Advanced Bending and Torsion
Unsymmetric Bending of Beams

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09 October 2018

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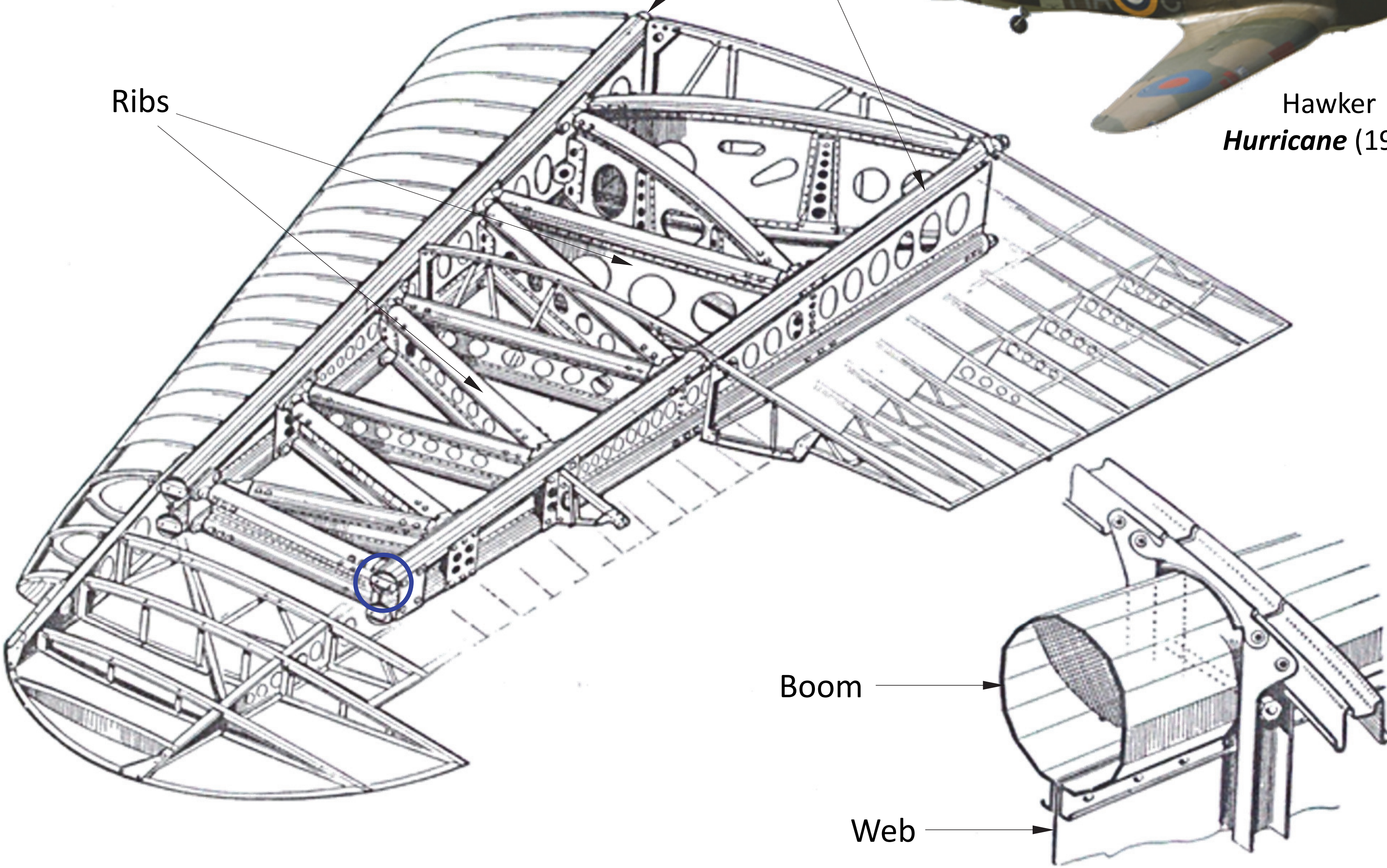
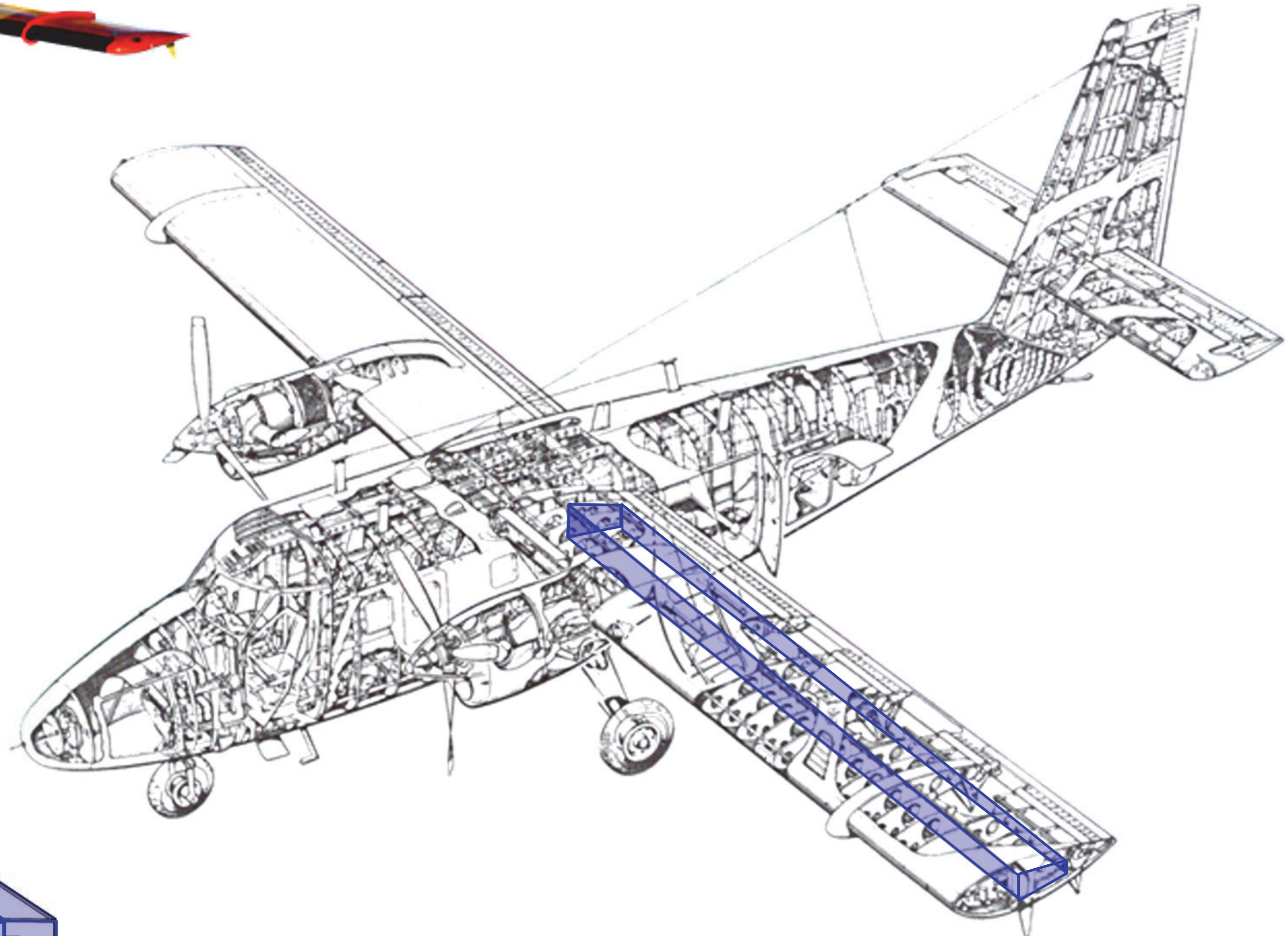
- ‘Beam’ Wing Structure



