UNIVERSITY OF BRISTOL
FACULTY OF ENGINEERING

First Year Examination for the Degree of
Master of Engineering

MAY/JUNE 2013 3 Hours

FLUIDS 1

AENG11101

This paper contains *two* sections

SECTION 1
Answer *all* questions in this section
This section carries *40 marks*.

SECTION 2
This section has *five* questions.
Answer *three* questions.
All questions in this section carry *20 marks* each.
The maximum for this paper is *100 marks.*

Calculators may be used.

For air, assume R = 287 J/kgK. Take 0^{o}C as 273 K.
Use a gravitational acceleration of 9.81m/s^2
1 bar = 10^5 N/m^2

TURN OVER ONLY WHEN TOLD TO START WRITING

Useful Equations

The volume of a sphere: $\frac{4}{3}\pi r^3$ Area of a circle: πr^2

Roots of a quadratic: $ax^2 + bx + c = 0 \quad \rightarrow \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The equation of state for a perfect gas is:

$$p = \rho RT$$

Drag equation: $\text{Drag} = \text{Area} \times C_D \times \frac{1}{2}\rho V^2$

Equation of a streamline in 2D flow

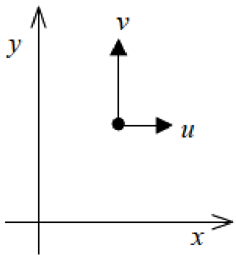
$$\frac{dy}{dx} = \frac{v}{u}$$

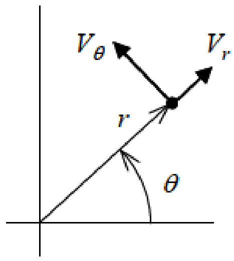
Change between Polar and Cartesian coordinate systems

$$r = \sqrt{x^2 + y^2}, \quad \theta = \tan^{-1}\left(\frac{y}{x}\right), \quad x = r\cos\theta, \quad y = r\sin\theta,$$

$$u = V_r\cos\theta - V_\theta\sin\theta, \qquad v = V_r\sin\theta + V_\theta\cos\theta$$

$$V_r = u\cos\theta + v\sin\theta, \qquad V_\theta = -u\sin\theta + v\cos\theta$$





2D Potential Flow

Velocity components in two-dimensional irrotational flow, in terms of the stream function and potential are given by

$$\underbrace{V_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} \qquad V_\theta = -\frac{\partial\psi}{\partial r} \qquad V_r = \frac{\partial\phi}{\partial r} \qquad V_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta}}_{\text{polar coordinates}} \qquad \underbrace{u = \frac{\partial\psi}{\partial y} \qquad v = -\frac{\partial\psi}{\partial x} \qquad u = \frac{\partial\phi}{\partial x} \qquad v = \frac{\partial\phi}{\partial y}}_{\text{Cartesian coordinates}}$$

The stream function & velocity potential in Polar coordinates and the velocity distribution for

i) A uniform flow U_∞ parallel to the x axis:

$$\psi = U_\infty r\sin\theta, \quad \phi = U_\infty r\cos\theta, \quad V_r = U_\infty\cos\theta, \quad V_\theta = -U_\infty\sin\theta, \quad u = U_\infty, \quad v = 0$$

ii) A source, of strength Λ at the origin:

$$\psi = \frac{+\Lambda\theta}{2\pi}, \quad \phi = \frac{+\Lambda}{2\pi}\ln r, \quad V_r = \frac{+\Lambda}{2\pi r}, \quad V_\theta = 0, \quad u = \frac{+\Lambda}{2\pi}\frac{x}{(x^2 + y^2)}, \quad v = \frac{+\Lambda}{2\pi}\frac{y}{(x^2 + y^2)}$$

iii) A doublet, of strength κ at the origin:

$$\psi = \frac{-\kappa}{2\pi}\frac{\sin\theta}{r}, \qquad \phi = \frac{+\kappa}{2\pi}\frac{\cos\theta}{r}, \qquad V_r = \frac{-\kappa}{2\pi r^2}\cos\theta, \qquad V_\theta = \frac{-\kappa}{2\pi r^2}\sin\theta,$$

