galois

Lab: Salsa20

Salsa20 is a symmetric stream cipher built on a pseudorandom function utilizing 32-bit addition mod 2³², constant-distance rotation operations <<< on an internal state of 16 32-bit words arranged as a 4x4 matrix, and XOR operations on 32-bit words. The design avoids possible timing attacks in some software implementations. The internal state is comprised of 8 words of key (64 key bytes), 2 words of stream position, 2 words of nonce, and 4 fixed words that spell out "expand 32-byte k". The initial state looks like this:

```
"expa" Key Key Key
Key "nd 3" Nonce Nonce
Pos Pos "2-by" Key
Key Key Key "te k"
```

The 4 sections of the state are colored for easier reading.

A core operation is function quarterround, shown below implemented in Cryptol, that maps a 4 word input to a 4 word output:

```
quarterround : [4][32] -> [4][32] quarterround [y0, y1, y2, y3] = [z0, z1, z2, z3] where  z1 = y1 \land ((y0 + y3) <<< 0x7) \\ z2 = y2 \land ((z1 + y0) <<< 0x9) \\ z3 = y3 \land ((z2 + z1) <<< 0xd) \\ z0 = y0 \land ((z3 + z2) <<< 0x12)
```

This function is invertible.

Salsa20, like many crypto algorithms, is round-based. Odd rounds apply quarterround to four columns of the state matrix and even rounds apply quarterround to the four rows of the state matrix like this, in Cryptol:

```
columnround : [16][32] -> [16][32]
columnround [x0, x1, x2, x3, x4, x5, x6, x7, x8, x9, x10, x11, x12, x13, x14, x15] =
   [y0, \ y1, \ y2, \ y3, \ y4, \ y5, \ y6, \ y7, \ y8, \ y9, \ y10, \ y11, \ y12, \ y13, \ y14, \ y15]
  where
      [y0, y4, y8, y12] = quarterround [x0, x4, x8, x12]
      [ y5, y9, y13, y1] = quarterround [ x5, x9, x13, x1]
      [y10, y14, y2, y6] = quarterround [x10, x14, x2, x6]
      [y15, y3, y7, y11] = quarterround [x15, x3, x7, x11]
rowround: [16][32] -> [16][32]
rowround [y0, y1, y2, y3, y4, y5, y6, y7, y8, y9, y10, y11, y12, y13, y14, y15] =
   [z0, z1, z2, z3, z4, z5, z6, z7, z8, z9, z10, z11, z12, z13, z14, z15]
  where
      [z0, z1, z2, z3] = quarterround [y0, y1,
                                                      y2,
      [z5, z6, z7, z4] = quarterround [y5, y6,
                                                          y4]
                                                     y7,
      [z10, z11, z8, z9] = quarterround [y10, y11, y8, y9]
      [z15, z12, z13, z14] = quarterround [y15, y12, y13, y14]
```

Two consecutive rounds are called a double-round and are implemented like this:

```
doubleround : [16][32] -> [16][32]
doubleround(xs) = rowround(columnround(xs))
```