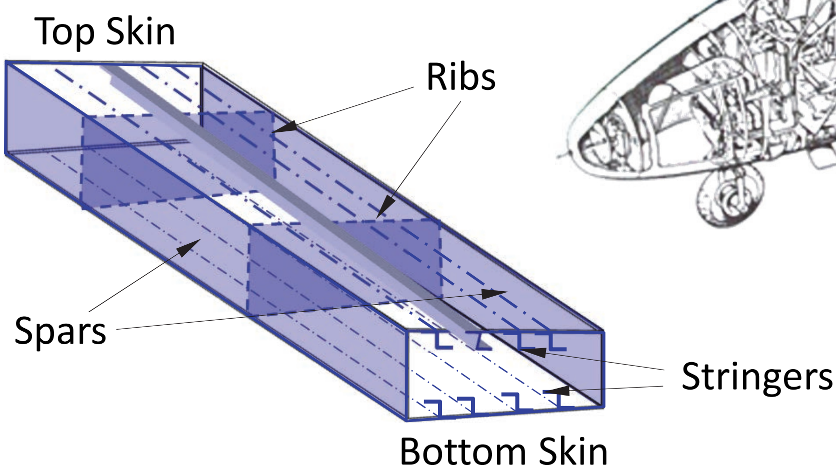
Hawker
Hurricane (1935)

- 2-Spar Wing Box Structure

de Havilland
Canada Twin Otter (1966)

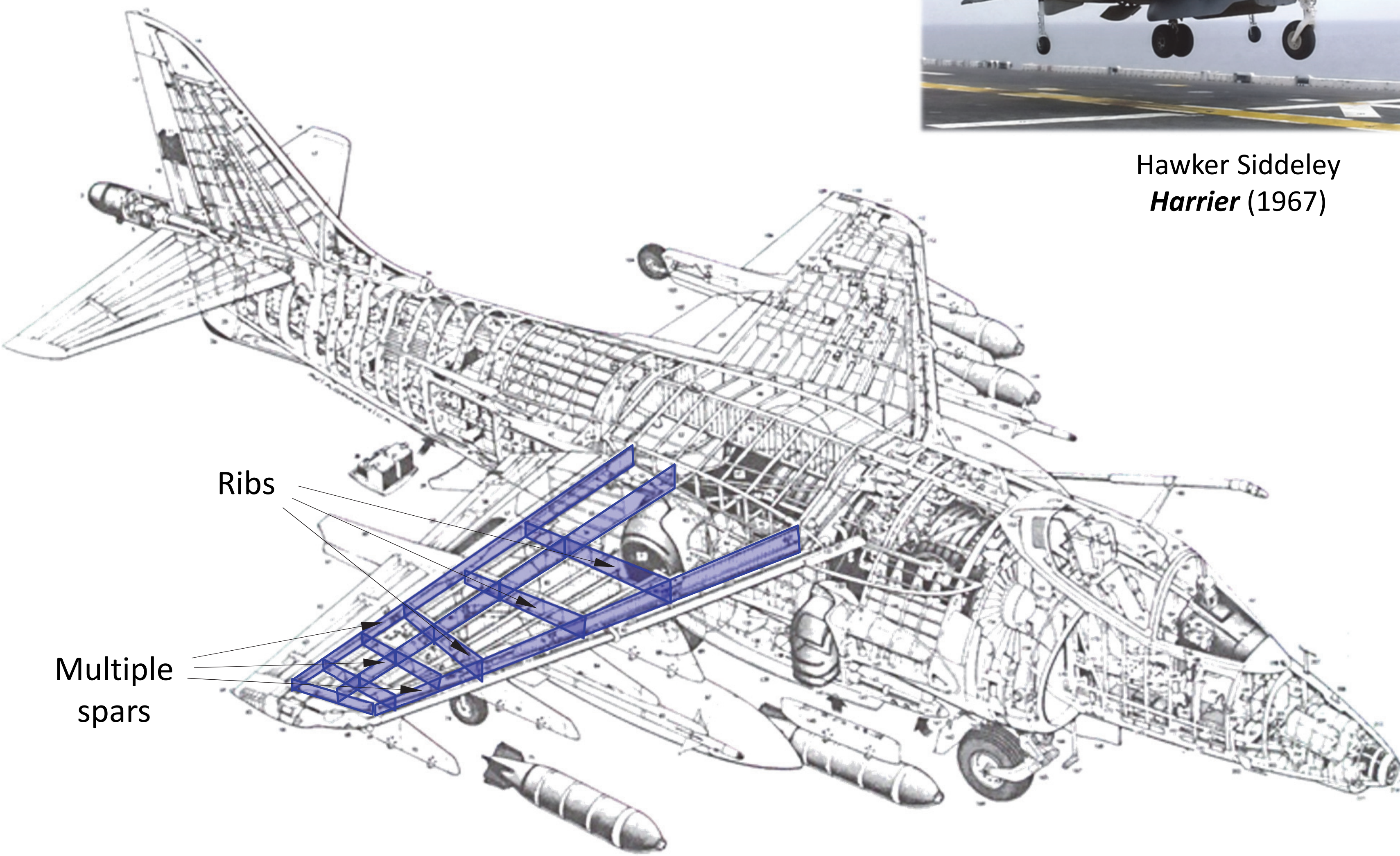


'Torsion Box'
concept



- Multi-Spar Wing Box Structure

Hawker Siddeley
Harrier (1967)

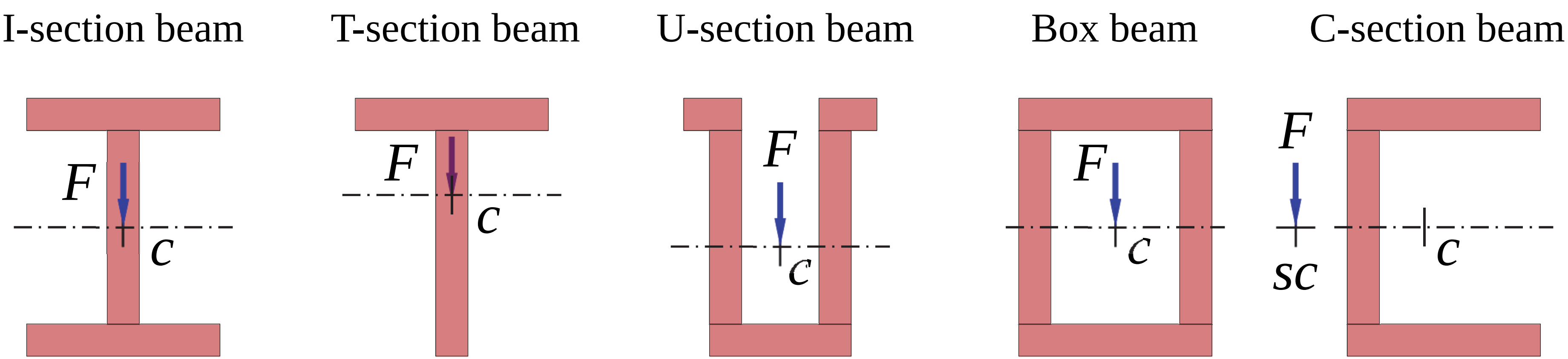


Compound Cross-Sections (Recap)

