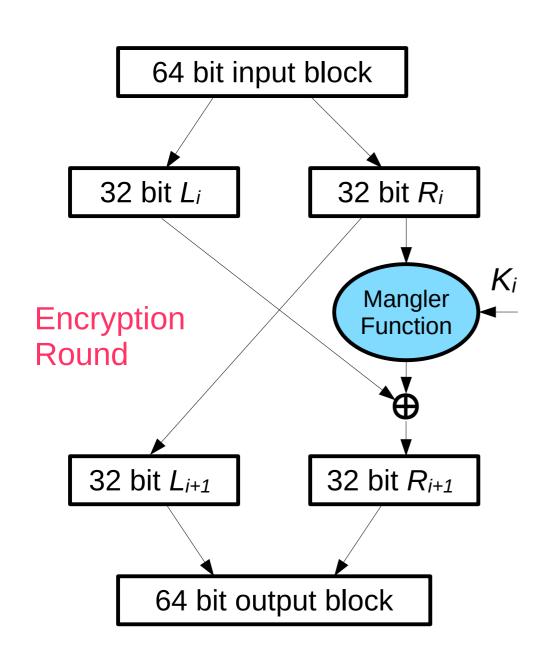
Secret Key Systems

Encrypting a small block of text (say 64 bits)

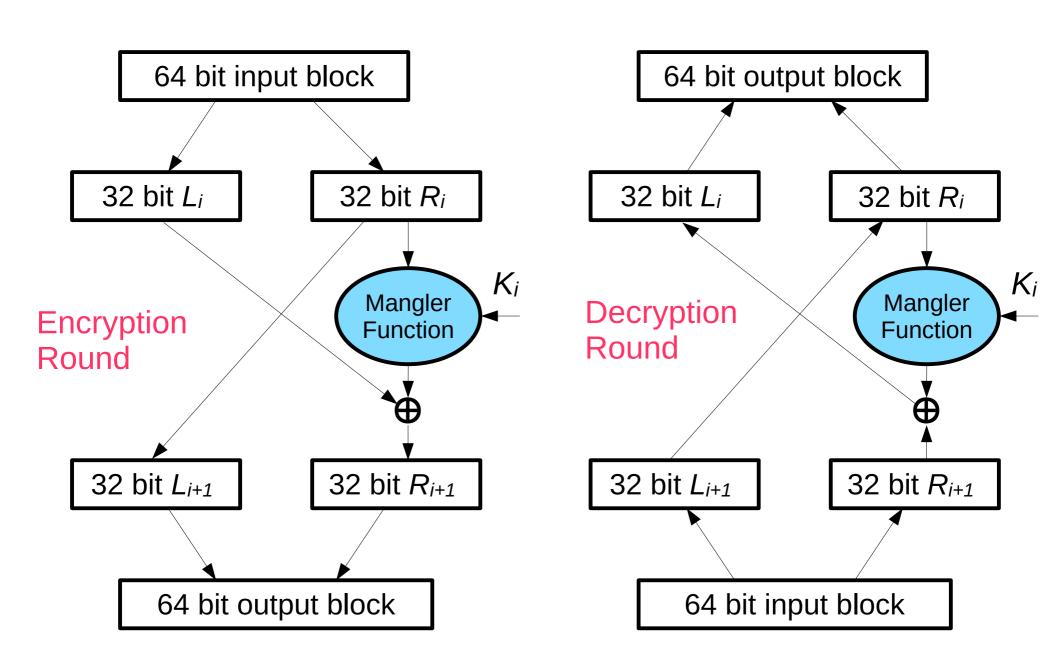
General Considerations:

- Encrypted data should look random.
 - As though someone flipped a fair coin 64 times and heads means 1 and tails 0.
 - Any change in one bit of output corresponds to a huge change in the input (bits are uncorrelated).
 - Should be about as many 1's as 0's usually.
- Try to spread the influence of each input bit to all output bits and change in one input bit should have 50% chance of changing any of the output bits (hence many rounds).
- Operations should be invertible hence xor and table lookup.
 Use of one key for both encryption and decryption.
- Attacks may be mitigated if they rely on operations that are not efficiently implemented in hardware yet allow normal operation to complete efficiently, even in software (permute).

