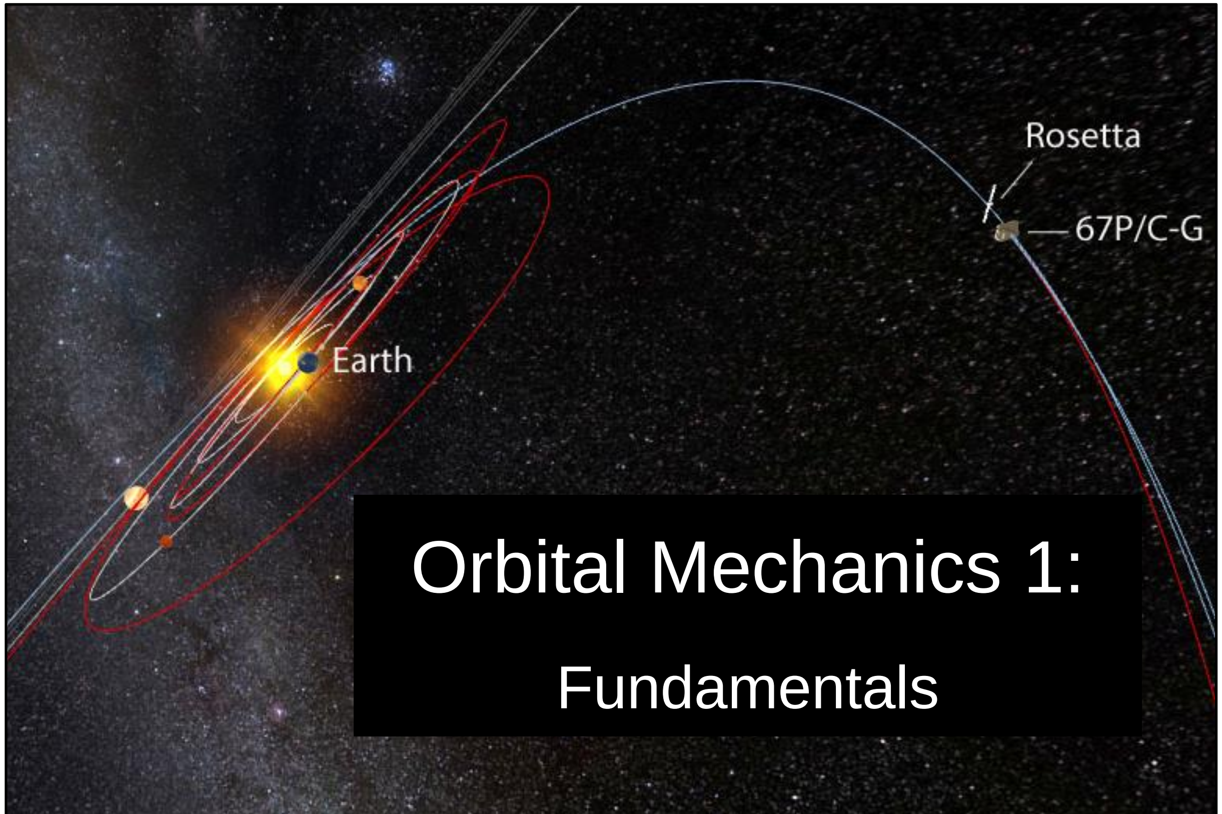


Orbital Mechanics 1:

Fundamentals

L.Berthoud

Learning Objectives

1. Why is it important?
2. Who were the main players?
3. Kepler's laws
4. Vocabulary and geometry for ellipses
5. Newton's laws of motion and gravitation
6. Conservation of energy

Explain reason for orbital mechanics
Know main players in development and what they did
Describe and explain the planetary laws of Kepler
Know vocabulary and geometry for ellipses
Describe and use Newton's laws of motion and gravitation
Conservation of energy

Introduction

Why is orbital mechanics important?

- different orbits enable satellites to perform different missions
- necessary to know the position (and velocity) of a satellite at all times
- trajectory of a satellite is determined by its initial conditions (position and velocity after launch)

and…

You know already that the payload drives the mission, so the orbital needs of the payload will drive the mission design eg: how much fuel it needs, power requirements etc.
The satellite will need to be navigated to where it needs to be and then we usually need to know where it is at all times. We also need to launch it correctly in terms of direction and time. Hence the concept of 'launch window' – the window of time within which we can launch a spacecraft for it to be able to fulfil its mission.

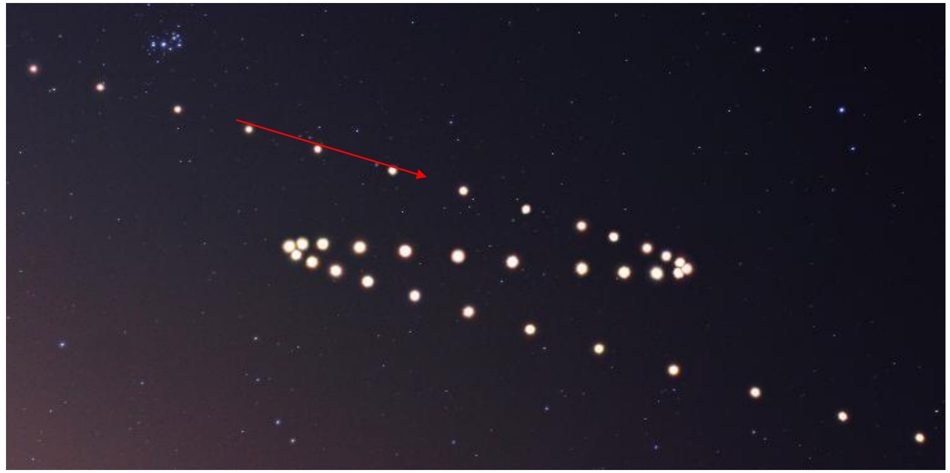
Introduction

Why is orbital mechanics important?

– how to get from A to B in space!

All good spaceships have a navigator. What are they doing? They are calculating how to get from one point to another in space. To do this they need to be able to predict the motion of the body. If there are other bodies in the vicinity the spaceship will be affected by their gravity.

Planetary Motion

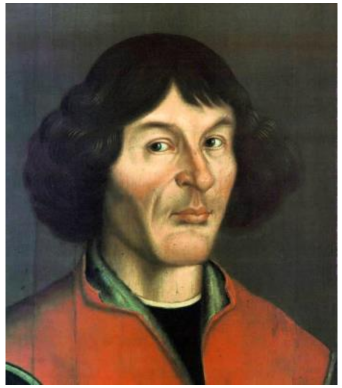


What does this tell us about the structure of the solar system?

