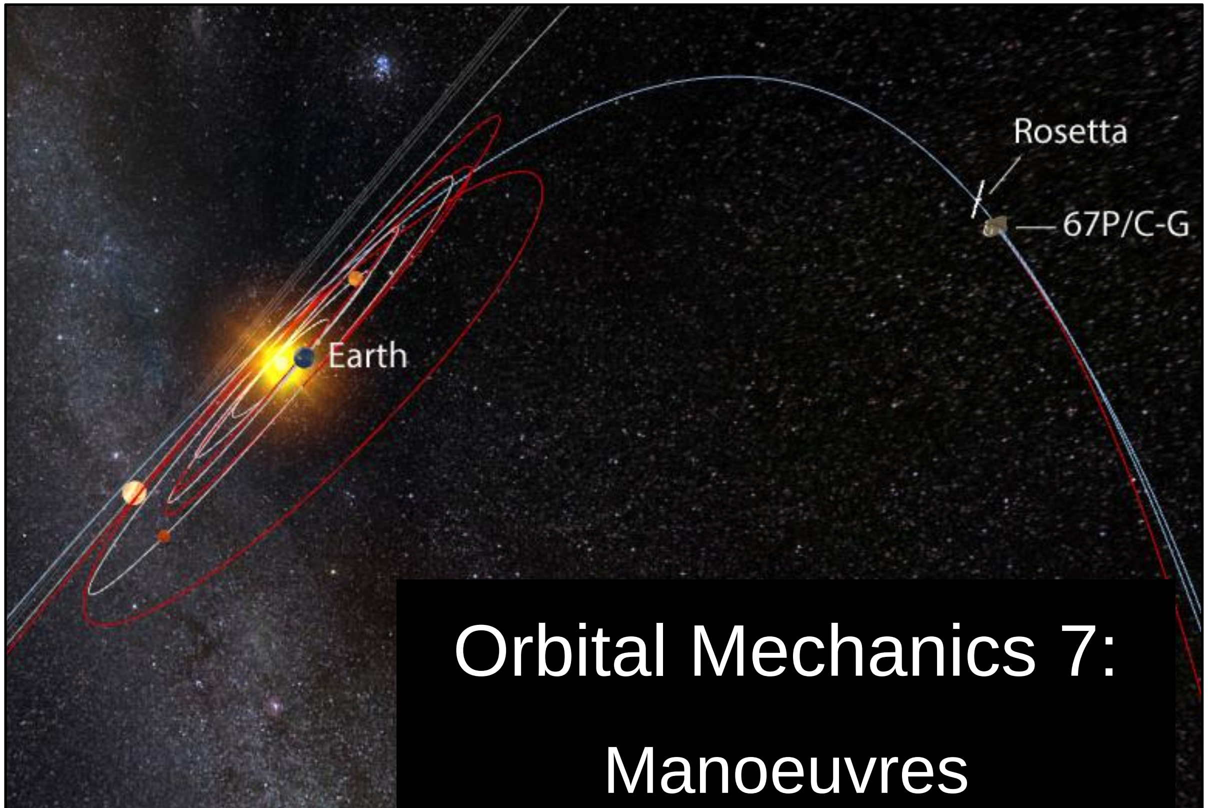

Orbital Mechanics 7: Manoeuvres

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Reminder: Summary Orbits 6

1. We have derived the Vis-viva equation to calculate velocities for circular, elliptical, parabolic orbits:

   ellipses parabola

   $$\varepsilon = \frac{1}{2}v^2 - \frac{\mu}{r} = -\frac{\mu}{2a} \quad \text{or } 0$$

2. Kepler equation: $M = n(t - t_0) = E - esinE$

   $$tan\frac{\theta}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2} \qquad n = \sqrt{\frac{\mu}{a^3}}$$

3. Bonus delta V due to Earth's rotation is worth up to 0.5km/s
4. Launch losses can be due to gravity, drag and steering losses

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Reminder: The Vis-Viva Equation

- Given:

$$\varepsilon = \frac{1}{2}v^2 - \frac{\mu}{r} = -\frac{\mu}{2a}, \qquad (6\text{-}3)$$

Rearranging:

$$v = \sqrt{\mu\left[\frac{2}{r} - \frac{1}{a}\right]} \qquad (6\text{-}4)$$

- This is a very useful equation that enables us to quickly calculate "delta velocity budgets"

These are both different forms of the vis-viva. I find the second one more useful practically. But if you were asked in an interview or exam you could write either. Note that the velocity changes as you go around the orbit, but it wouldn't cost any delta V to stay in the orbit in this ideal case. In reality for an Earth orbiting satellite there are luni-solar perturbations, solar radiation pressure etc. which mean that staying in an orbit (station-keeping) requires a small amount of delta V to maintain its orbit.

Learning Objectives

1. General principles
2. Changes in energy due to altitude
3. 3 main manoeuvre types
4. Inclination changes
5. Hohmann transfer
6. Rendez-vous

1. Be familiar with general principles of manoeuvres in space
2. Calculate changes in energy due to altitude
3. Be familiar with the 3 main manoeuvre types
4. Be able to calculate Delta V for an inclination change
5. Be able to describe and calculate a Hohmann transfer
6. Be able to describe the process of rendez-vous

Orbital manoeuvres

1. Launch (and effect of Earth's rotation)

2. In-plane

Change in a.

3. Out-of-Plane

Change in i, Ω, ω

These lead to…

- Hohmann Transfer
- Fast transfer
- Spiral transfer
- Rendezvous
- Gravitational assist

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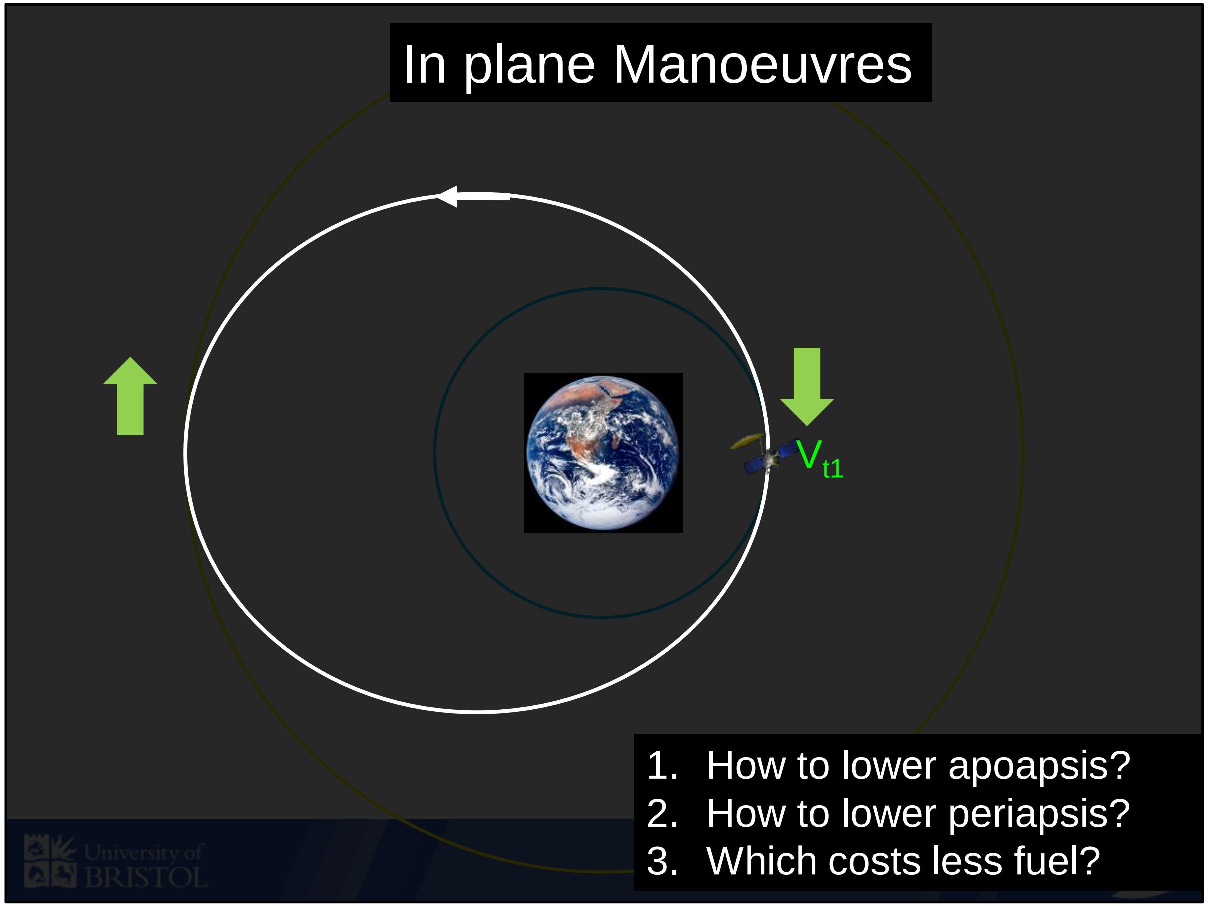
There are 3 main manoeuvre types: launch, in plane (change of size or a) and out of plane (change of inclination, Ω, ω). Knowledge of these enables us to perform a host of space manoeuvres including a Hohmann transfer, fast transfer (quicker than Hohmann), Spiral transfer (a low thrust transfer with electric propulsion), rendezvous and gravitational assists. You do not need to know anything about those in grey.

