Part 2: Test of Homogeneity, Fisher Exact Test, and McNemar's Test for Matched Pairs

Test of Homogeneity

In Part 1 of this section, we focused on the test of *independence* between the row and column variables in a contingency table. In Part 1, the sample data are from one population, and individual sample results are categorized with the row and column variables. In a *chi-square test of homogeneity,* we have samples randomly selected from different populations, and we want to determine whether those populations have the same proportions of some characteristic being considered. (The word *homogeneous* means "having the same quality," and in this context, we are testing to determine whether the proportions are the same.) Section 9-2 presented a procedure for testing a claim about *two* populations with categorical data having two possible outcomes, but a chi-square test of homogeneity allows us to use two or more populations with outcomes from several categories.

DEFINITION A **chi-square test of homogeneity** is a test of the claim that *different populations* have the same proportions of some characteristics.

Sampling Plan In a typical test of independence as described in Part 1 of this section, sample subjects are randomly selected from one population (such as people treated for stress fractures in a foot bone) and values of two different variables are observed (such as success/failure for people receiving different treatments). In a typical chi-square test of homogeneity, subjects are randomly selected from the different populations separately.

Procedure In conducting a test of homogeneity, we can use the same notation, requirements, test statistic, critical value, and procedures given in Part 1 of this section, with this exception: Instead of testing the null hypothesis of independence between the row and column variables, we test the null hypothesis that *the different populations have the same proportions of some characteristics*.

Example 5 Do the Four Treatment Populations Have the Same Success Rate?

Example 3 used the sample results from Table 11-6 to test for *independence* between treatment and success. If we want to use the same data from Table 11-6 in a test of the null hypothesis that the four populations corresponding to the four different treatment groups have the same proportion of success, we could use the chi-square test of homogeneity. The test statistic, critical value, and *P*-value are the same as those found in Example 3. Based on those values, we should reject the null hypothesis that the four treatment populations have the same success rate.

Fisher Exact Test

The procedures for testing hypotheses with contingency tables have the requirement that every cell must have an expected frequency of at least 5. This requirement is necessary for the χ^2 distribution to be a suitable approximation to the exact distribution of the χ^2 test statistic. The *Fisher exact test* is often used for a 2×2 contingency table with one or more expected frequencies that are below 5. The Fisher exact test provides an *exact P*-value and does not require an approximation technique. Because the calculations are quite complex, it's a good idea to use technology when using the Fisher exact test. STATDISK, Minitab, XLSTAT, and StatCrunch all have the ability to perform the Fisher exact test.