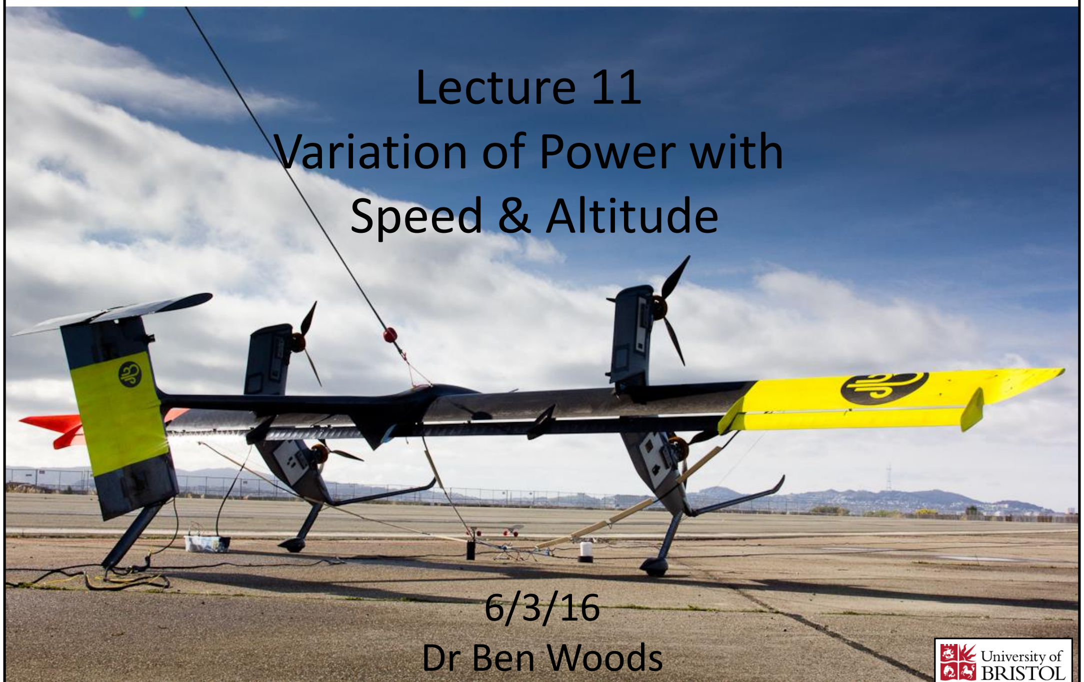
Aeronautics & Mechanics AENG11301



Lecture 11
Variation of Power with Speed & Altitude

6/3/16

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Makani AWT

The Makani Airborne Wind Turbine is a tethered wing that generates power by flying in large circles where the wind is stronger and more consistent.
This is 30 kW prototype (8 m wing span), but have plans for 5 MW (65 m wing span)
It eliminates 90% of the material used in conventional wind turbines
hybrid rotors that let them generate energy as a turbine or apply thrust like a propeller.
The rotors are used as propellers to keep the wing aloft during short lapses in the wind, allowing the wing to stay aloft if the wind dies.
http://www.makanipower.com/

Outline for today

- Power required in steady level flight
- Power required with altitude
- Minimum power required
- Minimum power vs minimum drag
- Examples

Aims for today

- Be able to calculate power required in steady level flight
- Know effects of changing altitude on power required
- Be able to calculate minimum power velocity
- Appreciate why minimum power and minimum drag speeds are different

Forces in steady flight

Equivalent air speed

Drag equation

Factors effecting drag

Examples

Power Required in S&L Flight

- the power required for steady level flight P or P_r is

$$\boxed{P = TV = DV}$$

(power = force $\times$ velocity)

