Aerodynamics 2- Rotorcraft Aerodynamics

Lecture 3 (notes)

Axial flight

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Aerodynamics of the Rotor in Axial Flight

For most cases of non-translational flight the helicopter will be in the hover. For climbing or descending it will be more efficient if the helicopter has some translational velocity. Never-the-less the helicopter does have the ability to operate in a number of axial flight conditions so an understanding of the associated flow states of the rotor is necessary. The helicopter's main rotor will need to operate in the more extreme of these flow states during a recovery from engine failure. The tail rotor is required to operate in all of theses flow states quite routinely.

It should be remembered that at all times the rotor is producing positive thrust and therefore a flow, v is induced through the rotor in the opposite sense to the thrust vector. If the flow through the rotor (due to the vertical velocity of the helicopter) V_v is considered to be positive in the climb condition then the total flow through the rotor $U = V_v + v$. There are basically four states of flow through the rotor:

NORMAL WORKING STATE - *from infinite climb to hover*

	Climb	Hover
$V_v =$	+ve (V_v is down)	0
$v =$	$< v_h$	v_h
$U =$	$> v$	v

This is the normal operating regime for a helicopter, the flow state is suitable for analysis using the momentum equations and it is therefore similar to the aeroplane propeller where static thrust is analogous to hover and forward flight to climb. Unfortunately this is where the similarity must end because for reasons that will be explained later, a rather different nomenclature has been adopted.

VORTEX RING STATE - *from hover* (ie. $V_v = 0$) *to* $-V_v = v$

	Slow Descent
$V_v =$	-ve (V_v is small to moderate)
$v =$	$> v_h$
$U =$	$< v$

Although not considered a "normal working state" the helicopter routinely operates at descent rates to $V_v = (-v_h/2)$ and the momentum theory still gives surprisingly good results. For $V_v \leq (-v_h/2)$ momentum theory is not valid and cannot be used.

TURBULENT WAKE STATE - *from* $-V_v = v$ *to* $-V_v = 2v$

	Moderate Descent
$V_v =$	-ve (V_v is moderate to large)
$v =$	$> \mathbf{v_h}$
$U =$	≤ 0

Still a very turbulent flow state but there is much less recirculation through the rotor than in the "vortex ring state". Autorotation exists when $V_v + v \leq 0$ (ideal autorotation when $V_v + v = 0$). The momentum theory is not valid and cannot be used

WINDMILL BRAKE STATE - *from* $-V_v = 2v$ *to* $-V_v = \infty$

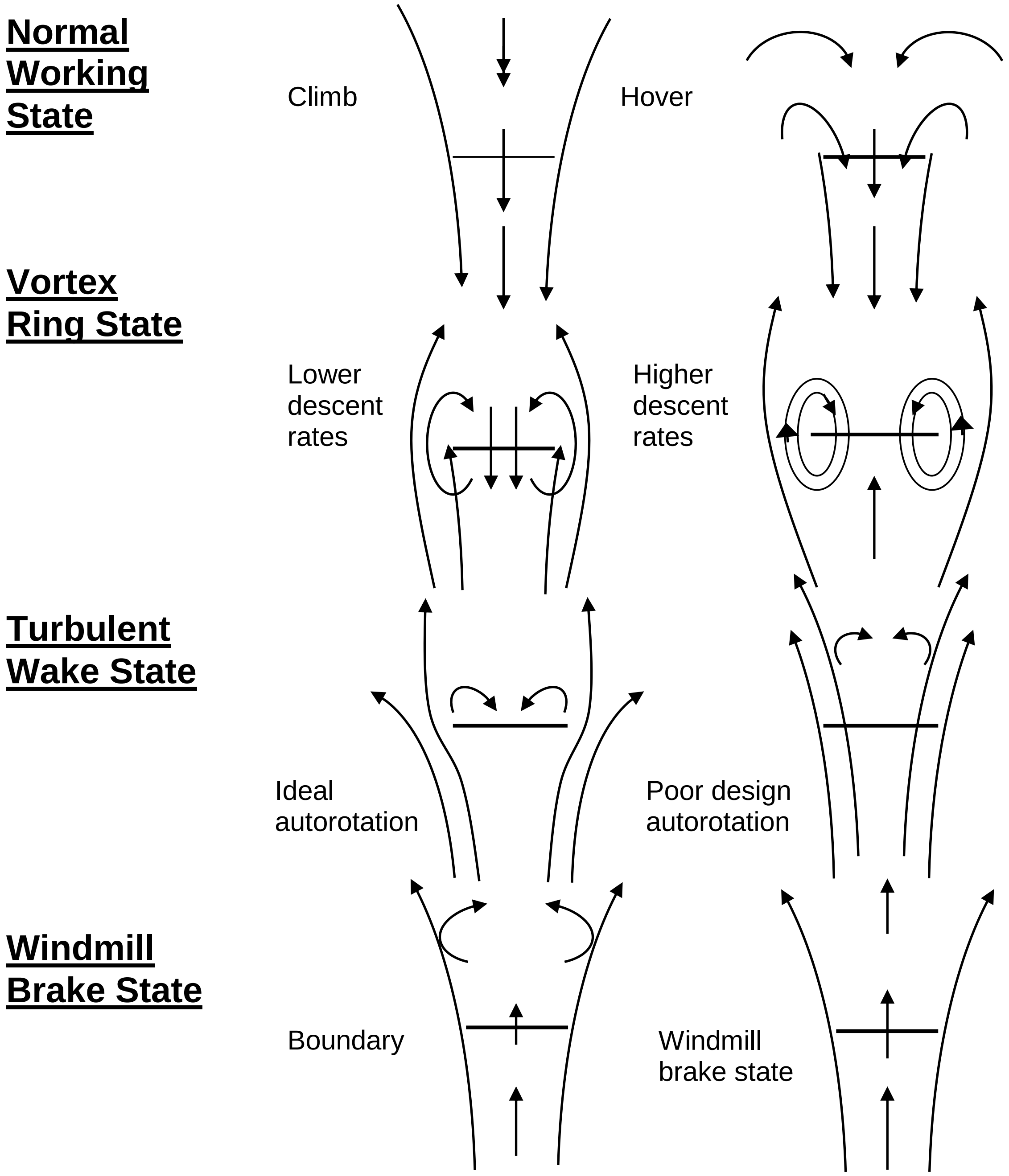
	Rapid Descent
$V_v =$	-ve (V_v is large)
$v =$	$\leq v_h$
$U =$	>> 0 (up)

Now the flow is a definite slipstream as it is in the "normal working state" but the flow is everywhere upwards, (opposite to the rotor climb). The momentum theory is again valid and yields good results.

Visualisation of the Four Axial Flow States

The four rotor working states are best visualised as shown below:

It should be noted that the two extreme cases, climb and fast descent have almost identical streamlines albeit of opposite sign. These well defined stream tubes are therefore suited to analysis by momentum consideration. The adjacent states of the aforementioned are also very similar, the entry into the "windmill brake state" had an induced velocity in the order of v_h and the downstream wake stagnates in a similar manner to the inflow for the hover.



The Universal Induced Velocity Diagram

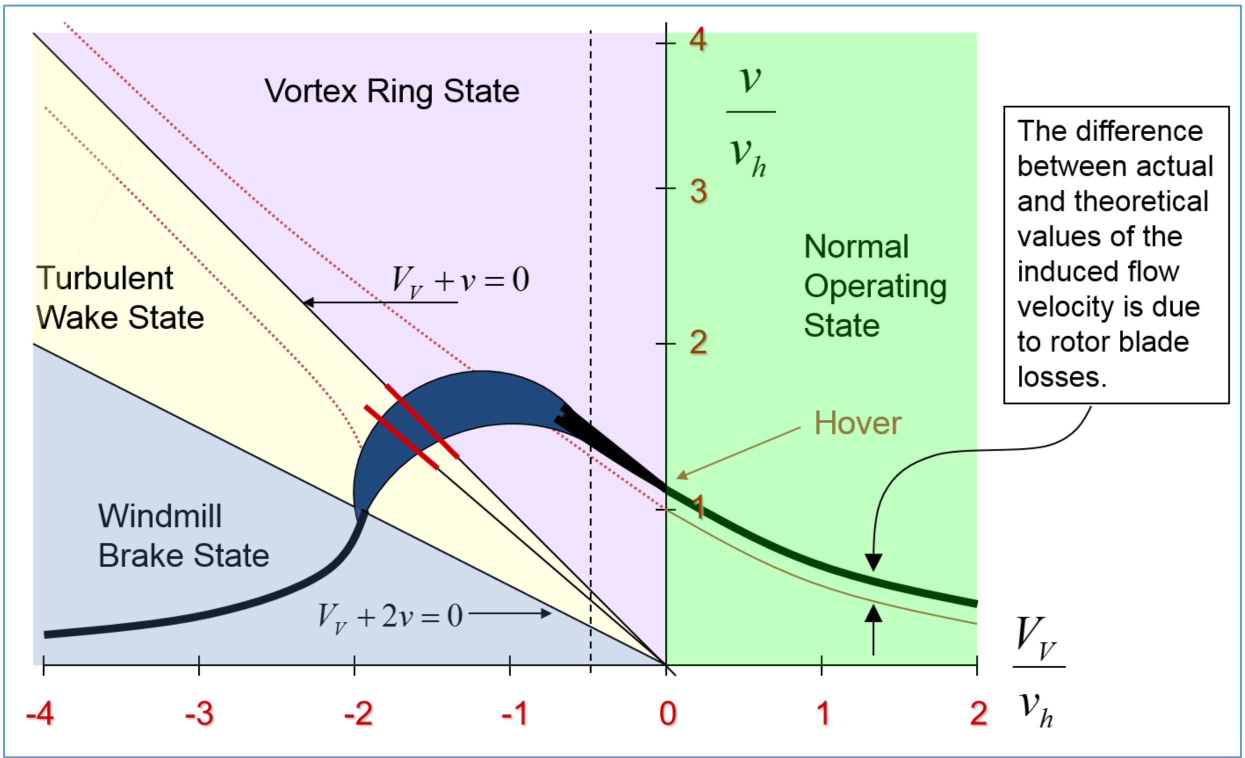
A graph showing the validity of the momentum theory of analysis for the rotor flow states is given below. The Induced velocity v and the climb velocity V_v have been non-dimensionalised by dividing by the induced velocity in the hover v_h.

The continuous lines indicate the relationship between induced velocity at the rotor and the climb / descent velocity where the momentum equations are valid. The dotted lines are projections of these lines into the vortex ring and turbulent wake state where the momentum equations are not valid.

It can be seen that the line at 45^o through the origin is the line along which <u>ideal autorotation</u> can occur since $V+v=0$ and therefore zero power condition results.

[*remembering that* $P = T(V+v)$ *and since* $V+v=0$, *then* $P=0$]

Whilst $P=0$, $T \neq 0$ as $TV = Tv = Wv$ where W is the weight of the helicopter for a steady state autorotative descent. This is however for ideal autorotation and in reality that cannot exist. Some power is required to keep the main rotor turning, overcome the transmission losses and drive the tail rotor. The tail rotor is intrinsically linked to the main rotor but will not be required to produce thrust for that rotor while it autorotates. Thus the line for actual autorotation will be further into the Turbulent Wake state at a lower angle of say 42^o, where $P = T(V+v) + P_0 = 0$



In general (and this very much depends upon rotor diameter and helicopter weight), helicopters settle down to an autorotational descent rates such that $V/v_h \approx -1.7$

Now in a steady autorotative descent, $T = W =$ thrust in the hover

and therefore $\quad T = 2\rho A v_h^2$

The rotor has no net flow through it so it can be likened to a solid disc of diameter A.

and it's Flat Plate Drag $D = 1/2\,\rho V^2 A C_D$

Equating the Thrust and Drag equations $T = D$ gives:

$$2\rho A v_h^2 = 1/2\,\rho V^2 A C_D$$

$$C_D = \frac{4}{(V/v_h)^2}$$

For $V/v_h = -1.7$, $C_D = 1.38$ which is the effective drag coefficient of a parachute.

It is obvious that flight in the region of autorotative descent is too important to be left without some knowledge of the induced velocity values. Becausethese cannot be determined by momentum considerations, other methods have had to be used. Flight testing in these areas allowed the rotor power and rotor thrust to be measured at various axial velocities.

From the values of thrust and power coefficients (and an estimate of C_{P_0}) the induced velocity was determined. These values have been used to complete the induced velocity curve across the range of flow states. This is known as the "**Universal Induced Velocity Curve**". This semi-empirical chart is used to predict helicopter autorotational performance.

In the hover and in the climb the measured induced velocity is higher than the momentum theory result by a small but relatively constant factor. This is because the momentum takes no account for losses due to a non-uniform inflow and tip effects. These losses must be kept to a minimum if the primary role of the helicopter is for hover or axial flight. A "**Figure of Merit**" is used to gauge the efficiency of the hovering rotor and this will be discussed in the next lecture.

Rotor Efficiency - Figure of Merit

The propeller efficiency $\eta_p = TV/P$ is not suitable for the helicopter as the rotor has not been optimised for propulsion (TV) but for sustaining flight (Tv) where v represents the induced velocity flow, the only flow through the rotor in hover, which can be accelerated to produce the force required to oppose the aircraft weight.

A more appropriate measure of efficiency is to compare the actual power required to produce a given thrust with the minimum possible power required to produce that thrust.Here again, the momentum analysis serves to provide a measure of ultimate operating efficiency.

so, $\eta_r = M = Tv/P$ where M = Figure of Merit

$$M = \frac{T}{P}\frac{1}{\sqrt{2}}\sqrt{\frac{T}{\rho\pi R^2}}, \qquad \text{since } v = \sqrt{\frac{T}{2\rho A}}$$

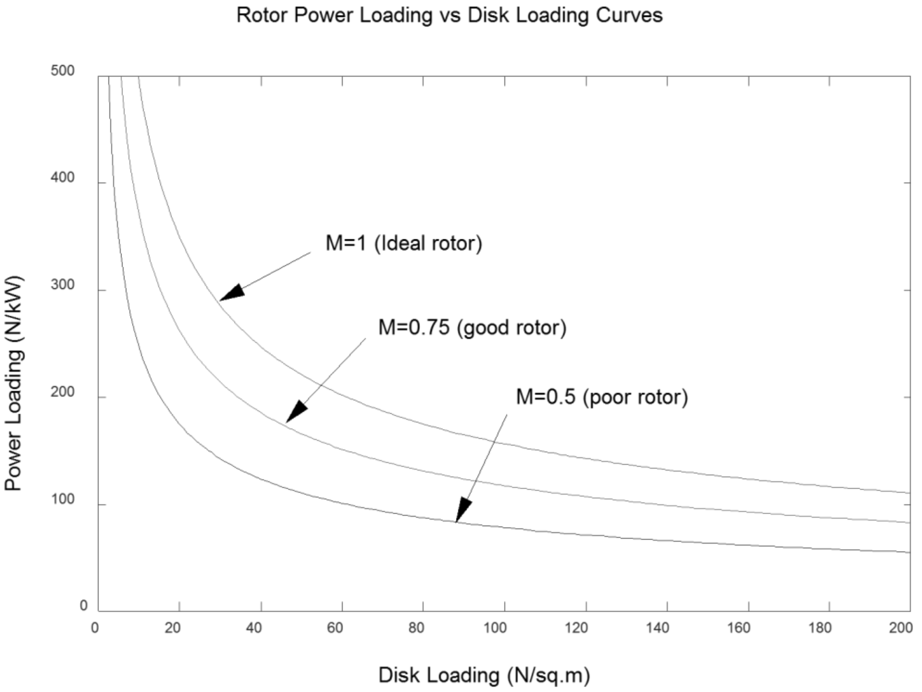
It should be noted that the Figure of Merit is inversely proportional to the rotor diameter and comparative studies therefore should be limited to rotors of the same diameter.

Letting $\frac{T}{P} = \mathbf{PL}$, (Power Loading) and $\frac{T}{A} = \mathbf{DL}$, (Disk Loading)

then, $\mathbf{PL} = 1.565M \frac{1}{\sqrt{\mathbf{DL}}}$

If the Figure of Merit is known, then for a given disk loading the power loading may be found from the graph.

A Figure of Merit value in excess of 0.7 is considered to be very good. The non-uniform induced velocity, blade drag and blade tip effects are the primary sources of efficiency loss.



Rotor Performance Coefficients

The general thrust coefficient $T_c = \frac{T}{\rho V^2 D^2}$ has limited value for helicopter rotors as it has an infinite value at $V = 0$. A more suitable coefficient is $C_T = \frac{T}{\rho A (\Omega R)^2}$ which gives a thrust coefficient based upon rotor tip speed ΩR (m/s) and the rotor disk area A.

(We refer to parameter ΩR as the reference velocity where rotational speed Ω is in rads/sec)

It is also normal to express the forward speed (V) of the helicopter relative to the rotor tip speed parameter ΩR and this is called the Advance Ratio μ

Thus $\mu = \frac{V}{\Omega R}$

In a similar way, the flow through the rotor (v in the hover but $V_V + v$ otherwise) is non-dimensionalised by ΩR and this is called the inflow ratio λ.

Thus $\lambda = \frac{V_V + v}{\Omega R}$

<u>A note of caution</u>:

Whilst $C_T = \dfrac{T}{\rho A(\Omega R)^2}$ has become the accepted form (**This is the expression used in these lectures**), some text books may show:

$C_T = \dfrac{T}{1/2\,\rho V_T^2 A}$ where $V_T = \Omega R$ (blade tip speed)

<u>or</u> $C_T = \dfrac{T}{\rho \sigma A(\Omega R)^2}$ where $\sigma = \dfrac{Nc}{\pi R}$ (solidity)

(where N is Number of blades)

Similarly, Torque Coefficient $C_Q = \dfrac{Q}{\rho A R(\Omega R)^2}$

and Power Coefficient $C_P = \dfrac{P}{\rho A(\Omega R)^3}$

and therefore $C_P = C_Q$

The Figure of Merit as previously defined can be more conveniently expressed in terms of non-dimensional quantities using the thrust and power coefficients above:

The induced velocity $v = \sqrt{\dfrac{T}{2\rho A}} = \sqrt{\dfrac{C_T \rho A(\Omega R)^2}{2\rho A}} = \Omega R\sqrt{\dfrac{C_T}{2}}$

But since $\lambda = \dfrac{V_V + v}{\Omega R}$, then for $V_V = 0$, $\lambda = \sqrt{\dfrac{C_T}{2}}$

$$M = \frac{Tv}{P} = \frac{C_T \rho A(\Omega R)^2 v}{C_P \rho A(\Omega R)^3} = \frac{C_T}{C_P}\frac{v}{\Omega R} = \frac{C_T}{C_P}\lambda$$

$$= \frac{1}{\sqrt{2}}\frac{C_T^{3/2}}{C_P}$$

or more commonly $\boxed{M = 0.707\dfrac{C_T^{3/2}}{C_Q}}$

The ideal rotor (Figure of Merit equal to unity) assumes no rotor profile losses and an even distribution on the induced velocity across the rotor disk. This of course cannot be achieved in practice. In order to minimise the profile and induced losses across the rotor, the combination of blade element analysis and momentum theory can be used to determine the correct blade twist.