

Example 1 Effect of Lead on IQ Score

Table 13-6 lists performance IQ scores from samples of subjects with low lead exposure, medium lead exposure, and high lead exposure (from Data Set 5 in Appendix B). Use a 0.05 significance level to test the claim that the three samples come from populations with medians that are all equal.

Solution

Requirement check (1) Each of the three samples is a simple random independent sample. (2) Each sample size is at least 5. The requirements are satisfied. ✓

The null and alternative hypotheses are as follows:

H_0 : The median performance IQ score is the same for the population of subjects with low lead exposure, the population with medium lead exposure, and the population with high lead exposure.

H_1 : The three populations have median performance IQ scores that are not all the same.

Test Statistic In determining the value of the H test statistic, we first rank all of the data, then we find the sum of the ranks for each category. In Table 13-6, ranks are shown in parentheses next to the original sample values. Next we find the sample size (n) and sum of ranks (R) for each sample. Those values are shown at the bottom of Table 13-6. Because the total number of observations is 19, we have $N = 19$. We can now evaluate the test statistic as follows:

$$\begin{aligned} H &= \frac{12}{N(N+1)} \left(\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \cdots + \frac{R_k^2}{n_k} \right) - 3(N+1) \\ &= \frac{12}{19(19+1)} \left(\frac{86^2}{8} + \frac{50.5^2}{6} + \frac{53.5^2}{5} \right) - 3(19+1) \\ &= 0.694 \end{aligned}$$

Critical Value Because each sample has at least five observations, the distribution of H is approximately a chi-square distribution with $k - 1$ degrees of freedom. The number of samples is $k = 3$, so we have $3 - 1 = 2$ degrees of freedom. Refer to Table A-4 to find the critical value of 5.991, which corresponds to 2 degrees of freedom and a 0.05 significance level (with an area of 0.05 in the right tail). Figure 13-4 on the next page shows the test statistic of $H = 0.694$ does not fall within the critical region bounded by 5.991, so we fail to reject the null hypothesis of equal population medians.

Figure 13-4 shows the test statistic of $H = 0.694$ and the critical value of 5.991. (The chi-square distribution has the general shape shown in Figure 13-4 whenever the number of degrees of freedom is 1 or 2.) The test statistic does not fall in the critical region, so we fail to reject the null hypothesis of equal medians.

Interpretation

There is not sufficient evidence to reject the claim that performance IQ scores from subjects with low lead exposure, medium lead exposure, and high lead exposure all have the same median. The population medians do not appear to be different.