Testing Claims About σ or σ^2

Objective

Conduct a hypothesis test of a claim made about a population standard deviation σ or population variance σ^2.

Notation

$n =$ sample size

$s =$ *sample* standard deviation

$s^2 =$ *sample* variance

$\sigma =$ *claimed* value of the *population* standard deviation

$\sigma^2 =$ claimed value of the *population* variance

Requirements

1. The sample is a simple random sample.

2. The population has a normal distribution. (Instead of being a loose requirement, this test has a fairly strict requirement of a normal distribution.)

Test Statistic for Testing a Claim About σ or σ^2

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$ (round to three decimal places, as in Table A-4)

***P*-values:** Use technology or use Table A-4 with degrees of freedom given by df $= n - 1$. (Table A-4 is based on *cumulative areas from the right*.)

Critical values: Use Table A-4 with degrees of freedom given by df $= n - 1$. (Table A-4 is based on *cumulative areas from the right*.)

CAUTION The χ^2 (chi-square) test of this section is not *robust* against a departure from normality, meaning that the test does not work well if the population has a distribution that is far from normal. The condition of a normally distributed population is therefore a much stricter requirement when testing claims about σ or σ^2 than tests of claims about a population mean μ (Section 8-4).

The chi-square distribution was introduced in Section 7-4, where we noted the following important properties.

Properties of the Chi-Square Distribution

1. All values of χ^2 are nonnegative, and the distribution is not symmetric (see Figure 8-9).

2. There is a different χ^2 distribution for each number of degrees of freedom (see Figure 8-10).

3. The critical values are found in Table A-4 using

$$\textbf{degrees of freedom} = n - 1$$

If using Table A-4 for finding critical values, note the following design feature of that table:

In Table A-4, each critical value of χ^2 in the body of the table corresponds to an area given in the top row of the table, and each area in that top row is a *cumulative area to the right* of the critical value.