Lecture 5 - Strip Theory Continued

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February 12, 2019

Effect of a Control Surface Deflection

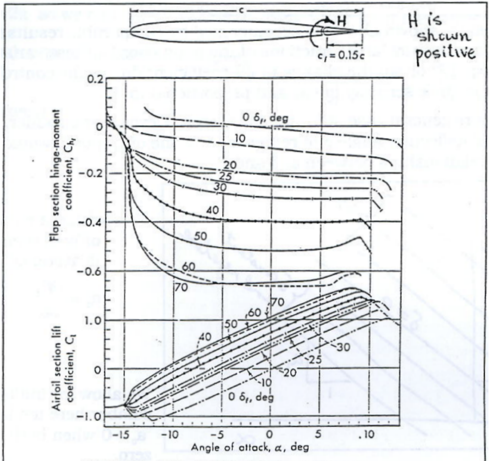
- Control surface deflection can give non-zero lift, even when the primary surface is still at zero incidence. On the following slide:

- The lower set of curves for the lift coefficient show successive increases in C_L at $\alpha=0$ as the surface deflection δ increases.

- The upper set of curves shows successively greater negative hinge moments as δ increases, again at $\alpha=0$ for example.

- Over the range of incidences $-5^{o}<\alpha<5^{o}$, both sets of curves appear linear with α. What we need to do in addition is cross-plot from the lower set, keeping α = constant = 0^{o} and see how the section lift is influence by δ alone.

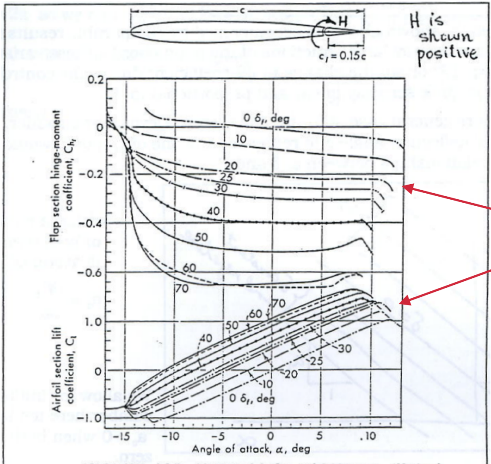
Effect of a Control Surface Deflection



NACA 0009 airfoil, with 0.15c plain flap and 0.005c gap. (*National Advisory Committee for Aeronautics.*)

- Data is given here for a control surface that could be a plain flap or perhaps an aileron with a chord (aft of the hinge) which is 15% of the full aerofoil chord.

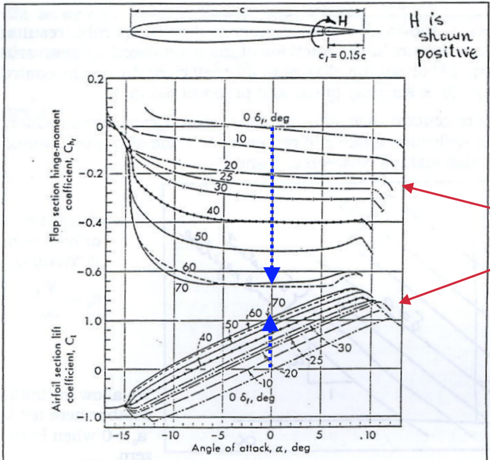
Effect of a Control Surface Deflection



NACA 0009 airfoil, with 0.15c plain flap and 0.005c gap. *(National Advisory Committee for Aeronautics.)*

- Data is given here for a control surface that could be a plain flap or perhaps an aileron with a chord (aft of the hinge) which is 15% of the full aerofoil chord.
  - Hinge Moment
  - Lift Coefficient
  - Consider a change in incidence α

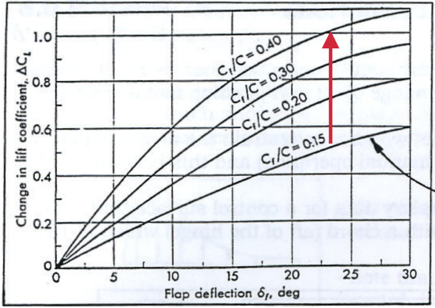
Effect of a Control Surface Deflection



NACA 0009 airfoil, with 0.15c plain flap and 0.005c gap. (*National Advisory Committee for Aeronautics.*)

