Example 1 At Least One Defective DVD

Topford Development, Ltd., supplies X-Data DVDs in lots of 50, and they have a reported defect rate of 0.5%, so the probability of an individual disk being defective is 0.005. It follows that the probability of a disk being good is 0.995. If a quality control engineer wants to carefully analyze a defective disk, what is the probability of her getting at least one defective disk in a lot of 50? Is the probability high enough that the engineer can be reasonably sure of getting a defective disk that can be used for her analysis?

Solution

Step 1: Let A = at least 1 of the 50 disks is defective.

Step 2: Identify the event that is the complement of A.

$$\begin{aligned}\overline{A} &= \textit{not} \text{ getting at least 1 defective disk among 50} \\ &= \text{all 50 disks are good}\end{aligned}$$

Step 3: Find the probability of the complement by evaluating $P(\overline{A})$.

$$\begin{aligned}P(\overline{A}) &= P(\text{all 50 disks are good}) \\ &= 0.995 \cdot 0.995 \cdot \cdots \cdot 0.995 \\ &= 0.995^{50} = 0.778\end{aligned}$$

Step 4: Find $P(A)$ by evaluating $1 - P(\overline{A})$.

$$P(A) = 1 - P(\overline{A}) = 1 - 0.778 = 0.222$$

Interpretation

In a lot of 50 DVDs, the engineer has a 0.222 probability of getting at least 1 defective DVD. This probability is not very high, so if a defective DVD is required for analysis, the engineer should start with more than one lot consisting of 50 DVDs.

Conditional Probability

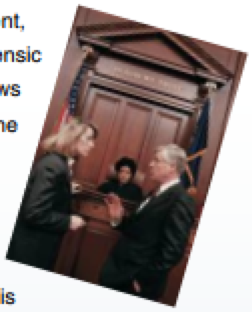
We now consider the second application of this section, which is based on the principle that the probability of an event is often affected by knowledge that some other event has occurred. For example, the probability of a golfer making a hole in one is $1/12{,}000$ (based on past results), but if you have the additional knowledge that the selected golfer is a touring professional, the probability changes to $1/2375$ (based on data from *USA Today*). In general, a *conditional probability* of an event is used when the probability is calculated with the knowledge that some other event has occurred.

DEFINITION A **conditional probability** of an event is a probability obtained with the additional information that some other event has already occurred. $P(B \mid A)$ denotes the conditional probability of event B occurring, given that event A has already occurred. $P(B \mid A)$ can be found by dividing the probability of events A and B both occurring by the probability of event A:

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

Prosecutor's Fallacy

The *prosecutor's fallacy* is misunderstanding or confusion of two different conditional probabilities: (1) the probability that a defendant is innocent, given that forensic evidence shows a match; (2) the probability that forensics shows a match, given that a person is innocent. The prosecutor's fallacy has led to wrong convictions and imprisonment of some innocent people.



Lucia de Berk was a nurse who was convicted of murder and sentenced to prison in the Netherlands. Hospital administrators observed suspicious deaths that occurred in hospital wards where de Berk had been present. An expert testified that there was only 1 chance in 342 million that her presence was a coincidence. However, mathematician Richard Gill calculated the probability to be closer to 1/150, or possibly as low as 1/5. The court used the probability that the suspicious deaths could have occurred with de Berk present, given that she was innocent. The court should have considered the probability that de Berk is innocent, given that the suspicious deaths occurred when she was present. This error of the prosecutor's fallacy is subtle and can be very difficult to understand and recognize, yet it can lead to the imprisonment of innocent people.