Chapter 7

Digital filters

Digital Filtering

- Introduction
- Frequency response
- Fir Filters
- Fir design

Introduction

- Digital filters are commonplace and an essential elements of everyday electronics such as digital radios, MP3 players, cellphones, and stereo receivers.

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Type of filters

- There are two types of filters:

1. Analogue filters.
2. Digital filters.

Analogue filters use discrete components such as resistors, capacitors and inductors.

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Analogue filters

There are also two types of analogue filters:

1. Passive filters: analogue filters that use discrete components such as resistors, capacitors and inductors.
2. Active filters: analogue filters that uses resistors, capacitors and operational amplifiers, (op-amp).

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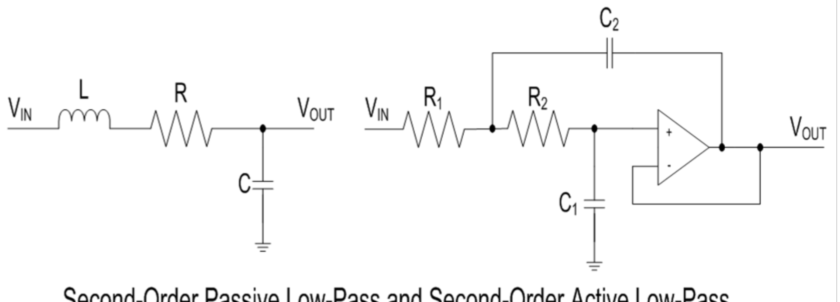
Analogue filters, examples



Second-Order Passive Low-Pass and Second-Order Active Low-Pass

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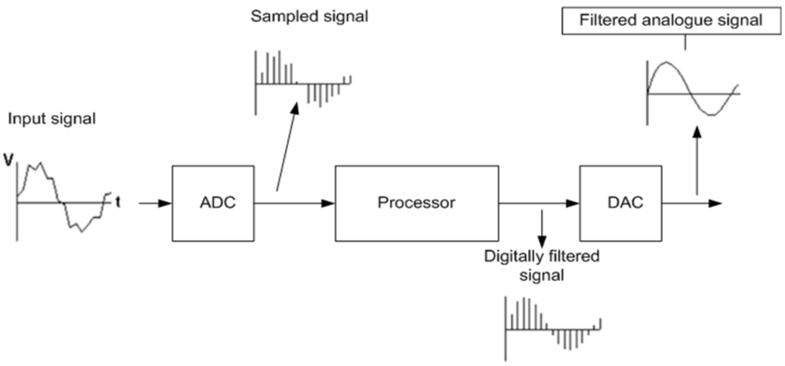
Digital Filters

Digital filters use digital processors to perform numerical operations on the sampled waveform.



Types of digital filters

There are two main types of digital filters, recursive and non-recursive:

1. FIR filters (Finite Impulse Response Filters): These are mainly used for there linear phase response. (non-recursive).

2. IIR Filters (Infinite Impulse Response Filters): These are mainly used for the low number of “tapes” but have non linear phase response. (recursive).

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Advantage of an FIR filter

- Amongst all the obvious advantages that digital filters offer, the FIR filter can guarantee linear phase characteristics.
- Neither analogue or IIR filters can achieve this.
- There are many commercially available software packages for filter design. However, without basic theoretical knowledge of digital filters, it will be difficult to use them.

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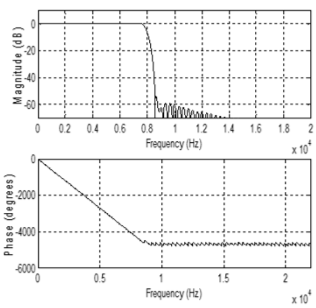
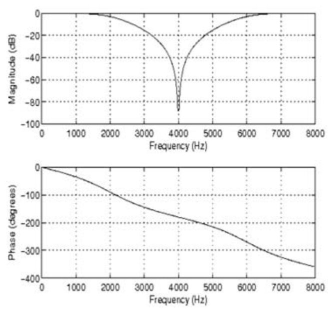
Type of digital filters

- Phase response of an FIR filter:



Phase response of an IIR filter:



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Advantages and disadvantages in using Digital Filters

- Can have linear phase response.
- Digital filters can be reprogrammable.
- They are low cost.
- They consume low power.
- The have small size (implemented in software)
- Easy to design.
- Have a very precise transfer function.
- Unlike the analogue filters, digital Filters don't:
  - Drift
  - Age
  - Change with temperature

