

Homework 4

● Graded

1 Day, 3 Hours Late

Student

John Stephen Gutam

Total Points

94 / 100 pts

Question 1

AdaBoost

46 / 50 pts

✓ + 50 pts Correct

+ 25 pts Implementation

+ 25 pts Writeup

+ 0 pts Missing solution

– 4 pts Point adjustment

1 The 3v5 task seems to be consistently more difficult compared to the 1v3 task. Why might that be? It could possibly be because 3 v 5 is more similar compared to 1 v 3 which causes the model to perform worse. Some hypothesis was expected [-4 points]

Question 2

Bias-Variance 1

10 / 10 pts

✓ + 10 pts Correct

+ 7 pts Most of the way there. Some minor details are missing or wrong.

+ 5 pts About halfway there. Some significant progress has been made in the right direction.

+ 3 pts Some attempt made towards progress in the right direction.

+ 1 pt Plausible attempt at an answer.

+ 0 pts Incorrect/Missing answer.

Question 3

Bias-Variance 2

13 / 15 pts

+ 15 pts Correct

✓ + 4.5 pts Correct part (a)

✓ + 6 pts Correct values [+1.5 points each]

✓ + 4.5 pts Correct values [+1.5 points each]

💬 - 2 pts Check annotations for the wrong parts. Check the answer key for the correct solution.

2 [-1 point]

3 [-1 point]

Question 4

Backprop

25 / 25 pts

+ 25 pts Correct

✓ + 2.5 pts Correct part (a)

✓ + 12.5 pts [+2.5 points] each for the 5 sub-questions

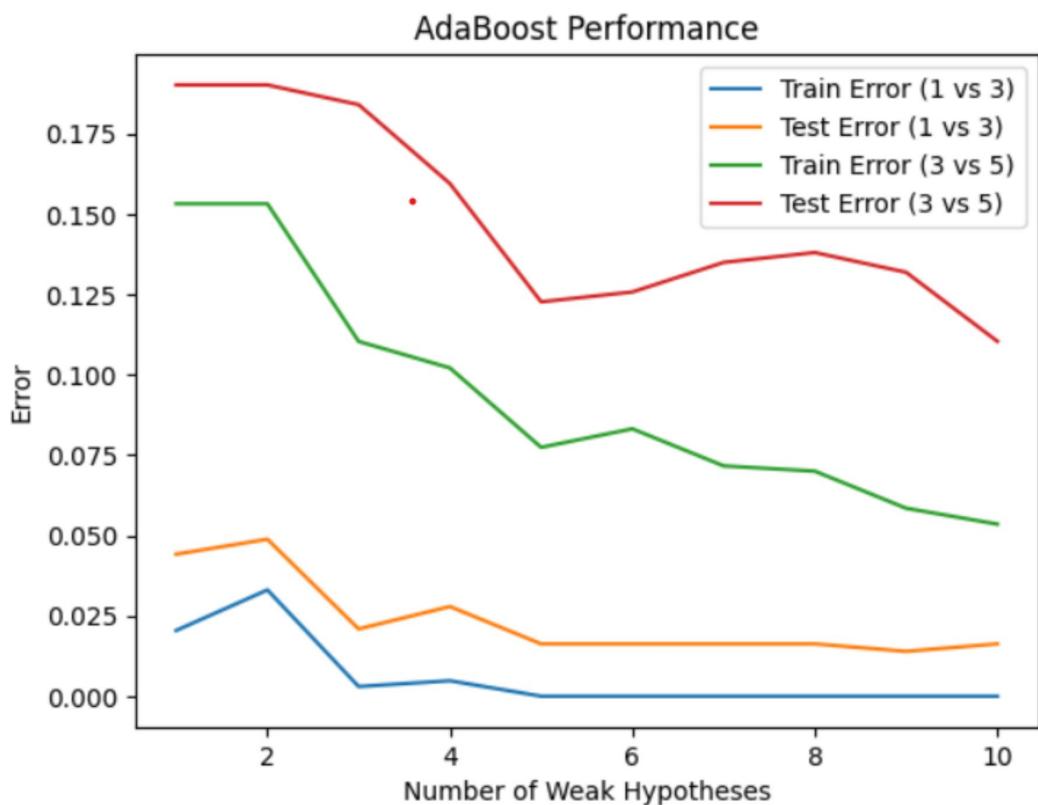
✓ + 10 pts [+2.5 points] each for part i. and ii. [+5 points] for part iii.

