Poisson distributions are used for describing behavior such as radioactive decay, arrivals of people in a line, eagles nesting in a region, patients arriving at an emergency room, crashes on the Massachusetts Turnpike, and Internet users logging onto a web site. For example, suppose your local hospital experiences a mean of 2.3 patients arriving at the emergency room on Fridays between 10:00 P.M. and 11:00 P.M. We can use a Poisson distribution to find the probability that for a randomly selected Friday, exactly four patients arrive at the ER between 10:00 P.M. and 11:00 P.M.

DEFINITION A **Poisson distribution** is a discrete probability distribution that applies to occurrences of some event *over a specified interval.* The random variable x is the number of occurrences of the event in an interval. The interval can be time, distance, area, volume, or some similar unit. The probability of the event occurring x times over an interval is given by Formula 5-9.

Formula 5-9 Poisson Probability Distribution

$$P(x) = \frac{\mu^x \cdot e^{-\mu}}{x!}$$

where $e \approx 2.71828$

μ = mean number of occurrences of the event over the intervals

Requirements for the Poisson Distribution

1. The random variable x is the number of occurrences of an event *over some interval.*
2. The occurrences must be *random.*
3. The occurrences must be *independent* of each other.
4. The occurrences must be *uniformly distributed* over the interval being used.

Parameters of the Poisson Distribution

- The mean is μ.
- The standard deviation is $\sigma = \sqrt{\mu}$.

A Poisson distribution differs from a binomial distribution in these fundamental ways:

1. A particular binomial distribution is determined by the sample size n and the probability p, but a Poisson distribution is determined only by the mean μ.
2. In a binomial distribution, the possible values of the random variable x are 0, 1, . . . , n, but a Poisson distribution has possible x values of 0, 1, 2, . . . , with no upper limit.

Example 1 Hurricanes

For a recent period of 100 years, there were 530 Atlantic hurricanes (based on data from the University of Maryland Department of Geography and Environmental Systems). Assume that the Poisson distribution is a suitable model.

continued