Let us explore an alternative formulation which is applicable in the case where the $\mathbf{X}_i$'s are invertible matrices. As we will show below this case would apply when the number of parameters are the same as the number of measurements and the transformation from parameter space to measurement space, represented by $Y = \mathbf{X}\beta$, is one-to-one and onto. Consider the following Matrix Identity

$$\mathbf{R}^{-1} + \mathbf{S}^{-1} = \mathbf{R}^{-1}(\mathbf{R}+\mathbf{S})\mathbf{S}^{-1} = \mathbf{S}^{-1}(\mathbf{R}+\mathbf{S})\mathbf{R}^{-1}$$
$$[\mathbf{R}^{-1} + \mathbf{S}^{-1}]^{-1} = \mathbf{S}(\mathbf{R}+\mathbf{S})^{-1}\mathbf{R} = \mathbf{R}(\mathbf{R}+\mathbf{S})^{-1}\mathbf{S} \quad \text{by symmetry}$$

Using the above formulation for this special case, and letting $\mathbf{R} = \mathbf{A}_N = \big(\sum_{i=1}^{N} \mathbf{X}_i^T \mathbf{Cov}_i^{-1}\mathbf{X}_i\big)^{-1}$, $\mathbf{S} = \big(\mathbf{X}_{N+1}^T \mathbf{Cov}_{N+1}^{-1}\mathbf{X}_{N+1}\big)^{-1}$ and $\hat\beta_{N+1} = \mathbf{A}_{N+1}\big(\sum_{i=1}^{N+1} \mathbf{X}_i^T \mathbf{Cov}_i^{-1} Y_i\big)$, we can write

$$\begin{aligned}
\mathbf{A}_{N+1} &= \big(\sum_{i=1}^{N+1} \mathbf{X}_i^T \mathbf{Cov}_i^{-1}\mathbf{X}_i\big)^{-1} \quad \text{by definition}\\
&= \mathbf{A}_N\big(\mathbf{A}_N + \mathbf{X}_{N+1}^{-1}\mathbf{Cov}_{N+1}(\mathbf{X}_{N+1}^T)^{-1}\big)^{-1}\mathbf{X}_{N+1}^{-1}\mathbf{Cov}_{N+1}(\mathbf{X}_{N+1}^T)^{-1} \quad \text{using the above Identity}\\
&= \mathbf{X}_{N+1}^{-1}\mathbf{Cov}_{N+1}(\mathbf{X}_{N+1}^T)^{-1}\big(\mathbf{A}_N + \mathbf{X}_{N+1}^{-1}\mathbf{Cov}_{N+1}(\mathbf{X}_{N+1}^T)^{-1}\big)^{-1}\mathbf{A}_N \quad \text{alternate form true by symmetry}\\
\hat\beta_{N+1} &= \mathbf{A}_{N+1}\big(\sum_{i=1}^{N+1} \mathbf{X}_i^T \mathbf{Cov}_i^{-1} Y_i\big) \quad \text{by definition}
\end{aligned}$$

$$\begin{aligned}
\hat\beta_{N+1} &= \big(\sum_{i=1}^{N} \mathbf{X}_i^T \mathbf{Cov}_i^{-1}\mathbf{X}_i + \mathbf{X}_{N+1}^T\mathbf{Cov}_{N+1}^{-1}\mathbf{X}_{N+1}\big)^{-1}\big(\sum_{i=1}^{N} \mathbf{X}_i^T \mathbf{Cov}_i^{-1} Y_i + \mathbf{X}_{N+1}^T\mathbf{Cov}_{N+1}^{-1}Y_{N+1}\big) \quad \text{by definition}\\
&= \mathbf{X}_{N+1}^{-1}\mathbf{Cov}_{\mathbf{N+1}}(\mathbf{X}_{N+1}^T)^{-1}\big(\mathbf{A}_N + \mathbf{X}_{N+1}^{-1}\mathbf{Cov}_{N+1}(\mathbf{X}_{N+1}^T)^{-1}\big)^{-1}\mathbf{A}_N\sum_{i=1}^{N} \mathbf{X}_i^T \mathbf{Cov}_i^{-1} Y_i \quad \text{separating the terms}\\
&+ \mathbf{A}_N\big(\mathbf{A}_N + \mathbf{X}_{N+1}^{-1}\mathbf{Cov}_{N+1}(\mathbf{X}_{N+1}^T)^{-1}\big)^{-1}\mathbf{X}_{N+1}^{-1}\mathbf{Cov}_{N+1}(\mathbf{X}_{N+1}^T)^{-1}\mathbf{X}_{N+1}^T\mathbf{Cov}_{N+1}^{-1}Y_{N+1} \quad \text{and using symmetry}
\end{aligned}$$

