

- Evaluate the margin of error $E = z_{\alpha/2} \sqrt{\hat{p}\hat{q}/n}$.
- Using the value of the calculated margin of error E and the value of the sample proportion \hat{p} , find the values of the *confidence interval limits* $\hat{p} - E$ and $\hat{p} + E$. Substitute those values in the general format for the confidence interval:

$$\hat{p} - E < p < \hat{p} + E$$

or

$$\hat{p} \pm E$$

or

$$(\hat{p} - E, \hat{p} + E)$$

- Round the resulting confidence interval limits to three significant digits.

Example 3 Constructing a Confidence Interval: Poll Results

In the Chapter Problem we noted that a Pew Research Center poll of 1007 randomly selected U.S. adults showed that 85% of the respondents know what Twitter is. The sample results are $n = 1007$ and $\hat{p} = 0.85$.

- Find the margin of error E that corresponds to a 95% confidence level.
- Find the 95% confidence interval estimate of the population proportion p .
- Based on the results, can we safely conclude that more than 75% of adults know what Twitter is?
- Assuming that you are a newspaper reporter, write a brief statement that accurately describes the results and includes all of the relevant information.

Solution

Requirement check We first verify that the necessary requirements are satisfied. (1) The polling methods used by the Pew Research Center result in samples that can be considered to be simple random samples. (2) The conditions for a binomial experiment are satisfied, because there is a fixed number of trials (1007), the trials are independent (because the response from one person doesn't affect the probability of the response from another person), there are two categories of outcome (subject knows what Twitter is or does not), and the probability remains constant. (3) With 85% of the respondents knowing what Twitter is, the number who know is 856 (or 85% of 1007). If 856 of the 1007 subjects know what Twitter is, the other 151 do not know, so the number of successes (856) and the number of failures (151) are both at least 5. The check of requirements has been successfully completed. ✓

Technology The confidence interval and margin of error can be easily found using technology. From the STATDISK display on the next page we can see the required entries on the left and the results displayed on the right. Like most technologies, STATDISK requires a value for the number of successes, so we simply find 85% of 1007 and round the result of 855.95 to the whole number of 856. The results show that the margin of error is $E = 0.022$ (rounded) and the confidence interval is $0.828 < p < 0.872$ (rounded). (The Wilson Score confidence interval is discussed near the end of this section.)