Design, Build, Test: Aerodynamics

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Note: this document is a first draft, so please notify of errors/omissions/typos.

1 Introduction

1.1 Task

The aerodynamic objectives are to achieve the:

- Minimum value of C_{D_0} (drag at zero lift) - this gives highest top speed ('dash')
- Highest value of $\frac{C_L^{\frac{3}{2}}}{C_D}$, at $C_L = 0.5$ (i.e. minimum C_D at that C_L) - this gives maximum endurance ('loiter')
- Highest value of maximum lift coefficient $C_{L_{max}}$ - this gives the slowest landing speed ('land')

For wings of the size you will make, these requirements are what you might expect for a 10Kg UAV cruising at ≈70mph and landing at ≈20mph.

The method of scoring will be through ranking all entries according to the objectives above. The rankings will then be summed for each team, and the team with the lowest summed ranking will win (ties are permitted).

Notice that the objectives above conflict. A high value of $C_{L_{max}}$ requires a deep aerofoil with a large radius leading edge, but this is unlikely to give the lowest C_{D_0}.

The ranking method is intentionally a sliding scale depending on peer performance; you are competing between groups to achieve a good compromise in the above areas. This is how the real world operates!

1.2 Analysis

The planform and % chord flap and droop lengths are fixed at 30% each. The variables you are free to adjust are the aerofoil section itself, the % thickness (subject to a minimum limit set by manufacturing requirements) and the droop and flap positions. The wing will be tested in three different droop/flap configurations, each time running an α sweep in incidence and recording lift, drag and pitching moment.

The starting point for calculations is the aerofoil. The parameters you need are C_{d_0}, $C_{l_{max}}$ and α_0. It is probably wise to start from a good set of experimental data at or close to the R_e at which the tests are to be conducted, which will be $\approx 670,000$. If you wish to choose an aerofoil for which you cannot find reliable experimental data, this is up to you, however, you will need to estimate C_{d_0}, $C_{l_{max}}$ and α_0 using ESDU data sheets or otherwise. One of the best sources of section data is *Theory of Wing Sections*.[1]

ESDU[1] produces easy to use accurate data sheets for estimating a wide range of parameters in aerodynamics. They will be of great use to you in future years. The relevant ones here are:

1. **94027/94028 -** 2D increments in lift and maximum lift for droop/flap $\Delta C_{l_{0_l}}$, $\Delta C_{l_{0_t}}$, $\Delta C_{l_{max_l}}$, $\Delta C_{l_{max_t}}$

[1] If you're wondering, the history of ESDU is aptly summarised by *Wikipedia*: 'In 1940 with World War II raging the British aircraft industry was rapidly expanding. Engineers from other industries as diverse as bicycle manufacture and piano making who lacked the specialised knowledge required for aircraft design were being drafted into the war effort to assist with the design and construction of aircraft. To meet this challenge, the government asked the Royal Aeronautical Society to form a Technical Department with the aim of producing easy-to-use design guides, data sheets and analysis methodologies.'

One Mars bar for the best sketch of an aircraft designed by a bicycle manufacturer and piano maker working in close collaboration!

2. **89034 -** plain 3D wing $C_{L_{max}}$
3. **70011 -** plain 3D wing lift gradient $\frac{dC_L}{d\alpha}$
4. **92031 -** 3D wing droop lift increment $\Delta C_{L_{0_l}}$
5. **91014 -** 3D wing flap lift increment $\Delta C_{l_{0_t}}$
6. **74035 -** induced drag factor for 3D wings δ
7. **Aero F.02.01.08 -** induced drag correction factor K for 3D wings with flaps
8. **06014 -** profile drag increment $\Delta C_{D_{0_t}}$ due to a full span flap for 3D wings
9. **Aero F.02.01.07 -** profile drag correction due to a part span flap for 3D wings

This document is written using the nomenclature of these sheets as closely as possible. The `.zip` provided gives the most important sheets, whilst any others can be downloaded from ESDU. On occasion we will make simplifications that reduce the calculation workload; one example being that we will keep $\frac{dC_L}{d\alpha}$ unchanged when the flap or droop is deployed.

The only drawback to these sheets is that they are sometimes too broad. You're designing a rectangular wing with a plain flap and droop, not a swept, tapered, cranked, wing with a transonic section, double and single slotted flaps and slats! So, what you need to do is simpler than at first sight.

Here's a rough guide.

