Important Applications of the Multiplication Rule

The following two examples illustrate practical applications of the multiplication rule. Example 4 gives us some insight into *hypothesis testing* (which is introduced in Chapter 8), and Example 5 illustrates the principle of *redundancy*, which is used to increase the reliability of many mechanical and electrical systems.

Example 4 *Hypothesis Testing:* Effectiveness of Gender Selection

MicroSort's XSORT gender-selection technique is designed to increase the likelihood that a baby will be a girl.

a. Assume that in a preliminary test of the XSORT technique, 20 couples gave birth to 20 babies, and all 20 babies were girls. If we were to assume that the XSORT method has no effect, what is the probability of getting 20 girls in 20 births by random chance? What does the result suggest?

b. Here are actual results of the XSORT gender-selection technique: Among 945 babies born, 879 were girls (based on data from the Genetics & IVF Institute). The probability of these results occurring by random chance with no effect from the XSORT method is calculated to be 0+, where 0+ denotes a positive probability that is so close to 0 that we can consider it to be 0 for all practical purposes. Does the probability of 0+ provide strong evidence to support a claim that the XSORT method is effective in increasing the likelihood that a baby will be a girl?

Solution

a. We want to find P(all 20 babies are girls) with the assumption that the XSORT method has no effect so that the probability of any individual offspring being a girl is 0.5. Because separate couples were used, we treat the events as being independent. We get this result:

$$\begin{aligned} &P(\text{all 20 babies are girls}) \\ &\quad = P(\text{1st is girl and 2nd is girl and 3rd is girl} \cdots \text{and 20th is girl}) \\ &\quad = P(\text{girl}) \cdot P(\text{girl}) \cdot \cdots \cdot P(\text{girl}) \\ &\quad = 0.5 \cdot 0.5 \cdot \cdots \cdot 0.5 \\ &\quad = 0.5^{20} = 0.000000954 \end{aligned}$$

Because the probability of 0.000000954 is so small, it appears that random chance is a poor explanation. The more reasonable explanation is that use of the XSORT technique makes babies more likely to be girls.

b. The probability of getting 879 girls in 945 births is 0+, which is a small positive number that is almost 0. This shows that these results are nearly impossible if boys and girls are equally likely. These results do provide strong evidence to support a claim that the XSORT method is effective in increasing the likelihood that a baby will be a girl.

Redundancy

Reliability of systems can be greatly improved with redundancy of critical components. Race cars in the NASCAR Winston Cup series have two ignition systems so that if one fails, the other will keep the car running. Airplanes have two independent electrical systems, and aircraft used for instrument flight typically have two separate radios. The following is from a *Popular Science* article about stealth aircraft: "One plane built largely of carbon fiber was the Lear Fan 2100 which had to carry two radar transponders. That's because if a single transponder failed, the plane was nearly invisible to radar." Such redundancy is an application of the multiplication rule in probability theory. If one component has a 0.001 probability of failure, the probability of two independent components both failing is only 0.000001.