

# Aerospace Vehicle Design And System Integration 3

## AENG30013

Stability Analysis using XFLR5

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# Contents

## 1. Static Stability

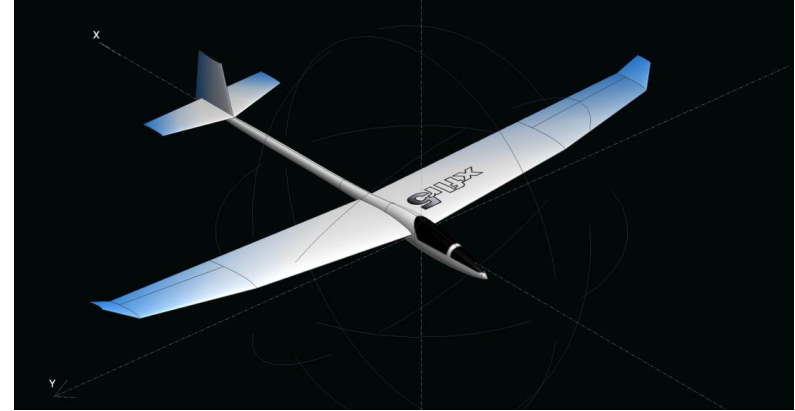
1. Review – including neutral point, static margin.
2. XFLR example – ‘sample project’

## 2. Dynamic Stability

1. Review – aircraft modes.
2. XFLR example– ‘sample project’

## 3. Lab Exercise





## Static & Dynamic Stability Analysis XFLR5

<http://www.xflr5.com/xflr5.htm>

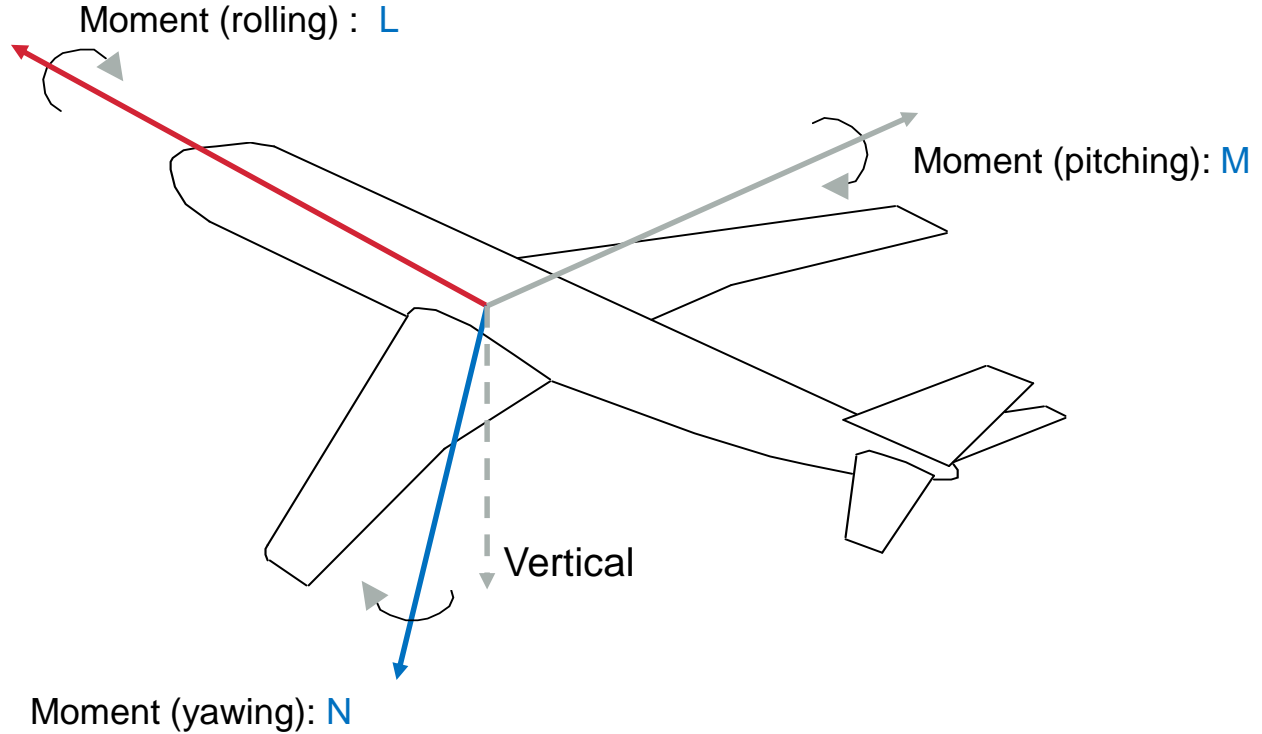
<https://sourceforge.net/projects/xflr5/files/>

(latest version) Sample\_Project.zip

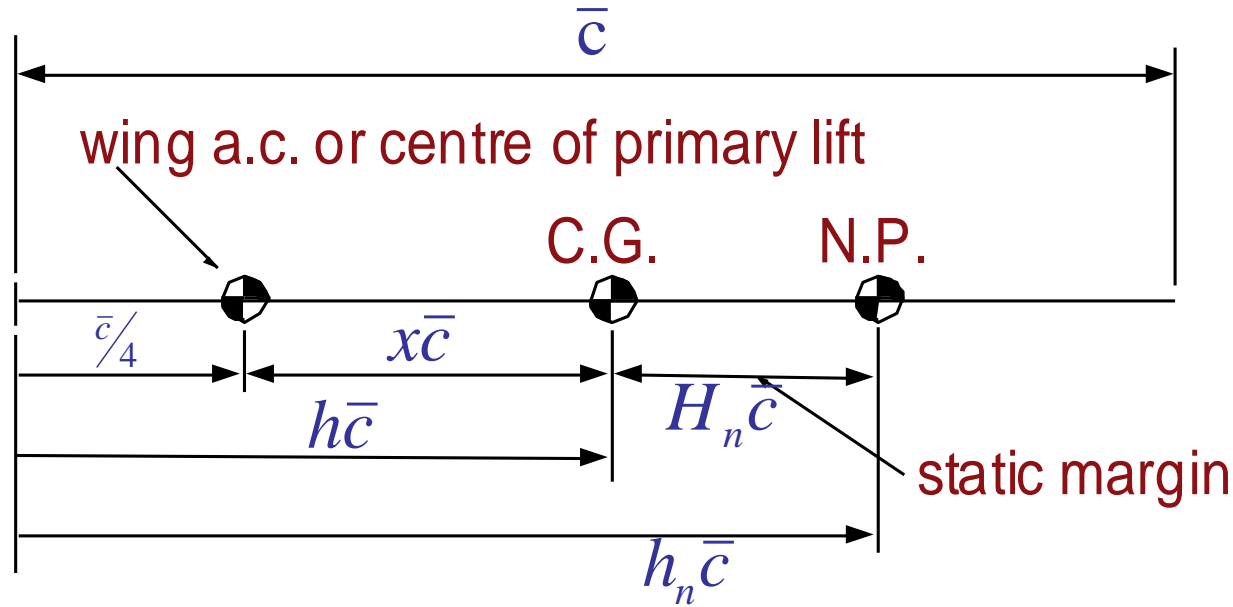
[http://www.xflr5.com/docs/XFLR5\\_and\\_Stability\\_analysis.pdf](http://www.xflr5.com/docs/XFLR5_and_Stability_analysis.pdf)

[http://www.xflr5.com/docs/XFLR5\\_Mode\\_Measurements.pdf](http://www.xflr5.com/docs/XFLR5_Mode_Measurements.pdf)

# Body Axes Notation and Sign Conventions



# The Neutral Point and Static Margin



The **neutral point** is the **rearmost position of the c.g.** before an unstable condition occurs.

