UNIVERSITY OF BRISTOL
FACULTY OF ENGINEERING

First Year Examination for the Degree of Master of Engineering

MAY/JUNE 2011 3 Hours

AENG11100

FLUIDS I

Solutions

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Q 1

$$p - p_a = \rho g h = 2\times10^5 = 1000\times9.81\times h$$
$$h = 20.39m$$

(4 *marks*)

Q 2

$$B = \tfrac{4}{3}\pi r^3 \rho_{air} \times 9.81 = \left(50 + \tfrac{4}{3}\pi r^3 \rho_{gas} + M\right)\times 9.81$$
$$M = \tfrac{4}{3}\pi r^3\left(\rho_{air} - \rho_{gas}\right) - 50 = \tfrac{4}{3}\pi 3^3 (0.7) - 50 = 29.17Kg$$

(4 *marks*)

Q 3 Along a streamline: Steady, incompressible, inviscid.
Inside ducts must also assume 1D (or Quasi-1D) flow

(4 marks)

Q 4 Use continuity so

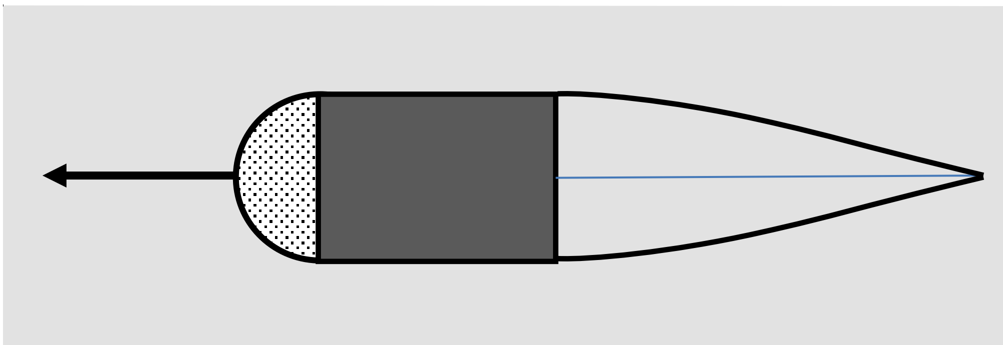
$$A_1V_1 = A_2V_2 \quad \rightarrow \quad = \frac{A_1}{A_2} = \frac{V_2}{V_1} = \frac{0.1}{0.4} = 4$$
$$P_{0_1} = p_1 + \tfrac{1}{2}\rho V_1^2 \quad \rightarrow \quad \tfrac{1}{2}\rho V_1^2 = \left(P_{0_1} - p_1\right)$$

Density of air is small so we neglect hydrostatic terms & use

$$P_{0_2} = P_{0_1} = 1.02\times10^5\, Nm^{-2}$$
$$p_1 + \tfrac{1}{2}\rho V_1^2 = p_2 + \tfrac{1}{2}\rho V_2^2 \quad \rightarrow \quad p_2 = p_1 + \tfrac{1}{2}\rho V_1^2\left(1 - \frac{V_2^2}{V_1^2}\right)$$
$$p_2 = p_1 + \left(P_{0_1} - p_1\right)\left(1 - 4^2\right) = 1.01\times10^5 - 0.01\times10^5\times15 = 0.86\times10^5\,\mathrm{Nm}^{-2}$$

(4 *marks*)

Q 5 (a)Increase in skin friction drag caused by the increased surface area is outweighed by the reduction in form drag due to the reduction (or elimination) of separated flow.



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(b) Galilean transformation , pressure and temperature remain unchanged

(4 *marks*)

Q 6

$$\mathrm{Re}_x = \frac{Vx}{\nu} \quad \rightarrow \quad x = \frac{\mathrm{Re}_x\,\nu}{V} = \frac{5\times10^5\times1.14\times10^{-6}}{5} = 0.114\mathrm{m}$$

Decelerating flow lowers the Reynolds number of transition.
Surface roughness & increased freestream turbulence produces early .

