Stress, Strain and Deformation
Axially Loaded Members

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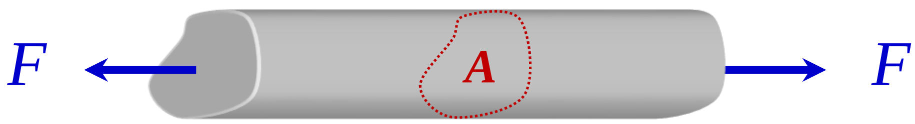
Normal Stress and Strain

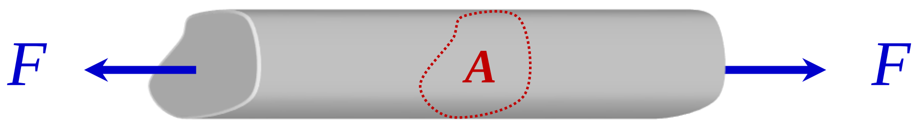
- Consider a bar subjected to an axial force:



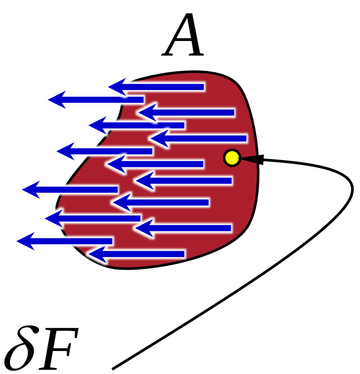
- We need to know:
  - the distribution of the force in the material
    - i.e. the "**stress**"
  - the extension of the bar per unit length
    - i.e. the "**strain**"

Normal Stress

- Normal stress = Force/Area → $\boxed{\sigma = F/A}$ σ = "sigma"
  - where the force is normal (perpendicular) to the area



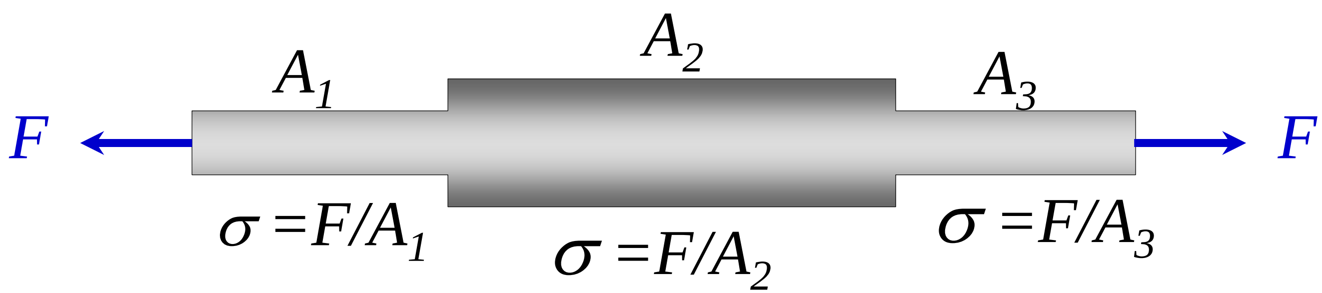
- Can be considered at a **point**
- Can be **tensile** or **compressive**
- Units of stress = force per unit area
  - e.g. 1 N/m^2 = 1 Pa (Pascal)
  - or 1 N/mm^2 = 10^6 N/m^2 = 10^6 Pa = 1MPa



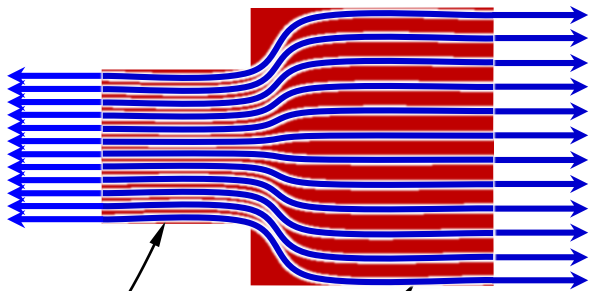
$$\sigma = \lim_{\delta A \to 0} \frac{\delta F}{\delta A}$$

Changes in Cross-Sectional Area

- Consider a change in x-section area, *e.g.*:



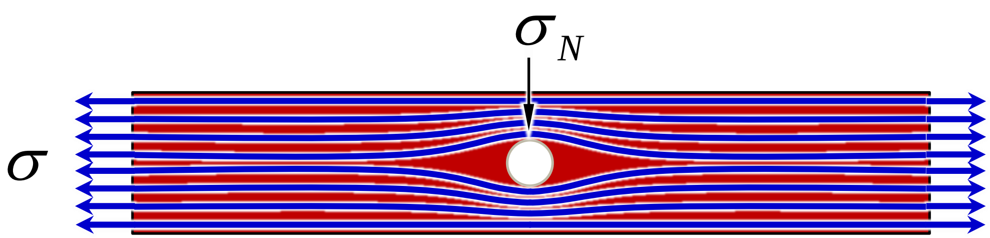
- Average stress = F/A
  - Valid away from discontinuity
- But at the discontinuity?
  - Consider "lines of internal loading"
  - Note areas of close spacing: higher stresses
  - and areas of wide spacing: lower stresses



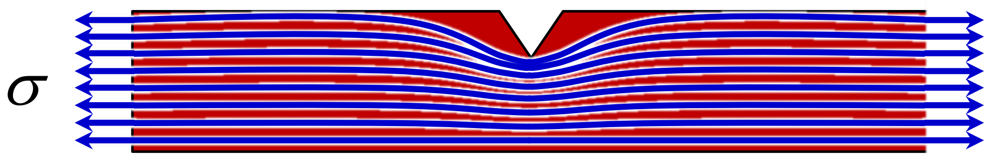
- For example at a hole, notch or crack:

$$\boxed{\sigma_N = K_T \cdot \sigma}$$

$$(\text{max. local stress}) = (\text{stress concentration factor}) \cdot (\text{far field stress})$$



$K_T = 3$ for round hole



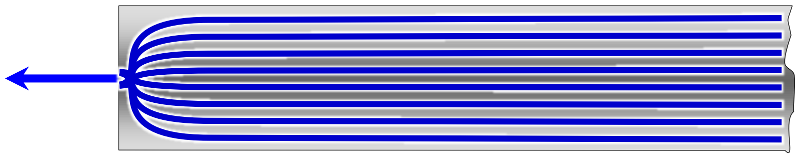
$K_T > 3$ for sharp notch or corner

- Tabulated solutions for standard cases usually account for the area reduction plus the stress concentration effect.

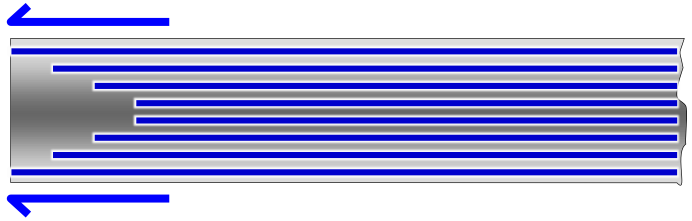
Load Input

- Local stress distribution will depend on the **details of load introduction**

- Consider a bar where the load is introduced (or reacted) at:
  - a point on its end