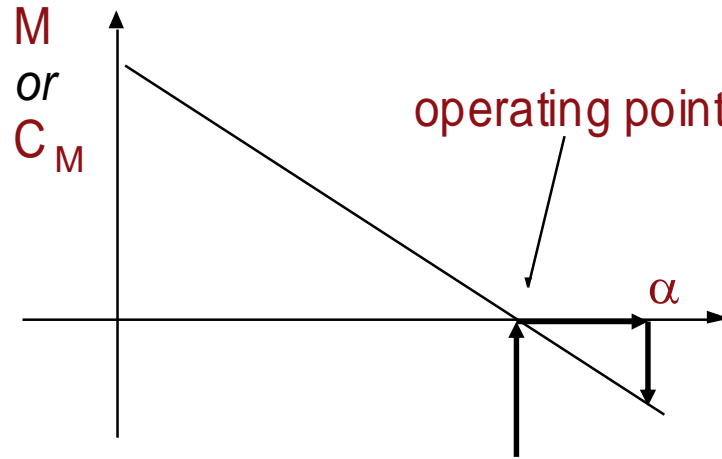
# Static Stability

- Consider a perturbation  $\Delta\alpha$  away from a trimmed, straight and level operating point? +ve or -ve slope desirable?
- This implies that whichever sign there is for  $\Delta\alpha$  away from the equilibrium flight angle  $\alpha$ , the pitching moment caused by  $\Delta\alpha$  will be of such a sign as to counteract  $\Delta\alpha$  and restore the flight to its equilibrium incidence.

The slope given by  $\partial M / \partial \alpha$  must therefore be negative for stability.

# Static Stability

- The "system" is the **pitch response** of an **aircraft**; the "input" is a pitch displacement  $\Delta\alpha$  and the "output" is the consequent pitching moment  $\Delta M$ .
- For stable flight we need  $\Delta M < 0$  for  $\Delta\alpha > 0$  so that there will be a restoring action.



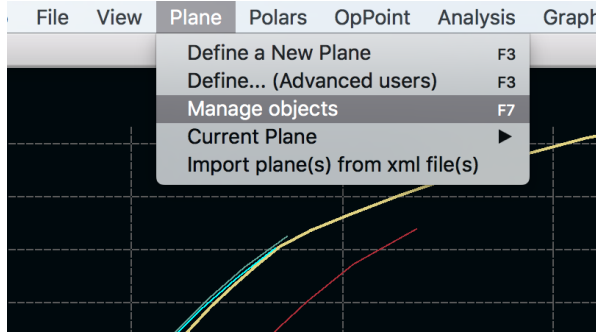
# Static Stability

**and thus in general**

$$\begin{array}{ll} \frac{1}{a_1} \frac{\partial C_M}{\partial \alpha} \text{ or } \frac{\partial C_M}{\partial C_L} > 0 & \text{unstable} \\ & = 0 \quad \text{neutrally stable} \\ & < 0 \quad \text{stable} \end{array}$$



# XFLR5 Example

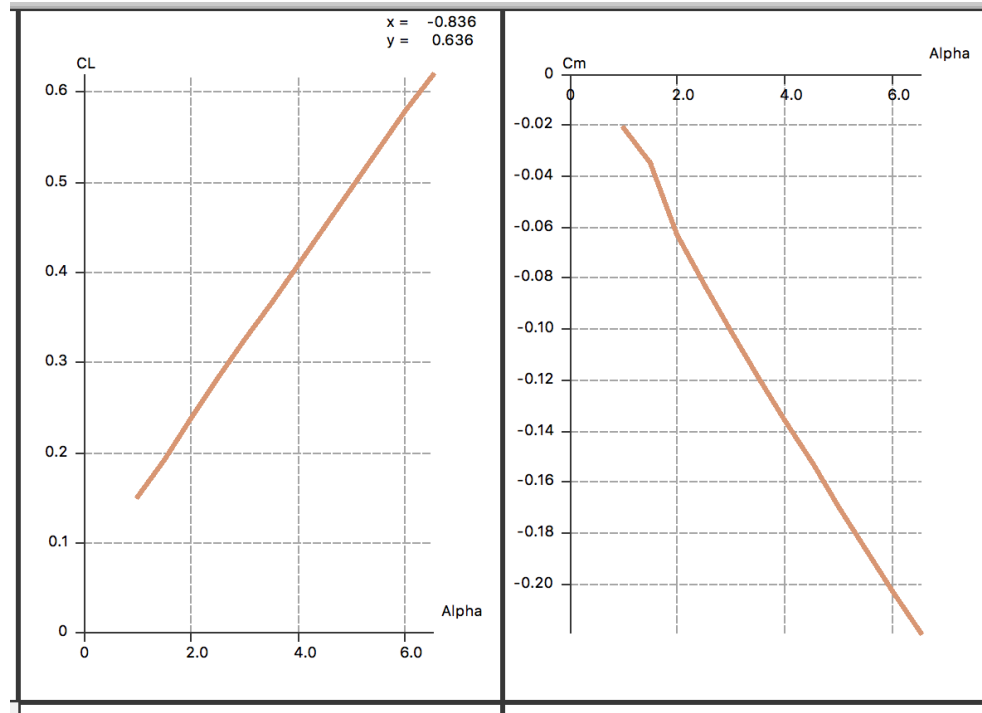


- Remove additional 'objects' – keeping 'Sample Plane'
- *Note: examples given in this lecture are exaggerated*

A screenshot of the 'Plane Object Management' dialog box. It contains a table with 9 columns: Name, Span (mm), Area (m²), M.A.C. (mm), AR, TR, It-Tip Sweep (°), and Tail Volume. There are four rows of data. The fourth row, 'Sample Plane with body', is highlighted in blue. To the right of the table are 'Rename' and 'Delete' buttons.

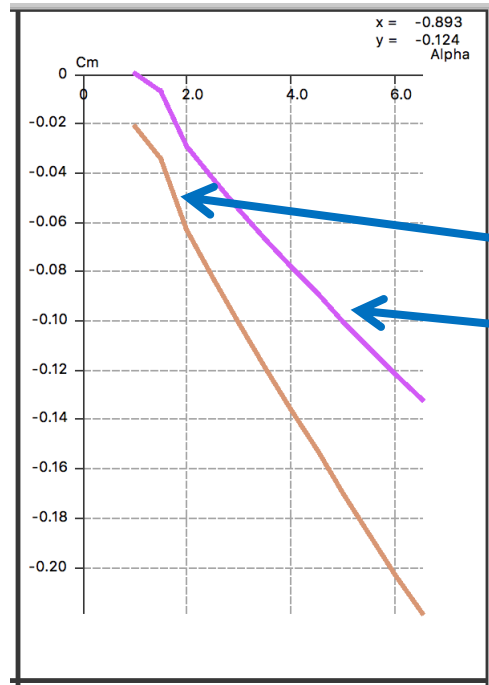
	Name	Span (mm)	Area (m²)	M.A.C. (mm)	AR	TR	It-Tip Sweep (°)	Tail Volume
1	Flying Wing	2260.000	0.445	215.791	11.48	3.75	27.8	0.00
2	Sample Plane no body	1500.000	0.210	147.426	10.73	4.50	8.0	0.46
3	Sample Plane no body asym	1500.000	0.210	147.426	10.73	4.50	8.0	0.46
4	Sample Plane with body	1500.000	0.210	147.426	10.73	4.50	8.0	0.46

# XFLR5 Example



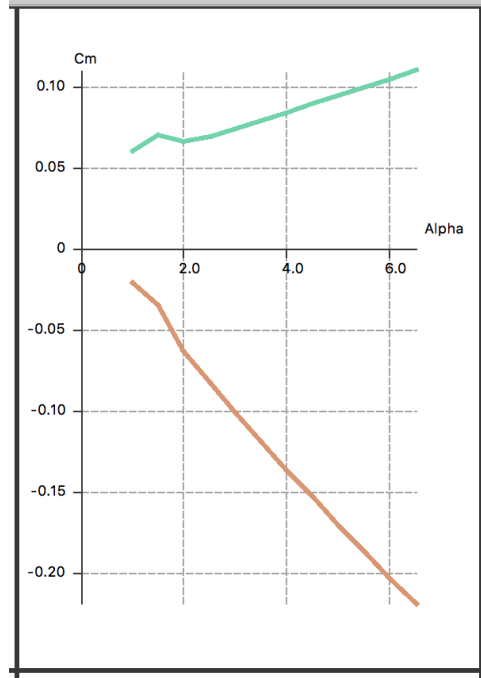
- $C_m$  vs  $\alpha$  with CoG.x = 58.44 mm

# XFLR5 Example



- $C_m$  vs  $\alpha$  with
- $CoG.x = 58.44$  mm &
- $CoG.x = 80.00$  mm

# XFLR5 Example



- $C_m$  vs  $\alpha$  with
- $CoG.x = 58.44$  mm &
- $CoG.x = 140.00$  mm

# Derivatives of the Pitching Moment Equation

- The pitching moment expression which we obtained was:

$$C_M = C_{M_0} - \bar{V} a_{1_T} i_T - C_{L_{WFP}} \left[ \bar{V} \frac{a_{1_T}}{a_1} (1-k) - x \right] - \bar{V} a_{2_T} \eta$$

For trimmed flight  $C_M = 0$

$$\bar{V} = \frac{S_T l_T}{S \bar{c}}$$

- Now consider a disturbance in pitch ( $\Delta\alpha$ ), perhaps caused by an upward gust, and this  $\Delta\alpha$  leads to an associated disturbance  $\Delta C_L > 0$ .