(4 marks)

Q 7 From continuity we know that the average velocity downstream of the sudden expansion must be

$$A_1V_1 = A_2V_2 \quad \rightarrow \quad \frac{V_2}{V_1} = \frac{A_1}{A_2} = \frac{1}{4}$$

Bernoulli, with pressure loss term

$$p_1 + \tfrac{1}{2}\rho V_1^2 + \rho g h_1 = p_2 + \tfrac{1}{2}\rho V_2^2 + \rho g h_2 + \Delta p_{loss}$$

$$p_2 = p_1 + \tfrac{1}{2}\rho V_1^2\left(1-\frac{V_2^2}{V_1^2}\right) + \rho g\left(h_1 - h_2\right) - \tfrac{1}{2}\rho V_1^2\left(1-\frac{A_1}{A_2}\right)^2 = p_1 + \tfrac{1}{2}\rho V_1^2\left(2\frac{A_1}{A_2} - 2\frac{A_1^2}{A_2^2}\right) + \rho g\left(h_1 - h_2\right)$$

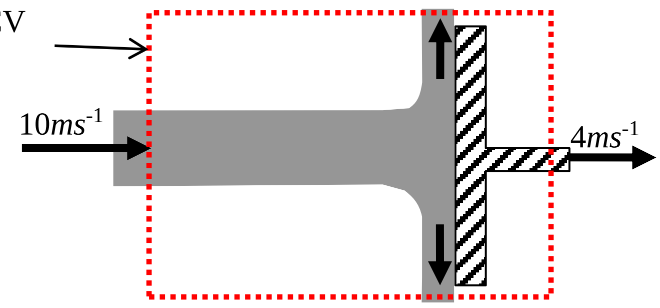
$$p_2 = 2\times10^5 + 1000\times4^2\left(\frac{3}{16}\right) + 1000\times9.81\times3 = 232430\text{N/m}^2$$

(4 marks)

Q 8 Consider a control volume fixed relative to the plate. The net horizontal force on the control volume equals the rate of change of momentum in that direction. We also assume that atmospheric pressure acts through the jet diameter so there is no contribution to the net horizontal force from the jet entry into the CV.

$$F_{CVx} = \dot{m}\left(V_2 - V_1\right) = \pi r^2 V\rho\left(0 - V\right) = -\pi\times0.05^2\times6\times1000\times6 = -282.74N$$

$$F = -F_{CVx} = 282.74N$$



(4 marks)

Q 9

$$c_p = \frac{p - p_\infty}{\frac{1}{2}\rho U_\infty^2}$$

p : local static pressure

p_∞ : freestream static pressure

ρ : density

U_∞ : freestream velocity

From Bernoulli's equation applied between the freestream and the local point

$$p_1 - p_\infty = \frac{1}{2}\rho\left(U_\infty^2 - V_1^2\right)$$

$$c_p = \frac{\frac{1}{2}\rho\left(U_\infty^2 - V_1^2\right)}{\frac{1}{2}\rho U_\infty^2} = 1 - \frac{V^2}{U_\infty^2}$$

(4 marks)

Q 10 From given equations, source provides a horizontal & vertical velocities:

$$u = \frac{+\Lambda}{2\pi}\frac{x}{\left(x^2 + y^2\right)} \qquad v = \frac{+\Lambda}{2\pi}\frac{y}{\left(x^2 + y^2\right)}$$

Total velocities therefore

$$U = 4 + 0 = 4ms^{-1} \qquad V = \frac{5}{2\pi}\frac{10}{\left(0^2 + 10^2\right)} = 0.0796ms^{-1}$$

(4 marks)

Q11

a) Length of base of dam

$$l = \frac{H}{\tan(\alpha)}$$

Vertical force, F_v, equals weight of water above dam surface

$$F_v = \rho_w g \times \tfrac{1}{2} \times \frac{H}{\tan(\alpha)} \times H \times L = \rho_w g \frac{H^2 L}{2\tan(\alpha)}$$

