Aeronautics & Mechanics
AENG11301
Lecture 7
Wave Drag & Pitching Moments

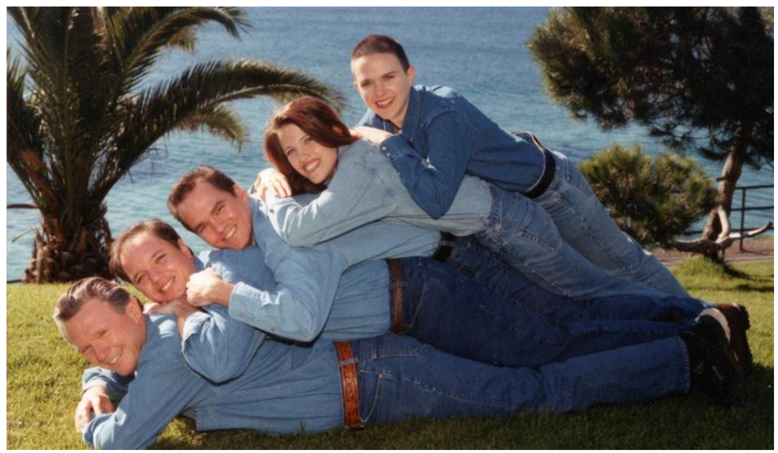
06/2/18

Dr Ben Woods

Department of Aerospace Engineering
University of Bristol

In class test – Friday Feb 23th

- 11am in Biomedical Building E29
- Right after class, so we can all walk over as one big happy family


- In the class before (Tues 20th) we will go through some example problems

Outline for today

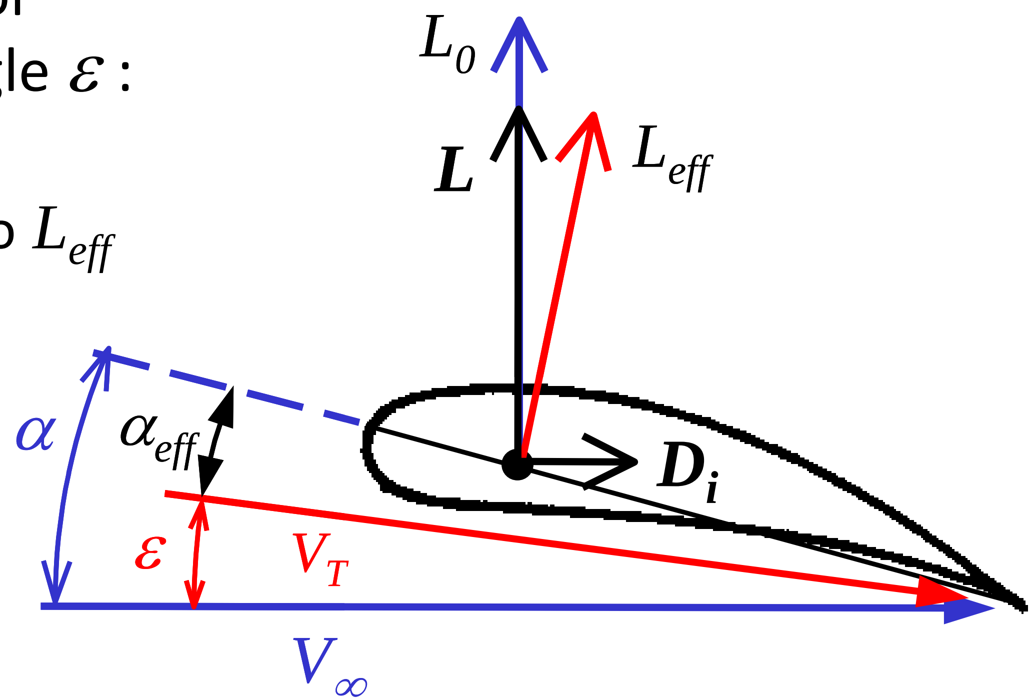
- Lift slope of a finite wing
- Wave drag
- Wing sweep
- Pitching moments

Aims for today

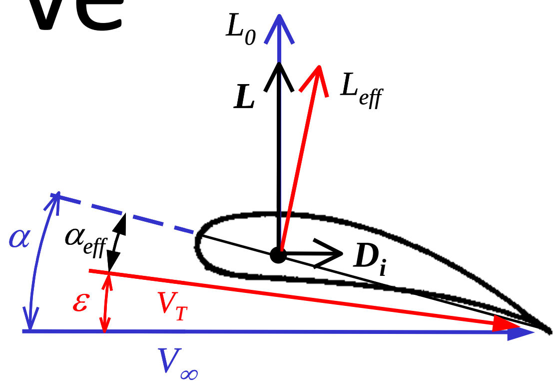
- Be able to state how the lift curve for a finite wing differs from a 2D wing
- Understand the impact of supersonic flow over an aerofoil
- Appreciate why high speed aircraft have swept wings
- Understand why pitching moments are defined around the ¼ chord point

