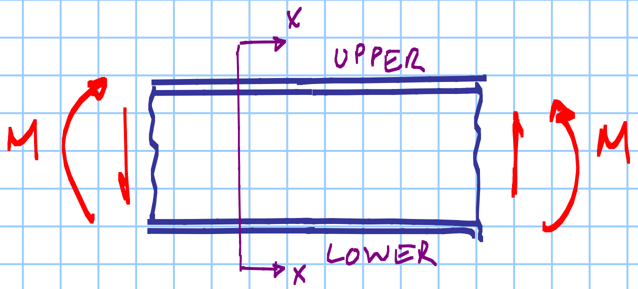
INTERNAL "CRUSHING" LOADS IN THIN WALL BEAMS (BRAZIER)

IRF
26.4.2010

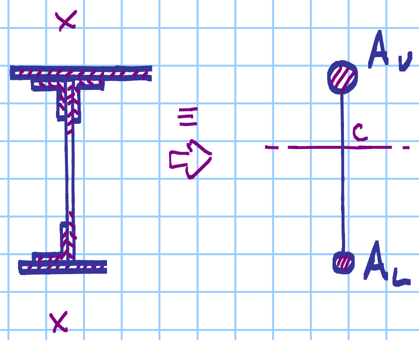
Note Title 18/12/2009

①

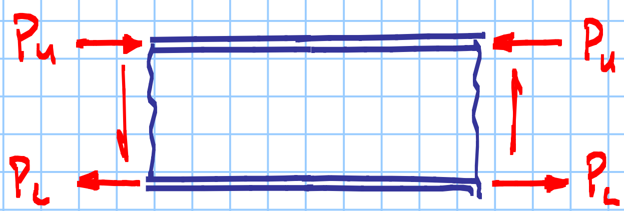


For thin wall beams we can simplify our analysis.

Assuming all bending reacted by flanges as point areas and all shear reacted by web.



Moment reaction can be simplified to tension and compression couple loads in the flanges.

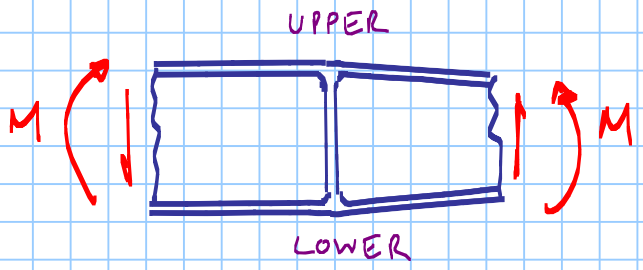


Note couple loads will be equal and opposite in 'upper' and 'lower' booms irrespective of relative areas since balanced by effective centroid and N/A position.

ie $P_U = P_L$

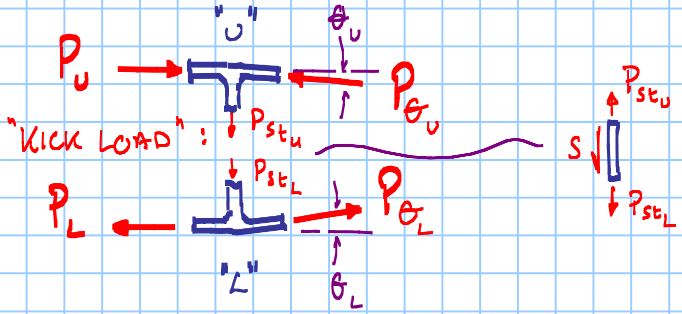
CRUSHING LOADS IN KINKED BEAMS

Bruhn D3.11



I.e. flanges with local changes in direction.

Kinked flanges cause "kick" loads in the web which should be accomodated by extra stiffening.