# Static Stability

- Thus for the case where the elevator is locked and  $\eta = \text{constant}$  (but not necessarily zero), we have for static stability

$$\frac{1}{a_1} \frac{\partial C_M}{\partial \alpha} = \frac{\partial C_M}{\partial C_L} = - \left[ \bar{V} \frac{a_{1T}}{a_1} (1 - k) - x \right] < 0$$

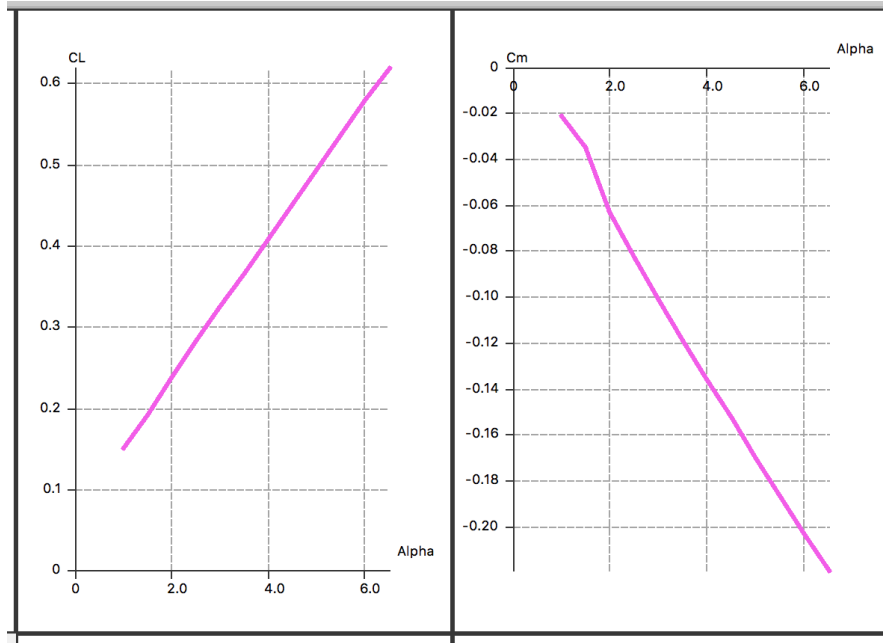
and the factor in the square brackets must therefore be  
positive for a stable aircraft.

## Elevator-Angle-to-Trim – *For Reference*

- *and thus the elevator angle required to obtain this trim is:*

$$\eta_{trim} = \frac{1}{\bar{V} a_{2_T}} \left\{ C_{M_0} - \bar{V} a_{1_T} i_T - C_{L_{WFP}} \left[ \bar{V} \frac{a_{1_T}}{a_1} (1-k) - x \right] \right\}$$

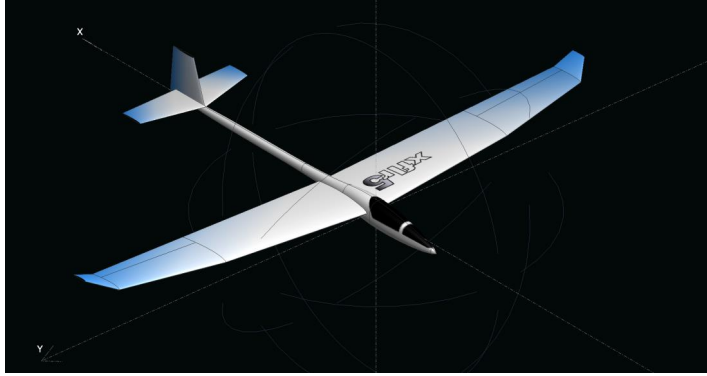
# XFLR5 Example



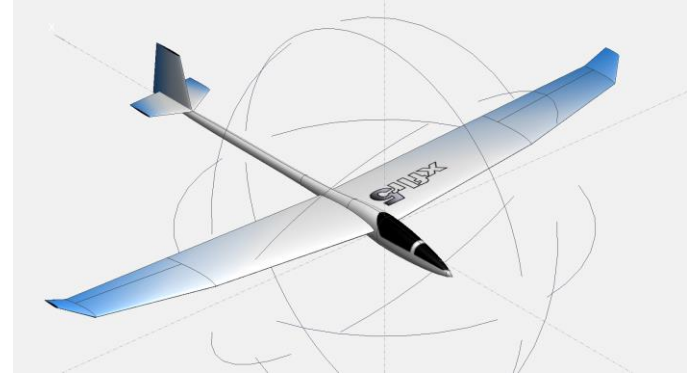
- $C_m$  vs  $\alpha$  with CoG.x = 58.44 mm



# XFLR5 Example

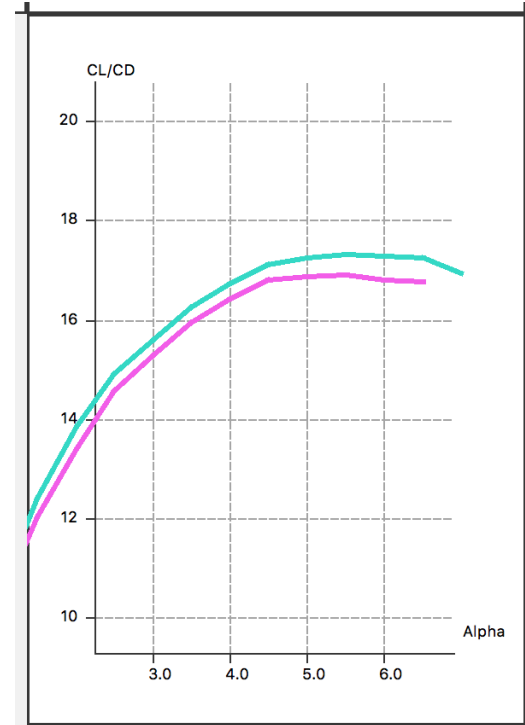
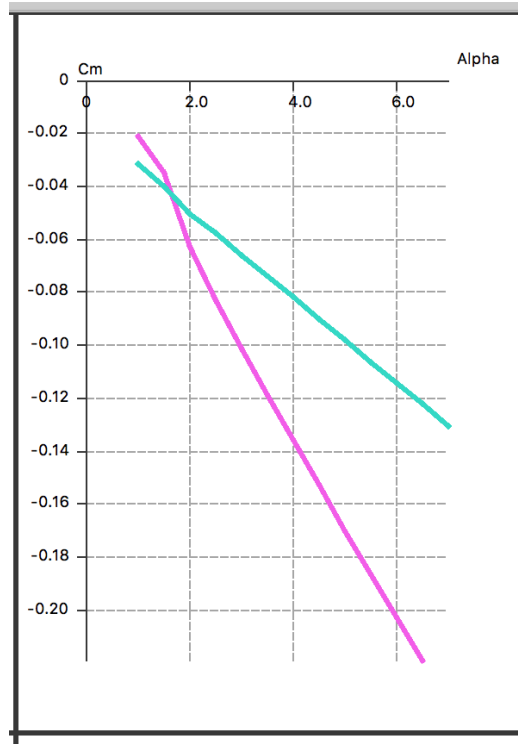


160 mm elevator semi-span



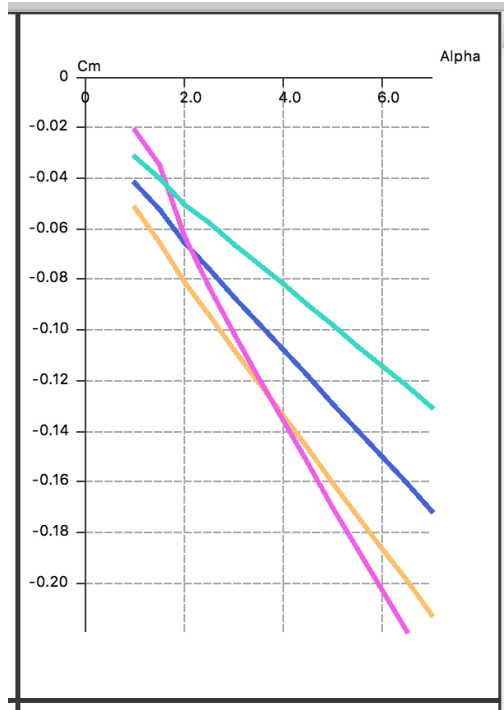
80 mm elevator semi-span

# XFLR5 Example



- Reduced tail size vs original

# XFLR5 Example

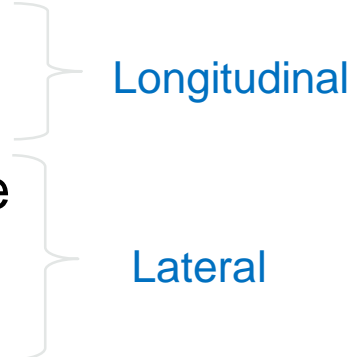


- Reduced tail size vs original
- New tail size with C.of.G. at 48.44mm and 38.44 mm
- Effect on static stability & static margin?
- Trim requirement?
- Control requirement?



