Lecture 9
Longitudinal Balance

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Longitudinal Balance

Introduction

- The longitudinal plane (the plane of symmetry) will contain the lift vectors for both wing and tail.
- There will also be aerodynamic moments which act in the pitch sense.
- For zero roll angle, the longitudinal plane also contains the gravity (weight) vector.
- We will consider:

  - an equilibrium or balance among the forces, and

  - the stability of motion following a disturbance.

Note: You need to be able to distinguish properly between flight balance and flight stability … often a source of confusion!

Longitudinal Balance

Balanced Flight

Simplest version of flight balance:

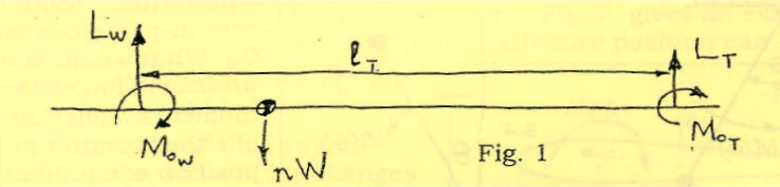
- we need enough lift to hold us up;
- we need to distribute the aerodynamic forces such that overall pitching moment is zero.

Challenge:

- To arrange for both of these to be achieved simultaneously, and to retain that balance throughout flight (in the simplest cases by careful adjustments to the throttle and elevator) .

- We will consider straight and level flight; however we could easily extend the analysis to include steady climbing flight. i.e. non-zero, constant flight path angle γ.

Longitudinal Balance



Note: always include a force/moment diagram!

- Hence equation (1) where a balance of forces is sought to ensure that:

$$\sum F_{vertical} = 0 \tag{1}$$

Longitudinal Balance

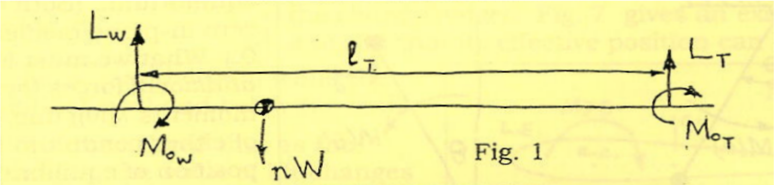


Fig. 1

- A need for balance in pitch leads to:

$$\sum M_{pitch} = 0 \qquad (2)$$

(positive nose-up, about any point):

- Since the moment $L_T l_T$ is very large compared with M_{o_T} it is normal to ignore M_{o_T} relative to $L_T l_T$, though M_{o_W} must be retained.

Longitudinal Balance

- An alternative configuration:

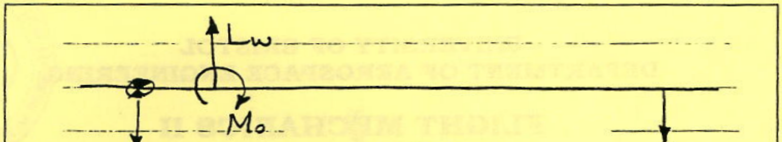


Fig. 2

- where the wing must produce:

Efficient?

$$L_W = nW + L_T \,, \tag{3}$$

Longitudinal Balance

- For the first configuration:

$$L_W = qSC_{L_W}$$
$$L_T = qS_T C_{L_T}, \qquad (4)$$

  and yet we must retain $(L_W + L_T) = nW$

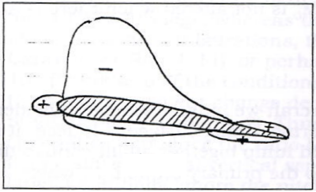
- The simple expression $L = qSa_1\alpha$ suggests that if speed (or q) drops, we must simply increase the incidence to keep L constant
- *but* the two separate lift coefficients C_{L_W} and C_{L_T} will be seen later to be separately related to α and we cannot simply alter the aircraft incidence and hope to have both L_W and L_T altered by the same factor.


Aerodynamic Centre

The Aerodynamic Centre

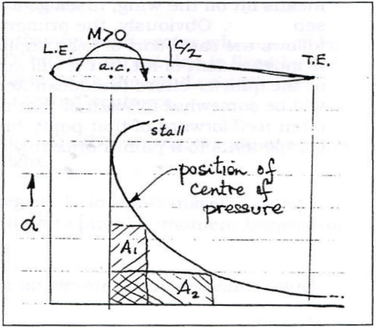
Need to consider:

- the primary lift vector (wing and fuselage)
- the pitching moment M_0 that is taken to be independent of lift while being a consequence of pressures on the same surfaces.



