INTRODUCTION TO 3D WING FLOWS

AIMS

- To explain how the flow over a 3D wing differs from the flow over an aerofoil.
- To introduce vortex filaments and the Biot-Savart law
- To describe a simple horse shoe vortex model for wing flow

1 INTRODUCTION

The 2D flows over aerofoils considered so far can be regarded as flows over rectangular wings of infinite span. When finite wings are considered the wing geometry is three-dimensional and the flow structure is also three-dimensional with a flow component in the spanwise direction. This difference in the basic flow structure for 3D finite wings needs to be understood before potential models of the flow can be developed.

2 BASIC FLOW STRUCTURE FOR WINGS

An aerofoil can be regarded as a section of a rectangular wing of infinite span. For these infinite wings the pressure difference between the upper and lower surface of the aerofoil section does not vary along the span, hence the lift and circulation produced by the section is constant across the span. If the span is then made finite, the high pressure air beneath the wing spills out around the wing tips towards the low pressure air above the wing. As a result of the tendency of the flow to equalize the upper and lower surface pressures near the tips, the pressure difference between the lower and upper surfaces, the lift and circulation all vary along the span, see Figure 1 which shows a representative load distribution for a rectangular wing in subsonic flight.

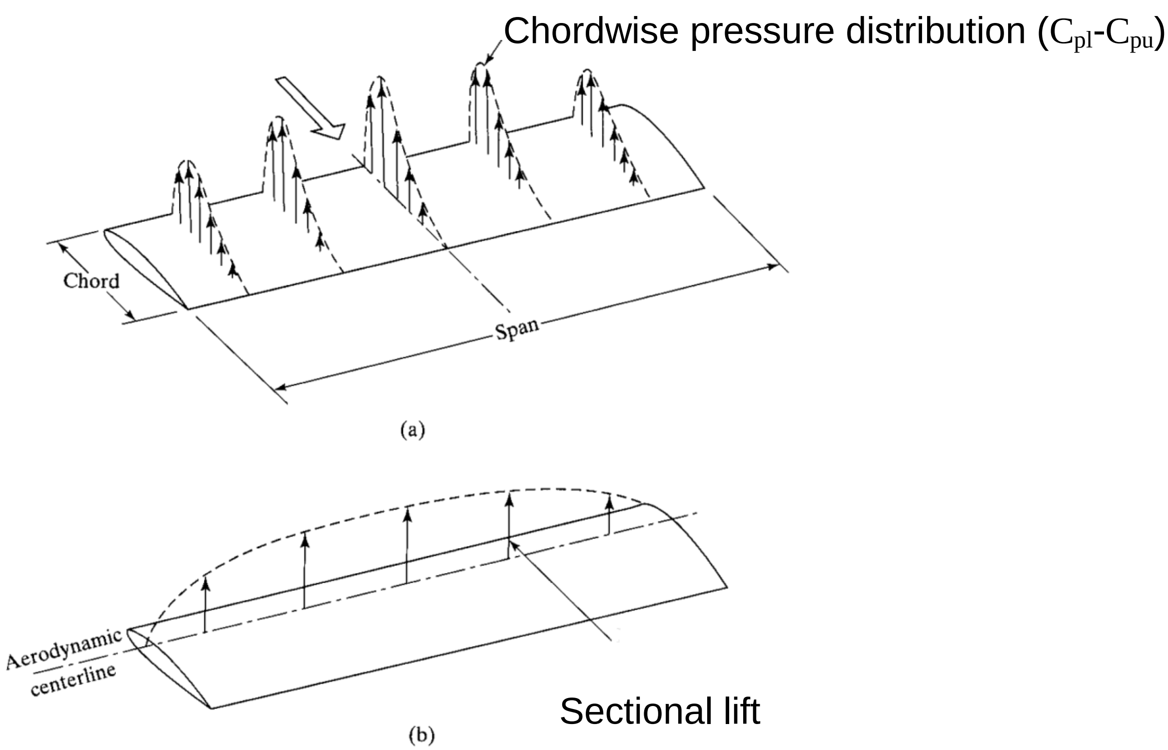


Figure 1

The spanwise variation in pressure, the air on the lower surface moves outboard and on the upper surface moves inboard, see Figure 2.