Clockwise moment from vertical thrust given by

$$M_v = -F_v \frac{2}{3}\frac{H}{\tan(\alpha)} = \rho_w g \frac{H^3 L}{3\tan^2(\alpha)}$$

Weight of dam

$$F_D = \rho_D g \times \tfrac{1}{2} \times \frac{H}{\tan(\alpha)} \times H \times L = \rho_D g \frac{H^2 L}{2\tan(\alpha)}$$

Clockwise moment from dam weight given by

$$M_D = -F_D \frac{1}{3}\frac{H}{\tan(\alpha)} = \rho_D g \frac{H^3 L}{6\tan^2(\alpha)}$$

Horizontal component of thrust from the left is given by

$$F_{HL} = \rho_w g \times \frac{H}{2} \times H \times L - = \rho_w g \frac{H^2 L}{2}$$

While the horizontal component of thrust from the left is given by

$$F_{HR} = \rho_w g \times \frac{h}{2} \times h \times L - = \rho_w g \frac{h^2 L}{2}$$

The vertical forces act at depths

$$h_{HL} = \frac{H}{2} + \frac{I_{xx}}{\frac{H}{2} \times H \times L} = \frac{H}{2} + \frac{LH^3}{12 \times \frac{H}{2} \times H \times L} = \frac{H}{2} + \frac{H}{6} = \frac{2H}{3}$$

$$h_{HR} = \frac{2h}{3}$$

Clockwise moment from the left and right horizontal thrust is given by

$$M_H = F_{HL} \times \left(H - \tfrac{2}{3}H\right) - F_{HR} \times \left(h - \tfrac{2}{3}h\right) = \rho_w g L\left(\frac{H^2}{2} \times \frac{H}{3} - \frac{h^2}{2} \times \frac{h}{3}\right) = \rho_w g L\left(\frac{H^3}{6} - \frac{h^3}{6}\right)$$

The total moment is therefore

$$M = M_D + M_V + M_H = -\rho_D g \frac{H^3 L}{6\tan^2(\alpha)} - \rho_w g \frac{H^3 L}{3\tan^2(\alpha)} + \rho_w g L\left(\frac{H^3}{6} - \frac{h^3}{6}\right)$$

$$M = \frac{gH^3 L}{6}\left(\rho_w\left(1 - \frac{h^3}{H^3}\right) - \frac{\rho_D}{\tan^2(\alpha)} - \frac{2\rho_w}{\tan^2(\alpha)}\right)$$

$$M = \frac{gH^3 L}{6}\left(\rho_w\left(1 - \frac{h^3}{H^3}\right) - \frac{\rho_D}{\tan^2(\alpha)} - \frac{2\rho_w}{\tan^2(\alpha)}\right) = 0 \quad \rightarrow \quad \rho_w\left(1 - \frac{h^3}{H^3}\right) = \frac{1}{\tan^2(\alpha)}\left(\rho_D + 2\rho_w\right)$$

$$\tan(\alpha) = \sqrt{\frac{(\rho_D + 2\rho_w)}{\rho_w}\frac{H^3}{(H^3 - h^3)}}$$

(13 marks)

(b)

Lightest prism has the largest value of α that doesn't tip so from (a)

$$\tan(\alpha) = \sqrt{\frac{(2.5\rho_w + 2\rho_w)}{\rho_w}\frac{5^3}{(5^3 - 0^3)}} = \sqrt{4.5} \quad \rightarrow \quad \alpha = 64.76^0$$

Also from part (a)

$$Mass = \rho_D \frac{H^2 L}{2\tan(\alpha)} = 2.5 \times 1000 \frac{5^2 \times 10}{2\sqrt{4.5}} = 147314 Kg$$

(7 marks)

Q12

Lower reservoir fluid just in throat when

$$p_t = p_a + \rho_v gh$$

Applying continuity between throat & exit

$$A_e V_e = A_t V_t \quad \rightarrow \quad \frac{V_t}{V_e} = \frac{A_e}{A_t} = \left(\frac{D_e}{D_t}\right)^2$$

