Notation for Hypothesis Tests with Two Variances or Standard Deviations

s_1^2 = *larger* of the two sample variances

n_1 = size of the sample with the *larger* variance

σ_1^2 = variance of the population from which the sample with the *larger* variance was drawn

The symbols s_2^2, n_2, and σ_2^2 are used for the other sample and population.

Requirements

1. The two populations are *independent.*
2. The two samples are simple random samples.
3. Each of the two populations must be *normally distributed,* regardless of their sample sizes. This F test is *not robust* against departures from normality, so it performs poorly if one or both of the populations has a distribution that is not normal. The requirement of normal distributions is quite strict for this F test.

Explore the Data! Because the F test requirement of normal distributions is quite strict, be sure to examine the distributions of the two samples using histograms and normal quantile plots, and confirm that there are no outliers. (See "Assessing Normality" in Section 6-6.)

Test Statistic for Hypothesis Tests with Two Variances

$F = \dfrac{s_1^2}{s_2^2}$ (where s_1^2 is the *larger* of the two sample variances)

***P*-values:** P-values are automatically provided by technology. If technology is not available, use the computed value of the F test statistic with Table A-5 to find a range for the P-value.

Critical values: Use Table A-5 to find critical F values that are determined by the following:

1. The significance level α (Table A-5 includes critical values for $\alpha = 0.025$ and $\alpha = 0.05$.)
2. **Numerator degrees of freedom $= n_1 - 1$** (determines *column* of Table A-5)
3. **Denominator degrees of freedom $= n_2 - 1$** (determines *row* of Table A-5) For significance level $\alpha = 0.05$, refer to Table A-5 and use the right-tail area of 0.025 or 0.05 depending on the type of test, as shown below:
   - *Two-tailed test:* Use Table A-5 with 0.025 in the right tail. (The significance level of 0.05 is divided between the two tails, so the area in the right tail is 0.025.)
   - *One-tailed test:* Use Table A-5 with $\alpha = 0.05$ in the right tail.

Find the critical F value for the right tail: Because we are stipulating that the larger sample variance is s_1^2, all one-tailed tests will be right-tailed and all two-tailed tests will require that we find only the critical value located to the right. (We have no need to find the critical value at the left tail, which can be tricky. See Exercise 21.)

F Distribution

For two normally distributed populations with equal variances ($\sigma_1^2 = \sigma_2^2$), the sampling distribution of the test statistic $F = s_1^2/s_2^2$ is the ***F* distribution** shown in Figure 9-5 (provided that we have not yet imposed the stipulation that the larger sample variance is s_1^2). If you repeat the process of selecting samples from two normally distributed populations with equal variances, the distribution of the ratio s_1^2/s_2^2 is the F distribution.

See Figure 9-5 and note these properties of the F distribution:

- The F distribution is not symmetric.
- Values of the F distribution cannot be negative.
- The exact shape of the F distribution depends on the two different degrees of freedom.