Review and Replication of Lengnick (2013)'s Macroeconomic Agent-Based Model

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1 Introduction

In this report, I review and replicate Lengnick (2013)'s macroeconomic agent-based model in Python with the Mesa library (Ter Hoeven et al. 2025).[1] I am able to reproduce endogenous business cycles in an economy without growth or shocks implemented at the aggregate level. Most aggregate phenomena, such as Phillips and Beveridge curves, resemble those found in the original paper. Though there are some deviations, particularly in the frequency and severity of downturns, and in the magnitude and frequency of firm-level decisions such as price changes.

There are several core reasons to model macroeconomic phenomena with agent-based computation. Compared to structural VAR or traditional DSGE models, ABMs are one of a few approaches that permits heterogeneity in agents. Compared with heterogeneous agent models such as HANK models, ABMs assume neither perfect foresight among agents nor equilibrium conditions in aggregate that are necessary to solve these alternative approaches.

Among macroeconomic ABMs, the Lengnick (2013) model is relatively influential despite its simplicity (Dawid and Delli Gatti 2018). Lengnick (2013) intended the model to be a *baseline* that is absolutely necessary to generate business cycles without shocks. This has the advantages of, first, being computationally less intensive than other macroeconomic ABMs.[2] Second, the supposedly smaller parameter space makes the model easier to estimate for applied use.[3]

This project also makes several small contributions. First, it converts the Lengnick (2013)'s original model, which was written in Java, to Python in an open-source repository.[4] Second,

[1] Code is available on https://github.com/johnthwong/mabm.

[2] A 600-year run (with each step being a day) takes about eight minutes to complete on my 2020 Macbook Air with an 8-core CPU and 8 GB of RAM.

[3] Though I will argue later that the parameter space is not that small.

[4] Though I should acknowledge that there are at least two other attempts on Github to replicate this exact model in Python, both with Mesa functionality. I found Linnen (2025) early into my attempt. Their model could not replicate the endogenous business cycles that were core to the Lengnick (2013) model, so I decided to start from scratch. When I was wrapping up the project, that's when I encountered Wilson (2025)'s

I clarify ambiguities and inconsistencies in the model. Finally, I also added rudimentary dashboard capabilities with Mesa's Solara-based modules. Though they are too slow to update to be practical to use for these 100,000-step runs.

The report is structured as follows. In Section 2, I will first review the key components in the Lengnick (2013) macroeconomic ABM. In Section 3, I discuss issues I encountered when implementing the model, including ambiguities in Lengnick (2013) and assumptions I have to make in response. **?@sec-output** compares the output of my model to the original model. **?@sec-discuss** discusses further issues with the model.

2 Model

The Lengnick (2013) macroeconomic ABM consists of two agent types—households and firms—interacting in two markets—that for consumer goods and labor. The model is best described from (i) how it is initialized and (ii) what a full step comprises. Table 1 summarizes all known parameters.

Table 1: Known parameters.

Parameter	Description	Value
H	Number of households.	1000
F	Number of firms.	$H/10$
n	Number of firms that household maintains in sellers network.	7
γ	Months of labor market slack until wage cut.	24
δ	Upper-bound of wage adjustment.	0.019
$\underline{\phi}$	Minimum desirable inventory (multiplier on units of goods demanded).	0.25
$\overline{\phi}$	Maximum desirable inventory (multiplier on units of goods demanded).	1
$\underline{\rho}$	Minimum desirable price (multiplier on "marginal cost").	1.025
$\overline{\rho}$	Maximum desirable price (multiplier on "marginal cost").	1.15
θ	Probability firm considers changing price.	0.75
η	Upper-bound of price adjustment.	0.02
ψ_p	Probability household tries to switch seller for price.	0.25
ψ_y	Probability household tries to switch seller for inventory.	0.25
ξ	Minimum price decrease required to switch seller.	0.01
β	Number of firms visited when unemployed.	5
π	Probability underpaid worker seeks new job.	0.1
α	Parameter for maximum consumption.	0.9
λ	Production technology parameter.	3
χ	Parameter for precautionary buffer (multiplier on total payroll).	0.1

attempt of the same project. I have not tried running it, but they claim to have successfully replicated Lengnick (2013)'s results.

2.1 Initialization

Each step represents a day. One month (denoted as t) elapses after every 21 days.

H **singleton** households (indexed h) are first initialized,[5] and then F firms (indexed f). Each household forms a network of n firms, from which the household will buy consumption goods when the model runs. Here I should note that all households and firms are permanent; there is no births or deaths for households, or entries or exits for firms.

Each household has a (monthly) reservation wage w_{ht} that changes with time. This refers to the wage that households must be offered to take a job. For clarity, it is **not** an amount that households receive for being unemployed. The reservation wage is initially w_{h0}.

Households are endowed with money $m_{ht}|_{t=0} = m_{h0}$. Each firm initially sets per-good price m_{f0}. and monthly wage w_{f0}. There is no borrowing, i.e., agents cannot spend more than the money they have.

2.2 One full step

At the start of each month (i.e., on day s where $s \,\%\, 21 = 1$), the following actions are taken before the day begins:

2.2.1 Firm adjusts wage

If a firm had a vacancy in the previous month, it adjusts wages upwards. Conversely, if it has had **no** vacancies for the previous γ months, it adjusts wages downwards. The symmetric adjustment is:

$$\begin{aligned} w_{ft} = w_{f,t-1}&(1 \pm \mu) \\ &\mu \sim U_{0,\delta}. \end{aligned}$$

2.2.2 Firm adjusts inventory

A firm increases (reduces) the number of employees l_{ft}, i.e., households it employs by one employee if inventory i_{ft} falls below (exceeds) $\underline{\phi}$ ($\overline{\phi}$) times previous month's demand, i.e.,

$$\begin{aligned} i_{ft} < \underline{\phi} \cdot d_{t-1} &\implies l_{ft} = l_{f,t-1} + 1 \\ i_{ft} > \overline{\phi} \cdot d_{t-1} &\implies l_{ft} = l_{f,t-1} - 1. \end{aligned}$$

Firings happen with a one-month delay upon being planned. Within the month, the household will still work at the firm.

[5] All households are singleton. They are called households throughout Lengnick (2013). I adopt the same language.

