Rationale for the Confidence Interval If we obtain simple random samples of size n from a normally distributed population with variance σ^2, there is a probability of $1 - \alpha$ that the statistic $(n - 1)s^2/\sigma^2$ will fall between the critical values of χ_L^2 and χ_R^2. (In Figure 7-9, the confidence level of 95% corresponds to $\alpha = 0.05$, and there is a 0.95 probability that the χ^2 test statistic falls between χ_L^2 and χ_R^2.) It follows that there is a $1 - \alpha$ probability that both of the following are true:

$$\frac{(n-1)s^2}{\sigma^2} < \chi_R^2 \quad \text{and} \quad \frac{(n-1)s^2}{\sigma^2} > \chi_L^2$$

If we multiply both of the preceding inequalities by σ^2 and divide each inequality by the appropriate critical value of χ^2, the two preceding inequalities can be expressed in these equivalent forms:

$$\frac{(n-1)s^2}{\chi_R^2} < \sigma^2 \quad \text{and} \quad \frac{(n-1)s^2}{\chi_L^2} > \sigma^2$$

The two preceding inequalities can be combined into one inequality to get the format of the confidence interval used in this section:

$$\frac{(n-1)s^2}{\chi_R^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_L^2}$$

Determining Sample Size The procedures for finding the sample size necessary to estimate σ^2 are much more complex than the procedures given earlier for means and proportions. Instead of using very complicated procedures, we will use Table 7-2.

Table 7-2 Finding Sample Size

σ		σ^2	
To be 95% confident that s is within	**of the value of σ, the sample size n should be at least**	**To be 95% confident that s^2 is within**	**of the value of σ^2, the sample size n should be at least**
1%	19,205	1%	77,208
5%	768	5%	3,149
10%	192	10%	806
20%	48	20%	211
30%	21	30%	98
40%	12	40%	57
50%	8	50%	38
To be 99% confident that s is within	**of the value of σ, the sample size n should be at least**	**To be 99% confident that s^2 is within**	**of the value of σ^2 the sample size n should be at least**
1%	33,218	1%	133,449
5%	1,336	5%	5,458
10%	336	10%	1,402
20%	85	20%	369
30%	38	30%	172
40%	22	40%	101
50%	14	50%	68