1.2.1 Below Stall

l denotes leading edge droop, and t denotes the trailing edge flap. Lower case subscripts denote 2D (aerofoil section, defined on chord) variables, with upper case used for the wing (3D, defined on area) variables. A dash $'$ indicates the modified chord, which includes a small extension that results from an offset droop hinge. This is used in the data sheets, but is subsequently scaled by $\frac{c'}{c}$ back to the reference (unextended) chord.

1.2.1.1 Lift To find lift as a function of incidence you must know the zero lift incidence of the wing and the lift curve gradient (rate of change of lift with respect to incidence. Then use

$$C_L = \frac{dC_L}{d\alpha}\left(\alpha - \alpha_{0_{3D}}\right) \tag{1}$$

You will have the section zero lift angle. Although not precise, assume

$$\alpha_{0_{3D}} = \alpha_{0_{2D}} \tag{2}$$

The value of $\frac{dC_L}{d\alpha}$ can be found from figure 1e on sheet 70011. It will be near to 4.2, and this doesn't change greatly even if camber, flaps or droops are used.

When the droop or flap is deployed there is a change in C_L such that

$$C_L = \frac{dC_L}{d\alpha}\left(\alpha - \alpha_{0_{2D}}\right) + \Delta C_{L_{0_l}} + \Delta C_{L_{0_t}} \tag{3}$$

You can calculate these increments from data sheets 94027 and 94028, then use 96032 and 97011 to find the 3D values. Don't forget that these increments depend on the deflection angles.

1.2.1.2 Drag

Drag of the clean wing may be found from

$$C_D = C_{D_0} + \frac{(1+\delta)C_L^2}{\pi A_R} \tag{4}$$

where δ is from sheet 74035 and will be quite close to 0.016 for wings of our aspect ratio with no sweep. Assume

$$C_{D_0} = C_{D_{0_{2D}}} \tag{5}$$

where $C_{D_{0_{2D}}}$ comes from your section data.

When the flap is deployed there is an increment $\Delta C_{D_{0_t}}$. This may be found for a full-span flap from sheet 06014 figure 1a/b, and then adjusted for a part-span flap from sheet Aero F.02.01.07. There is also an induced drag correction K to use from figure 1a on sheet Aero F.02.01.07, giving

$$C_D = C_{D_0} + \Delta C_{D_{0_t}} + \frac{(1+\delta)C_L^2}{\pi A_R} + K\Delta C_{L_{0_t}}^2 \tag{6}$$

The effect of the droop on profile drag is not well documented. The flow will usually be attached near this region, so any change in zero lift drag would be small. There will be an influence on induced drag, which is included because the induced drag depends on the total lift coefficient C_L and the droop gives an increment in this value. So, assume the zero lift drag does not depend on the droop angle (satisfactory for for small angles, but dubious for larger ones).

1.2.2 At Stall

With your 2D aerofoil choice made, the next step is to adjust the 2D properties to allow for a drooped leading edge and a plain trailing edge flap. After this, these 2D results can be adjusted into 3D results.

1.2.2.1 Droop: 94027

Go to section 3.2; equation (3.10) gives the increment in $C_{l_{max}}$ due to a droop. K_g and K_L come from the graphs 2a and 1a.

K_g depends on the leading edge radius of the aerofoil. Interestingly, it reaches a maximum around $\frac{\rho_l}{c} = 0.011$. This may be worth noticing for design; however, $C_{l_{max}}$ for aerofoils generally continues to increase with leading edge radius, so the decision is not necessarily clear cut. Also, the flap effectiveness continues to increase with increases in leading edge radius.

K_l points to a decreasing return on droop deflections above 23^o.

Note that there is a small change in the effective chord if the hinge for the droop is offset from the aerofoil chord. As the hinge will usually be below this line, the chord increases slightly as the droop deploys.

Finally, the data used for validating the ESDU method only went down to $R_e = 4.5 \times 10^6$. This is well above where we are operating, but the slat and Krüger data does extend to our range, so although a risk it is not unreasonable to use this approach.

1.2.2.2 Plain Flap: 94028

The increment due to a plain flap is slightly more elaborate as it involves first finding the offset in the lift curve caused by the flap (this is a constant upwards offset across the whole α range). This comes from equation (4.5). It would also be possible to find this from a 2D panel code, although it would be unlikely to yield an improvement in accuracy.

Equation (4.8) then gives the final result needed. K_G is a factor that increases with leading edge radius and K_t is fixed. The parameter T is the most complicated factor, but B and C are zero as you can assume that separation takes place at the leading edge. T is then only a function of the % chord flap length.

