IRF 16.2.2009

ASD2

DATA + EXAMPLES

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Refer to full texts for details.

REFERENCES

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Conmilit Press Ltd. 1991 ISBN 962-7128-04-X

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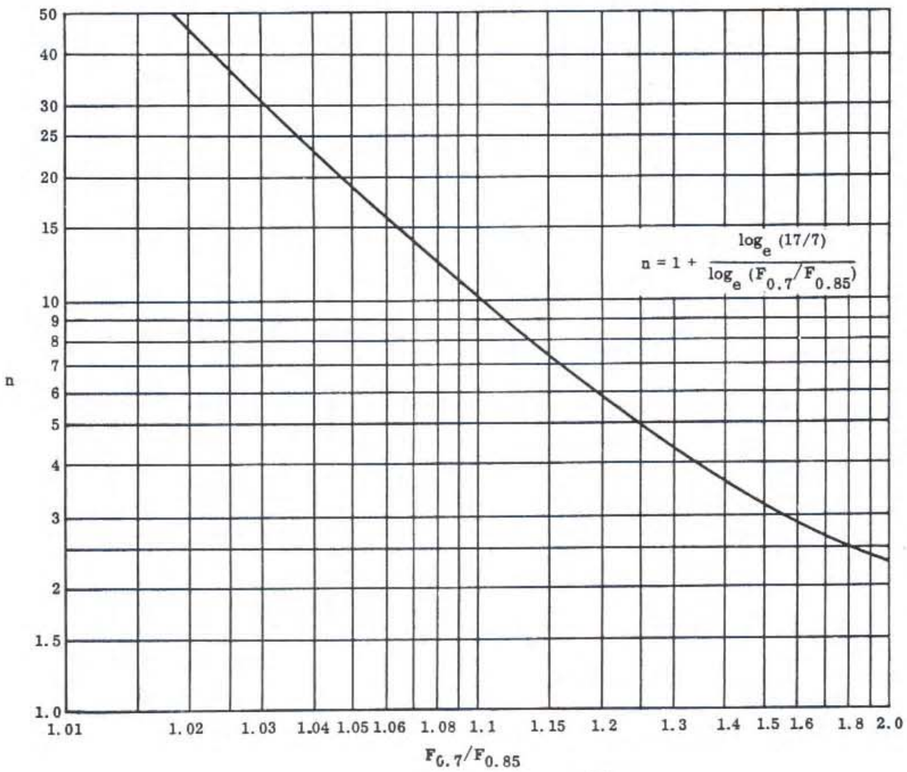
Hong Kong Conmilit Press Ltd 1997. ISBN 962-7128-07-4

Bruhn B1.9

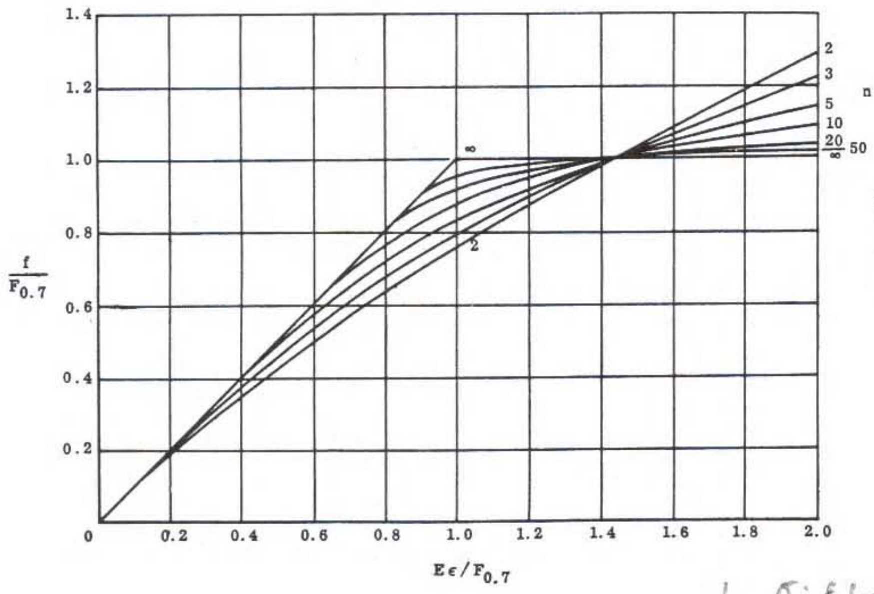
RAMSBERG + OSGOOD Inelastic σ-ε curve approximation

Fig. B1.13



Exponent n
(May be available from material tables.
E.g. see Bruhn B1.10 below

Fig. B1.14



*

FOR KNOWN
E, n, $F_{0\cdot7}$
VALUES

↳ σ : ε for inelastic region.

Bruhn B1.10

Note

RAMSBERG + OSGOOD

e σ_{TU} σ_{CU} E_c $\sigma_{0.7}$ $\sigma_{0.85}$ n

Table B1.1 Values of F_{tu}, F_{cy}, E_c, $F_{0.7}$, $F_{0.85}$, n, for Various Materials Under Room & Elevated Temperatures (From Ref. 6)

MATERIAL	Temp. Exp. Hr.	Temp. °F	e, %	F_{tu}, ksi	F_{cy}, ksi	E_c 10^6psi	$F_{0.7}$ ksi	$F_{0.85}$ ksi	n
STAINLESS STEEL									
AISI 301 1/4 Hard Sheet	1/2	RT	25	125	80	27.0	73	63	6.9
Transverse Compression									
Longitudinal Compression	1/2	RT	25	125	43	26.0	28.2	23	5.2
AISI 301 1/2 Hard Sheet	1/2	RT	15	150	118	27.0	116.5	105	9.2
Transverse Compression	1/2	400		118	108.5	23.2	108.5	97	8.6
	1/2	600		110	107.5	20.9	108.5	96.5	8.2
	1/2	1000		86	86	16.2	94.5	83.5	8.0
Longitudinal Compression	1/2	RT	15	150	58	26.0	48	37	4.4
	1/2	400		118	53.3	22.4	45.5	36	4.7
	1/2	600		110	52.8	20.1	44	31	3.5
	1/2	1000		86	45.2	15.6	40	30.5	4.3
AISI 301 3/4 Hard Sheet	1/2	RT	12	175	160	27.0	163.5	151.5	13.2
Transverse Compression	1/2	400		148	148	24.1	153	142.5	13.2
	1/2	600		138	138	22.4	152	140	11.2
	1/2	1000		112	112	18.9	127	121	19.2
Longitudinal Compression	1/2	RT	12	175	76	26.0	70	61.5	7.6
	1/2	400		148	71	23.3	65	56	6.8
	1/2	600		138	70.3	21.6	65.5	56.5	6.8
	1/2	1000		112	59.3	18.2	55	46	5.9
AISI 301 Full Hard Sheet	1/2	RT	8	185	179	27.0	183	172	16
Transverse Compression	1/2	400		168	168	25.1	174	164	16
	1/2	600		159	159	23.8	172	162	16
	1/2	1000		131	130	21.6	141.5	135.5	21.5
Longitudinal Compression	1/2	RT	8	185	85	26.0	77.5	63	5.2
	1/2	400		168	80.8	24.2	74	59.5	5
	1/2	600		159	79.9	22.9	74	58	4.6
	1/2	1000		131	66.3	20.8	58	42.5	3.9
17-4 PH Bar & Forgings	1/2	RT	6	180	165	27.5	166	160	24
	1/2	400		162	135	25.3	137	129	16
	1/2	700		146	105.5	23.1	106	97	11
	1/2	1000		88	62.6	21.2	60	52	7.1
17-7 PH (TH1050) Sheet, Strip & Plate, t = .010 to .125 in.	1/2	RT		180	162	29.0	166	145	7.4
	1/2	400		169	144	27.8	146	126	6.8
	1/2	700		144	118	24.9	117	104	8.4
	1/2	1000		88	61.5	20.3	56	47	6
17-7 PH (RH950) Sheet, Strip & Plate, t = .010 to .125 in.	1/2	RT		210	205	29.0	208	196	16.4
19-9DL (AMS 5526) & 19-9DX (AMS 5538), Sheet, Strip & Plate	1/2	RT	30	95	45	29.0	36.5	32	7.6
19-9DL (AMS 5527) & 19-9DX (AMS 5539) Sheet, Strip & Plate	1/2	RT	12	125	90	29.0	85	74	7.2
PH15-7Mo (TH1050) Sheet & Strip, t = .020 to .187 in.	1/2	RT	5	190	170	28.0	171	164	22.5
PH15-7Mo (RH950) Sheet & Strip, t = .020 to .187 in.	1/2	RT	4	225	200	28.0	218	189	7.3
LOW CARBON & ALLOY STEELS									
AISI 1023 & 1025 Tube, Sheet & Bar, Cold Finished		RT	22	55	36	29.0	32.7	31.5	24
AISI 4130 Normalized, t > .188 in.	1/2	RT	23	90	70	29.0	61.5	53	6.8
	1/2	500		81	61.5	27.3	55	48	7.3
	1/2	800		68	46.2	23.8	40	32.5	5.2
	1/2	1000		46	30.8	20.6	28	22	4.7
AISI 4130, 4140, 4340 Heat Treated	1/2	RT	23	125	113	29.0	111	102	10.9
	1/2	500		113	98.3	27.3	96	88	10.9
	1/2	850		88	68.9	23.2	66.5	61.5	12
	1/2	1000		64	49.7	20.6	45.5	41	9.2
AISI 4130, 4140, 4340 Heat Treated	1/2	RT	18.5	150	145	29.0	145	140	25
	1/2	500		135	126	27.3	126	122	29
	1/2	850		105	88.5	23.2	88	83.5	18.5
	1/2	1000		76	63.8	20.6	62	57	10.9
AISI 4130, 4140, 4340 Heat Treated	1/2	RT	15	180	179	29.0	179	176	50
	1/2	500		162	156	27.3	156	153	46
	1/2	850		126	109.3	23.2	109.4	105	22
	1/2	1000		92	77	20.6	75	68	9.8
AISI 4130, 4140, 4340 Heat Treated	1/2	RT	13.5	200	198	29.0	198	196	90
	1/2	500		180	170	27.3	172.5	169	46
	1/2	850		140	121	23.2	121.5	117	25
	1/2	1000		104	87.1	20.6	87	83	19
HEAT RESISTANT ALLOYS									
A-286 (AMS 5725A) Sheet, Plate & Strip	1/2	RT	15	140	95	29.0	93	87	14
	1/2	600		129	88.4	24.4	87	81	13.5
	1/2	1000		115	81.7	19.8	81	75	12.5
	1/2	1400		52	50.3	14.2	50	47	15.3
K-MONEL Sheet, Age Hardened	1/2	RT	15	125	90	26.0	88	82	13.5
MONEL Sheet, Cold Rolled & Annealed	1/2	RT	35	70	28	26.0	20	17	6.4
INCONEL-X	1/2	RT	20	155	105	31.0	104	100	23.5
	1/2	400		152	95.6	28.9	94	89	17
	1/2	800		141	90.2	26.4	88.6	84	18.5
	1/2	1200		104	83	23.2	82	78.6	21