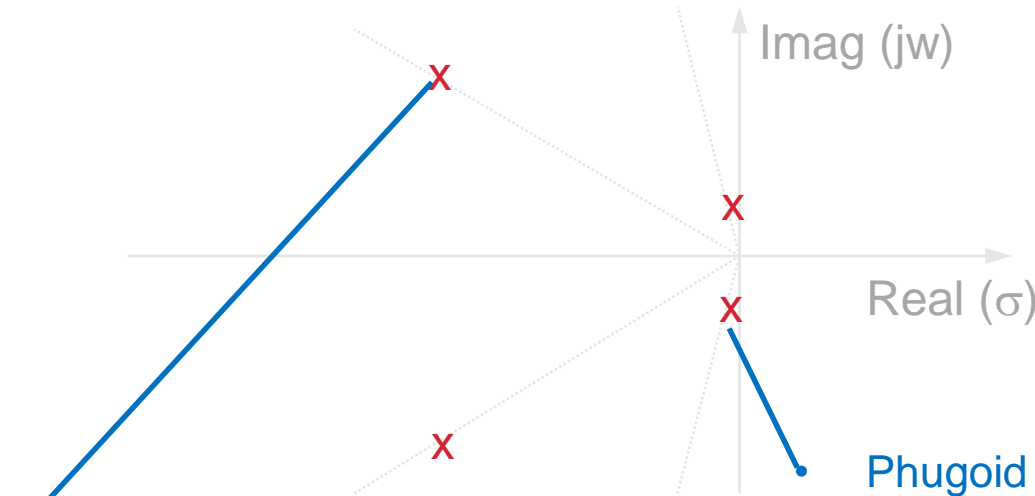
# Aircraft Modes

# Aircraft Modes

- Short Period
  - Phugoid
  - Roll Subsidence
  - Spiral
  - Dutch Roll
- 
- The diagram uses curly braces to group the modes. The first two modes, 'Short Period' and 'Phugoid', are grouped under the 'Longitudinal' label. The next three modes, 'Roll Subsidence', 'Spiral', and 'Dutch Roll', are grouped under the 'Lateral' label.
- Longitudinal
- Lateral

# Typical Longitudinal Responses

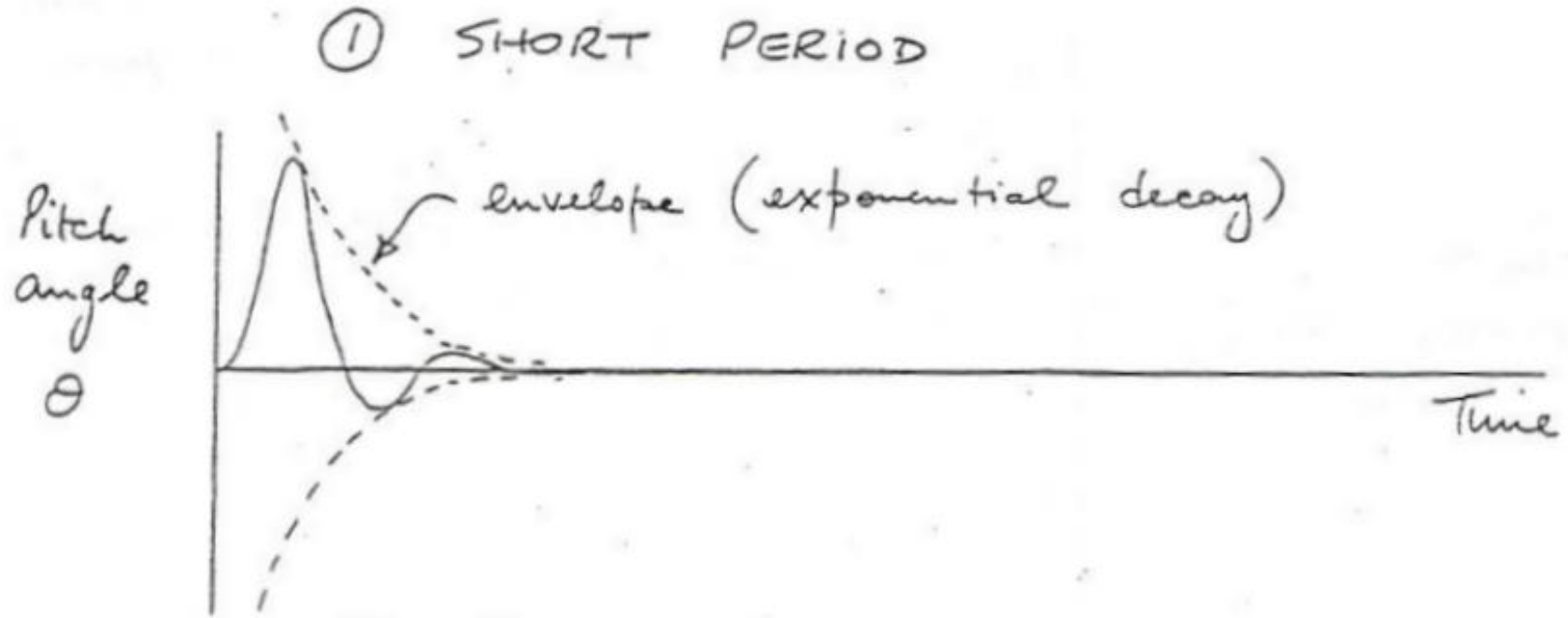
- For a conventional aircraft configuration, the characteristic roots will normally occur in **two complex pairs**, as shown below.



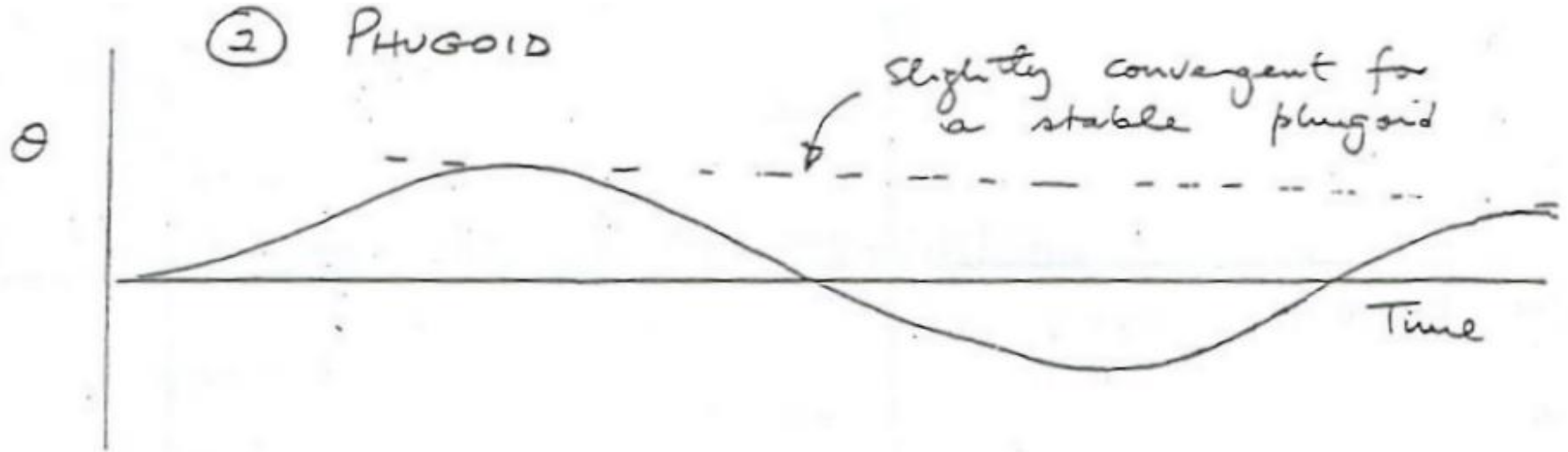
**Short-period** pair (higher frequency and more heavily damped)  $\sigma$  is much more negative

**Phugoid pair** (low frequency and low damping ratio) i.e.  $\sigma$  is not large and can even be positive for some flight conditions.