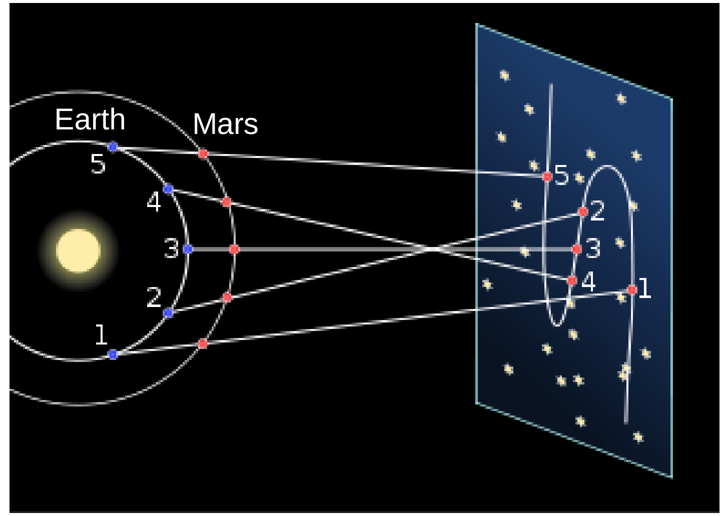
Our ancestors have always looked at the stars, Moon and planets. They used the stars to navigate and they even worked out when to plant their crops according to the Moon and time of year. Stonehenge was used to record the Vernal equinox vector which we will look at next lecture. If we look at the planets in the night sky, usually they move West to East compared to the background stars. However, early astronomers noticed that Mars sometimes moved East to West (and all the planets do this). This is known as the retrograde loop of Mars and is caused by its heliocentric motion.

As you know it is hard to guess from Earth that we are orbiting around the Sun, which is why everyone used to think the Sun moved around the Earth.

Planetary Motion



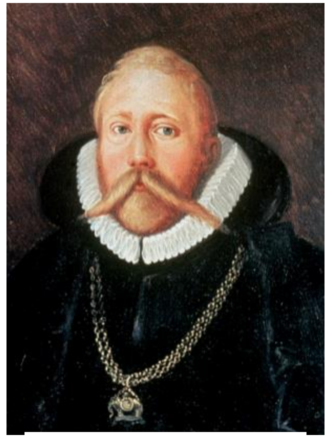
Nicolaus Copernicus (1473 - 1543)



Copernicus proposed that the Sun was at the centre of the Universe instead of the Earth

Our understanding of the elliptical motion of planets about the Sun spanned several years and included contributions from many scientists. Copernicus was the first to try and work out from Mars' motion what was going on. If you look at the figure, you can see that the only thing that made sense to him was to put the Sun at the centre of the Universe instead of Earth.

Planetary Motion



Tycho Brahe
(1546 - 1601)

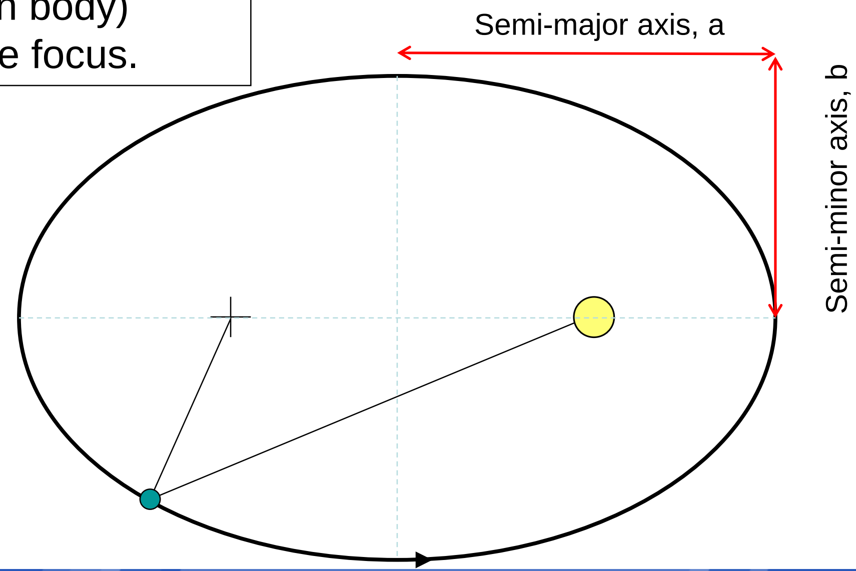
Johannes
Kepler
(1571 - 1630)

Brahe was an astronomer who collected astronomical data on planets. Kepler worked on the data to produce laws.

Brahe was an astronomer who did lots of naked eye astronomy in order to collect astronomical data on planets. Kepler then took on the long and difficult task of analyzing the data to produce some laws. It is important to realise that Kepler's laws were based on observation, not on calculations. Newton then developed mathematical explanations of the laws.

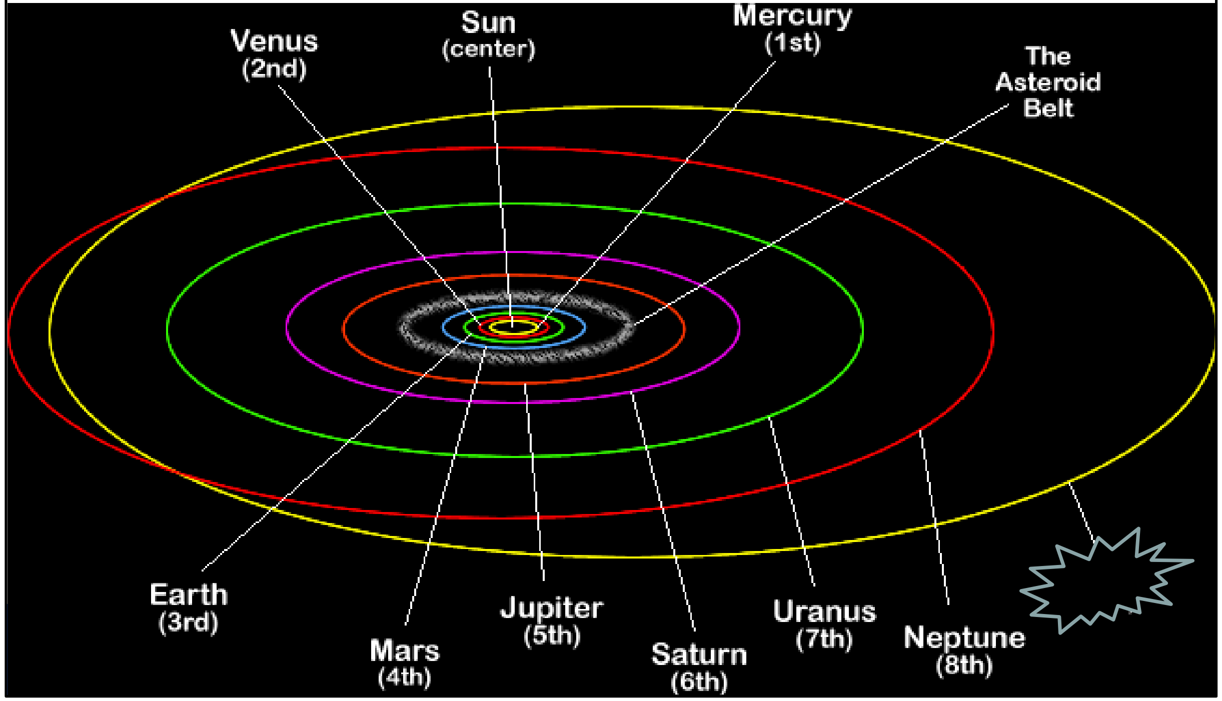
Kepler's First Law

K1: The orbits of the planets are ellipses with the Sun (main body) located at one focus.



