$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

becomes

$$P(\text{positive test result}|\text{subject uses drugs})$$
$$= \frac{P(\text{subject uses drugs and had a positive test result})}{P(\text{subject uses drugs})}$$
$$= \frac{44/1000}{50/1000} = 0.88$$

By comparing the intuitive approach to the use of the formula, it should be clear that the intuitive approach is much easier to use, and it is also less likely to result in errors. The intuitive approach is based on an *understanding* of conditional probability, instead of manipulation of a formula, and understanding is so much better.

b. Here we want P(subject uses drugs | positive test result). This is the probability that the selected subject uses drugs, *given that the subject had a positive test result.* If we assume that the subject had a positive test result, we are dealing with the 134 subjects in the first column of Table 4-1. Among those 134 subjects, 44 use drugs, so

$$P(\text{subject uses drugs}|\text{positive test result}) = \frac{44}{134} = 0.328$$

Again, the same result can be found by applying the formula for conditional probability, but we will leave that for those with a special fondness for manipulations with formulas.

Interpretation

The first result of P(positive test result | subject uses drugs) $= 0.88$ indicates that a subject who uses drugs has a 0.88 probability of getting a positive test result. The second result of P(subject uses drugs | positive test result) $= 0.328$ indicates that for a subject who gets a positive test result, there is a 0.328 probability that this subject actually uses drugs.

Confusion of the Inverse

Note that in Example 2, P(positive test result | subject uses drugs) $\neq$ P(subject uses drugs | positive test result). This example proves that in general, $P(B|A) \neq P(A|B)$. (There could be individual cases where $P(A|B)$ and $P(B|A)$ are equal, but they are generally not equal.) To incorrectly think that $P(B|A)$ and $P(A|B)$ are equal or to incorrectly use one value for the other is called *confusion of the inverse*.

Example 3 Confusion of the Inverse

Consider the probability that it is dark outdoors, given that it is midnight: P(dark | midnight) $= 1$. (We conveniently ignore the Alaskan winter and other such anomalies.) But the probability that it is midnight, given that it is dark outdoors, is almost zero. Because P(dark | midnight) $= 1$ but P(midnight | dark) is almost zero, we can clearly see that in this case, $P(B|A) \neq P(A|B)$. Confusion of the inverse occurs when we incorrectly switch those probability values or think that they are equal.

One study showed that physicians often give very misleading information when they confuse the inverse. They tended to confuse P(cancer | positive test result) with P(positive test result | cancer). About 95% of physicians estimated P(cancer | positive test result) to be about 10 times too high, with the result that patients were given diagnoses that were very misleading, and patients were unnecessarily distressed by the incorrect information. See Exercise 35.

DNA Evidence Misused

Micheal Bobelian wrote "DNA's Dirty Little Secret" that was published in *Washington Monthly.* He describes the use of DNA in the conviction of John Puckett for the murder of a young nurse. Thirty years after the murder, police used DNA found on the victim to identify John Puckett as the assailant, and the jury was told that there was only one chance in 1.1 million that a random person's DNA would match the DNA found on the victim. Although the DNA was degraded, this was the only physical evidence linking Puckett to the crime. Michael Bobelian wrote that when old and degraded DNA is used in a search through a large database, the "odds are exponentially higher" of getting a match with someone who is not actually guilty. He wrote that "in Puckett's case the actual chance of a false match is a staggering one in three, according to the formula endorsed by the FBI's DNA advisory board and the National Research Council." Yet Puckett's attorneys were not allowed to mention that. A serious consequence of this process is that the use of DNA evidence in similar cases might result in the conviction of innocent people. Puckett was convicted and sentenced to life in prison, but is he actually guilty?