LECTURE 2

Aircraft Propulsion
AENG 31102

Aircraft Gas Turbine Performance & Design

Recap on Fundamentals

Objectives ~ Lecture 2

- To describe the International Standard Atmosphere and its significance.
- To show the ideal efficiency of the Joule or Brayton Cycle.
- To calculate the main characteristics of a practical turbojet.

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International Standard Atmosphere ISA
Gravity

- The term "acceleration due to gravity" is more correctly "the acceleration in free fall" due to the combined effects of gravitational attraction and the Earth's rotation, which varies with latitude.

- Variations with latitude are inconvenient for comparing aircraft and engine performance and a standard value at sea level e.g. 9.80665 m/s^2 is used.

- For many calculations, it is convenient to keep the sea level value constant with altitude, by defining geopotential altitude (see next slide).

International Standard Atmosphere ISA
Altitudes

- Geometric height Z: The actual height above mean-sea-level.

- Geopotential height H: The height in a uniform gravitational field *(g constant with altitude)* which gives the same potential energy as exists in the actual, variable gravitational field.

- Pressure height: Aircraft normally fly at altitudes defined by barometric means. The pressure height in any atmosphere is the geopotential height in the standard atmosphere giving the same pressure.

International Standard Atmosphere ISA

- The International Standard Atmosphere is based on an idealised, mean-annual, steady-state model, assuming a period of moderate solar activity at a latitude of 45° N.

Non-standard atmospheres:

- Both temperature and pressure are independent functions of geopotential height.

- Thus ISA + 10C may be obtained by adding 10C to the ISA temperatures with the pressures remaining constant.

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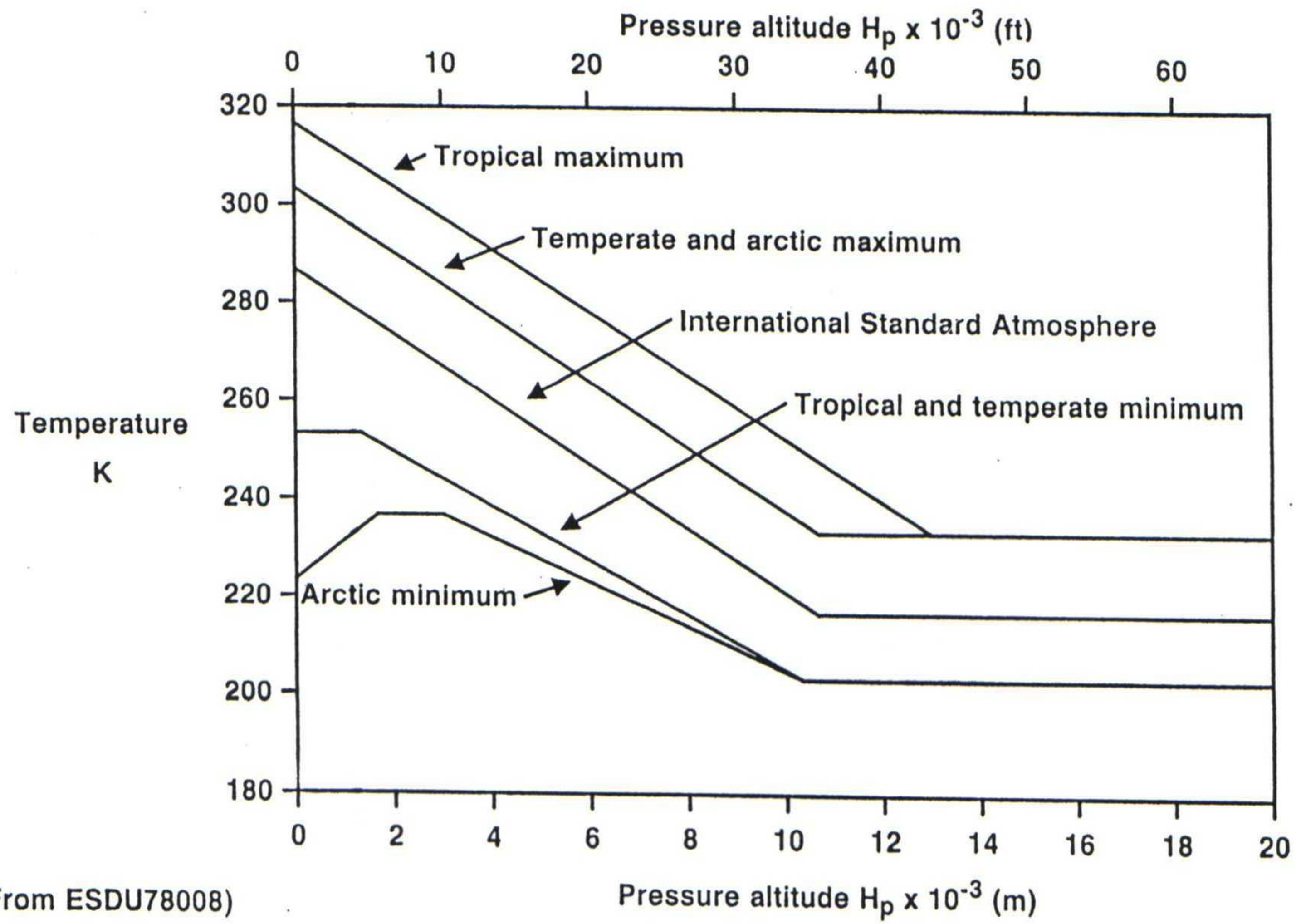
International Standard Atmosphere ISA

