6-5 The Central Limit Theorem

Key Concept In Section 6-4 we saw that the sampling distribution of sample means tends to be a normal distribution as the sample size increases. In this section we use the sampling distribution of sample means as we introduce and apply the *central limit theorem* that allows us to use a normal distribution for some very meaningful applications.

> **Central Limit Theorem**
>
> For all samples of the same size n with $n > 30$, the sampling distribution of $\bar{x}$ can be approximated by a normal distribution with mean μ and standard deviation $\sigma/\sqrt{n}$.

According to the central limit theorem, the original population can have *any* distribution (uniform, skewed, and so on), but the distribution of sample means $\bar{x}$ can be approximated by a normal distribution when $n > 30$. (There are some special cases of very nonnormal distributions for which the requirement of $n > 30$ isn't quite enough, so the number 30 must be higher, but those cases are relatively rare.)

Example 1 Normal, Uniform, and U-Shaped Distributions

Table 6-7 illustrates the central limit theorem at work. The top dotplots in Table 6-7 show an approximately normal distribution, a uniform distribution, and a distribution with a shape resembling the letter *U*. In each column, the second dotplot shows the distribution of sample means for samples of size $n = 10$, and the bottom dotplot shows the distribution of sample means for samples of size $n = 50$. As we proceed down each column of Table 6-7, we can see that the distribution of sample means is approaching the shape of a normal distribution. That characteristic is included among the following observations that we can make from Table 6-7.

- As the sample size increases, the sampling distribution of sample means tends to approach a normal distribution.

Table 6-7 Sampling Distributions