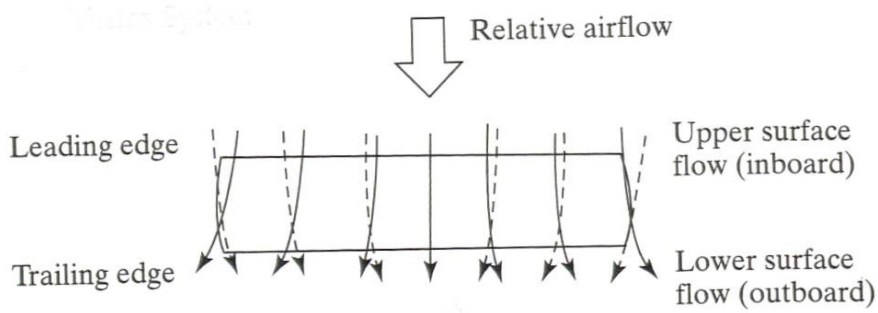


Figure 2 **TOP VIEW -PLANFORM**

Figure 3 shows a sketch of the flow over a section of the wing taken in the spanwise direction, the movement of air from the lower to the upper surface is shown. This means that at a particular spanwise station there is a difference in velocity between the upper and lower surfaces.

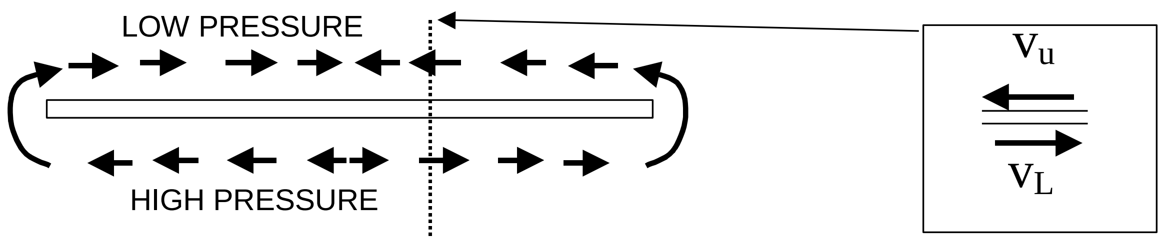


Figure 3 **FRONT VIEW-SPANWISE SECTION**

This difference or jump in velocity is maintained towards the trailing edge of the wing and hence a vortex wake shear layer is shed from the trailing edge of the wing. If this shear layer is quite thin then it can be approximated as an infinitely thin surface of discontinuity in velocity i.e. a vortex sheet.

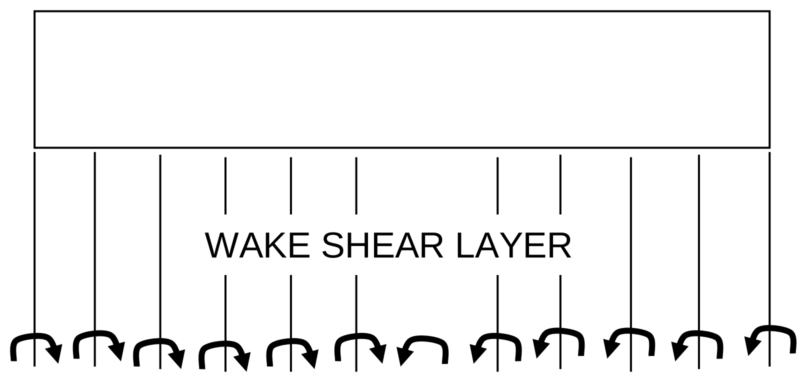


Figure 4

The wake shear layer is unstable and tends to roll up at its edges to form tip vortices, see Figure 5.

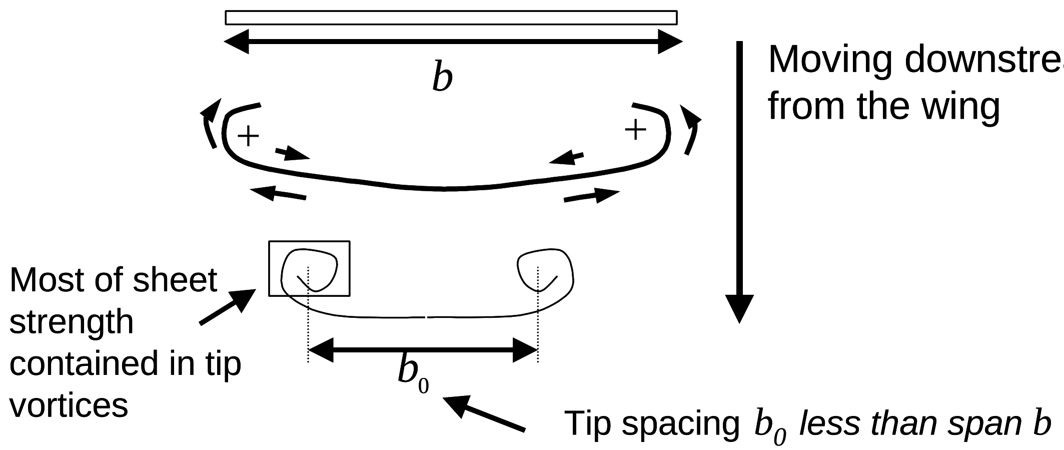


Figure 5

Far downstream most of the vorticity is contained in tip vortices. The spanwise distance between the tip vortices is b_0 which is less than the span of the wing, *b*. The presence of the tip vortices leads to *induced drag*. Insight into how tip vortices lead to the generation of induced drag can be gained via a simple horseshoe vortex model, however vortex filaments must be introduced before it can be developed.