Standard Atmosphere & Non Standard Atmosphere



(From ESDU78008)

International Standard Atmosphere

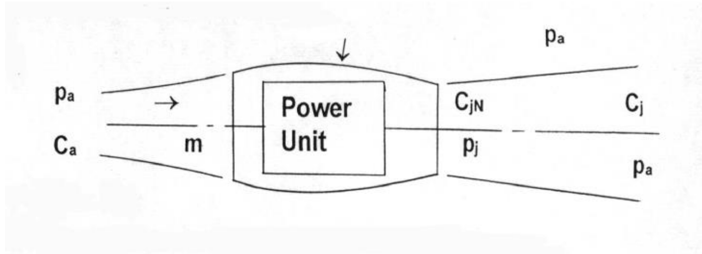
International Standard Atmosphere

z [m]	p [bar]	T [K]	ρ/ρ_0	a [m/s]
0	1·01325	288·15	1·0000	340·3
500	0·9546	284·9	0·9529	338·4
1 000	0·8988	281·7	0·9075	336·4
1 500	0·8456	278·4	0·8638	334·5
2 000	0·7950	275·2	0·8217	332·5
2 500	0·7469	271·9	0·7812	330·6
3 000	0·7012	268·7	0·7423	328·6
3 500	0·6578	265·4	0·7048	326·6
4 000	0·6166	262·2	0·6689	324·6
4 500	0·5775	258·9	0·6343	322·6
5 000	0·5405	255·7	0·6012	320·5
5 500	0·5054	252·4	0·5694	318·5
6 000	0·4722	249·2	0·5389	316·5
6 500	0·4408	245·9	0·5096	314·4
7 000	0·4111	242·7	0·4817	312·3
7 500	0·3830	239·5	0·4549	310·2
8 000	0·3565	236·2	0·4292	308·1
8 500	0·3315	233·0	0·4047	306·0
9 000	0·3080	229·7	0·3813	303·8
9 500	0·2858	226·5	0·3589	301·7
10 000	0·2650	223·3	0·3376	299·5

z [m]	p [bar]	T [K]	ρ/ρ_0	a [m/s]
10 500	0·2454	220·0	0·3172	297·4
11 000	0·2270	216·8	0·2978	295·2
11 500	0·2098	216·7	0·2755	295·1
12 000	0·1940	216·7	0·2546	295·1
12 500	0·1793	216·7	0·2354	295·1
13 000	0·1658	216·7	0·2176	295·1
13 500	0·1533	216·7	0·2012	295·1
14 000	0·1417	216·7	0·1860	295·1
14 500	0·1310	216·7	0·1720	295·1
15 000	0·1211	216·7	0·1590	295·1
15 500	0·1120	216·7	0·1470	295·1
16 000	0·1035	216·7	0·1359	295·1
16 500	0·09572	216·7	0·1256	295·1
17 000	0·08850	216·7	0·1162	295·1
17 500	0·08182	216·7	0·1074	295·1
18 000	0·07565	216·7	0·09930	295·1
18 500	0·06995	216·7	0·09182	295·1
19 000	0·06467	216·7	0·08489	295·1
19 500	0·05980	216·7	0·07850	295·1
20 000	0·05529	216·7	0·07258	295·1

Density at sea level $\rho_0 = 1{\cdot}2250\ \mathrm{kg/m^3}$.
Extracted from: ROGERS G F C and MAYHEW Y R *Thermodynamic and Transport Properties of Fluids* (Blackwell 1995)

THRUST & PROPULSIVE EFFICIENCY ~ 1



THRUST is equal to rate of change of momentum:

$$\boldsymbol{F} = \dot{m}(C_j - C_a) = \dot{m}(C_{jN} - C_a) + A_j\,(P_j - Pa)$$

Fuel flow $= \dot{m}_f$ ***Net calorific value of fuel*** $= Q_{net}$

Specific Fuel Consumption is equal to the *fuel flow / unit of thrust:*

$$\boldsymbol{SFC} = \frac{\dot{m}_f}{F}$$

OVERALL EFFICIENCY is equal to useful work / energy supplied by fuel: $$\boldsymbol{\eta_{overall}} = \frac{F \cdot C_a}{\dot{m}_f \cdot Q_{net}}$$

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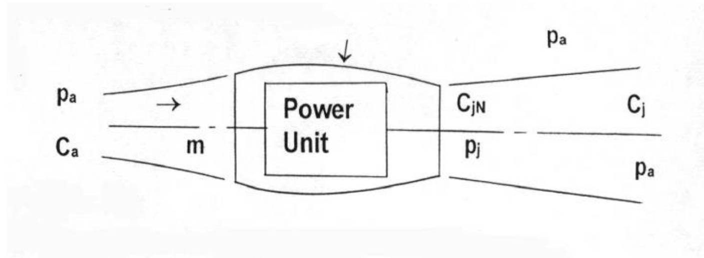
THRUST & PROPULSIVE EFFICIENCY ~ 2