A hash function, Salsa20 below, is used in counter mode as a stream cipher: it encrypts a 64 byte block of plaintext by hashing the key, nonce, and block number and xoring the result with the plaintext.

```
// Creates a little-endian word of 32 bits from 4 bytes pointed to by b
   littleendian : [4][8] -> [32]
   littleendian b = join(reverse b)
   // Moves the little-endian word into the 4 bytes pointed to by b
   littleendian_inverse : [32] -> [4][8]
   littleendian_inverse b = reverse(split b)
  // Creates two copies of the state in little-endian format. The First
  // copy is hashed together. The second copy is added to the first, word-by-word.
  // This becomes a 64 byte keystream block.
  Salsa20 : [64][8] -> [64][8]
  Salsa20 xs = join ar
     where
        ar = [ littleendian_inverse words | words <- xw + zs@10 ]</pre>
        xw = [ littleendian xi | xi <- split xs ]</pre>
        zs = [xw] # [doubleround zi | zi <- zs]
The result of Salsa20 is a 64 byte key stream block in seq.
   // The 16-byte (128-bit) or 32-byte (256-bit) key expansion function.
  Salsa20_expansion : {a} (a >= 1, 2 >= a) => ([16*a][8], [16][8]) -> [64][8]
  Salsa20_expansion(k, n) = z
     where
         [s0, s1, s2, s3] = split "expand 32-byte k" : [4][4][8]
         [t0, t1, t2, t3] = split "expand 16-byte k" : <math>[4][4][8]
        x = if(a == 2) then s0 # k0 # s1 # n # s2 # k1 # s3
                        else t0 # k0 # t1 # n # t2 # k0 # t3
        z = Salsa20(x)
        [k0, k1] = (split(k#zero)):[2][16][8]
Finally
   // Performs up to 2^32-1 bytes of encryption or decryption under a
  // 128- or 256-bit key.
  Salsa20_encrypt : \{a,l\} (a >= 1, 2 >= a, l <= 2^^70) => ([16*a][8], [8][8], [1][8]) -> [1][8]
  Salsa20_encrypt(k, v, m) = c
        salsa = take(join[Salsa20_expansion(k, v#(reverse(spliti))) | i <- [0, 1...]])
        c = m \wedge salsa
Exercise 1:
Create Salsa20.cry from the above.
What is quarterround [0xd3917c5b, 0x55f1c407, 0x52a58a7a, 0x8f887a3b]?
What is rowround [0x08521bd6, 0x1fe88837, 0xbb2aa576, 0x3aa26365,
                    0xc54c6a5b, 0x2fc74c2f, 0x6dd39cc3, 0xda0a64f6,
                    0x90a2f23d, 0x067f95a6, 0x06b35f61, 0x41e4732e,
                    0xe859c100, 0xea4d84b7, 0x0f619bff, 0xbc6e965al?
What is columnround [0x08521bd6, 0x1fe88837, 0xbb2aa576, 0x3aa26365,
                       0xc54c6a5b, 0x2fc74c2f, 0x6dd39cc3, 0xda0a64f6,
                       0x90a2f23d, 0x067f95a6, 0x06b35f61, 0x41e4732e,
                       0xe859c100, 0xea4d84b7, 0x0f619bff, 0xbc6e965a]?
What is littleendian [86, 75, 30, 9]?
```

```
What is Salsa20 [88, 118, 104, 54, 79, 201, 235, 79, 3, 81, 156, 47, 203, 26, 244, 243,
                  191, 187, 234, 136, 211, 159, 13, 115, 76, 55, 82, 183, 3, 117, 222, 37,
                   86, 16, 179, 207, 49, 237, 179, 48, 1, 106, 178, 219, 175, 199, 166, 48,
                  238, 55, 204, 36, 31, 240, 32, 63, 15, 83, 93, 161, 116, 147, 48, 113]?
What is Salsa20_encrypt (k,v,m)
          where
             k = [0x23, 0x12, 0x14, 0x72, 0xEE, 0xEa, 0x45, 0x23,
                   0x4A, 0x2A, 0x6D, 0x55, 0xF2, 0xCC, 0xCA, 0xC2];
             V = [0x11, 0x78, 0x8E, 0x3B, 0x77, 0x63, 0x3A, 0x3C];
             m = [0xDD, 0x34, 0x67, 0x33, 0x23, 0xC4, 0xD3, 0xEE]?
What is Salsa20 encrypt (k,v,m)
          where
             k = [0x23, 0x12, 0x14, 0x72, 0xEE, 0xEa, 0x45, 0x23,
                   0x4A, 0x2A, 0x6D, 0x55, 0xF2, 0xCC, 0xCA, 0xC2];
             V = [0x11, 0x78, 0x8E, 0x3B, 0x77, 0x63, 0x3A, 0x3C];
             m = [0x21, 0xC1, 0x66, 0xCB, 0x24, 0x58, 0x7E, 0x34]?
```

Exercise 2:

Prove Salsa20_encrypt(k, v, Salsa20_encrypt(k, v, m)) is equal to m

Prove doubleround x1 is not equal to doubleround x2 if x1 is not equal to x2

Prove columnround is the transpose of rowround

```
Define
```

```
rowround_opt : [16][32] -> [16][32]
rowround_opt ys = join [(quarterround (yi<<<i)))>>>i | yi <- split ys | i <- [0 .. 3]]
Prove rowround x is equal to rowround opt x
```

Prove littleendian is invertible ■

The above will be the Salsa20 "gold standard" specification. A c implementation of Salsa20 is given in salsa20.c. SAW can be used to prove that the implementation is functionally identical to the specification supplied by Salsa20.cry. This is a little difficult due to the fact that the implementation has functions that take pointers as input. Some built-ins help with this:

<u>llvm_alloc</u>: specifies that a function expects a particular pointer to refer to an allocated region appropriate for a specific type. Most functions that operate on pointers expect that certain pointers point to allocated memory before they are called. In the initial state, llvm_alloc specifies that the function expects a pointer to allocated space to exist. In the final state, it specifies that the function itself performs an allocation. This command takes one argument: the llvm type for which an allocation is to be made.

<u>llvm_alloc_readonly</u>: this works like llvm_alloc except that writes to allocated space are forbidden.

<u>llvm_points_to</u>: takes two arguments: the first must be a pointer, and states that the memory specified by that pointer should contain the value given in the second argument. Pointers returned by llvm_alloc don't, initially, point to anything. So if a pointer is directly passed into a function that tries to dereference it, symbolic execution will fail with a message

about an invalid load. The llvm_points_to command is used to state that a pointer points to some specific value, thereby avoiding this problem.

<u>llvm fresh var</u>: creates a new variable given a llvm type and reference name as arguments. Fresh variables are used to prove function properties for a class of inputs, or all inputs, not for a concrete value.

The following <u>alloc init ty v</u> function for SAW returns a pointer to memory allocated and initialized to a value v of type ty. The <u>alloc init readonly</u> function for SAW does the same, except the memory allocated cannot be written to. The $llvm_term v$ expression identifies v as a cryptol object and becomes a variable of llvm type.

```
let alloc_init ty v = do {
   p <- llvm_alloc ty;
   llvm_points_to p (llvm_term v);
   return p;
};
let alloc_init_readonly ty v = do {
   p <- llvm_alloc_readonly ty;
   llvm_points_to p (llvm_term v);
   return p;
};</pre>
```

The following functions create fresh llvm variables for which memory is allocated and pointers to that memory. The return value of ptr to fresh is a pair (x, p) where x is the fresh symbolic variables and p is the pointer to x. The type of x is given by argument ty.

```
let ptr_to_fresh n ty = do {
    x <- llvm_fresh_var n ty;
    p <- alloc_init ty x;
    return (x, p);
};
let ptr_to_fresh_readonly n ty = do {
    x <- llvm_fresh_var n ty;
    p <- alloc_init_readonly ty x;
    return (x, p);
};</pre>
```

The following function <u>oneptr update func n ty f</u> specifies the behavior of a function that takes a single pointer, with name n, to memory containing a value of type ty and changes the contents of that memory to the value given by the application of f to the value in that memory before execution. The expression in double braces ($\{\{\ldots\}\}$) is a Cryptol expression.

```
let oneptr_update_func n ty f = do {
   (x, p) <- ptr_to_fresh n ty;
   llvm_execute_func [p];
   llvm_points_to p (llvm_term {{ f x }});
};</pre>
```

Verification will be in stages: a C function f will be verified against a corresponding Cryptol function, then functions dependent on f will be verified and so on until the entire C module is verified. The SAW command for this llvm_verify.

<u>llvm verify</u>: inputs are an llvm module obtained using clang, the name of the C function to verify, a list of already-verified C functions to use for compositional verification, a true or false indicating whether path satisfiability checking is to be done, the name of the SAW function to verify against, the name of the solver to use for the verification. An example of its use is the following:

```
Outside of SAW — make the llvm module salsa20.bc:
    clang -g -00 -c salsa20.c -o salsa20.bc

Within SAW:
    mm <- llvm_load_module "salsa20.bc";
    qr <- llvm_verify mm "s20_quarterround" [] false quarterround_setup abc;
```