2.2.3 Firm adjusts price

Prices are only raised (reduced) conditional on unsatisfactorily low (high) inventory as described above. In addition, there are two other conditions. Second, the price must be too low (high) relative to $\underline{\rho}$ ($\overline{\rho}$) times marginal cost c_t, i.e.,

$$p_{f,t-1} < \underline{\rho} c_t$$
$$p_{f,t-1} > \overline{\rho} c_t.$$

The third condition is that a firm only considers changing price with θ probability, i.e., Calvo pricing.

The price adjustment is randomly drawn and as follows:

$$\begin{aligned} p_{ft} &= p_{f,t-1}(1+v) \\ v &\sim U_{0,\eta}. \end{aligned}$$

2.2.4 Household changes sellers for price

There is a ψ_p probability that each household takes this action. Each household draws a random new firm to pair with. The odds of drawing the new firm is proportional to the number of employees the firm has. The household compares this to a random existing connection. If the former offers a relative decrease in price greater than ξ, it replaces the latter as a connected seller.

2.2.5 Household changes sellers for inventory

There is a ψ_y probability that each household takes this action. Among existing connections that could not fully fulfill the household's demand last month, a random one is replaced with a new firm.

2.2.6 Household seeks new job if unemployed

If a household is unemployed in the previous month, it visits β firms. If the firm has a vacancy and their wage offer exceeds the household's reservation wage, the household takes the job and ends the search.

Note that because firings happen with a delay, households that are fired might refuse to engage in job search even though they will lose their employer.

2.2.7 Household seeks new job if employed but underpaid

A household can be paid less than their reservation wage (due to a downward wage adjustment, as previously discussed). They do not quit. Instead, with probability π, they will seek **one** new random firm. If that firm has a vacancy and their wage offer exceeds the household's current wage, the household will change jobs. Note that the new wage could still be below reservation.

2.2.8 Household plans consumption

Household plans how much to consume over the course of a month according to the following equation:

$$c_{ht} = \min[(\frac{m_{ht}}{\overline{p}_{ht}})^{\alpha}, \frac{m_{ht}}{\overline{p}_{ht}}],$$

where $\overline{p}_{ht}$ is the average price of all sellers in the household h's network and $0 < \alpha < 1$. Note that α makes the derivative of consumption to the money-price ratio higher for lower values, i.e., consumption increases with additional income more when a household is poor in a real sense.

Also note that the consumption equation only determines the maximum, which may or may not be binding. Consumption occurs daily. If the household matches with higher-priced connections more, they may end up buying less than c_{ht}.

The following actions are undertaken daily.

2.2.9 Household buys (and consumes)

Households buy $c_{ht}/21$ from connected firms, visited in a random order, each day. If the firm visited cannot satisfy all of the household's demand, it moves to the next firm. The household stops the visit loop if (i) 95 percent or more of its demand is satisfied or (ii) the household has no more money.

2.2.10 Firm produces

Firms produce daily according to the following linear production function:

$$\begin{aligned} y_{fs} &= \lambda l_{ft}, \\ \implies y_{ft} &= 21 \cdot \lambda l_{ft}. \end{aligned}$$

where λ is a technology parameter and l_{ft} is the number of employees employed that month. Thus, each household implicitly supplies one unit of labor daily. The output is added to the firm's inventory.

At the end of each month (i.e., on day s where $s \ \% \ 21 = 0$), the following actions are taken after the end of day.

2.2.11 Firm pays wages

The firm pays each employee the same wage. Total payroll is:

$$\min(l_{ft}w_{ft}, m_{ft}),$$

i.e., employees can be underpaid.

2.2.12 Firm retains buffer

This amount is retained as the firm's money rather than returned to shareholders (see below). This is a precautionary retained profit for future wages and is determined by the multiplier χ on total payroll. The rule is:

$$\min(\chi l_{ft}w_{ft}, m_{ft}).$$

2.2.13 Firm pays shareholders

Firm pays all remaining money back to shareholders as dividends. Each firm is owned by all households. For any given firm, the household's ownership share is equal to that household's money over the economy's total money, i.e., richer households receive more dividends.

2.2.14 Household adjusts reservation wage

If a household was unemployed this month, their reservation wage is lowered by 10 percent. If they were employed and their payment was higher than their reservation wage, they set their reservation wage equal to their latest wage.

3 Implementation issues

Omissions in the original paper make recreation impossible. The first issue has to do with the initial step. Notice that initial day's consumption requires that firms have pre-existing inventory. Similarly, headcount and price adjustments require information about inventory and employees that do not exist initially. For simplicity, I did not assume a starting inventory. Instead, I have all households search for jobs as if they are unemployed, and firms are assumed to have unlimited openings on day 1 only. After pairing, households skip consumption and produce output for firms. Since households do not consume, firms will not make money, and thus households are unpaid for their labor on day 1. All planning steps other than wage-setting is skipped on day 1 (all firms will raise their wage once since they have vacancies that day).

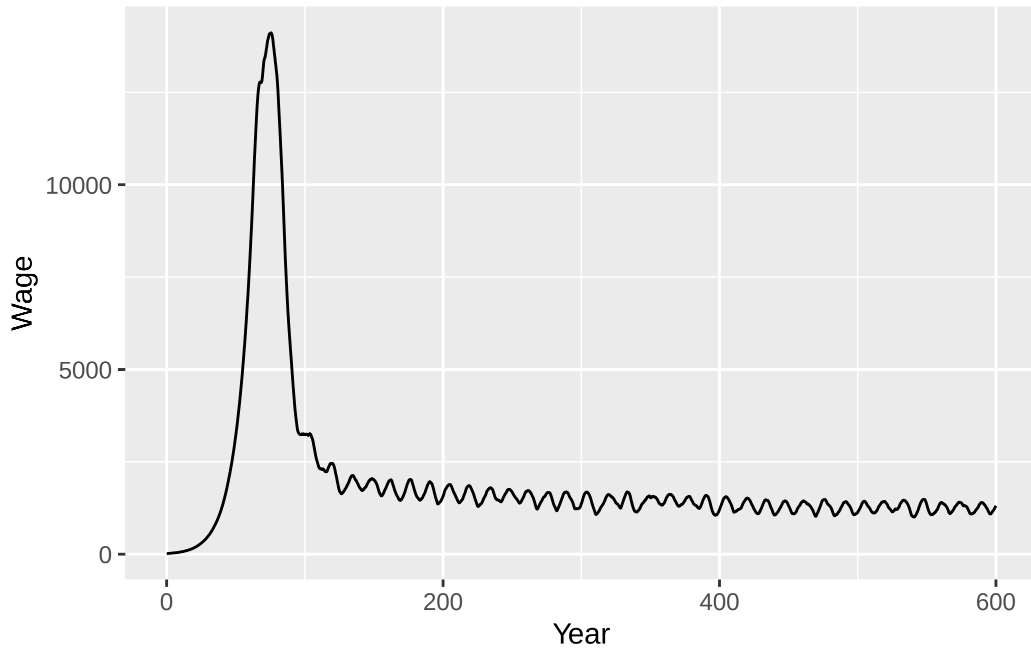


Figure 1: Mean wage across the entire run.

