UNIVERSITY OF BRISTOL
FACULTY OF ENGINEERING

First Year Examination for the Degree of Master of Engineering

MAY/JUNE 2017 2 Hours

AENG11100

FLUIDS I

Solutions

.

Q 1

$$p_G = \rho_{water} g \mathrm{h}_{\mathrm{water}} = 1030 \times 9.81(\mathrm{h}_{\mathrm{water}}) = 4 \times 10^5 N/m^2$$
$$\mathrm{h}_{\mathrm{water}} = 39.59m$$

(2 *marks*)

Q 2

For vertical component use "weight of water above", in this case that means weight of water in the cylinder (minus the hemisphere) directly above the dome. For horizontal force use the pressure at the centre of the projected area times the projected area. In both cases use gauge pressure as the atmospheric pressure acts on the other side of the windows.

$$F_v = \left(-\tfrac{2}{3} \times \pi \times 0.5^3 + 3 \times \pi \times 0.5^2\right) \times 1000 \times 9.81 = \left(-\tfrac{1}{3} + 3\right) \times \pi \times 2452.5 =$$
$$= 20546N$$
$$F_h = 2 \times 1000 \times 9.81 \times \pi \times 0.5^2 = 15409.5N$$

(3 *marks*)

Q 3

Steady, incompressible, inviscid,1D flow

(3 marks)

Q 4

Drag coefficient must be assumed the same between the wind tunnel and free air

$$D_{tunnel} = A_{tunnel} \tfrac{1}{2} \rho U_{tunnel}^2 C_D$$
$$D_{free} = A_{free} \tfrac{1}{2} \rho U_{free}^2 C_D$$
$$D_{free} = 16 A_{tunnel} \tfrac{1}{2} \rho 4 U_{tunnel}^2 C_D = 64 D_{tunnel} = 192N$$

(3 *marks*)

Q 5

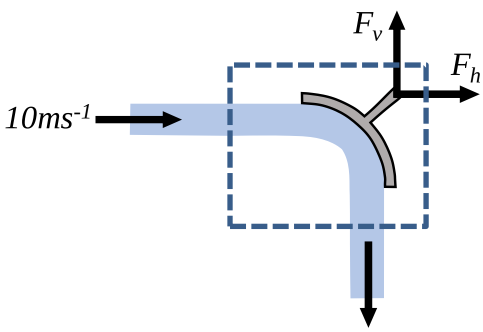
Applying Bernoulli (set vertical zero at downstream location)

$$A_1 V_1 = A_2 V_2$$
$$V_2 = \tfrac{A_1}{A_2} V_1 = \tfrac{5}{4} = 1.25$$
$$p_1 + \tfrac{1}{2}\rho V_1^2 + \rho g h_1 = p_2 + \tfrac{1}{2}\rho V_2^2 + \rho g h_2 + \Delta p_{loss}$$
$$p_2 = p_1 + \tfrac{1}{2}\rho\left(V_1^2 - V_2^2\right) + \rho g\left(h_1 - h_2\right) - \tfrac{1}{2}\rho V_1^2\left(1 - \tfrac{A_1}{A_2}\right)^2$$
$$p_2 = p_1 + \tfrac{1}{2}\rho V_1^2\left(1 - \left(\tfrac{A_1}{A_2}\right)^2\right) + \rho g\left(h_1 - h_2\right) - \tfrac{1}{2}\rho V_1^2\left(1 - \tfrac{A_1}{A_2}\right)^2$$
$$p_2 = p_1 + \rho V_1^2\left(1 - \tfrac{A_1}{A_2}\right)\tfrac{A_1}{A_2} + \rho g\left(h_1 - h_2\right)$$
$$p_2 = 2 \times 10^5 + 1000 \times 5^2\left(\tfrac{3}{16}\right) - 1000 \times 9.81 \times 3 = 1.752575 bar$$

(3 *marks*)

Q 6

Consider a control volume fixed relative to the plate. The net horizontal force on the control volume equals the rate of change of momentum in that direction. We also assume that atmospheric pressure acts through the jet diameter so there is no contribution to the horizontal force from the jet entry into the CV



$$F_{CV-x} = \dot{m}(V_2 - V_1) = -\pi \times 0.05^2 \times 1000 \times 10^2 = -785.4N$$
$$F = -F_{CV-x} = 785.4N$$
$$F_{CV-y} = \dot{m}(V_2 - V_1) = -\pi \times 0.05^2 \times 1000 \times 10^2 = -785.4N$$
$$F = -F_{CV-y} = 785.4N$$

(*4 marks*)

Q 7 sum the velocity components at the top of the cylinder

$$u = U + \frac{+\Gamma}{2\pi}\frac{y}{(x^2+y^2)} + \frac{-\kappa}{2\pi}\frac{(x^2-y^2)}{(x^2+y^2)^2}$$
$$u = U + \frac{+\Gamma}{2\pi}\frac{1}{0.5} + \frac{\kappa}{2\pi}\frac{1}{(0.5)^2} = 10 + \frac{5\pi}{2\pi}\frac{1}{0.5} + \frac{5\pi}{2\pi}\frac{1}{(0.5)^2} = 10 + 5 + 10 = 25m/s$$

(2 marks)

Q8

(a) Applying Bernoulli's equation between the surface and the exit of the siphon, taking the vertical height as zero at the siphon exit. Note that the exit pressure is atmospheric and the surface velocity is assumed zero

$$p_a + \rho g H = p_a + \tfrac{1}{2}\rho V_e^2$$

$$V_e = \sqrt{2gH}$$

$$\dot{m} = \rho A\sqrt{2gH}$$

Applying Bernoulli's equation between the exit and the highest point of the siphon

$$p_h + \rho g(H+h) + \tfrac{1}{2}\rho V_h^2 = p_a + \tfrac{1}{2}\rho V_e^2$$

From continuity

$$AV_h = AV_e \qquad \rightarrow \qquad V_h = V_e$$

Substituting back into Bernoulli's equation

$$p_h + \rho g(H+h) = p_a + \tfrac{1}{2}\rho V_e^2(1-1) = p_a$$

$$h = \frac{p_a - p_h}{\rho g} - H$$

At the maximum point $p_h = p_v$

$$h_{\max} = \frac{p_a - p_v}{\rho g} - H$$

Note: Do not use the static equations to get $h_{\max}$, fluid statics seems to work but that is a coincidence because the area of the pipe doesn't change and the dynamic terms cancel. Consider what would happen if the area at the exit were half or twice the area at $h_{\max}$.

(13 marks)

(b) Applying values to equation for maximum height

$$2 = \frac{1.015\times10^5 - 3400}{1000\times9.81} - H \qquad \rightarrow \qquad H = 8$$

From previously, velocity at the exit given by

$$V_e = \sqrt{2gH} = \sqrt{2\times9.81\times8} = 12.528$$

Applying Bernoulli's equation between the exit and a point 1m above the exit ($p=p_a$ throughout)

$$p_a + \rho g\times4 + \tfrac{1}{2}\rho V_1^2 = p_a + \tfrac{1}{2}\rho V_e^2$$

$$g\times8 + V_1^2 = V_e^2$$

$$V_1 = \sqrt{V_e^2 - 8g} = \sqrt{12.528^2 - 8\times9.81} = 8.858ms^{-1}$$

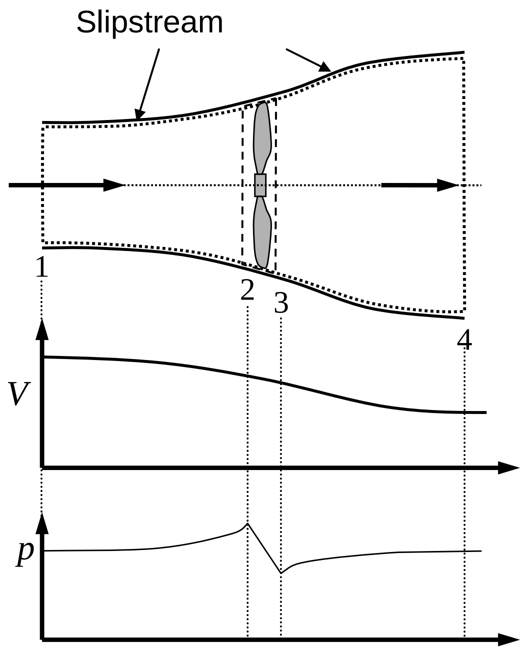
Applying continuity

$$A_1V_1 = A_eV_e \qquad \rightarrow \qquad A_1 = \frac{A_eV_e}{V_1} = 0.001\times\frac{12.528}{8.858} = 0.001414m^2$$

(7 marks)

Q9

(a) Use the actuator disc theory for an ideal windmill, see figure below



Assumptions: Frictionless & incompressible, Steady 1D flow (neglect rotation and variation across the disc radius). Actuator disc is thin so $A_2=A_3=A_d$ & $V_2=V_3=V_d$. $p=p_a$ at all points on slipstream boundary & 1 & 4

Continuity: $Q=V_dA_d$

Bernoulli's equation for CV 1-2 & CV 3-4

$$\begin{aligned} p_1+\tfrac{1}{2}\rho V_1^2 &= p_2+\tfrac{1}{2}\rho V_d^2 \\ p_3+\tfrac{1}{2}\rho V_d^2 &= p_4+\tfrac{1}{2}\rho V_4^2 \end{aligned} \quad \rightarrow \quad p_3-p_2=\tfrac{1}{2}\rho\left(a^2V^2-V^2\right)$$

Steady Flow momentum for CV 2-3:

$$(p_2-p_3)A_d+F_{CV}=\rho Q(V_d-V_d)=0 \quad \rightarrow \quad F_{CV}=(p_3-p_2)A_d$$

Where F is the force on the control volume

Applying results from Bernoulli's equation above

$$F_{CV}=\tfrac{1}{2}\rho A_dV^2\left(a^2-1\right)=\tfrac{\pi}{8}\rho d^2V^2\left(a^2-1\right)$$

Force on the windmill is equal and opposite to the force on the CV so

$$F=\rho\tfrac{\pi}{8}d^2V^2\left(1-a^2\right)$$

(8 marks)

(b) Steady Flow momentum for CV 1-4:

$$0+F_{CV}=\rho Q(V_4-V_1) \quad \rightarrow \quad F_{CV}=\rho V_dA_d(V_4-V_1)$$

From momentum & continuity

$$(p_3-p_2)A_d=\rho V_dA_d(V_4-V_1)$$

Eliminating (p_3-p_2) using Bernoulli's equation above

$$\rho V_d\left(aV - V\right) = \tfrac{1}{2}\rho V^2\left(a^2 - 1\right)$$
$$V_d V\left(a - 1\right) = \tfrac{1}{2}V^2\left(a + 1\right)\left(a - 1\right)$$
$$V_d = \tfrac{1}{2}V\left(a + 1\right)$$

The power drawn from the air by the disc is

$$P_{\text{disc}} = -F_{CV}V_d = -\rho Q\left(V_4 - V_1\right)V_d = \tfrac{1}{2}\rho A_d V_d\left(V_1 - V_4\right)V_d = \tfrac{1}{4}\rho A_d\left(aV + V\right)\left(V^2 - a^2V^2\right)$$

Power in the wind passing through the disc area, if the disc were not present.

$$P_{\text{wind}} = \tfrac{1}{2}\dot{m}V_1^2 = \tfrac{1}{2}\rho A_d V_1\, V_1^2 = \tfrac{1}{2}\rho A_d V^3$$

The efficiency of the turbine is therefore

$$\eta = \frac{P_{\text{disc}}}{P_{\text{wind}}} = \frac{\tfrac{1}{4}\rho A_d\left(aV + V\right)\left(V^2 - a^2V^2\right)}{\tfrac{1}{2}\rho A_d V^3} = \frac{\left(aV + V\right)\left(V^2 - a^2V^2\right)}{2V^3} = \frac{1}{2}\left(a + 1\right)\left(1 - a^2\right) = \frac{1 + a - a^2 - a^3}{2}$$

(6 marks)

(c) Use a Galilean transformation so that we consider the force and power relative to a frame moving with the vehicle at constant speed v

$$V_d = \tfrac{1}{2}\left(V + v\right)\left(a + 1\right)$$

The power drawn from the air by the disc is

$$P_{\text{disc}} = -F_{CV}V_d$$

The power required to drive the electric motor at a speed v

$$P_{\text{motor}} = -F_{CV}v$$

Assuming $P_{disc}=P_{motor}$ moving upwind

$$\eta = \frac{P_{\text{disc}}}{P_{\text{motor}}} = \frac{V_d}{v} = \frac{(V + v)\left(1 + a\right)}{2v} = \frac{1}{2}\left(\frac{V}{v} + 1\right)\left(1 + a\right) = 1$$
$$v = \frac{\left(1 + a\right)}{\left(1 - a\right)}V$$

Similarly downwind

$$\eta = \frac{P_{\text{disc}}}{P_{\text{motor}}} = \frac{V_d}{v} = \frac{(V - v)\left(1 + a\right)}{2v} = \frac{1}{2}\left(\frac{V}{v} - 1\right)\left(1 + a\right) = 1$$
$$\frac{V}{v} = \frac{2}{\left(1 + a\right)} + 1 = \frac{\left(3 + a\right)}{\left(1 + a\right)} \qquad\qquad v = \frac{\left(1 + a\right)}{\left(3 + a\right)}V$$

Note: Maximum and minimum efficiency at

$$\frac{d\eta}{da} = \frac{0 + 1 - 2a - 3a^2}{2} = 0$$
$$3a^2 + 2a - 1 = 0$$ ignore the other solution (a=-1)
$$a = \frac{-2 + 4}{6} = \frac{1}{3}$$

So maximum speed is into wind given by $v = \dfrac{\left(\frac{4}{3}\right)}{\left(\frac{2}{3}\right)}V = 2V$ *(6 marks)*

Q10 (a) The stream function is given by

$$\boxed{\psi = U_\infty y - \frac{\kappa}{2\pi}\frac{\sin\theta}{r} = U_\infty r\sin\theta\left(1-\frac{\kappa}{2\pi U_\infty r^2}\right)}$$

The velocity components are given by

$$V_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \left(1-\frac{\kappa}{2\pi U_\infty r^2}\right)U_\infty\cos\theta,\quad V_\theta = -\frac{\partial\psi}{\partial r} = -\left(1+\frac{\kappa}{2\pi U_\infty r^2}\right)U_\infty\sin\theta$$

Now the cylinder is a stream line of the flow so there is no flow normal to the cylinder i.e.

$$V_r = 0$$

This means that

$$\left(1-\frac{\kappa}{2\pi U_\infty r^2}\right)U_\infty\cos\theta = 0$$

for all θ so the circulation must be given by

$$\kappa = 2\pi U_\infty R^2$$

The velocity components are then

$$V_r = \left(1-\frac{R^2}{r^2}\right)U_\infty\cos\theta,\quad V_\theta = -\left(1+\frac{R^2}{r^2}\right)U_\infty\sin\theta$$

On the cylinder

$$V_r = 0,\quad V_\theta = -2U_\infty\sin\theta$$

The pressure coefficient on the cylinder is given by

$$\boxed{C_p = 1-\left(\frac{V}{U_\infty}\right)^2 = 1-4\sin^2\theta}$$

The pressure distribution on the cylinder is then

$$p(\theta) = p_\infty + \frac{1}{2}\rho U_\infty^2(1-4\sin^2\theta)$$

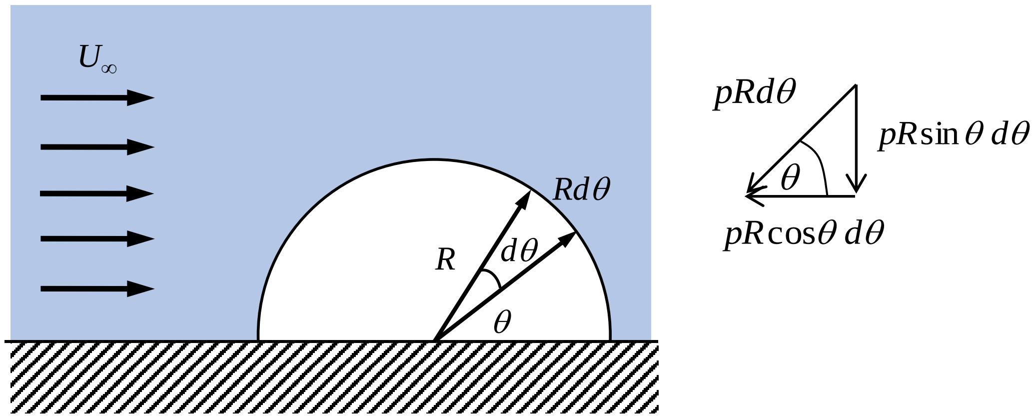
(6 marks)

(b) Consider a small arc of the outer surface, as shown in the sketch, of size $Rd\theta$. The force acting on this element per unit depth is given by $p(\theta)Rd\theta$

this acts normal to the surface and must be resolved to get the components (see above). Furthermore the net pressure at each point is in fact

$p(\theta) - p_0$

because of the static pressure inside the greenhouse. Hence the force in the vertical direction over the entire upper surface per unit length is given by



$$l = -\int_0^\pi (p(\theta) - p_0) R \sin\theta d\theta$$

So the net lift is given by

$$l = -\int_0^\pi (p(\theta) - p_0) R \sin\theta d\theta = p_0 R \int_0^\pi \sin\theta d\theta - \int_0^\pi p(\theta) R \sin\theta d\theta$$

$$= 2 p_0 R - \int_0^\pi p(\theta) R \sin\theta d\theta$$

where

$$p_0 = p_\infty + \frac{1}{2}\rho U_\infty^2$$

(6 marks)

Now using the results from part (a)

$$p(\theta) = p_\infty + \frac{1}{2}\rho U_\infty^2 (1 - 4\sin^2\theta)$$

find that

$$l = 2 p_\infty R + \rho U_\infty^2 R - \int_0^\pi [p_\infty + \frac{1}{2}\rho U_\infty^2 (1 - 4\sin^2\theta)] R \sin\theta d\theta$$

$$= 2 p_\infty R + \rho U_\infty^2 R - R \int_0^\pi [(p_\infty + \frac{1}{2}\rho U_\infty^2)\sin\theta - 2\rho U_\infty^2 \sin^3\theta)] d\theta$$

$$= 2 p_\infty R + \rho U_\infty^2 R - R(p_\infty + \frac{1}{2}\rho U_\infty^2) \int_0^\pi \sin\theta d\theta + 2\rho U_\infty^2 R \int_0^\pi \sin^3\theta d\theta$$

$$= 2 p_\infty R + \rho U_\infty^2 R - 2R(p_\infty + \frac{1}{2}\rho U_\infty^2) + 2\rho U_\infty^2 R \frac{4}{3}$$

So finally the net lift force acting on the greenhouse is equal to

$$l = \frac{8}{3}\rho_\infty U_\infty^2 R$$

(8 *marks)*