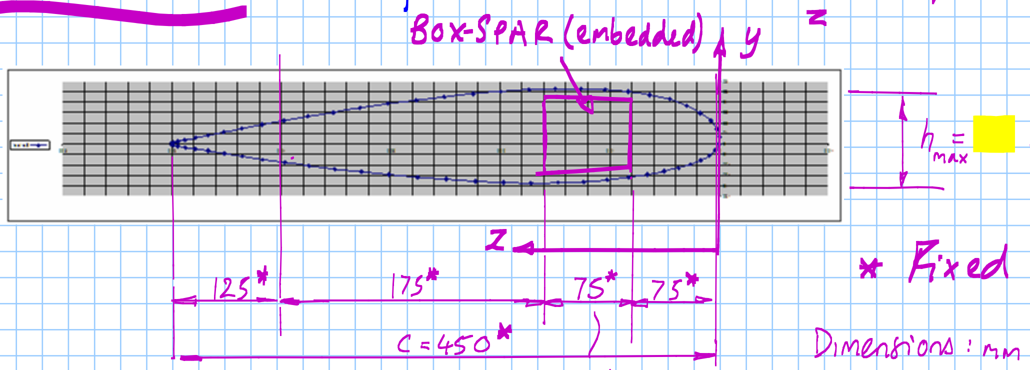
# ASD2 DBT Wing Initial Sizing 2016-17 - Calc Illustration JRP

Note Title 13/09/2016

28.9.2017

## 1. TRIAL SCHEME  Aerofoil :  h/c  %





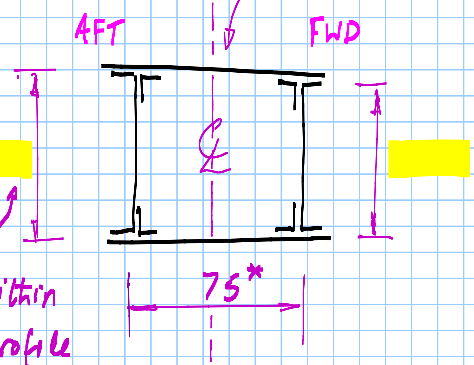

* Fixed dimensions!

Dimensions : mm

Starboard wing!

Box Spar:

dimensions obtained from chosen aerofoil geometry. At least 5mm within top/btm aero profile




* Angles and spar flanges must be 15mm

* Use this scheme!

i.e. C-section spars with angle supports at upr+lwr spar caps.

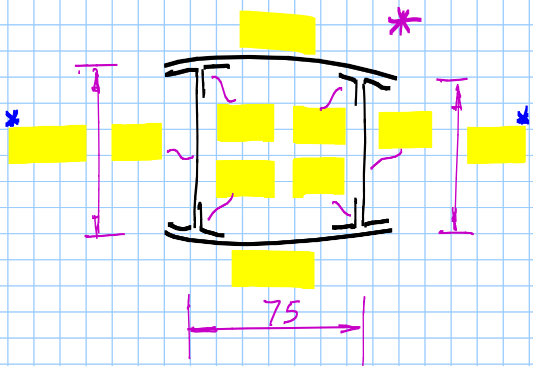
Angle and spar thicknesses should be the same either side of the vertical ℄



Trial dimensions:

Section @ $x$ = #

eg. root wing bay etc.



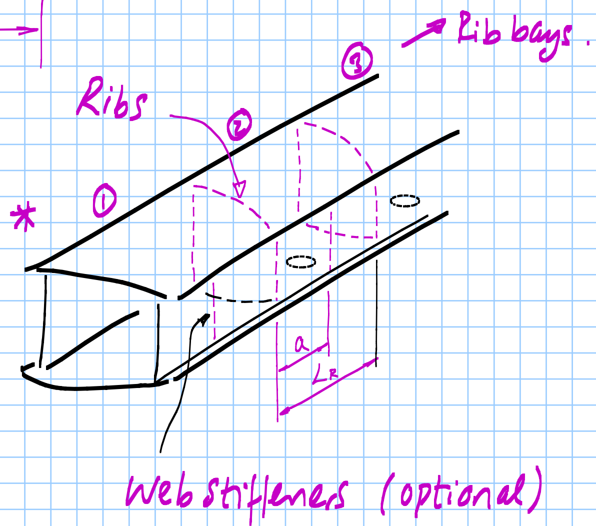

* Box-spar outline geometry 5mm within chosen aerofoil profile

Also select:

Rib pitch $L_R$ = mm

Spar web stiffener pitch $a$ = mm

* Note sketches show curved skins but for the latest embedded box-spar design the skins can be flat.

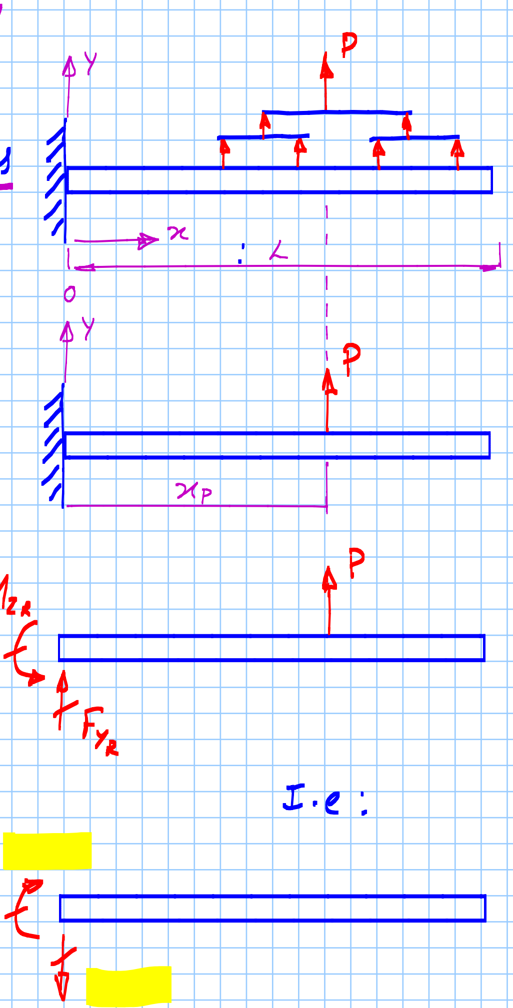



Note, at the detail design stage reinforcement will be added to the root bay section to carry joint loading but for now we will deal with just the basic section.

## 2. PART MODELS

*Designing for the test load in this exercise.*

### 2a Cantilever beam with idealised "wiffle tree" loads




Assuming symmetric loading at chordwise shear centre and initially simplifying as the total load at its line of application (reasonable for inboard checks).

For reactions @ root: FBD:

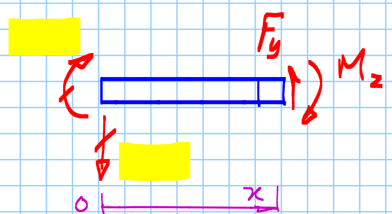
Equilibrium:

$\Sigma \uparrow^{+} = 0$: $\quad = 0 \quad$ : $F_{y_R} =$

$\Sigma \circlearrowleft_{0}^{+} = 0$: $\quad = 0 \quad$ : $M_{z_R} =$



For internal loads + mmts: Partial FBD:

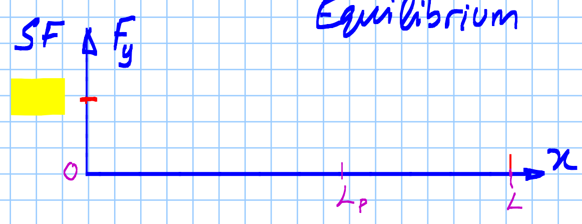



Internal sign convention: ($Y_R$ 2)

Equilibrium:

