Lectures 6 & 7
The General Rigid-Body Equations of Motion

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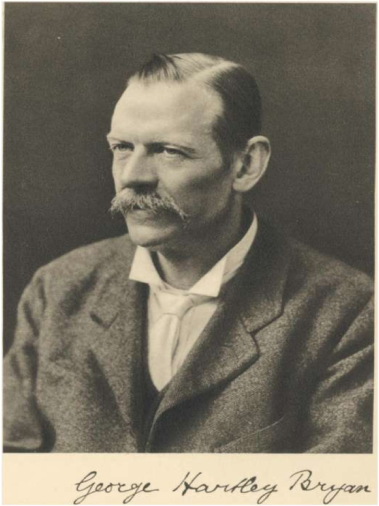
References

1. *Flight Dynamics Principles* - M V Cook, 1997, 2007 or 2013
2. *Aircraft Control and Simulation* - Stephens & Lewis, 2003
3. *Dynamics of Flight: Stability and Control* - Etkin & Reid, 1995
4. (*Aircraft Handling Qualities* - Hodgkinson, 1999)


N169EX
N111XT
Equations of Motion

George Hartley Bryan



George Hartley Bryan

- George Bryan addressed the problem of stability and control in the first decade of the 20^{th} century.
- Bryan published "Stability in Aviation" in 1911.
- His treatment, with very few changes, is still in everyday use.
- Bryan developed the general equations of motion of a rigid body with six degrees of freedom to describe successfully the motion of the aircraft.

Why do we need a mathematical model?

- The Equations of Motion form a mathematical model - for a rigid body aircraft, in our case.
- The model provides a complete description of the response to controls (e.g. aileron, rudder and elevator), subject only to modelling limitations.
- Output can be measured in terms of displacement, velocity and acceleration.
- The mathematical model can include equations which describe control system and sensor dynamics.

Why do we need a mathematical model?

Why do we need a mathematical model?

- Simulation - e.g. step & piloted response
- Stability analysis - linear and nonlinear
- Handling qualities
- Aircraft design & development
- Control system design
- Flight clearance
- Pilot training
- UAS simulation
- Air accident analysis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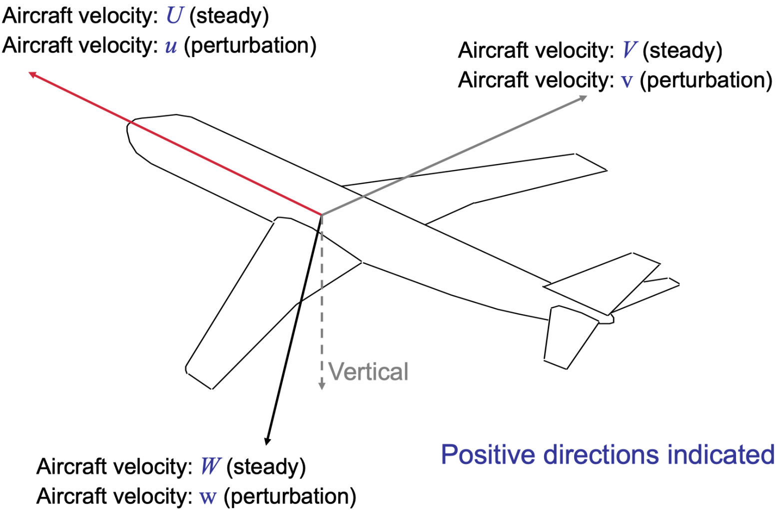
Equations of Motion

Introduction to the Equations of Motion

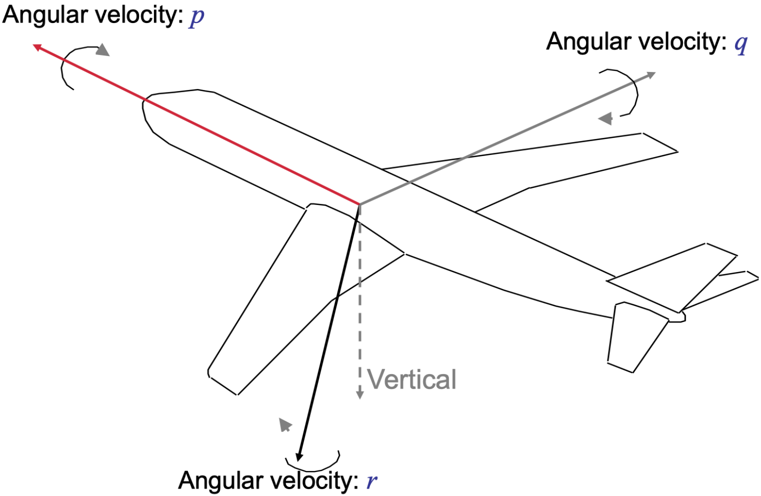
- You are NOT required to memorise a full derivation of the equations but understand their structure and the terms involved.
- The derivation of the equations is from Cook 2013.
- The general equations are given first, and small disturbance equations presented later in the course.
- *You do need to learn the notation from the first lectures!*

Body Axes Notation and Sign Conventions

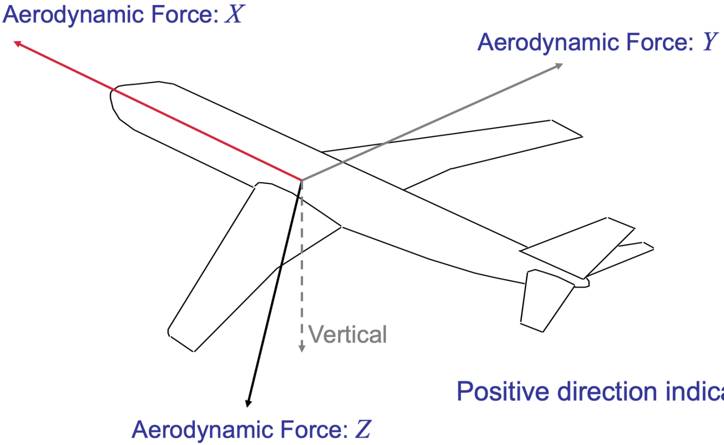


Positive directions indicated

Body Axes Notation and Sign Conventions

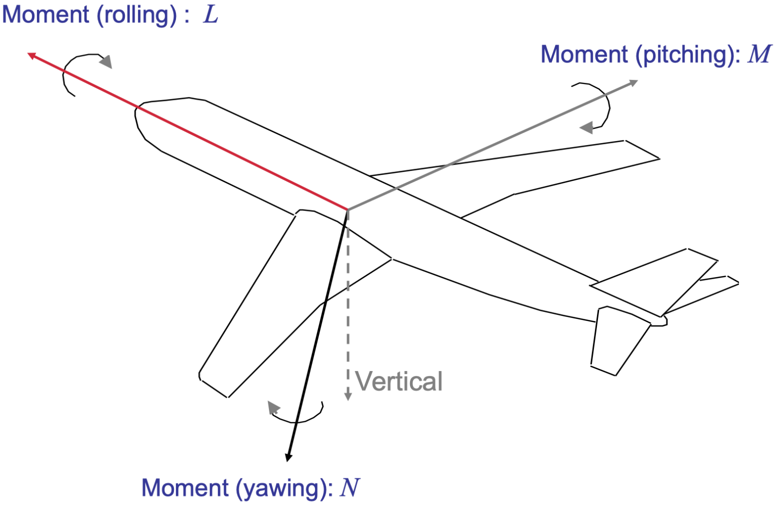


Body Axes Notation and Sign Conventions



Positive direction indicated

Body Axes Notation and Sign Conventions



Derivation of the Equations of Motion

Generalised Newton's Second Law applied to rigid body:

rate of change of linear momentum of a rigid body in a particular direction	=	sum of the components of the external forces acting on the body in that direction

and:

rate of change of angular momentum of a rigid body about a particular fixed axis	=	sum of the moments of the external forces acting on the body about that axis

Derivation of the Equations of Motion

These are commonly interpreted as:

Force = mass × inertial acceleration $\boxed{ma_i = \sum F_i}$

Moment = moment of inertia × angular accel. $\boxed{I_j \alpha_j = \sum M_j}$

where: m = mass of the body

a_i = acceleration in direction i

F_i = the set of external forces in direction i

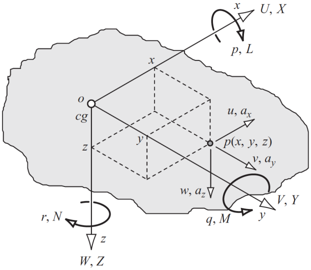
I_j = moment of inertia of the body about chosen axis j

α_j = angular acceleration about axis j

M_j = the set of external moments about axis j.

Components of inertial acceleration

We start by defining the inertial accelerations arising from the components of external force acting on the aircraft.
Consider an orthogonal set of body axes with origin fixed at the CG of the rigid body shown in the figure; these axes are in motion relative to an external inertial or earth frame.



- p is an arbitrary point within the body axes at position (x,y,z).
- Local components of vel. and accel. at p rel. to body axes are:
- (u, v, w) and (a_x, a_y, a_z).

