

4-8 Bayes' Theorem

The concept of *conditional probability* is introduced in *Elementary Statistics*. We noted that the conditional probability of an event is a probability obtained with the additional information that some other event has already occurred. We used $P(B|A)$ to denote the conditional probability of event B occurring, given that event A has already occurred. The following formula was provided for finding $P(B|A)$:

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

In addition to the above formal rule, the textbook also included this “intuitive approach for finding a conditional probability”:

The conditional probability of B given A can be found by assuming that event A has occurred and, working under that assumption, calculating the probability that event B will occur.

In this section we extend the discussion of conditional probability to include applications of *Bayes' theorem* (or *Bayes' rule*), which we use for revising a probability value based on additional information that is later obtained. One key to understanding the essence of Bayes' theorem is to recognize that we are dealing with *sequential* events, whereby new additional information is obtained for a subsequent event, and that new information is used to revise the probability of the initial event. In this context, the terms *prior probability* and *posterior probability* are commonly used.

DEFINITIONS

A **prior probability** is an initial probability value originally obtained before any additional information is obtained.

A **posterior probability** is a probability value that has been revised by using additional information that is later obtained.

Example 1

The Gallup Organization randomly selects an adult American for a survey about credit card usage. Use subjective probabilities to estimate the following.

- What is the probability that the selected subject is a male?
- After selecting a subject, it is later learned that this person was smoking a cigar during the interview. What is the probability that the selected subject is a male?
- Which of the preceding two results is a prior probability? Which is a posterior probability?

Solution

- Roughly half of all Americans are males, so we estimate the probability of selecting a male subject to be 0.5. Denoting a male by M , we can express this probability as follows: $P(M) = 0.5$.
- Although some women smoke cigars, the vast majority of cigar smokers are males. A reasonable guess is that 85% of cigar smokers are males. Based on this additional subsequent information that the survey respondent was smoking a