Where qr is a top level method specification, [] means nothing has been verified yet, s20_quarterround is the C function, quarterround_setup encapsulates the Cryptol specification corresponding to the C function, and the abc solver is used for the verification. The quarterround_setup function has three parts: a specification of the initial state of quarterround before execution; a description of how to call the Cryptol code to be executed; and a specification of the final state of quarterround. The s20_quarterround function in C takes four pointers as input and changes the values of locations they point to: the new values are the resulting output of the function. Then, the quarterround_setup function looks like this:

```
let quarterround_setup = do {
   y0 <- llvm_fresh_var "y0" (llvm_int 32);
                                               // create 4 fresh variables
   y1 <- llvm_fresh_var "y1" (llvm_int 32);
                                               // of type 32 bit int and
   y2 <- llvm_fresh_var "y2" (llvm_int 32);
                                               // allocate space for each
   y3 <- llvm_fresh_var "y3" (llvm_int 32);
    p0 <- alloc_init (llvm_int 32) {{ y0 }};
                                               // with pointers to each
    p1 <- alloc_init (llvm_int 32) {{ y1 }};
                                               // space
    p2 <- alloc_init (llvm_int 32) {{ y2 }};
    p3 <- alloc_init (llvm_int 32) {{ y3 }};
    llvm_execute_func [p0, p1, p2, p3];
                                               // how to call quarterround
    let zs = {{ quarterround [y0,y1,y2,y3] }}; // quarterround called
    llvm_points_to p0 (llvm_term {{ zs@0 }});
    llvm_points_to p1 (llvm_term {{ zs@1 }});
                                               // pointers point to the
    llvm_points_to p2 (llvm_term {{ zs@2 }}); // results of the execution
    llvm_points_to p3 (llvm_term {{ zs@3 }});
                                              // of quarterround
};
```

Exercise 3:

Run clang -g -00 -c salsa20.c -o salsa20.bc if you haven't already. Use a text editor to create file s1.saw. Make the first line 'import "Salsa20.cry";'. Add function alloc_init, then quarterround_setup, and finally the following:

```
let main = do {
   mm <- llvm_load_module "salsa20.bc";
   qr <- llvm_verify m "s20_quarterround" [] false quarterround_setup yices;
   print "Done!";
};</pre>
```

Make sure that 'main' is the bottommost function in s1.saw. Run saw s1.saw What is the result? ■

Next add the verification of s20_rowround to s1. saw: place the line

```
rr <- llvm_verify mm "s20_rowround" [] false rowround_setup yices;</pre>
```

right after qr <- ... in function main. Since s20_rowround of salsa.c takes an array of 16 32 bit ints as input the rowround setup function will be written as

```
let rowround_setup =
  oneptr_update_func "y" (llvm_array 16 (llvm_int 32)) {{ rowround }};
```

which requires oneptr_update_func to be added to s1.saw in addition to rowround_setup. Since oneptr_update_func uses ptr_to_fresh, that must also be added to s1.saw. At this point it is straightforward to add verification of s20_columnround, s20_doubleround, and s20_hash as the lines to add are similar to what was added for the verification of s20_rowround. The three additional setup functions are:

```
let columnround_setup =
    oneptr_update_func "x" (llvm_array 16 (llvm_int 32)) {{ columnround }};
let doubleround_setup =
    oneptr_update_func "x" (llvm_array 16 (llvm_int 32)) {{ doubleround }};
let salsa20_setup =
    oneptr_update_func "seq" (llvm_array 64 (llvm_int 8)) {{ Salsa20 }};
and the lines to add in main are:
    cr <- llvm_verify mm "s20_columnround" [] false columnround_setup yices;
    dr <- llvm_verify mm "s20_doubleround" [] false doubleround_setup yices;
    s20 <- llvm_verify mm "s20_hash" [] false salsa20_setup yices;</pre>
```

Exercise 4:

Add the above to s1. saw and run it. What is the result? ■

Whoa. The computation probably did not finish. This is where the [] brackets come in. Since s20_hash depends on s20_doubleround, add the verified dr object to [] so verification of s20_hash can make use of the verification of s20_doubleround. Thus, the line in main looks like this:

```
s20 <- llvm_verify mm "s20_hash" [dr] false salsa20_setup yices;</pre>
```

Exercise 5:

Modify the s20 line as above and try again ■

The function s20 expand32 has prototype:

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This function's verification follows patterns seen above. The prototype for the function has a pointer, k, to an array of 32 bytes, an array, n, of 16 bytes, and a keystream array. The Cryptol function used to verify this is Salsa20_expansion which has k and n as arguments. The Cryptol variable k references 32 bytes with a=2 in the type signature. To verify the C function s20_expand32, which uses k as a pointer, a llvm_fresh_variable is defined and a pointer to 32 bytes of space is allocated. Similarly for the Cryptol variable n which references an array of 16 bytes. Since the C function should only write to keystream, memory allocated to k and n can be assured to be unaffected by using llvm_alloc_readonly instead of llvm_alloc in s1.saw, salsa20_expansion_32 (creation of which is the next exercise), like this:

```
k <- llvm_fresh_var "k" (llvm_array 32 (llvm_int 8));
pk <- llvm_alloc_readonly (llvm_array 32 (llvm_int 8));
llvm_points_to pk (llvm_term k);
n <- llvm_fresh_var "n" (llvm_array 16 (llvm_int 8));
pn <- llvm_alloc_readonly (llvm_array 16 (llvm_int 8));
llvm_points_to pn (llvm_term n);</pre>
```

where pk and pn are the k and n pointers. Memory allocated to keystream looks like this:

```
pks <- llvm_alloc (llvm_array 64 (llvm_int 8));</pre>
```

