UNIVERSITY OF BRISTOL
FACULTY OF ENGINEERING

First Year Examination for the Degree of Master of Engineering

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MAY/JUNE 2016 2 Hours

AENG11100

FLUIDS I

Solutions

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Q 1

$$p_G = \rho_{water} g \mathrm{h}_{\mathrm{water}} = 1000 \times 9.81(\mathrm{h}_{\mathrm{water}}) = 2 \times 10^5 N/m^2$$

$$\mathrm{h}_{\mathrm{water}} = \frac{200}{9.81} m$$

$$h = 20.39m$$

(2 *marks*)

Q 2 Horizontal force given by gauge pressure at C.G times the area (use gauge because atmospheric pressure acts on both sides). The vertical force equals the weight of water required to balance the vertical forces (again atmosphere acts on both sides)

$$F_v = \left(\tfrac{1}{2} \times 4 \times \tan 30^o \times 4 \times 6\right) \times 1000 \times 9.81 = 271863N$$

$$F_h = 1000 \times 9.81 \times 2 \times 6 \times 4 = 470880N$$

(3 *marks*)

Q 3 Steady, incompressible, inviscid,1D flow

(3 marks)

Q 4 a)Must match the Reynolds Number, Mach number and Euler number (or pressure coefficient b) Neglect the Mach number because compressibility is negligible for low speed air flows

(*3 marks*)

Q 5 Applying Bernoulli (set vertical zero at downstream location)

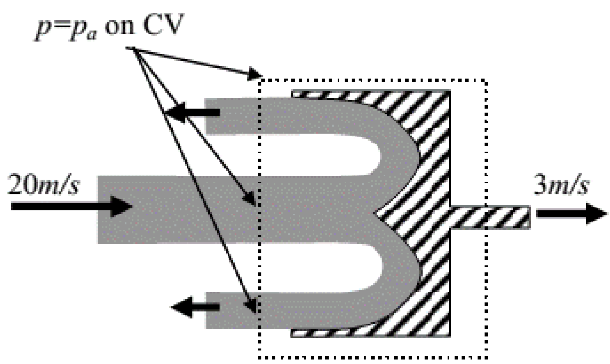
$$p_1 + \tfrac{1}{2}\rho V_1^2 + \rho g h_1 = p_2 + \tfrac{1}{2}\rho V_2^2 + \rho g h_2$$

$$p_2 = p_1 + \tfrac{1}{2}\rho\left(V_1^2 - V_2^2\right) + \rho g\left(h_1 - h_2\right)$$

$$p_2 = 2 \times 10^5 + \frac{1}{2} \times 1000 \times \left(4^2 - 2^2\right) - 1000 \times 9.81 \times 1 = 1.962 bar$$

(*3 marks*)

Q 6 Consider a control volume fixed relative to the plate. The net horizontal force on the control volume equals the rate of change of momentum in that direction. We also assume that atmospheric pressure acts through the jet diameter so there is no contribution to the horizontal force from the jet entry into the CV



$$F_{CV-x} = \dot{m}\left(V_2 - V_1\right) = -\pi \times 0.1^2 \times 1000 \times 2 \times 17^2 = -18158.4N$$

$$F = -F_{CV-x} = 18158.4N$$

(*4 marks*)

Q 7 The flow over an oval is modelled as the sum of a free stream plus a source and a sink of equal strength *(2 marks)*

Q8

(a) Δh is due to the difference in static pressure between the wind tunnel pressure tapping and ambient pressure. Applying Bernoulli's equation between ambient and working section locations and neglecting hydrostatic terms as air density is small

$$p_a = p_w + \tfrac{1}{2}\rho_a V_w^2$$

Applying the hydrostatic equation across the manometer fluid surfaces

$$p_a = p_w + \rho_m g \Delta h$$

Rearranging

$$\rho_m g \Delta h = \tfrac{1}{2}\rho_a V_w^2 \qquad \rightarrow \Delta h = \frac{\rho_a}{2\rho_m g} V_w^2$$

(8 marks)

(b) From continuity and assuming the flow is incompressible, we know that the volume flow rate at the exit must be the same as at the working section.

$$A_e V_e = A_w V_w \qquad \rightarrow \quad V_e = \frac{A_w}{A_e} V_w$$

Similarly the average velocity downstream of the fan must be the same as in the working section (the cross section is constant).

Taking Bernoulli's equation from just downstream of the fan (subscript fd) to the exit, where the pressure is equal to the ambient pressure

$$p_{fd} + \tfrac{1}{2}\rho_a V_{fd}^2 = p_a + \tfrac{1}{2}\rho_a V_e^2$$

Applying continuity

$$p_{fd} + \tfrac{1}{2}\rho_a V_w^2 = p_a + \tfrac{1}{2}\rho_a \left(\frac{A_w}{A_e}\right)^2 V_w^2 \qquad \rightarrow \quad p_{fd} = p_a + \tfrac{1}{2}\rho_a \left(\left(\frac{A_w}{A_e}\right)^2 - 1\right) V_w^2$$

