Assessment A - Energy Methods

(StM1 - Dr Luiz Kawashita)

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1 Assessment Details

This is an **individual exercise** which accounts for 70% of 'Assessment A' in *AENG 11200 - Structures and Materials 1* (*i.e.* it is worth 7% of the entire unit, or 1.4 credit points).

You will need to submit your results through Blackboard by **Friday 2nd of March 2018** (23:59). An online form will be made available on Blackboard, where you will be able to enter your solution in the form of equations and numerical values.

2 Question

The plane pin-jointed structure in Figure 1 is made of **four** axial members which are pin-jointed at Q and individually pinned to four other supports. The exact location of each support is unique for each student, as shown in Table 1. For example, the particular configuration shown in Figure 1 is the first entry in Table 1, *i.e.* utilising joints $CGKO$.

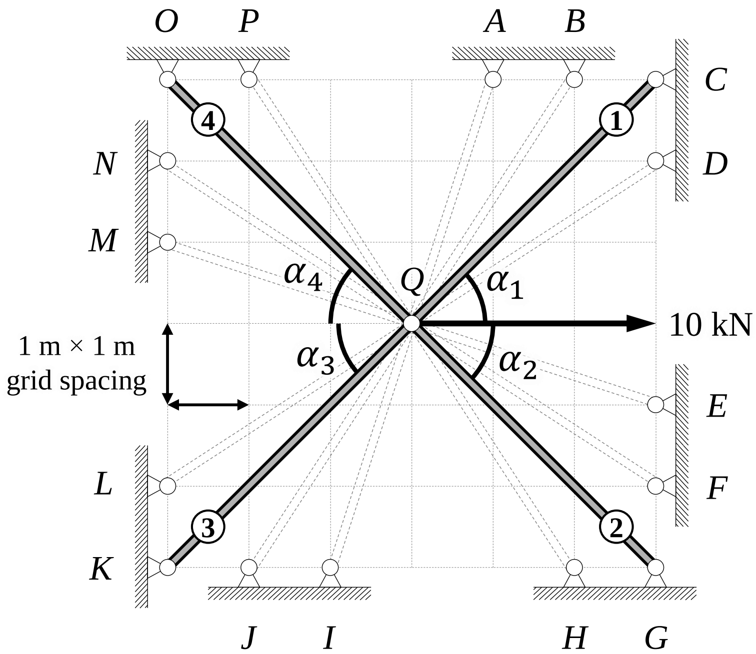

Figure 1: A statically indeterminate plane pin-jointed structure made of **four** axial members. Note that all members are pin-jointed at node Q, and that the grid spacing is 1 m × 1 m.

A horizontal force of 10 kN is applied to the central joint Q as shown. All members have a solid square cross-section measuring 30 mm × 30 mm and are made of aluminium alloy with $E = 70$ GPa. Note that the structure has 'degree of redundancy' 2. When working your solutions, you **must** take the two unknowns, X and Y, to be the internal forces of the two members specified in Table 1.

Calculate the horizontal and vertical components of the displacement of joint Q due to the applied load of 10 kN for your particular configuration, as listed in Table 1.

3 Individual configurations

Table 1: Individual pin joint configurations.