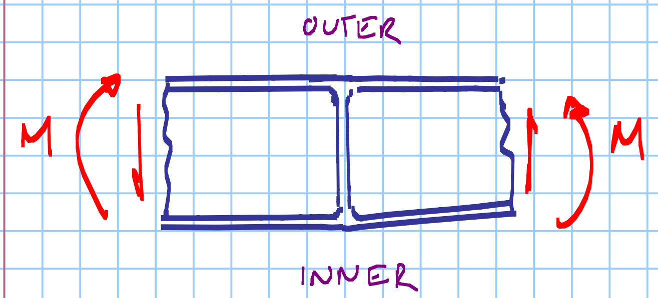
Where $P_u = P_L = P$ .ie "couple load"

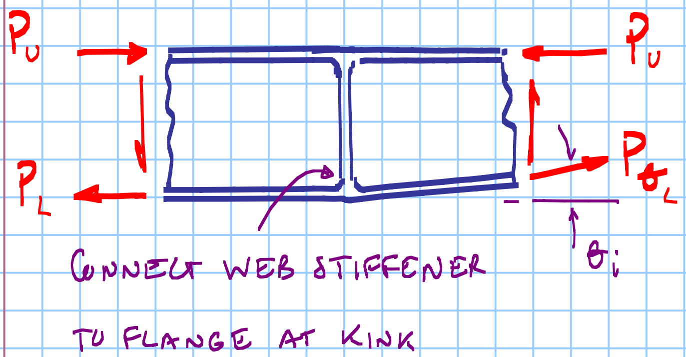
Flanges at an angle must resolve to provide couple loads. I.e. wrt upper or lower flanges:

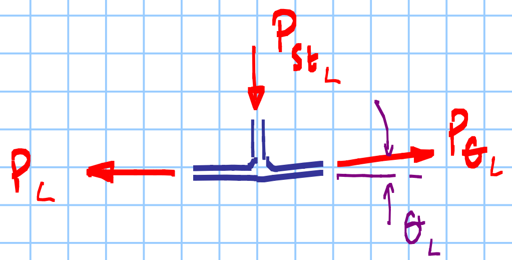
$$P_\theta \cos\theta = P$$
$$\hookrightarrow P_\theta = P/\cos\theta$$

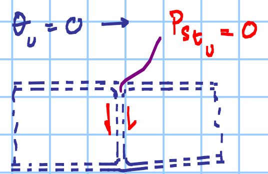
For $P = P_U$ or P_L and $P_\theta = P_{\theta_U}$ or P_{θ_L}

Note, angled flange loads are greater in order to provide the necessary equilibriating couple (ie. $\cos\theta < 1$).









E.g. DBT wing: kink on lower flange only.

Kick load at bottom flange must be dissipated into web as shear as we approach the top flange.

A web stiffener with good connection to the kinked flange is essential to avoid local overloading of the web at the kink. Static equilib'm:

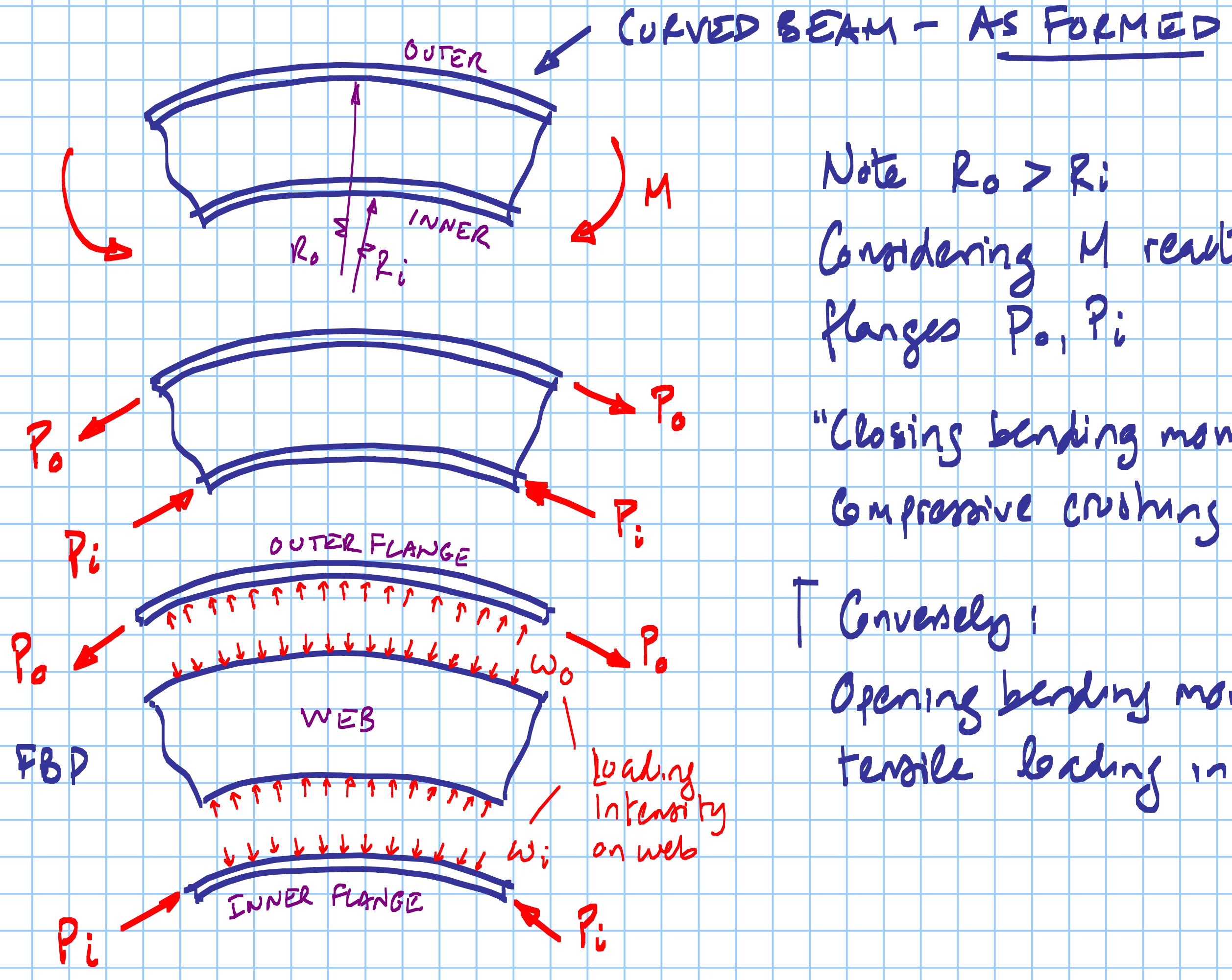
$$\Sigma \rightarrow = 0: \; -P_L + P_{\theta_L}\cos\theta_L = 0: \quad P_{\theta_L} = P_L / \cos\theta_L$$