The second issue has to do with unknown parameters. Table 2 summarizes them. All omitted parameters are starting values. While it should be noted that starting parameters likely do not have an effect on general patterns (this is because the model does equilibrate, see Figure 1; and because Lengnick (2013) drops the first 100 years—or 25200 ticks—of results in a 600-year run), they do matter for replication. In lieu, I assume only households start with money and in equal amounts. Money stock does not matter as prices will adjust proportionally. Accordingly, I simply assume that initial wages is a low share of monthly output, and that the reservation wage is an even lower share to encourage initial employment.

Table 2: Unknown parameters.

Parameter	Description	Value used in lieu
m_{h0}	Initial household money.	10000
w_{h0}	Initial reservation wage (monthly).	$(21 \cdot \lambda) \cdot 0.1$
m_{f0}	Initial firm money (monthly).	0
p_{f0}	Initial price (per good).	1
w_{f0}	Initial wage (monthly).	$21 \cdot \lambda \cdot \min(0, N[0.3, 0.1])$
	Randomization seed.	1000
ψ	Unknown parameter.	0.25

It should also be noted that there is a parameter θ in Lengnick (2013)'s parameter list, but that variable is not mentioned again or explained upon at all. Perhaps most important of all, I also do not know the seed that recreated the result—though this would not be helpful since the original model was written in Java.

The third issue has to do with definitions, some of which are very integral to the model.

i. Section 2.2.2 mentions demand. But it is never clarified whether demand refers to last month's consumers' full demand, which is what the consumer planned to consume that day or what they can afford (whichever lower), or consumers' demand that the firm was able to fulfill. I assume the latter because the former definition can lead to over-counting of one person's demand at multiple firms.
ii. Section 2.2.3 mentions marginal cost for wage calculation, but marginal cost is never defined. We know that the marginal cost of ***63 consumption goods*** $(21 \cdot \lambda)$ is the cost of one worker, i.e., the firm's wage. But it is unclear how you can convert that to the per-good marginal cost. I simply divide wage by 63 to approximate this variable.
iii. It is unclear whether Section 2.2.4 and Section 2.2.5 draw new sellers the same way. It is explicitly mentioned that for the former, the probability of a draw is scaled by firm's number of employees. But this is not addressed for the latter. I have assumed the probability for drawing non-connected firms is also scaled by number of employees in the latter case.
iv. I have assumed consumption goods and money to be continuous. Discretization is not mentioned in Lengnick (2013).

The fourth issue is that some downward adjustments of wages and price are asymptotic to zero. I put a floor on these values of 1e-9 to ensure the firm does not permanently crash in either goods or labor market.

Finally, some of the equations are inconsistent. As mentioned in Section 2.2.8, putting a min on max consumption is unnecessary because consumers are already bounded by a hard budget constraint. Unemployed households are also modeled to be more selective than underpaid ones, since the latter will take a job below their reservation wage, but the former will not.

4 Results

In this section, I replicate Lengnick (2013)'s attempt to replicate stylized macroeconomic facts. I simulate an economy with a seed of 1000 for 600 years, and drop the first 100.

Figure 2 shows the last 50-year period within this window. I am able to replicate the core emergent phenomenon of the model: the endogenous generation of business cycles that are driven by coordination failures. For example, a firm can raise price too much and reduce the demand for its goods in the following period; this causes a cascade of the firm laying off a worker, that household consuming less, and another firm within the household's network cutting another worker, and so forth.

There are differences between the replication and the original: first, business cycles are far more frequent in Lengnick (2013) (eight in a 50-year period). Second, they are less severe in magnitude, with employed households in the period dropping to about 955 at most (about 4.5 percent unemployment), while my model generated a 10-percent unemployment rate within the period. In terms of matching reality, however, less frequent but more severe downturns seem more plausible.

