We can proceed with the construction of the confidence interval by first finding the critical value $z_{\alpha/2}$. With a 95% confidence level, we have $\alpha = 0.5$, and we get $z_{\alpha/2} = 1.96$ (as shown in Example 2 from Section 7-2). Using $z_{\alpha/2} = 1.96$, $\sigma = 4.1$, and $n = 12$, we find the value of the margin of error E:

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$= 1.96 \cdot \frac{4.1}{\sqrt{12}} = 2.31979$$

Using the 12 sample values listed in Example 2, we find that $\bar{x} = 60.7$. With $\bar{x} = 60.7$ and $E = 2.31979$, we find the 95% confidence interval as follows:

$$\bar{x} - E < \mu < \bar{x} + E$$

$$60.7 - 2.31979 < \mu < 60.7 + 2.31979$$

$$58.4 < \mu < 63.0$$ (rounded to one decimal place more than the original sample values)

Remember, this example illustrates the situation in which the population standard deviation σ is known, which is rare. The more realistic situation with σ unknown is considered in Part 1 of this section.

Power of Small Samples

The Environmental Protection Agency (EPA) had discovered that Chryslers had malfunctioning carburetors, with the result that carbon monoxide emissions were too high. Chryslers with 360- and 400-cubic-inch displacements and two-barrel carburetors were involved. The EPA ordered Chrysler to fix the problem, but Chrysler refused, and the case of *Chrysler Corporation vs. The Environmental Protection Agency* followed. That case led to the conclusion that there was "substantial evidence" that the Chryslers produced excessive levels of carbon monoxide. The EPA won the case and Chrysler was forced to recall and repair 208,000 vehicles. In discussing this case in an article in *AMSTAT News*, Chief Statistician for the EPA Barry Nussbaum wrote this: "Sampling is expensive, and environmental sampling is usually quite expensive. At the EPA, we have to do the best we can with small samples or develop models. . . . What was the sample size required to affect such a recall (of the 208,000 Chryslers)? The answer is a mere 10. It is both an affirmation of the power of inferential statistics and a challenge to explain how such a (small) sample could possibly suffice."

Choosing the Appropriate Distribution

When constructing a confidence interval estimate of the population mean μ, it is important to use the correct distribution. Table 7-1 summarizes the key points to consider. Table 7-1 shows that when we have a small sample ($n < 30$) drawn from a distribution that differs dramatically from a normal distribution, we can't use the methods of this chapter. In such cases, we might use nonparametric methods (see Chapter 13) or bootstrap resampling methods. (The bootstrap method is described in the Technology Project at the end of this chapter.) Remember that in reality, σ is rarely known, so estimates of μ typically involve the Student t distribution, provided that its requirements are met.

Table 7-1 Choosing between Student t and z (Normal) Distributions

Conditions	Method
σ not known and normally distributed population *or* σ not known and $n > 30$	Use Student t distribution.
σ known and normally distributed population *or* σ known and $n > 30$ (In reality, σ is rarely known.)	Use normal (z) distribution.
Population is not normally distributed and $n \le 30$.	Use a nonparametric method or the bootstrapping method.