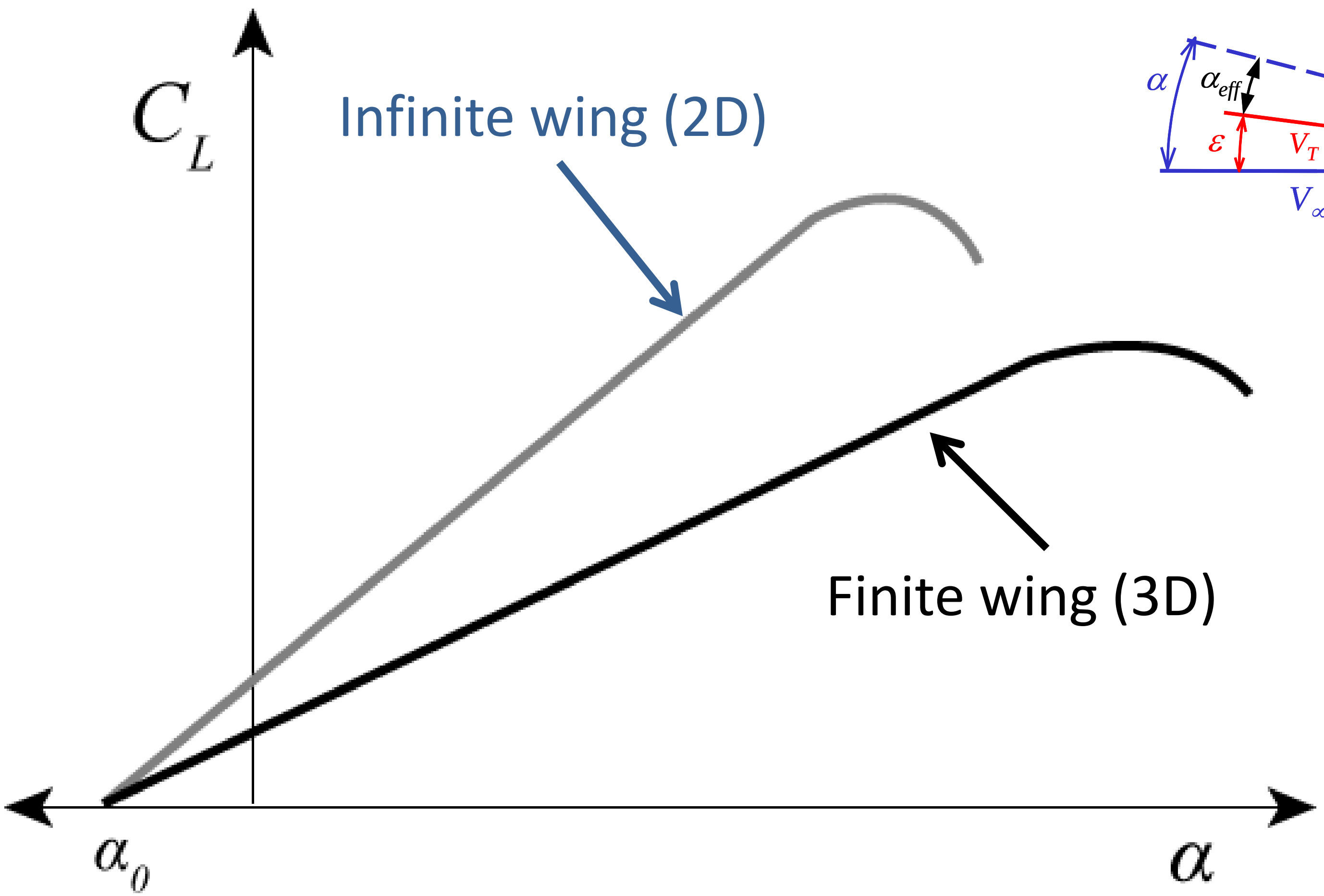
Recap: Induced Drag

- In 2D flow we only have profile drag
- In 3D flow an additional drag component
  - **lift-induced drag**
- tip vortices induce **downwash** , a downward flow component (w) over the wing span
- rotates local velocity vector downwards by a small angle ε :
  a) reduces incidence to α_{eff} and hence *reduces* lift to L_{eff}
  b) rotates lift vector L_{eff} rearwards by angle ε