The calling spec for the C function is this:

```
llvm_execute_func [pk, pn, pks];
```

The Cryptol specification for the s20 expand32 C function is this, for 32 bytes:

```
let rks = {{ Salsa20_expansion`{a=2}(k, n) }};
llvm_points_to pks (llvm_term rks);
```

where, looking at the type signature for Salsa20_expansion in Salsa20.cry, a=2 makes the Cryptol function monomorphic with signature ([32][8] -> [16][8]) -> [64][8], thus 32 byte k.

Exercise 6:

Write and add function salsa20_expansion_32 to s1.saw. Also add the following to main to be able to verify the C function salsa20_expansion for 32 bytes.

```
s20e32 <- llvm_verify mm "s20_expand32" [s20] false salsa20_expansion_32 yices;</pre>
```

where salsa20_expansion_32 refers to the above setup function that must be inserted into s1.saw. Use all of the above. Note that line s20 is listed as a dependency in line s20e32. Without it a run will not complete, at least not in a reasonable time. Run s1.saw. What happens? ■

Finally, the C function s20_crypt32 is verified against Cryptol function Salsa20_encrypt. The C function has 5 arguments. The first two and the fourth are arrays named key, nonce, and buf. The third and fifth arguments are 32 bit integers. A setup function for s20_crypt32 is as follows.

Name the SAW setup function s20_encrypt32 and let it have parameter n which specifies buffer size. For the three arrays input to the C function construct the translation from the

Cryptol function Salsa20_encrypt whose arguments are k (for key), v (for nonce), m (for buffer):

```
key <- llvm_fresh_var "key" (llvm_array 32 (llvm_int 8));
pkey <- llvm_alloc (llvm_array 32 (llvm_int 8));
llvm_points_to pkey (llvm_term key);

v <- llvm_fresh_var "nonce" (llvm_array 8 (llvm_int 8));
pv <- llvm_alloc (llvm_array 8 (llvm_int 8));
llvm_points_to pv (llvm_term v);

m <- llvm_fresh_var "buf" (llvm_array n (llvm_int 8));
pm <- llvm_alloc (llvm_array n (llvm_int 8));
llvm_points_to pm (llvm_term m);</pre>
```

The C function will be run with inputs pkey, pv, and pm, similar to what has been done above, but also with Cryptol input, 32 bit 0, for the nonce and the value of n, coming from the input to the setup function, for the buffer size. This looks like the following:

```
llvm_execute_func [ pkey, pv, llvm_term {{ 0:[32] }}, pm, llvm_term {{ `n:[32] }}];
```

The function s20_crypt32 has a return value that is a 32 bit status. This can be taken care of with the following:

```
llvm_return (llvm_term {{ 0 : [32] }});
```

The output of the Cryptol function Salsa20_encrypt will be the contents of the buffer in the C function. This is stated in the setup function as follows:

```
llvm_points_to pm (llvm_term {{ Salsa20_encrypt (key, v, m) }});
```

Exercise 7:

Create and develop setup function s20_encrypt32 to verify C function s20_crypt32 against Cryptol function Salsa20_encrypt for given buffer length. Use the above. Add the following line to main:

```
s20encrypt_64 <- llvm_verify mm "s20_crypt32" [s20e32] false (s20_encrypt32 64) yices;
```

The parens are needed because the setup function s20_encrypt32 takes an argument. The reference to line s20s32 provides faster output. Run s1.saw. What happens? ■

Note: other dependences can be inserted for faster results: line rr depends on line qr as does line cr. Line dr depends on lines rr and cr. Adding these dependencies makes the computation much faster.

Note: all lines in main have branch satisfiability checking turned off (false after []). Please check the SAW manual for information about when it is advantageous to turn branch satisfiability checking on.

Note: ptr_to_fresh and ptr_to_fresh_readonly may be used instead of llvm_fresh_var, llvm_alloc, llvm_alloc_readonly, and llvm_points_to for one line of SAW code to replace three lines.