Therefore

$$p_{fd} - p_w = p_a + \tfrac{1}{2}\rho_a \left(\left(\frac{A_w}{A_e}\right)^2 - 1\right) V_w^2 - p_a + \tfrac{1}{2}\rho_a V_w^2$$

(8 marks)

$$\Delta p_{fan} = \tfrac{1}{2}\rho_a \left(\frac{A_w}{A_e}\right)^2 V_w^2$$

(c) If the volume flow rate is fixed then

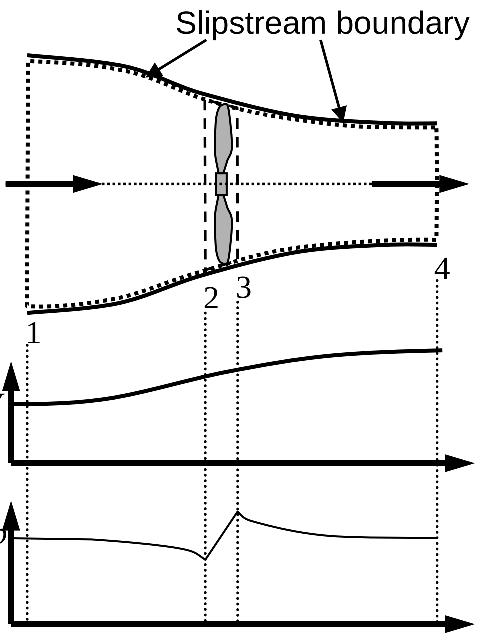
$$\Delta p_{fan2} = \tfrac{1}{2}\rho_a \left(\frac{A_w}{A_{e2}}\right)^2 V_w^2$$

$$\frac{\Delta p_{fan}}{\Delta p_{fan2}} = \tfrac{1}{2}\rho_a \left(\frac{A_w}{A_e}\right)^2 V_w^2 \bigg/ \tfrac{1}{2}\rho_a \left(\frac{A_w}{A_{e2}}\right)^2 V_w^2 = \left(\frac{A_{e2}}{A_e}\right)^2$$

(4 marks)

Q9

(a) Use the actuator disc theory for an ideal propeller, see figure below



Consider the Galilean transformation so that the propeller is fixed. In this case the inflow velocity is now $V_1 = V + v$

Assumptions: Frictionless & incompressible, Steady 1D flow (neglect rotation and variation across the disc radius). Actuator disc is thin so $A_2 = A_3 = A_d$ & $V_2 = V_3 = V_d$. $p = p_a$ at all points on slipstream boundary & 1 & 4

Continuity: $Q = V_d A_d$

Bernoulli's equation for CV 1-2 & CV 3-4

$$\begin{aligned} p_1 + \tfrac{1}{2}\rho V_1^2 &= p_2 + \tfrac{1}{2}\rho V_d^2 \\ p_3 + \tfrac{1}{2}\rho V_d^2 &= p_4 + \tfrac{1}{2}\rho V_4^2 \end{aligned} \quad \rightarrow \quad p_3 - p_2 = \tfrac{1}{2}\rho\left(V_4^2 - V_1^2\right)$$

Steady Flow momentum for CV 2-3:

$$\left(p_2 - p_3\right)A_d + F_{CV} = \rho Q\left(V_d - V_d\right) = 0 \qquad \rightarrow \quad F_{CV} = \left(p_3 - p_2\right)A_d$$

Applying results from Bernoulli's equation above

$$F = \tfrac{1}{2}\rho A_d\left(V_4^2 - V_1^2\right) = \tfrac{1}{2}\rho A_d\left(V_4^2 - (V + v)^2\right)$$

(6 marks)

(b)

Steady Flow momentum for CV 1-4:

$$0 + F_{CV} = \rho Q\left(V_4 - V_1\right)$$

From momentum & continuity

$$\left(p_3 - p_2\right)A_d = \rho V_d A_d\left(V_4 - V_1\right)$$

Eliminating (p_3-p_2) using Bernoulli's equation above

$$\rho V_d(V_4 - V_1) = \tfrac{1}{2}\rho\left(V_4^2 - V_1^2\right) = \tfrac{1}{2}\rho(V_4 - V_1)(V_4 + V_1)$$
$$V_d = \tfrac{1}{2}(V_4 + V_1)$$
$$V_4 = 2V_d - V_1$$

Continuity from 1 to 2

$$A_1V_1 = A_dV_d$$
$$V_d = \frac{A_1V_1}{A_d}$$
$$\frac{A_1}{A_d} = a + 1 \quad \rightarrow \quad V_d = (a+1)V_1$$
$$V_d = (a+1)V_1 = \tfrac{1}{2}(V_4 + V_1) \quad \rightarrow \quad V_4 = (2a+1)V_1$$

Substituting back in gives

$$F = \tfrac{1}{2}\rho A_d\left(V_4^2 - V_1^2\right) = \tfrac{1}{2}\rho A_d V_1^2\left((2a+1)^2 - 1\right) = \rho A_d V_1^2\, 2a(a+1)$$
$$F = 2\rho A_d (V+v)^2 a(a+1)$$