General Principles of Manoeuvres

1. Burning prograde (forwards) increases ‘a’,
2. Burning retrograde (backwards) decreases ‘a’.
3. Initial and final orbits intersect at point where impulse is applied
4. Need to get to right point at right time
5. Separate manoeuvres may be combined vectorially
6. All in-plane manoeuvres done at periapsis or apoapsis

It is necessary to either wait for the right conditions to initiate orbit transfer, or to perform a phasing manoeuvre, identical to the rendezvous manoeuvre, to get to the correct point at the correct time.
All manoeuvres take place at periapsis or apoapsis. An impulse at pericentre changes the apocentre distance; an impulse at apocentre changes the pericentre distance - i.e. to change the orbit geometry, you need to always think half an orbit ahead.



1. To lower apoapsis, burn retrograde at periapsis

Raise apoapsis, burn prograde at periapsis

2. Lower periapsis by burning retrograde at apoapsis

Raise periapsis by burning prograde at apoapsis

3. Burns at apoapsis cost less fuel.

Reminder: Orbital Energy

- Gravitational potential energy increases as distance increases
- Kinetic energy decreases as distance increases
- So, why do spacecraft in higher orbits need more energy?

$$E = K + U$$

$$\boxed{E = \frac{1}{2}mv^2 - \frac{GMm}{r}} = \frac{GMm}{2r} - \frac{GMm}{r} \qquad (3\text{-}5)$$

$$E = -\frac{GMm}{2r} \qquad (3\text{-}6)$$

U is defined as 0 at infinity, so gravitational energy is always negative

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The last equation means that if r is large then, as E is negative, E getting smaller means more energy ie: -1/4 > -1/2!

Changing Orbit Altitudes

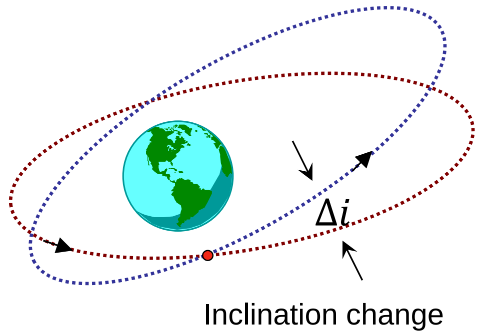
A news station wants to move their weather satellite (12000kg) from an orbit of 650km above the surface of the Earth to an 800km orbit above Earth's surface. How much energy will this procedure take?

$$E_1 = -\frac{GMm}{2r} = \frac{-(6.67x10^{-11})(5.98x10^{24})(12000)}{2(6.378x10^{6} + 6.5x10^{5})}$$

$$E_2 = \frac{-(6.67x10^{-11})(5.98x10^{24})(12000)}{2(6.378x10^{6} + 8x10^{5})}$$

$$E_2 - E_1 = \Delta E = 7.116x10^{9}\,J$$

Plane Change Manoeuvres

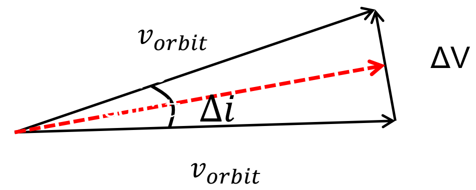


- Burn must take place where the initial and target planes intersect (the nodes).
- Even a small amount of plane change requires *lots* of ΔV
- Less ΔV required at higher altitudes (e.g., slower orbital velocities).
- Often combined with Hohmann transfer or rendezvous manoeuvre.

As inclination is measured at ascending node, we ensure that the two planes intersect at the ascending and descending nodes. If the craft is passing through the ascending node, where the angle is positive, it needs to burn antinormal (South from an equatorial orbit), while if it's passing through the descending node, and the angle is negative, it should burn normal (North from an equatorial orbit). Due to the lower delta V at higher altitudes, it can be more efficient to boost to a higher altitude, do the plane change and then drop back down to a lower altitude.

Plane Change Manoeuvre

- ‘Simple’/ ‘Pure’ Plane Change = no other elements change
- No ‘Hohmann component’ = magnitude of v remains constant



$$\Delta V = 2\,V \sin\left(\frac{\Delta i}{2}\right) \qquad (7\text{-}1)$$

What velocity penalty does an equatorial to polar change carry?

Simple/Pure plane change means that no other elements change.
No Hohmann component means that no combined manoeuvres are performed.
Maximum velocity penalty is 2V*sin45=1.414V=SQRT(2)V

Plane change numerical example

Question:
A spacecraft with an orbital velocity of 7km/s needs to make a simple plane change of 30°, how much delta V will it need?

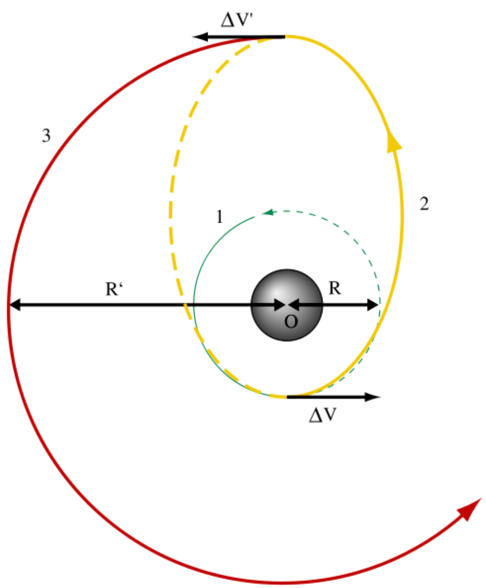
Answer:

$$\Delta V = 2\,V \sin\left(\frac{\Delta i}{2}\right) \qquad (7\text{-}1)$$

$\Delta V = 2 \times 7000 \times \sin(30 * \pi / 2 * 180)$

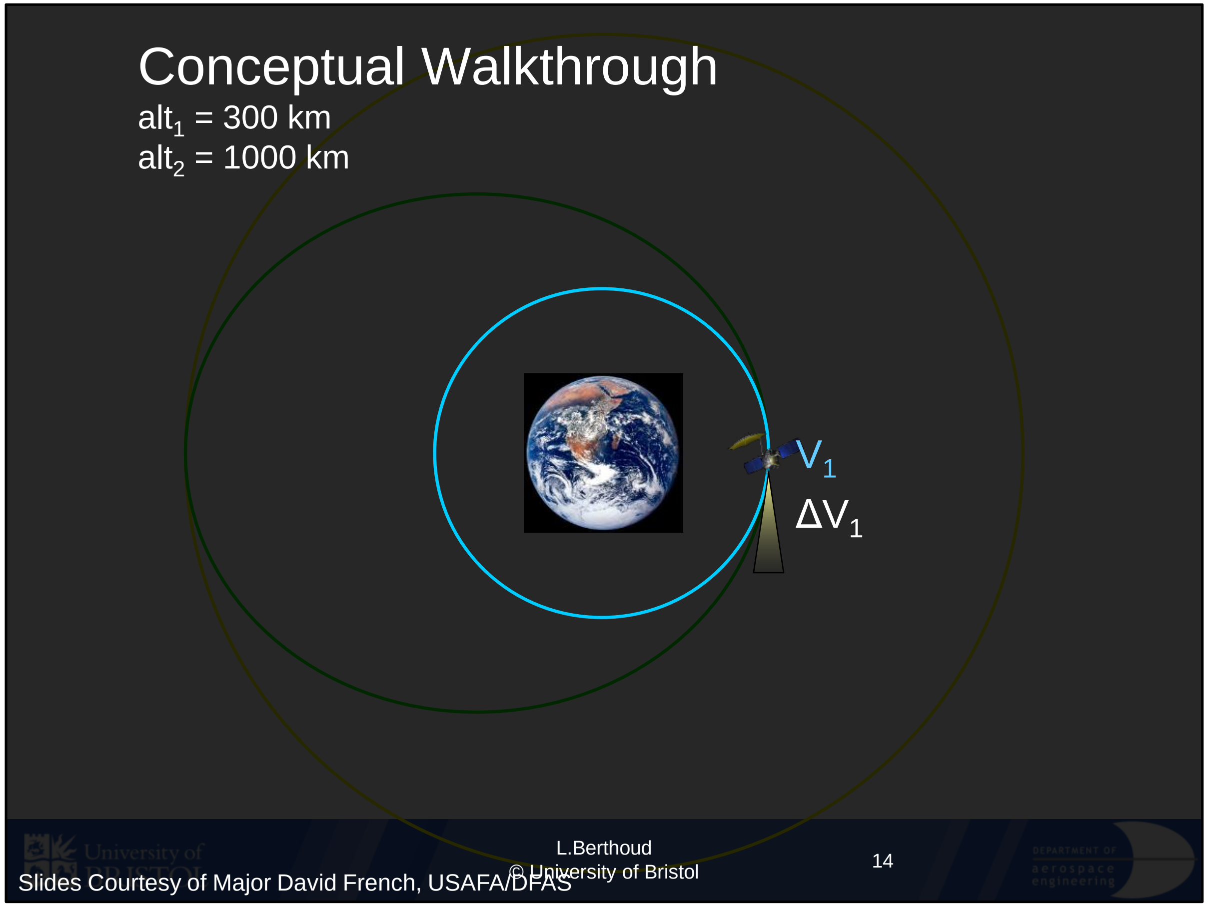
$\Delta V = 3623\text{m/s}$

Hohmann Transfer

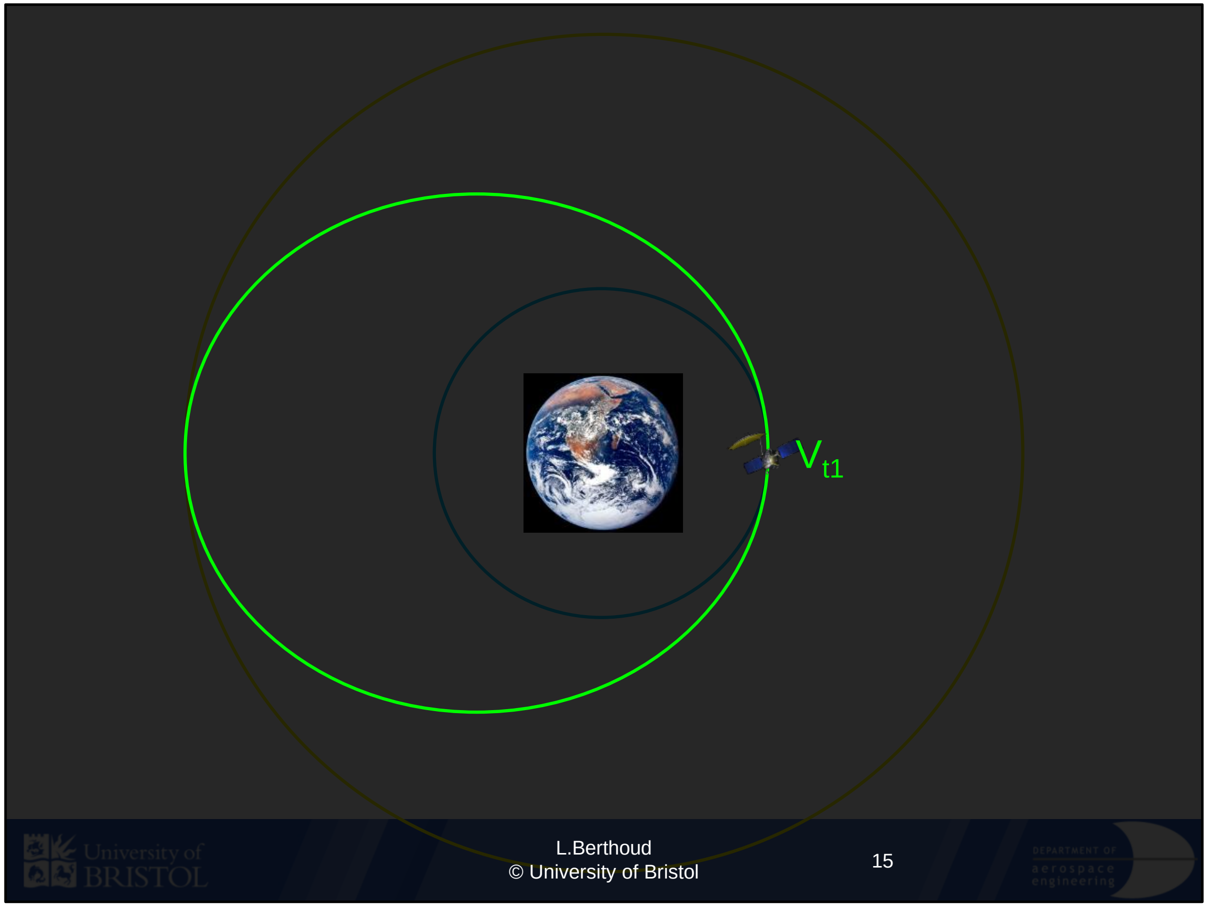


- Can be used to reduce or increase orbit altitudes.
- Used with coplanar circular orbits.
- There are 2 burns.
- 1st: at periapsis to increase apoapsis
- 2nd: at apoapsis, to circularise.

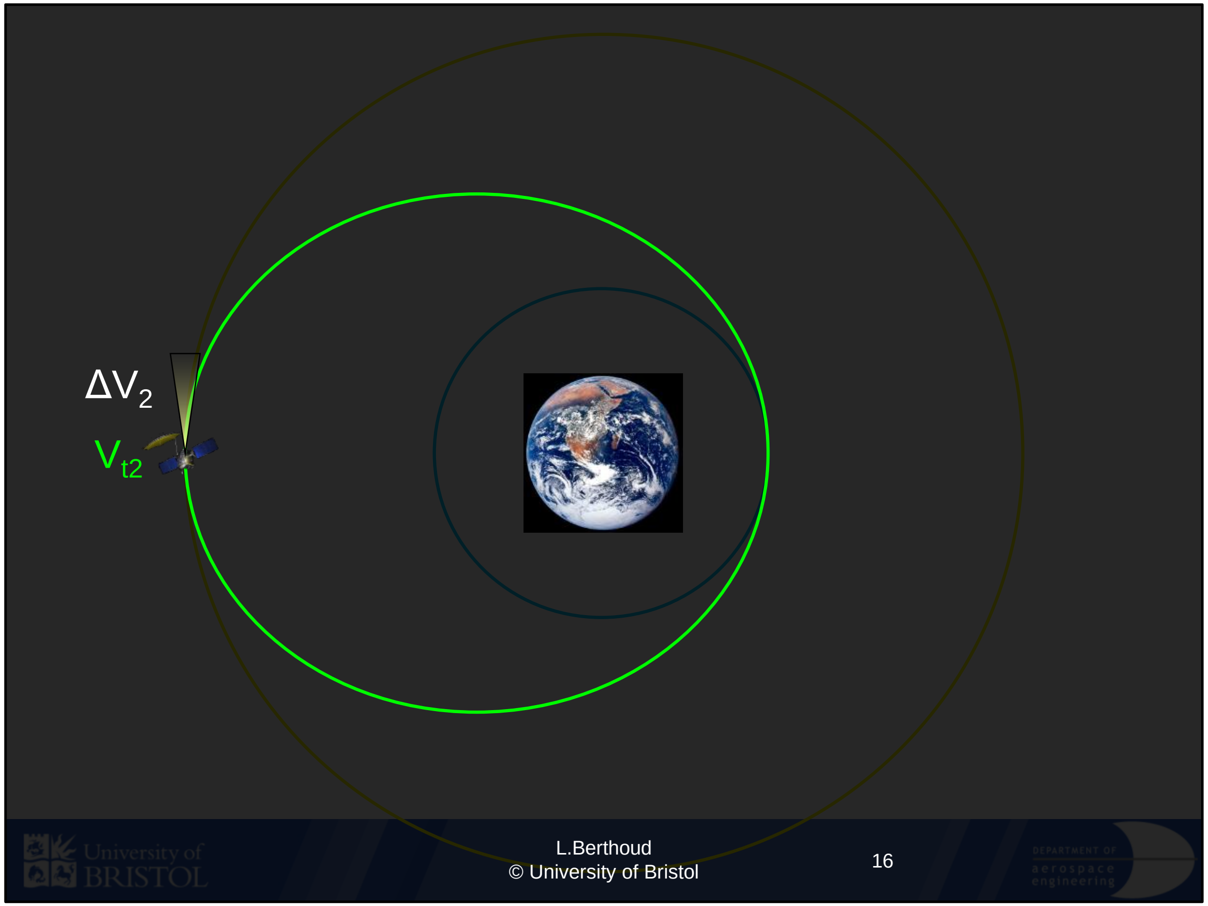
It is by far the most common orbital maneuver because it provides the minimum delta V to travel between different semimajor axes. It requires co-planar initial and ending orbits.



