Effects of Information Demand on the Probability of Positive Daily Asset Index Excess Returns

EC981-7-FY-CO

Financial Econometrics Masters Dissertation

University of Essex

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September 3, 2019

Abstract

This thesis addresses well established return forecasting challenges via frameworks that focus on the sign of the change in asset index excess returns using a family of GARCH models. It investigates them in the literature's original S&P 500 index and applies them to the FTSE 100 to study the predictive power of information demand proxied by Google's internet search vector index and finds evidence suggesting that an efficient trading strategy stemming from this study can be constructed.

Contents

1 Introduction

Since Tinbergen (1933)'s modelling work on business cycles post World War I, many econometric methods were developed with the aim of constructing ever better economic and financial forecasts. The subject evolved through statistical linear and non-linear relationships with Autoregressive (AR) (Yule (1927)), Moving Average (MA) (N. and Wold (1939)), Autoregressive Moving Average (ARMA) (Whittle (1951)), Autoregressive Conditional Heteroskedasticity (ARCH) (Engle (1982)), Generalized ARCH (GARCH) (Bollerslev (1986)), Exponential GARCH (EGARCH) (Nelson (1991)), threshold nonlinear ARMA (Tong (1990)), and Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) (Glosten, Jagannathan, and Runkle (1993))) models amongst others.

Their use in financial time-series analysis have grown to become staple tools in academia; but many came upon what are now well established asset return forecasting challenges: Stylized Facts of Financial Return (SFFR) - a set of properties found in most asset return datasets (Booth and Gurun (2004); Harrison (1998); Mitchell, Brown, and Easton (2002)). Many SFFR exist, but only a few are of interest in this instance: asset returns' (i) non-Gaussian distribution (unconditional leptokurtosis/excess-kurtosis and negative-skewness/asymmetry), (ii) weak serial correlation (*id est* (*i.e.*): the lack of correlation between returns on different days), (iii) volatility clustering (heteroskedasticity), as well as (iv) serial correlation in squared returns, and (v) leverage effects (the tendency for variance to rise more following large negative price changes than positive ones of similar magnitude). (Taylor (2005))

Numerous time-series processes fail to encapsulate these SFFR, *exempli gratia* (*e.g.*): AR, MA and ARMA models may necessitate the assumption of homoscedasticity (*i.e.*: constant variance in error terms) - failing SFFR (iii), (iv) and (v) - which is a useful characteristic to include as *"large changes* [in asset prices] *tend to be followed by large changes-of either sign-and small changes tend to be followed by small changes"* (Mandelbrot (1963), p.418).

In this thesis, I address these challenges using aforementioned models' forecasts in the framework outlined in Christoffersen and Diebold (2006) (C&D hereon) to focus on asset return sign changes instead of their level changes as highlighted in Pesaran and Timmermann (2000). My investigation expands on the work of Chronopoulos, Papadimitriou, and Vlastakis (2018) (CPV hereon) - replicating their research on the United States (U.S.)' Standard & Poor's 500 Index (SPX) - extends it to the United Kingdom (U.K)'s Financial Times Stock Exchange 100 Index (FTSE), and studies its feasible financial impact by constructing and assessing investment strategies stemming from findings.

Evidence that an efficient trading strategy may be constructible for the FTSE are found.

1.1 Literature Review

Whilst a family of GARCH models are used in this thesis to remedy the particular issue of heteroscedasticity modelling, the task of predicting asset prices remains notoriously difficult; often, *"even the best prediction models have no out-of-sample forecasting powers"* (Bossaerts and Hillion (1999), p.405) (*e.g.*: Kostakis, Magdalinos, and Stamatogiannis (2014)). This is what motivated C&D to challege such difficulties by demonstrating how correlations between asset price volatilities and returns arise given non-zero expected returns irrespective of the shape of their distribution. When expected returns are zero, Christoffersen, Diebold, Mariano, Tay, and Tse (2006) (CDMTT hereon) highlighted that sign prediction was still possible with asymmetrically distributed returns - in alignment with SFFR (i).

Market participants try to inform themselves optimally prior to trading (Simon (1955)). Therefore, risk aversion and information demand are closely related, it is thus so for investment activities too, that in turn cause asset price changes. While model specifications such as C&D's are naturally of paramount importance, numerous economists also investigated the explanatory powers of exogenous variables in forecasting models since Keynes' study on Business Cycles (*e.g.*: Fama and French (1996), famously).

CPV novelly incorporated such a variable - information demand - in these models to study their forecasting powers. While a plethora of studies investigated how asset prices are affected/correlated with information demand indexed via Wikipedia (Moat et al. (2013); Rubin and Rubin (2010)), online message boards (Antweiler and Frank (2004)), *et cetera* (Vlastakis and Markellos (2012)), Google Trends stood out as particularly useful in finance (Da, Engelberg, and Gao (2011); Preis, Moat, and Stanley (2013)). The prevalence of Google as the internationally dominant search-engine[1] allowed for the use of Google Trends in academia with varying degrees of accuracy in (science (Pelat, Turbelin, Bar-Hen, Flahault, and Valleron (2009)), economics (Askitas and Zimmermann (2009)), and) finance (Choi and Varian (2012); Curme, Preis, Stanley, and Moat (2014), and Jiang (2016)). Vlastakis and Markellos (2012) found that financial information-networks (*e.g.*: Reuters, Bloomberg, ...) tend to share articles outside their platforms and lead to information arriving in traditional and non-traditional information-channels at approximately the same time; Google Trends can thus be used to measure signals of investor interests in certain topics/assets and that there is a relationship between the information demand of an asset and its return's volatility.

CPV thus used Google Trends data as a proxy for information demand in the form of a Search Vector Index (SVI). They found evidence of distribution asymmetry in the SPX excess returns - allowing for the application of CDMTT's work - as well as a clear and statistically-significant relationship between those returns and the first difference in SVI - ΔSVI - using a family of GARCH models which were then used in C&D's framework. This proved highly

[1] Google is not strictly dominant in all countries. Notable exceptions exist in South Korea (with Naver), Russia (with Yandex) and China (with Baidu).

successful: when testing their findings using Granger and Pesaran (2000)'s framework[2] over the time period extending from the 1^{st} of January 2004 to the 31^{st} of December 2016 (01/01/2004 - 31/12/2016), their scenario analysis resulted in higher Sharpe-ratios (Sharpe (1966)) when implementing SVIs in an active strategy compared to a naïve or a buy-and-hold (B&H) strategy for a utility maximising investor acting with a Constant Absolute Risk Aversion (*i.e.*: with a negative exponential utility function) with and without short-selling. It is also interesting to note that short- and long-term interest rates were shown to have smaller - but still significant - predictive powers when using monthly data (comparing CPV's findings to Nyberg (2011) and Chevapatrakul (2013)'s) suggesting that higher frequency data - such as ΔSVI - may be of better use.

1.2 Breakdown

In this thesis, I primarily study the frameworks outlined in CPV to investigate the explanatory and forecasting powers of information demand on the FTSE. To compare and contrast with CPV, I also replicate their work with the SPX.

The choice of the FTSE is not random. Google is most prevalent and far reaching in Anglo-Saxon countries, one may thus expect a close relation between the SPX and FTSE (Silk (2012)) that - if caused by similar variables - would allow for significant findings akin to CPV's.

Financial applications of my models are studied via the perspective of a single profit-maximising investor in a manner akin to CPV (Kandel and Stambaugh (1996)). In the interest of time, I only look for the optimal risk aversion level for profit maximisation via graphical analyses as opposed to CPV who empirically investigated the economic significance of their findings to find this optimal risk aversion level.

Section 2 outlines the sources of all data used in my research. It also expresses derivations of variables used in subsequent sections. Section 3 formally specifies the mathematical models/frameworks at play. Section 4 displays results concisely, leaving details to the supplementary material[3], but expanding on their meaning and effects in my study. It focuses on FTSE results as they are primary to this study, although it outlines the reproducing work underwent. Section 5 concludes before - finally - Section 6 presents all endnotes used throughout this thesis.

[2] Granger and Pesaran (2000)'s framework allowed for transaction cost considerations including spread-and-commission (Anand, Irvine, Puckett, and Venkataraman (2011)), short-sale, immediacy- and price-impact-cost (Lesmond, Ogden, and Trzcinka (1999)) components.

[3] Supplementary material can be found at:
`https://1drv.ms/b/s!AlXQLwgrepbXgP8EqBoW6dy-a_O8Vw?e=7IaYZr`.

2 Data

2.1 Samples

In this thesis, $\left\{\begin{array}{c}\text{‘in-sample’}\\ \text{‘out-of-CPV-sample’}\\ \text{‘out-of-sample’}\\ \text{‘full-CPV-sample’}\\ \text{‘full-sample’}\end{array}\right\}$ refers to the time period $\left\{\begin{array}{c}01/01/2004 - 31/12/2004\\ 01/01/2005 - 15/03/2017\\ 01/01/2005 - 13/03/2019\\ 01/01/2004 - 15/03/2017\\ 01/01/2004 - 13/03/2019\end{array}\right\}$.

2.2 SVI

Alphabet Inc.'s Google Trends tool provides the number of times a specified term was searched on their website - for specified world regions and time-periods - normalised from 0 to 100, forming the SVI. In this thesis, SVI was gathered as outlined in CPV's Appendix A, normalising all results such that the day the term was searched the most in the period of choice is valued as 100 and the least as 0.

In the interest of ease of comparability with CPV, the worldwide searched term used as an SVI keyword in this thesis is the same as in their research - "s&p 500" - rendering SVI^{SPX}. This is in line with Vlastakis and Markellos (2012) and their study on the best terms to use in such investigations. They used several variations of the index term in addition to ones proposed by Wordtracker[4] and suggested picking the one with greatest search volume in the sample period. This is - naturally - in addition to picking no terms that may easily be misconstrued and have other meanings (*e.g.*: the listed company name 'Apple' which may also refer to the apple fruit). In line with this method, I use worldwide searches of "ftse 100" for my study of the FTSE - rendering SVI^{FTSE}.