SI Conversion: 1 psi = 6'895 Pa = $6{\cdot}895 \times 10^{-3}$ N/mm²

1 ksi = 6·895 N/mm²

Bruhn B1.11

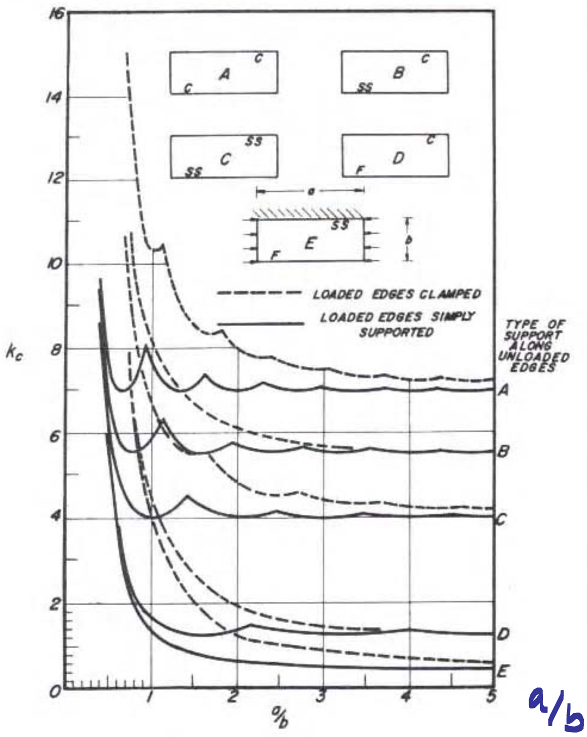
Table B1.1 Values of F_{tu}, F_{cy}, E_c, $F_{0.7}$, $F_{0.85}$, n, for Various Materials Under Room & Elevated Temperatures (From Ref. 6) (Continued)

MATERIAL	Temp. Exp. Hr.	Temp. °F	e, %	F_{tu}, ksi	F_{cy}, ksi	E_c 10^6psi	$F_{0.7}$ ksi	$F_{0.85}$ ksi	n
ALUMINUM ALLOYS									
2014-T6 Extrusions	2	RT	7	60	53	10.7	53	50.3	18.5
t ≤ 0.499 in.	2	300		51	42.5	10.2	41.5	40	24
	2	450		28	21	9.2	20.5	19.5	25
	2	600		10	8.0	7.4	5.5	4.5	5.4
	1/2	300		51	43.5	10.2	44.0	42.5	25
	1/2	450		31	26	9.2	26	25.2	29
2014-T6 Forgings	2	RT	7	62	52	10.7	52.3	50	20
t ≤ 4 in.	2	300		53	41	10.2	40.5	38.5	19
	2	450		29	22	9.2	21.5	20	12.6
	2	600		10	7.5	7.4	4.5	3.0	3.2
	1/2	300		53	43	10.2	42.5	40	15.8
	1/2	450		32	25.5	9.2	25.0	23.5	15.6
2024-T3 Sheet & Plate,	2	RT	12	65	40	10.7	39	36	11.5
Heat Treated, t ≤ .250 in.	2	300			37	10.3	35.7	33.5	15
	2	500			26	8.4	24.8	22.8	10.9
	2	700			7.5	6.4	6.2	5.5	8.2
2024-T4 Sheet & Plate,	2	RT	12	65	38	10.7	36.7	34.5	15.6
Heat Treated, t ≤ 0.50 in.	2	300			34	10.3	32.5	30.5	14.6
	2	500			24	8.4	23	21	10.2
	2	700			7	6.4	60	5.7	18.5
2024-T3 Clad Sheet & Plate,	2	RT	12	60	37	10.7	35.7	33	12
Heat Treated, t = .020 to .062 in.	2	300			34	10.3	33	30.3	11
	2	500			24.5	8.4	22.7	20	7.9
	2	700			6.5	6.4	5.8	5.5	18.5
2024-T6 Clad Sheet & Plate,	2	RT	8	62	49	10.7	49	45	11
Heat Treated, t ≥ 0.063 in.	2	300			45	10.3	44.3	40.7	11
	2	500			22	8.4	31.5	28	8.3
	2	700			6	6.4	7.0	6.0	6.6
2024-T6 Clad Sheet & Plate,	2	RT	8	60	47	10.7	47	43	10.6
Heat Treated, t < 0.063 in.	2	300			43.2	10.3	42.3	38.7	10.8
	2	500			21	8.4	29.5	26	7.8
	2	700			6	6.4	5.00	4.0	4.9
2024-T81 Clad Sheet, Heat Treated,	2	RT	5	62	55	10.7	56	51.6	11.2
t < 0.064 in.	2	300			50.5	10.3	51.2	46.5	10
	2								
	2								
6061-T6 Sheet, Heat Treated & Aged,	1/2	RT	10	42	35	10.1	35	34	31
t < 0.25 in.	1/2	300			29.5	9.5	29	28	26
	1/2	450			20.5	8.5	19.3	17.7	10.9
	1/2	600			7.5	7.0	6.6	6.2	15.2
7075-T6 Bare Sheet & Plate,	2	RT	7	76	67	10.5	70	63	9.2
t ≤ 0.50 in.	2	300			54	9.4	55.8	52.5	15.6
	2	425			25.5	8.1	25.4	23.5	12.1
	2	600			8	5.3	7.2	5.2	3.7
	1/2	425			30	8.1	34.5	32.5	16
7075-T6 Extrusions,	2	RT	7	75	70	10.5	72	68	16.6
t ≤ 0.25 in.	2	300			54	9.4	58.5	54.5	13.4
	2	450			22.5	7.8	21.3	18.5	7.2
	2	600			8	5.3	6.5	4.3	3.2
	1/2	450			25	7.8	29	26	8.8
7075-T6 Die Forgings,	2	RT	7	71	58	10.5	58.5	55.1	15.2
t ≤ 2 in.	2	300			47.6	9.4	47.8	45	15.6
	2	450			18.5	7.8	17.3	16	12
	2	600			7.0	5.3	5.0	3.7	3.9
	1/2	450			23	7.8	24	22	10.9
7075-T6 Hand Forgings,	2	RT	4	72	63	10.5	63.8	61.5	25
Area ≤ 16 sq. in.	2	300			51.6	9.4	52.2	50	21.5
	2	450			20.2	7.8	20.3	19	13.7
	2	600			7.6	5.3	6.0	5.0	5.8
	1/2	450			24	7.8	26.5	25.3	19.5
7075-T6 Clad Sheet & Plate,	2	RT	8	70	64	10.5	64.5	61.6	19.5
t ≤ 0.50 in.	2	300			50	9.4	54	51.7	20
	2	450			20.5	7.8	19.7	17.5	4.6
	2	600			7.7	5.3	7.7	5.5	3.6
	1/2	450			23	7.8	27.2	25.3	12.4
7079-T6 Hand Forgings,	1/2	RT	4	67	59	10.5	59.5	57.5	26
t ≤ 6.0 in.	1/2	300			47	9.4	46.5	45	29
	1/2	450			21	7.8	20	18.5	12
	1/2	600			7.0	5.3	5.5	3.5	3.0
MAGNESIUM ALLOYS									
AZ61A Extrusions, t ≤ 0.249 in.		RT	8	38	14	6.3	12.9	12.3	19
HK31A-0 Sheet	1/2	RT	12	30	12	6.5	10	8.4	6
t = 0.016 to 0.250 in.	1/2	300		20	11.1	6.16	8.9	6.9	4.5
	1/2	500		15	9.3	4.94	7.5	5.6	4.2
	1/2	600		10	4.9	3.77	3.3	1.6	2.2
HK31A-H24 Sheet,	1/2	RT	4	34	19	6.5	17.3	14.6	6.2
t < 0.250 in.	1/2	300		22	17.7	6.2	15.6	12.6	5.1
	1/2	500		17	14.8	4.9	13.1	10.5	4.9
	1/2	600		11	7.8	3.8	6.7	5.2	4.5
TITANIUM ALLOYS									
Ti-8Mn Annealed Sheet, Plate & Strip	1000	RT	10	120	110	15.5	119.5	102	13.7
Ti-6Al-4V Annealed Bar	1/2	RT	10	130	126	16.0	127	124.5	43
& Sheet, t ≤ .187 in.	1/2	400		105	96	14.1	97	93	22
	1/2	600		99	84.5	13.0	85.5	82	22
	1/2	800		87	79.4	11.8	80.5	77	21.5
	1/2	1000		70	60.6	7.7	61	59.5	36

Bruhn C5.2

$$\sigma_{c_{CR}} = \frac{k_c \pi^2 \eta_c E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2$$

k_c



a/b

Fig. C5.2 (Ref. 1) Compressive-buckling coefficients for flat rectangular plates.

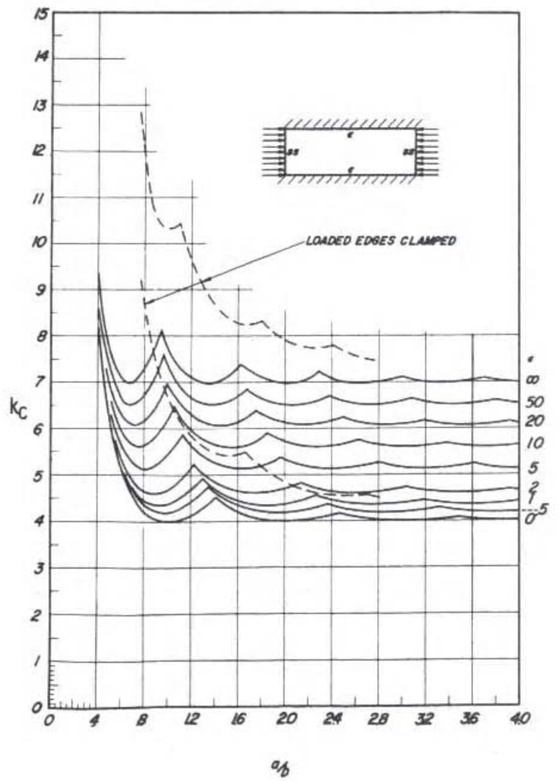


Fig. C5.3 (Ref. 1) Compressive-buckling-stress coefficient of plates as a function of a/b for various amounts of edge rotational restraint.

