Call Option Prices Assuming a Log-normal Distribution

Stephen P. Smith
July 2017

Definitions

P_0 = stock price at option purchase
D = days to expiration
v = daily implied volatility on log scale
S = strike price

Assumptions

The stock price is assumed to possess independent increments and follow the log-normal distribution. That is, the stock price D days past purchase date (denoted by P_D) is assume to follow a log-normal distribution with $\mu=\log_e(P_0)$ and $\sigma^2=D\times v$.

The value of the call option D days past expiration, denoted by O_D, is

$$O_D = \begin{cases} P_D - S, & if\ P_D > S \\ 0, & otherwise \end{cases}$$

Option Expectation

The expectation of O_D is evaluated from the probability density of a truncated log-normal distribution.

$$E[O_D] = \int_{\log(S)} (e^x - S)\frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}dx$$

This expectation is evaluated using an identity described by Tallis (1961) for the moment generating function of the truncated normal distribution, producing the following:

$$E[O_D] = e^{\mu+\frac{\sigma^2}{2}}\left[\frac{1-\Phi\left(\frac{\log_e S-\mu}{\sigma}-\sigma\right)}{1-\Phi\left(\frac{\log_e S-\mu}{\sigma}\right)}\right] - S\times\Phi\left(\frac{\log_e S-\mu}{\sigma}\right)$$

where Φ(T) is the commutative probability distribution function for the standard normal distribution; i.e., $\Phi(T)=pr(z<T)$, $z \sim N(0,1)$.

References

Tallis, G.M., 1961, The moment generating function of the Truncated Multi-Normal Distribution, *Journal of the Royal Statistical Society, Series B*, 23: 223-229.