Example 3.1.1

Figure 1 shows a plane, pin-jointed truss which is supported at A and B and carries a vertical load of 10 kN at F as shown. All six members have a cross-sectional area of 300 mm^2 and are made of steel with $E = 200$ GPa. Note that the structure is *statically determinate*.

i) Calculate the reactions at A and B.

ii) Calculate the internal forces in all six members of the truss.

iii) Using an energy method, calculate the vertical deflection at joint F.

iv) Using an energy method, calculate the horizontal deflection at joint F.

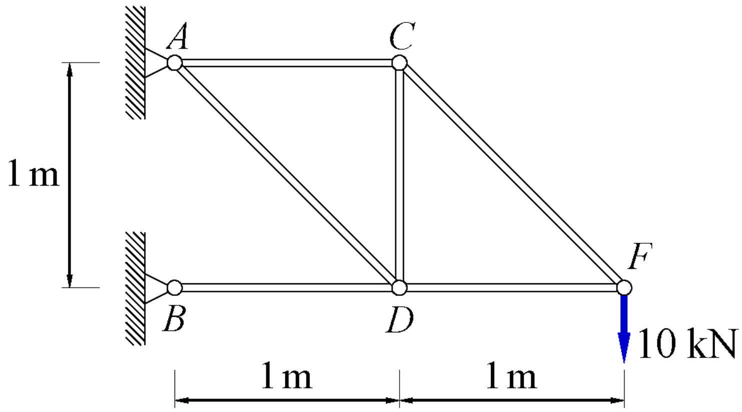


Figure 1: A plane pin-jointed truss.

i) Support reactions:

A global balance of moments about point B gives:

$$\begin{aligned}
& \sum M_B && = 0 \\
(10000)(2.0) \; & + R_{A,x}(1.0) && = 0 \\
20000 \; & + R_{A,x} && = 0 \\
& R_{A,x} && = -20\,\text{kN (i.e. 20 kN to the left)}
\end{aligned}$$

And a global balance of forces gives:

$$R_{B,x} = 20\,\text{kN (to the right)}$$

$$R_{B,y} = 0$$

$$R_{A,y} = 10\,\text{kN (upwards)}$$

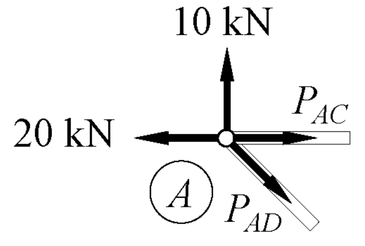
ii) Internal forces

The characteristic angle between members is $\theta = 45^0$.

Joint *B*:

$$P_{BD} = -R_{B,x} = -20\,\text{kN}$$

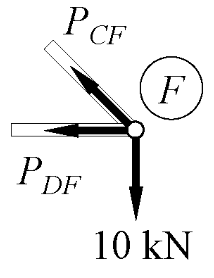
Joint *A*:



$$\begin{array}{lll} & \sum F_y & = 0 \\ 10\text{ kN} & -P_{AD}\sin\theta & = 0 \\ 10\text{ kN} & -P_{AD}\left(\sqrt{2}/2\right) & = 0 \\ & P_{AD} & = 14142\,\text{N} \end{array}$$

$$\begin{array}{lllll} & & \sum F_x & = 0 & \\ 20\,\text{kN} & -P_{AD}\cos\theta & -P_{AC} & = 0 & \\ & & P_{AC} & = -(14142\,\text{N})\cos\theta & +20\,\text{kN} \\ & & P_{AC} & = 10\,\text{kN} & \end{array}$$

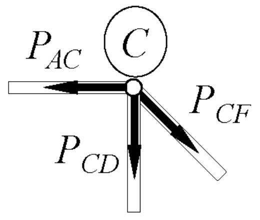
Joint *F*:



$$\begin{array}{lll} & \sum F_y & = 0 \\ 10\text{ kN} & -P_{CF}\sin\theta & = 0 \\ 10\text{ kN} & -P_{CF}\left(\sqrt{2}/2\right) & = 0 \\ & P_{CF} & = 14142\,\text{N} \end{array}$$

$$\begin{array}{lll} & \sum F_x & = 0 \\ P_{CF}\cos\theta & +P_{DF} & = 0 \\ & P_{DF} & = -P_{CF}\cos\theta \\ & P_{CF} & = -10\text{ kN} \end{array}$$

Joint *C*:



$$\begin{array}{lll} & \sum F_y & = 0 \\ P_{CD} & +P_{CF}\sin\theta & = 0 \\ & P_{CD} & = -P_{CF}\sin\theta \\ & P_{CD} & = -10\text{ kN} \end{array}$$

iii) Vertical deflection at F

Member	Li / m	Pi / N	Pi'	Pi Pi' Li / Nm
AC	1	10000		
AD	1.414214	14142.14		
BD	1	-20000		
CD	1	-10000		
CF	1.414214	14142.14		
DF	1	-10000		

Applying Castigliano's theorem:

$$\left(\delta_y\right)_F = \sum_{i=1}^{m} \frac{P_i \, P_i' \, L_i}{A_i \, E_i}$$

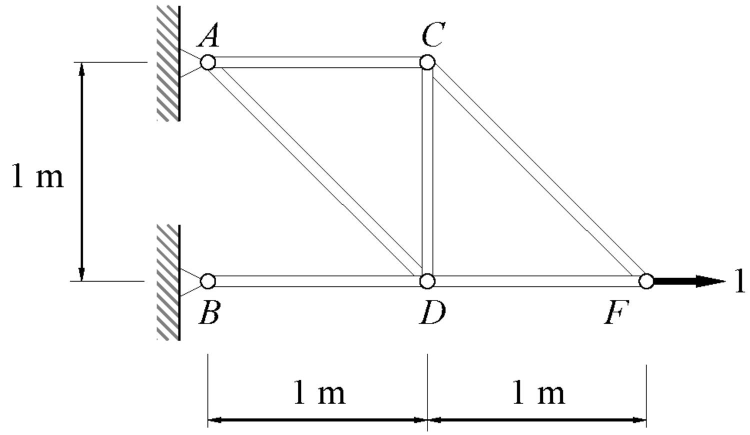
$$\left(\delta_y\right)_F = \frac{\quad}{\quad}$$

$$\left(\delta_y\right)_F =$$

i.e.

iv) Horizontal deflection at F

Apply a unit load at the joint of interest with the correct orientation:



Now perform a new 'static equilibrium' analysis to obtain P_i' values.

Member	Li / m	Pi / N	Pi'	Pi Pi' Li / Nm
AC	1	10000		
AD	1.414214	14142.14		
BD	1	-20000		
CD	1	-10000		
CF	1.414214	14142.14		
DF	1	-10000		

Applying Castigliano's theorem:

$$(\delta_x)_F = \sum_{i=1}^{m} \frac{P_i \, P_i' \, L_i}{A_i \, E_i}$$

$$(\delta_x)_F = \frac{\qquad\qquad}{}$$

$$(\delta_x)_F =$$

i.e.