- Data is given here for a control surface that could be a plain flap or perhaps an aileron with a chord (aft of the hinge) which is 15% of the full aerofoil chord.
  - Hinge Moment
  - Lift Coefficient
  - Consider a change in incidence α
  - Consider a flap deflection – note symmetric aerofoil

Effect of a Control Surface Deflection

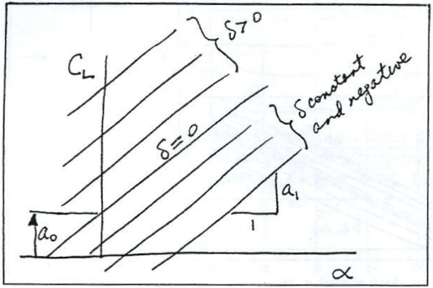


- only this one is shown on the previous page

- Results are shown above for control surfaces of increasing fractions of the main chord.
- These variations show that for $|\delta|<15^o$ or so, the change in lift coefficient due to *relatively* small control surface deflections appears linear and is proportional to δ.

Effect of a Control Surface Deflection

- The more general approximation, allowing for aileron, elevator and rudder, and using the general deflection angle δ to represent any one of ξ, η or ζ, would show the following relationships between α, δ and C_L.



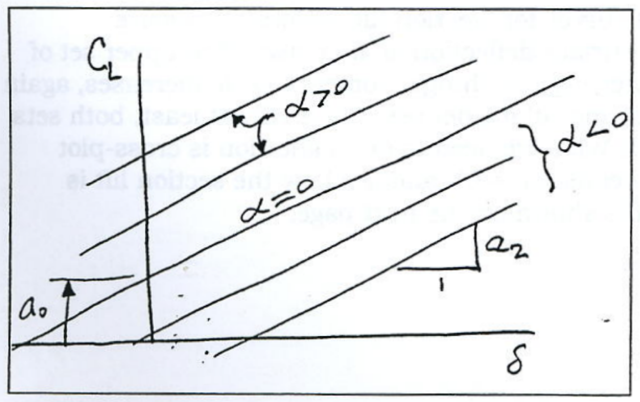
- within a reasonable range of both α and δ the slope is assumed constant at

$$a_1 = \left. \frac{\partial C_L}{\partial \alpha} \right|_{\delta = const.}$$

- We allow for initial aerofoil camber here too by showing $a_0 \neq 0$ when both α and δ are zero.

Effect of a Control Surface Deflection

Similarly, we expect to find, for several constant values of α:



- again, with the assumption of a linear response, we can define the slope in this case as:

$$a_2 = \left.\frac{\partial C_L}{\partial \delta}\right|_{\alpha=const.}$$

Effect of a Control Surface Deflection

- Using the two coefficients given on the previous slides we can define the lift coefficient as:

$$C_l = a_0 + a_1\alpha + a_2\delta$$

where the general parameter δ can be replaced by one of the control surface deflection angles ξ, η or ζ.

- Theoretical thin-aerofoil theory predicts for a 2D wing with full-span control surface the value:-

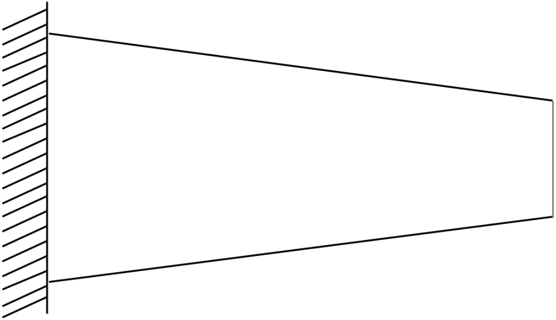
$a_1 = 2\pi$ so we can expect values of order 5 to 6

for α measured in radians or

$a_1 \approx 0.1$ for α measured in degrees.

