High Cost of Low Quality

Solution

The centerline for the control chart is located by the value of $\bar{p}$:

$$\bar{p} = \frac{\text{total number of defective quarters from all samples combined}}{\text{total number of quarters sampled}}$$

$$= \frac{8 + 7 + 12 + \cdots + 24}{20 \cdot 10{,}000} = \frac{252}{200{,}000} = 0.00126$$

Because $\bar{p} = 0.00126$, it follows that $\bar{q} = 1 - \bar{p} = 0.99874$. Using $\bar{p} = 0.00126$, $\bar{q} = 0.99874$, and $n = 10{,}000$, we find the positions of the centerline and the control limits as follows:

Centerline: $\bar{p} = 0.00126$

Upper control limit:

$$\bar{p} + 3\sqrt{\frac{\bar{p}\bar{q}}{n}} = 0.00126 + 3\sqrt{\frac{(0.00126)(0.99874)}{10{,}000}} = 0.002324$$

Lower control limit:

$$\bar{p} - 3\sqrt{\frac{\bar{p}\bar{q}}{n}} = 0.00126 - 3\sqrt{\frac{(0.00126)(0.99874)}{10{,}000}} = 0.000196$$

Having found the values for the centerline and the control limits, we can proceed to plot the proportions of defective quarters. The Minitab control chart for p is shown in the accompanying display.

CONTROL CHART FOR p: DEFECTIVE QUARTERS
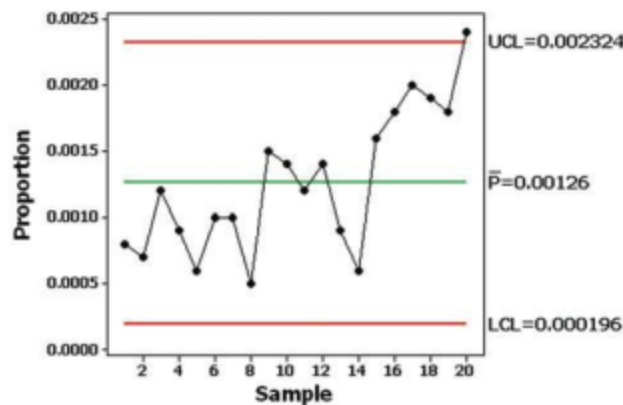
Interpretation

We can interpret the control chart for p by considering the three out-of-control criteria listed in Section 14-2. Using those criteria, we conclude that this process is out of statistical control for these reasons: The Run of 8 Rule is violated because there are 8 consecutive points lying below the centerline, and there is a point lying beyond the upper and lower control limits. This is based on a new process that is being tested, and the control chart indicates that this process requires corrective action so that it is brought within statistical control.