Stress, Strain and Deformation
Double Integration Method – The Heaviside Function

Dr Luiz Kawashita

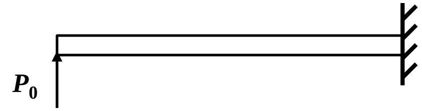
Luiz.Kawashita@bristol.ac.uk

22 November 2017

University of BRISTOL

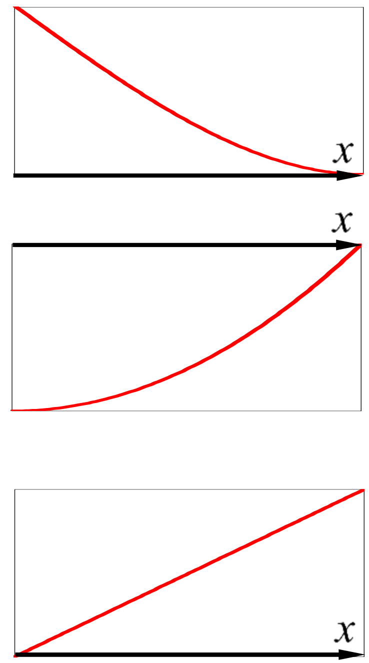
Double Integration Method (Recap)

- Take for example a tip-loaded cantilever:
  - Cantilever with tip load:



differentiate

  - Deflection: $v_{(x)}$
  - Slope: $\phi_{(x)} = \dfrac{\mathrm{d}\,v}{\mathrm{d}\,x}$
  - Curvature: $\kappa_{(x)} = \dfrac{\mathrm{d}^2\,v}{\mathrm{d}\,x^2}$



integrate

→ This is what we wish to find out

→ First derivative of deflection

→ Second derivative of deflection

  - Moreover, the Engineer's theory of bending states that:

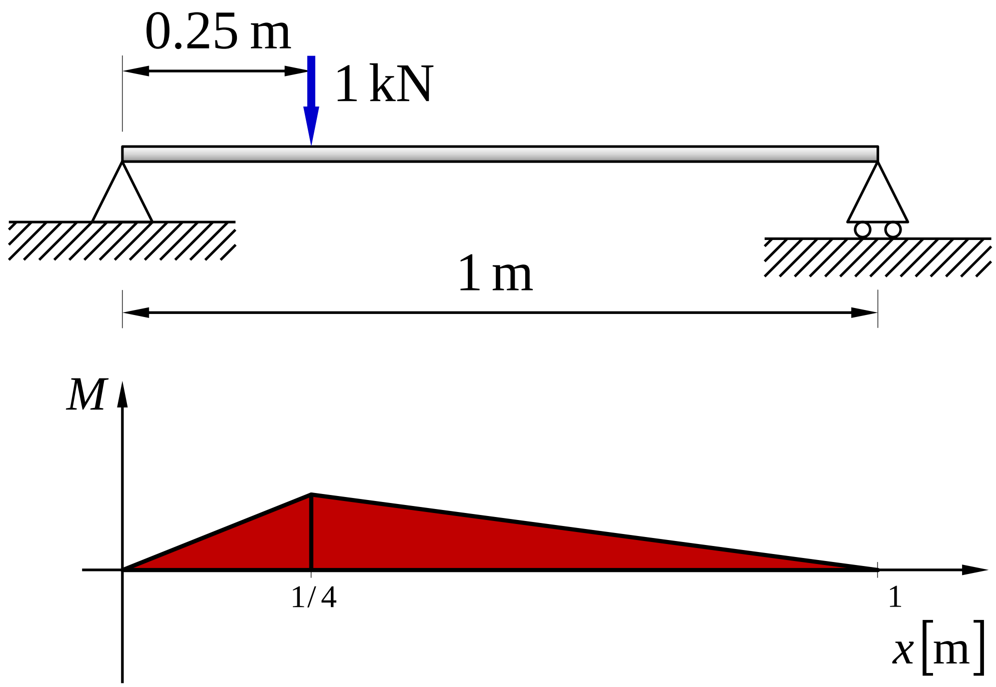
$$M_{(x)} = \frac{1}{R_{(x)}}\,E\,I = \kappa_{(x)}\,E\,I \quad \therefore \quad \boxed{M_{(x)} = EI\,\frac{\mathrm{d}^2\,v}{\mathrm{d}\,x^2}}$$

→ So we obtain deflections by **<u>integrating twice</u>** the moment equation!