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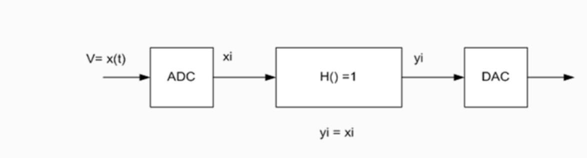
How non-recursive digital filters work

Eg 1: Filter with a unite gain



$$x_i : x_0, x_1, x_2, x_3, x_4, ...x_N$$

T: the sampling period

These digital filter has no effect on the input signal

$$\begin{aligned} y_0 &= x_0, \\ y_1 &= x_1, \\ y_2 &= x_2, \\ y_3 &= x_3, \\ y_4 &= x_4, \\ &\dots \\ &\dots \\ y_N &= x_N \end{aligned}$$

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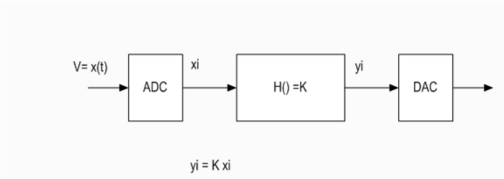
How digital filters work

Filter with a gain, K



$$x_i : x_0, x_1, x_2, x_3, x_4, \ldots\ldots\ldots x_N$$

The digital filter act as an amplifier

$$\begin{aligned} y_0 &= K\ x_0, \\ y_1 &= K\ x_1, \\ y_2 &= K\ x_2, \\ y_3 &= K\ x_3, \\ y_4 &= K\ x_4, \\ &\ldots \\ &\ldots \\ y_N &= K\ x_N \end{aligned}$$

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How digital filters work

Filter with a gain, K

K value	Effect of the digital filter
0 or 1	Switch
-1	Inverter
0 < K < 1	Attenuator
-1< K<0	Inverting attenuator
K > 1	Amplificator
K < -1	Inverting amplificator

$$\begin{aligned} y_0 &= K x_0, \\ y_1 &= K x_1, \\ y_2 &= K x_2, \\ y_3 &= K x_3, \\ y_4 &= K x_4, \\ &\ldots \\ &\ldots \\ y_N &= K x_N \end{aligned}$$

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How digital filters work

Eg 2: Two-term filter

$$y_i = \frac{x_i + x_{i-1}}{2}$$

$$y_0 = \frac{x_0 + x_{-1}}{2} \qquad y_3 = \frac{x_3 + x_2}{2}$$

$$y_1 = \frac{x_1 + x_0}{2} \qquad y_4 = \frac{x_4 + x_2}{2}$$

$$y_2 = \frac{x_2 + x_1}{2} \qquad y_5 = \frac{x_5 + x_3}{2}$$

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How digital filters work

Eg 3: Three-term filter

$$y_i = \frac{x_i + x_{i-1} + x_{i-2}}{3}$$

$$y_0 = \frac{x_0 + x_{-1} + x_{-2}}{3} \qquad y_3 = \frac{x_3 + x_2 + x_1}{3}$$

$$y_1 = \frac{x_1 + x_0 + x_{-1}}{3} \qquad y_4 = \frac{x_4 + x_3 + x_2}{3}$$

$$y_2 \frac{x_2 + x_1 + x_0}{3} \qquad y_5 = \frac{x_5 + x_4 + x_3}{3}$$

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How digital filters work

- N-term filter

$$y_i = \frac{x_i + x_{i-1} + ... + x_{i-(N-1)}}{N}$$

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Digital filter coefficients

- General form of a digital filter:

$$y_i = a_0 x_i + a_1 x_{i-1} + a_2 x_{i-2} + ... + a_N x_{i-(N-1)}$$

Eg:

$$y_i = 3x_i - x_{i-1} + 2x_{i-6} + 5x_{i-(106)}$$

What is the order of this filter? 106!

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Order of digital filters

The order of digital filter is the number of previous inputs that need to be stored in memory

Zero Order:

$$y_i = a_0 x_i$$

First order

$$y_i = a_0 x_i + a_1 x_{i-1}$$

Second order

$$y_i = a_0 x_i + a_1 x_{i-1} + a_2 x_{i-2}$$

N order

$$\boxed{y_i = a_0 x_i + a_1 x_{i-1} + a_2 x_{i-2} + \ldots + a_N x_{i-(N-1)}}$$

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How recursive digital filters work?

General form

$$\begin{aligned} y_i = \; & b_0 x_i + b_1 x_{i-1} + \ldots + b_N x_{i-N} \\ & + a_0 y_i + a_1 y_i + \ldots + a_N y_{i-N} \end{aligned}$$

Eg:

$$y_i = 2x_i - x_{i-1} + 3x_{i-2} + y_{i-1}$$

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What is digital filter design?

- The design of a digital filter is carried out in three steps:
  1. **Specifications:** they are determined by the applications
  2. **Approximations:** once the specification are defined, we use various concepts and mathematics that we studied so far to come up with a filter description that approximates the given set of specifications.
  3. **Implementation:** The product of the above step is a filter description in the form of either a difference equation, or a system function H(z), or an impulse response h(n). From this description we implement the filter in hardware or through software on a computer.

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Specifications

- Specifications are required in the frequency-domain in terms of the desired **magnitude** and **phase** response of the filter.
- Generally a **linear phase response** in the passband is desirable.
  – In the case of **FIR** filters, It is possible to have exact linear phase.
  – In the case of **IIR** filters, a linear phase in the passband is not achievable.

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Filter Types

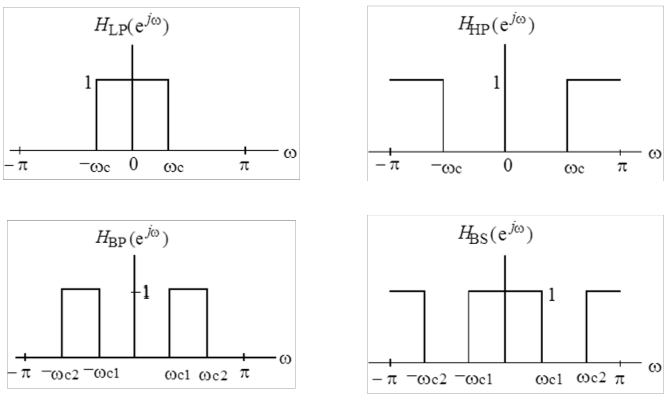
- Frequency responses for four basic types of **ideal filters**

LOW PASS

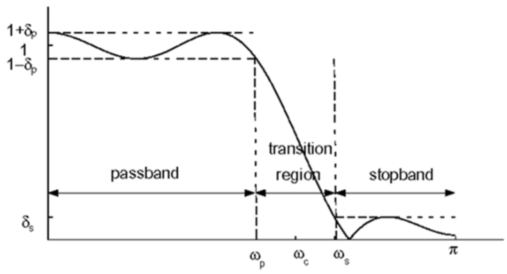
HIGH PASS

BAND PASS

BAND STOP



Filter Specification



δ_p passband ripple

δ_s stopband ripple

passband: $|\omega| < \omega_p$ in passband: $\left|H\left(e^{j\omega}\right) - 1\right| \le \delta_p$

stopband: $|\omega| > \omega_s$ in stopband: $\left|H\left(e^{j\omega}\right)\right| \le \delta_s$

transition band: $\omega_p < |\omega| < \omega_s$

Properties of an FIR Filter

- Filter coefficients:

$$y[n] = \sum_{k=0}^{N-1} b_k \cdot x[n-k]$$

x[n] **represents the filter input,**
b_k **represents the filter coefficients,**
y[n] **represents the filter output,**
N **is the number of filter coefficients (order of the filter).**

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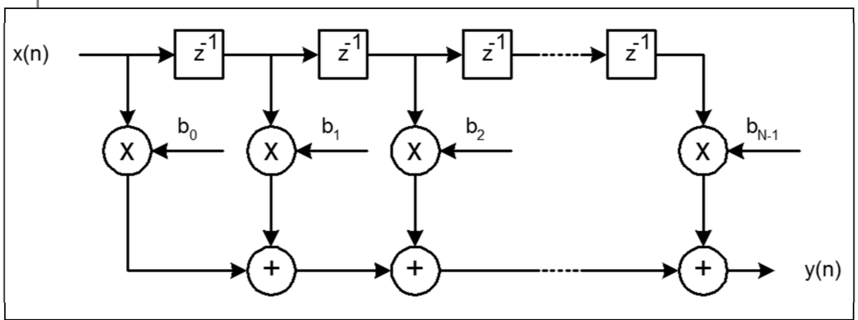
FIR Filter Structure

$$y[n] = \sum_{k=0}^{N-1} b_k \cdot x[n-k]$$

FIR equation



FIR Filter structure

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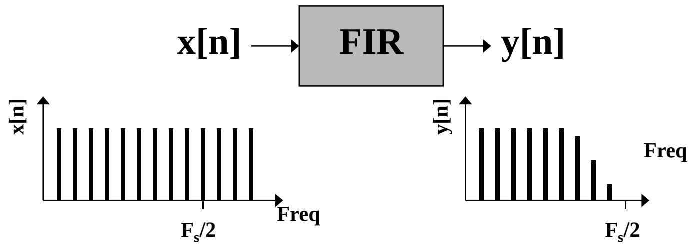
Frequency Response of an FIR Filter

- Frequency response: $H\left(e^{j\omega+2k\pi}\right)=H\left(e^{j\omega}\right)$



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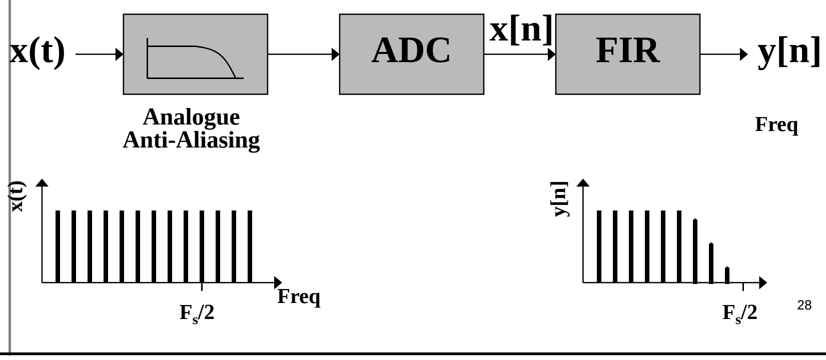
Frequency Response of an FIR Filter

- Solution: Use an anti-aliasing filter.



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Design Procedure

- To fully design and implement a filter five steps are required:
  (1) Filter specification.
  (2) Coefficient calculation.
  (3) Structure selection.
  (4) Simulation (optional).
  (5) Implementation.

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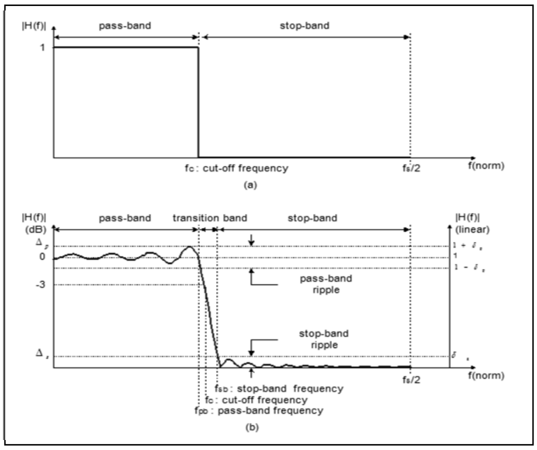
Filter Specification - Step 1



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Coefficient Calculation - Step 2

- There are several different methods available, the most popular are:
  – Window method.
  – Frequency sampling.
  – Parks-McClellan.
- We will just consider the window method.

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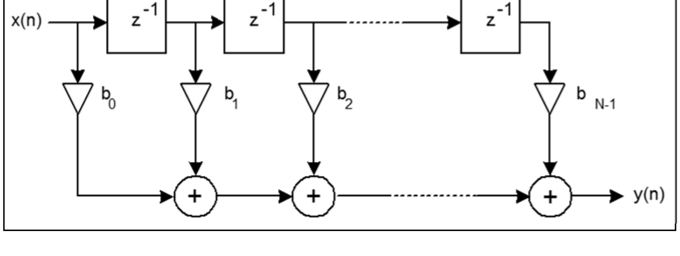
Realisation Structure Selection - Step 3

$$\boxed{H(z) = \sum_{k=0}^{N-1} b_k z^{-k}}$$

$$\boxed{Y(z) = H(z) \cdot X(z)} \qquad \boxed{y(n) = b_0 x(n) + b_1 x(n-1) + \ldots + b_{N-1} x(n-N+1)}$$



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Implementation - Step 5

b_0
b_1
b_2
b_3

x_0
x_1
x_2
x_3

y0 = b0*x0 + b1*x1 + b2*x2 + b3*x3

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Implementation - Step 5

b_0
b_1
b_2
b_3

x_4
x_1
x_2
x_3

y0 = b0*x0 + b1*x1 + b2*x2 + b3*x3

y1 = b0*x4 + b1*x1 + b2*x2 + b3*x3

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Implementation - Step 5

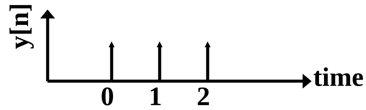
b_0
b_1
b_2
b_3

x_4
x_5
x_2
x_3

y0 = b0*x0 + b1*x1 + b2*x2 + b3*x3

y1 = b0*x4 + b1*x1 + b2*x2 + b3*x3

y2 = b0*x4 + b1*x5 + b2*x2 + b3*x3



Digital filters

- End -