3. Kinematics and Principle of Virtual Work

Design 2
AENG21350

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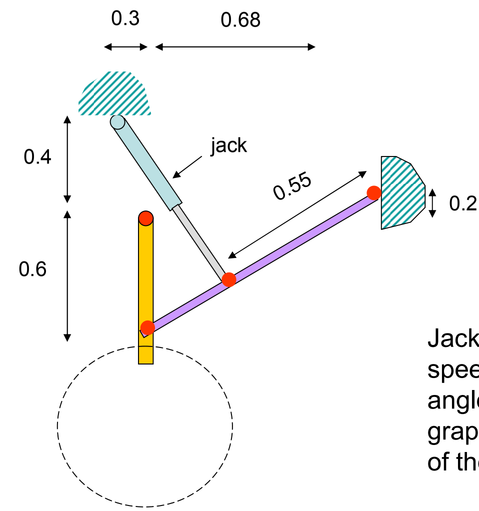
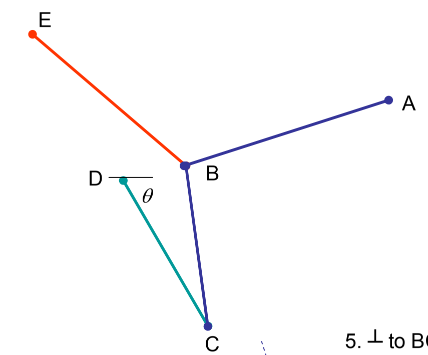
3.1 Kinematics of an undercarriage -1

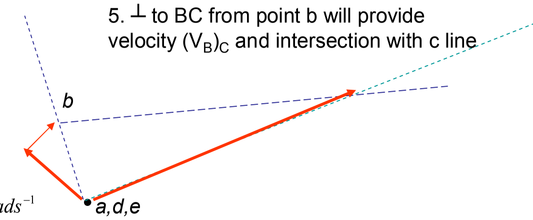


Jack moves at constant speed of 5 cm/s. For leg angle of 60° determine graphically the rate of rotation of the leg.

3.1 Kinematics of an undercarriage -2



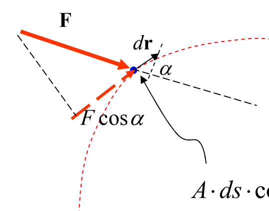
1. Fixed points: D, A, E.
2. Absolute velocities on c and b lines (⊥ to DC and AB respectively)
3. Jack velocity known in direction and magnitude
4, Projection on jack velocity on b line will identify point b
5. ⊥ to BC from point b will provide velocity $(V_B)_C$ and intersection with c line



6. Velocity point C identified. In scale: $V_C = 19.5$ cm/s

$$\omega_{leg} = \frac{V_C}{CD} = \frac{19.5}{60} = 0.325 \quad rads^{-1}$$

3.1 Work of a force and a couple



$A \cdot ds \cdot \cos\alpha = |d\mathbf{r}|\cos\alpha$

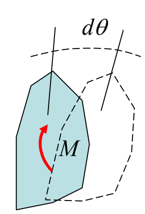
Infinitesimal work of a force:

$$dU = \mathbf{F} \cdot d\mathbf{r}$$

$$dU = F \cdot ds \cdot \cos\alpha$$

General definition (Cartesian coordinates)

$$dU = \left(\mathbf{i}F_x + \mathbf{j}F_y + \mathbf{k}F_z\right) \bullet \left(\mathbf{i}dx + \mathbf{j}dy + \mathbf{k}dz\right) = F_x dx + F_y dy + F_z dz$$



Infinitesimal work of a couple:

$$dU = M \cdot d\theta$$

Integral forms:

$$U = \int F \cos\alpha \cdot ds$$

$$U = \int M \cdot d\theta$$

3.2 Principle of virtual work (PVW)

The virtual work done by external active forces on a ideal mechanical system in equilibrium is zero for any and all virtual displacements consistent with the constraints

Constraint: restriction of motion by the supports.

$$\delta U = 0$$

δU = **total virtual work done on the system by all active forces during a virtual displacement**

2 advantages:
•Not necessary to dismember ideal systems to establish relations between the active forces
•We may determine the relations between the active forces directly without reference to reactive forces

PVW requires that internal friction forces do negligible work during any virtual displacement. If not, PVW cannot be used unless work done by internal friction forces is included

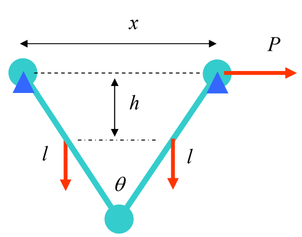
When using PVW, draw a diagram which isolates the system under consideration – only active forces (it is not a FBD!)

