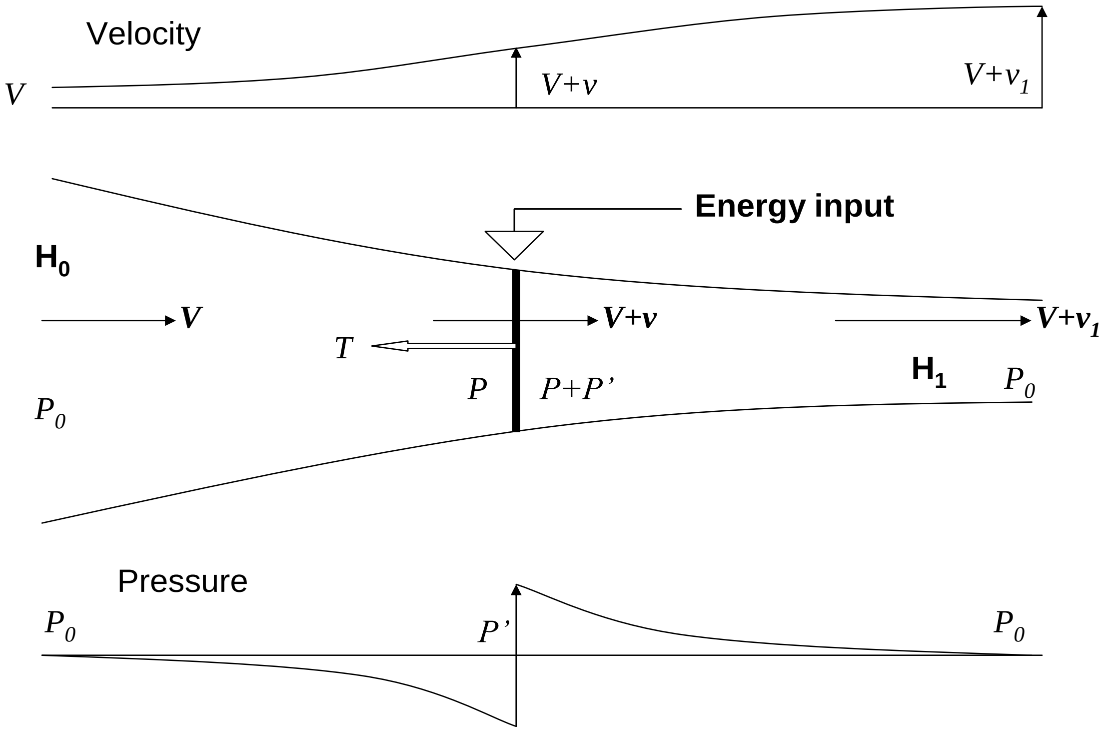
Analysis of Propeller Aerodynamics

1. Momentum analysis including the swirl

Momentum analysis based on actuator disk theorem can be extended to include swirl, though strictly speaking it can no longer be referred to as an actuator disk.



Considering the flow states in the actuator disk analogy of flow through the rotor (see lecture 1)

$$H_0 = P_0 + \tfrac{1}{2}\rho V^2 = P + \tfrac{1}{2}\rho (V+v)^2$$

$$H_1 = P_0 + \tfrac{1}{2}\rho (V+v_1)^2 = P + P' + \tfrac{1}{2}\rho (V+v)^2$$

$$\frac{T}{A} = \rho (V+v) v_1 = P' = \rho v_1 (V + \frac{v_1}{2})$$

$$\frac{v_1}{2} = v \qquad \text{and} \qquad T = 2\rho A (V+v) v$$

More generally in both H_0 and H_1 states, we can re-write the formula as:

$$\overline{P} + \tfrac{1}{2}\rho \overline{V}^2 = \text{contant, or } \frac{\overline{P}}{\rho} + \tfrac{1}{2}\overline{V}^2 = \text{constant}$$

Adding the effect of swirl gives:

$$\frac{\overline{P}}{\rho} + \tfrac{1}{2}\overline{V}^2 + \frac{K^2}{A} = \text{constant, where } K = \frac{\tau}{\dot{m}}$$

"*K*" *is a* "swirl parameter" based on a free vortex, τ is propeller torque and $\dot{m}$ is the mass flow rate.

