

## Testing a Claim About a Population Proportion

### Objective

Conduct a formal hypothesis test of a claim about a population proportion  $p$ .

### Notation

$n$  = sample size or number of trials

$\hat{p} = \frac{x}{n}$  (*sample* proportion)

$p$  = population proportion ( $p$  is the value used in the statement of the null hypothesis)

$q = 1 - p$

### Requirements

1. The sample observations are a simple random sample.
2. The conditions for a *binomial distribution* are satisfied. (There is a fixed number of independent trials having constant probabilities, and each trial has two outcome categories of “success” and “failure.”)
3. The conditions  $np \geq 5$  and  $nq \geq 5$  are both satisfied, so **the binomial distribution of sample proportions can be approximated by a normal distribution with  $\mu = np$  and  $\sigma = \sqrt{npq}$**  (as described in Section 6-7). Note that  $p$  used here is the *assumed* proportion used in the claim, not the sample proportion  $\hat{p}$ .

### Test Statistic for Testing a Claim About a Proportion

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

**P-values:**  $P$ -values are automatically provided by technology. If technology is not available, use the

standard normal distribution (Table A-2) and refer to Figure 8-1.

**Critical values:** Use the standard normal distribution (Table A-2).

### CAUTION

Reminder: Don't confuse a  $P$ -value with a proportion  $p$ .

- $P$ -value = probability of getting a test statistic at least as extreme as the one representing the sample data, assuming that the null hypothesis  $H_0$  is true
- $p$  = population proportion

The test statistic above does not include a correction for continuity (as described in Section 6-7), because its effect tends to be very small with large samples.

### Example 1 Reality Check

Based on information from the National Cyber Security Alliance, 93% of computer owners believe that they have antivirus programs installed on their computers. In a random sample of 400 scanned computers, it is found that 380 of them (or 95%) actually have antivirus programs. Use the sample data from the scanned computers to test the claim that 93% of computers have antivirus programs.

**Requirement check** We first check the three requirements.

1. The 400 computers are randomly selected.
2. There is a fixed number (400) of independent trials with two categories (computer has an antivirus program or does not).