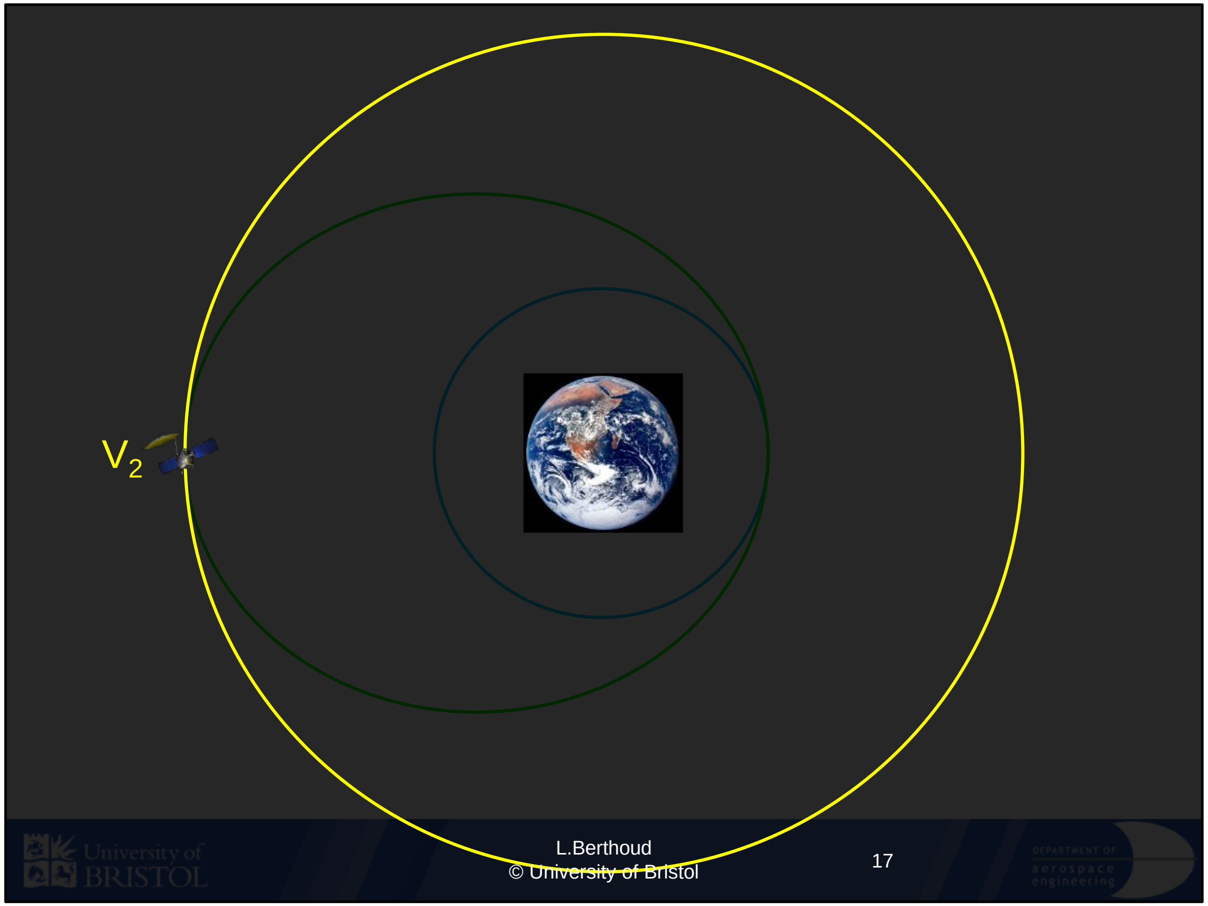
First burn delta V1 at what will become periapsis.



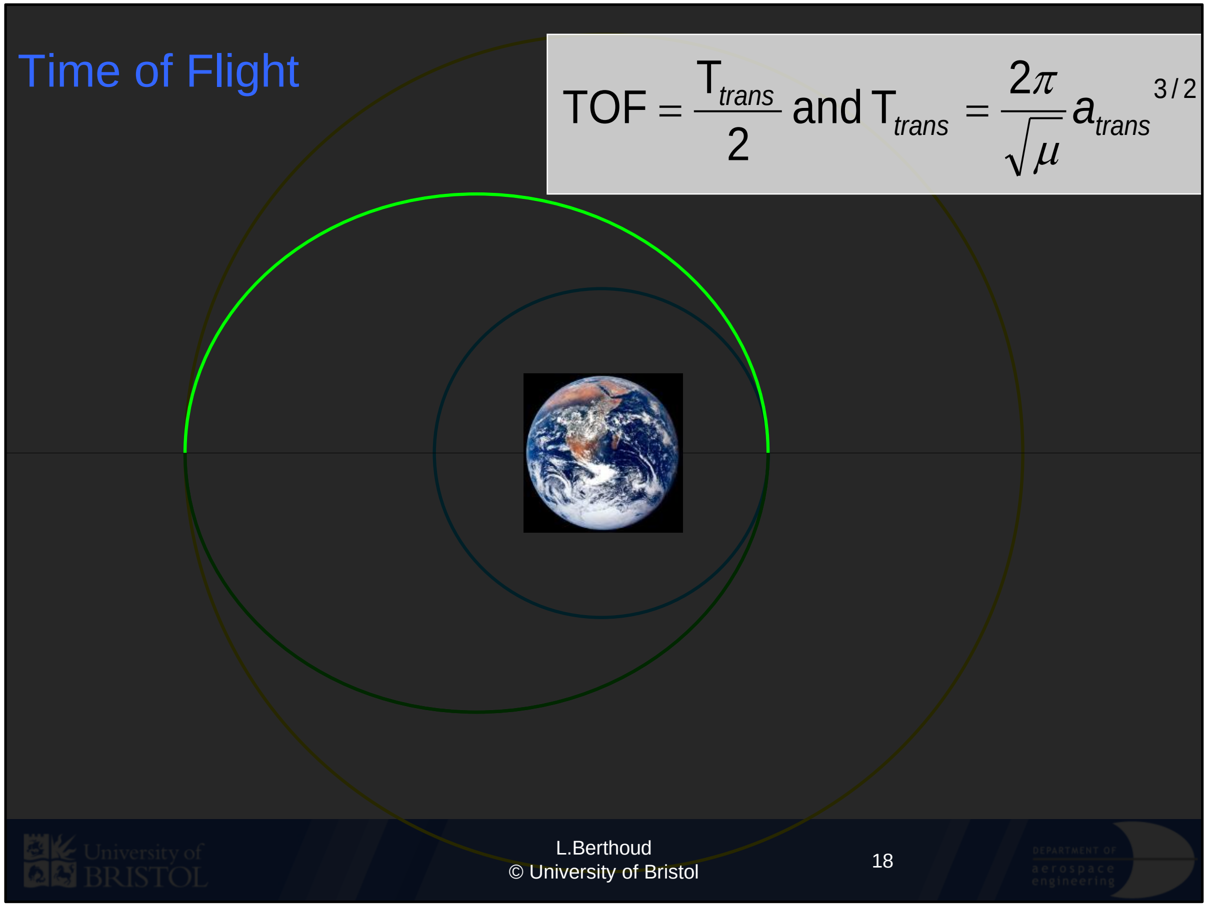
Delta V1 makes the orbit become more elliptical.



The second burn delta V2 at apoapsis will boost the periapsis up to the same altitude as the apoapsis.



So we end up with a circular orbit at the desired altitude.



The time of flight is half the period of the orbit. We can calculate the period from Kepler's 3rd law.

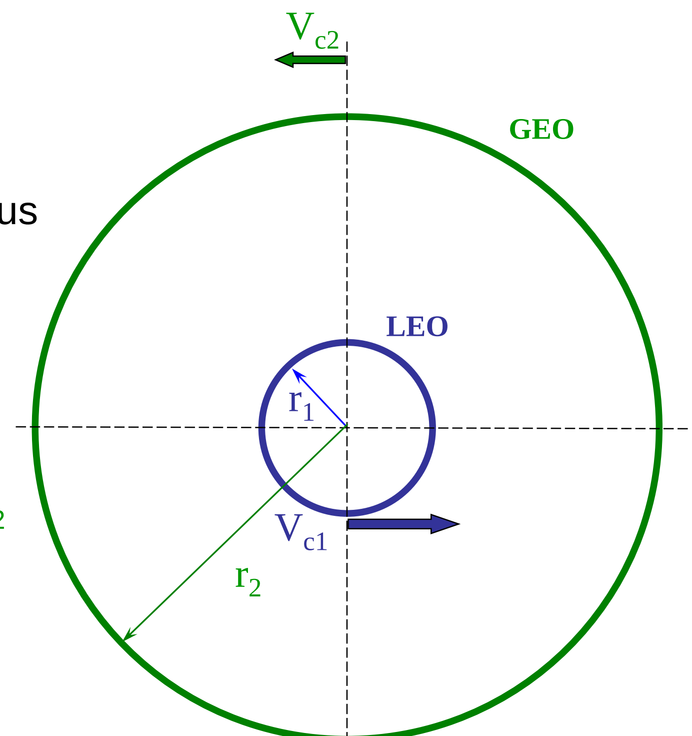
Hohmann transfer 1

- We want to move from LEO→GEO
- Initial LEO orbit has radius r_1 and velocity V_{c1}

$$V_{c1} = \sqrt{\frac{\mu}{r_1}}$$

- Desired GEO orbit has radius r_2 and velocity V_{c2}

$$V_{c2} = \sqrt{\frac{\mu}{r_2}}$$



Here are the calculations broken down step by step.
It is useful to think of the velocities in terms of energy, ie: Vc2 will have greater energy than Vc1