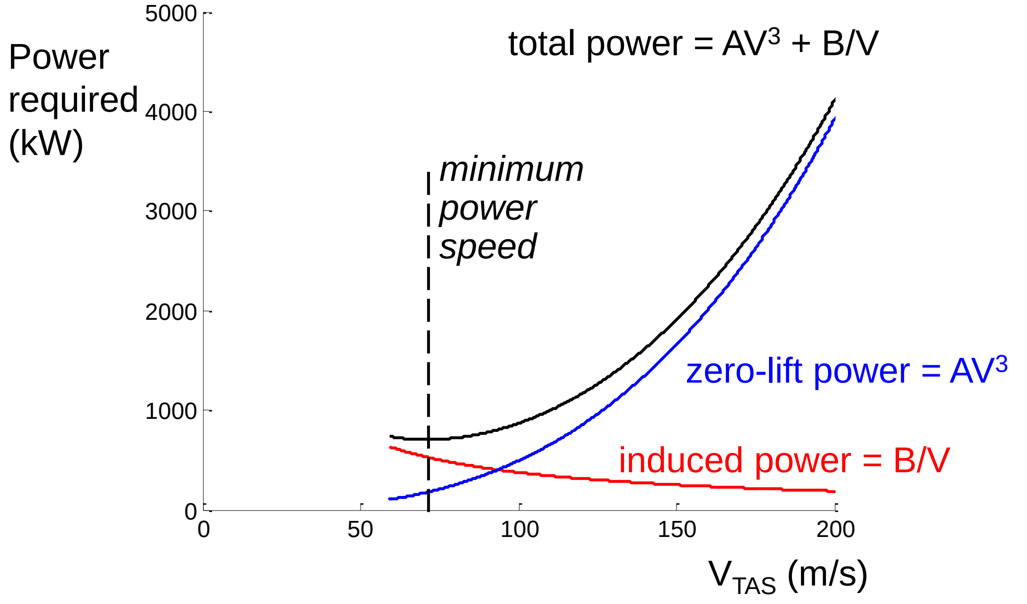
$$\Rightarrow \quad P = \left(AV^2 + \frac{B}{V^2} \right) V$$

$$\Rightarrow \quad \boxed{P = AV^3 + \frac{B}{V}}$$

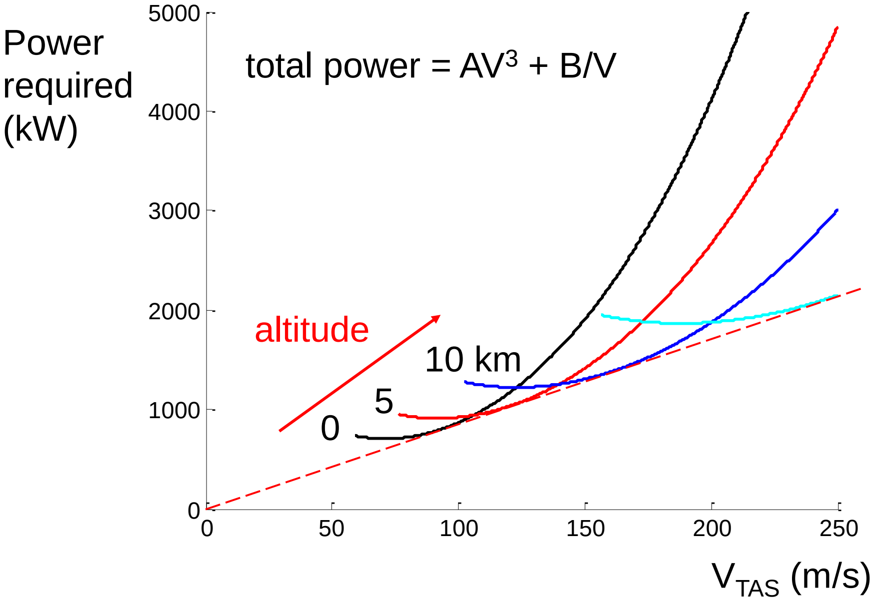
- where A and B are functions of altitude / density as defined previously

$$\boxed{A = C_{D_0} \tfrac{1}{2} \rho S \ , \quad B = \frac{KW^2}{\tfrac{1}{2} \rho S}}$$

Power required at Sea Level



Power required at Altitude



Features of Power Curves

- combination of V^3 and $1/V$ terms gives a minima in the total power required curve at the **minimum power speed** V_{MP}
  - at low speed the induced power required term B dominates – take-off, landing & air combat
  - at high speed the profile power required term A dominates – cruise conditions
- effect of altitude/density is to shift curves upwards and to the right
  - V_{MP} and stall speed V_{stall} increase with altitude
  - minimum power P_{min} also increases
- if plotted in terms of $P\sqrt{\sigma}$ and V_{EAS} instead then variation with altitude disappears

