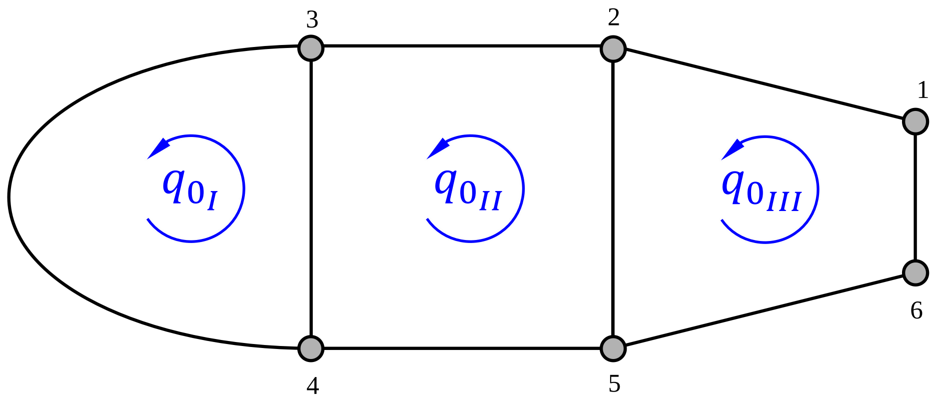
Assignment B – Solution

- This assignment requires the analysis of **two different problems**:
  - Shear loading (for shear centre)
  - Pure torsion (for stiffness)
- There are **at least three different ways** to solve them:
  - Method 1
    - Shear: apply S_y **off** the shear centre $\xrightarrow{4\times4}$ find q_{0_j} and $\mathrm{d}\theta/\mathrm{dz}$
    - Torsion: apply same $\mathrm{d}\theta/\mathrm{dz}$ $\xrightarrow{4\times4}$ find q_j and e_x
  - Method 2
    - Shear: apply S_y **at** the shear centre $\xrightarrow{3\times3}$ find q_{0_j} and q_i^{closed} $\xrightarrow{\sum M=0}$ find e_x
    - Torsion: apply arbitrary $\mathrm{d}\theta/\mathrm{dz}$ $\xrightarrow{4\times4}$ find q_j and T
  - Method 3
    - Shear: apply S_y **at** the shear centre, $\mathrm{d}\theta/\mathrm{dz} = 0$, $e_x \neq 0$ $\xrightarrow{4\times4}$ find q_{0_j} and e_x
    - Torsion: apply arbitrary $\mathrm{d}\theta/\mathrm{dz}$ $\xrightarrow{4\times4}$ find q_j and T

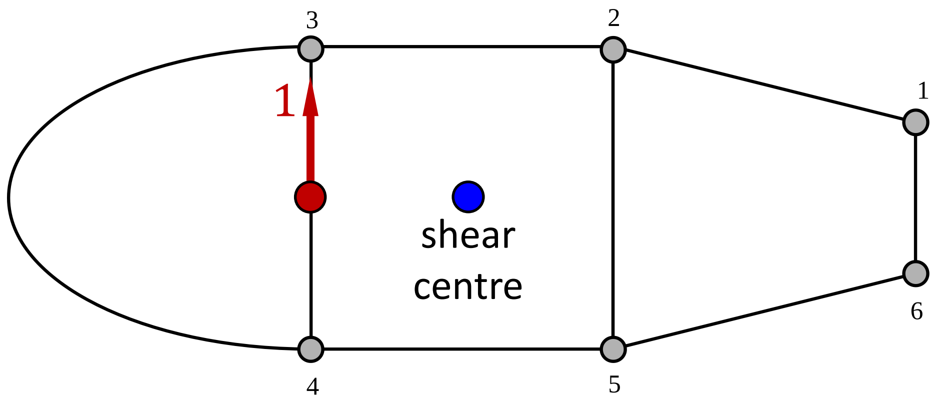
Assignment B – Method **1** (1/3)

- Always assume **counter clockwise** as **positive** (for moments and twist)



