Step 2: If $p_1 < p_2$ is false, then $p_1 \geq p_2$.

Step 3: Because the claim of $p_1 < p_2$ does not contain equality, it becomes the alternative hypothesis. The null hypothesis is the statement of equality, so we have

$$H_0: p_1 = p_2 \quad H_1: p_1 < p_2 \text{ (original claim)}$$

Step 4: The significance level is $\alpha = 0.05$.

Step 5: We will use the normal distribution (with the test statistic given earlier in this section) as an approximation to the binomial distribution. We estimate the common value of p_1 and p_2 with the pooled sample estimate $\bar{p}$ calculated as shown below, with extra decimal places used to minimize rounding errors in later calculations.

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{12 + 27}{46 + 43} = 0.438202$$

With $\bar{p} = 0.438202$, it follows that $\bar{q} = 1 - 0.438202 = 0.561798$.

Step 6: Because we assume in the null hypothesis that $p_1 = p_2$, the value of $p_1 - p_2$ is 0 in the following calculation of the test statistic:

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\dfrac{\bar{p}\bar{q}}{n_1} + \dfrac{\bar{p}\bar{q}}{n_2}}}$$

$$= \frac{\left(\dfrac{12}{46} - \dfrac{27}{43}\right) - 0}{\sqrt{\dfrac{(0.438202)(0.561798)}{46} + \dfrac{(0.438202)(0.561798)}{43}}}$$

$$= -3.49$$

This is a left-tailed test, so the P-value is the area to the left of the test statistic $z = -3.49$ (as indicated by Figure 8-4). Refer to Table A-2 and find that the area to the left of the test statistic $z = -3.49$ is 0.0002, so the P-value is 0.0002. The test statistic and P-value are shown in Figure 9-1(a).

Step 7: Because the P-value of 0.0002 is less than the significance level of $\alpha = 0.05$, we reject the null hypothesis of $p_1 = p_2$.

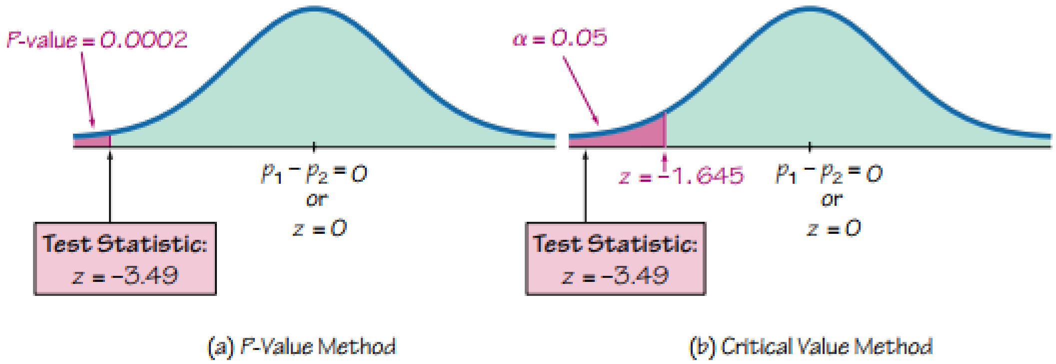

Figure 9-1 Hypothesis Test with Two Proportions

Author as a Witness

The author was asked to testify in New York State Supreme Court by a former student who was contesting a lost reelection to the office of Dutchess County Clerk. The author testified by using statistics to show that the voting behavior in one contested district was significantly different from the behavior in all other districts. When the opposing attorney asked about results of a confidence interval, he asked if the 5% error (from a 95% confidence level) could be added to the three percentage point margin of error to get a total error of 8%, thereby indicating that he did not understand the basic concept of a confidence interval. The judge cited the author's testimony, upheld the claim of the former student, and ordered a new election in the contested district. That judgment was later overturned by the appellate court on the grounds that the ballot irregularities should have been contested before the election, not after.