$$u = \frac{-\kappa}{2\pi}\frac{\left(x^2 - y^2\right)}{\left(x^2 + y^2\right)^2}, \qquad v = \frac{-\kappa}{2\pi}\frac{2xy}{\left(x^2 + y^2\right)^2}$$

iv) A vortex, of strength Γ́, at the origin:

$$\psi = \frac{+\Gamma}{2\pi}\ln r, \quad \phi = \frac{-\Gamma}{2\pi}\theta,$$

$$V_r = 0, \quad V_\theta = \frac{-\Gamma}{2\pi r}, \qquad u = \frac{+\Gamma}{2\pi}\frac{y}{\left(x^2 + y^2\right)}, \quad v = \frac{-\Gamma}{2\pi}\frac{x}{\left(x^2 + y^2\right)}$$

Useful integrals

$$\int \sin^2(ax)\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a} + C$$

$$\int \cos^2(ax)\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a} + C$$

turn over …

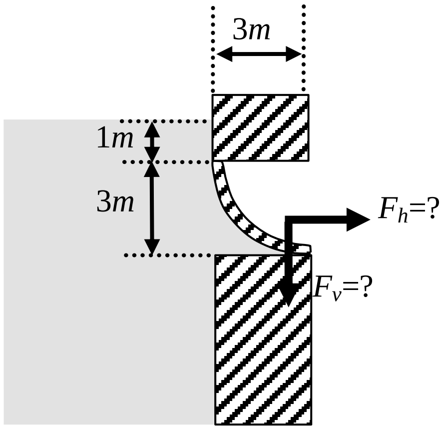
SECTION 1
Answer all questions in this section

Q1 Two pressure transducer are lowered 2*m* into two reservoirs. The first reservoir contains water, the second reservoir contains a light oil. If the difference in gauge pressure is 4000Nm^{-2} and the density of the water is 1000 $kg\ m^{-3}$, what is the specific gravity of the oil?

(4 marks)

Q2 A dam has a sluice gate whose cross section is a quarter circle of radius 3*m*. The gate is 4*m* wide. If the water just reaches a height of 1*m* above the top of the gate, find the vertical and horizontal components of thrust on the gate. The water density can be taken as 1000 $kg\ m^{-3}$

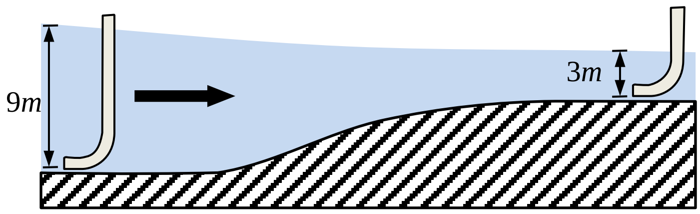
(4 marks)



Q3 State the assumptions that must be made for Bernoulli's equation to be valid along the streamline A-A shown. Write an equation for the static pressure at C in terms of the static pressure and velocity at B (p_B V_B) as well as the incompressible density ρ, if the fluid is a gas.

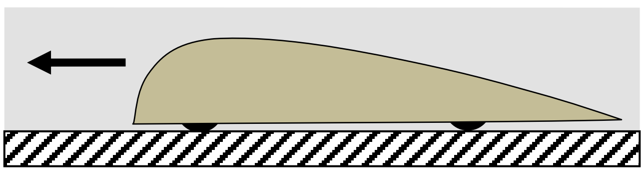
(4 marks)

Q4 Water flows down a river with slowly varying depth. The width can be assumed constant and the cross section rectangular. At a certain point where the depth of water is 9m, a pitot probe measures a gauge pressure of $0.95\text{x}10^5$ Nm^{-2} at the bottom of the river. At a point further downstream, where the height of the water is 3*m*, find the velocity of the water and the gauge pitot pressure measured at the bottom. The density of the water as 1000 $kg\ m^{-3}$.



(4 marks)

Q5 (a) A wheeled vehicle designed for maximum fuel efficiency is shown in the diagram. Explain this design in terms of form drag and skin friction.



(b) What is the transformation that allows wind tunnel testing of moving air over a static body? Which fundamental flow variables remain unchanged by this transformation?