$\Sigma \uparrow^{+} = 0$: $\quad = 0 \quad$ : $F_y =$

@LIM

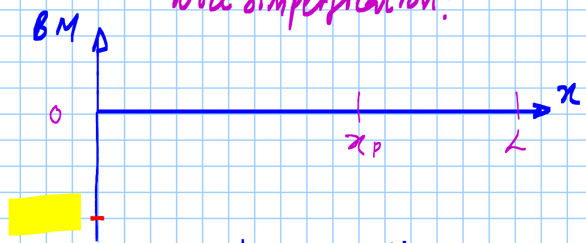



*Note simplification!*

$\Sigma \circlearrowleft_{@x}^{+} = 0$: $\quad = 0$ : $M_z =$

@ $x = 0$: $M_z = \quad = $

@ $x \geq x_p$: $M_z =$




Note use units: N, mm

@LIM

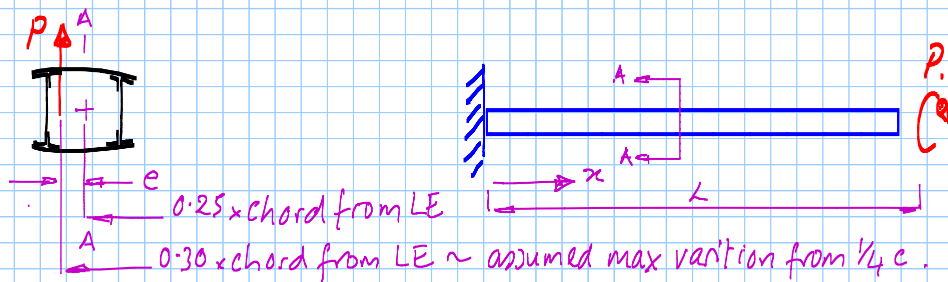
$\times 1.5 \rightarrow$ ULT

# 2b Shaft subject to torsion from offset tip load.



Assuming effective shear centre @ ℄

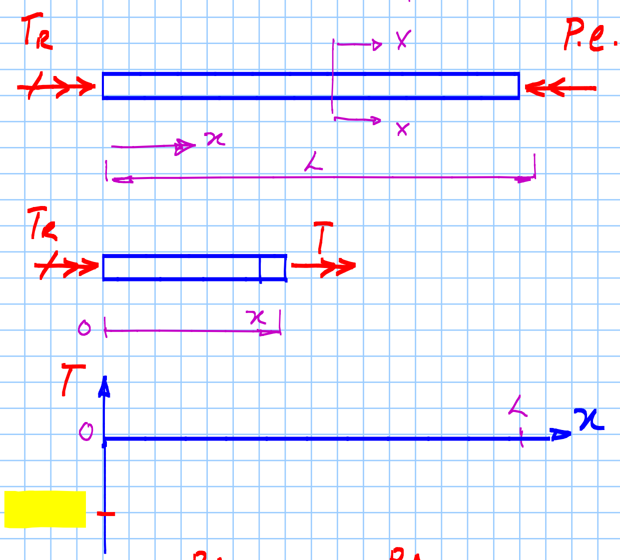
Torque = P.e




Equilibrium:

For reactions: whole FBD:

$\Sigma \overset{+}{\twoheadrightarrow} = 0$ : = 0 : $T_R$ =



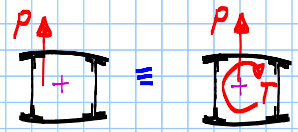

For internal torque: partial FBD:

Internal sign convention: ←← [+] →→

$\Sigma \overset{+}{\twoheadrightarrow} = 0$ : = 0 : T =

Finally, assuming linearity.

add the results of the two models by superposition:

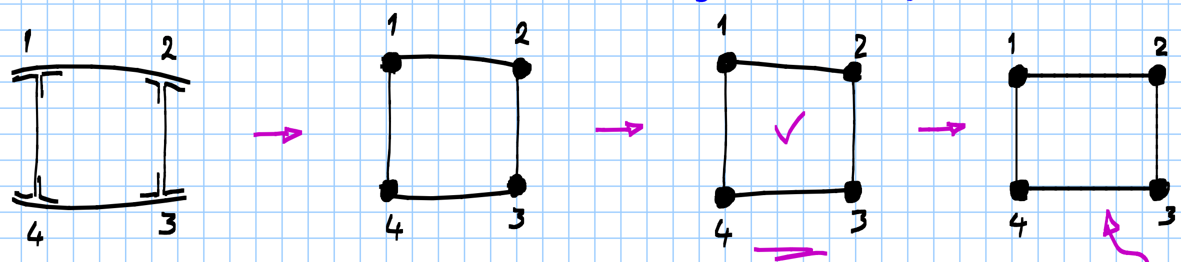



# 3 SECTION MODEL



Idealised Section model: For thin-wall structure

"Boom + skin" idealisation { Booms carry end loading only
Skins carry shear only




We could simplify to a rectangular box of average height but a basic trapezoid should be simple enough for our structure.

The new embedded box can actually be designed as a flat sided rectangular box!

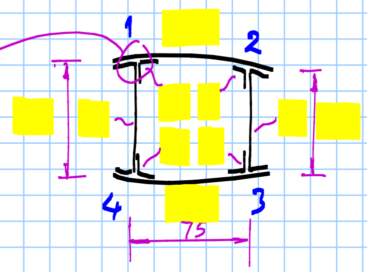
Booms are defined according to effective areas of the section.

For simplicity we will approximate the corners of the box as rectangular and ignore the slope and curve of the skins as a first approximation here.

Considering booms 1-4 in order:

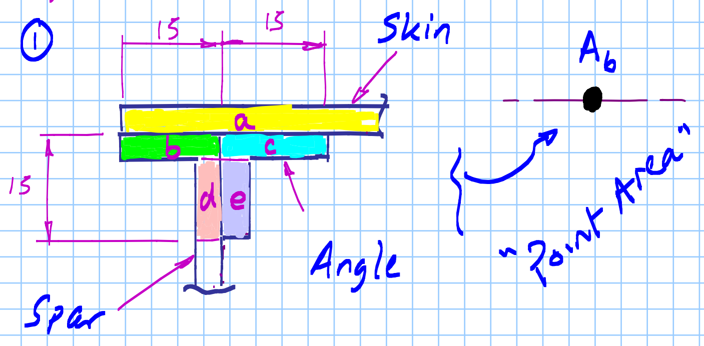
3a **Boom area properties.** 

As a simplification here estimate the effective boom areas from the "doubled" areas at the box corners.




E.g.

①




$A_{b_1}$ = a $2 \times 15 \times$ Skin

b $15 \times$ spar flange

c $15 \times$ angle flange

d* $\frac{2}{3}(15-\ )$ spar web

e* $\frac{2}{3}(15-\ )$ angle flange

$\Sigma$

etc.

$\hookrightarrow A_{b_1} =$ mm² Similarly:

$A_{b_2} =$ mm²

$A_{b_3} =$ mm²

$A_{b_4} =$ mm²

* Reduce the areas d, e by ⅓ to avoid overestimating their 2nd mmt of area contribution at the outer skin line.

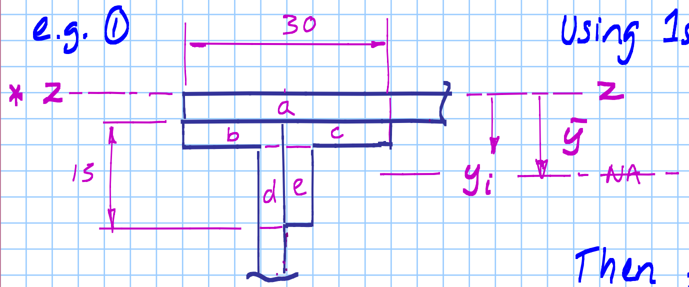
**Boom 2nd mmt of area :** 

We need the 2nd mmt area for the upper booms to check "column stability".

As an approximation use the assumed components of areas for the booms but do not apply the $\frac{1}{3}$ reduction for the web and angle blade components here.

First find the boom centroid:

e.g. ①

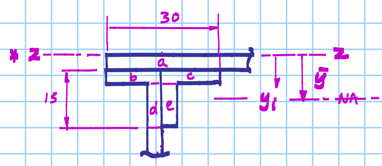



Using 1st mmt of area about chosen reference axis:

$$\bar{y} = \frac{\Sigma(A_i y_i)}{\Sigma A_i}$$

Then find the 2nd mmt of area about the neutral axis using the // axis theorem. Choosing the outer skin line as the reference zz axis proceed by tabulation:

| Element | $A_i$ | $y_i$ | $A_i y_i$ | $bd^3/12$ | $y_i^2$ | $A_i y_i^2$ |
|---|---|---|---|---|---|---|
| a | | | | | | |
| b | | | | | | |
| c | | | | | | |
| d | | | | | | |
| e | | | | | | |
| $\Sigma$ | | | | | | |

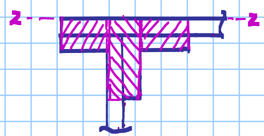



$$\hookrightarrow \quad \bar{y} = \frac{\Sigma (A_i y_i)}{\Sigma A_i} = \frac{\quad}{\quad} = \quad \text{mm}$$

$$I_{zz} = \Sigma \left( \frac{bd^3}{12} + A_i y_i^2 \right) = \quad + \quad \approx \quad \text{mm}^4$$

$$I_{NA_{①}} = I_{zz} - \Sigma A_i \cdot \bar{y}^2 = \quad - \quad \times \quad \approx \quad \text{mm}^4$$

Note alternative method (often easier!)
- used in illustration xls.




$\rightarrow I_{zz} = \Sigma \frac{bd^3}{3}$ etc

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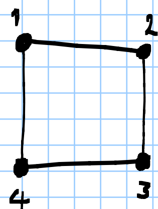
## 3 b Box section area properties.

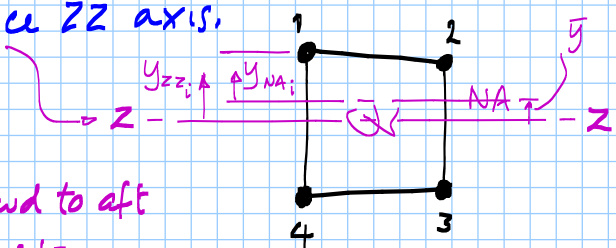
Box 2nd mmt of area :




We need the box 2nd mmt of area to find the deflection and stresses due to bending. Using a boom + skin approximation we only need to consider the effective boom areas.