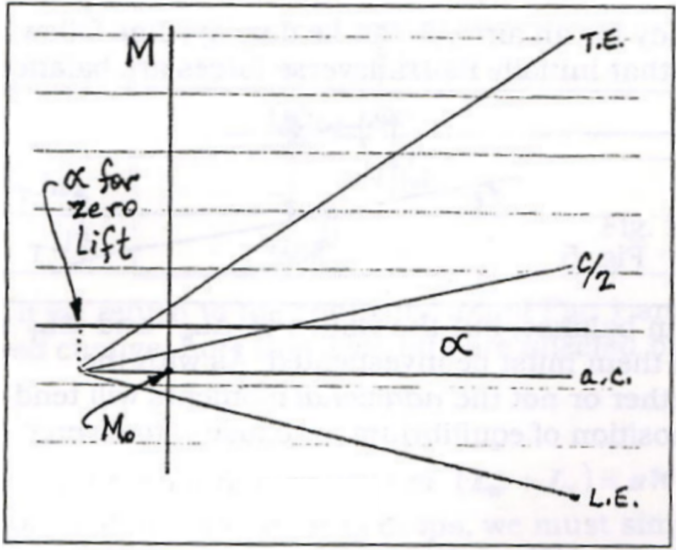
- The figure above shows a typical pressure distribution on an aerofoil but the shape of this distribution changes quickly with incidence such that the centre of pressure moves quite noticeably over the aft part of the chord.

The Aerodynamic Centre



- Lift × moment arm about a.c. = constant
- α proportional to lift
- x proportional to moment arm
- Implies that areas A_1 and A_2 are equal.

The Aerodynamic Centre



The Aerodynamic Centre

- As the incidence changes, the value of lift changes and the centre of its action also changes (moment arm from c.p. to any chosen point).
- Clearly, for most values of α there must be a positive moment about the T.E. and a negative moment about the L.E.
- What also becomes evident is that for some position near $c/4$ the pitching moment can be independent of α though not necessarily of zero value.

The Aerodynamic Centre

- Since the moment about the aerodynamic centre (M_0) is taken as constant, we are implying that

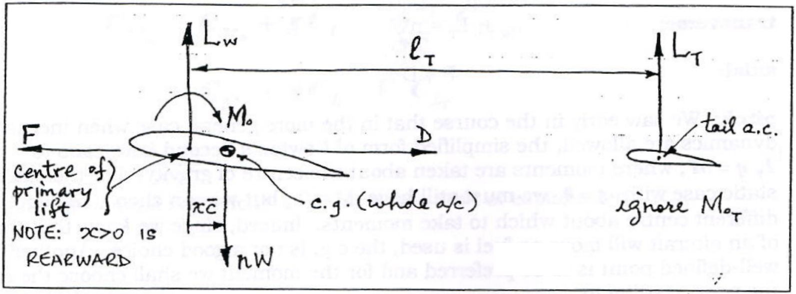
$$\text{lift force} \times \text{arm about a.c.} = \text{constant.} \tag{5}$$

i.e. define the aerodynamic centre as:

- *The point through which additional lift acts when incidence changes, i.e. M_0 is not altered when there is a ΔLift.*

Simple Model for Longitudinal Balance

The Simple Model for Longitudinal Balance



The Simple Model for Longitudinal Balance

The assumptions we make in using this simple model are:

- The pitching moments due to thrust F and all drag components D are often considered to be quite small, compared with other moments;
- i.e. we initially ignore the pitching effect of F and D.
- The two lift forces act normal to the horizontal wind vector which itself is aligned with F and D. Clearly, this is true for D, but the thrust line is fixed by the engine installation and will not in general be aligned with the wind vector. The difference of alignment will not be large, hence the assumption given above.

The Simple Model for Longitudinal Balance

- The weight vector nW also acts vertically. In other words, we are confining the analysis to horizontal flight, not climbing flight.

- Note that whilst we might choose to neglect L_T in the transverse equation, if $L_T << L_w$, we cannot ignore its major contribution to pitching moment because of its long moment arm l_T.

- The tail is not *shown* to have an elevator but we shall include such effects in any expression for C_{L_T}.

The Simple Model for Longitudinal Balance

The three equations valid in the plane of symmetry are :

- transverse: $L_w + L_T = nW$ (6)
- axial: $F = D$ (7)
- pitch: for the moment we shall choose to take moments about the a.c. of the wing, leading to:

$$
\begin{aligned}
M_{ac} &= M_0 + nWx\bar{c} - L_T l_T \\
&= 0 \quad \text{for equilibrium}
\end{aligned}
\tag{8}
$$

- We can non-dimensional the factors by using qS for forces and $qS\bar{c}$ for moments (with w implicit on S).