3 VORTEX FILAMENTS

3.1 Extending Potential Solutions from 2D to 3D

Potential methods for modelling 2D flows over lifting aerofoils have already been described in this course. These methods can be extended to yield three dimensional models.

The simplest models can be constructed using *vortex filaments* which are the 3D flow equivalent to the 2D point vortex. The circulation around any path enclosing the vortex filament is a constant Γ.

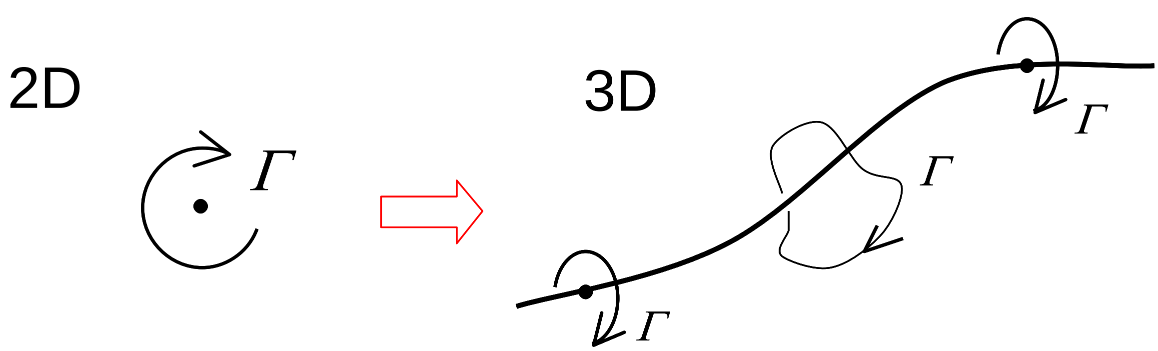


Figure 6

Other models use the *3D vortex sheet*, which is the equivalent of the 2D vortex sheet seen earlier.

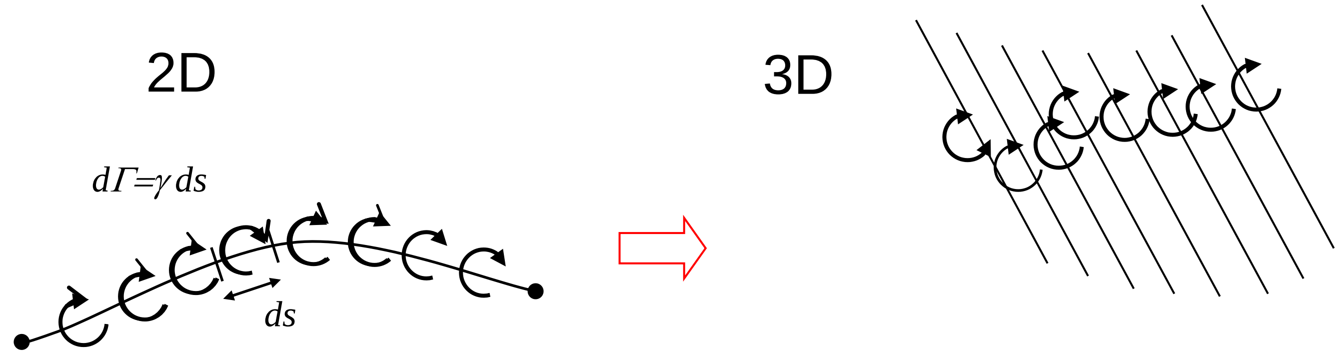


Figure 7

3.2 Helmholtz Theorems

The Helmholtz theorems of vortex filaments are:

(1) The strength of a vortex filament is constant along its length
(2) A vortex filament cannot start or end in a fluid. It must either form a closed path, or extend to the fluid boundaries including infinity.

