Lecture 4 - Strip Theory

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Preview - Equations of Motion

The translational equations can be reduced to the following, even allowing for non-zero values of V, W, and a climb angle θ:

- Fore/ Aft: $$m(\dot{U} - rV + qW) = X - mg\,\mathbf{sin}\,\theta$$
- Lateral: $$m(\dot{V} - pW + rU) = Y + mg\,\mathbf{cos}\,\theta\,\mathbf{sin}\,\phi$$
- Transverse: $$m(\dot{W} - qU + pV) = Z + mg\,\mathbf{cos}\,\theta\,\mathbf{cos}\,\phi$$

Strip Theory Aerodynamics

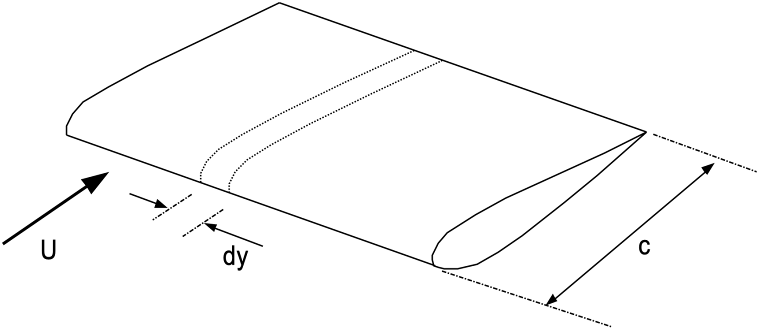
Introduction

- Later in your studies of theoretical aerodynamics you will see different ways of analysing the lift and drag on wings of various shapes.
- These will be far more comprehensive than what is presented here; e.g. CFD. However the thoroughness of such methods implies complexity.
- There are great advantages in employing the method below for many routine computations - Simple equations (AVDASI Design Project)

Wing Aerodynamics

- It is more difficult to calculate overall basic lift distribution than to calculate additional lift components.
- These additional lift components can be due to control surface deflections, due to aircraft movements away from steady level flight, or due to gusts. It is in these cases that strip theory is most useful.
- The difficulties are centered on C_l , i.e. how to estimate the true local lift coefficient at any station y from root to tip.
- Some components of the total effective incidence α can easily be described whereas others depend on the lift that is currently being created, i.e. local incidence depends on current lift while current lift depends on the distribution of incidence.

Strip Theory Aerodynamics



Strip Theory Aerodynamics

- The essence of the method is to describe the differential element of lift that is generated on a single differential strip of a lifting surface.
- The underlying assumption of this method is that:
  "the lift on any strip is independent of the aerodynamics of all neighbouring strips".
- Every strip is treated independently and its contribution to lift expressed accordingly - *distributed lift*.

Strip Theory Aerodynamics

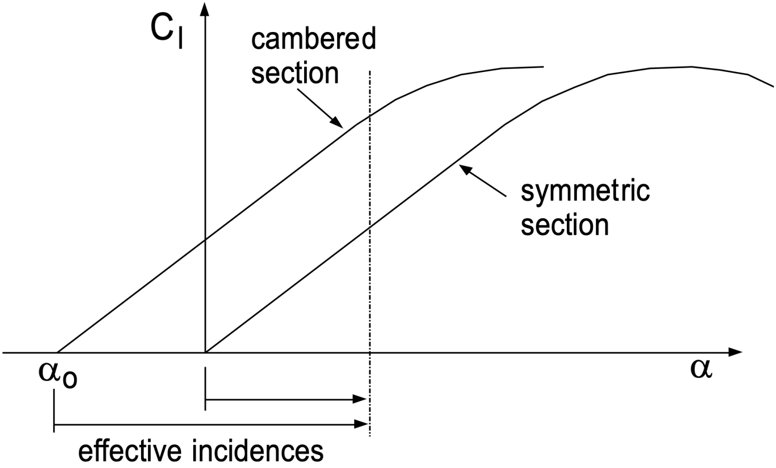
Introduction

- For a single strip of an aerofoil (which, for the moment, has no camber so $a_0 = 0$) we can write:

dLift	=	1/2 ρ U^2	c dy	C_l .
↑		↑	↑	↑
differential lift		pressure	differential area	coefficient of proportionality to allow for shape and orientation to the flow