Applying Bernoulli's equation between the upper reservoir surface and the exit (assuming the reservoir is large enough to ignore the dynamic contribution at the surface)

$$p_a + \rho_w gH = p_a + \tfrac{1}{2}\rho_w V_e^2 \quad \rightarrow \quad V_e^2 = 2gH$$

Applying Bernoulli's equation between the throat and the exit

$$p_t + \tfrac{1}{2}\rho_w V_t^2 = p_a + \tfrac{1}{2}\rho_w V_e^2 \quad \rightarrow \quad p_a - p_t = \tfrac{1}{2}\rho_w\left(V_t^2 - V_e^2\right) = \tfrac{1}{2}\rho_w V_e^2\left(\frac{V_t^2}{V_e^2} - 1\right)$$

Applying continuity

$$p_a - p_t = \tfrac{1}{2}\rho_w V_e^2\left(\left(\frac{D_e}{D_t}\right)^4 - 1\right)$$

Applying exit velocity relation

$$p_a - p_t = \rho_w gH\left(\left(\frac{D_e}{D_t}\right)^4 - 1\right)$$

Equating to static condition from lower reservoir

$$p_a - p_t = \rho_w gH\left(\left(\frac{D_e}{D_t}\right)^4 - 1\right) = \rho_v gh$$

Minimum value of H given by

$$H_{\min} = \frac{\rho_v}{\rho_w}\frac{h}{\left(\left(\frac{D_e}{D_t}\right)^4 - 1\right)} = h\frac{\rho_v}{\rho_w}\frac{D_t^4}{\left(D_e^4 - D_t^4\right)}$$

(13 marks)

(b)

$$H = SG\frac{hD_t^4}{(D_e^4 - D_t^4)} \quad \rightarrow \quad SG = \frac{H}{h}\frac{(D_e^4 - D_t^4)}{D_t^4} = \frac{30}{1}\frac{(0.1^4 - 0.05^4)}{0.05^4} = 450$$

From previously

$$V_e^2 = 2gH$$

$$V_e = \sqrt{2 \times 9.81 \times 30} = 24.26 ms^{-1}$$

(7 marks)

Q13

(a) From the equation of state for the compressed air, assuming the temperature does not change

$$p \times (H-h)A = rp_a \times (H - sH)A$$

$$p = rp_a \times \frac{(H-sH)}{(H-h)}$$

Ignoring the hydrostatic effect applied to the air (because the density of air is small relative to water and the dynamic pressure effects are large) and applying bernoullis equation between the instantaneous surface of the water and the exit:

$$p_e + \tfrac{1}{2}\rho_w V_e^2 = p + \tfrac{1}{2}\rho_w V_h^2 + \rho_w gh$$

Where V_h is the velocity of the water surface and p is the pressure in the compressed air. Using the previous pressure relation and that the jet is at atmospheric pressure

$$p_e + \tfrac{1}{2}\rho_w V_e^2 = p + \tfrac{1}{2}\rho_w V_h^2 + \rho_w gh \quad \rightarrow \quad p_a + \tfrac{1}{2}\rho_w V_e^2 = rp_a\left(\frac{H-sH}{H-h}\right) + \tfrac{1}{2}\rho_w V_h^2 + \rho_w gh$$

Applying continuity we have

$$AV_h = A_e V_e$$

Therefore

$$p_a + \tfrac{1}{2}\rho_w V_e^2 = rp_a\left(\frac{H-sH}{H-h}\right) + \tfrac{1}{2}\rho_w\left(\frac{A_e}{A}\right)^2 V_e^2 + \rho_w gh$$