Efficiency of Energy Conversion is equal to useful mechanical energy / energy supplied by fuel:

$$\eta_e = \frac{1}{2}\dot{m}\,\frac{(Cj^2 - Ca^2)}{Q_{net}\cdot\dot{m}_f}$$

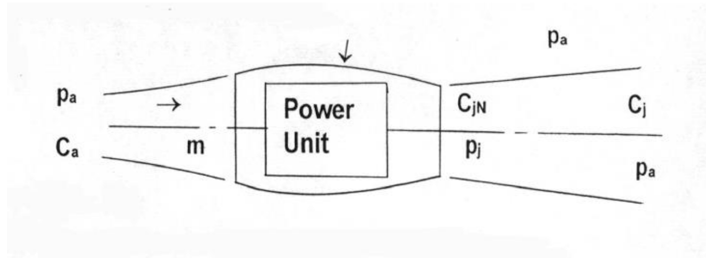
Propulsive (Froude) Efficiency is equal to useful work / useful work + unused KE in jet:

$$\eta_p = \frac{F\cdot C_a}{F\cdot C_a + \frac{1}{2}\dot{m}(C_j - C_a)^2} = \frac{2}{(1 + C_j/C_a)}$$

OVERALL EFFICIENCY is the product of efficiency of energy conversion and propulsive efficiency:

$$\eta_{overall} = \eta_e \cdot \eta_p$$

THRUST & PROPULSIVE EFFICIENCY ~ 3



Overall Efficiency: $$\eta_{overall} = \frac{F \cdot C_a}{\dot{m}_f \cdot Q_{net}}$$

Specific Fuel Consumption: $SFC = \frac{\dot{m}_f}{F}$

Hence overall efficiency (for a given fuel) ∝ flight speed / SFC:

$$\eta_{overall} = \frac{C_a}{SFC \cdot Q_{net}}$$

That means that SFC is a measure of overall efficiency

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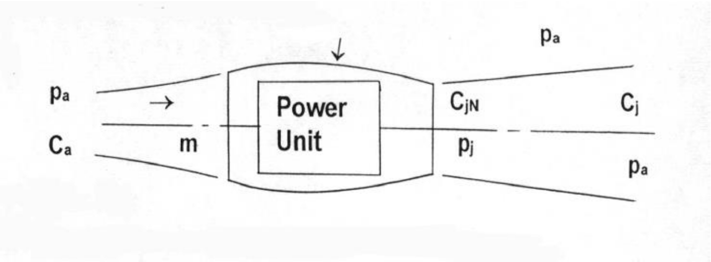
THRUST & PROPULSIVE EFFICIENCY ~ 4



Fuel Air Ratio (FAR):

$$\boldsymbol{FAR} = \frac{fuel\ flow}{air\ flow} = \frac{\dot{m}_f}{\dot{m}}$$

Specific Thrust (ST) is equal to Thrust / unit Mass Flow:

$$\boldsymbol{ST} = \frac{F}{\dot{m}} = (C_j - C_a)$$

That means that:

$$\boldsymbol{SFC} = \frac{FAR}{ST} = \frac{\dot{m}_f}{F}$$

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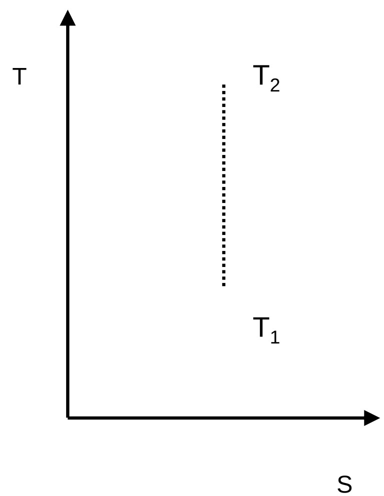
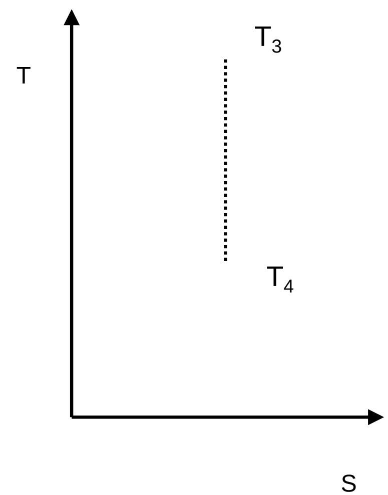
Isentropic Compression and Expansion



Compression

Increase in entropy due to temperature rise exactly balances decrease due to volume effect.

In an isentropic compression from state 1 to state 2 increases in entropy are exactly balanced by decreases due to the volume effect.



Expansion

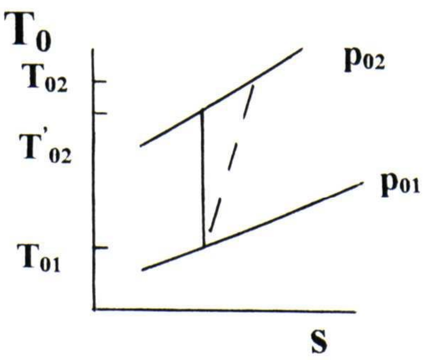
Decrease in entropy due to temperature effect exactly balances increase in entropy due to volume effect

In an isentropic expansion from state 3 to state 4 decreases in entropy are exactly balanced by increases due to the volume effect.