Motion referred to generalized body axes (Cook)

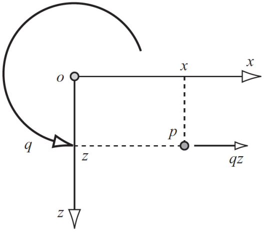
Components of inertial acceleration

Velocity components at $p(x, y, z)$ relative to origin o are:

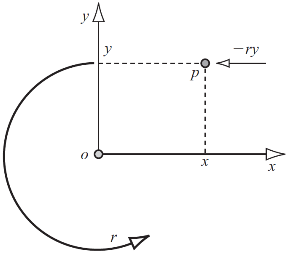
$$u = \dot{x} - ry + qz$$
$$v = \dot{y} - pz + rx$$
$$w = \dot{z} - qx + py$$

We see that in addition to the linear terms in each equation, there are two additional terms due to rotary motion - the *tangential velocity* components. Their origin is illustrated by example for the u equation in the figure on the next slide.

Components of inertial acceleration



Looking into axes
system along y axis



Looking into axes
system along z axis

Velocity terms due to rotary motion (Cook)

Since the body (the aircraft) is assumed to be rigid, $\dot{x} = \dot{y} = \dot{z} = 0$

Therefore the velocity equations (1) reduce to:

$$u = qz - ry$$

$$v = rx - pz$$

$$w = py - qx$$

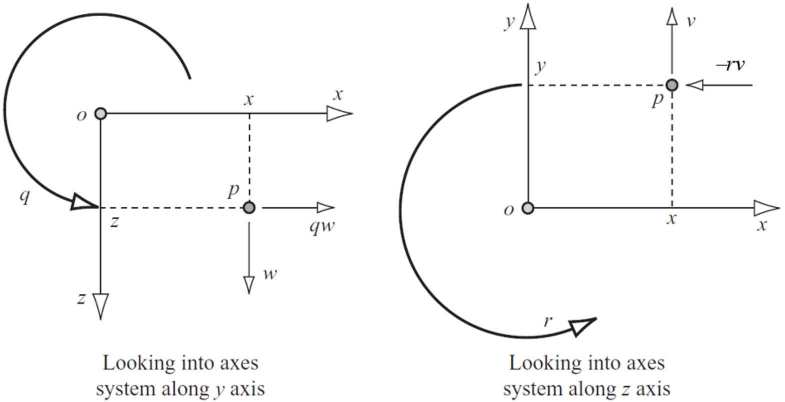
Components of inertial acceleration

The corresponding components of acceleration at $p(x, y, z)$ relative to o are:

$$
\begin{aligned}
a_x &= \dot{u} - rv + qw \\
a_y &= \dot{v} - pw + ru \qquad (2) \\
a_z &= \dot{w} - qu + pv
\end{aligned}
$$

Again, in addition to the linear terms in each equation, there are two additional terms due to rotary motion – the *tangential acceleration* components. Their origin is illustrated by example for the a_x equation in the figure on the next slide.

Components of inertial acceleration



Acceleration terms due to rotary motion (Cook)

Components of inertial acceleration

Now, the CG has velocity components (U, V, W).
Superimposing this onto the local velocity components (u, v, w), the absolute – or inertial – velocity components (u', v', w'), of the point $p(x, y, z)$ are described:

$$
\begin{aligned}
u' &= U + u = U - ry + qz \\
v' &= V + v = V - pz + rx \\
w' &= W + w = W - qx + py
\end{aligned}
\qquad (3)
$$

We need the velocities and accelerations in inertial axes as Newton's 2nd Law applies only in the inertial frame.

Components of inertial acceleration

Similarly, the components of inertial acceleration $\left(a'_x, a'_y, a'_z\right)$, of the point $p(x, y, z)$ can be found by substituting the expressions for components $\left(u', v', w'\right)$ from eqns. (3) for (u, v, w) in eqns. (2):

$$
\begin{aligned}
a'_x &= \dot{u}' - rv' + qw' \\
a'_y &= \dot{v}' - pw' + ru' \\
a'_z &= \dot{w}' - qu' + pv'
\end{aligned}
\tag{4}
$$

Components of inertial acceleration

Differentiating eqns. (3), recalling that $\dot{x} = \dot{y} = \dot{z} = 0$, we find that:

$$\begin{aligned} \dot{u}' &= \dot{U} - \dot{r}y + \dot{q}z \\ \dot{v}' &= \dot{V} - \dot{p}z + \dot{r}x \\ \dot{w}' &= \dot{W} - \dot{q}x + \dot{p}y \end{aligned} \qquad (5)$$

Then, substituting eqns. (3) and (5) into (4), the inertial acceleration components of the point $p(x,y,z)$ in body axes can be written:

$$\begin{aligned} a_x' &= \dot{U} - rV + qW - x\left(q^2 + r^2\right) + y\left(pq - \dot{r}\right) + z\left(pr - \dot{q}\right) \\ a_y' &= \dot{V} - pW + rU + x\left(pq + \dot{r}\right) - y\left(p^2 + r^2\right) + z\left(qr - \dot{p}\right) \\ a_z' &= \dot{W} - qU + pV + x\left(pr - \dot{q}\right) + y\left(qr + \dot{p}\right) - z\left(p^2 + q^2\right) \end{aligned} \qquad (6)$$

Generalised force equations

Consider incremental mass δm at point $p(x,y,z)$ in the rigid body.

