

# CS440 Assignment2

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## P1

A)

All five True =  $P(A,B,C,D,E) = P(A)P(B)P(C)P(D|A+B)P(E|B+C)$

$P(A) = 0.2$

$P(B) = 0.5$

$P(C) = 0.8$

$P(D|A+B) = 0.1$

$P(E|B+C) = 0.3$

$P(A,B,C,D,E) = 0.2*0.5*0.8*0.1*0.3 = 0.0024$

B)

All five false =  $P(!A,!B,!C,!D,!E) = P(!A)P(!B)P(!C)P(!D|!A+!B)P(!E|!B+!C)$

$P(!A) = 0.8$

$P(!B) = 0.5$

$P(!C) = 0.2$

$P(!D|!A+!B) = 0.1$

$P(!E|!B+!C) = 0.8$

$P(!A,!B,!C,!D,!E) = 0.8*0.5*0.2*0.1*0.8 = 0.0064$

C)

$P(!A|B+C+D+E) = P(!A+B+C+D+E) / P(B+C+D+E)$

$P(!A+B+C+D+E) = P(!A)P(B)P(C)P(D|!A+B)P(E|B+C) = 0.8*0.5*0.8*0.6*0.3$   
 $= 0.0576$

$P(B+C+D+E) = P(B)P(C)P(D|B)P(E|B+C) = 0.5*0.8*0.7*0.3 = 0.084$

$P(!A|B+C+D+E) = 0.0576 / 0.084 = 0.6857$

## P2

A)

$P(\text{Burglary}|\text{JohnCalls} + \text{MaryCalls}) = P(B,J,M,A,E) + P(B,J,M,A,!E) +$   
 $P(B,J,M,!A,E) + P(B,J,M,!A,!E) / P(B,J,M) + P(!B,J,M)$

$$\begin{aligned}
& P(B,J,M,A,E) = P(B)P(J|A)P(M|A)P(A|B+E)P(E) = 0.001*0.9*0.7*0.95*0.002 \\
& = 0.000001197 \\
& P(B,J,M,A,!E) = P(B)P(J|A)P(M|A)P(A|B+!E)P(!E) = 0.001*0.9*0.7*0.94*0.998 \\
& = 0.000591 \\
& P(B,J,M,!A,E) = P(B)P(J|!A)P(M|!A)P(-A|B+E)P(E) = 0.001*0.05*0.01*0.05*0.002 \\
& = 0.0000000000005 \\
& P(B,J,M,!A,!E) = P(B)P(J|!A)P(M|!A)P(!A|B+!E)P(!E) = 0.001*0.05*0.01*0.06*0.998 \\
& = 0.00000000299 \\
& P(B,J,M) = P(B)(P(J|A)+P(J|!A))(P(M|A)+P(M|!A)) = 0.001*(0.9+0.05)*(0.7+0.01) \\
& = 0.0006745 \\
& P(-B,J,M) = P(-B)(P(J|A)+P(J|!A))(P(M|A)+P(M|!A)) = 0.999*(0.9+0.05)*(0.7+0.01) \\
& = 0.674 \\
& P(\text{Burglary}|\text{JohnCalls} + \text{MaryCalls}) = (0.000001197 + 0.000591 + 0.0000000000005 \\
& + 0.00000000299)/(0.0006745 + 0.674) = 0.001
\end{aligned}$$

B) enumeration tree would be a straight line. Therefore worst case is that all n variables must be checked.

complexity using enumeration =  $O(n)$

complexity using variable elimination =  $O(n)$

## P3

A)

OC : card holder owns a computer or smart phone.

Fraud : current transaction is fraudulent.

Trav : card holder is currently travelling.

FP : current transaction is a foreign purchase.

IP : current purchase is an internet purchase.

