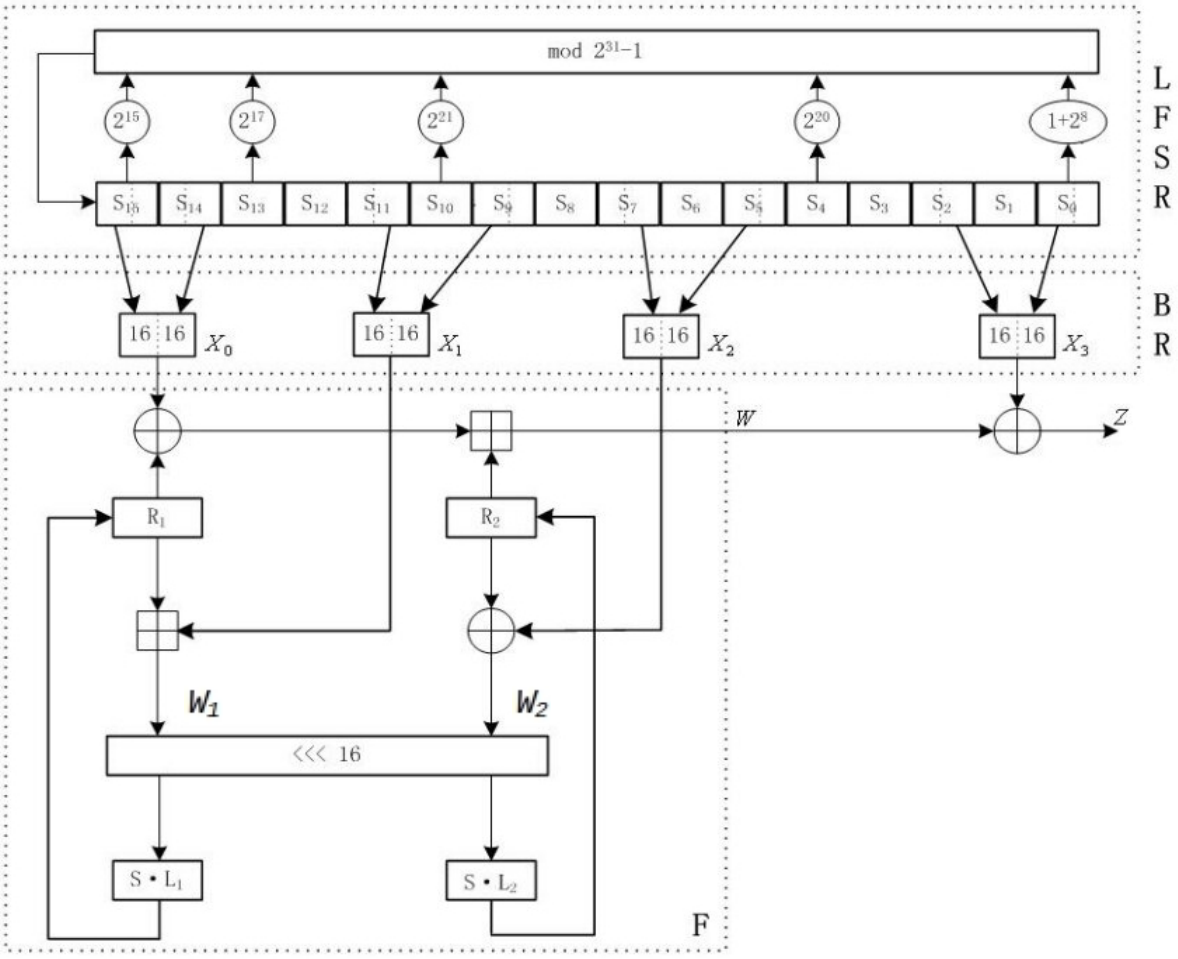
Lab: ZUC specification in Cryptol

ZUC is a word-oriented stream cipher intended for cell phone encryption and decryption. It takes a 128-bit initial key and a 128-bit initial vector as input and outputs a keystream of 32-bit words. The Figure below shows the schematic diagram from which ZUC is derived.



The section labeled `LFSR` shows a Linear Feedback Shift Register. This register has 16 stages labeled S_0 to S_{15}. Each stage is 31 bits wide. For all stages, a clock moves data from input to output, simultaneously, from S_i to S_{i-1} for `i` from 15 to 1. Input to S_{15} comes from the output of a function of values from stages S_{15}, S_{13}, S_{10}, S_4, and S_0. This function is defined below during an exercise. The section labeled `BR` shows the Bit Reorganization registers that contribute to inputs in the Mangler function `F` and to the output keystream `Z`. There are 4 such registers, X_0, X_1, X_2, X_3, taking values from half (16 bits) of stages S_{15}, S_{14}, S_{11}, S_9, S_7, S_5,

S_2, and S_0. The Mangler function F takes input from the BR registers and outputs a 32 bit wide stream that is xored with BR register X_3 to provide the keystream output Z.