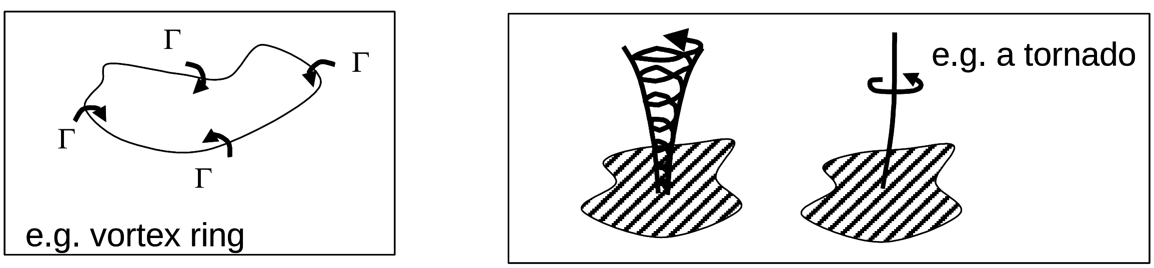


Figure 8

(3) The fluid that forms a vortex filament will continue to form a vortex filament. A vortex filament therefore moves with the fluid (unless ‘bound’).

3.3 The velocity induced by a vortex filament

The velocity induced by a vortex filament is given by the Biot-Savart Law. A general form is available for curved filaments, but in this unit only the special case for straight vortex filaments is required, see ***Appendix 1*** on the Biot-Savart Law.

4 THE HORSESHOE VORTEX MODEL

4.1 Basis for model

The Helmholtz theorems together with knowledge of the flow structure over finite wings leads to a simple first model of 3D wing flow. This is based on extending the 2D ‘lumped vortex’ model into 3D. The aerofoil is modelled by a bound vortex of strength Γ, together with a shed starting vortex of strength –Γ. When the starting vortex is far enough downstream, its influence on the flow near the aerofoil can be neglected.

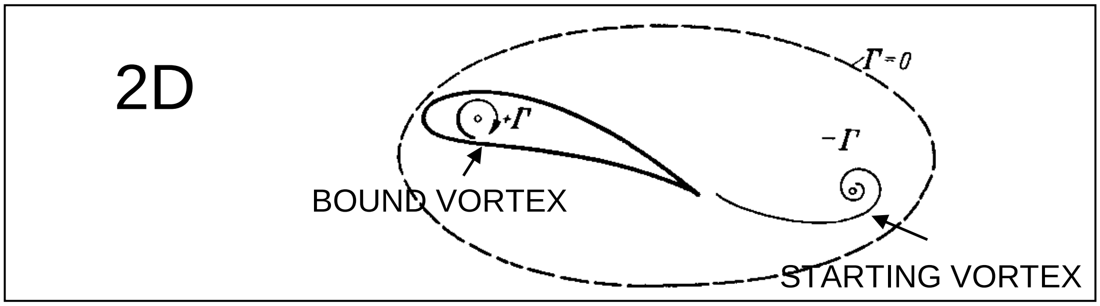


Figure 9

Extending this basic idea to 3D, start with a bound vortex filament along the wing span.

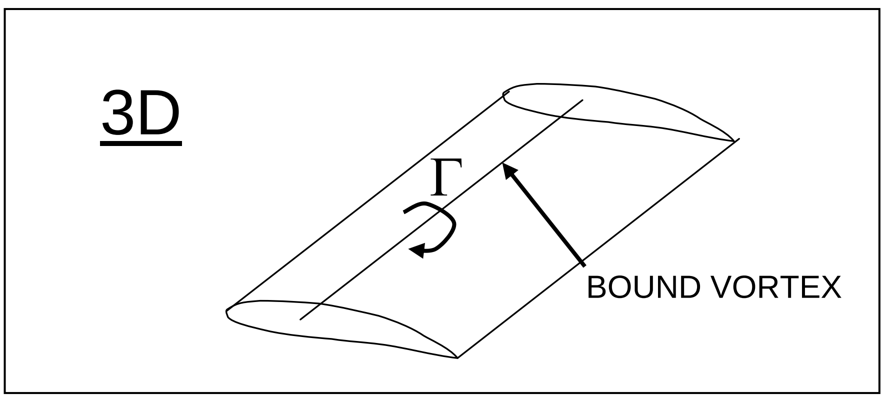


Figure 10

Due to Helmholtz theorem the bound' vortex cannot end at the wing tips. Therefore, based on observations of real wing flows, the vortex filament is continued as 'trailing' or 'tip' vortices. The circuit is then completed by a 'starting'vortex.

