equal to $\sqrt{s_1^2/n_1 + s_2^2/n_2}$. This last expression for the standard deviation is based on the property that the variance of the *differences* between two independent random variables equals the variance of the first random variable *plus* the variance of the second random variable.

Part 2: Alternative Methods

Part 1 of this section dealt with situations in which the two population standard deviations are unknown and are not assumed to be equal. In Part 2 we address two other situations:

1. The two population standard deviations are both known.
2. The two population standard deviations are unknown but are assumed to be equal.

Alternative Method Used When σ_1 and σ_2 are Known

In reality, the population standard deviations σ_1 and σ_2 are almost never known, but if they are somehow known, the test statistic and confidence interval are based on the normal distribution instead of the t distribution. See the summary box below.

Inferences About Means of Two Independent Populations, with σ_1 and σ_2 Known

Requirements

1. The two population standard deviations σ_1 and σ_2 are both known.
2. The two samples are *independent*.
3. Both samples are *simple random samples*.
4. Either or both of these conditions is satisfied: The two sample sizes are both *large* (with $n_1 > 30$ and $n_2 > 30$) or both samples come from populations having normal distributions. (For small samples, the normality requirement is loose in the sense that the procedures perform well as long as there are no outliers and departures from normality are not too extreme.)

Hypothesis Test

Test statistic:

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

***P*-values:** *P*-values are automatically provided by technology. If technology is not available, refer to the normal distribution in Table A-2. Use the procedure summarized in Figure 8-4 from Section 8-2.

Critical values: Refer to Table A-2.

Confidence Interval Estimate of $\mu_1 - \mu_2$

$$(\bar{x}_1 - \bar{x}_2) - E < (\mu_1 - \mu_2) < (\bar{x}_1 - \bar{x}_2) + E$$

where

$$E = z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$