$$\begin{aligned}
\hat\beta_{N+1} &= \mathbf{X}_{N+1}^{-1}\mathbf{Cov}_{\mathbf{N+1}}(\mathbf{X}_{N+1}^T)^{-1}\big(\mathbf{A}_N + \mathbf{X}_{N+1}^{-1}\mathbf{Cov}_{N+1}(\mathbf{X}_{N+1}^T)^{-1}\big)^{-1}\hat\beta_N\\
&+ \mathbf{A}_N\big(\mathbf{A}_N + \mathbf{X}_{N+1}^{-1}\mathbf{Cov}_{N+1}(\mathbf{X}_{N+1}^T)^{-1}\big)^{-1}\mathbf{X}_{N+1}^{-1}Y_{N+1}
\end{aligned}$$

Adding and subtracting a term of the form: $\mathbf{A}_N\big(\mathbf{A}_N + \mathbf{X}_{N+1}^{-1}\mathbf{Cov}_{N+1}(\mathbf{X}_{N+1}^T)^{-1}\big)^{-1}\hat\beta_N$ we have

$$\begin{aligned}
\hat\beta_{N+1} &= \big(\mathbf{A}_N + \mathbf{X}_{N+1}^{-1}\mathbf{Cov}_{\mathbf{N+1}}(\mathbf{X}_{N+1}^T)^{-1}\big)\big(\mathbf{A}_N + \mathbf{X}_{N+1}^{-1}\mathbf{Cov}_{N+1}(\mathbf{X}_{N+1}^T)^{-1}\big)^{-1}\hat\beta_N\\
&- \mathbf{A}_N\big(\mathbf{A}_N + \mathbf{X}_{N+1}^{-1}\mathbf{Cov}_{N+1}(\mathbf{X}_{N+1}^T)^{-1}\big)^{-1}\hat\beta_N\\
&+ \mathbf{A}_N\big(\mathbf{A}_N + \mathbf{X}_{N+1}^{-1}\mathbf{Cov}_{N+1}(\mathbf{X}_{N+1}^T)^{-1}\big)^{-1}\mathbf{X}_{N+1}^{-1}Y_{N+1}
\end{aligned}$$

which we can combine with the new measurement Y_{N+1}

$$\hat\beta_{N+1} = \hat\beta_N + \mathbf{A}_N\big(\mathbf{A}_N + \mathbf{X}_{N+1}^{-1}\mathbf{Cov}_{N+1}(\mathbf{X}_{N+1}^T)^{-1}\big)^{-1}\mathbf{X}_{N+1}^{-1}\big[Y_{N+1} - \mathbf{X}_{N+1}\hat\beta_N\big]$$

This formulation suggests that a second form for Gain Matrix in the case of invertible $\mathbf{X}$ matrices be defined as

$$\mathbf{K}_{N+1} = \mathbf{A}_N\big(\mathbf{A}_N + \mathbf{X}_{N+1}^{-1}\mathbf{Cov}_{N+1}(\mathbf{X}_{N+1}^T)^{-1}\big)^{-1}\mathbf{X}_{N+1}^{-1}.$$

Using this expression we can now write

$$\begin{aligned}
\hat\beta_{N+1} &= \hat\beta_N + \mathbf{K}_{N+1}\big[Y_{N+1} - \mathbf{X}_{N+1}\hat\beta_N\big] \quad \text{with Updated Covariance given by}\\
\mathbf{A}_{N+1} &= \mathbf{A}_N\big(\mathbf{A}_N + \mathbf{X}_{N+1}^{-1}\mathbf{Cov}_{N+1}(\mathbf{X}_{N+1}^T)^{-1}\big)^{-1}\mathbf{X}_{N+1}^{-1}\mathbf{Cov}_{N+1}(\mathbf{X}_{N+1}^T)^{-1}\\
&= \mathbf{K}_{N+1}\mathbf{Cov}_{N+1}(\mathbf{X}_{N+1}^T)^{-1}
\end{aligned}$$