Basic Relationships ~ 4

Isentropic efficiency* of (say) a compressor:

$$\eta_{isen} = \frac{Isentropic\ Total\ Temperature\ rise}{Actual\ Total\ Temperature\ rise} = \frac{T'_{o2} - T_{o1}}{T_{o2} - T_{o1}}$$



Total Temperature: $T_0 = T + \frac{C^2}{2C_p}$

$$T_0 = T(1 + ½\,(\gamma - 1)M^2)$$

Sonic Velocity: $a = \sqrt{\gamma RT}$ Mach Number: $\mathbf{M} = \frac{C}{\sqrt{\gamma RT}}$

Isentropic Pressure - Temperature Relationship:

$$\frac{T_{02}}{T_{01}} = \left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}$$

Note there is another way of defining the efficiency of a process: Polytropic Efficiency ~ see lecture on Compressors:

$$\frac{T_{02}}{T_{01}} = \left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma\cdot\eta_{poly}}}$$

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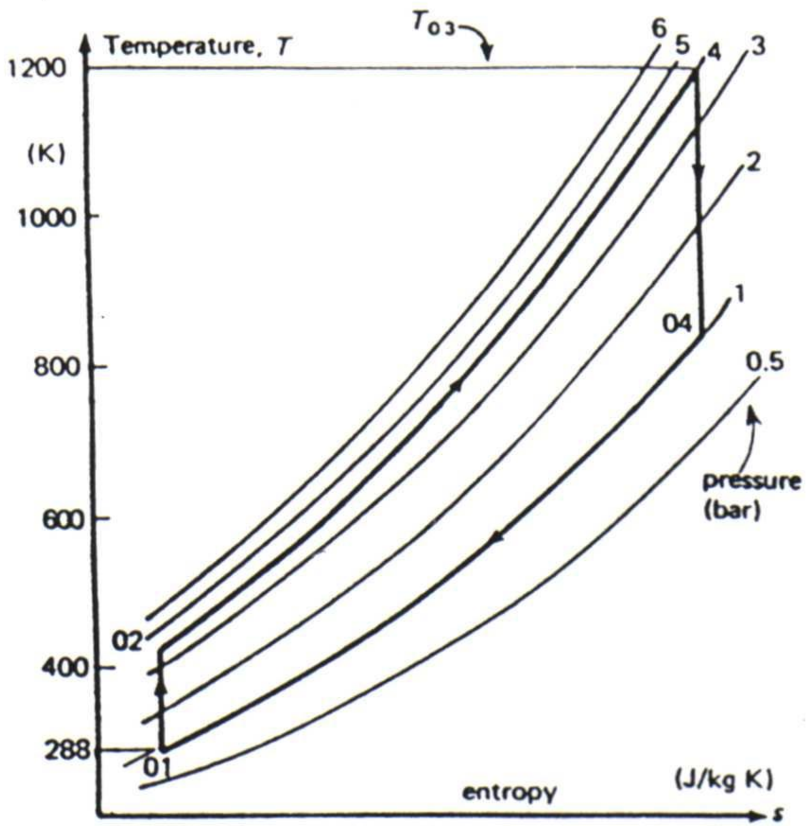
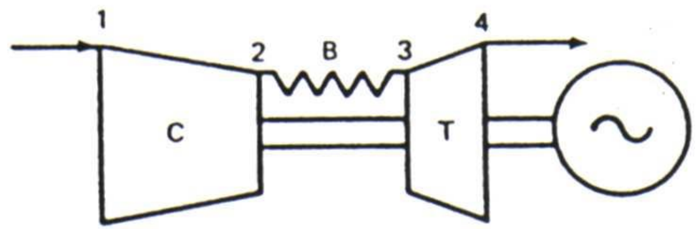
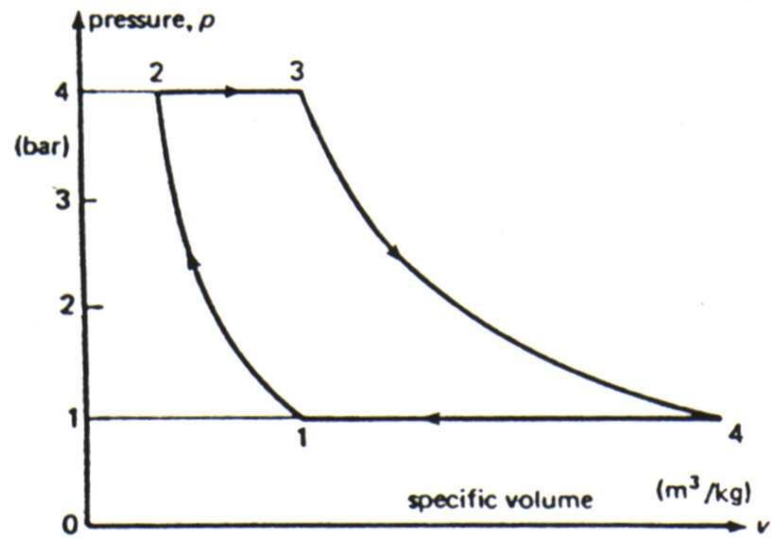
Joule or Brayton Cycle for a Gas Turbine







$$\eta = 1 - \{1/(p_2/p_1)\}^{\gamma - 1/\gamma}$$

Derivation of Ideal Efficiency

Assumptions for Ideal Cycle for gas turbine (Joule or Brayton):

- Compression & expansion processes are reversible adiabatic i.e. isentropic;
- Change in K E between inlet & outlet of each component is negligible;
- Pressure losses are negligible;
- Working fluid is a perfect gas with constant specific heats;
- Mass flow constant throughout the cycle;
- Heat transfer i.e. combustion is complete.