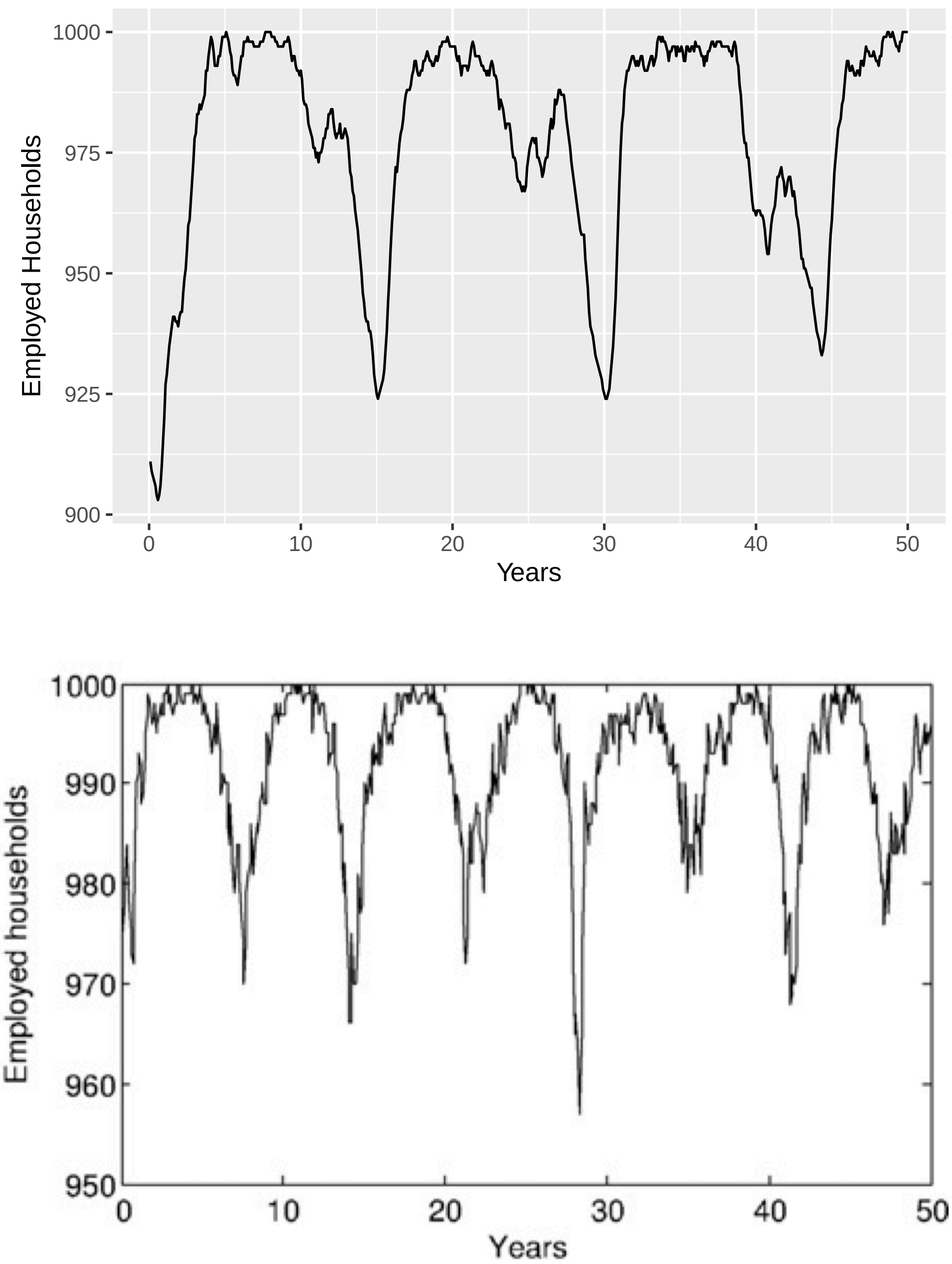


Figure 2: Business cycles in employment.

The next figure compares unsatisfied demand in both models. Specifically, one observation is the 1 - (aggregate fulfilled demand over aggregate planned demand) in a month.[6] Note that while I am able to replicate a similar mode, my probability density function has fatter tails. But this could be due to original graph being truncated at 0.2.

Up next, I am also able to replicate aggregate correlations between price changes (year-over-year) and unemployment levels (Phillips curve, Figure 4); and between vacancies and unemployment levels (Beveridge curve, Figure 5). Each data point is one month with household- or firm-level statistics aggregated.[7] Prices are first averaged across firms, and then first-differenced.

While patterns are largely the same, there are substantial deviations. First, as noted already, unemployment levels in my model are higher by about three times. Second, price changes in my model across year are mostly within one dollar, while many of Lengnick (2013)'s reported price changes fall across [-4, 4] dollars. Third, my model produces less vacancies (consistent with more unemployment)—with maximum vacancies within a month to be no more than 20. Meanwhile, Lengnick (2013) reports more than 80 vacancies in some months.

Figure 6 shows that I am able to replicate the distribution of firm sizes (measured by their fulfilled demand in the last step), though firms in my model exhibit higher variance by this measure. However, I am unable to replicate the distribution of "firms changing price" Figure 7. While there is some ambiguity as to what the running variable means,[8] I interpreted each data point as the share of firms that changed their price (binary) within a month. Note that my distribution is multimodal, with a second peak at about 75 percent—which is consistent with the price adjustment probability parameter θ.

Finally, I am unable to fully replicate the cross-correlation coefficients between output and the k-th lag of price levels. Figure 8 shows that within the range of lags Lengnick (2013) considers (eight quarters, or 24 months), (real) output is negatively correlated with lags of the price level, up to 18 months; whereas in Lengnick (2013), output is negatively correlated with price level for up to 2 lagged quarters (6 months) only.

5 Discussion

I would like to discuss the properties of the Lengnick (2013) model that are unrelated to the implementation issues discussed in Section 3.

First, while the model is intended to be a *baseline* macroeconomic MABM, the parameter space is quite large and difficult to parameterize. The large number of parameters (19 known + 6 unknown) makes conducting a parameter sweep computationally unfeasible. It is unlikely that some parameters, like ψ_p, ψ_y, ξ can be structurally estimated with real data. The parameters such as $\underline{\phi}, \overline{\phi}, \underline{\rho}, \overline{\rho}$ appear to presume equilibrium relationships between variables

[6] It is not entirely clear if this is how unsatisfied demand is defined in the original.

[7] Like the original paper, I add noise to discrete unemployment and vacancy levels of $U[-0.5, 0.5]$ for better visualization.

[8] I also tried plotting the densities of relative change from last month's price, where each data point is (i) a firm in a month and (ii) a firm in the last run. Neither plots resemble the original graph.