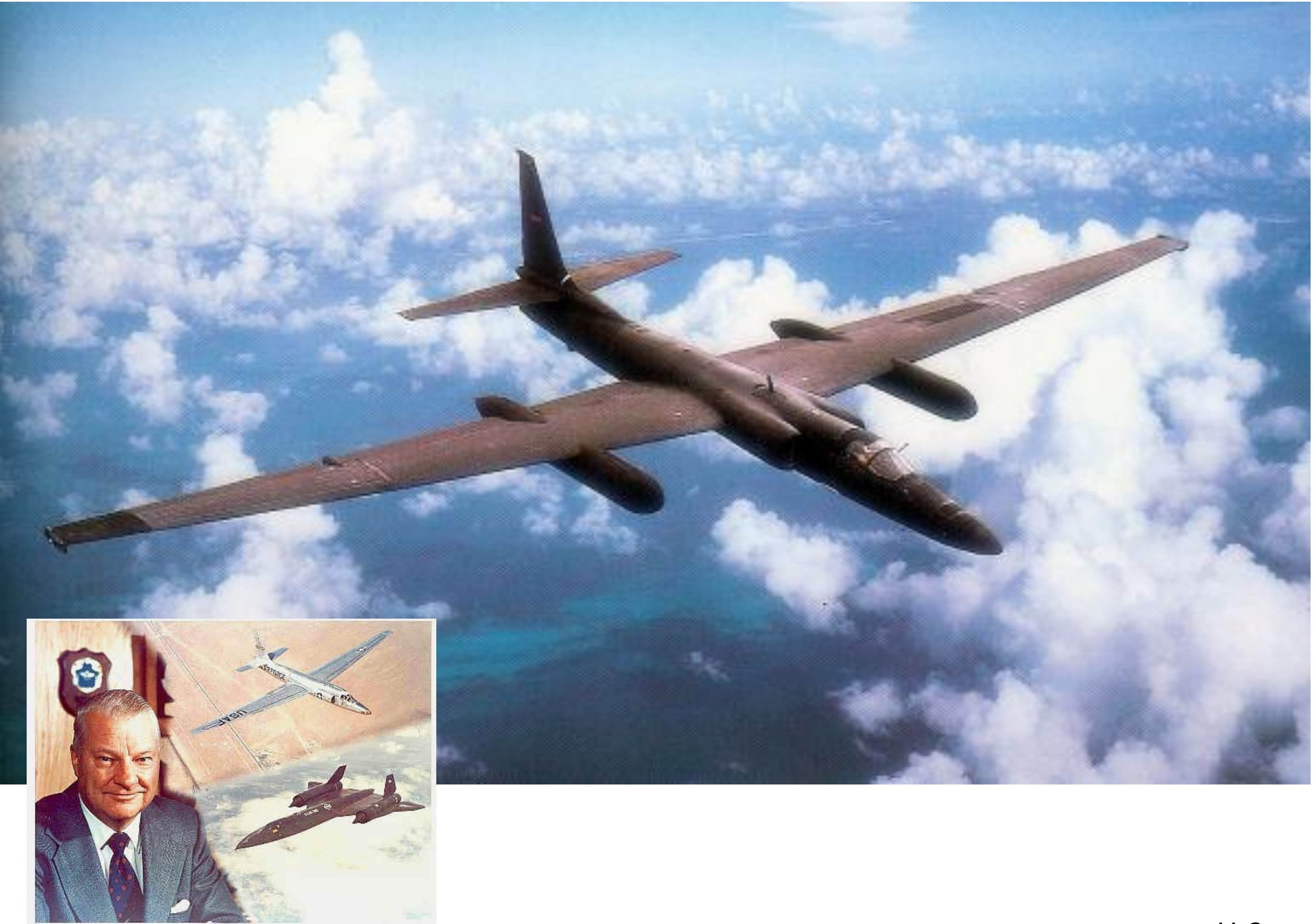


Finite wing lift curve





The effect of a finite wing is to reduce the lift curve slope

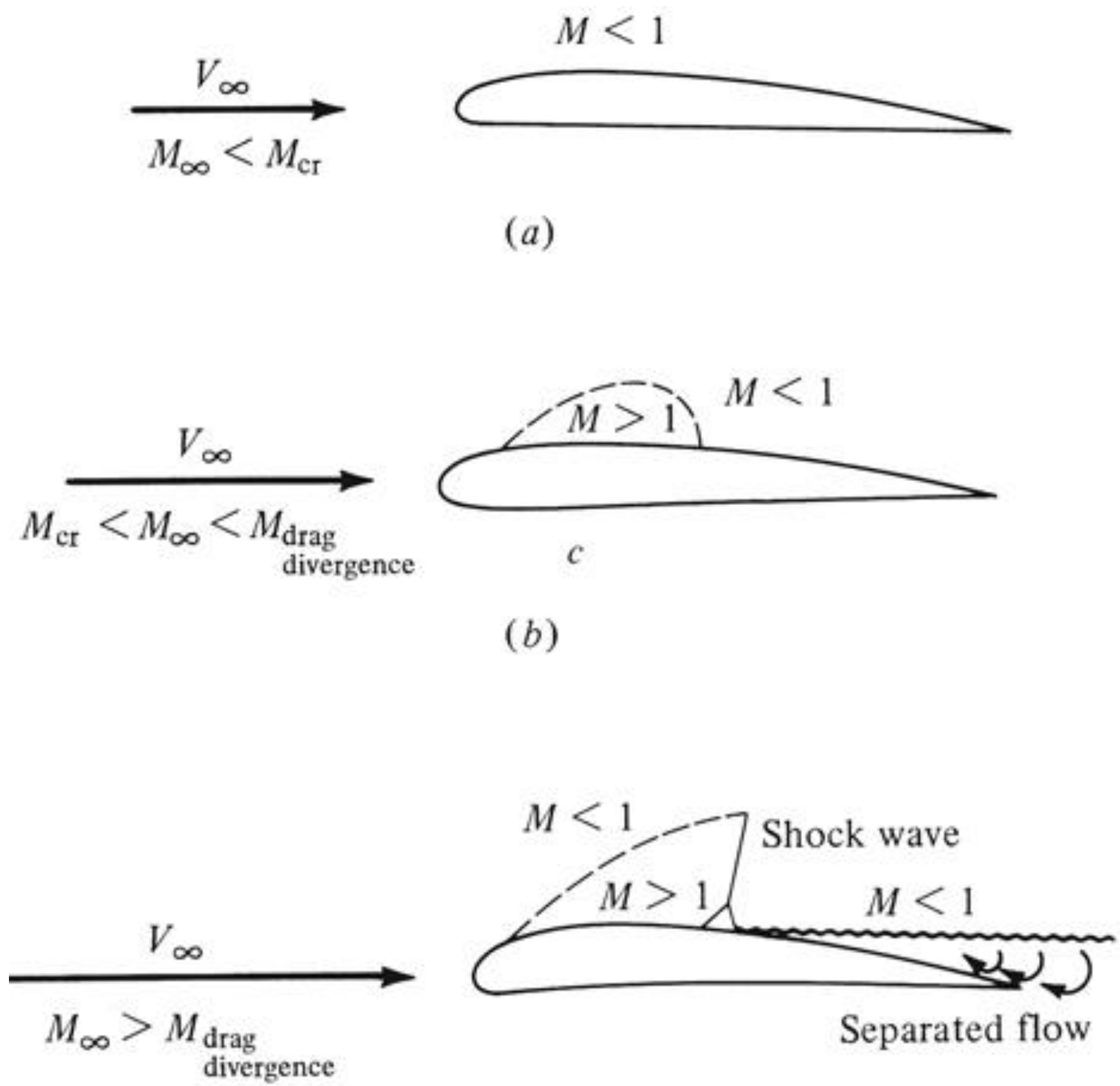


U-2

Kelly Johnson – the coolest aerospace engineer ever? → Yes

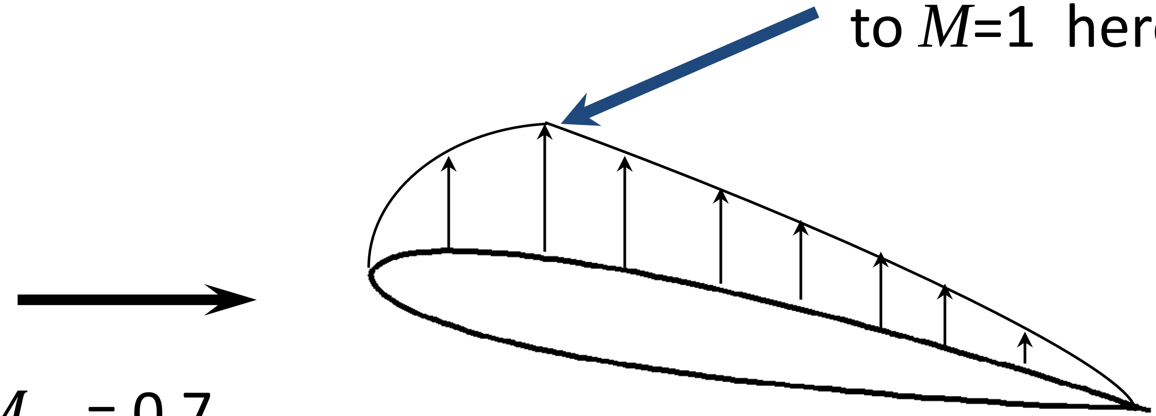
Critical Mach number

The **critical Mach number** (M_{cr}) is the lowest free stream Mach number (M_∞)at which the airflow over any part of the aerofoil reaches the speed of sound.