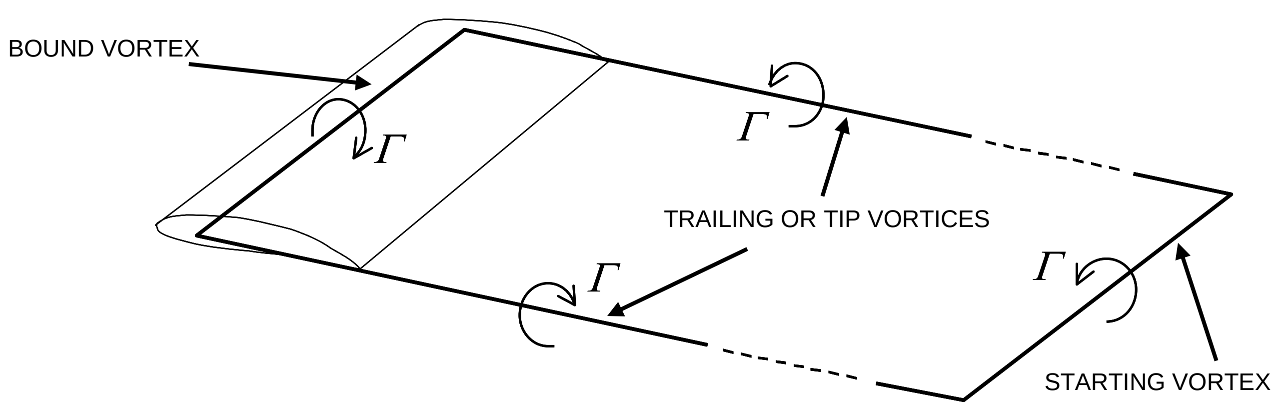


Figure 11

The starting vortex is convected downstream and its effect on the wing can be neglected, with the tip vortices effectively extending to infinity. This three-sided vortex is called a *horseshoe vortex*.

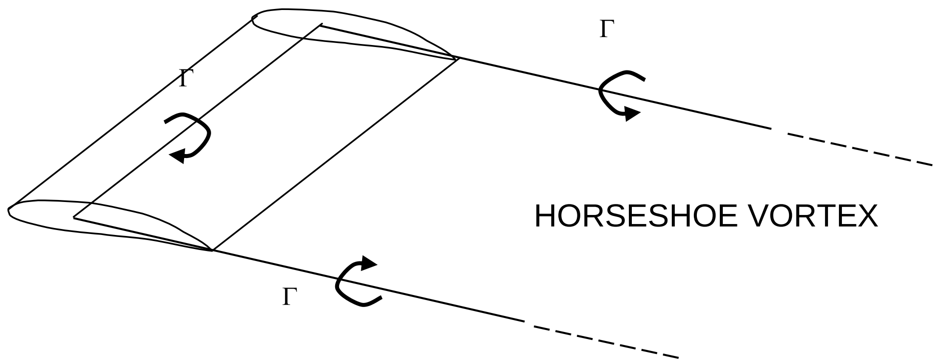


Figure 12

This is the simplest solution that satisfies the requirement for shed wake to be force free and leave the thin wing “smoothly.”

4.2 Circulation and Lift Distribution

For the horseshoe vortex model the bound circulation distribution along the span is a constant see Figure 12

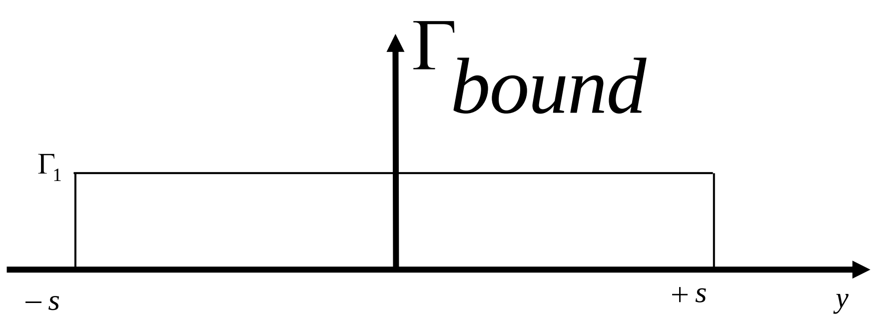


Figure 13

and hence the lift distribution is also constant across the span, which differs from the representative loading for a real wing, see figure 13.