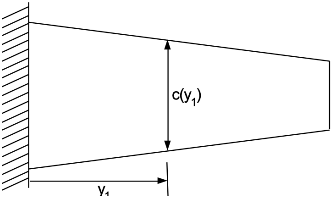
Functions That Vary Across The Span

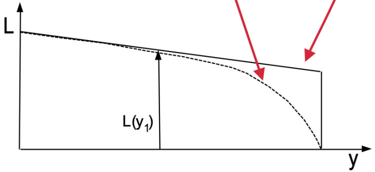
- Sometimes it is convenient to make one or more of the coefficients variable (i.e. $a_i(y)$). Take a simple tapered wing as shown below:



Functions That Vary Across The Span

- If the only component of incidence were the flight incidence that is uniform across the span, then the integrand would be all constants except for $c(y)$, and the distribution of lift across the span would reflect the distribution of chord length.
- However we know that true lift distributions drop to zero at the tip, and if we want to be more realistic we can alter the differential to display that reality.





Functions That Vary Across The Span

- One approximation to the spanwise distribution of lift, is to consider the product $a_1\alpha$ to give a reasonable variation of the product without trying to portray either factor accurately.
- We might be able to estimate components of α other than that due to downwash, whereas the spanwise variation of the loss in incidence due to downwash would be more difficult to describe.
- In principle, a_1 is almost constant but we could "shape" it by using a function such:

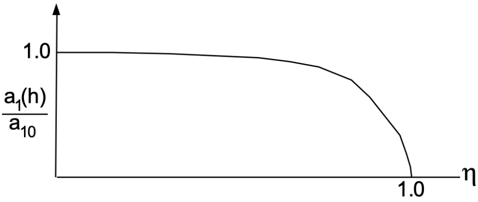
$$a_1(\eta) = a_{10}\,(1-\eta^2)^{1/2}$$

$$(\ \eta = y/s\ )$$

(η is the fractional semi-span)

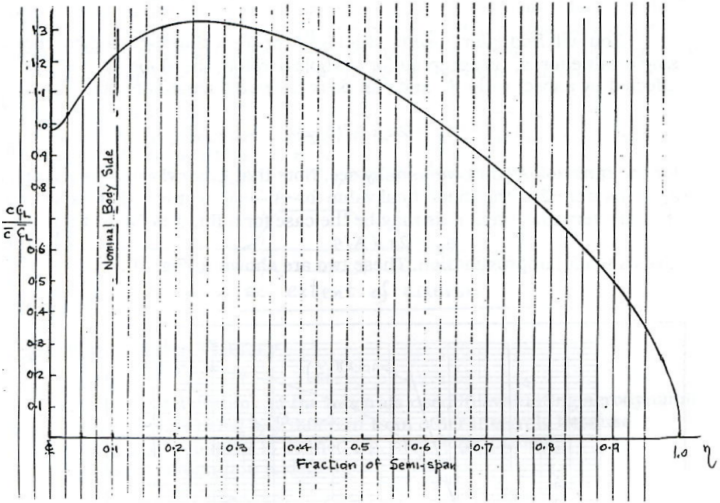
Note: *Given if needed*

Functions That Vary Across The Span



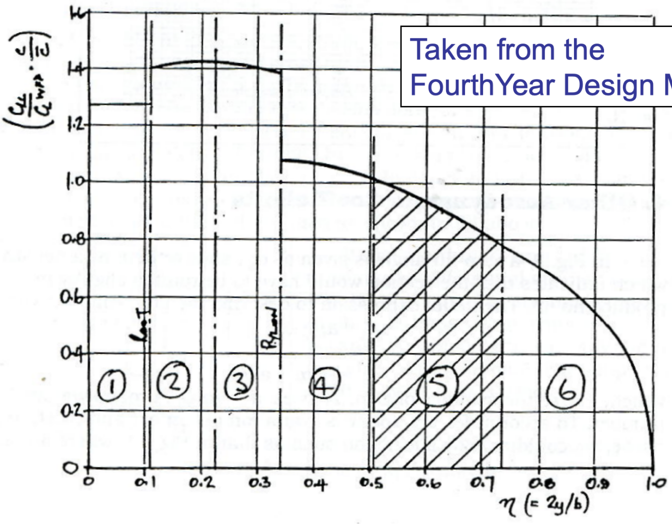
- The product $a_{10}\,(1-\eta^2)^{1/2}\,\alpha(y)$ will drop to zero at the tip even if using a function $\alpha(y)$ that correctly represents camber, constructed twist and flight incidence.
- The elliptical shape given by Eqn.3 is "*about right*". There remains the task of selecting a value for a_{10} the apparent value of the lift-curve slope for the root section.