First we need to find the box centroid and N/A position then we simply add the $A_i y_{NA_i}^2$ values for each boom to obtain $I_{NA_{BOX}}$

E.g. choosing the central axis as the reference ZZ axis.
Proceed by tabulation!




Note, you are expected to use the same booms fwd to aft but not top to bottom. ie 1 = 2 , 3 = 4 BUT 1 ≠ 4 , 2 ≠ 3

| Boom | $A_i$ | $y_{zz_i}$ | $A_i\, y_{zz_i}$ | $y_{NA_i}$ | $A_i\, y^2_{NA_i}$ |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| $\Sigma$ | | | | | |

$\hookrightarrow \; \bar{y} = \dfrac{\Sigma A_i\, y_{zz_i}}{\Sigma A_i} = \dfrac{\quad}{\quad} = \quad$ mm i.e. centroid and N/A mm above ℄.

$I_{NA_{Box}} = \Sigma A_i\, y^2_{NA_i} \simeq \quad$ $mm^4$

Now we have the area properties we can proceed to evaluate the stresses at the critical stations along the wing and then perform checks of stiffness, strength and stability.



# 4 Stress

## 4a Direct stress due to bending - E.g. @ station $x$ along the span.

$\sigma = \dfrac{M\,y}{I}$ where $M = M_z = \quad$ @ $x = \ldots$, $I = I_{NA_{Box}}$, $y = y_{NA_i}$

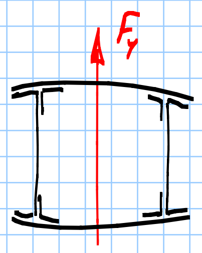
Calculate stresses at limit and ultimate at stations along the span:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $x$ | mm | | | | | | | etc. |
| | | LIM | ULT | LIM | ULT | LIM | ULT | → further critical stations |
| $M_z$ | Nmm | | | | | | | |
| Boom $i$ | $y_{NA_i}$ mm | $\sigma$ $N/mm^2$ | $\sigma$ $N/mm^2$ | $\sigma$ $N/mm^2$ | $\sigma$ $N/mm^2$ | $\sigma$ $N/mm^2$ | $\sigma$ $N/mm^2$ | |
| 1 | | | | | | | | |
| 2 | | | | | | | | |
| 3 | | | | | | | | |
| 4 | | | | | | | | |

Note rounding of values!

NOTE ESTIMATES IN RIB BAY 1 NEGLECTING OFFLOAD TO FIXTURE REINFORCEMENTS V. CONSERVATIVE!

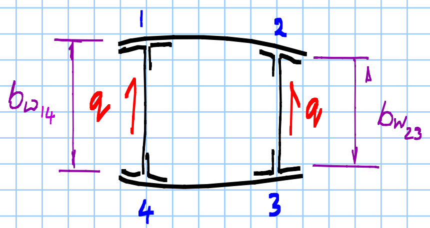
## 4b Shear flow due to transverse load




Detailed integration methods will be covered in StM2 but for design expediency we can perform a simple estimate of shear flow* and thence shear stress.

* Shear flow = shear force per unit length $q$ N/mm

Here we assume the shear flow acts through the "shear centre" and deal with the offset as torsion separately.



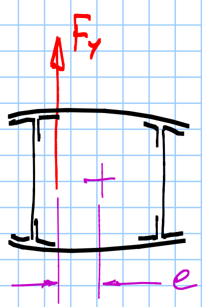

As a thin wall section we can assume that the webs primarily carry the resultant shear flow and estimate as:

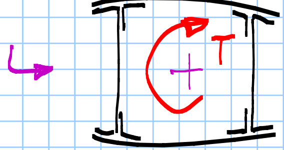
$$q \approx \frac{F_y}{b_{w_{14}} + b_{w_{23}}} = \frac{\quad}{\quad} \approx \quad \text{N/mm @ LIM}$$

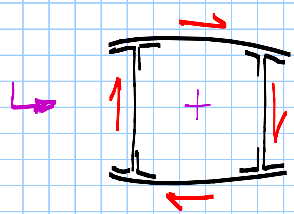
$$\hookrightarrow \times 1.5 = \quad \text{N/mm @ ULT}$$



## 4c Shear flow due to torsion



Resultant shear flow constant along span!

The transverse load may be eccentric, i.e. it is not aligned with the shear centre of the box. This creates a torque and a resultant shear flow which is uniformly carried around the closed section walls. Where:

$$T = F_y . e = \quad \times \quad = \quad \text{Nmm @ LIM}$$

$$\hookrightarrow \times 1.5 = \quad \text{Nmm @ ULT}$$

Using the Bredt Batho torsion formula (StM2):

$$q = \frac{T}{2A} = \frac{\quad}{\quad} = \quad \text{N/mm @ LIM}$$

$$\times 1.5 \downarrow$$

$$= \quad \text{N/mm @ ULT}$$

A = enclosed box area

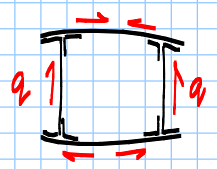

– estimate from simple straight-walled trapezoid.

4d

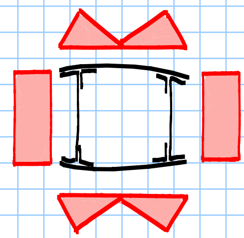
## Transverse + Torsional shear flow superposition



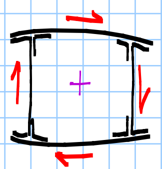
The transverse shear flow continues around the box section but dissipates to zero at free edges and cancels towards the middle of the upper and lower skins, i.e.:



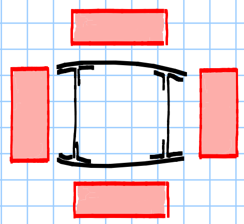

– giving shear flow distribution ~



The torsional shear flow is constant around the closed section



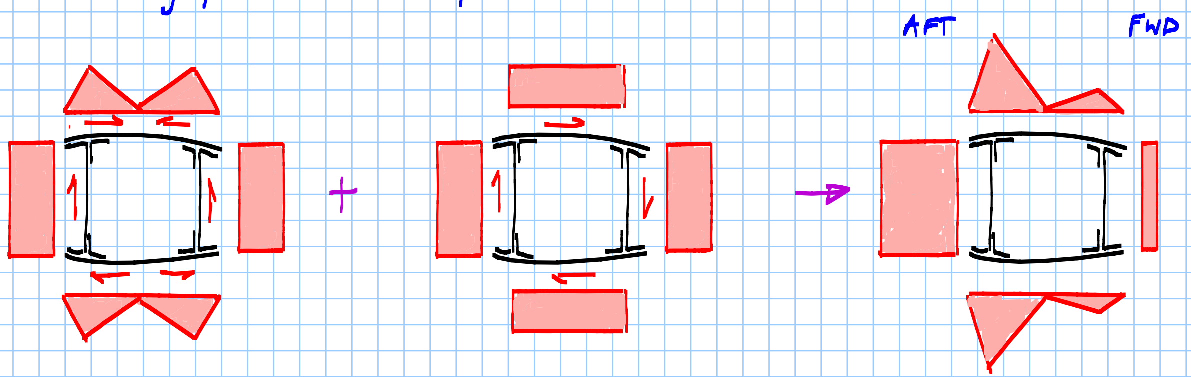

– giving shear flow distribution ~



By superposition we can add these shear flows:



Accounting for the sense of shear:




For the maximum shear flow at the rear spar simply add

ie.: $q_\Sigma = q_{F_y} + q_T = \quad + \quad = \quad$ N/mm @ LIM

$\hookrightarrow \times 1.5 = \quad$ N/mm @ ULT

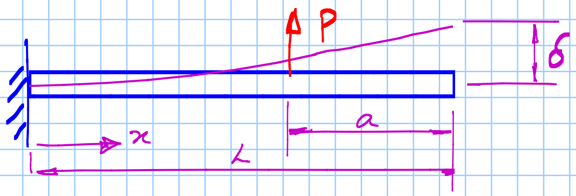
For the shear stress simply divide the shear flow by the thickness of the wall that carries it. E.g. in rear spar $\tau = \frac{q}{t} = \frac{\quad}{\quad} = \quad$ N/mm² @LIM

$\hookrightarrow \times 1.5 = \quad$ N/mm² @ULT

# 5 STIFFNESS. (at limit) 

## 5a Bending deflection.

Initially, considering a "typical" prismatic section along the inner span we can make a rough estimate of deflection using a standard cantilever equ<sup></sup>n for the major wiffle tree load at its line of loading.




Initial estimate using resultant test loading @ limit:

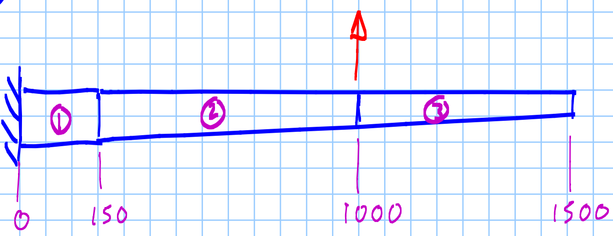
Ref Roark.

$$\delta = \text{"}k\frac{PL^3}{EI}\text{"} = \frac{1}{6}\frac{P(2L^3 - 3L^2a + a^3)}{EI} = \frac{1}{6}\times\frac{\square(\square)}{\square\times\square} = \square\ \text{mm}$$

Note, above limit load the elastic prediction will underestimate the deflection due to non-linearities. No deflection limit has been specified but excessive deflection may affect the alignment of mechanism hinge lines.

