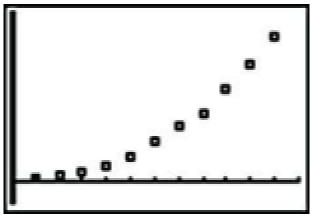
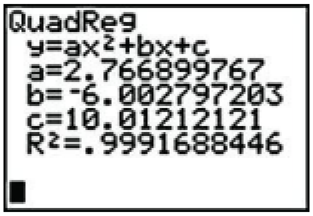
TI-83/84 PLUS





Solution

First, we "code" the year values by using 1, 2, 3, . . . , instead of 1800, 1820, 1840, ... The reason for this coding is to use values of x that are much smaller and much less likely to cause computational difficulties.

1. *Look for a pattern in the graph.* Examine the pattern of the data values in the TI-83/84 Plus display (shown in the margin), and compare that pattern to the generic models shown earlier in this section. The pattern of those points is clearly not a straight line, so we rule out a linear model. Good candidates for the model appear to be the quadratic, exponential, and power functions.

2. *Find and compare values of R^2.* The TI-83/84 display for the quadratic model is shown in the margin. For the quadratic model, $R^2 = 0.9992$ (rounded), which is quite high. Table 10-8 includes this result with results from two other potential models. Comparing the values of the coefficient R^2, it appears that the quadratic model is best because it has the highest value of 0.9992. If we select the quadratic function as the best model, we conclude that the equation $y = 2.77x^2 - 6.00x + 10.01$ best describes the relationship between the year x (coded with $x = 1$ representing 1800, $x = 2$ representing 1820, and so on) and the population y (in millions).

   Based on its R^2 value of 0.9992, the quadratic model appears to be best, but the other values of R^2 are also quite high. Our general knowledge of population growth might suggest that the exponential model is most appropriate. (With a constant birth rate and no limiting factors, population will grow exponentially.)

   Table 10-8 Models for the Population Data

Model	R^2	Equation
Quadratic	**0.9992**	$y = 2.77x^2 - 6.00x + 10.01$
Exponential	0.9631	$y = 5.24(1.48^x)$
Power	0.9764	$y = 3.35x^{1.77}$

   To predict the U.S. population for the year 2020, first note that the year 2020 is coded as $x = 12$ (see Table 10-7). Substituting $x = 12$ into the quadratic model of $y = 2.77x^2 - 6.00x + 10.01$ results in $y = 337$, which indicates that the U.S. population is estimated to be 337 million in the year 2020.

3. *Think.* The forecast result of 337 million in 2020 seems reasonable. (As of this writing, the latest figures from the U.S. Bureau of the Census use much more sophisticated methods to project that the U.S. population in 2020 will be 325 million.) However, there is considerable danger in making estimates for times that are beyond the scope of the available data. For example, the quadratic model suggests that in 1492, the U.S. population was 671 million, which is a result statisticians refer to as *ridiculous.* The quadratic model appears to be good for the available data (1800–2000), but other models might be better if it is absolutely necessary to make future population estimates.