IBM/NSA 1977 - 64 bit blocks, 56 bit key, 8 bits parity, 16 rounds



IBM/NSA 1977 - 64 bit blocks, 56 bit key, 8 bits parity, 16 rounds



Generating per round keys $K_1 K_2 ... K_{16}$ from the 56 bit Key + 8 parity bits

Key bits:

1...8

9...16

17...24

25...32

33...40

41...48

49...56

57...64

Generating per round keys $K_1 K_2 \dots K_{16}$ from the 56 bit Key + 8 parity bits

```
Key bits: 1...8 9...16 17...24 25...32 33...40 41...48 49...56 57...64

Co: 57 49 41 33 25 17 9 1 58 50 42 34 26 18 10 2 59 51 43 35 27 19 11 3 60 52 44 36

Do: 63 55 47 39 31 23 15 7 62 54 46 38 30 22 14 6 61 53 45 37 29 21 13 5 28 20 12 4
```

Create two sequences of permutations of key bits.

Generating per round keys $K_1 K_2 \dots K_{16}$ from the 56 bit Key + 8 parity bits

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Do: 63 55 47 39 31 23 15 7 62 54 46 38 30 22 14 6 61 53 45 37 29 21 13 5 28 20 12 4
```

Create two sequences of permutations of key bits. In Cryptol:

The first 2 lines are C_0 , the next two lines are D_0

Generating per round keys $K_1 K_2 \dots K_{16}$ from the 56 bit Key + 8 parity bits

```
Key bits: 1...8 9...16 17...24 25...32 33...40 41...48 49...56 57...64

Co: 57 49 41 33 25 17 9 1 58 50 42 34 26 18 10 2 59 51 43 35 27 19 11 3 60 52 44 36

Do: 63 55 47 39 31 23 15 7 62 54 46 38 30 22 14 6 61 53 45 37 29 21 13 5 28 20 12 4
```

Create two sequences of permutations of key bits. In Cryptol:

But, since sequence indices start with 0, not 1, the following is needed:

```
zeroBasify XP = [i - 1 | i < - XP]
KPz = zeroBasify KP
```

Generating per round keys $K_1 K_2 \dots K_{16}$ from the 56 bit Key + 8 parity bits

Key bits: 1...8 9...16 17...24 25...32 57...64 33...40 41...48 49...56

Each round: K_i has 48 bits assembled in 2 halves permuted from 24 bits each of C_i and D_i , K_{i+1} is obtained by rotating C_i and D_i left to form C_{i+1} and D_{i+1} (rotation is 1 bit for rounds 1,2,9,16 and 2 bits for other rounds)

Permutations:

```
Left half C_i: (9,18,35,43 are missing)
 14 17 11 24 1 5 3 28 15 6 21 10 23 19 12 4 26 8 16 7 27 20 13 2
Right half D_i: (22,38,43,54 are missing)
  41 52 31 37 47 55 30 40 51 45 33 48 44 49 39 56 34 53 46 42 50 36 29 32
  CP: [48][6]
  CP = [14, 17, 11, 24, 1, 5, 3, 28, 15, 6, 21, 10,
           23, 19, 12, 4, 26, 8, 16, 7, 27, 20, 13, 2,
           41, 52, 31, 37, 47, 55, 30, 40, 51, 45, 33, 48,
```

44, 49, 39, 56, 34, 53, 46, 42, 50, 36, 29, 32]

CPz = zeroBasify CP

Generating per round keys $K_1 K_2 ... K_{16}$ from the 56 bit Key + 8 parity bits

Key bits: 1...8 9...16 17...24 25...32 33...40 41...48 49...56 57...64

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In Cryptol:

```
sums : [16][8] -> [16][8]
sums xs = ys
  where ys = [ x + y | x <- xs | y <- [0] # ys ]
expand: [2][28] -> [16][48]
expand [kl, kr] =
  [ ((kl <<< i) # (kr <<< i)) @@ CPz
  | i <- sums [1,1,2,2,2,2,2,2,1,2,2,2,2,2,2,1]]</pre>
```

expand needs to be supplied with C_0 and D_0 from key information This is done on the next slide.

Generating per round keys $K_1 K_2 ... K_{16}$ from the 56 bit Key + 8 parity bits

 Key bits:
 1...8
 9...16
 17...24
 25...32
 33...40
 41...48
 49...56
 57...64

Each round: K_i has 48 bits assembled in 2 halves permuted from 24 bits each of C_i and D_i , K_{i+1} is obtained by rotating C_i and D_i left to form C_{i+1} and D_{i+1} (rotation is 1 bit for rounds 1,2,9,16 and 2 bits for other rounds)

In Cryptol:

```
expandKey : [64] -> [16][48]
expandKey key = expand (split (key @@ KPz))
```

Example:

```
let key = 0x0001020304050607
let res1 = (key @@ KPz)
res1 is 0x0000000ccf0000 (56 bits)
let res2 = (split (key @@ KPz)):[2][28]
res2 is [0x0000000, 0xccf0000]
let keys = expand (split (key @@ KPz))
```

Generating per round keys $K_1 K_2 \dots K_{16}$ from the 56 bit Key + 8 parity bits

Key bits: 1...8 9...16 17...24 25...32 33...40 41...48 49...56 57...64

Each round: K_i has 48 bits assembled in 2 halves permuted from 24 bits each of C_i and D_i , K_{i+1} is obtained by rotating C_i and D_i left to form C_{i+1} and D_{i+1} (rotation is 1 bit for rounds 1,2,9,16 and 2 bits for other rounds)

Example:

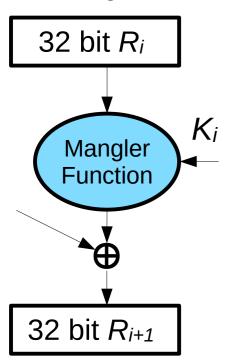
```
keys is
[0x000000102307,
                 0x000000340105,
                                   0x0000000220c6,
                                                    0x00000064a181,
0x00000022044b,
                 0x0000004e9102,
                                  0x000000044568,
                                                    0 \times 0000000489840,
0x000000488078,
                 0x00000081dc08,
                                  0x000000081630,
                                                    0x000000994824,
                                                    0x000000132282]
0x000000004a90,
                 0x000000912015,
                                  0x000000a30280,
```

These would be the 16 per round keys generated from the 56 bit key

