Hypothesis Test for Correlation (Using Test Statistic r)

Notation

n = number of pairs of sample data

r = linear correlation coefficient for a *sample* of paired data

ρ = linear correlation coefficient for a *population* of paired data

Requirements

The requirements are the same as those given in the preceding box from Part 1 of this section.

Hypotheses

$$H_0: \rho = 0 \quad \text{(There is no linear correlation.)}$$
$$H_1: \rho \neq 0 \quad \text{(There is a linear correlation.)}$$

Test Statistic: r

Critical values: Refer to Table A-6.

Conclusion

Consider critical values from Table A-6 as being both positive and negative, and draw a graph similar to Figure 10-3.

- **Correlation** If the computed linear correlation coefficient r lies in the left tail beyond the leftmost critical value or if it lies in the right tail beyond the rightmost critical value, reject H_0 and conclude that there is sufficient evidence to support the claim of a linear correlation.
- **No Correlation** If the computed linear correlation coefficient lies *between* the two critical values, fail to reject H_0 and conclude that there is not sufficient evidence to support the claim of a linear correlation.

The following criteria are equivalent to those given above.

- **Correlation** If $|r| >$ critical value from Table A-6, reject H_0 and conclude that there is sufficient evidence to support the claim of a linear correlation.
- **No Correlation** If $|r| \leq$ critical value, fail to reject H_0 and conclude that there is not sufficient evidence to support the claim of a linear correlation.

Example 7 Hypothesis Test Based on r

Use the paired shoe print lengths and heights in Table 10-1 to conduct a formal hypothesis test of the claim that there is a linear correlation between the two variables. Use a 0.05 significance level.

Solution

Requirement check The solution in Example 1 already includes verification that the requirements are satisfied. ✓

To claim that there is a linear correlation is to claim that the population linear correlation coefficient ρ is different from 0. We therefore have the following hypotheses:

$$H_0: \rho = 0 \quad \text{(There is no linear correlation.)}$$
$$H_1: \rho \neq 0 \quad \text{(There is a linear correlation.)}$$

The test statistic is $r = 0.591$ (from Examples 1, 2, and 3). The critical values of $r = \pm 0.878$ are found in Table A-6 with $n = 5$ and $\alpha = 0.05$. See Example 4