13-5 Kruskal-Wallis Test

Key Concept This section introduces the *Kruskal-Wallis test,* which uses ranks of data from three or more independent simple random samples to test the null hypothesis that the samples come from populations with the same median.

In Section 12-2 we used one-way analysis of variance (ANOVA) to test the null hypothesis that three or more populations have the same *mean,* but ANOVA requires that all of the involved populations have normal distributions. The Kruskal-Wallis test for equal *medians* does not require normal distributions.

DEFINITION The **Kruskal-Wallis test** (also called the ***H* test**) is a nonparametric test that uses ranks of simple random samples from three or more independent populations to test the null hypothesis that the populations have the same median. (The alternative hypothesis is the claim that the populations have medians that are not all equal.)

In applying the Kruskal-Wallis test, we compute the *test statistic H, which has a distribution that can be approximated by the chi-square distribution as long as each sample has at least five observations.* When we use the chi-square distribution in this context, the number of degrees of freedom is $k - 1$, where k is the number of samples. (For a quick review of the key features of the chi-square distribution, see Section 7-4.)

The H test statistic is basically a measure of the variance of the rank sums $R_1, R_2, \ldots, R_k$. If the ranks are distributed evenly among the sample groups, then H should be a relatively small number. If the samples are very different, then the ranks will be excessively low in some groups and high in others, with the net effect that H will be large. Consequently, only large values of H lead to rejection of the null hypothesis that the samples come from identical populations. *The Kruskal-Wallis test is therefore a right-tailed test.*

Kruskal-Wallis Test

Objective

Use the Kruskal-Wallis test with simple random samples from three or more independent populations for the following null and alternative hypotheses:

H_0: The samples come from populations with the same median.

H_1: The samples come from populations with medians that are not all equal.

Notation

N = total number of observations in all samples combined

k = number of different samples

R_1 = sum of ranks for Sample 1

n_1 = number of observations in Sample 1

For Sample 2, the sum of ranks is R_2 and the number of observations is n_2, and similar notation is used for the other samples.

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