Solution

We use the following notation:

A = ELT manufactured by Altigauge
B = ELT manufactured by Bryant
C = ELT manufactured by Chartair
D = ELT is defective
$\overline{D}$ = ELT is not defective (or it is good)

a. If an ELT is randomly selected from the general population of all ELTs, the probability that it was made by Altigauge is 0.8 (because Altigauge manufactures 80% of them).

b. If we now have the additional information that the ELT was tested and was found to be defective, we want to revise the probability from part (a) so that the new information can be used. We want to find the value of $P(A|D)$, which is the probability that the ELT was made by the Altigauge company given that it is defective. Based on the given information, we know these probabilities:

$P(A) = 0.80$ because Altigauge makes 80% of the ELTs
$P(B) = 0.15$ because Bryant makes 15% of the ELTs
$P(C) = 0.05$ because Chartair makes 5% of the ELTs
$P(D|A) = 0.04$ because 4% of the Altigauge ELTs are defective
$P(D|B) = 0.06$ because 6% of the Bryant ELTs are defective
$P(D|C) = 0.09$ because 9% of the Chartair ELTs are defective

Here is Bayes' theorem extended to include three events corresponding to the selection of ELTs from the three manufacturers (A, B, C):

$$P(A|D) = \frac{P(A)\cdot P(D|A)}{[P(A)\cdot P(D|A)] + [P(B)\cdot P(D|B)] + [P(C)\cdot P(D|C)]}$$
$$= \frac{0.80\cdot 0.04}{[0.80\cdot 0.04] + [0.15\cdot 0.06] + [0.05\cdot 0.09]}$$
$$= 0.703 \text{ (rounded)}$$

Intuitive Bayes' Theorem Now let's find $P(A|D)$ by using a table. Let's arbitrarily assume that 10,000 ELTs were manufactured. (The solution doesn't depend on the number selected, but it's helpful to select a number large enough so that the cells in the table are all whole numbers.) Because 80% of the ELTs are made by Altigauge, we have 8000 ELTs made by Altigauge, and 4% of them (or 320) are defective. Also, if 320 of the Altigauge ELTs are defective, the other 7680 are not defective. See the values of 320 and 7680 in the table below. The other values are found using the same reasoning.

	D (Defective)	$\overline{D}$ (Not Defective)	Total
A (Altigauge)	320	7680	**8,000**
B (Bryant)	90	1410	**1,500**
C (Chartair)	45	455	**500**
Total	**455**	**9545**	**10,000**