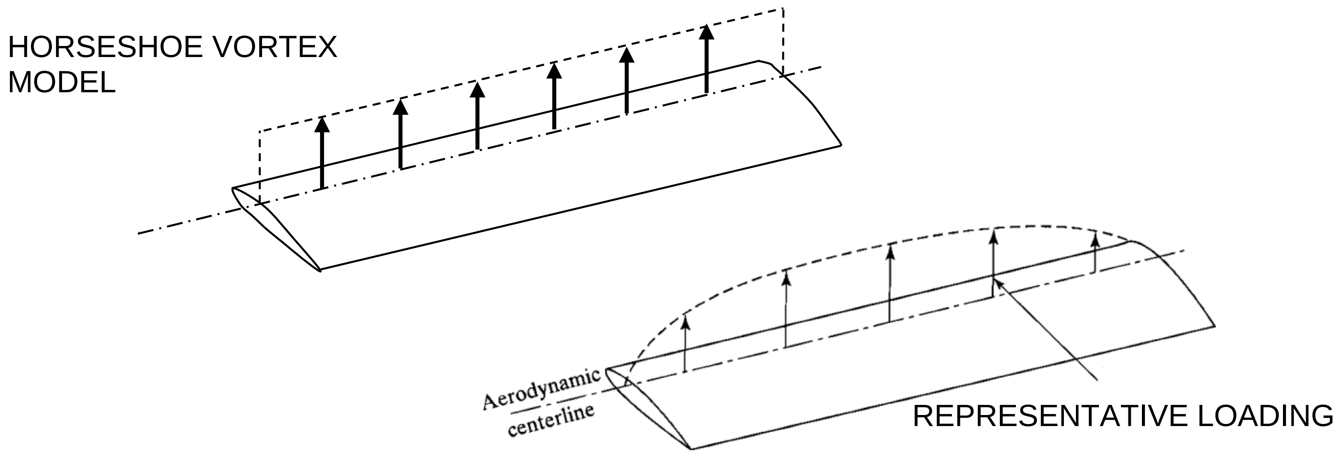


Figure 14

4.3 Downwash

The bound (lifting) wing vortex does not induce a velocity on itself. However the trailing or tip vortices induce a velocity on the bound vortex

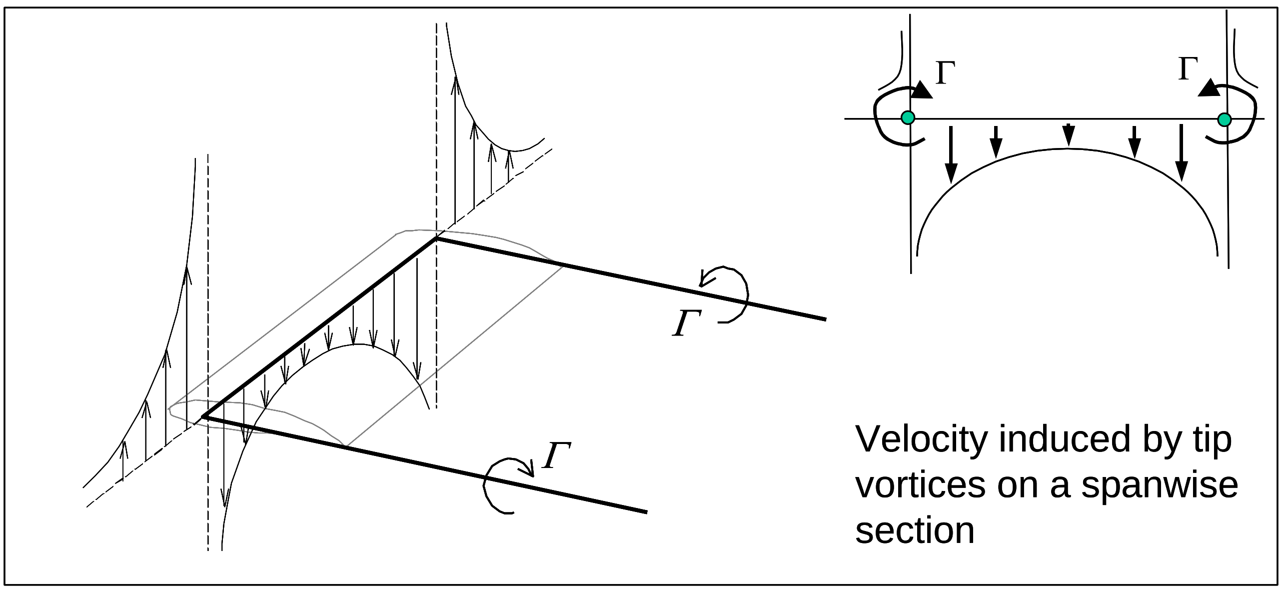


Figure 15

The velocity induced by tip vortices along the wing bound vortex is a pure *downwash* (i.e. is in the downwards direction). This magnitude of the downwash is proportional to the circulation Γ. The horseshoe vortex model predicts infinite velocities at the tips, this does not occur in the real flow. However, despite this deficiency, the model provides a useful insight into the effects of tip vortices and can be used for preliminary calculations where details of the flow near the wing are not required.

4.4 Effects of Tip Vortices-Induced Drag

The horseshoe vortex model can provide insight into how tip vortices lead to the generation of induced drag. It is necessary to consider the local flow over an infinitesimal element of the bound vortex.

