Table 6-9 Sitting Heights of 5-Year-Old Children

	Boys	Girls
μ	61.8 cm	61.2 cm
σ	2.9 cm	3.1 cm
Distribution	Normal	Normal

Solution

Requirement check We can use the normal distribution if the original population is normally distributed or $n > 30$. In parts (a) and (b), the original population is normally distributed, so the normal distribution can be used.

a. If the sitting height accommodates 95% of the boys, it accommodates the *lowest* 95%. Only the boys with sitting heights in the top 5% would not fit. We therefore want the sitting height that separates the lowest 95% from the top 5%, and this corresponds to the sitting height with a cumulative left area of 0.95 under the normal distribution curve. See Figure 6-17(a). Because we are working with individual students, we use the methods of Section 6-3. If using technology, we find that the shaded area in Figure 6-17(a) is bounded by the sitting height of 66.6 cm. If using Table A-2, we refer to that table to find that $z = 1.645$ corresponds to a cumulative left area of 0.95. Using $z = (x - \mu)/\sigma$, we substitute $z = 1.645$, $\mu = 61.8$, and $\sigma = 2.9$, then we solve to get $x = 66.6$ cm.

b. Because we are now working with means from samples of 25 boys, we must use these parameters:

$$\mu_{\bar{x}} = \mu = 61.8$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2.9}{\sqrt{25}} = 0.58$$

Figure 6-17(b) shows the shaded area corresponding to the lowest 95% of the means of sitting heights. We can find the sitting height that is at the rightmost boundary for that shaded area by using the same procedures developed in Section 6-3. If using technology, we find that the sitting height is 62.8 cm. If using Table A-2, we refer to the table to find that $z = 1.645$ corresponds to a cumulative left area of 0.95, and we get the following:

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} \text{ becomes } 1.645 = \frac{\bar{x} - 61.8}{0.58}$$

Solving for $\bar{x}$ in the equation above results in $\bar{x} = 62.8$ cm.

c. See the following interpretation of the results.

Interpretation

Parts (a) and (b) provide us with two different sitting heights: 66.6 cm and 62.8 cm. We now temporarily leave the world of statistical calculations and enter the world of common sense. A critical consideration is that we really don't care about the means from part (b). Each desk will be occupied by one individual child, not a group of 25 children, so we are concerned with the distribution of sitting heights of individuals, as in part (a). Next, Table 6-9 shows that boys have greater sitting heights than girls, so if we accommodate 95% of the boys, more than 95% of the girls will also be accommodated. We should therefore use desks designed to accommodate a sitting height of 66.6 cm.

The Fuzzy Central Limit Theorem

In *The Cartoon Guide to Statistics*, by Gonick and Smith, the authors describe the Fuzzy Central Limit Theorem as follows: "Data that are influenced by many small and unrelated random effects are approximately normally distributed. This explains why the normal is everywhere: stock market fluctuations, student weights, yearly temperature averages, SAT scores: All are the result of many different effects." People's heights, for example, are the results of hereditary factors, environmental factors, nutrition, health care, geographic region, and other influences which, when combined, produce normally distributed values.