Properties of Materials

Theme: Selection

Lecture 6: Materials Index

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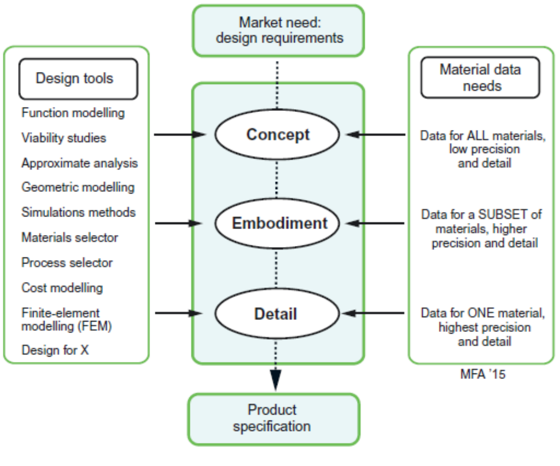
Typical Design Process



How can the vast range of material data be evaluated to give a designer the greatest freedom to consider alternatives?

- Material property charts (often referred to as Ashby plots)

Image: Ashby, Materials selection in Mechanical Design, 5th Ed, 2017.

Intended Learning Objectives

- Create Material Property Charts (Ashby Plots) to select materials for Engineering problems
- Translate a problem into a Materials Index
- Demonstrate how Shape can enhance structural efficiency of materials
- Select materials for wing elements

3

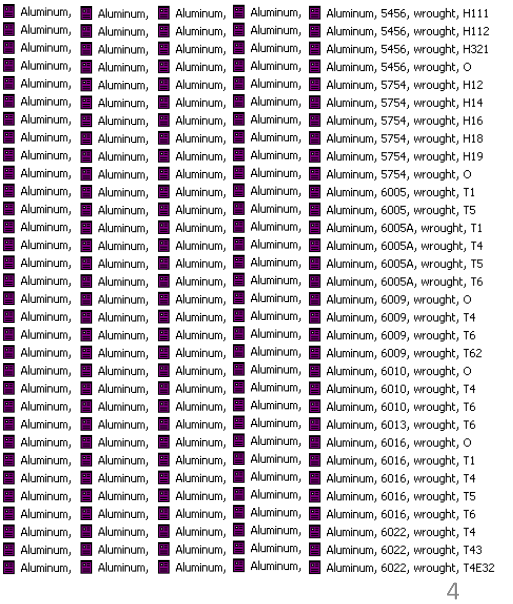
Material Selection

- What metal might we use for inexpensive, light weight applications?

Aluminium

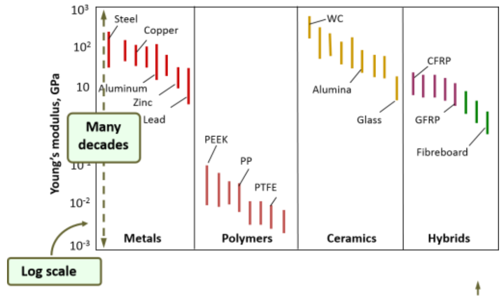
But which one?

- Difficult choice
  - Leads to experience dominated choices and/or specialisation



4

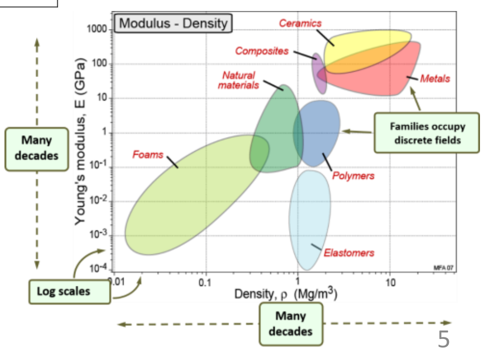
Selection Charts



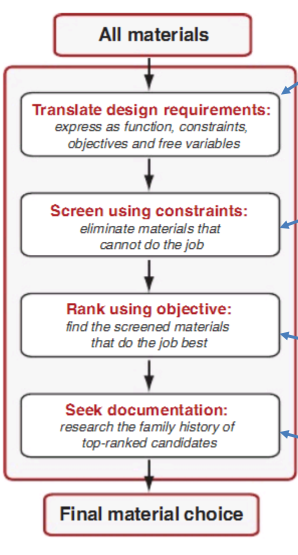
Charts of material properties used for comparison, screening and ranking materials for different applications

