AVDASI 3
Aerodynamics - Constraint Diagrams

During the initial design phase of a new aircraft concept, a number of processes occur. After mission parameters have been determined, historical trends are often used to begin to sketch a number of concepts. As part of this, there are a substantial number of requirements placed on the design and while not all of these are taken into account in initial design, analysis can be performed on major requirements that act as drivers to the overall design.

Two of the driving parameters that need to be determined in initial design are the size of the wing and the size of the engines. This tends to be done implicitly by deciding on two other parameters;

- Wing loading (aircraft weight as a fraction of wing area) - W/S
- Thrust loading (engine thrust as a fraction of aircraft weight) - T/W

For a number of conditions these are tightly coupled. For example, on take-off, having a large wing (low wing loading) means that a smaller engine is permitted to be able to accelerate the aircraft to a lower take-off speed. On the other hand, these two parameters may not be coupled at all, for example in determining the stall speed (which is often a driver for wing sizing).

Simple hand calculations are useful for determining if various requirements can be met by choosing different values for wing and thrust loading, however, it is often more useful to visualise things. This is where constraint diagrams come in. A constraint diagram is any diagram where design constraints can be plotted visually. In this case, the specific diagram in question is thrust loading against wing loading (figure 1).

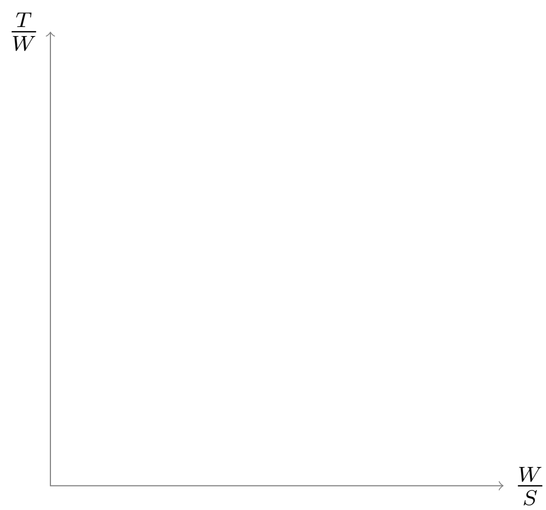

Figure 1: Constraint diagram

Clearly it is not suitable to just choose any combination of thrust and wing loading. Consideration of the design specification via mission parameters, design parameters and certification requirements is necessary. Hence, one specific design condition represents a constraint; choosing a thrust and wing loading outside of this constraint is not feasible. Due to the two variables that are considered, constraints need to be cast in the form of:

$$\frac{T_0}{W_{MTOW}} = f\left(\frac{W_{MTOW}}{S}\right) \tag{1}$$

where T_0 is the engine thrust at sea-level conditions, W_{MTOW} is the maximum take-off weight and S is the wing area. Equation 1 simply says that the constraint lines need to be cast into a form where thrust loading is some fraction of wing loading. For simplification, for the rest of this paper, the subscripts are dropped (i.e. $T \equiv T_0$ and $W \equiv W_{MTOW}$).

Even in the initial design phase, there are a substantial number of constraints that can be cast into the required form. In this paper, the important ones will be considered when designing a conventional transonic passenger jet airliner. These are:

1. Landing/stall speed
2. Take-off roll
3. Manoeuvre
4. Climb rate

Clearly, if a different aircraft type were being designed, then these requirements would change. For example, if designing a fighter aircraft, then minimum max speed may come into play, while for a reconnaissance aircraft, service ceiling may also be a design requirement (e.g. SR-71). The following sections will outline the constraints for the conventional passenger aircraft.