$$\Sigma \uparrow = 0: \; -P_{st_L} + P_{\theta_L}\sin\theta_L = 0: \hookrightarrow P_{st_L} = P_L \tan\theta_L$$

Note here $\theta_U = 0$ so $P_{st_U} = 0$, ie P_{st_L} dissipates into web shear towards top flange.

CRUSHING LOADS IN CURVED BEAMS

BRUHN D3.10



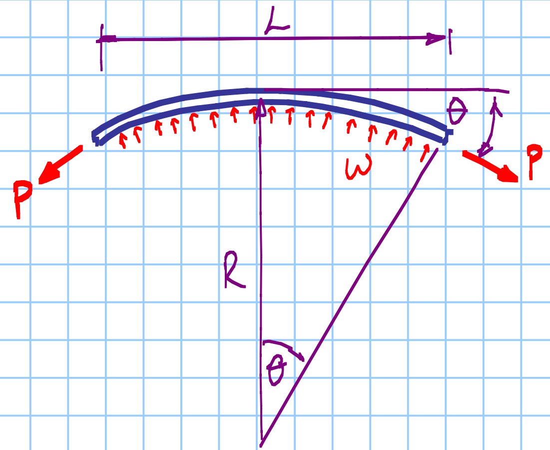
Note $R_o > R_i$

Considering M reacted by couple loads flanges P_o, P_i

"Closing bending moment" results in compressive crushing loading on web

Conversely:

Opening bending moment results in tensile loading in web

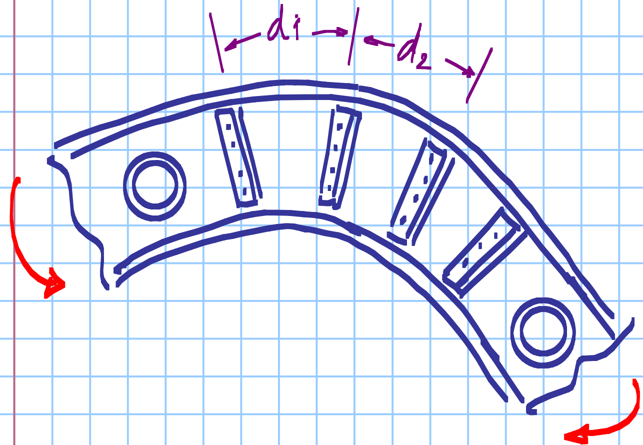


Eg. Considering equilibrium of flange FBD
Radius of curvature R.
Projected arc length L

