

CS440 Assignment2

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P1

A)

All five True = $P(A,B,C,D,E) = P(A)P(B)P(C)P(D|A+B)P(E|B+C)$

$P(A) = 0.2$

$P(B) = 0.5$

$P(C) = 0.8$

$P(D|A+B) = 0.1$

$P(E|B+C) = 0.3$

$P(A,B,C,D,E) = 0.2*0.5*0.8*0.1*0.3 = 0.0024$

B)

All five false = $P(!A,!B,!C,!D,!E) = P(!A)P(!B)P(!C)P(!D|!A+!B)P(!E|!B+!C)$

$P(!A) = 0.8$

$P(!B) = 0.5$

$P(!C) = 0.2$

$P(!D|!A+!B) = 0.1$

$P(!E|!B+!C) = 0.8$

$P(!A,!B,!C,!D,!E) = 0.8*0.5*0.2*0.1*0.8 = 0.0064$

C)

$P(!A|B+C+D+E) = P(!A+B+C+D+E) / P(B+C+D+E)$

$P(!A+B+C+D+E) = P(!A)P(B)P(C)P(D|!A+B)P(E|B+C) = 0.8*0.5*0.8*0.6*0.3$
 $= 0.0576$

$P(B+C+D+E) = P(A,B,C,D,E) + P(!A,B,C,D,E) = 0.0024 + 0.0576 = 0.06$

$P(!A|B+C+D+E) = 0.0576 / 0.06 = 0.96$

P2

A)

$P(\text{Burglary} | \text{JohnCalls} + \text{MaryCalls}) = P(B,J,M,A,E) + P(B,J,M,A,!E) +$
 $P(B,J,M,!A,E) + P(B,J,M,!A,!E) / P(B,J,M) + P(!B,J,M)$

$$\begin{aligned}
P(B,J,M,A,E) &= P(B)P(J|A)P(M|A)P(A|B+E)P(E) = 0.001*0.9*0.7*0.95*0.002 \\
&= 0.000001197 \\
P(B,J,M,A,!E) &= P(B)P(J|A)P(M|A)P(A|B+!E)P(!E) = 0.001*0.9*0.7*0.94*0.998 \\
&= 0.000591 \\
P(B,J,M,!A,E) &= P(B)P(J|!A)P(M|!A)P(!A|B+E)P(E) = 0.001*0.05*0.01*0.05*0.002 \\
&= 0.0000000000005 \\
P(B,J,M,!A,!E) &= P(B)P(J|!A)P(M|!A)P(!A|B+!E)P(!E) = 0.001*0.05*0.01*0.06*0.998 \\
&= 0.0000000299 \\
P(B,J,M) &= P(B,J,M,A,E) + P(B,J,M,A,!E) + P(B,J,M,!A,E) + P(B,J,M,!A,!E) \\
&= 0.000001197 + 0.000591 + 0.0000000000005 + 0.0000000299 = 0.000592 \\
P(!B,J,M) &= P(!B,J,M,A,E) + P(!B,J,M,A,!E) + P(!B,J,M,!A,E) + P(!B,J,M,!A,!E) \\
P(!B,J,M,A,E) &= P(!B)P(J|A)P(M|A)P(A|!B+E)P(E) = 0.999 * 0.9 * 0.7 * \\
0.29 * 0.002 &= 0.000365 \\
P(!B,J,M,A,!E) &= P(!B)P(J|A)P(M|A)P(A|!B+!E)P(!E) = 0.999 * 0.9 * 0.7 \\
* 0.001 * 0.998 &= 0.000628 \\
P(!B,J,M,!A,E) &= P(!B)P(J|!A)P(M|!A)P(!A|!B+E)P(E) = 0.999 * 0.05 * \\
0.01 * 0.71 * 0.002 &= 0.000000709 \\
P(!B,J,M,!A,!E) &= P(!B)P(J|!A)P(M|!A)P(!A|!B+!E)P(!E) = 0.999 * 0.05 * \\
0.01 * 0.999 * 0.998 &= 0.000498 \\
P(!B,J,M) &= 0.000365 + 0.000628 + 0.000000709 + 0.000498 = 0.00149 \\
P(Burglary|JohnCalls + MaryCalls) &= (0.000592)/(0.000592 + 0.00149) = \\
0.284
\end{aligned}$$

B) enumeration tree would be a straight line. Therefore worst case is that all n variables must be checked.

complexity using enumeration = $O(n)$

complexity using variable elimination = $O(n)$

P3

A)

OC : card holder owns a computer or smart phone.

