Fortunately, we need not deal directly with the least-squares property when we want to find the equation of the regression line. Calculus has been used to build the least-squares property into Formulas 10-3 and 10-4. Because the derivations of these formulas require calculus, we don't include the derivations in this text, and for that, we should be very thankful.

Residual Plots

In this section and the preceding section we listed simplified requirements for the effective analyses of correlation and regression results. We noted that we should always begin with a scatterplot, and we should verify that the pattern of points is approximately a straight-line pattern. We should also consider outliers. A *residual plot* can be another helpful tool for analyzing correlation and regression results and for checking the requirements necessary for making inferences about correlation and regression.

DEFINITION A **residual plot** is a scatterplot of the (x, y) values after each of the y-coordinate values has been replaced by the residual value $y - \hat{y}$ (where $\hat{y}$ denotes the predicted value of y). That is, a residual plot is a graph of the points $(x, y - \hat{y})$.

To construct a residual plot, draw a horizontal reference line through the residual value of 0, then plot the paired values of $(x, y - \hat{y})$. Because the manual construction of residual plots can be tedious, the use of computer software is strongly recommended. When analyzing a residual plot, look for a pattern in the way the points are configured, and use these criteria:

- The residual plot should not have any obvious pattern (not even a straight line pattern). (This confirms that a scatterplot of the sample data is a straight-line pattern and not some other pattern that is not a straight line.)
- The residual plot should not become much wider (or thinner) when viewed from left to right. (This confirms the requirement that for the different fixed values of x, the distributions of the corresponding y values all have the same standard deviation.)

Example 6 Residual Plot

The shoe print and height data from Table 10-1 are used to obtain the accompanying Minitab-generated residual plot. The first sample x value of 29.7 cm is substituted into the regression equation of $\hat{y} = 125 + 1.73x$ (found in Examples 1 and 2). The result is the predicted value of $\hat{y} = 176.4$ cm. For the first x value of 29.7 cm, the actual corresponding y value is 175.3 cm, so the value of the residual is

$$\text{observed } y - \text{predicted } y = y - \hat{y} = 175.3 - 176.4 = -1.1$$

(The result is -1.4 if we use greater precision in the calculations.) Using the x value of 29.7 cm and the residual of -1.1, we get the coordinates of the point $(29.7, -1.1)$, which is one of the points in the residual plot shown on the following page. This residual plot becomes thicker, suggesting that the regression equation might not be a good model.