Power Equation in Terms of V_{EAS} and σ

- substitute equivalent air speed V_E and (constant) density ρ_0 into power equation as before

$$P = V\left(A_1 V_E^2 + \frac{B_1}{V_E^2}\right) \Rightarrow P = \frac{V_E}{\sqrt{\sigma}}\left(A_1 V_E^2 + \frac{B_1}{V_E^2}\right)$$

Reminder:

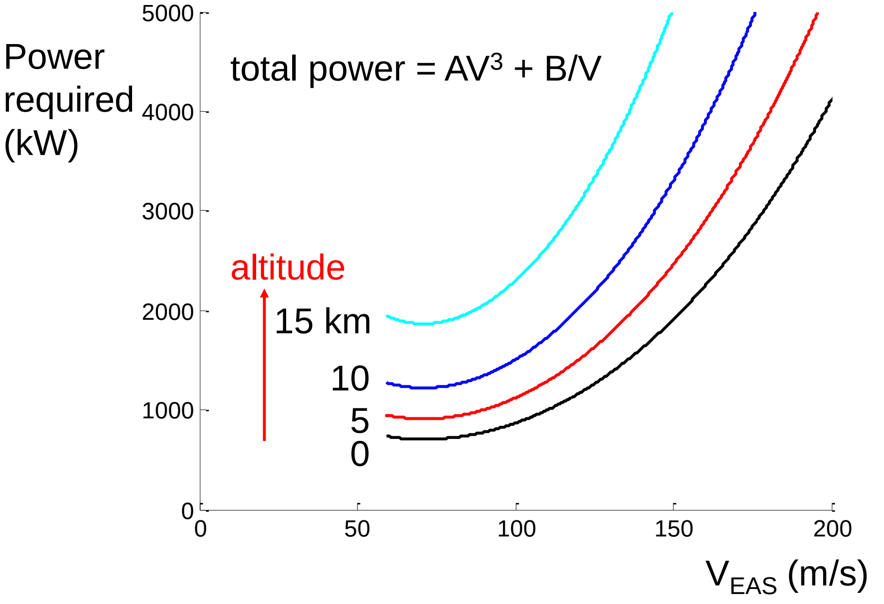
$$V_E = V\sqrt{\frac{\rho}{\rho_0}} = V\sqrt{\sigma}$$

$$\boxed{P\sqrt{\sigma} = V_E\left(A_1 V_E^2 + \frac{B_1}{V_E^2}\right)}$$

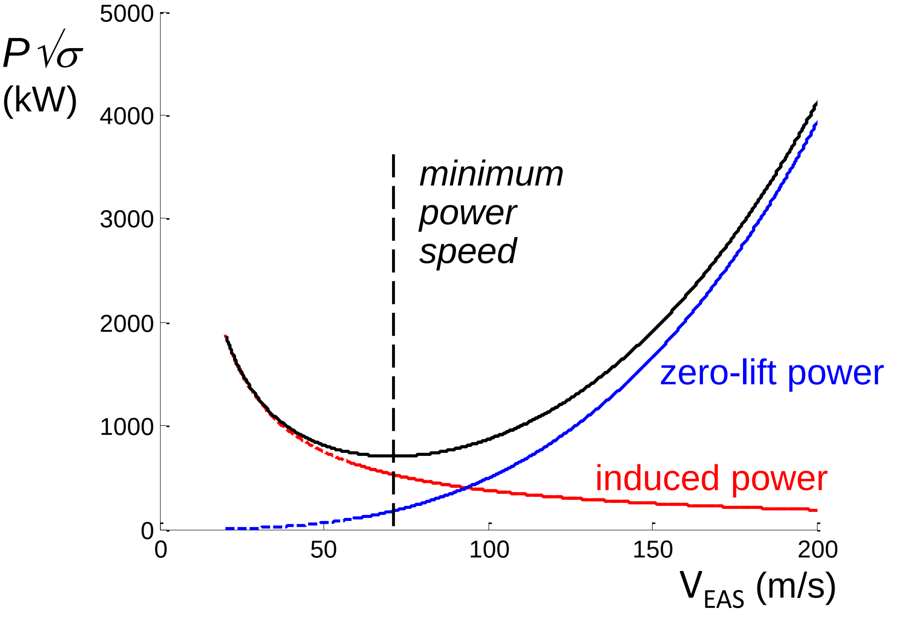
- where A_1 and B_1 are constants as defined previously

$$\boxed{A_1 = C_{D_0}\,\tfrac{1}{2}\rho_0 S\ , \quad B_1 = \frac{KW^2}{\tfrac{1}{2}\rho_0 S}}$$

Power required vs V_{EAS}



Single Power Required Curve



Minimum Power Required

- power given by $$\boxed{P = DV = W \times \left(\frac{D}{L}\right) \times V = W\left(\frac{C_D}{C_L}\right)\sqrt{\frac{W}{\frac{1}{2}\rho S C_L}}}$$

$$\Rightarrow \boxed{P \propto \frac{C_D}{C_L^{3/2}}}$$

  – therefore need to find minimum $C_D/C_L^{3/2}$

$$\frac{C_D}{C_L^{3/2}} = \frac{C_{D0} + KC_L^2}{C_L^{3/2}} = \frac{C_{D0}}{C_L^{3/2}} + KC_L^{1/2} \Rightarrow \frac{d\left(C_D/C_L^{3/2}\right)}{dC_L} = -\frac{3}{2}\frac{C_{D0}}{C_L^{5/2}} + \frac{1}{2}\frac{K}{C_L^{1/2}}$$

- so at minimum point $3C_{D0} = KC_L^2$

$$\Rightarrow \boxed{C_{D_{MP}} = C_{D0} + 3C_{D0} = 4C_{D0}}$$

$$\boxed{C_{L_{MP}} = \sqrt{3C_{D0}/K}}$$

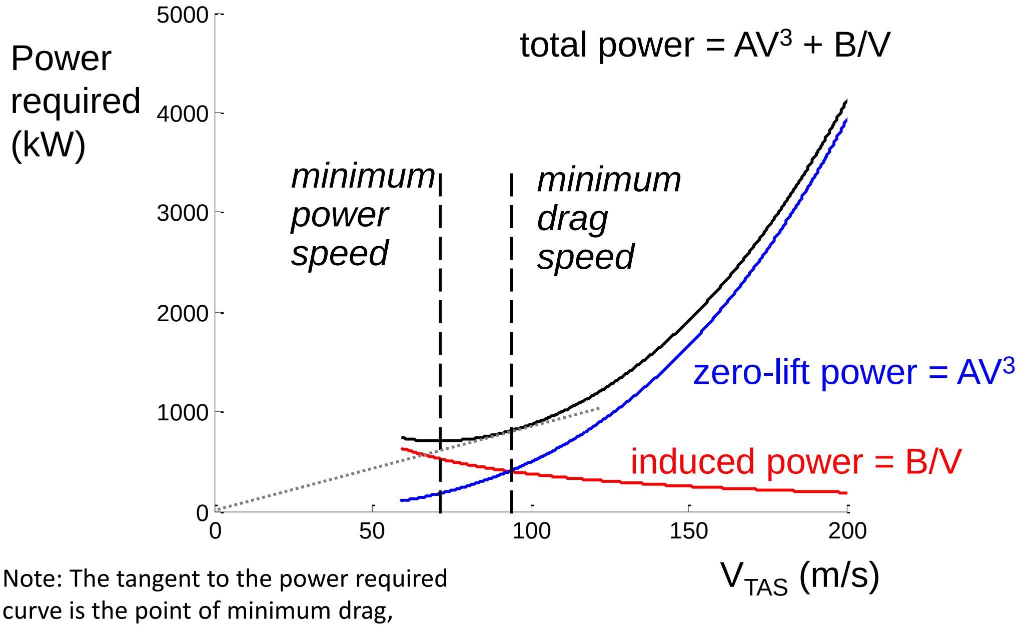
Minimum Power Required Speed

- lift at minimum power is $C_{L_{MP}} = \sqrt{\dfrac{3C_{D0}}{K}}$
- substitute into speed equation $V = \sqrt{\dfrac{W}{\frac{1}{2}\rho S C_L}}$

