sequence has 5 B's, so $n_1 = 5$. The sequence has 5 A's, so $n_2 = 5$. There are 2 runs, so $G = 2$.

Because $n_1 \leq 20$ and $n_2 \leq 20$ and the significance level is $\alpha = 0.05$, the test statistic is $G = 2$ (the number of runs), and we refer to Table A-10 to find the critical values of 2 and 10. Because $G = 2$ is less than or equal to the critical value of 2 (or it is greater than or equal to 10), we *reject the null hypothesis of randomness.* Because all of the values below the median occur in the beginning of the sequence and all values above the median occur at the end, it appears that there is an upward trend in the Dow Jones Industrial Average.

Example 3 Large Sample: Randomness of Study Subjects

Data Set 3 in Appendix B lists data from 107 study subjects. Let's consider the sequence of the listed genders indicated below. (The complete list of 107 genders can be seen in Data Set 3.) Use a 0.05 significance level to test the claim that the sequence is random.

M M M M M M M M M M M M M M M M M M F F . . . M M M M M

Solution

Requirement check (1) The data are arranged in order. (2) Each data value is categorized into one of two separate categories (male/female). The requirements are satisfied. ✓

The null and alternative hypotheses are as follows:

H_0: The sequence is random.

H_1: The sequence is not random.

Examination of the sequence of 107 genders results in these values:

$$n_1 = \text{number of males} = 92$$
$$n_2 = \text{number of females} = 15$$
$$G = \text{number of runs} = 25$$

Since $n_1 > 20$, we need to calculate the test statistic z. We must first evaluate μ_G and σ_G as follows:

$$\mu_G = \frac{2n_1n_2}{n_1 + n_2} + 1 = \frac{2(92)(15)}{92 + 15} + 1 = 26.7944$$

$$\sigma_G = \sqrt{\frac{(2n_1n_2)(2n_1n_2 - n_1 - n_2)}{(n_1 + n_2)^2(n_1 + n_2 - 1)}}$$

$$= \sqrt{\frac{(2)(92)(15)[2(92)(15) - 92 - 15]}{(92 + 15)^2(92 + 15 - 1)}} = 2.45633$$

We now find the test statistic:

$$z = \frac{G - \mu_G}{\sigma_G} = \frac{25 - 26.7944}{2.45633} = -0.73$$

Because the significance level is $\alpha = 0.05$ and we have a two-tailed test, the critical values are $z = -1.96$ and $z = 1.96$. The test statistic of $z = -0.73$ does not fall within the critical region, so we fail to reject the null hypothesis of randomness. The given sequence appears to be random.