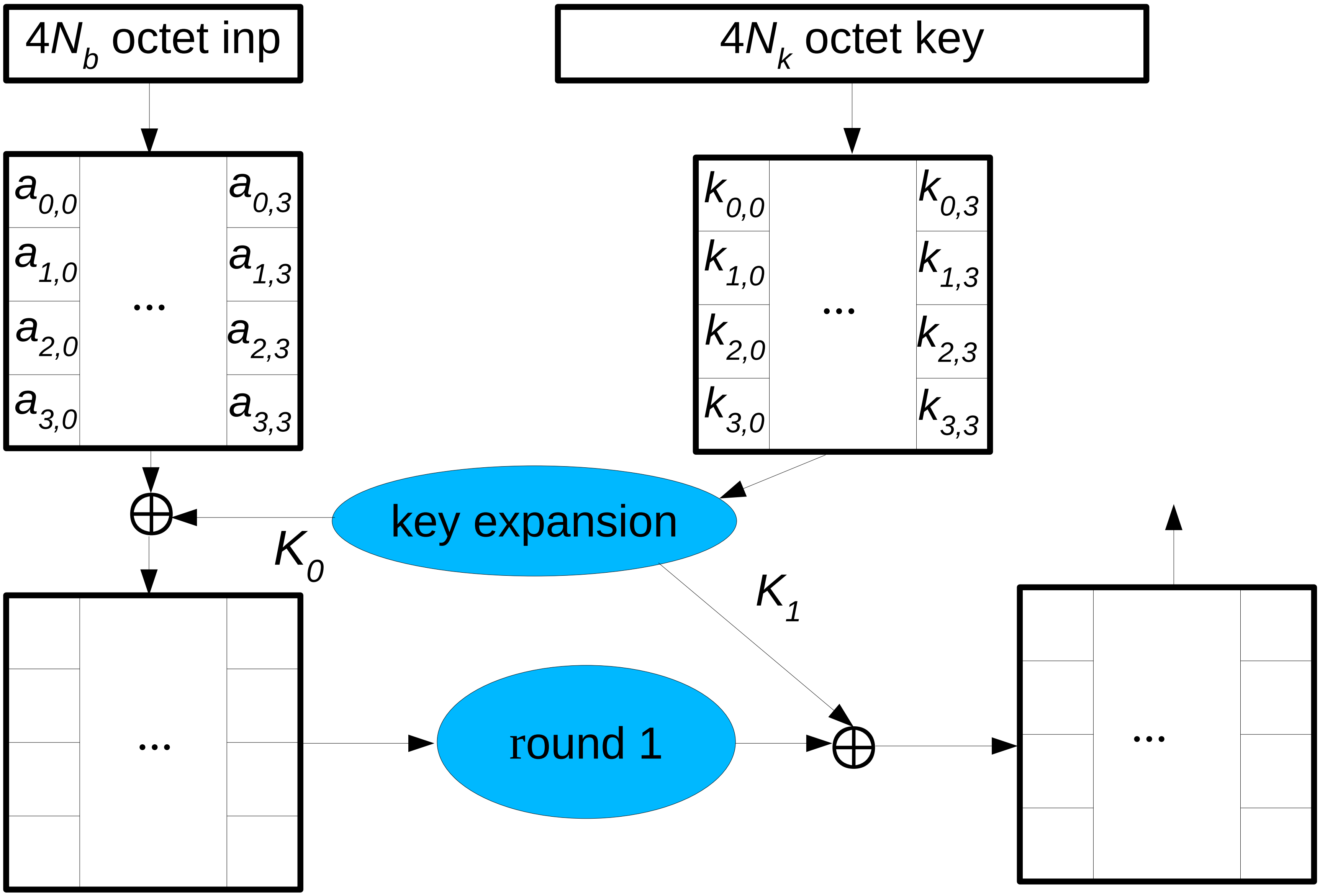
Secret Key Systems - AES

What is learned from this activity

1. What is AES
2. What are some benefits of the AES cipher

Secret Key Systems - AES

NIST (2001) parameterized key size (128 bits to 256 bits)



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The State: An array of four rows and N_b columns – each element is a byte.

Initially: next block of $4N_b$ input bytes.

Execution: all operations are performed on the State.

