

c. For each of the nine different possible samples of two values selected with replacement, find the variance by treating each sample as if it is a population (using the formula for population variance, which includes division by  $n$ ), then find the mean of those nine population variances.

d. Which approach results in values that are better estimates of  $\sigma^2$ : part (b) or part (c)? Why? When computing variances of samples, should you use division by  $n$  or  $n - 1$ ?

e. The preceding parts show that  $s^2$  is an unbiased estimator of  $\sigma^2$ . Is  $s$  an unbiased estimator of  $\sigma$ ? Explain.

**46. Mean Absolute Deviation** Use the same population of {2 min, 3 min, 8 min} from Exercise 45. Show that when samples of size 2 are randomly selected with replacement, the samples have mean absolute deviations that do not center about the value of the mean absolute deviation of the population. What does this indicate about a sample mean absolute deviation being used as an estimator of the mean absolute deviation of a population?

### 3-4

## Measures of Relative Standing and Boxplots

**Key Concept** This section introduces measures of relative standing, which are numbers showing the location of data values relative to the other values within the same data set. The most important concept in this section is the  $z$  score, which will be used often in following chapters. We also discuss percentiles and quartiles, which are common statistics, as well as a new statistical graph called a boxplot.

### Part 1: Basics of $z$ Scores, Percentiles, Quartiles, and Boxplots

#### $z$ Scores

A  $z$  score (or standardized value) is found by converting a value to a standardized scale, as given in the following definition. This definition shows that a  $z$  score is the number of standard deviations that a data value is away from the mean. We will use  $z$  scores extensively in Chapter 6 and later chapters.

**DEFINITION** A  $z$  score (or **standardized value**) is the number of standard deviations that a given value  $x$  is above or below the mean. The  $z$  score is calculated by using one of the following:

Sample		Population
$z = \frac{x - \bar{x}}{s}$	or	$z = \frac{x - \mu}{\sigma}$

#### Round-Off Rule for $z$ Scores

Round  $z$  scores to two decimal places (such as 2.31).

This round-off rule is motivated by the format of standard tables in which  $z$  scores are expressed with two decimal places. See Table A-2 in Appendix A, which is a typical table of  $z$  scores, and notice that Table A-2 has  $z$  scores expressed with two decimal places. Example 1 illustrates how  $z$  scores can be used to compare values, even if they come from different populations.



#### Example 1 Comparing a Count and a Weight

Example 8 in Section 3-3 used the coefficient of variation to compare the variation among numbers of chocolate chips in cookies to the variation among weights of