winning by selecting Super Saver to win and Ice Box to finish second (as they did)? Do all of the different possible exacta bets have the same chance of winning?

Solution

We have $n = 20$ horses available, and we must select $r = 2$ of them without replacement. The number of different sequences of arrangements is found as shown:

$$_nP_r = \frac{n!}{(n-r)!} = \frac{20!}{(20-2)!} = 380$$

There are 380 different possible arrangements of 2 horses selected from the 20 that are available. If one of those arrangements is randomly selected, there is a probability of $1/380$ that the winning arrangement is selected. There are 380 different possible exacta bets, but not all of them have the same chance of winning, because some horses tend to be faster than others. (A correct \$2 exacta bet in this race won \$152.40.)

Example 4 Permutations Rule (with Some Identical Items): Designing Surveys

Here are two tricks that pollsters use when designing surveys: (1) Give different subjects rearrangements of the questions so that order does not have an effect; (2) as a check to see if a subject is thoughtlessly spewing answers just to finish the survey, repeat a question with some rewording and check to see if the answers are consistent. For one particular survey with 10 questions, 2 of the questions are the same, and 3 other questions are also identical. For this survey, how many different arrangements are possible? Is it practical to survey enough subjects so that every different possible arrangement is used?

Solution

We have 10 questions with 2 that are alike and 3 others that are alike, and we want the number of permutations. Using the rule for permutations with some items identical to others, we get

$$\frac{n!}{n_1!n_2!\cdots n_k!} = \frac{10!}{2!3!} = \frac{3{,}628{,}800}{2\cdot 6} = 302{,}400$$

Interpretation

There are 302,400 different possible arrangements of the 10 questions. For typical surveys, the number of subjects is around 1000. For the vast majority of typical surveys, it is not practical to use 302,400 subjects; that is far too many.

Example 5 Combinations Rule: Lottery

In the Pennsylvania Match 6 Lotto, winning the jackpot requires that you select six different numbers from 1 to 49, and the same six numbers must be drawn in the lottery. The winning numbers can be drawn in any order, so order does not make a difference. Find the probability of winning the jackpot when one ticket is purchased.

How to Choose Lottery Numbers

Many books and suppliers of computer programs claim to be helpful in predicting winning lottery numbers. Some use the theory that particular numbers are "due" (and should be selected) because they haven't been coming up often; others use the theory that some numbers are "cold" (and should be avoided) because they haven't been coming up often; and still others use astrology, numerology, or dreams. Because selections of winning lottery number combinations are independent events, such theories are worthless. A valid approach is to choose numbers that are "rare" in the sense that they are not selected by other people, so that if you win, you will not need to share your jackpot with many others. The combination of 1, 2, 3, 4, 5, 6 is a poor choice because many people tend to select it. In a Florida lottery with a \$105 million prize, 52,000 tickets had 1, 2, 3, 4, 5, 6; if that combination had won, the prize would have been only \$1000. It's wise to pick combinations not selected by many others. Avoid combinations that form a pattern on the entry card.