Landing/stall speed

This is the simplest to cast into the required form. In fact, the resulting constraint is independent of the thrust loading. The important information here is that regulations dictate the minimum landing speed, V_{min}, is some multiple ($\times 1.1 - 1.3$ depending on the aircraft type) of the minimum stall speed. Hence, the requirement comes from requiring that the wing lift maintain the weight of the aircraft. In this analysis, the maximum take-off weight is considered. Often, for undercarriage design reasons, the maximum landing weight will be less that the MTOW and if this were the case, then a multiplier can be introduced and the following analysis redone. Keeping with the MTOW, lift must equal weight, and from the lift equation:

$$L = W = \frac{1}{2}\rho V_{min}^2 S C_L^{max}$$

where ρ is the density at sea-level and C_L^{max} is the maximum lift coefficient which is usually obtained using high-lift devices. In initial design, this is obtained by considering historical trends but can also act as a design parameter in itself. The final constraint is then given by:

$$\frac{W}{S} = \frac{1}{2}\rho V_{min}^2 C_L^{max} \tag{2}$$

So this is a vertical line on the constraint diagram and is driven by the ability to obtain a high C_L^{max}. Since C_L^{max} is the maximum C_L that the aircraft can possibly achieve, this is a constraint on the upper bound of wing loading (the infeasible region is shaded in figure 2).

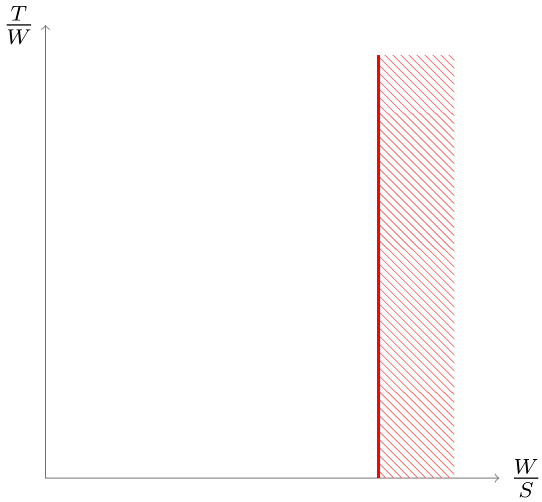

Figure 2: Constraint diagram with landing constraint

Take-off roll

The analysis of the take-off phase can become very in-depth, and includes all aspects from the take-off roll, rotation and initial climb. This analysis is only going to consider the take-off roll to give a flavour of the types of trade-offs required when taking into account take-off. Furthermore, there are a number of quite sweeping assumptions that underpin this analysis, so an understanding of what these mean in terms of design is essential.

First, the resistance forces acting on the aircraft are drag, D, and the rolling resistance, $F = \mu R$, where μ is the friction coefficient (typically 0.03-0.05 for asphalt) and R is the reaction force of the ground on the aircraft. This is given the its weight minus the lift currently produced. Work done is then equated to the change in kinetic energy, hence:

$$\int_0^x T - (D + \mu(W - L))ds = \frac{1}{2}\frac{W}{g}V_{TO}^2$$

where x is the take-off distance and V_{TO} is the velocity at take-off. It is now assumed that the terms inside the integral are constant throughout the roll (which is obviously not correct but done for ease of analysis) but to be conservative, they are assumed to be the largest they can be. This is where drag is maximum, which is at the take-off speed, and lift is minimum, which is at zero. Therefore:

$$(T - D_{TO} - \mu W)x = \frac{1}{2}\frac{W}{g}V_{TO}^2$$

Rearranging and dividing by the weight:

$$\begin{aligned}\frac{T}{W} &= \frac{V_{TO}^2}{2gx} + \frac{D_{TO}}{W} + \mu \\ &= \frac{V_{TO}^2}{2gx} + \frac{D_{TO}}{W} + \mu\end{aligned}$$