The execution of ZUC has two stages: initialization stage and working stage. In the first stage, a key/IV initialization is performed, i.e., the cipher is clocked without producing output. The second stage is a working stage. In this stage, with every clock pulse, produces a 32-bit word of output.

Exercise 1:
Write a function `plus (a,b)` that takes two 31 bit integers `a` and `b` as input, adds them `mod` $(2^{31}-1)$, and returns the resulting 31 bit integer.

Exercise 2:
Write a function `add xs` that takes a sequence `xs` of 31 bit integers as input and adds them all `mod` $(2^{31}-1)$. Use the result of Exercise 1 in a comprehension.

Exercise 3:
Write a function `v ss` that takes as input a sequence `ss` of 16, 31 bit integers, and returns

$(S_{15}<<<_{31}15)+(S_{13}<<<_{31}17)+(S_{10}<<<_{31}21)+(S_4<<<_{31}20)+(S_0<<<_{31}8)+S_0 \bmod (2^{31}-1)$

Use the result of Exercise 2 on a comprehension.

Exercise 4:
In the Initialization mode, the LFSR receives a 31-bit input word u, which is obtained by removing the rightmost bit from the 32-bit output W of the nonlinear function F, i.e., u=W>>1. More specifically, according to the ZUC specification, the initialization mode works as follows:

```
LFSRWithInitialisationMode (u) {
  v=(S15<<<31 15)+(S13<<<31 17)+(S10<<<31 21)+(S4<<<31 20)+(S0<<<31 8)+S0 mod (2^31-1)
  S16 = (v+u) mod (2^31-1)
  if S16 == 0, then set S16 = 2^31-1;
  (S1,S2, …,S15,S16) -> (S0,S1, …,S14,S15).
}
```

Write `LFSRWithInitializationMode` as a Cryptol function using the results of Exercises 1 to 3. You will need to add `ss`, representing the 16 stage sequence of the LFSR, to the argument list of the function. Note that the addition of `v` and `u` is a `mod` $(2^{31}-1)$ addition.

Exercise 5:
In Work mode LFSR operates like this according to the specification:

```
LFSRWithWorkMode () {
 s16=(S15<<<31 15)+(S13<<<31 17)+(S10<<<31 21)+(S4<<<31 20)+(S0<<<31 8)+S0 mod (2^31-1)
 if s16==0, then set s16=2^31-1;
 (s1,s2, …,s15,s16) -> (s0,s1, …,s14,s15).
}
```

Write `LFSRWithWorkMode` as a Cryptol function using the results of Exercises 1 to 3. You will need to add `ss` to the argument list of the function.

Exercise 6:
The BR section extracts 128 bits from the LFSR stages and forms 4, 32-bit words, where the first three words are used by the nonlinear function F in the bottom layer, and the last word is involved in producing the keystream. According to the ZUC specification this may be expressed as:

```
Bitreorganization () {
  X0 = S15H || S14L;
  X1 = S11L || S9H;
  X2 = S7L || S5H;
  X3 = S2L || S0H.
}
```

where the || operation is concatenation, and the S_i are 31-bit integers, so S_iH means bits 30...15 and not 31...16 of S_i, for $0 \leq i \leq 15$. Write the Bitreorganization function in Cryptol. You will need to add ss to the argument list of the function.

Exercise 7:
The 32×32 S-box S is composed of 4 juxtaposed 8×8 S-boxes, i.e., S=(S0,S1,S2,S3), where S0=S2, S1=S3. The definitions of S0 and S1 can be found below as S0Box and S1Box, respectively.

