

(Please email me both script and report as attached files at depalle@music.mcgill.ca. Please include your name in filenames).

Part 1:

The goal is to build a simple synthesizer in MATLAB, which synthesizes a periodic signal from the definition and control of its waveform spectrum. While it could be used for any spectral shape, it is proposed to study and implement this synthesizer structure in the simple case of periodic rectangular waveform sound synthesis. Let's recall the Fourier representation of a rectangular signal:

$$S(f) = \exp(-j * \pi * f * T) * \sin(\pi * f * T) / (\pi * f)$$

where T is the duration of the non-zero part of the signal expressed in seconds, and f is the frequency expressed in Hz.

The required control parameters of the synthesizer are:

- 1) Fundamental frequency f_0 ,
- 2) Relative duration of non-zero values $R = T/T_0$ (where $T_0 = 1 / f_0$),
- 3) Initial phase of the rectangular waveform, expressed as a proportion of the period,
- 4) Duration of the signal in seconds.

Hint: Proceed in two steps - First generate a waveform regardless of the fundamental frequency used; - second generate a periodic signal at a given fundamental frequency by a commonly used strategy in sound synthesis.

The script should be functional as soon as it is loaded, and include a function with the required control parameters as arguments.

Rmk0: The idea is NOT to use additive synthesis, but to start from the spectrum expression of the reference period.

Rmk1: Aliasing should be minimized, which requires in particular interpolation.

Rmk2: R is often called PWM, which stands for Pulse Wave Modulation.

Part 2:

Let's consider the signal $s(t)$, with $\alpha > 0$,

$$\begin{cases} s(t) = e^{-\alpha t} & \text{for } t \geq 0 \\ s(t) = 0 & \text{for } t < 0 \end{cases}$$

which Fourier transform is:

$$S(f) = \frac{1}{\alpha + 2j\pi f}$$

- 1) Demonstrate, by only using concepts developed in the course, that the spectrum of:

$$x(t) = e^{-\alpha|t|}$$

is:

$$X(f) = \frac{2\alpha}{\alpha^2 + 4\pi^2 f^2}$$

- 2) Give the analytical expression of the signal of the 'periodized' version of $x(t)$ at period T_0 .
- 3) Derive the amplitudes of the harmonic components of $x(t)$ from the previous result.
- 4) Give at least one advantage of the proposed method to get the harmonic components' amplitudes of $x(t)$.