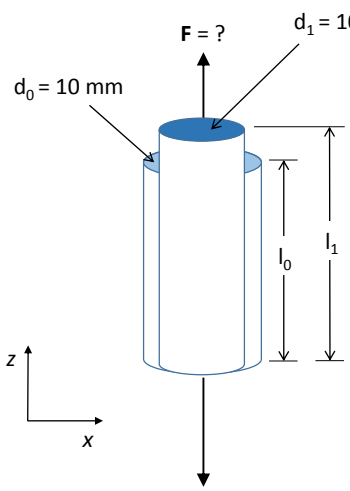
Example
Elastic Material Deformation

A tensile stress is applied to a brass rod that has a 10 mm diameter. Determine the load required to produce a 2.5 x 10^{-3} mm reduction in diameter if the deformation is purely elastic.

Step 1: Schematic and Properties



Metal	Young's Modulus	Shear Modulus	Poisson's Ratio
Brass	97 GPa	37 GPa	0.34

Step 2: Calculation Plan

We were asked to find the **Force** required to cause the reduction in diameter.

Eng. Stress:

$$\sigma = \frac{F}{A_0} \xrightarrow{\text{Rearrange for Force}} F = \sigma A_0 = \sigma \left(\frac{d_0}{2}\right)^2 \pi$$

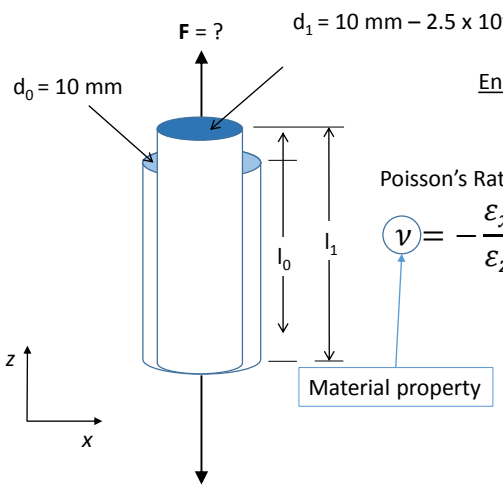
Given 10 mm

The stress is computed from the **axial strain** and the **Young's Modulus**

$$\sigma = \varepsilon_z E$$ ← Given 97 GPa

We need to find ε_z

Step 3: Calculate Axial Strain



Eng. Strain:

$$\epsilon_z = \frac{\Delta l}{l_0} = \frac{l_1 - l_0}{l_0}$$

Poisson's Ratio

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z}$$

Material property

$$\epsilon_x = \frac{\Delta d}{d_0} = \frac{d_1 - d_0}{d_0}$$

$$\epsilon_x = \frac{-2.5 \times 10^{-3} mm}{10\ mm}$$

$$\epsilon_x = -2.5 \times 10^{-4}$$

$$\epsilon_z = -\frac{\varepsilon_x}{\nu} = -\frac{-2.5 \times 10^{-4} mm}{0.34} = 7.35 \times 10^{-4}$$

Step 4: Solve for Force

First calculate stress:

$$\sigma = \varepsilon_z E = 7.35 \times 10^{-4}(9.7 \times 10^{10} Pa) = 7.13 \times 10^{7}\ Pa$$

Finally calculate force:

$$F = \sigma A_0$$

$$F = \sigma \left(\frac{d_0}{2}\right)^2 \pi$$

$$= \left(7.13 \times 10^{7} \frac{N}{m^2}\right) \left(\frac{1 \times 10^{-2}\ m}{2}\right)^2 \pi$$

$$= 5600\ N$$