Functions That Vary Across The Span



Wing Spanwise Load Distribution due to Incidence

Functions That Vary Across The Span



Taken from the FourthYear Design Manual

A Lift-curve Slope for The Whole Wing

- Where a single value for a_1 is accepted for an aerofoil, the slope $a_1 = \delta C_L/\delta\alpha$ is affected by aspect ratio A and can be represented approximately by:

$$a_{10} = 2\pi \times A/(A+2)$$

Note: *even when A is as high as 8, a_1 drops from 2π (the theoretical value for $A \rightarrow \infty$) to only 5.*

- For aspect ratios that are very low ($A \leq 2$), as may be the case for a fin, it is found that the following gives a useful approximation:

$$a_{10} = 2\pi \times A/4$$

A Lift-curve Slope for The Whole Wing

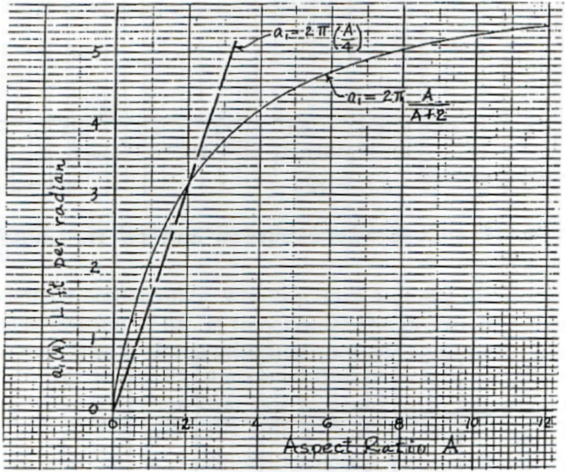


Fig. 10

A Lift-curve Slope for The Whole Wing

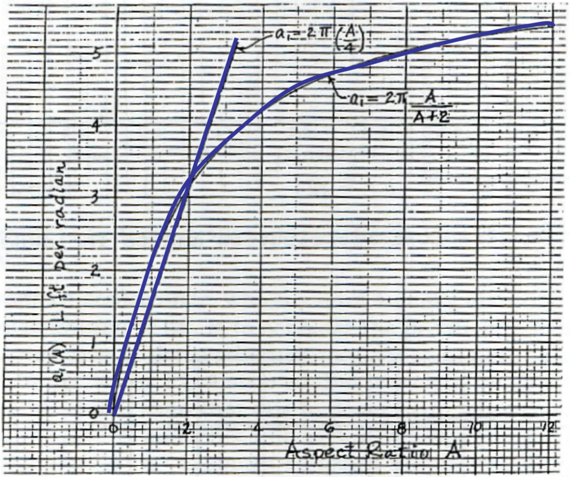


Fig. 10

Other Aerodynamic Coefficients

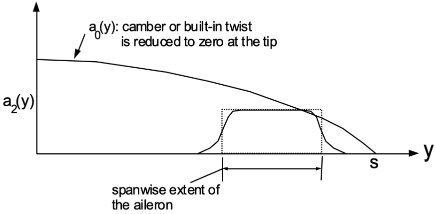
- If the zero-lift angle is taken as α_0 , this is usually a negative value which indicates that the section would have to be rotated slightly nose-down to produce no lift.
- An expression for the lift coefficient could then be given as:

$$
\begin{aligned}
C_l &= a_1 (\alpha - \alpha_0) \\
&= -a_1\alpha_0 + a_1\alpha
\end{aligned}
$$

which gives $a_0 = -a_1\alpha_0$, probably a positive number.

Effect of Control Surface Deflection

- To account for the spanwise variation of camber and twist, as described above, we could employ a function such as that shown below, where a realistic shape is shown for a_0 and also a typical shape for a_2 , the coefficient related to the control surface deflection.
- "Pure" strip theory would dictate that lift cannot be changed by such a surface outside its own range and therefore a_2 should be zero outside the aileron extent, but in reality there will be some ‘spill’.