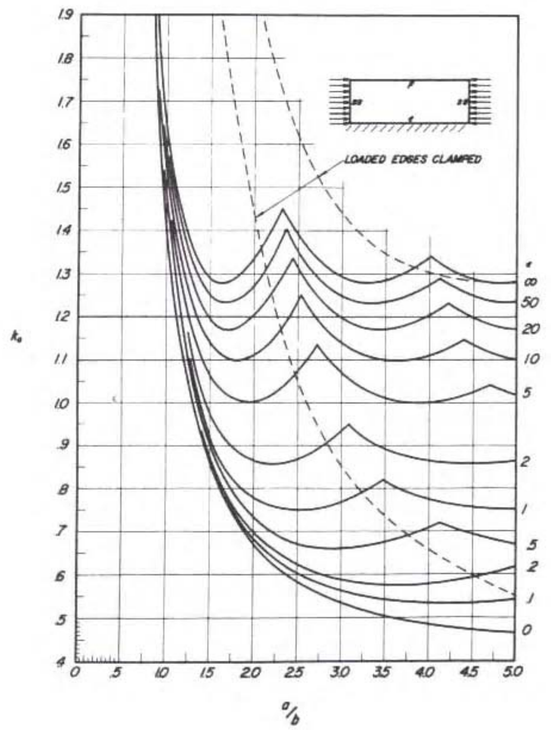


Fig. C5.4 (Ref. 1) Compressive-buckling-stress coefficient of flanges as a function of a/b for various amounts of edge rotational restraint.

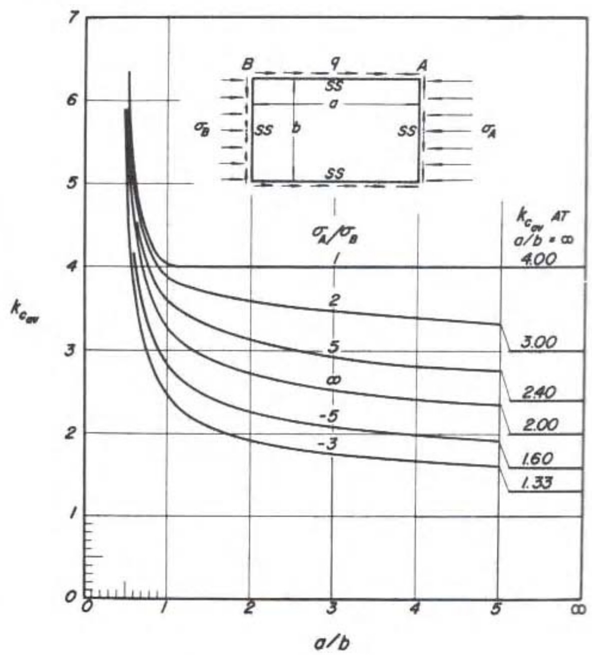
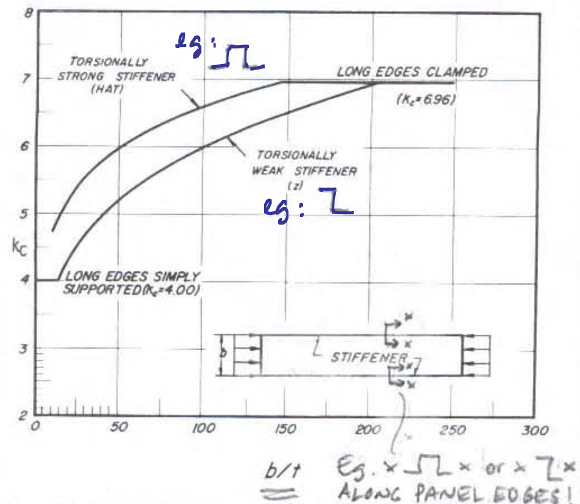


Fig. C5.5 (Ref. 1) Average compressive-buckling-stress coefficient for rectangular flat plate of constant thickness with linearly varying axial load. $\sigma_{av} = \frac{k_{c_{av}} \pi^2 E}{12(1-\nu_e^2)} \left(\frac{t}{b}\right)^2$.

In this course we will either assume ideal end conditions i.e: either free, simply supported or fixed (clamped), or assume edge conditions for torsionally strong or weak edge support (Fig C5.6 below)

Bruhn C.S.3

which is a closed section and, therefore, a relatively torsionally strong stiffener. Fig. C5.6a gives the compression buckling coefficients k_c for isosceles triangular plates.



k_c

Fig. C5. 6 (Ref. 1) Compressive-buckling coefficient for long rectangular stiffened panels as a function of b/t and stiffener torsional rigidity.

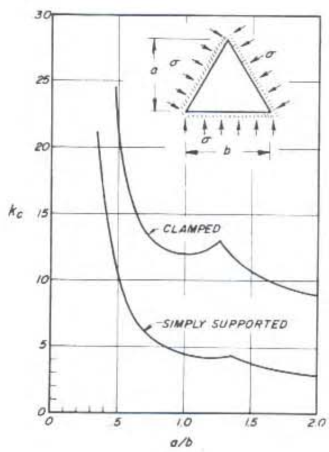


Fig. C5. 6a (Ref. 1) Uniform Compression.

Note

Illustrative Problem. Find the compressive buckling stress for a sheet panel with (a) = 10 and b = 5 inches, thickness t = .04 and all edges are simply supported. Material is 2024-T3 aluminum alloy.

Solution: E = 10,700,000. ν_e = 0.3, a/b = 10/5 = 2. The boundary or edge condition corresponds to Case (c) in Fig. C5.2. Thus using curve (c) for a/b = 2, we read k_c = 4.0.

Substituting in Eq. C5.1,

$$\sigma_{cr} = \frac{\pi^2 \times 4.0 \times 10{,}700{,}000}{12\,(1 - .3^2)} \left(\frac{.04}{5}\right)^2 = 2480 \text{ psi.} = 17.1\ \text{N/mm}^2$$

This stress is below the proportional limit stress for the material, thus equation C5.1 applies and needs no plasticity correction.

C5. 4 Equation for Inelastic Buckling Strength of Flat Sheet in Compression.

If the buckling or instability occurs at a stress in the inelastic or plastic stress range, then E and ν are not the same as for elastic buckling, thus a plasticity correction factor is required and equation C5.1 is written,

$$\sigma_{cr} = \frac{\eta \pi^2 k_c E}{12(1-\nu_e^2)} \left(\frac{t}{b}\right)^2 \quad \text{- - - - - - - - - - (C5.2)}$$

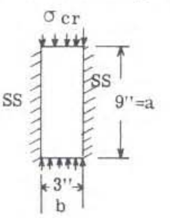
Where η is the plasticity reduction factor and equals σ_{cr} plastic/σ_{cr} elastic.

The values of k_c and ν_e are always the elastic values since the coefficient η contains all changes in those terms resulting from inelastic behavior.

A tremendous amount of theoretical and experimental work has been done relative to the value of the so-called plasticity correction factor. Possibly the first values used by design engineers were $\eta = E_t/E$ or $\eta = E_{sec}/E$. Whatever the expression for η it must involve a measure of the stiffness of the material in the inelastic stress range and since the stress-strain relation in the plastic range is non-linear, a resort must be made to the stress-strain curve to obtain a plasticity correction factor. This complication is greatly simplified by using the Ramberg and Osgood equations for the stress-strain curve which involves 3 simple parameters. (The reader should refer to Chapter B1 for information on the Ramberg-Osgood equations.) Thus using the Ramber-Osgood parameters (Ref.1) presents Figs. C5.7 and C5.8 for finding the compressive buckling stress for flat sheet panels with various boundary conditions for both elastic and inelastic buckling or instability.

Note

C5. 5 Simple Problems to Illustrate Use of Curves in Figs. C5. 7 and C5. 8.



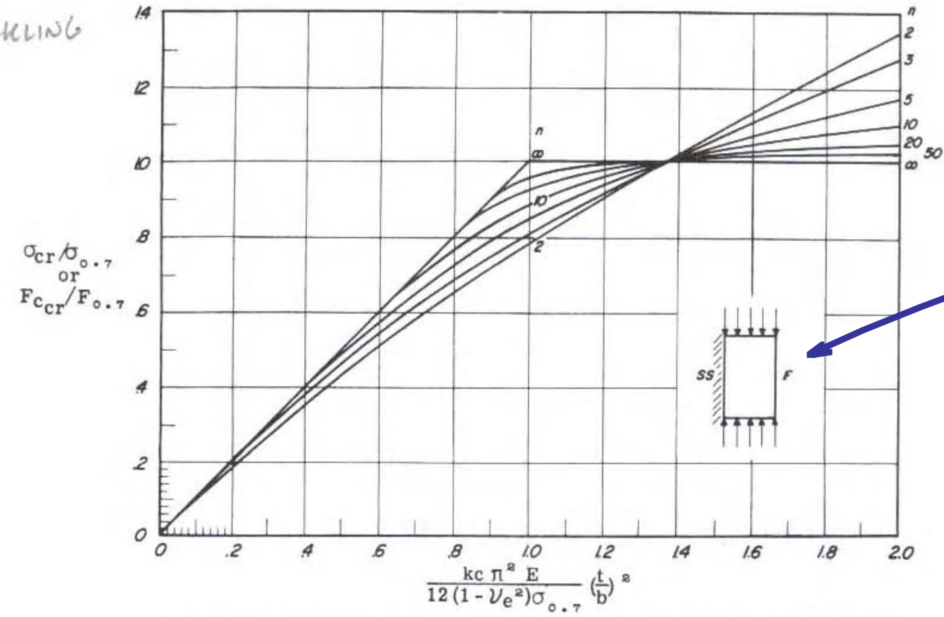
The sketch shows a 3 x 9 inch sheet panel. The sides are simply supported. The material is aluminum alloy 2024-T3. The thickness is .094". E = 10,700,000.

SI Conversions 1 inch = 25.4 mm

1 psi = 6895 Pa = 6.895×10^{-3} N/mm²

Bruhn C5.4

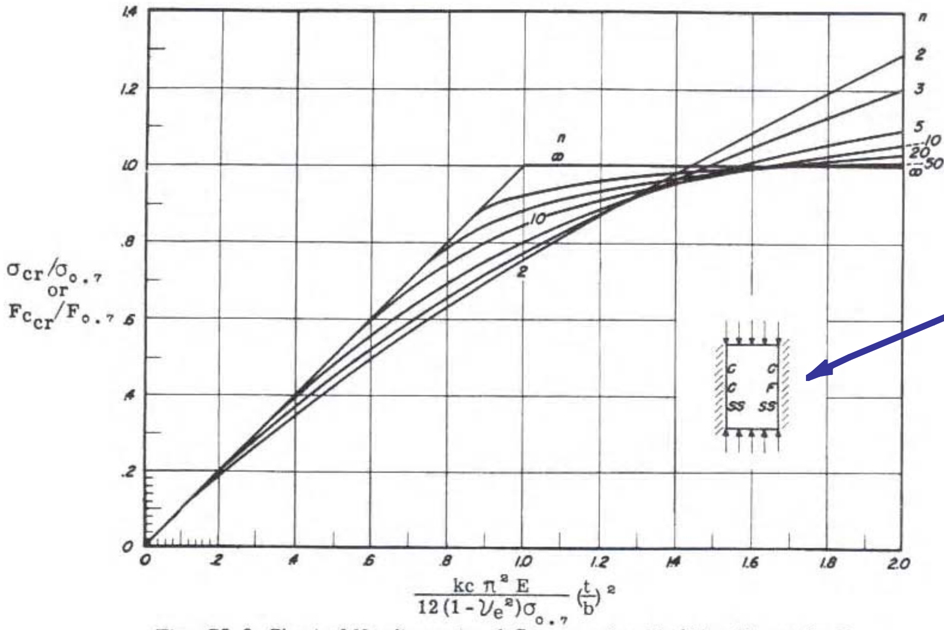
Plastic Buckling



Note edge restraints

Fig. C5.7 Chart of Nondimensional Compressive Buckling Stress for Long Hinged Flanges. $\eta = (E_s/E)(1-\nu_e^2)/(1-\nu^2)$.