Hohmann transfer 2

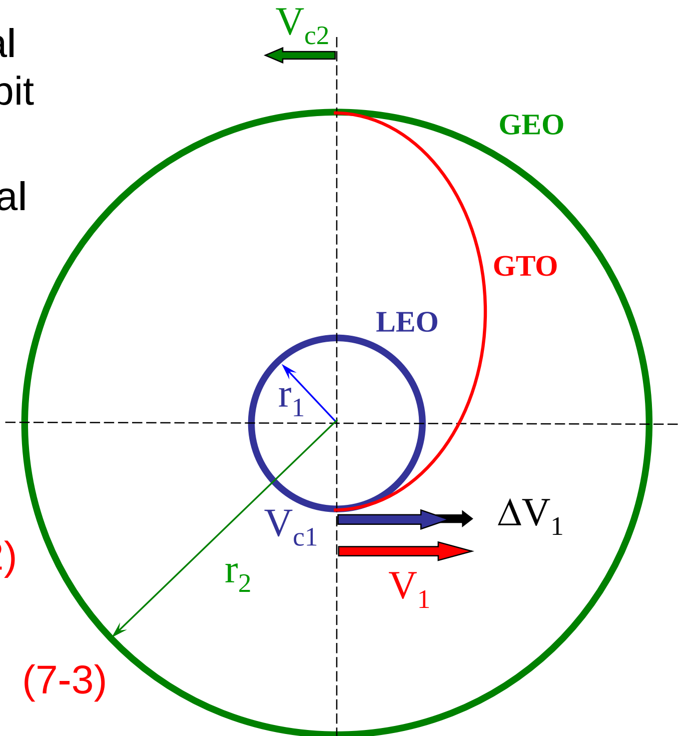
- Impulsive ΔV_1 for elliptical geostationary transfer orbit (GTO) at perigee:
- Leave LEO (r_1) with a total velocity of V_1. We have:

$$a = \frac{r_1 + r_2}{2}$$

$$V_1 = \sqrt{\mu \left[\frac{2}{r_1} - \frac{1}{a} \right]}$$

$$\Delta V_1 = V_1 - V_{c1} \qquad (7\text{-}2)$$

$$\Delta V_1 = \sqrt{\mu \left[\frac{2}{r_1} - \frac{1}{a} \right]} - \sqrt{\frac{\mu}{r_1}} \qquad (7\text{-}3)$$



Delta V1 (black), Vc1 (blue)

Hohmann transfer 3

- Arrive at GEO with V_2

$$V_2 = \sqrt{\mu\left[\frac{2}{r_2} - \frac{1}{a}\right]}$$

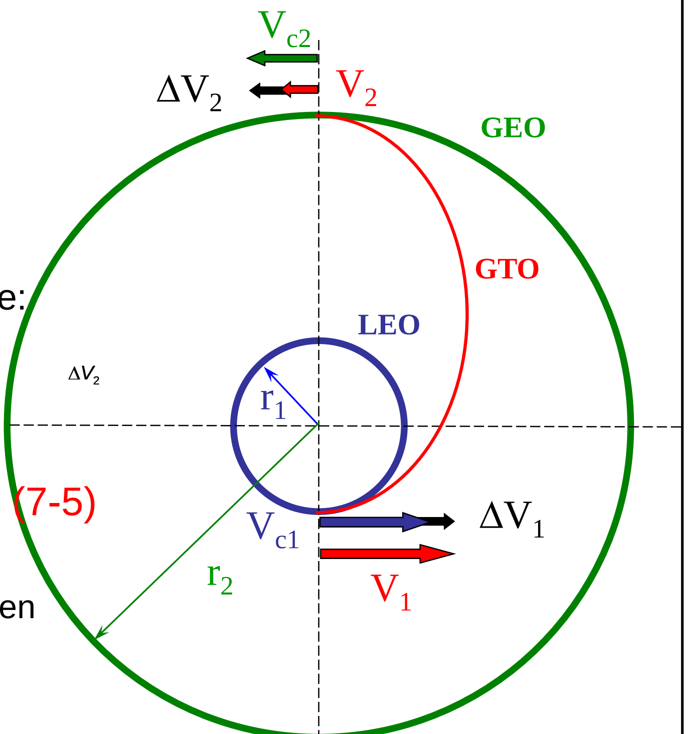
When arriving at GEO, apply some ΔV_2 in order to circularize:

$$\Delta V_2 = V_{c2} - V_2 \qquad (7\text{-}4)$$

$$\Delta V_2 = \sqrt{\frac{\mu}{r_2}} - \sqrt{\mu\left[\frac{2}{r_2} - \frac{1}{a}\right]} \qquad (7\text{-}5)$$

- The total Hohmann ΔV will then be:

$$\Delta V = \Delta V_1 + \Delta V_2 \qquad (7\text{-}6)$$



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It is most fuel efficient because the transfers are tangential, any orbit transfers which are not tangential require additional propulsion to realign the orbit path.

Numerical example 1: LEO 622km to GEO

1. First calculate 'a' for transfer ellipse:

$$a = \frac{r_1 + r_2}{2} = \frac{7x10^6 + 42x10^6}{2} = 24.5x10^6 ms^{-1}$$

2. Next calculate V_{c1} and V_{c2}:

$$V_{c1} = \sqrt{\frac{\mu}{r_1}} = 7546ms^{-1} \qquad V_{c2} = \sqrt{\frac{\mu}{r_2}} = 3080ms^{-1}$$

3. Then calculate V_1:

$$V_1 = \sqrt{\frac{2\mu}{r_1} - \frac{\mu}{a}} = \sqrt{\frac{2x3.986e14}{7e6} - \frac{3.986e14}{24.5e6}}$$

$$V_1 = 9883m/s$$

μ for Earth = 3.986x10^{14}m^3/s^2

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Assuming μ for Earth = 3.9886x10^{14}m^3/s^2

Numerical example – LEO to GEO 2

4. Then calculate V_2:

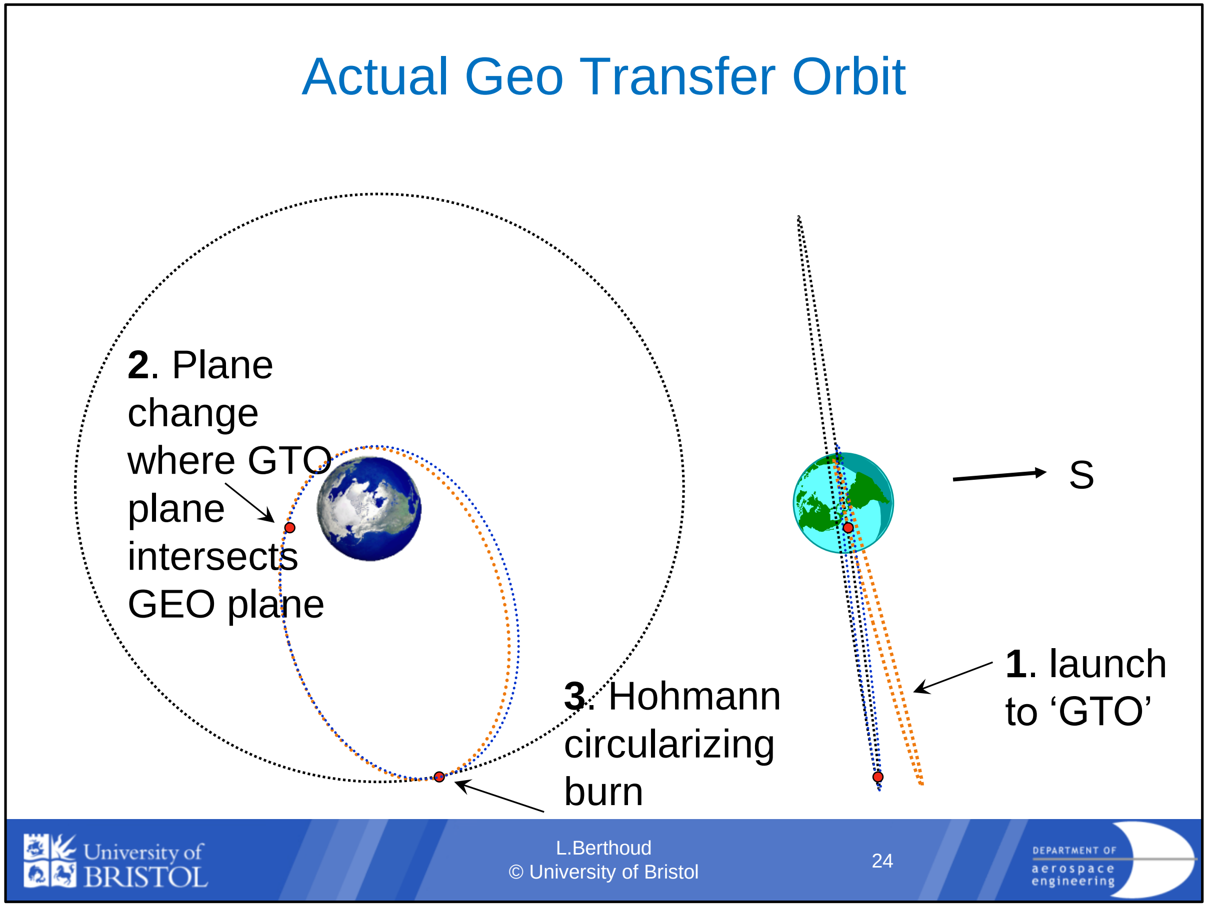
$$V_2 = \sqrt{\frac{2\mu}{r_2} - \frac{\mu}{a}} = \sqrt{\frac{2x3.986e14}{42e6} - \frac{3.986e14}{24.6e6}}$$