1.2.2.3 Plain Wing: 89034 This is conceptually straightforward. When we move from 2D to 3D, the maximum lift coefficient of the aerofoil is not reproduced on the 3D wing, but is reduced by a factor μ_p. This is because the load distribution varies along the wing, dropping off towards the wingtips, and the stall will begin at the most highly loaded spanwise station, and the local lift coefficient at that point will be close to the maximum C_l of the 2D aerofoil.

So, we just need the factor μ_p and we're done. Unfortunately, as sketch 6.1 shows, you need to know the location and value of the ratio peak local lift coefficient:wing lift coefficient along the wing. You could labour through data sheets 84026 and 83040, but I'll save you some effort.

Figure 1 shows the local C_l along a rectangular wing of your aspect ratio for a $C_L = 0.367$. The root $C_l = 0.435$ so $\mu = \frac{0.435}{0.367}$. This is just for an uncambered section, and was calculated using AVL. Doing the same calculation for a cambered wing is unlikely to alter μ_p greatly but you may wish to check yourselves. μ_p is independent of incidence.

Finally, from the local C_l graph it looks like stall of the clean wing is likely to start at the root section. That's no bad thing; it's safer for flying as smaller assymetric rolling moments will be produced during a stall (rolling+stalling is only a yaw away from spinning). In addition the separated flow from the root section would normally buffet the tailplane and elevator, providing warning feedback to the pilot through the controls (assuming they're connected directly to the surfaces, otherwise a stick-shaker might be used).

Also note that if you had a constant C_l along the span, then the 3D maximum lift would be the same as the 2D section maximum lift, and the wing would stall simultaneously all the way along the span. This could be achieved using an elliptical chord distribution, and would lead to a fairly treacherous aircraft! [2]

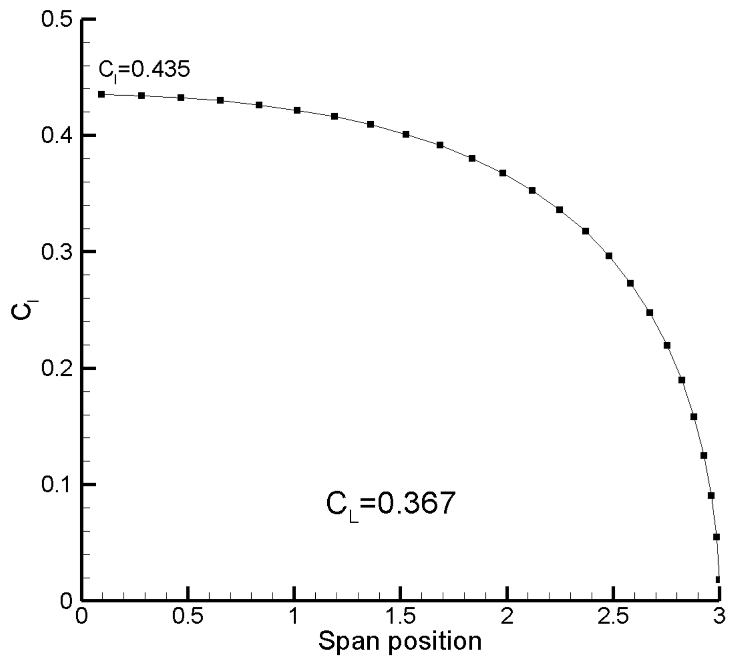


Figure 1: $A_R = 6$ spanload at $\alpha = 5^o$ for a rectangular untwisted uncambered wing

[2]You may be thinking of the Spitfire's 'elliptical' wing. However, the chord distribution was not actually elliptical, and the load per unit span distribution *certainly* was not, with at least a couple of degrees of nose-down twist at the tip.

1.2.2.4 Wing with Flap: 91014 Go straight to equation (6.3). $K_f = 1$ and F_R is the Reynolds number correction. You need to know the 2D maximum lift coefficient increment (from sheet 94028), as well as the factor μ_p used on sheet 89034. The variables Φ_o and Φ_i allow for the outboard and inboard limits of the flap extend, so it permits analysis of a flap that doesn't extend along the whole wing, just like the one used here. Figure 3a gives these values; remember that there is no sweep or taper, so on this figure you should be using the -8λ line. The net effect is that the maximum wing lift flap increment is almost scaled down in proportion to how much of the span is flapped, but actually the effect is slightly better than a proportional scaling.

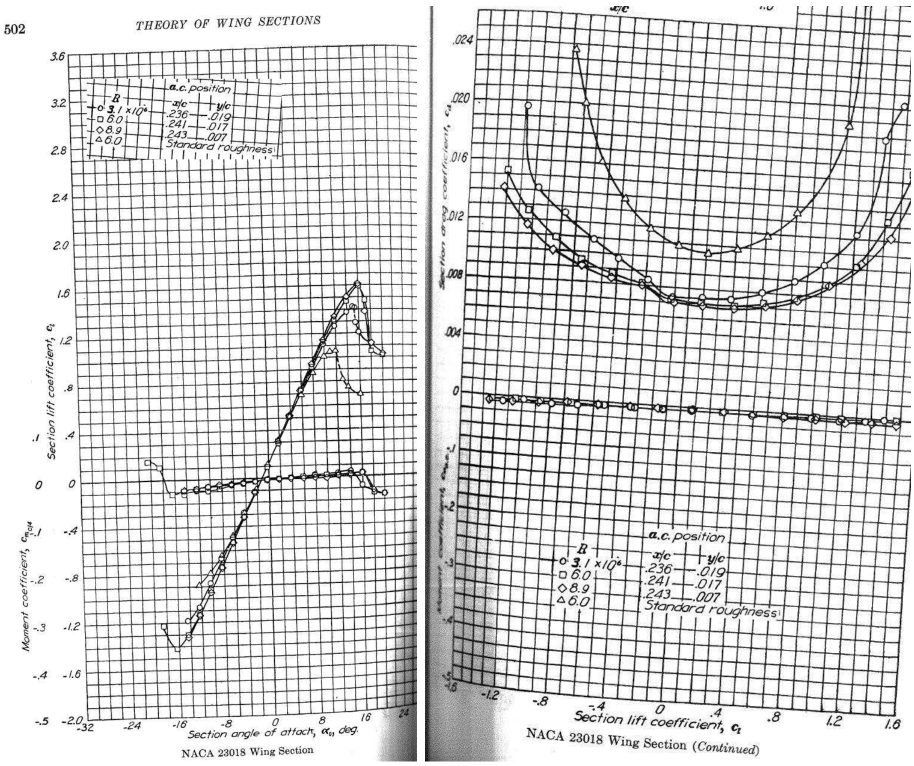
1.2.2.5 Wing with Droop and Flap: 92031 Equation (6.5) is requires the 2D increment in maximum lift from sheet 94027, the μ_p factor from sheet 89034 and a new variable Ψ_i. However, your droop extends along the complete span, so this is 1. Again, the Reynolds number factor F_R is used.

At last you can use equations (6.2) and (6.3) to get the maximum lift coefficient of the 3D wing with droop and flap, using the results from 89034, 91014 and 92031.

2 Example

We'll consider an example using NACA 23018. This is not necessarily a good choice, it is just an example! From figure 2, $C_{l_{max}} = 1.0$ for a rough surface (rivets etc mean the result for a rougher surface is a more accurate choice) and $C_{d_0} = 0.01$.

You can also refer to the example spreadsheet.



(a) Lift (b) Pitching moment and Drag

Figure 2: NACA 23018 lift curve at varying R_e and roughness, from Abbott and von Doenhoff[1]

2.1 Below Stall

C_{d_0} must come from the aerofoil data you have, and you can assume the value C_{D_0} for the wing is the same as for the aerofoil. When the flap is deployed there is a change ΔC_{D_0} due to added profile drag, as well as a change in induced drag due to alteration of the span loading.

The induced drag factor $1+\delta$ for the clean wing can be found from 74035; for a wing of $A_R = 6$ this comes in at around 1.016 (this might surprise you; the loading distribution is not too bad for a rectangular wing of this A_R). 74035 only applies to uncambered wings, but since our camber will be the same along the span and is roughly equivalent to a change in incidence, we shall not adjust the value further.

From 06014 for a 30% full span flap at 30^o then $\Delta C_{D_0} = 0.075$. Flaps 02.01.07 allows this to be scaled as $\Delta C_{D_0} = 0.075\frac{2}{3} = 0.05$ to allow for the part span effect.

This is a zero lift drag, so we need to use Flaps 02.01.08 to correct for the effect on induced drag. Roughly, $K = 0.175$ from figure 1b, allowing for 20% span un-flapped at root and tip (you may wish to linearly interpolate for better accuracy). Now use

$$C_D = C_{D_0} + \Delta C_{D_{0_t}} + \frac{(1+\delta)C_L^2}{\pi A_R} + K\Delta C_{L_{0_t}}^2 \tag{7}$$

where

$$C_L = \frac{dC_L}{d\alpha}(\alpha - \alpha_{0_{2D}}) + \Delta C_{L_{0_l}} + \Delta C_{L_{0_t}} \tag{8}$$

$\Delta C_{L_{0_t}}$ comes from 97011 with $K_{f0} = 1.02$, $\Delta C_{l_{0_t}} = 1.09$ (see below), $J_p = 0.74$ and $\Phi_o = 0.82$, $\Phi_i = 0.29$, giving $\Delta C_{L_{0_t}} = 0.597$.

$\Delta C_{L_{0_l}}$ comes from 96032 and gives $\Delta C_{L_{0_l}} = -0.117$.

Remember, when you input C_L in equation (7) this is the total lift coefficient including the droop and flap increments. The correction constant K is used because the flap deflection modifies the spanwise loading distribution from what was assumed when estimating δ.

2.2 At Stall

We shall consider a wing using NACA 23018. For this section $C_{l_{max}} = 1.0$ and $\alpha_{0_{2D}} = -1^o$. Fundamentally $C_{l_{max}}$ is a viscous quantity and must usually come from experimental data, while $\alpha_{0_{2D}}$ is an inviscid quantity that could equally well be found using, for example, a panel method.

First find the 2D increments due to a 30% droop and flap.

2.2.1 Droop: 94027

Using $z_h = 0.04$, $c_l = 0.3$ then $c_l' = 0.31$ and $c' = 1.014$. Then $\frac{c_{el}}{c'} = 0.303$ and $K_0 = 1$ so if $\delta_l = \frac{\pi}{180} \times 20$ then $\Delta C_{l_{0_l}} = -0.174$.

The maximum lift increment with $K_e = K_g = K_l = 1$ is $\Delta C_{l_{max_l}} = 0.580$.

2.2.2 Flap: 94028

Using $J_p = 0.54$ (for roughly a 30^o deflection) and $\delta_t = \frac{\pi}{180} \times 30$ with $\frac{c_t}{c'} = 0.296$ (the droop alters the chord a little if the hinge is offset from datum line) the gives $\Delta C_{l_{0_t}} = 1.18$.

NACA 4 and 5 digit aerofoils share the same thickness distributions (but have difference camber distributions). For these designs,[1] $\frac{\rho_l}{t} = 1.1019 \left(\frac{t}{c}\right)^2 \frac{c}{t}$.

For the maximum lift increment $\frac{\rho_l}{t} = 0.198$ (you'll need to check this carefully for the aerofoil you use) with $K_G = 2.122$, $K_t = 0.8$, $A = 0.794$, $T = 0.442$ then $\Delta C_{l_{max_t}} = 0.792$.

Finally, that's a $C_{l_{max}} = 2.37$ for the aerofoil, with the high-lift devices providing boost of 110%.

2.2.3 Clean Wing: 89034

Maximum clean wing lift coefficient is $\frac{1.2}{1.186} = 0.843$.

2.2.4 Wing with Flap: 91014

$K_f = 1$, $\Phi_o = 0.82$, $\Phi_i = 0.29$ then $\Delta C_{L_{max_t}} = 0.354$.

2.2.5 Wing with Droop: 92031

$\psi_i = 1$ (full span) so $\Delta C_{L_{max_l}} = 0.489$.

Finally $C_{L_{max}} = 1.69$ for the wing, with the high lift devices giving a boost of 86%.

3 Tips on Design

The 'dash' requirement for a low C_{d_0} will tend to drive towards a thin symmetric section. At high speed the lift coefficient is low, which means the bulk of the drag is profile rather than induced drag. For 2D sections camber can be thought of as a somewhat more efficient technique for producing lift than incidence, which is why sections are often cambered. However, if the target lift coefficient is low (i.e. high speed) then this is not so important.

The 'loiter' requirement is the opposite; it would be expected that a cambered section could produce a lower drag at a fixed positive lift.

Whichever section you choose, you will need to estimate how it performs with the flap and droop deployed. Thicker sections with a larger radius leading edge will be more effective even before the devices are added, and will subsequently show larger increments due to the flap/droop. Thus, the improvement of a thicker section is compounded. In terms of mechanical behaviour, thin sections will be heavier for a given bending or torsion stiffness.

A sensible way to proceed would be to select aerofoils over a range of thicknesses and camber, analyse these and compare the end results. From this you will be able to see the 'exchange rate' between the quantities and make an informed decision about which to use. It is likely that you will want to use the largest possible deflections of the droop and flap for the 'land' case.

References

[1] I.H. Abbot and A.E. Von Doenhoff. *Theory of Wing Sections*. Dover Publications Inc., 1959. Standard Book Number 486-60586-8.