The Simple Model for Longitudinal Balance

- With $q = \frac{1}{2}\rho U^2 = \frac{1}{2}\rho_0 U_E{}^2$, from Eqn. (6):

$$\frac{L_W}{qS} + \frac{S_T}{S}\frac{L_T}{qS_T} = \frac{nW}{qS}$$

or

$$C_{L_W} + \frac{S_T}{S} C_{L_T} = \frac{nW}{qS} \tag{9}$$

- and from Eqn. (8)

$$\frac{M_{ac}}{qS\bar{c}} = \frac{M_0}{qS\bar{c}} + \frac{nWx\bar{c}}{qS\bar{c}} - \frac{S_T}{S}\frac{L_T l_T}{qS_T\bar{c}} \tag{10}$$

The Simple Model for Longitudinal Balance

- If we distinguish between a total C_L (no subscript on L) and the wing coefficient C_{L_W}, i.e. if we choose to use, for convenience:

$$C_L = \frac{(L_W + L_T)}{qS} \tag{11}$$

- then Eqn. (10) becomes:

$$\begin{aligned} C_{M_{ac}} &= C_{M_0} + xC_L - \frac{S_T l_T}{S\bar{c}} C_{L_T} \\ &= C_{M_0} + xC_L - \bar{V} C_{L_T} \end{aligned} \tag{12}$$

- where $\bar{V} = \frac{S_T l_T}{S\bar{c}}$ (13)

is the "tail volume coefficient".

Tail Volume, Tail Efficiency, Horizontal Tail Sizing

Recalling Equation 8:

$$M_{ac} = M_0 + nWx\bar{c} - L_T l_T = 0 \text{ for equilibrium.}$$

- The term $L_T l_T$ is a measure of the 'leverage' or pitching moment capability of the horizontal tail. We could write:

$$L_T l_T = q S_T C_{L_T} l_T \qquad (14)$$

$q \Rightarrow \rho, U$	$C_{L_T} \Rightarrow \alpha, \delta$
$S_T \Rightarrow$ area	$l_T \Rightarrow$ moment arm

- i.e. the product $S_T l_T$ is key when considering longitudinal balance and must therefore be chosen carefully at an early stage - *Design project!*

Tail Efficiency Factor

- We should also allow for the fact that airspeed at the tail will have been disturbed from the value at the wing.
- A large field of air is affected by prior passage of the wing and fuselage, so that it is not only deflected downward but is 'generally slower' when it reaches the tail. We should really consider:

$$q_T = \frac{1}{2}\rho U_T^{\,2}, \text{ not } q = \frac{1}{2}\rho U^2$$

- which would give the second term in Eqn. (9) as:

$$\frac{q_T}{q}\frac{S_T}{S}\frac{L_T}{q_T S_T} \rightarrow \eta_T \frac{S_T}{S} C_{L_T} \qquad (15)$$

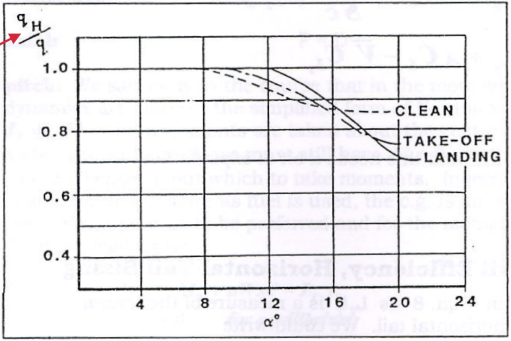
$$Note: \text{prop-wash} \Rightarrow \eta_T > 1$$

- where C_{L_T} is $\dfrac{L_T}{q_T S_T}$ and where $\eta_T = \dfrac{q_T}{q}$ is the "*tail efficiency*".

Tail Efficiency Factor

- Airbus Design Project version of η_T is as shown in the figure below.
- For most purposes it can be ignored because it is close to 1.0 for most practical situations. Unless the incidence is high, the apparent disturbance to q_H is small.

Horizontal Tail



Cruise vs take-off and landing

Tail Volume

- It is only because the <u>dimensions</u> for "tail volume" $S_T l_T$ are (length)^3 that this product has the given name.
- S_T is related to the force that can be produced and l_T is the lever arm that gives moment-capacity.
- Therefore l_T needs to be considered in conjunction with S_T for design considerations; in Eqn. (12) note that we use:

$$\bar{V} = \text{ tail volume coefficient } = \frac{S_T l_T}{S\bar{c}} \quad (16)$$

which is just a non-dimensional version.