- Cross-sections of more complex shape, seen as an assembly of simpler sections – most often rectangular sections

- All 'parts' are perfectly bonded together – *i.e.* via welding or adhesive bonding

Classic compound cross-sections:



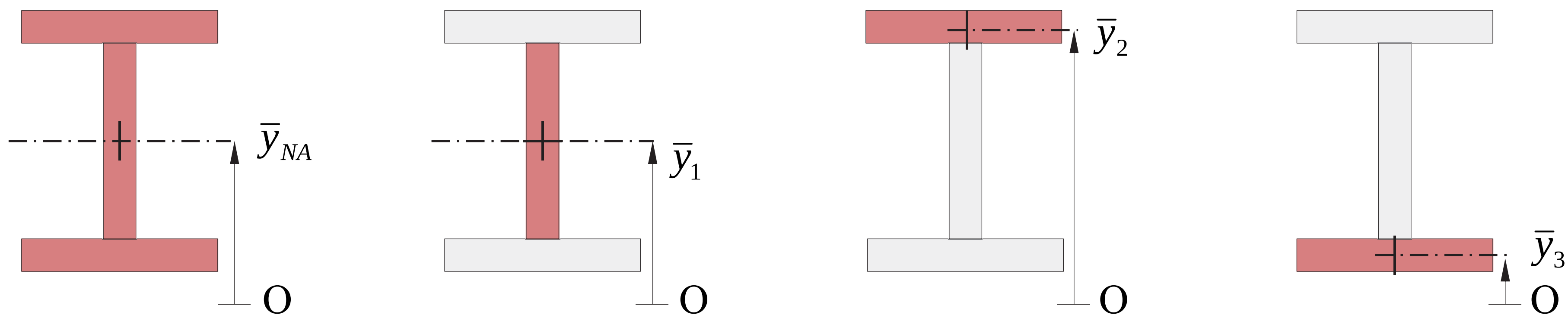
- **Webs** are elements **parallel** with main transverse loading

- **Flanges** are elements **normal** to main transverse loading

Position of Neutral Axis (Recap)

- The **centroid of the compound section** is found by considering individual centroids w.r.t a 'vertical' coordinate $\bar{y}$ of arbitrary origin:

Compound section	Part 1	Part 2	Part 3



Total area A	Area A_1	Area A_2	Area A_3
Centroid at $\bar{y}_{NA}$	Centroid at $\bar{y}_1 (= \bar{y}_{NA})$	Centroid at $\bar{y}_2$	Centroid at $\bar{y}_3$

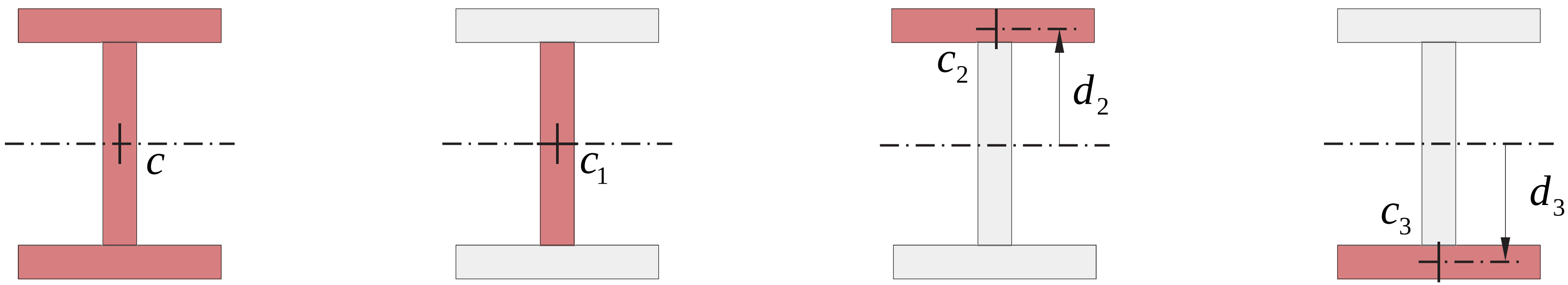
- The first moment of area gives:

$$A\,\bar{y}_{NA} = \sum_i (A_i\,\bar{y}_i) \quad \therefore \quad \boxed{\bar{y}_{NA} = \frac{\sum_i (A_i\,\bar{y}_i)}{\sum_i (A_i)}}$$

Parallel Axis Theorem (Recap)

- We decompose the compound cross section into simple sections identified by an index *i*:

Compound section | Part 1 | Part 2 | Part 3



	Part 1	Part 2	Part 3
	Area A_1	Area A_2	Area A_3
	Centroid c_1	Centroid c_2	Centroid c_3
	SMA I_1	SMA I_2	SMA I_3
	Distance $d_1 (= 0)$	Distance d_2	Distance d_3

- The contribution of each ‘section’ *i* to the compound I_{NA} is:

$$I_{i,ZZ} = I_i + A_i (d_i)^2$$

- And the final compound I_{NA} is:

$$\boxed{I_{NA} = \sum_i (I_{i,ZZ}) = \sum_i \left[I_i + A_i (d_i)^2 \right]}$$

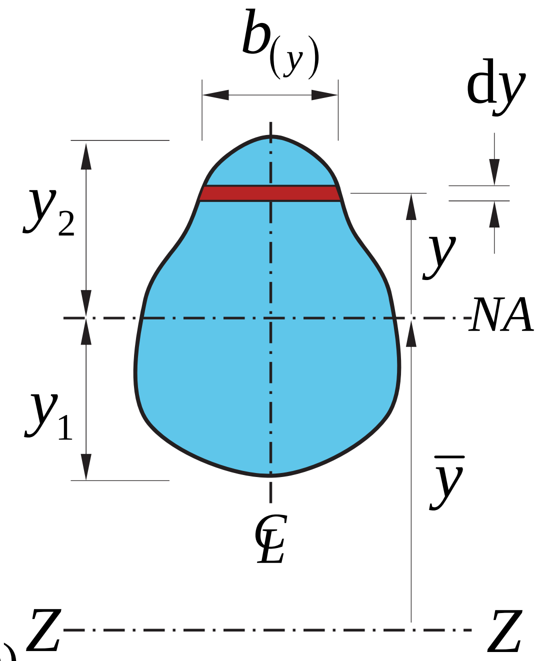
Parallel Axis Theorem – Derivation

- The theorem is derived directly from the **2nd moment of area**:

(Imagine that this shape is now only one 'part' *i* of a compound section)

$$I_i = \int_{y_1}^{y_2} y^2\, b_{(y)}\, dy$$

(2nd moment of area of a 'part' w.r.t. its own centroid)



$$I_{ZZ} = \int_{y_1}^{y_2} \left(y + \bar{y}\right)^2\, b_{(y)}\, dy$$

(2nd moment of area of a 'part' w.r.t. the compound centroid)

$$I_{ZZ} = \int_{y_1}^{y_2} \left(y^2 + 2\,y\,\bar{y} + \bar{y}^2\right) b_{(y)}\, dy$$

(Origin of $\bar{y}$, for convenience)

$$I_{ZZ} = \underbrace{\int_{y_1}^{y_2} y^2\, b_{(y)}\, dy}_{I_i} + 2\,\bar{y} \underbrace{\cancelto{0}{\int_{y_1}^{y_2} y\, b_{(y)}\, dy}}_{\text{1st moment of area}} + \underbrace{\bar{y}^2}_{d_i^{\,2}} \underbrace{\int_{y_1}^{y_2} b_{(y)}\, dy}_{A_i}$$

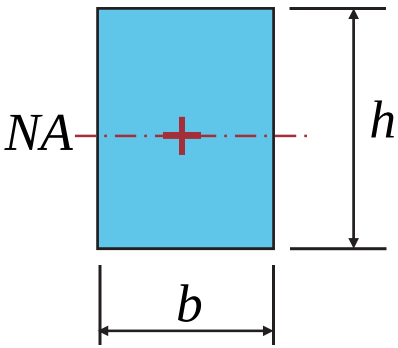
$$\boxed{I_{i,ZZ} = I_i + A_i\left(d_i\right)^2}$$

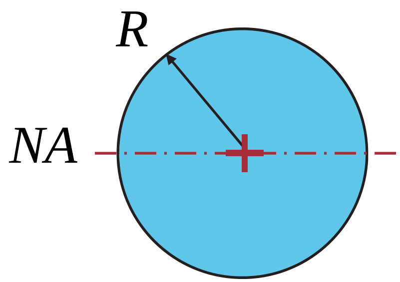
Bending Stresses and Strains – Formulae

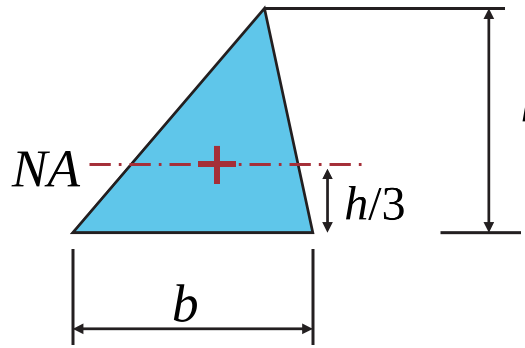
Engineer's theory of bending

$$\boxed{-\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}}$$

Basic 2nd moments of area







$$\boxed{I_{NA} = \frac{b\,h^3}{12}}$$

$$\boxed{I_{NA} = \frac{\pi\,R^4}{4}}$$

$$\boxed{I_{NA} = \frac{b\,h^3}{36}}$$

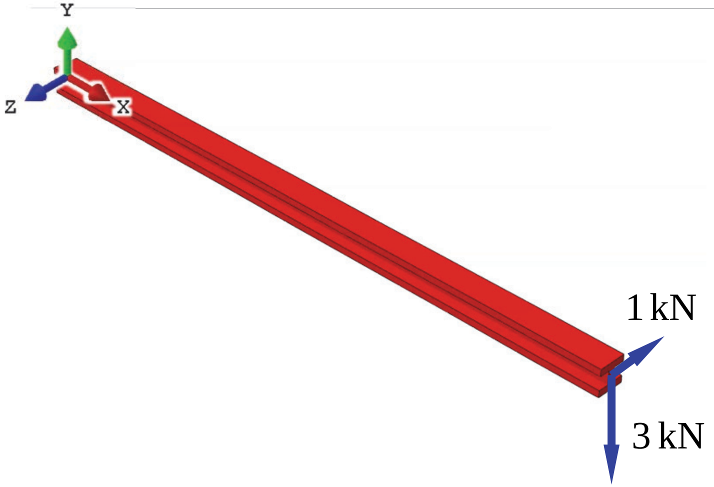
Parallel axis theorem

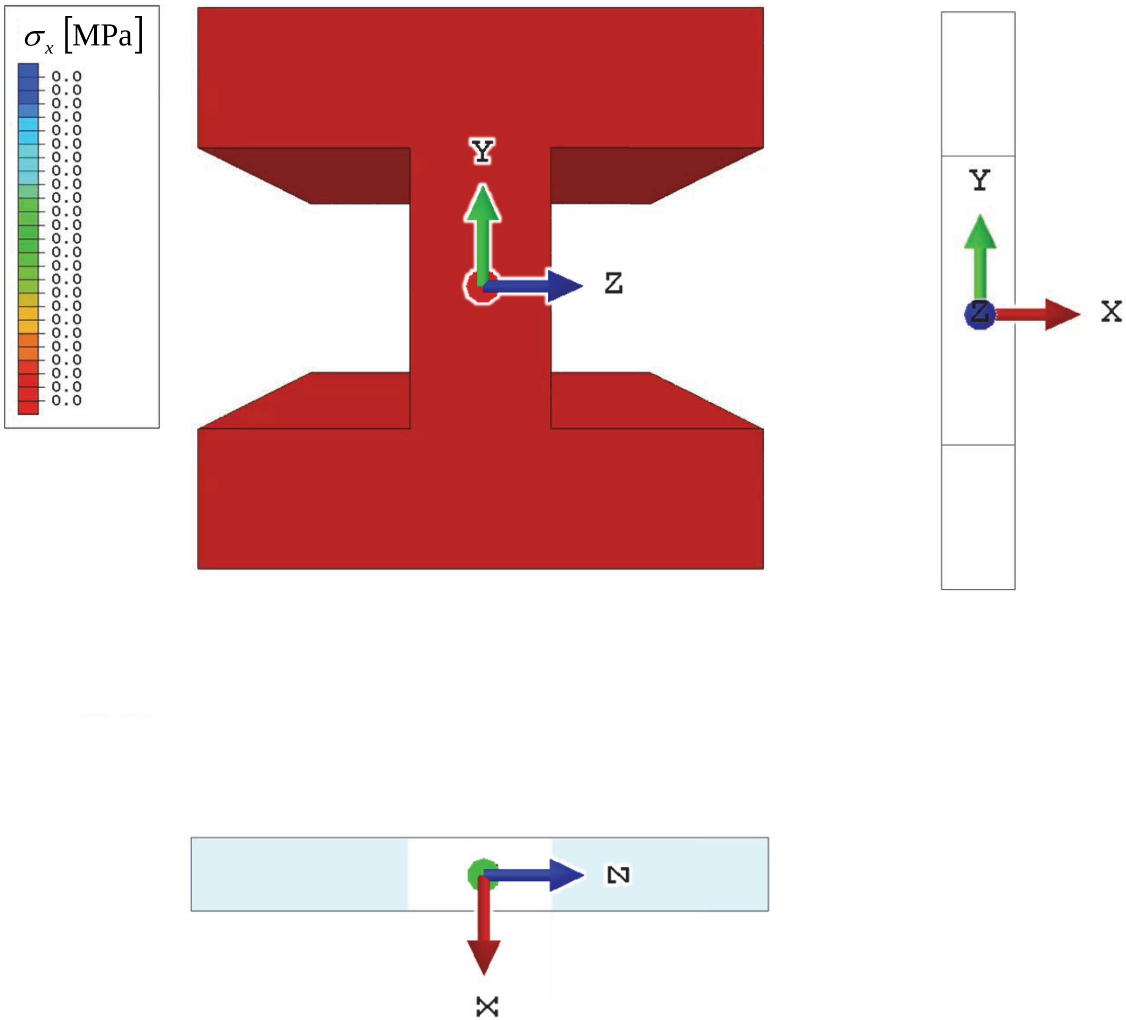
$$\boxed{\bar{y}_{NA} = \frac{\sum_i (A_i\,\bar{y}_i)}{\sum_i (A_i)}}$$