Applying Strip Theory

- Work from the basics
- Do not try to remember the developed expressions
- Explain the assumptions you make and the approximations you use

Applying Strip Theory

- There is no need to think of the integrals used in evaluating aerodynamic forces and moments as being any different from other integrals :

  (a) a differential element is defined e.g. dForce or dMoment,

  (b) a range of integration is defined

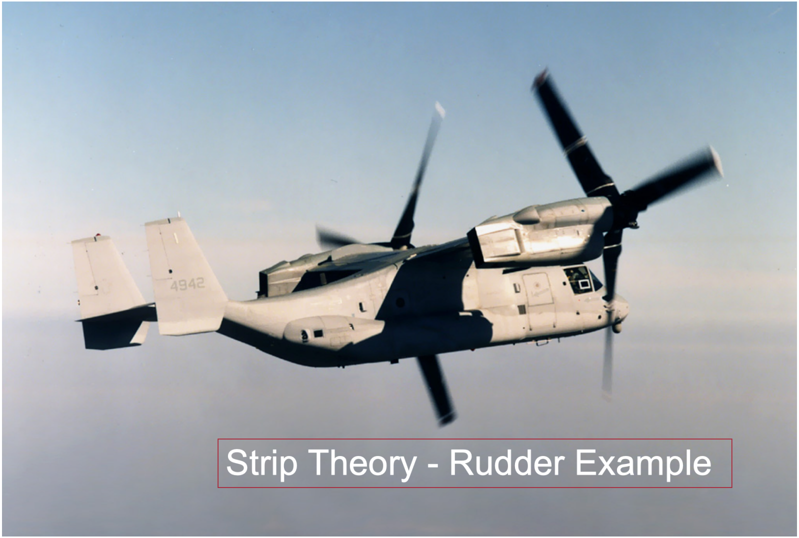
$$\int_0^s = \int_0^{y_1} + \int_{y_1}^{y_2} + \int_{y_2}^{y_3} + \cdots + \int_{y_i}^{s},$$

  (c) the integral is evaluated in the usual way.

Applying Strip Theory

The task is to express each differential force or moment by using all necessary factors to make up a force, and then possibly also using a moment arm. When selecting the ranges for the integrations you need to consider:

- (d) the full range (e.g. 0 to s) must encompass all contributions to the force or moment which is described by the differential expression, *including control surfaces*,
- (e) there may be a dependence on individual factors in the integrand, *i.e.* these factors may not all have the same range, so the integral may have to be broken into sections such as shown in (b) above.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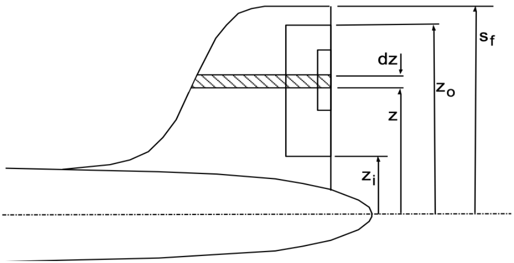
Strip Theory - Rudder Example

Strip Theory - Rudder Example

- For example, take a typical expression for C_l :

$$C_l = a_0 + a_1\alpha + a_2\zeta + a_3\beta$$

$$\begin{array}{cccc} \uparrow & \uparrow & \uparrow & \uparrow \\ \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} \end{array}$$



Strip Theory - Rudder Example

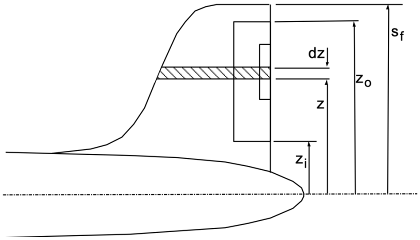
- For example, take a typical expression for C_l :

$$C_l = \underset{\substack{\uparrow \\ \mathbf{1}}}{a_0} + \underset{\substack{\uparrow \\ \mathbf{2}}}{a_1\alpha} + \underset{\substack{\uparrow \\ \mathbf{3}}}{a_2\zeta} + \underset{\substack{\uparrow \\ \mathbf{4}}}{a_3\beta}$$

Term 1 We would usually have $a_0=0$ for the symmetric aerofoil which acts as a fin, but a_0 will not generally be zero.

Strip Theory - Rudder Example

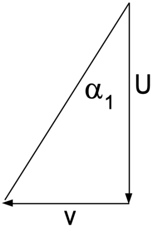
Term 2 We need to know what actions are causing an incidence (lateral) on the fin and this will include any action that leads to a transverse fluid velocity relative to the fin.