(1) If there were no tip vortices

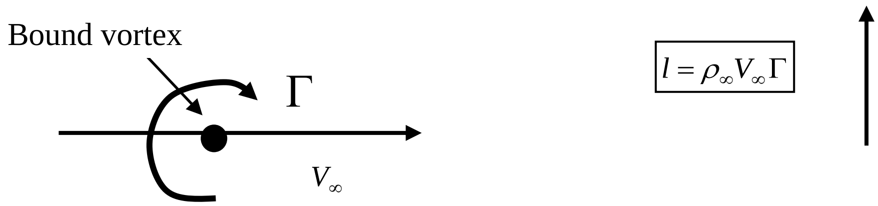


Figure 16

There is no drag and the lift per unit section is given using Kutta Joukowski by $l = \rho_\infty V_\infty \Gamma$.

(2) If there are tip vortices
The downwash effect of the tip vortices can be superimposed on top of the above result (potential solutions of Laplace's equation)

NB vertical velocity
w is +ve upwards
So downwash
corresponds to a
negative value for is w

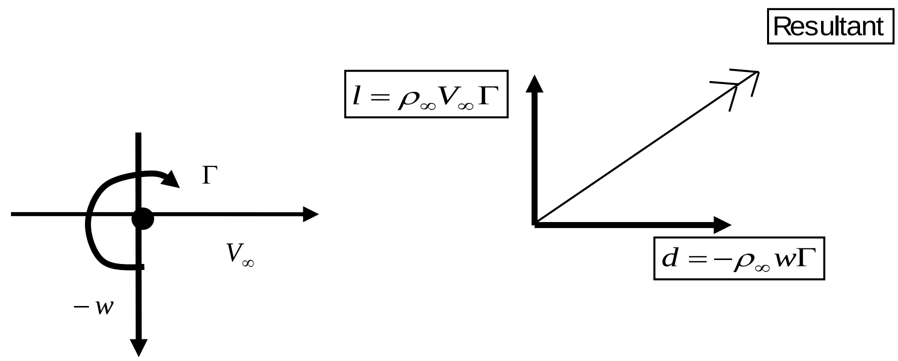


Figure 17

This results in a drag component. This drag due to the presence of tip vortices is called *INDUCED DRAG*.

The effect of the tip vortices can be thought of as rotating the free stream velocity through an angle α_i which is called the induced angle of attack

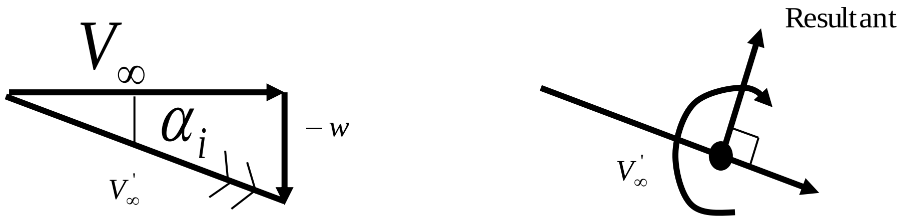


Figure 18

i.e. the effective angle of attack of the wing is reduced by α_i.

The downwash velocity $-w$ is directly proportional to the tip vortex strength Γ so

$$-w \propto \Gamma \quad \rightarrow \quad -w \propto l$$
$$D_i \propto \Gamma^2 \quad \rightarrow \quad D_i \propto l^2$$

where the subscript *i* has been introduced because to indicate that this is induced drag. Induced drag is also referred to as lift dependent drag.

D'Alembert's paradox does not apply to 3D lifting systems, because such a system will generate trailing vortices which induce a downwash and hence induced drag.

4.5 Uses of the Model

The most important use of the horseshoe vortex model is that it provides a simple demonstration of the fundamental reason for existence of induced drag. For 3D wing calculations it does not lead to good <u>quantitative</u> predictions of lift and drag, however it can also be used to give first order estimates of interactions between lifting surfaces (i.e. wing & tail, biplanes, formation flight, ground effect).

For these calculations it is usually used in a modified form where the total lift associated with the horseshoe vortex is the same as that for the actual wing and the total vortex strength is the same. This gives a better representation of the real flow. In the modified model the bound vortex does not extend to the wing tips.

4.6 Deficiencies of the model

The horseshoe vortex model has a number of deficiencies for wing flows:

(1) It predicts non-physical flow in vicinity of the wing, i.e. infinite velocities near wing tips.