Example: $N_b = 4$

in_0	in_4	in_8	in_{12}
in_1	in_5	in_9	in_{13}
in_2	in_6	in_{10}	in_{14}
in_3	in_7	in_{11}	in_{15}

$s_{0,0}$	$s_{0,1}$	$s_{0,2}$	$s_{0,3}$
$s_{1,1}$	$s_{1,2}$	$s_{1,3}$	$s_{1,0}$
$s_{2,2}$	$s_{2,3}$	$s_{2,0}$	$s_{2,1}$
$s_{3,3}$	$s_{3,0}$	$s_{3,1}$	$s_{3,2}$

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Addition: modulo 2 addition (xor) of polynomials of maximum degree 7

Examples:

$(x^6+x^4+x^2+x^1+1) + (x^7+x^1+1) = x^7+x^6+x^4+x^2$ (polynomial)

$01010111 \oplus 10000011 = 11010100$ (binary notation)

$0x57 \oplus 0x83 = D4$ (hexadecimal)

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Multiplication of two degree 7 polynomials (bytes):

Just like ordinary multiplication except mod $m(x) = (x^8+x^4+x^3+x^1+1)$

Reason: for each byte there will be an inverse: $a \times a^{-1} = 1 \bmod m(x)$

Basis: $x \times b = b_7x^8 + b_6x^7 + b_5x^6 + b_4x^5 + b_3x^4 + b_2x^3 + b_1x^2 + b_0x$

Shift b left by 1, if result has a degree 8 bit, xor with $m(x)$

This operation is called **xtime(x) = (x << 1) ⊕ (((x >> 7) & 1) * 0x11b)**

Example:

xtime($x^7 + x^5 + x^3 + x^2 + x$) = $x^6 + x^2 + x + 1$ or

101011100 ⊕ 100011011 = 01000111 or xtime(0xAE) = 0x47

Example:

$(x^6 + x^4 + x^2 + x^1 + 1) \otimes (x^4 + x + 1)$ = 0x57 ⊗ 0x13

$(x^6 + x^4 + x^2 + x^1 + 1) \otimes x$ = xtime(0x57) = 0xAE

$(x^6 + x^4 + x^2 + x^1 + 1) \otimes x^2$ = xtime(0xAE) = 0x47

$(x^6 + x^4 + x^2 + x^1 + 1) \otimes x^3$ = xtime(0x47) = 0x8E

$(x^6 + x^4 + x^2 + x^1 + 1) \otimes x^4$ = xtime(0x8E) = 0x7

0x57 ⊗ 0x13 = 0x7 ⊕ 0xAE ⊕ 0x57 = 0xFE

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Multiplication of two degree 7 polynomials (bytes):

Find the inverse of a polynomial:

$$a(x) \otimes b(x) \quad \oplus \quad m(x) \otimes c(x) \; = \; 1$$

Example:

0x57 ⊗ 0xBF = 1   so 0xBF is the inverse of 0x57

S-Box number:

Apply the transformation on the right to the inverse

Example:

getSbox(0x57) = 0x5B

$$\begin{bmatrix} b_0' \\ b_1' \\ b_2' \\ b_3' \\ b_4' \\ b_5' \\ b_6' \\ b_7' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix} \oplus \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

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The S-Box:

		y															
		0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
x	0	63	7c	77	7b	f2	6b	6f	c5	30	01	67	2b	fe	d7	ab	76
	1	ca	82	c9	7d	fa	59	47	f0	ad	d4	a2	af	9c	a4	72	c0
	2	b7	fd	93	26	36	3f	f7	cc	34	a5	e5	f1	71	d8	31	15
	3	04	c7	23	c3	18	96	05	9a	07	12	80	e2	eb	27	b2	75
	4	09	83	2c	1a	1b	6e	5a	a0	52	3b	d6	b3	29	e3	2f	84
	5	53	d1	00	ed	20	fc	b1	5b	6a	cb	be	39	4a	4c	58	cf
	6	d0	ef	aa	fb	43	4d	33	85	45	f9	02	7f	50	3c	9f	a8
	7	51	a3	40	8f	92	9d	38	f5	bc	b6	da	21	10	ff	f3	d2
	8	cd	0c	13	ec	5f	97	44	17	c4	a7	7e	3d	64	5d	19	73
	9	60	81	4f	dc	22	2a	90	88	46	ee	b8	14	de	5e	0b	db
	a	e0	32	3a	0a	49	06	24	5c	c2	d3	ac	62	91	95	e4	79
	b	e7	c8	37	6d	8d	d5	4e	a9	6c	56	f4	ea	65	7a	ae	08
	c	ba	78	25	2e	1c	a6	b4	c6	e8	dd	74	1f	4b	bd	8b	8a
	d	70	3e	b5	66	48	03	f6	0e	61	35	57	b9	86	c1	1d	9e
	e	e1	f8	98	11	69	d9	8e	94	9b	1e	87	e9	ce	55	28	df
	f	8c	a1	89	0d	bf	e6	42	68	41	99	2d	0f	b0	54	bb	16