To refine the estimate consider an incremental cantilever model: 

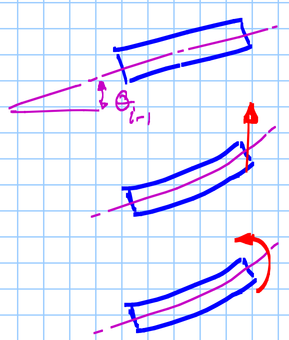
E.g. consider three increments and use the average section in each increment




[A solution could be obtained by integration but the varying I value due to taper and thickness changes would make this rather difficult.]

Sum the tip deflections cumulatively along the span

See APM!



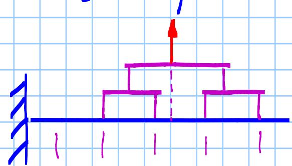

Tip rigid body deflection

Tip load deflection

Tip moment deflection

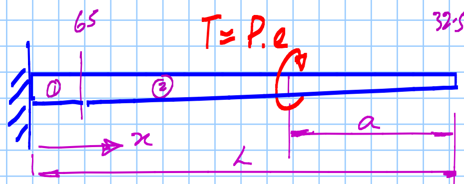
Also add the shear deflection of each element

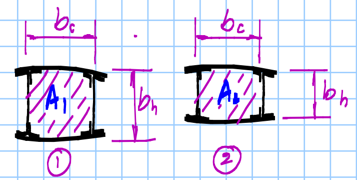
For further refinement further elements could be used to account for the wiffle tree test load increments along the span.



## 5b Torsional deflection, ie. twist.

As an initial estimate consider elements and their representative properties inboard of the major wiffle tree loading line. E.g. the root prismatic section and the tapered section up to the major loading line. Then simply add the twist for each element length.




$b_{h②}$ = mm

Twist: $\theta = \sum \frac{TL_i}{GJ_i}$ ; $\theta = \frac{\quad}{\quad} + \frac{\quad}{\quad}$ = rad

= deg.

Where $G = \frac{E}{2(1+\nu)} = \frac{\quad}{\quad} =$ N/mm²

And $J = \frac{4A^2}{\int \frac{ds}{t}} = \frac{4A^2}{\sum b_i/t_i}$ ; $J_1 = \frac{\quad}{\quad} =$ and $J_2 = \frac{\quad}{\quad} =$

To refine use an incremental model to account for particular sections and wiffle tree loading components and their offsets along the span.



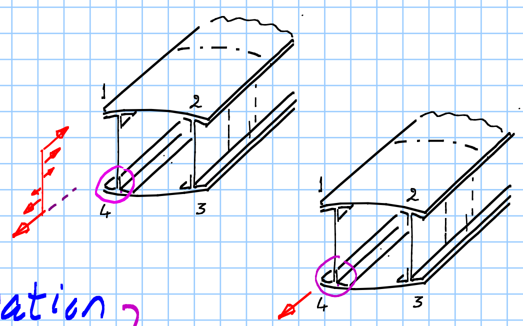
## 6. STRENGTH (at ultimate)

### 6a(i) Tension / shear material failure (at ULT)

E.g. @ boom ◯ @ $x =$ (Mid bay )

Considering direct stress due to bending :

$$\sigma = \frac{My}{I}$$




Neglecting joint reinforcements at this location } Conservative!

The stress in the lower booms will be higher since further from the NA.

Note, this stress will be increased slightly due to shear lag and web offloading when panel buckling occurs.

From previous stress tabulation, boom ④ ULT stress @ $x =$ , $\sigma =$

c/w 2014a-T3 ultimate strength $\sigma^* =$ N/mm² ∴ $\boxed{RF = \frac{\quad}{\quad} = \quad}$

## Combined stress check :

Although secondary we should also account for the shear stress.

We can account for the combined direct and shear stresses by using an "interactive failure criterion". Here we will use the "Energy of distortion" failure criterion :

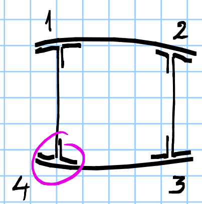
$$\left(\frac{\sigma_x}{\sigma_0}\right)^2 + 3\left(\frac{\tau_{xy}}{\sigma_0}\right)^2 = FI$$

where $\sigma_0 = \sigma_{ult}^*$ ie the ultimate tensile strength

$FI$ = Failure index and $RF = \frac{1}{\sqrt{FI}}$

e.g.

Shear stress @ boom ○ , $\tau_{xy} = \frac{q}{t}$




Refering to the thinnest element at boom ○

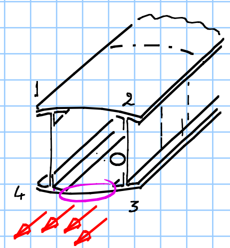
↳ $t$ = mm : $\tau_{xy}$ = = N/mm²

EoD failure criteria : $\left(\frac{\quad}{\quad}\right)^2 + 3\left(\frac{\quad}{\quad}\right)^2 =$ → $\boxed{RF = \frac{1}{\sqrt{\quad}} =}$



6a(ii) Tension material failure @ inspection hole (at ult)

Eg. @ Skin 3-4 @ x = 225 (mid rib bay 2)




– Neglecting shear since reducing towards box mid chord.

The lower skin can be assumed to carry some direct stress in tension.

As a conservative estimate we will take the average of stresses in booms 3 and 4, but this will be conservative since "shear lag" tends to reduce the stress in the skins compared with that in the bounding stiffeners which represent a "stiffer" load path.

– Also note the effective stiffness of the panel is significantly reduced in the region of the inspection hole.

– Converseley, the hole represents a stress concentration factor of 3 i.e. the field stress in the viscinity of the hole will be magnified ×3 !

As a crude estimate here we will estimate the skin stress as half the average boom stress in that region.

From previous stress tabulation:
from stresses in booms ③, ④ at ultimate @ $x = 225$ mm

$$\sigma \approx \frac{\quad + \quad}{\quad} \Big/ 2 = \qquad \text{N/mm}^2$$

Applying $K_T = 3$ : stress at hole $= 3 \times \quad = \quad$ N/mm²

c/w $\sigma^* = \quad$ N/mm² ↳ $\boxed{RF = \frac{\quad}{\quad} = \quad}$

Note! This is only a rough conservative check!

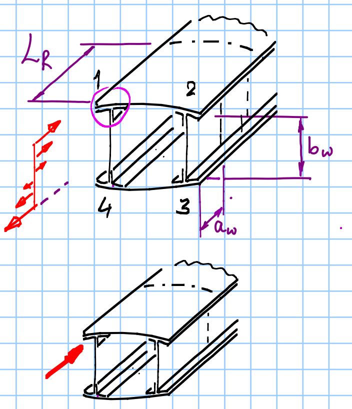


# 7 STABILITY (at Limit or Ultimate)

## 7a Column Buckling: (at ULT)

E.g. @ Boom  @ $x =$  (mid rib bay ) – neglecting joint reinforcement in bay 1 at this stage.

Make an initial buckling check of the effective column using the Euler equation with allowance for "plastic buckling".




$$\frac{\sigma^*_{CRIT}}{\eta} = K \frac{\pi^2 E I}{L^2} \Big/ A$$

where $K$ = buckling constant for end fixity
$E$ = Elastic Modulus
$\eta$ = Plastic buckling correction factor.
$L = L_R$ = rib pitch
$I = I_{NA_i}$
$A = A_i$

$K = 1$ for simply supported ends

$$\frac{\sigma^*_{CRIT}}{\eta} = \frac{\qquad}{\qquad} \Big/ \qquad$$

$$= \qquad \text{N/mm}^2 \rightarrow \sigma_p^* \; !?$$

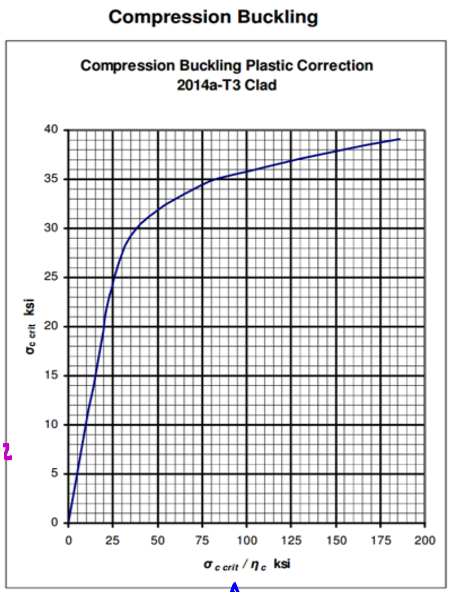
If the value approaches the material proof stress we must correct for plasticity.