To produce a stationary variable, the 1^{st} difference in SVI is used. Moreover, several different permutations of SVI data were chosen to test their resemblance with CPV's such that (:)

$$\Delta SVI_t^{Index} = SVI_t^{Index} - SVI_{t-1}^{Index}$$

where $Index = (SPX_1, SPX_2, SPX_{CPV}, SPX_{CPV-US}, FTSE)$ and $t \in \mathbb{Z}$ denotes only trading days[5].

SVI^{SPX_1}, SVI^{SPX_2} and SVI^{FTSE} were gathered over the full-sample time period, while $SVI^{SPX_{CPV}}$ and $SVI^{SPX_{CPV-US}}$ over the full-CPV-sample one.

All SVI^{Index} data represented Worldwide results bar $SVI_t^{SPX_{CPV-US}}$ that was only over

[4] Wordtracker offers Search Engine word search reports that include related search terms.

[5] For completeness, I repeated my work relaxing SVI_{t-1}^{Index}'s time set to any day, not only trading days, while keeping SVI_t^{Index}'s strictly to trading days to allow for situations where, *e.g.*, t would denote a Monday and $t-1$ a Sunday.

Equally, I repeated my work on all SVI^{SPX} moving averages from 2 days to one week.

the U.S. region. This was to test the finding that *"strategies based on global search volume data are less successful than strategies based on U.S. search volume data in anticipating movements of the U.S. market"* (Preis et al. (2013), p.5).

No two draw of SPX's ΔSVI ultimately yielded different results, I thus focus on ΔSVI^{SPX_1} to outline my findings. (The supplementary material and R code[6] provide more details if one wishes to dig deeper.) As a result, $Index = (SPX, FTSE)$ where $SPX = SPX_1$ from hereon.

One can identify several key dates at peaks in Figures 1 to 5 that graphically show my collected SVI data.[7] Interestingly, SPX's data (Figures 2 to 5) shows three peaks that did not show in CVP: 06/02/2018 and 09/02/2018, the days following the (more than) doubling of the the fear-gauge Chicago Board Options Exchange Volatility Index (Economist (8th of Febuary 2018)); and 26/12/2018, when tensions between the U.S. Federal Bank Chair and the Government's President were revealed as impactful. Turning to the FTSE data, the two days with the highest SVI results were 24/06/2016 and 27/06/2016, in the midst of the U.K's European Union withdrawal referendum result repercussion.

Table 2 shows that ΔSVI^{FTSE} and ΔSVI^{SPX} are stationary, which is essential in this case.[8] Indeed, ΔSVI^{Index}'s Augmented Dickey-Fuller (ADF) (Dickey and Fuller (1979)) Phillips-Perron (PP) (Phillips and Perron (1988)) tests indicate a rejection of the null in preference for the alternative of stationarity at the 99% Confidence Level (CL).

It is important to note several caveats with SVIs. First, SVI data has randomised elements so as to stop any individual form identifying any one particular person or group's internet searches. Figures 1 to 5 expose just this point. The differences from the separate draws for the same search term ("s&p 500"), time period (full-sample) and region (Worldwide) in Figures 2 and 3 especially highlights these contrasts; the former seem to have higher variance and values at any one time, as confirmed by Table 1.

Second, we must not forget that Google's implementation of caffeine (Dave Davis (2017)) on 08/06/2010 may have distorted SVI data little enough for CPV's work in 2017/2018, but may have accumulated by 13/03/2019, rendering this dataset less consistent from year to year.

2.3 Realized Volatilities and Index Excess Returns

As per Barndorff-Nielsen and Shephard (2002), Realised Volatility (Andersen, Bollerslev, Diebold, and Ebens (2001)) has been shown to be an accurate, useful and model-free representation of volatility in the literature. I therefore use it as a benchmark against which to assess

[6]The R code can be found at:
`https://github.com/johnukfr/IDaSRP/blob/master/R%20code`

[7]Note that the figures 1 to 5 are blue to aid referencing and comparison with CPV if one wishes to compare them against their SVI graph.

[8]For more on the necessity of stationary variables in research such as this one, see Verbeek (2008) Chapters 8 to 10.

Table 1: SVI Descriptive Statistics §

Variable	Statistic				
	Mean	Standard Deviation	Median	Skewness	Kurtosis
SVI^{FTSE}	2.764727	2.916147	2	8.9068	236.1216
SVI^{SPX_1}	11.8463	7.164055	10	3.135406	18.8146
SVI^{SPX_2}	9.723473	6.673175	8	3.064971	19.41453
$SVI^{SPX_{CPV}}$ *	9.24238	4.948923	8	2.994608	31.55793
$SVI^{SPX_{CPV-US}}$ *	10.22849	5.953334	9	3.050486	25.5173

Table 2: Stationarity Statistics §

Variable	Statistic	
	ADF	PP
ΔSVI^{FTSE}	-56.4	-61.073
ΔSVI^{SPX_1}	-86.5	-114.8
ΔSVI^{SPX_2}	-67.9	-79.023
$\Delta SVI^{SPX_{CPV}}$ *	-68	-82.101
$\Delta SVI^{SPX_{CPV-US}}$ *	-65.2	-78.147

All statistics have p-values of 0.01 or less.

