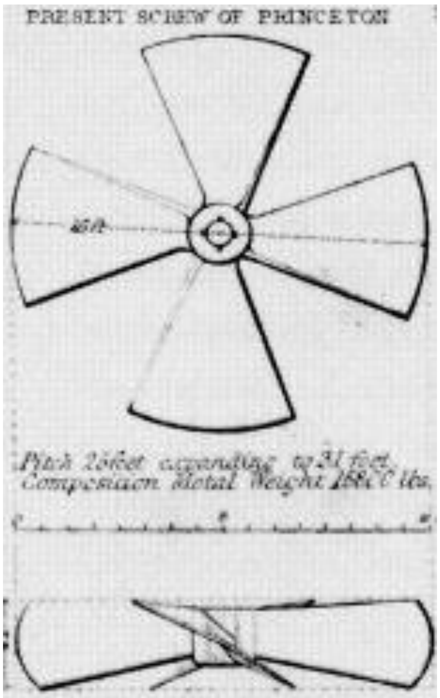
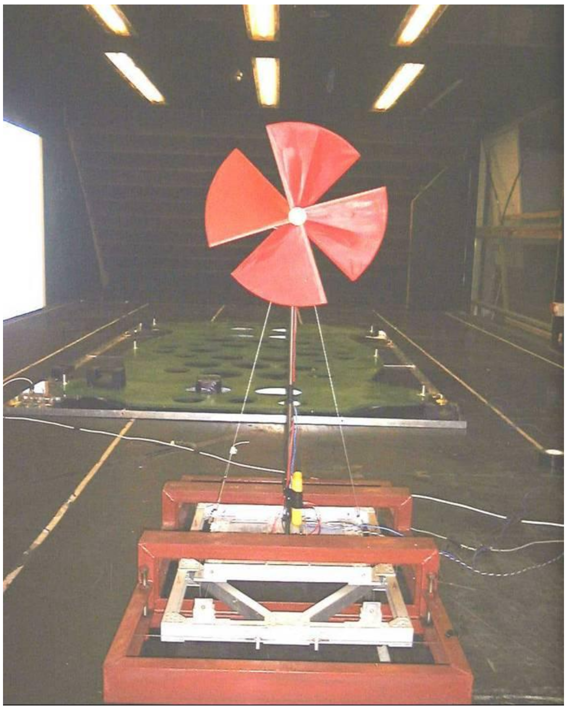
Introduction to Propellers (Air Screws)

1. A Bit of History



▲ The first propellers operated in water, not air. This is the "prop" of the first propeller driven steam ship – **Brunel's "S.S.Great Britain"**

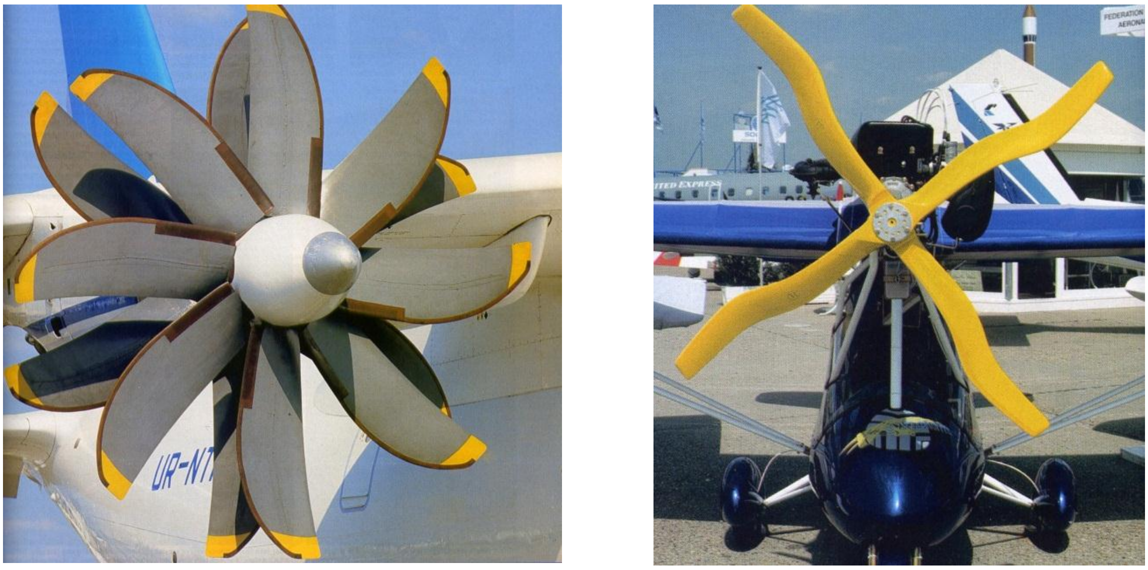


▲ The propeller of **Stringfellow's "Flyer"** model aircraft of 1848 is very similar in form, as can be seen from this replica on test in the Bristol University wind tunnel in 2003.

"Wright Flyer" (circa 1903)

"Supermarine S6b" (circa 1931)


Different types of propellers


Some new thoughts of propellers

2. Introduction - The Fundamentals

It is conventional to assess the performance of a propeller as the ratio of the rate of useful work (output) to the power supplied (input)

$$\eta_p = \frac{\boldsymbol{TV}}{\boldsymbol{P}} \quad \textit{(which we can call propeller efficiency)}$$

In a simple, perfectly trimmed aircraft application, the thrust (***T***) will be opposing the aircraft drag force (***D***) and this is the “useful work” being done. However, even for the ideal propeller (with no losses) the efficiency can never be unity as in producing thrust, some energy (in the form of kinetic energy) is “wasted” by increasing the momentum of the air.