Steady Flow Energy Equation (per unit mass):

$$\boldsymbol{Q} = (h_2 - h_1) + 1/2\ (C_2{}^2 - C_1{}^2) + W$$

Hence:

$$\boldsymbol{W_{12}} = -(h_2 - h_1) = -C_p\,(T_2 - T_1)$$

$$\boldsymbol{Q_{23}} = (h_3 - h_2) = C_p\,(T_3 - T_2)$$

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$$\boldsymbol{W_{34}} = (h_3 - h_4) = C_p\,(T_3 - T_4)$$

Derivation of Ideal Efficiency

Cycle efficiency equals net work output / heat supplied:

$$\boldsymbol{\eta} = \frac{C_p\,[(T_3 - T_4) - (T_2 - T_1)]}{C_p\,(T_3 - T_2)}$$

Using the isentropic Pressure & Temperature relationship:

$$\frac{T_{02}}{T_{01}} = \left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_3}{T_4}$$

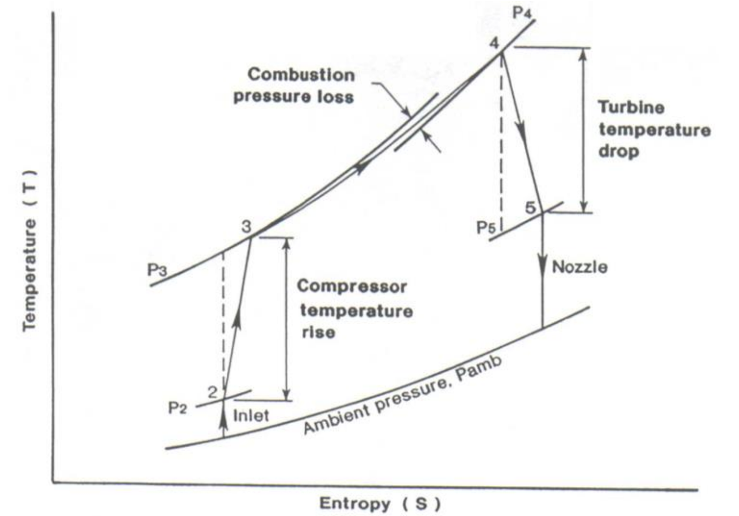
$$\eta = 1 - \left[\frac{1}{\frac{P_2}{P_1}}\right]^{\frac{\gamma-1}{\gamma}}$$

Thus the efficiency depends only on the pressure ratio and the nature of the gas.

The Practical Turbojet Cycle

Ambient Conditions:

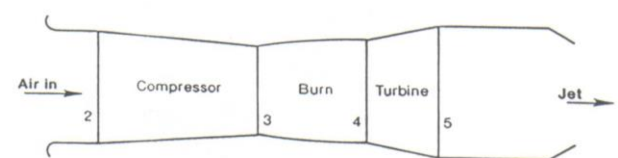
$\boldsymbol{M} = 0.8$ @ $\boldsymbol{h} = 10km$ ISA Conditions
Pressure $\boldsymbol{P_a} = 0.265\ bar$
Temperature $\boldsymbol{T_a} = 223.3K$
Speed of Sound $\boldsymbol{a} = 299.5\ m/s$
Flight Speed $\boldsymbol{C_a} = 239.6\ m/s$



Cycle Details:

No Intake Loss: $\frac{\boldsymbol{P_{o2}}}{\boldsymbol{P_{o1}}} = 1$

Compression Ratio of 8: $\frac{\boldsymbol{P_{o3}}}{\boldsymbol{P_{o2}}} = 8$



With an Isentropic Efficiency of 0.87: $\boldsymbol{\eta_{isen}} = 0.87$ (compressor)

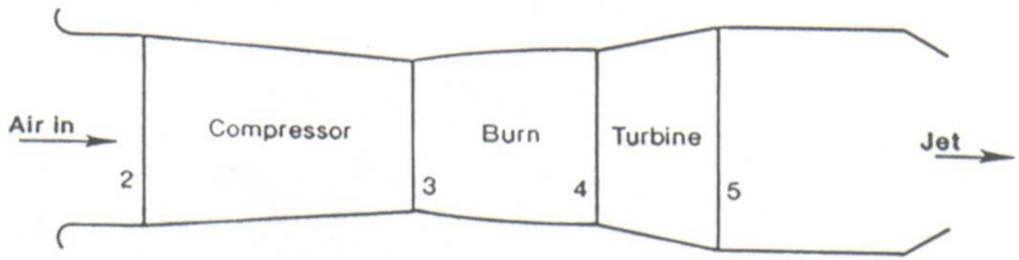
Combustion Chamber Temperature: $\mathbf{T_{o4}} = 1200K$

Turbine Isentropic Efficiency of 0.9: $\boldsymbol{\eta_{isen}} = 0.9$ (turbine)

Combustion Pressure Loss is 4%: $\frac{\boldsymbol{P_{o4}}}{\boldsymbol{P_{o3}}} = 0.96$

Transmission Efficiency is 99%: $\boldsymbol{\eta_{transmission}} = 0.99$

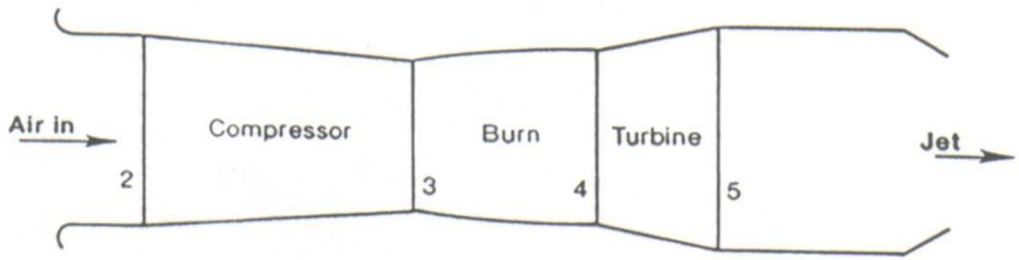
The Practical Turbojet Cycle



- **<u>Step 1: Total Temperature & Pressure at entry</u>**
  - We find the stagnation properties at the fan:
  - T_{o1} & P_{o1}