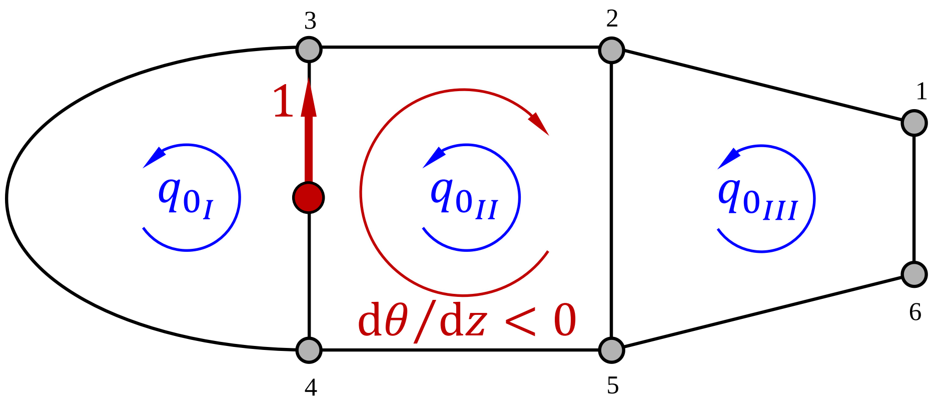
Shear

- Assume a vertical unit load being applied halfway between joints **3** and **4**
- Web **34** is to the **left of the shear centre**, so expect **negative** twist and torque



Assignment B – Method **1** (2/3)

- Following **Example 3.5**, solve the 4×4 system of equations to find the unknowns q_{0_j} and $\mathrm{d}\theta/\mathrm{d}z$ (which should be **negative**)



Torsion

- Write the 3 torsion equations entering your **known** value of $\mathrm{d}\theta/\mathrm{d}z$:

$$\left(\frac{\mathrm{d}\theta}{\mathrm{d}z}\right)_j = \frac{1}{2\,A_j\,G}\sum_{i\in j}\left(q_i\,\frac{b_i}{t_i}\right)$$

unknowns, written in terms of q_j

- Last equation → balance of moments about the applied load:

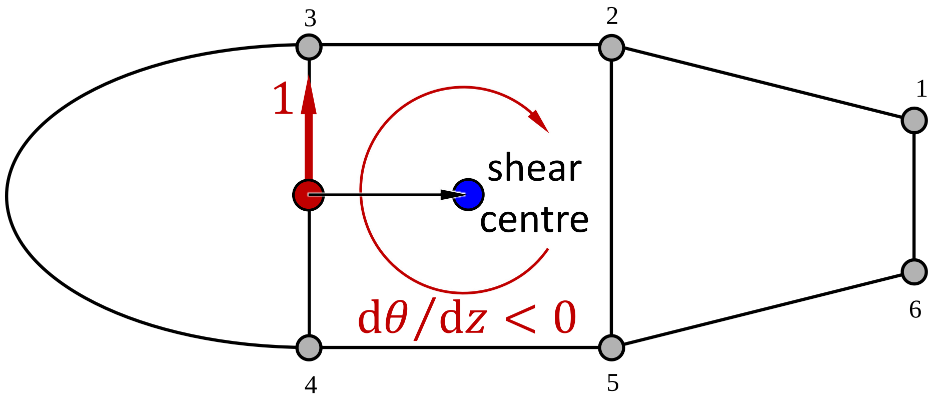
$$S_y\,e_x - \cancel{S_x}^{\,0}\,\cancel{e_y}^{\,0} = \sum_{\text{all } i}(q_i\,r_i\,b_i)$$

unknown

unknowns, written in terms of q_j

Assignment B – Method **1** (3/3)

- Solution of the 4×4 system gives q_j and e_x



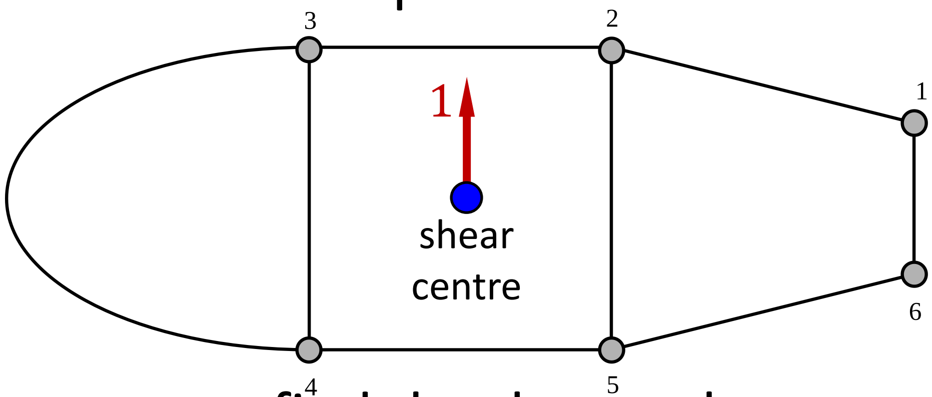
- Note: If you find a negative e_x it means that the shear flow is generating **negative torque**, so the radii r_i should be made **negative**
- If in doubt: the shear centre **must** be to the **right** of web **34** (*i.e.* within cell II) and close to the centroid of the section
- Finally, the torsional stiffness is

$$\frac{T}{\theta} = \frac{S_y\, e_x}{(\mathrm{d}\theta/\mathrm{dz})\, L}$$

Tip: you should get results in the order of ~ 10^9 (N mm)/rad

Shear

- In this method we assume loading through the shear centre, so that $\mathrm{d}\theta/\mathrm{d}z = 0$ and we have 3 equations and 3 unknowns