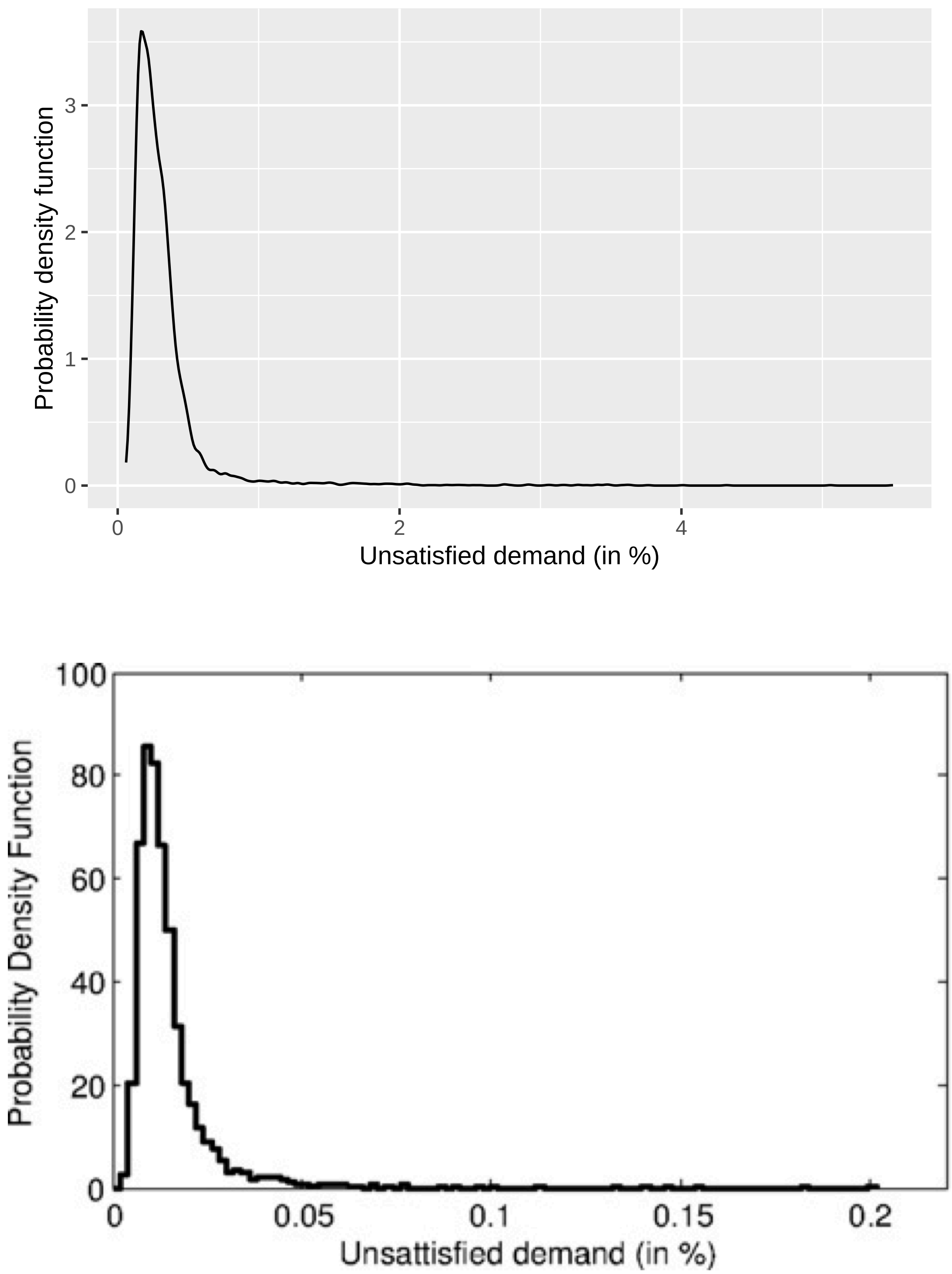


Figure 3: Aggregate unsatisfied demand. Each data point is one month.

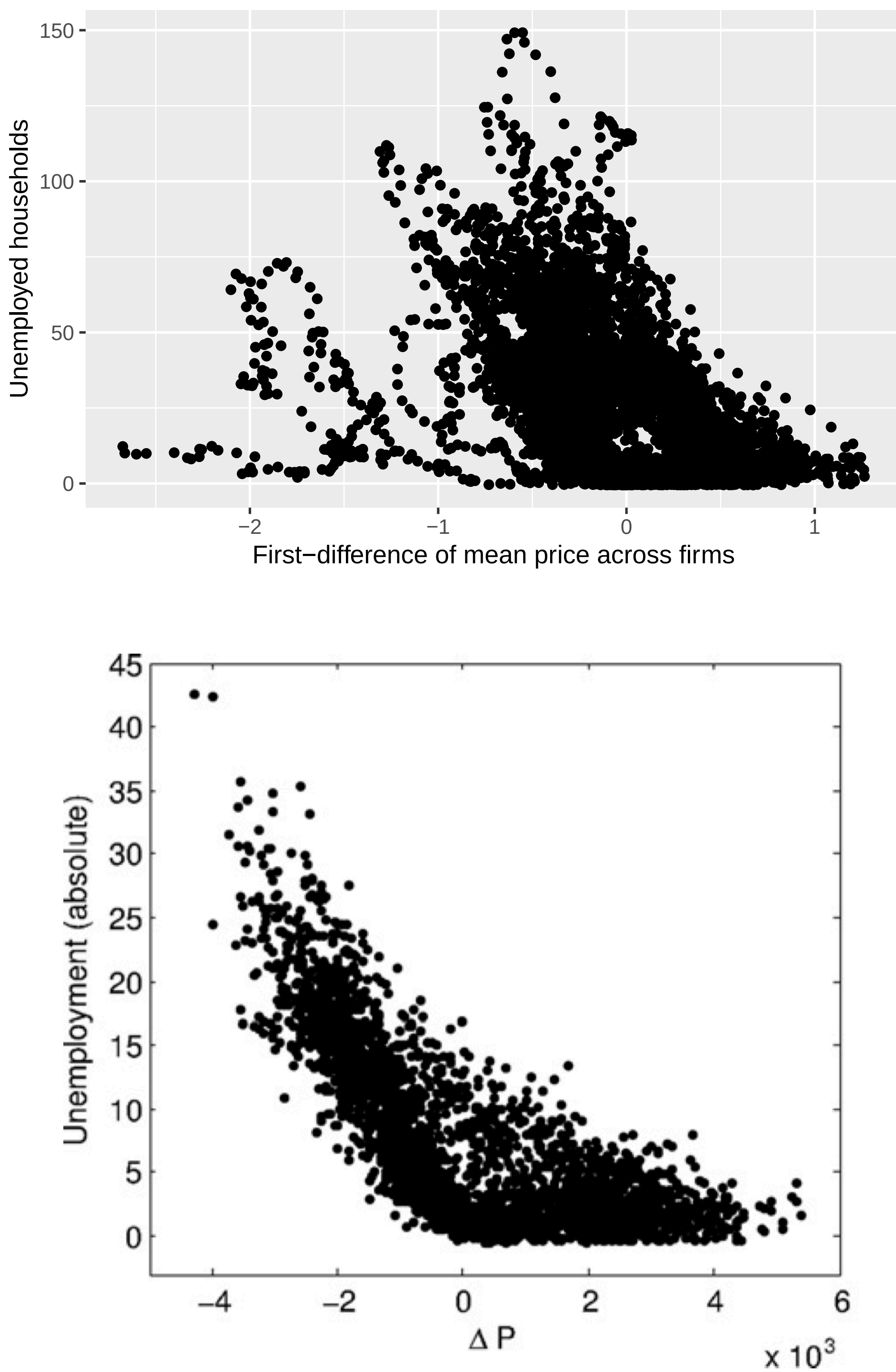


Figure 4: Phillips curve. Each data point is one month.

