P_1

$X_t \in \{A, B, C, D, E, F\}$ $\quad X_1 = A$

$E_t \in [hot, cold]$

$X_t \backslash X_{t+1}$		A	B	C	D	E	F
X_1	A	0.2	0.8	0	0	0	0
X_2	B	0	0.2	0.8	0	0	0
X_3	C	0	0	0.2	0.8	0	0
X_4	D	0	0	0	0.2	0.8	0
X_5	E	0	0	0	0	0.2	0.8
X_6	F	0	0	0	0	0	0.2

$X_t \backslash e_t$	hot	cold
A	1	0
B	0	1
C	0	1
D	1	0
E	0	1
F	0	1

transition $P(X_t \mid X_{t-1})$

observation $P(e_t \mid X_t)$

· (1) filtering

$$P(X_3 \mid hot_1, cold_2, cold_3) \propto P(cold_3 \mid X_3) \sum_{X_2} P(X_3 \mid X_2) \underline{P(X_2 \mid hot_1, cold_2)}$$

$$P(X_2 \mid hot_1, cold_2) \propto P(cold_2 \mid X_2) \sum_{X_1} P(X_2 \mid X_1) \underline{P(X_1 \mid hot_1)}$$

$$P(X_1 \mid hot_1) \propto P(hot_1 \mid X_1) P(X_1)$$

$$\propto [P(hot_1 \mid X_1 = A), P(hot_1 \mid X_1 = B), \cdots C, \cdots D, \cdots E, \cdots F] \cdot$$
$$[P(X_1 = A), P(X_1 = B), \cdots C, \cdots D, \cdots E, \cdots F]$$

$$\propto [1 \cdot 1, 0, 0, 0, 0, 0]$$

$$= [1, 0, 0, 0, 0, 0]$$

$$P(cold_2 \mid X_2) = \begin{array}{cccccc} A & B & C & D & E & F \\ \end{array}$$
$$[0, 1, 1, 0, 1, 1]$$

We found out that only entry of B, C, E, F is not multiplied by zero.

$$P(cold_2 \mid X_2 = B) \cdot \left[P(X_2 = B \mid X_1 = A) P(X_1 = A \mid hot_1) + 0 + \dots + 0 \right]$$

$$= 1 \cdot [0.8 \cdot 1]$$

$$= 0.8$$

$$P(cold_2 \mid X_2 = C) \cdot \left[P(X_2 = C \mid X_1 = A) P(X_1 = A \mid hot_1) + 0 \dots + 0 \right]$$

$$= 1 \cdot [0 \cdot 1]$$

$$= 0$$

Therefore, according to observation, only $X_2 = B$ is not multiplied by zero.

$$P(X_2 \mid hot_1, cold_2) = \begin{array}{c} \begin{matrix} A & B & C & D & E & F \end{matrix} \\ \begin{bmatrix} 0, & 1, & 0, & 0, & 0, & 0 \end{bmatrix} \end{array}$$

$$P(cold_3 \mid X_3) = \begin{array}{c} \begin{matrix} A & B & C & D & E & F \end{matrix} \\ \begin{bmatrix} 0, & 1, & 1, & 0, & 1, & 1 \end{bmatrix} \end{array}$$

We found out that only entry of B, C, E, F is not multiplied by zero.

$$P(cold_3 \mid X_3 = B) \left[0 + P(X_3 = B \mid X_2 = B) P(X_2 = B \mid hot_1, cold_2) + 0 + \dots + 0 \right]$$

$$= 1 \cdot [0.2 \cdot 1]$$

$$= 0.2$$

$$P(cold_3 \mid X_3 = C)\,[\,0 + P(X_3 = C \mid X_2 = B)\,P(X_2 = B \mid hot_1, cold_2) + 0 + \dots + 0\,]$$

$$= 1 \cdot [\,0.8 \cdot 1\,]$$

$$= 0.8$$

$$P(cold_3 \mid X_3 = E)\,[\,0 + P(X_3 = E \mid X_2 = B)\,P(X_2 = B \mid hot_1, cold_2) + 0 + \dots + 0\,]$$

$$= 0$$

$$P(X_3 \mid hot_1, cold_2, cold_3) = [\,0,\ 0.2,\ 0.8,\ 0,\ 0,\ 0\,]$$