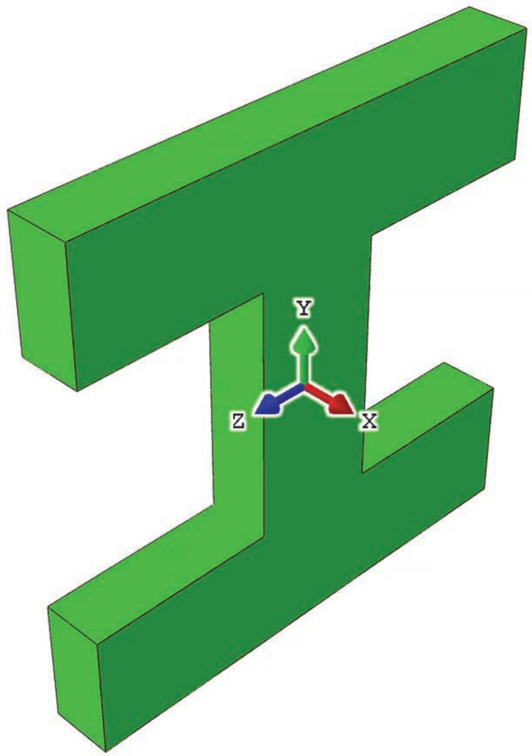
$$\boxed{d_i = \bar{y}_i - \bar{y}_{NA}}$$

$$\boxed{I_{NA} = \sum_i \left[I_i + A_i\,(d_i)^2 \right]}$$

I-section beam in combined bending







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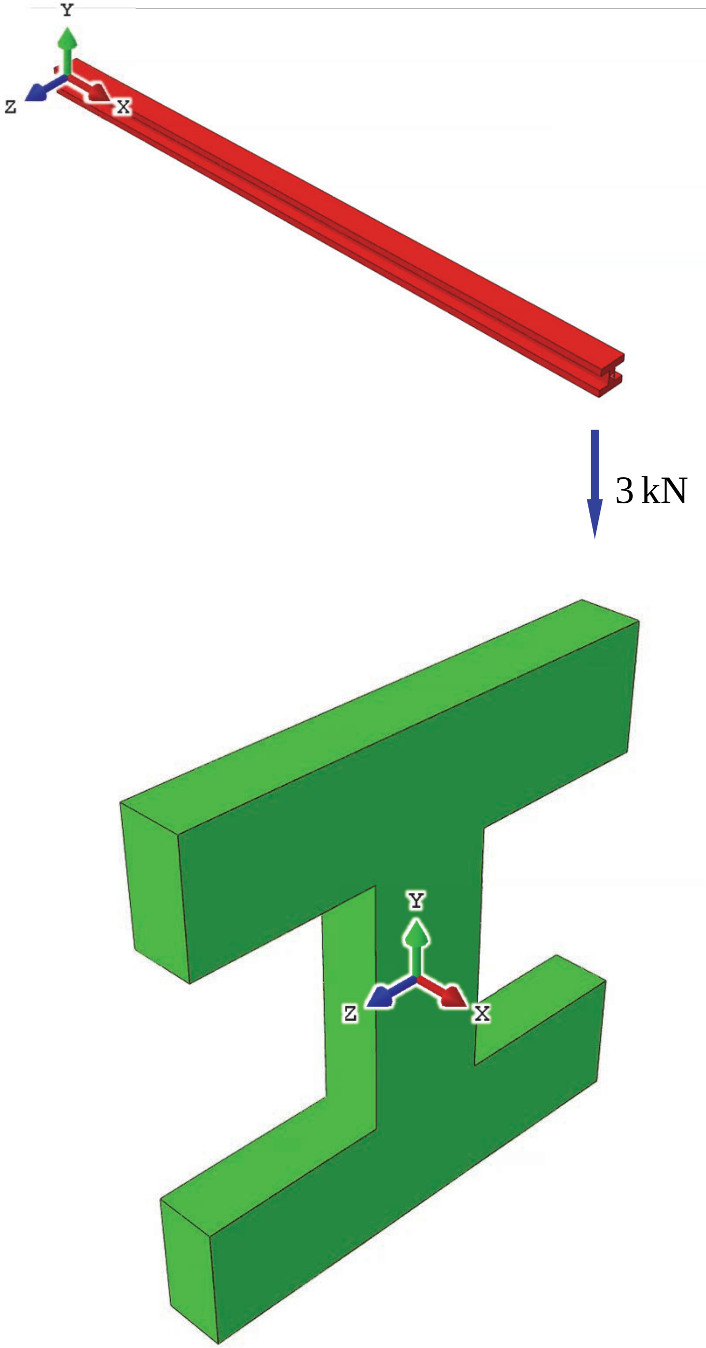