To correct for plasticity we refer to "plastic buckling correction" charts or "tangent modulus" curves.

Usually these are based on imperial units

eg. "ksi" kilo pounds per sq' inch

Note $1\,ksi = 6{\cdot}895\ N/mm^2$ (MPa)




Convert the predicted elastic buckling strength to ksi then read the corrected value from chart and convert back to $N/mm^2$ (MPa)

ksi

$\times\, 6{\cdot}895$

$= \quad N/mm^2$

$\dfrac{\quad}{6{\cdot}895} = \quad ksi$

Finally, compare with the applied stress at ultimate From previous stress tabulation

eg. Ult stress in boom ① @ $x =$ (mid rib bay ): $\sigma =$

Note, a reasonable margin is needed here

$$\boxed{RF = \frac{\quad}{\quad} = \quad}$$



## 7b Panel buckling. (at limit or ultimate)

Panel buckling is not necessarily critical provided that the bounding stiffeners are not overloaded when the panel buckles.

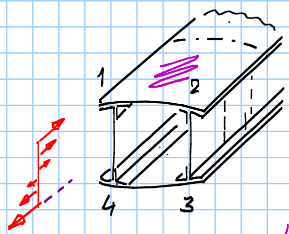
Elastic panel buckling can be allowed below ultimate or even below limit load provided that adequate edge stiffening is present and that panel distortions do not significantly affect the aerodynamic performance.

Due to offloading to the bounding spar caps the stress in the skin will never exceed its initial buckling stress. Even so, we need to know what stress the skin buckles at in order to check if it is acceptable. For our structure we will aim to avoid panel buckling below half of the limit load. I.e. panel buckling above half of limit load is deemed acceptable here.

7b(i) **Compression panel buckling:** (at limit)

E.g. @ Upper skin panel 1-2 @ $x =$ ___ (Mid rib bay ___) neglecting reinforcement for root joint. Conservative!

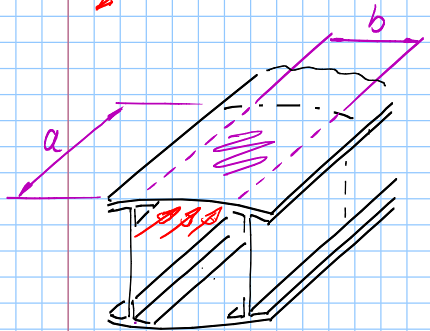
Flat panel buckling equation:




$$\frac{\sigma_{c_{cr}}}{\eta} = K_c E \left(\frac{t}{b}\right)^2$$

where $K_c$ = buckling constant for end compression
$E$ = Elastic Modulus
$\eta$ = Plastic buckling correction factor.
$t$ = panel thickness*
$b$ = panel width

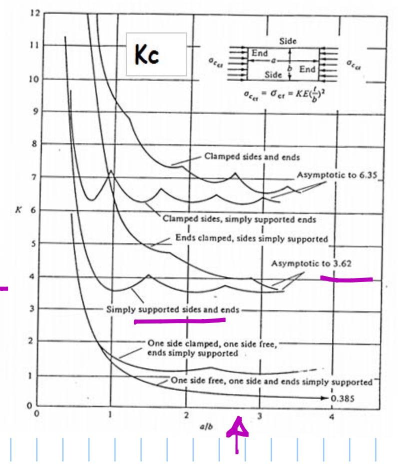
$$\frac{a}{b} = \frac{\text{___}}{\text{___}} = \text{___}$$




Take the effective panel width 'b' as the distance between the effective boom widths.

Read $K_c$ from buckling chart for a/b value and all edges simply supported:

$\hookrightarrow K_c =$ ___






For thickness, $t$, we must account for 'cladding' of the aluminium alloy.

Using a cladding correction factor of 0.9: $t =$ ___ $\times 0.9 =$ ___ mm

Note, only apply cladding correction to t for buckling calc. I.e. not to areas for stress calc

$$\hookrightarrow \quad \frac{\sigma_{c_{cr}}}{\eta} = \text{___} \times \text{___} \left(\frac{\text{___}}{\text{___}}\right)^2 = \text{___ N/mm}^2$$

$\hookrightarrow \sigma_p$ ?

$\hookrightarrow$ ___ ksi $\rightarrow$ Plastic correction chart etc.

$\div 6.895$

Accounting for curvature

$$\sigma_{c_{rad}} = K_r E \frac{t}{R}$$

Use $K_r = 0.25$

Use clad corrected value for $t$

$$\hookrightarrow \quad \sigma_{c_{rad}} = \text{___} \times \text{___} \times \frac{\text{___}}{\text{___}}$$

$$= \text{___ N/mm}^2$$

As a simple estimate of upper skin curvature in the spar box region take $R \approx C = 450$ mm

Giving a "curvature assisted" buckling strength:

$$\sigma_{c_{CRIT}} = \sigma_{c_{cr}} + \sigma_{c_{rad}} = \text{___} + \text{___} = \text{___ N/mm}^2$$

Omit for flat skins

Compare with the stress at limit in upper spar booms taken as the average stress in booms ① + ② @ $x =$ mm (mid rib bay 1)

From previous stress tabulation:

from stresses in booms ①, ② at limit, @ $x =$ mm

$$\sigma \approx \frac{\quad + \quad}{2} = \quad \text{N/mm}^2 \text{ at limit.}$$

So the panel will buckle at $\sim \frac{\quad}{\quad} = \quad$ of limit.

Remember $> 0.5 \times$ limit acceptable here.

Note, there will be a skin doubler at the root end of rib bay 1, so a check at this position will be conservative.

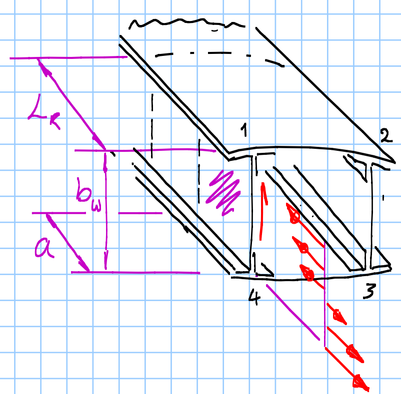


7b(ii) In-plane bending/shear panel buckling: (at limit)

E.g. @ Rear Spar web @ $x =$ (mid rib bay )

Accounting for in-plane bending and shear loading
Consider separately then combine:

Rear spar more critical since transverse and torsional shears add and largest depth web




- In plane bending:

Panel buckling equ'n:

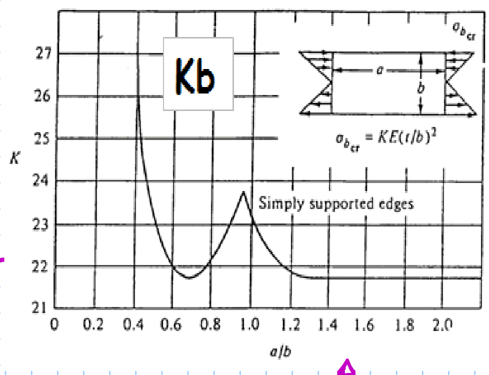
$$\frac{\sigma_{b_{CR}}}{\eta} = K_b E \left(\frac{t}{b}\right)^2$$

$K_b$ = Panel bending buckling constant for in-plane bending.

$$\frac{a}{b} = \frac{\quad}{\quad} = $$

Take the effective panel width 'b' as the distance between the effective boom widths.

Read K from buckling chart for a/b value and all edges simply supported: $\Rightarrow K_b =$



Use cladding correction factor for $t$. → $t = 0.9 \times \square = \square$

↳ $\frac{\sigma_{b_{cr}}}{2} = \square \times \square \left(\frac{\square}{\square}\right)^2$

$= \square$ N/mm²

c/w $\sigma_p^* = \square$ N/mm²

and check plastic buckling:

↳ $\square$ ksi

$\div 6.895$

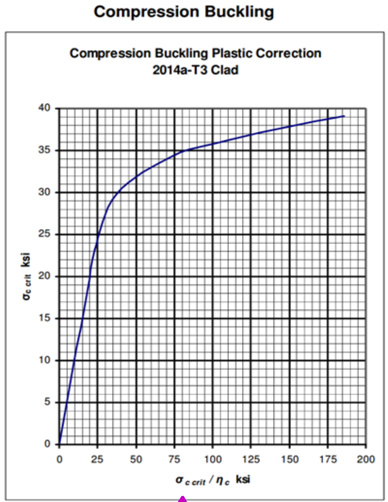
then compare with

peak compressive stress

27 ksi ←

$\times 6.895$

$= \square$ N/mm²




$\frac{\square}{\square} = \square$ ksi

From previous stress tabulation:

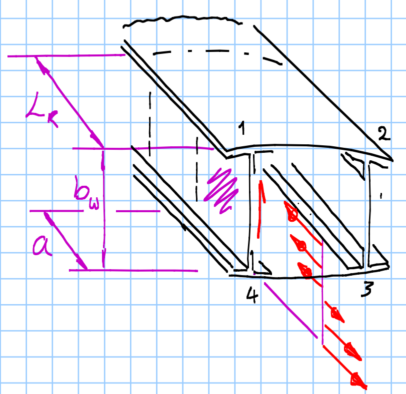
taking stress in boom ( ) @ mid rib bay ___ as the

peak compressive stress in the web: $\sigma = \square$ N/mm²

↳ Web will buckle under in-plane bending alone @:

$\frac{\square}{\square} = \square$ of limit - remember > 0.5 acceptable.






