Example 7 Bone Density Test

Using the same bone density test described in Example 3, we have a standard normal distribution with a mean of 0 and a standard deviation of 1. Find the bone density test score that separates the bottom 2.5% and find the score that separates the top 2.5%.

Solution

The required z scores are shown in Figure 6-10. Those z scores can be found using technology. If using Table A-2 to find the z score located to the left, we search the *body of the table* for an area of 0.025. The result is $z = -1.96$. To find the z score located to the right, we search *the body of Table A-2* for an area of 0.975. (Remember that Table A-2 always gives cumulative areas from the *left*.) The result is $z = 1.96$. The values of $z = -1.96$ and $z = 1.96$ separate the bottom 2.5% and the top 2.5%, as shown in Figure 6-10.

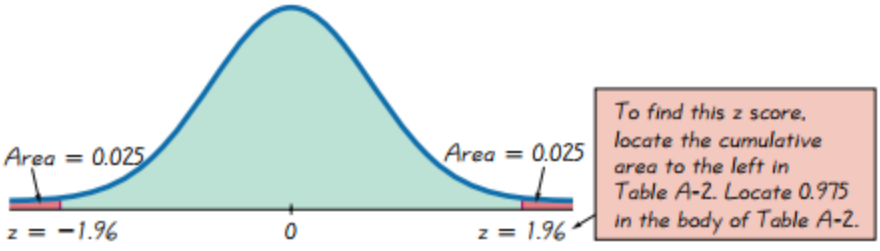

Figure 6-10 Finding z Scores

Interpretation

For the population of bone density test scores, 2.5% of the scores are equal to or less than −1.96 and 2.5% of the scores are equal to or greater than 1.96. Another interpretation is that 95% of all bone density test scores are between −1.96 and 1.96.

Critical Values For a normal distribution, a *critical value* is a z score on the borderline separating the z scores that are *likely* to occur from those that are unlikely. Common critical values are $z = -1.96$ and $z = 1.96$, and they are obtained as shown in Example 7. In Example 7, the values below $z = -1.96$ are unlikely, because only 2.5% of the population have scores below −1.96, and the values above $z = 1.96$ are unlikely because only 2.5% of the population have scores above 1.96. The reference to *critical values* is not so important in this chapter, but will become extremely important in following chapters. The following notation is used for critical z values found by using the standard normal distribution.

DEFINITION For the standard normal distribution, a **critical value** is a z score separating unlikely values from those that are likely to occur.

Notation

The expression z_α denotes the z score with an area of α to its right. (α is the Greek letter alpha.)