# Typical Aircraft Response

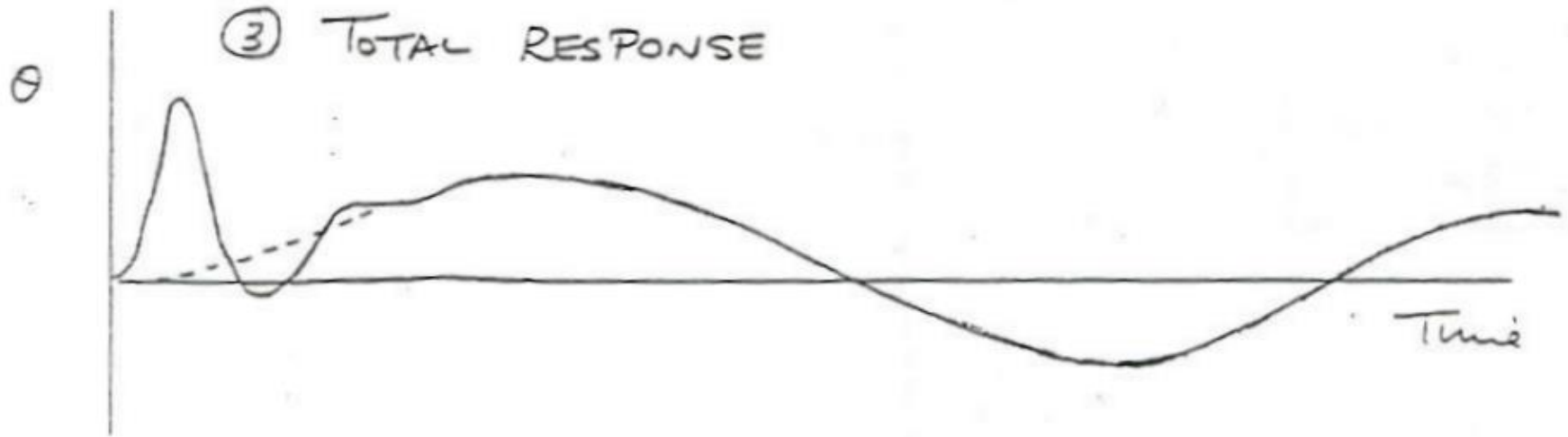


# Typical Aircraft Response



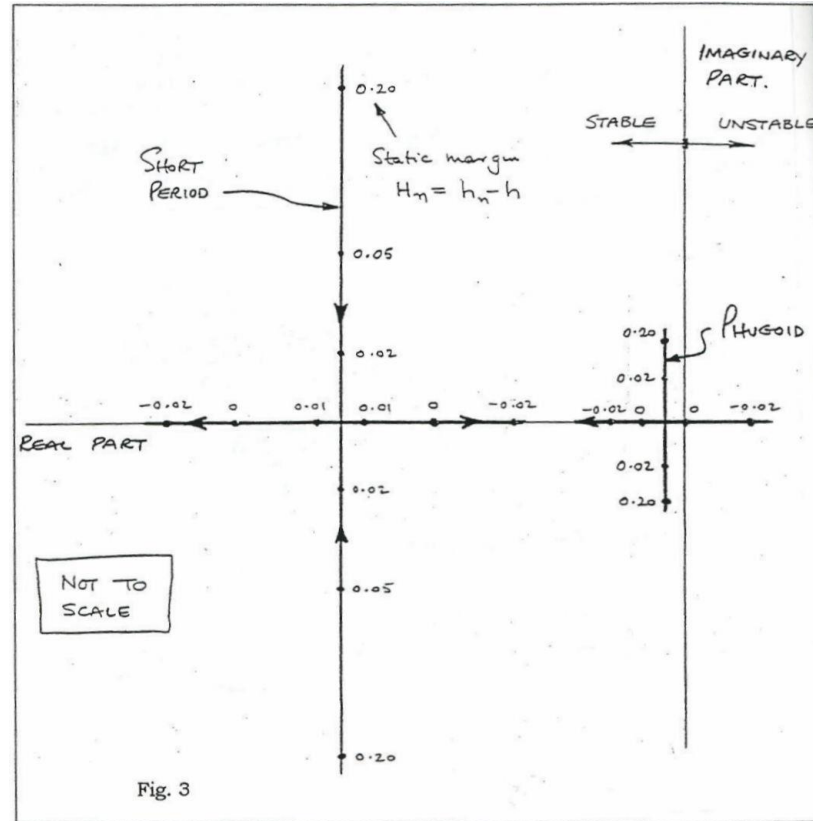


# Typical Aircraft Response

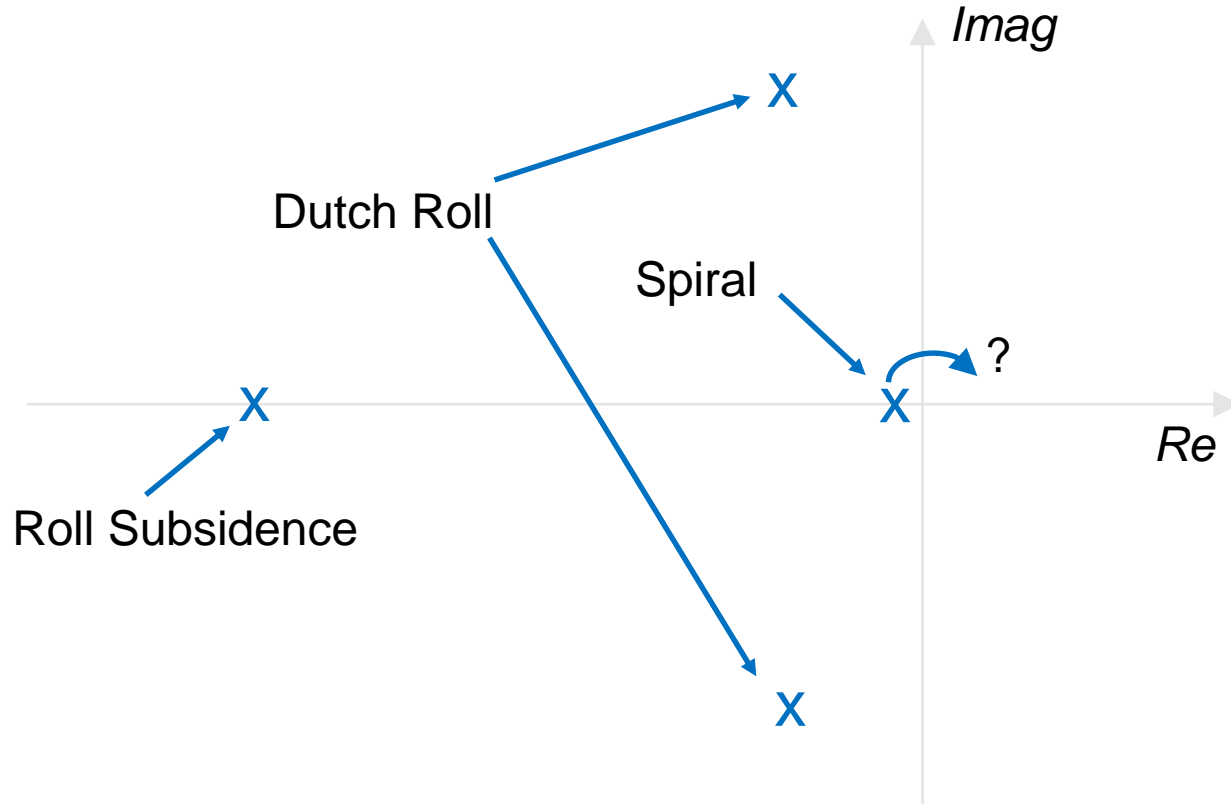


# Root locus as CG is shifted aft

- $h$  is the CG position relative to the leading edge; values shown on the loci are for static margin  $H_n$ .
- Instability occurs when  $H_n=0$ , i.e. starting when the first root crosses into the right half plane.
- As this occurs here at the origin it corresponds to static instability.
- The CG position affects the derivative  $M_w$  fairly significantly, although other derivatives may also vary.