Let x be an 8-bit input to S0 (or S1). Write x into two hexadecimal digits as x=h||l, then the entry at the intersection of the h^{th} row and the l^{th} column in S0Box or S1Box is the output of S0 (or S1). So, define function S X, where X is a 32 bit integer, to be the concatenation of the SBox lookups for the 4 bytes of X. That is, the high order byte lookup is in S0Box, the next is in S1Box, then S0Box and S1Box. Write this function, and all supporting functions, in Cryptol.

```
S0Box =
  [0x3E, 0x72, 0x5B, 0x47, 0xCA, 0xE0, 0x00, 0x33, 0x04, 0xD1, 0x54,
   0x98, 0x09, 0xB9, 0x6D, 0xCB, 0x7B, 0x1B, 0xF9, 0x32, 0xAF, 0x9D,
   0x6A, 0xA5, 0xB8, 0x2D, 0xFC, 0x1D, 0x08, 0x53, 0x03, 0x90, 0x4D,
   0x4E, 0x84, 0x99, 0xE4, 0xCE, 0xD9, 0x91, 0xDD, 0xB6, 0x85, 0x48,
   0x8B, 0x29, 0x6E, 0xAC, 0xCD, 0xC1, 0xF8, 0x1E, 0x73, 0x43, 0x69,
   0xC6, 0xB5, 0xBD, 0xFD, 0x39, 0x63, 0x20, 0xD4, 0x38, 0x76, 0x7D,
   0xB2, 0xA7, 0xCF, 0xED, 0x57, 0xC5, 0xF3, 0x2C, 0xBB, 0x14, 0x21,
   0x06, 0x55, 0x9B, 0xE3, 0xEF, 0x5E, 0x31, 0x4F, 0x7F, 0x5A, 0xA4,
   0x0D, 0x82, 0x51, 0x49, 0x5F, 0xBA, 0x58, 0x1C, 0x4A, 0x16, 0xD5,
   0x17, 0xA8, 0x92, 0x24, 0x1F, 0x8C, 0xFF, 0xD8, 0xAE, 0x2E, 0x01,
   0xD3, 0xAD, 0x3B, 0x4B, 0xDA, 0x46, 0xEB, 0xC9, 0xDE, 0x9A, 0x8F,
   0x87, 0xD7, 0x3A, 0x80, 0x6F, 0x2F, 0xC8, 0xB1, 0xB4, 0x37, 0xF7,
   0x0A, 0x22, 0x13, 0x28, 0x7C, 0xCC, 0x3C, 0x89, 0xC7, 0xC3, 0x96,
   0x56, 0x07, 0xBF, 0x7E, 0xF0, 0x0B, 0x2B, 0x97, 0x52, 0x35, 0x41,
   0x79, 0x61, 0xA6, 0x4C, 0x10, 0xFE, 0xBC, 0x26, 0x95, 0x88, 0x8A,
   0xB0, 0xA3, 0xFB, 0xC0, 0x18, 0x94, 0xF2, 0xE1, 0xE5, 0xE9, 0x5D,
   0xD0, 0xDC, 0x11, 0x66, 0x64, 0x5C, 0xEC, 0x59, 0x42, 0x75, 0x12,
   0xF5, 0x74, 0x9C, 0xAA, 0x23, 0x0E, 0x86, 0xAB, 0xBE, 0x2A, 0x02,
   0xE7, 0x67, 0xE6, 0x44, 0xA2, 0x6C, 0xC2, 0x93, 0x9F, 0xF1, 0xF6,
   0xFA, 0x36, 0xD2, 0x50, 0x68, 0x9E, 0x62, 0x71, 0x15, 0x3D, 0xD6,
   0x40, 0xC4, 0xE2, 0x0F, 0x8E, 0x83, 0x77, 0x6B, 0x25, 0x05, 0x3F,
   0x0C, 0x30, 0xEA, 0x70, 0xB7, 0xA1, 0xE8, 0xA9, 0x65, 0x8D, 0x27,
   0x1A, 0xDB, 0x81, 0xB3, 0xA0, 0xF4, 0x45, 0x7A, 0x19, 0xDF, 0xEE,
   0x78, 0x34, 0x60]
```

```
S1Box =
  [0x55, 0xC2, 0x63, 0x71, 0x3B, 0xC8, 0x47, 0x86, 0x9F, 0x3C, 0xDA,
   0x5B, 0x29, 0xAA, 0xFD, 0x77, 0x8C, 0xC5, 0x94, 0x0C, 0xA6, 0x1A,
   0x13, 0x00, 0xE3, 0xA8, 0x16, 0x72, 0x40, 0xF9, 0xF8, 0x42, 0x44,
   0x26, 0x68, 0x96, 0x81, 0xD9, 0x45, 0x3E, 0x10, 0x76, 0xC6, 0xA7,
   0x8B, 0x39, 0x43, 0xE1, 0x3A, 0xB5, 0x56, 0x2A, 0xC0, 0x6D, 0xB3,
   0x05, 0x22, 0x66, 0xBF, 0xDC, 0x0B, 0xFA, 0x62, 0x48, 0xDD, 0x20,
   0x11, 0x06, 0x36, 0xC9, 0xC1, 0xCF, 0xF6, 0x27, 0x52, 0xBB, 0x69,
   0xF5, 0xD4, 0x87, 0x7F, 0x84, 0x4C, 0xD2, 0x9C, 0x57, 0xA4, 0xBC,
   0x4F, 0x9A, 0xDF, 0xFE, 0xD6, 0x8D, 0x7A, 0xEB, 0x2B, 0x53, 0xD8,
   0x5C, 0xA1, 0x14, 0x17, 0xFB, 0x23, 0xD5, 0x7D, 0x30, 0x67, 0x73,
   0x08, 0x09, 0xEE, 0xB7, 0x70, 0x3F, 0x61, 0xB2, 0x19, 0x8E, 0x4E,
   0xE5, 0x4B, 0x93, 0x8F, 0x5D, 0xDB, 0xA9, 0xAD, 0xF1, 0xAE, 0x2E,
   0xCB, 0x0D, 0xFC, 0xF4, 0x2D, 0x46, 0x6E, 0x1D, 0x97, 0xE8, 0xD1,
   0xE9, 0x4D, 0x37, 0xA5, 0x75, 0x5E, 0x83, 0x9E, 0xAB, 0x82, 0x9D,
   0xB9, 0x1C, 0xE0, 0xCD, 0x49, 0x89, 0x01, 0xB6, 0xBD, 0x58, 0x24,
   0xA2, 0x5F, 0x38, 0x78, 0x99, 0x15, 0x90, 0x50, 0xB8, 0x95, 0xE4,
   0xD0, 0x91, 0xC7, 0xCE, 0xED, 0x0F, 0xB4, 0x6F, 0xA0, 0xCC, 0xF0,
   0x02, 0x4A, 0x79, 0xC3, 0xDE, 0xA3, 0xEF, 0xEA, 0x51, 0xE6, 0x6B,
   0x18, 0xEC, 0x1B, 0x2C, 0x80, 0xF7, 0x74, 0xE7, 0xFF, 0x21, 0x5A,
   0x6A, 0x54, 0x1E, 0x41, 0x31, 0x92, 0x35, 0xC4, 0x33, 0x07, 0x0A,
   0xBA, 0x7E, 0x0E, 0x34, 0x88, 0xB1, 0x98, 0x7C, 0xF3, 0x3D, 0x60,
   0x6C, 0x7B, 0xCA, 0xD3, 0x1F, 0x32, 0x65, 0x04, 0x28, 0x64, 0xBE,
   0x85, 0x9B, 0x2F, 0x59, 0x8A, 0xD7, 0xB0, 0x25, 0xAC, 0xAF, 0x12,
   0x03, 0xE2, 0xF2]
```