$$V_2 = 1667 m/s$$

5. Then calculate $\Delta V_1 = V_1 - V_{c1} = 9883 - 7546 = 2337 m/s$ (7-2)

6. Then calculate $\Delta V_2 = V_{c2} - V_2 = 3080 - 1667 = 1413 m/s$ (7-4)

7. Total $\mathbf{\Delta V = \Delta V_1 + \Delta V_2 = 3750 m/s}$



1. In brown orbit when it is launched to the GTO
2. The plane is changed at one of the nodes
3. The burn is circularized at apogee.

So a launch into GTO always requires a plane change as well as a Hohmann.

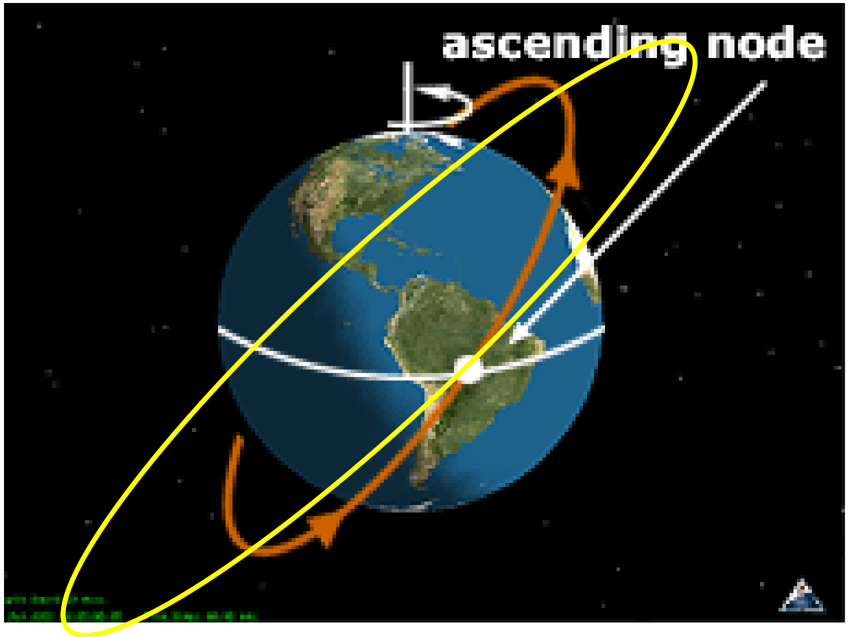
Less plane changing is required when launched from near the equator.

Rendezvous Manoeuvres

1. Launch into an orbit with similar plane
2. Match inclination at nodes
3. Move apoapsis of your orbit to target orbit
4. Change periapsis to match target orbit. You should now be in same orbit but out of phase.
5. Use Hohmann to change phase (coorbital rendezvous)
6. Last 50m done at very low velocities (1-5m/s)

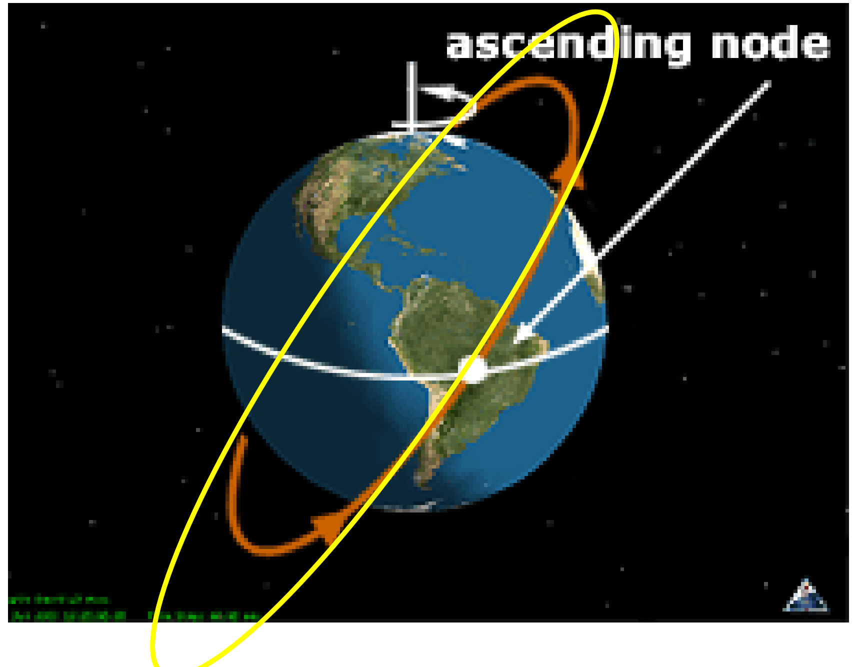
After number 5, relative velocities need to be almost zero and position separated by less than a km. This applies to all rendezvous including getting into orbit around planets and moons.

1. Launch into similar plane



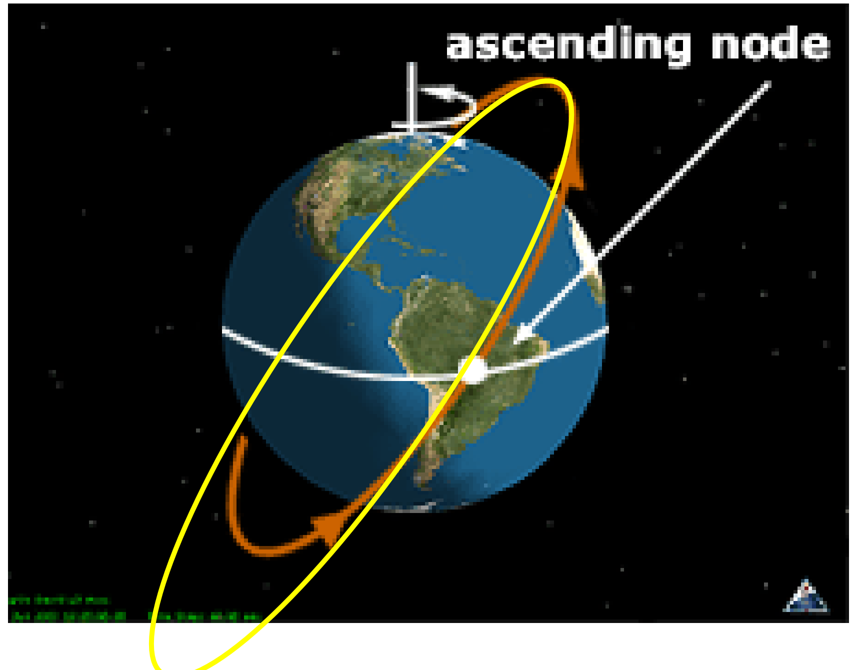
Yellow plane is the one we have launched into, and we are trying to get into the orange plane.
First we try to launch into an orbit with a similar plane, then we try and match inclination at the nodes.