Use of these curves gives σ_{cr} directly (Don't need to calc η)



Note edge restraints

Fig. C5.8 Chart of Nondimensional Compressive Buckling Stress for Long Clamped Flanges and for Supported Plates with Edge Rotational Restraint.

$$\eta = (E_s/2E)\left\{1 + 0.5\left[1 + (3E_t/E_s)\right]^{1/2}\right\}(1-\nu_e^2)/(1-\nu^2).$$

Bruhn C5.5

PROBLEM C5.5 Continued

$\nu_e = 0.3$. Find the buckling stress σ_{cr}.

Solution: We use Fig. C5.8 since it covers the boundary conditions of our problem. The parameter for bottom scale is,

$$\frac{k_c \pi^2 E}{12(1-\nu_e^2)\sigma_{0.7}}\left(\frac{t}{b}\right)^2 \quad \text{- - - - - - - - - - - (A)}$$

For a/b = 9/3 = 3, we find k_c from curve (c) of Fig. C5.2 equals 4.0.

The use of Fig. C5.8 involves the use of $\sigma_{0.7}$ and n the Ramberg-Osgood parameters. Referring to Table B1.1 of Chapter B1, we find for 2024-T3 aluminum alloy that $\sigma_{0.7}$ = 39000 and the shape factor n = 11.5.

Substituting in (A):-

$$\frac{4.0\,\pi^2 \times 10{,}700{,}000}{12\,(1-.3^2)\,39{,}000}\left(\frac{.094}{3}\right)^2 = .98$$

From Fig. C5.8 using .98 on bottom scale and n = 11.5 curve, we read on left hand scale that $\sigma_{cr}/\sigma_{0.7}$ = .84.

Then σ_{cr} = 39000 x .84 = 32800 psi.

If we neglected any plasticity effect, then we would use equation C5.2 with η = 1.0, or,

$$\sigma_{cr} = \frac{\pi^2 \times 4.0 \times 10{,}700{,}000}{12\,(1-.3^2)}\left(\frac{.094}{3}\right)^2 = \underline{38400} \text{ psi}$$

Whereas the actual buckling stress was 32800, or in this case the plasticity correction factor is 328/384 = .854.

The sheet thickness used in this example of .094 is relatively large. If we change the sheet thickness to .051 inches the results would be practically no correction within the accuracy of reading the curves, and the buckling stress σ_{cr} would calculate to be 11200 psi, which is below the proportional limit stress and thus no plasticity correction.

NOTE

C5. 6 Cladding Reduction Factors.

Aluminum alloy sheet is available with a thin covering of practically pure aluminum and is widely used in aircraft structures. Such material is referred to as alclad or clad aluminum alloy. The mechanical strength properties of this clad material is considerably lower than the core material. Since the clad is located at the extreme fibers of the alclad sheet, it is located where the strains attain their highest value when buckling takes place. Fig. C5.9 shows make up of an alclad sheet and Fig. C5.10 shows the stress-strain curves for cladding, core and alclad combinations.

Thus a further correction must be made for alclad sheets because of the lower strength clad covering material. Thus the buckling stress for alclad sheets can be written:

$$\bar{\sigma}_{cr} = \bar{\eta}\,\sigma_{cr} \quad \text{- - - - - - - - - - - - - - (C5.3)}$$

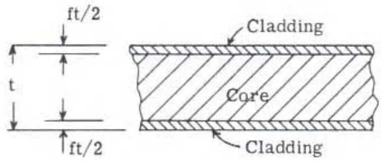


Fig. C5. 9

Reference 1 gives simplified cladding reduction factors as summarized in Table C5.1. Thus the buckling stress for alclad sheets is determined for the primary strength properties as normally listed for such materials as illustrated in the two previous example problems. The resulting σ_{cr} is then reduced by use of equation C5.3, using values of $\bar{\eta}$ from Table C5.1.

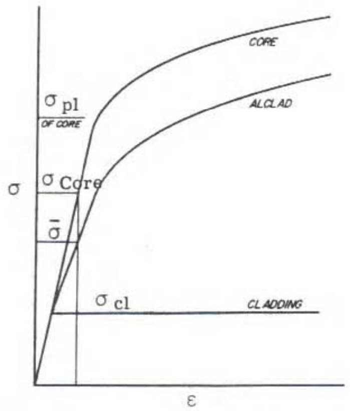


Fig. C5. 10 (Ref. 1) Stress-strain Curves for Cladding, Core, and Alclad Combinations. $\bar{\sigma}/\sigma_{core} = 1 - f + \beta f$; $\beta = \sigma_{cl}/\sigma_{core}$.

Table C5. 1 (Ref. 1) Summary of Simplified Cladding Reduction Factors

Loading	$\sigma_{cl} < \bar{\sigma}_{cr} < \sigma_{pl}$	$\bar{\sigma}_{cr} > \sigma_{pl}$
Short plate columns	$\frac{1 + (3\beta f/4)}{1 + 3f}$	$\frac{1}{1 + 3f}$
Long plate columns	$\frac{1}{1 + 3f}$	$\frac{1}{1 + 3f}$
Compression and shear panels	$\frac{1 + 3\beta f}{1 + 3f}$	$\frac{1}{1 + 3f}$

Bruhn C5.6

BUCKLING UNDER SHEAR LOADS

C5.7 Buckling of Flat Rectangular Plates Under Shear Loads.

The critical elastic shear buckling stress for flat plates with various boundary conditions is given by the following equation:

$$\tau_{cr} = \frac{\pi^2 k_s E}{12(1-\nu_e^2)} \left(\frac{t}{b}\right)^2 \quad \text{-------- (C5.4)}$$

Where (b) is always the shorter dimension of the plate as all edges carry shear. k_s is the shear buckling coefficient and is plotted as a function of the plate aspect ratio a/b in Fig. C5.11 for simply supported edges and clamped edges.

If buckling occurs at a stress above the proportional limit stress, a plasticity correction must be included and equation C5.4 becomes

$$\tau_{cr} = \frac{\eta_s \pi^2 k_s E}{12(1-\nu_e^2)} \left(\frac{t}{b}\right)^2 \quad \text{-------- (C5.5)}$$

Test results compare favorably with the results of equation C5.5 if $\eta_s = G_s/G$ where G is the shear modulus and G_s the shear secant modulus as obtained from a shear stress-strain diagram for the material.

A long rectangular plate subjected to pure shear produces internal compressive stresses on planes at 45 degrees with the plate edges and thus these compressive stresses cause the long panel to buckle in patterns at an angle to the plate edges as illustrated in Fig. C5.12, and the buckle patterns have a half wave length of 1.25b.

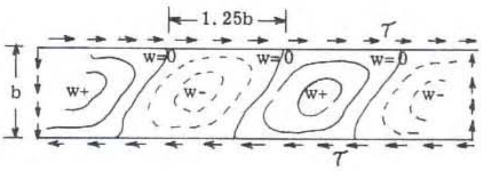


Fig. C5.12 (Ref. 7)

Fig. C5.13 is a chart of non-dimensional shear buckling stress for panels with various edge rotational restraint. This chart is similar to the chart in Figs. C5.7 and C5.8 in that the values $\sigma_{0.7}$ and n must be known for the material before the chart can be used to find the shear buckling stress.

BUCKLING UNDER BENDING LOADS

C5.8 Buckling of Flat Plates Under Bending Loads.

The equation for bending instability of flat plates in bending is the same as for compression and shear except the buckling coefficient k_b is different from k_c or k_s. When a plate in bending buckles, it involves relatively short wave length buckles equal to 2/3 b for long plates with simply supported edges (see Fig. C5.14). Thus the smaller buckle patterns cause the buckling coefficient k_b to be larger than k_c or k_s.

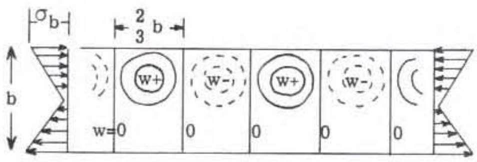


Fig. C5.14 (Ref. 7) Bending Buckle Patterns

For bending elastic buckling the equation is,

$$\sigma_{cr} = \frac{\pi^2 k_b E}{12(1-\nu_e^2)} \left(\frac{t}{b}\right)^2 \quad \text{--------- (C5.6)}$$

For bending inelastic buckling,

$$\sigma_{cr} = \frac{\eta_b \pi^2 k_b E}{12(1-\nu_e^2)} \left(\frac{t}{b}\right)^2 \quad \text{--------- (C5.7)}$$

Where k_b is the buckling coefficient and is obtained from Fig. C5.15 for various a/b ratios and edge restraint ε against rotation. In the a/b ratio the loaded edge is (b).

The plasticity reduction factor can be obtained from Fig. C5.8 using simply supported edges.

BUCKLING OF FLAT SHEETS UNDER COMBINED LOADS

The practical design case involving the use of thin sheets usually involves a combined load system, thus the calculation of the buckling strength of flat sheets under combined stress systems is necessary. The approach used involves the use of inter-action equations or curves (see Chapter C1, Art. C1.15 for explanation of inter-action equations).

C5.9 Combined Bending and Longitudinal Compression.

The interaction equation that has been widely used for combined bending and longi-

Bruhn C5.7

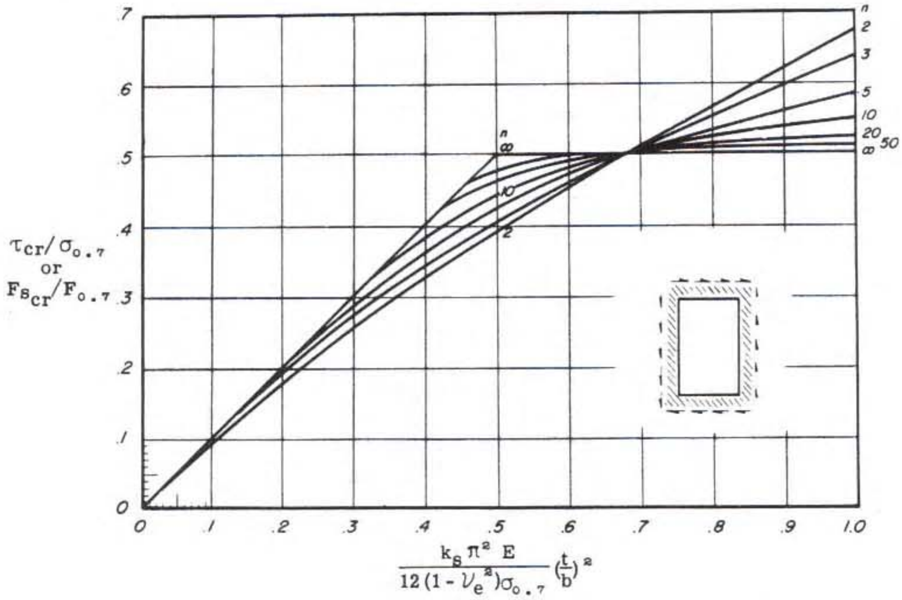


Fig. C5.13 (Ref. 1) Chart of Nondimensional Shear Buckling Stress for Panels With Edge-Rotational Restraint. $\eta = (E_s/E)(1-\nu_e^2)/(1-\nu^2)$.