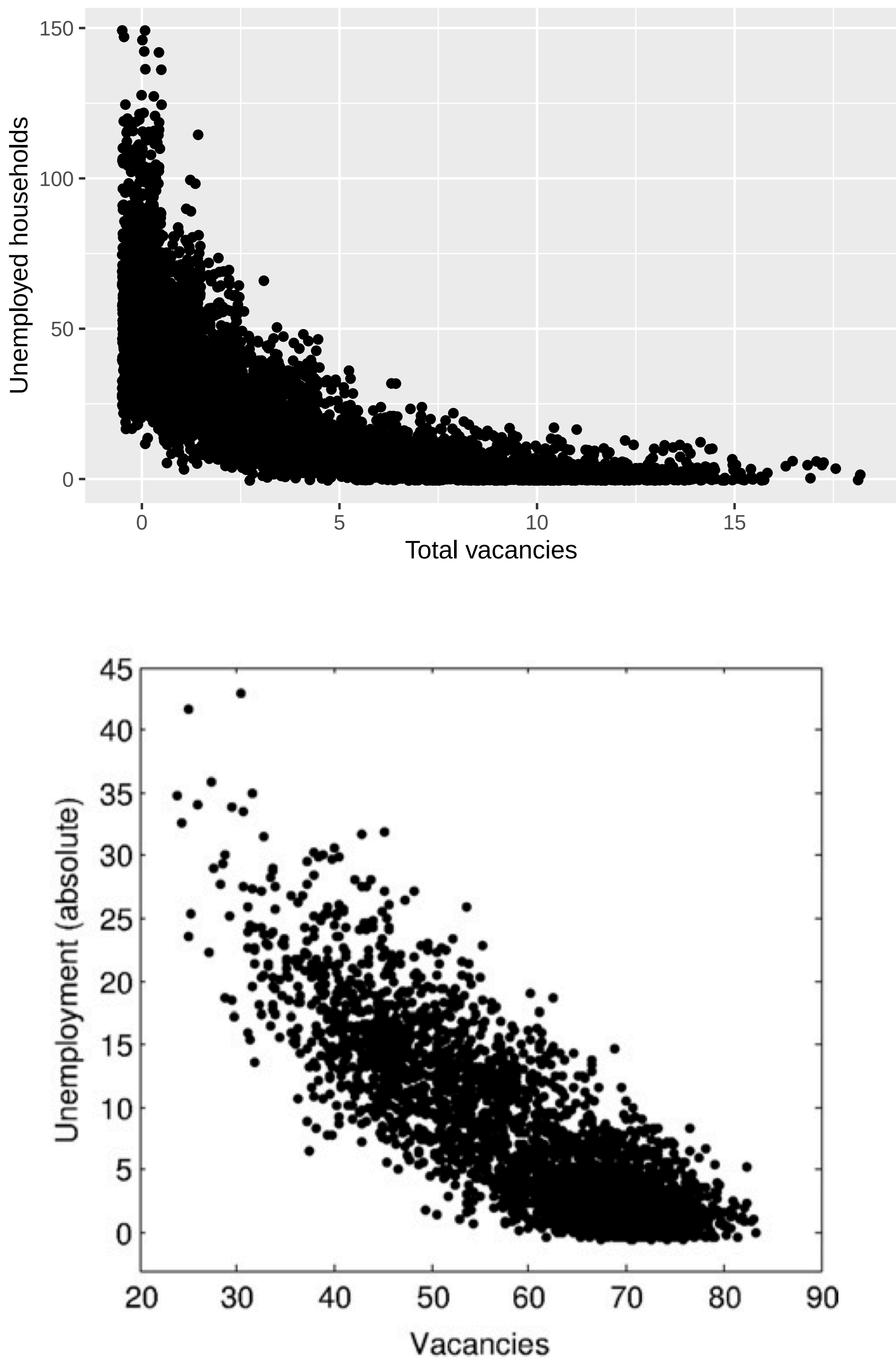


Figure 5: Beveridge curve. Each data point is one month.

