)^2}{47.478} + \cdots + \frac{(5 - 6.174)^2}{6.174}$$
$$= 58.393$$

XLSTAT

Chi-square (Observed value)	58.3933
Chi-square (Critical value)	7.8147
DF	3
p-value	< 0.0001
alpha	0.05

***P*-Value** If using technology, results typically include the χ^2 test statistic and the P-value. For example, see the accompanying XLSTAT display showing the test statistic is $\chi^2 = 58.393$ and the P-value is less than 0.0001. Because the P-value is less than the significance level of 0.05, we reject the null hypothesis of independence between success and treatment.

Critical Value If using the critical value method of hypothesis testing, the critical value of $\chi^2 = 7.815$ is found from Table A-4 with $\alpha = 0.05$ in the right tail and the number of degrees of freedom given by $(r - 1)(c - 1) = (4 - 1)(2 - 1) = 3$. The test statistic and critical value are shown in Figure 11-4. Because the test statistic does fall within the critical region, we reject the null hypothesis of independence between success and treatment.