Types of Hypothesis Tests: Two-Tailed, Left-Tailed, Right-Tailed

The test statistic alone usually does not give us enough information to make a decision about the claim being tested. For that decision, we can use either the *P*-value approach summarized in Figure 8-1 or the critical value approach summarized in Figure 8-2. Both approaches require that we first determine whether our hypothesis test is two-tailed, left-tailed, or right-tailed.

The **critical region** (or **rejection region**) corresponds to the values of the test statistic that cause us to reject the null hypothesis. Depending on the claim being tested, the critical region could be in the two extreme tails, it could be in the left tail, or it could be in the right tail.

- **Two-tailed test:** The critical region is in the two extreme regions (tails) under the curve (as in the top graph in Figure 8-3).
- **Left-tailed test:** The critical region is in the extreme left region (tail) under the curve (as in the middle graph in Figure 8-3).
- **Right-tailed test:** The critical region is in the extreme right region (tail) under the curve (as in the bottom graph in Figure 8-3).

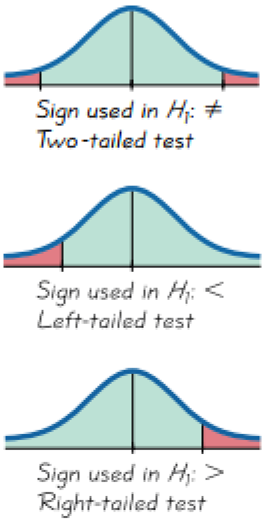

Figure 8-3 Critical Region in Two-Tailed, Left-Tailed, and Right-Tailed Tests

HINT To determine whether a test is two-tailed, left-tailed, or right-tailed, look at the alternative hypothesis and identify the region that supports that alternative hypothesis and conflicts with the null hypothesis. A useful check is summarized in Figure 8-3. *See that the inequality sign in H_1 points in the direction of the critical region.* The symbol $\neq$ is sometimes expressed in programming languages as <>, and this reminds us that an alternative hypothesis such as $p \neq 0.5$ corresponds to a two-tailed test.

Example: With H_0: $p = 0.5$ and H_1: $p > 0.5$, we reject the null hypothesis and support the alternative hypothesis only if the sample proportion is greater than 0.5 by a significant amount, so the hypothesis test in this case is *right-tailed.*

Interpreting the Test Statistic: Using the *P*-Value or Critical Value

After determining whether the hypothesis test is two-tailed, left-tailed, or right-tailed, we can proceed with either the *P*-value approach (summarized in Figure 8-1) or the critical value approach (summarized in Figure 8-2). Because technology typically provides a *P*-value in a hypothesis test, the *P*-value method is now much more common than the method based on critical values.

_P_-Value (or _p_-Value or Probability Value) Method Find the *P*-value, which is the probability of getting a value of the test statistic that is *at least as extreme* as the one representing the sample data, assuming that the null hypothesis is true. To find the *P*-value, first find the area beyond the test statistic, then use the procedure given in Figure 8-4. That procedure can be summarized as follows:

Critical region in the left tail: *P*-value = area to the *left* of the test statistic
Critical region in the right tail: *P*-value = area to the *right* of the test statistic
Critical region in two tails: *P*-value = *twice* the area in the tail beyond the test statistic

EXAMPLE The test statistic of $z = 1.60$ has an area of 0.0548 to its right, so a right-tailed test with test statistic $z = 1.60$ has a *P*-value of 0.0548. (See the different technology