Name	Joints	X	Y
Figure 1	C G K O	1	2
Kawashita, Luiz	A E J O	1	2
Abass, Hussain	A E J P	2	3
Allen, Ben	A E K N	3	4
Ami Azrul, Insyirah	A E K P	4	1
Andraous, Joseph	A E L N	1	2
Attygalle, Damin	A E L O	2	3
Avigdor, Joel	A F I O	3	4
Ayinaparthi, Anudeep	A F I P	4	1
Banu, Shenaz	A F J O	1	2
Barnes, Lauryn	A F J P	2	3
Barra, Nick	A F K M	3	4
Beattie, James	A F K N	4	1
Beech, Theo	A F K O	1	2
Beggan, Sean	A F K P	2	3
Best, Kim	A F L M	3	4
Billyeald, Dillon	A F L N	4	1
Bolos, Quim	A F L O	1	2
Bourland, Xavier	A F L P	2	3
Bruno, Beatriz	A G I N	3	4
Catmull, Jamie	A G I P	4	1
Changpingnga, Pakapoom	A G J M	1	2
Charnley, Blaine	A G J N	2	3
Chen, Yue	A G J O	3	4
Chin, Jun Man	A G J P	4	1
Chit, Thaw Htin	A G K N	1	2
Chivers, Emma	A G K P	2	3
Chung, Lok Wai	A G L M	3	4
Cichecki, Mikolaj	A G L N	4	1
Cosimini, Aidan	A G L O	1	2
Coventry, Jake	A G L P	2	3
Daws, Sebastian	A H I N	3	4
Dennis, Jack	A H I O	4	1
Doherty, Pete	A H J M	1	2
Drennan, Aengus	A H J N	2	3
Ephraim, Wilfred	A H J O	3	4
Fan, Yang	A H J P	4	1
Fenemore, Tom	A H K M	1	2
Fogarassy, Sebastian	A H K N	2	3
Forth, Matthew	A H K O	3	4
Gale, Will	A H K P	4	1
Ganniclifft, Eden	A H L N	1	2
Ge, Junting	A H L O	2	3
Georgiou, Eleni	B E I O	3	4
Gill, Matthew	B E I P	4	1
Gjergji, Celestin	B E J O	1	2
Gladwin, Josh	B E J P	2	3
Gleed, Hannah	B E K M	3	4
Gomes De Freitas, Tiago	B E K N	4	1
Greasley, Ben	B E K O	1	2
Grice, Miller	B E K P	2	3
Grieve, Alex	B E L M	3	4
Haddican, Finley	B E L N	4	1
Hadjipanteli, Michail	B E L O	1	2
Haigh, William	B E L P	2	3
Halim, Jovin	B F I O	3	4
Han, Yifei	B F I P	4	1
Hart, Danny	B F K M	1	2
Hernandez Segura, Guillermo	B F K P	2	3
Hetherington, Harry	B F L M	3	4
Hoerter, Thomas	B F L O	4	1
Huang, Shih-Keng	B G I M	1	2
Hughes, Ben	B G I N	2	3
Iordache, Georgiana	B G I O	3	4
Iordan, Andrei	B G I P	4	1
Jayatilake, Sanuja	B G J M	1	2
Jerome, Matthew	B G J P	2	3
Kelley, Dan	B G K M	3	4
Kharkhalis, Marko	B G K P	4	1
Lalvani, Shaan	B G L M	1	2
Lam, Tsz	B G L N	2	3
Li, Jeffery	B G L O	3	4
Li, Xinyue	B G L P	4	1
Li, Yueheng	B H I M	1	2
Lin, Pi-Che	B H I N	2	3
Liu, Robert	B H I O	3	4
Lloyd, Evan	B H I P	4	1
Lucaci, Dacian	B H J M	1	2
Lupusoru, Alex	B H J O	2	3
Lyu, Yangkai	B H K M	3	4
Moreira, Antonio Jose	B H K N	4	1
Mareo, Jason	B H K O	1	2
McClure, Luke	B H K P	2	3
McDonnell, Callum	B H L M	3	4
Milne, Ashford	B H L O	4	1
Mina, Katerina	C E I N	1	2
Mistry, Shiv	C E I P	2	3
Mourant, Charles	C E J M	3	4
Mowat, Ben	C E J N	4	1
Nemolyaev, Stan	C E J O	1	2
Neofet, Theodor	C E J P	2	3
O'Dowd O'Rourke, Alan	C E K N	3	4
Omiat, Humphrey	C E K P	4	1
Pagona, Marietta	C E L M	1	2
Paine, Louis	C E L N	2	3
Parmenter, Steven	C E L O	3	4
Pike, Oliver	C E L P	4	1
Pringle, Gabriel	C F I M	1	2
Pringle, Luke	C F I N	2	3
Pudney, Richard	C F I O	3	4
Purves, James	C F I P	4	1
Purwadi, Johan David	C F J M	1	2
Quyyam, Omair	C F J P	2	3
Raduta, Mircea-Teodor	C F K M	3	4
Rashid, Mamunur	C F K P	4	1
Ratkoceri, Dardan	C F L M	1	2
Ripley, Julian	C F L N	2	3
Rochaix Yamamoto, Anthelm	C F L O	3	4
Rollett, Emelye	C F L P	4	1
Saliu, Oladotun	C G I N	1	2
Scupham, Jason	C G I P	2	3
Searle, Sam	C G J M	3	4
Shah, Sooruj	C G J P	4	1
Sharma, Tanuj	C G L M	1	2
Sharp, Bart	C G L N	2	3
Sheppard, Ben	C H I M	3	4
Sivasuthan, Tiana	C H I N	4	1
Small, Sebastian	C H I O	1	2
Soh, Zhen Yang	C H I P	2	3
Standring, Arran	C H J M	3	4
Stephenson, Timothy	C H J N	4	1
Stubbs, Will	C H J O	1	2
Syriac, Baptin	C H J P	2	3
Tai, Anson	C H K M	3	4
Tam, Aidan	C H K N	4	1
Tang, Jonathan	C H L M	1	2
Taylor, Liam	C H L N	2	3
Thom, Ben	D E I N	3	4
Thomas, Luke	D E I O	4	1
Thompson, Kyle	D E J M	1	2
Thompson, Matthew	D E J N	2	3
Tijbosch, Guillermo	D E J O	3	4
Timperley, Louis	D E J P	4	1
Tirmzi, Zammurd	D E K M	1	2
Trascinelli, Leone	D E K N	2	3
Tsumura, Yasunori	D E K O	3	4
Uzzell, James	D E K P	4	1
Viljoen, John	D E L N	1	2
Virk, Jagjeevan	D E L O	2	3
Wang, Jing	D F I M	3	4
Whitfield, Ben	D F I N	4	1
Woods Ballard, Will	D F I O	1	2
Xing, Gaoyue	D F I P	2	3
Yap, Li-Toong	D F J M	3	4
Yi, Christopher	D F J O	4	1
Zeng, Xue	D F K M	1	2
Zhang, Rai	D F K N	2	3
Zheng, Yiling	D F K O	3	4