- We can use a sectional lift coefficient C_l , i.e. that which exists at a single spanwise station, as opposed to a wing lift coefficient C_L
- We can use the variation (with span) of this coefficient to generate expressions for the change in overall lift generated.

Strip Theory Aerodynamics



Strip Theory Aerodynamics

- In practice we can allow for all normal local lift-producing mechanisms, including a trim tab:

$$C_l = a_0 + a_1 \alpha + a_2 \delta + a_3 \beta$$

- where the generalized control surface angle δ will specifically be one of ξ, η, ζ.
- Note: β here is the control tab deflection

Past Exam Question

Q1. One of the requirements for roll control is that the ailerons be able to counteract the imbalance when, for example, fuel is taken entirely from one wing. The geometry for one wing of an aircraft is given in Fig. Q1 and the rolling moment developed by emptying its outer two tanks is 6 x 10^5 Nm.

(a) You are to use strip-theory to estimate the deflection angle required for the ailerons ξ if the aircraft is to fly level. You are to suggest and if necessary use a sensible value for $a_1 = \dfrac{\partial C_L}{\partial \alpha}$ for the wing section and also use $a_2 = \dfrac{\partial C_L}{\partial \xi} = 0.047$ /degree. Determine the necessary angle for each of the flight conditions:

250 knots (TAS) cruise at 6 km altitude,
130 knots approach at sea level.

(15 marks)

Past Exam Question

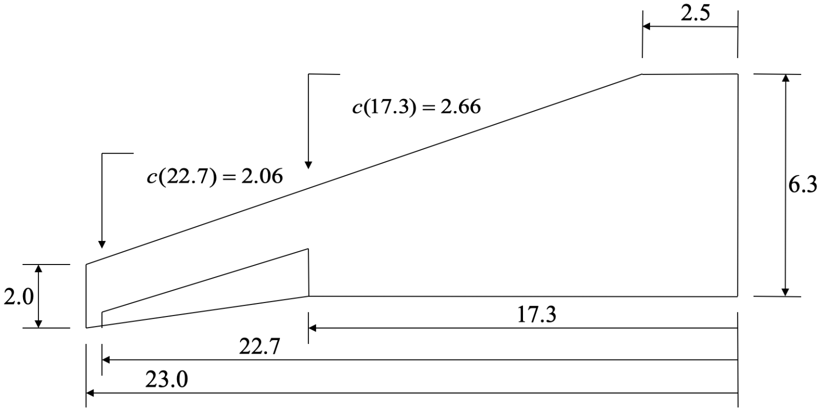
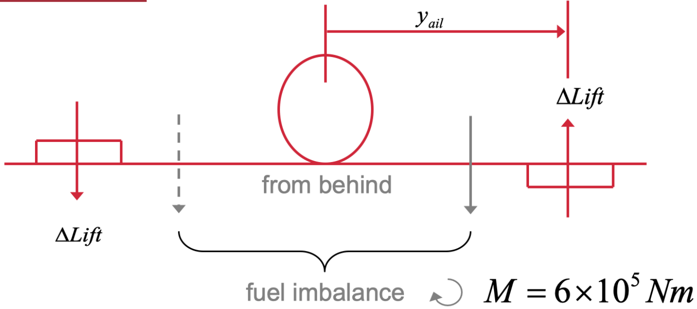


Fig. Q1 (dimensions in metres)