- Shear:

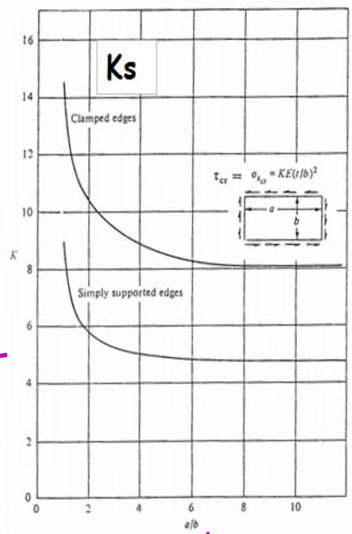
Panel buckling equ'n:

$$\frac{\tau_{cR}}{2} = K_s E \left(\frac{t}{b}\right)^2$$

$$\frac{a}{b} = \frac{\square}{\square} = \square$$

Where

$K_s$ = panel shear buckling constant for shear




Take the effective panel width 'b' as the distance between the effective boom widths.

Read $K_s$ from buckling chart for a/b value and all edges simply supported: ↳ $K_s = \square$

Use cladding correction factor for $t$. → $t = 0.9 \times \square = \square$

↳ $\frac{\tau_{cr}}{2} = \square \times \square \left(\frac{\square}{\square}\right)^2 = \square$ N/mm²

$< \tau_p^*$ ?

Compare against shear stress at limit

From previous stress calcs, shear stress at limit @ mid bay 1

$$\tau = q/t = \frac{\quad}{\quad} = \quad \text{N/mm}^2$$

↳ Web will buckle under shear alone @:

$$\frac{\quad}{\quad} = \quad \times \text{limit.}$$

Considering combined in-plane bending + shear

Use the failure criterion:

$$\left(\frac{\sigma_b}{\sigma_{b_{cr}}}\right)^2 + \left(\frac{\tau}{\tau_{cr}}\right)^2 = FI \quad \text{and} \quad RF = \frac{1}{\sqrt{FI}}$$

$$\hookrightarrow \left(\frac{\quad}{\quad}\right)^2 + \left(\frac{\quad}{\quad}\right)^2 = \quad \rightarrow \boxed{RF = \frac{1}{\sqrt{\quad}} = \quad}$$

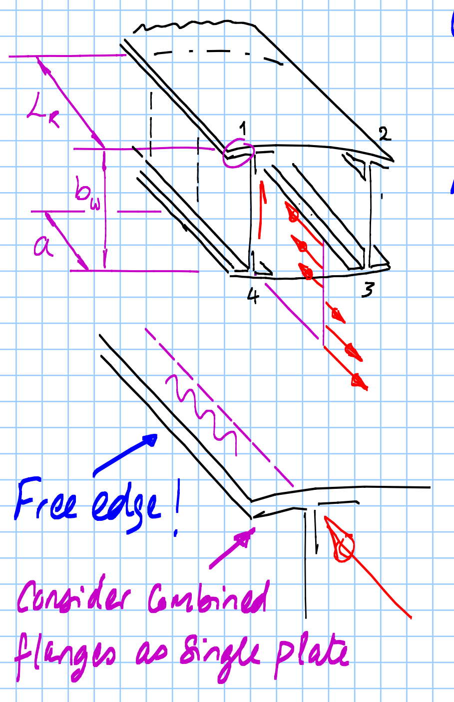
↳ Web will buckle under combined bending + shear @ ___ × limit

Remember > 0.5 × limit ok here.



7b(iii) Compression panel "blade" buckling (at ultimate)

E.g. @ upper aft rear spar flange @ $x$ = ___ (mid rib bay ___)




Considering axial compression only

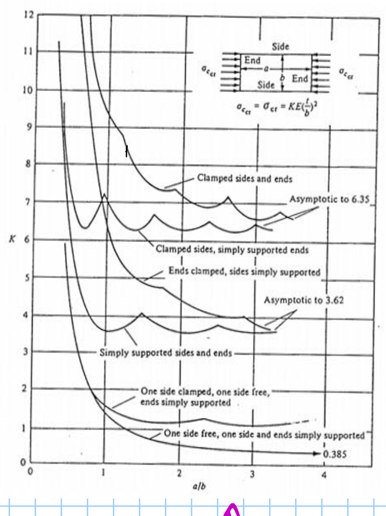
Note the spar flange has a free edge and ideally we need to avoid buckling before reaching ultimate.

Apply buckling formula:

$$\frac{\sigma_{c_{cr}}}{\eta} = K_c E \left(\frac{t}{b}\right)^2$$

Read $K_c$ from buckling chart for simply supported sides and ends but one side free ←

and $\frac{a}{b} = \frac{\quad}{\quad} = \quad$




Neglecting reinforcement for joint in rib bay 1

↳ $K_c$ = ___

$t$ = combined thicknesses of flange elements (corrected for cladding).

$= 0{\cdot}9 \times ( \quad + \quad ) =$

$b$ = flange width = 15 mm

$\hookrightarrow \quad \dfrac{\sigma_{c_{cr}}}{2} = \quad \times \quad \left(\dfrac{\quad}{\quad}\right)^2 = \quad$ N/mm²

Plastic correction

$\hookrightarrow \quad \sigma_{c_{cr}} = \quad$ N/mm²

c/w boom 1 stress @ ult @ mid rib bay 1

Previous stress tabulation: $\sigma = \quad$ N/mm²

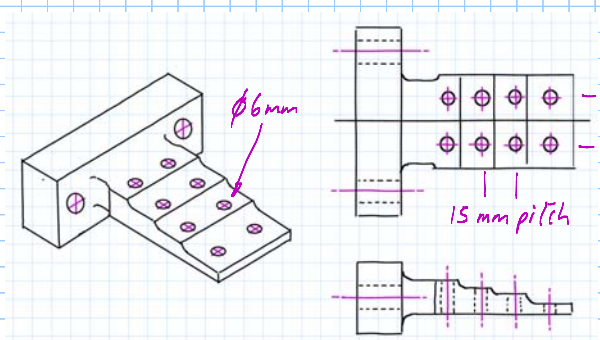
$\hookrightarrow$ $\boxed{RF = \dfrac{\quad}{\quad} = \quad}$

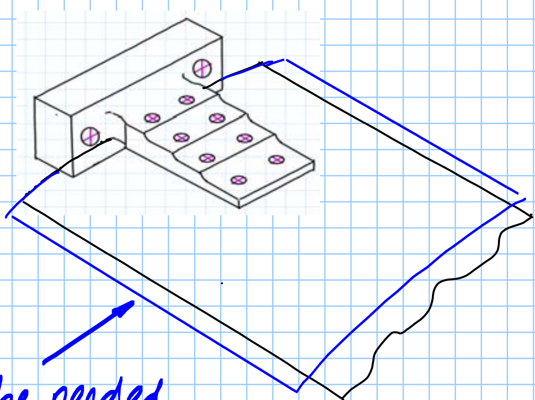


# 8. JOINT STRENGTH (@ ultimate)

## 8a Root fixture end-loaded joint

The root joint will be made by attachment to upper and lower steel fittings through pairs of 6mm bolts in four rows (eight bolts in total) in each fitting.

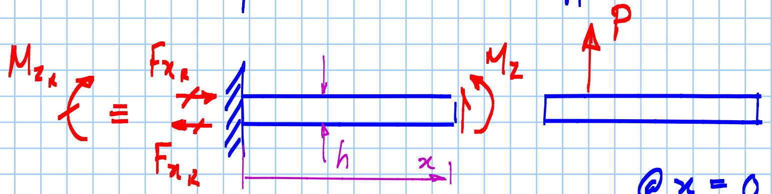


Note a doubler will be needed.

Initially check without a doubler to assess the sizing needed.

**Loading:** The bending moment at the root must be reacted as a couple load at the upper and lower root fixtures.




Where $h$ is the average height of the box spar

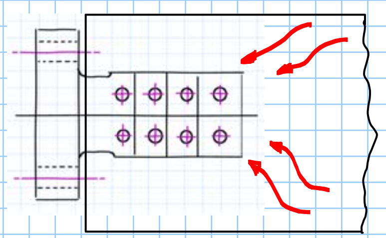
@ $x = 0$ (conservative)

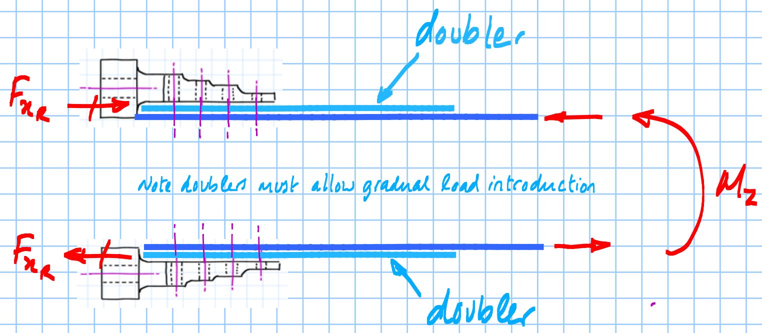
Calculating at ultimate:

$M_{z_R} \equiv F_{x_R} h$ : $F_{x_R} \simeq \dfrac{M_{z_R}}{h}$ where $M_{z_R} = P.L = \quad \times \quad = \quad$ Nmm

$\hookrightarrow$ $F_{x_R} = \dfrac{\quad}{\quad} = \quad$ kN @ ULT.