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Problem 1:

Graphical report of the training set error and the test set error as a function of the number of weak hypotheses.



The graph includes four lines, each representing different scenarios.

1. Train error (1 vs 3): The blue line represents the error on the training set when distinguished between classes labeled 1 and 3. As the number of weak hypotheses increases, the error decreases, indicating that the model is learning and improving its predictions on the training data.
2. Test error (1 vs 3): The orange line represents the error on the test set for the same classes 1 and 3. Similar to the training error, the test error also decreases as more weak hypotheses are added, suggesting that the model is generalizing well to unseen data.
3. Train error (3 vs 5): The green line shows the training error for a different pair of classes 3 and 5. The error decreases as more weak hypotheses are included, but it starts to plateau

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towards the end, indicating that adding more weak hypotheses is yielding a diminishing return in terms of reducing the training error.

4. Test error (3 vs 5): The red line represents the test error for classes 3 and 5. The error decreases initially as more weak hypotheses are added but then starts to increase slightly after reaching the minimum. This could be an indication of overfitting, where the model performs very well on the training data but less on the test data.

Summary:

The graph demonstrates that AdaBoost can effectively reduce both training and test errors by combining multiple weak hypotheses. However, there is a point beyond which adding more weak hypotheses does not necessarily lead to better performance on the test set. This is an important consideration to avoid overfitting.

For classes 1 and 3, both training and test errors decrease consistently as more weak hypotheses are added, indicating good model performance and generalization. However, for classes 3 and 5, while the training error decreases with more weak hypotheses, the test error starts to increase after reaching a minimum. This suggests that the model may be overfitting to the training data, leading to poorer performance on the test data.

In conclusion, while adding more weak hypotheses can improve the model's performance, care must be taken to avoid overfitting, which can negatively impact the model's ability to generalize the unseen data.

1

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Problem 2:

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CS 688: Homework 4

2. In the regression problem where the labels are noisy, that is $y_i^o = f(x_i^o) + \epsilon$. Given ϵ is a zero-mean variable with variance σ^2 .

Bias - Variance decomposition:

On the hypothesis 'g' of a training set (D). The error is defined

$$E_{\text{out}}(g^{(D)}) = \mathbb{E}[(g^{(D)}(x) - y)^2] \quad \text{where } y = f(x) + \epsilon$$

$$\mathbb{E}_D[L(g^{(D)})] = \mathbb{E}_D[E_{\text{out}}(g^{(D)})] = \mathbb{E}_D[\underbrace{\mathbb{E}_x[(g^o(x) - y)^2]}_{\text{Let this be } Z}]$$

Let's solve 'Z'

$$Z = \mathbb{E}_D[(g^o(x) - y)^2] = \mathbb{E}_D[(g^o(x) - (f(x) + \epsilon))^2]$$

$$= \mathbb{E}_D[g^o(x)^2 + (f(x) + \epsilon)^2 - 2g^o(x) \cdot (f(x) + \epsilon)]$$

$$= \mathbb{E}_D[g^o(x)^2 + f(x)^2 + \epsilon^2 + 2f(x)\epsilon - 2 \cdot g^o(x) \cdot f(x) - 2g^o(x)\epsilon]$$

$$= \mathbb{E}_D[g^o(x)^2] + f(x)^2 + \mathbb{E}(\epsilon^2) + 2f(x)\mathbb{E}(\epsilon) - 2\mathbb{E}g^o(x)f(x) - 2\mathbb{E}g^o(x)\mathbb{E}(\epsilon)$$

given $E(\epsilon) = 0$, $E(\epsilon^2) = \sigma^2$

$$\text{let } \bar{g}(x) = \mathbb{E}[g^o(x)]$$

$$= \mathbb{E}_D[g^o(x)^2] + f(x)^2 + \sigma^2 + 0 - 2\mathbb{E}g^o(x)f(x) - 0$$

$$= \mathbb{E}_D[g^o(x)^2] + f(x)^2 + \bar{g}(x)^2 - \bar{g}(x)^2 - 2\bar{g}(x) \cdot f(x) + \sigma^2$$

