Confidence Interval for Estimating a Population Proportion p

Objective

Construct a confidence interval used to estimate a population proportion p.

Notation

p = population proportion

$\hat{p}$ = sample proportion

n = number of sample values

E = margin of error

$z_{\alpha/2}$ = z score separating an area of $\alpha/2$ in the right tail of the standard normal distribution

(*Note:* The symbol π is sometimes used to denote the population proportion. Because π is so closely associated with the value of 3.14159…, this text uses p to denote the population proportion.)

Requirements

1. The sample is a simple random sample. (*Caution:* If the sample data have been obtained in a way that is not suitable, the estimate of the population proportion may be very wrong.)
2. The conditions for the binomial distribution are satisfied. That is, there is a fixed number of trials, the trials are independent, there are two categories of outcomes, and the probabilities remain constant for each trial. (See Section 5-3.)
3. There are at least 5 successes and at least 5 failures. (With the population proportions p and q unknown, we estimate their values using the sample proportion, so this requirement is a way of verifying that $np \geq 5$ and $nq \geq 5$ are both satisfied, so the normal distribution is a suitable approximation to the binomial distribution. There are procedures for dealing with situations in which the normal distribution is not a suitable approximation, as in Exercise 40.)

Confidence Interval

$$\hat{p} - E < p < \hat{p} + E \quad \text{where} \quad E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

The confidence interval is often expressed in the following equivalent formats:

$$\hat{p} \pm E$$

or

$$(\hat{p} - E, \hat{p} + E)$$

Round-Off Rule for Confidence Interval Estimates of p

Round the confidence interval limits for p to three significant digits.

Confidence intervals can be easily created by using technology or Table A-2 with the following procedure:

Procedure for Constructing a Confidence Interval for p

1. Verify that the requirements in the preceding box are satisfied.
2. Use technology or Table A-2 to find the critical value $z_{\alpha/2}$ that corresponds to the desired confidence level.