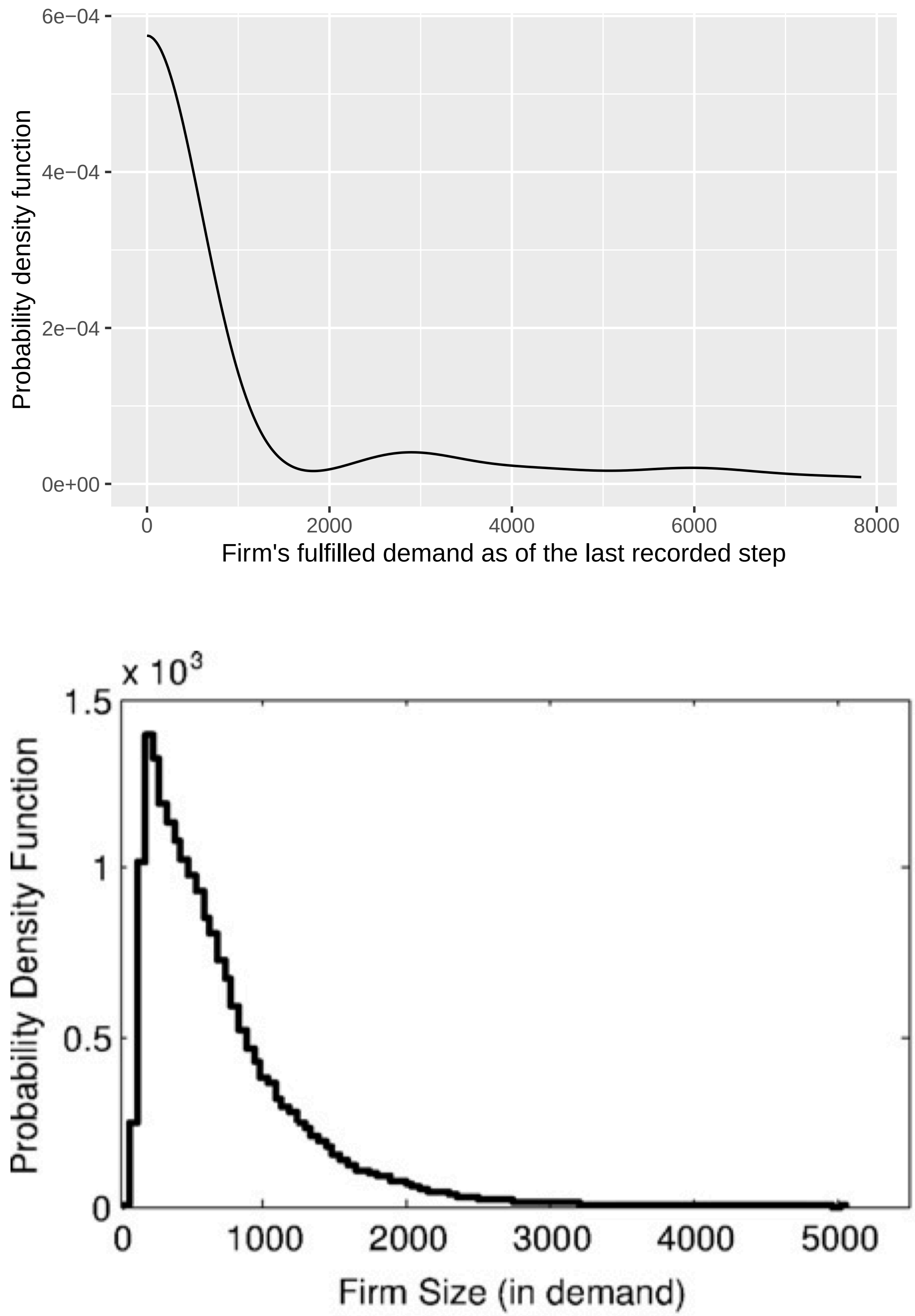
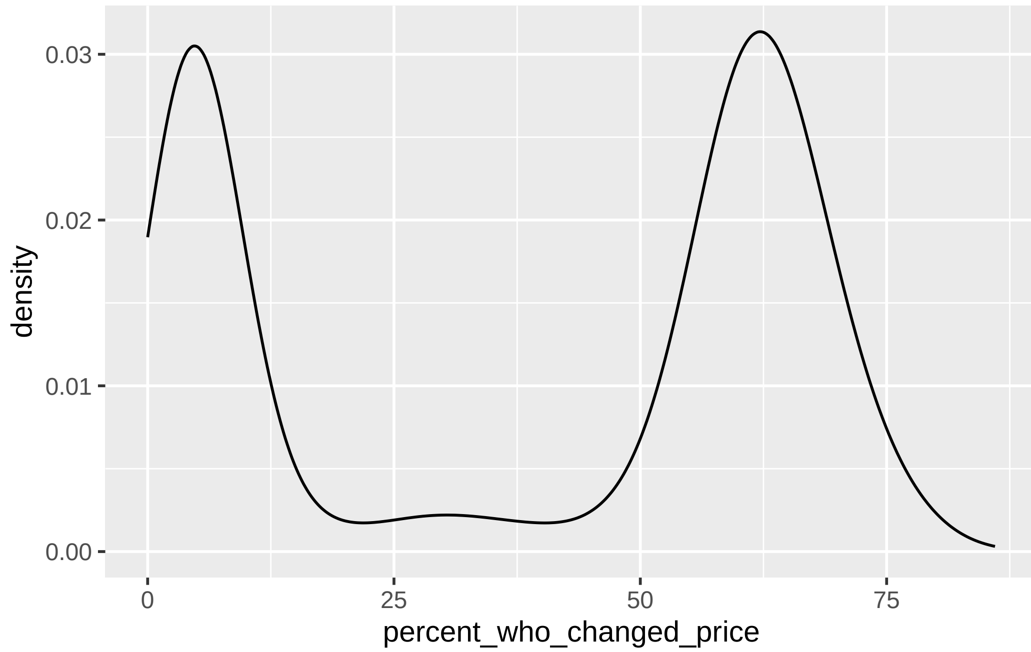


Figure 6: Firm's fulfilled demand. Each data point is one firm the last recorded step.



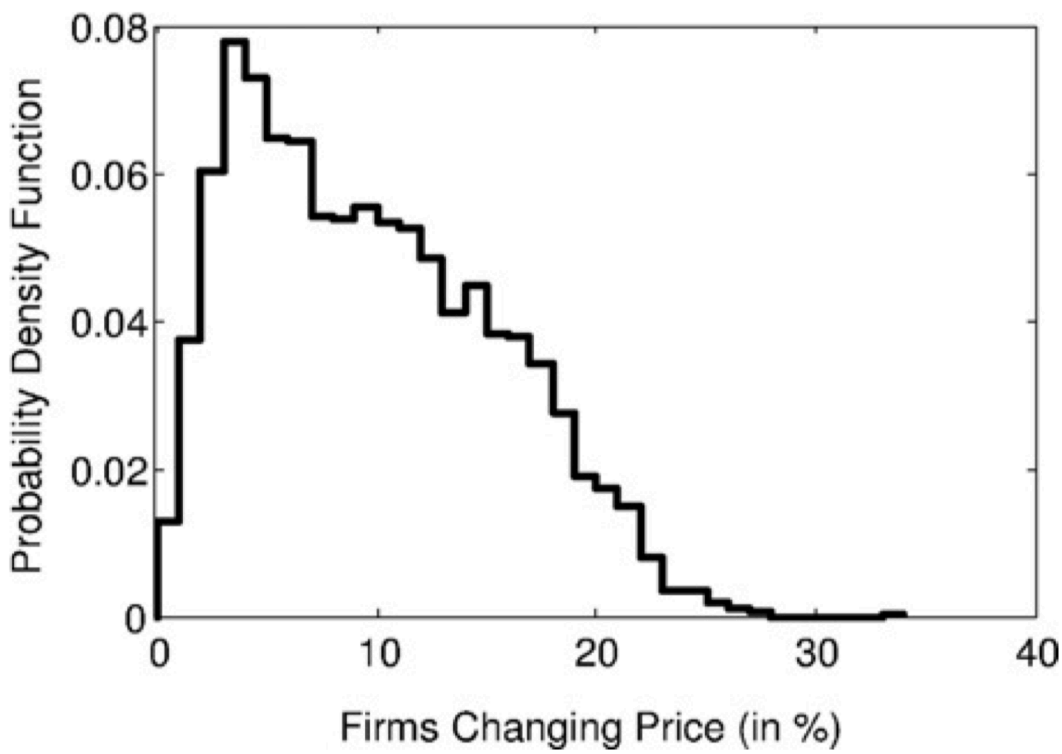
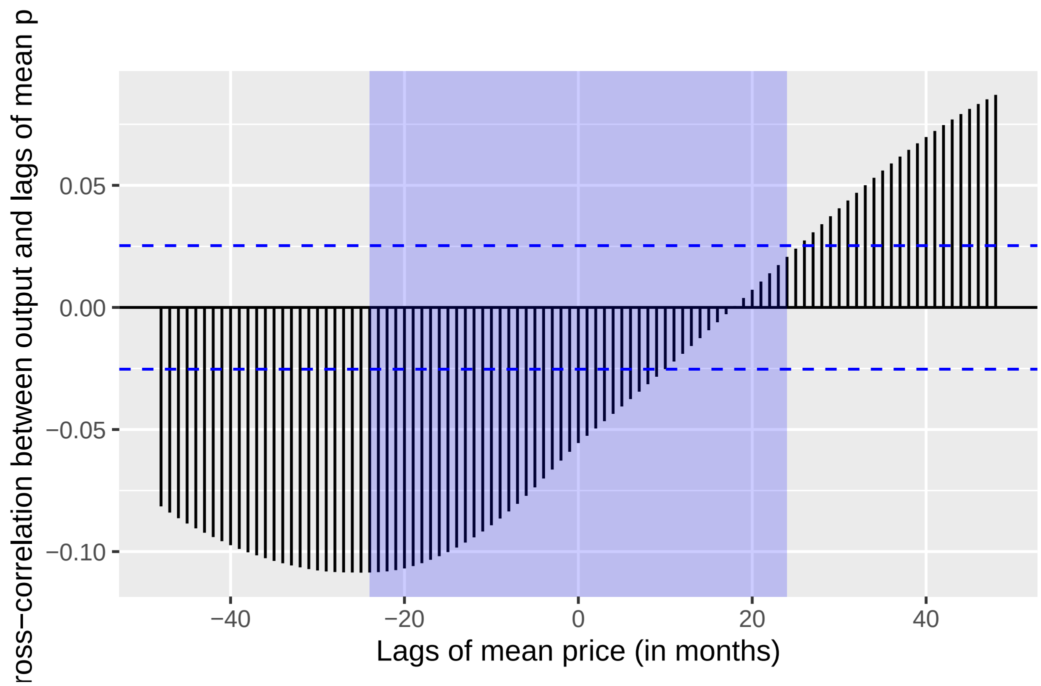