Now, applying $F=ma$ to the incremental mass, the incremental components of force acting on the mass are given by $\left(\delta m.a'_x, \delta m.a'_y, \delta m.a'_z\right)$.

The components of total force acting on the body are then:

$$
\begin{aligned}
\sum \delta m.a'_x &= X \\
\sum \delta m.a'_y &= Y \\
\sum \delta m.a'_z &= Z
\end{aligned}
\tag{7}
$$

Generalised force equations

If we substitute the expressions for components of inertial acceleration $\left(a'_x, a'_y, a'_z\right)$ from eqns. (6) and note that the axes origin is the CG, we get:

$$\sum \delta m.x = \sum \delta m.y = \sum \delta m.z = 0 \qquad (8)$$

The resultant components of total force acting on the rigid body are then:

$$\boxed{\begin{aligned} m\left(\dot{U} - rV + qW\right) &= X \\ m\left(\dot{V} - pW + rU\right) &= Y \\ m\left(\dot{W} - qU + pV\right) &= Z \end{aligned}} \qquad (9)$$

m is the total mass of the body (aircraft). Note that eqn. (9) describes motion of the CG (need extra terms if origin $\neq$ CG).

Generalised moment equations

Consider the incremental moments produced by forces acting on the incremental mass δm at point $p(x,y,z)$ in the rigid body. Summing these over the body yields the moment equations.

For example, the total rolling moment, L, about the ox axis is:

$$\sum \delta m\left(y.a_z' - z.a_y'\right) = L \qquad (10)$$

Substituting a_y' and a_z' from eqns. (6) into (10) and recalling eqn. (8), eqn. (10) can be written as:

$$\left(\begin{array}{l} \dot{p}\sum \delta m\left(y^2 + z^2\right) + qr\sum \delta m\left(y^2 - z^2\right) + \left(r^2 - q^2\right)\sum \delta m.yz - \\ \left(pq + \dot{r}\right)\sum \delta m.xz + \left(pr - \dot{q}\right)\sum \delta m.xy \end{array}\right) = L \qquad (11)$$

Generalised moment equations

The terms in the summation signs in eqn. (11) are moments of inertia. We define:

$$\begin{aligned} I_{xx} &= \sum \delta m\left(y^2+z^2\right) & I_{xy} &= \sum \delta m.xy \\ I_{yy} &= \sum \delta m\left(x^2+z^2\right) & I_{xz} &= \sum \delta m.xz \\ I_{zz} &= \sum \delta m\left(x^2+y^2\right) & I_{yz} &= \sum \delta m.yz \end{aligned} \qquad (12)$$

Eqn. (11) can therefore be written as:

$$I_{xx}\dot{p}-\left(I_{yy}-I_{zz}\right)qr+I_{xy}\left(pr-\dot{q}\right)-I_{xz}\left(pq+\dot{r}\right)+I_{yz}\left(r^2-q^2\right)=L \qquad (13)$$

Generalised moment equations

Similarly, the total moments M and N about the oy and the oz axes are:

$$\begin{aligned} \sum \delta m\left(z.a'_x - x.a'_z\right) &= M \\ \sum \delta m\left(x.a'_y - y.a'_x\right) &= N \end{aligned} \tag{14}$$

Substituting a'_x, a'_y and a'_z from eqns. (6) into (14), recalling eqn. (8) and using the above inertia definitions, we get:

$$I_{yy}\dot{q} - \left(I_{xx} - I_{zz}\right)pr + I_{yz}\left(pq - \dot{r}\right) - I_{xz}\left(p^2 - r^2\right) - I_{xy}\left(qr + \dot{p}\right) = M \tag{15}$$

$$I_{zz}\dot{r} - \left(I_{xx} - I_{yy}\right)pq - I_{yz}\left(pr + \dot{q}\right) + I_{xz}\left(qr - \dot{p}\right) + I_{xy}\left(q^2 - p^2\right) = N \tag{16}$$

Generalised moment equations

These equations are the generalized rigid body moment equations for the rotational motion about the orthogonal axes through the CG, since the axes origin is at the CG.

In conventional fixed-wing aircraft, x-z is a plane of symmetry in terms of the mass distribution so that $I_{xy} = I_{yz} = 0$.

