- P_1

$X_t \in (A, B, C, D, E, F)$ $\quad X_1 = A$

$E_t \in [hot, cold]$

$X_t \backslash X_{t+1}$		A	B	C	D	E	F
X_1	A	0.2	0.8	0	0	0	0
X_2	B	0	0.2	0.8	0	0	0
X_3	C	0	0	0.2	0.8	0	0
X_4	D	0	0	0	0.2	0.8	0
X_5	E	0	0	0	0	0.2	0.8
X_6	F	0	0	0	0	0	0.2

$X_t \backslash e_t$	hot	cold
A	1	0
B	0	1
C	0	1
D	1	0
E	0	1
F	0	1

- (1) filtering

$$P(X_3 \mid hot_1, cold_2, cold_3) \propto P(cold_3 \mid X_3) \sum_{X_2} P(X_3 \mid X_2) P(X_2 \mid hot_1, cold_2)$$

$$P(X_2 \mid hot_1, cold_2) \propto P(cold_2 \mid X_2) \sum_{X_1} P(X_2 \mid X_1) P(X_1 \mid hot_1)$$

$$P(X_1 \mid hot_1) \propto P(hot_1 \mid X_1) P(X_1)$$

$$\propto [P(hot_1 \mid X_1 = A), P(hot_1 \mid X_1 = B), \cdots C, \cdots D, \cdots E, \cdots F] \cdot$$
$$[P(X_1 = A), P(X_1 = B), \cdots C, \cdots D, \cdots E, \cdots F]$$

$$\propto [1 \cdot 1, 0, 0, 0, 0, 0]$$

$$= [1, 0, 0, 0, 0, 0]$$

$$P(cold_2 \mid X_2) = \begin{array}{c} \begin{matrix} A & B & C & D & E & F \end{matrix} \\ \begin{bmatrix} 0, & 1, & 1, & 0, & 1, & 1 \end{bmatrix} \end{array}$$

We found out that only entry of B, C, E, F is not multiplied by zero.

$$P(cold_2 \mid X_2 = B) \cdot \left[P(X_2 = B \mid X_1 = A)\, P(X_1 = A \mid hot_1) + 0 + \dots + 0 \right]$$

$$= 1 \cdot [\, 0.8 \cdot 1 \,]$$

$$= 0.8$$

$$P(cold_2 \mid X_2 = C) \cdot \left[P(X_2 = C \mid X_1 = A)\, P(X_1 = A \mid hot_1) + 0 \dots + 0 \right]$$

$$= 1 \cdot [\, 0 \cdot 1 \,]$$

$$= 0$$

Therefore, according to observation, only $X_2 = B$ is not multiplied by zero.

$$P(X_2 \mid hot_1, cold_2) = \begin{array}{c} \begin{matrix} A & B & C & D & E & F \end{matrix} \\ \begin{bmatrix} 0, & 1, & 0, & 0, & 0, & 0 \end{bmatrix} \end{array}$$

$$P(cold_3 \mid X_3) = \begin{array}{c} \begin{matrix} A & B & C & D & E & F \end{matrix} \\ \begin{bmatrix} 0, & 1, & 1, & 0, & 1, & 1 \end{bmatrix} \end{array}$$

We found out that only entry of B, C, E, F is not multiplied by zero.

$$P(cold_3 \mid X_3 = B) \left[0 + P(X_3 = B \mid X_2 = B)\, P(X_2 = B \mid hot_1, cold_2) + 0 + \dots + 0 \right]$$

$$= 1 \cdot [\, 0.2 \cdot 1 \,]$$

$$= 0.2$$

$P(cold_3 \mid X_3 = C)\,[0 + P(X_3 = C \mid X_2 = B)\, P(X_2 = B \mid hot_1, cold_2) + 0 + \dots + 0]$

$= 1 \cdot [0.8 \cdot 1]$

$= 0.8$

$P(cold_3 \mid X_3 = E)\,[0 + P(X_3 = E \mid X_2 = B)\, P(X_2 = B \mid hot_1, cold_2) + 0 + \dots + 0]$