$\Sigma F^{\uparrow} = 0$: $wL - 2P.\sin\theta = 0$

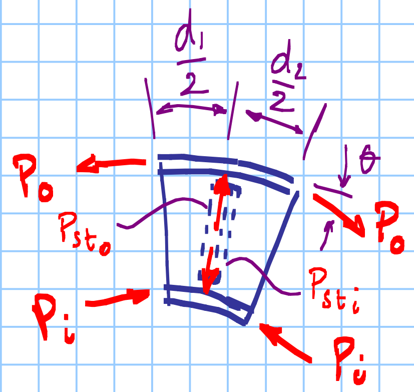
: $wL - 2P.\frac{L/2}{R} = 0$: $\underline{\underline{w = \frac{P}{R}}}$

(ie. analogous to PV analysis!)



Note difference in values of w_o, w_i due to different R_o, R_i will be taken up as shear in the web.

Note beams with significant curvature will need stiffeners to support the web.

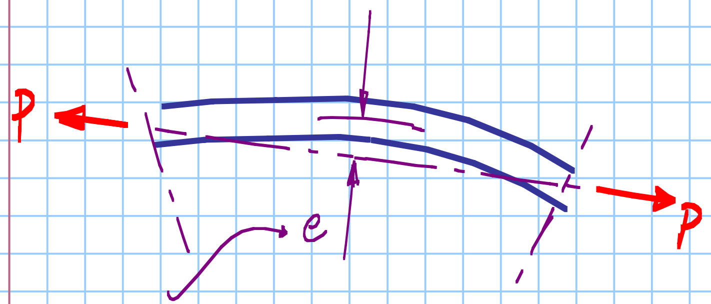


Stiffener loads can be estimated from static equilib'm of FBD. E.g. Consider FBD to mid bay either side of a stiffener.

$$\Sigma \uparrow^{+} = 0 : \quad P_{st_o} = 2 P_o \tan \theta_o$$

$$P_{st_i} = 2 P_i \tan \theta_i$$

Again, any difference in value between the load on the outer and inner side of the stiffener will be taken up as shear in the web.



Eccentricity between stiffener positions due to curvature

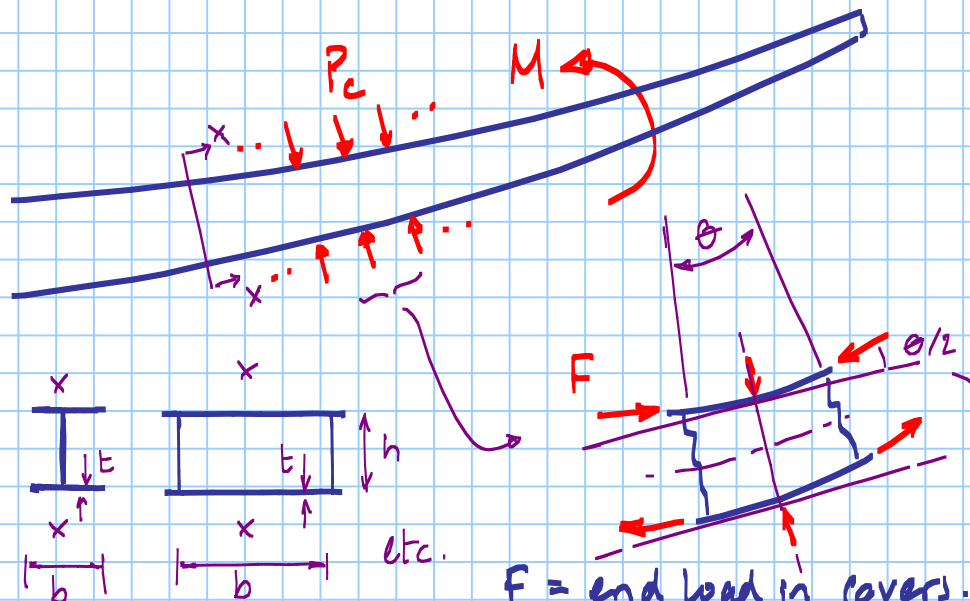
The flanges are designed for end loads P and some allowance for local bending can also be accounted for as $M \approx P.e/2$ at the stiffener positions but this may be negligible.

