- **Student *t* Distribution** If a population has a normal distribution, then the distribution of

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

is a **Student *t* distribution** for all samples of size $n$. A Student $t$ distribution is commonly referred to simply as a ***t* distribution.**

- **Degrees of Freedom** Finding a critical value $t_{\alpha/2}$ requires a value for the **degrees of freedom** (or **df**). In general, the number of degrees of freedom for a collection of sample data is the number of sample values that can vary after certain restrictions have been imposed on all data values. (*Example:* If 10 test scores have the restriction that their mean is 80, then their sum must be 800, and we can freely assign values to the first 9 scores, but the 10th score would then be determined, so in this case there are 9 degrees of freedom.) For the methods of this section, the number of degrees of freedom is the sample size minus 1.

$$\textbf{degrees of freedom} = n - 1$$

- **Finding Critical Value $t_{\alpha/2}$** A critical value $t_{\alpha/2}$ can be found using technology or Table A-3. Technology can be used with any number of degrees of freedom, but a $t$ distribution table can be used for select numbers of degrees of freedom only.

- **Using Table A-3 when a Number of Degrees of Freedom Is Not Included** If using Table A-3 to find a critical value of $t_{\alpha/2}$, but the table does not include the number of degrees of freedom, you could use the closest value, or you could be conservative by using the next lower number of degrees of freedom found in the table, or you could interpolate.

## Example 1 Finding a Critical *t* Value

A sample of size $n = 12$ is a simple random sample selected from a normally distributed population (as in Example 2 that follows). Find the critical value $t_{\alpha/2}$ corresponding to a 95% confidence level.

### Solution

Because $n = 12$, the number of degrees of freedom is given by $n - 1 = 11$. The 95% confidence level corresponds to $\alpha = 0.05$, so there is an area of 0.025 in each of the two tails of the $t$ distribution, as shown in Figure 7-4 on the next page. Technology or a $t$ distribution table can be used to find that for 11 degrees of freedom and an area of 0.025 in each tail, the critical value is $t_{\alpha/2} = 2.201$. We could also express this as $t_{0.025} = 2.201$.

To find $t_{\alpha/2} = 2.201$ using Table A-3, locate the 11th row (because df = 11) by referring to the column at the extreme left, then use the column with 0.05 for the "Area in Two Tails" (or use the same column with 0.025 for the "Area in One Tail").

### Largest Inauguration Ever?

When Barack Obama was inaugurated as the first African American president, a huge crowd filled the region from the Capitol building to the Lincoln Memorial. The National Park Service estimated that 1.8 million people attended the inauguration. Instead of making somewhat casual estimates of the size of a crowd, aerial photographs can be used to develop reasonably good estimates based on crowd density. Allison Puccioni, an analyst for IHS Jane's, studied a satellite photograph, and she estimated that the crowd size was between 1.031 million and 1.411 million people—about 22% less than the estimate made by the National Park Service.

In another case, a celebration parade for the Boston Red Sox was attended by 3.2 million fans (according to Boston city officials), or 1 million fans (according to Boston police), or at most 400,000 fans (according to Boston University Professor Farouk El-Baz, who analyzed photos). MIT physicist Bill Donnelly said that "it's a serious thing if people are just putting out any number. It means other things aren't being vetted that carefully."