Example 1 **Fundamental Counting Rule: Computer Design**

Computers are typically designed so that the most basic unit of information is a *bit* (or binary digit), which represents either a 0 or a 1. Letters, digits, and punctuation symbols are represented as a *byte,* which is a sequence of eight bits in a particular order. For example, the ASCII coding system represents the letter *A* as 01000001 and the number 7 is represented as 00110111. How many different characters are possible if they are all to be represented as bytes?

Solution

The byte is a sequence of eight numbers, and there are only two possible numbers (0 or 1) for each of them. By applying the fundamental counting rule, the number of different possible bytes is

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^8 = 256$$

Interpretation

There are 256 different characters (letters, digits, punctuations) that can be represented with different bytes. The author's keyboard has 47 keys for characters, and each of those keys is used for two different characters, so the byte system is more than adequate for such keyboards.

Example 2 **Factorial Rule: Chronological Order of Presidents**

A history pop quiz has one question in which students are asked to arrange the following presidents in chronological order: Hayes, Taft, Polk, Taylor, Grant, Pierce. If an unprepared student makes random guesses, what is the probability of selecting the correct chronological order?

Solution

The factorial rule tells us that six different items have 6! different possible rearrangements.

$$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$$

Interpretation

Because only one of the 720 possible arrangements is correct, the probability of getting the correct chronological order with random guessing is 1/720, or 0.00139. With such a low probability, it is highly unlikely that a student will get the correct answer with random guessing. (The correct chronological order can be found from Data Set 12 in Appendix B.)

Example 3 **Permutations Rule (with Different Items): Exacta Bet**

In horse racing, a bet on an *exacta* in a race is won by correctly selecting the horses that finish first and second, and you must select those two horses in the correct order. The 136th running of the Kentucky Derby had a field of 20 horses. If a bettor randomly selects two of those horses for an exacta bet, what is the probability of

Choosing Personal Security Codes

All of us use personal security codes for ATM machines, computer Internet accounts, and home security systems. The safety of such codes depends on the large number of different possibilities, but hackers now have sophisticated tools that can largely overcome that obstacle. Researchers found that by using variations of the user's first and last names along with 1800 other first names, they could identify 10% to 20% of the passwords on typical computer systems. When choosing a password, *do not* use a variation of any name, a word found in a dictionary, a password shorter than seven characters, telephone numbers, or social security numbers. Do include nonalphabetic characters, such as digits or punctuation marks.

Go Figure

43,252,003,274,489,856,000: Number of possible positions on a Rubik's cube.