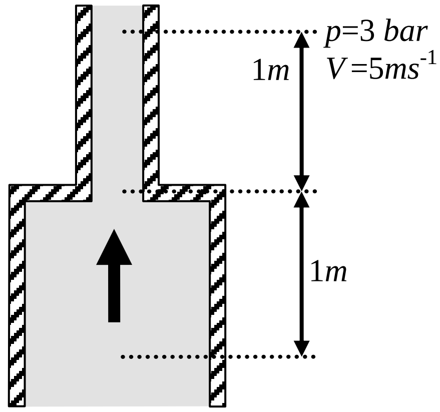
(4 marks)

Q6 Draw an aerofoil at zero incidence and then draw and label regions where you would expect the viscous forces to be important or the flow to be essentially inviscid. What would you expect to happen to these regions if: the speed were increased but the incidence remained zero; the incidence were increased to above 20^o?

(4 marks)

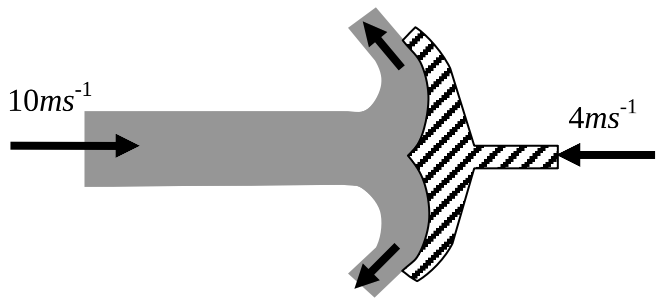
Q7 Oil, with a specific gravity of 1.1, flows up a straight smooth vertical pipe with a circular cross-section. At a particular point, 1m downstream of a sudden contraction, the pressure and velocity at the centre of the cross section are 3 *bar* and $5ms^{-1}$ respectively.

Find the pressure at the centre of the cross section 1m upstream of the contraction if the area ratio across the contraction is 4 and the velocity profiles are assumed to have the same scaled shape at each location.



(4 marks)

Q8 A horizontal circular water jet of diameter 20 *cm* and speed 10 ms^{-1} hits a plate moving with a constant velocity of 4 ms^{-1} into the jet. By using a suitable control volume, find the horizontal force that must be applied to the plate if all the water leaving the plate is turned through an angle of 130^o to the plate with no loss of speed. Assume the water has a density of 1000 $kg\ m^{-3}$.

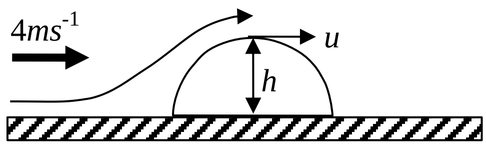


(4 marks)

Q9 If the fluid velocity components in a potential flow field are given by
$u = -\omega\, y \qquad v = \omega\, x,$
where ω is a constant, find an equation for the streamlines.

(4 marks)

Q10 An air flow over a body with a semi-circular cross-section can be modelled as a horizontal onset flow of 4 ms^{-1} and a doublet of strength $128\pi\ m^2s^{-1}$. Find the height, *h*, of the body and the velocity, *u*, at the highest point.



(4 marks)

turn over…

SECTION 2

Answer *three* questions in this section

Q11 (a) A spherical hydrogen balloon is used to carry meteorological equipment. If it is assumed that the pressure and temperature within the balloon stay in equilibrium with the atmosphere as the balloon rises, show that the balloon radius is related to the atmospheric density by

$$\frac{\rho}{\rho_{sl}} = \frac{r_{sl}^3}{r^3}$$

where the subscript "sl" represents values at sea level before release. Hence show that the net upward force (buoyancy minus weight) does not change with altitude and is given by

$$F = \tfrac{4}{3}\pi r^3 \rho g - m_B g$$

where m_B is the total mass of the balloon instruments and hydrogen.

(6 marks)

(b) The atmospheric temperature decreases linearly with altitude (at a rate λ) so that the pressure and temperature at an altitude, *z*, are given by

$$p = p_{sl}\left(1 - \lambda z / T_{sl}\right)^{\frac{g}{R\lambda}} \quad \text{and} \quad T = T_{sl} - \lambda z$$

where "sl" denotes the value at sea level (z=0), g is the acceleration due to gravity and R is the specific gas constant of air.