Rearranging

$$\tfrac{1}{2}\rho_w V_e^2 - \tfrac{1}{2}\rho_w\left(\frac{A_e}{A}\right)^2 V_e^2 = rp_a\left(\frac{H-sH}{H-h}\right) - p_a + \rho_w gh$$

$$V_e^2\left[\tfrac{1}{2}\frac{\rho_w}{A^2}\left(A^2 - A_e^2\right)\right] = p_a\left(r\left(\frac{H-sH}{H-h}\right) - 1\right) + \rho_w gh$$

$$V_e^2 = A^2\left\{\frac{p_a\left(r\left(\frac{H-sH}{H-h}\right) - 1\right) + \rho_w gh}{\tfrac{1}{2}\rho_w\left(A^2 - A_e^2\right)}\right\}$$

$$V_e = A\left\{\frac{p_a\left[r\left(\frac{H-sH}{H-h}\right) - 1\right] + \rho_w gh}{\tfrac{1}{2}\rho_w\left(A^2 - A_e^2\right)}\right\}^{\frac{1}{2}}$$

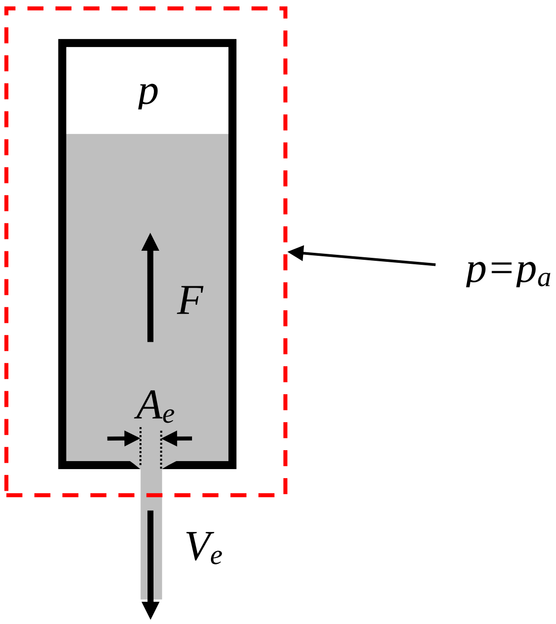
(10 marks)

(b) From control volume analysis for the volume shown below we can write

$$F = \dot{m} \times (V_{in} - V_{out}) = A_e V_e \, \rho_w \times \left(0 - (-V_e)\right) = A_e \rho_w V_e^2$$

$$F = A_e A^2\left\{\frac{p_a\left[r\left(\frac{H-sH}{H-h}\right) - 1\right] + \rho_w gh}{\tfrac{1}{2}\rho_w\left(A^2 - A_e^2\right)}\right\}\rho_w = 2A_e A^2\left\{\frac{p_a\left[r\left(\frac{H-sH}{H-h}\right) - 1\right] + \rho_w gh}{\left(A^2 - A_e^2\right)}\right\}$$

From the terms in the numerator we can see that they decrease with h, therefore the maximum force is found initially, so



$$F_{\max} = 2A_e A^2 \left\{ \frac{p_a[r-1] + \rho_w gsH}{\left(A^2 - A_e^2\right)} \right\}$$

$$= 2\times\left(\pi\times 0.003^2\right)\times\left(\pi\times 0.05^2\right)^2\times\left\{\frac{1.013\times 10^5\times[4] + 1000\times 9.81\times 0.2}{\left(\pi\times 0.05^2\right)^2 - \left(\pi\times 0.003^2\right)^2}\right\}$$

$$= 2\times\left(\pi\times 0.003^2\right)\times\left\{\frac{1.013\times 10^5\times[4] + 1000\times 9.81\times 0.2}{1-\left(\frac{0.3}{5}\right)^4}\right\} = 23.025N$$