Now, expand out the drag term noting that $C_D = C_{D_0} + \frac{C_L^2}{\pi ARe}$ where C_{D_0} is the coefficient of drag represented by all other forms of drag, AR is the aspect ratio of the wing, e is the Oswald efficiency factor, so the constraint line is given by:

$$\frac{T}{W} = \frac{V_{TO}^2}{2gx} + \frac{1}{W/S}\left(\frac{1}{2}\rho V_{TO}^2 C_{D_0} + \frac{\frac{1}{2}\rho V_{TO}^2 C_{L_{TO}}^2}{\pi ARe}\right) + \mu \tag{3}$$

Depending on the aircraft type, the take-off speed will be specified as some multiple of the stall speed (typically $V_{TO} = 1.1V_{stall}$). Therefore, $C_{L_{TO}} = C_L^{max}/1.1^2$. The constraint is a minimum restriction on the amount of thrust required to ensure the aircraft can take-off, hence this may be visualised by figure 3.

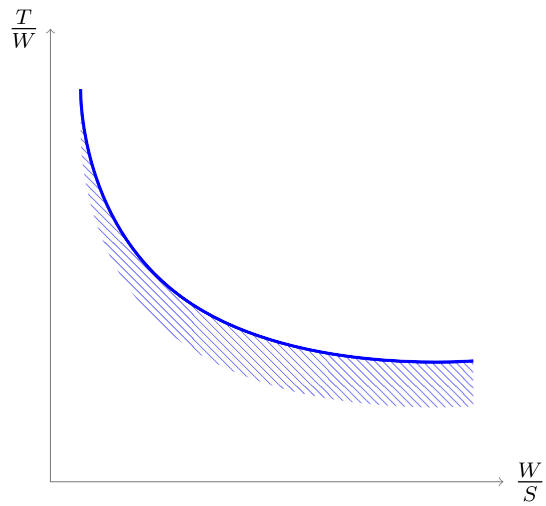

Figure 3: Constraint diagram with take-off roll constraint

Manoeuvre

The n-g manoeuvre requires that the aircraft is able to perform a manoeuvre at n-g (where n is the loading factor) at altitude. To maintain the speed, $T = D$ and to maintain flight, $L = nW$. However, engine performance deteriorates with altitude, which can be approximated as:

$$T_h \approx T_0\sigma \equiv T\sigma$$

where $\sigma = (20 - H)/(20 + H)$ so this must be taken into account. Drag is made up of induced drag and all other drag, hence:

$$T_h \approx T\sigma = \frac{1}{2}\rho_h V^2 S C_{D_0} + \frac{1}{2}\rho_h V^2 S \frac{C_L^2}{\pi A R e}$$

where the density at altitude is $\rho_h = \rho\sigma$. Rearranging $L = nW$ to get C_L, substituting the result in and dividing by W:

$$\begin{aligned} \frac{T}{W}\sigma &= \frac{\frac{1}{2}\rho_h V^2 C_{D_0}}{W/S} + \frac{1}{2}\rho_h V^2 S \frac{C_L^2}{W\pi A R e} \\ &= \frac{\frac{1}{2}\rho_h V^2 C_{D_0}}{W/S} + \frac{n^2 W/S}{\frac{1}{2}\rho_h V^2 \pi A R e} \\ \frac{T}{W} &= \frac{\frac{1}{2}\rho V^2 C_{D_0}}{W/S} + \frac{n^2 W/S}{\frac{1}{2}\rho\sigma^2 V^2 \pi A R e} \end{aligned} \tag{4}$$

The constraint is a minimum requirement constraint so anything below this is not acceptable (figure 4).

