replace p_1 and p_2 by $\bar{p}$ and replace q_1 and q_2 by $\bar{q} = 1 - \bar{p}$, the variance above leads to the following standard deviation:

$$\sigma_{(\hat{p}_1 - \hat{p}_2)} = \sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}$$

We now know that the distribution of $p_1 - p_2$ is approximately normal, with mean $p_1 - p_2$ and standard deviation as shown above, so the z test statistic has the form given earlier.

The form of the confidence interval requires an expression for the variance different from the one given above. When constructing a confidence interval estimate of the difference between two proportions, we don't assume that the two proportions are equal, and we estimate the standard deviation as

$$\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

In the test statistic

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}}$$

use the positive and negative values of z (for two tails) and solve for $p_1 - p_2$. The results are the limits of the confidence interval given in the box near the beginning of this section.

using TECHNOLOGY

STATDISK Select **Analysis** from the main menu bar, then select either **Hypothesis Testing** or **Confidence Intervals.**

Claim: p1 < p2

Pooled proportion: 0.4382022
Test Statistic, z: -3.4874
Critical z: −1.6449
P-Value: 0.0002

90% Confidence interval:
−0.5284103 < p1-p2 < −0.2056645

Select the menu item of **Proportion-Two Samples.** Enter the required items in the dialog box, then click on the **Evaluate** button. The accompanying display is from Example 1 in this section.

MINITAB Select **Stat** from the main menu bar, then select **Basic Statistics,** then **2 Proportions.** Click on the button for **Summarized data** and enter the sample values. Click on the **Options** bar and enter the desired confidence level. (Enter 95 for a hypothesis test with a 0.05 significance level.) If testing a hypothesis, enter 0 for the claimed value of $p_1 - p_2$, then select the format for the alternative hypothesis, and click on the box to use the pooled estimate of p for the test. Click **OK** twice.

In **Minitab 16,** you can also click on **Assistant,** then **Hypothesis Tests,** then select the case for **2-Sample % Defective.** Fill out the dialog box, then click **OK** to get three windows of results that include the *P*-value and much other helpful information.

EXCEL **Hypothesis Test:** Use XLSTAT. Click on **XLSTAT** at the top. Click on **Parametric tests,** then select **Test for two proportions.** In the dialog box that appears, enter the frequency and sample size for each of the two samples. For the "Data format" options, select **Frequencies.** Be sure that there is a checkmark in the box next to "z test." Click on the **Options** tab and select the type of test; for a two-tailed test, select the case including the symbol $\neq$, for a left-tailed test, select the case including $<$; and for a right-tailed test, select the case including $>$. Enter a value in the "Significance level (%)" box. For example, enter 5 for a 0.05 significance level. For the options listed under "Variance," select pq(1/n1 + 1/n2). Click **OK** to get results that include the test statistic and *P*-value.

Confidence Interval: Use XLSTAT. Click on **XLSTAT** at the top. Click on **Parametric tests,** then select **Test for two proportions.** In the dialog box that appears, enter the frequency and sample size for each of the two samples. For the "Data format" options, select **Frequencies.** Be sure that there is a checkmark in the box next to "z test." Click on the **Options** tab. For the alternative hypothesis, select the format of a two-tailed test. Enter a value in the "Significance level (%)" box. For example, enter 5 for a 95% confidence interval. For the options listed under "Variance," select p1q1/n1 + p2q2/n2. Click **OK** to get results that include the confidence interval.

TI-83/84 PLUS The TI-83/84 Plus calculator can be used for hypothesis tests and confidence intervals. Press STAT and select **TESTS.** Then choose the option of **2-PropZTest** (for a hypothesis test) or **2-PropZInt** (for a confidence interval). When