$= 0$

$P(X_3 \mid hot_1, cold_2, cold_3) = [0, 0.2, 0.8, 0, 0, 0]$

(2) Smoothing

$P(X_2 \mid hot_1, cold_2, cold_3) \propto P(X_2 \mid hot_1, cold_2)\, P(cold_3 \mid X_2)$

According to (1), $P(X_2 \mid hot_1, cold_2) = \begin{array}{c} \begin{matrix} A & B & C & D & E & F \end{matrix} \\ \begin{bmatrix} 0, & 1, & 0, & 0, & 0, & 0 \end{bmatrix} \end{array}$

$P(cold_3 \mid X_2) = \sum_{X_3} P(cold_3 \mid X_3)\, P(X_3 \mid X_2)$

Through observation, we can see only entry "B" would not be multiplied by zero.

$P(cold_3 \mid X_2 = B) = P(cold_3 \mid X_3 = A)\, P(X_3 = A \mid X_2 = B) + P(cold_3 \mid X_3 = B)\, P(X_3 = B \mid X_2 = B)$
$+ P(cold_3 \mid X_3 = C)\, P(X_3 = C \mid X_2 = B) + P(cold_3 \mid X_3 = D)\, P(X_3 = D \mid X_2 = B)$
$+ 0 + \dots + 0$

$= 0 + 1 \cdot 0.2 + 1 \cdot 0.8 + 0$

$= 1$

$P(X_2 \mid hot_1, cold_2, cold_3) = [0, 1, 0, 0, 0, 0, 0]$

(3) Prediction

$$P(hot_4 \mid hot_1, cold_2, cold_3) = \sum_{X_3} P(X_3 \mid hot_1, cold_2, cold_3)\, P(hot_4 \mid X_3)$$

According to (1), $P(X_3 \mid hot_1, cold_2, cold_3) = \begin{array}{cccccc} A & B & C & D & E & F \\ \end{array} [0, 0.2, 0.8, 0, 0, 0]$

$$P(hot_4 \mid X_3) = \sum_{X_4} P(hot_4 \mid X_4)\, P(X_4 \mid X_3)$$

Through observation, we observe that only entry "B" "C" have non-zero value.

$$\begin{aligned} P(hot_4 \mid X_3 = B) &= P(hot_4 \mid X_4 = A)\, P(X_4 = A \mid X_3 = B) + P(hot_4 \mid X_4 = B)\, P(X_4 = B \mid X_3 = B) \\ &+ P(hot_4 \mid X_4 = C)\, P(X_4 = C \mid X_3 = B) + P(hot_4 \mid X_4 = D)\, P(X_4 = D \mid X_3 = B) \\ &+ 0 + \dots + 0 \\ &= 0 + 0 + 0 + \dots + 0 \\ &= 0 \end{aligned}$$

$$\begin{aligned} P(hot_4 \mid X_3 = C) &= 0 + 0 + P(hot_4 \mid X_4 = C)\, P(X_4 = C \mid X_3 = C) \\ &+ P(hot_4 \mid X_4 = D)\, P(X_4 = D \mid X_3 = C) + P(hot_4 \mid X_4 = E)\, P(X_4 = E \mid X_3 = C) \\ &+ 0 \\ &= 0 + 1 \cdot 0.8 + 0 \\ &= 0.8 \end{aligned}$$

$$P(hot_4 \mid hot_1, cold_2, cold_3) = 0.64$$

(4) Predilction

$$P(X_4 \mid hot_1, cold_2, cold_3) = \sum_{X_3} P(X_3 \mid hot_1, cold_2, cold_3)\, P(X_4 \mid X_3)$$

According to (1), $P(X_3 \mid hot_1, cold_2, cold_3) = \begin{array}{cccccc} A & B & C & D & E & F \end{array}$ $[0, 0.2, 0.8, 0, 0, 0]$

Through observation, we observe that only entry "B" "C" have non-zero value.

$$P(X_3 = B \mid hot_1, cold_2, cold_3)\, P(X_4 = A \mid X_3 = B) + P(X_3 = C \mid hot_1, cold_2, cold_3)\, P(X_4 = A \mid X_3 = C)$$