Fraud : current transaction is fraudulent.

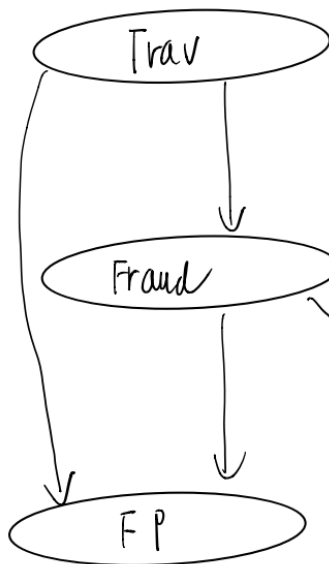
Trav : card holder is currently travelling.

FP : current transaction is a foreign purchase.

IP : current purchase is an internet purchase.

CRP : a computer related purchase was made in the past week

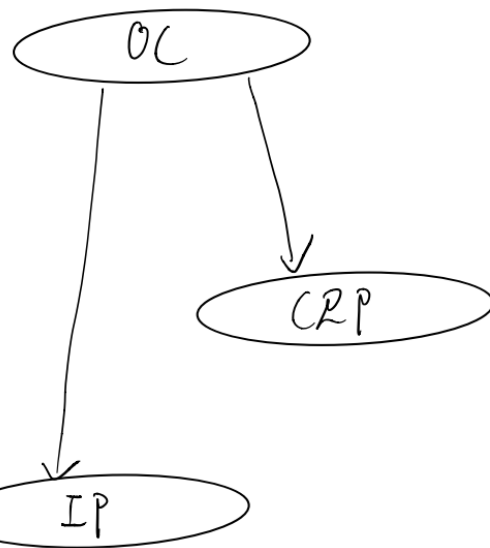
$$P(\text{trav}) = 0.05$$



Trav	P(Fraud)
t	0.01
f	0.004

Trav	Fraud	P(FP)
t	t	0.9
t	f	0.9
f	t	0.1
f	f	0.01

$$P(OC) = 0.75$$



OC	P(CRP)
t	0.1
f	0.001

OC	Fraud	P(IP)
t	t	0.02
t	f	0.01
f	t	0.011
f	f	0.001

$$\begin{aligned}
& B) \\
& P(\text{Fraud}) = (P(\text{Fraud} + \text{Trav}) + P(\text{Fraud} + !\text{Trav})) / (P(\text{Fraud}) + P(!\text{Fraud})) \\
& P(\text{Fraud}) = (P(\text{Fraud}|\text{Trav})P(\text{Trav}) + P(\text{Fraud}|\text{!Trav})P(!\text{Trav})) / (1) \\
& P(\text{Fraud}) = 0.01 * 0.05 + 0.004 * 0.95 \\
& P(\text{Fraud}) = 0.0043 \\
& P(\text{Fraud} | \text{FP} + !\text{IP} + \text{CRP}) = (P(\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP} + \text{Trav} + \text{OC}) \\
& + P(\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP} + \text{Trav} + !\text{OC}) + P(\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP} \\
& + !\text{Trav} + \text{OC}) + P(\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP} + !\text{Trav} + !\text{OC})) / (P(\text{Fraud} + \\
& \text{FP} + !\text{IP} + \text{CRP}) + P(!\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP} + \text{Trav} + \text{OC})) \\
& P(\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP} + \text{Trav} + \text{OC}) = P(\text{Fraud}|\text{Trav})P(\text{FP}|\text{Trav}+\text{Fraud}) \\
& P(!\text{IP}|\text{OC}+\text{Fraud})P(\text{CRP}|\text{OC})P(\text{Trav})P(\text{OC}) \\
& = 0.01 * 0.9 * 0.98 * 0.1 * 0.05 * 0.75 \\
& = 0.00003308 \\
& P(\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP} + \text{Trav} + !\text{OC}) = P(\text{Fraud}|\text{Trav})P(\text{FP}|\text{Trav}+\text{Fraud}) \\
& P(!\text{IP}|\text{OC}+\text{Fraud})P(\text{CRP}|\text{!OC})P(\text{Trav})P(!\text{OC}) \\
& = 0.01 * 0.9 * 0.989 * 0.001 * 0.05 * 0.25 = 0.0000001112 \\
& P(\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP} + !\text{Trav} + \text{OC}) = P(\text{Fraud}|\text{!Trav})P(\text{FP}|\text{!Trav}+\text{Fraud}) \\
& P(!\text{IP}|\text{OC}+\text{Fraud})P(\text{CRP}|\text{OC})P(!\text{Trav})P(\text{OC}) \\
& = 0.004 * 0.1 * 0.98 * 0.1 * 0.95 * 0.75 = 0.00002793 \\
& P(\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP} + !\text{Trav} + !\text{OC}) = P(\text{Fraud}|\text{!Trav})P(\text{FP}|\text{!Trav}+\text{Fraud}) \\
