Correction for a Finite Population

In applying the central limit theorem, our use of $\sigma_{\bar{x}} = \sigma/\sqrt{n}$ assumes that the population has infinitely many members. When we sample with replacement (that is, put back each selected item before making the next selection), the population is effectively infinite. Yet many realistic applications involve sampling without replacement, so successive samples depend on previous outcomes. In manufacturing, quality control inspectors typically sample items from a finite production run without replacing them. For such a finite population, we may need to adjust $\sigma_{\bar{x}}$. Here is a common rule of thumb:

> **When sampling without replacement and the sample size n is greater than 5% of the finite population size N (that is, $n > 0.05N$), adjust the standard deviation of sample means $\sigma_{\bar{x}}$ by multiplying it by the *finite population correction factor:***

$$\sqrt{\frac{N - n}{N - 1}}$$

Except for Exercises 23, 24, and 25 the examples and exercises in this section assume that the finite population correction factor does *not* apply, because we are sampling with replacement, or the population is infinite, or the sample size doesn't exceed 5% of the population size.

6-5 Basic Skills and Concepts

Statistical Literacy and Critical Thinking

1. Standard Error of the Mean The population of current statistics students has ages with mean μ and standard deviation σ. Samples of statistics students are randomly selected so that there are exactly 40 students in each sample. For each sample, the mean age is computed. What does the central limit theorem tell us about the distribution of those mean ages?

2. Small Sample Heights of adult females are normally distributed. Samples of heights of adult females, each of size $n = 3$, are randomly collected and the sample means are found. Is it correct to conclude that the sample means cannot be treated as a normal distribution because the sample size is too small? Explain.

3. Notation The population of distances that adult females can reach forward is normally distributed with a mean of 60.5 cm and a standard deviation of 6.6 cm (from the Federal Aviation Administration). If samples of 36 adult females are randomly selected, what do $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$ represent, and what are their values?

4. Lottery Numbers In each drawing for the Texas Pick 3 lottery, three digits between 0 and 9 inclusive are randomly selected. What is the distribution of the selected digits? If the mean is calculated for each drawing, can the distribution of the sample means be treated as a normal distribution?

Using the Central Limit Theorem. *In Exercises 5–10, use this information about the overhead reach distances of adult females: $\mu = 205.5$ cm, $\sigma = 8.6$ cm, and overhead reach distances are normally distributed (based on data from the Federal Aviation Administration). The overhead reach distances are used in planning assembly work stations.*

5. a. If 1 adult female is randomly selected, find the probability that her overhead reach is less than 222.7 cm.

b. If 49 adult females are randomly selected, find the probability that they have a mean overhead reach less than 207.0 cm.