b. Based on the additional given information, we have the following:

$P(M) = 0.51$ because 51% of the adults are males

$P(\overline{M}) = 0.49$ because 49% of the adults are females (not males)

$P(C|M) = 0.095$ because 9.5% of the males smoke cigars (That is, the probability of getting someone who smokes cigars, given that the person is a male, is 0.095.)

$P(C|\overline{M}) = 0.017$ because 1.7% of the females smoke cigars (That is, the probability of getting someone who smokes cigars, given that the person is a female, is 0.017.)

Let's now apply Bayes' theorem by using the preceding formula with M in place of A, and C in place of B. We get the following result:

$$\begin{aligned} P(M|C) &= \frac{P(M) \cdot P(C|M)}{[P(M) \cdot P(C|M)] + [P(\overline{M}) \cdot P(C|\overline{M})]} \\ &= \frac{0.51 \cdot 0.095}{[0.51 \cdot 0.095] + [0.49 \cdot 0.017]} \\ &= 0.85329341 \\ &= 0.853 \text{ (rounded)} \end{aligned}$$

Before we knew that the survey subject smoked a cigar, there is a 0.51 probability that the survey subject is male (because 51% of the adults in Orange County are males). However, after learning that the subject smoked a cigar, we revised the probability to 0.853. There is a 0.853 probability that the cigar-smoking respondent is a male. This makes sense, because the likelihood of a male increases dramatically with the additional information that the subject smokes cigars (because so many more males smoke cigars than females).

Intuitive Bayes' Theorem

The preceding solution illustrates the application of Bayes' theorem with its calculation using the formula. Unfortunately, that calculation is complicated enough to create an abundance of opportunities for errors and/or incorrect substitution of the involved probability values. Fortunately, here is another approach that is much more intuitive and easier:

Assume some convenient value for the total of all items involved, then construct a table of rows and columns with the individual cell frequencies based on the known probabilities.

For the preceding example, simply assume some value for the adult population of Orange County, such as 100,000, then use the given information to construct a table, such as the one shown below.

Finding the number of males who smoke cigars: If 51% of the 100,000 adults are males, then there are 51,000 males. If 9.5% of the males smoke cigars, then the number of cigar-smoking males is 9.5% of 51,000, or $0.095 \times 51{,}000 = 4845$. See the entry of 4845 in the table. The other males who do *not* smoke cigars must be $51{,}000 - 4845 = 46{,}155$. See the value of 46,155 in the table.

Finding the number of females who smoke cigars: Using similar reasoning, 49% of the 100,000 adults are females, so the number of females is 49,000. Given that 1.7% of the