TOPIC IV - The Second Law
Entropy (Lecture 3/4)

<u>Contents</u>

Gives background to the concept of entropy.

New fluid property. Yields efficiency of reversible engines with >2 reservoirs

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8. The Clausius Inequality

Corollary 6. Whenever an engine undergoes a cycle,

$$\oint \frac{dQ}{T} \leq 0 \tag{9}$$

The cyclic integral is zero if the cycle is reversible and negative if irreversible. T refers to fluid that supplies heat.

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Proof

Carnot efficiency $$1-\eta=\frac{-Q_2}{Q_1}=\frac{T_2}{T_1}$$

Irreversible case $$\frac{-Q_2}{Q_1}\geq\frac{T_2}{T_1}$$

X $-Q_1/T_2$
+ Q_1/T_1

$$\frac{Q_1}{T_1}+\frac{Q_2}{T_2}\leq 0$$

Cycles with different reservoir temps

$$\underbrace{\frac{Q_1}{T_1}+\frac{Q_2}{T_2}}_{1st\ cycle} + \underbrace{\frac{Q_3}{T_3}+\frac{Q_4}{T_4}}_{2nd\ cycle}+\ldots\leq 0$$

Small heat additions $$\frac{\delta Q_1}{T_1}+\frac{\delta Q_2}{T_2}+\ldots\leq 0$$

In the limit $\delta Q_1 \rightarrow 0$ $$\oint\frac{dQ}{T}\leq 0 \qquad (9b)$$

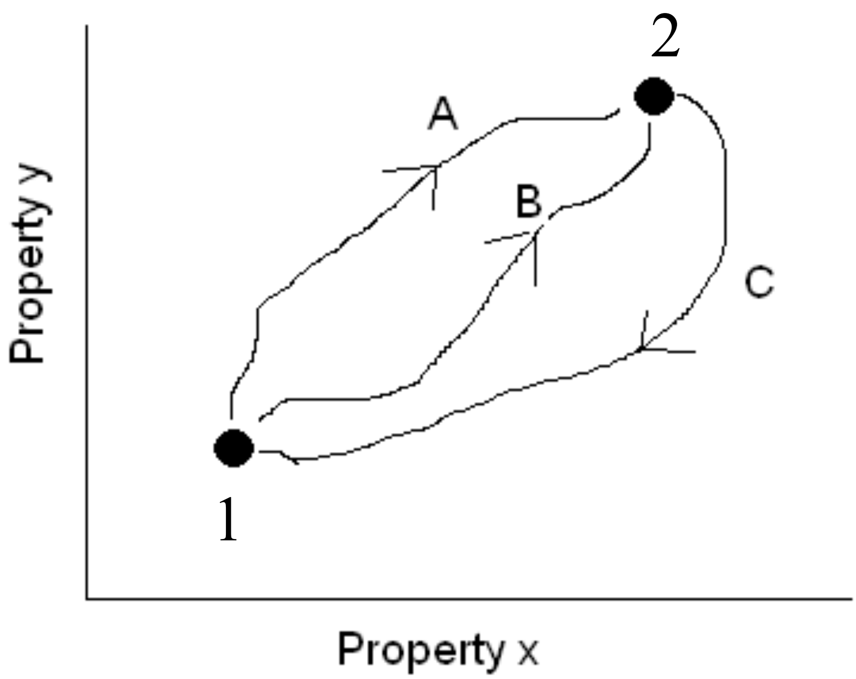
9. Entropy

Corollary 7: There exists a property of equilibrium state of a closed system such that a change in its value is equal to

$$\Delta S = \int_1^2 \frac{dQ_{rev}}{T} \qquad (10)$$

for any reversible process undergone by the system between state 1 and 2.

Proof



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<u>Proof</u>

Write Clausis inequality (Eqn 9) as

$$\int_1^2 \frac{dQ_{rev}}{T} \qquad + \qquad \int_2^1 \frac{dQ_{rev}}{T} \quad = \quad 0$$

$$\textit{outward path} \qquad\qquad\qquad \textit{return path}$$

Substitute definition (Eqn 10) - cycles AC, BC

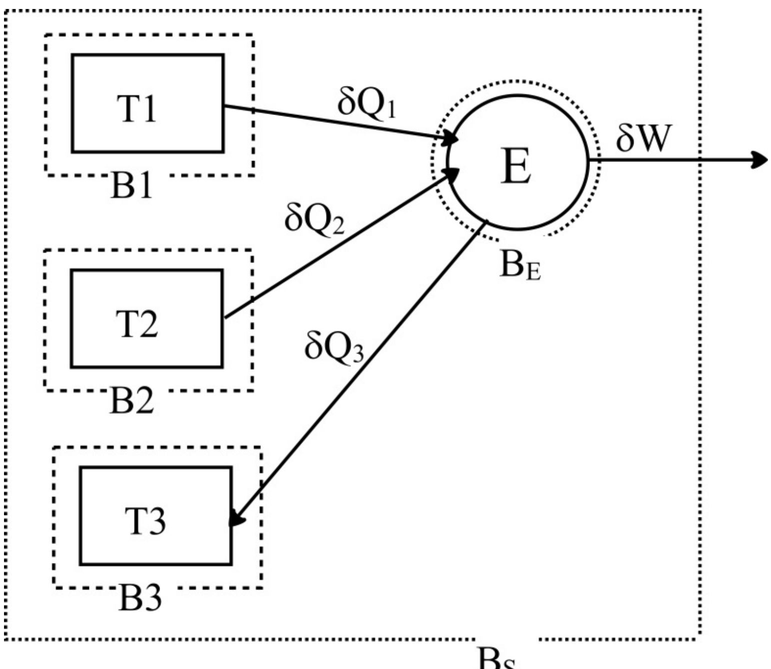
$$\Delta S_{1,2,A} + \Delta S_{2,1,C} = 0 \qquad \text{(if reversible)}$$

$$\Delta S_{1,2,B} + \Delta S_{2,1,C} = 0 \text{ (if reversible)}$$

ΔS_{12} has same value for paths A and B, is independent of path and S is a property.

Corollary 8 The entropy of any closed system which is thermally isolated from the surroundings either increases or, if the process undergone by the system is reversible, remains constant.

Proof



Fluid packets 1, 2, 3 pass small amounts of heat to engine E during one cycle. From Clausius inequality, <u>engine viewpoint</u>

$$\frac{\delta Q_1}{T_1} + \frac{\delta Q_2}{T_2} + \quad \ldots \quad \leq \qquad 0 \qquad (9b)$$

From fluid viewpoint each δQ changes sign

$$\frac{\delta Q_1}{T_1} + \frac{\delta Q_2}{T_2} + \ldots \geq 0$$

limit $\delta Q_1 \rightarrow 0$ and integrate from start to end time

$$\int_{start}^{end} \frac{dQ_1}{T_1} + \int_{start}^{end} \frac{dQ_2}{T_2} + \ldots \geq 0$$

Each term represents entropy

$$\Delta S_1 + \Delta S_2 + \ldots \geq 0$$

(Worked example concerning four reservoirs follows.)

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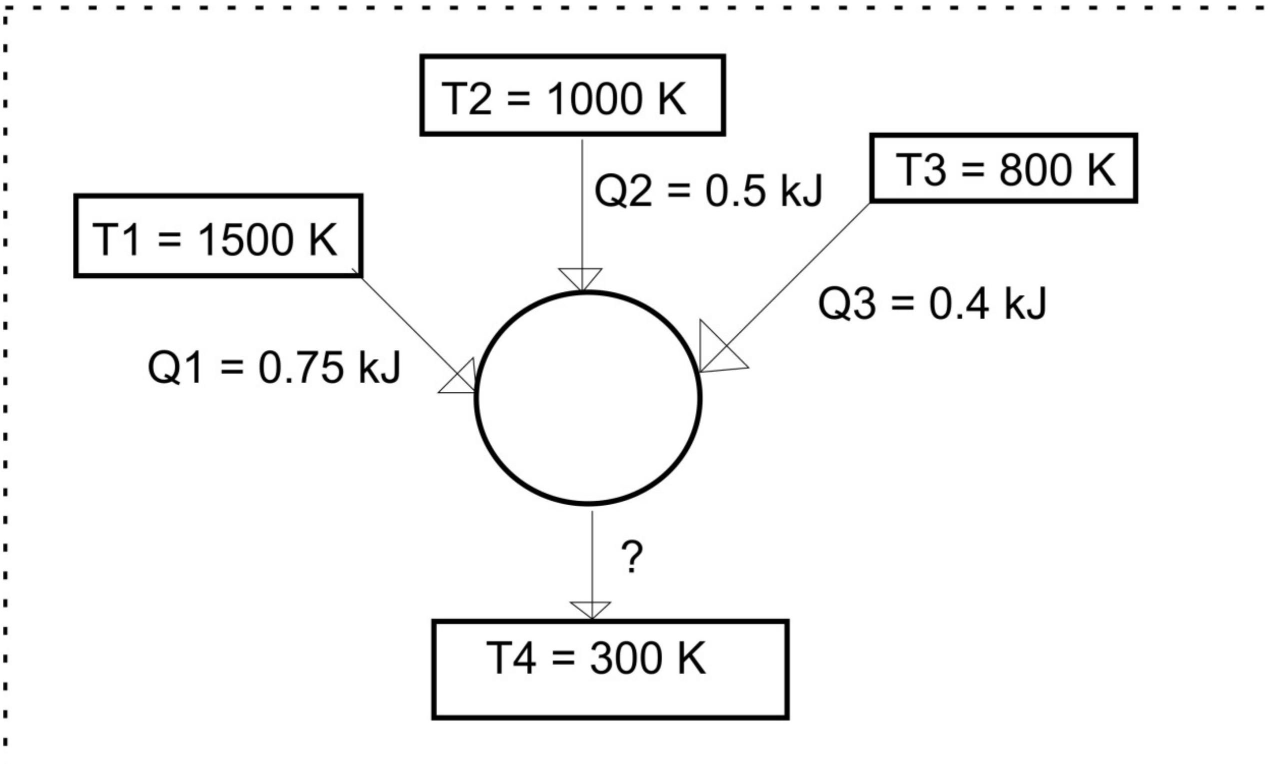
Find the minimum value of heat rejection Q_4



Items 1 to 4 are reservoirs for each of which

$$\Delta S = \int dQ_{rev}/T = Q/T$$

Total entropy change is

$$-0.75/1500 \;\; -0.5/1000 \;\; -0.4/800 + Q_4/300 \geq 0$$

$Q_4 \geq 0.45$ kJ   also $W \leq 1.2$ kJ (1^{st} law)

Conclusion

Through the Clausius inequality, we have demonstrated a property, entropy.

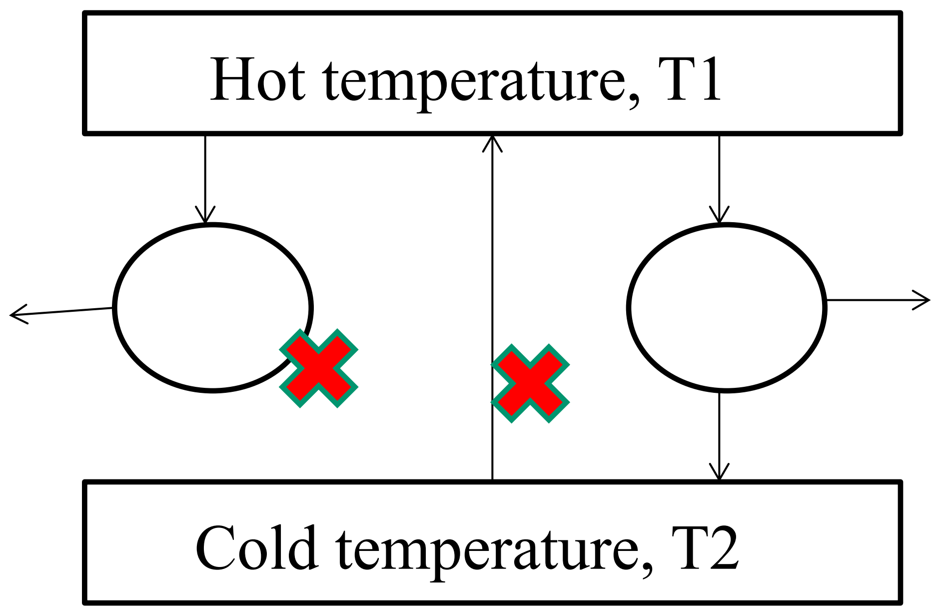
The entropy of a thermally isolated system increases when irreversible processes take place.

We can find entropy by integrating reversible heat addition.

$$\Delta S = \int_1^2 \frac{dQ_{rev}}{T} \qquad (10)$$

Here it indicates the minimum heat loss from a four-reservoir-engine.

Second Law – Recap



KP – No perfect engine exists.

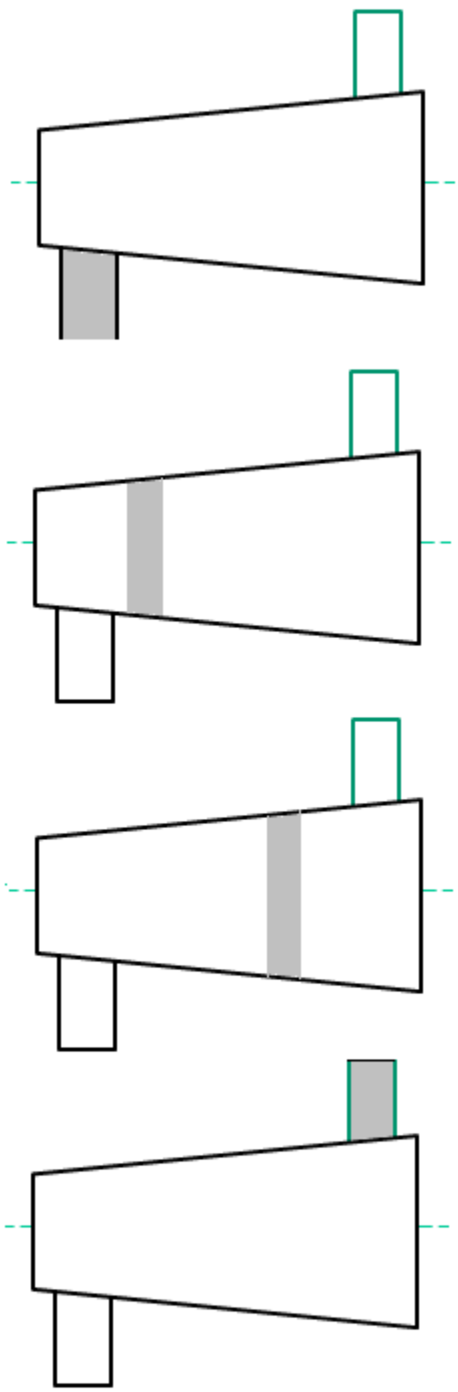
Clausius – No perfect heat pump exists.

Reverisible engine exhibits max. efficiency.

Define Thermodynamic Temperature scale so that

$$\eta = 1 - \frac{|Q_2|}{|Q_1|} = 1 - \frac{|T_2|}{|T_1|}$$

Moving boundary/ closed system



E.g turbine – a packet of fluid (grey) experiences no added heat. If frictionless, then $\Delta S = \int dQ/T = 0$ and the flow is isentropic.