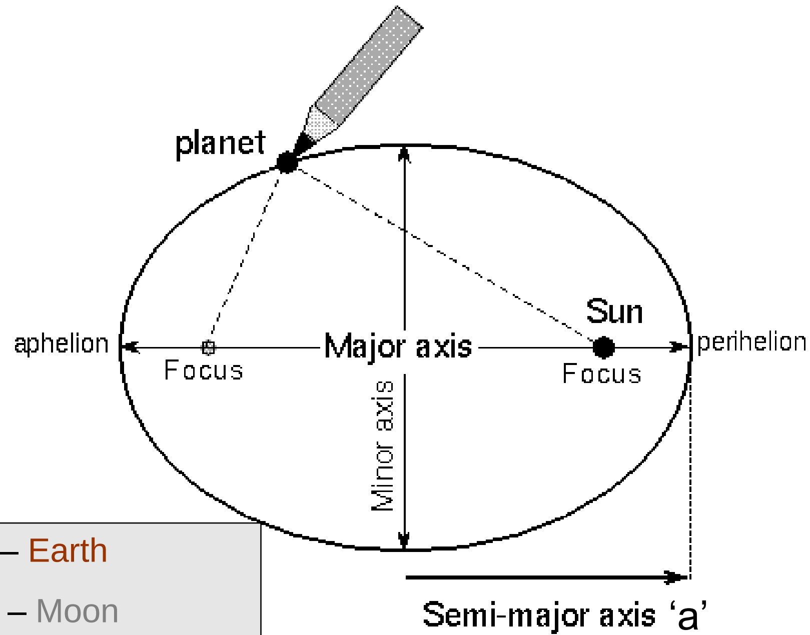
To make an ellipse – take a piece of string, pencil, 2 pins. There are two useful applications of this law: Sun/planets (as Kepler stated) and also Earth/satellites.
Note that the other focus is empty.
A circle is a special case of ellipse (with both foci in same spot).

K1:'The planets trace ellipses around the Sun'



OK, we all know that Pluto isn't a planet any more…just a 'dwarf planet'.

Ellipses – a reminder



Apo/Perigee – Earth

Apo/Perilune – Moon

Apo/Perihelion – Sun

Apo/Periapsis – non-specific

Apo means 'far' and this is the furthest point from the body.
Peri means 'near' and this is the nearest point to the body.
The major axis is 2a in dimension
The minor axis is 2b in dimension

Definition of orbit

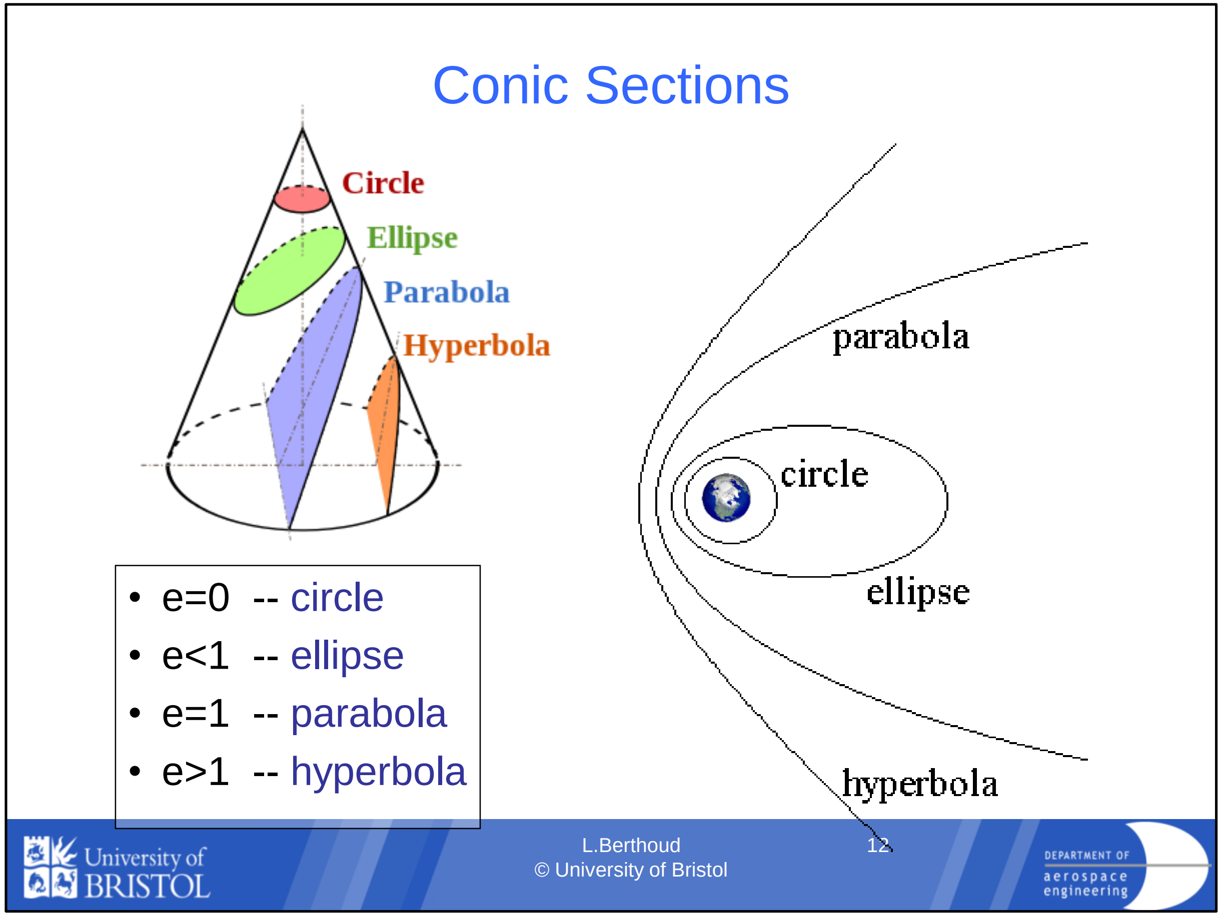
- Orbit is a closed or recurring path that a spacecraft or planet follows around a body.

