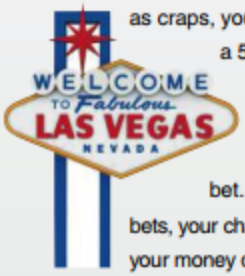
To Win, Bet Boldly

The New York Times published an article by Andrew Pollack in which he reported lower than expected earnings for the Mirage casino in Las Vegas. He wrote that "winnings for Mirage can be particularly volatile, because it caters to high rollers, gamblers who might bet $100,000 or more on a hand of cards. The law of averages does not work as consistently for a few large bets as it does for thousands of smaller ones . . ." This reflects the most fundamental principle of gambling: To win, place one big bet instead of many small bets! With the right game, such as craps, you have just under a 50% chance of doubling your money if you place one big bet. With many small bets, your chance of doubling your money drops substantially.



Example 5 Airbus 310: *Redundancy* for Better Safety

Modern aircraft are now highly reliable, and one design feature contributing to that reliability is the use of *redundancy*, whereby critical components are duplicated so that if one fails, the other will work. For example, the Airbus 310 twin-engine airliner has three independent hydraulic systems, so if any one system fails, full flight control is maintained with another functioning system. For this example, we will assume that for a typical flight, the probability of a hydraulic system failure is 0.002.

a. If the Airbus 310 were to have one hydraulic system, what is the probability that it would work for a flight?

b. Given that the Airbus 310 actually has three independent hydraulic systems, what is the probability that on a typical flight, control can be maintained with a working hydraulic system?

Solution

a. The probability of a hydraulic system failure is 0.002, so the probability that it does *not* fail is 0.998. That is, the probability that flight control can be maintained is as follows:

$$P(\text{1 hydraulic system } \textit{does not fail}) = 1 - P(\text{failure}) = 1 - 0.002 = 0.998$$

b. With three independent hydraulic systems, flight control will be maintained provided that the three systems do not all fail. The probability of all three hydraulic systems failing is $0.002 \cdot 0.002 \cdot 0.002 = 0.000000008$. It follows that the probability of maintaining flight control is as follows:

$$P(\text{it does } \textit{not} \text{ happen that all three hydraulic systems fail}) = 1 - 0.000000008 = 0.999999992$$

Interpretation

With only one hydraulic system we have a 0.002 probability of failure, but with three independent hydraulic systems, there is only a 0.000000008 probability that flight control cannot be maintained because all three systems failed. By using three hydraulic systems instead of only one, risk of failure is decreased not by a factor of 1/3, but by a factor of 1/250,000. By using three independent hydraulic systems, risk is dramatically decreased and safety is dramatically increased.

Rationale for the Multiplication Rule

To see the reasoning that underlies the multiplication rule, consider a pop quiz consisting of (1) a true/false question and (2) a multiple-choice question with five possible answers (a, b, c, d, e). We will use the following two questions:

1. True or false: A pound of feathers is heavier than a pound of gold.
2. Who said that "By a small sample, we may judge of the whole piece"?
   (a) Judge Judy; (b) Judge Dredd; (c) Miguel de Cervantes;
   (d) George Gallup; (e) Gandhi

The answer key is T (for "true") and c. (The first answer is true, because weights of feathers are given in avoirdupois units, but weights of gold and other precious metals