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$$z = \left(E_D [g^{(0)}(x)^2] - \bar{g}(x)^2 \right) + \left(\bar{g}(x)^2 - 2\bar{g}(x)f(x) + f(x)^2 \right) + \sigma^2$$

$$z = E_D [(g^{(0)}(x) - \bar{g}(x))^2] + (\bar{g}(x) - f(x))^2 + \sigma^2$$

Follow
 $E[x - \bar{x}]^2 = E[x^2] - \bar{x}^2$

Note

$$\begin{aligned} E_D [L(g^{(0)})] &= E_D [E_{\text{out}}(g^{(0)})] = E_D [E_x (g^D(x) - f(x))^2] \\ &= E_x [E_D (g^D(x) - f(x))^2] \\ &= E_x [z] \end{aligned}$$

$$E_D [L(g^{(0)})] = E_x \left[\underbrace{E_D [(g^{(0)}x - \bar{g}(x))^2]}_{\text{variance}} + \underbrace{(\bar{g}(x) - f(x))^2}_{\text{bias}} + \sigma^2 \right]$$

so. $E_D [L(g^{(0)})] = \text{bias} + \text{variance} + \sigma^2$

Hence proved.

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Problem 3:

3. The true target function $y = x^2$, the hypothesis space H has linear functions $h(x) = ax + b$, and each training set we have two examples $(x_1, x_1^2), (x_2, x_2^2)$

(a) For a linear function $h(x) = ax + b$, the mean hypothesis $\bar{g}(x)$ will have (\bar{a}, \bar{b}) and the equation be

$$\boxed{\bar{g}(x) = \bar{a}x + \bar{b}}$$

$$a = \text{slope of the line} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{x_2^2 - x_1^2}{x_2 - x_1} = \frac{(x_2 + x_1)(x_2 - x_1)}{(x_2 - x_1)}$$

$$\boxed{a = (x_1 + x_2)} \Rightarrow \boxed{\bar{a} = \mathbb{E}(x_1 + x_2)}$$

$$b = y \text{ intercept} = y - ax = x_1^2 - ax_1 = x_1^2 - (x_1 + x_2)x_1 \\ \text{at } (x_1, x_1^2) = x_1^2 - x_1^2 - x_1 x_2$$

$$\boxed{b = -x_1 x_2} \Rightarrow \boxed{\bar{b} = \mathbb{E}(-x_1 x_2)}$$

$$\text{Therefore } \bar{g}(x) = \bar{a}x + \bar{b} = (\mathbb{E}(x_1 + x_2))x + \mathbb{E}(-x_1 x_2).$$

(b) In uniform distribution

$$\text{pdf (probability density function)} = P_x[a \leq x \leq b] = \int_a^b f(x) dx$$

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \leq x \leq b \\ 0 & \text{for } x < a \text{ and } x > b \end{cases}$$

$$E[x] = \int_{-\infty}^{\infty} x \frac{1}{b-a} = \frac{b^2 - a^2}{2(b-a)} = \frac{b+a}{2} \quad \left(\text{from wikipedia} \right)$$

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$$E(x^2) = \int_a^b x^2 \frac{dx}{b-a} = \frac{b^3 - a^3}{3(b-a)}.$$

$$\begin{aligned}\bar{a} &= E(x_1 + x_2) = E(x_1) + E(x_2) \\ &= \left(\frac{-1+1}{2}\right) + \left(\frac{-1+1}{2}\right) \\ &= 0 + 0 \\ &= 0.\end{aligned}$$

as x_1 and x_2 are independent
 $\therefore \frac{[-1, 1]}{a b}$
 $\therefore E[x] = \frac{b+a}{2}$

$$\bar{b} = E(-x_1 x_2) = -E(x_1 x_2) = -E(x_1) E(x_2)$$

$$\begin{aligned}&= -\left[\frac{(1)^3 - (-1)^3}{3(1 - (-1))}\right] \left[\frac{(1)^3 - (-1)^3}{3(1 - (-1))}\right] E[x] = \frac{b+a}{2} \\ &= \left(\frac{-1+1}{2}\right) \cdot \left(\frac{-1+1}{2}\right) \\ &= 0.\end{aligned}$$