- Solve the 3×3 system to find the three values $q_{0\,j}$
- Calculate individual values of 'closed' (*i.e.* real) shear flow

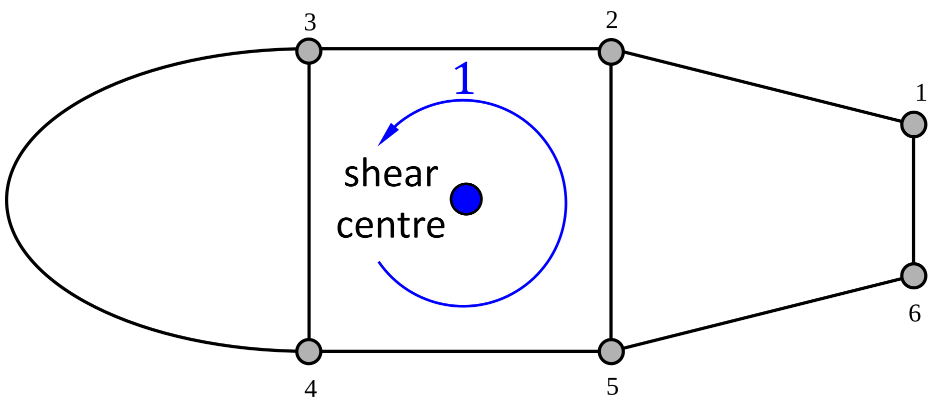
$$q_i^{\text{closed}} = q_i^{\text{open}} + q_{0_j}$$

- Then balance moments about any reference point:

$$S_y\, e_x - \cancelto{0}{S_x}\, \cancelto{0}{e_y} = \sum_{\text{all } i} \left(q_i^{\text{closed}}\, r_i\, b_i\right)$$

Torsion

- Follow **Example 3.4**, *i.e.* apply a unit torque T and find $\mathrm{d}\theta/\mathrm{d}z$ (or vice-versa)



- Solve the 4×4 system to find the three values q_j and the unknown $\mathrm{d}\theta/\mathrm{d}z$

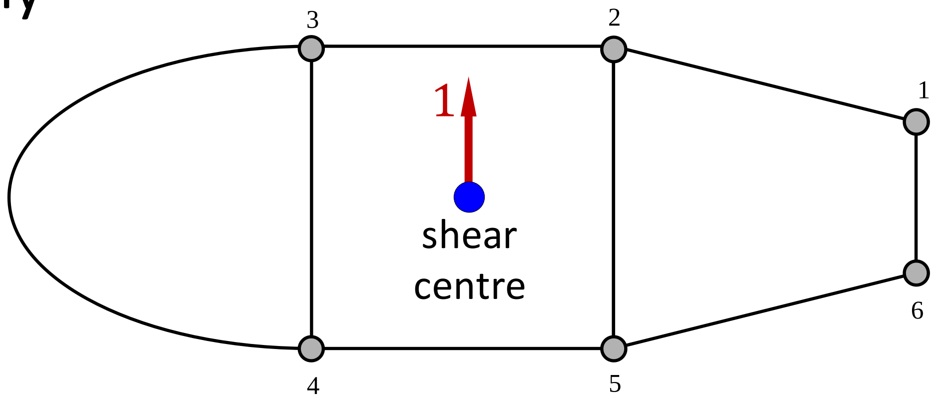
- Finally, the torsional stiffness is

$$\frac{T}{\theta} = \frac{T}{(\mathrm{d}\theta/\mathrm{d}z)\,L}$$

Assignment B – Method **3** (1/1)

Shear

- Method 3 is virtually identical Method 2, but consists in writing the 'balance of moments' as a **fourth equation** and solving the 4×4 system to find q_{0_j} and e_x directly



Torsion

- For the torsional stiffness we follow the same procedure as before, *i.e.* solve a pure torsion problem with unit torque