

When testing a claimed value of a median for a population of individual values, we create matched pairs by pairing each sample value with the claimed median, so the same procedure is used for both of the applications above.

### Claims Involving Matched Pairs

The sign test (Section 13-2) can be used with matched pairs, but the sign test uses only the signs of the differences. By using ranks instead of signs, the Wilcoxon signed-ranks test takes the magnitudes of the differences into account, so it includes and uses more information than the sign test and therefore tends to yield conclusions that better reflect the true nature of the data.

## Wilcoxon Signed-Ranks Test

Objective: Use the Wilcoxon signed-ranks test for the following tests:

- **Matched Pairs:** Test the claim that a population of matched pairs has the property that the matched pairs have differences with a median equal to zero.
- **One Population of Individual Values:** Test the claim that a population has a median equal to some claimed value. (By pairing each sample value with the claimed median, we again work with matched pairs.)

### Notation

$T$  = the smaller of the following two sums:

1. The sum of the positive ranks of the nonzero differences  $d$
2. The absolute value of the sum of the negative ranks of the nonzero differences  $d$

(Details for evaluating  $T$  are given in the procedure following this box.)

### Requirements

1. The data are a simple random sample.
2. The population of differences has a distribution that is approximately *symmetric*, meaning that the left half of its histogram is roughly a mirror image of its right half. (For a sample of matched pairs, obtain differences by subtracting the second value from the first

value in each pair; for a sample of individual values, obtain differences by subtracting the value of the claimed median from each sample value.)

*Note:* There is *no* requirement that the data have a normal distribution.

### Test Statistic

If  $n \leq 30$ , the test statistic is  $T$ .

$$\text{If } n > 30, \text{ the test statistic is } z = \frac{T - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}$$

### P-Values

$P$ -values are often provided by technology or  $P$ -values can be found using the  $z$  test statistic and Table A-2.

### Critical Values

1. If  $n \leq 30$ , the critical  $T$  value is found in Table A-8.
2. If  $n > 30$ , the critical  $z$  values are found in Table A-2.