} Don't need to calc η

E_s ?
ν_e

$$\tau_{cr} = \frac{k_s \pi^2 \eta_s E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$$

$$\sigma_{b_{cr}} = \frac{k_b \pi^2 \eta_b E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$$

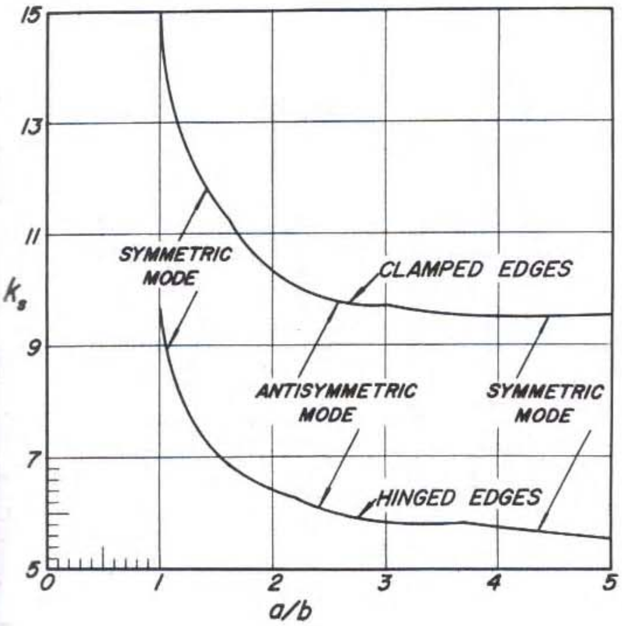


Fig. C5.11 (Ref. 1) Shear-Buckling-Stress Coefficient of Plates as a Function of a/b for Clamped and Hinged Edges.

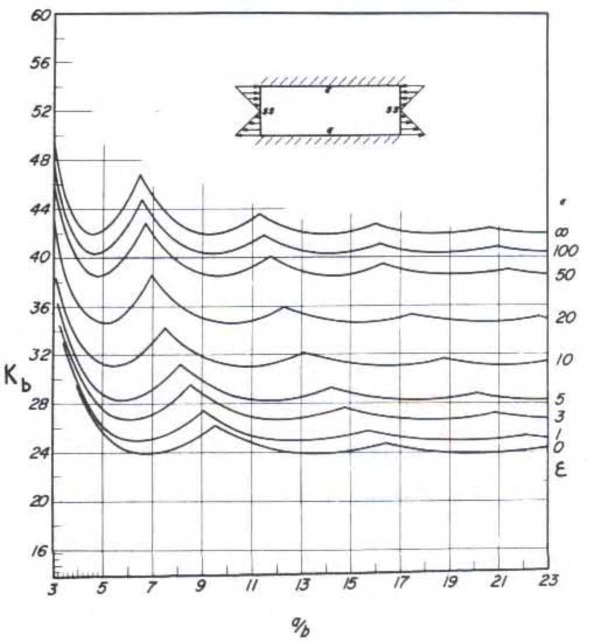


Fig. C5.15 Bending-Buckling Coefficient of Plates as a Function of a/b for Various Amounts of Edge Rotational Restraint.

Niu Stress p456

(B) PLASTIC REDUCTION FACTORS

Since the values of E and μ are not constant in the plastic range as they are in the elastic range, the plastic reduction factor (η_p) as defined below is used:

(a) Compression stress:

$$\eta_c = \left(\frac{E_t}{E}\right)^{\frac{1}{2}} = \left[\frac{1}{1 + \left(\frac{0.002\,E\,n}{F_{cy}}\right)\left(\frac{F_{c.cr}}{F_{cy}}\right)^{n-1}}\right]^{\frac{1}{2}} \qquad \text{Eq. 11.2.7}$$

where: n – Material shape parameter

(b) Shear stress:

$$\eta_s = \left(\frac{G_t}{G}\right)^{\frac{1}{2}} = \left[\frac{1}{1 + \left(\frac{0.002\,G\,n}{F_{sy}}\right)\left(\frac{F_{s.cr}}{F_{sy}}\right)^{n-1}}\right]^{\frac{1}{2}} \qquad \text{Eq. 11.2.8}$$

where: $G = \frac{E}{2(1-\mu^2)}$ – Modulus of rigidity

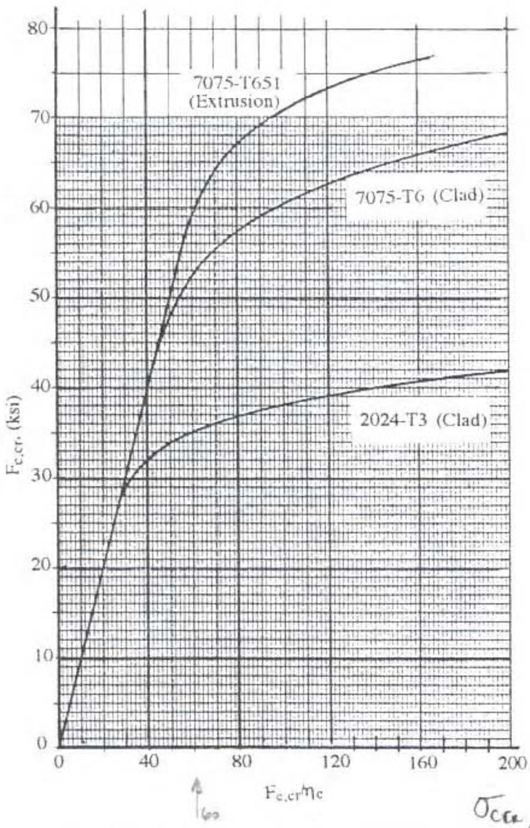
$F_{sy} = 0.55\,F_{cy}$

A tremendous amount of theoretical and empirical work has been done to obtain these values and Eq. 11.2.7 and Eq. 11.2.8 can be applied for any plate boundary condition. A simple method for obtaining the buckling stresses, $F_{c.cr}$, and $F_{s.cr}$ is shown in Fig. 11.2.4 and Fig. 11.2.5, respectively.

For Any bc's

$\sigma_{c\,CRIT}$ E_T

ksi =



→ Asymptote ?

→ Asymptote ?

$F_{ccr} = \eta_c K E \left(\frac{t}{b}\right)^2$

↳ $\frac{F_{ccr}}{\eta_c} = KE\left(\frac{t}{b}\right)^2$

ie use this elastic value to represent F_{ccr}/η_c

Fig. 11.2.4 Compression Bucking Stress $F_{c.cr}$ σ_{ccr}/η_c

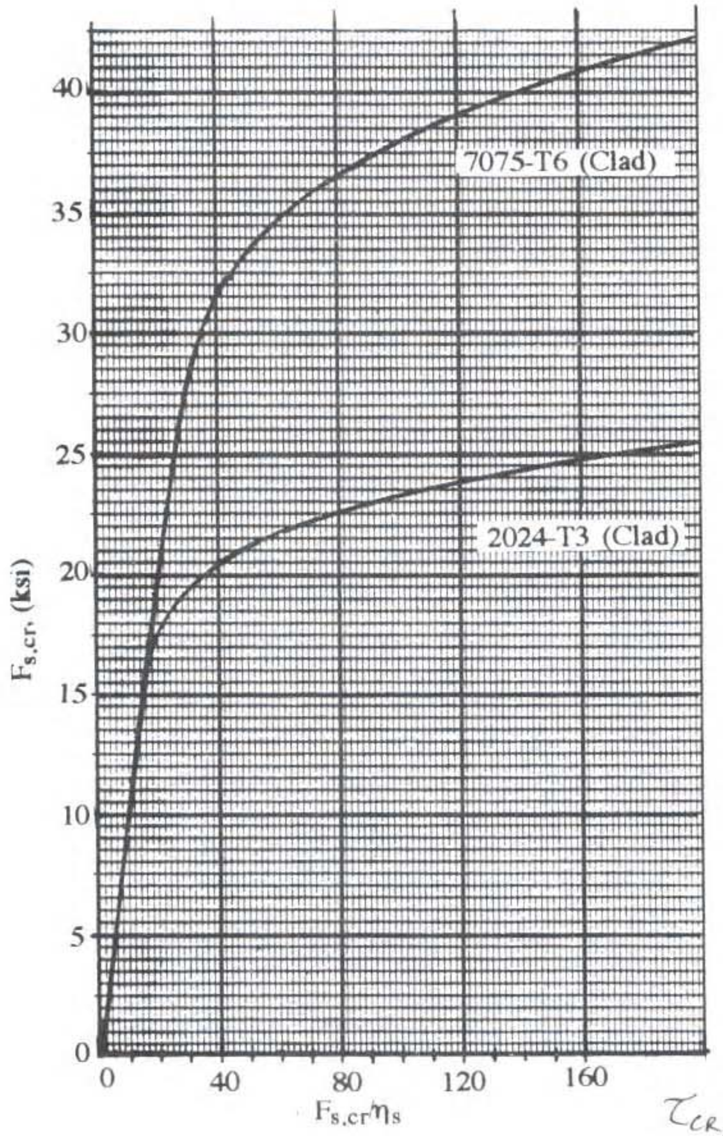
But what if outside range of axis?
– I guess you just use the asymptotic value?

Niu Stress 457

For any bc's

τ_{CRIT} e_T

ksi



τ_{CR}/η_s

Fig. 11.2.5 Shear Buckling Stress $F_{s,cr}$

If design curves are not available they can be constructed for specific materials by following procedure:

- Use Eq. 11.2.7 or Eq. 11.2.8 to construct the curves
- E, F_{cy} and n values can be obtained from Ref. 4.1
- If an n value is not available, use n = 20 which is a good approximation for most materials
- Assume a $F_{c,cr}$ value and calculate the η_p value several times and the plot them on graph paper as shown in Fig. 11.2.4
- Sheasr curves, such as shown in Fig. 11.2.5, can be constructed in the same manner

See Bruhn

Niu Stress p 458

*

(C) CLADDING EFFECT

Cladding will reduce the plate buckling stress (F_{cr}) of an aluminum clad material. A simple method for accounting the presence of the cladding is to use bare material properties and reduce the material thickness used in Eq. 11.2.3 and Eq. 11.2.4. The actual thickness used to calculate buckling stress is indicated in Fig. 11.2.6 by the cladding reduction factor (λ).

