Figure 1 shows the cross-section of a thin-walled torsion member. The member is 1.2 m in length and is fabricated from sheet steel of thickness 2 mm. The member is subjected to a twisting moment of magnitude 5.0 kNm.

a) Obtain the maximum shear stress in the section.

b) Obtain the total angular twist of the section.

c) It is required to reduce the shear stress in the vertical web BC to zero by changing the thickness of the three sections BE, EF and FC to a new value (h), whilst retaining the four sections AB, BC, CD, and DA at a thickness of 2 mm. Determine the required thickness h.

Assume G = 80 GPa.

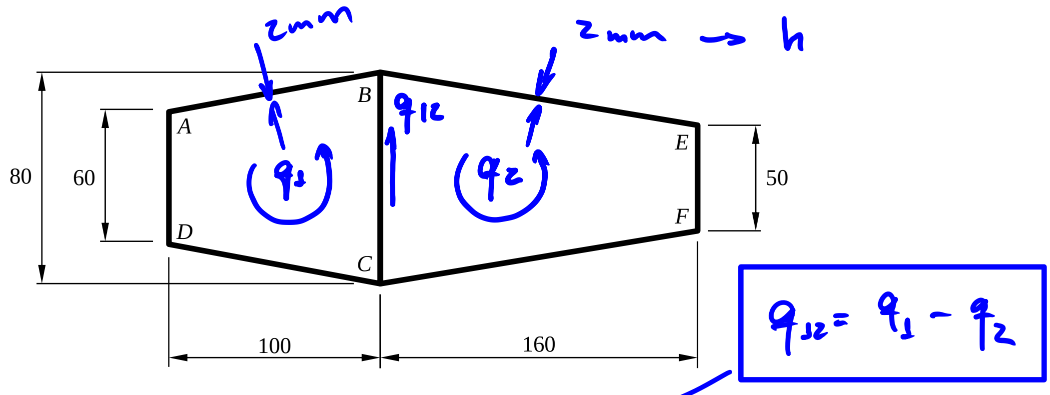


Figure 1 (Dimensions in mm)

Given: $$\left(\frac{\theta}{L}\right) = \frac{q}{2AG}\oint\frac{ds}{t}$$

Preliminaries:

Lengths.

$$AB = CD = \sqrt{100^2 + \left(\frac{80-60}{2}\right)^2} = 100.5 \text{ mm}$$

$$BE = CF = \sqrt{160^2 + \left(\frac{80-50}{2}\right)^2} = 160.7 \text{ mm}$$

Areas:

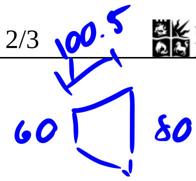
$$A_1 = (100)\left(\frac{80+60}{2}\right) = 7000 \text{ mm}^2$$

$$A_2 = (160)\left(\frac{80+50}{2}\right) = 10400 \text{ mm}^2$$

$$\left(\frac{\theta}{L}\right)_1 = \frac{q_1}{2A_1G}\oint_1\frac{ds}{t} - \frac{q_2}{2A_1G}\int_{12}\frac{ds}{t}$$

closed contour along the web

$$\left(\frac{\theta}{L}\right)_2 = \frac{q_2}{2A_2G}\oint_2\frac{ds}{t} - \frac{q}{2A_2G}\int_{12}\frac{ds}{t}$$



$$\left(\frac{\theta}{L}\right)_1 = \left(\frac{\theta}{L}\right)_2$$

Integrals:

$$\oint_1 \frac{ds}{t} = \frac{60 + 80 + (2)(100.5)}{2} = \boxed{170.5}$$

$$\oint_2 \frac{ds}{t} = \frac{50 + 80 + (2)(160.7)}{2} = \boxed{225.7}$$

$$\int_{12} \frac{ds}{t} = \frac{80}{2} = \boxed{40}$$

Therefore.

$$\frac{q_1}{\cancel{2}(7000)\cancel{G}}(170.5) - \frac{q_2}{\cancel{2}(7000)\cancel{G}}(40) = \frac{q_2}{\cancel{2}(10400)\cancel{G}}(225.7) - \frac{q_1}{\cancel{2}(10400)\cancel{G}}(40)$$

$$\boxed{q_2 = 1.028\, q_1} \qquad (\text{eq. } 1)$$

Torque equation: $T = 2Aq$

$$T = 2A_1 q_1 + 2A_2 q_2 \qquad T = 5\,\text{kNm} = 5 \times 10^6\ \text{Nmm}$$

$$\boxed{5 \times 10^6 = (2)(7000)q_1 + (2)(10400)q_2} \qquad (\text{eq. } 2)$$

Substituting eq. 1 in eq. 2:

$$q_1 = 141.3\ \text{N/mm}$$

$$q_2 = 145.2\ \text{N/mm}$$

$$q_{12} = q_1 - q_2 = -3.9\ \text{N/mm}$$

Finally: $q = \tau \cdot t \;\therefore\; \tau = \dfrac{q}{t}$

$$\tau_{max} = \frac{q_2}{t} = \boxed{72.6\ \text{MPa}}$$

b) Angle of twist $G = 80\ GPa = 80{,}000\ MPa$

$$\left(\frac{\theta}{L}\right)_1 = \frac{q_1}{2A_1G}\oint_1 \frac{ds}{t} - \frac{q_2}{2A_1G}\int_{12}\frac{ds}{t}$$

$$= \frac{(141.3)(170.5)}{2(7000)(80\times10^3)} - \frac{(145.2)(40)}{2(7000)(80\times10^3)}$$

$$= 1.63\times10^{-5}\ rad/mm$$

However $L = 1.2\ m$ therefore

$$\theta = (1.63\times10^{-5})(1200) = \boxed{0.0196\ rad = 1.12^\circ}$$

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c) New thickness h for $q_{12} = 0$ $\quad q_{12} = q_1 - q_2$

$q_1 = q_2$

$$\oint_2 \frac{ds}{t} = \frac{(2)(160.7)+50}{h} + \frac{80}{2}$$

$$= \frac{371.4}{h} + 40$$

The new equation 1 is:

$$\frac{170.5}{7000} - \frac{40}{7000} = \frac{1}{10400}\left[\frac{371.4}{h} + 40\right] - \frac{40}{10400}$$

Finally: $\boxed{h = 1.92\ mm}$