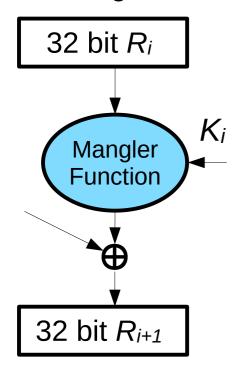
```
key @@ KPz = 0x0000000ccf0000
```

Of course, this is a bad key but was used to simplify your observations

The Mangler function: mixes 32 bit input with 48 bit key to produce 32 bits



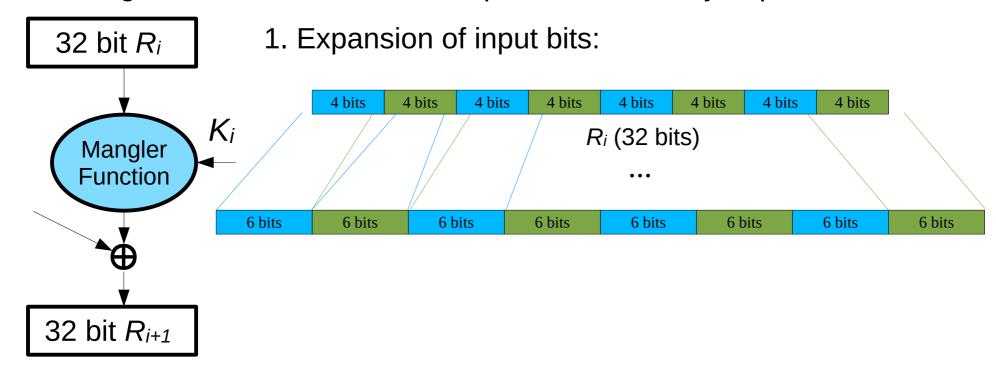
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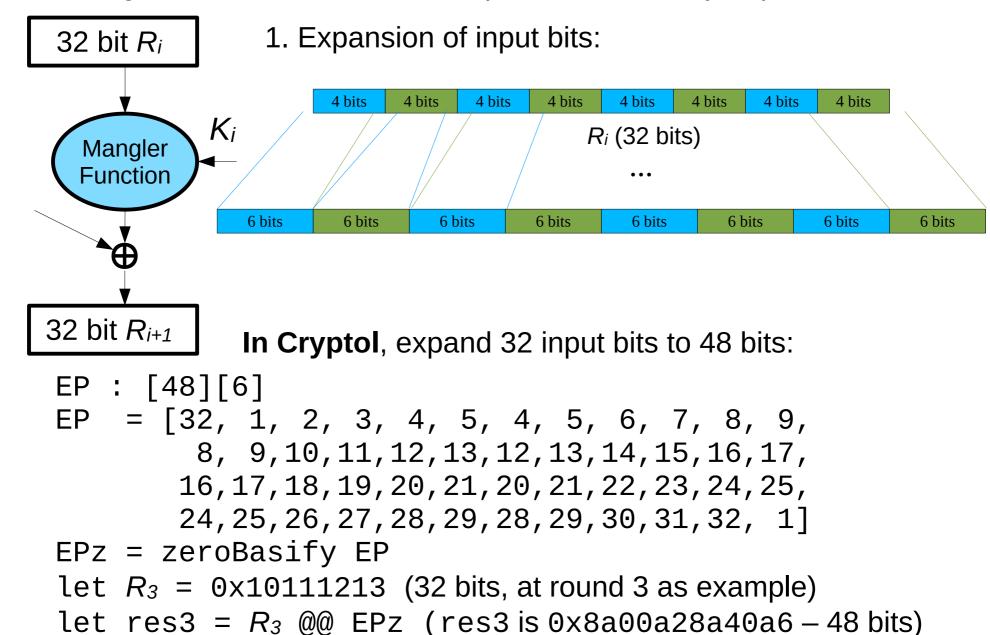
1. Expansion of input bits:



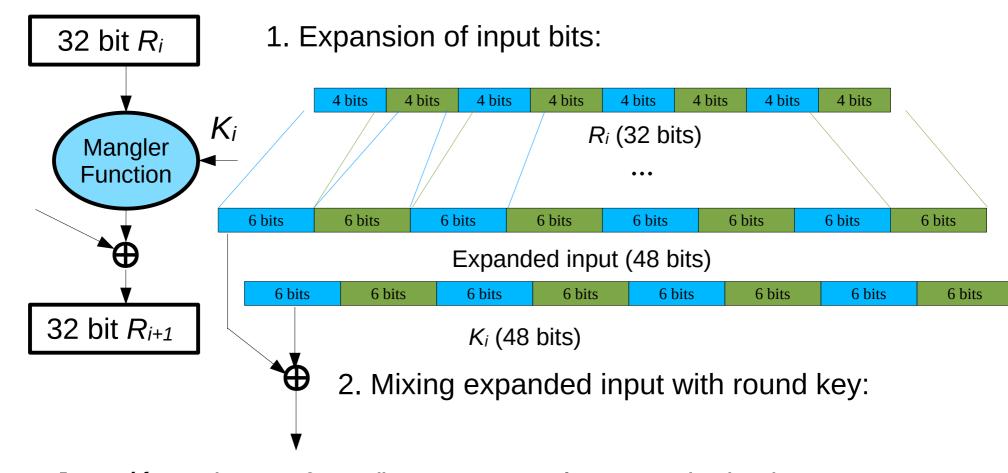
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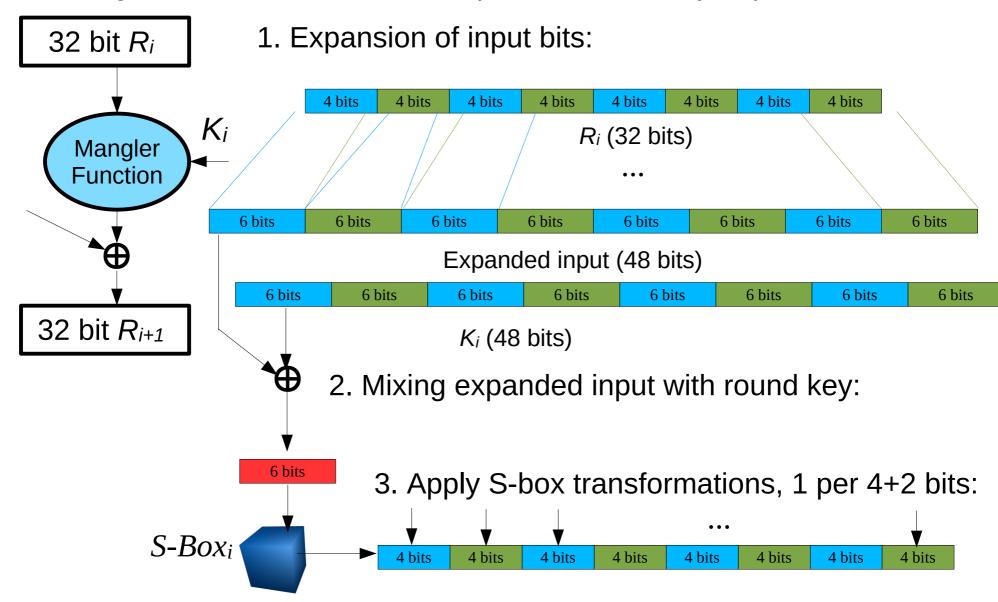


The Mangler function: mixes 32 bit input with 48 bit key to produce 32 bits



let K_3 = keys @ 3 (just an example – round 3 key) K_3 is $0 \times 00000064a181$ (48 bits) let res4 = (R_3 @@ EPz) ^ K_3 (xor of expanded R_3 and K_3) res4 is $0 \times 8a00a2eee127$ (48 bits)

The Mangler function: mixes 32 bit input with 48 bit key to produce 32 bits



The S-Box: maps 6 bit blocks to 4 bit sections

S- Box_1 (first 6 bits):

Input bits 2,3,4,5

	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
00	1110	0100	1101	0001	0010	1111	1011	1000	0011	1010	0110	1100	0101	1001	0000	0111
01	0000	1111	0111	0100	1110	0010	1101	0001	1010	0110	1100	1011	1001	0101	0011	1000
10	0100	0001	1110	1000	1101	0110	0010	1011	1111	1100	1001	0111	0011	1010	0101	0000
11	1111	1100	1000	0010	0100	1001	0001	0111	0101	1011	0011	1110	1010	0000	0110	1101

Input bits 1 and 6