Find the maximum altitude the balloon will reach if it bursts when it reaches a diameter three times greater than that at sea level. You may assume that R=287 $J\ kg^{-1}\ K$ and λ=0.0065$^{o}Km^{-1}$, while the temperature at sea level is 20^{o}C and the instruments have negligible volume.

(7 marks)

(c) Find the terminal velocity of the balloon just as it bursts if the total mass of the balloon instruments and hydrogen is 30*kg*. You may assume that the density of the air at sea level is 1.2 $kg\ m^{-3}$, the balloon radius at sea level is 2*m* and the Drag coefficient (C_D), based on balloon cross-sectional area is 0.45.

(7 marks)

Q12 Water is siphoned from a large tank as shown in figure Q12. The highest point of the pipe is h above the water level in the tank, while the exit of the pipe is at a height H below the water level in the tank. The constant internal cross sectional area of the pipe is A.

(a) Show that the mass flow rate, $\dot{m}$, when the height of the water surface is H above the exit of the pipe is

$$\dot{m} = \rho A\sqrt{2gH}$$

and the maximum height of the siphon is given by

$$h_{\max} = \frac{(p_a - p_v)}{\rho g} - H$$

where p_a and p_v are the atmospheric and water vapour pressure respectively.

(13 marks)

(b) For the siphon system defined above, find the velocity and cross sectional area of the jet 1m below the exit of the siphon for the maximum height of siphon above the water surface (h=h_{max}). The siphon height (h) is 3m, the atmospheric pressure is 1.02x10^5Pa, the vapour pressure is 2330Pa, the cross sectional area of the siphon is 0.0013m^2 and the density of water is 1000 $kg\ m^{-3}$.

(13 marks)

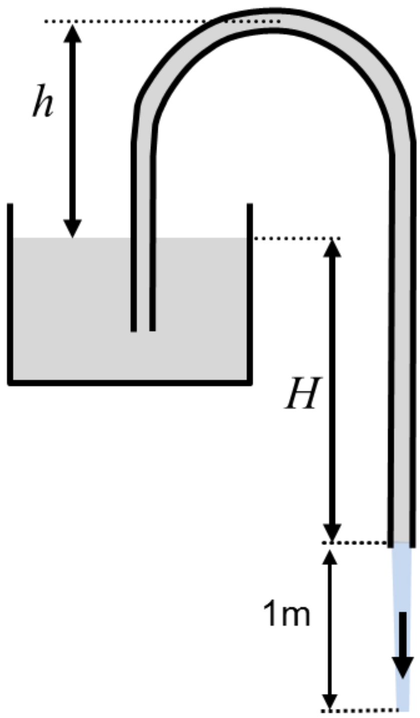


***Figure Q12**: Diagram of siphon system used in question 12.*

turn over…

Q13 An axisymmetric body is being tested in a wind tunnel with a circular test section as shown in the figure Q13. The air in the tunnel has a density of 1.2 $kg\ m^{-3}$ and the test section radius is R=0.5m. Measurements at location A indicate that the incoming velocity is uniform with U_A= 10 ms^{-1} while the downstream velocity, measured at location B, takes the form:

$$U_B = U_{\max}\left(\frac{r^3}{R^3} - 3\frac{r^2}{R^2} + 3\frac{r}{R}\right) ,$$

where r is measured from the tunnel's centreline. A U-tube manometer filled with water (with a density of 1000 $kg\ m^{-3}$) is used to measure the pressure difference between sections A and B (the pressure is assumed uniform at each section). The difference in height between the two legs of the manometer is h=0.003 m, as shown in the figure.

