cigar, we estimate the probability of this person being a male as 0.85. Denoting a male by M and denoting a cigar smoker by C, we can express this result as follows: $P(M|C) = 0.85$.

c. In part (a), the value of 0.5 is the initial probability, so we refer to it as the prior probability. Because the probability of 0.85 in part (b) is a revised probability based on the additional information that the survey subject was smoking a cigar, this value of 0.85 is referred to a posterior probability.

The Reverend Thomas Bayes [1701 (approximately)–1761] was an English minister and mathematician. Although none of his work was published during his lifetime, later (posterior?) publications included the following theorem (or rule) that he developed for determining probabilities of events by incorporating information about subsequent events.

Bayes' Theorem

The probability of event A, given that event B has subsequently occurred, is

$$P(A|B) = \frac{P(A) \cdot P(B|A)}{[P(A) \cdot P(B|A)] + [P(\overline{A}) \cdot P(B|\overline{A})]}$$

That's a formidable expression, but we will simplify its calculation. See the following example, which illustrates use of the expression above, but also see the alternative method based on a more intuitive application of Bayes' theorem.

Example 2

In Orange County, 51% of the adults are males. (It doesn't take too much advanced mathematics to deduce that the other 49% are females.) One adult is randomly selected for a survey involving credit card usage.

a. Find the prior probability that the selected person is a male.

b. It is later learned that the selected survey subject was smoking a cigar. Also, 9.5% of males smoke cigars, whereas 1.7% of females smoke cigars (based on data from the Substance Abuse and Mental Health Services Administration). Use this additional information to find the probability that the selected subject is a male.

Solution

Let's use the following notation:

$M =$ male $\qquad \overline{M} =$ female (or not male)

$C =$ cigar smoker $\qquad \overline{C} =$ not a cigar smoker

a. Before using the information given in part (b), we know only that 51% of the adults in Orange County are males, so the probability of randomly selecting an adult and getting a male is given by $P(M) = 0.51$.