Lecture 2 - Axes, Notation and Conventions

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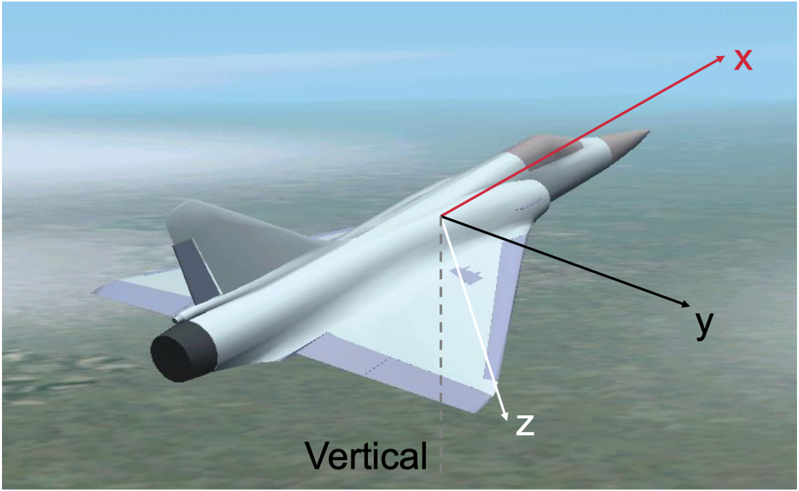
Reference Axes

Reference Axes

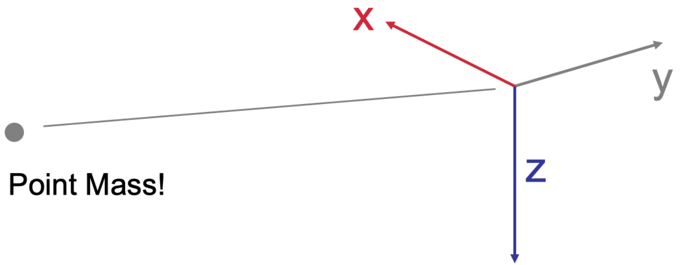


Reference Axes

- Body Axes
- Earth Axes
- Wind Axes
- Stability Axes

Reference Axes

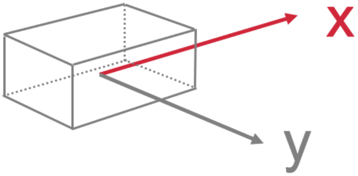
- Movements of a point mass do not require a local set of axes attached to the mass.
- Movements of a rigid body having distributed mass do require a local set attached to the body with the second set for reference.



Reference Axes

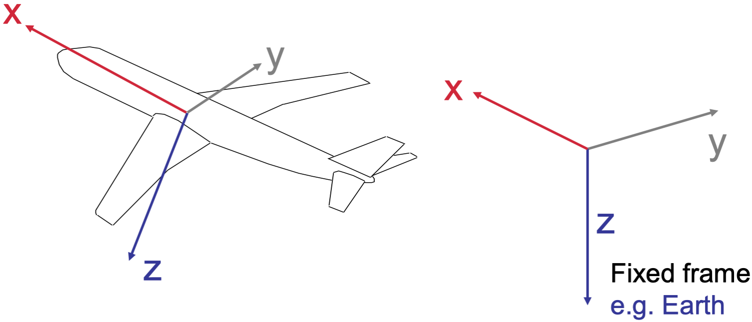
The need for reference axes arises from:

- Any use of rotational inertias implies distribution of mass.
- The development of equations that include rotational inertias will require axes in order to define them.



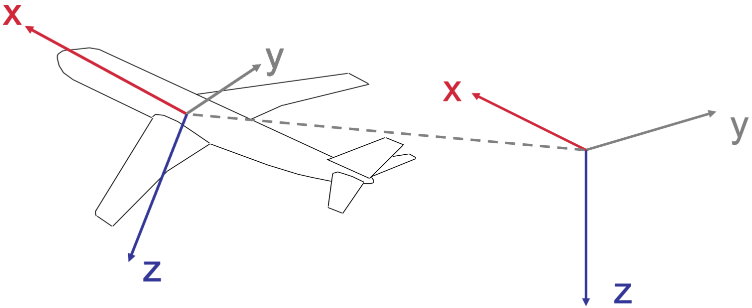
Reference Axes

The six motions of a rigid body - three translational, three rotational - are all defined *relative* to a second fixed frame.



