Solution

Requirement check (1) The two populations are independent of each other. The two samples are not matched in any way. (2) Given that the design for the study was carefully planned, we assume that the two samples can be treated as simple random samples. (3) The journal article describing the study did not discuss the distributions of the data, so we proceed by assuming that the samples are from normally distributed populations, but we note that the validity of the results depends on that assumption.

TI-83/84 PLUS

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2-SampFTest
 σ1≠σ2
 F=2.370622323
 p=.0129276814
 Sx1=.97
 Sx2=.63
↓n1=35
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Technology Computer programs and calculators usually provide a *P*-value, so the *P*-value method is used. See the accompanying TI-83/84 Plus calculator results showing the test statistic of $F = 2.3706$ (rounded), and the *P*-value of 0.0129 (rounded). Because the *P*-value of 0.0129 is less than the significance level of $\alpha = 0.05$, we reject the null hypothesis. See the interpretation at the end of this example.

Instead of using the sample standard deviations to test the claim of equal population standard deviations, we use the sample variances to test the claim of equal population variances, but we can state conclusions in terms of standard deviations. Because we stipulate in this section that the larger variance is denoted by s_1^2, we let $s_1^2 = 0.97^2$ and $s_2^2 = 0.63^2$. We now proceed to use the critical value method of testing hypotheses as outlined in Figure 8-2 from Section 8-2.

Step 1: The claim of equal standard deviations is equivalent to a claim of equal variances, which we express symbolically as $\sigma_1^2 = \sigma_2^2$.

Step 2: If the original claim is false, then $\sigma_1^2 \neq \sigma_2^2$.

Step 3: Because the null hypothesis is the statement of equality and because the alternative hypothesis cannot contain equality, we have

$$H_0\colon \sigma_1^2 = \sigma_2^2 \text{ (original claim)} \quad H_1\colon \sigma_1^2 \neq \sigma_2^2$$

Step 4: The significance level is $\alpha = 0.05$.

Step 5: Because this test involves two population variances, we use the *F* distribution.

Step 6: The test statistic is

$$F = \frac{s_1^2}{s_2^2} = \frac{0.97^2}{0.63^2} = 2.3706$$

For the critical values in this two-tailed test, refer to Table A-5 for the area of 0.025 in the right tail. Because we stipulate that the larger variance is placed in the numerator of the *F* test statistic, we need to find only the right-tailed critical value. The number of degrees of freedom for the numerator is $n_1 - 1 = 34$, and for the denominator we have $\text{df} = n_2 - 1 = 35$. From Table A-5 we see that the critical value of *F* is between 1.8752 and 2.0739. Technology can be used to find that the critical *F* value is 1.9678.

Step 7: Figure 9-6 shows that the test statistic $F = 2.3706$ does fall within the critical region, so we reject the null hypothesis of equal variances. There is sufficient evidence to warrant rejection of the claim of equal standard deviations.