- **<u>Step 1-2: Intake</u>**
  - For this example, there is no loss at the intake, and hence:
  - $P_{o2} = P_{o1}$ & $T_{o2} = T_{o1}$

The Practical Turbojet Cycle



- **Step 2-3: Compression**
  - We calculate P_{o3}, the Isentropic Temperature Rise T'_{o3} and the Real temperature Rise T_{o3}
  - We then calculate the Specific Power required to drive the compressor $\frac{P_{ow}}{\dot{m}}$

- **Step 3-4: Combustion – Heat Addition**
  - We calculate P_{o4}

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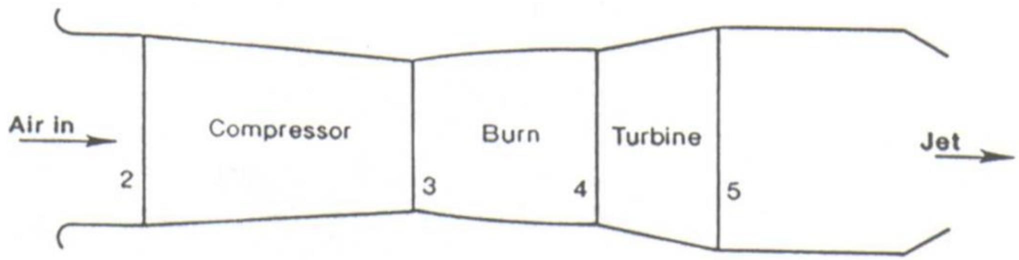
The Practical Turbojet Cycle



- <u>Step 4-5: Expansion through turbine</u>
  - Using the transmission efficiency, we can find how much power the turbine is producing to drive the compressor, and so we can find T_{o5}, T'_{05} and P_{05}

- <u>Step 5A: Fully expanded in the Ideal Con-Di Nozzle</u>
  - Since there are no losses, $T_{o5} = T_{oN}$ & $P_{o5} = P_{oN}$.
  - We calculate the temperature of the fully expanded jet T_{FE}, and the velocity of it C_{FE}
  - We can then calculate the Specific Thrust ST.

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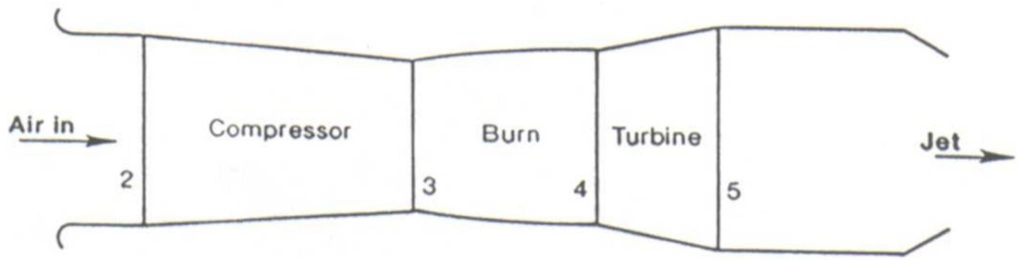
The Practical Turbojet Cycle



- **Step 5B: Expansion in a Convergent Nozzle**
  - Since there are no losses, $T_{o5} = T_{oN}$ & $P_{o5} = P_{oN}$.
  - We need to check if the nozzle is choked. That will happen when $\frac{\mathbf{P}_{oN}}{\mathbf{P}_a} > \frac{\mathbf{P}_{oN}}{\mathbf{P}_{N^*}}$
  - Since nozzle is choked, the exit conditions of the jet will be those at $M = 1$: $\boldsymbol{T_{N^*}}$, & $\boldsymbol{P_{N^*}}$
  - We calculate the speed of sound at the exit temperature C_{N^*}, and we also find the density of the flow $\boldsymbol{\rho_{N^*}}$
  - We can now calculate the ratio of jet area A_N to mass flow $\dot{m}$, and once we know that we can find the Specific Thrust ST

