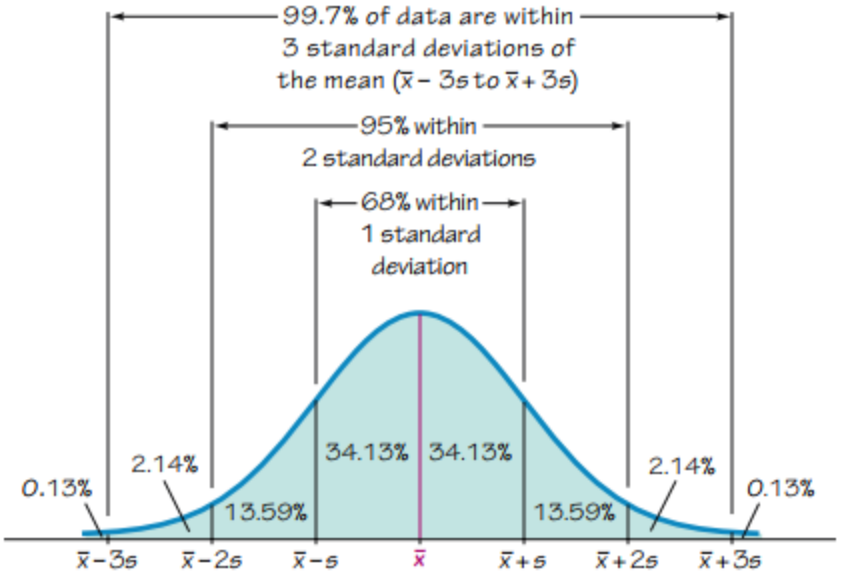
Therefore, 2 standard deviations from the mean is

$$100 - 30 = 70$$
$$\text{or} \quad 100 + 30 = 130$$

The empirical rule tells us that about 95% of all values are within 2 standard deviations of the mean, so about 95% of all IQ scores are between 70 and 130.

Figure 3-3
The Empirical Rule



Another concept helpful in understanding or interpreting a value of a standard deviation is **Chebyshev's theorem.** The empirical rule applies only to data sets with bell-shaped distributions, but Chebyshev's theorem applies to *any* data set. Unfortunately, results from Chebyshev's theorem are only approximate. Because the results are lower limits ("at least"), Chebyshev's theorem has limited usefulness.

Chebyshev's Theorem

The proportion (or fraction) of any set of data lying within K standard deviations of the mean is always *at least* $1 - 1/K^2$, where K is any positive number greater than 1. For $K = 2$ and $K = 3$, we get the following statements:

- At least $3/4$ (or 75%) of all values lie within 2 standard deviations of the mean.
- At least $8/9$ (or 89%) of all values lie within 3 standard deviations of the mean.

Example 7 Chebyshev's Theorem

IQ scores have a mean of 100 and a standard deviation of 15. What can we conclude from Chebyshev's theorem?

Solution

Applying Chebyshev's theorem with a mean of 100 and a standard deviation of 15, we can reach the following conclusions:.

- At least $3/4$ (or 75%) of IQ scores are within 2 standard deviations of the mean (between 70 and 130).
- At least $8/9$ (or 89%) of all IQ scores are within 3 standard deviations of the mean (between 55 and 145).