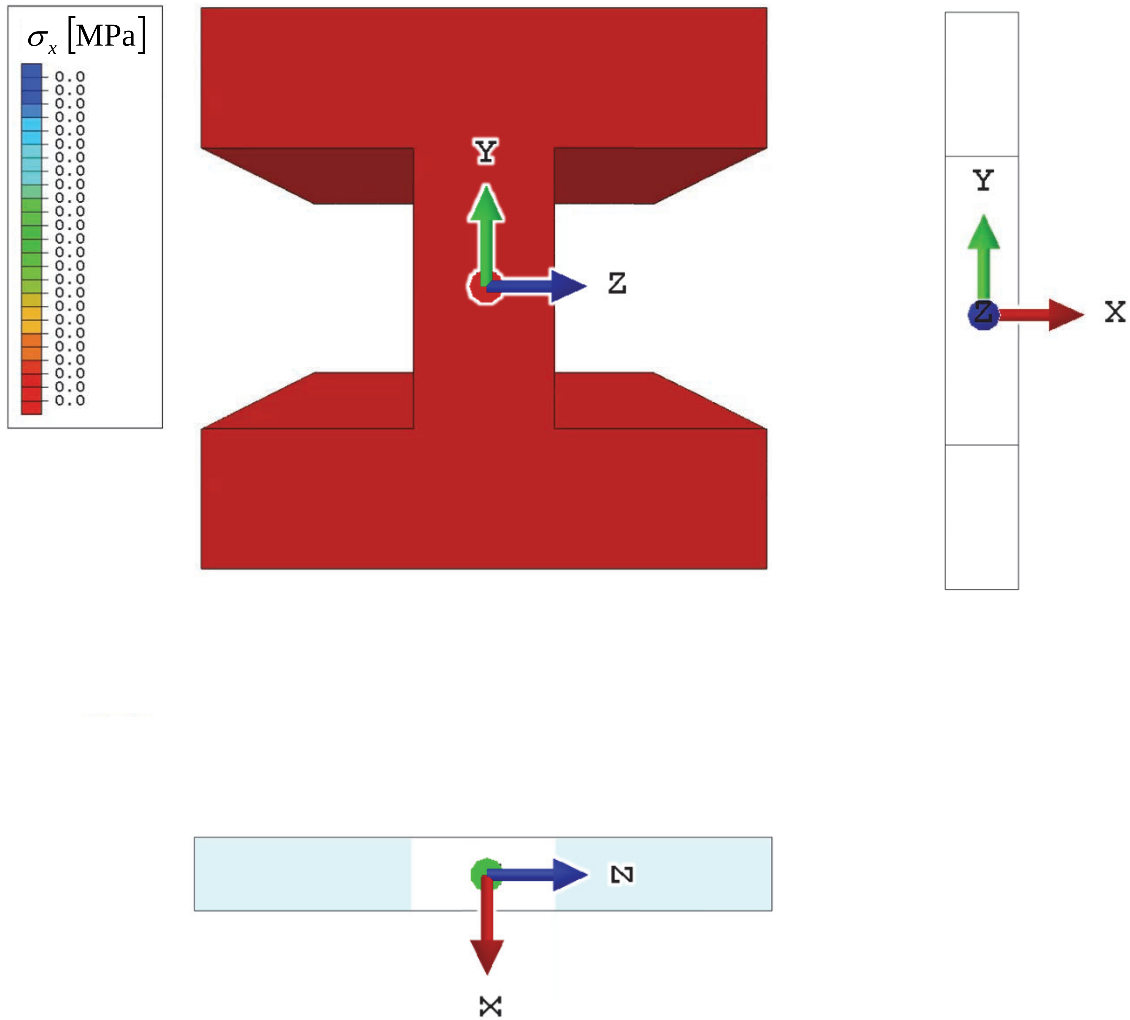

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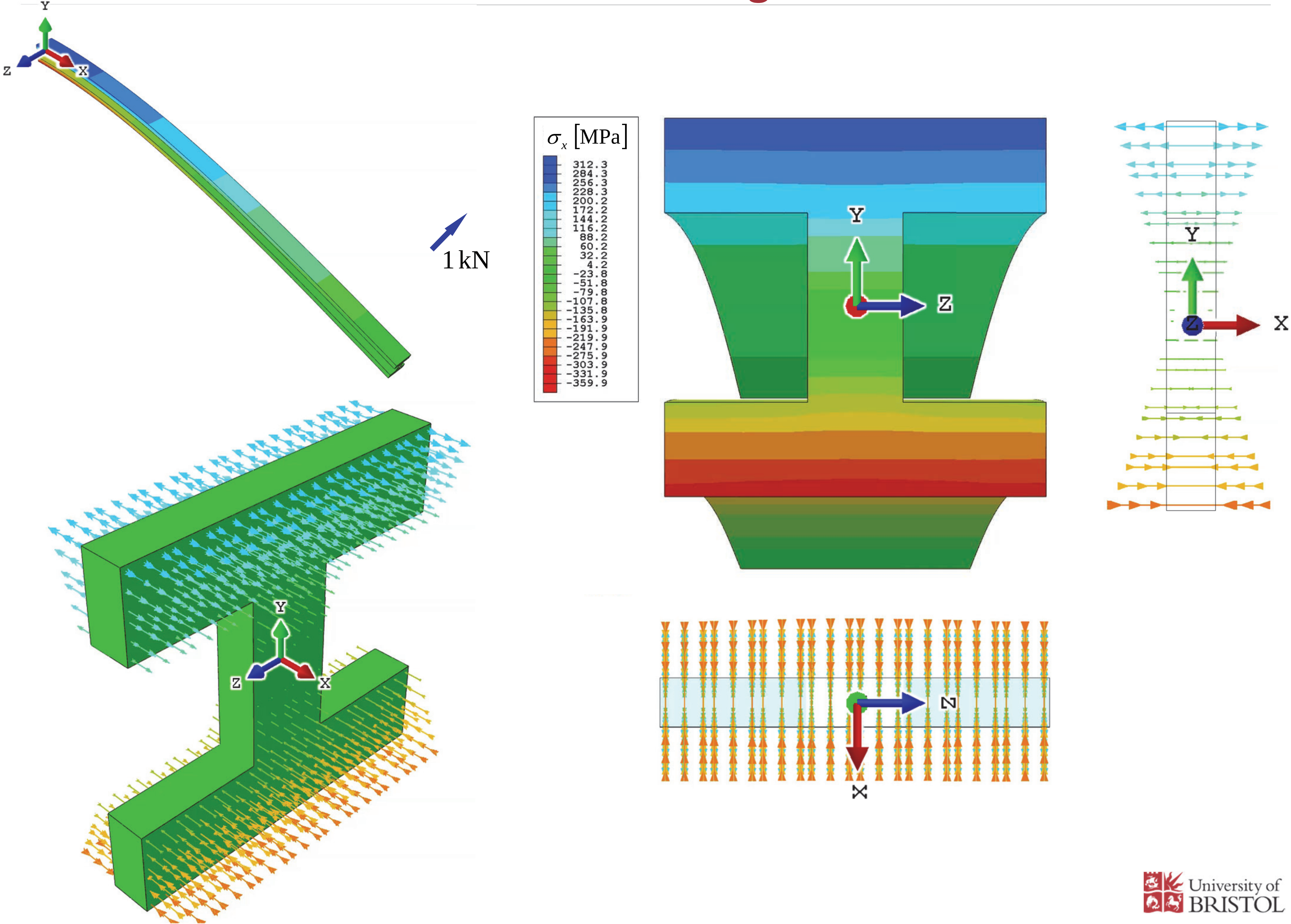