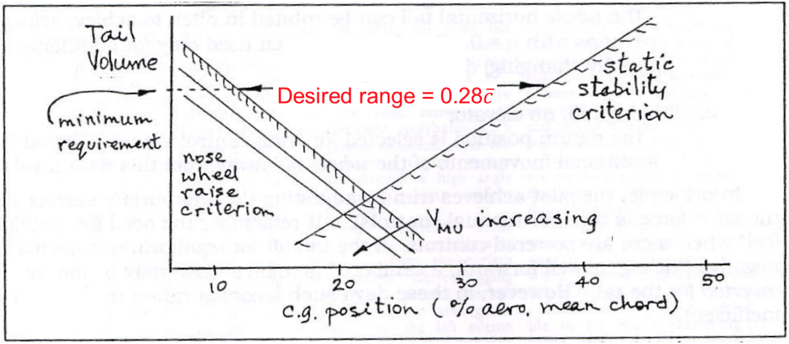
S_T : tailplane area

l_T : moment arm

S : wing area

$\bar{c}$: mean chord

Tail Sizing



Tail Volume
minimum requirement
nose wheel raise criterion
Desired range = $0.28\bar{c}$
static stability criterion
V_{MU} increasing
10
20
30
40
50
c.g. position (% aero. mean chord)

Tail Sizing

- Need to ensure that the tail can meet all the requirements whilst the c.g. is in any position within the range required.
- In a simple sizing diagram as in the previous figure, the various limiting criteria being set by the various tasks for the tail.
- The farther aft the c.g. goes, the larger is the tail capacity required to return the orientation to $\theta = 0$, thus retaining a positive "aerodynamic stiffness". i.e. consider ΔC_m.
- The sizing diagram shows how to determine the minimum tail volume necessary to allow a c.g. range of $0.28\bar{c}$, i.e. the lowest level the vector can fit *both sets* of slanted lines, is the level which defines a minimum for $S_T l_T$.

Example: Longitudinal Balance

Example: Longitudinal Balance

So far we have considered:

- Lift forces
- Overall pitching moment
- Note: Even if we satisfy the balance criteria we may not have a *stable* aircraft

Balance vs Stability!

- For the moment we no consider stability and look only at the necessary tail lift to satisfy the balance criteria.

Example: Longitudinal Balance

- Can we calculate the *variation* in tail lift as the speed changes in steady level flight, for an aircraft with the following characteristics?

span $b = 34\text{m}$
mean chord $\bar{c} = 4.2\text{m}$
mass $m = 60{,}000\text{kg}$
tail arm $l_T = 18\text{m}$

c.g. pos'n $x = 0.05, 0.11, 0.17$
fixed moment $C_{M_0} = -0.05, -0.15$
lift coeff.(max) $C_{L_{stall}} = 1.48$
wing area $S = 125\ \text{m}^2$

(cruise, landing)

- From eqn. (8), pitch balance is given by:

$$M_{ac} = M_0 + W x\bar{c} - L_T l_T = 0 \qquad (17)$$

Example: Longitudinal Balance

- We need to:
  - solve for the value of L_T
  - ensure that primary lift is equal to weight (eqn. (6) or (9))

- If we increase incidence to increase lift, note that neither of the two moments M_0 or $Wx\bar{c}$ would be affected.

- Thus re-arrange (8) to find L_T and we have:

$$L_T = \frac{M_0 + Wx\bar{c}}{l_T} \qquad (18)$$

Example: Longitudinal Balance

- In order to investigate the variation of tail lift with forward speed we can expand this and present it as:

$$L_T = \frac{\rho_0 S\bar{c}}{2l_T} C_{M_0} U_E^2 + \frac{W\bar{c}}{l_T} x \tag{19}$$

- Into which we can insert the range of values for the basic aerodynamic moment, the speed and the c.g. position:

$$\begin{aligned} L_T &= \frac{1.225 \times 125 \times 4.2}{2 \times 18} C_{M_0} U_E^2 + \frac{60000 \times 9.81 \times 4.2}{18} x \\ &= 17.9 \times C_{M_0} U_E^2 + 13.7 \times 10^4 x \end{aligned} \tag{20}$$

Example: Longitudinal Balance

- Note that *eqn. (20)* does not say anything about the distribution of lift between the two aerofoils.
- At this point we can simply state that with correct adjustment of the flight incidence we would arrange to have

$$L_W = W - L_T \tag{21}$$

- Ideally we would have positive tail lift so that the wing lifts the aircraft weight and does not have to counteract negative tail lift.

