populations have the same mean, why not simply pair them off and test two at a time by testing H_0: $\mu_1 = \mu_2$, H_0: $\mu_2 = \mu_3$, and H_0: $\mu_1 = \mu_3$? For the data in Table 12-1, the approach of testing equality of two means at a time requires three different hypothesis tests. If we use a 0.05 significance level for each of those three hypothesis tests, the actual overall confidence level could be as low as 0.95^3 (or 0.857). In general, as we increase the number of individual tests of significance, we increase the risk of finding a difference by chance alone (instead of a real difference in the means). The risk of a type I error—finding a difference in one of the pairs when no such difference actually exists—is far too high. The method of analysis of variance helps us avoid that particular pitfall (rejecting a true null hypothesis) by using *one test* for equality of several means, instead of several tests that each compare two means at a time.

CAUTION When testing for equality of three or more populations, use analysis of variance. (Using multiple hypothesis tests with two samples at a time could wreak havoc with the significance level.)

Part 2: Calculations and Identifying Means That Are Different

Calculations with Equal Sample Sizes *n*

Table 12-2 can be very helpful in understanding the methods of ANOVA. In Table 12-2, compare Data Set A to Data Set B to see that Data Set A is the same as Data Set B with this notable exception: the Sample 1 values each differ by 10. If the data sets all have the same sample size (as in $n = 4$ for Table 12-2), the following calculations aren't too difficult, as shown on the next page.

Table 12-2 Effect of a Mean on the *F* Test Statistic

	A			B (add 10 to A Sample 1)		
	Sample 1	Sample 2	Sample 3	Sample 1	Sample 2	Sample 3
	7	6	4	17	6	4
	3	5	7	13	5	7
	6	5	6	16	5	6
	6	8	7	16	8	7
	$n_1 = 4$	$n_2 = 4$	$n_3 = 4$	$n_1 = 4$	$n_2 = 4$	$n_3 = 4$
	$\bar{x}_1 = 5.5$	$\bar{x}_2 = 6.0$	$\bar{x}_3 = 6.0$	$\bar{x}_1 = 15.5$	$\bar{x}_2 = 6.0$	$\bar{x}_3 = 6.0$
	$s_1^2 = 3.0$	$s_2^2 = 2.0$	$s_3^2 = 2.0$	$s_1^2 = 3.0$	$s_2^2 = 2.0$	$s_3^2 = 2.0$
Variance between samples	$ns_{\bar{x}}^2 = 4(0.0833) = 0.3332$			$ns_{\bar{x}}^2 = 4(30.0833) = 120.3332$		
Variance within samples	$s_p^2 = \dfrac{3.0 + 2.0 + 2.0}{3} = 2.3333$			$s_p^2 = \dfrac{3.0 + 2.0 + 2.0}{3} = 2.3333$		
F test statistic	$F = \dfrac{ns_{\bar{x}}^2}{s_p^2} = \dfrac{0.3332}{2.3333} = 0.1428$			$F = \dfrac{ns_{\bar{x}}^2}{s_p^2} = \dfrac{120.3332}{2.3333} = 51.5721$		
P-value (found from Excel)	*P*-value = 0.8688			*P*-value = 0.0000118		