Aerofoil at Critical Mach

Air accelerates
to M=1 here

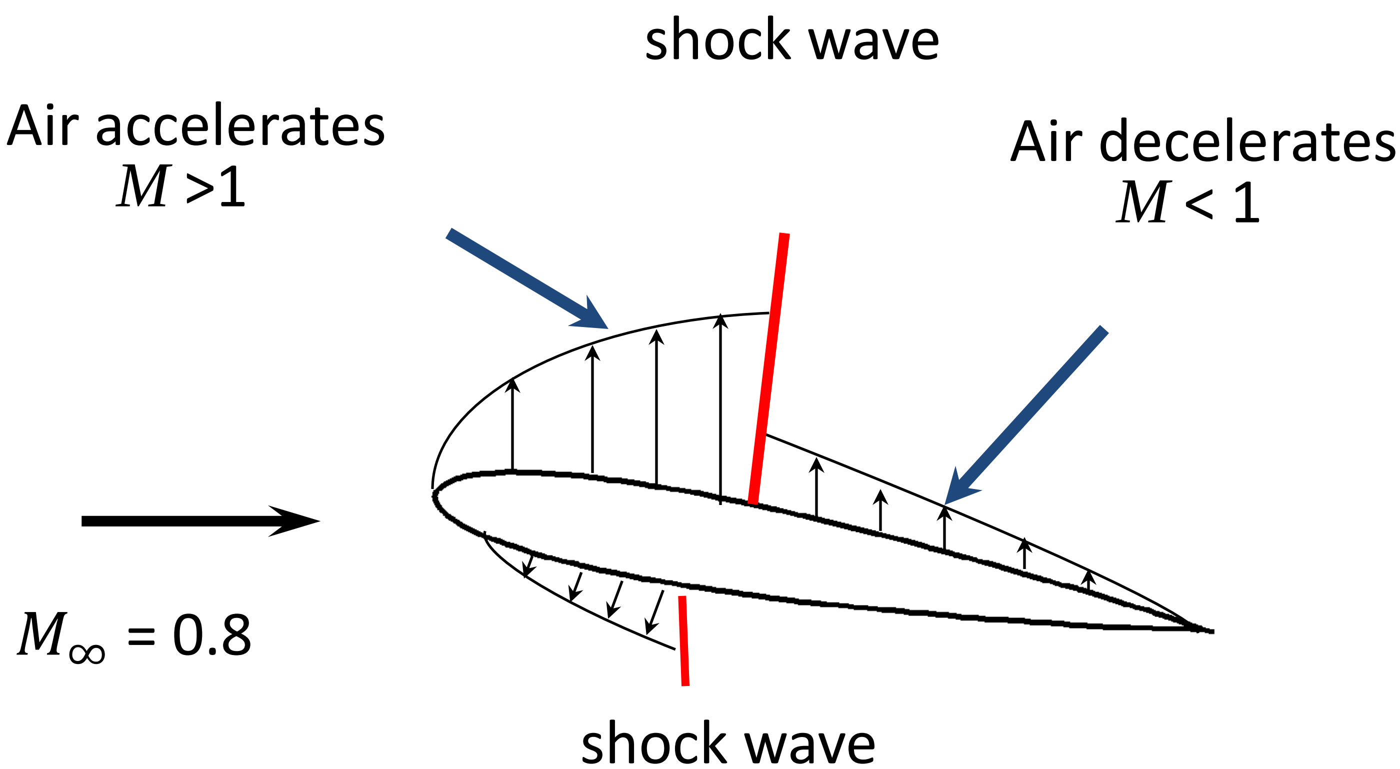


M_∞ = 0.7

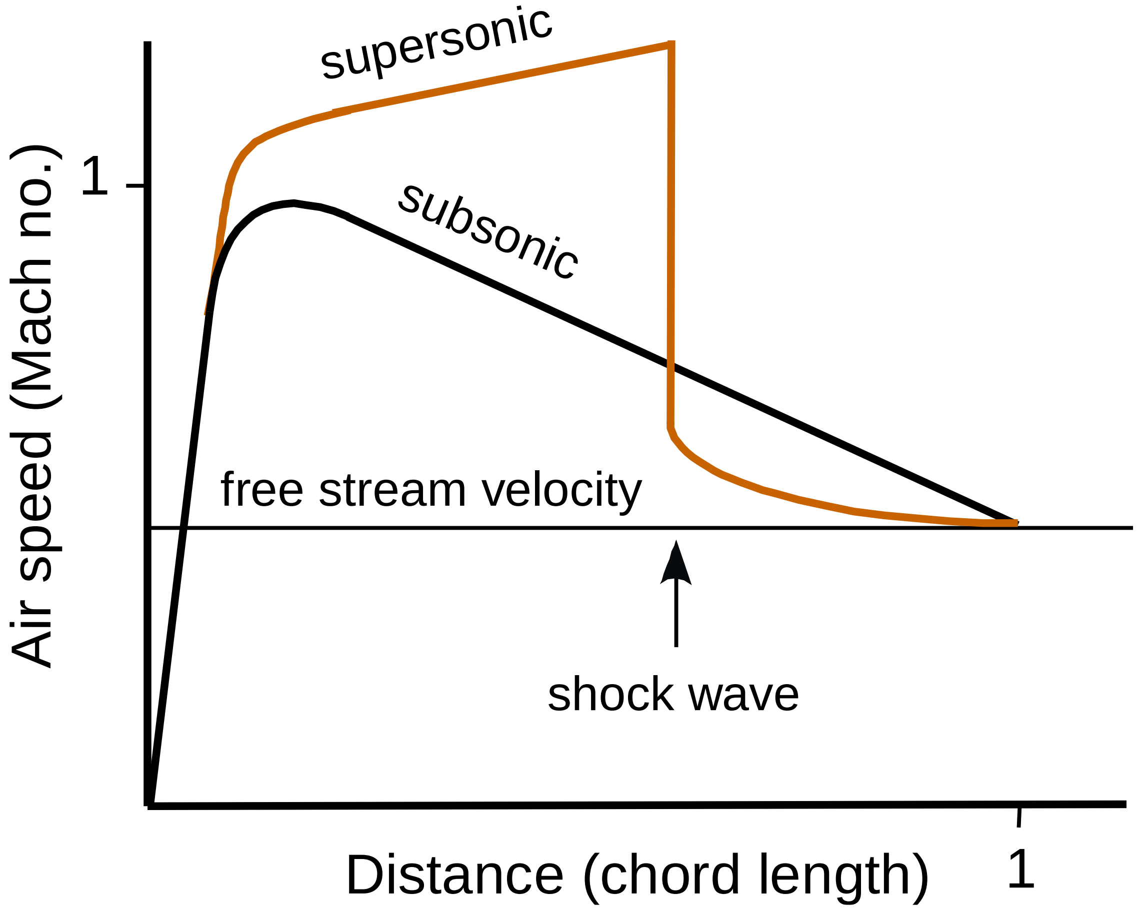
Air is subsonic below wing

In this example the critical Mach number is 0.7

Aerofoil above Critical Mach

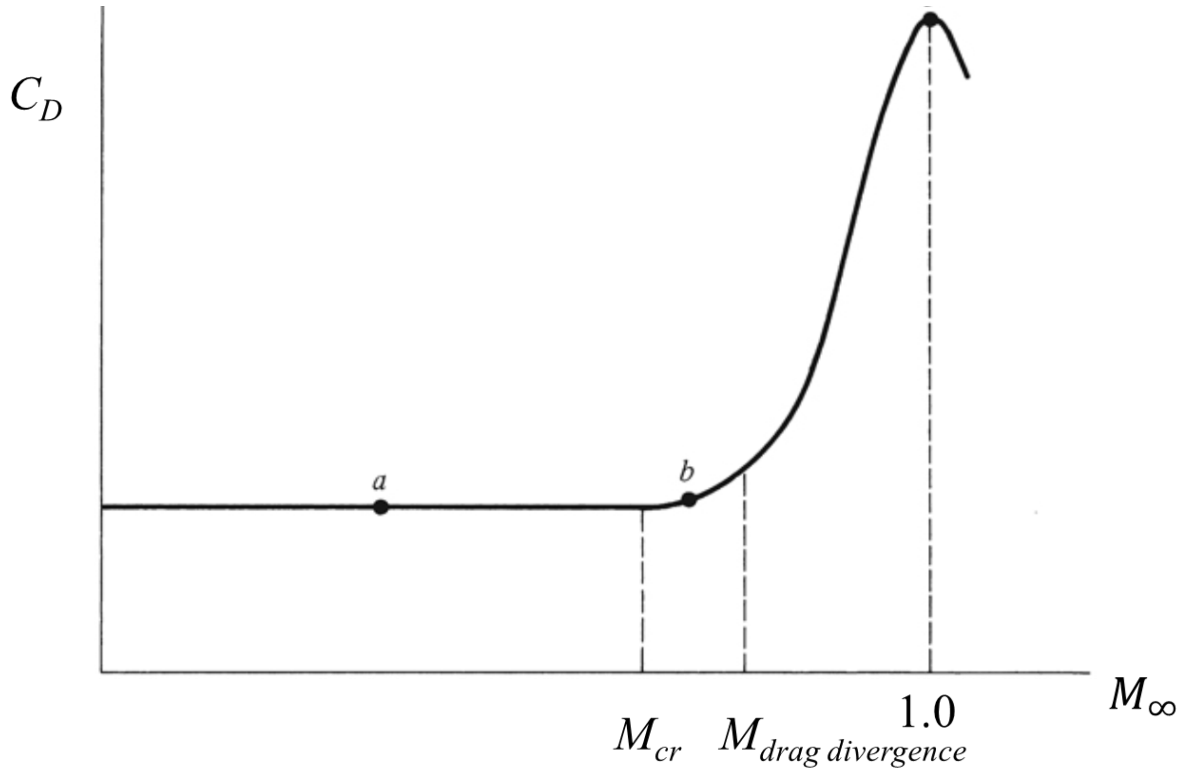


Different velocity distributions



Drag divergence

Drag divergence Mach number (M_{dd}) is the free-stream Mach number where C_D begins to increase rapidly due to **wave drag** caused by shock waves

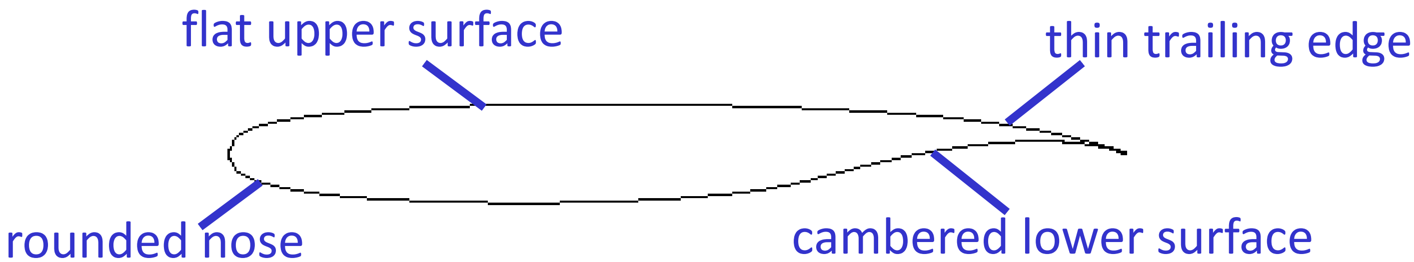


Two common solutions to increasing M_{dd}

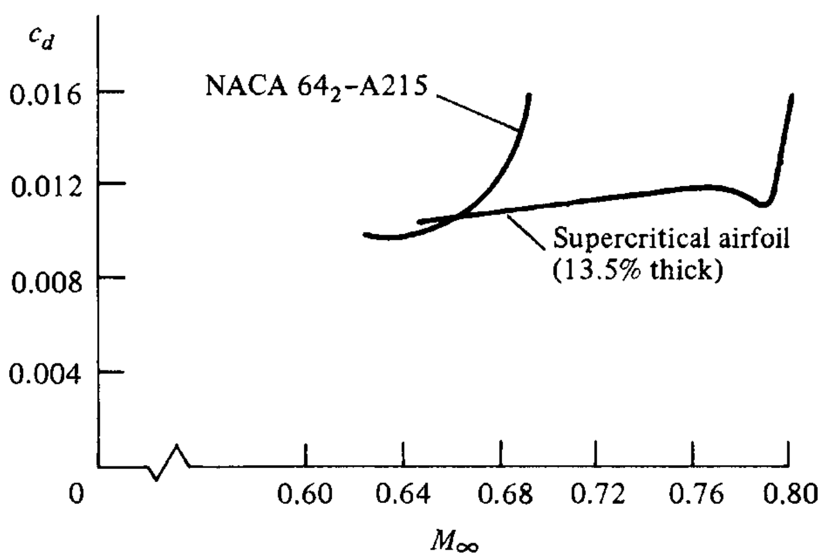
- Aerofoil level = supercritical design
- Wing level = sweep