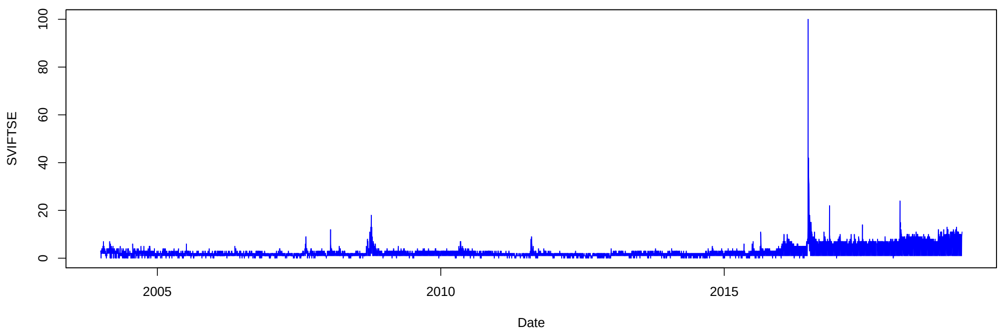

Figure 1: SVI_t^{FTSE}

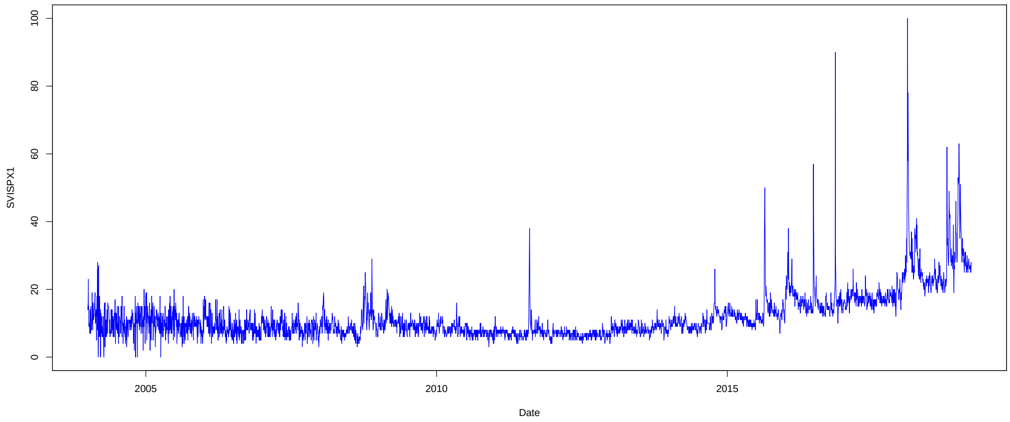

Figure 2: $\text{SVI}_t^{SPX_1}$

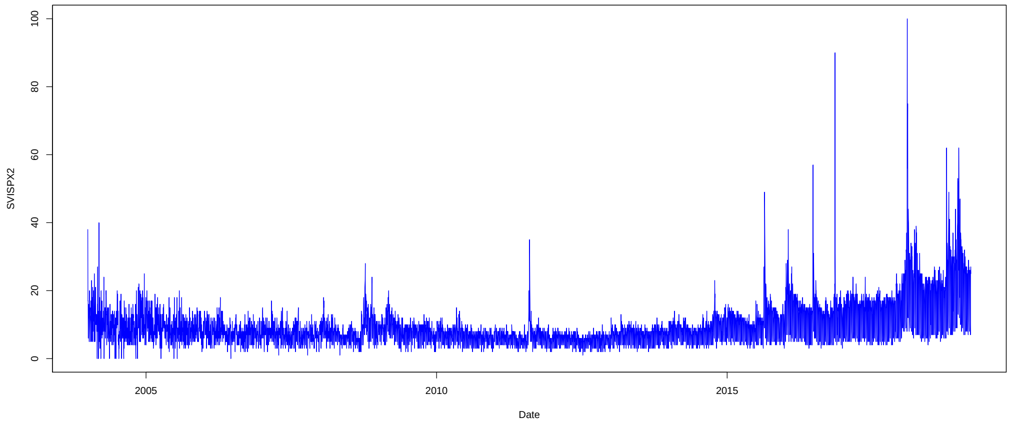

Figure 3: $\text{SVI}_t^{SPX_2}$

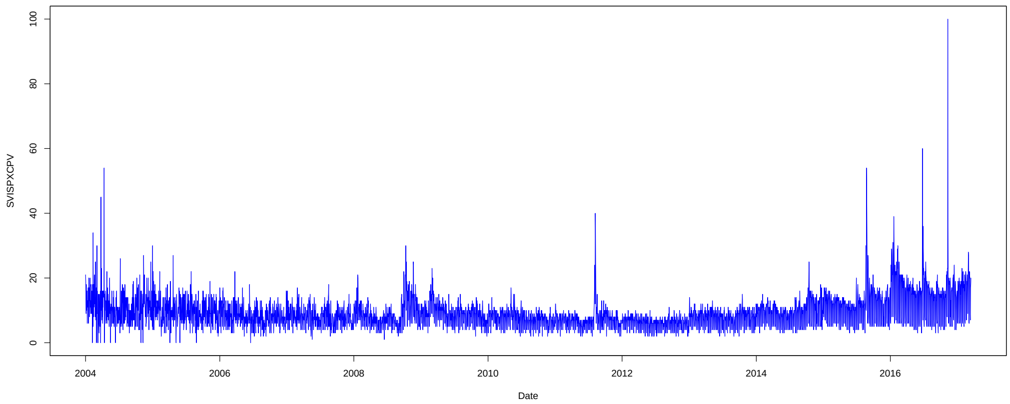

Figure 4: $\text{SVI}_t^{SPX_{CPV}}$

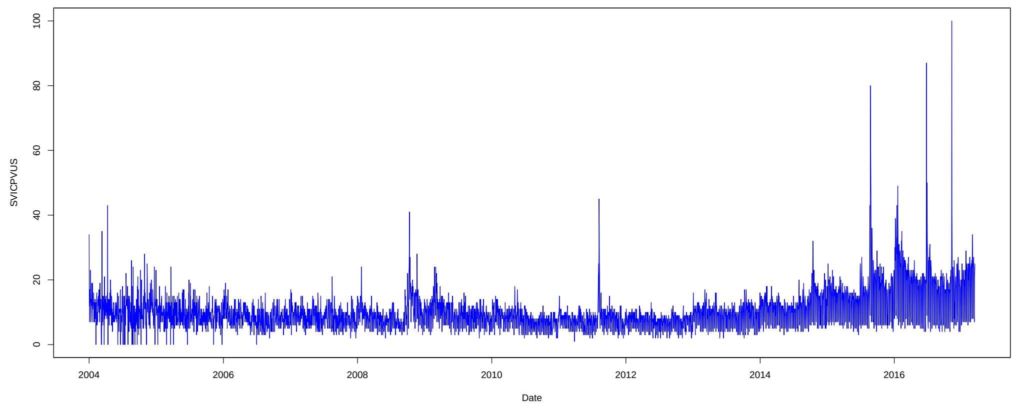

Figure 5: $\text{SVI}_t^{SPX_{CPV-US}}$

volatility forecasts constructed as per Section 3.

The SPX and FTSE close-prices (P^{SPX} and P^{FTSE} respectively) as well as their 5 minute subsampling Realised-Variances ($RVar^{SPX}$ and $RVar^{FTSE}$ respectively) are collected from the Realized Library of the Oxford-Man Institute. Realised Volatilities at time t are computed as

$$RV_t^{Index} = \sqrt{RVar_t^{Index}} \; .$$

Index excess returns at time t are computed as per CPV, Pesaran and Timmermann (1995) and Chevapatrakul (2013) for each index **:**

$$R_t^{Index} = \frac{P_t^{Index} - P_{t-1}^{Index}}{P_{t-1}^{Index}} - r_{f,t}^{Index} \tag{1}$$

where the risk free asset's daily return - $r_{f,t}^{Index}$ - is the One Month U.S. TreasuryBill Daily Return when $Index = SPX$ and the British Government 5Year Inflation ZeroCoupon Daily Return when $Index = FTSE$.

Figure 6 shows the Empirical Probability Distribution Functions (EPDFs) of R_t^{FTSE} and R_t^{SPX} for the full-sample. It allows us to discern their leptokurtoticity of approximately 8 and 11 respectively as per Table 3 [9] and as anticipated by SFFR (i). One may also note their skewness of approximately -0.015 and -0.20 that allows for the application of CDMTT's work.

Table 3: Descriptive Statistics

Variable	Statistic				
	Mean	Standard Deviation	Median	Skewness	Kurtosis
R^{FTSE}	0.000131821	0.01169291	0.000534463	-0.01498459	8.390776
R^{SPX}	0.000311301	0.01142788	0.000660012	-0.1953126	11.30544

[9] A fuller table of Descriptive Statistics is available in the supplementary material

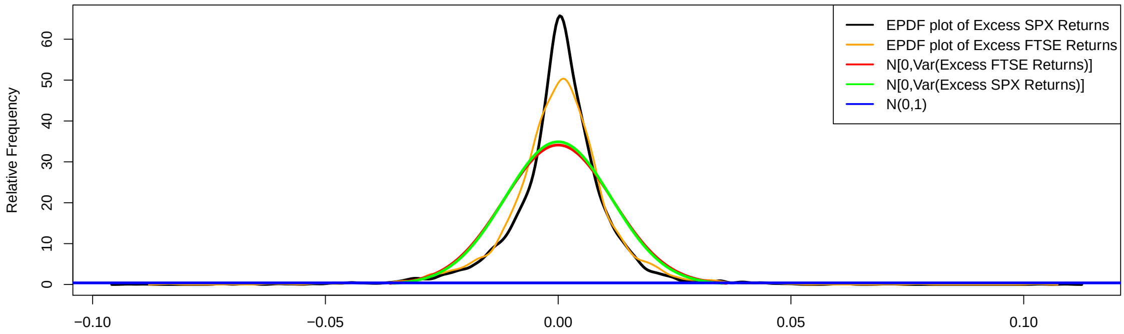

Figure 6: R_t^{Index} EPDFs and relevant Probability Distribution Functions (PDFs)

3 Methodology

3.1 Modelling Level Changes

Let $\{R_t\}_{t\in\mathbb{Z}}$ be the level change in index excess return $:$ $R_t \in \mathbb{R}$ at time t as per Section 2. Then one may attempt to encapsulate the movement in R_t via the discrete-time stochastic process

$$R_t = C + \phi_1 R_{t-1} + \epsilon_t \tag{2}$$

where $C \in \mathbb{R}$ is an unknown constant, ϕ_1 is the cofactor of the regressor R_{t-1} - the 1^{st} lag of R_t, and the error term $\epsilon_t \overset{iid}{\sim} N(0, \sigma^2)$ has standard deviation $\sigma \in \mathbb{R}$. This is an AR process of order one - AR(1). It is then easy to incorporate SVI - enabling us to study its effect on R_t - with the cofactor $\phi_2 \in \mathbb{R}$ via the AR(1)-SVI model:

$$R_t = C + \phi_1 R_{t-1} + \phi_2 \Delta SVI_{t-1} + \epsilon_t \ . \tag{3}$$

These two models will be used throughout this thesis to model the level change in index excess returns.

3.2 Modelling Non-Linear Variances

3.2.1 ARCH Models

As aforementioned, the lack of time-dependence in σ results in the AR(1) model outlined in Equation 2 and 3 failing to represent SFFR (iii). To account for this heteroscedasticity, ARCH models of order $q \in \mathbb{Z}$ where $q > 0$ can be used. These ARCH(q) models render σ into time-dependent σ_t $:$ $\epsilon_t \overset{iid}{\sim} N(0, \sigma_t^2)$. To do so, let Ω_t denote the information set at time t; let the conditional mean and variance of R_{t+1} respectively be $\mu_{t+1|t} = E(R_{t+1}|\Omega_t)$ and $\sigma^2_{t+1|t} = Var(R_{t+1}|\Omega_t)$ where E and Var are respectively the expectation and variance operators; let

$$\epsilon_t = \sigma_{t|t-1}\eta_t$$

where $\eta_t \perp\!\!\!\perp \Omega_t$ and $\eta_t \overset{iid}{\sim} N(0,1)$; and let

$$\sigma^2_{t|t-1} = c + \sum_{i=1}^{q} \alpha_i \epsilon^2_{t-i}$$

where $c, \alpha_i \in \mathbb{R}$, $c > 0$ and $\alpha_i \geq 0 \quad \forall i = (1, 2, ..., q)$ to allow only for positive - thus valid - variances.

Several expansions were applied to the ARCH model; the ones of interest here are namely the GARCH, GJRGARCH and EGARCH models of order one.

3.2.2 GARCH models

GARCH(1,1) account for SFFR (iv) and (v) in addition to (iii) and non-normal unconditional distributions in levels, which works in parallel with SFFR (i). To do so, an MA component is added to the ARCH(1):

$$\sigma^2_{t|t-1} = c + \alpha\epsilon^2_{t-1} + \beta\sigma^2_{t-1}$$

where elements are as aforementioned, $\beta \in \mathbb{R}$ is positive and in this specific case: $(\alpha + \beta) < 1$ for stationarity. I implement the SVI factor with positive $\delta \in \mathbb{R}$ - enabling us to study its effect on R_t's variance - via:

$$\sigma^2_{t|t-1} = c + \alpha\epsilon^2_{t-1} + \beta\sigma^2_{t-1} + \delta\Delta SVI_{t-1}$$

Whilst GARCH models are a natural fit in this study by the virtues mentioned, "*in general, models that incorporate volatility asymmetry such as EGARCH and GJR-GARCH perform better*" (Poon and Granger (2003), p.507). Indeed, GARCH models cannot account for SFFR (v) of leverage effects, whereas GJRGARCH and EGACH models may; CPV therefore considered them too - as do I. The thicker bottom left tail of Figure 6's EPDFs suggest such leverage effects.