The swirl power is given as $P_{\text{swirl}} = \dfrac{\dot{m}K^2}{A}(watts)$

In a subsequent lecture on "Ducted Fans" it will be shown that whilst angular momentum is conserved this is not the case for the rotational kinetic energy resulting in further reductions, or indeed, increases in static pressure.

For the real flow applications that follow in the Blade Element Analysis, the swirl is shown to be the result of combined blade viscous effects and in-plane contributions from the lift induced velocity at the rotor plane.

It should be noted that the swirl exists only in the downstream wake. Upstream of the rotor, the flow is irrotational. This is of course why in wind tunnel design the working section is always upstream of a non-contra-rotating fan.

2. Blade Element Theory (BET) for propellers

The simple momentum theory is useful as an indication of the potential performance of a rotor system in ideal conditions. In practice such performance levels could not be achieved because the rotor is working in a real flow that has viscosity.

In order to obtain a more detailed knowledge of the behaviour of a rotor it is necessary to analyse the forces on the rotor blades. Due to the rotation, the blade has to be considered as a number of separate aerofoil elements which are then integrated to represent the characteristics of the whole propeller.

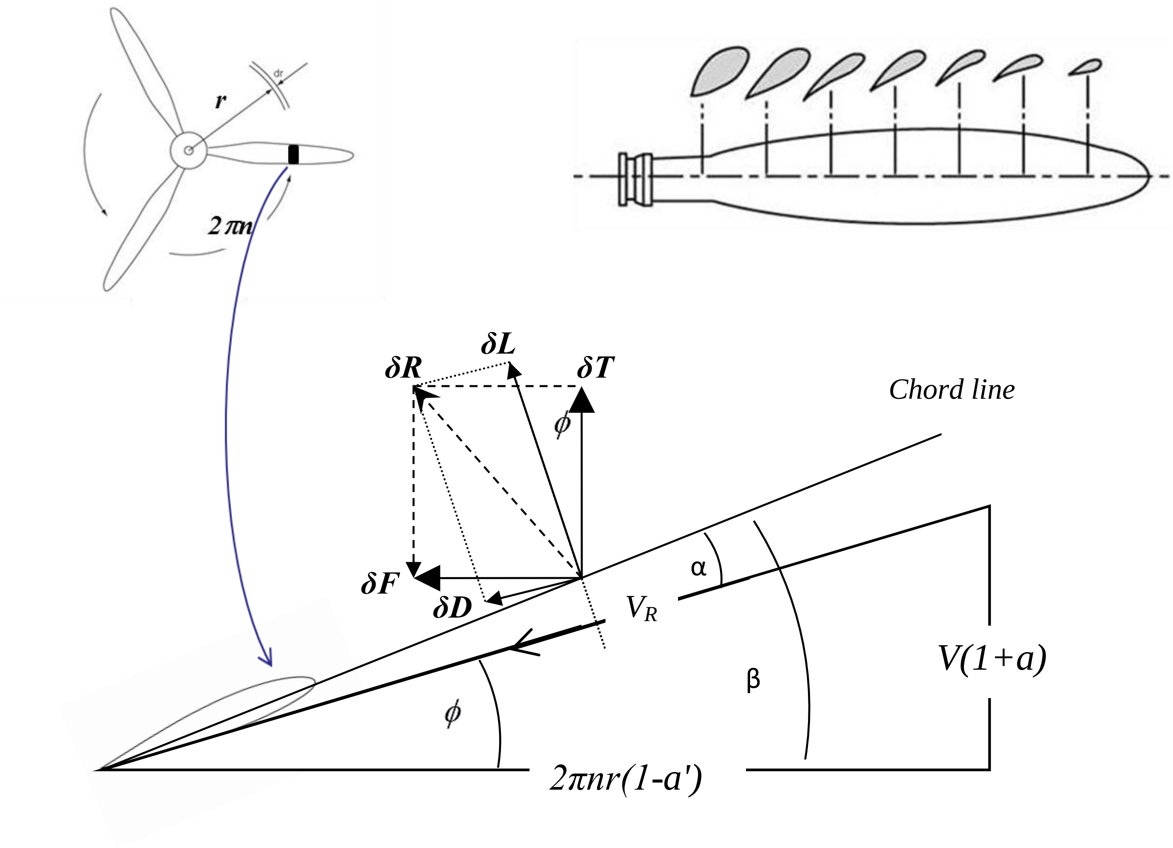
Interference Flows

These are the axial and rotational flows that are in addition to those of the rotor. The axial interference flow (also referred to as the "induced" velocity $\boldsymbol{v}$) has been determined from momentum analysis. The rotational interference flow (which we shall call $\boldsymbol{w}$) can be determined from vortex theory and is physically represented by the swirl of the rotor wake. It is sufficient to say that it has a value w at the rotor and $2w$ in the far wake.