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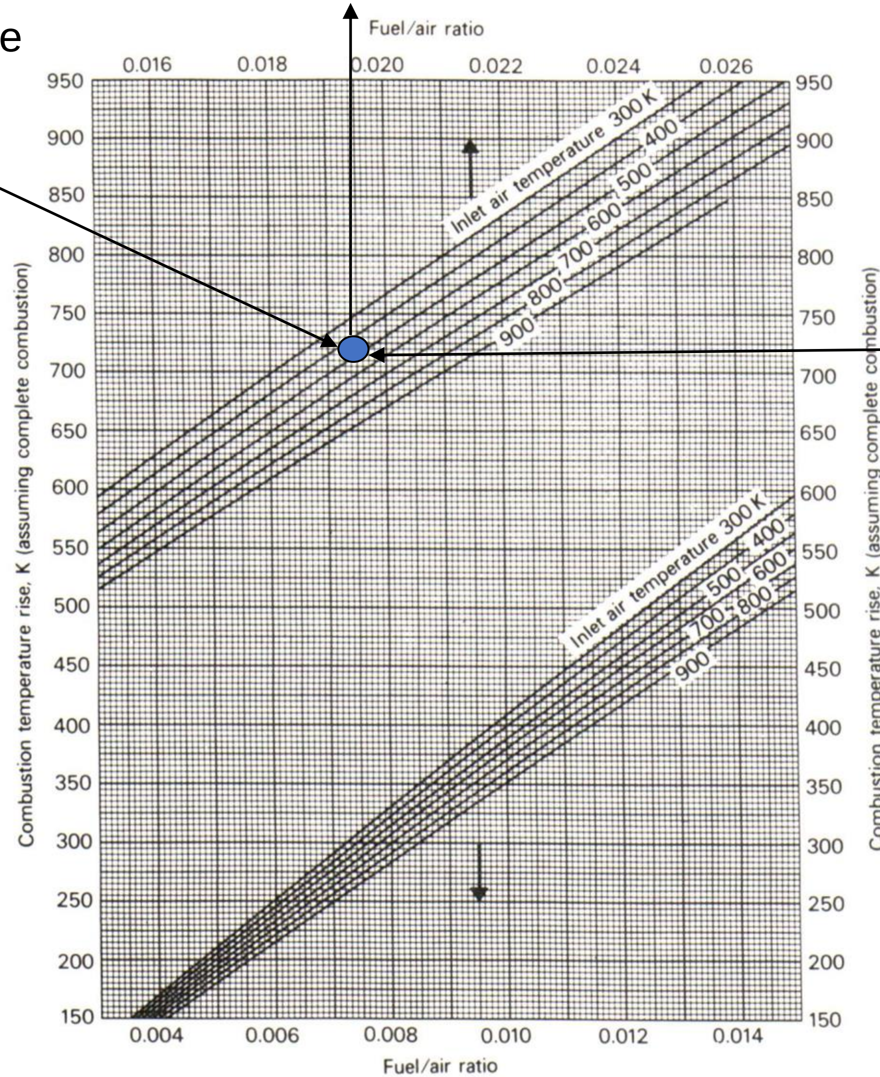
Combustion ~ Heat addition

Fuel Air Ratio = 0.0194

Inlet Air temperature
T_{03} = 487K

Combustion Temperature Rise
$(T_{04} - T_{03})$ = 713K



SPECIFIC THRUST CALCULATION

Thrust = $m(C_N - C_a) + A_N(P_N - P_a)$

Specific Thrust = Thrust/Mass Flow

SPECIFIC FUEL CONSUMPTION

Fuel Air Ratio *(from Chart of Combustion Temperature Rise v Fuel Air Ratio)*

Inlet Air temperature = T_{03}

Combustion Temperature Rise = $(T_{04} - T_{03})$

Fuel Air Ratio (from Chart)

Fuel Flow = Fuel Air Ratio x Engine Mass Flow

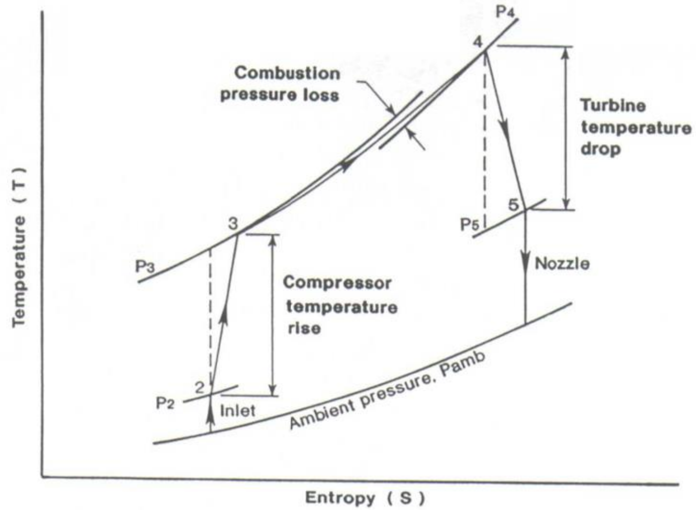
Specific Fuel Consumption SFC = Fuel Flow / Thrust