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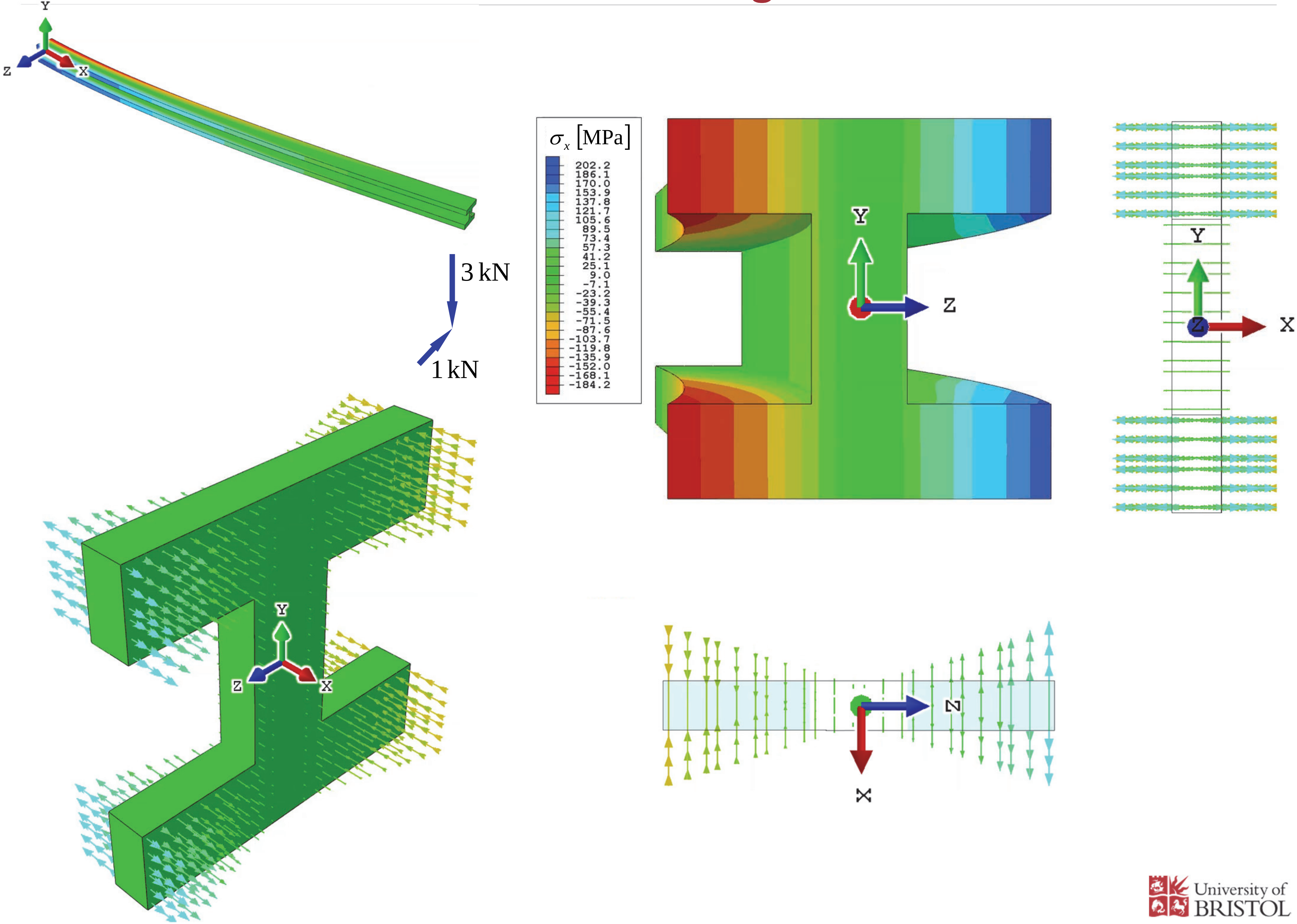

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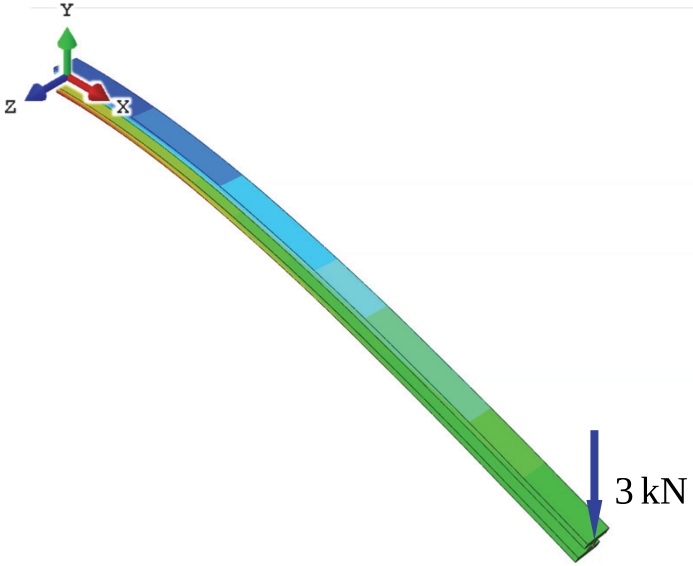

