When finding probabilities with the relative frequency approach, we obtain an *approximation* instead of an exact value. As the total number of observations increases, the corresponding approximations tend to get closer to the actual probability. This property is stated as a theorem commonly referred to as the *law of large numbers.*

Law of Large Numbers

As a procedure is repeated again and again, the relative frequency probability of an event tends to approach the actual probability.

The law of large numbers tells us that relative frequency approximations tend to get better with more observations. This law reflects a simple notion supported by common sense: A probability estimate based on only a few trials can be off by a substantial amount, but with a very large number of trials, the estimate tends to be much more accurate.

CAUTION The law of large numbers applies to behavior over a large number of trials, and it does not apply to one outcome. Don't make the foolish mistake of losing large sums of money by incorrectly thinking that a string of losses increases the chances of a win on the next bet.

Probability and Outcomes That Are Not Equally Likely One common mistake is to incorrectly assume that outcomes are equally likely just because we know nothing about the likelihood of each outcome. When we know nothing about the likelihood of different possible outcomes, we cannot necessarily assume that they are equally likely. For example, we should not conclude that the probability of passing the next statistics test is 1/2, or 0.5 (because we either pass the test or do not). The actual probability depends on factors such as the amount of preparation and the difficulty of the test.

Example 2 Relative Frequency Probability: Smoking

A recent Harris Interactive survey of 1010 adults in the United States showed that 202 of them smoke. Find the probability that a randomly selected adult in the United States is a smoker.

Solution

We use the relative frequency approach as follows:

$$P(\text{smoker}) = \frac{\text{number of smokers}}{\text{total number of people surveyed}} = \frac{202}{1010} = 0.200$$

Note that the classical approach cannot be used since the two outcomes (smoker, not a smoker) are not equally likely.

Example 3 Classical Probability: Positive Test Result

Refer to Table 4-1 included with the Chapter Problem. Assuming that one of the 1000 subjects included in Table 4-1 is randomly selected, find the probability that the selected subject got a positive test result.

Solution

The sample space consists of results from 1000 subjects listed in Table 4-1. Among the 1000 results, 134 of them are positive test results (found from 44 + 90). Because

Winning the Lottery

In the New York State Lottery Mega Millions game, you select five numbers from 1 to 56, then you select another "Mega Ball" number from 1 to 46. To win, you must get the correct five numbers *and* the correct Mega Ball number. The chance of winning this lottery with one ticket is 1/175,711,536, even though commercials for this lottery state that "all you need is a little bit of luck." The probability of 1/175,711,536 is not easily perceived by many people, so let's consider a helpful analogy developed by Brother Donald Kelly of Marist College.

A stack of 175,711,536 dimes is about 154 miles high. Commercial jets typically fly about 7 miles high, so this stack of dimes is about 22 times taller than the height of a commercial jet when it is at cruising altitude. The chance of winning the Mega Millions lottery game is equivalent to the chance of randomly selecting *one* specific dime from that pile of dimes that is 154 miles high. Any of us who spend money on this lottery should understand that the chance of winning the grand prize is very, very, very close to zero.