FLUIDS I
Example sheet 6: Potential Flow

1. (a) Sketch the semi-infinite body and surrounding flowfield formed by the combination of a uniform freestream and a point source. If the stream functions for the freestream and source are

$$\psi_{freestream} = U_{\infty} y\,, \quad \psi_{source} = \frac{\Lambda}{2\pi} \tan^{-1}\left(\frac{y}{x}\right)$$

respectively, determine the position of the stagnation point at the nose of this body relative to the point source, and hence the value of the stream function for the dividing streamline. Show that the maximum width of the body is Λ/U_{∞}

(b) A steady wind of velocity 10m/s is blowing straight in off the sea on to a rounded cliff face which extends laterally for several kilometres. The flow over the cliff face can be assumed to be two-dimensional and to be represented by the flow due to a point source at sea level. The ultimate height of the cliff face is 100m. A glider is flying straight & level along the length of the cliff face, directly above the foot of the cliff. If the sinking speed of the glider is 1.5m/s, at what altitude is it flying? If the glider moves inland, how much higher could it fly at the same sinking speed and still maintain altitude ?

Ans: (a) $\Lambda/2\pi U_{\infty}$ forward of the source, $\Lambda/2$ (b) 207m, 5m higher

2. An airflow velocity sensor is to be mounted either in front, or above, the nose of the wing of a light aircraft, in order to give an indication of the speed of the aircraft. To obtain some feel for the situation, assume that the local flow is two-dimensional (ie the wing is straight, and of large aspect ratio) and that the nose of the wing is symmetrical and at zero incidence. In this case, the wing leading-edge is represented by a point source in a uniform flow, and the ultimate thickness of the body so obtained is taken to be the maximum thickness of the wing section (which in this case is 0.3m). The stream function for this flow is

$$\psi = U_{\infty} y + \frac{\Lambda}{2\pi} \tan^{-1}\left(\frac{y}{x}\right), \quad \psi_{surface} = \frac{\Lambda}{2}$$

(a) If the sensor is mounted on the centreline of the section, how far forward of the nose of the section must it be for the velocity error to be (i) 1% , and (ii) 10% ?

(b) If the sensor is mounted above the wing, at a chordwise position where the section thickness is half the maximum thickness, how far above the surface must it be for the error to be 1% ?

(c) If a pressure transducer is connected between a surface pressure tapping positioned at the same chordwise position, and another at the wing nose, what factor would need to be applied to the pressure differential reading to obtain the freestream dynamic pressure ?

Ans: (a) 4.73m, 0.43m (b) 0.262m (c) 0.711

3. (a) Given the stream function for a cylinder of radius R in a uniform flow

$$\psi = U_\infty r \sin\theta \left(1 - \frac{R^2}{r^2}\right)$$

show that the surface pressure coefficient distribution is given by $C_p = 1 - 4\sin^2\theta$.

(b) A yawmeter consists of a circular cylinder with its axis normal to the flow, in which there are two static pressure tappings 60° apart in the same plane normal to the axis. The angle of the incident flow is found by turning the cylinder until the pressures at the two holes are equal (at which point they are 30° either side of the front stagnation point. Assuming inviscid flow, calculate the sensitivity of the yawmeter (ie rate of change of pressure differential with yaw angle) in the vicinity of the null position (where the pressures are equal).

What angle between the holes would give the maximum sensitivity, and what is the value of this sensitivity ?

Ans: 0.121 per degree, 90°, 0.140 per degree

4. (a) Given the stream function for a cylinder in a uniform flow with circulation

$$\psi = U_\infty r \sin\theta \left(1 - \frac{R^2}{r^2}\right) + \frac{\Gamma}{2\pi} \ln\left(\frac{r}{R}\right)$$

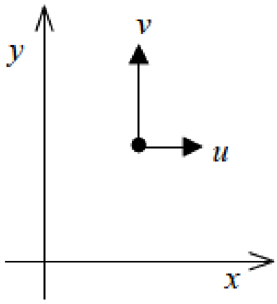
sketch the resulting flowfield and determine the angular position of the front and rear stagnation points.

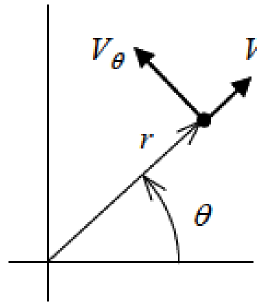
(b) The circulation on a lifting cylinder is such that the rear stagnation point is displaced by 30° downwards. Calculate the magnitudes of the flow velocities at the top and bottom of the cylinder and the corresponding surface pressure coefficients. Using the Kutta-Joukowski law, evaluate the lift coefficient per unit span.

Ans: (a) $\sin\theta_{SP} = -\Gamma/4\pi U_\infty R$ (b) $3U_\infty$, U_∞, -8, 0, 2π

COORDINATE SYSTEMS

$r = \sqrt{x^2 + y^2} \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right) \qquad x = r\cos\theta \quad y = r\sin\theta$





VELOCITY COMPONENTS

$u = V_r\cos\theta - V_\theta\sin\theta \qquad v = V_r\sin\theta + V_\theta\cos\theta$

$V_r = u\cos\theta + v\sin\theta \qquad V_\theta = -u\sin\theta + v\cos\theta$

$$u = +\frac{\partial\phi}{dx} \quad v = +\frac{\partial\phi}{dy} \qquad V_r = +\frac{\partial\phi}{dr} \qquad V_\theta = +\frac{1}{r}\frac{\partial\phi}{d\theta}$$

$$u = +\frac{\partial\psi}{dy} \quad v = -\frac{\partial\psi}{dx} \qquad V_r = +\frac{1}{r}\frac{\partial\psi}{d\theta} \quad V_\theta = -\frac{\partial\psi}{dr}$$

<u>Freestream velocity (parallel to x axis)</u>

$$\phi = V_\infty x \qquad \psi = V_\infty y$$

$$u = V_\infty \qquad v = 0 \qquad V_r = V_\infty\cos\theta \qquad V_\theta = -V_\infty\sin\theta$$

<u>Source</u> (reverse sign of Λ for sink flow)

$$\phi = +\frac{\Lambda}{2\pi}\ln r \qquad \psi = +\frac{\Lambda}{2\pi}\theta \qquad V_r = +\frac{\Lambda}{2\pi r} \qquad V_\theta = 0$$

<u>Doublet</u>

$$\phi = +\frac{\kappa}{2\pi}\frac{\cos\theta}{r} \qquad \psi = -\frac{\kappa}{2\pi}\frac{\sin\theta}{r} \qquad V_r = -\frac{\kappa}{2\pi r^2}\cos\theta \qquad V_\theta = -\frac{\kappa}{2\pi r^2}\sin\theta$$

<u>Vortex</u>

$$\phi = -\frac{\Gamma}{2\pi}\theta \qquad \psi = +\frac{\Gamma}{2\pi}\ln r \qquad V_r = 0 \qquad V_\theta = -\frac{\Gamma}{2\pi r}$$