Material	Cladding	Plate thickness (in.)	Reduction facotr (λ)
2014	6053	t < 0.04	0.8
		t > 0.04	0.9
2024	1230	t < 0.064	0.9
		t > 0.064	0.95
7075	7072	All thickness	0.92

Fig. 11.2.6 Cladding Reduction Factor (λ) for Aluminum Clad Materials (Ref. 11.7)

11.3 FLAT PLATES

(A) COMPRESSION LOAD

The initial buckling stress for a flat plate under an in-plane compression load is:

$$F_{c,cr} = \frac{k_c \eta_c \pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2 \qquad \text{Eq. 11.3.1}$$

Note k_c, K_c on charts below!

$$\text{or } F_{c,cr} = K_c \eta_c E\left(\frac{t}{b}\right)^2 \qquad \text{Eq. 11.3.2}$$

where: η_p – Plasticity reduction factor in compression load

Fig. 11.3.1 shows flat plate buckling coefficients (K_c) for in-plane compression loads and Fig. 11.3.2 through Fig. 11.3.4 shows flat plate buckling coefficients (k_c) for in-plane compression loads.

K_c

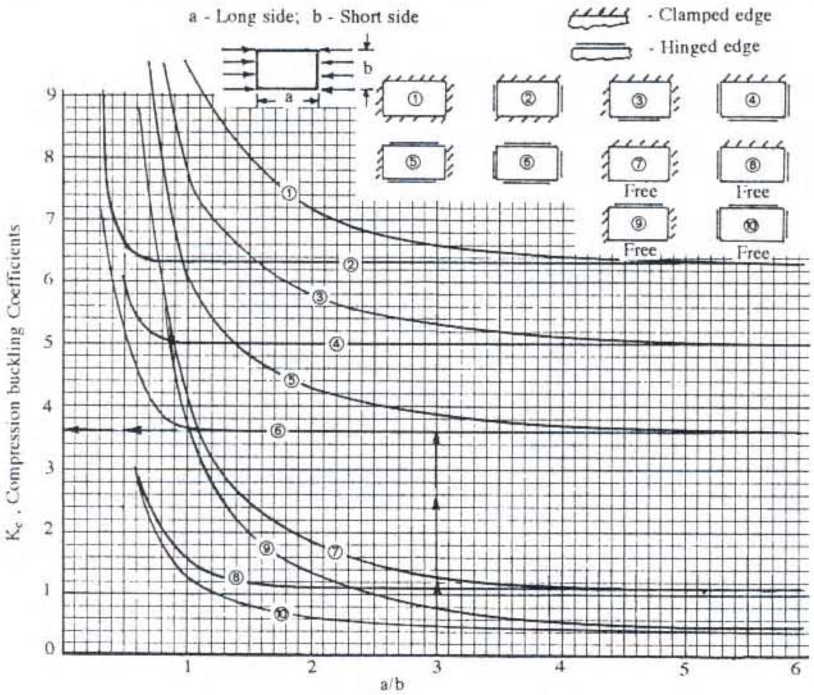


Fig. 11.3.1 K_c Coefficients (Compression)

Niu Stress p459

kc

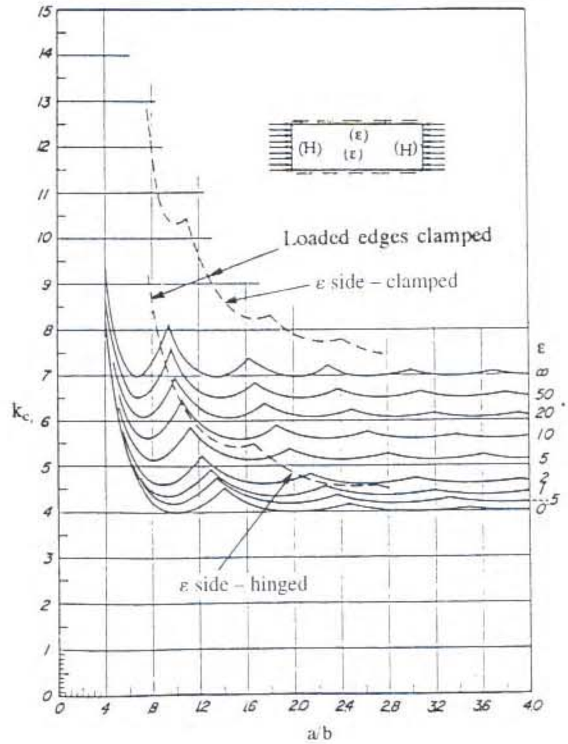


Fig. 11.3.2 k_c Coefficients with Various Edge Rotational Restraints (Compression)

kc

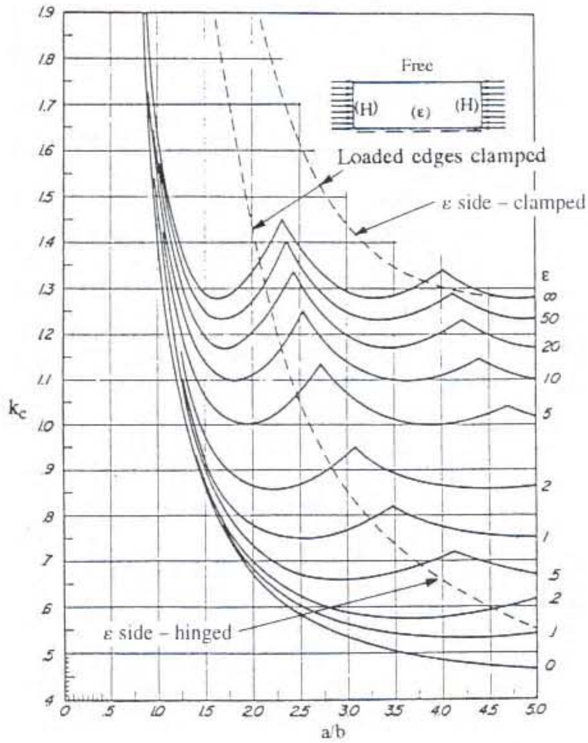
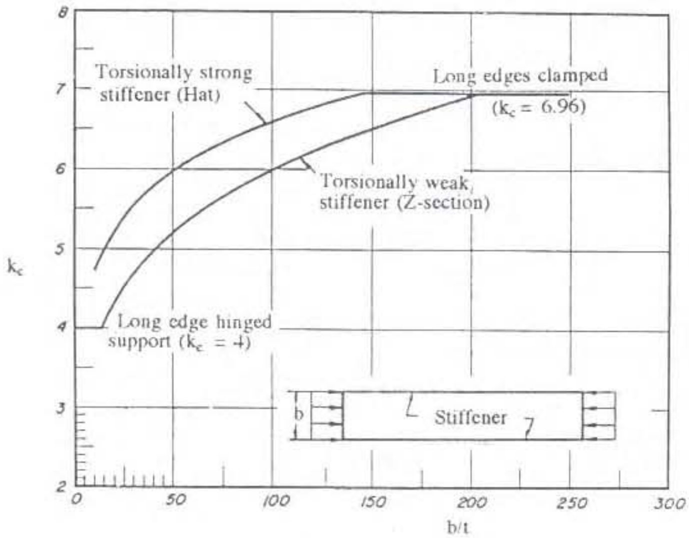


Fig. 11.3.3 k_c Coefficients of Free Flange with Various Edge Rotational Restraints (Compression)

Niu Stress p460

k_c



STIFFENERS ALONG
PLATE EDGE
eg

Fig. 11.3.4 k_c Coefficients for a Skin with Stiffeners on Two Sdies (Compression) (Ref. 11.7)

(B) SHEAR LOAD

The initial buckling stress for a flat plate under in-plane shear load is

$$F_{s,cr} = \frac{k_s \eta_s \pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2 \qquad \text{Eq. 11.3.3}$$

$$\text{or } F_{s,cr} = K_s \eta_s E\left(\frac{t}{b}\right)^2 \qquad \text{Eq. 11.3.4}$$

Fig. 11.3.5 shows flat plate buckling coefficients (K_s) for in-plane shear.

K_s

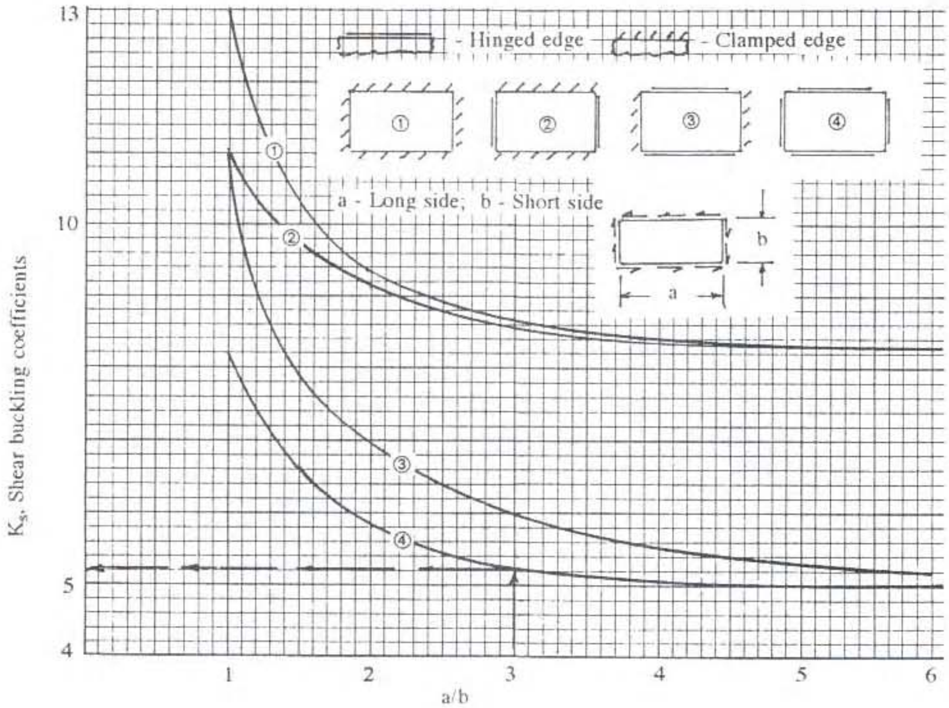


Fig. 11.3.5 K_s Coefficients (Shear)

Niu Stress p461

(C) BENDING LOAD

The initial buckling stress for a flat plate under in-plane bending load is

$$F_{b,cr} = \frac{k_b \eta_c \pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2 \qquad \text{Eq. 11.3.5}$$

$$\text{or} \quad F_{b,cr} = K_b \eta_c E\left(\frac{t}{b}\right)^2 \qquad \text{Eq. 11.3.6}$$

Fig. 11.3.6 shows flat plate buckling coefficients, K_b, for in-plane bending loads.