I.e.:

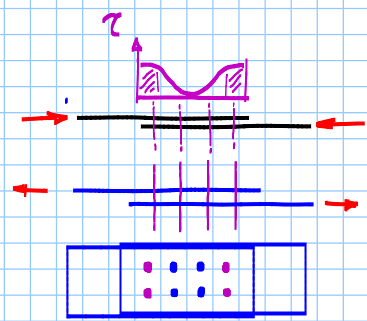







**Loading distribution:**

The fitting is stepped to promote even load sharing amongst the rows of bolts but, even so, we will assume that the two outer rows carry 60% of the total load to be transferred.




Check upper/lower bolted root joint (wing side only)

tfr load = Couple load @ Root $F_{x_R} = \dfrac{M_z}{h}$ @ $x = 0$

– assume the outer two rows of two bolts transfer 60% of the total load

$\hookrightarrow$ $P_{BOLT} = 0.6 \times \dfrac{F_{x_R}}{4} = \quad \times \dfrac{\quad}{\quad} = \quad$ kN

Note RF's > 1.5 required

i.e. to allow for a bolted joint "fitting factor" of 1.5.

You will probably need to use a skin doublers here!

E.g. @ lower bolted root joint:

- Plate bearing $\sigma_{br} = \dfrac{P_{BOLT}}{dt} = \dfrac{\quad}{\quad} = \quad N/mm^2$

  c/w $\sigma^*_{br_{ULT}} = \quad N/mm^2 \hookrightarrow$ $\boxed{RF = \dfrac{\quad}{\quad} = \quad}$

- Plate tension $\sigma_t = \dfrac{P_{BOLT}}{(w-d)t} = \dfrac{\quad}{\quad} = \quad N/mm^2$

  where $w$ = effective width
  - use widthwise pitch of fitting

  c/w $\sigma^*_{t_{ULT}} = \quad N/mm^2 \hookrightarrow$ $\boxed{RF = \dfrac{\quad}{\quad} = \quad}$

- Plate shear out $\tau_{so} = \dfrac{P_{BOLT}}{2et} = \dfrac{\quad}{\quad} = \quad N/mm^2$

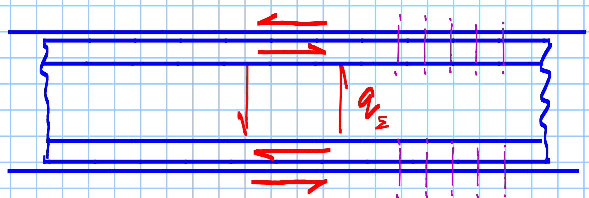
  where $e$ = shear-out distance
  - use lengthwise pitch of fitting.

  c/w $\tau^*_{ULT} = \quad N/mm^2 \hookrightarrow$ $\boxed{RF = \dfrac{\quad}{\quad} = \quad}$

Similarly check upper.



## 8b Spar cap - skin "shear-loaded" joints




Loading:

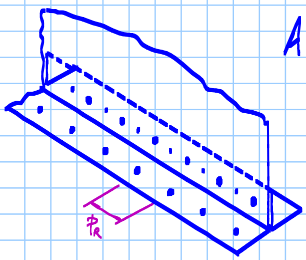
The transverse and torsional shear must be transferred between the spar cap, angle and skin as complimentary shear. As a shear loaded joint the distribution can be assumed to be uniform

$$q_\Sigma = q_{F_y} + q_T$$ i.e. adding at the aft spar.

Loading distribution: assuming half of the shear transfers between the spar cap and skin (the other half through the angle).

## E.g. Spar cap - skin joint






Assuming, as above, half shear transferred between spar cap and skin the other half through the angle

For rivet pitch $p_R =$ mm :

- Rivet shear load: $P_{RIVET} = \frac{q_\Sigma}{2} \cdot p_R = \quad = \quad N$

c/w $P^{*}_{RIVET\ SHEAR} = \quad$ : $\boxed{RF = \frac{\quad}{\quad} = \quad}$

Aim for minimum RF > 3 here to allow for the variability of a pop rivet joint!

Note, the rivet strength has been obtained from riveted lap joint tests in 2014a T3 sheets where rivet shear has been the critical failure mode. So here we will assume that a rivet shear check covers other possible failure modes.

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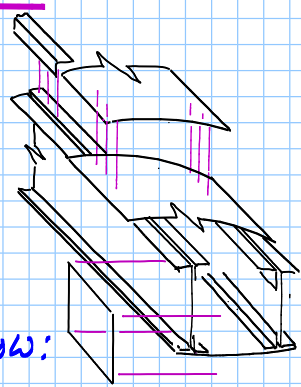
## 8c-e Mid wing - Upper / lower "end loaded" lap and butt strap joints

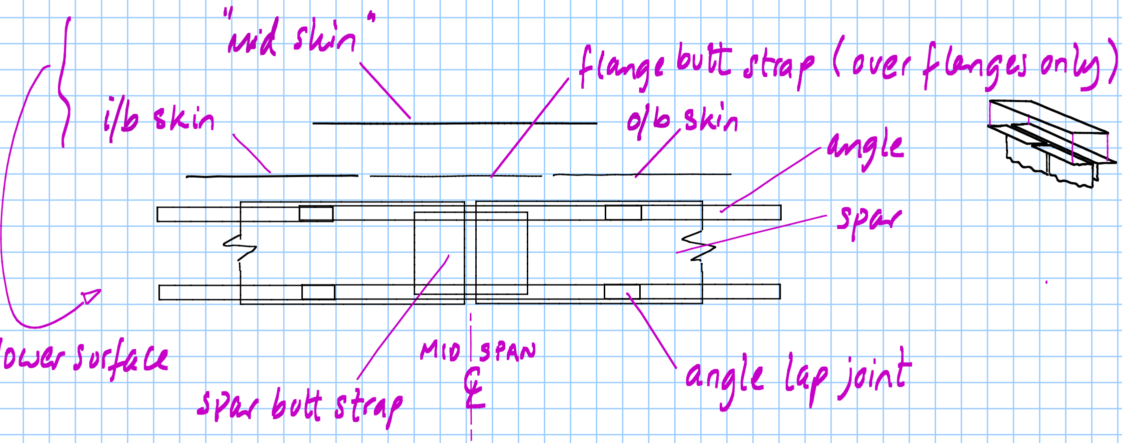


The skin, angle and spar joints need to be offset to allow some of the load to be bypassed by the strapping effect of continuous elements across the joint.



A proposed joint configuration @ mid span region is given below:

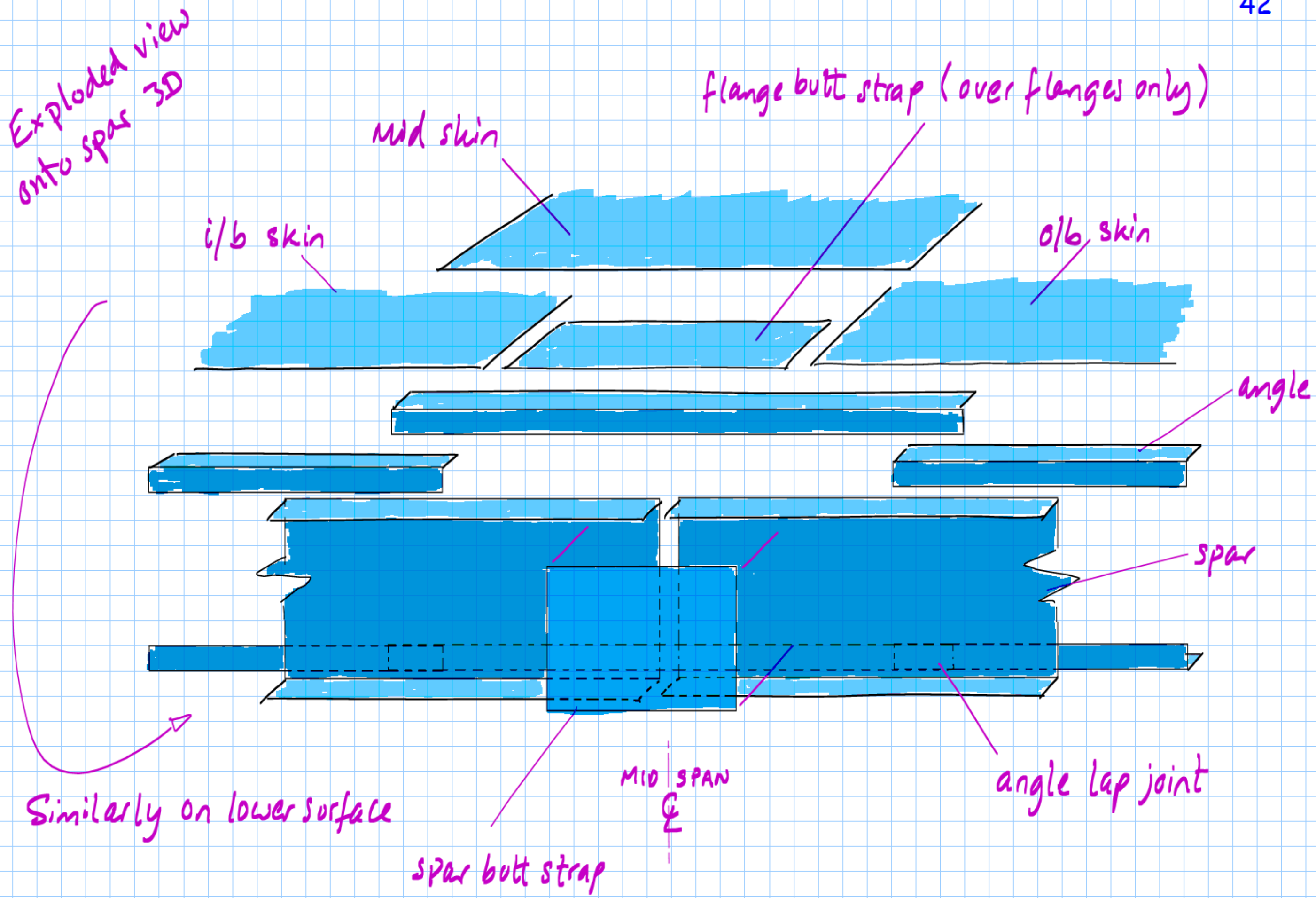
Exploded view onto spar 2D.




Similarly on lower surface

Aim for ≥ 50t offset between joints

See over for 3D sketch.






Loading:

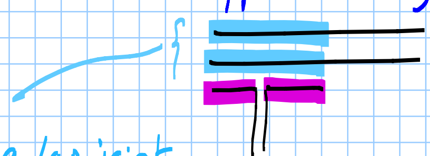
The upper and lower "end-load" joints transfer the couple loading due to bending carried by the booms which we can estimate as: $P_i = \sigma_i \times A_i$

where: $\sigma_i = \dfrac{M_z \, y_{NA_i}}{I_{NA_{BOX}}}$

Note, as a check the pair of boom loads, either on the top or bottom, should equate to the couple load.

Load Transfer and bypass across the joint:

Assuming all the load is transferred in the boom flange regions the proportions of load transferred or bypassed can be estimated approximately in proportion to their relative cross section areas. E.g.:



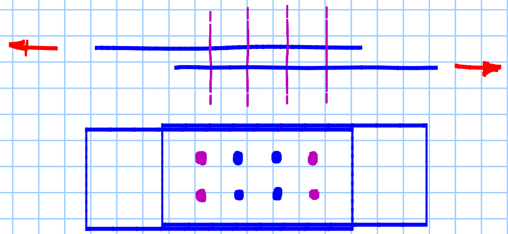
Note, just claim the average of the joined thicknesses of a lap joint for the load transfer area (not the combined lap thickness).

## Load distribution:

The load distribution will vary significantly across an untailored end-loaded joint such that the majority of the load will be transferred across the ends of the joint.

As an initial conservative estimate here proceed as follows:

For riveted "end-load" joints with a minimum of 4 rows of rivets across the lap at 4d spacing assume that the two outer rows carry 60% of the joint load.



The upper + lower end loaded skin lap joints 'c' are considered in detail below

The angle lap and spar cap butt strap joints can be considered similarly.



Eg. Upper, lower riveted skin lap joints @ $x =$

Loading: $M_z =$ Nmm, assuming the same spanwise position as the upper joint

@ boom 4 : Load $P_4 = \sigma_4 . A_4$ where $\sigma_4 = \dfrac{M_z \, y_{NA_4}}{I} = \dfrac{\quad}{\quad} = \quad N/mm^2$

← higher stress c/w boom 3

and $A_4 = \quad mm^2 \rightarrow P_4 = \quad kN$

Tfr + bps – similar to upper

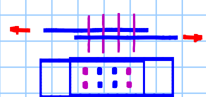
– Check rivet transfer load:

tfr load: $P_{tfr} = P_1 \times \dfrac{A_{tfr}}{\Sigma A} = P_1 \times \dfrac{2 b_f . t^*_{SKN}}{b_f . t_{SPR} + b_f \times t_{ANG} + 2 b_f \, t^*_{SKN}} = \quad \times \dfrac{\quad}{\quad}$

$\Sigma A$ ↖ $A_{tfr} + A_{bps}$

$= \quad kN$

So for a pair of rivets in each outer row in the boom flange regions we have 4 rivets carrying 60% of the transfer load:



↳ $P_{RIVET} = 0{\cdot}6 \times \dfrac{\quad}{4} = \quad kN$ c/w $P^*_{rivet} = \quad kN$ ∴ $\boxed{RF = \dfrac{\quad}{\quad} = \quad}$

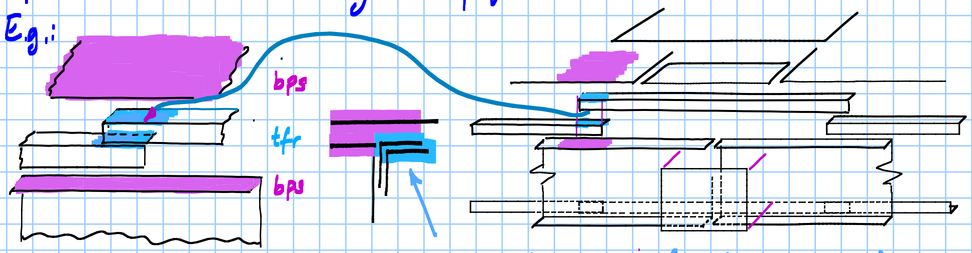
– Check strap bypass stress ··· etc.

8d,e **Upper and lower angle and spar cap lap and butt strap joints.** 

Proceed as above, accounting for appropriate tfr and bps areas. E.g.:

d. Upper and lower angle lap joints

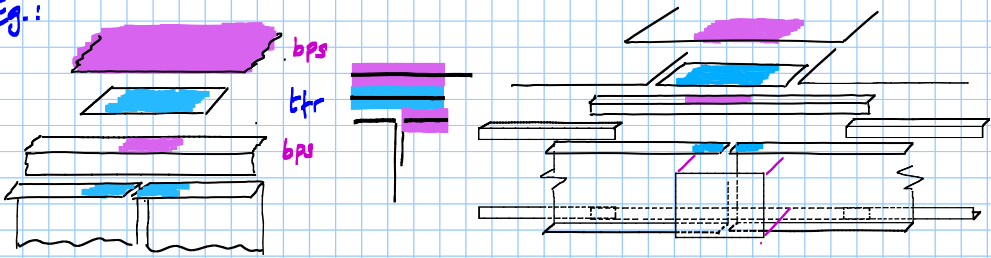
Eg.:




skin bps width = $2b_f$
angle tfr " = $b_f$
spar bps " = $b_f$

e. Upper and lower spar cap butt strap joints.

Eg.:

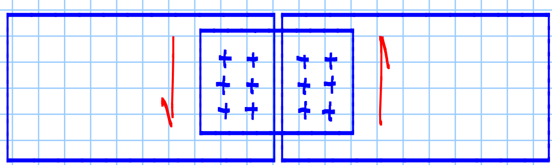



Note:

skin bps width = $2b_f$
butt strap " = $2b_f$
angle bps " = $b_f$

Consider each side of the spar cap-butt strap joint as a lap joint of minimum developed length.
I.e. $50t_{STR}$ either side of butt etc.

8f **Spar web "shear-loaded" butt strap joint** 



Loading:

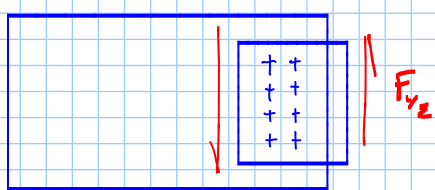
Shear due to transverse load and torsion at ultimate:

$q_{\Sigma} = q_{F_Y} + q_T =$ ___ N/mm $\rightarrow$ $F_{Y_{\Sigma}} = q_{\Sigma} \cdot h =$ ___ $\times$ ___ $=$ ___ N

$h$ = spar height

Configuration: two rows of three or four rivets each side of joint

– Check rivet transfer load:




Assume inner row carries 60% of load uniformly

$$P_{RIVET} = 0.6 \times \frac{\_\_\_}{\_\_\_} = \_\_\_ \text{ N}$$

c/w $P^*_{RIVET} =$ ___ N/mm² $\rightarrow$ $RF = \frac{\_\_\_}{\_\_\_} = \_\_\_$

* Note, the stability of the butt strap will also be an issue!