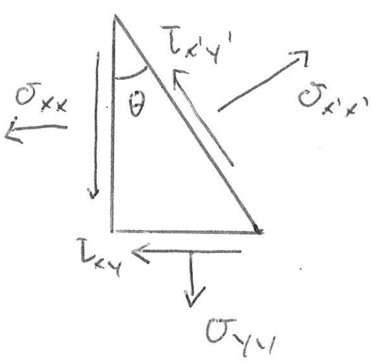
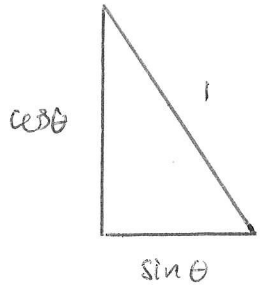


Force equilibrium, so need area over which stress acts



$$\Sigma F_{\parallel} : \quad \tau_{x'y'} = (\sigma_{yy}\sin\theta + \tau_{xy}\cos\theta)\cos\theta - (\sigma_{xx}\cos\theta + \tau_{xy}\sin\theta)\sin\theta$$

$$\Sigma F_{\perp} : \quad \sigma_{x'x'} = (\sigma_{yy}\sin\theta + \tau_{xy}\cos\theta)\sin\theta + (\sigma_{xx}\cos\theta + \tau_{xy}\sin\theta)\cos\theta$$

combine into matrix:

$$\begin{bmatrix} \sigma_{x'x'} \\ \sigma_{y'y'} \\ \tau_{x'y'} \end{bmatrix} = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix}$$

where $\sigma_{y'y'}$ was found using $\theta + \pi/2$.