K_b

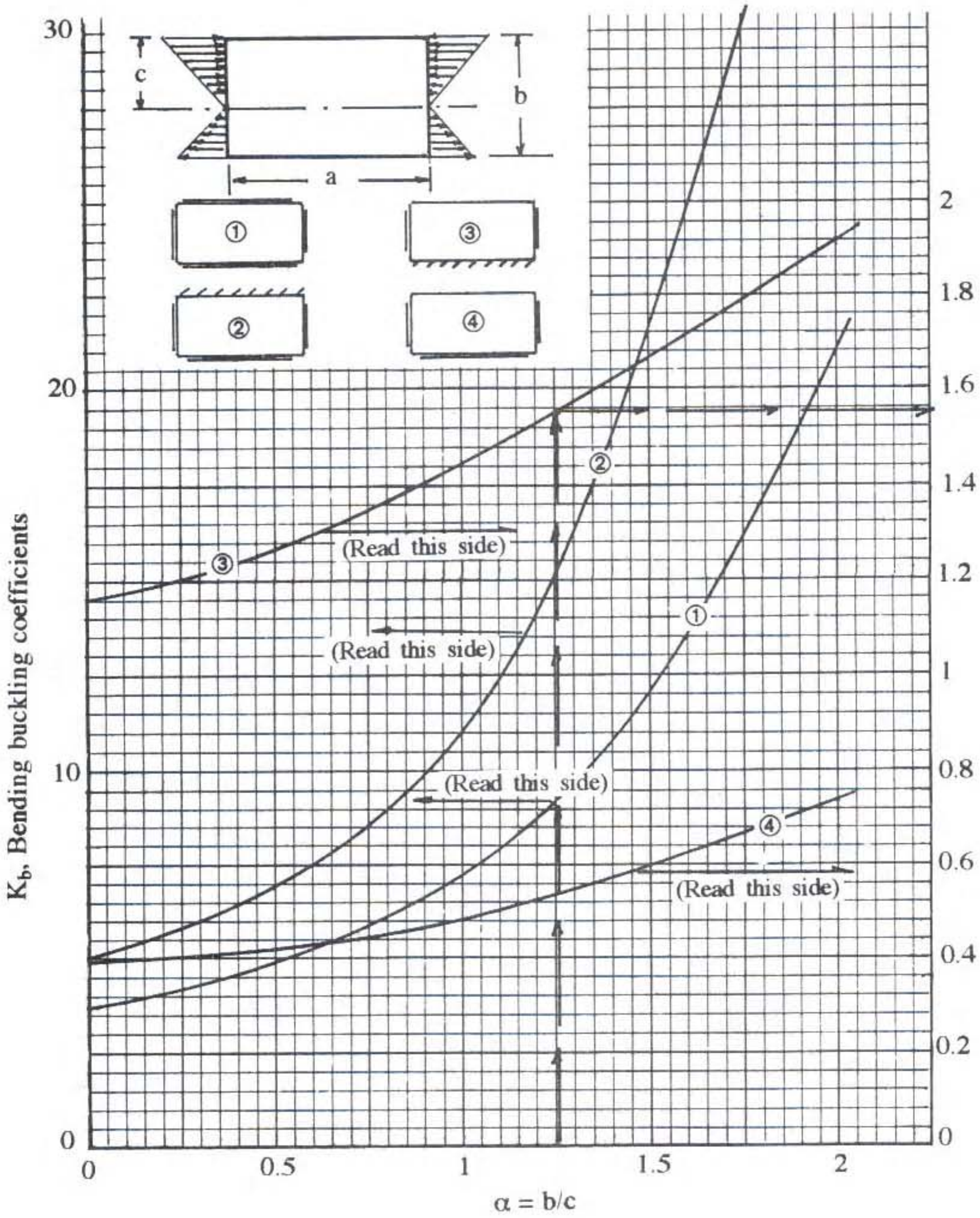


Fig. 11.3.6 K_b Coefficients (Bending) – $\frac{a}{b} > 1.0$

Niu Stress p462

Example 1:

Given a plate with four edges, hinged as shown, determine buckling stress (F_{cr}) under various boundary conditions.

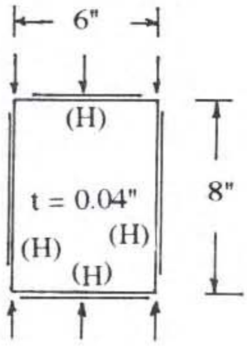
Material: 2024-T3 bare and $E = 10^7$ psi

$a = 8''$ ~200 mm

$b = 6''$ (short side) ~150 mm

$t = 0.04''$

$\frac{a}{b} = \frac{8}{6} = 1.33$



(1) Compression:

$K_c = 3.6$ (see Fig. 11.3.1, Case ⑥)

From Eq. 11.3.2

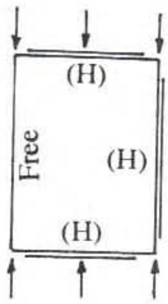
$$\frac{F_{c.cr}}{\eta_c} = K_c E (\frac{t}{b})^2$$

$$\frac{F_{c.cr}}{\eta_c} = 3.6 \times 10^7 (\frac{0.04}{6})^2$$

$= 1{,}600$ psi = 1.6 ksi

On linear portion of curves Fig 11.2.4 ∴ elastic calc valid

From Fig. 11.2.4, the true buckling stress, $F_{c.cr} = 1{,}600$ psi



(2) Compression – one edge free:

$K_c = 0.9$ (see Fig. 11.3.1, Case ⑩)

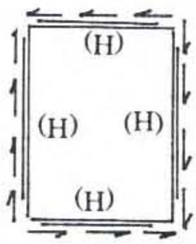
From Eq. 11.3.2

$$\frac{F_{c.cr}}{\eta_c} = K_c E (\frac{t}{b})^2$$

$$\frac{F_{c.cr}}{\eta_c} = 0.9 \times 10^7 (\frac{0.04}{6})^2$$

$= 400$ psi = 0.4 ksi

From Fig. 11.2.4, the true buckling stress, $F_{c.cr} = 400$ psi



(3) Shear:

$K_s = 6.8$ (see Fig. 11.3.5, Case ④)

From Eq. 11.3.4

$$\frac{F_{s.cr}}{\eta_s} = K_s E (\frac{t}{b})^2$$

$$\frac{F_{s.cr}}{\eta_s} = 6.8 \times 10^7 (\frac{0.04}{6})^2$$

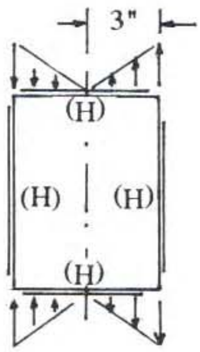
$= 3{,}022$ psi = 3.0 ksi

From Fig. 11.2.5, the true buckling stress, $F_{c.cr} = 3{,}022$ psi

Niu Stress p 463

(4) Bending:

$K_b = 21.6$ (see Fig. 11.3.6, $\alpha = \frac{b}{c} = \frac{6}{3} = 2.0$)



From Eq. 11.3.6

$$\frac{F_{b,cr}}{\eta_c} = K_b E(\frac{t}{b})^2$$

$$\frac{F_{b,cr}}{\eta_c} = 21.6 \times 10^7(\frac{0.04}{6})^2$$

$$= 9{,}600 \text{ psi}$$ ≈ 9.6 ksi

From Fig. 11.2.4, the true buckling stress, $F_{b,cr} = F_{c,cr} = 9{,}600$ psi

Example 2:

Given a plate with edges, hinged as shown, determine buckling stress ($F_{c,cr}$) under an in-plane compression loading.

Material: 2024-T3 clad

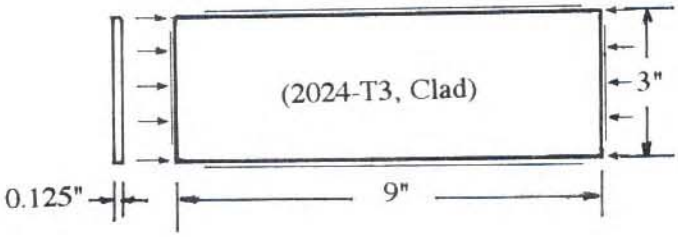
$E = 10.5 \times 10^6$ psi

$F_{cy} = 39{,}000$ psi

$a = 9$ in.

$b = 3$ in. (short side)

$\frac{a}{b} = \frac{9}{3} = 3$



(1) $t = 0.125$ in.:

From Fig. 11.2.6, the cladding reduction factor, $\lambda = 0.95$

The correct $t = 0.125 \times 0.95 = 0.119''$

$K_c = 3.6$ (see Fig. 11.3.1, Case 6)

From Eq. 11.3.2

$$F_{c,cr} = K_c \eta_c E(\frac{t}{b})^2$$

$$\frac{F_{c,cr}}{\eta_c} = 3.6 \times 10.5 \times 10^6(\frac{0.119}{3})^2$$

$$= 59{,}476 \text{ psi}$$ ≈ 59.5 ksi

From Fig. 11.2.4, obtain the true compression buckling stress, $F_{c,cr} = 35{,}000$ psi ✓

(2) $t = 0.063$ in.:

From Fig. 11.2.6, the cladding reduction factor, $\lambda = 0.9$

The correct, $t = 0.063 \times 0.9 = 0.0567''$

$$\frac{F_{c,cr}}{\eta_c} = 3.6 \times 10.5 \times 10^6(\frac{0.0567}{3})^2$$

$$= 13{,}502 \text{ psi}$$

From Fig. 11.2.4, obtain $F_{c,cr} = 13{,}502$ psi (in elastic range and $\eta_c = 1.0$)

Niu Stress p464

Example 3:

Use the same plate given in Example 2 and determine buckling stress ($F_{s,cr}$) under an in-plane shear loading.

$K_s = 5.25$ (see Fig. 11.3.5, Case ④) with $\frac{a}{b} = 3$

From Eq. 11.3.4

$$F_{s,cr} = K_s \eta_s E(\frac{t}{b})^2$$

$$\frac{F_{s,cr}}{\eta_s} = 5.25 \times 10.5 \times 10^6 (\frac{0.119}{3})^2$$

$$= 86{,}736 \text{ psi}$$

From Fig. 11.2.5, obtain the true shear buckling stress, $F_{s,cr} = 22{,}800$ psi

Example 4:

Given a T-shaped extrusion as shown below, find the MS under an in-plane moment loading of M = 3,900 in.-lbs.