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The Inverse S-Box:

		y															
		0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
	0	52	09	6a	d5	30	36	a5	38	bf	40	a3	9e	81	f3	d7	fb
	1	7c	e3	39	82	9b	2f	ff	87	34	8e	43	44	c4	de	e9	cb
	2	54	7b	94	32	a6	c2	23	3d	ee	4c	95	0b	42	fa	c3	4e
	3	08	2e	a1	66	28	d9	24	b2	76	5b	a2	49	6d	8b	d1	25
	4	72	f8	f6	64	86	68	98	16	d4	a4	5c	cc	5d	65	b6	92
	5	6c	70	48	50	fd	ed	b9	da	5e	15	46	57	a7	8d	9d	84
	6	90	d8	ab	00	8c	bc	d3	0a	f7	e4	58	05	b8	b3	45	06
x	7	d0	2c	1e	8f	ca	3f	0f	02	c1	af	bd	03	01	13	8a	6b
	8	3a	91	11	41	4f	67	dc	ea	97	f2	cf	ce	f0	b4	e6	73
	9	96	ac	74	22	e7	ad	35	85	e2	f9	37	e8	1c	75	df	6e
	a	47	f1	1a	71	1d	29	c5	89	6f	b7	62	0e	aa	18	be	1b
	b	fc	56	3e	4b	c6	d2	79	20	9a	db	c0	fe	78	cd	5a	f4
	c	1f	dd	a8	33	88	07	c7	31	b1	12	10	59	27	80	ec	5f
	d	60	51	7f	a9	19	b5	4a	0d	2d	e5	7a	9f	93	c9	9c	ef
	e	a0	e0	3b	4d	ae	2a	f5	b0	c8	eb	bb	3c	83	53	99	61
	f	17	2b	04	7e	ba	77	d6	26	e1	69	14	63	55	21	0c	7d

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Four term polynomials with 4 bit coefficients:

$$a(x) = a_3x^3 + a_2x^2 + a_1x + a_0$$
$$b(x) = b_3x^3 + b_2x^2 + b_1x + b_0$$

Addition: $a(x) + b(x) = (a_3 \oplus b_3)\, x^3 + (a_3 \oplus b_3)\, x^3 + (a_3 \oplus b_3)\, x^3 + (a_3 \oplus b_3)\, x^3$

Multiplication:

1. create $c(x) = c_6x^6 + c_5x^5 + c_4x^4 + c_3x^3 + c_2x^2 + c_1x + c_0$
   where

$$c_0 = a_0 \otimes b_0$$
$$c_1 = (a_1 \otimes b_0) \oplus (a_0 \otimes b_1)$$
$$c_2 = (a_2 \otimes b_0) \oplus (a_1 \otimes b_1) \oplus (a_0 \otimes b_2)$$
$$c_3 = (a_3 \otimes b_0) \oplus (a_2 \otimes b_1) \oplus (a_1 \otimes b_2) \oplus (a_0 \otimes b_3)$$
$$c_4 = (a_3 \otimes b_1) \oplus (a_2 \otimes b_2) \oplus (a_1 \otimes b_3)$$
$$c_5 = (a_3 \otimes b_2) \oplus (a_2 \otimes b_3)$$
$$c_6 = a_3 \otimes b_3$$

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Four term polynomials with 4 bit coefficients:

Multiplication: $a(x) \otimes b(x)$

$$d(x) = d_3x^3 + d_2x^2 + d_1x + d_0$$

where

$$d_0 = (a_0 \otimes b_0) \oplus (a_3 \otimes b_1) \oplus (a_2 \otimes b_2) \oplus (a_1 \otimes b_3)$$
$$d_1 = (a_1 \otimes b_0) \oplus (a_0 \otimes b_1) \oplus (a_3 \otimes b_2) \oplus (a_2 \otimes b_3)$$
$$d_2 = (a_2 \otimes b_0) \oplus (a_1 \otimes b_1) \oplus (a_0 \otimes b_2) \oplus (a_3 \otimes b_3)$$
$$d_3 = (a_3 \otimes b_0) \oplus (a_2 \otimes b_1) \oplus (a_1 \otimes b_2) \oplus (a_0 \otimes b_3)$$