The moment equations (13), (15) and (16) then simplify to:

$$\boxed{\begin{aligned} I_{xx}\dot{p} - \left(I_{yy} - I_{zz}\right)qr - I_{xz}\left(pq + \dot{r}\right) &= L \\ I_{yy}\dot{q} - \left(I_{xx} - I_{zz}\right)pr - I_{xz}\left(p^2 - r^2\right) &= M \\ I_{zz}\dot{r} - \left(I_{xx} - I_{yy}\right)pq + I_{xz}\left(qr - \dot{p}\right) &= N \end{aligned}} \qquad (17)$$

Kinematic equations for rotation

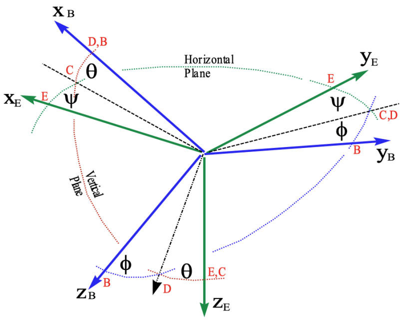
Recall the definition of the Euler angles in Lecture 2, where the aircraft body axes were related to the inertial axes via a sequence of rotations about non-orthogonal axes.

To go from inertial to body axes:

1. Rotate through ψ about z_E to intermediate frame F_C

2. Rotate through θ about y_c to intermediate frame F_D

3. Rotate through ϕ about x_B to body axes frame F_B.



Kinematic equations for rotation

We can find the relationship between body axes rates (p,q,r) and inertial axes rates $\left(\dot{\phi},\dot{\theta},\dot{\psi}\right)$ by carrying out the necessary trigonometric transformations according to the sequences referred to in the previous slide.

This yields:

$$\boxed{\begin{aligned}\dot{\phi} &= p + \tan\theta\left(q\sin\phi + r\cos\phi\right)\\ \dot{\theta} &= q\cos\phi - r\sin\phi\\ \dot{\psi} &= \left(q\sin\phi + r\cos\phi\right)/\cos\theta\end{aligned}} \qquad (18)$$

Kinematic equations for translation

We can also use the Euler angles to transform between body axes velocities (U, V, W) and inertial axes velocities $(\dot{x}_E, \dot{y}_E, \dot{z}_E)$.

These form the so-called navigation equations (not derived here):

$$\boxed{\begin{aligned}
\dot{x}_E &= U\cos\theta\cos\psi + V(\sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi) + \\
&\quad W(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi) \\
\dot{y}_E &= U\cos\theta\sin\psi + V(\sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi) + \\
&\quad W(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi) \\
\dot{z}_E &= -U\sin\theta + V\sin\phi\cos\theta + W\cos\phi\cos\theta
\end{aligned}} \tag{19}$$

6DOF Equations of Motion

Equations (9), (17), (18) and (19) form the 12 equations of motion for general atmospheric 6 degree-of-freedom flight.

They are all first-order differential equations that can be expressed in nonlinear state-space form as:

$$\dot{x} = f\left(x, \delta\right) \tag{20}$$

where $x = \left[U, V, W, p, q, r, \psi, \theta, \phi, x_E, y_E, z_E\right]'$ (state vector)

and δ is the vector of input parameters of interest. These are often aerodynamic or propulsion system inputs such as aileron, elevator, rudder deflection or thrust (upon which the formulation of the forces X, Y and Z will depend).

Coordinates and Forces

- The motions that are described are a consequence of the applied forces and moments from outside the body, being:

  - aerodynamic (including control forces and gusts)
  - propulsive (all additional forces due to propulsion)
  - gravitational

- The list above must be all inclusive, e.g. drag due lowered undercarriage.

- Aerodynamic forces are typically modelled using aerodynamic data tables.

Coordinates and Forces

- The three general forces X, Y, Z in the directions of the co-ordinate axes are taken to be forces other than of gravitational origin.
- When the CG is taken as the axes origin there are no gravity-induced moments so the three moments L, M, N also have no gravity terms present.
- For virtually all our work the propulsive force will be constant so the thrust F will often not appear; it will generally be in equilibrium with drag which does not usually appear either.
- Different forms of the equations will be used to address different problems.

Notation

- The steady velocities are U, V, W (upper case).
- Later, when we consider perturbations from the steady state condition, these short-term variations from steady values will be given the symbols u, v, w (lower case).
- We will sometimes consider flight with a forward speed U but with $V=W=0$, hence we will see only v and w in the perturbation equations, plus U and u.
- The steady angular velocities are written p, q, r.
- This is the same when considering perturbations from the steady state as we usually regard the steady state as having zero angular rate.