CRUSHING LOADS DUE TO BENDING CURVATURE

18.12.09

Refs: STM3 "Light Aircraft" - Brazier Effect / Niu "Airframe Stress Analysis + Sizing"

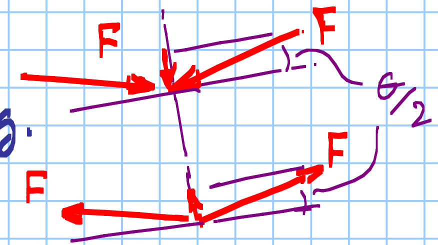
Internal crushing loads are developed within a beam due to curvature under bending. This is usually ignored in solid beams but can be significant in thin wall beams E.g. I beam or box beam (wing box):



F = end load in covers.

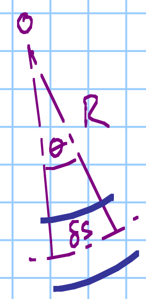
Essentially, the tension and compression end loads in the flanges or covers of the deflected beam resolve into crushing loads across the depth of the beam

E.g.



$$P_c = 2F\sin\frac{\theta}{2}$$

So: $P_c = 2F\sin\left(\frac{\theta}{2}\right) \approx \cancel{2}F\frac{\theta}{\cancel{2}} = F\theta$ ① where θ = rad, ie:



For small angles θ rad $\approx \sin\theta \approx \tan\theta = \frac{\delta s}{R}$

Bending theory: $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$: $\frac{1}{R} = \frac{M}{EI}$ $\rightarrow$ $\theta = \frac{M\,\delta s}{EI}$

① $\rightarrow$ $P_c = F\frac{M}{EI}\delta s$: $\underline{\underline{\frac{P_c}{\delta s} = F\frac{M}{EI}}}$ ② ie crushing load per unit length along beam. "Brazier loading"

Also: $F = \sigma . A$, where σ represents stress in the flanges or covers and A represents their cross sectional areas, eg. b.t

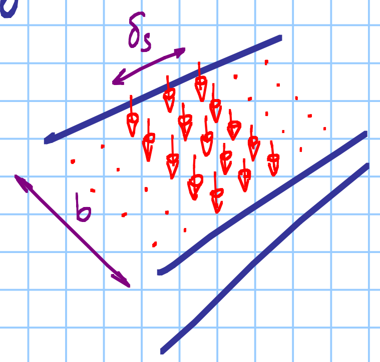
and $\sigma = \frac{My}{I}$

② $\rightarrow$ $\frac{P_c}{\delta s} = \frac{MyA}{I}\frac{M}{EI}$: $\underline{\underline{\frac{P_c}{\delta s} = \frac{M^2Ay}{EI^2}}}$ etc.

Other references may continue to derive expressions assuming

$y \approx \frac{h}{2}$, $I \approx 2bt\left(\frac{h}{2}\right)^2$, $A = bt$ and may be based on $\frac{P_c}{\delta A}$ where $\delta A = b.\delta s$

ie as a planar compressive stress created by the flanges or covers and reacted by the webs and ribs within the beam. E.g.:



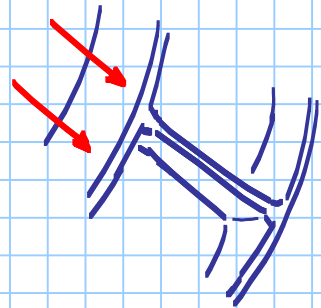
$$\frac{P_c}{\delta s} = \frac{M^2 A y}{EI^2} = \frac{M^2 bt\left(\frac{h}{2}\right)}{EI^2}$$

$$\hookrightarrow \frac{P_c}{\underbrace{\delta_s b}_{\delta A}} = \frac{M^2 t h}{2EI^2} \quad \text{etc.}$$

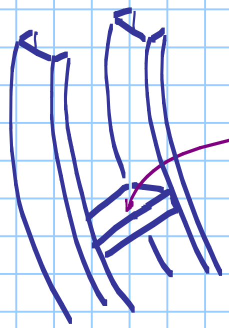
Further support

Flanges subjected to local loading causing bending across the flange should be supported by web stiffening

E.g. flange support at joint position



Long deep thin wall beams should be supported by "intercostals" to prevent lateral buckling.



E.g. Intercostals between fus' frames.