Let $a(x) = 3x^3 + x^2 + x + 2$; $a^{-1}(x) = 11x^3 + 13x^2 + 9x + 14$

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```
void Cipher () {             // Nr is the number of rounds
    int i, j, round=0;

    // Copy the input PlainText to state array.
    state = in;

    AddRoundKey(0);

    for (round=1 ; round < Nr ; round++) {
        SubBytes();
        ShiftRows();
        MixColumns();
        AddRoundKey(round);
    }

    SubBytes();
    ShiftRows();
    AddRoundKey(Nr);

    // Copy the state array to the Output array.
    out = state;
}
```

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SubBytes ( ):

Make substitutions from the S-Box

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ShiftRows ( ):

$$
\begin{array}{|c|c|c|c|}
\hline
s_{0,0} & s_{0,1} & s_{0,2} & s_{0,3} \\ \hline
s_{1,0} & s_{1,1} & s_{1,2} & s_{1,3} \\ \hline
s_{2,0} & s_{2,1} & s_{2,2} & s_{2,3} \\ \hline
s_{3,0} & s_{3,1} & s_{3,2} & s_{3,3} \\ \hline
\end{array}
\longrightarrow
\begin{array}{|c|c|c|c|}
\hline
s_{0,0} & s_{0,1} & s_{0,2} & s_{0,3} \\ \hline
s_{1,1} & s_{1,2} & s_{1,3} & s_{1,0} \\ \hline
s_{2,2} & s_{2,3} & s_{2,0} & s_{2,1} \\ \hline
s_{3,3} & s_{3,0} & s_{3,1} & s_{3,2} \\ \hline
\end{array}
$$

N_b \ *Row*	1	2	3
4	1	2	3
6	1	2	3
8	1	3	4

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MixColumns ( ) :

$$\begin{bmatrix} s'_{0,c} \\ s'_{1,c} \\ s'_{2,c} \\ s'_{3,c} \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 & 1 \\ 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 3 \\ 3 & 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} s_{0,c} \\ s_{1,c} \\ s_{2,c} \\ s_{3,c} \end{bmatrix}$$

$$tmp = s_{0,c} \oplus s_{1,c} \oplus s_{2,c} \oplus s_{3,c}$$
$$s_{0,c} = \text{xtime}(s_{0,c} \oplus s_{1,c}) \oplus tmp \oplus s_{0,c}$$
$$s_{1,c} = \text{xtime}(s_{1,c} \oplus s_{2,c}) \oplus tmp \oplus s_{1,c}$$
$$s_{2,c} = \text{xtime}(s_{2,c} \oplus s_{3,c}) \oplus tmp \oplus s_{2,c}$$
$$s_{3,c} = \text{xtime}(s_{0,c} \oplus s_{3,c}) \oplus tmp \oplus s_{3,c}$$

Replace state columns by matrix multiplication ($\times$ and $\oplus$) above

Columns are considered as polynomials over GF(2^8) and multiplied mod x^4+1 by a fixed polynomial given by

$$3x^3 + x^2 + x + 2$$

Example:

$$\begin{bmatrix} \text{0xD4} \\ \text{0xBF} \\ \text{0x5D} \\ \text{0x30} \end{bmatrix} \rightarrow \begin{bmatrix} \text{0x04} \\ \text{0x66} \\ \text{0x81} \\ \text{0xE5} \end{bmatrix}$$

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AddRoundKey ( ):

$s_{0,0}$	$s_{0,1}$	$s_{0,2}$	$s_{0,3}$