The power supplied to the disc is

$$P_{disc} = FV_d$$

Power output

$$P_{out} = FV_1$$

The efficiency of the rotor is therefore (remember efficiency for turbines and propellers is not the same)

$$\eta = \frac{P_{\text{out}}}{P_{\text{disc}}} = \frac{FV_1}{FV_d} = \frac{V_1}{V_d} = \frac{1}{a+1}$$

(9 marks)

(a) From parts a and b we have

$$F = \tfrac{1}{2}\pi\rho d^2 V^2 a(a+1)$$
$$9000 = \tfrac{1}{2}\pi \times 1.21 \times d^2 \times 90^2 \times 0.21(1.21) = \pi \times d^2 \times 90^2 \times 0.105(1.21)^2$$
$$d^2 = \frac{100}{\pi \times 90 \times 0.105 \times (1.21)^2} \quad \rightarrow \quad d = \frac{10}{3 \times 1.21}\sqrt{\frac{1}{10\pi \times 0.105}} = 1.517$$
$$FV_d = \frac{FV}{\eta}$$
$$\eta = \frac{1}{1.21}$$
$$FV_d = FV \times 1.21 = 9000 \times 90 \times 1.21 = 980100W$$

(5 marks)

Q10 (a) Using Bernoulli's equation for incompressible flow gives

$$p_\infty + \frac{1}{2}\rho_\infty U_\infty^2 = p + \frac{1}{2}\rho_\infty U^2$$

since density is constant everywhere.
Then by definition of the pressure coefficient

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty U_\infty^2} = 1 - \frac{U^2}{U_\infty^2}$$

b) Using Galilean transformation we can analyse a stationary cylinder into the onset flow. Using cylindrical polar coordinates, from given equations the stream function is given by

$$\psi = U_\infty r\sin\theta - \frac{\kappa}{2\pi}\frac{\sin\theta}{r} = U_\infty r\sin\theta\left(1 - \frac{\kappa}{2\pi U_\infty r^2}\right)$$

The velocity components are given by (could go straight from given equations)

$$V_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \left(1 - \frac{\kappa}{2\pi U_\infty r^2}\right)U_\infty\cos\theta, \quad V_\theta = -\frac{\partial\psi}{\partial r} = -\left(1 + \frac{\kappa}{2\pi U_\infty r^2}\right)U_\infty\sin\theta$$

Now the cylinder is a stream line of the flow so there is no flow normal to the cylinder i.e. $V_r = 0$. This means that

$$\left(1 - \frac{\kappa}{2\pi U_\infty R^2}\right)U_\infty\cos\theta = 0$$

for all θ so the doublet strength must be given by

$$\kappa = 2\pi U_\infty R^2$$

The velocity components are then

$$V_r = \left(1 - \frac{R^2}{r^2}\right)U_\infty\cos\theta, \quad V_\theta = -\left(1 + \frac{R^2}{r^2}\right)U_\infty\sin\theta$$

On the cylinder

$$V_r = 0, \quad V_\theta = -2U_\infty\sin\theta$$

The pressure coefficient on the cylinder therefore given by

$$C_p = 1 - \left(\frac{V_\theta}{U_\infty}\right)^2 = 1 - 4\sin^2\theta$$

And the pressure on the cylinder is

$$p(\theta) = p_\infty + \frac{1}{2}\rho U_\infty^2(1 - 4\sin^2\theta)$$

(9 marks)

c) Assume the flow can be modelled as the summation of the velocity due to the same cylinder plus an equal image cylinder. Along the line from the surface to the nearest point the vertical velocity will be zero so we need only consider horizontal velocity. Writing the horizontal velocity in Cartesian coordinates, with the origin at the surface,

$$u = U_\infty + \frac{-\kappa}{2\pi}\frac{\left(x^2 - (y-h)^2\right)}{\left(x^2 + (y-h)^2\right)^2} + \frac{-\kappa}{2\pi}\frac{\left(x^2 - (y+h)^2\right)}{\left(x^2 + (y+h)^2\right)^2}$$

From the previously found measure of the doublet strength and setting *x=0* & *y=0*, we can write

$$u = U_\infty - U_\infty R^2 \frac{\left(-(-h)^2\right)}{\left((-h)^2\right)^2} - U_\infty R^2 \frac{\left(-(h)^2\right)}{\left((h)^2\right)^2} = U_\infty + U_\infty R^2 \left(\frac{1}{(h)^2} + \frac{1}{(h)^2} \right)$$

$$u = U_\infty + 2U_\infty R^2 \left(\frac{1}{h^2} \right)$$

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty U_\infty^2} = 1 - \frac{u^2}{U_\infty^2} = 1 - \frac{U_\infty^2 + 4U_\infty^2 R^2 \left(\frac{1}{h^2} \right) + 4U_\infty^2 R^4 \left(\frac{1}{h^2} \right)^2}{U_\infty^2} = 1 - 1 - 4R^2 \left(\frac{1}{h^2} \right) - 4R^4 \left(\frac{1}{h^2} \right)^2$$

$$C_p = -4 \frac{R^2}{h^2} \left(1 + \frac{R^2}{h^2} \right)$$

(9 marks)