



Figure 7-3 Finding $z_{\alpha/2}$ for a 95% Confidence Level

Example 2 showed that a 95% confidence level results in a critical value of $z_{\alpha/2} = 1.96$. This is the most common critical value, and it is listed with two other common values in the table that follows.

| Confidence Level | α | Critical Value, $z_{\alpha/2}$ |
|------------------|----------|--------------------------------|
| 90% | 0.10 | 1.645 |
| 95% | 0.05 | 1.96 |
| 99% | 0.01 | 2.575 |

Margin of Error

When we collect sample data that result in a sample proportion, such as the Pew Research Center poll given in Example 1, we can identify the sample proportion \hat{p} . Because of random variation in samples, the sample proportion \hat{p} is typically different from the population proportion p . The difference between the sample proportion and the population proportion can be thought of as an error. We now define the *margin of error* E as follows.

DEFINITION When data from a simple random sample are used to estimate a population proportion p , the **margin of error**, denoted by E , is the maximum likely difference (with probability $1 - \alpha$, such as 0.95) between the observed sample proportion \hat{p} and the true value of the population proportion p . The margin of error E is also called the *maximum error of the estimate* and can be found by multiplying the critical value and the standard deviation of sample proportions, as shown in Formula 7-1.

Formula 7-1

$$E = z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}} \quad \text{margin of error for proportions}$$

For a 95% confidence level, $\alpha = 0.05$, so there is a probability of 0.05 that the sample proportion will be in error by more than E . This property is generalized in the following box.