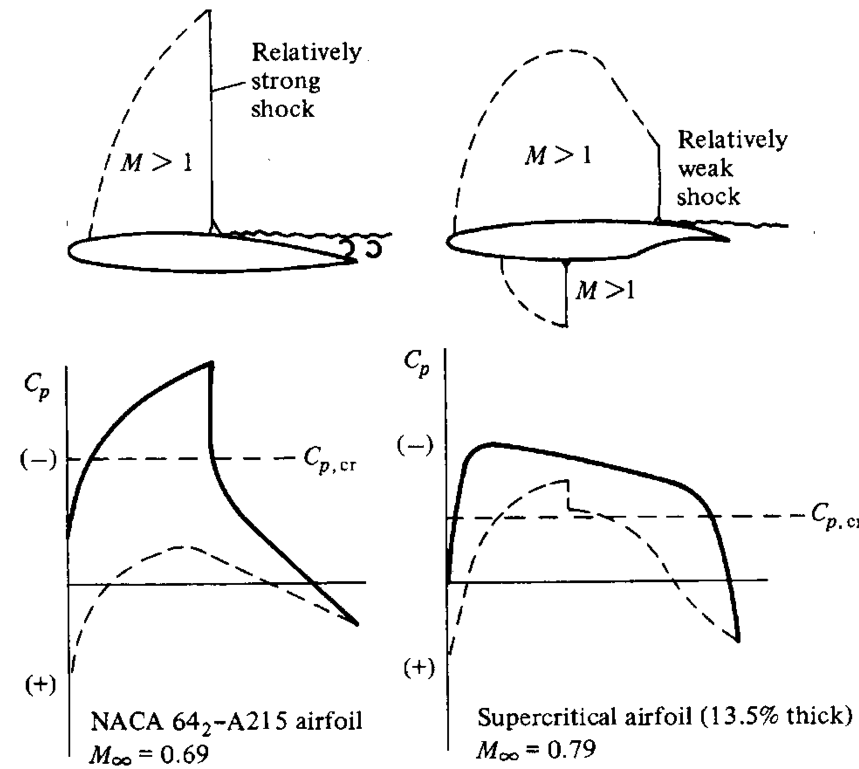
Supercritical Aerofoils

- significant lower surface curvature

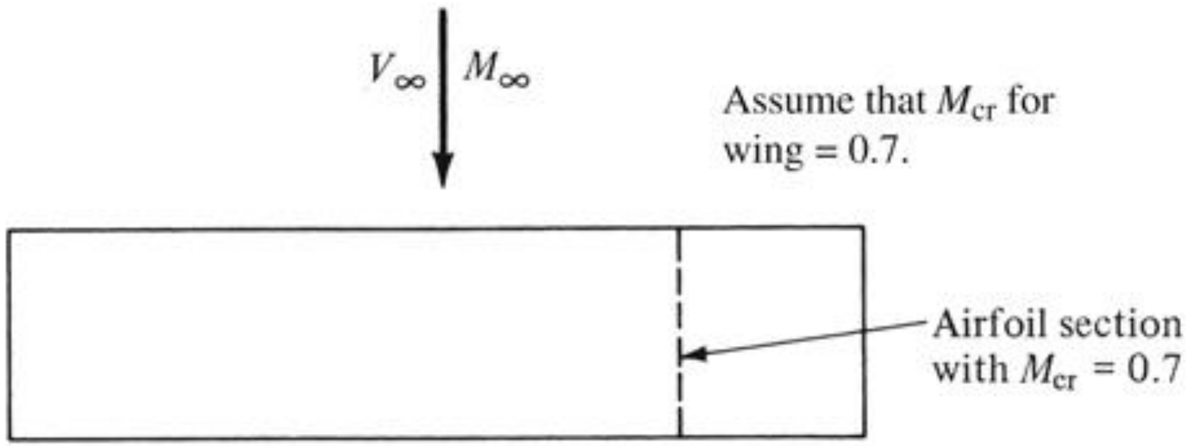


- rather similar to modern laminar flow sections
  - good structural shape!



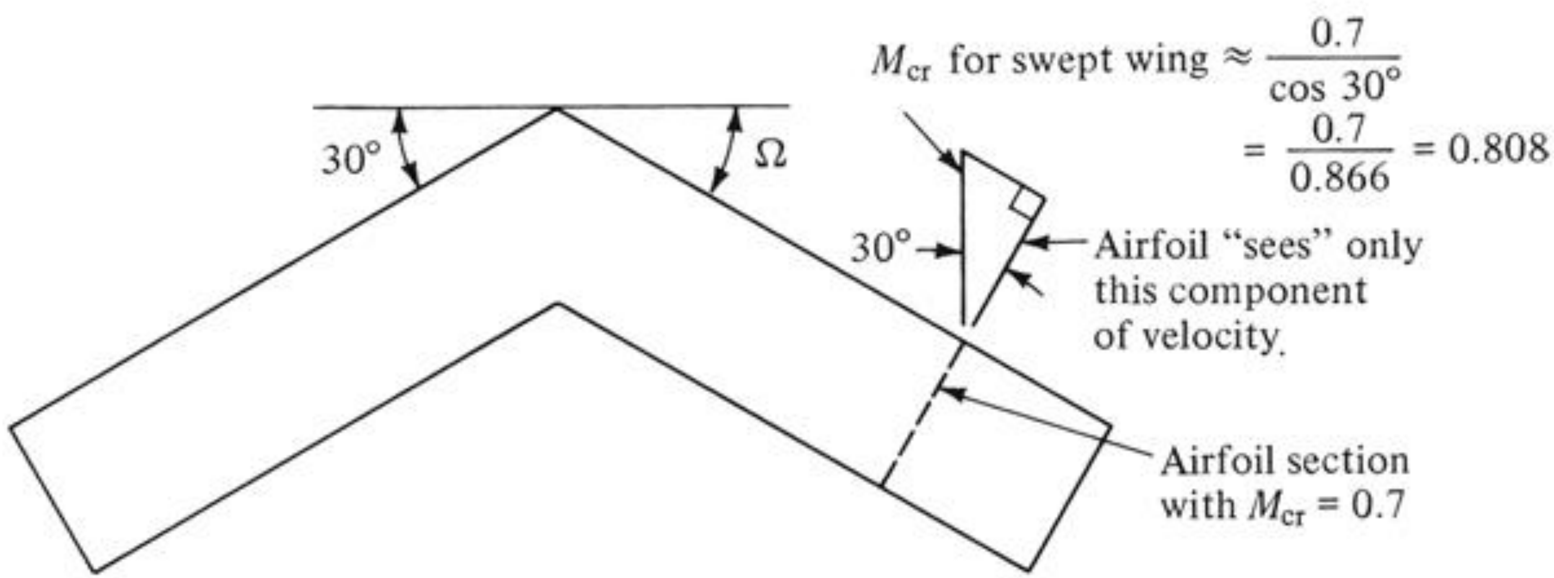


Swept wings – delay M_{dd}

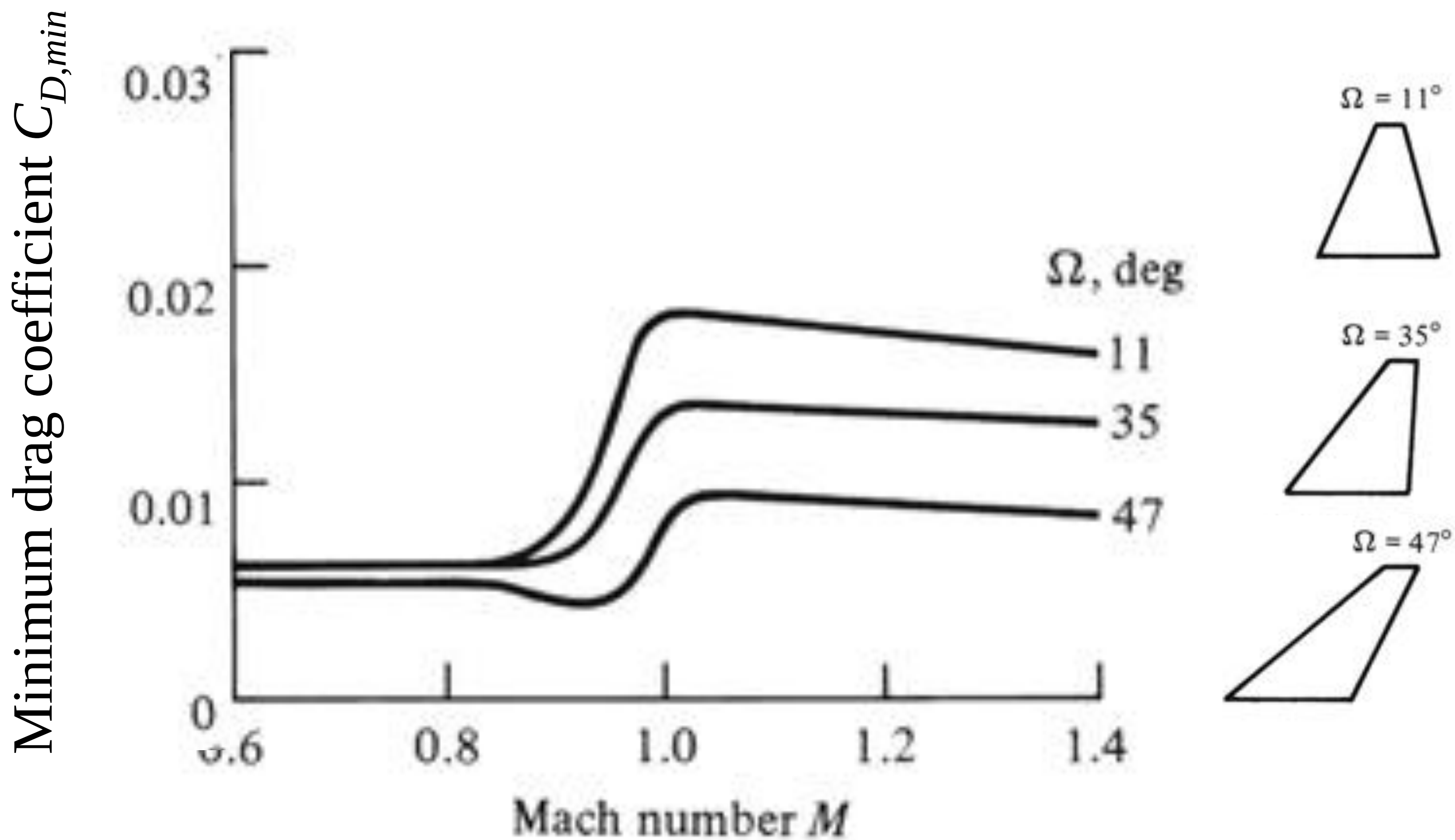


Now sweep the same wing by 30°.

(*a*)



Impact of sweep on wing drag



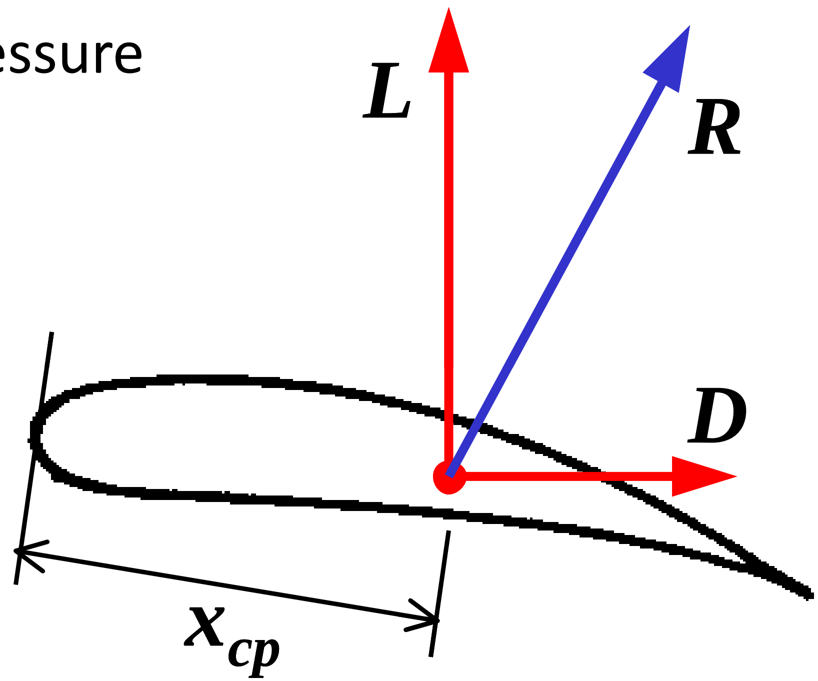
Pitching Moment

Balancing the aircraft

- F-14

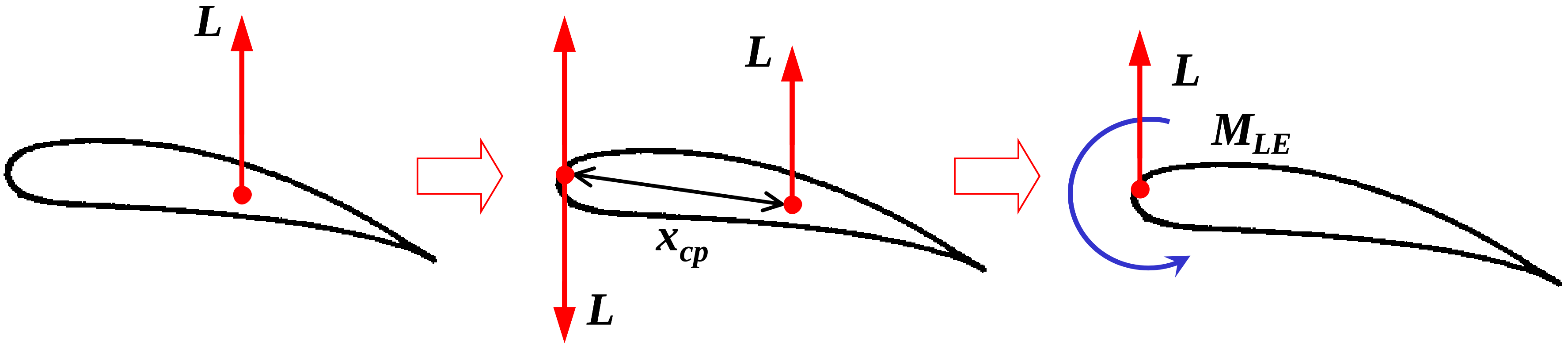
Centre of Pressure (1)

- lift L and drag D can be combined into a **single resultant aerodynamic force** R
- line of action of R crosses the chord-line at the **centre of pressure** x_{cp}
  - there is *<u>no pitching moment</u>* about this point!
- unfortunately, the centre of pressure is not fixed
  - changes with C_L and hence with incidence
  - rate of variation depends on camber



Moment About a Fixed Axis

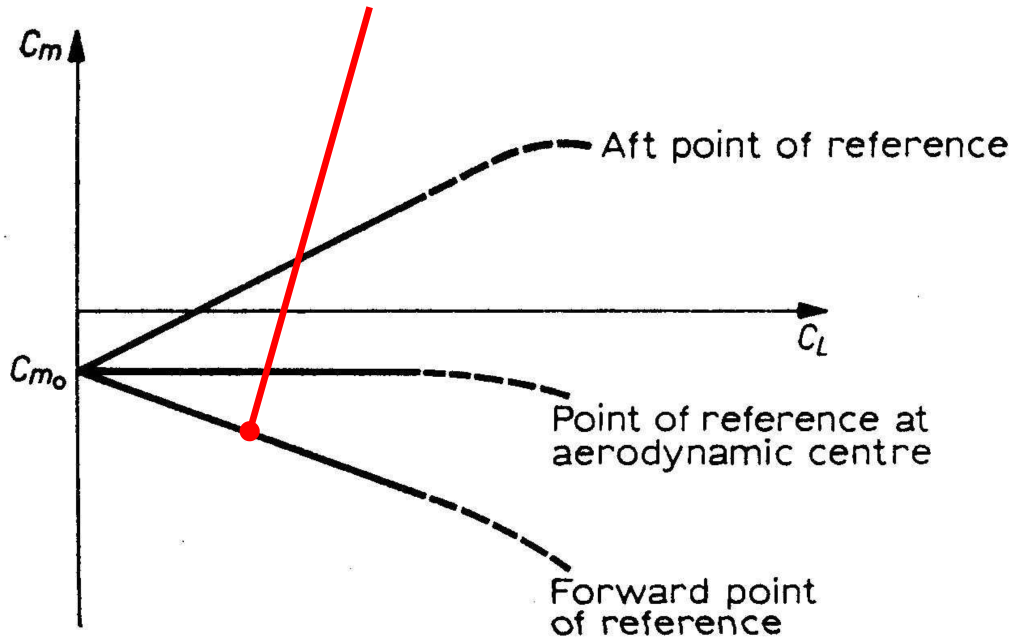
- movement of cp makes balance & stability calculations rather awkward
  - would rather have a fixed reference point … but where?
- one option is the leading-edge
  - resolve lift at CP into lift and pitching moment at LE
  - ignore drag as << lift (& moment arm is small)



$$M_{LE} = -x_{cp} L \quad \Rightarrow \quad C_{M_{LE}} = -\left(\frac{x_{cp}}{c}\right) C_L$$

Effect of Moment Axis Position

- but now $C_{M_{LE}}$ varies (linearly) with lift / incidence



$$C_{M_{LE}} = \underbrace{C_{M_0}}_{\text{zero-lift pitching moment}} + \underbrace{b}_{dC_{M_{LE}}/dC_L} C_L$$

– not really an improvement!
– what about a different moment axis position?

- general equation for pitching moment at fixed axis x

$$C_{Mx} = -\left(\frac{x_{cp}}{c} - \frac{x}{c}\right) C_L = C_{M_{LE}} + \left(\frac{x}{c}\right) C_L$$

The Aerodynamic Centre

- slope of general pitching moment curve at x

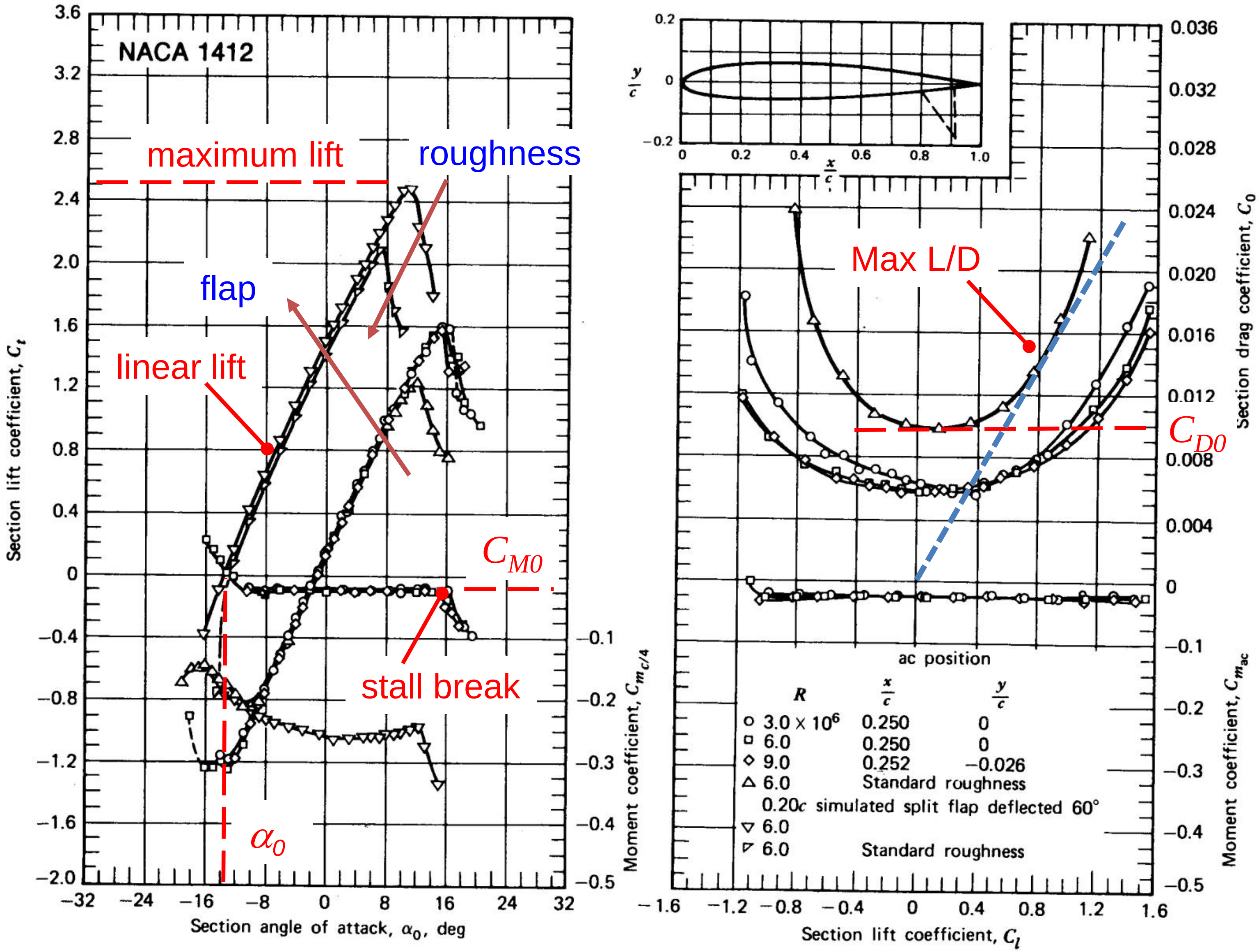
$$\frac{dC_{Mx}}{dC_L} = \frac{dC_{M_{LE}}}{dC_L} + \left(\frac{x}{c}\right)$$

- chose x so that pitching moment is **constant**
  - i.e. $\dfrac{dC_{Mx}}{dC_L} = 0 \quad \Rightarrow \quad \left(\dfrac{x_{ac}}{c}\right) = -\dfrac{dC_{M_{LE}}}{dC_L} = -b$

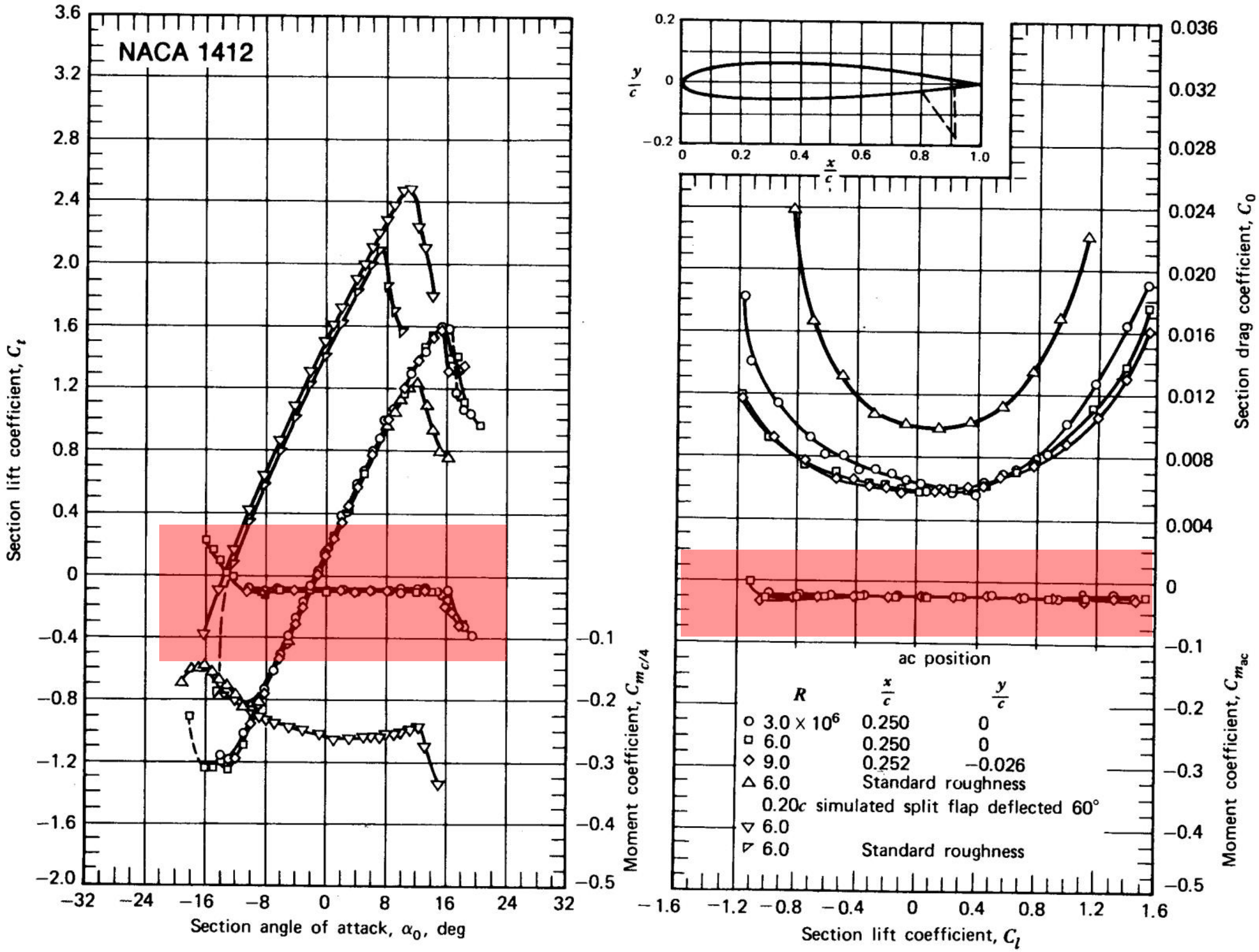
- this is the **aerodynamic centre** (x_0 or x_{ac})
  - for 2D thin aerofoils the aerodynamic centre is at the quarter-chord point

$$\Rightarrow \quad \boxed{C_{M_{ac}} = C_{M_0}\,, \quad \frac{x_{ac}}{c} = 0.25}$$

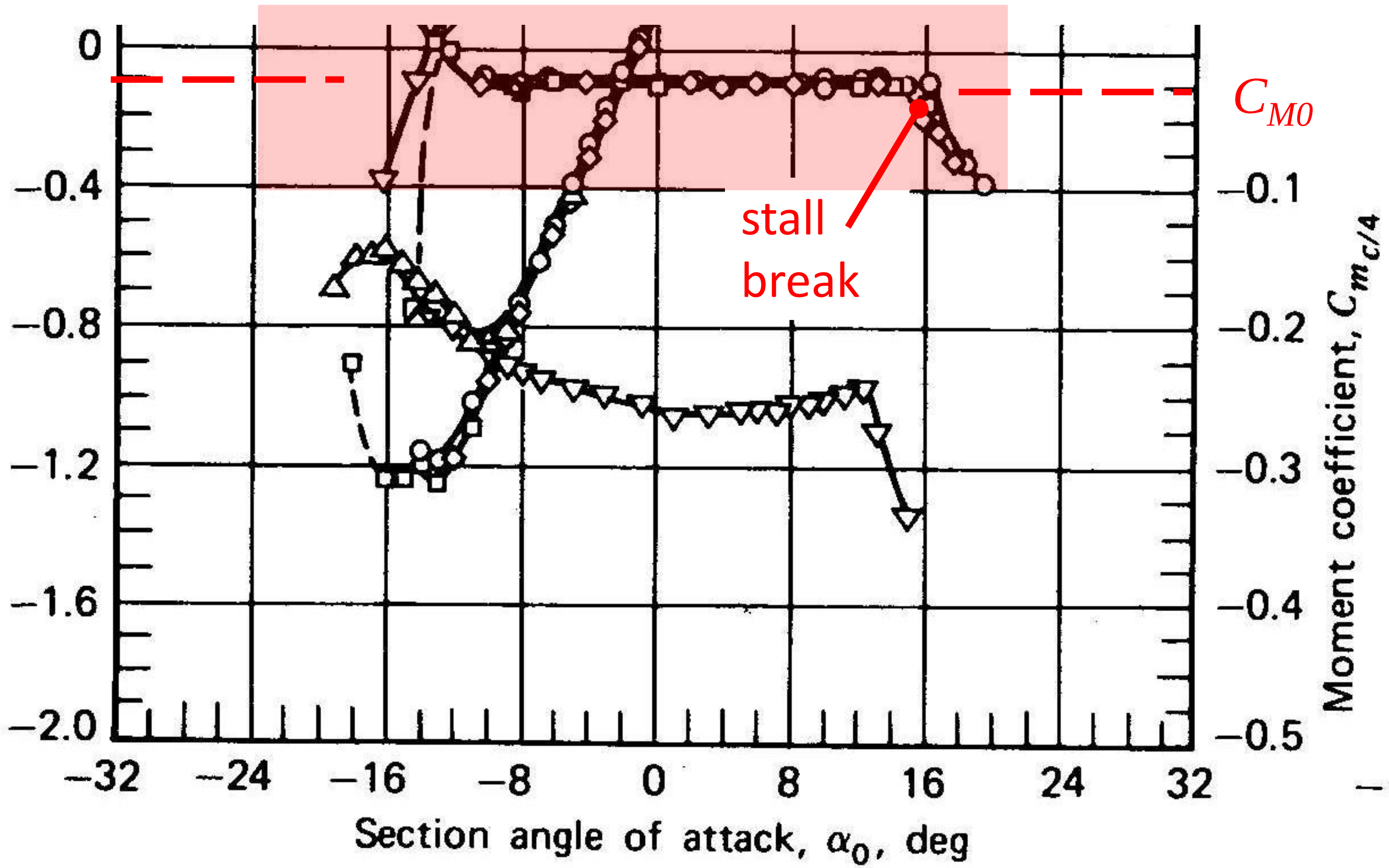
Typical 2D Aerofoil Characteristics



Typical 2D Aerofoil Characteristics



Typical 2D Aerofoil c/4 Moment



Summary

- The effect of a finite wing is to reduce the lift curve slope
- **Drag divergence Mach number** is the free-stream Mach number where C_D begins to increase rapidly due to wave drag
- High speed aircraft have swept wings to reduce wave drag
- The **aerodynamic centre** is where the pitching moment is constant and does not vary with angle of attack
- For 2D thin aerofoils the aerodynamic centre is at the quarter-chord point

$$\boxed{C_{M_{ac}} = C_{M_0}\ , \quad \frac{x_{ac}}{c} = 0.25}$$

Follow-up materials

To help with exam:

- Introduction to Flight
  - 5.15 change in lift slope
  - 5.10 drag divergence
  - 5.16 swept wings
  - 7.3 moments