Confidence Interval for Estimating a Population Mean with σ Not Known

Objective

Construct a confidence interval used to estimate a population mean.

Notation

μ = population mean

$\bar{x}$ = sample mean

n = number of sample values

E = margin of error

Requirements

1. The sample is a simple random sample.

2. Either or both of these conditions is satisfied: The population is normally distributed or $n > 30$.

Confidence Interval

$$\bar{x} - E < \mu < \bar{x} + E$$

or

$$\bar{x} \pm E$$

or

$$(\bar{x} - E, \bar{x} + E)$$

where the margin of error E is found from the following:

$$E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} \qquad (\text{Use df} = n - 1.)$$

where $t_{\alpha/2}$ = critical t value separating an area of $\alpha/2$ in the right tail of the Student t distribution, and df $= n - 1$ is the number of degrees of freedom. (Find $t_{\alpha/2}$ using technology or Table A-3.)

Round-Off Rule

1. When constructing a confidence interval from the *original set of data* values, round the confidence interval limits to one more decimal place than is used for the original set of data.

2. When constructing a confidence interval from *summary statistics* (n and $\bar{x}$ and s), round the confidence interval limits to the same number of decimal places used for the sample mean.

used to construct the confidence interval. As in Section 7-2, α is the complement of the confidence level. For a 0.95 (or 95%) confidence level, $\alpha = 0.05$.

Student t Distribution

If σ is not known, but the relevant requirements are satisfied, we use a *Student t distribution* (instead of a normal distribution), as developed by William Gosset (1876–1937). Gosset was a Guinness Brewery employee who needed a distribution that could be used with small samples. The Irish brewery where he worked did not allow the publication of research results, so Gosset published under the pseudonym "Student." (In the interest of research and better serving his readers, the author visited the Guinness Brewery and sampled some of the product. This is true author dedication.) Gosset first referred to his distribution as a z distribution, but it was modified and he later called it a t distribution. Here are some key points about the Student t distribution: