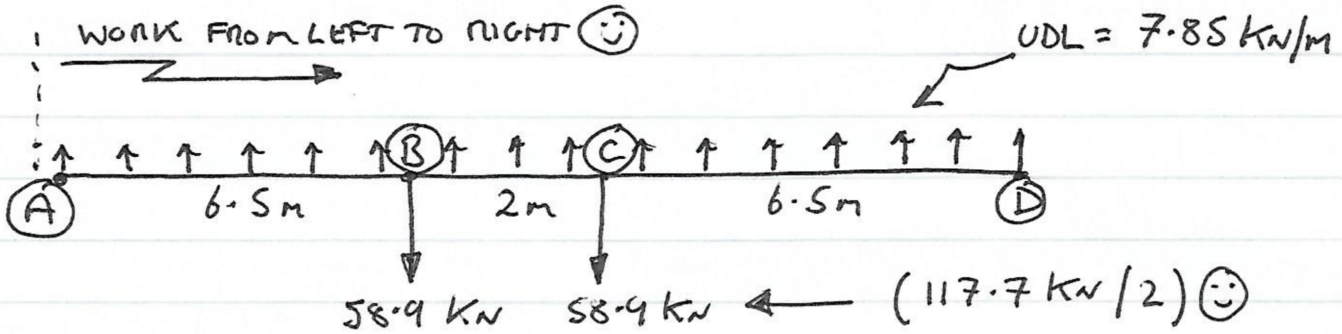
IAS LECTURE 5 - WORKED EXAMPLES

EXAMPLE 1

BUSINESS JET, AUM = 4500 Kg, WINGSPAN 15 m,
WING STRUCTURE MASS = 500 Kg,
WING FUEL PAYLOAD MASS = 1000 Kg,
SYMMETRIC GUST, LOAD FACTOR, n = 4 (ie. 4g)

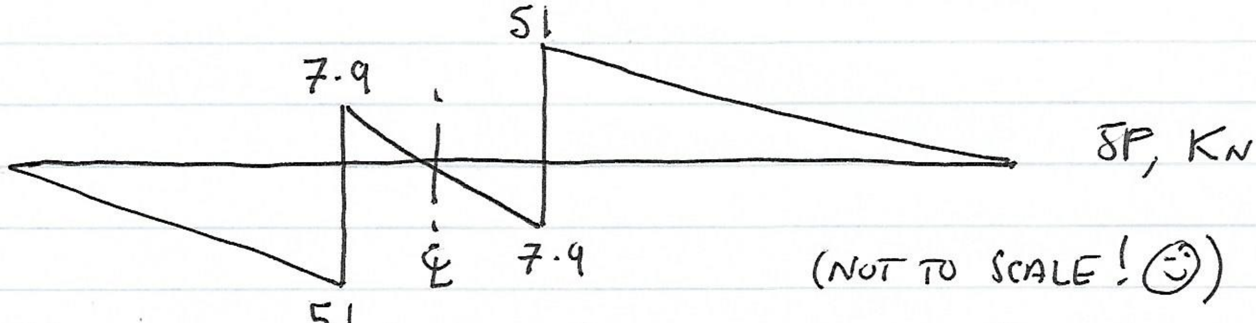
↑ LIFT LOADING ON WING = 4500 × 9.81 × 4 = 176.6 KN
↓ WING + FUEL INERTIA = (1000 + 500) × 9.81 × 4 = 58.9 KN
↓ NET LOAD FROM FUSELAGE = 176.6 − 58.9 = 117.7 KN
NET DISTRIBUTED LOAD ON WING = 117.7 / 15 = 7.85 KN/m

∴ WING LOADING DIAGRAM BECOMES:



NOW, TO DRAW THE SHEAR FORCE DIAGRAM......
AT POINT A (WINGTIP): SF = <u>0</u>
AT POINT B (FUSELAGE ATTACHMENT POINT): SF = 6.5 × 7.85 = <u>51.0 KN</u>
AT POINT B: SF = (6.5 × 7.85) − 58.9 KN = <u>−7.9 KN</u>
AT MID POINT BETWEEN B & C, MIDSPAN SF = ZERO
(BALANCED IN FLIGHT FOR SYMMETRIC CASE)

∴ SHEAR FORCE DIAGRAM BECOMES;



EXAMPLE 1 (CONTINUED)

NOW, TO DRAW THE BENDING MOMENT DIAGRAM .....

AT POINT A : BM = <u>0</u>

AT POINT B : SUM MOMENTS ACTING TO LEFT OF POINT B

↺ ABOUT POINT B : BM = (TOTAL FORCE) x (CENTROID OF ACTION OF THAT FORCE)

$= (7.85 \times 6.5) \times (6.5/2)$

$= \underline{165.8}$ kNm

AT CENTRE SPAN (℄) - MIDWAY BETWEEN POINTS B & C ;

[NOTE : SYMBOL ℄ IS A COMMON NOTATION FOR 'CENTRE LINE' OR MID-SPAN ON WING LOADING DIAGRAMS]

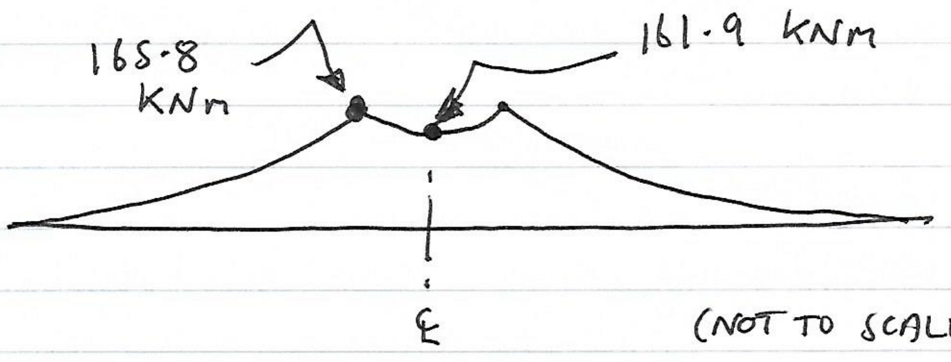
AT ℄ : BM = MOMENTS ACTING ↺ ABOUT ℄ <u>MINUS</u> MOMENTS ACTING ↻ ABOUT ℄

$$\therefore BM = \underbrace{\left[(7.85 \times 7.5) \times (7.5/2)\right]}_{\text{'CLOCKWISE' MOMENTS}} - \underbrace{(58.9 \times 1)}_{\text{'ANTI-CLOCKWISE' MOMENTS}}$$

$\therefore BM = 220.8$ kNm $- 58.9$ kNm

$\therefore BM = \underline{161.9}$ kNm

$\therefore$ BENDING MOMENT DIAGRAM BECOMES ;

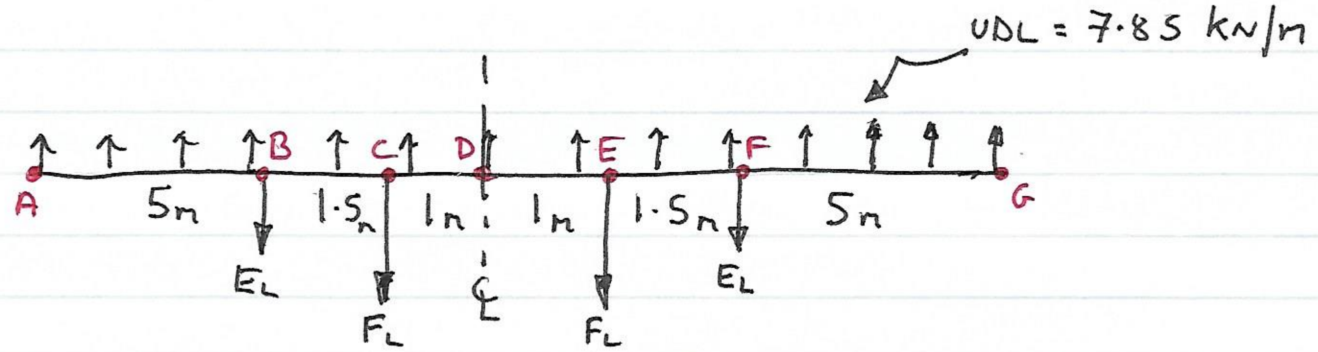


(NOT TO SCALE!)

EXAMPLE 2

BUSINESS JET AS IN EXAMPLE 1 BUT WITH THE ENGINES NOW MOUNTED ON THE WINGS. EACH ENGINE HAS A MASS OF 500 Kg AND THE ENGINE ATTACHMENT POINTS ARE 2·5 m PORT & STARBOARD OF THE AIRCRAFT MID-SPAN CENTRE-LINE (℄).

∴ WING LOADING DIAGRAM BECOMES :



ENGINE LOAD, $E_L = 500 \times 9{\cdot}81 \times 4$

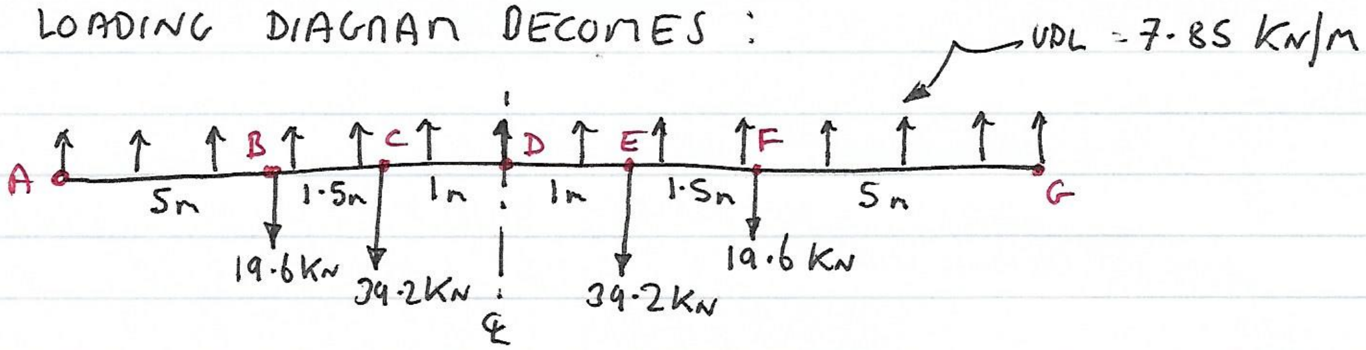
$\therefore E_L = \underline{19{\cdot}6 \text{ kN}}$

FUSELAGE LOAD, F_L = FUSELAGE ATTACHMENT LOAD FROM EXAMPLE 1 WHERE ENGINES ARE ON THE REAR FUSELAGE MINUS THE ENGINE LOAD E_L - PER SIDE!

$\therefore F_L = 58{\cdot}9 \text{ kN} - 19{\cdot}6 \text{ kN}$

$\therefore F_L = \underline{39{\cdot}2 \text{ kN}}$ (ROUNDED DOWN)

SO LOADING DIAGRAM BECOMES :



EXAMPLE 2 (CONTINUED)

NOW, TO DRAW THE SHEAR FORCE DIAGRAM .....

NOTE: START AT POINT A AND MOVE FROM LEFT TO RIGHT SUMMING UP ALL THE FORCES TO THE LEFT OF THE POINT YOU ARE CONSIDERING.

NOTE: REMEMBER THAT AT POINTS WHERE AN ADDITIONAL VERTICAL FORCE ACTS THE SF HAS TWO VALUES! ONE VALUE IS THAT IMMEDIATELY BEFORE THE ADDITIONAL FORCE INPUT, THE SECOND IS THAT IMMEDIATELY AT THE ADDITIONAL FORCE INPUT.

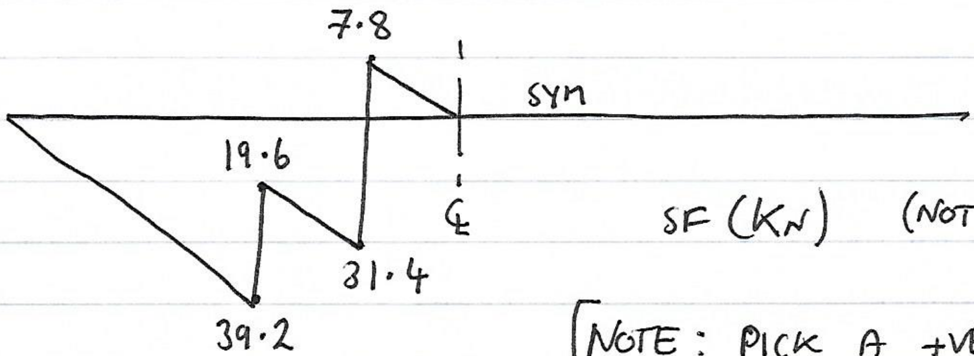
AT POINT A : SF = 0

AT POINT B : SF = 7.85 × 5 = 39.2 KN
SF = (7.85 × 5) − 19.6 = 19.6 KN

AT POINT C : SF = (7.85 × 6.5) − 19.6 = 31.4 KN
: SF = (7.85 × 6.5) − 19.6 − 39.2 = −7.8 KN

AT POINT D : SF = 0 AS LOAD CASE IS SYMMETRIC

SO THE SHEAR FORCE DIAGRAM BECOMES;



SF (KN) (NOT TO SCALE!)

[NOTE: PICK A +VE / −VE SIGN CONVENTION]

EXAMPLE 2 (CONTINUED)

NOW, TO DRAW THE BENDING MOMENT DIAGRAM:

AT POINT A : BM = 0

AT POINT B : BM = $(7.85 \times 5) \times (5/2) = \underline{98.1 \text{ kNm}}$

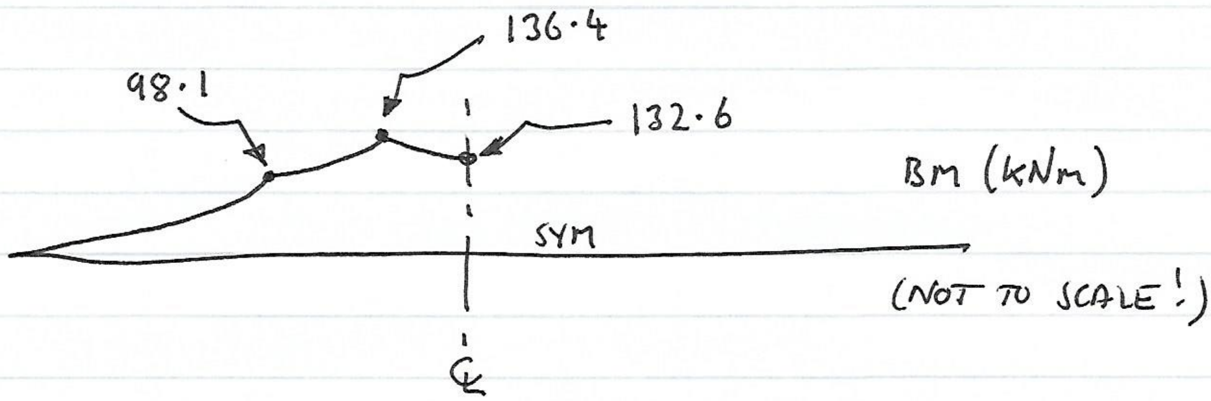
AT POINT C : BM = $\left[(7.85 \times 6.5) \times (6.5/2)\right] - (19.6 \times 1.5)$

$\therefore$ BM = $\underline{136.4 \text{ kNm}}$

AT POINT D : BM = $\left[(7.85 \times 7.5) \times (7.5/2)\right] - (19.6 \times 2.5) - (39.2 \times 1)$

$\therefore$ BM = $\underline{132.6 \text{ ~~kNm~~ KNM}}$

SO THE BENDING MOMENT DIAGRAM BECOMES;



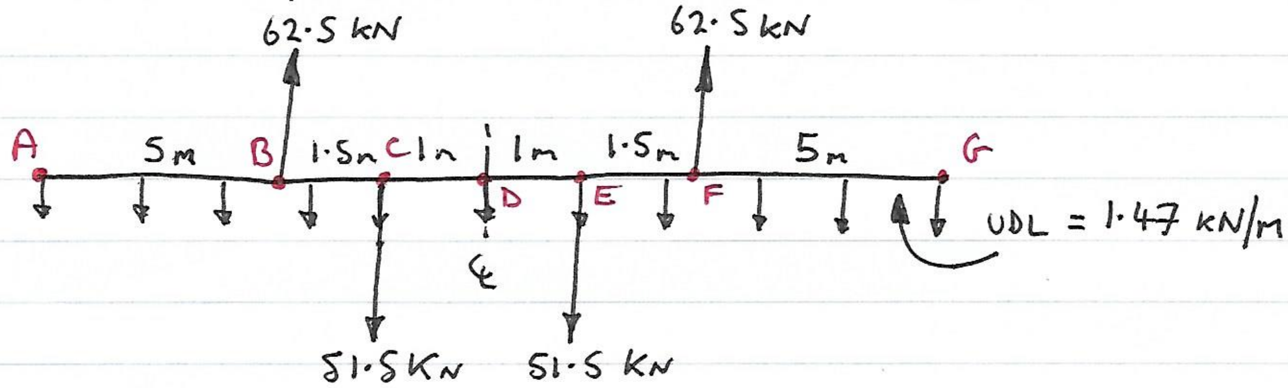
EXAMPLE 3

THIS IS THE EXAMPLE WHERE THE BUSINESS JET WITH THE REAR FUSELAGE MOUNTED ENGINES IS LANDING WITH A LOAD FACTOR OF 3·5

ONE THING TO NOTE HERE IS THAT THE DISTRIBUTED LOAD (UDL) ON THE WING IS NOT THE AERODYNAMIC LIFT AS IN THE OTHER EXAMPLES.

AS THE AIRCRAFT IS LANDING, THE UDL ON THE WING STRUCTURE IS THE DOWNWARD INERTIA LOAD FROM THE WING STRUCTURE AND FUEL PAYLOAD MINUS THE RESIDUAL LIFT LOADING ON THE WING AND THEREFORE ACTS DOWNWARDS AND NOT UPWARDS!

SO, TAKING THE WING LOADING DIAGRAM FROM THE LECTURE NOTES:



∴ TO DRAW THE SHEAR FORCE DIAGRAM......

AT POINT A : SF = 0

AT POINT B : $SF = 1.47 \times 5 = \underline{7.3 \text{ kN}}$

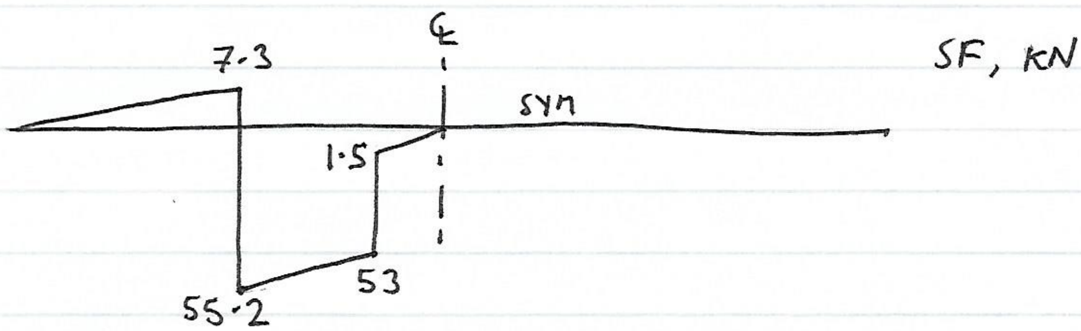
: $SF = (1.47 \times 5) - 62.5 = \underline{-55.2 \text{ kN}}$

AT POINT C : $SF = (1.47 \times 6.5) - 62.5 = \underline{-53 \text{ kN}}$

: $SF = (1.47 \times 6.5) - 62.5 + 51.5 = \underline{-1.5 \text{ kN}}$

AT POINT D : SF = 0 AS CASE IS SYMMETRIC

SO THE SHEAR FORCE DIAGRAM BECOMES;



CONSIDERING BENDING MOMENTS.......

AT POINT A : BM = 0

AT POINT B : $BM = (1.47 \times 5) \times (5/2) = \underline{18.4 \text{ kNm}}$

AT POINT C : $BM = \left[(1.47 \times 6.5) \times (6.5/2)\right] - (62.5 \times 1.5)$

$= \underline{62.7 \ \cancel{\text{kNm}} \ \text{kNm}}$

AT POINT D : $BM = \left[(1.47 \times 7.5) \times (7.5/2)\right] - (62.5 \times 2.5)$

$+ (51.5 \times 1)$

$= \underline{63.4 \text{ kNm}}$

GIVING THE BM DIAGRAM SHOWN IN THE LECTURE NOTES.