Independent Jet Engines

Soon after departing from Miami, Eastern Airlines Flight 855 had one engine shut down because of a low oil pressure warning light.

As the L-1011 jet turned to Miami for landing, the low pressure warning lights for the other two engines also flashed. Then an engine failed, followed by the failure of the last working engine. The jet descended without power from 13,000 ft to 4000 ft when the crew was able to restart one engine, and the 172 people on board landed safely. With independent jet engines, the probability of all three failing is only 0.0001^3, or about one chance in a trillion. The FAA found that the same mechanic who replaced the oil in all three engines failed to replace the oil plug sealing rings. The use of a single mechanic caused the operation of the engines to become dependent, a situation corrected by requiring that the engines be serviced by different mechanics.

Treating Dependent Events as Independent: 5% Guideline for Cumbersome Calculations

When calculations with sampling are very cumbersome and the sample size is no more than 5% of the size of the population., treat the selections as being *independent* (even if they are actually dependent).

Example 1 illustrates the basic multiplication rule, with independent events in part (a) and dependent events in part (b). Example 2 is another illustration of the multiplication rule, and part (c) of Example 2 illustrates use of the above 5% guideline for cumbersome calculations.

Example 1 Drug Screening

Let's use only the 50 test results from the subjects who use drugs (from Table 4-1), as shown below:

Positive Test Results:	44
Negative Test Results:	6
Total Results:	50

a. If 2 of the 50 subjects are randomly selected *with replacement*, find the probability that the first selected person had a positive test result and the second selected person had a negative test result.

b. Repeat part (a) by assuming that the two subjects are selected *without* replacement.

Solution

a. *With Replacement:* First selection (with 44 positive results among 50 total results):

$$P(\text{positive test result}) = \frac{44}{50}$$

Second selection (with 6 negative test results among the same 50 total results):

$$P(\text{negative test result}) = \frac{6}{50}$$

We now apply the multiplication rule as follows:

$$P(\text{1st selection is positive and 2nd is negative}) = \frac{44}{50} \cdot \frac{6}{50} = 0.106$$

b. *Without Replacement:* Without replacement of the first subject, the calculations are the same as in part (a), except that the second probability must be adjusted to reflect the fact that the first selection was positive and is not available for the second selection. After the first positive result is selected, we have 49 test results remaining, and 6 of them are negative. The second probability is therefore 6/49, as shown in the calculation below:

$$P(\text{1st selection is positive and 2nd is negative}) = \frac{44}{50} \cdot \frac{6}{49} = 0.108$$

The key point of part (b) in Example 1 is this: *We must adjust the probability of the second event to reflect the outcome of the first event.* Because selection of the second subject