Reference Axes

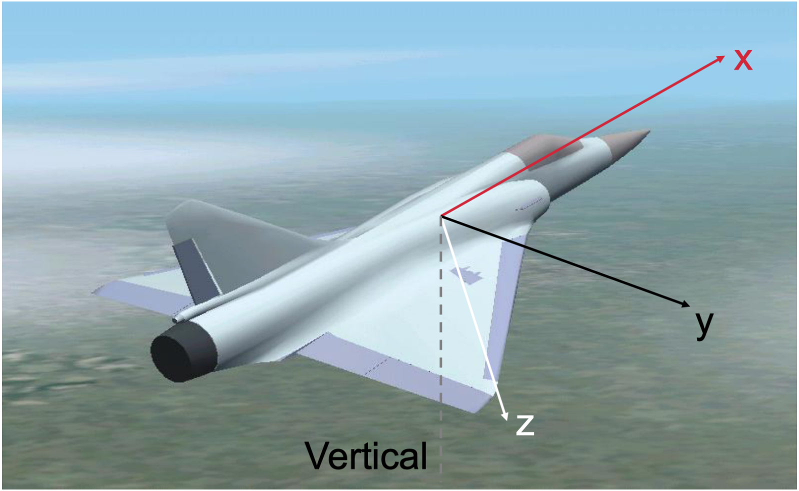
All angular motions are measured from the reference set to the local set thereby giving motion relative to where the local axes started, e.g. a local airport



Body Axes

- We need to be able to define the forces acting on the body.
- Most of our "external" forces will be of aerodynamic origin so we have to bear in mind the source of these.
- Hence we need a convenient set of orthogonal axes fixed in the vehicle; then we need to know where the wind vector is, relative to these in order to express the aerodynamic forces mathematically.

Body Axes



Body Axes

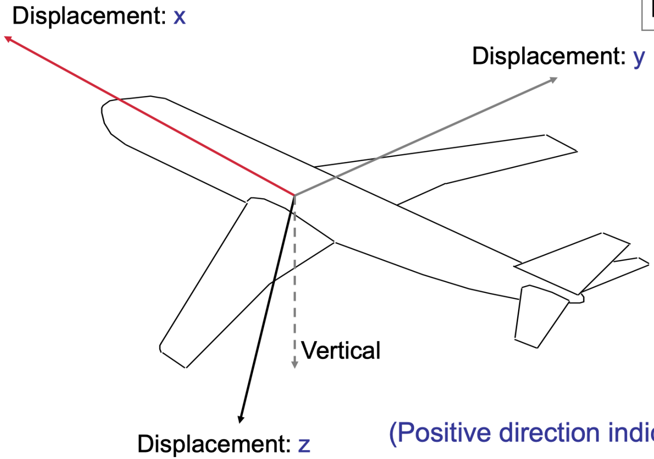
- Body axes (the set fixed to the body) are attached with the orientation being chosen for the convenience of defining inertias, especially rotational inertias.
- The CG could be chosen for the origin of the Body axes but the CG position may vary and so some other fixed but easily defined point is usually selected.

Body Axes

- The x-axis will be directed forward, usually along the fuselage centre-line (or at least parallel to the fuselage reference).
- The y-axis will be laterally outward (positive to starboard) and perpendicular to the aircraft's plane of symmetry, i.e. not likely to lie fully within the wing though the inner portion will be close to the wing.
- The z-axis is nominally down (perpendicular to both x and y).

Body Axes Notation and Sign Conventions

LEARN!



(Positive direction indicated)

Body Axes Notation and Sign Conventions

Sets!

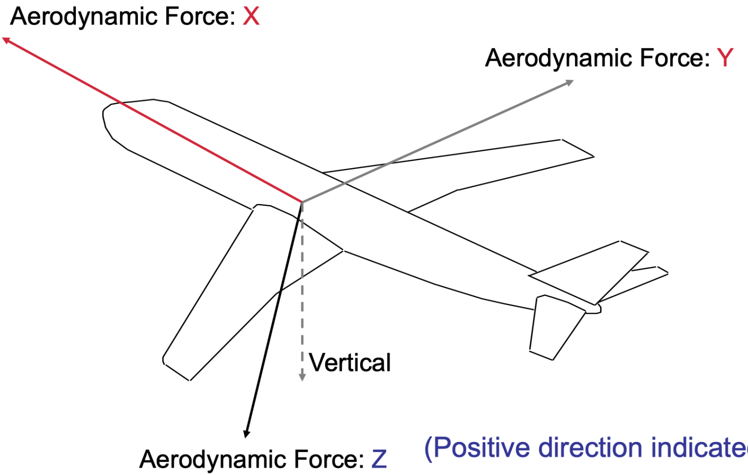
$x \longrightarrow y \longrightarrow z$

roll $\longrightarrow$ pitch $\longrightarrow$ yaw

$u \longrightarrow v \longrightarrow w$

$\phi \longrightarrow \theta \longrightarrow \psi$

Body Axes Notation and Sign Conventions



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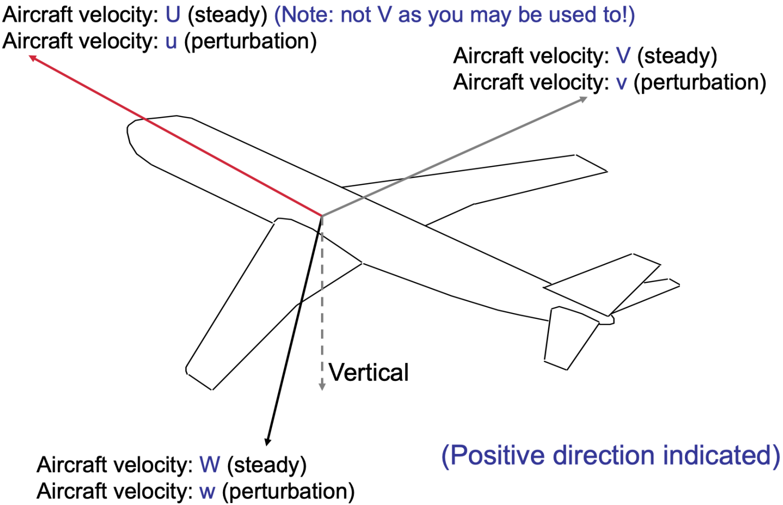
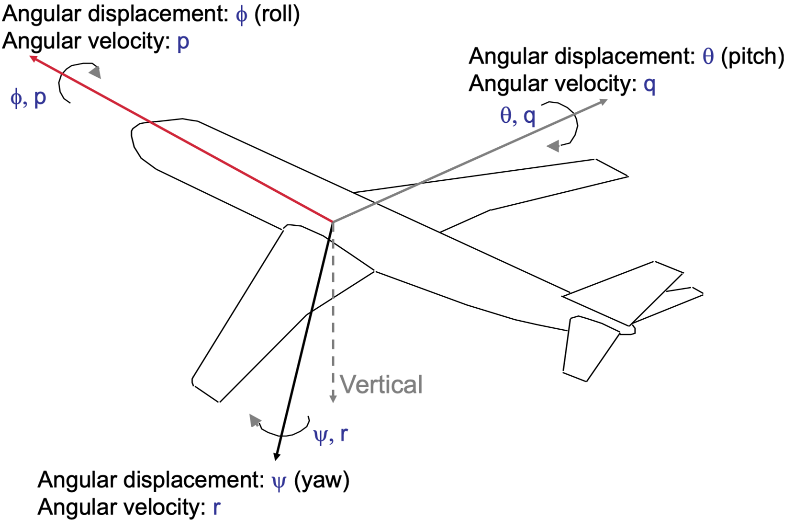
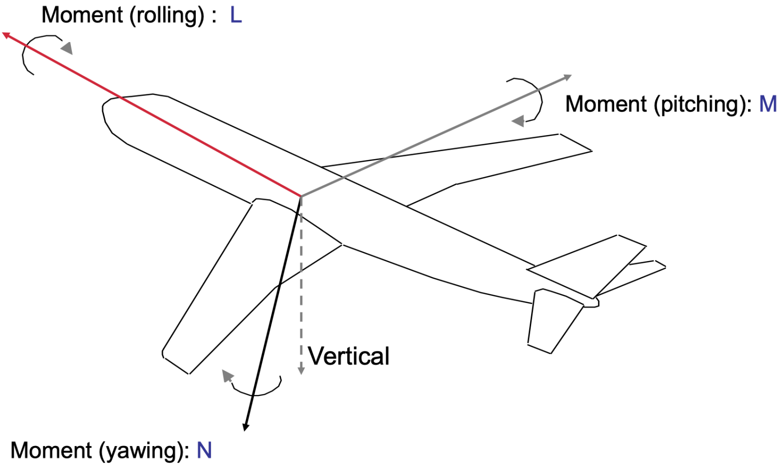
Body Axes Notation and Sign Conventions



