- The mean of all sample means is equal to the mean of the original population.
- As the sample size increases, the spans of the dotplots become narrower, showing that the standard deviations of sample means become smaller as the sample size increases.

The following key points form the foundation for estimating population parameters and hypothesis testing—topics discussed at length in the following chapters.

The Central Limit Theorem and the Sampling Distribution of $\bar{x}$

Given

1. The original population has mean μ and standard deviation σ.
2. Simple random samples of the same size n are selected from the population.

Practical Rules for Real Applications Involving a Sample Mean $\bar{x}$

Case 1: Original population *is* normally distributed.

For *any* sample size n: The distribution of $\bar{x}$ is a normal distribution with these parameters:

Mean of all values of $\bar{x}$: $\mu_{\bar{x}} = \mu$

Standard deviation of all values of $\bar{x}$: $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$

z score conversion of $\bar{x}$: $z = \dfrac{\bar{x} - \mu}{\dfrac{\sigma}{\sqrt{n}}}$

Case 2: Original population is *not* normally distributed.

For $n > 30$: The distribution of $\bar{x}$ is approximately a normal distribution with these parameters:

Mean of all values of $\bar{x}$: $\mu_{\bar{x}} = \mu$

Standard deviation of all values of $\bar{x}$: $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$

z score conversion of $\bar{x}$: $z = \dfrac{\bar{x} - \mu}{\dfrac{\sigma}{\sqrt{n}}}$

Distribution of Original Population	Distribution of Sample Means
Normal	Normal (for *any* sample size n)
Not normal and $n > 30$	Normal (approximately)
Not normal and $n \le 30$	*Not normal*

For $n \le 30$: The distribution of $\bar{x}$ cannot be approximated well by a normal distribution and the methods of this section do not apply. Use other methods, such as nonparametric methods or bootstrapping methods. (See the Technology Project for Chapter 7.)

Considerations for Practical Problem Solving

1. **Check Requirements:** When working with the mean from a sample, verify that the normal distribution can be used by confirming that the original population has a normal distribution or $n > 30$.
2. **Individual Value or Mean from a Sample?** Determine whether you are using a normal distribution with a single value x or the mean $\bar{x}$ from a sample of n values. See the following.

- **Individual value:** When working with an *individual* value from a normally distributed population, use the methods of Section 6-3 with $z = \frac{x - \mu}{\sigma}$.
- **Mean from a sample of values:** When working with a mean for some *sample* of n values, be sure to use the value of $\sigma / \sqrt{n}$ for the standard deviation of the sample means, so use $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$.