it suggests that all jet engines always work successfully. We need more precision to correctly reflect the true mean, such as the precision in the number 3.999714.

Expected Value

The mean of a discrete random variable x is the theoretical mean outcome for infinitely many trials. We can think of that mean as the *expected value* in the sense that it is the average value that we would expect to get if the trials could continue indefinitely. The uses of expected value (also called *expectation,* or *mathematical expectation*) are extensive and varied, and they play an important role in *decision theory*. (See Example 8 in Part 2 of this section.)

DEFINITION The **expected value** of a discrete random variable x is denoted by E, and it is the mean value of the outcomes, so $E = \mu$ and E can also be found by evaluating $\Sigma[x \cdot P(x)]$, as in Formula 5-1.

CAUTION An expected value need not be a whole number, even if the different possible values of x might all be whole numbers. We say that the expected number of girls in five births is 2.5, even though five specific births can never result in 2.5 girls. If we were to survey many couples with five children, we *expect* that the mean number of girls will be 2.5.

Example 5 Finding the Mean, Variance, and Standard Deviation

Table 5-1 describes the probability distribution for the number of girls in two births. Find the mean, variance, and standard deviation for the probability distribution described in Table 5-1 from Example 1.

Solution

In Table 5-4, the two columns at the left describe the probability distribution given earlier in Table 5-1; we create the three columns at the right for the purposes of the calculations required.

Using Formulas 5-1 and 5-2 and the table results, we get

Mean: $\mu = \Sigma[x \cdot P(x)] = 1.0$

Variance: $\sigma^2 = \Sigma[(x - \mu)^2 \cdot P(x)] = 0.5$

The standard deviation is the square root of the variance, so

Standard deviation: $\sigma = \sqrt{0.5} = 0.707107 = 0.7$ (rounded)

Table 5-4 Calculating μ and σ for a Probability Distribution

x	$P(x)$	$x \cdot P(x)$	$(x - \mu)^2 \cdot P(x)$
0	0.25	$0 \cdot 0.25 = 0.00$	$(0 - 1)^2 \cdot 0.25 = 0.25$
1	0.50	$1 \cdot 0.50 = 0.50$	$(1 - 1)^2 \cdot 0.50 = 0.00$
2	0.25	$2 \cdot 0.25 = 0.50$	$(2 - 1)^2 \cdot 0.25 = 0.25$
Total		1.00 ↑ $\mu = \Sigma[x \cdot P(x)]$	0.50 ↑ $\sigma^2 = \Sigma[(x - \mu)^2 \cdot P(x)]$