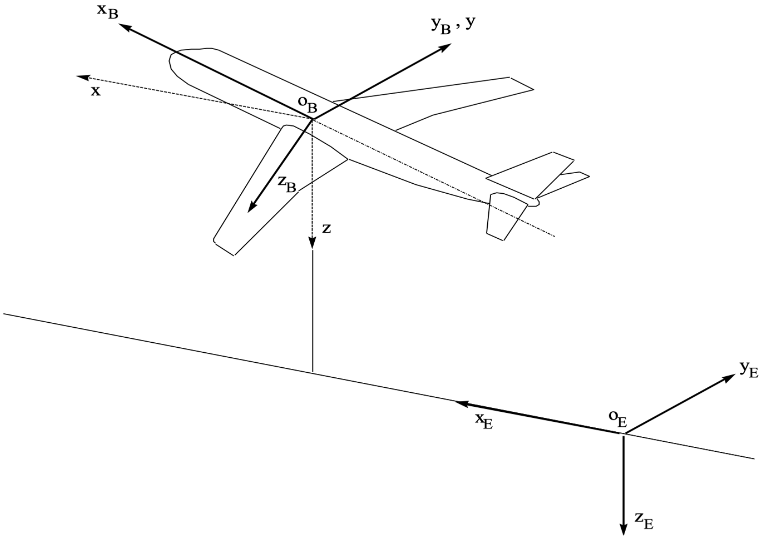
Body Axes Notation and Sign Conventions



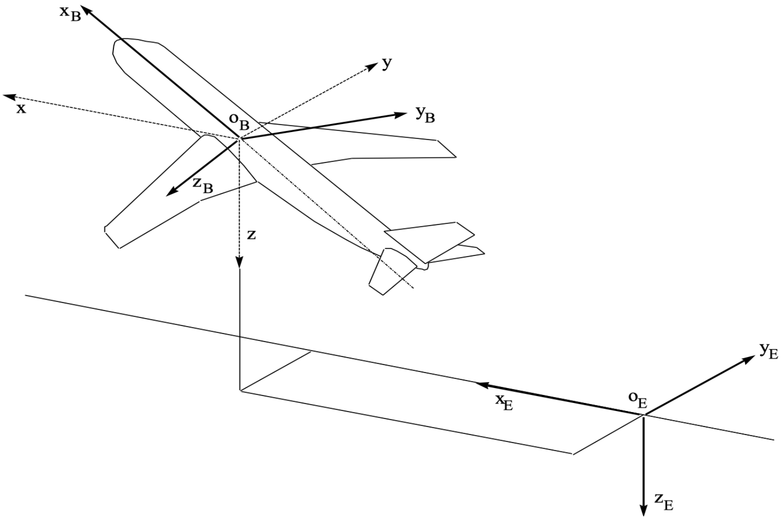
Body Axes Notation and Sign Conventions



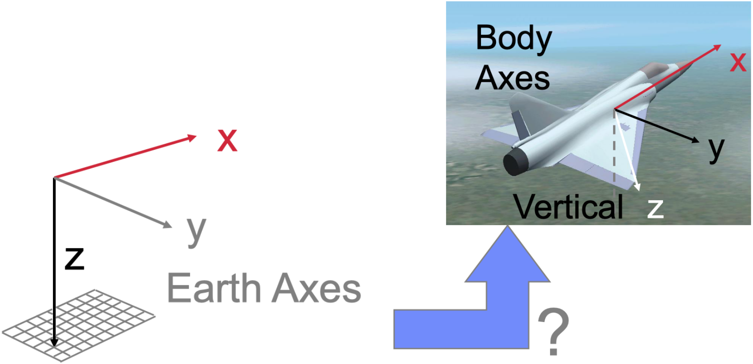
Longitudinal Flight



General Case



Reference Axes

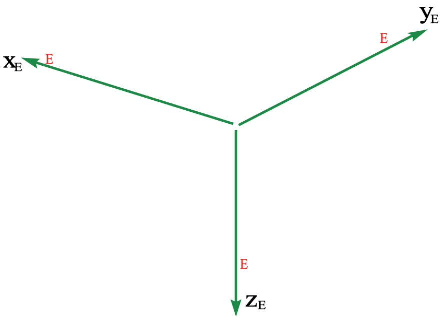


The Euler Angles

The following diagrams show four reference frames:

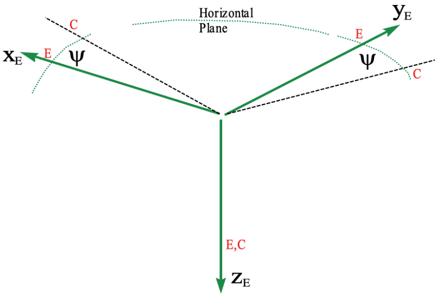
1. Earth Axes Frame F_E
2. Intermediate Frame F_C
3. Intermediate Frame F_D
4. Body Axes Frame F_B

The Euler Angles



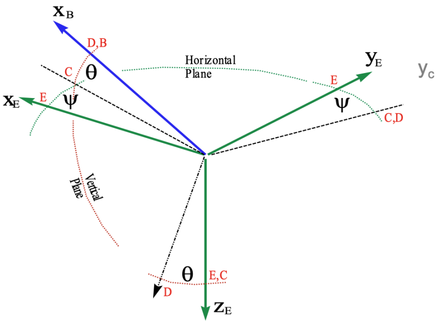
Earth Axes

The Euler Angles



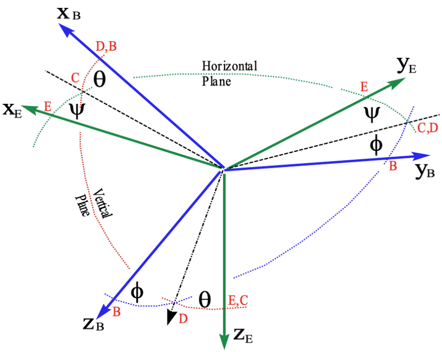
1. Rotation ψ about z_E to Intermediate frame F_C

The Euler Angles



2. Rotation θ about y_C to Intermediate frame F_D

The Euler Angles



3. Rotation ϕ about x_B to Intermediate frame F_B

The Euler Angles

The sequence of rotations is:

1. Rotation ψ about z_E to Intermediate frame F_C
2. Rotation θ about y_C to Intermediate frame F_D
3. Rotation ϕ about x_B to Intermediate frame F_B

check...

Wind Axes

- The orientation of the wind axes is usually chosen for the convenience of defining aerodynamic forces.
- As with the body axes, the origin is fixed at the aircraft reference point (probably different from the CG), but its x-axis points into the wind.

Aerodynamic incidence angles α and β

- The usual incidence angle α probably needs no further description - first year.
- You need be clear on the distinction between the two angles:

  ψ - yaw angle (psi)

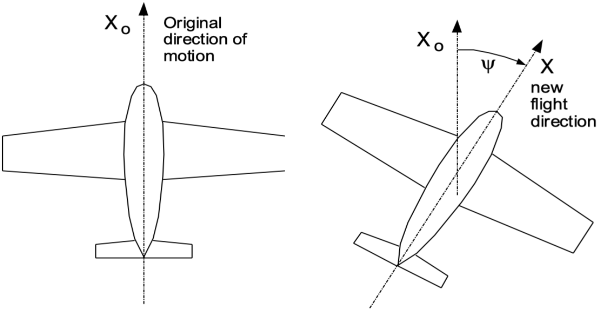
  and

  β - sideslip angle (beta)

  for it is the two incidence angles α and β on which aerodynamic forces are based, not ψ.

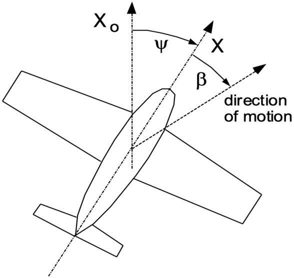
Sideslip angle β

Essentially, ψ is an orientation angle which indicates a pointing direction for the fuselage reference line compared with a previous direction, perhaps relative to a fixed set of fixed earth axes.



