

the formula and the table, it makes sense to use the table. However, given a choice that includes the formula, table, and technology, the use of technology is the way to go. Here is an effective and efficient strategy for choosing a method for finding binomial probabilities:

1. Use computer software or a T1-83/84 Plus calculator, if available.
2. If neither computer software nor a T1-83/84 Plus calculator is available, use Table A-1 (Binomial Probabilities) if possible.
3. If neither computer software nor the T1-83/84 Plus calculator is available and the probabilities can't be found using Table A-1, use the binomial probability formula.

Rationale for the Binomial Probability Formula

The binomial probability formula is the basis for all three methods presented in this section. Instead of accepting and using that formula blindly, let's see why it works.

In Example 2, we used the binomial probability formula to find the probability of getting exactly three adults who know Twitter when five adults are randomly selected. With $P(\text{knows Twitter}) = 0.85$, we can use the multiplication rule from Section 4-4 to find the probability that the first three adults know Twitter while the last two adults do not know Twitter. We get the following result:

$$\begin{aligned} P(3 \text{ adults know Twitter followed by 2 adults who do not know Twitter}) \\ &= 0.85 \cdot 0.85 \cdot 0.85 \cdot 0.15 \cdot 0.15 \\ &= 0.85^3 \cdot 0.15^2 \\ &= 0.0138 \end{aligned}$$

This result gives a probability of randomly selecting five adults and finding that the first three know Twitter and the last two do not. However, the probability of 0.0138 is not the probability of getting exactly three adults who know Twitter because it assumes a particular sequence for three people who know Twitter and two who do not. Other different sequences are possible.

In Section 4-6 we saw that with three subjects identical to each other (such as adults who know Twitter) and two other subjects identical to each other (such as adults who do not know Twitter), the total number of arrangements, or permutations, is $5! / [(5 - 3)!3!]$ or 10. Each of those 10 different arrangements has a probability of $0.85^3 \cdot 0.15^2$, so the total probability is as follows:

$$P(3 \text{ adults know Twitter among 5}) = \frac{5!}{(5 - 3)!3!} \cdot 0.85^3 \cdot 0.15^2 = 0.138$$

This particular result can be generalized as the binomial probability formula (Formula 5-5). That is, the binomial probability formula is a combination of the multiplication rule of probability and the counting rule for the number of arrangements of n items when x of them are identical to each other and the other $n - x$ are identical to each other. (See Exercises 13 and 14.)

The number of outcomes with exactly x successes among n trials The probability of x successes among n trials for any one particular order

$$P(x) = \frac{n!}{(n - x)!x!} \cdot p^x \cdot q^{n-x}$$