(2) The bound circulation Γ should vary with span

(3) It assumes the tip vortices are formed only from vorticity is shed into the wake at the tips. In real flows the 'tip' vortices do not originate just from vorticity shed at the wing tips, the 'bound' circulation is progressively shed along the span, from centreline to tips

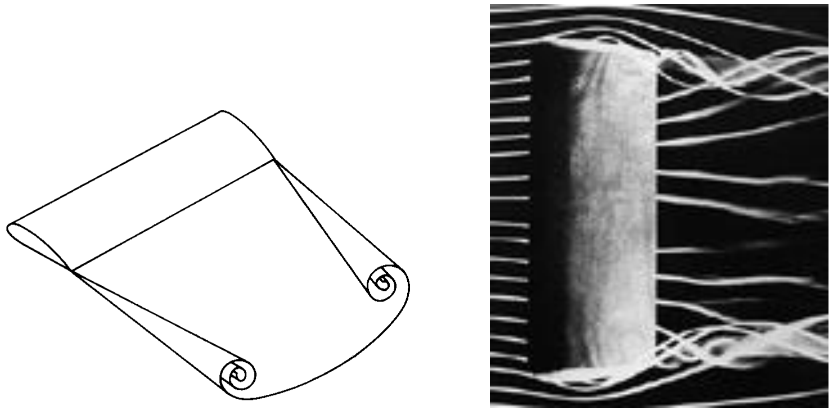


Figure 19

4.7 Improvements on the Model

Improved models are available which give better qualitative predictions of the flow than the simple horseshoe vortex model:

<u>Improved Model (1) lifting line method</u>
The bound vortex strength is allowed to vary with the spanwise position. Will be covered in Aero2.

<u>Improved Model (2) lifting surface method</u>
The wing surface is replaced by vortex sheet. This is a 3D version of thin aerofoil theory and will not be covered in Aero2.

REVISION OBJECTIVES

You should be able to:

- explain how the flow structure over a finite wing differs from that of an infinite rectangular wing (2D flow)
- understand what a vortex filament is and explain the Helmholz laws governing its behaviour
- quote the special form of the Biot-Savart law for a straight finite vortex filament and hence derive expressions for infinite and semi-infinite filaments
- explain using the horseshoe vortex model the fundamental mechanism for the generation of induced drag.

APPENDIX 1 THE BIOT-SAVART LAW FOR VORTEX INDUCED VELOCITIES

A1.1 Biot-Savart for a General Vortex Filament

The velocity induced by an element **dl** of a general vortex filament is given by

$$\mathbf{dV} = \frac{\Gamma}{4\pi}\frac{\mathbf{dl}\times\mathbf{r}}{|\mathbf{r}|^3}$$

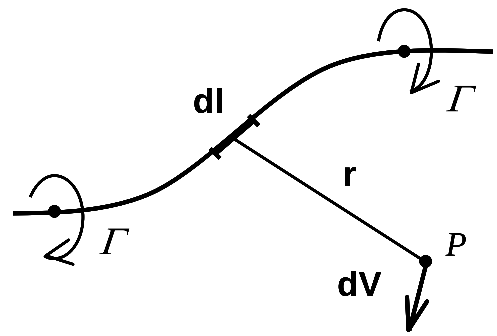


Figure A1

The total contribution of the entire vortex filament running from point a to point b can then be found via integration.

$$\mathbf{V} = \int_a^b \frac{\Gamma}{4\pi}\frac{\mathbf{dl}\times\mathbf{r}}{|\mathbf{r}|^3}$$

A1.2 Historical Notes (Interest Only)

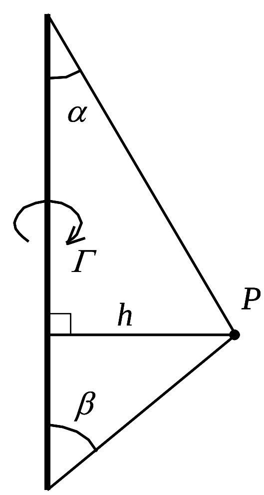
The following points about the Biot-Savart Law should be noted:

- This law was originally derived in electromagnetism, where it relates intensity of magnetic field in the vicinity of a conductor to the magnitude of the current.
- It applies to any field that can be described by potential theory
- In fluid mechanics the vortex filament takes the place of the conductor and the velocity and vortex strength are analogous to the magnetic field strength and electric current.

A1.3 Biot-Savart Law For A General Straight Filament

The general form of the equation can be simplified for a straight filament so that the total velocity induced by the filament at point P is given by

$$V = \frac{\Gamma}{4\pi h}(\cos\alpha + \cos\beta)$$



A1.4 Special Cases of Biot-Savart Law For A Straight Filament
There are three special cases of the straight vortex filament which you should be able to derive from the formula in A1.3.

(1) **Infinite Vortex Element,** $\alpha = \beta = 0^o$

$$V = \frac{\Gamma}{2\pi h}$$

(2) **Semi-infinite Vortex Element,** $\beta = 0^o$

$$V = \frac{\Gamma}{4\pi h}(1 + \cos\alpha)$$

(3) **Semi-infinite Vortex Element, *P*, Level with End of Vortex,** $\alpha = 90^o, \beta = 0^o$

$$V = \frac{\Gamma}{4\pi h}$$