```
sbox1 : [4][16][4] \\ sbox1 = [[14,4, 13, 1, 2,15,11, 8, 3,10, 6,12, 5, 9, 0, 7], \\ [0,15, 7, 4,14, 2,13, 1,10, 6,12,11, 9, 5, 3, 8], \\ [4, 1,14, 8,13, 6, 2,11,15,12, 9, 7, 3,10, 5, 0], \\ [15,12, 8, 2, 4, 9, 1, 7, 5,11, 3,14,10, 0, 6,13]]
```

This is just one of 8 S-Boxes, they are all different. Input to an S-box is 6 bits, output is 4 bits, determined by the coordinates of a row and column.

The *S-Box*: maps 6 bit blocks to 4 bit sections *S-Box*¹ (first 6 bits):

S-Box₁ (first 6 bits): Input bits 2,3,4,5

_																	
ı		0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
	00	1110	0100	1101	0001	0010	1111	1011	1000	0011	1010	0110	1100	0101	1001	0000	0111
	01	0000	1111	0111	0100	1110	0010	1101	0001	1010	0110	1100	1011	1001	0101	0011	1000
	10	0100	0001	1110	1000	1101	0110	0010	1011	1111	1100	1001	0111	0011	1010	0101	0000
	11	1111	1100	1000	0010	0100	1001	0001	0111	0101	1011	0011	1110	1010	0000	0110	1101