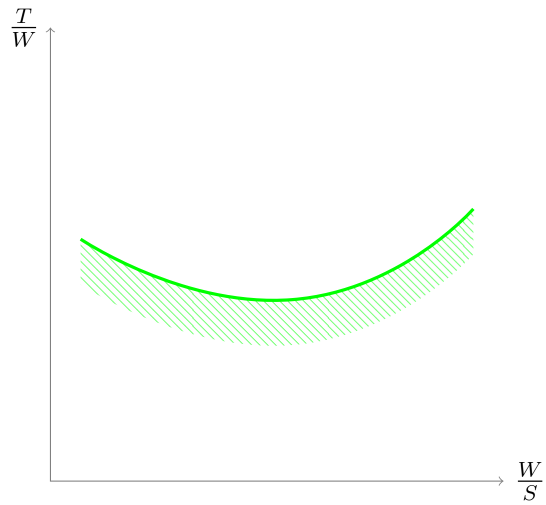

Figure 4: Constraint diagram with manoeuvre constraint

Climb rate

The climb is specified first in terms of climb angle, γ. It is assumed that the aircraft is travelling in the direction of the climb angle as in figure 5.

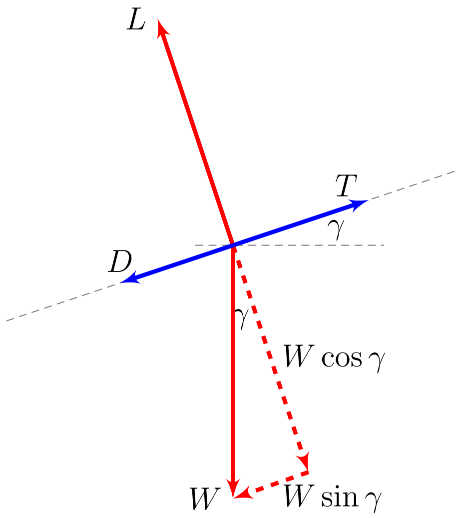

Figure 5: Forces on climbing aircraft

Resolving forces leads to $T = D + W\sin\gamma$ so assuming small angles, $\gamma \approx (T-D)/W$. The climb ratio, G, is introduced as the fraction of vertical height against horizontal distance. Again, assuming small angles, $G \approx \gamma$, so $G \approx (T-D)/W$. Hence:

$$\frac{T_h}{W} = \frac{D}{W} + G$$

So the same analysis can be performed as in the manoeuvre (except there is no load factor). Hence:

$$\frac{T}{W} = \frac{\frac{1}{2}\rho V^2 C_{D_0}}{W/S} + \frac{W/S}{\frac{1}{2}\rho\sigma^2 V^2 \pi A R e} + \frac{G}{\sigma} \tag{5}$$

The rate of climb is specified by certification requirements, hence the climb gradient can be calculated. As in the manoeuvre, this is a requirement on the minimum thrust from the engines, so anything below the constraint line is not acceptable - figure 6. Furthermore, since the difference between the manoeuvre and climb constraints is due to 1) an n^2 on the second term, and; 2) a vertical shift by G/σ, for lower values of wing loading, the climb constraint will be limiting whilst for higher wing loadings, the manoeuvre constraint is the limiting factor.

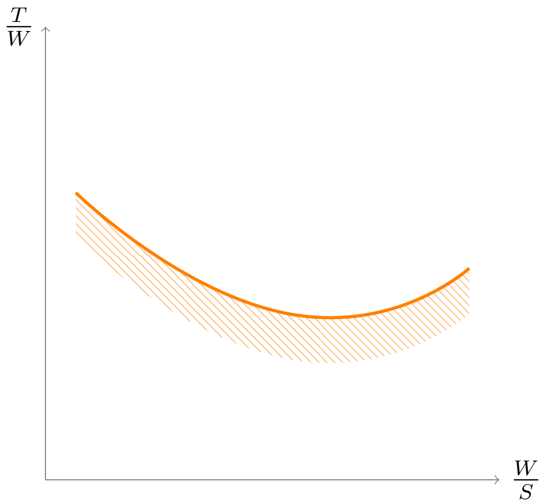

Figure 6: Constraint diagram with manoeuvre constraint

Choosing the design point

The point of the constraint diagram is to choose a thrust loading and wing loading that satisfies a number of mission and design constraints. In the four constraint lines (equations 2, 3, 4 and 5), there are a number of aircraft design constants (such as C_L^{max} and AR) that need to be decided before the lines can be plotted. In the first instance, it is usual to base these on historical trends. Later on in the process, once broad sizing has been performed, these can be refined using various analysis tools. Broadly, the final constraint diagram is given in figure 7, where the shaded areas denote the side of the lines that are infeasible.

