One-Way Analysis of Variance for Testing Equality of Three or More Population Means

Objective

Use samples from three or more different populations to test a claim that the populations all have the same mean.

Requirements

1. The populations have distributions that are approximately normal. (This is a loose requirement, because the method works well unless a population has a distribution that is very far from normal. If a population does have a distribution that is far from normal, use the Kruskal-Wallis test described in Section 13-5.)
2. The populations have the same variance σ^2 (or standard deviation σ). This is a loose requirement, because the method works well unless the population variances differ by large amounts. Statistician George E. P. Box showed that as long as the sample sizes are equal (or nearly equal), the variances can differ by amounts that make the largest up to nine times the smallest and the results of ANOVA will continue to be essentially reliable.
3. The samples are simple random samples of quantitative data.
4. The samples are independent of each other. (The samples are not matched or paired in any way.)
5. The different samples are from populations that are categorized in only one way.

Procedure for Testing H_0: $\mu_1 = \mu_2 = \mu_3 = \cdots = \mu_k$

1. Use STATDISK, Minitab, Excel, StatCrunch, a TI-83/84 Plus calculator, or any other technology to obtain results.
2. Identify the *P*-value from the display. (The ANOVA test is right-tailed because only large values of the test statistic cause us to reject equality of the population means.)
3. Form a conclusion based on these criteria that use the significance level α:
   - **Reject:** If the *P*-value $\le \alpha$, reject the null hypothesis of equal means and conclude that at least one of the population means is different from the others.
   - **Fail to Reject:** If the *P*-value $> \alpha$, fail to reject the null hypothesis of equal means.

Example 1 **Lead and Performance IQ Scores**

Use the performance IQ scores listed in Table 12-1 and a significance level of $\alpha = 0.05$ to test the claim that the three samples come from populations with means that are all equal.

Solution

Requirement check (1) Based on the three samples listed in Table 12-1, the three populations appear to have distributions that are approximately normal, as indicated by normal quantile plots. (2) The three samples in Table 12-1 have standard deviations that are not dramatically different, so the three population variances appear to be about the same. (3) Based on the careful design of the study, we can treat the samples as simple random samples. (4) The samples are independent of each other; the performance IQ scores are not matched in any way. (5) The three samples are from populations categorized according to the single factor of lead level (low, medium, high). The requirements are satisfied.

The null hypothesis and the alternative hypothesis are as follows:

H_0: $\mu_1 = \mu_2 = \mu_3$.

H_1: At least one of the means is different from the others.