```
Input bits 1 and 6
```

```
let sboxes = [sbox1, sbox2, ..., sbox8]
// n is the sbox, b is the 6 bit input to the sbox
// returns 4 bit translation
sbox (n,b) = (s @ [b1, b6] @ [b2, b3, b4, b5])
  Where
    s = sboxes @ (n-1)
    b1 = b @ 0
    . . .
    b6 = b @ 5
```

The *S-Box*: maps 6 bit blocks to 4 bit sections *S-Box*₄ (first 6 bits):

S-Box₁ (first 6 bits): Input bits 2,3,4,5

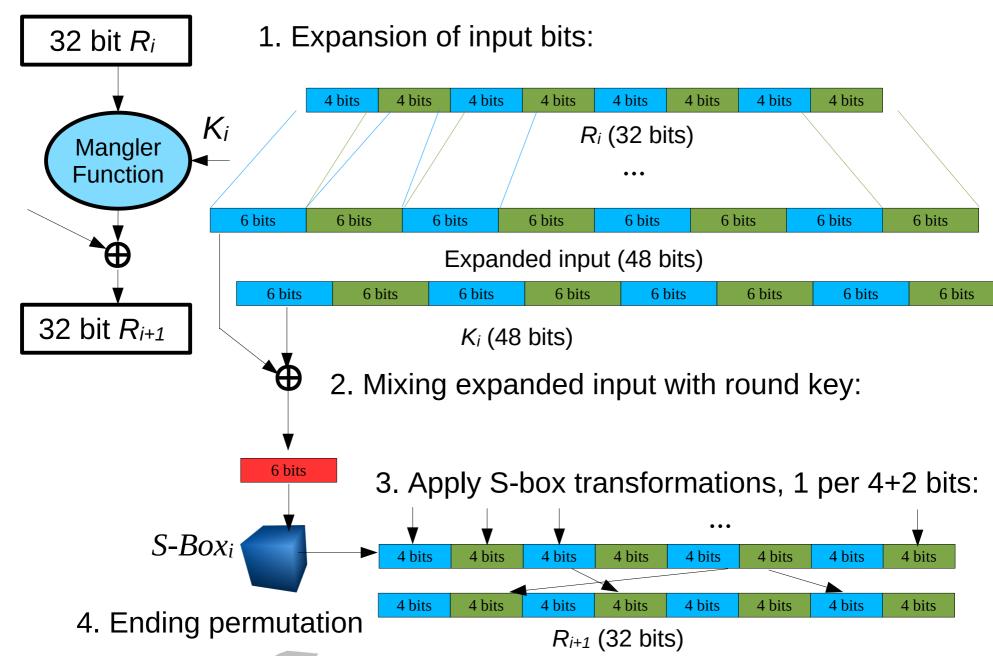
	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
00	1110	0100	1101	0001	0010	1111	1011	1000	0011	1010	0110	1100	0101	1001	0000	0111
01	0000	1111	0111	0100	1110	0010	1101	0001	1010	0110	1100	1011	1001	0101	0011	1000
10	0100	0001	1110	1000	1101	0110	0010	1011	1111	1100	1001	0111	0011	1010	0101	0000
11	1111	1100	1000	0010	0100	1001	0001	0111	0101	1011	0011	1110	1010	0000	0110	1101

Input bits 1 and 6