Strip Theory - Rudder Example

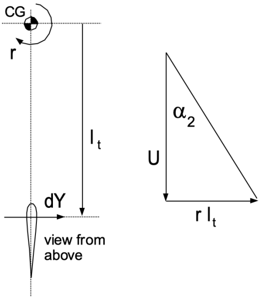
- The whole vehicle may be drifting to starboard at velocity v, causing an apparent negative incidence $\alpha_1 = -v/U$ uniformly over the fin.

  (*Note*: this sideslip angle v/U is normally refered to as β but we use α_1 here to distinguish it from the tab angle β and to make it consistent with the angles α_2 and α_3)



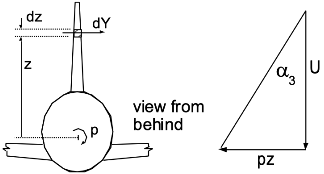
Strip Theory - Rudder Example

- The vehicle may have a yaw rate r that causes lateral motion of the fin of value rl_t (to port), this motion inducing a fluid velocity (from the port) such that incidence $\alpha_2 = rl_t/U$ (positive) is induced over the whole fin.



Strip Theory - Rudder Example

- The vehicle may have a roll rate p that induces a velocity pz of the *strip* (to starboard) and this causes a fluid velocity to be seen from the starboard which leads to $\alpha_3 = -pz/U$. Thus the total incidence at a fin strip due to these three would be:



$$\alpha_{fin} = \frac{r\,l_t - v - pz}{U}.$$

Strip Theory - Rudder Example

Term 3 Some strips dz pass through the rudder region and some do not, so the range of integration over which the term $a_2\zeta$ is included may be less than the full possible range 0 to s_f .

The expression for dLift within the rudder extent will have the term $a_2\zeta$, whereas the expression for positions z outside this will not.

Term 4 There may even be a tab on the control surface, and a fourth term could be legitimate over an even narrower range of z .

Strip Theory Application

1. Define Geometry and define ‘strip’ boundaries

2. Develop an expression for dForce in general terms:
   pressure × area × coefficient

3. If necessary, develop a dMoment (with the required moment arm).

4. Determine spanwise functions as needed.

5. Express the full integral with its proper limits. → Integrate

6. Differentiate to get an “aerodynamic derivative”?


Strip Theory – Rolling Power
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Aviation Investigation Report
Loss of Control on Take-off
PA-28-140 C-FXAY
Mascouche, Quebec
13 January 2001

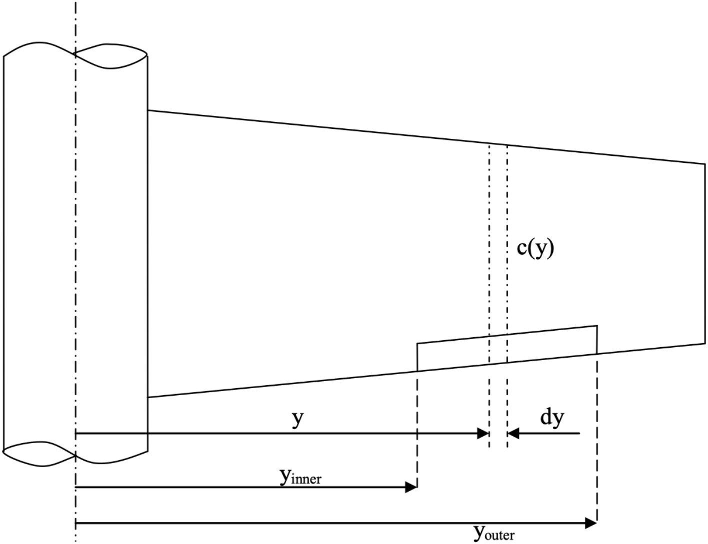
Note: the bell crank replacement work was completed in a hurry on Friday, 22 December 2000.

- During climbout, about 25 feet above ground level, the aircraft rolled to the left. The pilot flying, who was also the owner of the aircraft, applied right aileron to compensate for the turn, but the aircraft continued to turn left.
- The aircraft came to rest in a field on the other side of the highway. The two pilots evacuated the aircraft and were taken to hospital for minor injuries.
- Preliminary examination of the aircraft by the investigator at Mascouche Airport revealed that the bell cranks were installed backwards. By moving the ailerons from outside the aircraft, it was confirmed that the flight controls moved in the opposite direction.