$s_{1,0}$	$s_{1,1}$	$s_{1,2}$	$s_{1,3}$
$s_{2,0}$	$s_{2,1}$	$s_{2,2}$	$s_{2,3}$
$s_{3,0}$	$s_{3,1}$	$s_{3,2}$	$s_{3,3}$

$$\oplus$$

$w_{0,0}$	$w_{0,1}$	$w_{0,2}$	$w_{0,3}$
$w_{1,0}$	$w_{1,1}$	$w_{1,2}$	$w_{1,3}$
$w_{2,0}$	$w_{2,1}$	$w_{2,2}$	$w_{2,3}$
$w_{3,0}$	$w_{3,1}$	$w_{3,2}$	$w_{3,3}$

$$\rightarrow$$

$s_{0,0}$	$s_{0,1}$	$s_{0,2}$	$s_{0,3}$
$s_{1,1}$	$s_{1,2}$	$s_{1,3}$	$s_{1,0}$
$s_{2,2}$	$s_{2,3}$	$s_{2,0}$	$s_{2,1}$
$s_{3,3}$	$s_{3,0}$	$s_{3,1}$	$s_{3,2}$

first round only - generally it's

$$w_{i,r+c}$$

where c is the column and r is the round

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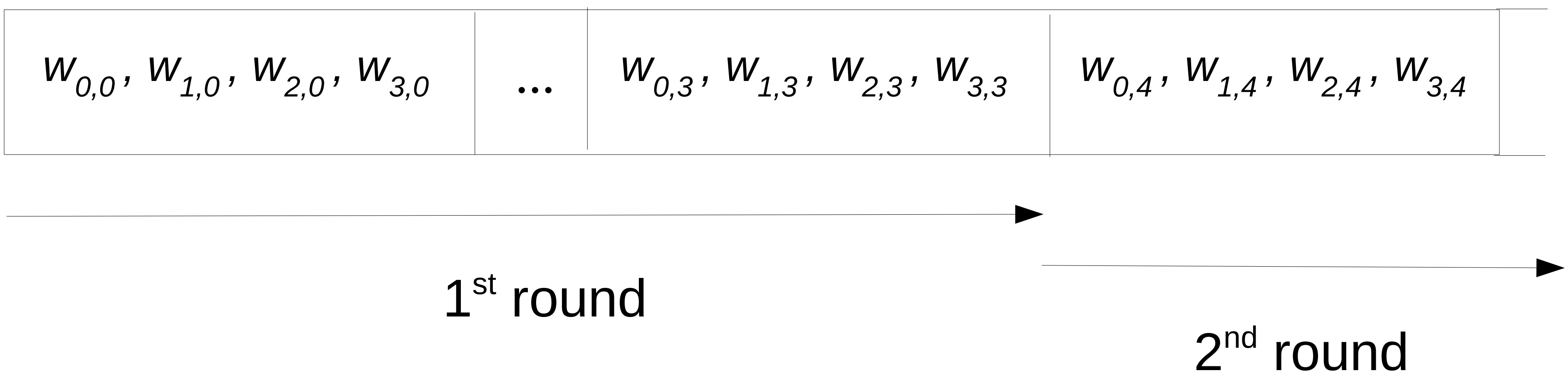
Key Schedule: example for N_k=4



All rounds: $32 * N_k$ bits for a round key

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Key Expansion: example for $N_k = 4$

RotWord ( ): changes [a_0 , a_1 , a_2 , a_3] to [a_3 , a_2 , a_1 , a_0]

Rcon[i]: the word [x^{i-1}, 0, 0, 0] mod $x^4 + 1$, where $x = 2$

SubWord ( ): maps each byte of [a_0 , a_1 , a_2 , a_3] using S-Box values

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Key Expansion: example for $N_k = 4$

RotWord ( ): changes $[\ a_0, a_1, a_2, a_3\ ]$ to $[\ a_3, a_2, a_1, a_0\ ]$

Rcon[i]: the word $[\ x^{i-1}, 0, 0, 0\ ]$ mod $x^4 + 1$, where $x = 2$

SubWord ( ): maps each byte of $[\ a_0, a_1, a_2, a_3\ ]$ using S-Box values

First Round: the original key (16 bytes if $N_k = 4$)

Other Rounds: $[\ w_{0,l}, w_{1,l}, w_{2,l}, w_{3,l}\ ]$ smallest l is N_k-1

$[\ w_{3,l}, w_{2,l}, w_{1,l}, w_{0,l}\ ]$ apply RotWord

$[\ S(w_{0,l}), S(w_{1,l}), S(w_{2,l}), S(w_{3,l})\ ]$ apply SubWord

$[\ S(w_{0,l}) \oplus \text{Rcon}[(l+1)/N_k], S(w_{1,l}), S(w_{2,l}), S(w_{3,l})\ ]$ use Rcon

$$[\ S(w_{0,l}) \oplus \text{Rcon}[(l+1)/N_k] \oplus w_{0,l}, S(w_{1,l}) \oplus w_{1,l}, S(w_{2,l}) \oplus w_{2,l}, S(w_{3,l}) \oplus w_{3,l}]$$

$[\ w_{0,l+1}, w_{1,l+1}, w_{2,l+1}, w_{3,l+1}\ ]$ next key word

Key: 2B 7E 15 16 28 AE D2 A6 AB F7 15 88 09 CF 4F 3C

i (dec)	temp	After RotWord()	After SubWord()	Rcon[i/Nk]	After XOR with Rcon	w[i–Nk]	w[i]= temp XOR w[i–Nk]
4	09cf4f3c	cf4f3c09	8a84eb01	01000000	8b84eb01	2b7e1516	a0fafe17
5	a0fafe17					28aed2a6	88542cb1
6	88542cb1					abf71588	23a33939
7	23a33939					09cf4f3c	2a6c7605
8	2a6c7605	6c76052a	50386be5	02000000	52386be5	a0fafe17	f2c295f2
9	f2c295f2					88542cb1	7a96b943
10	7a96b943					23a33939	5935807a
11	5935807a					2a6c7605	7359f67f
12	7359f67f	59f67f73	cb42d28f	04000000	cf42d28f	f2c295f2	3d80477d
13	3d80477d					7a96b943	4716fe3e