2. Match inclination at nodes



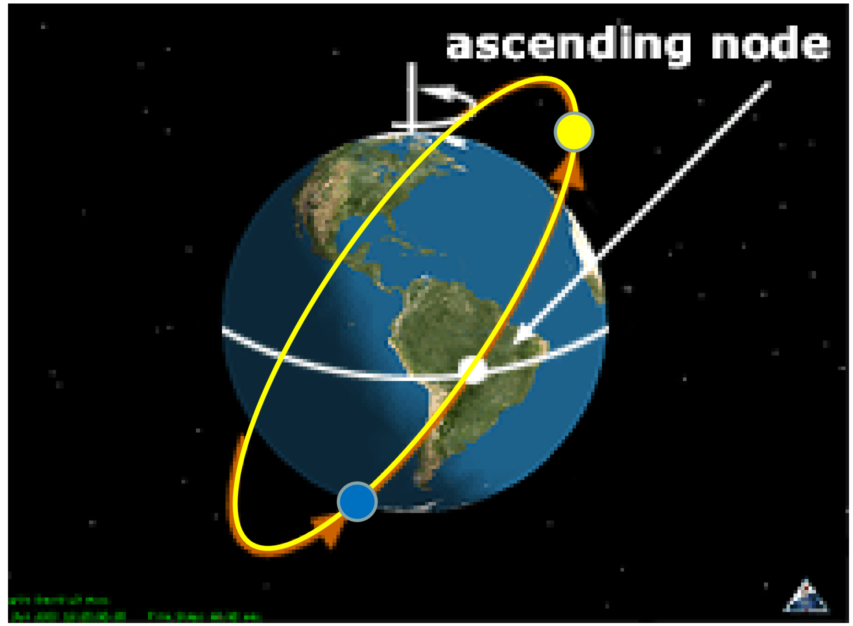
Here we have matched the inclination at one of the nodes.

3. Match apoapsis

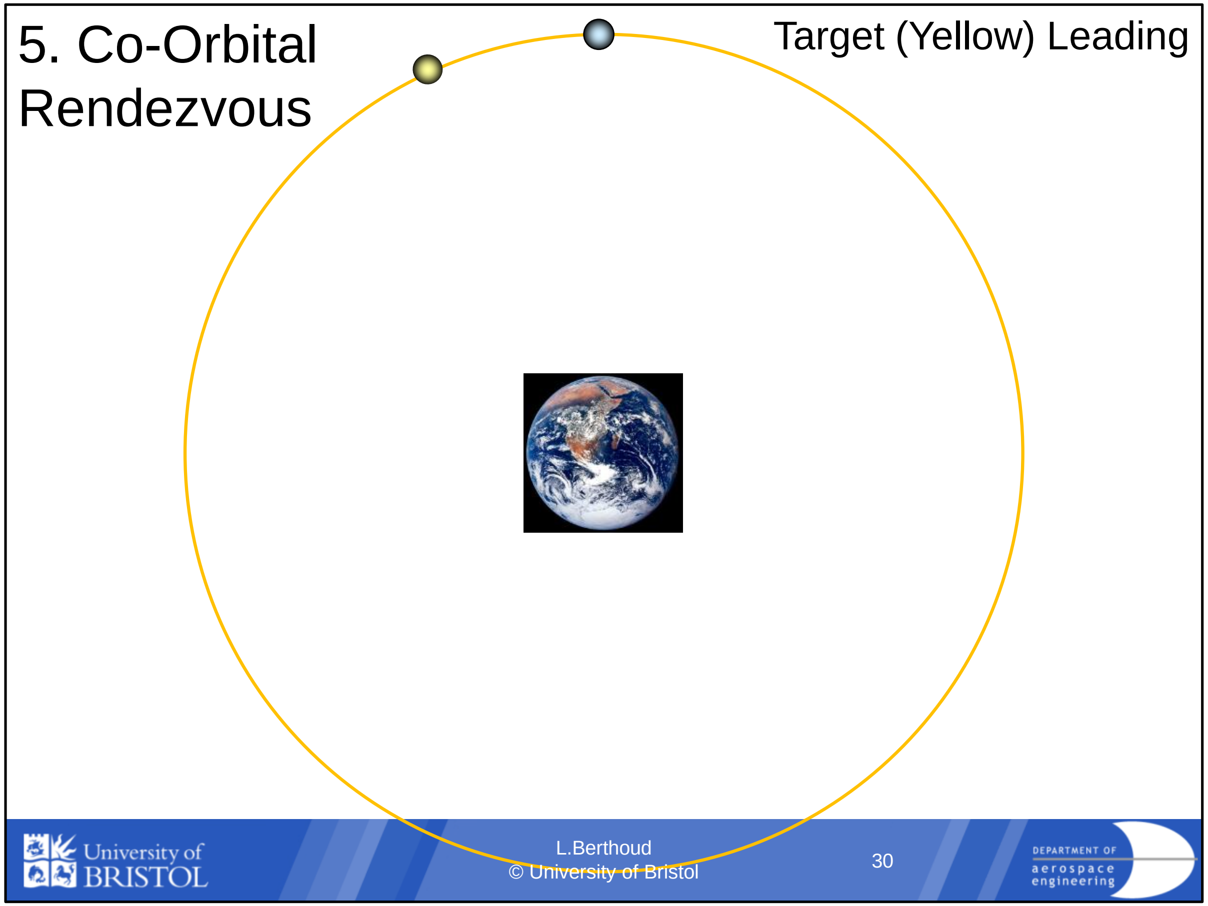


Then we match apoapsis. [These orbits look circular but let’s imagine they are ellipses.]