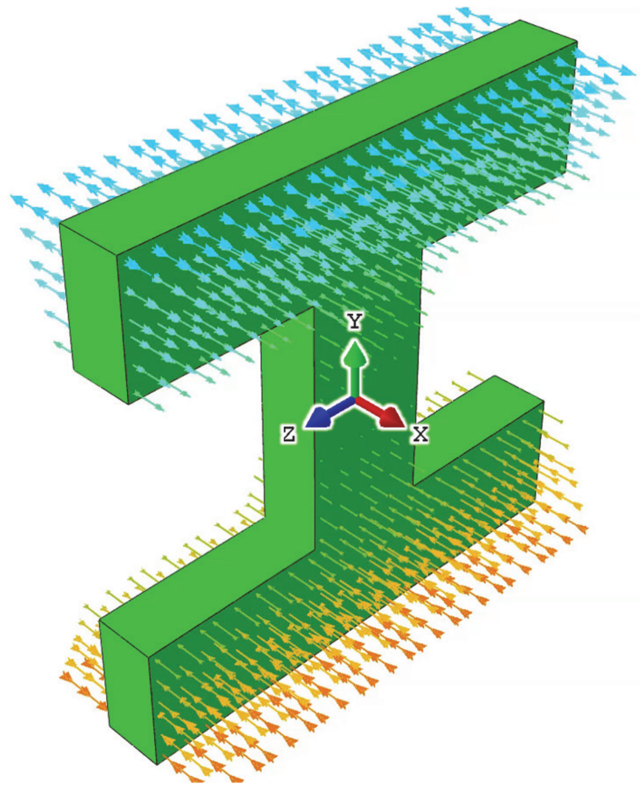

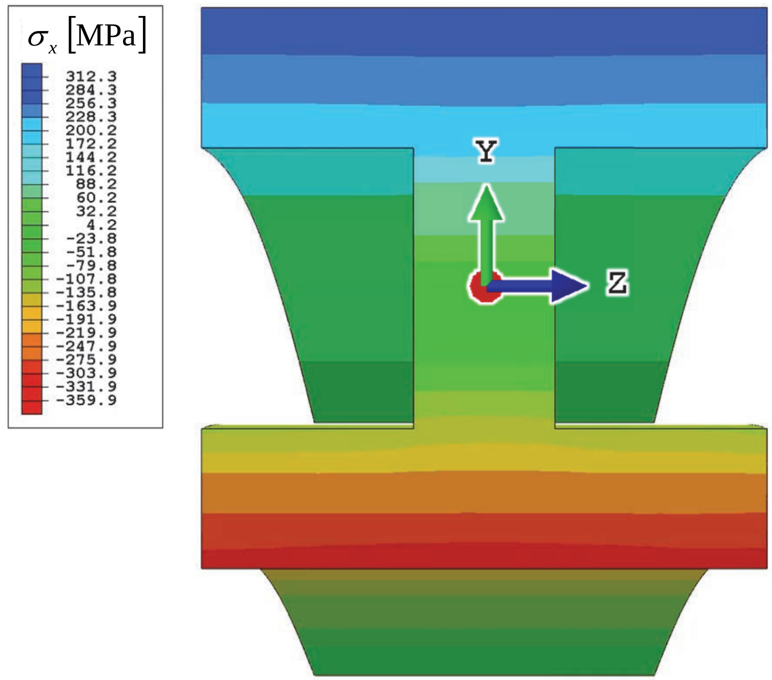

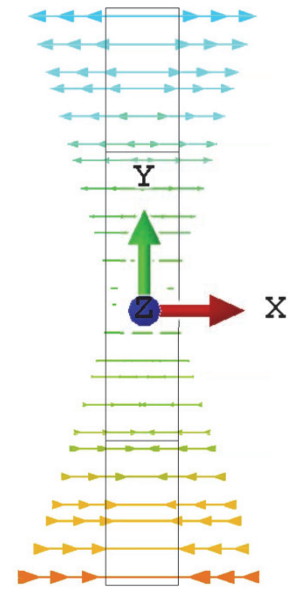

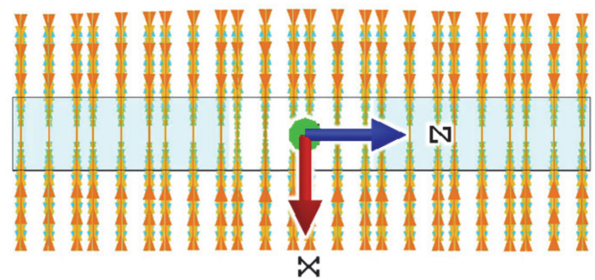

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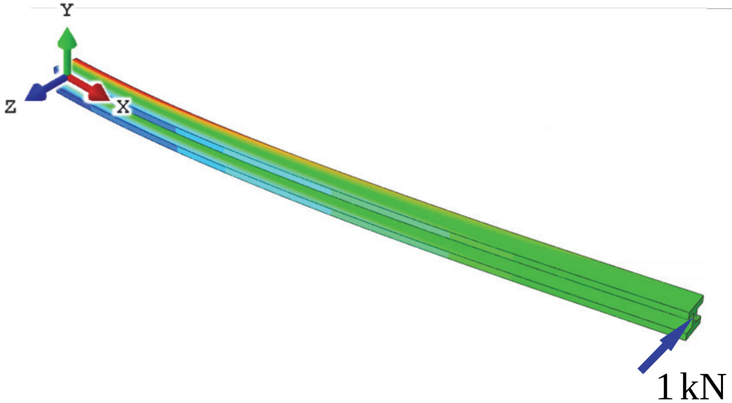

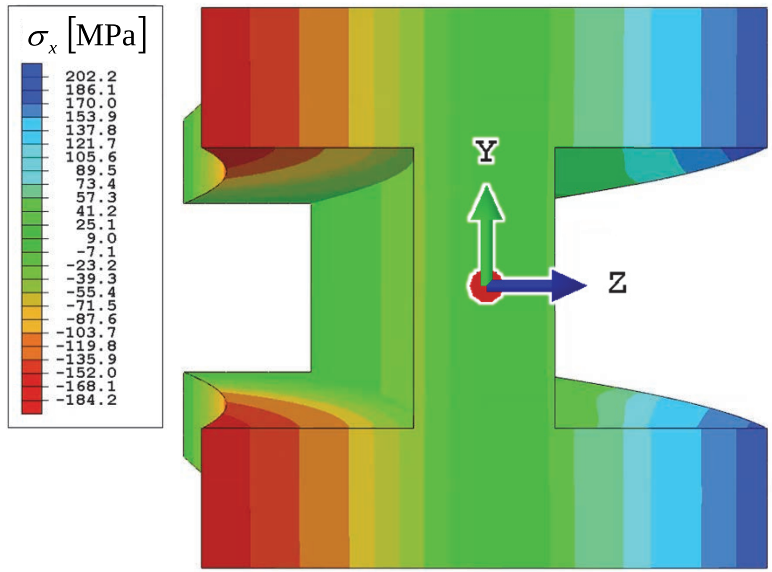

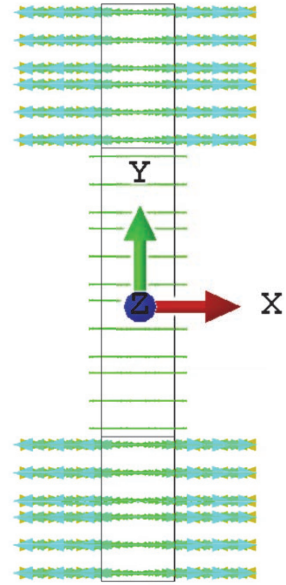

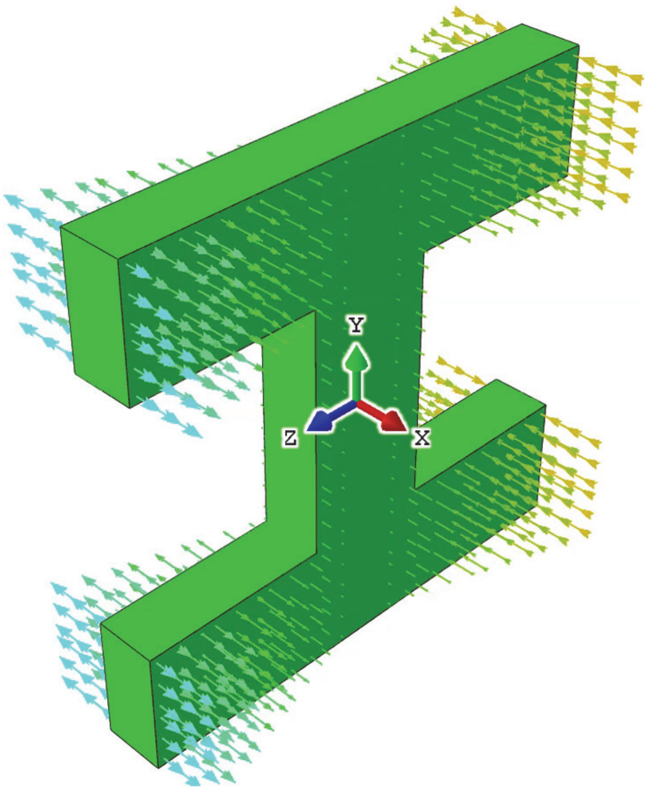

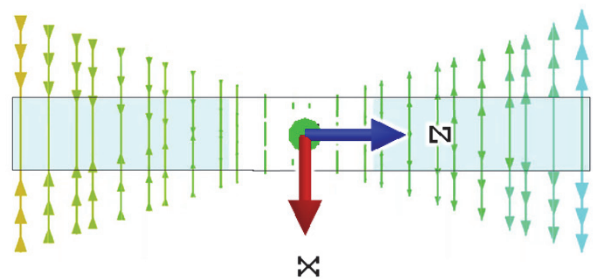

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