Notation

- You will have been used to expressions such as dynamic pressure $q = \frac{1}{2}\rho V^2$ (or $q = \frac{1}{2}\rho U^2$). Note, however, that q can also be pitch rate.
- We shall also run up against the problem of distinguishing between L=lift and L=rolling moment.
- Moments and products of inertia:

$$I_{xx} = \sum \delta m\left(y^2 + z^2\right) \qquad I_{xy} = \sum \delta m.xy$$

$$I_{yy} = \sum \delta m\left(x^2 + z^2\right) \qquad I_{xz} = \sum \delta m.xz$$

$$I_{zz} = \sum \delta m\left(x^2 + y^2\right) \qquad I_{yz} = \sum \delta m.yz$$

Learn the notation!

The Formal Equations

With the gravitational forces separated from the aerodynamic/ propulsive forces, the translational equations can be written as follows (allowing for non-zero values of *V, W*, and pitch angle θ:

- Fore/ Aft: $$m(\dot{U} - rV + qW) = X - mg\,\mathbf{sin}\,\theta$$
- Lateral: $$m(\dot{V} - pW + rU) = Y + mg\,\mathbf{cos}\,\theta\,\mathbf{sin}\,\phi$$
- Transverse: $$m(\dot{W} - qU + pV) = Z + mg\,\mathbf{cos}\,\theta\,\mathbf{cos}\,\phi$$

The Formal Equations

With the gravitational forces separated from the aerodynamic/ propulsive forces, the translational equations can be written as follows (allowing for non-zero values of V, W, and pitch angle θ:

- Fore/ Aft: $m(\dot{U} - rV + qW) = X - mg\sin\theta$
- Lateral: $m(\dot{V} - pW + rU) = Y + mg\cos\theta\sin\phi$
- Transverse: $m(\dot{W} - qU + pV) = Z + mg\cos\theta\cos\phi$

The Formal Equations

- Similarly, the rotational equations are expressed in a their complete form as:

Roll: $$I_{xx}\dot{p} - (I_{yy} - I_{zz})qr - I_{yz}(q^2 - r^2) - I_{zx}(\dot{r} + pq) - I_{xy}(\dot{q} - rp) = L$$

Pitch: $$I_{yy}\dot{q} - (I_{zz} - I_{xx})rp - I_{zx}(r^2 - p^2) - I_{xy}(\dot{p} + qr) - I_{yz}(\dot{r} - pq) \quad = M$$

Yaw: $$I_{zz}\dot{r} - (I_{xx} - I_{yy})pq - I_{xy}(p^2 - q^2) - I_{yz}(\dot{q} + rp) - I_{xz}(\dot{p} - qr) = N$$

(You are not expected to remember these *– see Cook for further reading*). Note: *Product terms & cross inertias.*

The Formal Equations

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Roll: $$I_{xx}\dot{p} - (I_{yy} - I_{zz})qr - I_{yz}(q^2 - r^2) - I_{zx}(\dot{r} + pq) - I_{xy}(\dot{q} - rp) = L$$

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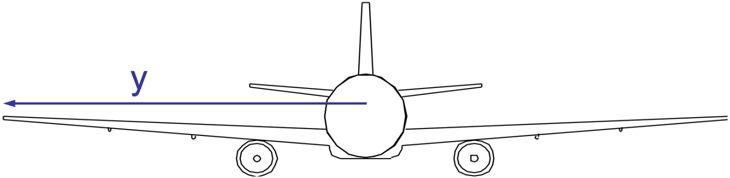
Yaw: $$I_{zz}\dot{r} - (I_{xx} - I_{yy})pq - I_{xy}(p^2 - q^2) - I_{yz}(\dot{q} + rp) - I_{xz}(\dot{p} - qr) = N$$

(You are not expected to remember these – *see Cook for further reading*). Note: *Product terms & cross inertias.*

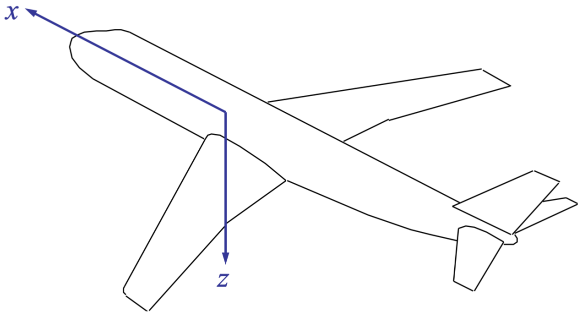
Reminder: the cross-inertias

- As mentioned previously, we usually adopt a simpler form of the equations on the previous slide because we assume the aircraft to have an inertial plane of symmetry. This results in I_{xy} and I_{zy} being zero, yielding eqn. (17).
- An interpretation of this is that for any dimension x away from the origin (fore/aft) there are equal masses at $\pm y$ which cancel in the inertia calculations.



Reminder: the cross-inertias

- Similarly, for a choice of some fixed position z away from the origin (vertically) there will be equal masses at $\pm y$.
- An obvious candidate that prevents I_{zx} being zero is the tail (fin): at aft positions x we do not have equal masses at $\pm z$ as there is no fin below the fuselage.