# Lateral Roots



# Lateral Modes

1. Large negative real root - the **roll convergence** (*or roll subsidence*).

This **implies** almost **pure rolling motion** and of course cannot last long in reality.

A more realistic attitude suggests that if a roll "pulse" hits the aircraft (e.g. a brief up-gust on one wing), the **consequent response** in **roll** would be **heavily damped**.

# Lateral Modes

2. Small real root, of either sign - the **spiral mode**.

This would be, for an **unstable** case, a slow **divergence in yaw** (say nose to starboard) while a roll angle built up (rolling to starboard) and thus also a sideslip would develop.

The later stage would be a tightening **spiral dive** with all three motion variables involved.

In practice, a **pilot** can control an unstable spiral mode (*slow!*).

# Lateral Modes

## 3. The complex pair - **Dutch Roll**.

Strictly speaking, all freedoms are active here, in an oscillatory sense, and out of phase with each other.

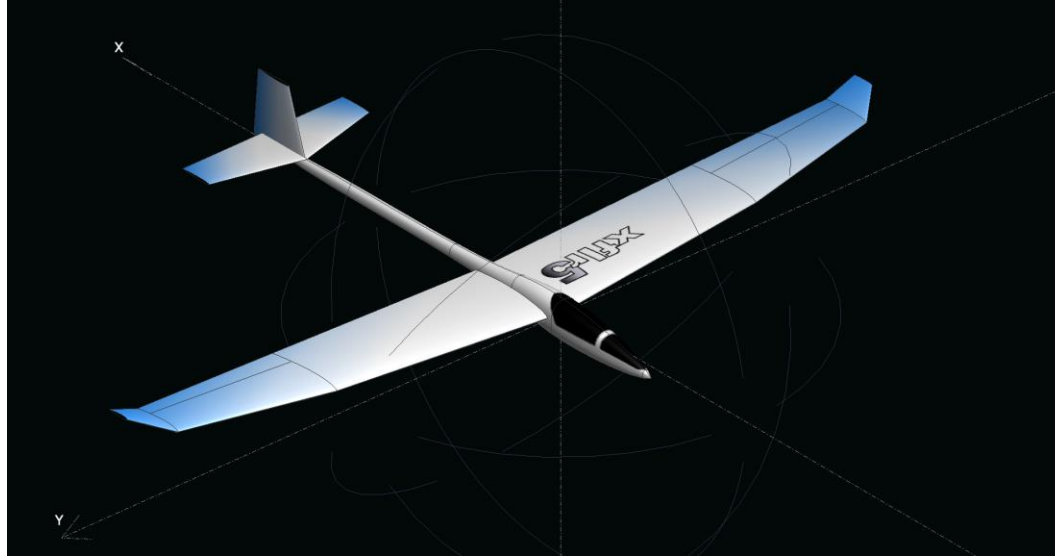
This mode can be badly (though positively) damped and will affect Handling Qualities. The frequency is probably lower (the period a bit longer) than the longitudinal short period mode.

Can be unpleasant if poorly damped! (nausea)

Often poorly damped on swept wing aircraft

Yaw damper

# XFLR example



(Using the baseline configuration)

