We gave three approaches to finding probabilities:

$$P(A) = \frac{\text{number of times that A occurred}}{\text{number of times trial was repeated}} \qquad \text{(relative frequency)}$$

$$P(A) = \frac{\text{number of ways A can occur}}{\text{number of different simple events}} = \frac{s}{n} \qquad \text{(for equally likely outcomes)}$$

$P(A)$ is *estimated* by using knowledge of the relevant circumstances. (subjective probability)

In Sections 4-3, 4-4, and 4-5 we considered compound events, which are events combining two or more simple events. We associated the word *or* with the addition rule and the word *and* with the multiplication rule.

Addition Rule for $P(A \text{ or } B)$

- $P(A$ or $B)$ denotes the probability that for a single trial, the outcome is event A or event B or both.
- The word *or* suggests addition, and when adding $P(A)$ and $P(B)$, we must be careful to add in such a way that every outcome is counted only once.

Multiplication Rule for $P(A \text{ and } B)$

- $P(A$ and $B)$ denotes the probability that event A occurs in one trial *and* event B occurs in another trial.
- The word *and* suggests multiplication, and when multiplying $P(A)$ and $P(B)$, we must be careful to ensure that the probability of event B takes into account the previous occurrence of event A.
- $P(B \mid A)$ denotes the conditional probability of event B occurring, given that event A has already occurred.
- $P($*at least one* occurrence of event $A) = 1 - P($*no* occurrences of event $A)$

Section 4-6 was devoted to the following five counting techniques, which are used to determine the total number of outcomes in probability problems:

Counting Rules

1. Fundamental Counting Rule

$m \cdot n =$ Number of ways that two events can occur, given that the first event can occur m ways and the second event can occur n ways.

2. Factorial Rule

$n! =$ Number of different *permutations* (order counts) of n different items when all n of them are selected.

3. Permutations Rule (When All of the Items Are Different)

$${}_nP_r = \frac{n!}{(n-r)!} = \text{Number of different } \textit{permutations} \text{ (order counts) when } n \text{ different}$$

items are available, but only r of them are selected *without replacement.*

4. Permutations Rule (When Some Items Are Identical to Others)

$$\frac{n!}{n_1!n_2!\cdots n_k!} = \text{Number of different } \textit{permutations} \text{ (order counts) when } n$$

items are available and all n are selected *without replacement* but some of the items are identical to others: n_1 are alike, n_2 are alike, . . . , and n_k are alike.