CAUTION Know that in this chapter, when we use a confidence interval to address a claim about a population proportion p, we simply make an *informal judgment* (that may or may not be consistent with formal methods of hypothesis testing introduced in Chapter 8).

Critical Values

The methods of this section (and many of the other statistical methods found in the following chapters) include reference to a standard z score that can be used to distinguish between sample statistics that are likely to occur and those that are unlikely. Such a z score is called a *critical value*. (Critical values were first presented in Section 6-2, and they are formally defined below.) Critical values are based on the following observations:

1. Under certain conditions, the sampling distribution of sample proportions can be approximated by a normal distribution, as shown in Figure 7-2.
2. A z score associated with a sample proportion has a probability of $\alpha/2$ of falling in the right tail of Figure 7-2.
3. The z score separating the right-tail region is commonly denoted by $z_{\alpha/2}$ and is referred to as a *critical value* because it is on the borderline separating z scores from sample proportions that are likely to occur from those that are unlikely.

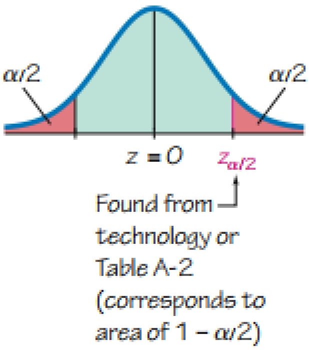

Figure 7-2
Critical Value $z_{\alpha/2}$ in the Standard Normal Distribution

DEFINITION A **critical value** is the number on the borderline separating sample statistics that are likely to occur from those that are unlikely. The number $z_{\alpha/2}$ is a critical value that is a z score with the property that it separates an area of $\alpha/2$ in the right tail of the standard normal distribution (as in Figure 7-2).

Example 2 Finding a Critical Value

Find the critical value $z_{\alpha/2}$ corresponding to a 95% confidence level.

Solution

A 95% confidence level corresponds to $\alpha/2 = 0.025$. Figure 7-3 shows that the area in each of the red-shaded tails is $\alpha/2 = 0.025$. We find $z_{\alpha/2} = 1.96$ by noting that the cumulative area to its left must be $1 - 0.025$, or 0.975. We can use technology or refer to Table A-2 to find that the cumulative left area of 0.9750 (found *in the body* of the table) corresponds to $z = 1.96$. For a 95% confidence level, the critical value is therefore $z_{\alpha/2} = 1.96$. To find the critical z score for a 95% confidence level, use a cumulative left area of 0.9750 (*not* 0.95).

Note: Many technologies can be used to find critical values. STATDISK, Excel, Minitab, StatCrunch, and the TI-83/84 Plus calculator all provide critical values for the normal distribution.