(a) Determine the maximum velocity, $U_{\max}$, at section B given that the mass flow rate at any section, with a velocity U, can be calculated using

$$\dot{m} = \int_{r=0}^{R}\int_{\theta=0}^{2\pi} \rho U r d\theta\, dr$$

(10 marks)

(b) Determine the drag force acting on the body

(10 marks)

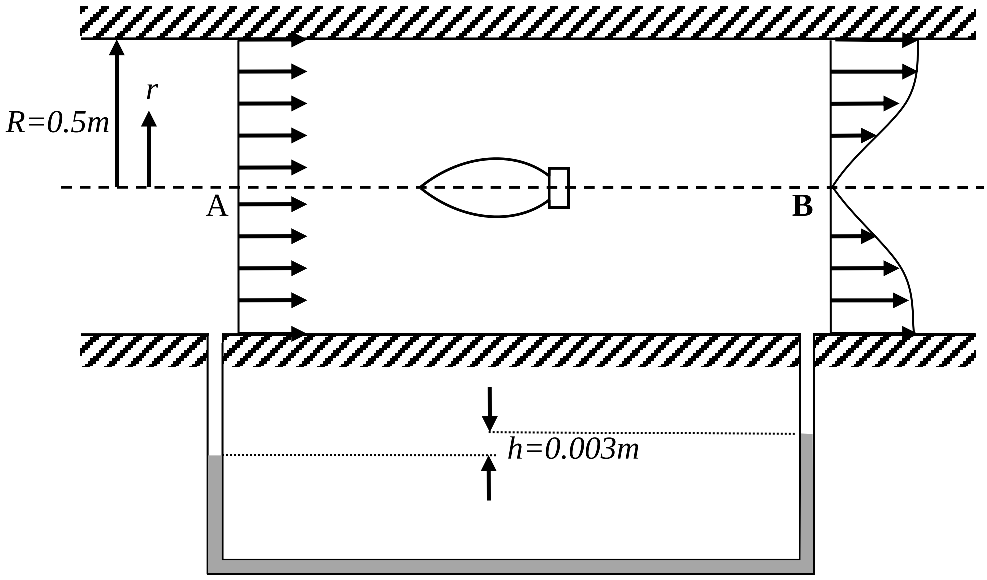


Figure Q13**: Diagram of the axisymmetric body in a wind tunnel with a circular cross section as used in question **Q13

Q14 A helicopter rotor sweeps out a circular disc of diameter *d* while climbing vertically through still air at a speed of $V\ ms^{-1}$.At a position underneath the rotor, where the pressure has returned to atmospheric, the velocity of the air relative to the ground is measured at $(a\text{-}1)V\ ms^{-1}$. a is a constant that represents the ratio of the relative downstream wind speed to the vertical velocity.

(a) Use the actuator disc theory for an ideal propeller to show from first principles that the force supplied by rotor can be written as.

$$F = \rho \tfrac{\pi}{8} d^2 V^2 \left(a^2 - 1\right)$$

where ρ is the density of the air. Clearly state all assumptions made during your derivation.

(8 marks)

(b) Further, show that the efficiency when in vertical motion is given by

$$\eta = \frac{2}{(1+a)}$$

(7 marks)

(c) A helicopter of mass 7000*kg* has a 14*m* diameter ideal rotor. If the air density is given by 1.2 $kg\ m^{-3}$, find the airspeed through the disc when the helicopter is in hover. Further, find the power required to maintain the hover.

(5 marks)

turn over...

Q15 (a) A free stream, a doublet and a vortex can be used to model the lifting flow over a rotating cylinder. Find the doublet strength, in terms of the cylinder radius. Hence find the velocity and pressure coefficient distributions on the cylinder surface. Then show that the pressure on the cylinder is given by

$$p(\theta) = p_\infty + \frac{1}{2}\rho U_\infty^2\left(1-4\sin^2\theta\right) - \left(\frac{\rho U_\infty \Gamma \sin\theta}{\pi R}\right) - \frac{1}{8}\rho\left(\frac{\Gamma}{\pi R}\right)^2$$

where Γ is the vortex strength

(6 marks)

(b) Consider a spinning cylinder whose radius is R. By considering the forces acting on a small element of the surface and assuming that the flow field is given by incompressible irrotational flow, show that the lift per unit length of cylinder is

$$l = \rho U_\infty \Gamma$$

(9 marks)

(c) Hence find the lift per unit length on a spinning cylinder of diameter 1*m* in a 4*ms*$^{-1}$ headwind if only a single stagnation point is on the surface of the cylinder.

(5 marks)