Typically use log scales due to large range of most properties

Available as PDF or *via* CES Edupack software



Materials Selection



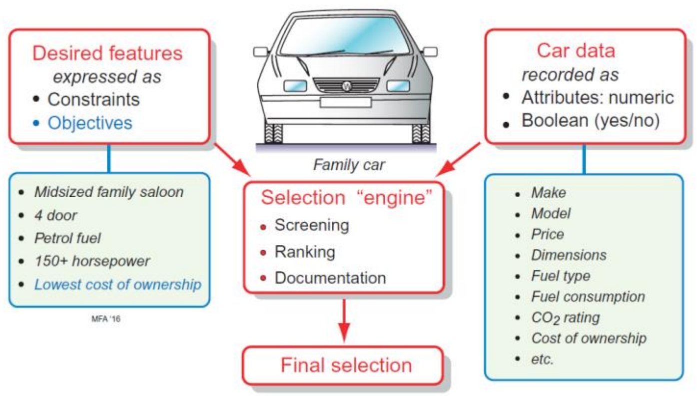
Turn the design into a small number of attributes that we can deal with

Remove as many materials as possible by identifying outright losers

Place candidates in order of performance (material index)

Get into the details of each candidate

Example: A new Family Car



Car data (right side) is screened to meet the constraints. The surviving candidates are ranked to meet the objective.

7

Activity

In groups of 2 or 3, select the best family saloon car.

Audi A4

BMW 3 series

Mercedes C-class

	Possible **Attributes** to consider.
Functions	Carry certain number occupants. Possessions.
Constraints	Colour. Fuel type. Power. Brand.
Objectives	Lowest monthly payment. Lowest emissions. Longest range.
Free Variables	Colour. Fuel type. Power. Brand.

8

Translation

What must a thing **do**?	What **attributes** must it have	What **properties** must it have?
• Bridge must not wobble • Drive shaft must not break • Heat sink must cool component	• Low deflection under load • Resist plastic strain/fracture • High heat transfer	• High elastic modulus E • High Strength or Toughness • High conductivity /heat capacity

Function	What must the component do?
Constraints	What non-negotiable conditions must be met? (Hard)
	What negotiable but desirable conditions? (Soft)
Objectives	What is to be maximised or minimised?
Free Variable	What parameters are we free to change?

9

Free Variables

- Attributes we are able to change

- We are usually free to choose structural properties such as thickness and cross-sectional areas
- Rarely the case that all dimensions are fixed or all free
- If something is fixed (say by a **design code**) we stop 'caring' about it

10

Attributes

Constraints

- Minimum attributes of a component
- Pass or fail
- No benefit to exceeding a constraint
  - Just adds cost
  - Reduces performance

Objectives

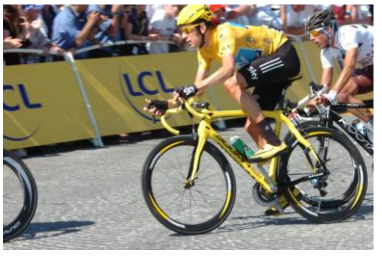
- Attribute to maximise or minimise
- Continuous scale from bad to good
- Always try to make an objective better
  - Lower price
  - Higher performance

11

Example: Bike frame

- Compare objectives and constraints for a bike intended for a professional vs day-to-day use

Simple analysis: Two attributes – mass and price



<u>Objective</u>: Light as possible
<u>Constraint</u>: Even teams have money limits

<u>Constraint</u>: Not so heavy it can't be used
<u>Objective</u>: Cheap as possible

12

Objective Functions

Equation describing the quantity to maximise or minimise

Mass = Density x Volume

$$m = \rho AL$$

Material property   Dimensions

Price = Cost per unit mass x Density x Volume

$$C = C_m \rho AL$$

Material properties

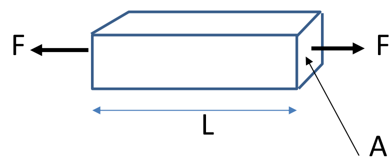
Dimensions

Deflection

$$\delta = \frac{1}{E}\frac{FL}{A}$$

Material property   Load and dimensions

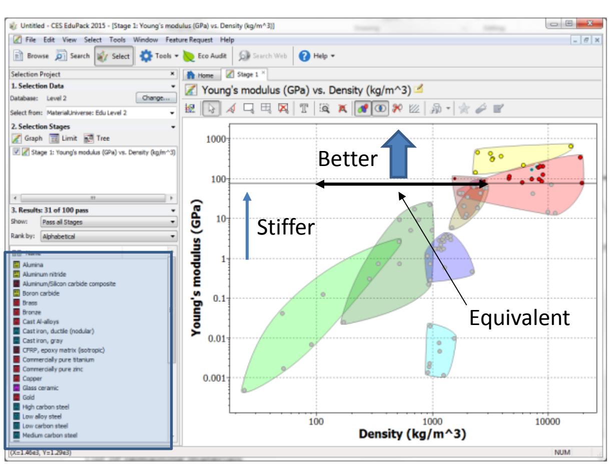
A tie in tension



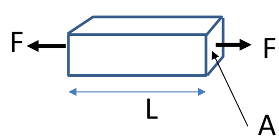
If we are to make rational choices we ideally want functions to describe objectives and constraints

Ranking Materials for Stiffness

If non-property attributes are fixed then ranking is very easy



List of remaining materials



If F, A, L are constant deflection only function of E

Higher E = higher ranking

Material Index

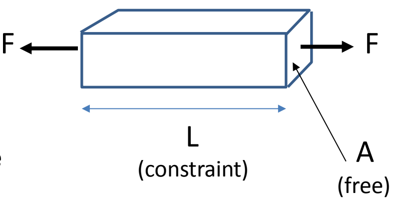
$$M = E$$

Coupled Constraints

Combine the objectives and constraints using the free variable (example: cross-section of tie)

Objective – minimise mass

Minimise density and area to minimise mass $m = \rho A L$



Constraint – deflection below a certain value

If we change density (material) we also change E so A needs to change to meet constraint.

$$\delta = \frac{1}{E}\frac{FL}{A}$$

Coupled constraints – Eliminate A

$$m \geq (F^*)\ (L)\ \boxed{\left(\frac{\rho}{E}\right)}$$

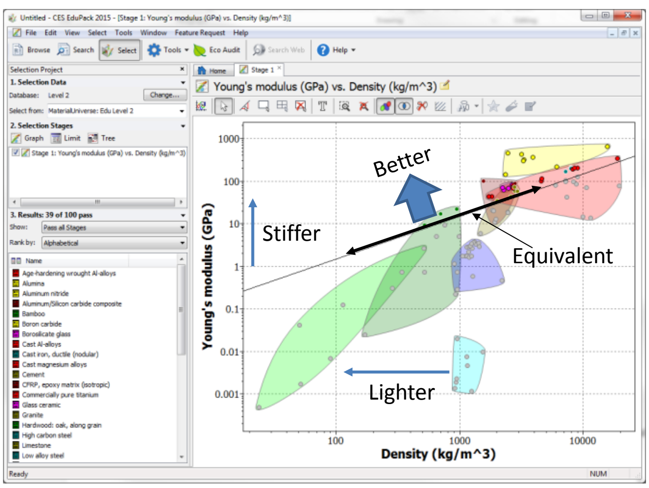
Material Properties

Functional Constraint
(Tensile Force up to deflection δ)

Geometric Constraint

Coupled Material Index

Appear as straight lines on bubble charts



$$M = \frac{E}{\rho}$$

Material Index
(invert Material Properties)

Not pass/fail

Greater distance from line = higher ranking (better material)

Material Indices

- Material indices are the **objective functions** to be minimised or maximised when conducting material selection (our **optimisation problem**)

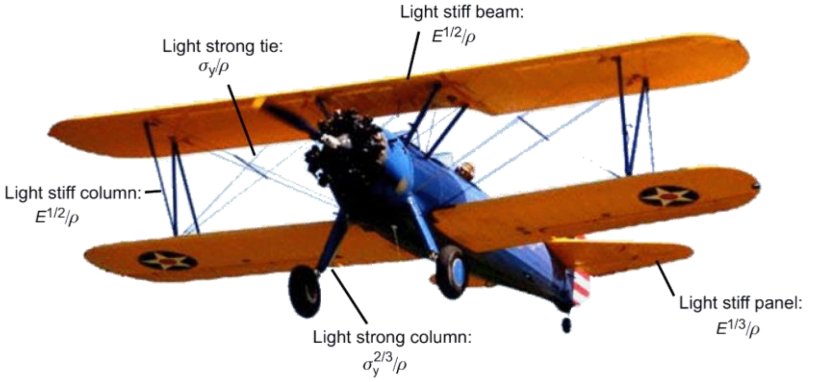


Image from: Ashby M – Materials Selection for Mechanical Design (4th Ed.), Elsevier, 2014

17

Multi-Property Index

Most material indices take the form

e.g. modulus

e.g. 1, 1/2, 1/3, 2/3

$$M = \frac{P_1^{\alpha}}{P_2}$$

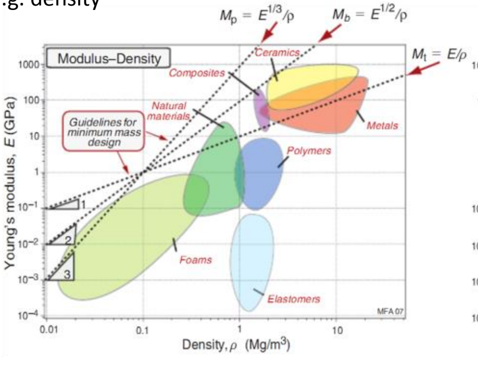
e.g. density

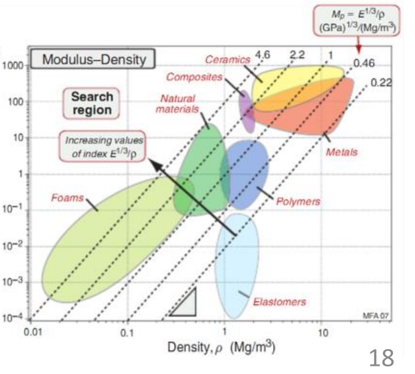
Bubble charts are usually log scale so:

$$\log(M) = \alpha \log P_1 + \log P_2$$

$$\log P_1 = \frac{1}{\alpha}(\log P_2 + \log M)$$

Straight line defines ranking line





18

Summary

A **Materials Index** can be developed for an Engineering problem to select the best material

1. Identify the objective (quantity to be maximised or minimised) , the constraints, and the free variables
2. Write an equation for the objective (tie example used mass, m).
3. If the objective equation has a free variable, eliminate the free variable (tie example, A was free)
4. Create a Materials Index for the combination of material properties to select a material(s) that maximise or minimise the objective (usually mass or cost).

19

Properties of Materials

Theme: Selection

Lecture 7: Material Selection for a Lightweight Wing Spar

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20

I'm working on a Project

Design and build a set of wings for a blimp

Every kg of mass requires m^3 of Helium for buoyancy

Where do you start? → The wings must be light.

21

22

Light, Stiff Beam

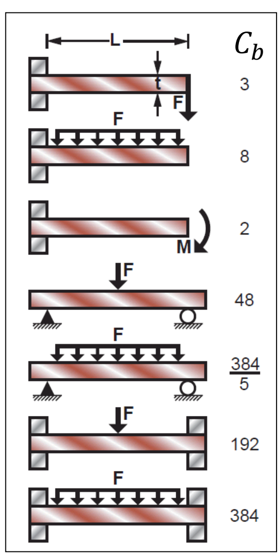
Stiffness

$$S = \frac{F}{\delta}$$

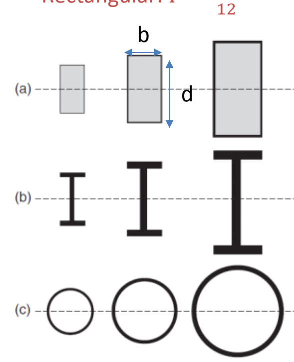
Second moment of area

$$S = C_b \frac{EI}{L^3}$$

Constant representing loading configuration and boundary conditions



Rectangular: $I = \frac{bd^3}{12}$



Thin walled tube $I = \pi r^3 t$

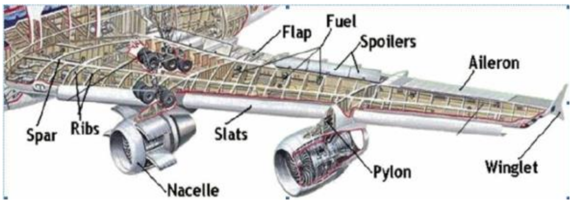
Let us assume **square cross section**

Light, Stiff Beam

Translation:

Function	Support load in bending
Constraints	Minimum stiffness, S^*
	Specified length, L
	Section shape square
Objectives	Minimise mass, m
Free Variable	Choice of material
	Choice of cross-section area A

Simple beams work even for complex shapes:



Objective equation

$$m = \rho A L = \rho d^2 L$$

Constraint equation

$$S = \frac{F}{\Delta} = \frac{C_b E I}{L^3}$$

$$I = \frac{bd^3}{12} = \frac{d^4}{12}$$

(Assume square cross-section, b=d)

$$S = \frac{C_b E d^4}{12 L^3}$$

Material index

$$d^2 = \left(\frac{12 S L^3}{C_b E}\right)^{1/2}$$

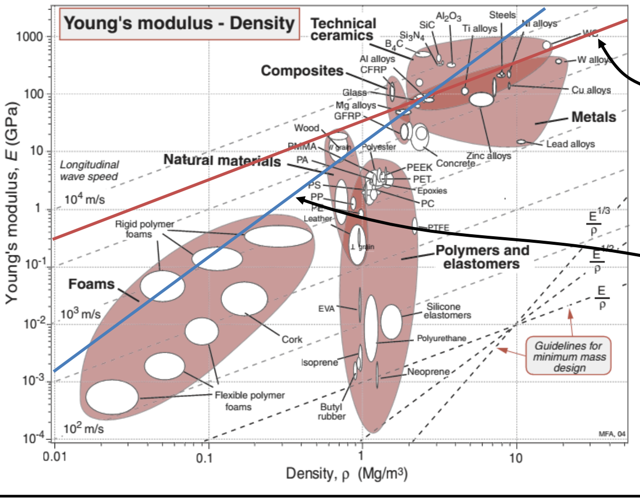
$$m = \rho \left(\frac{12 S L^3}{C_b E}\right)^{1/2} L$$

$$m = \left(\frac{12 S L^3}{C_b}\right) L \left(\frac{\rho}{E^{1/2}}\right)$$

$$M = \frac{E^{1/2}}{\rho}$$

Example: Light, Stiff Beam

$$M = \frac{E^{\frac{1}{2}}}{\rho} \Rightarrow \log E = 2 \log \rho + 2 \log M$$

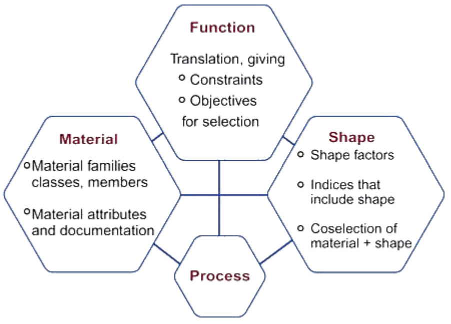


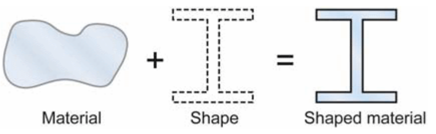
Tie (gradient 1)
Aluminium more or less equal to steel

Beam (gradient 2)
Aluminium much better than steel; woods and even foam become useful!

The Effect of **Shape**

- Shape also affects the performance of structural elements
- The best material-and-shape combination depends on the mode of loading
  - Axial loading
  - Bending
  - Torsion
  - Buckling



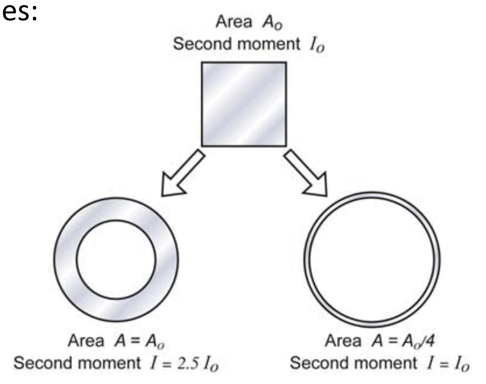


Note: Not all shapes are available for a given material (e.g. need to consider process)

25

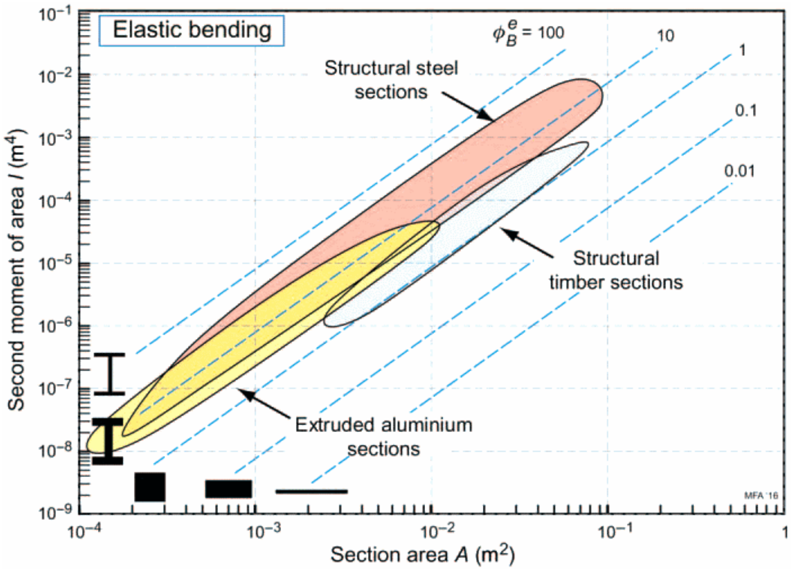
The Effect of **Shape**

- Let us describe the effect of shape for bending
- The bending stiffness S is the flexural modulus: $S = EI$
- Where $I = \int y^2 \, \mathrm{d}A$
- So we can get the same bending stiffness using different cross-section shapes:



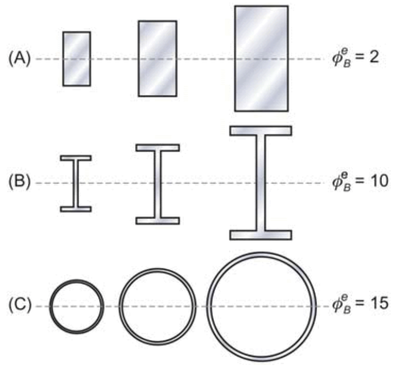
26

The Effect of **Shape**



Shape Factor

- The shape factor ϕ is defined as: $\phi = \dfrac{S}{S_0}$
- Where S_0 is the stiffness for a reference shape – in this case the solid square cross-section



(NB. constant proportions)

ϕ_B^e

elastic (stiffness)

bending

Shape Factors

ϕ_T^f ← f: failure, T: torsion

Table 10.3 Shape Efficiency Factors

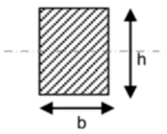
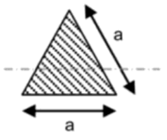
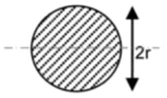
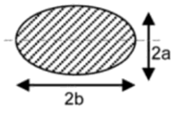
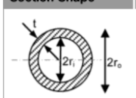
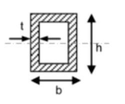
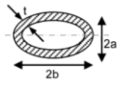
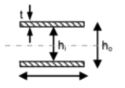
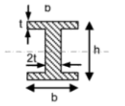
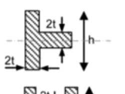
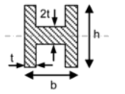
Section Shape	Bending Factor ϕ_B^e	Torsional Factor ϕ_T^e	Bending Factor ϕ_B^f	Torsional Factor ϕ_T^f
	$\frac{h}{b}$	$2.38\frac{h}{b}\left(1-0.58\frac{b}{h}\right)$ ($h>b$)	$\left(\frac{h}{b}\right)^{0.5}$	$1.6\sqrt{\frac{b}{h}}\frac{1}{\left(1+0.6\frac{b}{h}\right)}$ ($h>b$)
	$\frac{2}{\sqrt{3}}=1.15$	0.832	$\frac{3^{1/4}}{2}=0.658$	0.83
	$\frac{3}{\pi}=0.955$	1.14	$\frac{3}{2\sqrt{\pi}}=0.846$	1.35
	$\frac{3}{\pi}\frac{a}{b}$	$\frac{2.28ab}{(a^2+b^2)}$	$\frac{3}{2\sqrt{\pi}}\sqrt{\frac{a}{b}}$	$1.35\sqrt{\frac{a}{b}}$ ($a<b$)

Table 10.3 Shape Efficiency Factors *Continued*

Section Shape	Bending Factor ϕ_B^e	Torsional Factor ϕ_T^e	Bending Factor ϕ_B^f	Torsional Factor ϕ_T^f
	$\frac{3}{\pi}\left(\frac{r}{t}\right)$ ($r>>t$)	$1.14\left(\frac{r}{t}\right)$	$\frac{3}{\sqrt{2}\,\pi}\sqrt{\frac{r}{t}}$	$1.91\sqrt{\frac{r}{t}}$
	$\frac{1}{2}\frac{h}{t}\frac{(1+3b/h)}{(1+b/h)^2}$ ($h,b>>t$)	$\frac{3.57b^2\left(1-\frac{t}{h}\right)^4}{th\left(1+\frac{b}{h}\right)^3}$	$\frac{1}{\sqrt{2}}\sqrt{\frac{h}{t}}\frac{\left(1+\frac{3b}{h}\right)}{\left(1+\frac{b}{h}\right)^{3/2}}$	$3.39\sqrt{\frac{h^2}{b\,t}}\frac{1}{\left(1+\frac{b}{h}\right)^{3/2}}$
	$\frac{3}{\pi}\frac{a}{t}\frac{(1+3b/a)}{(1+b/a)^2}$ ($a,b>>t$)	$\frac{9.12(a\,b)^{5/2}}{t(a^2+b^2)(a+b)^2}$	$\frac{3}{2\sqrt{\pi}}\sqrt{\frac{a}{t}}\frac{\left(1+\frac{3b}{a}\right)}{\left(1+\frac{b}{a}\right)^{3/2}}$	$5.41\sqrt{\frac{a}{t}}\frac{1}{\left(1+\frac{a}{b}\right)^{3/2}}$
	$\frac{3}{2}\frac{h_o^2}{bt}$ ($h,b>>t$)	–	$\frac{3}{\sqrt{2}}\frac{h_o}{\sqrt{b\,t}}$	–
	$\frac{1}{2}\frac{h}{t}\frac{(1+3b/h)}{(1+b/h)^2}$ ($h,b>>t$)	$1.19\left(\frac{t}{b}\right)\frac{\left(1+\frac{4h}{b}\right)}{\left(1+\frac{h}{b}\right)^2}$	$\frac{1}{\sqrt{2}}\sqrt{\frac{h}{t}}\frac{\left(1+\frac{3b}{h}\right)}{\left(1+\frac{b}{h}\right)^{3/2}}$	$1.13\sqrt{\frac{t}{b}}\frac{\left(1+\frac{4h}{b}\right)}{\left(1+\frac{h}{b}\right)^{3/2}}$
	$\frac{1}{2}\frac{h}{t}\frac{(1+4bt^2/h^3)}{(1+b/h)^2}$ ($h,b>>t$)	$0.595\left(\frac{t}{h}\right)\frac{\left(1+\frac{8b}{h}\right)}{\left(1+\frac{b}{h}\right)^2}$	$\frac{3}{4}\sqrt{\frac{h}{t}}\frac{\left(1+\frac{4bt^2}{h^3}\right)}{\left(1+\frac{b}{h}\right)^{3/2}}$	$0.565\sqrt{\frac{t}{h}}\frac{\left(1+\frac{8b}{h}\right)}{\left(1+\frac{b}{h}\right)^{3/2}}$
	$\frac{1}{2}\frac{h}{t}\frac{(1+4bt^2/h^3)}{(1+b/h)^2}$ ($h,b>>t$)	$1.19\left(\frac{t}{h}\right)\frac{\left(1+\frac{4b}{h}\right)}{\left(1+\frac{b}{h}\right)^2}$	$\frac{3}{4}\sqrt{\frac{h}{t}}\frac{\left(1+\frac{4bt^2}{h^3}\right)}{\left(1+\frac{b}{h}\right)^{3/2}}$	$1.13\sqrt{\frac{t}{h}}\frac{\left(1+\frac{4b}{h}\right)}{\left(1+\frac{b}{h}\right)^{3/2}}$

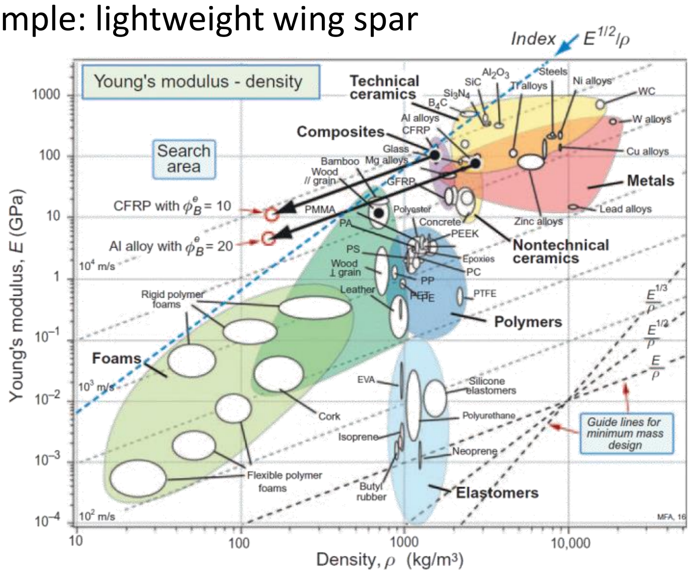
Material-Shape Co-Selection

- The shape factor can be combined with material properties in **co-selection**
- Example: lightweight wing spar $M = \dfrac{E^{1/2}}{\rho}$
- Accounting for shape: $M = \dfrac{(\phi E)^{1/2}}{\rho}$
- To generate bubble charts we need to modify the axes: $E^* = \dfrac{E}{\phi}$ $\rho^* = \dfrac{\rho}{\phi}$

31

Material-Shape Co-Selection

- Example: lightweight wing spar



32

Upper Limits for Shape Factors

- Difficulty making thin-walled tubes or I-sections
- Local buckling of very thin structures
- Loading mode influences shape factor

Table 10.4 Empirical Upper Limits for the Shape Factors $\phi_B^e, \phi_T^e, \phi_B^f$ and ϕ_T^f

Material	$(\phi_B^e)_{max}$	$(\phi_T^e)_{max}$	$(\phi_B^f)_{max}$	$(\phi_T^f)_{max}$
Structural steel	65	25	13	7
6061 aluminium alloy	44	31	10	8
GFRP and CFRP	39	26	9	7
Polymers (e.g., nylons)	12	8	5	4
Woods (solid sections)	5	1	3	1
Elastomers	<6	3	–	–

Note: Steel higher shape factor than aluminium in bending.
Opposite is true in torsion.

Summary

- The best choice of material often depends of which shapes it can be formed
- Shape factors describe the efficiency of materials in each loading mode
  - Materials performing better in Bending might not be the best option in Torsion (steel vs. aluminium)
  - Some shapes will also perform better in Bending than in Torsion (I-beam vs. hollow tube)

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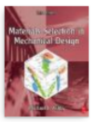

BOOK

Materials selection in mechanical design / Michael F. Ashby.

M. F. Ashby author. dawsonera.

Amsterdam : Elsevier Butterworth-Heinemann , Third edition.. , 2005

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