~~$E(x^2) = \frac{b^3 - a^3}{3(b-a)}$~~

$$\bar{a}^2 = E[(x_1 + x_2)^2] = E[x_1^2] + E[x_2^2] + 2E[x_1 x_2]$$

$$= \frac{(1)^3 - (-1)^3}{3(1 - (-1))} + \frac{(1)^3 - (-1)^3}{3(1 - (-1))} + 2(0)$$

F1 on

$$= \frac{2}{3(2)} + \frac{2}{3(2)} + 0 \quad E[x^2] = \frac{b^3 - a^3}{3(b-a)}$$

$$\boxed{\bar{a}^2 = \frac{2}{3}}$$

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$$\begin{aligned}\bar{b}^2 &= \mathbb{E}[(x_1 x_2)^2] = E[x_1^2 x_2^2] = E[x_1^2] E[x_2^2] \\ &= \frac{2}{3(2)} \cdot \frac{2}{3(2)} \\ &= \frac{1}{9}\end{aligned}$$

$$\boxed{\bar{b}^2 = \frac{1}{9}}$$

(c) bias = squared difference between mean hypothesis and the true function.

$$\begin{aligned}&= \mathbb{E}[(\bar{g}(x) - f(x))^2] \\ &= \mathbb{E}[(\bar{ax} + \bar{b} - x^2)^2] \\ &= \mathbb{E}[(0 + 0 - x^2)^2] \\ &= \mathbb{E}[x^4] \\ &= \int_{-1}^1 \frac{1}{(b-a)} (x^4) dx = \left\{ \frac{1}{2} [x^5/5] \right\}_{-1}^1 \\ &= \frac{1}{10} [(1)^5 - (-1)^5] = \frac{1}{10} [1 - (-1)] = \frac{2}{10}\end{aligned}$$

$$\boxed{\text{bias} = \frac{1}{5}} = 0.2$$

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$$\begin{aligned}
 \text{Variance} &= E[(g(x) - \bar{g}(x))^2] \\
 &= E[(\alpha x + b - (\bar{\alpha}x + \bar{b}))^2] \\
 &= E[(\alpha x + b - 0 + 0)^2] \\
 &= E[(\alpha x + b)^2] \\
 &= E[\alpha^2 x^2 + b^2 + 2\alpha x b] \\
 &= \alpha^2 E[x^2] + E[b^2] + 2ab E[x] \\
 &= \alpha^2 \left(\frac{(1)^3 - (-1)^3}{3(1 - (-1))} \right) + b^2 + 2ab \left(\frac{1 + (-1)}{2} \right) \\
 &= \alpha^2 \left(\frac{1}{3} \right) + b^2 \\
 &= (-1)^2 \left(\frac{1}{3} \right) + (1)^2
 \end{aligned}$$

$$\begin{aligned}
 E(x) &= \frac{b+a}{2} \\
 E(x^2) &= \frac{b^3 - a^3}{3(b-a)}
 \end{aligned}$$

$$\boxed{\text{Variance} = \frac{4}{3}} = 1.33$$

$\begin{bmatrix} -1, 1 \\ a, b \end{bmatrix}$

3

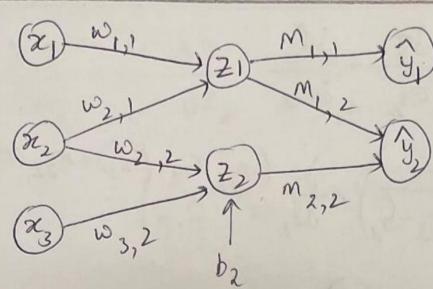
$$\begin{aligned}
 \text{Mean squared error (E)} &= \text{bias} + \text{variance} \quad \text{#} \\
 &= \frac{1}{5} + \frac{4}{3} \\
 &= \frac{3+20}{15} = \frac{23}{15} = 1.53
 \end{aligned}$$

2

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Problem 4:

(4)



$$\text{Given } z_1 = \text{ReLU}(\omega_{1,1}x_1 + \omega_{2,1}x_2)$$

$$z_2 = \omega_{2,2}z_2^2 + \omega_{3,2}x_3 + b_2$$

$$\hat{y}_1 = \sigma(m_{1,1}z_1^3 + c_1)$$

$$\hat{y}_2 = m_{1,2} \sin(z_1) + \\ m_{2,2} \cos(z_2)$$

$$J(\theta) = 2 \log(y_1 - \hat{y}_1) + 2 \log(y_2 - \hat{y}_2)$$

$$(a) \frac{\partial J}{\partial \hat{y}_1} = 2 \cdot \frac{1}{y_1 - \hat{y}_1} (-1) = \frac{-2}{y_1 - \hat{y}_1} \Rightarrow \boxed{\frac{\partial J}{\partial \hat{y}_1} = \frac{-2}{y_1 - \hat{y}_1}}$$

similarly $\boxed{\frac{\partial J}{\partial \hat{y}_2} = \frac{-2}{y_2 - \hat{y}_2}}$

Note:
 $\frac{d}{dx} \sigma(x) = \sigma(x)(1 - \sigma(x))$

$$(b) (i) \frac{\partial J}{\partial z_1} = \frac{\partial J}{\partial \hat{y}_1} \times \frac{\partial \hat{y}_1}{\partial z_1} + \frac{\partial J}{\partial \hat{y}_2} \times \frac{\partial \hat{y}_2}{\partial z_1}$$

$$= \left(\frac{-2}{y_1 - \hat{y}_1} \right) \frac{\partial (\sigma(m_{1,1}z_1^3 + c_1))}{\partial z_1} + \left(\frac{-2}{y_2 - \hat{y}_2} \right) \times \frac{\partial (m_{1,2} \sin(z_1) + m_{2,2} \cos(z_2))}{\partial z_1}$$

$$= -2 \left[\frac{\hat{y}_1(1 - \hat{y}_1)(m_{1,1}3z_1^2)}{y_1 - \hat{y}_1} + \frac{m_{1,2} \cos(z_1)}{y_2 - \hat{y}_2} \right]$$

$$\frac{\partial J}{\partial z_1} = -2 \left[\frac{3m_{1,1}z_1^2 \hat{y}_1(1 - \hat{y}_1)}{y_1 - \hat{y}_1} + \frac{m_{1,2} \cos(z_1)}{y_2 - \hat{y}_2} \right]$$

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$$\begin{aligned}
 \text{(ii)} \quad \frac{\partial J}{\partial z_2} &= \frac{\partial J}{\partial \hat{y}_1} \cdot \frac{\partial \hat{y}_1}{\partial z_2} + \frac{\partial J}{\partial \hat{y}_2} \cdot \frac{\partial \hat{y}_2}{\partial z_2} \\
 &= \left(\frac{-2}{y_1 - \hat{y}_1} \right) (0) + \left(\frac{-2}{y_2 - \hat{y}_2} \right) \cdot \frac{\partial}{\partial z_2} (m_{12} \sin(z_1) + m_{22} \cos(z_2)) \\
 &= \frac{-2}{y_2 - \hat{y}_2} m_{22} (-\sin(z_2))
 \end{aligned}$$

$$\frac{\partial J}{\partial z_2} = \frac{-2}{y_2 - \hat{y}_2} m_{22} (\sin(z_2))$$

$$\begin{aligned}
 \text{(iii)} \quad \frac{\partial J}{\partial m_{1,1}} &= \frac{\partial J}{\partial \hat{y}_1} \times \frac{\partial \hat{y}_1}{\partial m_{1,1}} = \frac{\partial J}{\partial \hat{y}_1} \cdot \frac{\partial (\sigma(m_{1,1} z_1^3 + c_1))}{\partial m_{1,1}} \\
 &= \left(\frac{-2}{y_1 - \hat{y}_1} \right) \cdot \hat{y}_1 (1 - \hat{y}_1) \cdot z_1^3
 \end{aligned}$$

