4-2 Basic Concepts of Probability

Key Concept Although this section presents three different approaches to finding the *probability* of an event, the most important objective of this section is to learn how to *interpret* probability values, which are expressed as values between 0 and 1. We should know that a small probability, such as 0.001, corresponds to an event that rarely occurs. In Part 2 of this section we discuss expressions of *odds* and how probability is used to determine the odds of an event occurring. The concepts related to odds are not needed for topics that follow, but odds are often used in some everyday situations, especially those that involve lotteries and gambling.

Part 1: Basics of Probability

In considering probability, we deal with procedures (such as answering a multiple-choice test question or undergoing a test for drug use) that produce outcomes.

DEFINITIONS

An **event** is any collection of results or outcomes of a procedure.

A **simple event** is an outcome or an event that cannot be further broken down into simpler components.

The **sample space** for a procedure consists of all possible *simple* events. That is, the sample space consists of all outcomes that cannot be broken down any further.

Example 1 illustrates the concepts defined above.

Example 1

In the following display, we use "b" to denote a baby boy and "g" to denote a baby girl.

Procedure	Example of Event	Sample Space (List of Simple Events)
Single birth	1 girl (simple event)	{b, g}
3 births	2 boys and 1 girl (bbg, bgb, and gbb are all simple events resulting in 2 boys and 1 girl)	{bbb, bbg, bgb, bgg, gbb, gbg, ggb, ggg}

With one birth, the result of 1 female is a *simple event* because it cannot be broken down any further. With three births, the event of "2 girls and 1 boy" is *not a simple event* because it can be broken down into simpler events, such as ggb, gbg, or bgg. With three births, the *sample space* consists of the eight simple events listed above. With three births, the outcome of ggb is considered a simple event, because it is an outcome that cannot be broken down any further. We might incorrectly think that ggb can be further broken down into the individual results of g, g, and b, but g, g, and b are not individual outcomes from three births. With three births, there are exactly eight outcomes that are simple events: bbb, bbg, bgb, bgg, gbb, gbg, ggb, and ggg.

We first list some basic notation, then we present three different approaches to finding the probability of an event.

Notation for Probabilities

P denotes a probability.

A, B, and C denote specific events.

$P(A)$ denotes the probability of event A occurring.

Probabilities That Challenge Intuition

In certain cases, our subjective estimates of probability values are dramatically different from the actual probabilities. Here is a classical example: If you take a deep breath, there is better than a 99% chance that you will inhale a molecule that was exhaled in dying Caesar's last breath. In that same morbid and unintuitive spirit, if Socrates' fatal cup of hemlock was mostly water, then the next glass of water you drink will likely contain one of those same molecules. Here's another less morbid example that can be verified: In classes of 25 students, there is better than a 50% chance that at least 2 students will share the same birthday (day and month).