CS440 Assignment2

Yeoun Chan Kim   John Strauser   Xuanang Wang

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P1

A)
All five True = P(A,B,C,D,E) = P(A)P(B)P(C)P(D|A+B)P(E|B+C)
P(A) = 0.2
P(B) = 0.5
P(C) = 0.8
P(D|A+B) = 0.1
P(E|B+C) = 0.3
P(A,B,C,D,E) = 0.2*0.5*0.8*0.1*0.3 = 0.0024

B)
All five false = P(!A,!B,!C,!D,!E) = P(!A)P(!B)P(!C)P(!D|!A+!B)P(!E|!B+!C)
P(!A) = 0.8
P(!B) = 0.5
P(!C) = 0.2
P(!D|!A+!B) = 0.1
P(!E|!B+!C) = 0.8
P(!A,!B,!C,!D,!E) = 0.8*0.5*0.2*0.1*0.8 = 0.0064

C)
P(!A|B+C+D+E) = P(!A+B+C+D+E) / P(B+C+D+E)
P(!A+B+C+D+E) = P(!A)P(B)P(C)P(D|!A+B)P(E|B+C) = 0.8*0.5*0.8*0.6*0.3 = 0.0576
P(B+C+D+E) = P(A,B,C,D,E) + P(!A,B,C,D,E) = 0.0024 + 0.0576 = 0.06
P(!A|B+C+D+E) = 0.0576 / 0.06 = 0.96

P2

A)
P(Burglary|JohnCalls + MaryCalls) = P(B,J,M,A,E) + P(B,J,M,A,!E) + P(B,J,M,!A,E) + P(B,J,M,!A,!E) / P(B,J,M) + P(!B,J,M)

P(B,J,M,A,E) = P(B)P(J|A)P(M|A)P(A|B+E)P(E) = 0.001*0.9*0.7*0.95*0.002 = 0.000001197

P(B,J,M,A,!E) = P(B)P(J|A)P(M|A)P(A|B+!E)P(!E) = 0.001*0.9*0.7*0.94*0.998 = 0.000591

P(B,J,M,!A,E) = P(B)P(J|!A)P(M|!A)P(-A|B+E)P(E) = 0.001*0.05*0.01*0.05*0.002 = 0.000000000005

P(B,J,M,!A,!E) = P(B)P(J|!A)P(M|!A)P(!A|B+!E)P(!E) = 0.001*0.05*0.01*0.06*0.998 = 0.0000000299

P(B,J,M) = P(B,J,M,A,E) + P(B,J,M,A,!E) + P(B,J,M,!A,E) + P(B,J,M,!A,!E) = 0.000001197 + 0.000591 + 0.000000000005 + 0.0000000299 = 0.000676

P(!B,J,M) = P(!B,J,M,A,E) + P(!B,J,M,A,!E) + P(!B,J,M,!A,E) + P(!B,J,M,!A,!E)

P(!B,J,M,A,E) = P(!B)P(J|A)P(M|A)P(A|!B+E)P(E) = 0.999 * 0.9 * 0.7 * 0.29 * 0.002 = 0.000365

P(!B,J,M,A,!E) = P(!B)P(J|A)P(M|A)P(A|!B+!E)P(!E) = 0.999 * 0.9 * 0.7 * 0.001 * 0.998 = 0.000628

P(!B,J,M,!A,E) = P(!B)P(J|!A)P(M|!A)P(!A|!B+E)P(E) = 0.999 * 0.05 * 0.01 * 0.71 * 0.002 = 0.000000709

P(!B,J,M,!A,!E) = P(!B)P(J|!A)P(M|!A)P(!A|!B+!E)P(!E) = 0.999 * 0.05 * 0.01 * 0.999 * 0.998 = 0.000498

P(!B,J,M) = 0.000365 + 0.000628 + 0.000000709 + 0.000498 = 0.00149

P(Burglary|JohnCalls + MaryCalls) = (0.000676)/(0.000676 + 0.00149) = 0.312

B) enumeration tree would be a straight line. Therefore worst case is that all n variables must be checked.

complexity using enumeration = O(n)

complexity using variable elimination = O(n)

P3

A)

OC : card holder owns a computer or smart phone.
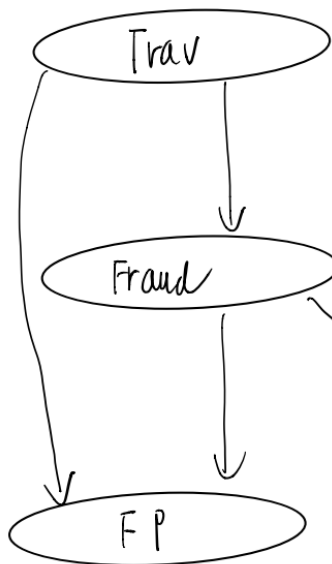
Fraud : current transaction is fraudulent.

Trav : card holder is currently travelling.

FP : current transaction is a foreign purchase.

IP : current purchase is an internet purchase.

