Confidence Interval Estimate of $\mu_1 - \mu_2$: Independent Samples

The confidence interval estimate of the difference $\mu_1 - \mu_2$ is

$$(\bar{x}_1 - \bar{x}_2) - E < (\mu_1 - \mu_2) < (\bar{x}_1 - \bar{x}_2) + E$$

where

$$E = t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

and the number of degrees of freedom df is as described above for hypothesis tests. (In this book, we use df = smaller of $n_1 - 1$ and $n_2 - 1$.)

CAUTION Before conducting a hypothesis test, consider the context of the data, the source of the data, and the sampling method, and explore the data with graphs and descriptive statistics. Be sure to verify that the requirements are satisfied.

Equivalent Methods

The *P*-value method of hypothesis testing, the critical value method of hypothesis testing, and confidence intervals all use the same distribution and standard error, so they are all equivalent in the sense that they result in the same conclusions.

Example 2 Are We More Creative with Blue?

In the Chapter Problem, we noted that researchers from the University of British Columbia conducted trials to investigate the effects of color on creativity. Subjects with a red background were asked to think of creative uses for a brick; other subjects with a blue background were given the same task. Responses were scored by a panel of judges and results from scores of creativity are given below. The researchers make the claim that "blue enhances performance on a creative task." Test that claim using a 0.01 significance level.

Creativity Scores

Red Background:	$n = 35, \bar{x} = 3.39, s = 0.97$
Blue Background:	$n = 36, \bar{x} = 3.97, s = 0.63$

Solution

Requirement check (1) The values of the two population standard deviations are not known and we are not making an assumption that they are equal. (2) The two samples are independent because different subjects were used for the two different color groups. (3) The samples are simple random samples because subjects were randomly assigned to each of the two different color groups. (4) Both samples are large (greater than 30), so we satisfy the requirement that "the two sample sizes are both *large* (with $n_1 > 30$ and $n_2 > 30$) or both samples come from populations having normal distributions." The requirements are all satisfied. ✔

Technology Computer programs and calculators usually provide a *P*-value, so the *P*-value method is used. See the accompanying TI-83/84 Plus calculator results showing the alternative hypothesis of $\mu_1 < \mu_2$ (from the claim that the second

TI-83/84 PLUS

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2-SampTTest
 μ1<μ2
 t=-2.978951111
 p=.0021077236
 df=58.11153827
 x̄1=3.39
↓x̄2=3.97
```