a. Find μ, the mean number of hurricanes per year.

b. If $P(x)$ is the probability of x Atlantic hurricanes in a randomly selected year, find $P(0)$, $P(2)$, and $P(9)$.

c. There were actually 2 years with no Atlantic hurricanes, 5 years with two Atlantic hurricanes, and 4 years with nine Atlantic hurricanes. How do these actual results compare to the probabilities found in part (b)? Does the Poisson distribution appear to be a good model in this case?

Solution

a. The Poisson distribution applies because we are dealing with the occurrences of an event (hurricanes) over some interval (a year). The mean number of hurricanes per year is

$$\mu = \frac{\text{number of hurricanes}}{\text{number of years}} = \frac{530}{100} = 5.3$$

b. Using Formula 5-9, the calculation for $x = 0$ hurricanes in a year is as follows (with μ replaced by 5.3 and e replaced by 2.71828):

$$P(0) = \frac{\mu^x \cdot e^{-\mu}}{x!} = \frac{5.3^0 \cdot 2.71828^{-5.3}}{0!} = \frac{1 \cdot 0.00499}{1} = 0.00499$$

The probability of exactly 0 hurricanes in a year is $P(0) = 0.00499$. We can use the same procedure to find that $P(2) = 0.0701$ and $P(9) = 0.0454$, as shown below.

$$P(2) = \frac{\mu^x \cdot e^{-\mu}}{x!} = \frac{5.3^2 \cdot 2.71828^{-5.3}}{2!} = \frac{28.09 \cdot 0.00499}{2} = 0.0701$$

$$P(9) = \frac{\mu^x \cdot e^{-\mu}}{x!} = \frac{5.3^9 \cdot 2.71828^{-5.3}}{9!} = \frac{3{,}299{,}763.592 \cdot 0.00499}{362{,}880} = 0.0454$$

c. The probability of $P(0) = 0.00499$ from part (b) is the likelihood of getting 0 Atlantic hurricanes in one year. So in 100 years, the expected number of years with 0 Atlantic hurricanes is $100 \times 0.00499 = 0.499$ years. The other expected values are included here:

Hurricanes in a Year	Actual Number of Years	Expected Number of Years (Using Poisson)
0	2	0.499
2	5	7.01
9	4	4.54

These expected frequencies don't differ dramatically from the actual frequencies, indicating that for these values of x, the Poisson distribution does a reasonably good job of describing the frequencies of Atlantic hurricanes. Instead of relying on a subjective judgment about the closeness of actual frequencies and expected frequencies, we can use more advanced methods to determine whether the Poisson distribution is a good model in this case. For example, we could use the method for testing "goodness-of-fit" in Section 11-2.