Good Experimental Design

Suppose we want to conduct an experiment to compare the effectiveness of two different types of fertilizer (one organic and one chemical). The fertilizers are to be used on 20 plots of land with equal area but varying soil quality. To make a fair comparison, we should divide each of the 20 plots in half so that one half is treated with organic fertilizer and the other half is treated with chemical fertilizer. The yields can then be matched by the plots they share, resulting in dependent data. The advantage of using matched pairs is that we reduce extraneous variation, which could occur if each plot were treated with one type of fertilizer rather than both—that is, if the samples were independent. This strategy for designing an experiment can be generalized by the following design principle:

> **When designing an experiment or planning an observational study, using dependent samples with paired data is generally better than using two independent samples.**

t Test There are no exact procedures for dealing with dependent samples, but the following approximation methods are commonly used.

Objectives

1. Use the differences from two dependent samples (matched pairs) to test a claim about the mean of the population of all such differences.
2. Use the differences from two dependent samples (matched pairs) to construct a confidence interval estimate of the mean of the population of all such differences.

Notation for Dependent Samples

d = individual difference between the two values in a single matched pair

μ_d = mean value of the differences d for the *population* of all matched pairs of data

$\bar{d}$ = mean value of the differences d for the paired *sample* data

s_d = standard deviation of the differences d for the paired *sample* data

n = number of *pairs* of sample data

Requirements

1. The sample data are dependent (matched pairs).
2. The samples are simple random samples.
3. Either or both of these conditions is satisfied: The number of pairs of sample data is large ($n > 30$) or the pairs of values have differences that are from a population having a distribution that is approximately normal. (These methods are robust against departures for normality, so for small samples, the normality requirement is loose in the sense that the procedures perform well as long as there are no outliers and departures from normality are not too extreme.)

Hypothesis Test Statistic for Dependent Samples

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} \quad \text{(For degrees of freedom use df} = n - 1.)$$

***P*-values:** *P*-values are automatically provided by technology. If technology is not available, refer to the t distribution in Table A-3. Use the procedure summarized in Figure 8-4 from Section 8-2.

Critical values: Use Table A-3 (t distribution) with degrees of freedom found by df $= n - 1$.