As fluid is just about to run out h becomes zero, therefore

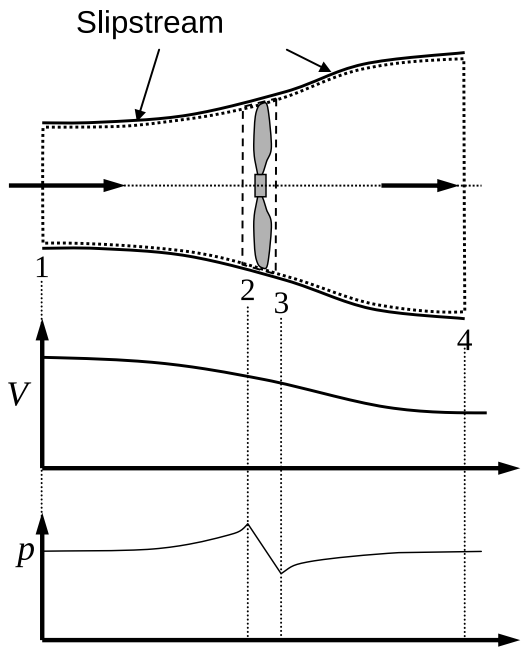
$$F_{final} = 2A_e A^2 \left\{ \frac{p_a[r(1-s)-1]}{\left(A^2 - A_e^2\right)} \right\}$$

$$= 2\times\left(\pi\times 0.003^2\right)\times\left\{\frac{1.013\times 10^5\times\left[5\left(1-\frac{2}{3}\right)\right]}{1-\left(\frac{0.3}{5}\right)^4}\right\} =$$

$$2\times\left(\pi\times 0.003^2\right)\times\left\{\frac{1.013\times 10^5\times\frac{5}{3}}{1-\left(\frac{0.3}{5}\right)^4}\right\} = 9.547N$$

(10 marks)

Q14 **(a)** Use the actuator disc theory for an ideal windmill, see figure below



Assumptions: Frictionless & incompressible, Steady 1D flow (neglect rotation and variation across the disc radius). Actuator disc is thin so $A_2=A_3=A_d$ & $V_2=V_3=V_d$. $p=p_a$ at all points on slipstream boundary & 1 & 4

Continuity: $Q=V_d A_d$

Bernoulli's equation for CV 1-2 & CV 3-4

$$\begin{aligned} p_1+\tfrac{1}{2}\rho V_1^2 &= p_2+\tfrac{1}{2}\rho V_d^2 \\ p_3+\tfrac{1}{2}\rho V_d^2 &= p_4+\tfrac{1}{2}\rho V_4^2 \end{aligned} \quad \rightarrow \quad p_3-p_2=\tfrac{1}{2}\rho\left(\frac{V^2}{a^2}-V^2\right)$$

Steady Flow momentum for CV 2-3:

$$(p_2-p_3)A_d+F_{CV}=\rho Q(V_d-V_d)=0 \qquad \rightarrow \quad F_{CV}=(p_3-p_2)A_d$$

Where F is the force on the control volume

Applying results from Bernoulli's equation above

$$F_{CV}=\tfrac{1}{2}\rho A_d V^2\left(\frac{1-a^2}{a^2}\right)=\tfrac{\pi}{8}\rho d^2V^2\left(\frac{1-a^2}{a^2}\right)$$

Force on the windmill is equal and opposite to the force on the CV so

$$F=\rho\tfrac{\pi}{8}d^2V^2\left(\frac{a^2-1}{a^2}\right)$$

(8 marks)

(b) Steady Flow momentum for CV 1-4:

$$0+F_{CV}=\rho Q(V_4-V_1) \qquad \rightarrow \quad F_{CV}=\rho V_d A_d V\left(\frac{1-a}{a}\right)$$

From momentum & continuity

$$(p_3 - p_2)A_d = \rho V_d A_d (V_4 - V_1)$$

Eliminating $(p_3$-$p_2)$ using Bernoulli's equation above

$$\rho V_d\left(\frac{V}{a} - V\right) = \tfrac{1}{2}\rho\left(\frac{V^2}{a^2} - V^2\right)$$

$$V_d V\left(\frac{1}{a} - 1\right) = \tfrac{1}{2}V^2\left(\frac{1}{a} + 1\right)\left(\frac{1}{a} - 1\right)$$

$$V_d = \tfrac{1}{2}V\left(\frac{1}{a} + 1\right)$$

The power drawn from the air by the disc is

$$P_{\text{disc}} = -F_{CV}V_d = -\rho Q(V_4 - V_1)V_d = \tfrac{1}{2}\rho A_d V_d\,(V_1 - V_4)V_d = \tfrac{1}{4}\rho A_d\left(\frac{V}{a} + V\right)\left(V^2 - \frac{V^2}{a^2}\right)$$

Power in the wind passing through the disc area, if the disc were not present.

$$P_{\text{wind}} = \tfrac{1}{2}\dot{m}V_1^2 = \tfrac{1}{2}\rho A_d V_1\,V_1^2 = \tfrac{1}{2}\rho A_d V^3$$

The efficiency of the turbine is therefore

$$\eta = \frac{P_{\text{disc}}}{P_{\text{wind}}} = \frac{\tfrac{1}{4}\rho A_d\left(\frac{V}{a} + V\right)\left(V^2 - \frac{V^2}{a^2}\right)}{\tfrac{1}{2}\rho A_d V^3} = \frac{\left(\frac{V}{a} + V\right)\left(V^2 - \frac{V^2}{a^2}\right)}{2V^3} = \frac{1}{2}\left(\frac{1+a}{a}\right)\left(\frac{a^2 - 1}{a^2}\right) = \frac{a^3 + a^2 - a - 1}{2a^3}$$

(6 marks)

(c) from the definition of efficiency in (b) we have

$$\eta = 0.5 = \frac{a^3 + a^2 - a - 1}{2a^3} \qquad \rightarrow \qquad a^2 - a - 1 = 0$$

$$a = \begin{cases} \frac{1-\sqrt{5}}{2} \\ \frac{1+\sqrt{5}}{2} \end{cases}$$

$a = \frac{\sqrt{5}+1}{2}$ is the only feasible solution

$$F = \rho\tfrac{\pi}{8}d^2V^2\left(\frac{a^2 - 1}{a^2}\right) = 1.2\times\tfrac{\pi}{8}\times 5^2 \times 16^2\left(\frac{\left(\frac{\sqrt{5}+1}{2}\right)^2 - 1}{\left(\frac{\sqrt{5}+1}{2}\right)^2}\right) = 1863.9N$$

$$V_d = \tfrac{1}{2}V\left(\frac{a+1}{a}\right) = \tfrac{1}{2}\times 16\times\left(\frac{\frac{\sqrt{5}+1}{2} + 1}{\frac{\sqrt{5}+1}{2}}\right) = \tfrac{1}{2}\times 16\times\left(\frac{\sqrt{5}+1+2}{\sqrt{5}+1}\right) = 12.944m/s$$

From Bernoulli's equations in (a)

$$p_2 - p_a = \tfrac{1}{2}\rho\left(V^2 - V_d^2\right) = \tfrac{1}{2}\times 1.2\times\left(16^2 - 12.944^2\right) = 53.07N/m^2$$

$$p_3 - p_a = \tfrac{1}{2}\rho\left(\frac{V^2}{a^2} - V_d^2\right) = \tfrac{1}{2}\times 1.2\times\left(2^2\,\frac{16^2}{\left(\sqrt{5}+1\right)^2} - 12.944^2\right) = -41.86N/m^2$$

(6 marks)

Q15

(a) The stream function is given by

$$\boxed{\psi = U_\infty y - \frac{\kappa}{2\pi}\frac{\sin\theta}{r} = U_\infty r\sin\theta\left(1-\frac{\kappa}{2\pi U_\infty r^2}\right)}$$

The velocity components are given by

$$V_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \left(1-\frac{\kappa}{2\pi U_\infty r^2}\right)U_\infty\cos\theta,\quad V_\theta = -\frac{\partial\psi}{\partial r} = -\left(1+\frac{\kappa}{2\pi U_\infty r^2}\right)U_\infty\sin\theta$$

Now the cylinder is a stream line of the flow so there is no flow normal to the cylinder i.e.

$$V_r = 0$$

This means that

$$\left(1-\frac{\kappa}{2\pi U_\infty r^2}\right)U_\infty\cos\theta = 0$$

for all θ so the circulation must be given by

$$\kappa = 2\pi U_\infty R^2$$

The velocity components are then

$$V_r = \left(1-\frac{R^2}{r^2}\right)U_\infty\cos\theta,\quad V_\theta = -\left(1+\frac{R^2}{r^2}\right)U_\infty\sin\theta$$

On the cylinder

$$V_r = 0,\quad V_\theta = -2U_\infty\sin\theta$$

The pressure coefficient on the cylinder is given by

$$\boxed{C_p = 1-\left(\frac{V}{U_\infty}\right)^2 = 1-4\sin^2\theta}$$

The pressure distribution on the cylinder is then

$$p(\theta) = p_\infty + \frac{1}{2}\rho U_\infty^2(1-4\sin^2\theta)$$

(6 marks)

(b) To find the net lift need lift forces on upper and lower surfaces. Now stagnation under the semi-cylinder

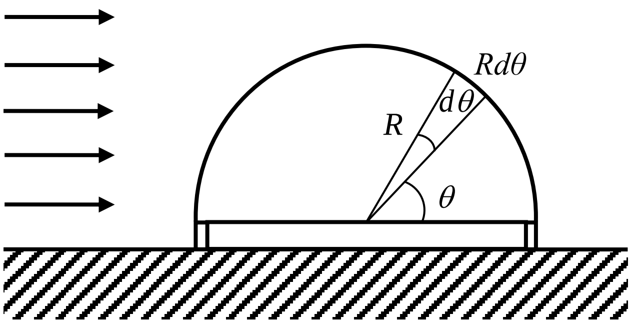
$$l_l = 2p_0 R$$

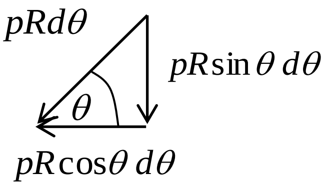
Where p_o is the total pressure, so need to find l_u

Upper Surface

Consider a small arc of the surface, as shown in the sketch, of size $Rd\theta$. The force acting on this element is given by

$$p(\theta)Rd\theta$$





this acts normal to the surface and must be resolved to get the components (see above)

So the force in the vertical direction over the entire upper surface is given by

$$-\int_0^{\pi} p(\theta) R \sin\theta d\theta$$

So the net lift is given by

$$l = 2p_0 R - \int_0^{\pi} p(\theta) R \sin\theta d\theta$$

where

$$p_0 = p_\infty + \frac{1}{2}\rho U_\infty^2$$

(6 marks)

Now using the results from part (a)

$$p(\theta) = p_\infty + \frac{1}{2}\rho U_\infty^2 (1 - 4\sin^2\theta)$$

find that

$$l = 2p_\infty R + \rho U_\infty^2 R - \int_0^{\pi} [p_\infty + \frac{1}{2}\rho U_\infty^2 (1 - 4\sin^2\theta)] R \sin\theta d\theta$$

$$= 2p_\infty R + \rho U_\infty^2 R - R\int_0^{\pi} [(p_\infty + \frac{1}{2}\rho U_\infty^2)\sin\theta - 2\rho U_\infty^2 \sin^3\theta)] d\theta$$

$$= 2p_\infty R + \rho U_\infty^2 R - R(p_\infty + \frac{1}{2}\rho U_\infty^2)\int_0^{\pi} \sin\theta d\theta + 2\rho U_\infty^2 R \int_0^{\pi} \sin^3\theta d\theta$$

$$= 2p_\infty R + \rho U_\infty^2 R - 2R(p_\infty + \frac{1}{2}\rho U_\infty^2) + 2\rho U_\infty^2 R \frac{4}{3}$$

So finally the net lift force acting on the greenhouse is equal to

$$l = \frac{8}{3}\rho_\infty U_\infty^2 R$$

(8 *marks)*