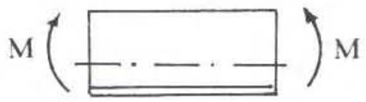
Material: 7075-T6 and $E = 10.7 \times 10^6$ psi

$b = 1.625 - 0.094 = 1.53$

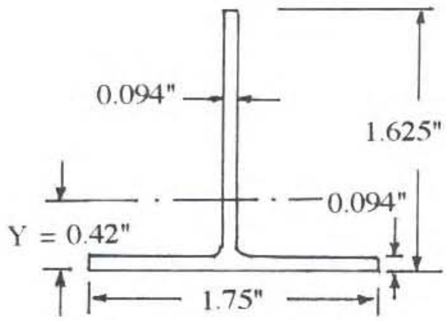
$c = 1.625 - 0.42 = 1.21$

$$\alpha = \frac{b}{c} = \frac{1.53}{1.21} = 1.26$$

$K_b = 1.55$ (see Fig. 11.3.6, Case ③)



(7075-T6, Extrusion)



$I_x = 0.0777$ in.^{4}

From Eq. 11.3.6:

$$F_{b,cr} = K_b \eta_c E(\frac{t}{b})^2$$

$$\frac{F_{b,cr}}{\eta_c} = 1.55 \times 10.7 \times 10^6 (\frac{0.094}{1.53})^2$$

$$= 62{,}602 \text{ psi}$$

From Fig. 11.2.4, obtain the true compression buckling stress, $F_{b,cr} = F_{c,cr} = 61{,}000$ psi

The bending stress,

$$f_b = \frac{Mc}{I_x} = \frac{3{,}900 \times 1.21}{0.0777} = 60{,}734 \text{ psi}$$

$$MS = \frac{61{,}000}{60{,}734} - 1 = \underline{0}$$ O.K.

11.4 CURVED PLATES

The initial buckling stress for a curved plate is the same as that of a flat plate, the same equations (see Eq. 11.3.1 and Eq. 11.3.3) can be used for curved plates except that the buckling coefficients for a curved plate, k_c' and k_s' are used instead of k_c and k_s.

To be considered later in the course.

Bruhn C5.8

Combined loading

tudinal compression is,

$$R_b^{1.76} + R_c = 1.0 \qquad \text{(C5.8)}$$

This equation was originally presented in Ref. 2 and the interaction curve from plotting this equation is found in many of the structures manuals of aerospace companies.

Fig. C5.15 is a plot of eq. C5.8. It also shows curves for various margin of safety values.

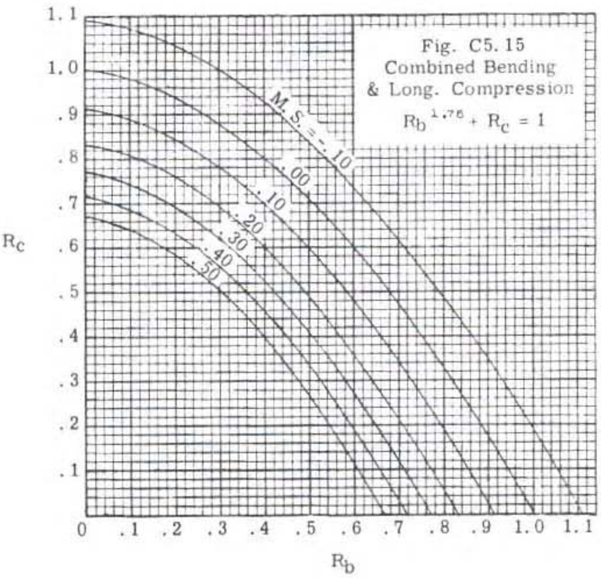


Fig. C5.15 Combined Bending & Long. Compression $R_b^{1.76} + R_c = 1$

C5.10 Combined Bending & Shear.

The interaction equation for this combined loading (Ref. 1 & 2) is,

$$R_b^2 + R_s^2 = 1 \qquad \text{(C5.9)}$$

The expression for margin of safety is,

$$M.S. = \frac{1}{\sqrt{R_b^2 + R_s^2}} - 1 \qquad \text{(C5.10)}$$

Fig. C5.16 is a plot of equation C5.9. Curves showing various M.S. values are also shown. R_s is the stress ratio due to torsional shear stress and R_{st} is the stress ratio for transverse or flexural shear stress.

C5.11 Combined Shear and Longitudinal Direct Stress. (Tension or Compression.)

The interaction equation is (Ref. 3,4)

$$R_L + R_S^2 = 1.0 \qquad \text{(C5.11)}$$

$$M.S. = \frac{2}{(R_L + \sqrt{R_L^2 + 4R_S^2})} - 1 \qquad \text{(C5.12)}$$

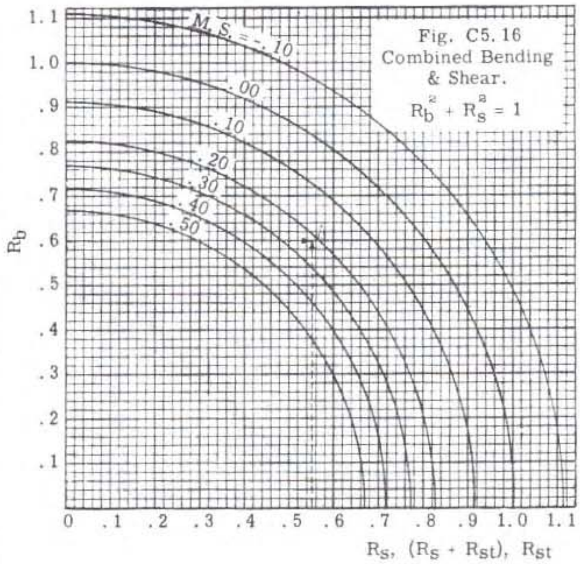


Fig. C5.16 Combined Bending & Shear. $R_b^2 + R_s^2 = 1$

Fig. C5.17 is a plot of equation C5.11. If the direct stress is tension, it is included on the figure as negative compression using the compression allowable.

C5.12 Combined Compression, Bending & Shear.

From Ref. 5, the conditions for buckling are represented by the interaction curves of Fig. C5.18. This figure tells whether the sheet will buckle or not but will not give the margin of safety. Given the ratios R_c, R_s and R_b:- if the value of the R_c curve defined by the given value of R_b and R_s is greater numerically than the given value of R_c, then the panel will buckle.

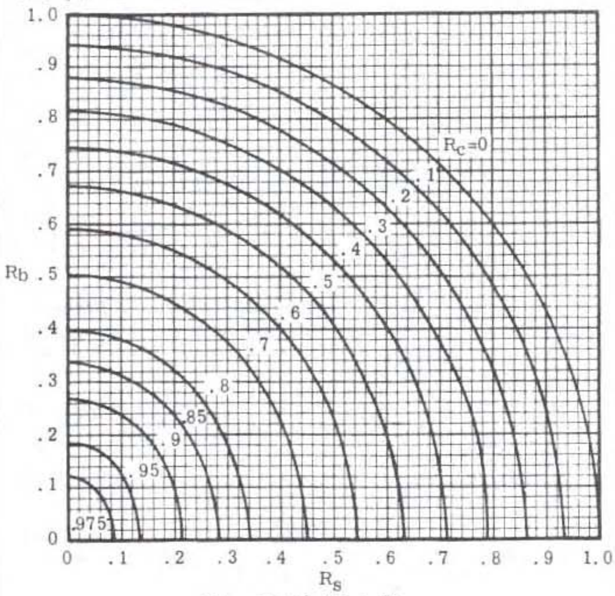


Fig. C5.18 (Ref. 5)

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Column Buckling
— To be considered later, with Crippling

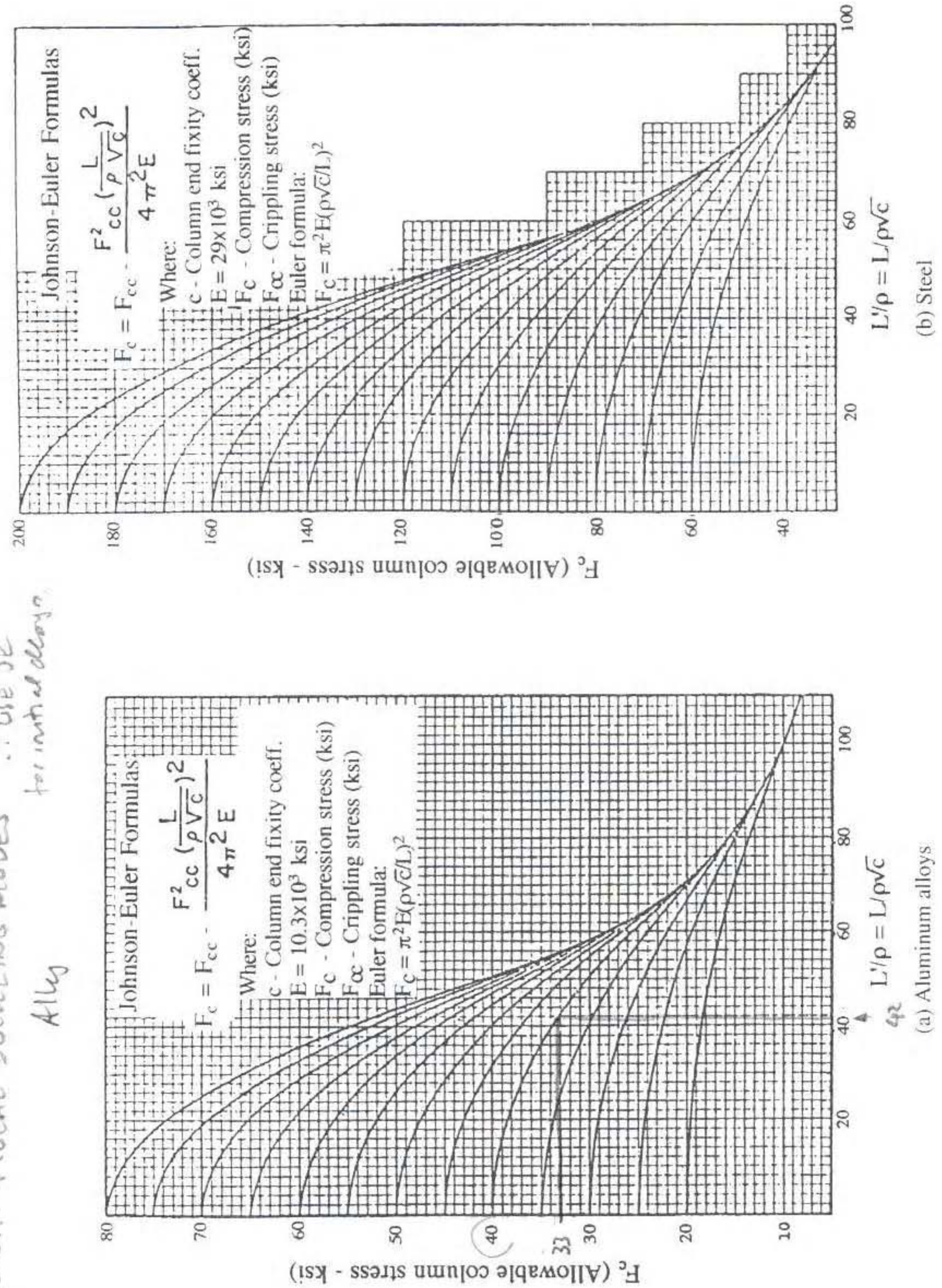

Fig. 10.8.2 Johnson-Euler Column Curves

More Conservative c/w EE Et
∴ use JE for initial designs

JE Curves allow for interaction of global + local buckling modes