testing is done only if the combined sample tests positive. Based on past results, there is a 0.005 probability of finding *E. coli* bacteria in a public swimming area. Find the probability that a combined sample from five public swimming areas will reveal the presence of *E. coli* bacteria. Is that probability low enough so that further testing of the individual samples is rarely necessary?

4-5 Beyond the Basics

33. Shared Birthdays Find the probability that of 25 randomly selected people,

a. no 2 share the same birthday.

b. at least 2 share the same birthday.

34. Unseen Coins A statistics professor tosses two coins that cannot be seen by any students. One student asks this question: "Did one of the coins turn up heads?" Given that the professor's response is "yes," find the probability that both coins turned up heads.

35. Confusion of the Inverse In one study, physicians were asked to estimate the probability of a malignant cancer given that a test showed a positive result. They were told that the cancer had a prevalence rate of 1%, the test has a false positive rate of 10%, and the test is 80% accurate in correctly identifying a malignancy when the subject actually has the cancer. (See *Probabilistic Reasoning in Clinical Medicine* by David Eddy, Cambridge University Press.)

a. Find $P(\text{malignant} \mid \text{positive test result})$. (*Hint:* Assume that the study involves 1000 subjects and use the given information to construct a table with the same format as Table 4-1.)

b. Find $P(\text{positive test result} \mid \text{malignant})$. (*Hint:* Assume that the study involves 1000 subjects and construct a table with the same format as Table 4-1.)

c. Out of 100 physicians, 95 estimated $P(\text{malignant} \mid \text{positive test result})$ to be about 75%. Were those estimates reasonably accurate, or did they exhibit confusion of the inverse? What would be a consequence of confusion of the inverse in this situation?

4-6 Counting

Key Concept Probability problems typically require that we know the total number of simple events, but finding that number often requires one of the five rules presented in this section. With the addition rule, multiplication rule, and conditional probability, we stressed intuitive rules based on understanding and we discouraged blind use of formulas, but this section requires much greater use of formulas as we consider different methods for counting the number of possible outcomes in a variety of different situations.

Permutations and Combinations: Does *Order* Count?

When using different counting methods, it is essential to know whether different arrangements of the same items are counted only once or are counted separately. The terms *permutations* and *combinations* are standard in this context, and they are defined as follows:

DEFINITIONS

Permutations of items are arrangements in which different sequences of the same items are counted separately. For example, with the letters {a, b, c}, the arrangements of abc, acb, bac, bca, cab, and cba are all counted separately as six different permutations.

Combinations of items are arrangements in which different sequences of the same items are *not* counted separately. For example, with the letters {a, b, c}, the arrangements of abc, acb, bac, bca, cab, and cba are all considered to be same combination.