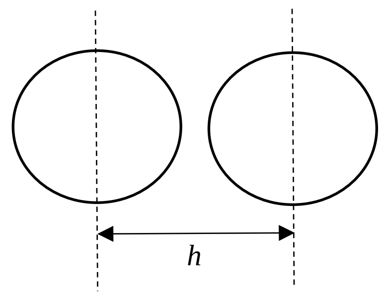
Theoretical tensile strength of a solid **S.J.Eichhorn 20/02/18**

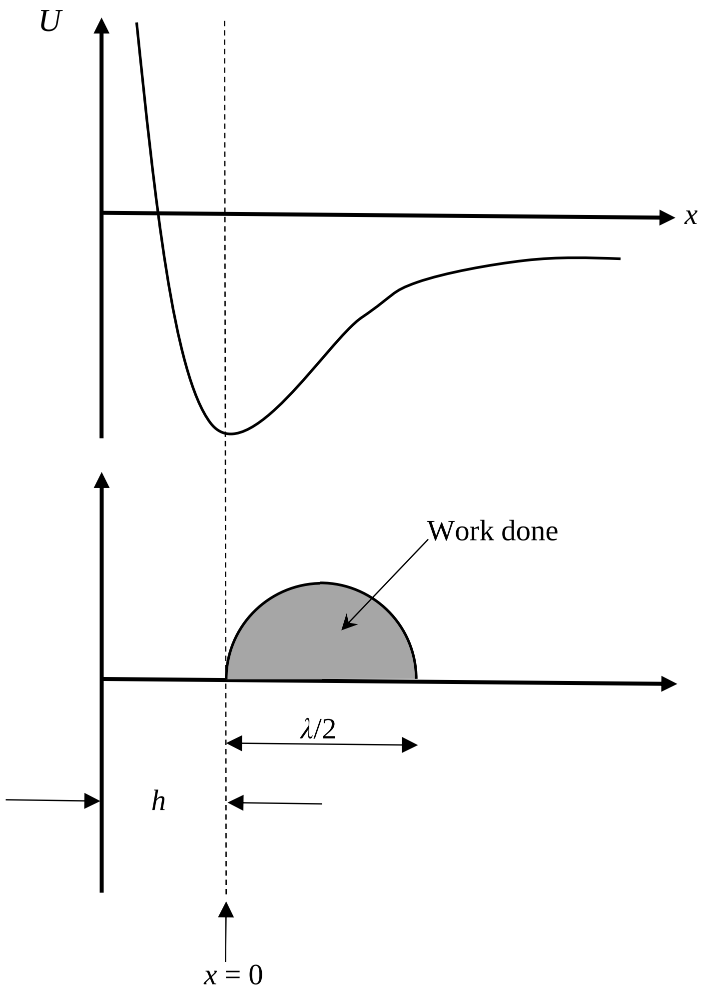
In the consideration of the fracture behaviour of materials it is useful to determine the strength of an ideal solid determined solely by the breaking of bonds.

Consider two neighbouring atoms in a solid



Let their equilibrium separation be h.

Consider the relationship between potential energy (U) and the separation of atoms x.



Force (F) between the atoms is given by

$$F = \frac{\mathrm{d}U}{\mathrm{d}x}$$

and,

$$\sigma = \text{Stress} = \frac{\frac{\mathrm{d}U}{\mathrm{d}x}}{A}$$

The dependence of stress upon the separation can be approximated to a Sine wave of wavelength λ. So the curve above can be approximated by an equation of the form

$$\sigma = \sigma_t \sin\left(\frac{2\pi x}{\lambda}\right) \tag{1}$$

At low strains or displacements

$$\sin\theta \sim \theta \tag{2}$$

Therefore,

$$\sigma = \sigma_t \left(\frac{2\pi x}{\lambda}\right) \tag{3}$$

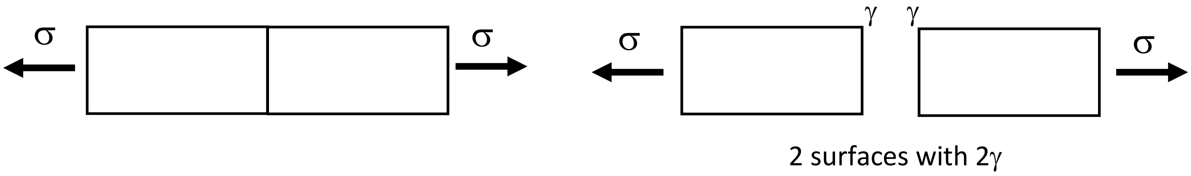
Also, at small strains Hooke's law is obeyed

$$\sigma = \text{modulus } \times \text{strain} = E\frac{x}{h} \tag{4}$$

Combining equations (3) and (4) gives

$$\sigma_{\mathrm{t}} = \frac{\lambda E}{2\pi h} \tag{5}$$

When a material is fractured in a brittle manner, two new surfaces are formed with surface energies of γ (J m^{-2}).



This will be equal to the work done in fracturing the material – area under the curve. So,

$$2\gamma = \int_0^{\lambda/2} \sigma_\mathrm{t} \sin\frac{2\pi x}{\lambda}\,\mathrm{d}x \qquad (6)$$

Therefore

$$2\gamma = \sigma_t \frac{\lambda}{\pi} \qquad (7)$$

But from (5) we find

$$\frac{\lambda}{\pi} = \frac{2\sigma_\mathrm{t} h}{E} \qquad (8)$$

Combining equations (7) and (8) we obtain

$$2\gamma = \frac{\sigma_\mathrm{t} 2\sigma_\mathrm{t} h}{E} \qquad (9)$$

And rearranging and solving for σ_t we obtain

$$\sigma_\mathrm{t} = \sqrt{\frac{E\gamma}{h}} \qquad (10)$$

Taking some real values of γ and h for real solids we find that

$$\sigma_t \approx \frac{E}{10} \qquad (11)$$