StM3 – Composite Laminate Analysis

Lecture 1 :
- introduction
- macromechanics of uni-directional lamina

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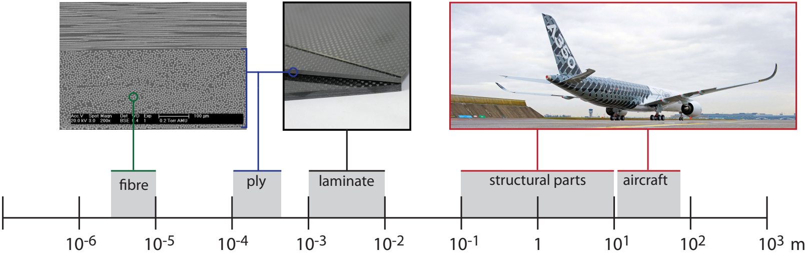
- Week 1 – 7, Tuesday 12:00 - 13:00
  Geography 1.1S Peel
- 6 lectures + 1 example class
- handouts are provided in lectures,
  lecture slides and example sheet on Blackboard
- any questions: Blackboard / e-mail / QB 0.108

Unit Content – I

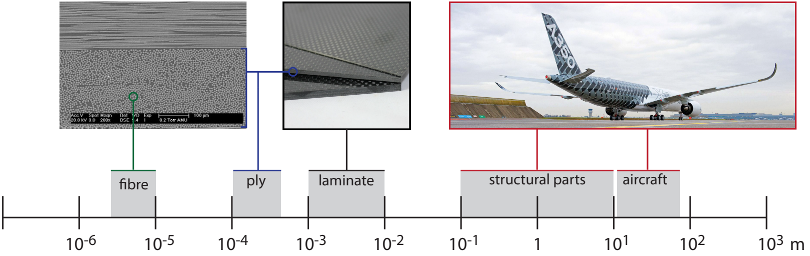
Composite Laminate Analysis (CLA)
is core knowledge for aerospace engineers

learning outcome: able to describe and analyse *structural* properties of a fibre reinforced composite laminate material



Unit Content – II

[video: Powers of Ten]



composite structural hierarchy:

- micro-mechanics of a ply (wk 3)
- macro-mechanics of a ply (wks 1-2)
- mechanics of a laminate (wks 4-6)

Unit Links

builds on StM1/StM2:

- material components in composites (fibres/matrix)
- 2D elasticity (stress/strain transformations)

prepares for:

- Composite Design and Manufacture (CDM)
- Advanced Composite Analysis (ACA)

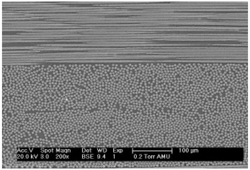
Text Books

- RM Jones (1998), "*Mechanics of Composite Materials*"
- AT Nettles (1994), "*Basic Mechanics of Laminated Composite Plates*", NASA Reference Publication 1351 (NASA-RP-1351)
- TW Clyne and D Hull (2019), "*An Introduction to Composite Materials*" 3rd edition, Cambridge University Press

Composite Materials – I

composite materials: composed of at least two materials, which combine to achieve enhanced properties and functionality

focus on Fibre Reinforced Polymer (FRP) composites; continuous, aligned fibres

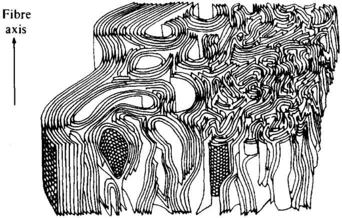


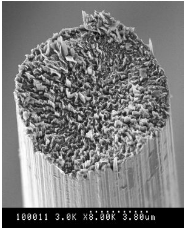
- fibre reinforcement:
  provides stiffness and strength
- polymer matrix: transfers load between fibres, provides toughness, and protects fibres

laminates: offer ability to tailor material/structural properties

Composite Materials – II

fibre reinforcement: carbon, glass, aramid, natural fibres





	ρ (kg/m^3)	E (GPa)	σ_* (GPa)	ε_* (%)
High Modulus carbon	1.95	380	2.4	0.6
High Strength carbon	1.75	230	3.4	1.1
E-glass	2.56	76	2.0	2.6
Kevlar 49	1.45	130	3.0	2.3

Composite Materials – III

polymer matrix: thermoset, thermoplastic

aerospace epoxy resins: $E \approx 5$ GPa

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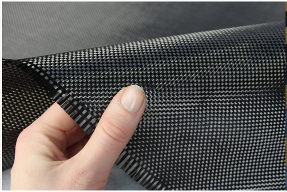
plies: unidirectional ply (UD), woven fabric, spread tow fabric

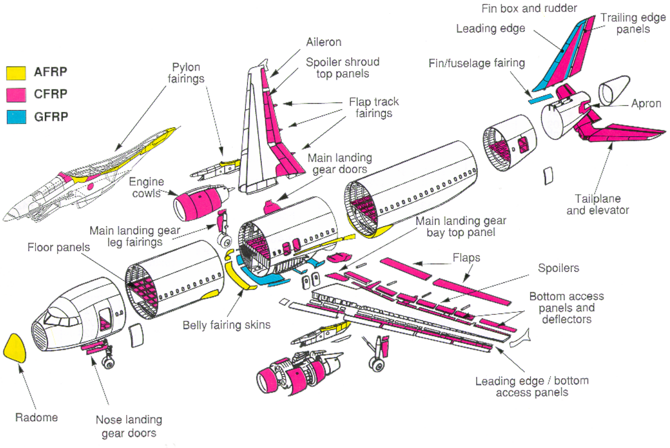

Composites in Aerospace: Passenger Aircraft – I

Airbus A350/Boeing 787, approx. 52%/50% weight composite

Q: differences in structural design of Boeing 787 and Airbus A350 composite fuselage?

Composites in Aerospace: Passenger Aircraft – II

A320 (designed 1980s) already had 15% weight composites



Composites in Aerospace: Passenger Aircraft – III

Q: Why use composites in (passenger) aircraft?

structural: high specific properties (reduced weight), good fatigue resistance, tailoring material properties

functional: corrosion resistance, low radar signature

operational: lower maintenance, reduced number of parts

Composites in Aerospace: Passenger Aircraft – IV

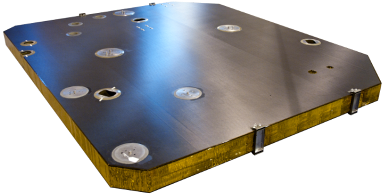
Q: Why **not** use composites in (passenger) aircraft?

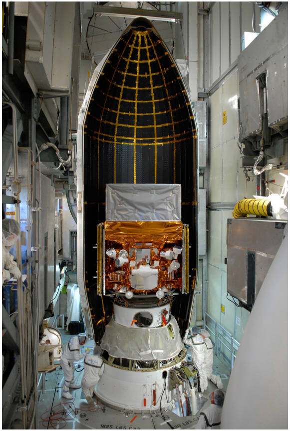
- cost
- manufacturing (learning new techniques)
- complexity of failure mechanisms + damage tolerance
- damage detectability + repairability
- unfamiliarity in design ('black aluminium') and design codes

Composites in Aerospace: Space Industry – I

lightweight: reduce launch costs

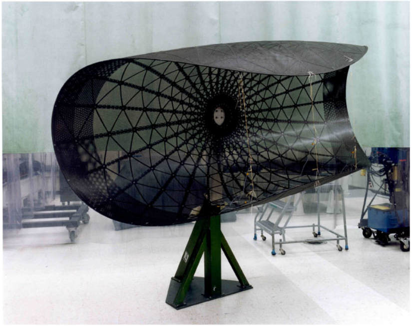
low CTE: thermal stability
(-30°C / + 50°C)





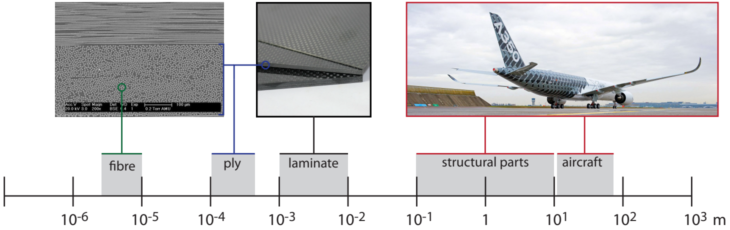
Composites in Aerospace: Space Industry – II

high strains: deployable space structures



flexible hinges, deployable masts, deployable reflectors, *etc*.

Lecture Outline

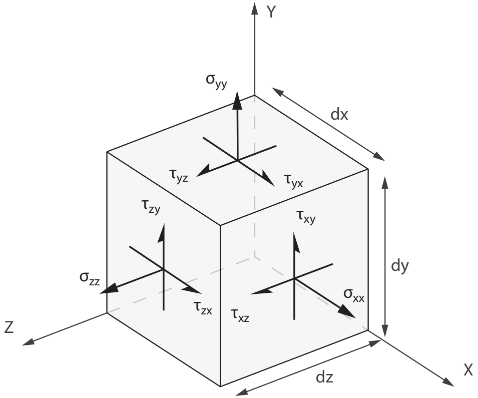


macromechanics of ply: constitutive equations for unidirectional composite ply, by modelling as *homogeneous* material

lecture outline:

- revise plane stress and strain conventions
- develop stress-strain relationship for a specially orthotropic ply

Revision: Stress – I

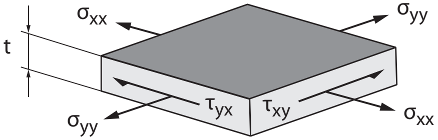


3D stress at a point:
Cauchy stress tensor

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{bmatrix}$$

Revision: Stress – II

plane stress: $\sigma_{zz} = \tau_{xz} = \tau_{yz} = 0$



Revision: Strain – I

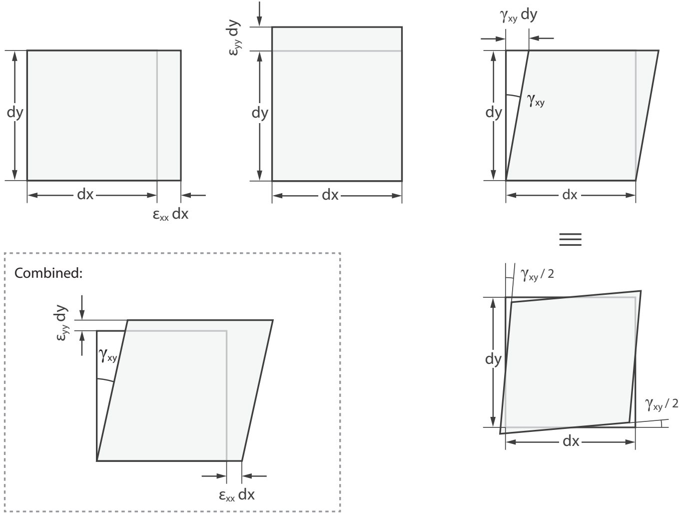
strain at a point also expressed as a tensor:

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz} \end{bmatrix}$$

mathematical (or *tensor*) shear strain $\varepsilon_{xy} = \gamma_{xy}/2$

plane strain: $\varepsilon_{zz} = \gamma_{xz} = \gamma_{yz} = 0$

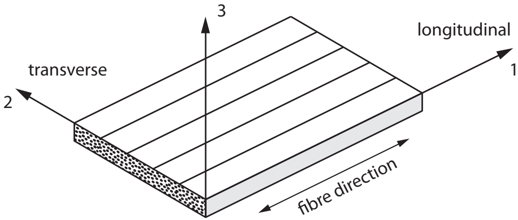
Revision: Strain – II



Composite Stress-Strain Relationship – I

macromechanics: consider laminate as homogeneous material

composite is **an**isotropic: mechanical properties differ along direction of fibres and perpendicular to fibres



sign convention: 123 refers to natural axes of the material, and *xyz* refers to structural axes

Composite Stress-Strain Relationship – II

general *anisotropic*, linear-elastic material:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{bmatrix}$$

note $C_{ij} = C_{ji}$; result of Maxwell's reciprocal theorem (see StM2)

Composite Stress-Strain Relationship – III

note the notation:

$$C_{ij} \qquad\qquad i, j = 1 \ldots 6$$

standardised numbering of components:

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

Composite Stress-Strain Relationship – IV

	ε_{11}	ε_{22}	ε_{33}	γ_{23}	γ_{13}	γ_{12}
σ_{11}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}
σ_{22}	C_{12}	C_{22}	C_{23}	C_{24}	C_{25}	C_{26}
σ_{33}	C_{13}	C_{23}	C_{33}	C_{34}	C_{35}	C_{36}
τ_{23}	C_{14}	C_{24}	C_{34}	C_{44}	C_{45}	C_{46}
τ_{13}	C_{15}	C_{25}	C_{35}	C_{45}	C_{55}	C_{56}
τ_{12}	C_{16}	C_{26}	C_{36}	C_{46}	C_{56}	C_{66}

Composite Stress-Strain Relationship – V

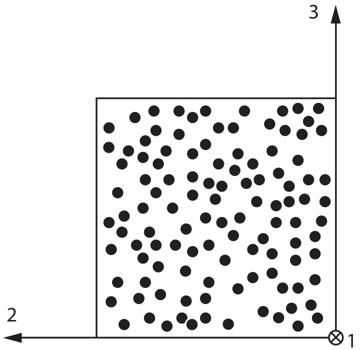
unidirectional composite: *orthotropic* material

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{bmatrix}$$

in natural axes: no extension-shear coupling

Composite Stress-Strain Relationship – VI

random fibre-packing: material properties transverse to fibre direction identical in all directions, *i.e.* isotropic



$$C_{22} = C_{33}$$

$$C_{13} = C_{12}$$

$$C_{55} = C_{66}$$

C_{44} no longer independent

$$C_{44} = \frac{C_{22} - C_{33}}{2}$$

Composite Stress-Strain Relationship – VII

transversely isotropic material model:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{12} & C_{23} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{(C_{22}-C_{23})}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{66} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{bmatrix}$$

Composite Stress-Strain Relationship – VIII

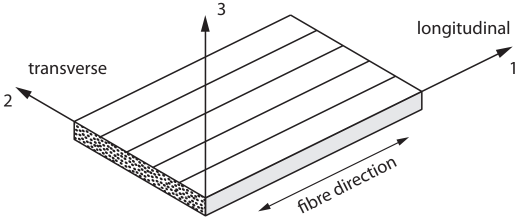
properties identical in all directions: *isotropic* material

$$C_{ij} = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{(C_{11}-C_{12})}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{(C_{11}-C_{12})}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{(C_{11}-C_{12})}{2} \end{bmatrix}$$

two independent material parameters (here, C_{11} and C_{12})

Composite Lamina: Plane Stress – I

composite is **specially orthotropic**: mechanical properties differ along direction of fibres and perpendicular to fibres



in composite laminate structures, each individual lamina is assumed to be loaded under **plane stress**

Composite Lamina: Plane Stress – II

lamina constitutive equations reduce to:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{bmatrix}$$

Q_{ij} components are known as **reduced stiffnesses**

$$Q_{ij} = C_{ij} - \frac{C_{i3}C_{j3}}{C_{33}} \qquad i,j = 1,2,6$$

found from C_{ij} by imposing the plane stress condition

Composite Lamina: Plane Stress – III

plane stress condition:

$$\sigma_{33} = C_{13}\varepsilon_{11} + C_{23}\varepsilon_{22} + C_{33}\varepsilon_{33} = 0$$

gives:

$$\varepsilon_{33} = -\frac{(C_{13}\varepsilon_{11} + C_{23}\varepsilon_{22})}{C_{33}}$$

substituting back into stiffness matrix:

$$\begin{aligned}\sigma_{11} &= C_{11}\varepsilon_{11} + C_{12}\varepsilon_{22} - C_{13}\frac{(C_{13}\varepsilon_{11} + C_{23}\varepsilon_{22})}{C_{33}} \\ &= \underbrace{\left(C_{11} - \frac{C_{13}C_{13}}{C_{33}}\right)}_{Q_{11}}\varepsilon_{11} + \underbrace{\left(C_{12} - \frac{C_{13}C_{23}}{C_{33}}\right)}_{Q_{12}}\varepsilon_{22}\end{aligned}$$

Summary

summary:

- revised plane stress and strain
- derived specially orthotropic material model for unidirectional composite lamina : reduced stiffness matrix $\boldsymbol{Q}$

next week:

- derive reduced stiffness matrix $\boldsymbol{Q}$ and compliance matrix $\boldsymbol{S}$
- calculate stress/strain for an angled composite lamina
- derive generally orthotropic material model ($\bar{\boldsymbol{Q}}$ and $\bar{\boldsymbol{S}}$)

Revision Objectives

Revision Objectives:

- describe constituents of fibre-reinforced polymer composites;
- recognise and use composite subscript conventions $(i, j = 1 \ldots 6)$;
- appreciate derivation of specially orthotropic material model;