Solution

Requirement check (1) The sample data are not randomly selected from a larger population, but we treat them as a random sample for the purpose of determining whether they are typical results that might be obtained from such a random sample. (2) The sample data do consist of frequency counts. (3) Each expected frequency is at least 5, as will be shown later in this solution. All of the requirements are satisfied.

Step 1: The original claim is that the leading digits fit the distribution given as Benford's law. Using subscripts corresponding to the leading digits, we can express this claim as $p_1 = 0.301$ and $p_2 = 0.176$ and $p_3 = 0.125$ and . . . and $p_9 = 0.046$.

Step 2: If the original claim is false, then at least one of the proportions does not have the value as claimed.

Step 3: The null hypothesis must contain the condition of equality, so we have

H_0: $p_1 = 0.301$ and $p_2 = 0.176$ and $p_3 = 0.125$ and . . . and $p_9 = 0.046$.

H_1: At least one of the proportions is not equal to the given claimed value.

Step 4: The significance level is not specified, so we use the common choice of $\alpha = 0.05$.

Step 5: Because we are testing a claim that the distribution of leading digits fits the distribution given by Benford's law, we use the goodness-of-fit test described in this section. The χ^2 distribution is used with the test statistic given earlier.

Step 6: Table 11-5 shows the calculations of the components of the χ^2 test statistic for the leading digits of 1 and 2. If we include all nine leading digits, we get the test statistic of $\chi^2 = 5.958$, as shown in the accompanying TI-84 Plus calculator display. The critical value is $\chi^2 = 15.507$ (found in Table A-4 with $\alpha = 0.05$ in the right tail and degrees of freedom equal to $k - 1 = 8$). The TI-84 Plus calculator display shows the value of the test statistic as well as the P-value of 0.652. (The bottom row of the display shows the expected values, which can be viewed by scrolling to the right. CNTRB is an abbreviated form of "contribution," and the values are the individual contributions to the total value of the χ^2 test statistic.)

Table 11-5 Calculating the χ^2 Test Statistic for Leading Digits in Table 11-4

Leading Digit	Observed Frequency O	Expected Frequency $E = np$	$O - E$	$(O - E)^2$	$\frac{(O-E)^2}{E}$
1	33	$120 \cdot 0.301 = 36.12$	−3.12	9.7344	0.2695
2	22	$120 \cdot 0.176 = 21.12$	0.88	0.7744	0.0367

Step 7: The P-value of 0.652 is greater than the significance level of 0.05, so there is not sufficient evidence to reject the null hypothesis. (Also, the test statistic of $\chi^2 = 5.958$ does not fall in the critical region bounded by the critical value of 15.507, so there is not sufficient evidence to reject the null hypothesis.)

Step 8: There is not sufficient evidence to warrant rejection of the claim that the 120 counties have populations with leading digits that fit the distribution given by Benford's law.

Safest Seats in a Commercial Jet

A study by aviation writer and researcher David Noland showed that sitting farther back in a commercial jet will increase your chances of surviving in the event of a crash. The study suggests that the chance of surviving is not the same for each seat, so a goodness-of-fit test would lead to rejection of the null hypothesis that every seat has the same probability of surviving. Records from the 20 commercial jet crashes that occurred since 1971 were analyzed. It was found that if you sit in business or first class, you have a 49% chance of surviving a crash, if you sit in coach over the wing or ahead of the wing you have a 56% chance of surviving, and if you sit in the back behind the wing you have a 69% chance of surviving.

In commenting on this study, David Noland stated that he does not seek a rear seat when he flies. He says that because the chance of a crash is so small, he doesn't worry about where he sits, but he prefers a window seat.

TI-84 PLUS