Exercise 8:
Cryptol specification of the nonlinear function F is a little tricky. There are two linear transformations L1 and L2 that are applied to an S input. These are:

$L1(X) = X \oplus (X <<<_{32} 2) \oplus (X <<<_{32} 10) \oplus (X <<<_{32} 18) \oplus (X <<<_{32} 24)$,
$L2(X) = X \oplus (X <<<_{32} 8) \oplus (X <<<_{32} 14) \oplus (X <<<_{32} 22) \oplus (X <<<_{32} 30)$.

Write Cryptol functions for these.

Exercise 9:
Build F in steps.

a) The output of F is W and is written as $(X_0 \oplus R_1) \boxplus R_2$ in the specification where $\oplus$ is exclusive or and $\boxplus$ is mod 2^{32} addition. R_1 and R_2 are shown in the Figure above. Write an expression for this.
b) See W_1 and W_2 in the Figure above and Section 3.4 in the ZUC specification. Write expressions for W_1 and W_2.
c) W_1 and W_2 get split into halves W_1L, W_1H, W_2L, W_2H. Then
$R_1 = S(L_1(W_1L || W_2H))$ and $R_2 = S(L_2(W_2L || W_1H))$
Write expressions to get the halves and use that to write R_1 and R_2.
d) Write the full expression for nonlinear function F. Notice that F takes $[X_0, X_1, X_2]$ and the sequence $[R_1, R_2]$ as input and outputs (W, [R1', R2']) where R1' and R2' are obtained in c) above.

Exercise 10:
Write a function that completes the key loading operation stated in Section 3.5. Call the function LoadKey, have it take arguments key:[128] and initialization vector iv:[128] and have it output values held in the 16 stages of the LFSR.