```
// Combine results from 8 sboxes - x is the 48 bit // input. returns the 32 bit output SBox : [48] \rightarrow [32] SBox x = join [ sbox (n, b) | n <- [1 \dots 8] | b <- split x ]
```

Since the type signature for sbox is $([4],[6]) \rightarrow [4]$ split knows to split 48-bit x into a sequence of 8 6-bit sections.

The Mangler function: mixes 32 bit input with 48 bit key to produce 32 bits



Ending permutation:

16 7 20 21 29 12 28 17 1 15 23 26 5 18 31 10 2 8 24 14 32 27 3 9 19 13 30 6 22 11 4 25

In Cryptol:

```
PP: [32][6] // Ending permutation applied after Sbox op
PP = [16, 7,20,21,29,12,28,17, 1,15,23,26, 5,18,31,10,
2, 8,24,14,32,27, 3, 9,19,13,30, 6,22,11, 4,25]
PPz = zeroBasify PP
```

The 48 bit result of xoring the per round key with expanded input:

```
(R_i @ @ EPz) \wedge K_i
```

The 32 bit result of SBox operations after the above operation:

```
SBox((R_i @@ EPz) \land K_i)
```

The 32 bit result after application of the Ending Permutation:

```
(SBox((R_i @@ EPz) \land K_i)) @@ PPz
```

As a function, to be applied on any of 16 rounds:

$$f(r,k) = (SBox((r@@EPz) ^ k) @@PPz$$

The operation of a round - r is right half of 64 bit input and 1 is the left half of the 64 bit input, k is the per-round 48 bit key:

```
round (k, [l, r]) = r # (l ^ f (r, k))
```

All 16 rounds, encryption and decryption. Plaintext is pt, IPz is the Initial Permutation on the 64 bit input (see below) keys is obtained from the expandKeys function, last round result is lst — the last operation is to swap the last block right half with the left half and apply Final Permutation FPz (see below):

Finally the encryption and decryption functions cleanly stated:

```
encrypt key pt = des pt (expandKey key)
decrypt key ct = des ct (reverse (expandKey key))
```

The two permutations not previously defined:

```
IP : [64][7] // Redo of CO & DO but applied to all 64 bits
IP = [58,50,42,34,26,18,10, 2,60,52,44,36,28,20,12, 4,
        62,54,46,38,30,22,14, 6,64,56,48,40,32,24,16, 8,
        57,49,41,33,25,17, 9, 1,59,51,43,35,27,19,11, 3,
        61,53,45,37,29,21,13, 5,63,55,47,39,31,23,15, 7]
IPz = zeroBasify IP

