Q1 Assuming the density to be constant

$$P = P_a + \rho g h = 101{,}300 + 1000 \times 9.81 \times 10 = 1.994 \times 10^5 \text{ Pa}$$

Note the approximate equivalence 1 atmosphere ≈ 10 m water.

Q2 a) $p = \rho g h + p_v = \rho_m g h \Rightarrow h = \dfrac{101300}{13560 \times 9.81} = 0.762\text{m}$

b) barometer reading remains unchanged, height unchanged by shape

c) $\rho_o g h + p_v = P \Rightarrow h = \dfrac{101300 - 5.7\times10^3}{790 \times 9.81} = 12.336\text{m}$

Q3 As the fluid interface in tube C rises, the heights of fluid in A and B also change, but not by the same amount as all 3 have different diameters (call these three heights $\Delta h_A, \Delta h_B$ and Δh_c)

$$\Delta h_A = \Delta h_c \cdot \frac{A_c}{A_A} = 0.06 \times \frac{70}{500} = 8.4\times10^{-3}\text{m}$$

$$\Delta h_B = \Delta h_c \cdot \frac{A_c}{A_B} = 0.06 \times \frac{70}{800} = 5.25\times10^{-3}\text{m}$$

(remember Δh_B represents a decrease in height)

At the undisturbed interface (where P_a = atmospheric pressure or some initial pressure applied to both A & B)

$$P_a + \rho_A g h_A = P_a + \rho_B g h_B \Rightarrow \rho_A g h_A = \rho_B g h_B$$

Once an additional pressure has been applied to B (ΔP say) the pressure at interface becomes

$$P_a + \rho_A g(h_A + \Delta h_A - \Delta h_c) = P_a + \Delta P + \rho_B g(h_B - \Delta h_B - \Delta h_c)$$

Using relation from undisturbed interface

$$\rho_A g(\Delta h_A - \Delta h_c) = \Delta P + \rho_B g(-\Delta h_B - \Delta h_c)$$

$$\Delta P = \left[800 \times (8.4\times10^{-3} - 0.06) - 900 \times (-5.25\times10^{-3} - 0.06)\right] \times 9.81$$

$$\Delta P = 171.13 \text{ N/m}^2$$

Q4 for the bubble $\frac{P_o V_o}{T_o} = \frac{P_s V_s}{T_s}$ (ideal gas equation)

where the subscripts o and s refer to its initial position at a depth of 9m and its final position at the surface.
Temperature is constant (isothermal process) so

$$\frac{P_o}{P_s} = \frac{V_s}{V_o} = \frac{d_s^3}{d_o^3}$$

hence $d_s^3 = \frac{P_o}{P_s} d_o^3$ and $d_s = \sqrt[3]{P_o/P_s}\, d_o$

$$= \sqrt[3]{\frac{1.013\times10^5 + 9\times9.81\times1000}{1.013\times10^5}} \times 0.004$$

$$= 4.93\times10^{-3}\,m = 4.93\,mm$$

Q6

Volume of the balloon $= \frac{4}{3}\pi(0.4)^3 = 0.2681\ m^3$

The effective density of the balloon & equipment $= \rho_{eff} = \frac{0.06+.1+M_H}{0.2681}$

where M_H is the mass of helium.
from Q5

$$p = 1.013\times10^5\left(1 - \frac{0.0065\times6000}{288.15}\right)^{9.81/287\times0.0065} = 47150.405\ N/m^2$$

$$T = T_{sl} - \lambda z = 288.15 - 0.0065\times6000 = 249.15^\circ K$$

$$\rho = \frac{P}{RT} = \frac{47150.405}{287\times249.15} = 0.6594\ kg/m^3$$

for equilibrium $\rho = \rho_{eff}$ $\frac{0.06+0.1+M_H}{0.2681} = 0.6594$

$$M_H = 0.0168\ kg$$

Q7 Gauge pressure $= \rho g h$ where $\rho = 910\ kg/m^3$ and $g = 9.81+30\ m/s^2$

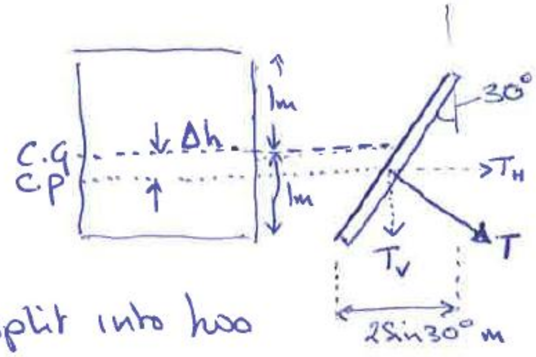
$$= 910\times39.81\times10 = 3.6227\times10^5\ N/m^2$$

Thrust = weight of fluid above $= \pi r^2 \times h \times \rho \times g$

$$= 4.552\times10^6\ N$$

Q8

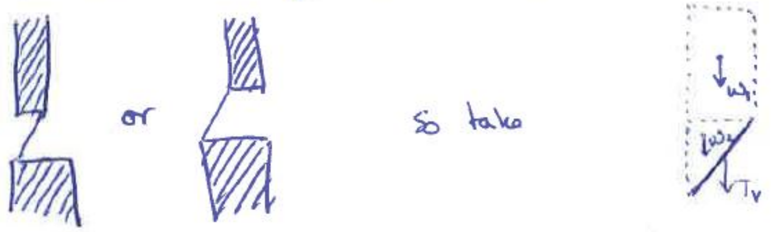
Case 1



Consider the total thrust to be split into two components: T_H and T_V

Consider T_V

The vertical thrust equals the "weight of water above" the gate. As you can see this should be the equivalent weight of water above, so that even though there is little water above the gate in the drawing there is no difference to the pressure & hence thrust if we had



$T_V = w_1 + w_2$

$$\text{so}\quad T_V = 18 \times 2\tan 30^\circ \times 2 \times 1000 \times 9.81 + \left(\tfrac{1}{2}\right) \times 2 \times 2\tan 30^\circ \times 1000 \times 9.81$$

$$= 2 \times 2 \times \tan 30^\circ \times 1000 \times 9.81 \times (18+1) = \qquad 430449.27\ \text{N}$$

Note we can leave out the weight of the air as the atmospheric pressure acts on the outside surface of the gate. Also we assume the atmospheric pressure does not change with the 20m change in height.

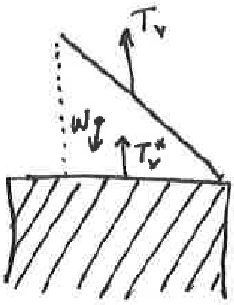
T_H = "pressure at the centre of gravity" × "vertical area"

$$= (p_a + \rho g h_{c.g})A_v - p_a A_v$$
$$= \rho g h_{c.g} A_v$$
$$= 1000 \times 9.81 \times 19 \times 2 \times 2$$
$$= 745560\ \text{N}$$

note that we can neglect atmospheric pressure as it acts on both sides of the gate

$$T = \sqrt{T_V^2 + T_H^2} = \qquad 8.609 \times 10^5\ \text{N}$$

Q8 Continued case 2



from the equality of vertical forces

$$T_V = T_V^* - W$$

where T_V^* is the resultant from the vertical thrust on the base given by

$$T_V^* = 20 \times 2\tan 30^\circ \times 2 \times 1000 \times 9.81$$

and W is the weight of the fluid

$$W = \left(\tfrac{1}{2}\right) \times 2 \times 2\tan 30^\circ \times 1000 \times 9.81$$

so

$$T_V = 2 \times 2 \times \tan 30 \times 1000 \times 9.81\,(20-1) = 430449 \quad \text{(as before)}$$

The horizontal thrust is unchanged hence $T_H = 745560\ \text{N}$

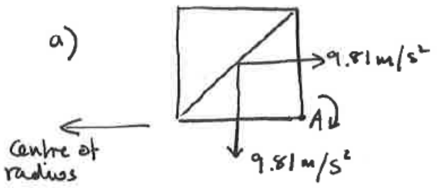
$$T = 8.609 \times 10^5\ \text{N}$$

Height of centre of pressure from the surface is (from notes)

$$h_{c.p} - h_{c.g} = \frac{I_{xx}}{h_{c.g} \times A} \Rightarrow h_{c.p} - 19 = \frac{4\,2.2^3}{12 \times 19 \times 2^2} \Rightarrow h_{cp} = 19.018$$

Q9

a)



Vertical thrust on base of fuel tank

$T_v = 850 \times 9.81 \times 1 \times 1 \times 0.5 = 4169.25$

acting downward (anticlockwise) at a distance $\frac{1}{3}$m

[Note Consider the C of G of the triangle]

horizontal thrust given by $T_H = \rho g h_{c.g} \times A$

[This is exactly the same as the horizontal thrust of a full fuel tank]

$$T_H = 850 \times 9.81 \times 0.5 \times 1 \times 1 = 4169.25$$

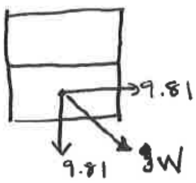
Line of action of the horizontal force: $h_{c.p} - h_{c.g} = \frac{I_{xx}}{h_{c.g} \times A} = \frac{1}{12} \frac{\times 1 \times 1^3}{0.5 \times 1 \times 1}$

$$h_{cp} = \left(\frac{1}{2} + \frac{1}{6}\right) m$$

So horizontal force acts at a height of $(1/3)$m from A in a clockwise direction

hence $\text{Moment}_A = \left\{4169.25 \times \frac{1}{3} - 4169.25 \times \frac{1}{3}\right\} = 0 \text{ Nm}$

b) Constrained fluid acts like a solid

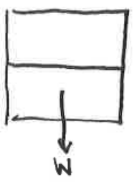


hence moment $= (\text{fluid mass}) \times 9.81 \times 0.25$
$- (\text{fluid mass}) \times 9.81 \times 0.5$

$= -0.5 \times 1 \times 1 \times 850 \times 9.81 \times 0.25$ Nm

$= -1042.3$ Nm

c)



moment $= -(\text{fluid mass}) \times 9.81 \times 0.5$

$= -0.5 \times 1 \times 1 \times 850 \times 9.81 \times 0.5$ Nm

$= -2084.6$ Nm

Note we see how the sloshing of the fluid means that the fluid is on the point of producing a "toppling" moment on the fuel tank.