is made *without* replacement of the first subject, the second probability must take into account the fact that the first selection removed a subject who tested positive, so only 49 subjects are available for the second selection, and 6 of them had a negative test result.

In Example 2, we consider three situations: (a) The items are selected *with* replacement; (b) the items are selected *without* replacement, but the required calculations are not too cumbersome; (c) the items are selected *without* replacement, but the required calculations are cumbersome and the "5% guideline for cumbersome calculations" can be used.

Convicted by Probability

A witness described a Los Angeles robber as a Caucasian woman with blond hair in a ponytail who escaped in a yellow car driven by an African-American male with a mustache and beard. Janet and Malcolm Collins fit this description, and they were convicted based on testimony that there is only about 1 chance in 12 million that any couple would have these characteristics. It was estimated that the probability of a yellow car is 1/10, and the other probabilities were estimated to be 1/10, 1/3, 1/10, and 1/1000. The convictions were later overturned when it was noted that no evidence was presented to support the estimated probabilities or the independence of the events. However, because the couple was not randomly selected, a serious error was made in not considering the probability of *other* couples being in the same region with the same characteristics.

Example 2 Airport Baggage Scales

Airport baggage scales can show that bags are overweight and high additional fees can be imposed. The New York City Department of Consumer Affairs checked all 810 scales at JFK and LaGuardia, and 102 scales were found to be defective and ordered out of use (based on data reported in *The New York Times*).

a. If 2 of the 810 scales are randomly selected *with replacement*, find the probability that they are both defective.

b. If 2 of the 810 scales are randomly selected *without replacement*, find the probability that they are both defective.

c. A larger population of 10,000 scales includes exactly 1259 defective scales. If 5 scales are randomly selected from this larger population without replacement, find the probability that all 5 are defective.

Solution

a. *With Replacement:* If the 2 scales are randomly selected *with replacement*, the two selections are independent because the second event is not affected by the first outcome. In each of the two selections there are 102 defective scales among the 810 scales available, so we get

$$\begin{aligned} P(\text{both scales are defective}) &= P(\text{first is defective } \textit{and} \text{ second is defective}) \\ &= P(\text{first is defective}) \cdot P(\text{second is defective}) \\ &= \frac{102}{810} \cdot \frac{102}{810} = 0.0159 \end{aligned}$$

b. *Without Replacement:* If the 2 scales are randomly selected *without replacement*, the two selections are dependent because the probability of the second event is affected by the first outcome. Being careful to adjust the second probability to reflect the result of a defect on the first selection, we get

$$\begin{aligned} P(\text{both scales are defective}) &= P(\text{first is defective } \textit{and} \text{ second is defective}) \\ &= P(\text{first is defective}) \cdot P(\text{second is defective}) \\ &= \frac{102}{810} \cdot \frac{101}{809} = 0.0157 \end{aligned}$$

c. With 1259 defective scales among 10,000, the exact calculation for getting 5 defective scales when 5 scales are randomly selected without replacement is cumbersome, as shown here:

$$\frac{1259}{10{,}000} \cdot \frac{1258}{9999} \cdot \frac{1257}{9998} \cdot \frac{1256}{9997} \cdot \frac{1255}{9996} = 0.0000314 \text{ (yuck!)}$$