CAUTION Table A-2 for the standard normal distribution provides cumulative areas from the *left*, but Table A-4 for the chi-square distribution uses cumulative areas from the *right*.

Example 1 Finding Critical Values of χ^2

A simple random sample of 22 IQ scores is obtained (as in Example 2 which follows). Construction of a confidence interval for the population standard deviation σ requires the left and right critical values of χ^2 corresponding to a confidence level of 95% and a sample size of $n = 22$. Find the critical value of χ^2 separating an area of 0.025 in the left tail, and find the critical value of χ^2 separating an area of 0.025 in the right tail.

Solution

With a sample size of $n = 22$, the number of degrees of freedom is $df = n - 1 = 21$. See Figure 7-9.

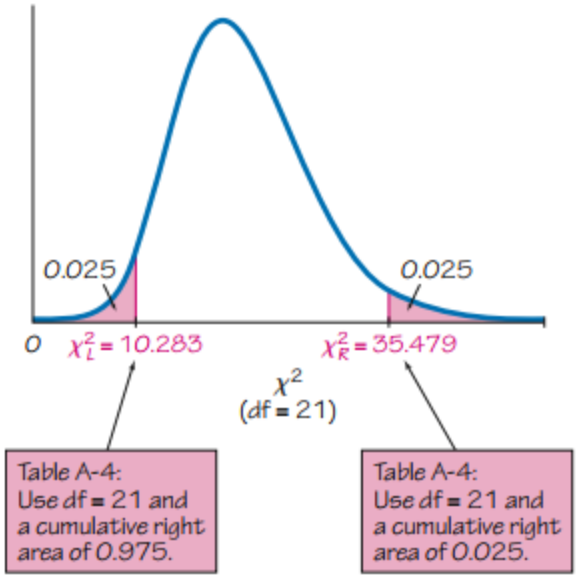

Figure 7-9 Finding Critical Values of χ_2

The critical value to the right ($\chi^2_R = 35.479$) is obtained in a straightforward manner by locating 21 in the degrees-of-freedom column at the left and 0.025 across the top row. The leftmost critical value of $\chi^2_L = 10.283$ also corresponds to 21 in the degrees-of-freedom column, but we must locate 0.975 (found by subtracting 0.025 from 1) across the top row because the values in the top row are always *areas to the right* of the critical value. Refer to Figure 7-9 and see that the total area to the right of $\chi^2_L = 10.283$ is 0.975. Figure 7-9 shows that, for a sample of 22 values taken from a normally distributed population, the chi-square statistic $(n - 1)s^2/\sigma^2$ has a 0.95 probability of falling between the chi-square critical values of 10.283 and 35.479.

Instead of using Table A-4, technology (such as STATDISK, Excel, Minitab, and StatCrunch) can be used to find critical values of χ^2. A major advantage of technology is that it can be used for any number of degrees of freedom and any confidence level, not just the limited choices included in Table A-4. Another major advantage of technology is that you can often generate the confidence interval without going through the steps of finding the critical values and manually calculating the confidence interval limits.

How Many People Do You Know?

Although the typical person cannot identify the number of people that he or she knows, methods of statistics can be used to estimate that number. The simple approach of simply asking someone how many people are known has been found to work poorly, due in large part to the same problems associated with voluntary response samples. Some other past approaches are based on the use of diaries and phonebooks. With one method, subjects are asked how many people they know in specific subpopulations. For example, subjects might be asked how many people they know who are named Adam, or how many people they know who have a twin brother or sister. Responses to such questions could be used to project the total number of people that are known. According to one estimate, the mean number of people known is 611, and the median is 472 (based on "How Many People Do You Know?: Efficiently Estimating Personal Network Size," by McCormick, Salganik, and Zheng, *Journal of the American Statistical Association,* Vol. 105, No. 489).