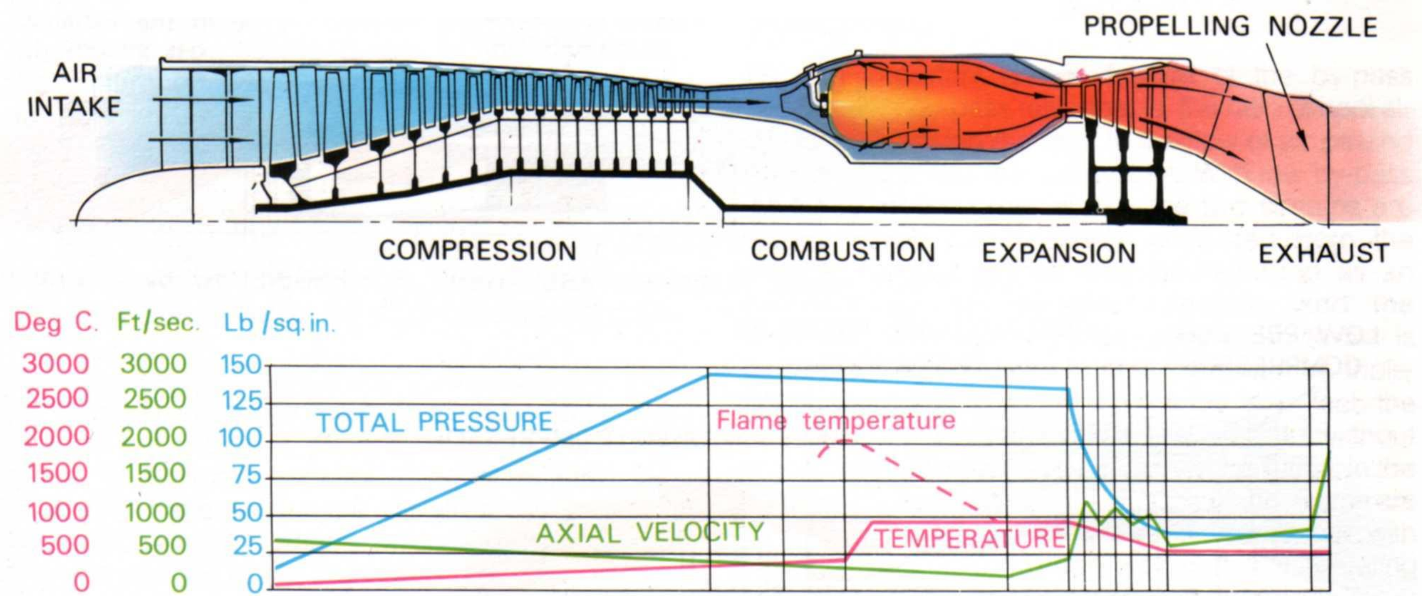
= Fuel Air ratio / Specific Thrust

Overall Efficiency = $C_a/SFC \times Q_{net}$

A detailed step by step calculation can be found on Blackboard.

The Practical Turbojet Cycle





Key Points from Lecture 2

- Relevance of the International Standard Atmosphere
- Basic Definitions of Thrust, Efficiency & Fuel Consumption
- Review of basic thermodynamic relationships for use in cycle calculations
- The method for the calculation of the performance of a simple turbojet

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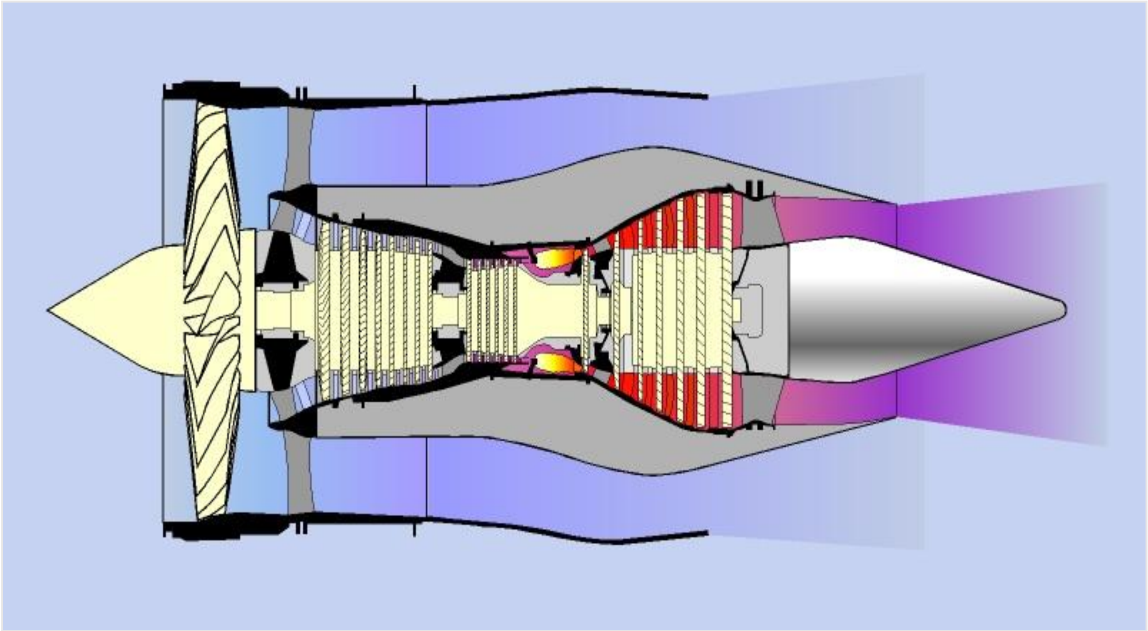
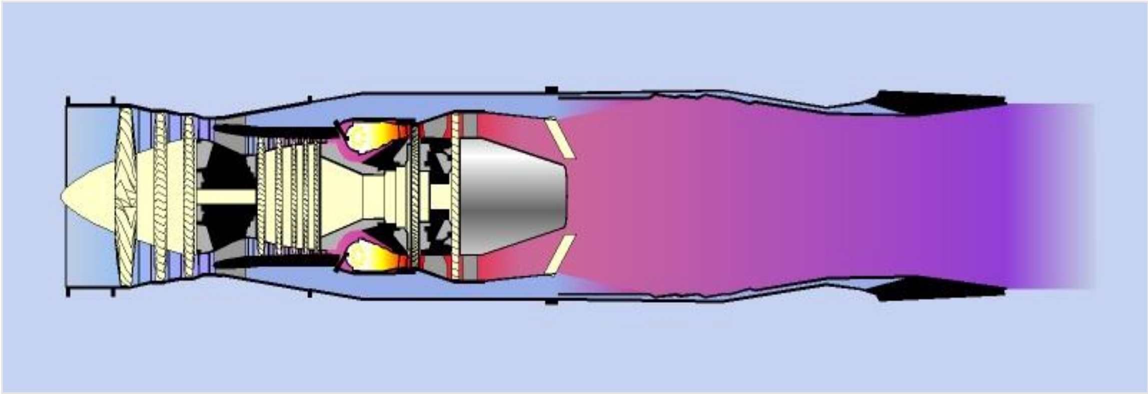
Different Turbofan Types

Civil Turbofan~ Trent



Military Turbofan ~ EJ200



Lecture 3

Design Point & Off-Design Performance

Objective ~ Lecture 3

To outline the way that the performance of a propulsion system can be characterised