

## Requirements

1. There are two independent simple random samples.
2. Each of the two samples has more than 10 values. (For samples with 10 or fewer values, special tables are available in reference books, such as *CRC Standard*

*Probability and Statistics Tables and Formulae*, published by CRC Press.)

*Note:* There is *no* requirement that the two populations have a normal distribution or any other particular distribution.

## Test Statistic

$$z = \frac{R - \mu_R}{\sigma_R}$$

where

$$\mu_R = \frac{n_1(n_1 + n_2 + 1)}{2}$$

$n_1$  = size of the sample from which the rank sum  $R$  is found

$$\sigma_R = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}$$

$n_2$  = size of the other sample

$R$  = sum of ranks of the sample with size  $n_1$

## P-Values

P-values can be found from technology or by using the  $z$  test statistic and Table A-2.

## Critical Values

Critical values can be found in Table A-2 (because the test statistic is based on the normal distribution).

**Table 13-5** Pulse Rates  
(Ranks in parentheses)

| Males         | Females       |
|---------------|---------------|
| 60 (4.5)      | 78 (14)       |
| 74 (11)       | 80 (17)       |
| 86 (19)       | 68 (9)        |
| 54 (1)        | 56 (2.5)      |
| 90 (20.5)     | 76 (12)       |
| 80 (17)       | 78 (14)       |
| 66 (7)        | 78 (14)       |
| 68 (9)        | 90 (20.5)     |
| 68 (9)        | 96 (22)       |
| 56 (2.5)      | 60 (4.5)      |
| 80 (17)       | 98 (23)       |
| 62 (6)        |               |
| $n_1 = 12$    | $n_2 = 11$    |
| $R_1 = 123.5$ | $R_2 = 152.5$ |

**Procedure for Finding the Value of the Test Statistic** To see how the following steps are applied, refer to the sample data listed in Table 13-5.

**Step 1:** Temporarily combine the two samples into one big sample, then replace each sample value with its rank. (The lowest value gets a rank of 1, the next lowest value gets a rank of 2, and so on. If values are tied, assign to them the mean of the ranks involved in the tie. See Section 13-1 for a description of ranks and the procedure for handling ties.)

*EXAMPLE:* In Table 13-5, the ranks of the 23 sample pulse rates are shown in parentheses. The rank of 1 is assigned to the lowest value of 54. The next lowest values are 56 and 56; because they are tied for the ranks of 2 and 3, we assign the rank of 2.5 to each of them.

**Step 2:** Find the sum of the ranks for either one of the two samples.

*EXAMPLE:* In Table 13-5, the sum of the ranks from the first sample is 123.5. (That is,  $4.5 + 11 + 19 + \cdots + 6 = 123.5$ .)

**Step 3:** Calculate the value of the  $z$  test statistic as shown in the preceding box, where either sample can be used as "Sample 1." (If both sample sizes are greater than 10, then the sampling distribution of  $R$  is approximately normal with mean  $\mu_R$  and standard deviation  $\sigma_R$ , and the test statistic is as shown in the preceding box.)

*EXAMPLE:* Calculations of  $\mu_R$  and  $\sigma_R$  and  $z$  are shown in Example 1, which follows.