Alternative Form of the Equations

TABLE 2.4-1. The Flat-Earth, Body Axes 6-DOF Equations

Force Equations

$$\begin{aligned}\dot{U} &= RV - QW - g_0' \sin\theta + \frac{F_x}{m}\\ \dot{V} &= -RU + PW + g_0' \sin\phi\cos\theta + \frac{F_y}{m}\\ \dot{W} &= QU - PV + g_0' \cos\phi\cos\theta + \frac{F_z}{m}\end{aligned} \qquad (2.4\text{-}2)$$

Kinematic Equations

$$\begin{aligned}\dot{\phi} &= P + \tan\theta(Q\sin\phi + R\cos\phi)\\ \dot{\theta} &= Q\cos\phi - R\sin\phi\\ \dot{\psi} &= \frac{Q\sin\phi + R\cos\phi}{\cos\theta}\end{aligned} \qquad (2.4\text{-}3)$$

Reference: Aircraft Control & Simulation, Stevens & Lewis, Wiley (First Edition!)

Alternative Form of the Equations

Moment Equations

$$\dot{P} = (c_1 R + c_2 P)Q + c_3 \overline{L} + c_4 N$$
$$\dot{Q} = c_5 PR - c_6(P^2 - R^2) + c_7 M \qquad (2.4\text{-}4)$$
$$\dot{R} = (c_8 P - c_2 R)Q + c_4 \overline{L} + c_9 N$$

Navigation Equations

$$\dot{p}_N = U\cos\theta\cos\psi + V(-\cos\phi\sin\psi + \sin\phi\sin\theta\cos\psi) + W(\sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi)$$
$$\dot{p}_E = U\cos\theta\sin\psi + V(\cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi) + W(-\sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi) \qquad (2.4\text{-}5)$$
$$\dot{h} = U\sin\theta - V\sin\phi\cos\theta - W\cos\phi\cos\theta$$

Reference: *Aircraft Control & Simulation*, Stevens & Lewis, *Wiley (First Edition!)*

Alternative Form of the Equations - Mass Properties

$$
\begin{aligned}
\Gamma c_1 &= (J_y - J_z)J_z - J_{xz}^2, & \Gamma c_2 &= (J_x - J_y + J_z)J_{xz} \\
\Gamma c_3 &= J_z, & \Gamma c_4 &= J_{xz} \\
c_5 &= \frac{J_z - J_x}{J_y}, & c_6 &= \frac{J_{xz}}{J_y} \\
c_7 &= \frac{1}{J_y}, & \Gamma c_8 &= J_x(J_x - J_y) + J_{xz}^2, \\
\Gamma c_9 &= J_x,
\end{aligned}
\tag{2.4-6}
$$

where

$$\Gamma = J_x J_z - J_{xz}^2 \qquad [\text{as in } (1.3\text{-}19b)].$$

Reference: Aircraft Control & Simulation, Stevens & Lewis, Wiley (First Edition!)

Alternative Form of the Equations

$$\dot{V}_T = \frac{U\dot{U} + V\dot{V} + W\dot{W}}{V_T}$$

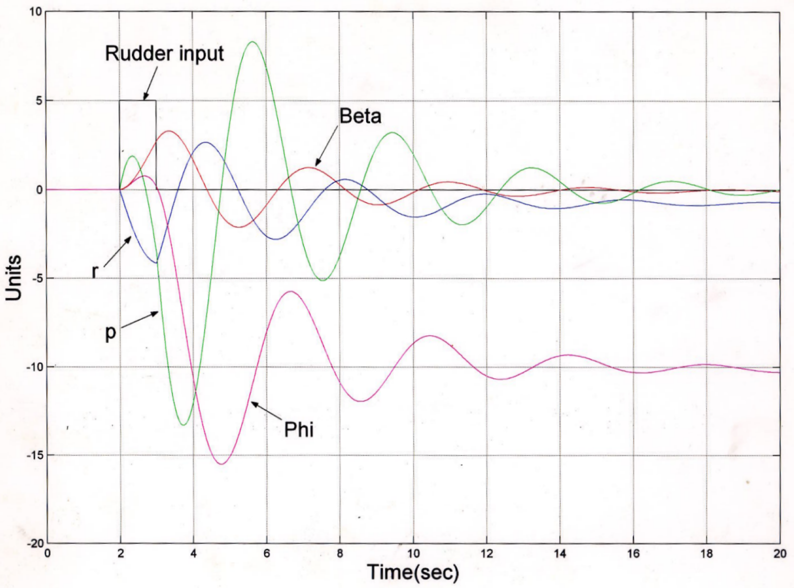
$$\dot{\beta} = \frac{\dot{V}V_T - V\dot{V}_T}{V_T^2 \cos\beta} \tag{2.4-8}$$