# XFLR example



Longitudinal modes

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Stability

Stability direction

☒ Longitudinal ☐ Lateral

Operating point modes

Mode Selection

☒ 1 ☐ 2 ☐ 3 ☐ 4

Mode properties

F = 5.639 Hz

F1 = 4.099 Hz

z = 0.945

t2 = s

t =

Eigenvalues

l= -24.3307- 25.7571i

v1= 0.03892+ 0.00000i

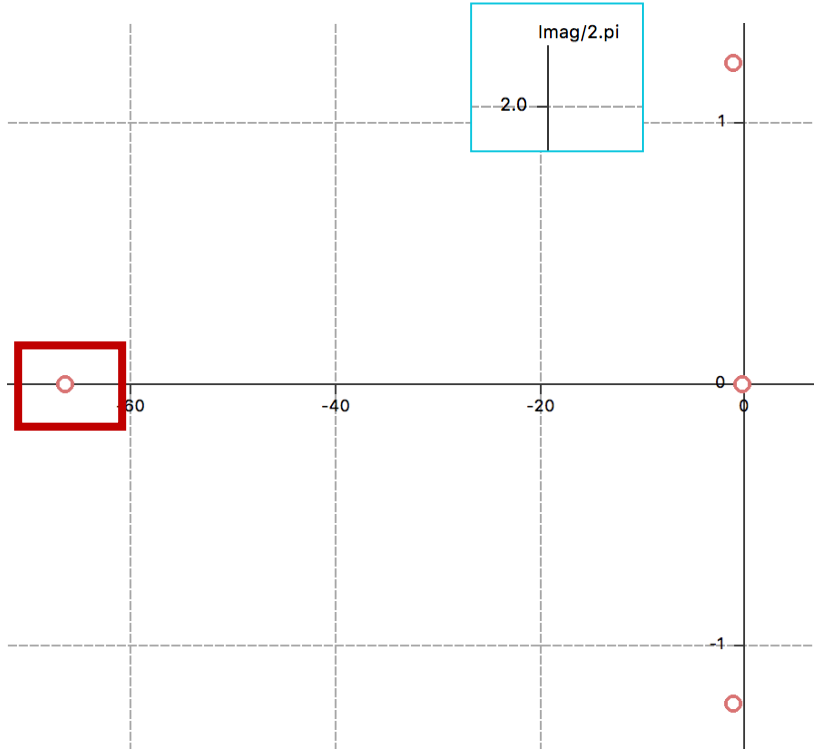
v2= 1.54740- 3.87757i

v3= -0.30261- 0.23748i

v4= 1.00000+ 0.00000i



# XFLR example



Lateral modes

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Stability

Stability direction

☐ Longitudinal ☒ Lateral

Operating point modes

Mode Selection

☒ 1 ☐ 2 ☐ 3 ☐ 4

Mode properties

F =  Hz

F1 =  Hz

z =

t2 =  0.010 s

t =  0.015

Eigenvalues

l=  -66.2168+ 0.0000i

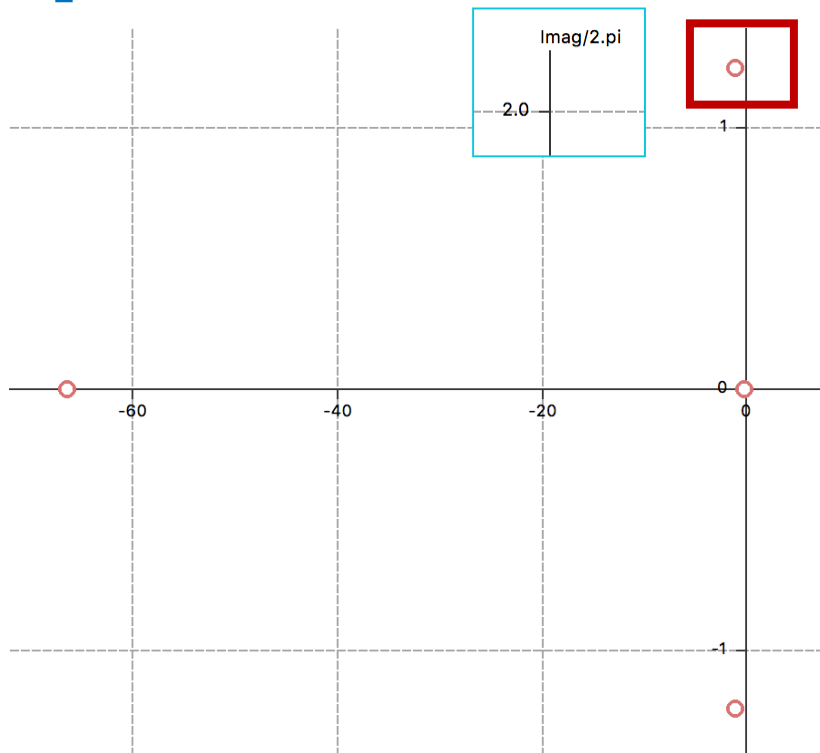
v1=  0.03892+ 0.00000i

v2=  16.74606+ 0.00000i

v3=  -0.30922+ 0.00000i

v4=  1.00000+ 0.00000i

# XFLR example



Lateral modes

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Stability

Stability direction

☐ Longitudinal ☒ Lateral

Operating point modes

Mode Selection

☐ 1 ☒ 2 ☐ 3 ☐ 4

Mode properties

F = 1.229 Hz

F1 = 1.219 Hz

z = 0.131

t2 = s

t =

Eigenvalues

I= -1.0046- 7.6581i

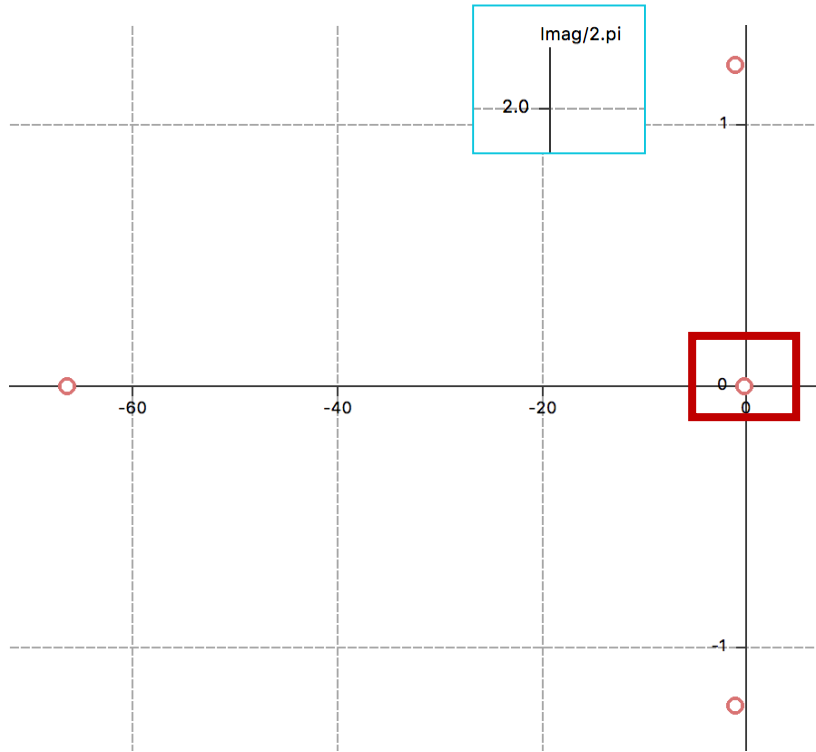
v1= 0.03892+ 0.00000i

v2= -0.00383- 0.00001i

v3= 0.00048+ 0.00865i

v4= 1.00000+ 0.00000i

# XFLR example



Lateral modes

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Stability

Stability direction

☐ Longitudinal ☒ Lateral

Operating point modes

Mode Selection

☐ 1 ☐ 2 ☐ 3 ☒ 4

Mode properties

F =  Hz

F1 =  Hz

z =

t2 =  65.185 s

t =  94.042

Eigenvalues

l=  -0.0106+ 0.0000i

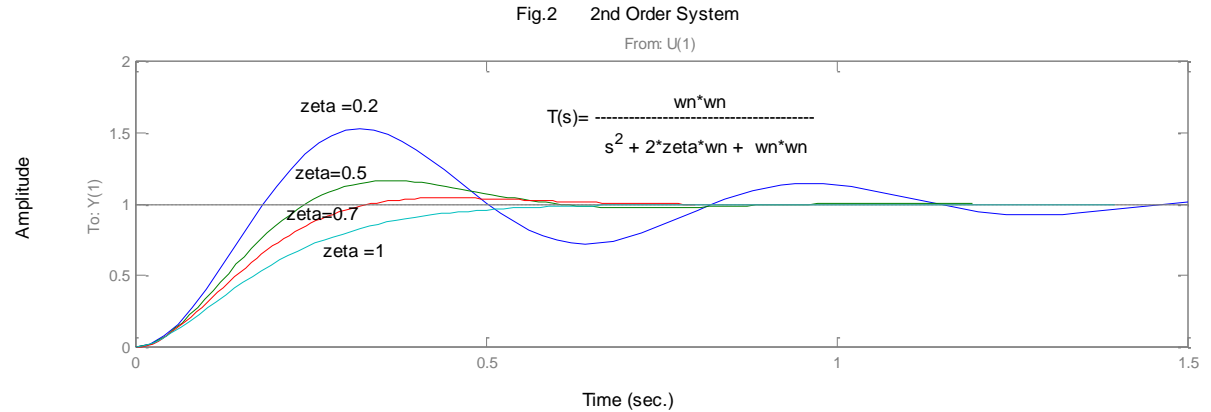
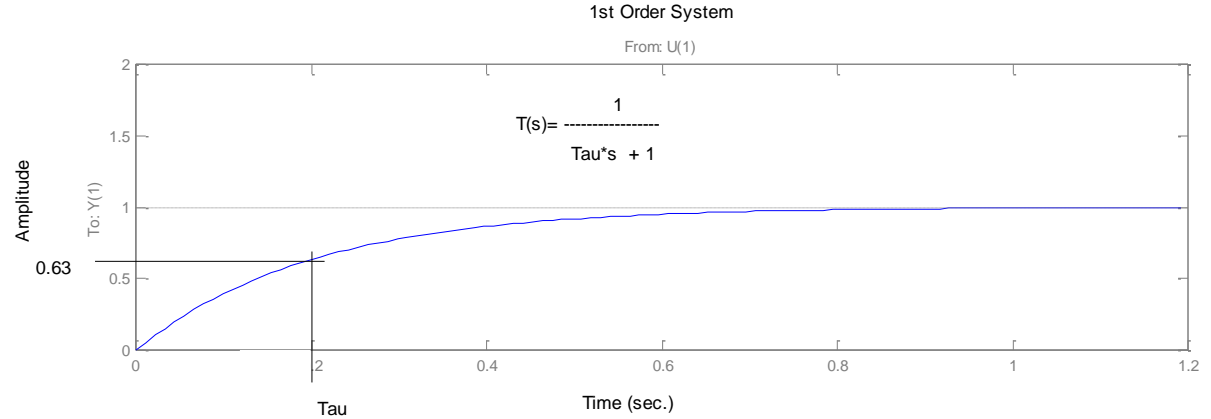
v1=  0.03892+ 0.00000i