3.2.3 GJRGARCH Models

GJRGARCH models account for asymmetry (SFFR (i)) via the last summand in

$$\sigma^2_{t|t-1} = c + \alpha\epsilon^2_{t-1} + \beta\sigma^2_{t-i} + \gamma I(\epsilon_{t-1} < 0)\epsilon^2_{t-1}$$

and second to last in

$$\sigma^2_{t|t-1} = c + \alpha\epsilon^2_{t-1} + \beta\sigma^2_{t-1} + \gamma I(\epsilon_{t-1} < 0)\epsilon^2_{t-1} + \delta\Delta SVI_{t-1}$$

where a leverage case would result in $\gamma > 0$, and $I(\cdot)$ is the indicator function returning 1 if its bracketed statement is true, 0 otherwise - in this case:

$$I(\epsilon_{t-1} < 0) = \begin{Bmatrix} 1 & \text{if } \epsilon_{t-1} < 0 \\ 0 & \text{otherwise} \end{Bmatrix} .$$

3.2.4 EGARCH Models

The use of the logarithm function insures non-negativity in EGARCH models

$$ln(\sigma^2_{t|t-1}) = c + \alpha \left[\frac{|\epsilon_{t-1}|}{\sigma_{t-1}} - \sqrt{\frac{2}{\pi}} \right] + \beta ln(\sigma^2_{t-1}) + \gamma \frac{\epsilon_{t-1}}{\sigma_{t-1}}$$

and

$$ln(\sigma^2_{t|t-1}) = c + \alpha \left[\frac{|\epsilon_{t-1}|}{\sigma_{t-1}} - \sqrt{\frac{2}{\pi}} \right] + \beta ln(\sigma^2_{t-1}) + \gamma \frac{\epsilon_{t-1}}{\sigma_{t-1}} + \delta \Delta SVI_{t-1}$$

where $ln(\cdot)$ and $|\cdot|$ denote the logarithm base e and the absolute value functions respectively and where $c, \alpha, \beta, \gamma \in \mathbb{R}$ are not constrained **:** $c + \alpha + \beta + \gamma > 0$. If $Var(R_t|\Omega_{t-1})$ and R_t react negatively to each other, β would be negative, and *vice versa.*

3.3 Sign Forecast Frameworks

3.3.1 Christopherson and Diebold Framework

Following from C&D, Christoffersen et al. (2006)'s working paper outlined how the conditional probability of a positive return in the next time period is

$$\mathbb{P}(R_{t+1} > 0|\Omega_t) = 1 - \mathbb{P}\left(\frac{R_{t+1} - \mu}{\sigma_{t+1|t}} \leq \frac{-\mu}{\sigma_{t+1|t}} \right) = 1 - F\left(\frac{-\mu}{\sigma_{t+1|t}} \right)$$

where $\mathbb{P}$ denotes probability, where there is no conditional mean predictability in returns **:** $\mu_{t+1|t} = \mu$, where $R_{t+1} \sim D(\mu, \sigma_{t+1|t})$ for a generic distribution D dependent only on μ and σ, and where F is the distribution function of the "standardized" return $\frac{R_{t+1}-\mu}{\sigma_{t+1|t}}$. Then R_{t+1}'s sign is predictable even if its conditional mean isn't, provided that $\mu \neq 0$. If F is asymmetric - as per SFFR(i) - then forecast constructions are still possible if $\mu = 0$ (Christoffersen et al. (2006)).

R_{t+1} sign forecasts are formulated as

$$\hat{\pi}^{C\&D_M}_{t+1} = \hat{\mathbb{P}}_{C\&D_M}(R_{t+1|t} > 0) = 1 - \frac{1}{t} \sum_{k=1}^{t} I\left(\frac{R_k - \hat{\mu}_{k|k-1}}{\hat{\sigma}_{k|k-1}} \leq \frac{-\hat{\mu}_{t+1|t}}{\hat{\sigma}_{t+1|t}} \right) \tag{4}$$

where M is the model used to compute recursive one-step-ahead out-of-sample forecasts $\hat{\mu}$ and $\hat{\sigma}$ ranging from GARCH to EGARCH-SVI[10], and where $k \in \mathbb{Z}$.

[10]Note that the model used to estimate $\hat{\mu}_{t+1|t}$ and $\hat{\sigma}_{t+1|t}$ are not specified on the right-hand-side of Equation 4 to keep the notation uncluttered. It is so throughout the paper.

3.3.2 Naïve Framework

A Naïve model of sign change is used as a benchmark to construct comparisons. It is formulated as

$$\hat{\pi}_{t+1}^{Naive} = \hat{\mathbb{P}}_{Naive}(R_{t+1|t} > 0) = \frac{1}{t}\sum_{k=1}^{t} I(R_k > 0) \ .$$

3.4 Error Functions

To asses the accuracy of volatility and positive sign probability forecasts, this sub-section specifies error functions.

Variance forecasts from the models GARCH to EGARCH-SVI (set M described in Section 3.3.1) are simply compared against realised volatilities to compute variance forecast errors as

$$\hat{\sigma}_{t|t-1} - RV_t \ .$$

Sign forecast errors are computed as

$$\hat{\pi}_{t+1}^{j} - \mathbb{P}(R_{t+1} > 0|\Omega_{t+\mathbf{1}}) \tag{5}$$

where $j = (C\&D_M, Naive)$. Note the bold '+1' in $\Omega_{t+\mathbf{1}}$ here to emphasise how $\mathbb{P}(R_{t+1} > 0|\Omega_{t+\mathbf{1}}) = I(R_{t+1} > 0)$.

3.5 Active Trading Strategy

To asses the financial repercussions of my forecasts, a trading strategy is studied whereby a profit maximising agent invests in the index if it is forecasted to increase in value and invests in the risk-free asset otherwise. At each time period, his/her incremental wealth is thus

$$R_{j,t}^{Index} = \omega_{j,t-1}^{Index} R_{m,t}^{Index} + (1 - \omega_{j,t-1}^{Index}) r_{f,t}^{Index}$$

where

$$\omega_{j,t} = I(\hat{\pi}_{t+1}^{j} > \psi)$$

for a probability threshold $\psi \in \mathbb{R} : 0 \leq \psi \leq 1$, and where

$$R_{m,t}^{Index} = \frac{P_t^{Index} - P_{t-1}^{Index}}{P_{t-1}^{Index}} \ .$$

Note that, in line with Equation 1, $R_{m,t}^{Index} = R_t^{Index} + r_{f,t}^{Index}$.

His/her return following a strategy is therefore cumulative and can be defined as

$$\mathcal{R}_{j,t}^{Index} = \prod_{i=1}^{t} R_{j,i}^{Index} \ . \tag{6}$$

For completeness, Sharpe-ratios of such $\mathcal{R}_{j,t}^{Index}$ are also provided. They are defined as:

$$\frac{\mathcal{R}_{j,t}^{Index} - \mathcal{R}_{f,t}^{Index}}{\sqrt{Var(\mathcal{R}_{j,t}^{Index})}}$$

where $\mathcal{R}_{f,t}^{Index}$ is $r_{f,t}^{Index}$ equivalent to $\mathcal{R}_{j,t}^{Index}$ in Equation 6 above for 'like-to-like' comparison **:**

$$\mathcal{R}_{f,t}^{Index} = \prod_{i=1}^{t} R_{f,i}^{Index} \ .$$

Similarly, the used Buy-and-Hold strategy may be considered a misnomer here as it is defined as the return from recursively investing previous-day-returns in the index every day, starting with one unit of currency (Great British Pound Sterling (£) and U.S. Dollar ($) for the FTSE and SPX respectively for ease) for compatibility, *i.e.*:

$$\prod_{i=1}^{t} R_{B\&H,i}^{Index} \ .$$

4 Results

Since this study uses variance forecasts to then use in further models, this section will not focus on mean forecasts. None the less, all model regressor coefficients are computed via Maximum-Log-Likelihood; one-step-ahead out-of-sample and out-of-CPV-sample cofactors are estimated as such and recursively.

4.1 SPX Results

4.1.1 SPX Models and Forecasts

In-sample parameters are displayed in Table 4. It shows mean and variance equation coefficients similar to CPV's in magnitude but drastically less statistically-significant from zero.

Table 5 shows coefficients computed from out-of-sample regressions following models outlined in Section 3; they are more akin to CPV's results: variance model estimates are all statistically-significantly different from zero (bar α). EGARCH models' α remain negative, but that may not necessarily distort my variance forecasts too far away from CPV's as their intercept estimate was negative (too).

Turning to forecasts, Table 6† shows recursive one-step-ahead out-of-sample result error statistics. They clearly favour SVI-free models over SVI-implementing ones. This is quantified well by the Diebold and Mariano (1995) (DM) statistic (that are all negative); it tests the predictive accuracy of two sets of forecasts against another (SVI-free against SVI-implementing ones here) comparing to true values (realised volatility here) with the null hypothesis that the second - SVI-implementing - set is more accurate than the first - SVI-free; a positive DM statistic is indicative of SVI-implementing model forecasts being more accurate. Results reject the null in favour of the alternative hypothesis that the SVI-implementing model is less accurate than the SVI-free one. Their p-value thus suggest SVI-implementing models to be statistically-significantly worst at the 93% CL (at least).

4.1.2 SPX Sign Probability Forecasts

As in Section 3.3, variance forecasts were used in the C&D framework using my family of GARCH models. They allowed for the computation of all of SPX's $\hat{\pi}_{t+1}^{j}$ that were measured against realised sign probabilities as per Equation 5. Table 7 displays results - including Brier Scores (Brier (1950)) - and show no statistically-significant findings[11] suggesting that SVI-implementing models are better forecasters of sign change than SVI-free ones - especially regarding GJRGARCH models. Further graphical analyses[12] from the perspective of a profit

[11] A finding is said 'statistically-significant' with a CL of 90% or more only.

[12] In the interest of space, SPX graphical results akin to Figures 7 and 10 are only shown in the supplementary material.

Table 4: SPX In-Sample Regression Coefficients‡

Model	AR Model Coefficients			Variance Model Coefficients				
	C	ϕ_1	ϕ_2	c	α	β	γ	δ
GARCH	0.000407	0.007451		$2.672989\text{x}10^{-08}$	0.000044	0.998944		
	(0.000445)	(0.063518)		(0.000001)	(0.000819)	(0.000870)		
	[0.36105]	[0.90662]		[0.96595]	[0.95730]	[0.00000]		
GARCH-	0.000402	0.007817	-0.000033	$9.812377\text{x}10^{-14}$	0.000690	0.998893		$2.103174\text{x}10^{-8}$
$\text{SVI}_1^{\text{SPX}}$	(0.000451)	(0.063667)	(0.000076)	(0.000001)	(0.000017)	(0.000656)		(0.000000)
	[0.37206]	[0.90229]	[0.66477]	[1.00000]	[0.00000]	[0.00000]		[0.00000]
GJRGARCH	0.000317	-0.007488		0.000004	$6.371049\text{x}10^{-10}$	0.885774	0.066545	
	(0.000442)	(0.064021)		(0.000000)	(0.013990)	(0.017150)	(0.046984)	
	[0.47272]	[0.90690]		[0.00000]	[1.00000]	[0.00000]	[0.15667]	
GJRGARCH-	0.000275	-0.013369	-0.000007	0.000004	$4.386507\text{x}10^{-05}$	0.875033	0.065633	0.000001
$\text{SVI}_1^{\text{SPX}}$	(0.000429)	(0.063647)	(0.000076)	(0.000000)	(0.008389)	(0.012803)	(0.005849)	(0.000000)
	[0.52133]	[0.83362]	[0.93094]	[0.00000]	[1.00000]	[0.00000]	[0.00000]	[0.00000]
EGARCH	0.000267	-0.000534		-4.436117	-0.305879	0.555263	-0.276841	
	(0.000443)	(0.051882)		(0.140141)	(0.046682)	(0.014156)	(0.090104)	
	[0.546929]	[0.991785]		[0.000000]	[0.000000]	[0.000000]	[0.002123]	
EGARCH-	0.000249	-0.008424	0.000020	-4.197116	-0.290930	0.579301	-0.263700	0.015024
$\text{SVI}_1^{\text{SPX}}$	(0.000395)	(0.054497)	(0.000072)	(0.036867)	(0.058299)	(0.006074)	(0.070405)	(0.017923)
	[0.528855]	[0.877150]	[0.783515]	[0.000000]	[0.000001]	[0.000000]	[0.000180]	[0.401888]

This tables shows coefficients estimated from models outlined in Section 3 under which are displayed their standard errors and p-values in brackets and square brackets respectively. Statistical-significance of the non-bracketed values (*i.e.* the coefficients) are to the CL of the difference between their attached square-bracketed values and one.

