Structural Loads in Trusses
Definitions and Conventions

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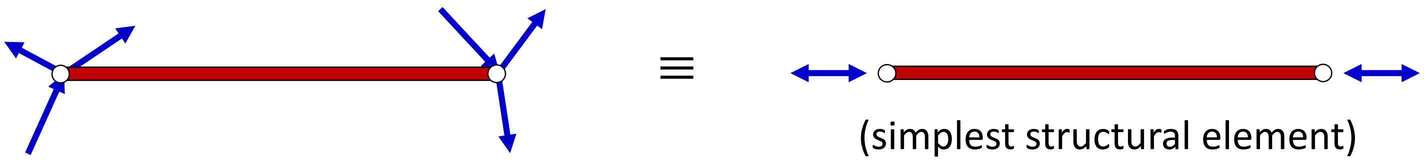
- Introduction
  - Assumptions & idealisations
  - Truss elements
  - Equilibrium
  - Sign conventions
  - Joints & supports
  - Redundancy
- Analysis methods
  - Method of Joints
  - Method of Sections
  - Method of Tension Coefficients

Assumptions and Idealisation

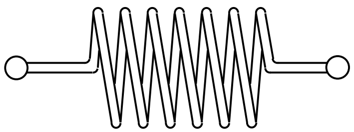
- Elements are ‘1D’ (thickness effects neglected) and straight
- Elements meet at a point
- Loads are applied at joints only
- Element weight is negligible compared with applied forces
- Joints are pinned, i.e. transmit forces but not moments
  - Or individual elements are much more flexible than the assembled structure, i.e. moments transmitted are negligible compared with axial forces)
- The truss structure is statically determinate or simply stiff

Truss Elements: Bars, Rods, Ties, Struts

- Definition - a structural member that carries **axial loads only**



- The resultant of any system of forces acting at the ends of a truss element must be **axial** - resulting in either **pure tension** or **compression**



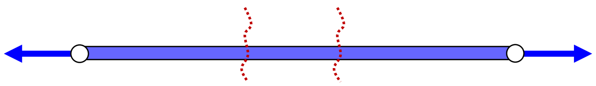
(i.e. no moments)

- Tension member = **Tie**

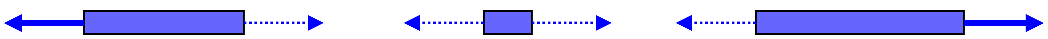
- Compression member = **Strut**

Truss Element Equilibrium

Example: bar in tension



- Consider cut member FBDs:



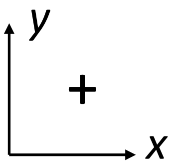
(internal forces shown dashed)

- All parts must be in equilibrium
- All parts must be in tension or compression
- Tensile force must be constant along length
- **Forces must be <u>equal</u> and <u>opposite</u> on each side**

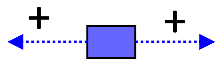
Sign Convention

There are **two** sign conventions to consider:

- **External** sign convention
  - Defined by positive direction of **reference axes**
  - e.g. displacement is positive when upwards and/or to the right:



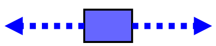
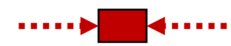
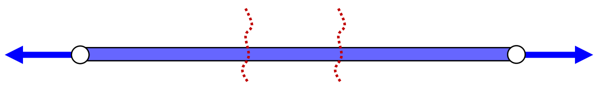
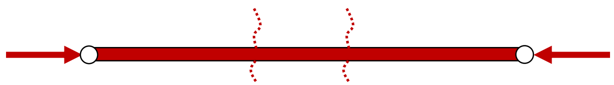
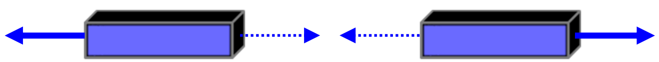
- **Internal** sign convention
  - Defined by **deformation** convention
  - e.g. **tensile** deformation is **positive**:



- Note: the choice of positive convention is **arbitrary**, but once chosen **all interpretations** must be **consistent**

Internal Sign Convention

KEY POINTS

Positive (+ve)	Negative (-ve)
Tension	**Compression**
	
	
	

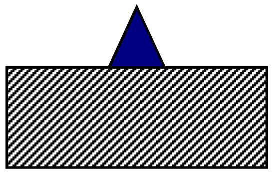
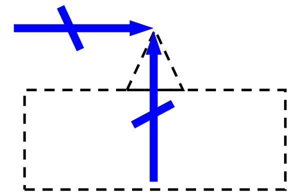
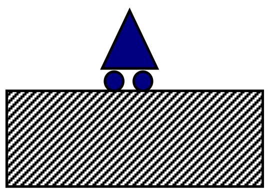
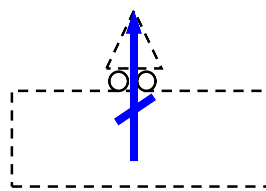
Watch out for internal force arrows drawn on complete elements:

Tension

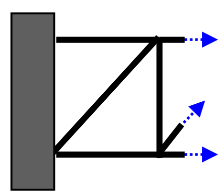
Compression

At first the arrows denoting internal load on an element may seem counter-intuitive; to understand them consider the **direction** of **internal forces** at **exposed faces** required for equilibrium of a cut element (as shown above)

Supports

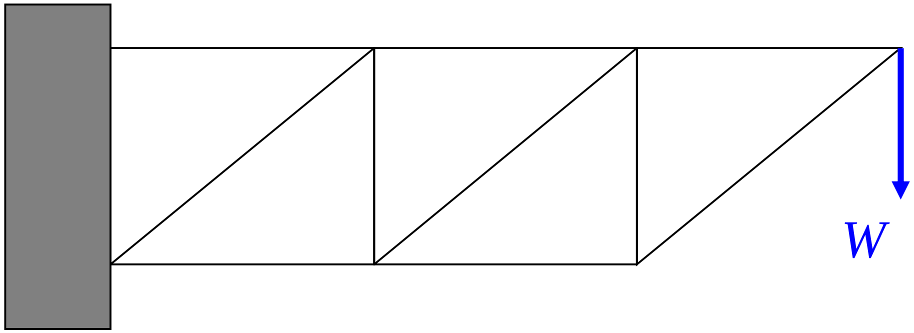
	Support	Possible reactions	Degrees of Freedoms
Pinned			**Rotational** freedom but **no translational** freedom
Roller			**Rotational** freedom + **translational** freedom in **direction of roller surface**

- **Pinned** condition is generally assumed for pin jointed truss structures. Symbols are often omitted in diagrams – members are simply shown as built-in:



Example

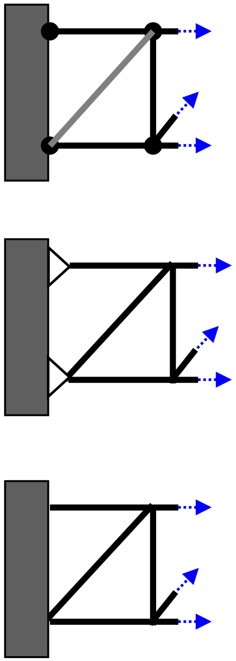
- Let us consider a light aircraft rear fuselage with a tail load:



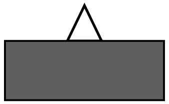
- It is made up from “truss” elements:
  - carrying axial forces only
- Ideal for modelling structures consisting of long slender members

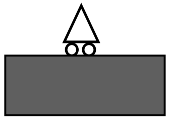
Joints and Supports

- All joints between elements & supports are assumed to be ‘**pinned**’
  - i.e. **constrained** in **translation** but with **rotational freedom**
- Can either be represented by:
  - Solid dots at joints within the structure
  - Triangles at supports
  - (or sometimes not represented but assumed)



- For translational freedom we use ‘**rollers**’:





Key concepts used in the analysis of trusses:

KEY POINTS

1. Static Equilibrium

- In static equilibrium all **forces** and **moments** acting on a structure are **balanced**

  In 2D:
  - Horizontal forces $\sum F_x = 0$
  - Vertical forces $\sum F_y = 0$
  - Moments/couples $\sum M = 0$

- If equilibrium is satisfied **globally,** then it is also satisfied **locally**
  - Global: entire structure
    - Forces & reactions cancel out
  - Local: components and ‘cut-outs’
    - ‘External’ and ‘internal’ forces cancel out

2. Pin-Jointed Trusses

- Members carry **only axial loads**
  - Tensile or compressive
- Pin-joints **do not transmit moments**
  - Ideal pins and/or very slender members
- **Forces** are applied at **joints only**
  - Members loaded at their extremities
  - Self-weight is neglected

3. Small Displacements

- Theory is only valid when **deformation** is **negligible** (i.e. loads are relatively small)
- **Angles** between members are assumed to **remain constant**

Assumptions and Idealisation (Recap)

KEY POINTS

- Elements are '**1D**' (thickness effects neglected) and **straight**
- Elements meet at a **point**
- **Loads** are applied at **joints only**
- Element **weight** is **negligible** compared with applied forces
- Joints are **pinned**, i.e. **transmit forces** but **not moments**
  - Or individual elements are much more flexible than the assembled structure, i.e. moments transmitted are negligible compared with axial forces)
- The truss structure is **statically determinate** or **simply stiff**

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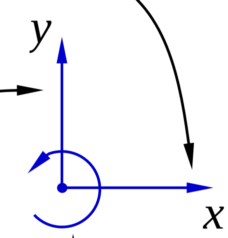
Equilibrium Equations

KEY POINTS

- We focus on **planar 2D** problems
- There are **two** orthogonal **coordinates**
- Therefore there are **three equations of equilibrium**:
  - Equilibrium of horizontal forces: $$\sum F_x = 0$$
  - Equilibrium of vertical forces: $$\sum F_y = 0$$
  - Equilibrium of moments/couples: $$\sum M = 0$$



- These three equations apply **locally** and **globally** within a **structure**
- However, when analysing **individual pin joints** the last equation is not very useful because **pin joints do not transmit moments**

Structural Loads in Trusses
Degree of Redundancy

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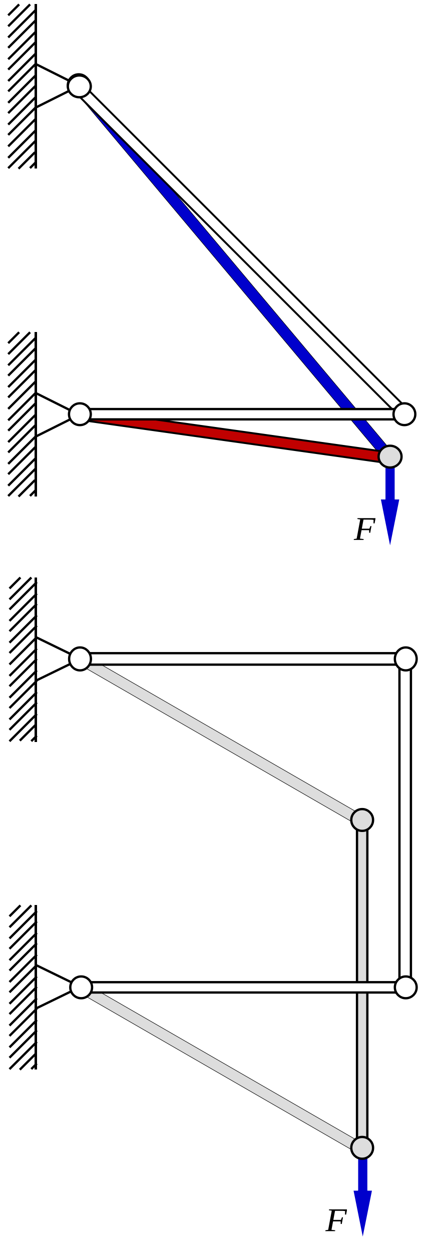
Deformation vs. Rigid Body Motion (of Truss Structures)

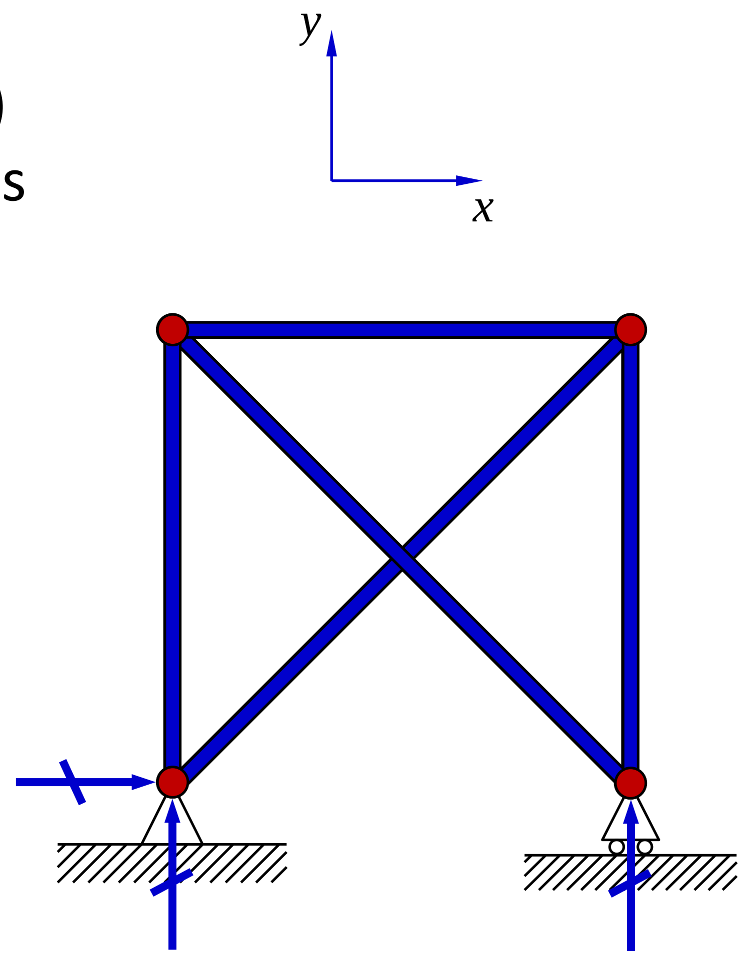
- Deformation
  - Joints move in space (1D, 2D or 3D)
  - Elements **shorten** or **elongate**
  - E.g. ‘elasticity’ problems
- Rigid Body Motion
  - Joints move in space (1D, 2D or 3D)
  - Lengths remain unchanged – elements simply rotate or translate in space (i.e. behave as ideal ‘**rigid bodies**’)
  - E.g. ‘rigid body dynamics’ problems



Degree of Redundancy

To determine the Degree of Redundancy (DoR) of a truss structure we need to find out:

- The number of Degrees of Freedom (DoF) per joint – for 2D pin-jointed trusses this is always **2**
- The total number of joints (including unconstrained and constrained)
- The number of unknown forces: element internal forces + reaction forces



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Degree of Redundancy

For 2D pinned frames:

KEY POINTS

N_u = N$^{o.}$ **unknowns** = N$^{o.}$ of reactions (N_r) + N$^{o.}$ of member forces (N_f)
(pinned=2, roller=1) (No. of truss elements)

N_e = N$^{o.}$ of **equations** = N$^{o.}$ joints (N_j) × N$^{o.}$ degrees of freedom per joint (N_{DoF})
(=2 for 2D trusses)

Degree of Redundancy (DoR) = $N_u - N_e$

< 0 : mechanism (unstable; kinematically indeterminate)

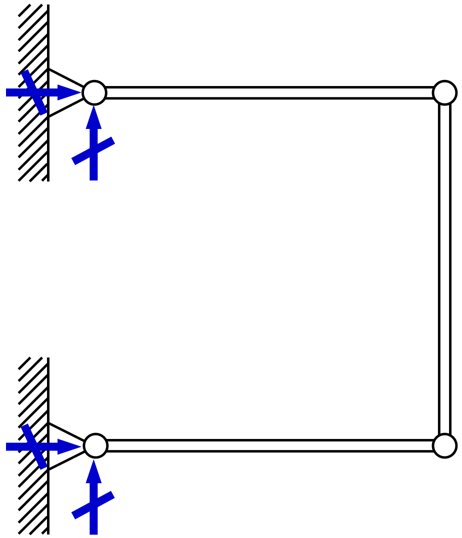
= 0 : statically determinate (simply stiff)

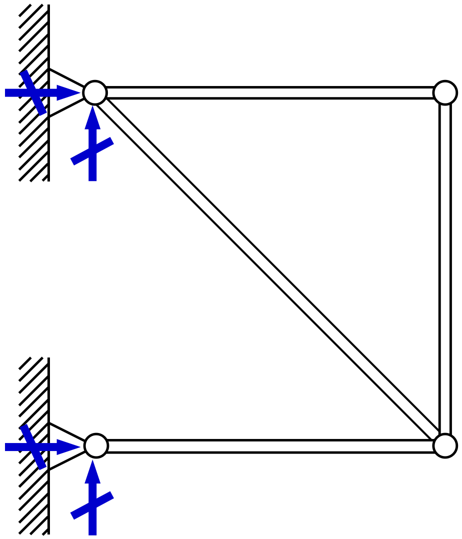
> 0 : statically indeterminate (redundant)

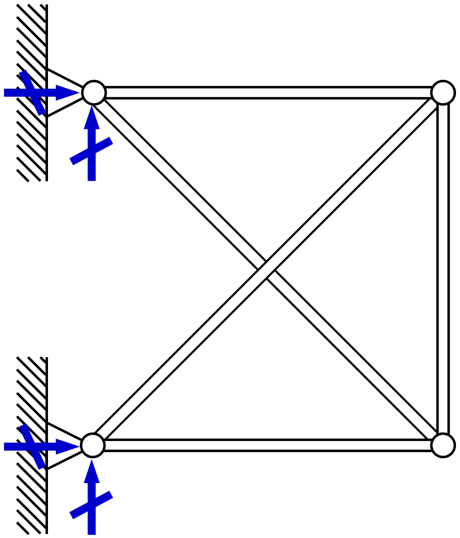
Note: this formula is not fool-proof – **you must still apply 'common sense'!**

- Structures which have redundancies and mechanisms
- Unloaded members
- Crossed members which are not joined
- Load at any joint in any direction

Degree of Redundancy – Examples







$N_u = N_r + N_f = 4 + 3 = \mathbf{7}$

$N_e = N_{DoF} \times N_j = 2 \times 4 = \mathbf{8}$

$DoR = N_u - N_e = 7 - 8 = \mathbf{-1}$

→ **Mechanism**

$N_u = N_r + N_f = 4 + 4 = \mathbf{8}$

$N_e = N_{DoF} \times N_j = 2 \times 4 = \mathbf{8}$

$DoR = N_u - N_e = 8 - 8 = \mathbf{0}$

→ **Statically Determinate**

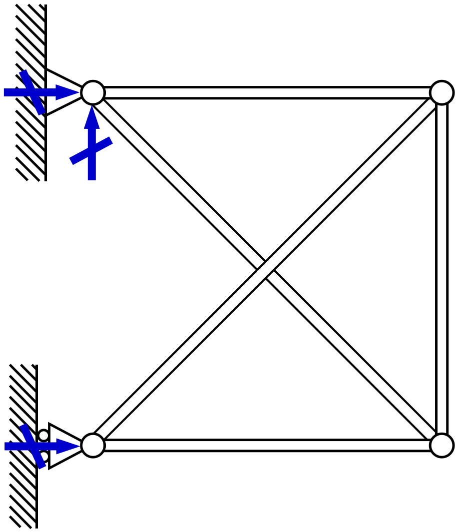
$N_u = N_r + N_f = 4 + 5 = \mathbf{9}$

$N_e = N_{DoF} \times N_j = 2 \times 4 = \mathbf{8}$

$DoR = N_u - N_e = 9 - 8 = \mathbf{1}$

→ **Statically indeterminate**

Degree of Redundancy – Examples

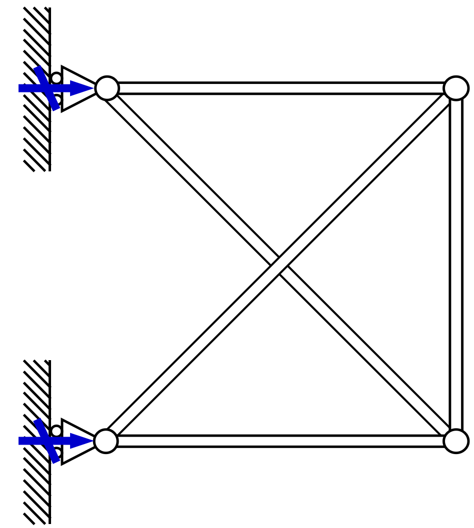


$N_u = N_r + N_f = 3 + 5 = \mathbf{8}$

$N_e = N_{DoF} \times N_j = 2 \times 4 = \mathbf{8}$

$DoR = N_u - N_e = 8 - 8 = \mathbf{0}$

$\rightarrow$ **Statically Determinate**



$N_u = N_r + N_f = 2 + 5 = \mathbf{7}$

$N_e = N_{DoF} \times N_j = 2 \times 4 = \mathbf{8}$

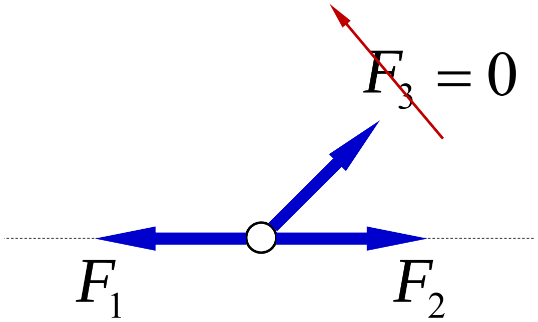
$DoR = N_u - N_e = 7 - 8 = \mathbf{-1}$

$\rightarrow$ **Mechanism**

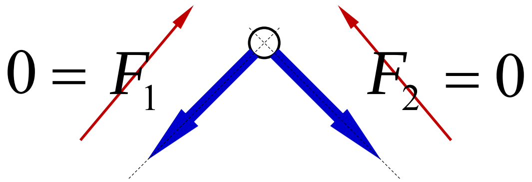
Unloaded Members

Some **internal forces** or some **reaction forces** in a pin-jointed truss might be **zero.** Often these 'zero loads' can be spotted early if we check for the **two collinearity rules**:

Rule 1:
If there are **exactly three forces** acting on a pin joint, an **two of these are collinear**, then the **non-collinear force must be zero**



Rule 2:
If there are **exactly two forces** acting on a pin joint and these are **not collinear**, then **both forces must be zero**



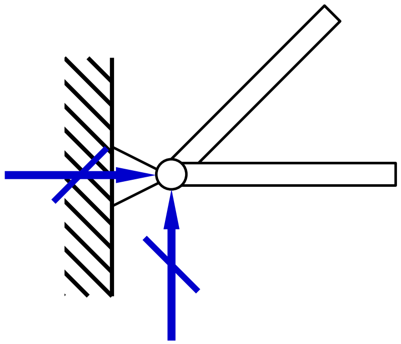
Default Load Directions or 'Sense'

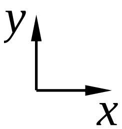
If the sense or direction of a force is unknown, **assume positive values**

- This applies to internal & external sign conventions

Reaction forces (external sign convention)**:**

- **Horizontal**: positive 'to the right'
- **Vertical**: positive 'upwards'





Member forces (internal sign convention):

- Assume **tension**
  - As if forces were 'flowing out' of each pin joint