- If e: eccentricity of an ellipse,

Then:

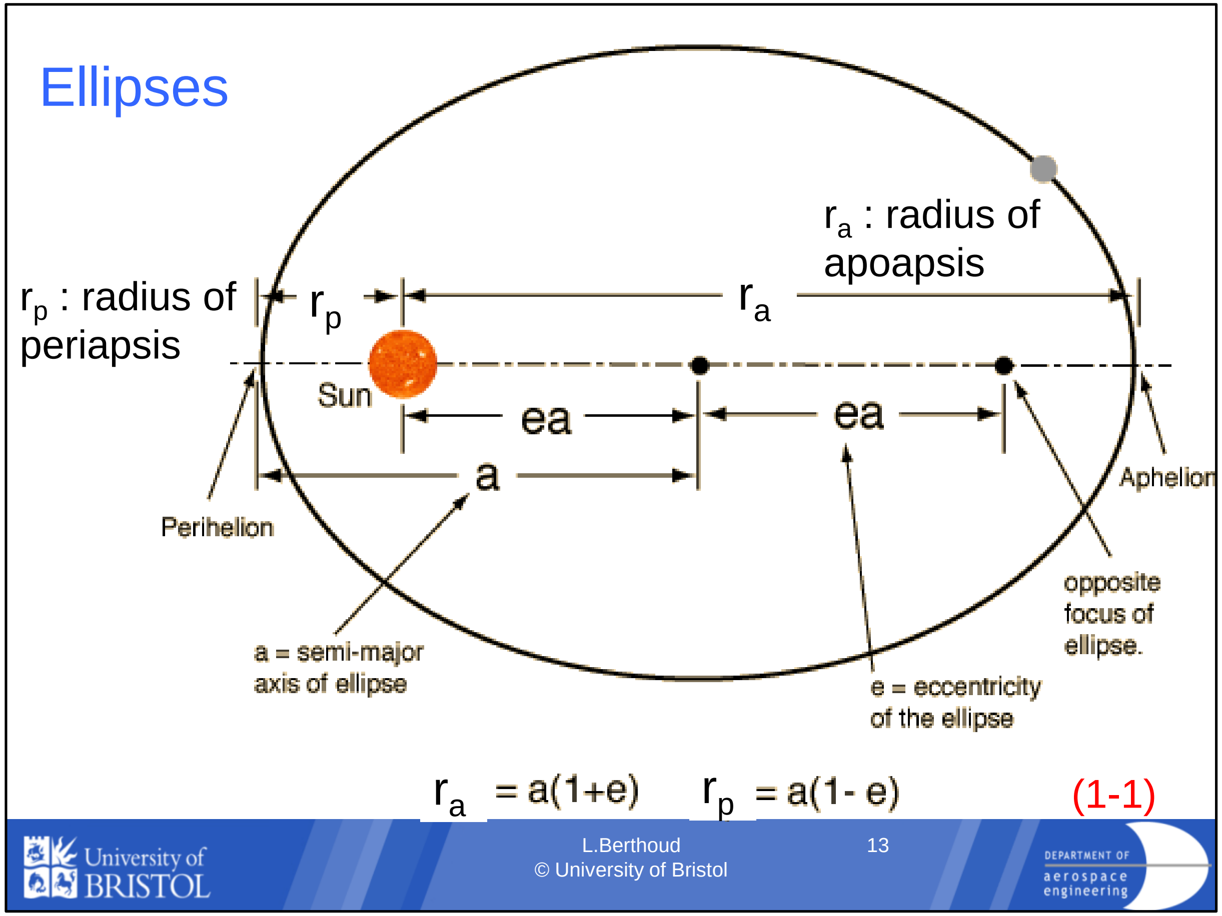
- e < 1 Orbit is 'closed' – recurring path (elliptical)
  e > 1 Not an orbit – passing trajectory (hyperbolic)
  e = 0 ?
  e = 1 ?

If you think of an ellipse as a 'squashed' circle, the eccentricity of the ellipse gives a measure of just how 'squashed' it is. For ellipses e is in the range 0 to 1.



We will see that all orbits form conic sections:
A circle when plane cuts cone perp to axis.
If intersection=unbounded curve then we have a parabola or hyperbola.
A parabola for when the plane cuts parallel to the generator line.
We have already said that circle is a special case of ellipses and now parabola is a special case of hyperbolae.

To go from one to another you have to add more velocity or ‘delta velocity’.

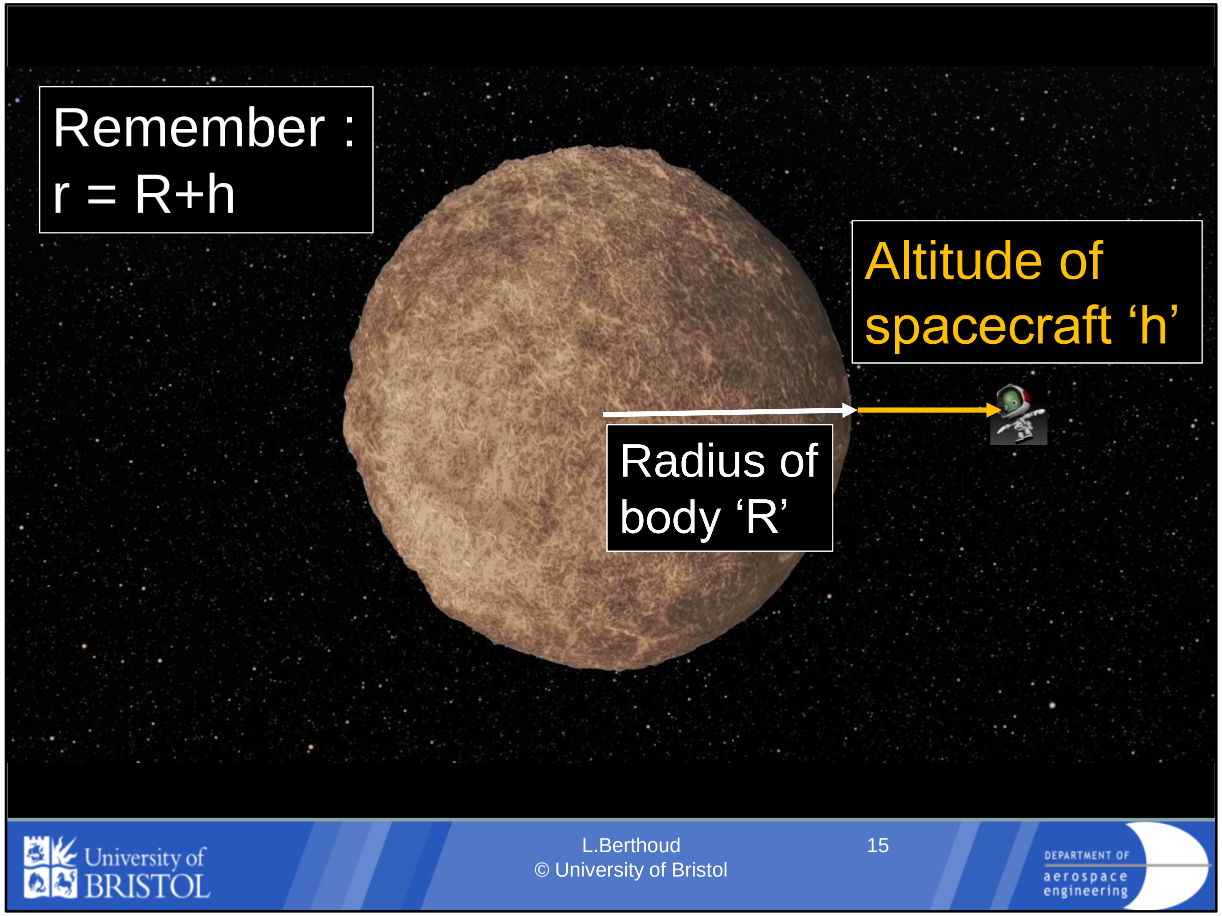


Here you can see that the orbital path around the Sun is the shape of an ellipse. Note that the radius of periapsis and apoapsis is measured to the CENTRE of the bodies.

Ellipse expressions

- periapsis distance $r_p = a(1 - e)$
- apoapsis distance $r_a = a(1 + e)$
- semi-major axis $a = (r_a + r_p)/2$ (1-2)
- eccentricity $e = ea/a = (r_a - r_p)/(r_a + r_p)$ (1-3)
- distance centre to focus $C_F = a - r_p = ae$ (1-4)
- area of ellipse πab (1-5)
- semi-minor axis $b = a\sqrt{1 - e^2}$ (1-6)
- equation of ellipse $\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$ (1-7)

Eccentricity is found by a formula that uses two measures of the ellipse. c is the distance from the center to a focus.
a is the distance from that focus to a vertex.
We will prove some of these soon in a few lectures. But it is good to be able to do a few basic calculations at this stage.



Note that radius ‘r’ consists of radius of body ‘R’ plus altitude or height ‘h’. Altitude means distance above surface, whereas radius means distance from middle of body. Watch out, it is easy to confuse altitude ‘h’ and radius ‘r’.

Numerical example

If a satellite is in Earth orbit with perigee at 200 km altitude and apogee at 2000 km, what are ‘a’ and ‘e’ for the orbit? [Radius of Earth=6378km]

Radius of Earth

Altitude of orbit

$r_p = R_e + h_p = 6378000+200000\ \text{m} = 6.578\text{x}10^6\text{m}$

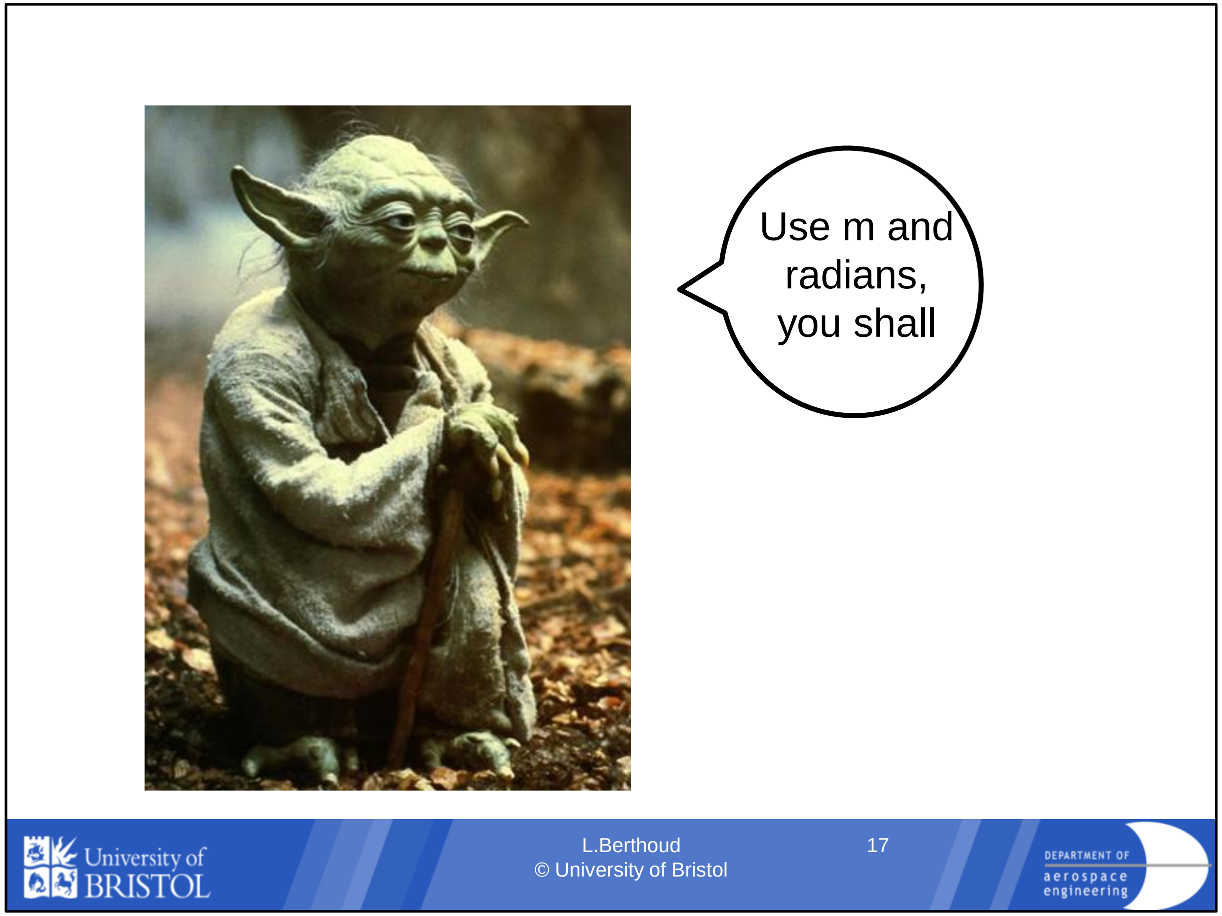
$r_a = R_e + h_a = 8.378\text{x}10^6\ \text{m}$

$a = (r_a + r_p)/2 = 7.478\ \text{x}10^6\ \text{m}$

$e = (r_a - r_p)/(r_a + r_p) = 0.1204$

L.Berthoud 16

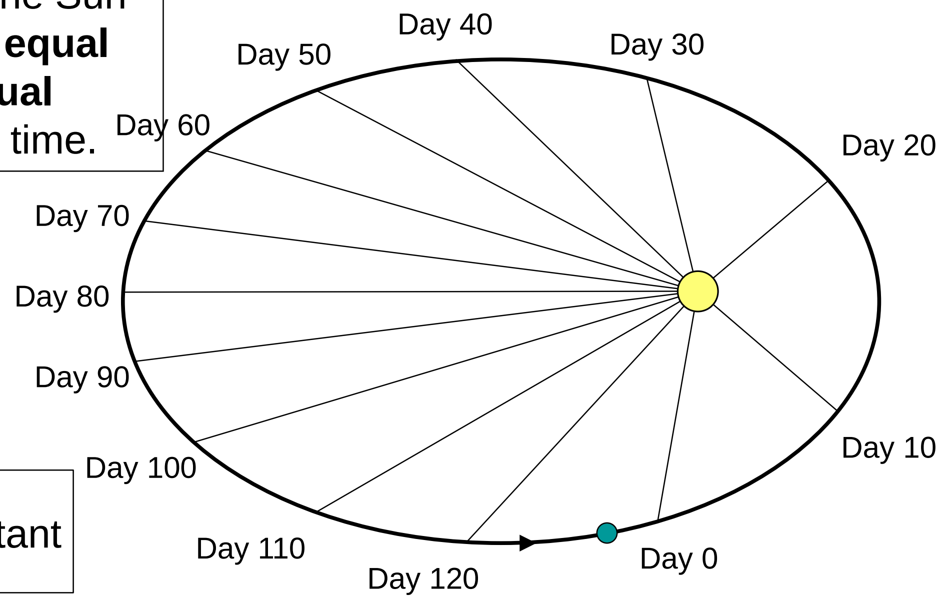
This is frequently the first part of a calculation, first calculate r and then calculate a and e.



We will use Yoda to try and remind ourselves of the necessity to: ‘always the units watch’. He might pop up now and then…

Kepler's Second Law

K2: The line joining a planet to the Sun sweeps out **equal areas** in **equal intervals** of time.



(1-8)

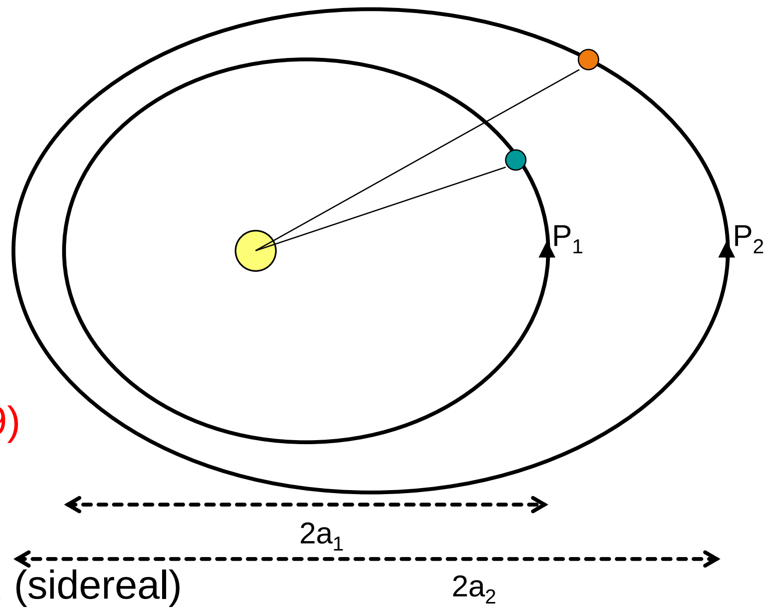
$$\frac{dA}{dt} = \text{constant}$$

Arises from conservation of angular momentum.
Where the satellite is travelling the fastest and what is this point called?

Kepler's Third Law

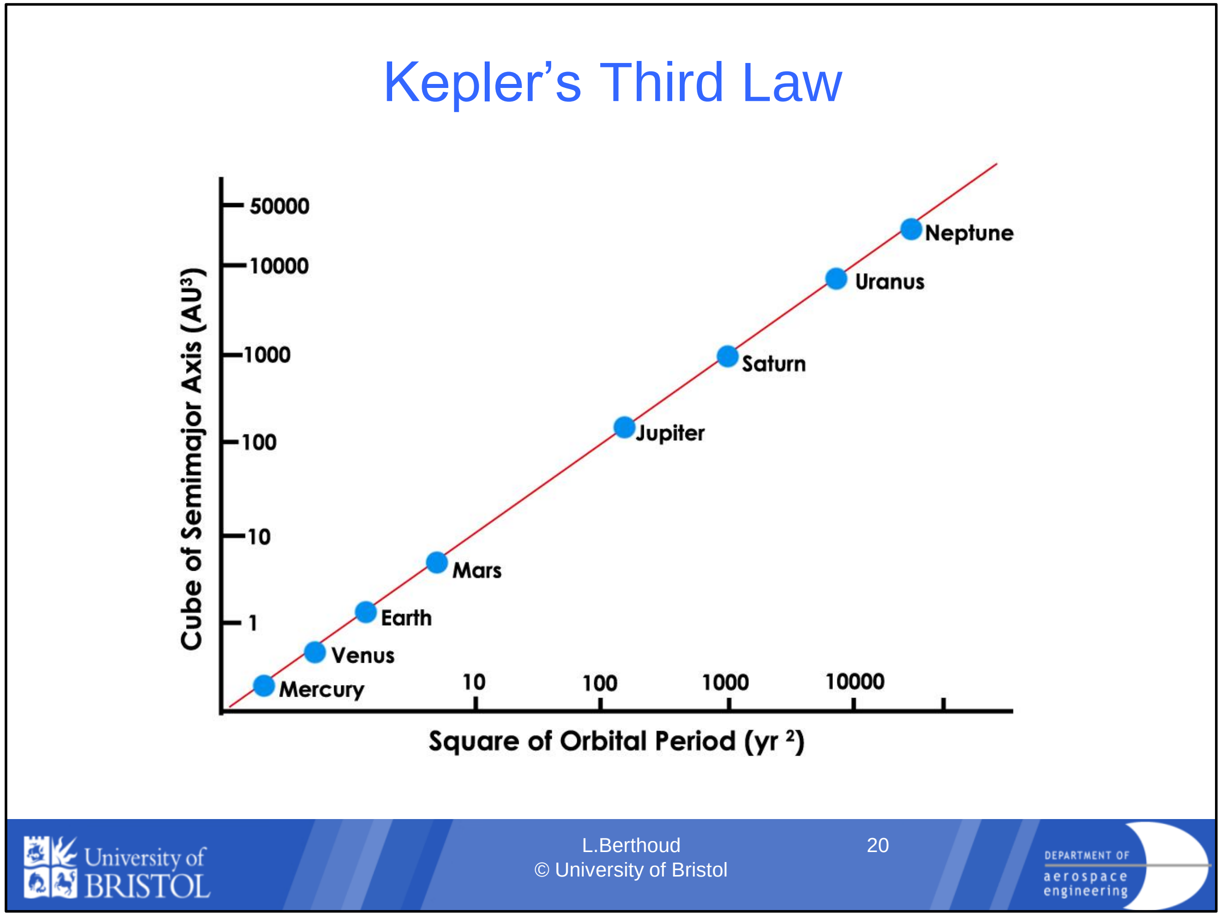
K3: The square of the orbital period of a planet is proportional to the cube of the semi-major axis of its orbit.

$$\boxed{\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3}} \quad (1\text{-}9)$$



T = Orbital Period (sidereal)
a = semi-major axis

Arises from gravitation. The period here is the sidereal period.
Also T2/a3 =k =1 if the period is in years and a in AU (dist bet Sun and Earth)
1 AU=149e6 km.
We will prove the more detailed formula for T in a few lectures time.



K3 is also known as the law of harmonies. Notice that there is a gap between Mars and Jupiter. What is between Mars and Jupiter? The asteroids.

Numerical example

Q: Galileo is often credited with the early discovery of four of Jupiter's many moons. One of the moons is called Io - its distance from Jupiter's center r_{io} is 4.2 units and its period T_{io} is 1.8 days. Another moon is called Ganymede; r_g =10.7 units. Make a prediction of the period of Ganymede T_g.

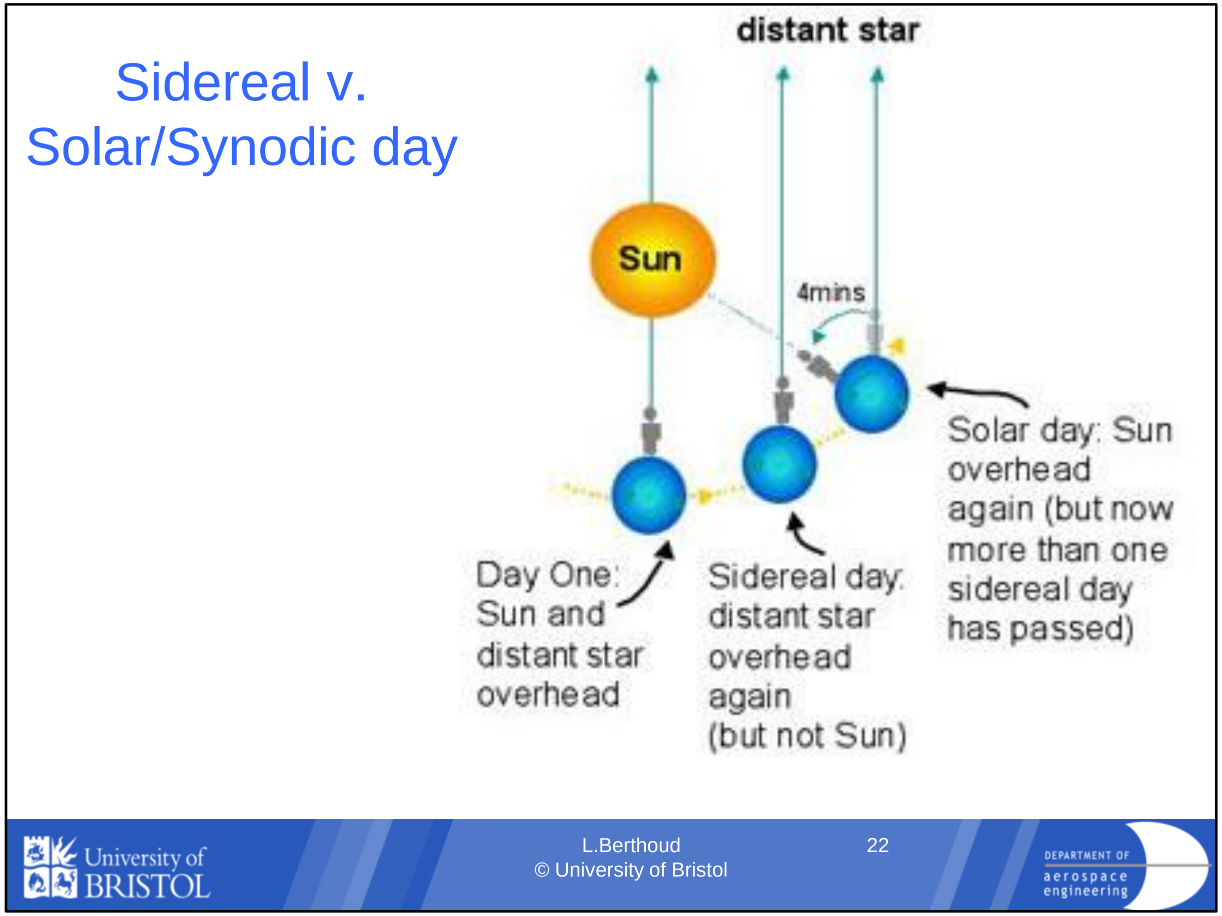
A: For a circular orbit a=r, then we use (1-9):

$T_{io}^2/r_{io}^3 = 1.8^2/4.2^3 = 0.04373 = T_g^2/r_g^3$

$T_g^2 = 0.04373.\ r_g^3$

$T_g^2 = 0.04373.10.7^3 = 53.57$ so $T_g = SQRT(53.57) = 7.32$ days

Io’s actual distance from Jupiter is 422,000km, Ganymede’s is 1,070,400km.



1 solar day = time taken for Sun to come back to the same place= 24hrs
1 sidereal day=time for Stars to come back to same position in sky = 23h 56min
The Earth rotates 360° in 23h 56’(Celestial or “Sidereal” Day)
Or put another way, it rotates ~361° in 24.000 hours(Synodic or “Solar” Day)
Satellites orbits are aligned to the Sidereal day – not the solar day

Newton's Laws of Motion

N1: When viewed in an inertial reference frame, an object either remains at rest or continues to move at a constant velocity, unless acted upon by an external force.

N2: **F** = m**a**. The vector sum of the forces **F** on an object is equal to the mass m of that object multiplied by the acceleration vector **a** of the object.

N3: Whenever a first body exerts a force **F** on a second body, the second body exerts a force **−F** on the first body. **F** and **−F** are equal in magnitude and opposite in direction.

Sir Isaac Newton (1643-1727), England, developed the laws of motion.
Note: force F and acceleration a are written in bold, i.e. they are vectors (magnitude + direction).

Gravity

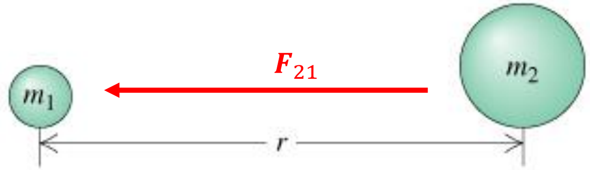
Courtesy of Warner Bros. Pictures

Bullock and Clooney’s velocity relative to station was zero. In orbit they are in free fall, so there are no forces on him and there was no weight for Bullock to bear. All she had to do was give the tether a gentle tug and Clooney would’ve been safely pulled toward her.

Newton's Universal Law of Gravitation

Force exerted on mass m_2 by m_1:

$$\boldsymbol{F}_{21} = -\frac{GM_1 m_2}{r^2}\hat{\boldsymbol{r}}_{12} = -\frac{GM_1 m_2}{r^3}\boldsymbol{r}_{12} \quad (1\text{-}10)$$



G: universal gravitational constant =6.67259×10^{-11} $m^3/kg/s^2$

$\hat{\mathbf{r}}$: unit vector from M_1 to m_2: $\hat{\boldsymbol{r}}_{12} = \frac{\boldsymbol{r}_{12}}{|\boldsymbol{r}_{12}|}$

$|\boldsymbol{r}_{12}|$ = r: magnitude of vector **r**, ie: distance between M_1 and m_2

The negative sign represents an attractive force.

We will stop using the magnitude lines $|\boldsymbol{r}_{12}|$ and just use r as the magnitude

Newton's Universal Law of Gravitation (magnitudes)

$$|\boldsymbol{F}| = m_2|\boldsymbol{a}| = \frac{GM_1 m_2}{r^2}$$

$$|\boldsymbol{a}| = \frac{GM_1}{r^2} \qquad (1\text{-}11)$$

Find accn of ISS due to Earth gravity:

Mass of Earth

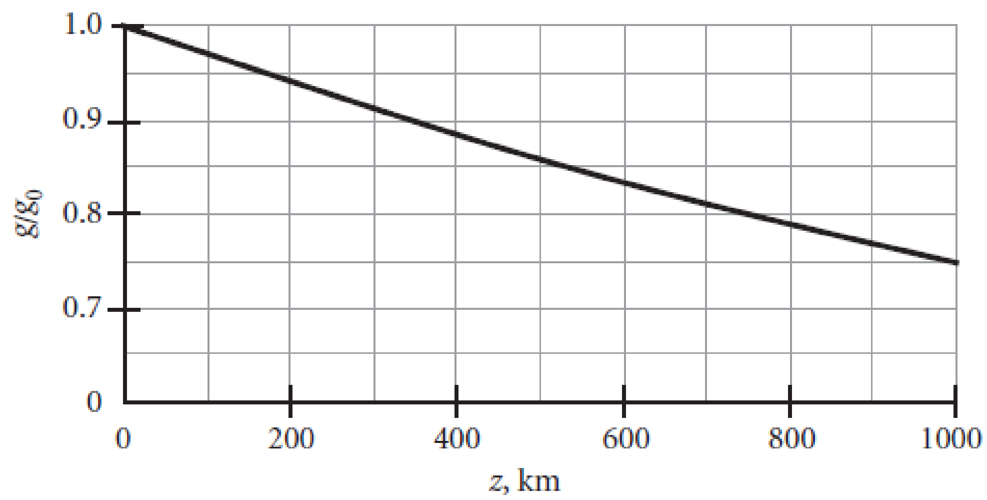
At sea level : $g = \frac{6.67259\times10^{-11} . 5.97219\times10^{24}}{(6378\text{x}10^3)^2} = 9.8\ \text{m}s^{-2}$

At ISS altitude: $g = \frac{6.67259\times10^{-11} . 5.97219\times10^{24}}{(6778\text{x}10^3)^2} = 8.67\ \text{m}s^{-2}$

400km

Using magnitudes or the "Euclidean Norm" of the vector, as you probably have done before.
The mass 2's cancel out.
Acceleration 'a' here, is the acceleration due to gravity, otherwise known as 'g'.

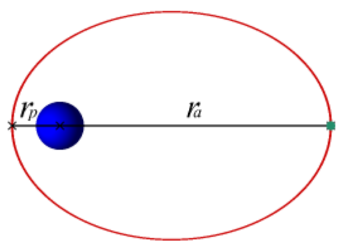
Variation of g

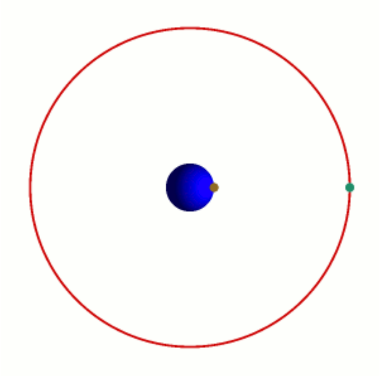


Airliners cruise at 10km, at this altitude g will only be 0.3% less than sea level, so we ignore the variation of g with altitude. But we cannot in space as the drop becomes more significant. However, you can see that it is not 'microgravity' or 'zero-g'.

Reminder: Conservation of Energy

- The total energy in a closed orbit is constant.
- However, the type of energy (form) can change.
- An elliptical orbit has a continual exchange between kinetic and potential energy but the sum does not change.





University of BRISTOL

L.Berthoud

DEPARTMENT OF aerospace engineering

If we are at periapsis, which energy is at a maximum and which is at a minimum? How about at apoapsis? How about for a circular orbit?

Orbital Energy

We know that the total Energy 'E' of a system does not change, we can write this:

$$E = KE + PE \qquad \text{(1-12)}$$

For space, this is often written:

$$E = K + U \qquad \text{(1-13)}$$

U is defined as 0 at infinity, so gravitational energy is always negative

L.Berthoud

Gravitational potential energy increases as distance from Earth's centre increases
Kinetic energy decreases as distance from Earth's centre increases
So, why do spacecraft in higher orbits need more energy? Let' see…

Orbital Energy

From centripetal force and N2, we have:

$$F = ma = \frac{GMm}{r^2} = \frac{mv^2}{r} \tag{1-14}$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\frac{GMm}{r} \tag{1-15}$$

So we have:

$$E = K + U$$

$$\boxed{E = \frac{1}{2}mv^2 - \frac{GMm}{r}} = \frac{GMm}{2r} - \frac{GMm}{r} \tag{1-16}$$

$$E = -\frac{GMm}{2r} \tag{1-17}$$

If you remember your circular motion equation for centripetal force it looks like (1-14) above. We can rearrange this so that we have an expression for the Kinetic energy of a circular orbit. At the end we can see that the total energy E is larger for larger radii (as it is negative).

Questions

If the Moon-Earth distance were to shrink, what would happen to the Moon’s kinetic energy?

The Moon’s KE increases.

Summary

- Orbital mechanics allows us to work out where the spacecraft is and to perform different missions.
- Brahe, Kepler and Newton developed the basics of orbital mechanics.
- Kepler's laws:
  - 1. Ellipses,
  - 2. $\frac{dA}{dt} = \text{constant}$
  - 3. $\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3}$
- Ellipse geometry

$r_p = a(1 - e)$
$r_a = a(1 + e)$
$a = (r_a + r_p)/2$
$e = (r_a - r_p)/(r_a + r_p)$

- Newton motion and gravitation: $\boldsymbol{F} = -\frac{GM_1 m_2}{r^2}\hat{\boldsymbol{r}}$

Test yourself!

1. Which scientist is credited with the collection of the data necessary to support the elliptical motion of planets?
2. Define the term ‘apohelion’
3. Explain why a solar day is longer than a sidereal day
4. If the apogee of a satellite’s orbit is 68000km, what is the altitude of the satellite at this point? Assume the radius of the Earth=6378km.
5. A satellite in earth orbit has a semi-major axis of 6,700 km and an eccentricity of 0.01. Calculate the satellite's altitude ‘h’ at both perigee and apogee.