FP : [64][7]
FP = [40, 8,48,16,56,24,64,32,39, 7,47,15,55,23,63,31,
        38, 6,46,14,54,22,62,30,37, 5,45,13,53,21,61,29,
        36, 4,44,12,52,20,60,28,35, 3,43,11,51,19,59,27,
        34, 2,42,10,50,18,58,26,33, 1,41, 9,49,17,57,25]
FPz = zeroBasify FP
```

Weak and semi-weak keys:

If key is such that C_0 or D_0 are:

- 1) all 0s;
- 2) all 1s;
- 3) alternating 1s and 0s,

then attack is easy. There are 16 such keys. Keys for which C_0 and D_0 are both 0 or both 1 are called *weak* (encrypting with key gives same result as decrypting).

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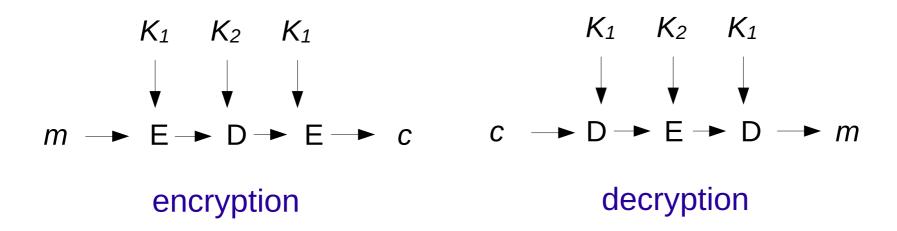
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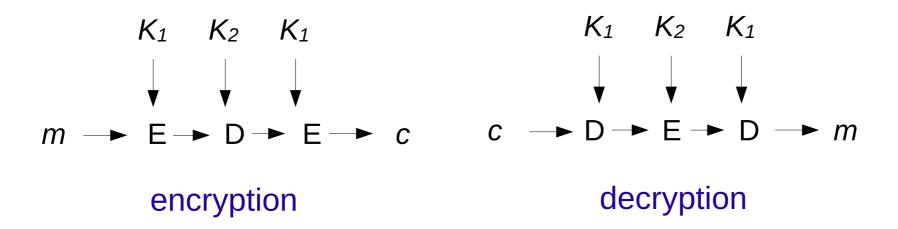
Discussion:

- 1. Not much known about the design not made public Probably attempt to prevent known attacks
- 2. Changing S-Boxes has resulted in provably weaker system

Two keys K_1 and K_2 :



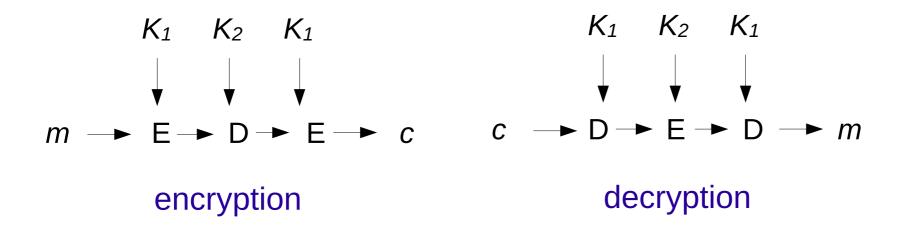
Two keys K_1 and K_2 :



Why not 2DES

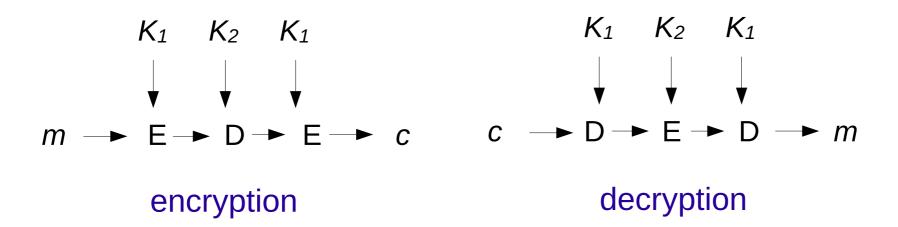
1. Double encryption with the same key still requires searching 2⁵⁶ keys

Two keys K_1 and K_2 :



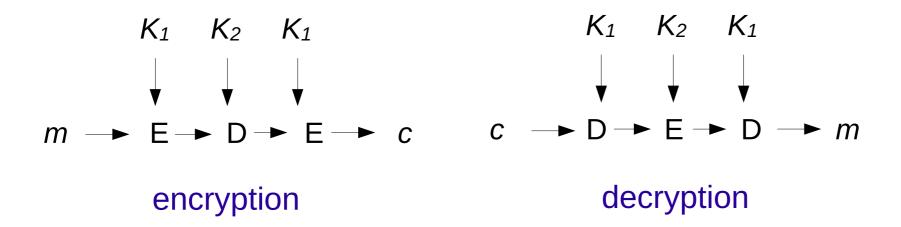
- 1. Double encryption with the same key still requires searching 2⁵⁶ keys
- 2. Double encryption with two different keys is just as vulnerable as DES due to the following, assuming some $\langle m,c \rangle$ pairs are known:

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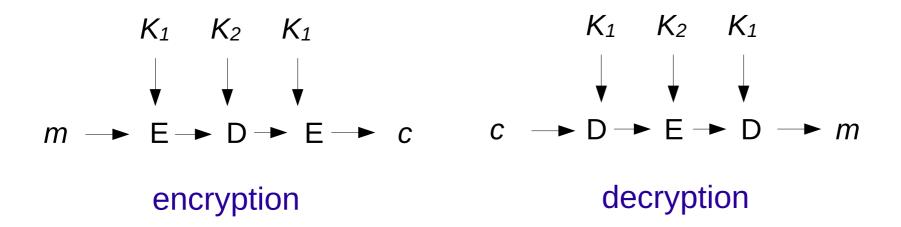
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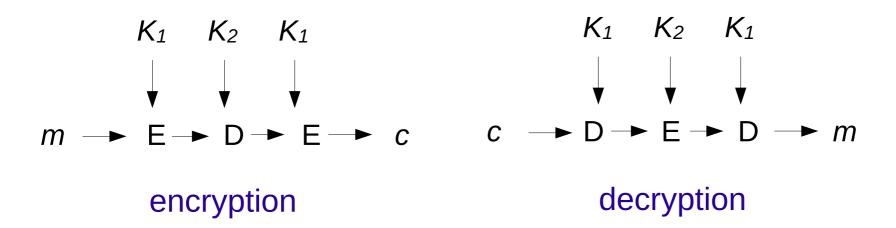
- 1. Double encryption with the same key still requires searching 2^{56} keys
- 2. Double encryption with two different keys is just as vulnerable as DES due to the following, assuming some $\langle m,c \rangle$ pairs are known:

Two keys K_1 and K_2 :



- 1. Double encryption with the same key still requires searching 2⁵⁶ keys
- 2. Double encryption with two different keys is just as vulnerable as DES due to the following, assuming some $\langle m,c \rangle$ pairs are known:

Two keys K_1 and K_2 :



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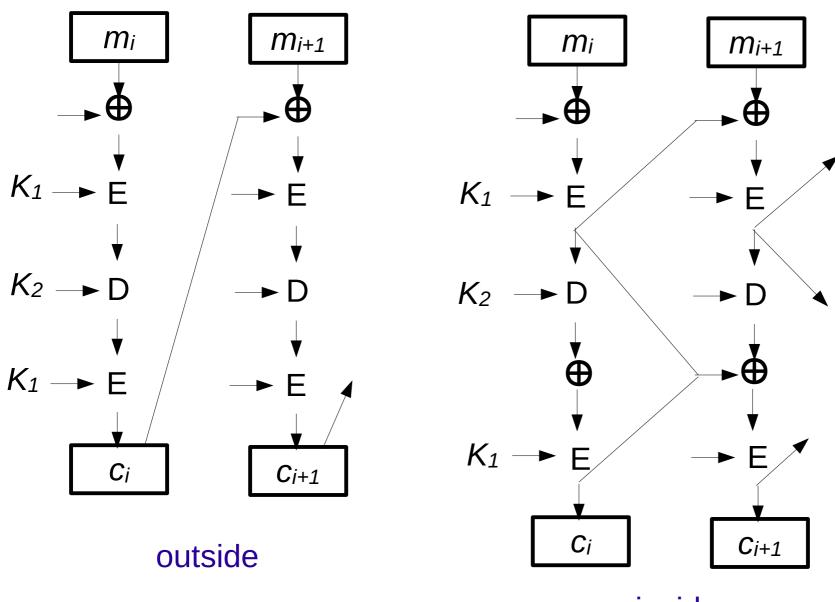
Why not 2DES

2. Double encryption with two different keys is just as vulnerable as DES due to the following, assuming some $\langle m,c \rangle$ pairs are known: How many $\langle m,c \rangle$ pairs do you need?

```
2^{64} possible blocks 2^{56} table entries each block has probability 2^{-8} = 1/256 of showing in a table probability a block is in both tables is 2^{-16} average number of matches is 2^{48} average number of matches for two < m,c> pairs is about 2^{32} for three < m,c> pairs is about 2^{16} for four < m,c> pairs is about 2^{0}
```

- 3. Triple encryption with two different keys
 - 112 bits of key is considered enough
 - straightforward to find a triple of keys that maps a given plaintext to a given ciphertext (no known attack with 2 keys)
 - EDE: use same keys to get DES, EEE: effect of permutations lost

CBC with 3DES:



inside

CBC with 3DES:

- 1. On the outside same attack as with CBC change a block with side effect of garbling another
- 2. On the inside attempt at changing a block results in all block garbled to the end of the message.
- 3. On the inside use three times as much hardware to pipeline encryptions resulting in DES speeds.
- 4. On the outside EDE simply is a drop-in replacement for what might have been there before.