University of BRISTOL

How to deal with 'sudden' bending moment changes?

- Example 2.3.2 (a):



For $0 \le x \le 0.25$ we have:

$$M_{(x)} = \frac{3}{4}x$$

But for $0.25 < x \le 1.0$ we have:

$$M_{(x)} = \frac{3}{4}x - \left(x - \frac{1}{4}\right)$$

How to deal with ‘sudden’ bending moment changes?

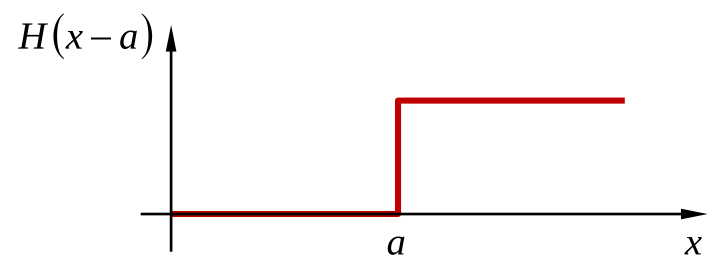
- Option 1: we can write it as an algorithm (*i.e.* an ‘if’ statement):

$$M_{(x)} = \begin{cases} \text{if } \left(0 \le x \le \dfrac{1}{4}\right), & \dfrac{3}{4}x \\ \text{else } \left(i.e.\ \dfrac{1}{4} < x \le 1\right), & \dfrac{3}{4}x - \left(x - \dfrac{1}{4}\right) \end{cases}$$

- Option 2: we can use an “algebraic ‘if’ statement”: the *Heaviside Step Function*:

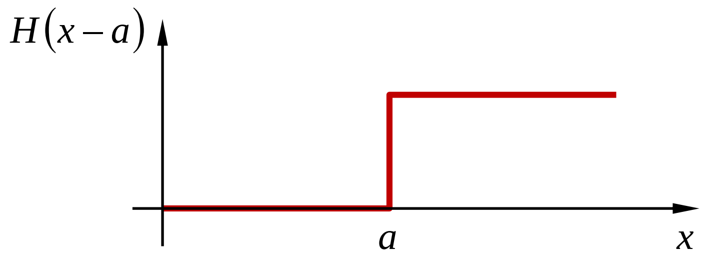
$$M_{(x)} = \frac{3}{4}x - \left(x - \frac{1}{4}\right)\underbrace{\left[H\left(x - \frac{1}{4}\right)\right]}_{\text{Heaviside step function}}$$

- Where $H(x-a) = \begin{cases} 0, & \text{for } x \le a \\ 1, & \text{for } x > a \end{cases}$



Heaviside Step Function

$$H(x-a)=\begin{cases}0, & \text{for } x \le a\\ 1, & \text{for } x > a\end{cases}$$



- The Heaviside step function is treated as an ‘**algebraic on/off switch**’

- It always follows the term being switched ‘on’ or ‘off’, but it is not affected by integration or differentiation

$$EI\frac{\mathrm{d}^2 v}{\mathrm{d}x^2}=\frac{3}{4}x^2-\left(x-\frac{1}{4}\right)\left[H\left(x-\frac{1}{4}\right)\right]$$

Integrating

$$EI\frac{\mathrm{d}v}{\mathrm{d}x}=\frac{3}{4}\frac{1}{2}x^2-\frac{1}{2}\left(x-\frac{1}{4}\right)^2\underbrace{\left[H\left(x-\frac{1}{4}\right)\right]}_{\text{Heaviside step function remains unchanged!}}+A$$

University of BRISTOL