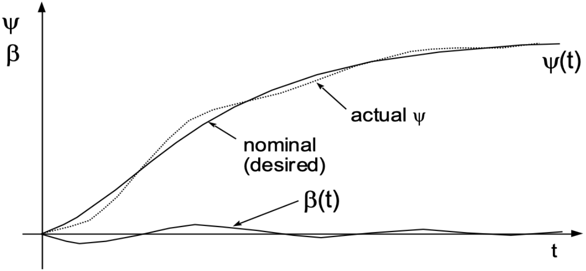
Sideslip angle β

- Sideslip implies that the instantaneous wind is not approaching from straight ahead, i.e. the motion of the aircraft is not in the direction of its own longitudinal centre-line.
- If the aircraft is apparently "skidding" to the right there is positive sideslip, and the sideslip incidence is given by $\beta = v/U$ (positive v gives positive β). This can be true for zero yaw angle, $\psi = 0$.



Sideslip angle β

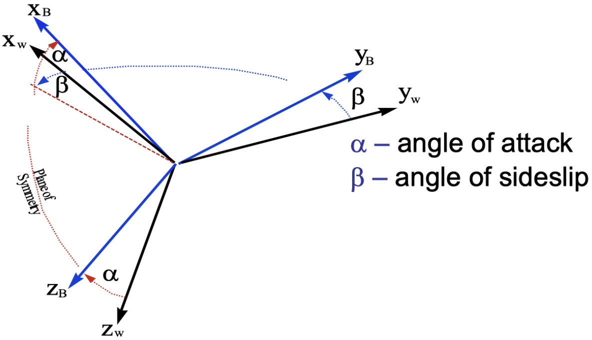
Small yawing oscillations of the aircraft during turning will cause $\psi(t)$ and $\beta(t)$ to be quite separate functions, as shown below, with $\psi(t)$ tending to a new steady positive value whereas $\beta(t)$ could be nearly zero throughout - i.e. a coordinated turn.



Wind axes

- If the atmosphere is steady and not moving relative to the earth then the axes are at a tangent to the flight trajectory of the aircraft relative to earth.
- The other constraint on this set of axes is that the z wind-axis remains in the aircraft plane of symmetry though it may tilt forward.
- Only pitch and yaw departures of the wind vector are necessary to define wind-induced forces.

Wind axes



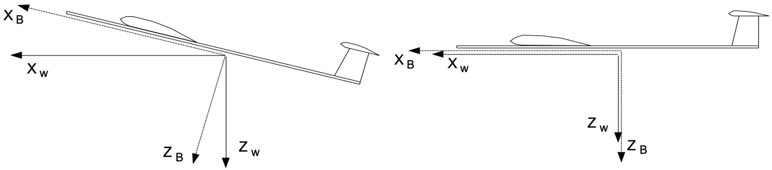
- α - is the angle between the x_B axis and the plane containing the y_B axis and the velocity vector.
- β - is the angle between the velocity vector and the plane of symmetry(x_B, z_B)

Stability Axes

- The relative motion of the body axes away from the wind axes is useful when defining a disturbance, or a response to a control deflection.
- It is convenient to have the two sets coincident just before the start of any study of the dynamics following a disturbance.

Stability Axes

- Think of two situations, low-speed (high incidence) and high-speed (low incidence) which suggest (exaggerated):



Stability Axes

- Clearly, at best there would be only one pair of values for pitch attitude and speed for which the most convenient body axes (X_B parallel to the fuselage centre-line) would also be aligned with the wind axes.
- i.e. at a fixed weight there will be one speed for which the wing incidence produces Lift = Weight when the body axis X_B is pointing into the wind.

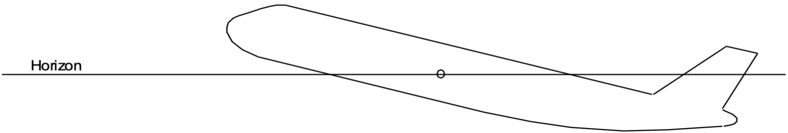
Stability Axes

- To allow for this likely initial mis-alignment of the X body-axis and the wind vector we can declare a special set of body axes, fixed to the body and initially aligned with the wind axes.
- This special set of axes, adhering to the requirement for initial alignment, is sometimes called the "stability axes".