v2=  -0.00137+ 0.00000i

v3=  0.04905+ 0.00000i

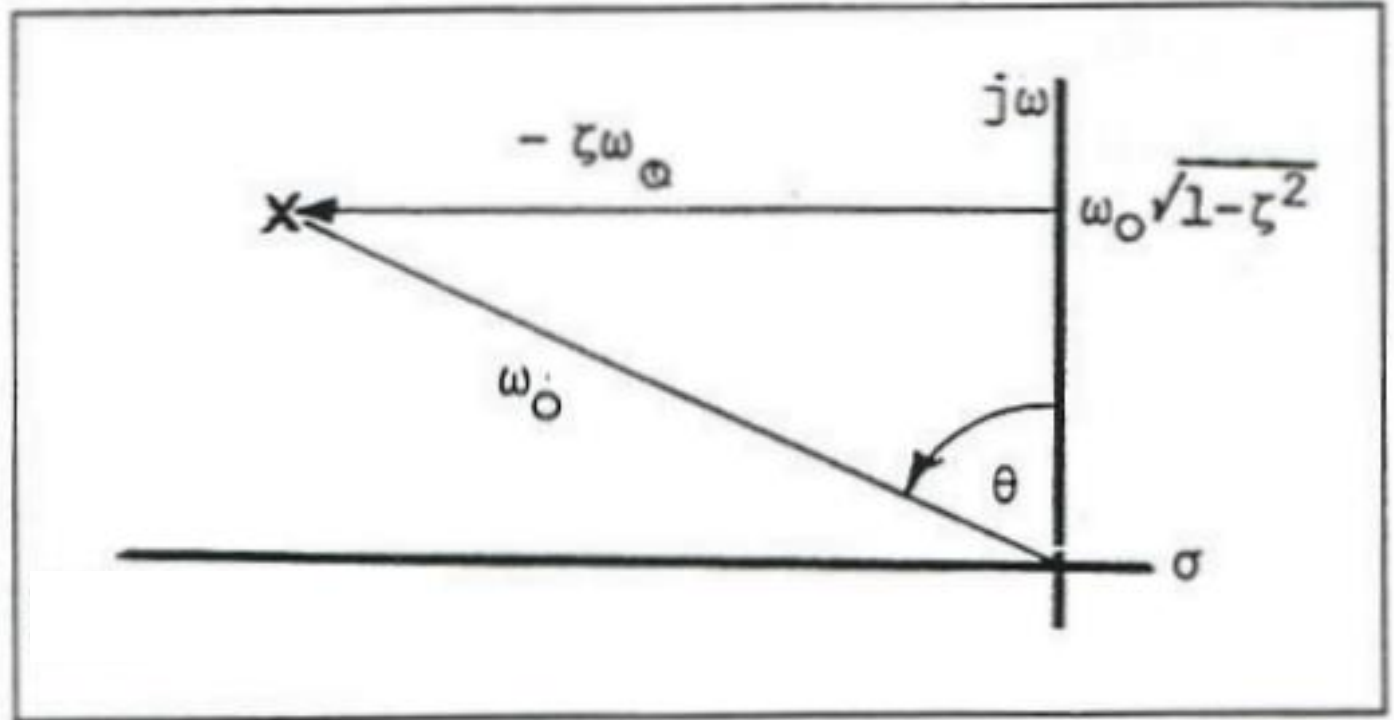
v4=  1.00000+ 0.00000i

# System Stability



# Constant Damping Ratio $\zeta$

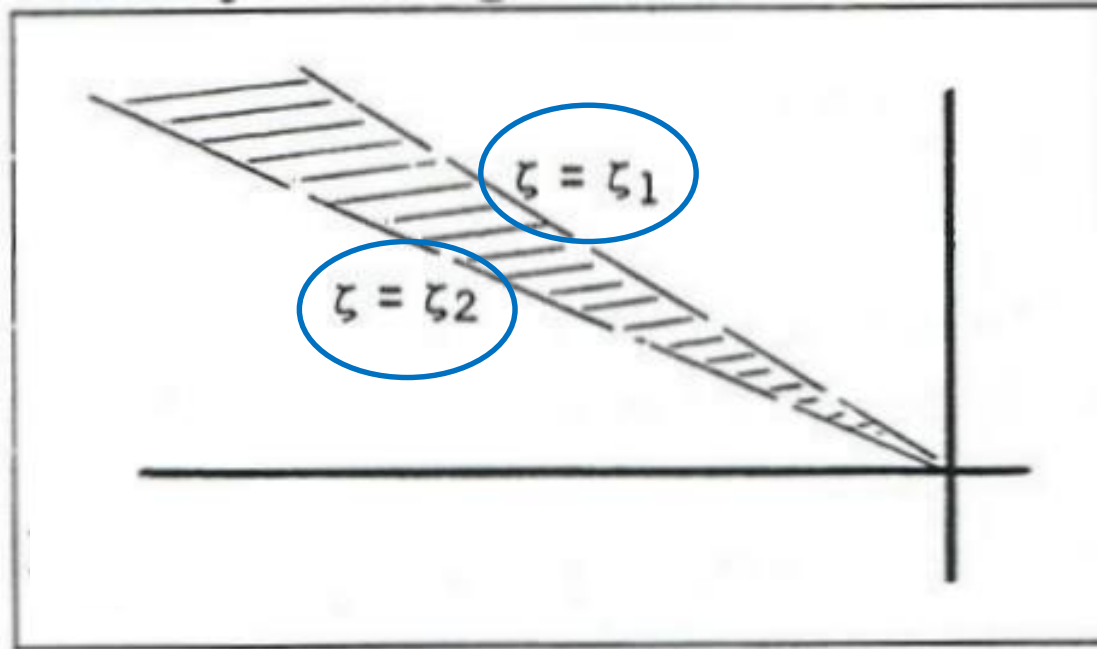
- Where  $\lambda = \sigma \pm j\omega$



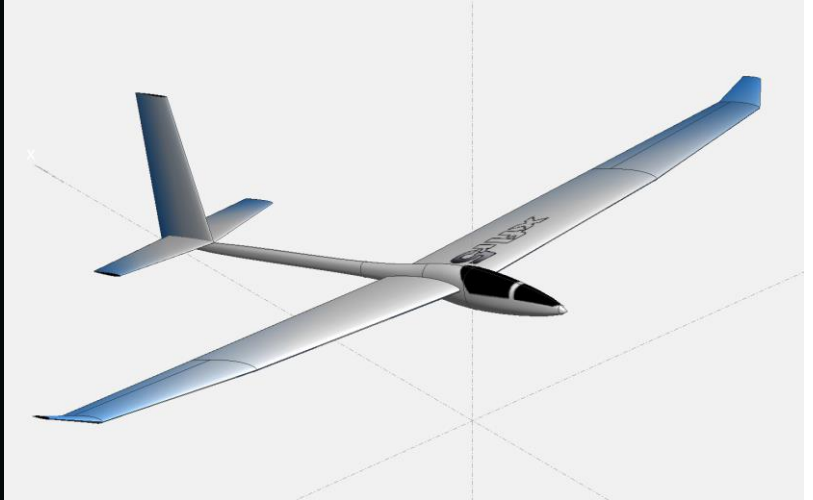
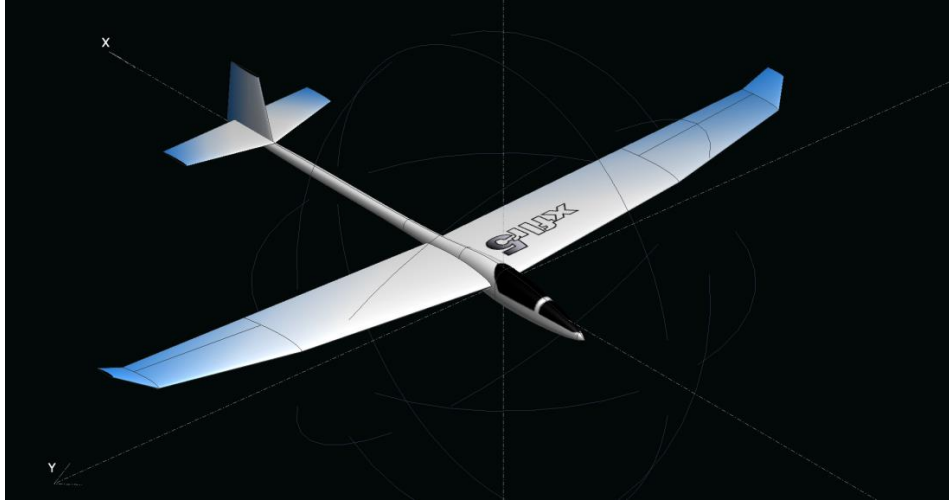
# Constant Damping Ratio $\zeta$

- We can show that:

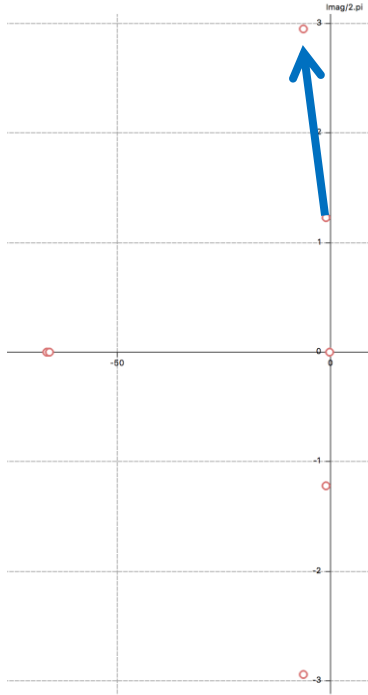
$$\zeta = \sin\theta$$



# XFLR example



# XFLR example



Lateral modes

bristol.ac.uk

Stability

Stability direction

☐ Longitudinal ☒ Lateral

Operating point modes

Mode Selection

☐ 1 ☒ 2 ☐ 3 ☐ 4

Mode properties

F =  Hz

F1 =  Hz

z =

t2 =  s

t =

Eigenvalues

l=

v1=

v2=

v3=

v4=



Stability

Stability direction

☐ Longitudinal ☒ Lateral

Operating point modes

Mode Selection

☐ 1 ☒ 2 ☐ 3 ☐ 4

Mode properties

F =  Hz

F1 =  Hz

z =

t2 =  s

t =

Eigenvalues

l=

v1=

v2=

v3=

v4=



# Coursework

- 2 Flight Mechanics Design
  - The stretch variant of the aircraft developed in part 1 (Aerodynamic Design) requires a horizontal and a vertical new tail, which you need to design and assess their effects on the aircraft trim and stability.
    - 2.1 Tail sizing and trim
    - 2.2 Static stability
    - 2.3 Dynamic stability
  - *Clear plots*
  - *Physical explanations*



Any Questions?

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