SOLUSI UTS
DSK - 2018/2019

1

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$y = \begin{bmatrix} 4 & 5 & 1 \end{bmatrix} x + \begin{bmatrix} 0 & 3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

a) To find $G_1(s) = \dfrac{y(s)}{u_1(s)}$, we have to find all matrices A, B, C, and D for that particular input-output pair :

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \; ; \; B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \; ; \; C_1 = \begin{bmatrix} 4 & 5 & 1 \end{bmatrix} \; ; \; D_1 = 0$$

$$G_1(s) = \frac{y(s)}{u_1(s)} = C_1 \left(sI - A_1 \right)^{-1} B_1 + D_1$$

Therefore we got :

$$G_1(s) = \frac{s^2 + 5s + 4}{s^3 + 6s^2 + 11s + 6}$$

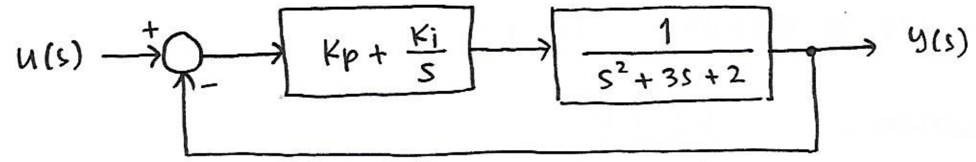
b) $$A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \; ; \; B_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \; ; \; C_2 = \begin{bmatrix} 4 & 5 & 1 \end{bmatrix} \; ; \; D_2 = 3$$

$$G_2(s) = \frac{y(s)}{u_1(s)} = C_2 \left(sI - A_2 \right)^{-1} B_2 + D_2$$

Therefore :

$$G_2(s) = \frac{3s^3 + 19s^2 + 38s + 22}{s^3 + 6s^2 + 11s + 6}$$

2



Transfer function : $$\frac{y(s)}{u(s)} = \frac{\left(K_p + \frac{K_i}{s}\right)\frac{1}{(s+1)(s+2)}}{1 + \left(K_p + \frac{K_i}{s}\right)\frac{1}{(s+1)(s+2)}} = \frac{K_p s + K_i}{s^3 + 3s^2 + (2 + K_p)s + K_i}$$

So, we have the characteristic eq : $s^3 + 3s^2 + (2 + K_p)s + K_i$

Routh's Table :

$$\begin{array}{c|c|c} s^3 & 1 & 2+K_p \\ s^2 & 3 & K_i \\ s^1 & \dfrac{3(2+K_p)-K_i}{3} & 0 \\ s^0 & K_i & 0 \end{array}$$

$\uparrow$ For stability this column should not have sign change

Therefore : $K_i > 0$

and $6 + 3K_p - K_i > 0 \Rightarrow K_p > \dfrac{K_i - 6}{3}$

$\boxed{3}$ The characteristic eq : $s^4 + 6s^3 + 2s^2 + s + 3$

Routh's Table :

$$\begin{array}{cccc} s^4 & 1 & 2 & 3 \\ s^3 & 6 & 1 & 0 \\ s^2 & \dfrac{-\begin{vmatrix} 1 & 2 \\ 6 & 1 \end{vmatrix}}{6} = \dfrac{11}{6} & \dfrac{-\begin{vmatrix} 1 & 3 \\ 6 & 0 \end{vmatrix}}{6} = \dfrac{18}{6} = 3 & 0 \\ s^1 & \dfrac{-\begin{vmatrix} 6 & 1 \\ 11/6 & 3 \end{vmatrix}}{11/6} = -\dfrac{102}{6} & \dfrac{-\begin{vmatrix} 6 & 0 \\ 11/6 & 0 \end{vmatrix}}{11/6} = 0 & 0 \\ s^0 & \dfrac{-\begin{vmatrix} 11/6 & 3 \\ -\frac{102}{6} & 0 \end{vmatrix}}{-\frac{102}{6}} = 3 & 0 & 0 \end{array}$$

a) Therefore :

$$\begin{array}{cc} s^4 & + \\ s^3 & + \\ s^2 & + \\ s^1 & - \\ 0 & + \end{array}$$

2 sign change means : unstable.

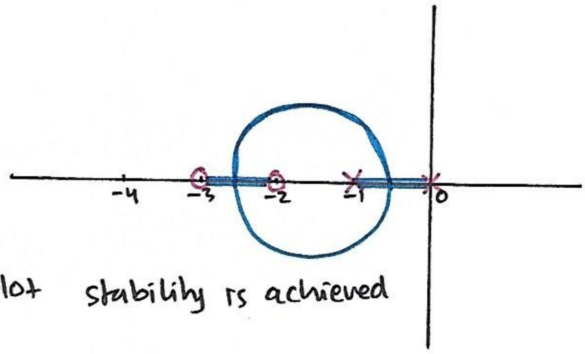
b) 2 sign change means 2 RHP poles. And since the system is 4^{th} order then there are 2 remaining LHP poles.

$\boxed{4}$ Transfer function :

$$G(s) = \frac{\dfrac{K(s+2)(s+3)}{s(s+1)}}{1 + \dfrac{K(s+2)(s+3)}{s(s+1)}}$$

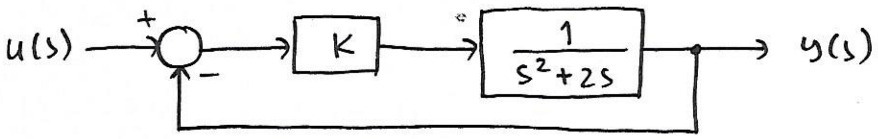
characteristic eq = $1 + KL(s) = 1 + \dfrac{K(s+2)(s+3)}{s(s+1)} \Rightarrow L(s) = \dfrac{(s+2)(s+3)}{s(s+1)}$

a) Root Locus Plot :



b) Based on root locus plot stability is achieved when $0 < K < \infty$

$\boxed{5}$



Transfer function : $\dfrac{y(s)}{u(s)} = \dfrac{\dfrac{K}{s(s+2)}}{1+\dfrac{K}{s(s+2)}} = \dfrac{K}{s^2+2s+K}$

So we got : $2\zeta\omega_n = 2.$ and $\omega_n^2 = K$

The fastest renspons while still having no overshoot $\rightarrow$ critically damped

Critically damped system has $\zeta = 1$, so :

$$2\zeta\omega_n = 2$$

$$2.1.\ \omega_n = 2 \rightarrow \omega_n = 1$$

And therefore $K = \omega_n^2$

$$K = 1.$$