Table 5: SPX Out-of-Sample Regression Coefficients‡

Model	AR Model Coefficients			Variance Model Coefficients				
	C	ϕ_1	ϕ_2	c	α	β	γ	δ
GARCH	0.000632	-0.058803		0.000002	0.117652	0.860377		
	(0.000112)	(0.017683)		(0.000001)	(0.010819)	(0.011693)		
	[0.000000]	[0.000883]		[0.005445]	[0.000000]	[0.000000]		
GARCH-	0.000480	-0.053840	-0.000002	0.000001	0.069613	0.917594		0.000002
$\text{SVI}_1^{\text{SPX}}$	(0.000097)	(0.002786)	(0.000033)	(0.000000)	(0.000448)	(0.001225)		(0.000000)
	[0.000001]	[0.000000]	[0.962353]	[0.000000]	[0.000000]	[0.000000]		[0.000000]
GJRGARCH	0.000307	-0.054967		0.000002	0.000001	0.875168	0.196215	
	(0.000103)	(0.017325)		(0.000000)	(0.002900)	(0.006883)	(0.014749)	
	[0.002807]	[0.001510]		[0.000000]	[0.999854]	[0.000000]	[0.000000]	
GJRGARCH-	0.000262	-0.051728	-0.000025	0.000001	$1.196655\text{x}10^{-8}$	0.906657	0.143491	0.000002
$\text{SVI}_1^{\text{SPX}}$	(0.000006)	(0.015578)	(0.000034)	(0.000000)	(0.000182)	(0.000227)	(0.001029)	(0.000000)
	[0.000000]	[0.000899]	[0.455235]	[0.000000]	[0.999947]	[0.000000]	[0.000000]	[0.000000]
EGARCH	0.000345	-0.054751		-0.274122	-0.156593	0.970680	0.156443	
	(0.000103)	(0.017157)		(0.001784)	(0.007590)	(0.000286)	(0.006404)	
	[0.000764]	[0.001417]		[0.000000]	[0.000000]	[0.000000]	[0.000000]	
EGARCH-	0.000303	-0.057591	0.000026	-0.128015	-0.111602	0.986324	0.108074	0.099365
$\text{SVI}_1^{\text{SPX}}$	(0.000092)	(0.017100)	(0.000038)	(0.001888)	(0.008018)	(0.000053)	(0.005033)	(0.007376)
	[0.001064]	[0.000757]	[0.486695]	[0.000000]	[0.000000]	[0.000000]	[0.000000]	[0.000000]

This tables shows coefficients estimated from models outlined in Section 3 under which are displayed their standard errors and p-values in brackets and square brackets respectively. Statistical-significance of the non-bracketed values (*i.e.* the coefficients) are to the CL of the difference between their attached square-bracketed values and one.

Table 6: SPX Out-of-Sample Recursive Variance Forecast Error Statistics ‡ ¶

$\hat{\sigma}_{t\|t-1}$ Model	Mean	Standard Deviation	RMSE	DM
GARCH	*0.001712773*	0.003939245	0.00429498	-2.2926
GARCH-SVI	0.001695439	0.003980292	0.004325824	[0.01097]
GJRGARCH	0.001586598	0.003562079	0.00389899	-1.9095
GJRGARCH-SVI	0.001571206	0.003594243	0.003922196	[0.02814]
EGARCH	**0.001296343**	**0.003459501**	**0.003693951**	-1.5185
EGARCH-SVI	0.001352416	*0.007389872*	*0.00751158*	[0.06449]

Under DM test statistics are located their p-values in squared brackets; their alternative hypothesis is that the SVI-implementing model is less accurate than the SVI-free one.

maximising agent - akin to in Section 4.2.3 - strongly suggested the use of SVI-implementing variance forecasts in C&D's framework to be extremely counterproductive.

Upon first inspection, one may find it compelling to see such evidence suggesting ΔSVI as a bad predictor, but it may instead be evidence of the lack of representation ΔSVI gives towards investors' information demand, reminiscent of the caveats outlined in Section 2.2.

Furthermore, the time sample used above (longer than CPV's) rendered results that are in line with the idea that once traders know of a successful forecasting model, it becomes readily adopted, which "*can then cause stock prices to move in a manner that eliminates the models' forecasting ability*" (Rapach and Zhou (2013), p.330). (Lo (2005); Timmermann and Granger (2004))

4.2 FTSE Results

4.2.1 FTSE Models and Forecasts

In-sample and full-sample parameters are displayed in Tables 10 and 11 respectively. Looking at in-sample results, notice that ΔSVI is only statistically-significantly different from zero in the EGARCH-SVI model but acts negatively, which is surprising. This could be because the in-sample period was too short for the model to correctly estimate δ.

Also, even though full-sample EGARCH-SVI results do not suggest ΔSVI to be negatively correlated to R^{FTSE}'s variance, it only appears to be statistically-significant to the 90% CL. The sole variance models' δ estimates comparable to CPV's are in the GARCH-SVI and GJRGARCH-SVI models that suggest positive correlations between ΔSVI and R^{FTSE}'s variance with statistical-significance (at least at the 99% CL).

As anticipated, GJRGARCH models' γ cofactors were estimated as positive. This is in line with observed leverage effects of excess returns (*i.e.*: R^{FTSE}'s EPDF negative skewness referring to SFFR(i)). Also note that both EGARCH's β estimates are positive, indicating a

Table 7: SPX Out-of-Sample Positive Excess Return Probability Forecast Error Statistics [‡] [¶]

$\hat{\sigma}_{t\|t-1}$ and $\hat{\mu}_{t\|t-1}$ Model	Mean	Standard Deviation	Brier Score	DM
GARCH	*0.009165994*	0.4990319	0.2490463	0.94406
GARCH-SVI	0.006523514	*0.4991192*	*0.2490918*	[0.1726]
GJRGARCH	**0.002551502**	0.4987796	0.2487174	-0.20004
GJRGARCH-SVI	0.002800718	0.4987847	0.2487237	[0.5793]
EGARCH	0.003155205	0.4987842	0.2487255	0.35842
EGARCH-SVI	0.003084877	**0.498753**	**0.2486939**	[0.36]

Under the DM test statistics are located their p-values in squared brackets; their alternative hypothesis is that the SVI-implementing model is <u>more</u> accurate than the SVI-free one.

positive relationship between excess returns and their variances; this confirms the logic that I may be able to exploit this relation for forecast model constructions.

Moving on to forecasts, Table 8 [†] displays FTSE related results - as Table 6 did SPX's - and similarly favours the SVI-free models over SVI-implementing ones, as emphasised by DM test statistics that are all negative. Note again that - as specified - their alternative hypothesis is that the SVI model is less accurate than the SVI-free one. Their p-value thus suggest SVI models to be statistically-significantly worst at the 98% CL (at least).

It is interesting to see here that even though Root Mean Squared Error (RMSE) statistics greatly disfavour the EGARCH-SVI model and favours the EGARCH one, the DM statistic is least harsh in rejecting the SVI-implementing model in the EGARCH case. It comes to show that several approaches need to be taken to study these results.

Table 8 also reveals that for SVI-free cases - according to RMSEs - more complex GARCH models encapsulating asymmetries perform better than the GARCH one, as per Awartani and Corradi (2005).

My results are in stark contrast with CPV's that suggested the implementation of SVI as significantly increasing the accuracy of volatility forecasts, again highlighting caveats outlined in Section 2.2.

4.2.2 FTSE Sign Probability Forecasts

Sign probability forecast error statistics are displayed in Table 9[†] as per Table 7. They show more anticipated results, in parallel with CPV. (Note that - as outlined - the DM test's alternative hypothesis is that the SVI-implementing model is more accurate than the SVI-free one this time.) Indeed, apart from the GJRGARCH case's mean, they indicate that variance models encapsulating asymmetries are preferable.

SVI-implementing cases - however - are only preferable according to the DM test at the 74%, 89% and 76% CLs for the GARCH, GJRGARCH and EGARCH models respectively - thus not statistically-significantly. This may be in line with Rapach and Zhou (2013)'s afore-

Table 8: FTSE Out-of-Sample Recursive Variance Forecast Error Statistics ‡ ¶

$\hat{\sigma}_{t\|t-1}$ Model	Mean	Standard Deviation	RMSE	DM
GARCH	0.001668278	0.004116172	0.004440866	-7.897438
GARCH-SVI	*0.001830709*	0.004119785	0.004507702	[1.879x10^{-15}]
GJRGARCH	0.001453083	0.003806531	0.004073952	-4.165269
GJRGARCH-SVI	0.001561568	0.003789451	0.0040981	[1.592x10^{-05}]
EGARCH	**0.00139185**	**0.003704772**	**0.003957115**	-2.265701
EGARCH-SVI	0.001710052	*0.004869707*	*0.00516059*	[0.01176]

Under DM test statistics are located their p-values in squared brackets; their alternative hypothesis is that the SVI-implementing model is <u>less</u> accurate than the SVI-free one.