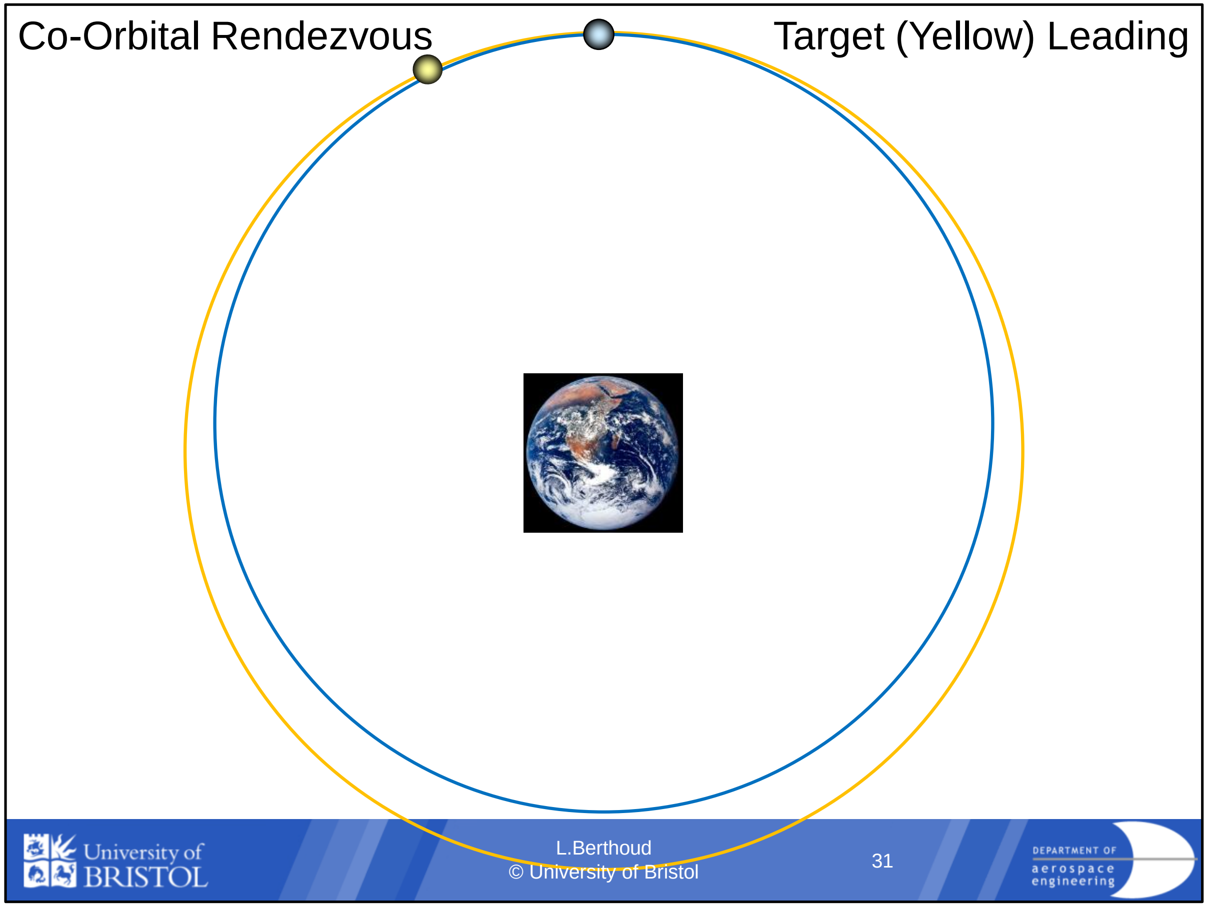
4. Match periapsis



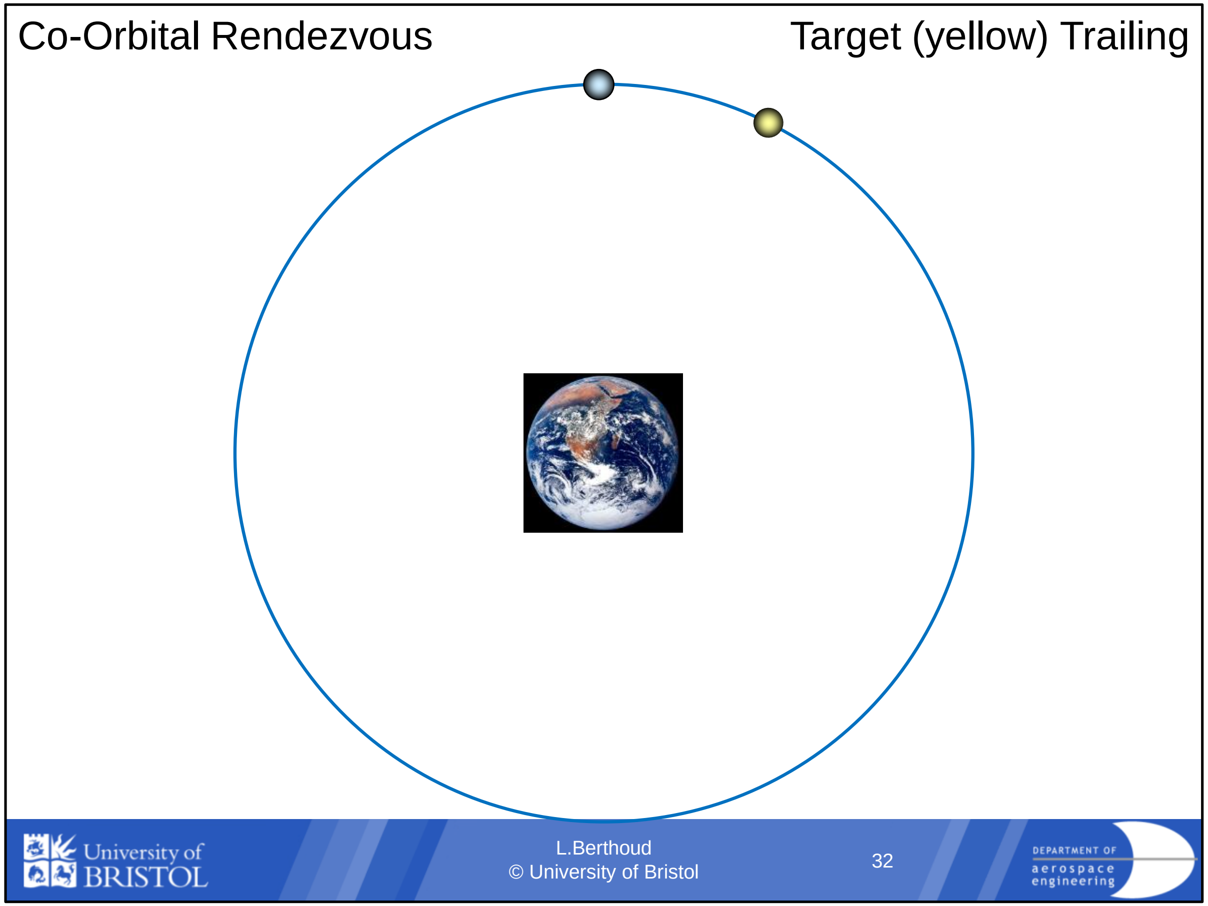
Then we match the periapsis. Our target is yellow and our chaser is blue. They are 'out of phase'. How does the spacecraft catch up with its target?



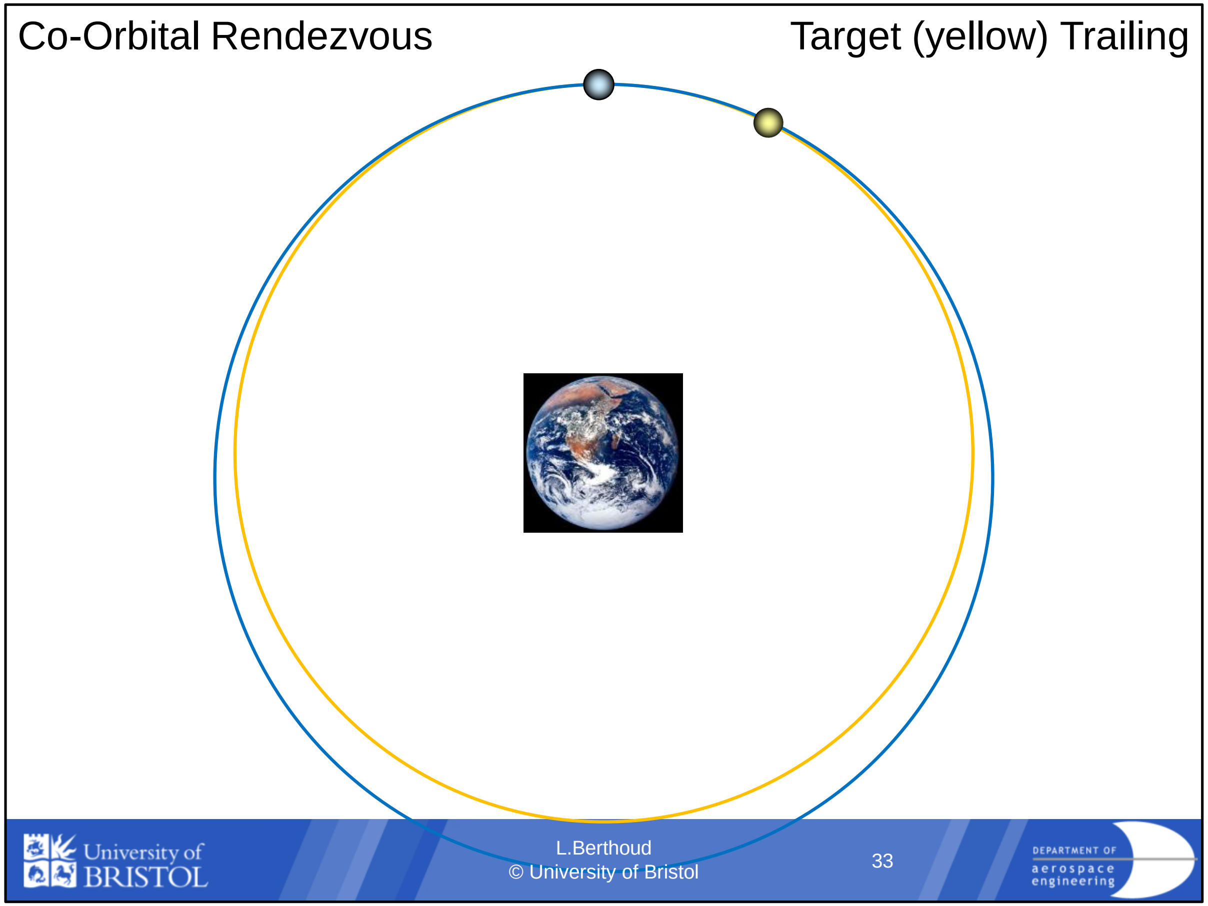
In this example the target eg: ISS is ahead of the supply spacecraft in the same plane with the same semimajor axis.
Launch when the orbital plane of the target vehicle crosses launch pad.
(Ideally) launch as the target vehicle passes straight overhead.
Course maneuvers designed to arrive in the same orbit at the same true anomaly.



Surprisingly we do not accelerate to catch up! The blue chaser retroburns to get into a smaller orbit. Smaller transfer orbits slowly overtake target (because of shorter orbit periods) and over the space of the orbit the blue chaser will gradually catch up with the target.



Now the yellow target is trailing behind the blue supply ship. What must it do to catch up?



We now need to slow the blue supply ship down, so we boost its apoapsis by burning prograde so that the period lengthens and the blue target and yellow chaser end up in the same place at the same time. This is called a ‘phasing manoeuvre’.

Example: Soyuz catching up with ISS

- Inclination burn to change plane
- 2 retroburns to increase velocity and lower ‘a’
- Catch up over several orbits
- 2 prograde burns to decrease velocity and raise ‘a’.

- Link to video

You will see this video in Lab II.

Summary of lecture

1. General principles of manoeuvres in space
2. 3 main manoeuvre types: launch, in plane change, out of plane change
3. ΔV for an inclination change: $\Delta V = 2\,V\sin\left(\frac{\Delta i}{2}\right)$
4. A Hohmann transfer uses 2 burns: $\Delta V_1 = V_1 - V_{c1}$
   $\Delta V_2 = V_{c2} - V_2$

4. Rendez-vous process: match inclination, move apoapsis and periapsis, phasing manoeuvre, slow visual approach.

Test Yourself! (Feedback)

1. After launch, what is the first step for a supply ship to rendezvous with a space station?
2. Describe how a supply ship catches up with a target space station which is trailing it? What is this manoeuvre called?
3. Where do you do an inclination burn in an elliptical orbit? Why?