Whilst it is similar to the axial interference flow in that the far wake velocity is twice that at the rotor, it has no value upstream of the rotor and it is not wholly induced, as it contains a contribution from the viscous drag of the rotor blades. Never-the-less it can be considered as a rotational inflow factor a' (and an outflow factor $2a'$).

Blade Element Efficiency

Considering an element of a propeller blade at a radius r and of width dr, the forces on a blade element can be resolved into an axial component δT and an in-plane component δF.



where, $\tan\phi = \dfrac{V(1+a)}{2\pi nr(1-a')}$

and the angle of incidence $\alpha = \beta - \phi$

$$\delta T = \delta L\cos\phi - \delta D\sin\phi$$
$$\delta F = \delta L\sin\phi + \delta D\cos\phi$$

The in-plane force δF act at a radius r and therefore $\delta F r$ is the rotational torque δQ.

thus $\delta Q = r(\delta L\sin\phi + \delta D\cos\phi)$

Considering δT and δQ in terms of lift and drag coefficients gives:

$$\delta T = \tfrac{1}{2}\rho V_R^2 c\delta r(C_L \cos\phi - C_D \sin\phi)$$

$$\delta Q = \tfrac{1}{2}\rho V_R^2 cr\delta r(C_L \sin\phi + C_D \cos\phi)$$

If the total number of blades is N, then for the propeller as a whole

$$\frac{dT}{dr} = N\ \tfrac{1}{2}\rho V_R^2 c(C_L \cos\phi - C_D \sin\phi)$$

and integrating over the whole disc gives:

$$T = \int_0^R N\tfrac{1}{2}\rho V_R^2 c(C_L \cos\phi - C_D \sin\phi)dr$$

where R is the propeller radius.

Similarly, ` $$Q = \int_0^R N\tfrac{1}{2}\rho V_R^2 cr(C_L \sin\phi + C_D \cos\phi)dr$$

If γ is the angle between the lift component and the resultant force;

$$\tan\gamma = \frac{D}{L} = \frac{C_D}{C_L}$$

and

$$\begin{aligned}\frac{dT}{dr} &= N\ \tfrac{1}{2}\rho V_R^2 cC_L(\cos\phi - \frac{\sin\gamma}{\cos\gamma}\sin\phi)\\ &= N\ \tfrac{1}{2}\rho V_R^2 cC_L \sec\gamma(\cos\phi\cos\gamma - \sin\phi\sin\gamma)\\ &= N\ \tfrac{1}{2}\rho V_R^2 cC_L \sec\gamma\cos(\phi+\gamma)\end{aligned}$$

and similarly $$\frac{dQ}{dr} = N\ \tfrac{1}{2}\rho V_R^2 crC_L \sec\gamma\sin(\phi+\gamma)$$

The ***solidity*** of the propeller (known as σ) is the ratio of propeller planform and the propeller disc area.

From the consideration of the elemental annulus , $\sigma = \dfrac{Nc}{2\pi r}$. and thus the elemental thrust and torque equations can be so modified:

$$\boxed{\begin{aligned}\frac{dT}{dr} &= \pi\rho V_R^2 r\sigma C_L \sec\gamma\cos(\phi+\gamma)\\ \frac{dQ}{dr} &= \pi\rho V_R^2 r^2\sigma C_L \sec\gamma\sin(\phi+\gamma)\end{aligned}}$$

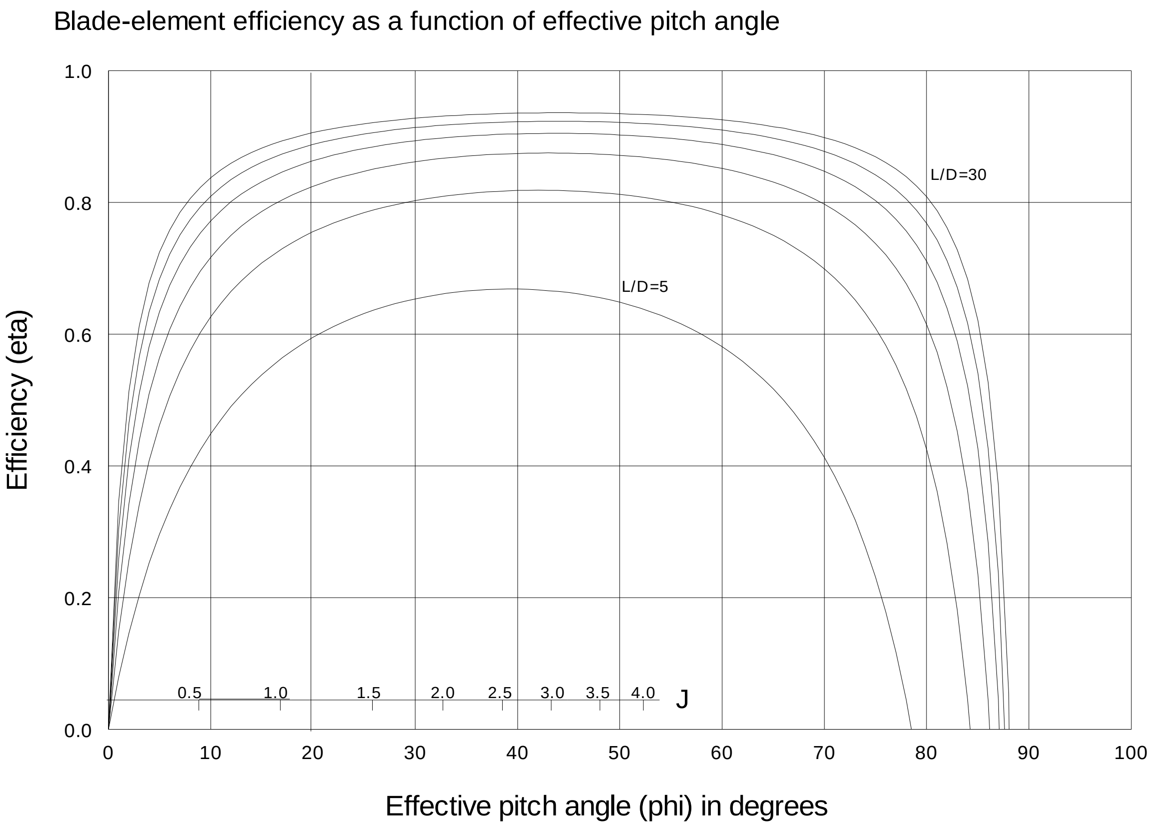
As before the propulsive efficiency is a measure of the work done for the energy used. Thus for the element of blade considered, the efficiency η' is given by:

$$\eta' = \frac{\delta T V}{\delta F\, 2\pi r n} = \frac{\delta R \cos(\phi + \gamma) V}{\delta R \sin(\phi + \gamma) 2\pi r n} = \frac{\tan\phi}{\tan(\phi + \gamma)}$$

{if a and a' are assumed to be small so that $\tan\phi = \dfrac{V}{2\pi r n}$.}

Recalling that $\tan\gamma = \dfrac{D}{L} = \dfrac{C_D}{C_L}$

then for a range of aerofoil lift~drag ratios (from 5 to 30) and across the full range of onset flow angles (from 0 to 90°), the blade element efficiency is plotted below:



It can be seen that the effective pitch should be in the region of 20° to 70° where the curves relatively flat.

The effect of aerofoil performance is quite noticeable, particularly at very low L/D ratios.

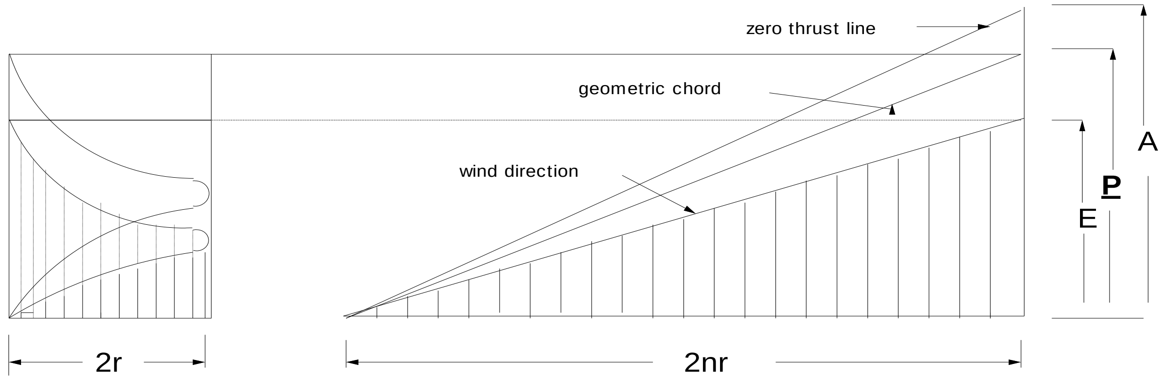
Blade Geometry

The **effective pitch** is the actual progress of the propeller through the air in one revolution and this will be the same for all the elements on the blade. The "effective pitch angle" ϕ must therefore decrease radially outward from the propeller hub to the tip.

The **geometric pitch** is the axial displacement of the propeller prescribed by the geometric chord in one revolution. This is analogous with the mechanical screw thread (which is why propellers were originally called "airscrews").

The "Geometric Pitch Angle"(β), cannot be the same for all radial elements, so it is usually quoted for 0.75 R. It is the <u>geometric pitch</u> of a propeller that defines the propeller setting and it <u>is what you would quote when specifying a propeller</u>.

Propeller Slip is the difference between the **geometric pitch** and the **effective pitch.**



E= Effective Pitch

<u>P= Geometric Pitch</u>

A= Aerodynamic Pitch

The **aerodynamic pitch** of the propeller is the axial displacement of the propeller prescribed by the zero lift chord line in one revolution. If the effective pitch and the aerodynamic pitch are equal the propeller thrust will fall to zero. If the effective pitch exceeds the aerodynamic pitch, the propeller will act as a brake.

It is clear that if a propeller is to perform efficiently over a range of aircraft speeds, i.e. over a large range of effective pitch angles, the geometric pitch cannot be constant.

Most modern propellers have the facility to adjust the geometric pitch angle β and these are known as **adjustable pitch propellers**. This can be as simple as setting a pitch angle prior to flight; in order to optimise one part of the flight phase (take-off, climb or cruise).

Variable pitch mechanisms that can change the pitch during flight are known as **controllable (*or* variable) pitch propellers** and these are well suited to sport aircraft.

A propeller fitted with an autonomous variable pitch mechanism that maintains constant propeller speed (and thus engine speed) irrespective of aircraft speed is known as a **constant speed propeller** and is used primarily on civil transport aircraft.

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