Steady Response in Roll

- This section looks at the steady-state response to an aileron deflection
- This implies pure roll, i.e. no other responses but rotation around the x-axis.
- This becomes unreal after any significant bank angle ϕ develops, because then sideslip v would develop also, and this would generate a yaw rate r because of its pressures on the fin; then we would have *three* active variables.
- In this example we ignore all other responses and seek the roll rate p that can be achieved by an aileron deflection of ξ

Rolling moment due to Ailerons



Rolling moment due to Ailerons

- Local lift coefficient on the strip can have the components:

$$C_l = a_0 + a_1 \alpha + a_2 \xi$$

- And thus the expression for the differential lift on the strip becomes:

$$dLift = \frac{1}{2} \rho U^2 c(y)\, dy\, C_l$$

- Differential rolling moment can be expressed as:

$$d\,L = -y\ d\,Lift$$

where the negative sign is necessary to display the fact that positive ξ produces a negative rolling moment.

Rolling moment due to Ailerons

- Just before the aircraft begins to roll, the only unbalanced components of lift from C_l which can produce a rolling moment are those from ξ.
- Even the effect of camber (a_0) will be symmetric on the two sides and can therefore be left out of the lift expression.
- As both ξ and y change sign from side to side we can integrate double the moment on one side i.e.:-

$$\boldsymbol{L}(\xi) = -2\int_0^{s} \frac{1}{2}\rho \boldsymbol{U}^2 \boldsymbol{y}\, \boldsymbol{c}(\boldsymbol{y})\, \boldsymbol{a}_2(\boldsymbol{y})\, \xi\, \boldsymbol{dy}$$

Note: There is a tendency to leap to here!

Rolling moment due to Ailerons

- Now, using pure strip theory we have:-

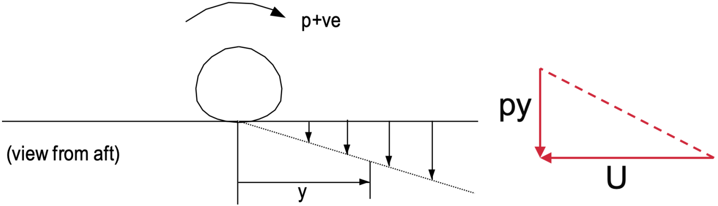
$$L(\xi) = -\rho U^2 a_2 \xi \int_{y_i}^{y_o} y \, c(y) \, dy$$

- This is the total, and yet unbalanced, rolling moment imposed by the ailerons.

- (if there were available a true function that represented the 'spill' of aileron-induced pressures near the aileron ends we could use that function $a_2(y)$ under the integral sign instead of using the constant implied here.)

Rolling Moment due to Response in Roll

- In order to develop a general expression we first assume that a steady roll rate p has already developed and then determine the rolling moment due to that rate p, even though it is clear that a positive aileron deflection will actually induce a negative value for p.



Rolling Moment due to Response in Roll

- The downward velocity (due to p) of a section at spanwise y will be py, thus inducing an (upward) incidence py/U and an upward lift (on the right side) as a consequence,
- i.e. the aerodynamic response to the rolling motion is a moment that opposes that motion. Similar to the above, we have:-

$$\boldsymbol{L}(\boldsymbol{p}) = -2\int_0^{s} \frac{1}{2}\rho \boldsymbol{U}^2 \; \boldsymbol{y}\, \boldsymbol{c}(\boldsymbol{y})\, \boldsymbol{a}_1(\boldsymbol{y})\, \frac{\boldsymbol{py}}{\boldsymbol{U}}\, \boldsymbol{dy}$$

$$= -\rho \boldsymbol{U}\, \boldsymbol{a}_1\; \boldsymbol{p}\int_0^{s} \boldsymbol{y}^2\, \boldsymbol{c}(\boldsymbol{y})\, \boldsymbol{dy}$$

- And subject to the assumption of *pure* roll, these two moments are the only “external” moments.

