UNIVERSITY OF BRISTOL
FACULTY OF ENGINEERING

First Year Examination for the Degree of Master of Engineering

MAY/JUNE 2011 3 Hours

AENG11100

FLUIDS I

Solutions

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Q 1

$$p = p_a + \rho g\mathrm{h} = 1.9\times10^5 = 1.023\times10^5 + 1000\times0.9\times9.81\times h$$
$$h = 9.93m$$

(4 *marks*)

Q 2

Horizontal force given by gauge pressure at C.G times the area (use gauge because atmospheric pressure acts on both sides, use C.G & areas of projected horizontal plane). The vertical force equals the weight of water supported (again atmosphere acts on both sides)

$$F_H = (p_{CG} - p_a)\times A_{gate} = \rho g\mathrm{h}\times A_{gate}$$
$$F_H = 1000\times9.81\times1\times6\times2 = 117720N$$
$$F_V = \rho g(\text{volume}) = \rho g(\pi\times r^2\times0.25\times6)$$
$$F_V = 1000.\times9.81\times(\pi\times4\times0.25\times6) = 184914N$$

(4 *marks*)

Q 3

Along a streamline: Steady, incompressible, inviscid.
Inside ducts must also assume 1D (or Quasi-1D) flow

(4 marks)

Q 4

Use continuity so

$$A_1V_1 = A_2V_2 \quad \rightarrow \quad = \frac{A_1}{A_2} = \frac{V_2}{V_1} = \frac{80}{5} = 16$$

Density of air is small, & the duct horizontal, so we neglect hydrostatic terms & use

$$p_1 + \tfrac{1}{2}\rho V_1^2 = p_2 + \tfrac{1}{2}\rho V_2^2 \quad \rightarrow \quad p_2 = p_1 + \tfrac{1}{2}\rho\left(V_1^2 - V_2^2\right)$$
$$p_2 = 0.98\times10^5 + 0.5\times1.2\times\left(5^2 - 80^2\right) = 94175\,\mathrm{N/m^2}$$

(*4 marks*)

Q 5

(a) Galilean transformation.
(b) Reynolds number defined as

$$\mathrm{Re}_{car} = \frac{V_{car}L_{car}}{\nu} = \frac{50\times L_{car}}{\nu} = \mathrm{Re}_{model} = \frac{V_{model}L_{model}}{\nu} = \frac{V_{model}\times {L_{car}}/{6}}{\nu}$$

$$\frac{V_{model}\times {L_{car}}/{6}}{\nu} = \frac{50\times L_{car}}{\nu} \quad \rightarrow \quad V_{model} = 300m/s$$

(c) The Mach number of the real car is 0.147 while the Mach number of the model is 0.88. While compressibility effects could be neglected for the real car, compressibility cannot be ignored for the model and comparisons to the flow in the wind tunnel cannot be made.

(*4 marks*)

Q 6

$$\mathrm{Drag} = \mathrm{Area}\times C_D\times\tfrac{1}{2}\rho V^2$$

So assuming drag coefficient doesn't change then

$$\mathrm{D}_1 = \mathrm{A}\times C_D\times\tfrac{1}{2}\rho V_1^2$$
$$\mathrm{D}_2 = \mathrm{A}\times C_D\times\tfrac{1}{2}\rho V_2^2 = \frac{\mathrm{D}_1}{V_1^2}V_2^2 = \frac{70}{16}\times49 = 214.4N$$

For a low speed flow and because we know this is a streamlined body we can assume that the flow structure has changed, ie from **separated to attached** flow. The cause of this is likely to be due to the **onset of turbulence**.

(4 marks)

Q 7 From continuity we know that the average velocity downstream of the sudden contraction must be

$$A_1V_1 = A_2V_2 \quad \rightarrow \quad V_2 = V_1 \frac{A_1}{A_2} = 1 \times \frac{4}{1} = 4m/s$$

For no pressure losses use Bernoulli

$$p_1 + \tfrac{1}{2}\rho V_1^2 + \rho g h_1 = p_{2\,\text{no loss}} + \tfrac{1}{2}\rho V_2^2 + \rho g h_2 \quad \rightarrow \quad p_{2\,\text{no loss}} = p_1 + \tfrac{1}{2}\rho\left(V_1^2 - V_2^2\right) + \rho g\left(h_1 - h_2\right)$$

$$p_{2\,\text{no loss}} = 2\times10^5 + 0.5\times1000\times\left(1^2 - 4^2\right) - 1000\times9.81\times2 = 172880 \text{ N/m}^2$$

$$\Delta p_2 = \tfrac{1}{2}\rho V_2^2 k = 0.5\times1000\times4^2\times0.4 = 4000 \text{ N/m}^2$$

$$p_2 = 172880 - 4000 = 168880 \text{ N/m}^2$$

Pressure lost to heat due to large viscous effects following the separation at the sudden contraction

(4 marks)

Q 8 From continuity

$$\dot{m} = \rho A V = 1000\times\pi\times0.15^2\times5 = 353.429\,kg/s$$

Steady flow x-momentum equation

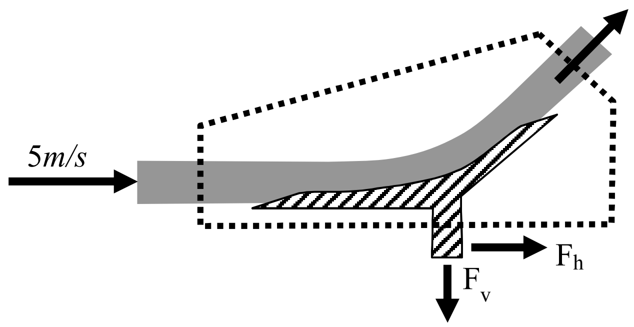
$$F_x = \dot{m}\left(V\cos30 - V\right) = \dot{m}\times5\times\left(\cos30 - 1\right) = -236.752\,N$$

$$F_h = 236.752\,N$$

Steady flow y-momentum equation

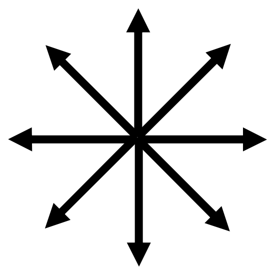
$$F_y = \dot{m}\left(V\sin\theta - 0\right) = \dot{m}\times5\times\sin\theta = 883.573\,N$$

$$F_v = 883.573\,N$$

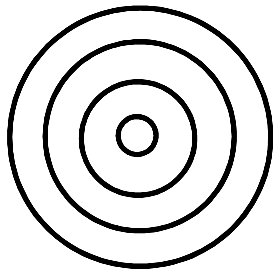


(4 marks)

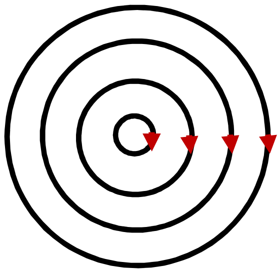
Q 9



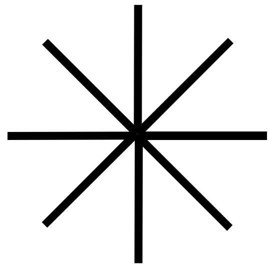
Source streamlines



Source equipotentials



Vortex streamlines



Vortex equipotentials

(4 marks)

Q 10 The non-lifting flow over an oval can be modelled as a combination of: plane onset flow, source and sink of equal strength.
The lifting flow over a cylinder can be modelled as a combination of: plane onset flow, a doublet and a vortex?

(4 marks)

Q11

a) At the surface, the density of the air in V_b and V_a is given by

$$\rho = \frac{p_s}{RT_s}$$

Volume and density of the air changes but the total mass of air in V_b and V_a is constant throughout the calculation and is given by

$$m = \rho(V_b + 2V_a) = \frac{p_s}{RT_s}(V_b + 2V_a)$$

The pressure in V_b at a depth h is given by

$$p_b = p_s + \rho_w gh$$

From the equation of state for an ideal gas we can write

$$p_b = \rho_b RT$$

At maximum depth the pressure in V_a is therefore rp_b and assuming an ideal gas and a constant temperature T the density in V_a is given by

$$\rho_a = r\rho_b$$

If V_h is the volume of air in the ballast chamber then the total mass of air is given by

$$m = \rho_b V_h + 2\rho_a V_a = \rho_b V_h + 2r\rho_b V_a = \rho_b(V_h + 2rV_a) = \frac{p_b}{RT}(V_h + 2rV_a)$$

Substituting in the relation for pressure with depth

$$m = \frac{p_b}{RT}(V_h + 2rV_a) = \frac{p_s + \rho_w gh_{\max}}{RT}(V_h + 2rV_a)$$

Equating the mass of air at the surface and at a depth h_{max}, then rearranging we have

$$m = \frac{p_s + \rho_w gh_{\max}}{RT}(V_h + 2rV_a) = \frac{p_s}{RT_s}(V_b + 2V_a)$$

$$V_h = \frac{T}{T_s}\frac{p_s}{p_s + \rho_w gh_{\max}}(V_b + 2V_a) - 2rV_a$$

Depth given when buoyancy balances weight

$$Mg = (V - V_b + V_h)\rho_w g$$

$$V_h = \frac{M}{\rho_w} - (V - V_b)$$

Equating volumes and rearranging gives

$$V_h = \frac{M}{\rho_w} - (V - V_b) = \frac{T}{T_s}\frac{p_s}{p_s + \rho_w gh_{\max}}(V_b + 2V_a) - 2rV_a$$

$$\frac{M - (V - V_b - 2rV_a)\rho_w}{\rho_w} = \frac{T}{T_s}\frac{p_s}{p_s + \rho_w gh_{\max}}(V_b + 2V_a)$$

$$p_s + \rho_w gh_{\max} = p_s\frac{T}{T_s}\frac{(V_b + 2V_a)\rho_w}{M - (V - V_b - 2rV_a)\rho_w}$$

$$h_{\max} = \frac{p_s}{\rho_w g}\left(\frac{T}{T_s}\frac{(V_b + 2V_a)\rho_w}{M - (V - V_b - 2rV_a)\rho_w} - 1\right)$$

(13 marks)

b)

The temperature T can be written as

$$T = 293 - \frac{16}{2000} h_{\max}$$

So

$$h_{\max} = \frac{1.02 \times 10^5}{1028 \times 9.81} \left(\frac{293 - \frac{16}{2000} h_{\max}}{293} \frac{(2.31 + 2 \times 0.01)1028}{4000 - (6.2 - 2.31 - 2 \times 2 \times 0.01)1028} - 1 \right)$$

$$h_{\max} = 10.11435 \left(\frac{293 - \frac{16}{2000} h_{\max}}{293} 56.7592 - 1 \right)$$

$$h_{\max} = \left(293 - \frac{16}{2000} h_{\max} \right) 1.95933 - 10.11435 = 563.9685 - 0.01567 h_{\max}$$

$$h_{\max} = \frac{563.9685}{1.01567h} = 555.3m$$

(7 marks)

Q12

(a) Applying Bernoulli's equation between the surface and the exit of the syphon, taking the vertical height as zero at the syphon exit. Note that the exit pressure is atmospheric and the surface velocity is assumed zero

$$p_a + \rho g(H + \delta) = p_a + \tfrac{1}{2}\rho V_e^2$$

$$V_e = \sqrt{2g(H + \delta)}$$

$$\dot{m} = \rho A_e \sqrt{2g(H + \delta)}$$

(5 marks)

(b) Applying Bernoulli's equation between the surface and the exit of the syphon, taking the vertical height as zero at the syphon exit. The velocity at the surface is given by V_s so

$$p_a + \rho g(H - \delta) + \tfrac{1}{2}\rho V_s^2 = p_a + \tfrac{1}{2}\rho V_e^2$$

Applying continuity

$$A_c V_s = A_e V_e$$

Substituting into Bernoulli's equation and rearranging

$$p_a + \rho g(H - \delta) + \tfrac{1}{2}\rho\left(\frac{A_e}{A_c}\right)^2 V_e^2 = p_a + \tfrac{1}{2}\rho V_e^2$$

$$V_e^2\left[1 - \left(\frac{A_e}{A_c}\right)^2\right] = 2g(H - \delta)$$

$$V_e = \frac{A_c}{\sqrt{A_c^2 - A_e^2}}\sqrt{2g(H - \delta)}$$

$$\dot{m} = \rho\frac{A_c A_e}{\sqrt{A_c^2 - A_e^2}}\sqrt{2g(H - \delta)}$$

(7 marks)

(c)

Applying Bernoulli's equation between the exit and the highest point of the syphon

$$p_h + \rho g(H + h) + \tfrac{1}{2}\rho V_h^2 = p_a + \tfrac{1}{2}\rho V_e^2$$

Applying continuity

$$A_h V_h = A_e V_e$$

Substituting back into Bernoulli's equation

$$p_h + \rho g(H + h) = p_a + \tfrac{1}{2}\rho V_e^2\left(1 - \left(\frac{A_e}{A_h}\right)^2\right)$$

$$p_h = p_a - \rho g(H + h) + \rho g(H + \delta)\left(\frac{A_h^2 - A_e^2}{A_h^2}\right)$$

Applying Bernoulli's equation between the exit and the highest point of the syphon

$$p_h + \rho g(H + h) + \tfrac{1}{2}\rho V_h^2 = p_a + \tfrac{1}{2}\rho V_e^2$$

$$A_h V_h = A_e V_e$$

$$p_h + \rho g\left(H + h\right) = p_a + \tfrac{1}{2}\rho V_e^2\left(1 - \left(\frac{A_e}{A_h}\right)^2\right)$$

$$p_h = p_a + \rho g\left(H - \delta\right)\left(\frac{A_c^2}{A_c^2 - A_e^2}\right)\left(\frac{A_h^2 - A_e^2}{A_h^2}\right) - \rho g\left(H + h\right)$$

Subtract the two values of pressure when δ=0

$$\Delta p_h = \rho g H\left(\frac{A_h^2 - A_e^2}{A_h^2}\right)\left(1 - \left(\frac{A_c^2}{A_c^2 - A_e^2}\right)\right) = -\rho g H\left(\frac{A_h^2 - A_e^2}{A_h^2}\right)\left(\frac{A_e^2}{A_c^2 - A_e^2}\right) = \rho g H\left(\frac{A_e^2 - A_h^2}{A_h^2}\right)\left(\frac{A_e^2}{A_c^2 - A_e^2}\right)$$

(8 marks)

Q13

(a) Δh is due to the difference in static pressure between the wind tunnel pressure tappings at the working and inlet sections. Applying Bernoulli's equation between locations and neglecting hydrostatic terms as air density is small

$$p_w + \tfrac{1}{2}\rho_a V_w^2 = p_i + \tfrac{1}{2}\rho_a V_i^2$$

Applying continuity we also have

$$A_w V_w = A_i V_i$$

Applying the hydrostatic equation across the manometer fluid surfaces

$$p_i = p_w + \rho_m g \Delta h$$

Rearranging

$$\rho_m g \Delta h = \tfrac{1}{2}\rho_a V_w^2 - \tfrac{1}{2}\rho_a V_i^2 \qquad \rightarrow \Delta h = \frac{\rho_a}{2\rho_m g} V_w^2 \left(1 - \frac{A_w^2}{A_i^2}\right)$$

(7 marks)

(b) From continuity and assuming the flow is incompressible, we know that the volume flow rate at the exit must be the same as at the working section.

$$A_e V_e = A_w V_w \qquad \rightarrow \quad V_e = \frac{A_w}{A_e} V_w$$

Similarly the average velocity downstream of the fan must be the same as in the working section (the cross section is constant).

Taking Bernoulli's equation from just downstream of the fan (subscript fd) to the exit, where the pressure is equal to the ambient pressure

$$p_{fd} + \tfrac{1}{2}\rho_a V_{fd}^2 = p_a + \tfrac{1}{2}\rho_a V_e^2$$

Applying continuity

$$p_{fd} + \tfrac{1}{2}\rho_a V_w^2 = p_a + \tfrac{1}{2}\rho_a \left(\frac{A_w}{A_e}\right)^2 V_w^2 \qquad \rightarrow \quad p_{fd} = p_a + \tfrac{1}{2}\rho_a \left(\frac{A_w}{A_e} - 1\right)^2 V_w^2$$

Therefore applying Bernoulli's equation from static atmospheric conditions to upstream of the fan gives

$$p_a = p_w + \tfrac{1}{2}\rho_a V_w^2$$

and therefore

$$p_{fd} - p_w = p_a + \tfrac{1}{2}\rho_a \left(\left(\frac{A_w}{A_e}\right)^2 - 1\right) V_w^2 - p_a + \tfrac{1}{2}\rho_a V_w^2$$

(7 marks)

$$\Delta p_{fan} = \tfrac{1}{2}\rho_a \left(\frac{A_w}{A_e}\right)^2 V_w^2$$

(c) Applying Bernoulli's equation from ambient conditions to just upstream of the model

$$p_a = p_i + \tfrac{1}{2}\rho_a V_i^2$$

$$p_i - p_w = \rho g \Delta h$$

$$p_w = -\rho_m g \Delta h + p_i = -\rho_m g \Delta h + p_a - \tfrac{1}{2}\rho_a V_i^2$$

Applying continuity and Bernoulli's equation downstream of the fan

$$p_{fd} + \tfrac{1}{2}\rho_a V_w^2 = p_a + \tfrac{1}{2}\rho_a \left(\frac{A_w}{A_e}\right)^2 V_w^2 \qquad \rightarrow \quad p_{fd} = p_a + \tfrac{1}{2}\rho_a \left[\left(\frac{A_w}{A_e}\right)^2 - 1\right] V_w^2$$

So the pressure change across the fan is now

$$\Delta p_f = \tfrac{1}{2}\rho_a\left[\left(\frac{A_w}{A_e}\right)^2 - 1\right]V_w^2 + \tfrac{1}{2}\rho_a V_i^2 + \rho_m g\Delta h = \tfrac{1}{2}\rho_a\left[\left(\frac{A_w}{A_e}\right)^2 - 1 + \left(\frac{A_w}{A_i}\right)^2\right]V_w^2 + \rho_m g\Delta h$$

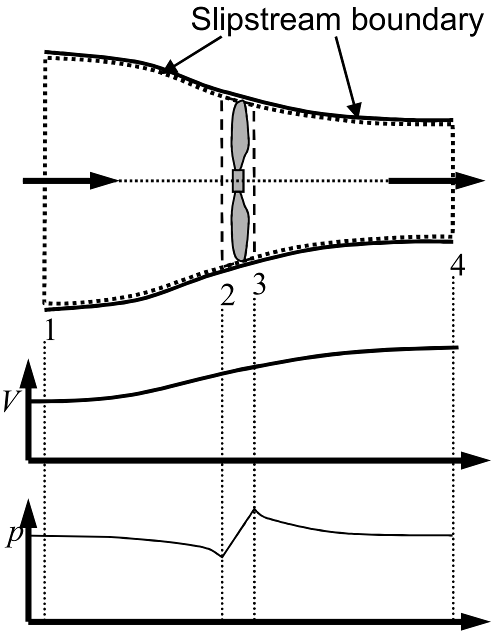
Equating original (no model) and final (model) pressure change

$$\Delta p = \tfrac{1}{2}\rho_a V_w^2\left(\frac{A_w}{A_e}\right)^2 - \tfrac{1}{2}\rho_a\left[\left(\frac{A_w}{A_e}\right)^2 - 1 + \left(\frac{A_w}{A_i}\right)^2\right]V_w^2 - \rho_m g\Delta h$$

$$\Delta p = \tfrac{1}{2}\rho_a\left[1 - \left(\frac{A_w}{A_i}\right)^2\right]V_w^2 - \rho_m g\Delta h$$

(6 marks)

Q14 **(a)** Use the actuator disc theory for an ideal propeller, see figure below



Assumptions: Frictionless & incompressible, Steady 1D flow (neglect rotation and variation across the disc radius). Actuator disc is thin so $A_2=A_3=A_d$ & $V_2=V_3=V_d$. $p=p_a$ at all points on slipstream boundary & 1 & 4

Continuity: $Q = V_d A_d$

Bernoulli's equation for CV 1-2 & CV 3-4

$$p_1 + \tfrac{1}{2}\rho V_1^2 = p_2 + \tfrac{1}{2}\rho V_d^2$$
$$p_3 + \tfrac{1}{2}\rho V_d^2 = p_4 + \tfrac{1}{2}\rho V_4^2$$

Using $V_1 = V$ & $V_4 = (a+1)V$

$$\rightarrow \quad p_3 - p_2 = \tfrac{1}{2}\rho\left((a+1)^2 V^2 - V^2\right)$$

Steady Flow momentum for CV 2-3:

$$(p_2 - p_3)A_d + F_{CV} = \rho Q(V_d - V_d) = 0 \qquad \rightarrow \quad F_{CV} = (p_3 - p_2)A_d$$

Where F is the force on the control volume

Applying results from Bernoulli's equation above

$$F = \tfrac{1}{2}\rho A_d V^2\left(a^2 + 2a + 1 - 1\right) = \tfrac{\pi}{2}\rho r^2 V^2 a(a+2)$$

(8 marks)

(b) Steady Flow momentum for CV 1-4:

$$0 + F = \rho Q(V_4 - V_1) \qquad \rightarrow \quad F = \rho V_d A_d V(a + 1 - 1)$$

From momentum & continuity

$$(p_3 - p_2)A_d = \rho V_d A_d (V_4 - V_1)$$

Eliminating $(p_3\text{-}p_2)$ using Bernoulli's equation above

$$\rho V_d\left((a+1)V - V\right) = \tfrac{1}{2}\rho\left((a+1)^2 V^2 - V^2\right)$$
$$V_d V a = \tfrac{1}{2}V^2 a(a+2)$$
$$V_d = \tfrac{1}{2}V(a+2)$$

The power supplied to the disc is

$$P_{disc} = F V_d = \rho Q V a V_d = \tfrac{1}{2}\rho Q V^2 a(a+2)$$

Power output

$$P_{out} = FV = \rho Q V^2 a$$

The efficiency of the rotor is therefore

$$\eta = \frac{P_{\text{out}}}{P_{\text{disc}}} = \frac{\rho Q V^2 a}{\frac{1}{2}\rho Q V^2 a(a+2)} = \frac{2}{a+2}$$

(7 marks)

(c) for a stationary rotor we have

$$V_1 = 0 \quad \rightarrow \quad V_4 = 2V_d$$

$$F = \rho A_d V_d (2V_d - 0) = 2\rho A_d V_d^2$$

$$V_d = \sqrt{F/2\rho A_d}$$

$$V_d = \sqrt{\frac{7000 \times 9.81}{2 \times 1.25 \times \pi \times 3^2}} = \sqrt{\frac{68670}{22.5 \times \pi}} = 31.17 m/s$$

(5 marks)

Q15 (a) Using Bernoulli's equation for incompressible flow gives

$$p_\infty + \frac{1}{2}\rho_\infty U_\infty^2 = p + \frac{1}{2}\rho_\infty U^2$$

since density is constant everywhere.
Then by definition of the pressure coefficient

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty U_\infty^2} = 1 - \frac{U^2}{U_\infty^2}$$

b) Using cylindrical polar coordinates, from given equations the stream function is given by

$$\psi = U_\infty r \sin\theta - \frac{\kappa}{2\pi}\frac{\sin\theta}{r} = U_\infty r \sin\theta \left(1 - \frac{\kappa}{2\pi U_\infty r^2}\right)$$

The velocity components are given by (could go straight from given equations)

$$V_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \left(1 - \frac{\kappa}{2\pi U_\infty r^2}\right) U_\infty \cos\theta, \quad V_\theta = -\frac{\partial\psi}{\partial r} = -\left(1 + \frac{\kappa}{2\pi U_\infty r^2}\right) U_\infty \sin\theta$$

Now the cylinder is a stream line of the flow so there is no flow normal to the cylinder i.e. $V_r = 0$. This means that

$$\left(1 - \frac{\kappa}{2\pi U_\infty R^2}\right) U_\infty \cos\theta = 0$$

for all θ so the doublet strength must be given by

$$\kappa = 2\pi U_\infty R^2$$

The velocity components are then

$$V_r = \left(1 - \frac{R^2}{r^2}\right) U_\infty \cos\theta, \quad V_\theta = -\left(1 + \frac{R^2}{r^2}\right) U_\infty \sin\theta$$

On the cylinder

$$V_r = 0, \quad V_\theta = -2U_\infty \sin\theta$$

The pressure coefficient on the cylinder therefore given by

$$C_p = 1 - \left(\frac{V_\theta}{U_\infty}\right)^2 = 1 - 4\sin^2\theta$$

And the pressure on the cylinder is

$$p(\theta) = p_\infty + \frac{1}{2}\rho U_\infty^2 (1 - 4\sin^2\theta)$$

c) Assume the flow can be modelled as the summation of the velocity due to the same cylinder plus an equal image cylinder. Along the line from the surface to the nearest point the vertical velocity will be zero so we need only consider horizontal velocity. Writing the horizontal velocity in Cartesian coordinates, with the origin at the surface,

$$u = U_\infty + \frac{-\kappa}{2\pi}\frac{\left(x^2 - (y-h)^2\right)}{\left(x^2 + (y-h)^2\right)^2} + \frac{-\kappa}{2\pi}\frac{\left(x^2 - (y+h)^2\right)}{\left(x^2 + (y+h)^2\right)^2}$$

From the previously found measure of the doublet strength and setting *x=0*, we can write

$$u = U_\infty - U_\infty R^2 \frac{\left(-(y-h)^2\right)}{\left((y-h)^2\right)^2} - U_\infty R^2 \frac{\left(-(y+h)^2\right)}{\left((y+h)^2\right)^2} = U_\infty + U_\infty R^2\left(\frac{1}{(y-h)^2} + \frac{1}{(y+h)^2}\right)$$

$$u = U_\infty + 2U_\infty R^2\left(\frac{y^2+h^2}{(y-h)^2(y+h)^2}\right) = U_\infty + 2U_\infty R^2\left(\frac{y^2+h^2}{(y-h)^2(y+h)^2}\right)$$

$$C_p = \frac{p-p_\infty}{\frac{1}{2}\rho_\infty U_\infty^2} = 1 - \frac{u^2}{U_\infty^2} = 1 - \frac{U_\infty^2 + 4U_\infty^2 R^2\left(\frac{y^2+h^2}{(y-h)^2(y+h)^2}\right) + 4U_\infty^2 R^4\left(\frac{y^2+h^2}{(y-h)^2(y+h)^2}\right)^2}{U_\infty^2}$$

$$C_p = -\left(4R^2\left(\frac{y^2+h^2}{(y-h)^2(y+h)^2}\right) + 4R^4\left(\frac{y^2+h^2}{(y-h)^2(y+h)^2}\right)^2\right)$$

$$C_p = -4R^2\frac{\left(y^2+h^2\right)}{\left(y^2-h^2\right)^2}\left(1 + R^2\frac{\left(y^2+h^2\right)}{\left(y^2-h^2\right)^2}\right)$$