$$\Rightarrow \quad V_{MP} = \sqrt{\frac{W\sqrt{K}}{\frac{1}{2}\rho S\sqrt{3C_{D0}}}}$$

$$\Rightarrow \quad \boxed{V_{MP} = \left(\frac{2W}{\rho S}\right)^{\frac{1}{2}}\left(\frac{K}{3C_{D0}}\right)^{\frac{1}{4}}}$$

Power Required Curve



Note: The tangent to the power required curve is the point of minimum drag, and hence maximum L/D

Minimum Power and Drag Compared

- minimum power required occurs at a lower speed than for minimum drag

$$\frac{V_{MP}}{V_{MD}} = \frac{\left(\frac{2W}{\rho S}\right)^{\frac{1}{2}}\left(\frac{K}{3C_{D0}}\right)^{\frac{1}{4}}}{\left(\frac{2W}{\rho S}\right)^{\frac{1}{2}}\left(\frac{K}{C_{D0}}\right)^{\frac{1}{4}}} = \frac{1}{3^{\frac{1}{4}}} = 0.76$$

  - requiring a higher lift coefficient, ie

$$C_{L_{MP}} = \sqrt{\frac{3C_{D0}}{K}} \quad \text{vs} \quad C_{L_{MD}} = \sqrt{\frac{C_{D0}}{K}}$$

C130 – “Fat Albert” – Jet Assisted Take Off (JATO) – Part of United States Navy “Blue Angles”
http://www.youtube.com/watch?v=97rSobuKBxI

Example – C-130J

Mass = 70,300 kg

Wing area, S = 162 m^2

Cruise velocity, V = 174 m/s

Cruise altitude, h = 8,500 m

Drag due to lift coefficient, K = 0.035

Zero lift drag coefficient, C_{D0} = 0.028

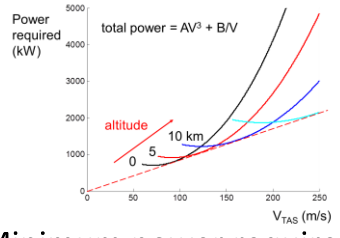
$$\sigma = \frac{\rho}{\rho_{SL}} \approx \frac{20-H}{20+H}$$

Calculate:

1) the power required at V_{TAS}=130 m/s at sea level
2) power required at cruise
3) minimum power speed at sea level
4) minimum power required at sea level
5) minimum power speed at cruise altitude
6) minimum power required at cruise altitude

Summary

- In steady level flight $\boxed{P = TV = DV}$

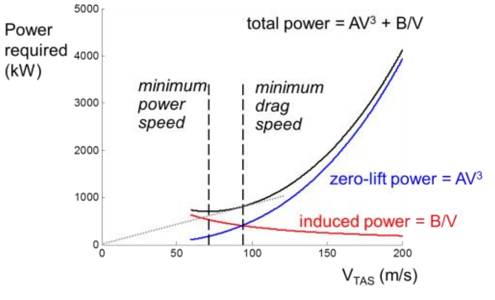


variation with altitude disappears when plot $P\sqrt{\sigma}$ and V_{EAS}

- Minimum power required condition

$$\boxed{C_{L_{MP}} = \sqrt{3C_{D0}/K}}$$

$$\boxed{C_{D_{MP}} = C_{D0} + 3C_{D0} = 4C_{D0}}$$



Be able to calculate power required in steady level flight

Know effects of changing altitude on power required

Be able to calculate minimum power velocity

Appreciate why minimum power and minimum drag speeds are different

Follow-up materials

To help with exam:

- Introduction to Flight – 6.5

To help with exam:
Introduction to Flight – 5.1-5.2

To aid in understanding:
Understanding flight – Chapter 1

For interest:
Introduction to Flight – 5.19 (explanation of lift)