$= 0 + 0$

$= 0$

$$P(X_3 = B \mid hot_1, cold_2, cold_3)\, P(X_4 = B \mid X_3 = B) + P(X_3 = C \mid hot_1, cold_2, cold_3)\, P(X_4 = B \mid X_3 = C)$$

$= 0.2 \cdot 0.2 + 0$

$= 0.04$

$$P(X_3 = B \mid hot_1, cold_2, cold_3)\, P(X_4 = C \mid X_3 = B) + P(X_3 = C \mid hot_1, cold_2, cold_3)\, P(X_4 = C \mid X_3 = C)$$

$= 0.2 \cdot 0.8 + 0.8 \cdot 0.2$

$= 0.16 + 0.16$

$= 0.32$

$$P(X_3 = B \mid hot_1, cold_2, cold_3)\, P(X_4 = D \mid X_3 = B) + P(X_3 = C \mid hot_1, cold_2, cold_3)\, P(X_4 = D \mid X_3 = C)$$

$$= 0 + 0.8 \cdot 0.8$$

$$= 0.64$$

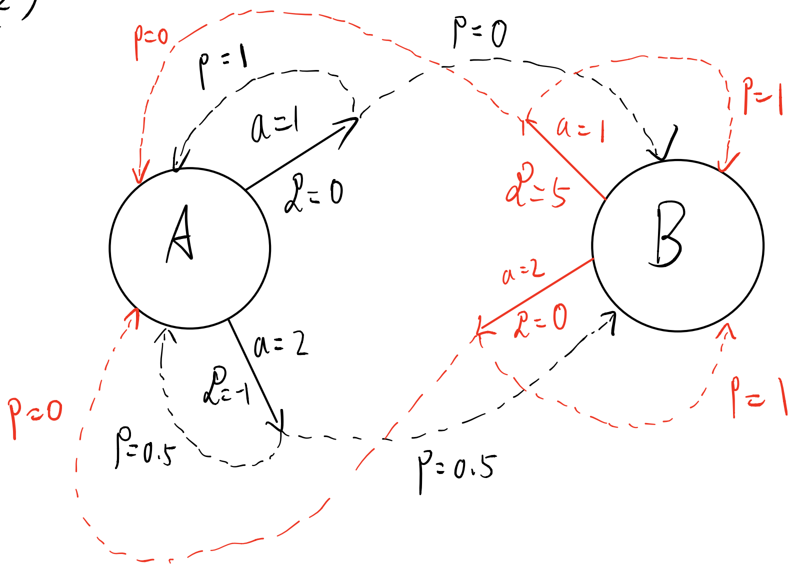
$$P(X_4 \mid hot_1, cold_2, cold_3) = [0,\ 0.04,\ 0.32,\ 0.64,\ 0,\ 0]$$

P_2

(1) $V_{k+1}(s) = R(s, a) + r \sum_{s' \in S} T(s, a, s') V_k(s')$

$V_{k+1}(s) = R(s, \pi_t(s)) + r \sum_{s' \in S} T(s, \pi_t(s), s') V_k(s')$

(2)



State	V_0^π	V_1^π	V_2^π
A	0	0	0
B	0	5	10

$0 + 1[0 \cdot 0 + 1 \cdot 0] = 0, \quad 5 + 1[0 + 1 \cdot 0] = 5$

$0 + 1[0 + 1 \cdot 0] = 0, \quad 5 + 1[0 + 1 \cdot 5] = 10$

(3) $Q(A, 1) = 0 + 1[0 + 0] = 0$

$Q(A, 2) = -1 + 1[0.5 \cdot 0 + 0.5 \cdot 10] = -1 + 5 = 4$

$\because 4 > 0$, $\pi(A) = 2$

$Q(B, 1) = 5 + 1[0 + 1 \cdot 10] = 5 + 10 = 15$

$Q(B, 2) = 0 + 1[0 + 1 \cdot 10] = 10$

$\because 10 < 15$, $\pi(B) = 1$ remain unchanged.