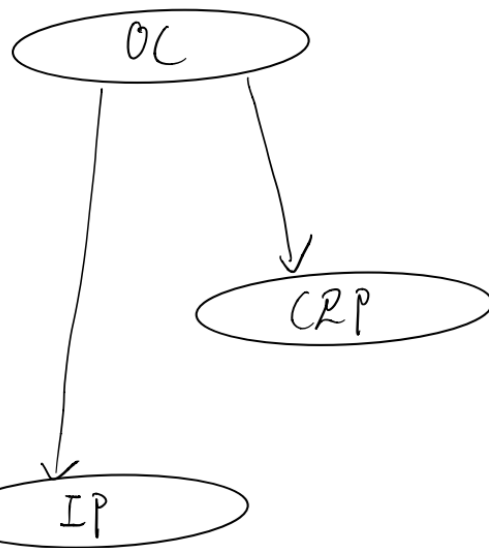CRP : a computer related purchase was made in the past week

$P(\text{trav}) = 0.05$

$P(OC) = 0.75$

Trav

OC

Trav	P(Fraud)
t	0.01
f	0.004

OC	P(CRP)
t	0.1
f	0.001

Fraud

CRP

FP

IP

Trav	Fraud	P(FP)
t	t	0.9
t	f	0.9
f	t	0.1
f	f	0.01

OC	Fraud	P(IP)
t	t	0.02
t	f	0.01
f	t	0.011
f	f	0.001

B)

P(Fraud) = (P(Fraud + Trav) + P(Fraud + !Trav)) / (P(Fraud) + P(!Fraud))

P(Fraud) = (P(Fraud|Trav)P(Trav) + P(Fraud|!Trav)P(!Trav)) / (1)

P(Fraud) = 0.01*0.05 + 0.004*0.95

P(Fraud) = 0.0043

P(Fraud | FP + !IP + CRP) = (P(Fraud + FP + !IP + CRP + Trav + OC) + P(Fraud + FP + !IP + CRP + Trav + !OC) + P(Fraud + FP + !IP + CRP + !Trav + OC) + P(Fraud + FP + !IP + CRP + !Trav + !OC))/(P(Fraud + FP + !IP + CRP) + P(!Fraud + FP + !IP + CRP + Trav + OC))

P(Fraud + FP + !IP + CRP + Trav + OC) = P(Fraud|Trav)P(FP|Trav+Fraud)P(!IP|OC+Fraud)P(CRP|OC)P(Trav)P(OC)

= 0.01 * 0.9 * 0.98 * 0.1 * 0.05 * 0.75

= 0.00003308

P(Fraud + FP + !IP + CRP + Trav + !OC) = P(Fraud|Trav)P(FP|Trav+Fraud)P(!IP|!OC+Fraud)P(CRP|!OC)P(Trav)P(!OC)

= 0.01 * 0.9 * 0.989 * 0.001 * 0.05 * 0.25 = 0.0000001112

P(Fraud + FP + !IP + CRP + !Trav + OC) = P(Fraud|!Trav)P(FP|!Trav+Fraud)P(!IP|OC+Fraud)P(CRP|OC)P(!Trav)P(OC)

= 0.004 * 0.1 * 0.98 * 0.1 * 0.95 * 0.75 = 0.00002793

P(Fraud + FP + !IP + CRP + !Trav + !OC) = P(Fraud|!Trav)P(FP|!Trav+Fraud)P(!IP|!OC+Fraud)P(CRP|!OC)P(!Trav)P(!OC)

= 0.004 * 0.1 * 0.989 * 0.001 * 0.95 * 0.25 = 0.000000093955

P(Fraud + FP + !IP + CRP) = P(Fraud + FP + !IP + CRP + Trav + OC) + P(Fraud + FP + !IP + CRP + Trav + !OC) + P(Fraud + FP + !IP + CRP + !Trav + OC) + P(Fraud + FP + !IP + CRP + !Trav + !OC)

= 0.00003308 + 0.0000001112 + 0.00002793 + 0.000000093955 = 0.000061215155

P(!Fraud + FP + !IP + CRP) = P(!Fraud + FP + !IP + CRP + Trav + OC) + P(!Fraud + FP + !IP + CRP + Trav + !OC) + P(!Fraud + FP + !IP + CRP + !Trav + OC) + P(!Fraud + FP + !IP + CRP + !Trav + !OC)

P(!Fraud + FP + !IP + CRP + Trav + OC) = P(!Fraud|Trav)P(FP|Trav+!Fraud)P(!IP|OC+!Fraud)P(CRP|OC)P(Trav)P(OC)

= 0.99 * 0.9 * 0.99 * 0.1 * 0.05 * 0.75 = 0.0033078375

P(!Fraud + FP + !IP + CRP + Trav + !OC) = P(!Fraud|Trav)P(FP|Trav+!Fraud)P(!IP|!OC+!Fraud)P(CRP|!OC)P(Trav)P(!OC)

= 0.99 * 0.9 * 0.999 * 0.001 * 0.05 * 0.25 = 0.000011126

P(!Fraud + FP + !IP + CRP + !Trav + OC) = P(!Fraud|!Trav)P(FP|!Trav+!Fraud)P(!IP|OC+!Fraud)P(CRP|OC)P(!Trav)P(OC)

= 0.996 * 0.01 * 0.99 * 0.1 * 0.95 * 0.75 = 0.00070255

P(!Fraud + FP + !IP + CRP + !Trav + !OC) = P(!Fraud|!Trav)P(FP|!Trav+!Fraud)P(!IP|!OC+!Fraud)P(CRP|!OC)P(!Trav)P(!OC)

= 0.996 * 0.01 * 0.999 * 0.001 * 0.95 * 0.25 = 0.000002363

P(!Fraud,FP,!IP,CRP) = 0.0033078375 + 0.000011126 + 0.00070255 + 0.000002363 = 0.0040238765

P(Fraud | FP + !IP + CRP) = (0.000061215155)/(0.000061215155 + 0.0040238765) = 0.01498501384