Figure 7: Share of firms that changed their prices. Each data point is one month.



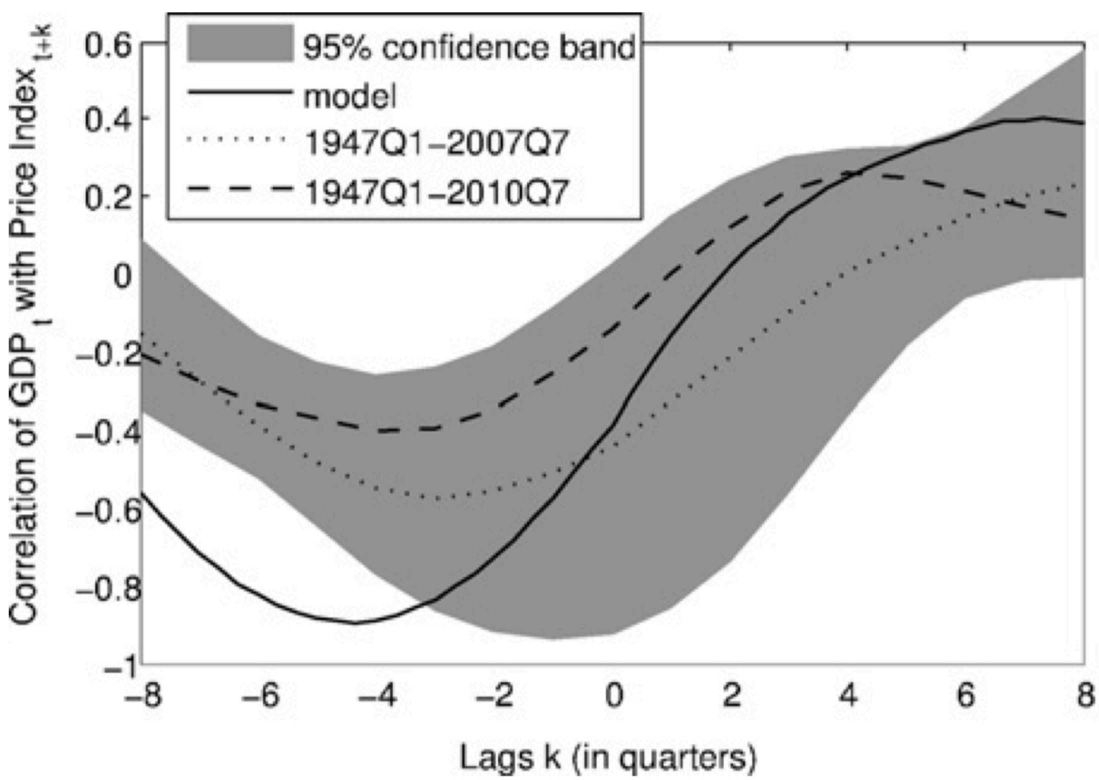


Figure 8: Cross correlations. Each correlation coefficient is calculated with a sample where each data point is a month. Shaded area in the top graph is defined by the x-axis range of the bottom graph. Dashed lines in the top graph indicate a range of bootstrapped estimates within ± 2 SD.

from the onset, but simply expand this equilibrium from a point to a range. Parameters like θ, ξ are likely redundant since they, respectively, have similar effects to $\underline{\rho}, \overline{\rho}$ and ψ_p.

Second, one potential addition to the model to have it generate policy-relevant conclusions (e.g., the models described in Richiardi, Ven, and Bronka (n.d.)) is to modify the search behaviors in Section 2.2.6 and Section 2.2.7 by having households compare their reservation wage to *after-tax* wage. Then, a policy shock can modify the effective average tax rate after the model has reached equilibrium without the shock (in about year 200), and we record the following 10-30 years of outcomes immediately after. That way forecasts of policy effects would include the coordination issues caused by policy shocks. By integrating an open-source calculator (e.g., PolicyEngine (n.d.)) that converts law-level parameters (e.g., standard deduction) to individual-specific average tax rates, one can potentially forecast the effect of real policy changes in a baseline environment where growth is absent.

Finally, the disequilibrium that is core to generating the model's endogenous business cycles makes integrating certain components difficult. I had mostly implemented model firm entries and exits with Axtell (2016)'s model of employees with heterogeneous ability optimizing efforts in the presence of increasing returns to firm size. But the lack of equilibrium in this model's goods market makes output-based compensation (which implicitly assumes all produced goods are sold, or no inventory) in Axtell (2016) impossible to justify.

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