Past Exam Question

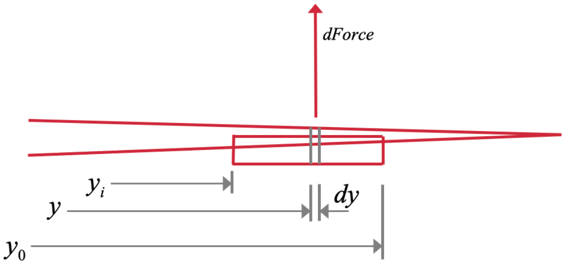
- Roll balance



Rolling moment needed from ailerons must be

$$\underbrace{2\times\Delta Lift\times y_{ail}}_{\text{need strip-theory equivalent of this}} = 6\times10^5\,Nm$$

Past Exam Question



From the aileron deflection there is developed

$$dForce = q\xi a_2 c(y)dy \qquad (1)$$

which leads to a differential rolling moment

$$dL = -q\xi a_2 yc(y)dy \qquad (2)$$

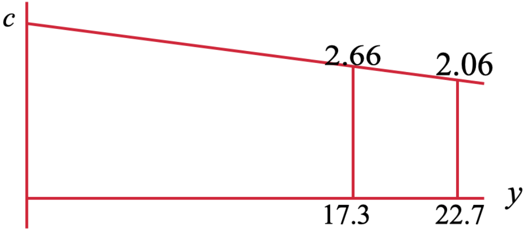
Past Exam Question

from which the total rolling moment developed would be

$$L_{TOT} = -2q\xi a_2 \int_{y_{inner}}^{y_{outer}} yc(y)dy \qquad (3)$$

two sides

There is an assumption here, with a_2 outside the integral, that a_2 is constant across the span of the aileron – reasonable. We need a function for $c(y)$.



Past Exam Question

for $$y = mx + b$$

$$m = \frac{2.06 - 2.66}{22.7 - 17.3}$$

$$= -0.6 / 5.4$$

$$= -0.111$$

therefore $$b = y - mx$$

$$= 2.66 + 0.111 \times 17.3 = 4.58$$

so $$c(y) = -0.111 y + 4.58$$

Past Exam Question

Also, needing *q* :

at 6 km $$\sigma = \frac{20-6}{20+6} = 0.538$$

at Sea level $$\rho = \rho_0 = 1.225$$

Speeds: 250kt $$= 250 \times \frac{1}{1.94} = 129$$ m/s

130kt $$= 130 \times \frac{1}{1.94} = 67$$ m/s

See front page of the exam

Past Exam Question

Therefore:

$$L_{250} = -2\times\frac{1}{2}\times 1.225\times 0.538\times 129^2\times\xi\times 0.047\times\int(\ldots)$$

$$L_{250} = -515\xi^o \underbrace{\int_{17.3}^{22.7} y(-0.111y+4.58)dy}_{-\frac{0.111}{3}y^3+\frac{4.58}{2}y^2\Big|_{17.3}^{22.7}=I}$$

$$I = \frac{-0.111}{3}\left(11.7\times 10^3 - 5.18\times 10^3\right)+2.29(515-299)$$

$$I = -241.2 + 495 = \boxed{253}$$

So $L_{250} = -1.3\times 10^5\xi^o$ Nm

Past Exam Question

Roll balance

$$-1.3\times10^5\xi^o + 6\times10^5 = 0$$

$$\xi^o = \frac{6\times10^5}{1.3\times10^5} = \boxed{4.6^o \quad \text{to counter for loss of fuel}}$$

Approach speed

$$L_{130} = -2\times\frac{1}{2}\times1.225\times67^2\times\xi^o\times0.047\times\int$$

$$= -258\times\xi^o\times253 = -0.653\times10^5\times\xi^o$$

same

Roll balance (Approach)

$$-0.653\times10^5\xi^o = -6\times10^5$$

$$\xi^o = \frac{6}{0.653}\,\text{deg} = \boxed{9.2^o}$$

Next Lecture

Strip Theory Continued