Stability Axes

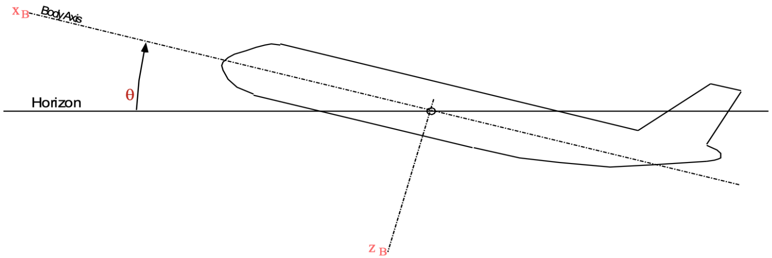
- One disadvantage with stability axes is that they would almost certainly be out of line with the principal inertia axes of the body - hence introducing cross-inertia terms in the resulting equations.
- Finally, every trimmed flight speed requires a new orientation for this special set of body-axes because the aircraft has a pitch attitude that is uniquely paired with forward speed for fixed weight.

Aircraft Axis System



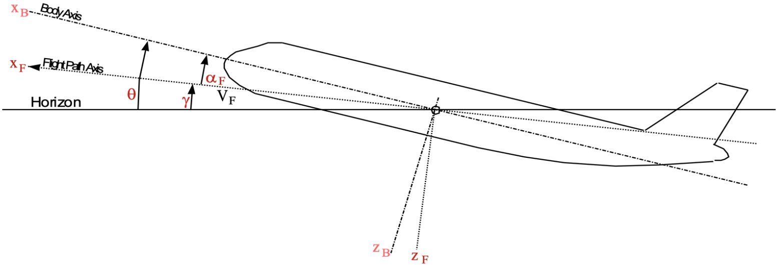
Aircraft + Horizon

Aircraft Axis System



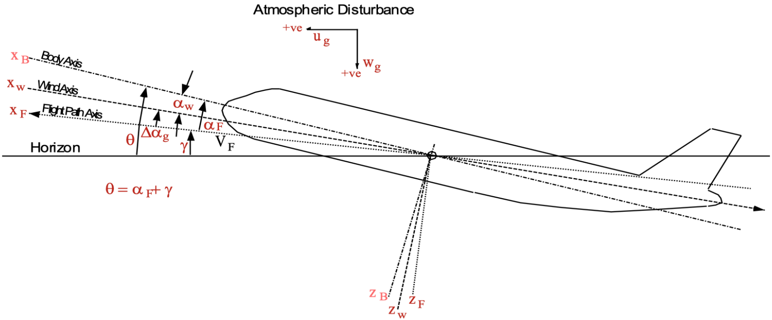
+ Body Axis

Aircraft Axis System



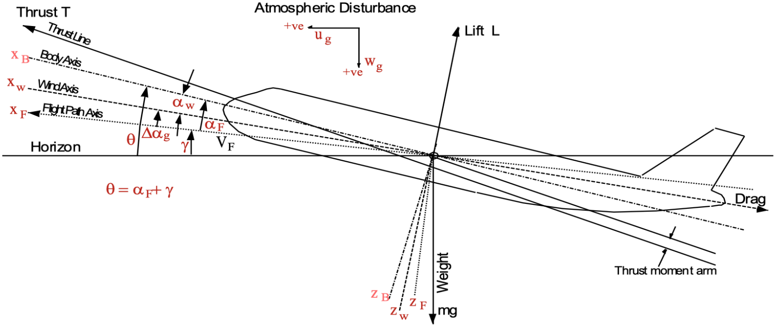
+ Flight Path Axis

Aircraft Axis System



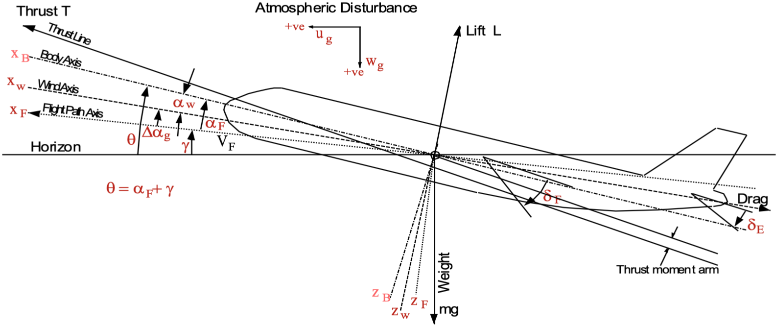
+ Wind Axis

Aircraft Axis System



+ Forces

Aircraft Axis System



+ Controls

Suggested Background Reading

Cook, M.V. *Flight Dynamics Principles*. Arnold, London, 2012.

Chapter 2 - *Systems of axes and notation - Controls Notation*

Next Lecture

Aircraft Controls