Example 3.1.2 – Figure 1 shows a plane, pin-jointed truss made of six members all of length 1 m. The truss is supported at D and F while a horizontal load of 5 kN is applied at point A as shown. All six members have a cross-sectional area of 400 mm^2 and are made of aluminium alloy with a Young's modulus E = 70 GPa.

a) Calculate the forces in all six members.
b) Using an energy method, calculate the **horizontal** deflection of point A.
c) Using an energy method, calculate the **vertical** deflection of point A.

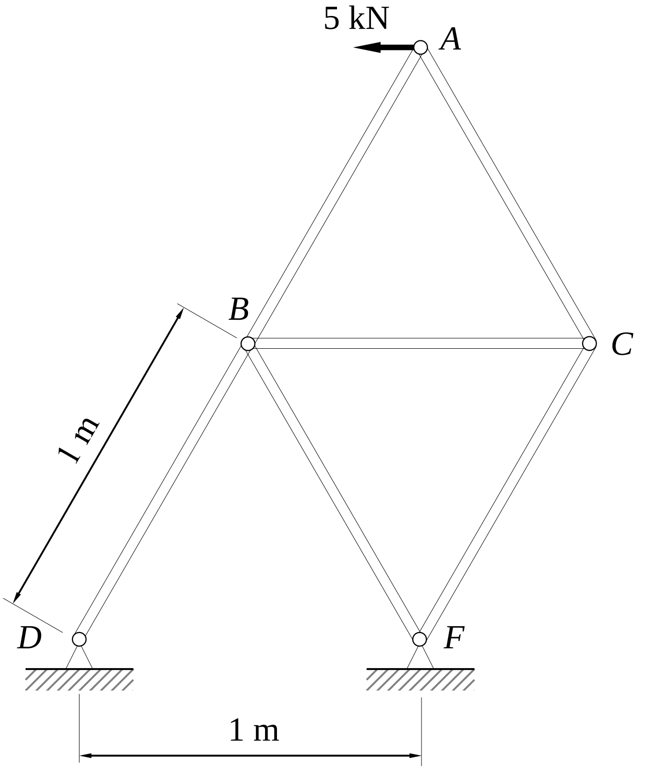
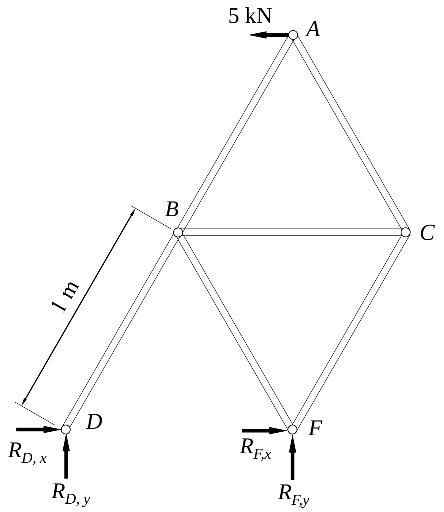


Figure 1: A pin-jointed truss.

a) Forces in all members:



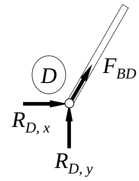
The characteristic angle between members is $\theta = 60^0$.

Balance of moments about joint F gives

$$\begin{aligned}
&\sum M_F &&= 0\\
(5000)(2)(1.0)(\sin\theta) \quad &- R_{D,y}(1.0) &&= 0\\
8660.25 \quad &- R_{D,y} &&= 0\\
&R_{D,y} &&= 8660.25\,\text{N}
\end{aligned}$$

Joint D

$$
\begin{array}{lll}
 & \sum F_y & = 0 \\
R_{D,y} & + F_{BD}\ \sin\theta & = 0 \\
8660.25 & + F_{BD}\ \sin\theta & = 0 \\
 & F_{BD} & = -10000\,\mathrm{N}
\end{array}
$$

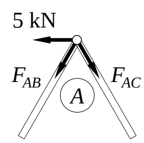


$$
\begin{array}{lll}
 & \sum F_x & = 0 \\
R_{D,x} & + F_{BD}\ \cos\theta & = 0 \\
 & R_{D,x} & = -F_{BD}\ \cos\theta \\
 & R_{D,x} & = 5000\,\mathrm{N}
\end{array}
$$

Joint A

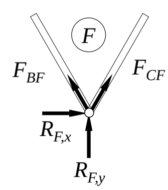
$$
\begin{array}{lll}
 & \sum F_y & = 0 \\
F_{AB}\sin\theta & + F_{AC}\ \sin\theta & = 0 \\
 & F_{AB} & = -F_{Ac}
\end{array}
$$

$$
\begin{array}{llll}
 & & \sum F_x & = 0 \\
5000 & + F_{AB}\cos\theta & - F_{AC}\cos\theta & = 0 \\
 & & 2\,F_{AB} & = \dfrac{-5000}{\cos\theta} \\
 & & F_{AB} & = -5000\,\mathrm{N} \\
 & & F_{AC} & = 5000\,\mathrm{N}
\end{array}
$$



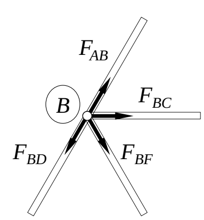
Joint F

$$
\begin{array}{lll}
 & \sum F_x & = 0 \\
-F_{BF}\cos\theta & + F_{CF}\cos\theta & = 0 \\
 & F_{BF} & = F_{CF}
\end{array}
$$



$$
\begin{array}{llll}
 & & \sum F_y & = 0 \\
R_{F,y} & + F_{BF}\sin\theta & + F_{CF}\sin\theta & = 0 \\
 & & 2\,F_{BF}\ \sin\theta & = 8660.25 \\
 & & F_{BF} & = 5000\,\mathrm{N} \\
 & & F_{CF} & = 5000\,\mathrm{N}
\end{array}
$$

Joint B



$$
\begin{array}{lllll}
 & & & \sum F_x & = 0 \\
-F_{BD}\cos\theta & + F_{BF}\cos\theta & + F_{AB}\cos\theta & + F_{BC} & = 0 \\
 & & & F_{BC} & = -\cos\theta\left(-F_{BD} + F_{BF} + F_{AB}\right) \\
 & & & F_{BC} & = -5000\,\mathrm{N}
\end{array}
$$

b) Horizontal deflection at A

Member	Li / m	Pi / N
AB	1	-5000
AC	1	5000
BC	1	-5000
BD	1	-10000
BF	1	5000
CF	1	5000

Applying Castigliano's theorem:

$$\frac{\partial P_i}{\partial Q} = P_i'$$

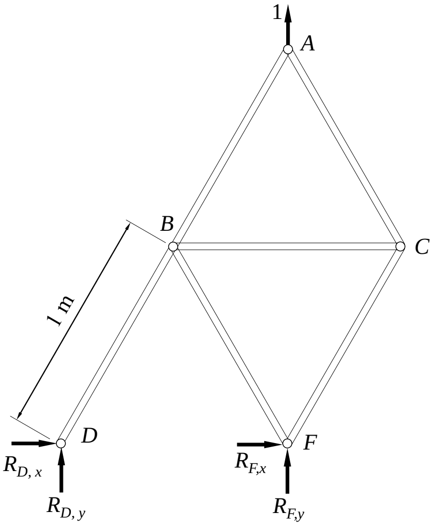
$$(\delta_x)_A = \sum_{i=1}^{m} \frac{P_i \; L_i}{A_i \; E_i} P_i'$$

$$(\delta_x)_A = \overline{\qquad\qquad\qquad}$$

$$(\delta_x)_A =$$

c) Vertical deflection at A

Apply a unit 'virtual' load:



Calculate 'virtual internal forces' in each member due to virtual load. Use these are the new P_i' values.

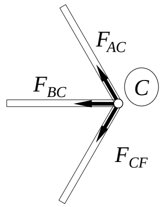
Joint A

$$\begin{aligned} & \sum F_y && = 0 \\ 1 \quad - F_{AC} \sin\theta \quad & - F_{AB} \sin\theta && = 0 \\ & 2\, F_{AC} \sin\theta && = 1 \\ & F_{AB} && = 0.577 \\ & F_{AC} && = 0.577 \end{aligned}$$

And due to vertical equilibrium:

$$
\begin{aligned}
F_{BF} &= 0.577 \\
F_{CF} &= 0.577 \\
F_{BD} &= 0
\end{aligned}
$$

Joint C



$$
\begin{aligned}
\sum F_x &= 0 \\
F_{BC} + F_{AC}\cos\theta + F_{CF}\cos\theta &= 0 \\
F_{BC} &= -\cos\theta\left(F_{AC} + F_{CF}\right) \\
F_{BC} &= -0.577
\end{aligned}
$$

Member	Li / m	Pi / N
AB	1	-5000
AC	1	5000
BC	1	-5000
BD	1	-10000
BF	1	5000
CF	1	5000

Applying Castigliano's theorem:

$$(\delta_y)_A = \sum_{i=1}^{m} \frac{P_i \; L_i}{A_i \; E_i} P_i'$$

$$(\delta_y)_A = \underline{\hspace{6cm}}$$

$$(\delta_y)_A =$$