

**38.** Construct the confidence interval for Exercise 12 assuming that  $\sigma$  is known to be 2.55 chocolate chips.

**39. Outlier Effect** If the first value of 3.0 in Exercise 21 is changed to 300, it becomes an outlier. Does this outlier have much of an effect on the confidence interval? How should you handle outliers when they are found in sample data sets that will be used for the construction of confidence intervals?

**40. Finite Population Correction Factor** If a simple random sample of size  $n$  is selected without replacement from a finite population of size  $N$ , and the sample size is more than 5% of the population size ( $n > 0.05N$ ), better results can be obtained by using the finite population correction factor, which involves multiplying the margin of error  $E$  by  $\sqrt{(N-n)/(N-1)}$ . For the sample of 100 weights of M&M candies in Data Set 20 from Appendix B, we get  $\bar{x} = 0.8565$  g and  $s = 0.0518$  g. First construct a 95% confidence interval estimate of  $\mu$  assuming that the population is large, then construct a 95% confidence interval estimate of the mean weight of M&Ms in the full bag from which the sample was taken. The full bag has 465 M&Ms. Compare the results.

**41. Confidence Interval for Sample of Size  $n = 1$**  Based on the article “An Effective Confidence Interval for the Mean with Samples of Size One and Two,” by Wall, Boen, and Tweedie (*American Statistician*, Vol. 55, No. 2), a 95% confidence interval for  $\mu$  can be found (using methods not discussed in this book) for a sample of size  $n = 1$  randomly selected from a normally distributed population, and it can be expressed as  $x \pm 9.68|x|$ . Use this result to construct a 95% confidence interval using only the first sample value listed in Exercise 21. How does it compare to the result found in Exercise 21 when all of the sample values are used?



## 7-4

## Estimating a Population Standard Deviation or Variance

**Key Concept** This section presents methods for using a sample standard deviation  $s$  (or a sample variance  $s^2$ ) to estimate the value of the corresponding population standard deviation  $\sigma$  (or population variance  $\sigma^2$ ). The methods of this section require that we use a *chi-square distribution*. (The Greek letter chi is pronounced “kigh.”) Here are the main concepts included in this section:

- **Point Estimate:** The sample variance  $s^2$  is the best *point estimate* (or single value estimate) of the population variance  $\sigma^2$ . The sample standard deviation  $s$  is commonly used as a point estimate of  $\sigma$  (even though it is a biased estimator). (Section 6-4 showed that  $s^2$  is an unbiased estimator of  $\sigma^2$ , but  $s$  is a biased estimator of  $\sigma$ .)
- **Confidence Interval:** When constructing a *confidence interval* estimate of a population standard deviation (or population variance), we construct the confidence interval using the  $\chi^2$  (*chi-square*) *distribution*.
- **Chi-Square Distribution:** We should know how to find critical values of  $\chi^2$ .

Here are some key points about the chi-square distribution:

**Chi-Square Distribution:** In a normally distributed population with variance  $\sigma^2$ , if we randomly select independent samples of size  $n$  and, for each sample, compute the sample variance  $s^2$  (which is the square of the sample standard deviation  $s$ ), the sample statistic  $\chi^2 = (n-1)s^2/\sigma^2$  has a sampling distribution called the **chi-square distribution**, so Formula 7-5 shows the format of the sample statistic.

Formula 7-5

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$