FLUIDS I
Example sheet 5: Potential Flow (short revision)

1. What are the fundamental principles used to derive the Navier-Stokes?
2. The usual form of Bernoulli's equation is found by integrating the 1D Euler equation. What assumptions are needed to first derive the 1D Euler equation $dp = \rho \mathbf{V} d\mathbf{V}$? What further assumption is necessary to integrate this equation to get the familiar form of Bernoulli's equation $p + 1/2\rho\mathbf{V}^2 = \text{ constant}$?
3. For what type of flow can a velocity potential be defined?
4. The velocity potential if it exists satisfies Laplace's equation. Give expressions for u, v and w in terms of the potential? Why is it advantageous to be able to solve Laplace's equation?
5. What is the name given to a line whose tangent an any point is in the direction of the local fluid velocity?
6. What is the equation for a streamline in 2D flow (assume that we are working in the x-y plane)?
7. Using the above equation obtain an equation for the streamlines of a flow which has $u=w\,x$ and $v=-w\,y$, where w is a constant. Also sketch the streamlines.
8. For the flow in question 7 find the expressions for the potential and streamfunction
9. Give expressions for the velocities u and v in terms of (a) the stream function ψ, (b) the potential ϕ.
10. For a uniform flow parallel to the x axis, derive expressions for the stream function and the potential
11. Give expressions for the radial and tangential velocity components v_r , v_θ in terms of the stream function and the potential
12. For a source $\phi_{source} = \dfrac{\Lambda}{2\pi}\ln r$ and $\psi_{source} = \dfrac{\Lambda}{2\pi}\theta$ Sketch the equipotentials and streamlines
13. What is the line integral equation for circulation?
14. For a vortex $\phi_{vortex} = -\dfrac{\Gamma}{2\pi}\theta$ and $\psi_{vortex} = \dfrac{\Gamma}{2\pi}\ln r$ Sketch the equipotentials and Streamlines.
15. Explain briefly in words how a doublet flow is obtained from a combination of elementary flows.
16. What elementary flows can be combined to produce a 2D potential model for a non-lifting flow over a circular cylinder
17. Derive an expression for the pressure coefficient Cp in terms of velocity from Bernoulli's equation.
18. Briefly describe the physical significance of the stream function ψ in incompressible flow. What are the limits on its application, compared with the potential function ϕ?
19. The non-lifting flow over a cylinder has a stream function given by $\psi = U_\infty y - \dfrac{\kappa}{2\pi}\dfrac{\sin\theta}{r}$

    (a) This flow is modelled as a combination of which two flows?

    (b) Put the stream function into consistent (r, θ) coordinates assuming that $R^2 = \dfrac{\kappa}{2\pi U_\infty}$

    (c) Find expressions for the radial and tangential velocity components v_r , v_θ

    (d) What is the pressure coefficient distribution for the cylinder which has radius R?
20. State D'Alembert's paradox.