occur by chance, assuming that the vaccine has no effect. We have two possible explanations for the results of this clinical experiment: (1) The vaccine has no effect and the results occurred by chance; (2) the vaccine has an effect, which explains why the treatment group had a much lower incidence of polio. Because the probability is so small (less than 0.001), the second explanation is more reasonable. We conclude that the vaccine appears to be effective.

The preceding example illustrates the "rare event rule for inferential statistics" given in Section 4-1. Under the assumption of a vaccine with no effect, we find that the probability of the results is extremely small (less than 0.001), so we conclude that the assumption is probably not correct. The preceding example also illustrates the role of probability in making important conclusions about clinical experiments. For now, we should understand that when a probability is small, such as less than 0.001, it indicates that the event is very unlikely to occur.

The following list summarizes some key notation and principles discussed so far in this section.

Important Principles and Notation for Probability

- **The probability of an event is a fraction or decimal number between 0 and 1 inclusive.**
- **The probability of an impossible event is 0.**
- **The probability of an event that is certain to occur is 1.**
- **Notation: The probability of event A is denoted by $P(A)$.**
- **Notation: The probability that event A does *not* occur is denoted by $P(\overline{A})$.**

Part 2: Beyond the Basics of Probability: Odds

Expressions of likelihood are often given as *odds,* such as 50:1 (or "50 to 1"). Because the use of odds makes many calculations difficult, statisticians, mathematicians, and scientists prefer to use probabilities. The advantage of odds is that they make it easier to deal with money transfers associated with gambling, so they tend to be used in casinos, lotteries, and racetracks. Note that in the three definitions that follow, the *actual odds against* and the *actual odds in favor* are calculated with the actual likelihood of some event, but the *payoff odds* describe the relationship between the bet and the amount of the payoff. The actual odds correspond to actual probabilities of outcomes, but the payoff odds are set by racetrack and casino operators. Racetracks and casinos are in business to make a profit, so the payoff odds will not be the same as the actual odds.

DEFINITIONS

The **actual odds against** event A occurring are the ratio $P(\overline{A})/P(A)$, usually expressed in the form of a:b (or "a to b"), where a and b are integers having no common factors.

The **actual odds in favor** of event A occurring are the ratio $P(A)/P(\overline{A})$, which is the reciprocal of the actual odds against that event. If the odds against A are a:b, then the odds in favor of A are b:a.

The **payoff odds** against event A occurring are the ratio of net profit (if you win) to the amount bet:

$$\text{payoff odds against event } A = (\text{net profit}):(\text{amount bet})$$

Probability of an Event That Has Never Occurred

Some events are possible, but are so unlikely that they have never occurred. Here is one such problem of great interest to political scientists: Estimate the probability that your single vote will determine the winner in a U.S. Presidential election. Andrew Gelman, Gary King, and John Boscardin write in the *Journal of the American Statistical Association* (Vol. 93, No. 441) that "the exact value of this probability is of only minor interest, but the number has important implications for understanding the optimal allocation of campaign resources, whether states and voter groups receive their fair share of attention from prospective presidents, and how formal 'rational choice' models of voter behavior might be able to explain why people vote at all." The authors show how the probability value of 1 in 10 million is obtained for close elections.