$$\begin{aligned}
 \frac{d}{dx} \sigma(x) \\
 = \sigma(x)(1 - \sigma(x)) \\
 = \hat{y}_1 (1 - \hat{y}_1)
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad \frac{\partial J}{\partial m_{1,2}} &= \frac{\partial J}{\partial \hat{y}_2} \cdot \frac{\partial \hat{y}_2}{\partial m_{1,2}} = \frac{\partial J}{\partial \hat{y}_2} \cdot \frac{\partial (m_{12} \sin(z_1) + m_{22} \cos(z_2))}{\partial m_{1,2}} \\
 &= \left(\frac{-2}{y_2 - \hat{y}_2} \right) \cdot \cancel{\sin(z_1)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(v)} \quad \frac{\partial J}{\partial c_1} &= \frac{\partial J}{\partial \hat{y}_1} \cdot \frac{\partial \hat{y}_1}{\partial c_1} = \frac{-2}{(y_1 - \hat{y}_1)} \cdot \frac{\partial (\sigma(m_{1,1} z_1^3 + c_1))}{\partial c_1} \\
 &= \frac{-2}{(y_1 - \hat{y}_1)} \hat{y}_1 (1 - \hat{y}_1) (1) = \frac{-2 y_1 (1 - \hat{y}_1)}{(y_1 - \hat{y}_1)}
 \end{aligned}$$

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(c)

$$(i) \frac{\partial J}{\partial \omega_{2,2}} = \frac{\partial J}{\partial z_2} \cdot \frac{\partial z_2}{\partial \omega_{2,2}}$$

$$= \left(\frac{2}{y_2 - \hat{y}_2} m_{2,2} (\sin(z_2)) \right) \cdot \frac{\partial}{\partial \omega_{2,2}} (\omega_{2,2} x_2^2 + \omega_{3,2} x_3 + b_2)$$

$$= \frac{2}{y_2 - \hat{y}_2} m_{2,2} (\sin(z_2)) \cdot x_2^2$$

$$\frac{\partial J}{\partial \omega_{2,2}} = \frac{2 x_2^2 m_{2,2} \sin(z_2)}{y_2 - \hat{y}_2}$$

$$(ii) \frac{\partial J}{\partial b_2} = \frac{\partial J}{\partial z_2} \cdot \frac{\partial z_2}{\partial b_2} = \left[\frac{2}{y_2 - \hat{y}_2} m_{2,2} (\sin(z_2)) \right] \frac{\partial (\omega_{2,2} x_2^2 + \omega_{3,2} x_3 + b_2)}{\partial b_2}$$

$$= \frac{2}{y_2 - \hat{y}_2} m_{2,2} (\sin(z_2)) \times (i)$$

$$\frac{\partial J}{\partial b_2} = \frac{2}{y_2 - \hat{y}_2} m_{2,2} (\sin(z_2))$$

$$(iii) \frac{\partial J}{\partial \omega_{1,1}} = \frac{\partial J}{\partial z_1} \times \frac{\partial z_1}{\partial \omega_{1,1}} = \frac{\partial J}{\partial z_1} \times \frac{\partial}{\partial \omega_{1,1}} (\text{ReLU}(\omega_{1,1} x_1 + \omega_{2,1} x_2))$$

where $\frac{\partial}{\partial x} \text{ReLU}(x) = \text{ReLU}'(x) = \begin{cases} 0 & , x < 0 \\ 1 & , \text{others.} \end{cases}$

$$= \frac{\partial J}{\partial z_1} \times \text{ReLU}'(\omega_{1,1} x_1 + \omega_{2,1} x_2) \cdot x_1$$

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$$\frac{\partial J}{\partial \omega_{1,1}} = \frac{\partial J}{\partial z_1} \times \text{ReLU}'(\omega_{11}x_1 + \omega_2 x_2) \times x_1$$

if $x < 0$, $\text{ReLU}'(x) = 0$

at $x < 0$, $\frac{\partial J}{\partial \omega_{1,1}} = \frac{\partial J}{\partial z_1} \cdot (0) \cdot x_1 = 0 \Rightarrow \boxed{\frac{\partial J}{\partial \omega_{11}} = 0 \text{ at } x < 0}$

at others, $\frac{\partial J}{\partial \omega_{1,1}} = \frac{\partial J}{\partial z_1} (1) \cdot x_1$ if x is others
 $\text{ReLU}'(x) = 1$

$$= \frac{\partial J}{\partial z_1} \cdot (x_1)$$

$$\frac{\partial J}{\partial z_1} = -2 \left[\frac{3m_{11}z_1^2 \hat{y}_1 (1-\hat{y}_1)}{y_1 - \hat{y}_1} + \frac{m_{12} \cos(z_1)}{y_2 - \hat{y}_2} \right] (x_1)$$

at $x \geq 0$

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