CRP : a computer related purchase was made in the past week

Trav | P(Fraud)

True 0.01

False 0.004

$P(\text{Trav}) = 0.05$

Trav Fraud | P(FP)

True True 0.9

True False 0.9

False True 0.1

False False 0.01

$P(\text{OC}) = 0.75$

OC Fraud | P(IP)

True True 0.01

True False 0.02

False True 0.001

False False 0.011

OC | P(CRP)

True 0.1

False 0.001

B)

$$P(\text{Fraud}) = (P(\text{Fraud} + \text{Trav}) + P(\text{Fraud} + !\text{Trav})) / (P(\text{Fraud}) + P(!\text{Fraud}))$$

$$P(\text{Fraud}) = (P(\text{Fraud}|\text{Trav})P(\text{Trav}) + P(\text{Fraud}|\text{!Trav})P(!\text{Trav})) / (1)$$

$$P(\text{Fraud}) = 0.01 * 0.05 + 0.004 * 0.95$$

$$P(\text{Fraud}) = 0.0043$$

$$P(\text{Fraud} | \text{FP} + !\text{IP} + \text{CRP}) = (P(\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP} + \text{Trav} + \text{OC}) + P(\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP} + \text{Trav} + !\text{OC}) + P(\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP} + !\text{Trav} + \text{OC}) + P(\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP} + !\text{Trav} + !\text{OC})) / (P(\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP}) + P(!\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP} + \text{Trav} + \text{OC}))$$

$$P(\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP} + \text{Trav} + \text{OC}) = P(\text{Fraud}|\text{Trav})P(\text{FP}|\text{Trav}+\text{Fraud})P(!\text{IP}|\text{OC}+\text{Fraud})P(\text{CRP}|\text{OC}) \\ = 0.01 * 0.9 * 0.99 * 0.1 * 0.05 * 0.75 = 0.0000334125$$

$$P(\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP} + \text{Trav} + !\text{OC}) = P(\text{Fraud}|\text{Trav})P(\text{FP}|\text{Trav}+\text{Fraud})P(!\text{IP}|\text{OC}+\text{Fraud})P(\text{CRP}|\text{!OC})P(\text{Trav})P(!\text{OC}) = 0.01 * 0.9 * 0.999 * 0.001 * 0.05 * 0.25 = 0.0000001124$$

$$P(\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP} + !\text{Trav} + \text{OC}) = P(\text{Fraud}|\text{!Trav})P(\text{FP}|\text{!Trav}+\text{Fraud})P(!\text{IP}|\text{OC}+\text{Fraud})P(\text{CRP}|\text{OC}) \\ = 0.004 * 0.1 * 0.99 * 0.1 * 0.95 * 0.75 = 0.000028215$$

$$P(\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP} + !\text{Trav} + !\text{OC}) = P(\text{Fraud}|\text{!Trav})P(\text{FP}|\text{!Trav}+\text{Fraud})P(!\text{IP}|\text{OC}+\text{Fraud})P(\text{CRP}|\text{!OC})P(!\text{Trav})P(!\text{OC}) = 0.004 * 0.1 * 0.999 * 0.001 * 0.95 * 0.25 = 0.0000000949$$

$$P(\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP}) = (P(\text{Fraud}|\text{Trav}) + P(\text{Fraud}|\text{!Trav}))(P(\text{FP}|\text{Trav}+\text{Fraud}) + P(\text{FP}|\text{!Trav}+\text{Fraud})) \\ = (0.01 + 0.004)(0.9 + 0.1)(0.99 + 0.999)(0.1 + 0.001) = 0.014 * 1 * 1.989 * 0.101 = 0.002812446$$

$$P(!\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP}) = (P(!\text{Fraud}|\text{Trav}) + P(!\text{Fraud}|\text{!Trav}))(P(\text{FP}|\text{Trav}+\text{!Fraud}) + P(\text{FP}|\text{!Trav}+\text{!Fraud})) \\ = (0.99 + 0.996)(0.9 + 0.01)(0.98 + 0.989)(0.1 + 0.001) = 1.986 * 0.91 * 1.969 * 0.101 = 0.3594079889$$

$$P(\text{Fraud} | \text{FP} + !\text{IP} + \text{CRP}) = (0.0000334125 + 0.0000001124 + 0.000028215 + 0.0000000949) / (0.002812446 + 0.3594079889) = 0.00017$$