4 Procedure

To solve this problem you will need to apply the Principle of Stationary Potential Energy (PSPE) and Castigliano's theorem **twice**.

4.1 Internal forces

Firstly, populate Table 2 with the correct geometrical parameters for your configuration.

Table 2: Geometrical parameters.

Member, i	**Length,** L_i **[m]**	**Angle,** α_i **[deg]**	$\sin \alpha_i$	$\cos \alpha_i$	$E_i A_i$ **[kN]**
1					
2					
3					
4					

Next, write the equations of equilibrium for joint Q and populate Table 3.

Table 3: Application of the PSPE to find internal forces F_i.

Member, i	**Length,** L_i **[m]**	**Force,** F_i **[kN]**	$\frac{\partial F_i}{\partial X}$	$\frac{\partial F_i}{\partial Y}$	$F_i L_i \frac{\partial F_i}{\partial X}$ **[kN·m]**	$F_i L_i \frac{\partial F_i}{\partial Y}$ **[kN·m]**
1						
2						
3						
4						

Then continue with the application of the PSPE to obtain the two energy-based equations, namely

$$\frac{\partial U}{\partial X} = \sum \frac{F_i L_i}{E_i A_i} \frac{\partial F_i}{\partial X} = 0 \quad \text{and} \quad \frac{\partial U}{\partial Y} = \sum \frac{F_i L_i}{E_i A_i} \frac{\partial F_i}{\partial Y} = 0. \tag{1}$$

You should now have **four** equations and **four** unknowns, so you can solve them (*e.g.* by substitution) and find the *real* (*i.e.* physical) internal forces F_i.

4.2 Horizontal displacement

We can now apply Castigliano's theorem to find the horizontal displacement at Q. This is straightforward; start by populating Table 4, then apply the theorem itself, *i.e.*

$$u_x = \sum \frac{F_i L_i}{E_i A_i} \frac{\partial F_i}{\partial Q}. \tag{2}$$

Table 4: Application of Castigliano's theorem to find the **horizontal** displacement at Q.

Member, i	**Length,** L_i **[m]**	**Force,** F_i **[kN]**	$\frac{\partial F_i}{\partial Q}$	$F_i L_i \frac{\partial F_i}{\partial Q}$ **[kN·m]**
1				
2				
3				
4				

4.3 Vertical displacement

To find the vertical displacement, we need to apply the 'unit force method' to find the following derivatives,

$$F_i' = \frac{\partial F_i}{\partial V}, \tag{3}$$

where V is a **unit vector** applied at Q and pointing **upwards**.

However, to find F_i' you will need to repeat the procedure described in Section 4.1, *i.e.* write two equilibrium equations based on vector V applied at Q, and then apply the PSPE again to find two energy-based equations involving F_i'. For the latter, start by populating Table 5.

Table 5: Application of the PSPE to find the derivatives F_i'.

Member, i	**Length,** L_i **[m]**	**'Virtual force',** F_i'	$\frac{\partial F_i'}{\partial X}$	$\frac{\partial F_i'}{\partial Y}$	$F_i' L_i \frac{\partial F_i'}{\partial X}$	$F_i' L_i \frac{\partial F_i'}{\partial Y}$
1						
2						
3						
4						

You should now have four equations involving the four unknowns F_i', which can be solved again by substitution. Once the values of F_i' have been found, you can apply Castigliano's theorem again to find u_y. First, populate Table 6.

Table 6: Application of Castigliano's theorem to find the **vertical** displacement at Q.

Member, i	**Length,** L_i **[m]**	**Force,** F_i **[kN]**	$F_i' = \frac{\partial F_i}{\partial V}$	$F_i L_i \frac{\partial F_i}{\partial V}$
1				
2				
3				
4				

Then, finally, apply the theorem again,

$$u_y = \sum \frac{F_i L_i}{E_i A_i} \frac{\partial F_i}{\partial V}. \tag{4}$$

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