Table 9: FTSE Out-of-Sample Positive Excess Return Probability Forecast Error Statistics ‡ ¶

$\hat{\sigma}_{t\|t-1}$ and $\hat{\mu}_{t\|t-1}$ Model	Mean	Standard Deviation	Brier Score	DM
Naïve	-0.5239242	0.499474	0.5239059	
GARCH	*0.01648532*	*0.5009519*	*0.2511544*	0.65944
GARCH-SVI	0.01609128	0.500874	0.2510634	[0.2548]
GJRGARCH	0.01140799	0.5008019	0.2508627	1.2713
GJRGARCH-SVI	0.01159474	0.5007111	0.2507758	[0.1018]
EGARCH	0.01095059	0.5006574	0.2507078	0.71382
EGARCH-SVI	**0.01083333**	**0.5004532**	**0.2505005**	[0.2377]

Under the DM test statistics are located their p-values in squared brackets; their alternative hypothesis is that the SVI-implementing model is <u>more</u> accurate than the SVI-free one.

mentioned theory that as traders catch wind of the effectiveness of a forecasting models, they decrease their predictive powers. Subsequent findings bellow - none-the-less - demonstrate that strategies developed above are of greater use in FTSE's case.

Table 10: FTSE In-Sample Regression Coefficients‡

Model	AR Model Coefficients			Variance Model Coefficients				
	C	ϕ_1	ϕ_2	c	α	β	γ	δ
GARCH	0.000273	-0.122103		$2.41853\text{x}10^{-08}$	$7.540898\text{x}10^{-05}$	0.998799		
	(0.000375)	(0.062686)		(0.000001)	(0.000740)	(0.000774)		
	[0.466779]	[0.051433]		[0.970382]	[0.918873]	[0.000000]		
GARCH-SVI	0.000294	-0.123094	-0.000094	$1.79235\text{x}10^{-13}$	0.000723	0.998593		$1.964113\text{x}10^{-08}$
	(0.000377)	(0.062723)	(0.000244)	(0.000001)	(0.000642)	(0.000724)		(0.000001)
	[0.434406]	[0.049704]	[0.701085]	[1.000000]	[0.260024]	[0.000000]		[0.978754]
GJRGARCH	0.000217	-0.127894		$7.711872\text{x}10^{-08}$	0.000100	0.988647	0.016066	
	(0.000372)	(0.063246)		(0.000001)	(0.023511)	(0.009502)	(0.033904)	
	[0.558910]	[0.043158]		[0.944303]	[0.996604]	[0.000000]	[0.635593]	
GJRGARCH-SVI	0.000250	-0.123736	-0.000078	$2.381989\text{x}10^{-12}$	0.000153	0.992976	0.011012	$2.477344\text{x}10^{-08}$
	(0.000387)	(0.062949)	(0.000244)	0.000001	(0.013542)	(0.002077)	(0.027472)	(0.000001)
	[0.518074]	[0.049338]	[0.747766]	0.999997	[0.990956]	[0.000000]	[0.688544]	[0.967623]
EGARCH	0.000256	-0.112421		-1.010648	-0.131410	0.899460	-0.028995	
	(0.000378)	(0.059081)		(0.011920)	(0.007552)	(0.000029)	(0.060242)	
	[0.498628]	[0.057064]		[0.000000]	[0.000000]	[0.000000]	[0.630304]	
EGARCH-SVI	0.000338	-0.123158	-0.000099	-0.362933	-0.043256	0.963858	-0.130684	-0.021257
	(0.000009)	(0.004053)	(0.000003)	(0.000002)	(0.000913)	(0.000117)	(0.000003)	(0.000022)
	[0.000000]	[0.000000]	[0.000000]	[0.000000]	[0.000000]	[0.000000]	[0.000000]	[0.000000]

This tables shows estimated coefficients under which are shown their standard errors and p-values in brackets and squared brackets respectively.

Table 11: FTSE Full-Sample Regression Coefficients‡

Model	AR Model Coefficients			Variance Model Coefficients				
	C	ϕ_1	ϕ_2	c	α	β	γ	δ
GARCH	0.000455	-0.029178		0.000002	0.110992	0.874124		
	(-0.000127)	(-0.017304)		(0.000001)	(0.017641)	(0.018653)		
	[0.000346]	[0.091765]		[0.090051]	[0.000000]	[0.000000]		
GARCH-SVI	0.000524	-0.027813	-0.000143	0.000002	0.112734	0.871592		6.571518x10^{-07}
	(0.000108)	(0.017292)	(0.000065)	(0.000001)	(0.012276)	(0.013168)		(1.541017x10^{-09})
	[0.000001]	[0.107746]	[0.026865]	[0.075077]	[0.000000]	[0.000000]		[0.000000]
GJRGARCH	0.0001	-0.021213		0.000002	0.002368	0.891332	0.170234	
	(0.000117)	(0.017197)		(0.000001)	(0.00176)	(0.00753)	(0.009376)	
	[0.392452]	[0.217398]		[0.000069]	[0.178353]	[0.000000]	[0.000000]	
GJRGARCH-	0.00021	-0.018892	-0.000229	0.000002	0.000898	0.893209	0.171042	3.750633x10^{-07}
SVI	(0.000132)	(0.017126)	(0.000058)	(0.000001)	(0.004896)	(0.012094)	(0.024371)	(4.151440x10^{-10})
	[0.110559]	[0.269969]	[0.000085]	[0.107655]	[0.854463]	[0.000000]	[0.000000]	[0.000000]
EGARCH	0.000036	-0.017757		-0.172217	-0.11993	0.981224	0.134368	
	(0.000037)	(0.005279)		(0.0025)	(0.009554)	(0.000139)	(0.010807)	
	[0.32876]	[0.000769]		[0.000000]	[0.000000]	[0.000000]	[0.000000]	
EGARCH-	0.00012	-0.015922	-0.000208	-0.172782	-0.120556	0.981384	0.129674	0.003898
SVI	(0.000062)	(0.015704)	(0.000014)	(0.007456)	(0.007187)	(0.00074)	(0.008584)	(0.002337)
	[0.051614]	[0.310644]	[0.000000]	[0.000000]	[0.000000]	[0.000000]	[0.000000]	[0.095355]

This tables shows estimated coefficients under which are shown their standard errors and p-values in brackets and squared brackets respectively.

4.2.3 FTSE Trading Strategy Results

This section investigates the financial implications of my work via the lens of a single profit maximising agent deciding on whether to invest in the risk-free rate of the index host's country (as defined in Section 2.3) or in the index itself. In this instance, the investor chooses between the r_f^{FTSE} and the R_m^{FTSE}.

First, results for $\psi = 0.5$ are investigated. $\psi = 0.5$ implies that the agent invests in the index if forecasts predict a higher probability of increase in its excess return than a decrease. It may seem the only logical ϕ choice for a purely profit maximising agent, but it - in fact - is not necessarily so for a utility maximising agent - as outlined by CPV - as one may be ready to take on more risk for the chance of greater rewards in accordance with their risk tolerance, or *vice versa*. Table 12 displays results' descriptives for such an agent applying the different strategies outlined.[13] Figure 7 shows out-of-sample performance of all FTSE strategies.

It clearly shows the Naïve and C&D$_{\text{GARCH}}$ strategies' poor performances, explaining their low Sharpe-ratios (even bellow B&H's).

It also displays how all active strategies seem to need a 3-year training period (01/01/2005-01/01/2008) before exceeding B&H's, at which point they anticipate the 2008 financial crisis and tower over B&H. This is reminiscent of the poor results the in-sample regression displayed in Table 10.

None of them - however - anticipated the late-2011/early-2012 European sovereign debt crisis - at which time the C&D$_{\text{GARCH}}$ fell short of even B&H and never outperformed it again - nor the 2015–16 stock market sell-off (close to the 2015 Greek debt default).

Moreover, when failing to keep restrained and only buying the risk free asset in turbulent times, active strategies' relative losses look greater than B&H's despite keeping above it in absolute terms; this is reflected in their high variance in Table 12. It must be noted that despite these shortcomings, all active strategies (bar C&D$_{\text{GARCH}}$) diverge above B&H as time goes, suggesting that they are working and predicting when index returns are positive on a regular enough basis.

What is also greatly interesting is C&D$_{\text{GJRGARCH-SVI}}$'s performance. Indeed, it hovers constantly as the best strategy post-2008, and remarkably outperforms C&D$_{\text{EGARCH-SVI}}$ - as quantified in its Sharpe-ratio - even though results from Table 9 favoured the latter. This could be due to sporadic and irrelevant single losses on the part of C&D$_{\text{EGARCH-SVI}}$, or single gains from C&D$_{\text{GJRGARCH-SVI}}$. Indeed - as outlined in Pesaran and Timmermann (2000) - the penalty for strategies to buy or not to buy may differ from one to the other. But C&D$_{\text{GJRGARCH-SVI}}$ does continue to diverge even from C&D$_{\text{EGARCH-SVI}}$, suggesting it to continuously outperform it.

The Naïve strategy performed the worst, as expected.

[13] Note that for Sharp-ratios, I consider the risk-free-rate to be the return from recursively investing in $r_{f,t}^{FTSE}$ daily over the full-sample time period (from $t = 1$ to $t = 3932$) for comparability. Here it is 1.228738, *i.e.* $r_f^{FTSE} = 1.228738$. This is very close to the straight forward difference in the risk free asset price at the start and end of the period of 0.229264108 (*i.e.*: $\frac{97.80099-79.5606}{79.5606}$).

