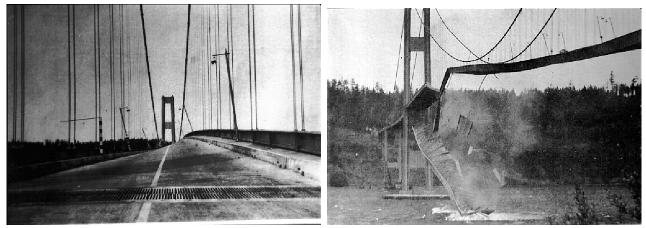
EMAT10100 Engineering Maths I
Lecture 1: Introduction

John Hogan & Alan Champneys

What is Engineering Mathematics?

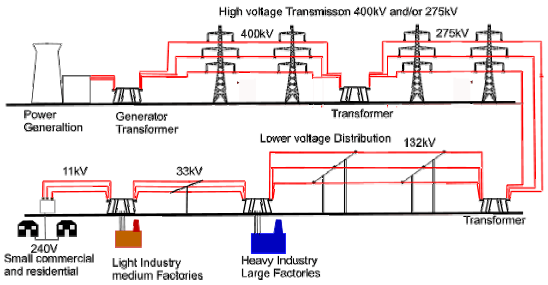
- Q: Why do we have to study Engineering Mathematics?
- A: Engineering is Mathematics, put into practice . . .
- e.g. Q. How to predict and understand unwanted instability?
  - . . . Tacoma Narrows bridge (USA) 1940



world's most flexible bridge starts to oscillate
  - after $\sim$ 1 day switches to torsional oscillation:
- A: Use theories from nonlinear dynamics

Future engineering challenges need maths

- e.g. Q: Future "Smart Grids", will they work?



Simplified UK Electical Power Transmission system

  - National grid is currently centrally controlled
  - "smart grid" is locally controlled, like Internet
  - but AC grid must synchronise, otherwise instability
- A: Needs ideas from network science

Engineering Mathematics Department

- Q. Why aren't we taught by 'real' engineers?
  - A1. If you were ill, would you go to the vet?
  - Would you ask a plumber to do your heart surgery?
  - Would you let a heart surgeon mend your leaking tap?
  - You pay the fees. You expect the best. You get the best.
  - Taught to the same high standard across the Faculty
    - with a common syllabus by UK's only Engineering Maths Dept
    - enables cross-fertilisation of engineering principles
  - A2. It's real, they pay us! other research includes
  - Control of helicopter rotors (*Westland Helicopters*)
  - Rattle in car engines (*Jaguar cars*)
  - Smoothing traffic flow (*UK Highways Agency*)
  - Flood forecasting (*Environment Agency*)
  - Stabilising landing gear (*Airbus*)
  - Placement of tidal energy devices (*DNV-GL*)
  - . . . and lots more!

What do we assume you know?

- A-level Curriculum:
  - core Mathematics modules C1, C2, C3, C4
- We assume that you have forgotten most things you learnt at school!!!
- See diagnostic test at end of this lecture for stuff we think you know. Most of these topics covered again (e.g. differentiation/integration techniques)
- If you've done Further Maths (or equivalent), most (but not all) of the syllabus will seem familiar, . . .
- . . . BUT we do things much more rapidly . . .
- . . . and in the computer age, graduate engineers need to understand principles, not just methods.
- We'll introduce a software package Maple that can do the calculus, algebraic manipulation, graph sketching etc.

Lectures: the core learning zone

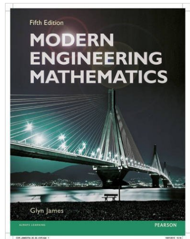
- When:
  - Mondays 11.00–11.50, 12.00–12.50
  - Thursdays 12.00–12.50
  - Nb: No lectures weeks 8,18 (reading weeks).
- Where:
  - Group 1. Aero, Eng Maths, Civil:
    Chemistry LT1
  - Group 2. Mechanical, Elec, EDeS, optional unit choice:
    Tyndale Lecture Theatre, Physics

For space & communication reasons:
Please attend lectures you're assigned to!