$\sigma_{xx} = 120 \text{ MPa}$

$\sigma_{yy} = -40 \text{ MPa}$

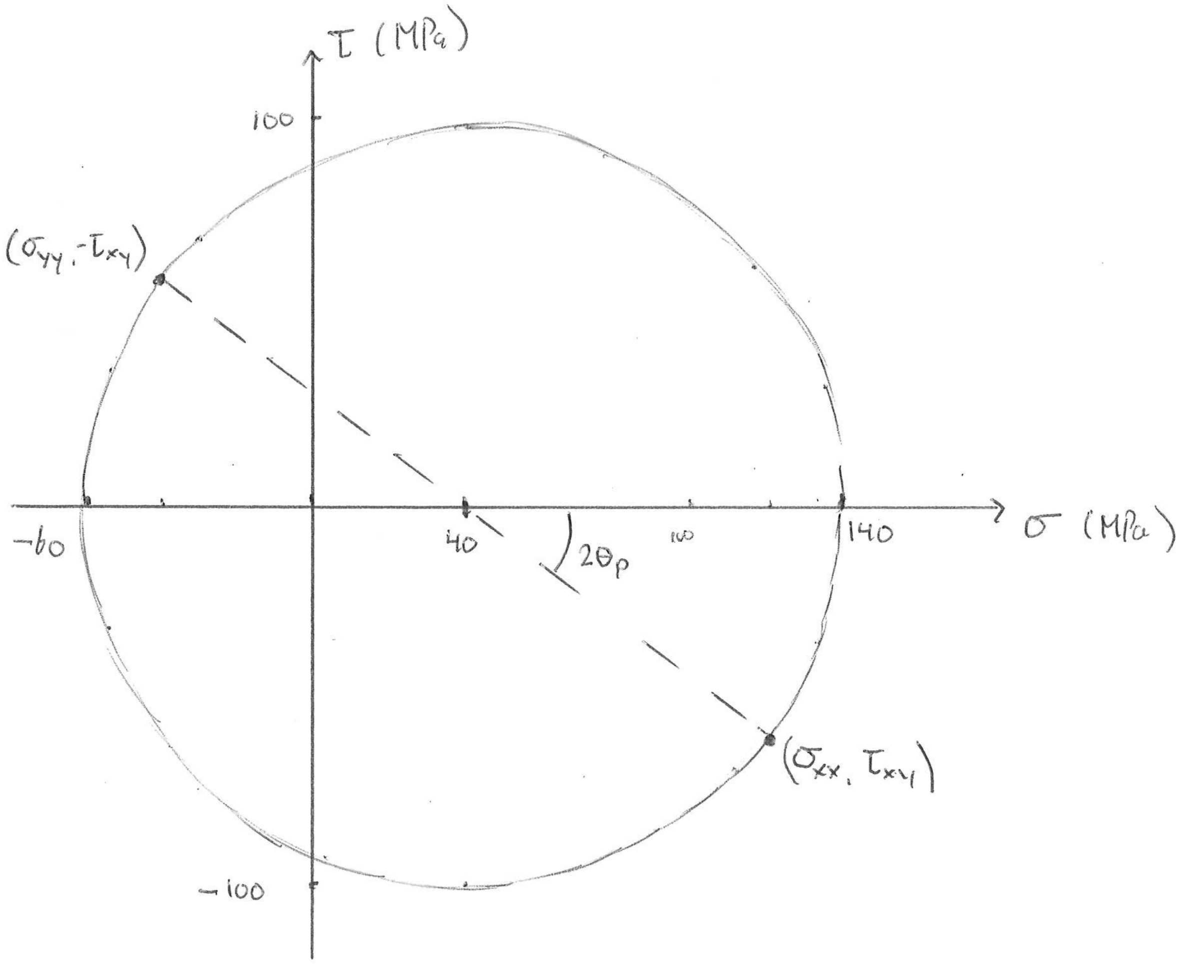
$\tau_{xy} = -60 \text{ MPa}$

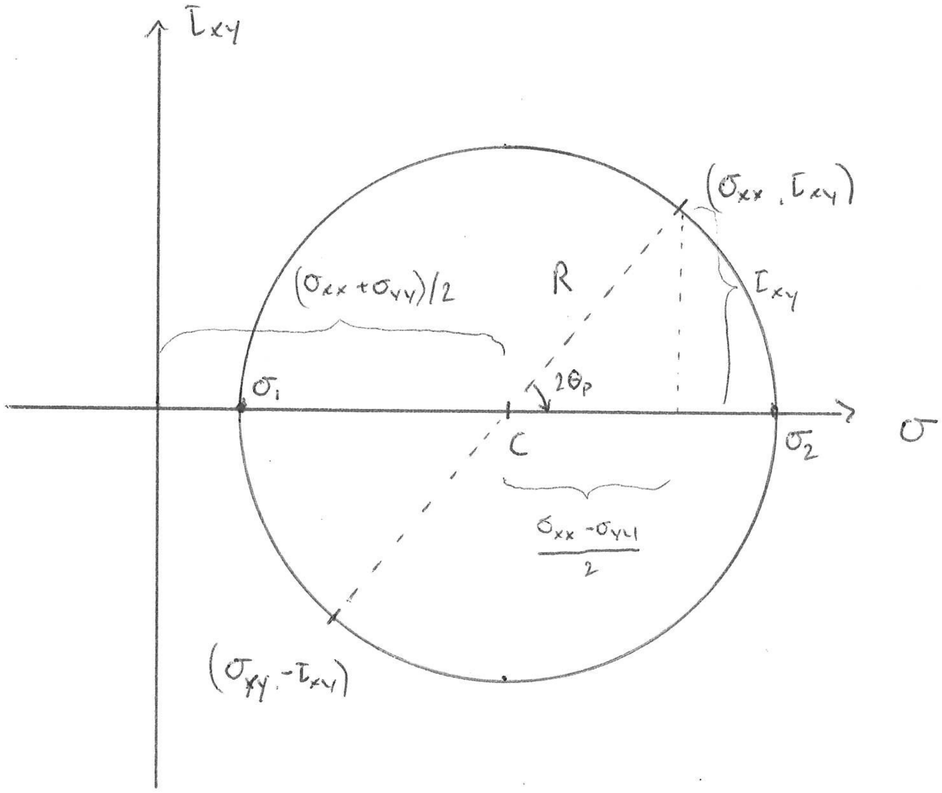
(i) $\sigma_{1,2} = \dfrac{\sigma_{xx} + \sigma_{yy}}{2} \pm \sqrt{\left(\dfrac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2} = 40 \pm 100 \text{ MPa}$

$\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_{xx} - \sigma_{yy}} \Rightarrow \theta_p = -18.4^\circ + 90^\circ$

(ii) $\tau_{max,min} = \pm \dfrac{\sigma_1 - \sigma_2}{2} = \pm 100 \text{ MPa}$

(iii)





$$C = \frac{\sigma_{xx} + \sigma_{yy}}{2} \qquad \tan 2\theta_p = \frac{2\tau_{xy}}{(\sigma_{xx} - \sigma_{yy})}$$

$$R = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = C \pm R$$

$$\tau_{max,min} = \pm R = \pm \frac{\sigma_1 - \sigma_2}{2}$$

strain gauge A : $\varepsilon_A = 50\,\mu\varepsilon$ $\theta_A = 0$

strain gauge B : $\varepsilon_B = -70\,\mu\varepsilon$ $\theta_B = 60^\circ$

strain gauge C : $\varepsilon_C = 130\,\mu\varepsilon$ $\theta_C = 120^\circ$

$$\varepsilon_A = \varepsilon_{xx} + 0 + 0$$

$$\varepsilon_B = \frac{\varepsilon_{xx}}{4} + \frac{3\varepsilon_{yy}}{4} + \frac{\sqrt{3}}{4}\gamma_{xy}$$

$$\varepsilon_C = \frac{\varepsilon_{xx}}{4} + \frac{3}{4}\varepsilon_{yy} - \frac{\sqrt{3}}{4}\gamma_{xy}$$

Using transformation matrix. Note that you must use $\gamma/2$.

Combine :

$$\varepsilon_{xx} = \varepsilon_A = 50\,\mu\varepsilon$$

$$\varepsilon_{yy} = \frac{-\varepsilon_A + 2\varepsilon_B + 2\varepsilon_C}{3} = 23{,}3\,\mu\varepsilon$$

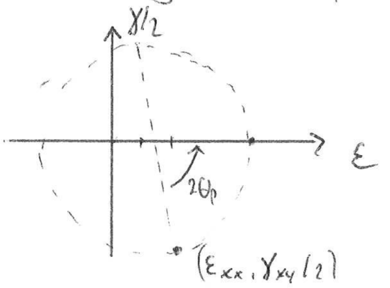
$$\gamma_{xy} = \frac{2\varepsilon_B - 2\varepsilon_C}{\sqrt{3}} = -231\,\mu\varepsilon$$

---

direction of principal strain

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_{xx} - \varepsilon_{yy}} \rightarrow \theta_p = -41{,}7^\circ + 90^\circ$$

to find which angle corresponds to maximum strain, sketch Mohr's circle



From inspection : $-41{,}7^\circ$

$\sigma_{xx} = -96$ MPa $\qquad E = 70$ GPa

$\sigma_{yy} = 72$ MPa $\qquad \nu = 0{,}3$

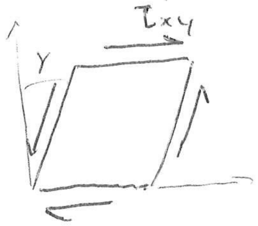
$\tau_{xy} = 34$ MPa

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} 1/E & -\nu/E & 0 \\ -\nu/E & 1/E & 0 \\ 0 & 0 & \frac{2(1+\nu)}{E} \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} -1680 \\ 1440 \\ 1263 \end{bmatrix} \mu\varepsilon$$

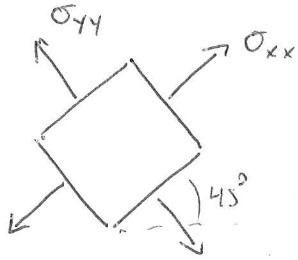
$$\varepsilon_{45} = \frac{\varepsilon_{xx}}{2} + \frac{\varepsilon_{yy}}{2} + \frac{\gamma_{xy}}{2} = 512\ \mu\varepsilon$$

using transformation matrix

shear modulus : equate strain energy for pure shear



$$U_s = \frac{1}{2} \tau_{xy}\, dx\, \gamma_{xy}\, dy = \frac{\tau_{xy}^2}{2G}\, dx\, dy$$



$$U_{bi} = \frac{1}{2} \sigma_{xx} \varepsilon_{xx}\, dx\, dy + \frac{1}{2} \sigma_{yy} \varepsilon_{yy}\, dx\, dy$$

$$= \frac{1}{2} \sigma_{xx} \frac{(\sigma_{xx} - \nu \sigma_{yy})}{E} dx\, dy + \frac{1}{2} \sigma_{yy} \frac{(\sigma_{yy} - \nu \sigma_{xx})}{E} dx\, dy$$

$$= \frac{1}{2E} \left(\sigma_{xx}^2 + \sigma_{yy}^2 - 2\nu \sigma_{xx} \sigma_{yy} \right) dx\, dy$$

for pure shear : $\sigma_{xx} = \tau_{xy}$, $\sigma_{xy} = -\tau_{xy}$

$$= \frac{1}{2E} \left(2\tau_{xy}^2 + 2\nu \tau_{xy}^2 \right) dx\, dy$$

equate :
$$\frac{\tau_{xy}^2}{2G} = \frac{\tau_{xy}^2}{E} (1 + \nu)$$

$$\therefore \boxed{G = \frac{E}{2(1+\nu)}}$$

bulk modulus

relates spherical stress to volumetric strain

$$V = (1+\varepsilon_{xx})(1+\varepsilon_{yy})(1+\varepsilon_{zz})\,dx\,dy\,dz$$ V is deformed volume

$$\approx (1+\varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz})\,dx\,dy\,dz$$

$$\frac{\Delta V}{V_0} = \varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}$$

$$= \frac{(\sigma_{xx}-\nu(\sigma_{yy}+\sigma_{zz}))}{E} + \frac{(\sigma_{yy}-\nu(\sigma_{xx}+\sigma_{zz}))}{E} + \frac{(\sigma_{zz}-\nu(\sigma_{xx}+\sigma_{yy}))}{E}$$

$$= \frac{(\sigma_{xx}+\sigma_{yy}+\sigma_{zz})(1-2\nu)}{E}$$

For spherical stress $\sigma_{xx}=\sigma_{yy}=\sigma_{zz}=\sigma$

$$= \frac{3\sigma(1-2\nu)}{E}$$

$$\sigma = K\frac{\Delta V}{V_0} \rightarrow \boxed{K = \frac{E}{3(1-2\nu)}}$$

---

both G and K must be positive and finite, and thus:

$$-1 < \nu < 0.5$$

$\varepsilon_{xx} = 1540\ \mu\varepsilon$ $\qquad E = 70\ GPa$

$\varepsilon_{yy} = -320\ \mu\varepsilon$ $\qquad v = 0.3$

$\gamma_{xy} = 632\ \mu\varepsilon$ $\qquad \sigma_Y = 240\ MPa$

(i) find stress

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix} = \frac{E}{1-v^2}\begin{bmatrix}1 & v & 0\\ v & 1 & 0\\ 0 & 0 & \frac{(1-v)}{2}\end{bmatrix}\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{bmatrix} = \begin{bmatrix}111\\ 11\\ 17\end{bmatrix} MPa$$

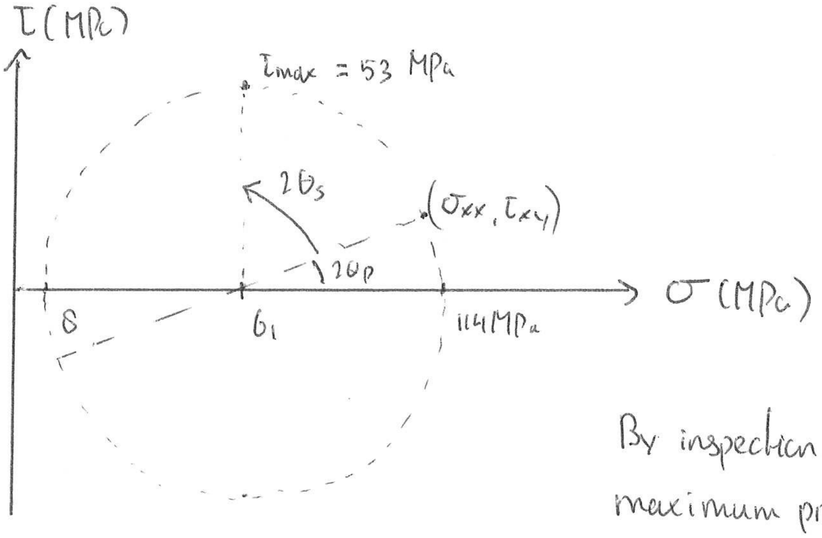
$$\sigma_{1,2} = \frac{\sigma_{xx}+\sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

$\sigma_1 = 114\ MPa \qquad \sigma_2 = 8\ MPa$

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = 53\ MPa$$

(ii)

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_{xx}-\sigma_{yy}} \qquad \theta_p = 9.4^\circ + 90^\circ$$



By inspection, direction of maximum principal stress: 9.4°

Maximum shear stress is at 45°: $9.4^\circ - 45^\circ = -35.6^\circ$

iii) using Von Mises:

$$\sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2} = 110 \text{ MPa} < \sigma_Y \quad \checkmark$$

using Tresca

$$|\sigma_1 - 0| = 114 \text{ MPa} < \sigma_Y \quad \checkmark$$

iv)

$\sigma_3 = 0$