Table 12: Descriptives of Cummulative Returns from Trading Strategies Aplied by an Agent with $\psi = 0.5$ to the FTSE ‡ ¶

Strategy j	Cumulative Return $\mathcal{R}_j^{Index}$	Return Standard Deviation $\sqrt{(Var(\mathcal{R}_j^{Index})}$	Sharpe Ratio
Naïve	0.215485	0.08690555	-0.15249889
Buy-and-hold	0.463295	0.1420475	1.651257502
C&D$_{\text{GARCH}}$	*0.262708*	**0.1143895**	*0.296967816*
C&D$_{\text{GARCH-SVI}}$	0.572055	0.1865034	1.840808264
C&D$_{\text{GJRGARCH}}$	0.562979	0.2003561	1.668234708
C&D$_{\text{GJRGARCH-SVI}}$	**0.83586**	*0.2837949*	**2.139298486**
C&D$_{\text{EGARCH}}$	0.648182	0.2241649	1.871140397
C&D$_{\text{EGARCH-SVI}}$	0.673478	0.2272068	1.957423809

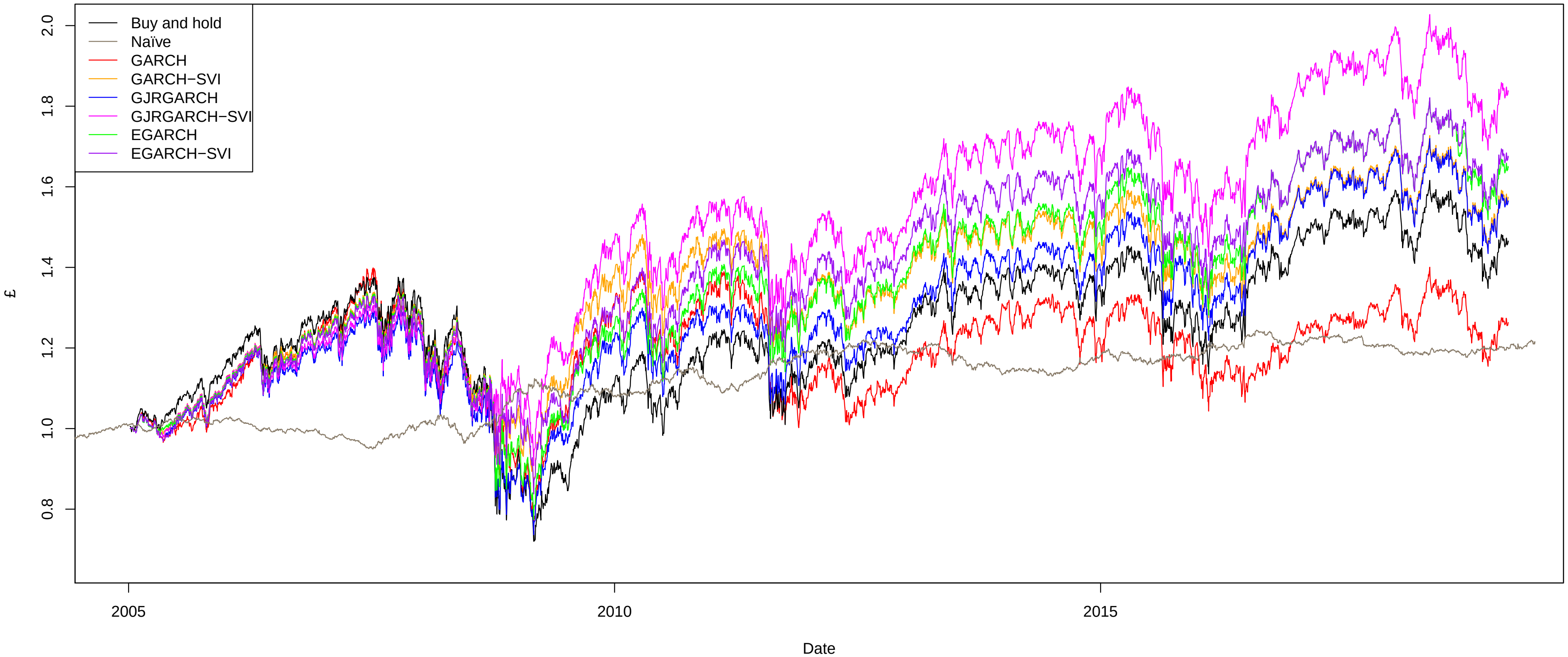

Figure 7: Cumulative Returns of Investment Strategies for an Investor with $\psi = 0.5$

This graph shows $\mathcal{R}_{j,t}^{FTSE}$ for the out-of-sample period - *i.e.*: for t in between 01/01/2005 and 13/03/2019. Notice that the Naïve strategy did not need variance model forecasts to produce forecasts itself. It also is not restricted by the availability of SVI data that ends on 13/03/2019. Therefore, the Naïve strategy results start before and ends after its counterparts.

4.2.3.1 Optimal Risk Analysis

One may now wonder if $\psi = 0.5$ truly is optimal. A three dimensional (3D) plot for each strategies as in Figure 7 for all ψs would allow for a wholistic view as to which one is indeed optimal. Such plots were made for all strategies[14], the most successful one - C&D$_{\text{GJRGARCH-SVI}}$ - for which are Figures 8 and 9. The second figure of the pair zooms into the ψ value range that allowed for highest strategy returns. It happens to be between 0.45 and 0.6 for each strategy (bar the C&D$_{\text{GARCH-SVI}}$, where it was between 0.4 and 0.55).

Low-resolution graphs such as Figure 8 rendered cumulative returns at ψ increments of 0.05 from 0 to 1. They show dates as time periods and ψ as 'Investment Threshold'. Their high-resolution counterparts - akin to Figure 9 - rendered cumulative returns at ψ increments of 0.0125 from 0.45 to 0.55; they exposed peaks and troughs previously unseen in low-resolution[15], and allowed for graphical analysis and approximation for the best ψ values for each strategy[16] - displayed in Figure 10 and analysed in Table 13.

The first revealing finding is that all strategies performed better with $\psi < 0.5$ apart from one using GJRGACHs' variance forecasts that did so when $\psi > 0.5$. This strongly suggests that - as aforementioned - the penalty for strategies to buy or not to buy may differ from one to the other, resulting in higher type II errors (when failing to predict a downturn in strategy return and investing in the index). This would mean that the investor should be relatively risk loving in fear of loosing out in order to maximise profits.

The second interesting finding comparing Figures 7 and 10 comes from the great improvement in C&D$_{\text{GARCH-SVI}}$'s performance, especially contrasting it with C&D$_{\text{GARCH}}$'s. SVI inclusion has proven very important (for both 0.5 and optimal ψ) in forecasts for which Table 9's DM statistic suggested it would have the smallest effect.

None-the-less, the strategy that proved optimal remained C&D$_{\text{GJRGARCH-SVI}}$, but this time with $\psi > 0.5$ suggesting to err on the side of caution and only invest when forecasts strongly indicate an increase in the index.

[14] 3D graphs can be found in the supplementary material.

[15] These high resolution 3D graphs show dates at time periods and ψ as 'Investment Threshold' starting from one meaning $\psi = 0.45$.

[16] Further more granular 2D graph inspections were done to workout more precise optimal ψ values.

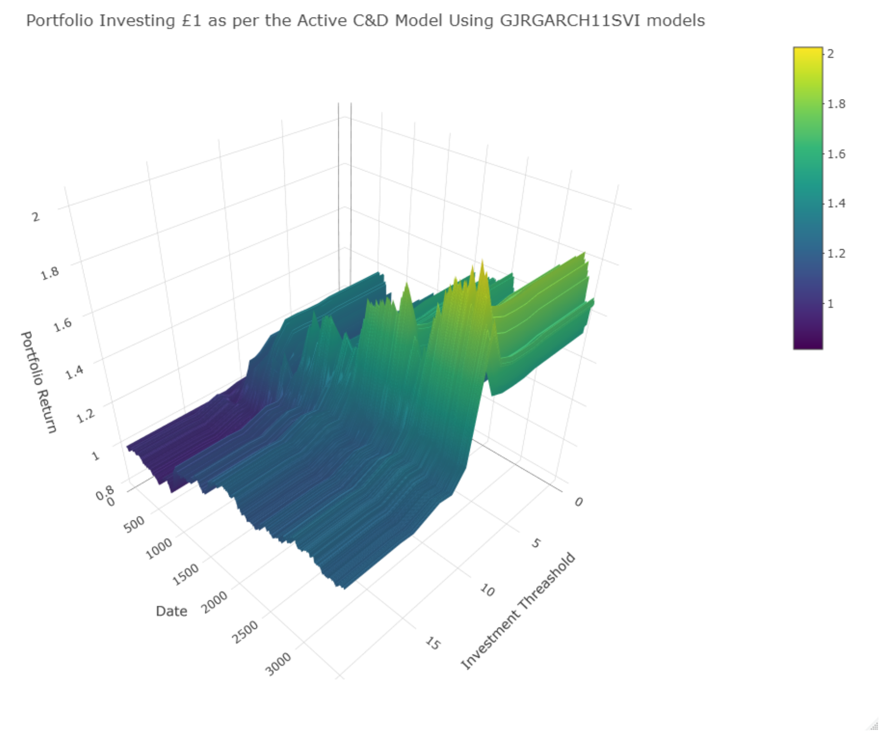

Figure 8: Cumulative Returns of Active Investment Strategy Following the C&D$_{\text{GJRGARCHSVI}}$ strategy for $0 \leq \psi \geq 1$

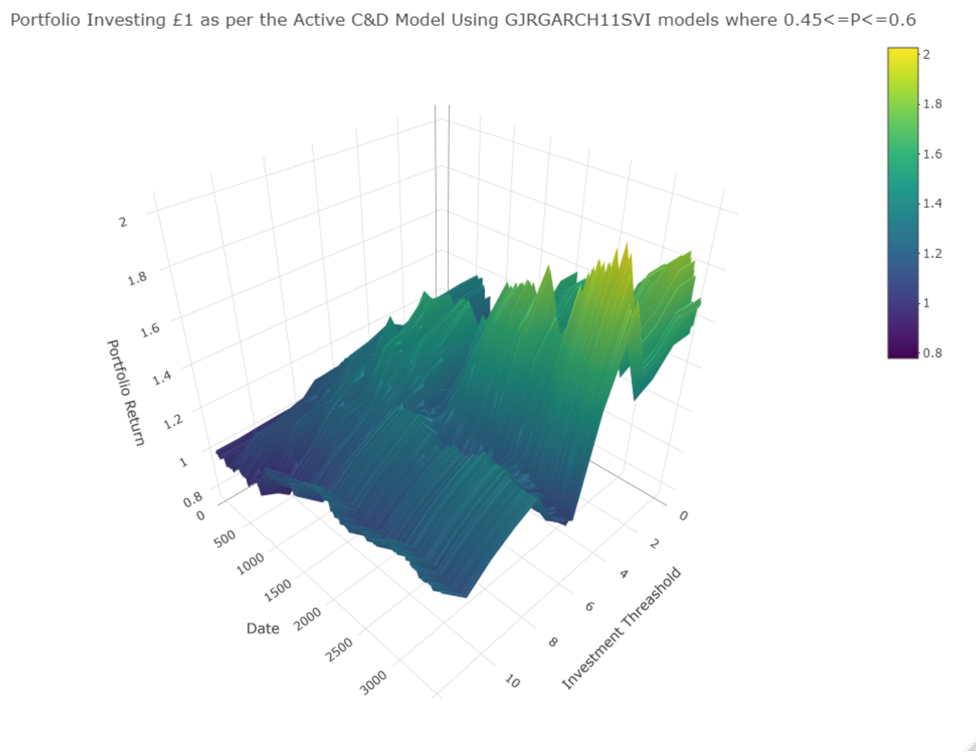

Figure 9: Cumulative Returns of Active Investment Strategy Following the C&D$_{\text{GJRGARCHSVI}}$ strategy for $0.45 \leq \psi \geq 0.6$