Balanced Moments While Rolling

- Newton's Second Law, in the rotational sense about the X-axis states that:-

$$I_{xx}\ \dot{p} = \sum externally\ \ applied\ \ moments$$

- And this is satisfied for $\dot{p} = 0$ (roll acceleration becomes zero when the sum of the applied moments acting on the aircraft from outside is zero) if:-

$$L(\xi) + L(p) = 0\ ,$$

or $$-\rho U^2 a_2\ \xi \int_{y_i}^{y_o} y\ c(y)\ dy - \rho U a_1\ p \int_0^s y^2\ c(y)\ dy = 0$$

Balanced Moments While Rolling

- Then the expression from which we determine the steady roll rate p as a function of ξ is:

$$p = -\frac{a_2 \int_{y_i}^{y_o} y \, c(y) \, dy}{a_1 \int_0^s y^2 \, c(y) \, dy} \xi U .$$

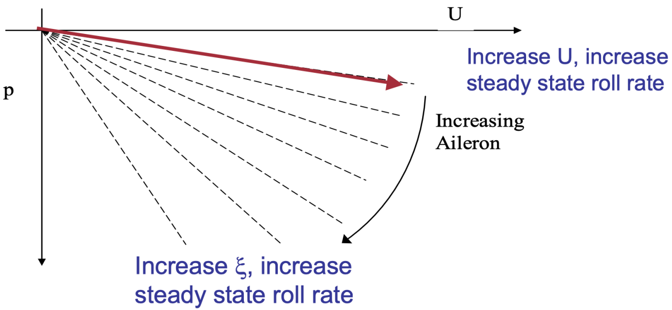
Balanced Moments While Rolling

- Then the expression from which we determine the steady roll rate p as a function of ξ is:

$$\boldsymbol{p} = -\frac{a_2 \int_{y_i}^{y_o} y\, c(y)\, dy}{a_1 \int_0^s y^2\, c(y)\, dy}\, \xi U \,.$$

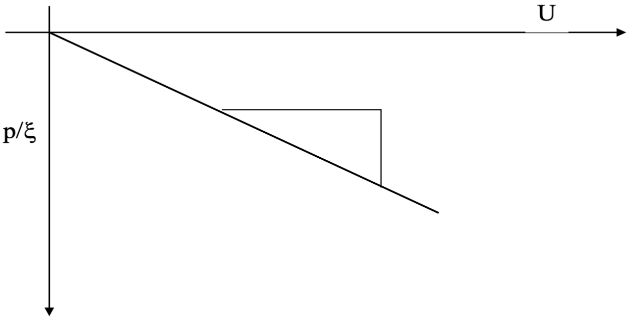
Balanced Moments While Rolling

- This can be expressed graphically as



Balanced Moments While Rolling

- or we plot "rolling power" as p/ξ



Balanced Moments While Rolling

- Looking at (5) note that the slope here is given by:

$$slope = -\frac{a_2 \int_{y_i}^{y_o}}{a_1 \int_0^s}$$

- and note that for constant ξ we have p directly proportional to U.


Aerodynamic Derivatives
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HB-2323

Aerodynamic Derivatives

Note: Mentioned here and used later on in the course

- If an aerodynamic force (or a moment) is known to be a function of several variables, each contributing linearly, i.e.,

$$\boldsymbol{T} = \boldsymbol{T}(\alpha, \beta, \gamma \ldots)$$

- then for small changes in each variable the total change in the aerodynamic function can be found using the Chain Rule:

$$\delta T = \frac{\partial T}{\partial \alpha}\delta\alpha + \frac{\partial T}{\partial \beta}\delta\beta + \frac{\partial T}{\partial \gamma}\delta\gamma + \ldots$$

Aerodynamic Derivatives

- Since we are often looking for expressions to represent the *departures* from a steady equilibrium flight condition we often want the total (but small) change in a force.

  - In the case we have been studying above, the rolling moment L is a function of p and ξ so we have:

$$\boldsymbol{L} = \boldsymbol{L}(\boldsymbol{p},\ \xi) = \frac{\partial \boldsymbol{L}}{\partial \boldsymbol{p}}\,\boldsymbol{p} + \frac{\partial \boldsymbol{L}}{\partial \xi}\,\xi$$

Aerodynamic Derivatives

- The partial derivatives by themselves are called "**aerodynamic derivatives**" expressed using the shorthand notation:

$$\frac{\partial T}{\partial u} = T_u$$

- These derivatives display the influence of a variable on a function, e.g. the influence of u on T and this implies the need to express T in terms of u before doing the partial differentiation.
- Note: using rolling power can be expressed as:

$$\frac{p}{\xi} = -\frac{L_\xi}{L_p}$$

Next Lecture

Equations of Motion