& P(!\text{IP}|\text{OC}+\text{Fraud})P(\text{CRP}|\text{!OC})P(!\text{Trav})P(!\text{OC}) \\
& = 0.004 * 0.1 * 0.989 * 0.001 * 0.95 * 0.25 = 0.000000093955 \\
& P(\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP}) = P(\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP} + \text{Trav} + \\
& \text{OC}) + P(\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP} + \text{Trav} + !\text{OC}) + P(\text{Fraud} + \text{FP} + !\text{IP} + \\
& \text{CRP} + !\text{Trav} + \text{OC}) + P(\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP} + !\text{Trav} + !\text{OC}) \\
& = 0.00003308 + 0.0000001112 + 0.00002793 + 0.000000093955 = 0.000061215155 \\
& P(!\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP}) = P(!\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP} + \text{Trav} + \\
& \text{OC}) + P(!\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP} + \text{Trav} + !\text{OC}) + P(!\text{Fraud} + \text{FP} + !\text{IP} \\
& + \text{CRP} + !\text{Trav} + \text{OC}) + P(!\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP} + !\text{Trav} + !\text{OC}) \\
& P(!\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP} + \text{Trav} + \text{OC}) = P(!\text{Fraud}|\text{Trav})P(\text{FP}|\text{Trav}+\text{!Fraud}) \\
& P(!\text{IP}|\text{OC}+\text{!Fraud})P(\text{CRP}|\text{OC})P(\text{Trav})P(\text{OC}) \\
& = 0.99 * 0.9 * 0.99 * 0.1 * 0.05 * 0.75 = 0.0033078375 \\
& P(!\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP} + \text{Trav} + !\text{OC}) = P(!\text{Fraud}|\text{Trav})P(\text{FP}|\text{Trav}+\text{!Fraud}) \\
& P(!\text{IP}|\text{OC}+\text{!Fraud})P(\text{CRP}|\text{!OC})P(\text{Trav})P(!\text{OC}) \\
& = 0.99 * 0.9 * 0.999 * 0.001 * 0.05 * 0.25 = 0.000011126 \\
& P(!\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP} + !\text{Trav} + \text{OC}) = P(!\text{Fraud}|\text{!Trav})P(\text{FP}|\text{!Trav}+\text{!Fraud}) \\
& P(!\text{IP}|\text{OC}+\text{!Fraud})P(\text{CRP}|\text{OC})P(!\text{Trav})P(\text{OC}) \\
& = 0.996 * 0.01 * 0.99 * 0.1 * 0.95 * 0.75 = 0.00070255 \\
& P(!\text{Fraud} + \text{FP} + !\text{IP} + \text{CRP} + !\text{Trav} + !\text{OC}) = P(!\text{Fraud}|\text{!Trav})P(\text{FP}|\text{!Trav}+\text{!Fraud}) \\
& P(!\text{IP}|\text{OC}+\text{!Fraud})P(\text{CRP}|\text{!OC})P(!\text{Trav})P(!\text{OC}) \\
& = 0.996 * 0.01 * 0.999 * 0.001 * 0.95 * 0.25 = 0.000002363 \\
& P(!\text{Fraud}, \text{FP}, !\text{IP}, \text{CRP}) = 0.0033078375 + 0.000011126 + 0.00070255 + 0.000002363 \\
& = 0.0040238765 \\
& P(\text{Fraud} | \text{FP} + !\text{IP} + \text{CRP}) = (0.000061215155) / (0.000061215155 + 0.0040238765) \\
& = 0.01498501384
\end{aligned}$$