Table 13: Cumulative Returns of Investment Strategies for an Investor with ψ optimal to the nearest 0.0005 for each strategy applied to the FTSE ‡ ¶

Strategy j	Cumulative Return $\mathcal{R}_j^{Index}$	Return Standard Deviation $\sqrt{(Var(\mathcal{R}_j^{Index})}$	Sharpe Ratio
Naïve	0.215485	0.08690555	-0.15249889
Buy-and-hold	0.463295	0.1420475	1.651257502
C&D$_{\text{GARCH}}$	*0.31429*	**0.1337679**	*0.639555529*
C&D$_{\text{GARCH-SVI}}$	0.867921	0.2854279	2.239385148
C&D$_{\text{GJRGARCH}}$	0.659908	0.2205491	1.954984174
C&D$_{\text{GJRGARCH-SVI}}$	**0.912214**	*0.2963181*	**2.306561766**
C&D$_{\text{EGARCH}}$	0.708553	0.2358739	2.034201325
C&D$_{\text{EGARCH-SVI}}$	0.732368	0.235332	2.140082947

The Naïve and B&H strategies are the same as in Table 12.

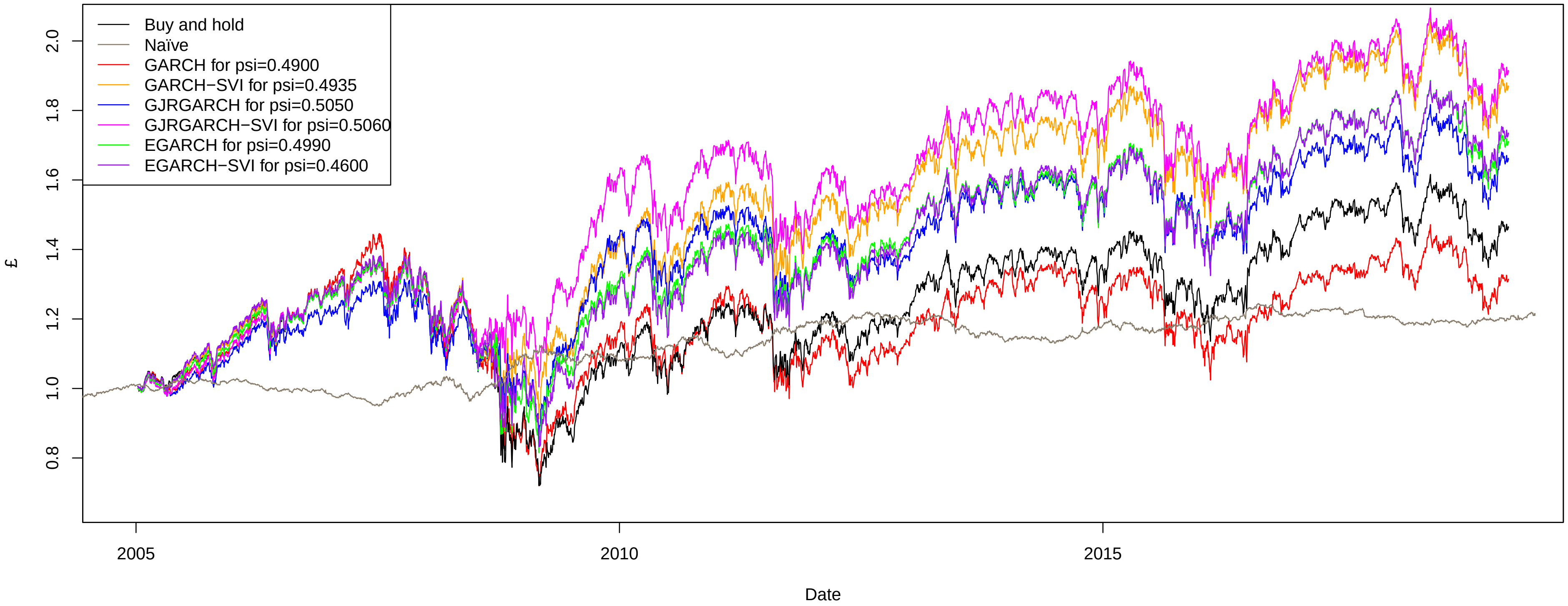

Figure 10: Cumulative Returns of Investment Strategies for an Investor with ψ optimal to the nearest 0.0005 for each strategy applied to the FTSE

5 Conclusion

This thesis replicates part of the work of CPV (Chronopoulos et al. (2018)) on the SPX and extends it to the FTSE. It empirically investigates the explanatory and predictive powers of Google Trends data as a proxy for information demand in a series of GARCH models and using the C&D (Christoffersen and Diebold (2006)) framework. In the interest of time, I empirically studied the financial implications of my findings, but not their economic significance, inferring them instead from financial and graphical analyses.

It shows how differences at each Google Trends draw (SVIs) - outlined in Section 2.2 - may impact results drastically. Indeed, despite using many external variable iterations of SPX SVI data, no efficient strategies were found above when applied to the SPX.

Findings above also highlighted how the inclusion of a single external variable in recursive models - namely ΔSVI - is extremely limiting as it does not allow for the inclusion of new variables that may become relevant due to structural changes in domestic economies. It is unrealistic to expect *a priori* knowledge of all variables affecting asset price change sign movements. One may attempt to use other factors in combination with dummy variables to anticipate when they should apply *ex ante* (Pesaran and Timmermann (2000)) such as in Vlastakis and Markellos (2012) where a dummy variable is attached to ΔSVI for low and high economic states (when returns dropped or rose by more than one standard deviation in a week).

This thesis' findings additionally indicate that investors clued into its forecasting models' effectiveness and reduced their performance over time by using it - in a similar fashion as in Simon (1955).

These particular predictive challenges - however - do not seem to be caused by the model constructions. Original tests on the use of Google Trends data in FTSE's excess return variance forecasts were positive. FTSE excess return sign forecasts compiled via C&D's framework also proved more successful to a profit maximising agent trading daily than comparable buy-and-hold and naïve model forecasts - almost doubling the former's return over the full-sample extending from 01/01/2005 to 13/03/2019.

One must - however - be careful when interpreting these results. Graphical analyses indicated that models tend not to foresee economic crashes (apart from 2008's), and that an application of my strategy may not be superior to buy-and-hold's at any time.

Moreover, inferences from the results in Tables 8 and 9 and Figures 7 and 10 differ wildly - further emphasising the shortcomings of graphical analyses over empirical ones.

Furthermore, FTSE SVI-implementing strategies worked remarkably well despite their variance forecasting models performing remarkably badly in comparison to their SVI-free counterparts (comparing Table 8 with 9 and 12). This could be indicative of SVI-implementation in C&D's models as useful in surprising ways - not in aiding variance forecasting, but in channelling its predictive powers via variance forecasting other than by improving it, possibly by reducing only Type II or Type I errors. Further studies on this aspect of results above would be enlightening.

In conclusion, further studies are needed to statistically-significantly propose that Google Trends can be used in forecasting the sign of index excess return at a daily frequency. Indeed, Google Trends data may have changed in minute but substantial ways - possibly since the introduction of caffeine on the 8^{th} of June 2010. It may be enlightening to look into structural changes in ΔSVI at that time before implementing it in this study.

Moreover, it would be compelling to continue the work above with a draw of SVI for the FTSE only for the U.K. country region. Extending the logic in Preis et al. (2013), one may find more robust findings this way.

Furthermore, only AR and GARCH models of order one were used in my (and CPV's) paper. It may be revealing to see how R_t or ΔSVI of further lags may pick up on cyclical trends (that can be seen in Figures 1 to 5). One may attempt and find their best orders via Information Criteria analyses akin to in Brooks and Burke (1998).

Following from the study in Fleming, Kirby, and Ostdiek (2003), models using Realised Volatility instead of the Moving Average components of the GARCH models (σ_{t-1}) could also be a novel implementation to improve volatility forecasts and see their repercussions in C&D's model.

Investigating the different forecasting models via the Henriksson and Merton (1981) test of market timing would be extremely interesting as well.

An attempt at the above profit maximising strategies over any period of time may also be revealing. They proved useful over our time periods, but that does not constitute of proof that it is the case for any period. These strategies only showed their use following the 2008 crisis and did not foresee significant economic crashes due to its short forecasting horizon. This leads to believe that longer horizons with lower frequency data - such as weekly data - could be easier to forecast and thus more accurate - as per Preis et al. (2013). This strategy would also allow for lower trading costs and improved overall performance.

Finally and most obviously, one ought to complete CPV's work on the FTSE and investigate the economic significance of the findings above in an empirical manner using Granger and Pesaran (2000)'s framework and implement trading costs in analyses.

6 Endnotes

† Results were shown in a manner akin to CPV to aid readers wishing to compare results.

‡ Significant figures differ as per their origin in the R code.

§ Tabulated statistics were computed from all full-sample trading days except the ones with asterisks (*) which were computed from full-CPV-sample trading days.

¶ Tabulated values indicating **best** and *worst* forecasts (excluding the Naïve and Buy-and-Hold strategies when applicable) are in **bold** and *italic* respectively.

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