Example 3.3

singly symmetric

Calculate the position of the shear centre for the closed-cell section in Figure 3, assuming that skins carry only shear stresses and booms carry only direct stresses.

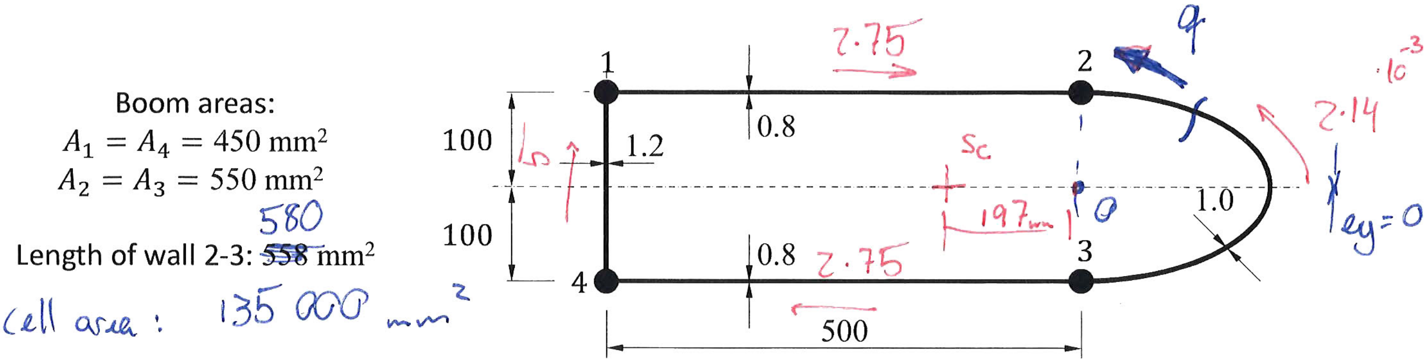

Figure 3: Closed-cell section. Assume skins to carry only shear and booms to carry all direct stresses.

$$-q_s^{open} = \frac{S_y}{I_{xx}} \sum y_i \underbrace{t_i b_i}_{A_i}$$

$$I_{xx} = \sum y_i^2 A_i \qquad I = \int y^2 dA$$

$$I_{xx} = 2(450\,mm^2)(100\,mm)^2 + 2(550\,mm^2)(100\,mm)^2$$

$$\boxed{I_{xx} = 20 \cdot 10^6\,mm^4}$$

$$q_s^{open} = -\frac{S_y}{20 \cdot 10^6} \sum y_i A_i = -5 \cdot 10^{-8} S_y \sum A_i y_i$$

Sectioning along 2-3 makes $q_{2-3}^{open} = 0$

$$q_{1-2}^{open} = -5 \cdot 10^{-8} S_y (550)(100) = \boxed{-2.75 \cdot 10^{-3} S_y}$$

$$q_{1-4}^{open} = -5 \cdot 10^{-8} S_y (450)(100) - 2.75 \cdot 10^{-3} S_y = \boxed{-5 \cdot 10^{-3} S_y}$$

$$q_{3-4}^{o} = -5 \cdot 10^{-8} S_y (450)(-100) - 5 \cdot 10^{-3} S_y = \boxed{-2.75 \cdot 10^{-3} S_y}$$

$$q_{2-3}^{open} = -5 \cdot 10^{-8} S_y (550)(-100) - 2.75 \cdot 10^{-3} S_y = \boxed{0}$$

From formula sheet: $q_0 = \dfrac{-\oint q_s^{open} \frac{ds}{t}}{\oint \frac{ds}{t}}$

$$-\oint q_s^{open} \frac{ds}{t} = -\sum q_i^{open} \frac{b_i}{t_i} = -2.75 \cdot 10^{-3} S_y \frac{(500)}{(0.8)} - 5 \cdot 10^{-3} S_y \frac{(200)}{(1.2)} - 2.75 \cdot 10^{-3} S_y \frac{(500)}{(0.8)} =$$

$$\oint \frac{ds}{t} = \sum \frac{b_i}{t_i} = \left(\frac{500}{0.8}\right)2 + \left(\frac{200}{1.2}\right) + \left(\frac{580}{1.0}\right) = 1996.7$$

$$q_0 = \frac{-4.271 \quad S_y}{1996.7} = \boxed{2.14 \cdot 10^{-3} S_y}$$

$$q_s^{closed} = q_s^{open} + q_0$$

$$\boxed{\begin{aligned} q_{32} &= 2.14 \cdot 10^{-3} S_y \\ q_{21} &= -0.61 \; 10^{-3} S_y \\ q_{14} &= -2.86 \; 10^{-3} S_y \\ q_{43} &= -0.61 \; 10^{-3} S_y \end{aligned}}$$

$$S_y e_x + \cancel{S_x e_y}^{\,0} = \int q\, r\, ds = \boxed{2Aq}$$

$$e_x = \sum q_i r_i b_i$$

$$= -0.61 \cdot 10^{-3} (100)(500)$$

$$+ 2.14 \cdot 10^{-3}\, 2(135000 - 200 \cdot 500)$$

$$- 0.61 \cdot 10^{-3} (100)(500)$$

$$- 2.86 \cdot 10^{-3} (200)(500)$$

$$\boxed{e_x = -197.2 \text{ mm}}$$