Light Aircraft Structures

Idealised Multi-Cell Sections – Shear

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Shear of Multi-Cell Sections

- Single-cell closed sections with up to three booms are statically determinate, *i.e.* they can be solved by equations of equilibrium alone (sufficient number of independent equilibrium equations)

- In a closed single-cell section with three booms we can solve for the three unknown shear flows using:

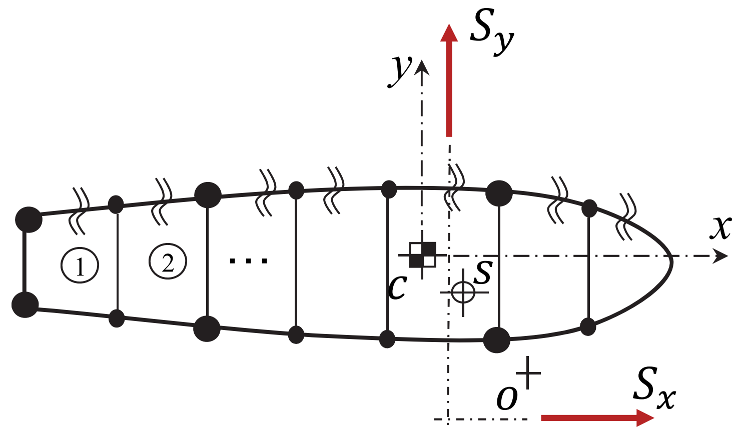
$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M = 0$$

- However for single-cell beams with more than three booms (and for multi-cell beams) we must supplement these with further arguments based on a constitutive relationship and compatibility

Shear of Multi-Cell Sections

- Consider an n-cell wing section subjected to shear loads S_x , S_y with lines of action not necessarily through the shear centre, so that we have a combination of transverse shear loading and torsion

c = centroid
s = shear centre
o = arbitrary point



- The method for determining the shear flow and rate of twist is simply an extension of the basic analysis, making a notional “cut” in each cell

- It is advisable to make the cut in the upper or lower skins for a well conditioned solution and reliable results

Shear of Multi-Cell Sections

- For cell j the complete shear flow distribution around the cell is given by the sum of the "open section" shear flow $q_{s_j}^{\text{open}}$ plus the "closed section" correcting shear flow q_{0_j}:

$$q_{s_j}^{\text{closed}} = q_{s_j}^{\text{open}} + q_{0_j}$$

- We can evaluate each cut section but we now also have an unknown value of q_{0_j} in each cell $j = 1 \ldots n$:

$$-q_{s_j}^{\text{closed}} = \left(\frac{S_x\, I_{xx} + S_y\, I_{xy}}{I_{xy}^2 - I_{xx}\, I_{yy}}\right) \sum_{i=1}^{n_s} x_i\, A_i + \left(\frac{S_y\, I_{yy} + S_x\, I_{xy}}{I_{xx}\, I_{yy} - I_{xy}^2}\right) \sum_{i=1}^{n_s} y_i\, A_i - q_{0_j}$$

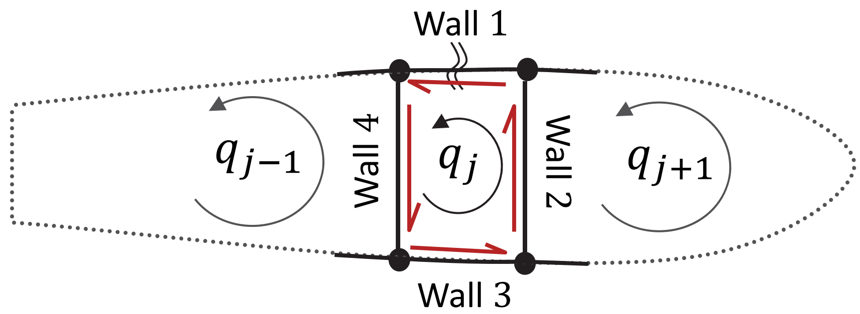
- The unknown rate of twist $\mathrm{d}\theta/\mathrm{d}z$ will be the same for each cell assuming an undistorted cross-section
  - Section shape is maintained by ribs at a reasonable pitch
- So we need n+1 equations

Shear of Multi-Cell Sections

- We can generate n equations by considering the rate of twist in each cell j:

$$\theta' = \frac{\mathrm{d}\theta}{\mathrm{d}z} = \frac{1}{2\,A_j\,G} \oint_j q_{sj} \frac{\mathrm{d}s}{t}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = \frac{1}{2\,A_j\,G} \oint_j \left(q_{s_j}^{\mathrm{open}} + q_{0_j}\right) \frac{\mathrm{d}s}{t}$$



Open-section shear flows at the j-th cell of an n-cell section subjected to shear

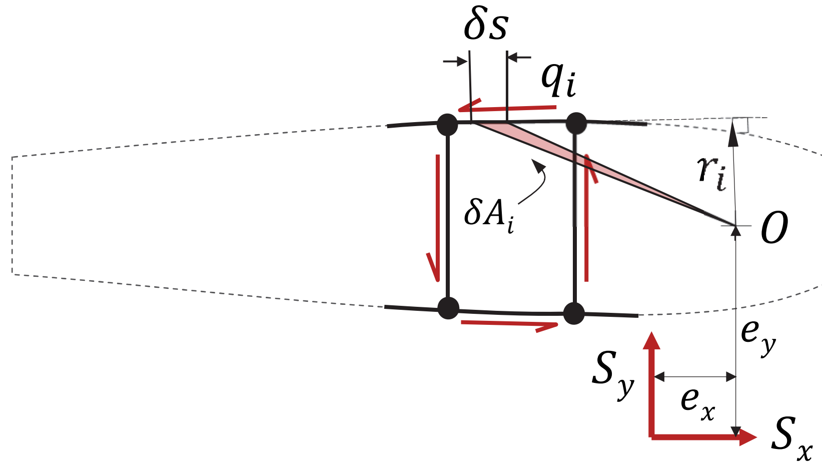
- Regarding $q_j = q_{s_j}^{\mathrm{open}} + q_{0_j}$ as a constant acting around cell j:

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = \frac{1}{2\,A_j\,G} \left[\overbrace{\oint_j q_{s_j}^{\mathrm{open}} \frac{\mathrm{d}s}{t}}^{\text{Open section solution}} + \overbrace{q_{0_j} \oint_j \frac{\mathrm{d}s}{t}}^{\text{Closed section constant}} - \overbrace{q_{j+1} \frac{b_2}{t_2}}^{\text{Wall 2}} - \overbrace{q_{j-1} \frac{b_4}{t_4}}^{\text{Wall 4}} \right]$$

- For discrete forms replace $\int$ with $\sum$, $\mathrm{d}s$ with b_i, t with t_i

Multi-cell Sections – Shear Centre

- As before, the shear centre can be found by picking a reference point O and balancing external moments with those generated by the shear flow in each cell:



$$S_y\, e_x - S_x\, e_y = \sum_{j=1}^{n} \oint_j q\, r\, \mathrm{ds}$$

$$S_y\, e_x - S_x\, e_y = \sum_{j=1}^{n} \left[\sum_{i=1}^{p} \int_i q\, r\, \mathrm{ds} \right]$$

$$S_y\, e_x - S_x\, e_y = \sum_{j=1}^{n} \left[\sum_{i=1}^{p} (q_i\, r_i\, b_i) \right]$$

- For walls of complex geometry (*e.g.* arcs) remember the area relationship:

$$\int_i q\, r\, \mathrm{ds} = 2\, A_i\, q_i$$