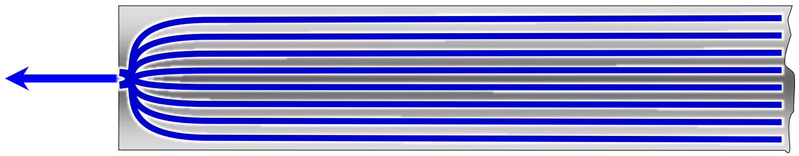


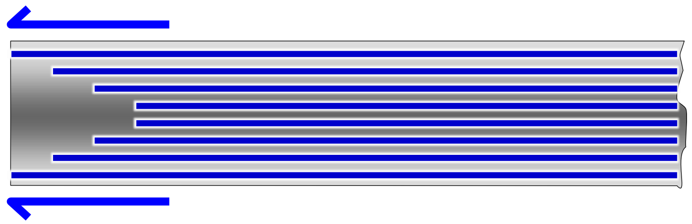
  - by gripping the sides near the end



→ Stress fields tend to become uniform as we move away from the discontinuity

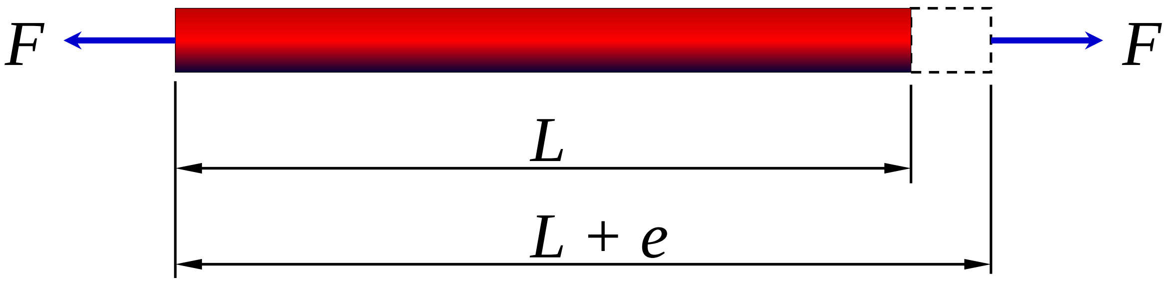
- For example, a ‘dogbone’ test specimen:





Normal Strain

- E.g. loaded bar:



$$\boxed{\varepsilon = \frac{e}{L}} \qquad \text{strain} = \frac{\text{extension}}{\text{original length}}$$

- Note that strain is non-dimensional, but is commonly stated as:
  - Percentage strain:
  - Micro strain:

$$\%\,\varepsilon = \varepsilon \cdot 100\%$$

$$\mu\varepsilon = \varepsilon \cdot 10^{-6}$$

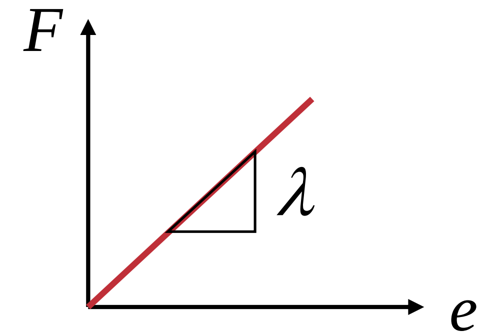
Stiffness and Modulus

KEY POINTS

Linear elastic behaviour:

$$\boxed{F = \lambda e}$$

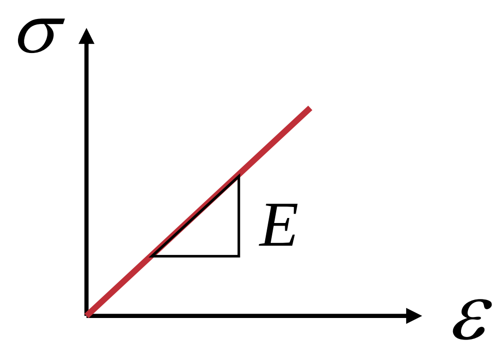
- where λ = "stiffness" λ = "lambda"
- units: force per unit displacement
- *e.g.* N/m, N/mm



In terms of stress and strain:

$$\boxed{\sigma = E\,\varepsilon}$$

- where E = "Young's modulus"
- units: stress per unit strain
- *i.e.* same as stress since strain is dimensionless
- *e.g.* N/mm^2 or N/m^2 or MPa or GPa



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Young's Modulus $E = \sigma / \varepsilon$

- Note, E is constant for a particular material and does not vary much within an alloy family, *e.g.*:

Steel:

E = 200,000 N/mm^2 = 200,000 MPa

= 200 kN/mm^2 = **200 GPa**

Titanium alloy:

E = 110,000 N/mm^2 = **110 GPa**

Aluminium alloy:

E = 70,000 N/mm^2 = **70 GPa**

(Note: 1kN/mm^2 = 1GPa)

Stiffness λ and Modulus E

- Relationship between stiffness and modulus:

$$\lambda = \frac{F}{e}$$

$$= \frac{\sigma A}{\varepsilon L} \quad \text{where } \sigma / \varepsilon = E$$

$$\boxed{\lambda = \frac{E A}{L}}$$