14	4716fe3e					5935807a	1e237e44
15	1e237e44					7359f67f	6d7a883b
16	6d7a883b	7a883b6d	dac4e23c	08000000	d2c4e23c	3d80477d	ef44a541
17	ef44a541					4716fe3e	a8525b7f
18	a8525b7f					1e237e44	b671253b
19	b671253b					6d7a883b	db0bad00
20	db0bad00	0bad00db	2b9563b9	10000000	3b9563b9	ef44a541	d4d1c6f8
21	d4d1c6f8					a8525b7f	7c839d87
22	7c839d87					b671253b	caf2b8bc
23	caf2b8bc					db0bad00	11f915bc

Key: 2B 7E 15 16 28 AE D2 A6 AB F7 15 88 09 CF 4F 3C

24	11f915bc	f915bc11	99596582	20000000	b9596582	d4d1c6f8	6d88a37a
25	6d88a37a					7c839d87	110b3efd
26	110b3efd					caf2b8bc	dbf98641
27	dbf98641					11f915bc	ca0093fd
28	ca0093fd	0093fdca	63dc5474	40000000	23dc5474	6d88a37a	4e54f70e
29	4e54f70e					110b3efd	5f5fc9f3
30	5f5fc9f3					dbf98641	84a64fb2
31	84a64fb2					ca0093fd	4ea6dc4f
32	4ea6dc4f	a6dc4f4e	2486842f	80000000	a486842f	4e54f70e	ead27321
33	ead27321					5f5fc9f3	b58dbad2
34	b58dbad2					84a64fb2	312bf560
35	312bf560					4ea6dc4f	7f8d292f
36	7f8d292f	8d292f7f	5da515d2	1b000000	46a515d2	ead27321	ac7766f3
37	ac7766f3					b58dbad2	19fadc21
38	19fadc21					312bf560	28d12941
39	28d12941					7f8d292f	575c006e
40	575c006e	5c006e57	4a639f5b	36000000	7c639f5b	ac7766f3	d014f9a8
41	d014f9a8					19fadc21	c9ee2589
42	c9ee2589					28d12941	e13f0cc8
43	e13f0cc8					575c006e	b6630ca6

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Example:

Input:

32	88	31	E0
43	5A	31	37
F6	30	98	07
A8	8D	A2	34

Key:

2B	28	AB	09
7E	AE	F7	CF
15	D2	15	4F
16	A6	88	3C

Secret Key Systems - AES

Example:

Input:

32	88	31	E0
43	5A	31	37
F6	30	98	07
A8	8D	A2	34

$\oplus$

19	A0	9A	E9
3D	F4	C6	F8
E3	E2	8D	48
BE	2B	2A	08

Key:

2B	28	AB	09
7E	AE	F7	CF
15	D2	15	4F
16	A6	88	3C

beginning of first round - input placed into the state and key has been added to state

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Example:

State:

19	A0	9A	E9
3D	F4	C6	F8
E3	E2	8D	48
BE	2B	2A	08

S-Box

D4	E0	B8	1E
27	BF	B4	41
11	98	5D	52
AE	F1	E5	30

ShiftRows

D4	E0	B8	1E
BF	B4	41	27
5D	52	11	98
30	AE	F1	E5

MixColumns

04	E0	48	28
66	CB	F8	06
81	19	D3	26
E5	9A	7A	4C

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Notes:

1. Many operations are table look ups so they are fast
2. Parallelism can be exploited
3. Key expansion only needs to be done one time until the key is changed
4. The S-box minimizes the correlation between input and output bits
5. There are no known weak keys

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Number of rounds:

N_k \ N_b	4	6	8
4	10	12	14
6	12	12	14
8	14	14	14