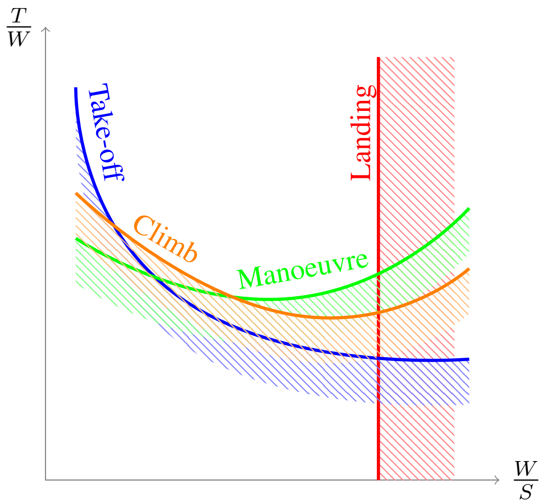

Figure 7: Overall constraint diagram

Following this, the art of choosing the best design point arrives. While it may immediately seem best to choose a design point at the intersection of some lines, it is worth taking a step back. In plotting figure 7, all that has been taken into account is some overall design requirements. What we have yet to do is consider what the function is that is trying to be design for when selecting thrust and wing loadings. Clearly, the function that would be optimised when designing an aircraft is a complex combination of a number of factors that cannot be simplified down to a function of two variables. As such, we have to select thrust and wing loadings that appear sensible given the constraints. Furthermore, we can also consider some broader consequences of thrust and wing loading such as structural weight, cost or cruise lift coefficient.

As an example, take two different selections of the design point (see figure 8). Design point 1 is a size based on the take-off and climb constraints. This is one where a low wing loading is coupled to a high thrust loading. Hence, engine size comes into play here, but the low wing loading means that the wing will be very large. This would allow a lower cruise lift coefficient, but the drag associated with the wing is likely to be large. On the other hand, design point 2 is at the intersection between the manoeuvre and landing constraints. The larger wing loading would lead to a low wing area but this has the disadvantage of, for example, requiring a larger cruise lift coefficient.

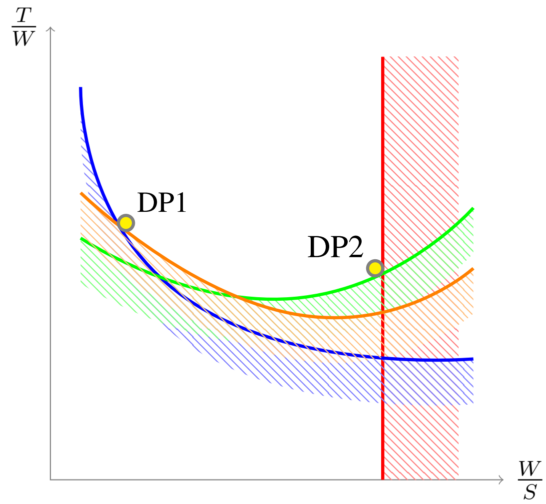

Figure 8: Constraint diagram with two design points

The selection of points is further complicated when considering sensitivity. For example, both of the design points in figure 8 are not great choices due to this. Specifically, take design point 2, which is right up against the landing constraint. This constraint is directly related to the available maximum lift coefficient of the aircraft which is prone to changes through the design process. As such, if design point two were to be selected and the maximum lift coefficient of the wing decreased during the design process, then the constraint line would move to the left and the design point would be infeasible. The wing loading would have to be reduced by increasing the wing area.