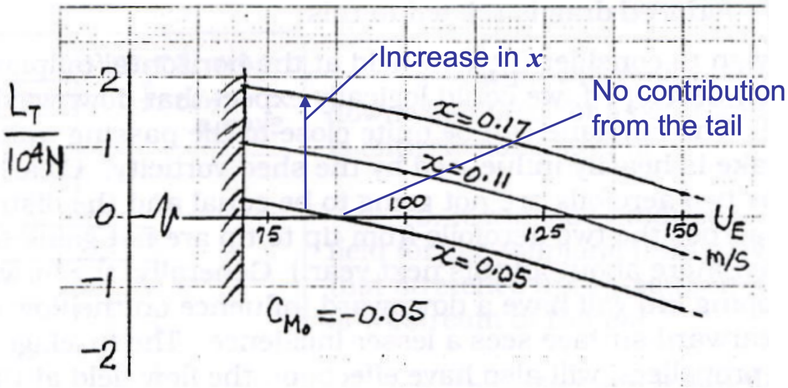
Example: Longitudinal Balance

- Before the plotting the results we can determine the lower end of the useful speed range for which we should believe the values of tail lift, i.e. we need the stall speed (*EAS*), available from:

$$U_{E_{stall}} = \sqrt{\frac{2W}{\rho_0 S C_{L_{stall}}}} = \sqrt{\frac{2\times 60000\times 9.81}{1.225\times 125\times 1.48}} = 72\, m/s \qquad (22)$$

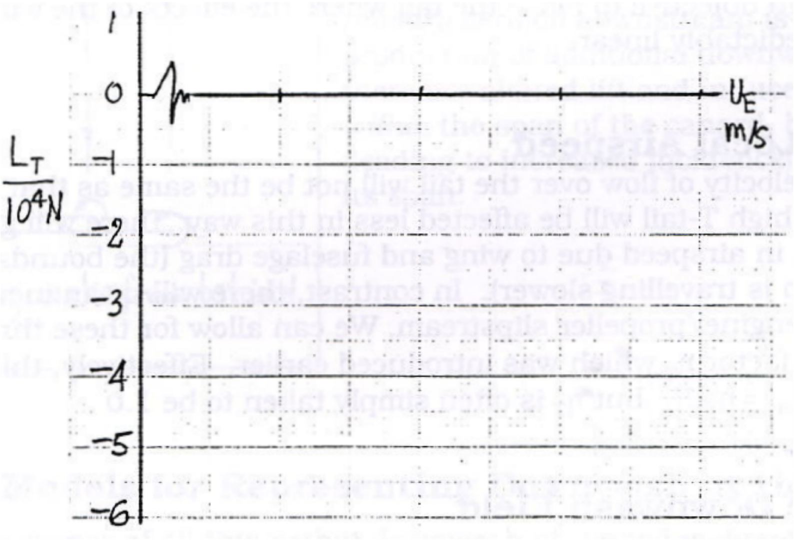
- The plot on the next slide shows the required tail lift, subject to this restriction.

Example: Longitudinal Balance



The plot is shown for the smaller value of C_{M_0} and the three values of c.g. position x.

Example: Longitudinal Balance



Example: Longitudinal Balance

- Exercise: insert a comparable set of curves for the other value of C_{M_0} , within the axes provided, and thus see the influence that this aerodynamic factor can have on the balance and the tail-sizing.

- Consider the size of tail implied here, and compare your estimate with the '*rule of thumb*' that S_T is often about 1/4 to 1/5 of S (wing). For a flight speed of 125 m/s determine the implied values of L_W and therefore also of the ratio L_T/L_W, for all six cases.

- Note also that as well as *ignoring stability considerations* we have not looked at implied stick forces that the pilot would have to resist (control surfaces).

Example: Longitudinal Balance

Note:

- The further back you put the c.g. (larger x), the higher the value for L_t.
- The points where L_t crosses the horizontal axis (=0). This is where the two moments M_0 and $W x \bar{c}$ produce an overall pitching moment=0 with no tail contribution.
- Remember that C_{M_0} is approximately = –0.05 for cruise and that with flaps down for landing you get about an order of magnitude greater, i.e. roughly $C_{M_0} = -0.5$
- So the value given for the second case corresponds to a take-off configuration.

Next Lecture

Elevator Angle to Trim