Coincidences?

John Adams and Thomas Jefferson (the second and third presidents) both died on July 4, 1826. President Lincoln was assassinated in Ford's Theater; President Kennedy was assassinated in a Lincoln car made by the Ford Motor Company. Lincoln and Kennedy were both succeeded by vice presidents named Johnson. Fourteen years *before* the sinking of the *Titanic*, a novel described the sinking of the *Titan*, a ship that hit an iceberg; see Martin Gardner's *The Wreck of the Titanic Foretold*? Gardner states, "In most cases of startling coincidences, it is impossible to make even a rough estimate of their probability."

Table 4-1 Pre-Employment Drug Screening Results*

	Positive Test Result (Drug Use Is Indicated)	**Negative Test Result** (Drug Use Is Not Indicated)
Subject Uses Drugs	**44** (True Positive)	**6** (False Negative)
Subject Is Not a Drug User	**90** (False Positive)	860 (True Negative)

**Numbers in bold correspond to positive test results or subjects who use drugs, and the total of those numbers is 140.*

Example 1 Pre-Employment Drug Screening

Refer to Table 4-1. If 1 subject is randomly selected from the 1000 subjects given a drug test, find the probability of selecting a subject who had a positive test result or uses drugs. This probability is denoted by P(positive test result or subject uses drugs).

Solution

Refer to Table 4-1 and carefully count the number of subjects who tested positive (first column) or use drugs (first row), but be careful to count subjects once, not twice. *When adding the frequencies from the first column and the first row, include the frequency of 44 only once.* In Table 4-1, there are 140 subjects who had positive test results or use drugs. We get this result:

$$P(\text{positive test result or subject uses drugs}) = 140/1000 = 0.140$$

In Example 1, there are several ways to count the subjects who tested positive or use drugs. Any of the following would work:

- Color the cells representing subjects who tested positive or use drugs; then add the numbers in those colored cells, being careful to add each number only once. This approach yields

$$44 + 90 + 6 = 140$$

- Add the 134 subjects who tested positive to the 50 subjects who use drugs, but the total of 184 involves double counting of 44 subjects, so compensate for the double counting by subtracting the overlap consisting of the 44 subjects who were counted twice. This approach yields a result of

$$134 + 50 - 44 = 140$$

- Start with the total of 134 subjects who tested positive, then add those subjects who use drugs and were not yet included in that total to get a result of

$$134 + 6 = 140$$

Example 1 illustrates that when finding the probability of an event A or event B, use of the word *or* suggests addition, and the addition must be done without double counting.

The preceding example suggests a general rule whereby we add the number of outcomes corresponding to each of the events in question: