Example 5 Sampling Distribution of the Sample Range

As in Example 1, consider samples of size n = 2 randomly selected from the population $\{4, 5, 9\}$.

- a. List the different possible samples along with the probability of each sample, then find the range for each sample.
- b. Describe the sampling distribution of the sample range in the format of a table summarizing the probability distribution.
- c. Based on the results, do the sample ranges target the population range, which is 9 4 = 5?
- d. What do these results indicate about the sample range as an estimator of the population range?

Solution

a. In Table 6-6 we list the nine different possible samples of size n = 2 selected with replacement from the population {4, 5, 9}. The nine samples are equally likely, so each has probability 1/9. Table 6-6 also shows the range for each of the nine samples.

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Sample	Sample Range	Probability
4, 4	0	1/9
4, 5	1	1/9
4, 9	5	1/9
5, 4	1	1/9
5, 5	0	1/9
5, 9	4	1/9
9, 4	5	1/9
9, 5	4	1/9
9, 9	0	1/9

Table 6-6 Sampling Distribution of Range

- b. The last two columns of Table 6-6 list the values of the range along with the corresponding probabilities, so the last two columns constitute a table summarizing the probability distribution. Table 6-6 therefore describes the sampling distribution of the sample range.
- c. The mean of the sample ranges in Table 6-6 is 20/9, or 2.2. The range of the population {4, 5, 9} is 9 4 = 5. Because the mean of the sample ranges (2.2) is not equal to the population range (5), the sample ranges do *not* target the value of the population range.
- d. Because the sample ranges do not target the population range, the sample range is a *biased estimator* of the population range.

Interpretation

We conclude that the sample range is a biased estimator of the population range. This shows that, in general, a sample range should not be used to estimate the value of the population range.