Example 2.2.8

The I-section cantilever beam shown in Figure 1a is 1 m long and subjected to two orthogonal loads, resulting in a downward vertical force of 3 kN (generating a negative moment about axis z) and a horizontal force of 1 kN (generating a negative moment about axis y) at the tip as shown. The cross-section is doubly-symmetric with the dimensions shown in Figure 1b. The beam is made of steel with E = 200 GPa. Assuming linear behaviour throughout, calculate the axial stresses at the four points a, b, c and d at the built-in end of the beam, as shown in Figure 1c.

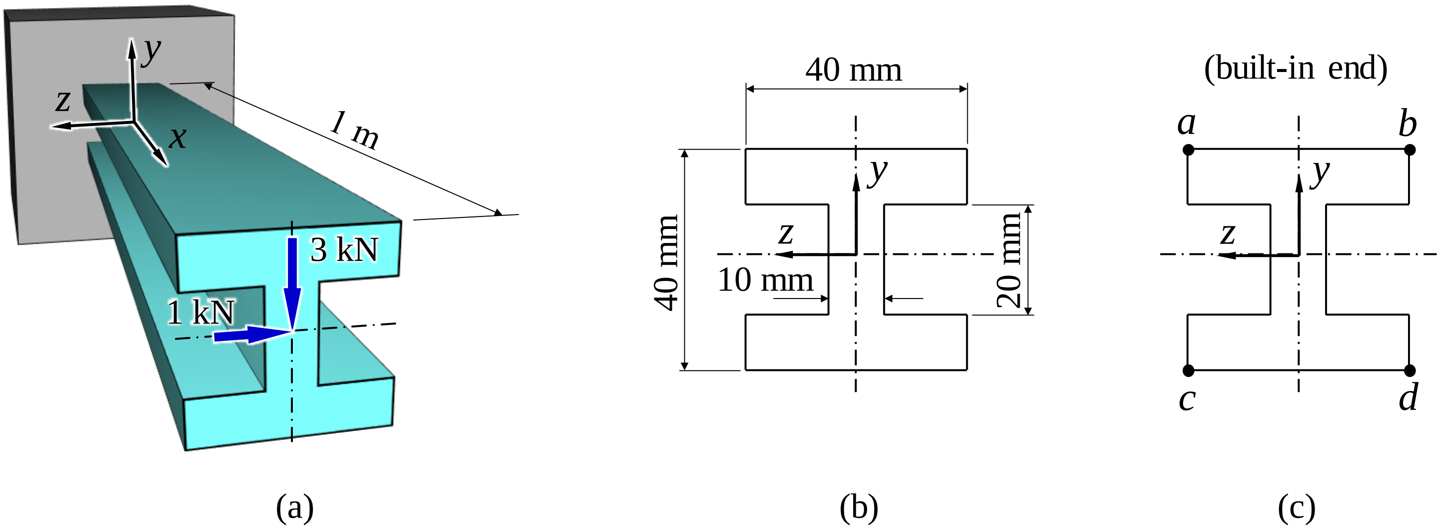


Figure 1: An I-section beam subjected to two orthogonal transverse loads.

The easiest way to compute I_{zz} is to 'subtract' the hollow portion from the outer contour:

$$I_{zz} = I_{contour} - I_{void} = \frac{40 \cdot 40^3}{12} - \frac{30 \cdot 20^3}{12}$$

$$\boxed{= 193\ 333 \ \text{mm}^4}$$

For I_{yy} we sum the central 'flange' with the two side 'webs':

$$I_{yy} = I_f + 2 I_w = \frac{20 \ 10^3}{12} + 2\left(\frac{10 \ 40^3}{12}\right)$$

$$\boxed{= 108\ 333 \ \text{mm}^4}$$

We then recall the engineer's bending formula:

$$-\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}$$

Note that bending stresses arise from bending about axes *z* **and** *y*. The assumptions of *linear elasticity* and of *small displacements* mean that we can consider the two stress fields independently, then sum their contributions.

Bending **about axis *z*** generates a linear stress distribution **along axis *y***, *i.e.*:

$$\sigma_{vert} = \frac{-M_{zz}}{I_{zz}} y \qquad -20 \text{ mm} < y < 20 \text{ mm}$$

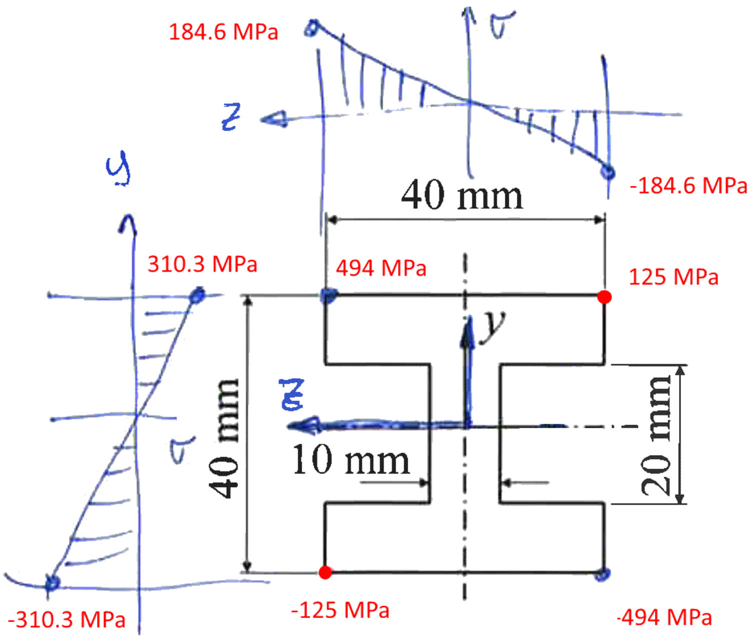
Along y: $\sigma = \pm 310.3$ MPa

Conversely, bending **about axis *y*** generates a linear stress distribution **along axis *z***, *i.e.*:

$$\sigma_{horiz} = \frac{-M_{yy}}{I_{yy}} z \qquad -20 \text{ mm} < z < 20 \text{ mm}$$

Along z: $\sigma = \pm 184.6$ MPa

The final stress distribution in the cross section is still linear, but is described by an inclined plane where this inclination is rotated about axis *x* (out-of-plane). This can be illustrated by plotting the two linear stress distributions (along *y* and along *z*), then summing the stresses at the four corners of the cross-section:



So the stresses at points *a*, *b*, *c* and *d* are 494 MPa, 125 MPa, -125 MPa and -494 MPa, respectively.