3.3 Considerations for mechanisms

1. Identify number of degrees of freedom for your system (Grubnel formula)

2. Apply PVW as many times as there are degrees of freedom. With each application, we allow only one independent coordinate to change at time while holding the others constant

3.4 Example



Each of the 2 uniform hinged bars has mass m and length l. For a given force P determine the angle θ for equilibrium

Ans.

Applying PVW:

$$P \cdot \delta x + 2mg \cdot \delta h = 0$$

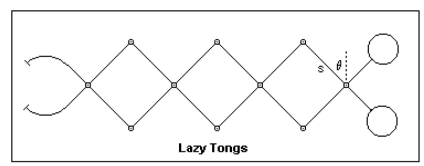
$$x = 2l \sin\frac{\theta}{2} \quad \Rightarrow \quad \delta x = l \cos\frac{\theta}{2} \cdot \delta\theta$$

$$h = \frac{l}{2}\cos\frac{\theta}{2} \quad \Rightarrow \quad \delta h = -\frac{l}{4}\sin\frac{\theta}{2} \cdot \delta\theta$$

Substituting:

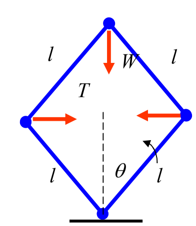
$$Pl\cos\frac{\theta}{2} \cdot \delta\theta - 2mg\frac{l}{4}\sin\frac{\theta}{2} \cdot \delta\theta = 0 \quad \Rightarrow \quad \boxed{\theta = 2\tan^{-1}\frac{2P}{mg}}$$

3.5 Lazy tongs



Current collectors of electric locomotive (pantograph), concertina mechanisms. If s is the length of a link in lazy tongs of N sections, then they will extend a distance of $x = 2Ns \sin\theta$, where θ is the angle shown in the figure. It is easy to find the velocity ratio $dx/d\theta$, and so the "leverage" of the tongs.

Example: car jack.
Load is applied by screw thread across horizontal diagonal. If the load on the jack is W, find the tension in the screw



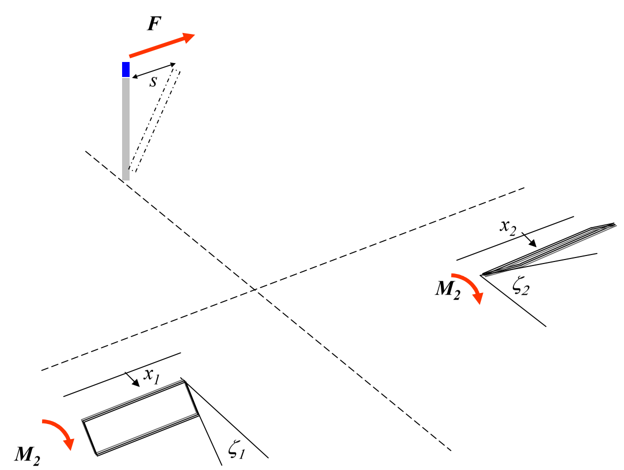
Ans. Application of PVW. Only θ is DOF

Work done by forces T: $2T \cdot l \cdot \cos\theta \cdot \delta\theta$

Work done by W: $-W \cdot 2 \cdot l \cdot \sin\theta \cdot \delta\theta$

$$2Tl\cos\theta \cdot \delta\theta - 2WL\sin\theta \cdot \delta\theta = 0 \quad \Rightarrow \quad \boxed{T = W\tan\theta}$$

3.6 Applications to aircraft control circuit



3.6 Applications to aircraft control circuit

Applying PVW:

$$F \cdot \delta s - M_1 \delta\zeta_1 - M_2 \delta\zeta_2 = 0$$

Pilot must counteract the combined work due to the restoring moments on the left and right aileron.

$$F = M_1\left(\frac{\partial \zeta_1}{\partial s}\right) + M_2\left(\frac{\partial \zeta_2}{\partial s}\right) = M_2\left(\frac{\partial \zeta_1}{\partial x_1}\frac{\partial x_1}{\partial s}\right) + M_2\left(\frac{\partial \zeta_2}{\partial x_2}\frac{\partial x_2}{\partial s}\right)$$

If $x_1 = x_2 = x$ → no differential applied to the system between control rod and point of application on each aileron:

$$F = M_2\left(\frac{\partial \zeta_1}{\partial x}\frac{\partial x}{\partial s}\right) + M_2\left(\frac{\partial \zeta_2}{\partial x}\frac{\partial x}{\partial s}\right)$$

Need to know control hinge moment at different speeds and information on relation between s and x

List of concepts to know

- Velocity diagrams for undercarriage
- PVW
- Applications to lazy tong and aircraft control systems