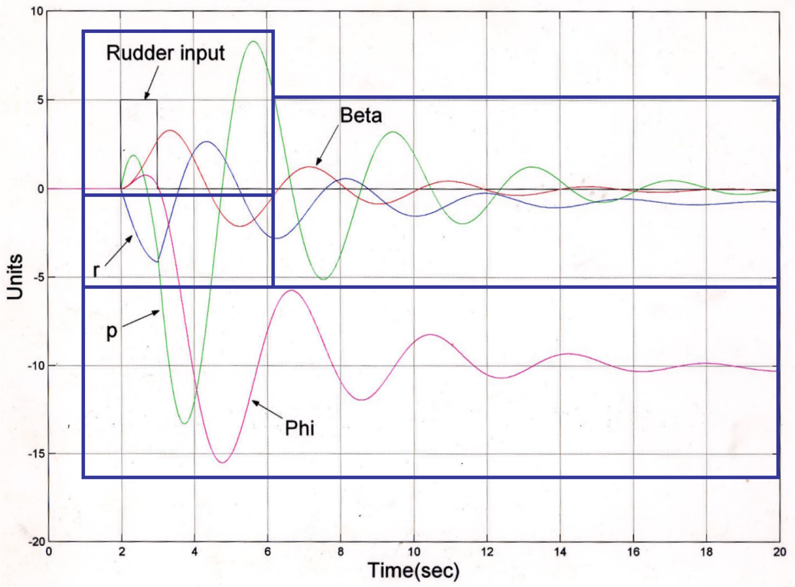
$$\dot{\alpha} = \frac{U\dot{W} - W\dot{U}}{U^2 + W^2}.$$

The new state vector is

$$X^{\mathrm{T}} = [V_T \quad \beta \quad \alpha \quad \phi \quad \theta \quad \psi \quad P \quad Q \quad R \quad p_N \quad p_E \quad h]. \tag{2.4-9}$$

Reference: Aircraft Control & Simulation, Stevens & Lewis, Wiley (First Edition!)


Rudder input
Beta
r
p
Phi
Units
Time(sec)
10
5
0
-5
-10
-15
-20
0
2
4
6
8
10
12
14
16
18
20


Rudder input
Beta
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10
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-10
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-20
0
2
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6
8
10
12
14
16
18
20

Coupled Motion

- In general, the motions are coupled and, for example, an impulsive force (moment) applied in roll will eventually disturb the motions in all 6 degrees of freedom and there will be resultant non-zero values for all 6 motion variables.

- However, for many applications it is standard practice, and sufficient, to separate the equations into two de-coupled sets of three freedoms each to provide what is called:

  - the longitudinal equations in U, W, q

  - the lateral-directional equations in V, p, r.

Coupled Motion

- In the form quoted above, the equations are not yet linearised and they show for example, products of the variables. However, after linearisation we shall find that the form of the full set of six can be displayed as follows:

$$\left[\begin{array}{c:c} Translational & equations \\ in upper & portion \\ \hdashline Rotational & equations \\ in lower & portion \end{array}\right]\left[\begin{array}{c} u \\ v \\ w \\ \hdashline p \\ q \\ r \end{array}\right] \; or \; \left[\begin{array}{c} x \\ y \\ z \\ \hdashline \phi \\ \theta \\ \psi \end{array}\right] = \left[\begin{array}{c} Fwd.force \\ Lateral\ force \\ Transv.force \\ \hdashline Roll.mom. \\ Pitch.mom. \\ Yaw.mom. \end{array}\right]$$

Coupled Motion

- In this form, all four partitions of the LHS matrix will have non-zero terms. If we change the order of the forces and the order of the variables to allow for the lateral/longitudinal split, namely to display

  - **longitudinal actions**: forces and motions which are within the plane of symmetry, and

  - **lateral-dirctional actions**: forces which act out of the plane of symmetry and consequent motions of that plane away from its normal vertical position,

  we can re-form the equations to display them as:

Coupled Motion

$$\left[\begin{array}{c:c} \begin{array}{c}\textit{Longitudinal}\\ \textit{equations}\end{array} & \\ \hdashline & \begin{array}{c}\textit{Lateral}\\ \textit{equations}\end{array} \end{array}\right]\left[\begin{array}{c} u\\ w\\ q\\ \hdashline v\\ p\\ r \end{array}\right] = \left[\begin{array}{c} \textit{Fwd. force}\\ \textit{Transv. force}\\ \textit{Pitch. mom.}\\ \hdashline \textit{Lat. force}\\ \textit{Roll. mom.}\\ \textit{Yaw. mom.} \end{array}\right]$$

both of these partitions are nominally null

Next Lecture

Moments of Inertia