What happens in lectures?

- Different lecturers:
  - wk 1-12: John Hogan (group 1), Alan Champneys & (group 2)
  - wk 12-24: Rosalyn Moran, Nikolai Bode, Lucia Marucci, Oscar Benjamin,
- Slides given as handout. Write on them during the lecture!
- All slides available before the lecture (on Blackboard)
- Each lecture: quick exercises done there and then.
  So bring paper, pen & calculator.
- Exercises & further reading in one core textbook:
  - Modern Engineering Mathematics by Glyn James
  - no need to bring it to lectures though

How do I get hold of a copy of James?



- We use fifth edition (2015)
- We don't use *MyMathLab*
- We also have Special Edition
  (same but no Ch 1,6,11,12)
- Several ways to get this book:
  - Special edition Blackwells (Richmond Bld) $\approx \pounds 36$
  - Amazon (etc): full book $\geq \pounds 43$, Kindle $\approx \pounds 29$
  - Find second-hand copy (4th edtn. v. similar to 5th)
  - Borrow from the library each week
- Recommend buying it, will be useful in future years.

The drop-in sessions

- We do not teach, just enable you to learn
  - you can only learn maths by doing
- You are in charge of your own learning
- If you do not understand something YOU MUST ASK FOR HELP!:
- Drop-in sessions 5 times per week:
  - EVENINGS: 5pm-6pm Mon, Tue, Thur MVB Foyer
    LUNCHTIMES: 1pm–2pm Tues QB 1.68, Friday QB 1.69
    starting tomorrow 26th Sept.
  - dedicated, trained postgrads and teaching fellows
  - aim to go to $\approx$ 1 session per week
- or, via Blackboard discussion forum (quick questions).

The syllabus - broad outline

- wk 1-6 Algebra
  - Complex numbers (2 weeks)
  - Vectors (1 week)
  - Matrices (3 weeks)
- wk 7, 9-11 Calculus
  - Functions and differentiation (2 weeks)
  - Integration (1.5 weeks)
  - Partial differentiation (1.5 weeks)
- wk 12 Revision
- wk 13-15 Probability
- wk 16-17,19 Ordinary differential equations
- wk 20-22 Numerical analysis
- wk 23-24 Revision

How is all this assessed?

- Summative assessment (for passing the unit):
  - 20$\%$: 1.5hr mid-sessional examination
    in January exam window — (on first term's material)
  - 80$\%$: 3hr main summer exam
    on whole syllabus from both terms
- Formative assessment (for quick feedback):
  - in-class test
    Monday 23rd Oct (Wk 5)
  - 2 marked homeworks
    given out in Week 7 & Week 17
  - weekly online *"Questionmark"* multiple-choice tests — can take multiple times (more about this on Weds).

How to study maths at University

- Total effort
  - Year 1 $=$ 120 credits
  - EMAT 10100 $=$ 20 credits
  - University assumes you work for 40 hours a week
- Time to study maths each week:

$$\frac{20}{120} \times 40 \qquad \simeq 7 \text{ hours}$$

  3 hours per week are timetabled lectures
- Therefore:
  You study maths 4 hours per week outside lectures!

Four hours of homework???!!!

- Re-read the notes from lectures ~ 15 mins
- Read relevant sections of James ~ 15 mins
- Do exercises from James ~ 1.5 hours
- Get stuck . . .
- Panic!
- Get help at drop in session ~ 30 mins
- Take online test ~ 15 mins
- More exercises from James ~ 1 hour
- Take the online test again ~ 15 mins
- EVERY WEEK!

Diagnostic test

These are the sorts of things we think you know (but many will be covered again!)

1. Find the radius and the co-ordinates of the centre of the circle with the equation
$$x^2 + y^2 + 4x - 6y = 3$$
2. Expand (a) $(x-3)^4$, (b) $(x+1/2)^3$
3. Express as a partial fraction
$$\frac{2x-1}{(x+1)(x-2)}$$
4. Find all the solutions for $0 \le x < 2\pi$ of
$$2\cos^2 x + 3\sin x = 3$$

5. (a) Simplify $e^{2\ln x}$
   (b) Express as a single logarithm $4\ln 2 - (1/2)\ln 25$
6. Differentiate
$$\frac{x}{x^2 + 5x + 6}$$
7. The equations $x = t\sin t$ and $y = t\cos t$, $0 \le t < \infty$ define a spiral. (a) Sketch the curve in the (x, y)-plane.
   (b) Find $\frac{dy}{dx}$ in terms of t.
8. Use integration by parts to evaluate
$$\int_0^{\pi/8} x\cos 2x\,dx$$

9. Evaluate the indefinite integral
$$\int \frac{1}{x^2 + 10x + 50}$$
10. Find the general solution of the differential equation
$$\frac{dy}{dx} = 6xy^2$$

Answers will appear on Blackboard after this lecture

with sections of James you can refer to for more help

Get less than $6/10 \Rightarrow$ you need to seek help
(at drop-in classes)

REMEMBER: Maths is a subject you need to keep working at every week

Lecture 2. What is a Complex Number?

- Complex $\not\equiv$ complicated!
- Square any 'normal' number: positive answer!
- Central idea:
  - define a special new number j such that $j^2 = -1$
  - so we might say $j = \sqrt{-1}$
- Notation:
  - some people use 'i' instead of 'j'
  - don't confuse j with $\mathbf{i}, \mathbf{j}, \mathbf{k}$ (unit coordinate vectors)
- This may all seem very odd

Do complex numbers 'exist'?

- Excuse me. $\sqrt{-1}$ means what?
  - I cannot have $\sqrt{-1}$ oranges
  - $\sqrt{-1}$ metres
  - $\sqrt{-1}$ kilograms
  - $\sqrt{-1}$ percent for Eng Maths exam
- However, I also cannot have
  - -1 oranges
  - -1 kilograms of oranges
  - -1 percent for Eng Maths exam
- BUT: negative numbers are useful
- Complex numbers also useful: 'existence' irrelevant.

Example application: roots of quadratics

- Consider the simple quadratic equation:

$$x^2 - 1 = 0$$

  - roots are ± 1
- Not so simple quadratic equation:

$$x^2 + 1 = 0$$

  - no 'real' solutions
  - but $\pm j$ are solutions
- We will return to this next lecture
- Seems irrelevant. Actually: Simple Harmonic Motion
- Also: waves, AC current, control theory, stability...

General form of complex numbers

- All numbers you ever need to know take form

$$z = x + jy$$

where
  - x, y are 'normal' real numbers
  - $j = \sqrt{-1}$
- z is called a complex number
  - e.g.

$$z = 1 + j\sqrt{2}, \quad \text{(or, equivalently } 1 + \sqrt{2}j\text{)}$$
$$z = \pi - j,$$
$$z = -1.31 + 10.7j \qquad \text{etc.}$$

Real and imaginary parts

- Take a complex number $z = x + jy$
- We often write

$$x = \mathrm{Re}(z)$$
$$y = \mathrm{Im}(z)$$

and say x, y are the real and imaginary parts of z

- If $x = 0$, then $z = jy$
  - z is called (purely) imaginary
- If $y = 0$, then $z = x$
  - z is called (purely) real
- If $x = 0$ and $y = 0$, we say $z = 0$

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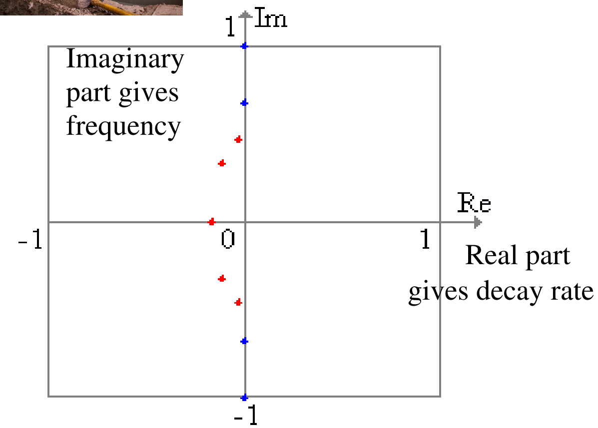
Engineering HOT SPOT

Structural testing:

Hit structure and record
what comes back at
each frequency
= sequence of complex numbers



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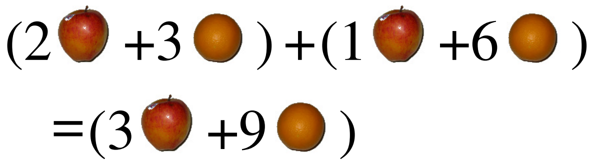
Add and subtract complex numbers?

- Basic principle:



- Same with complex numbers:

$$(x_1 + jy_1) + (x_2 + jy_2) = (x_1 + x_2) + j(y_1 + y_2)$$

$$(x_1 + jy_1) - (x_2 + jy_2) = (x_1 - x_2) + j(y_1 - y_2)$$

- Exercise: Let $z_1 = 2 + j$, $z_2 = 1 - 2j$. Find $z_1 + z_2$.

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Complex conjugate

- Take a complex number

$$z = x + jy$$

- Complex conjugate (or c.c.) is defined by

$$\overline{z} = x - jy$$

  - equivalent notation: z^* (used by Physicists & James)

- Note:
  - $\mathrm{Re}(\overline{z}) = \mathrm{Re}(z)$
  - $\mathrm{Im}(\overline{z}) = -\mathrm{Im}(z)$
  - $z + \overline{z}$ is real, $z - \overline{z}$ is imaginary

How to multiply complex numbers

- Basic principle
  - distributive: $a(b+c) = ab + ac$
- So:

$$\begin{aligned}(x_1 + jy_1)(x_2 + jy_2) &= x_1x_2 + jx_1y_2 + jy_1x_2 + j^2y_1y_2,\\ &= (x_1x_2 - y_1y_2) + j(x_1y_2 + x_2y_1)\end{aligned}$$

- DO NOT learn this formula
  - Learn how to apply the process
- Exercise: simplify $(2+j)(3-2j)$.
- Division: more difficult . . .

How to divide complex numbers

- Take complex numbers $z_1 = x_1 + jy_1$ and $z_2 = x_2 + jy_2$,

  how to find $\frac{z_1}{z_2} = \square + j\square$?.
- Trick: multiply top and bottom by $\overline{z}_2$:

$$\begin{aligned}\frac{z_1}{z_2} = \frac{z_1\overline{z_2}}{z_2\overline{z_2}} &= \frac{(x_1+jy_1)(x_2-jy_2)}{(x_2+jy_2)(x_2-jy_2)}\\ &= \left(\frac{x_1x_2+y_1y_2}{x_2^2+y_2^2}\right) + j\left(\frac{-x_1y_2+x_2y_1}{x_2^2+y_2^2}\right)\end{aligned}$$

- DO NOT learn this formula
  - Learn how to apply the process
- Exercise: simplify $\dfrac{2+j}{3-2j}$
- We will return to this next lecture.

Summary

- Complex numbers = "doing maths in stereo":

  $z = x + jy$, where $j^2 = -1$
- x is "real channel", y is "imaginary channel"
- Don't ask "do complex numbers exist?"

  . . . but "are complex numbers useful?"
- They are easy to add and subtract

  e.g. $(1+3j) + (2+5j) = (3+8j)$
- multiplication is harder:

  e.g. $(1+3j)(2+5j) = \ldots$ expand, remember $j^2 = -1$
- division is even harder, but there is a trick:

$$\text{e.g.} \quad \frac{(1+3j)}{(2+5j)} = \frac{(2-5j)}{(2-5j)} \cdot \frac{(1+3j)}{(2+5j)}$$

Homework

- Do the diagnostic test (from Lecture 1)
- Get hold of copy of *James* Modern Engineering Maths:
  - read *James*, Sections 3.1–3.2.2
  - attempt *James* Excercise 3.2.5 Qns 1,2,4-6
- if you get stuck:

  go to at least one of the drop-in sessions:
- Next time:
  - A geometric view of complex numbers
  - Polar form of a complex number