So there are two mechanisms at work;

1. the work done by the propeller on moving the aircraft forward.
2. the work done by the propeller on moving the air backwards.

Generally, for the propulsion engineer, the work done by the propeller in moving the aircraft forward should be maximised and the work done on the air should be minimised.

It is important to be clear of these objectives when designing a propeller.

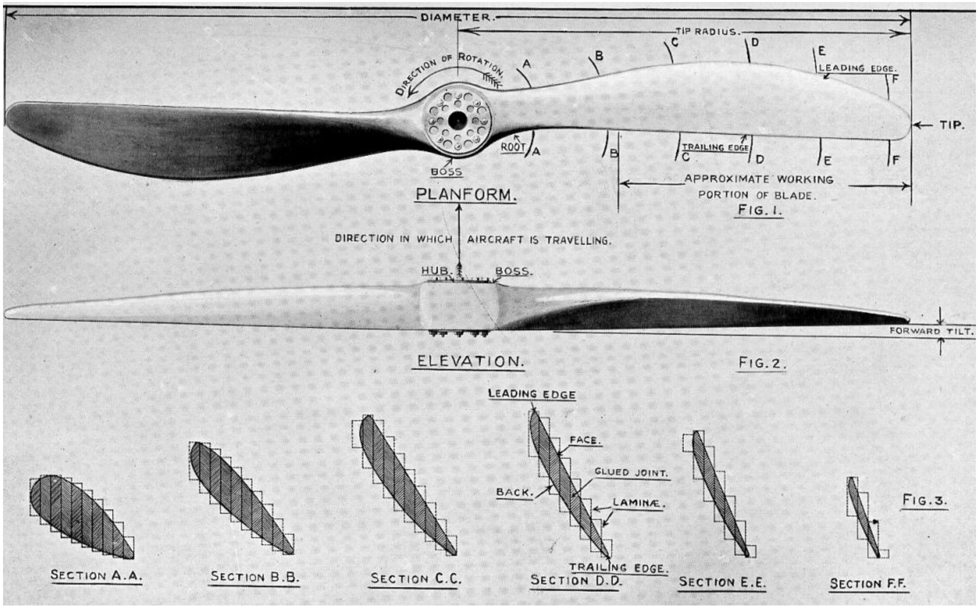
Before the propeller can be designed, the methods of energy exchange from the prime mover to the thrust generator must be fully understood and analysed. There are a number of ways to do this but generally one or more of the methods listed below are adopted:

Momentum Analysis - Actuator Disc Theory

The propeller can be considered as an actuator disc with an infinite number of blades operating in an ideal fluid. The flow is considered non-rotational and therefore swirl can be neglected. The actuator disc has an area (***A***) and it can support a pressure differential (P`). The basic theory is based on a consideration of the mean axial velocity in the slipstream and determines the thrust and torque of the propeller from the rate of increase of momentum of the fluid. The theory determines an upper limit to the efficiency of any propeller but gives no indication of the form which must be given to the propeller to achieve this result.

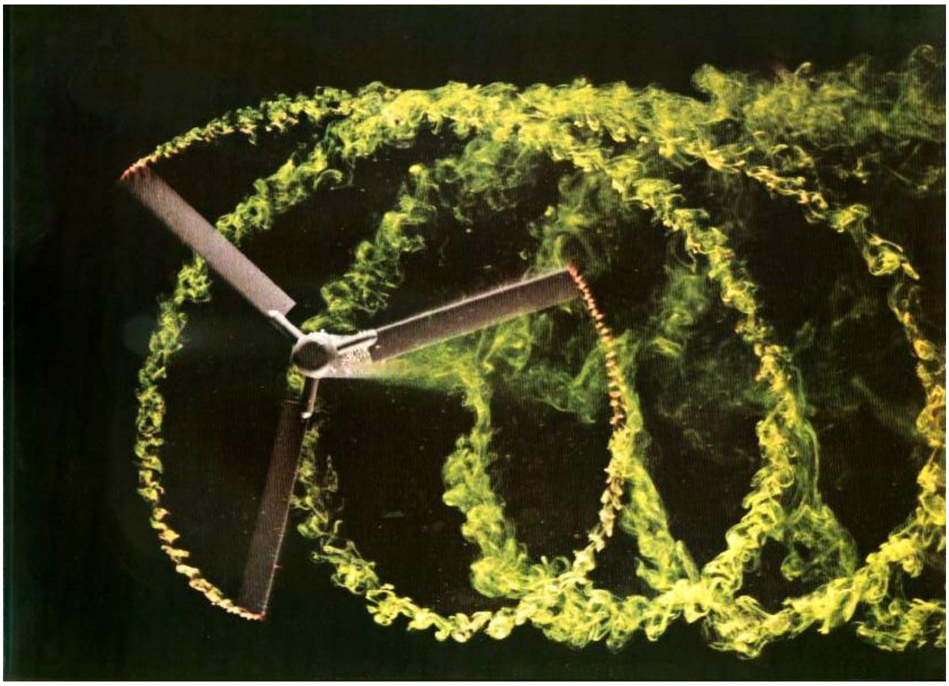
Blade Element Analysis (often referred to as strip theory)

The propeller blade is considered to be made up of a number of chordwise strips extending from the blade root to the blade tip. The theory is based upon the assumptions that the aerodynamic force acting on a blade element can be estimated from consideration of the element as an aerofoil acting in 2-D flow. The components of the lift and drag forces are resolved into the thrust and torque planes before summation along the blade span. In this way the blade elements can be optimised for a given design point resulting in a particular blade form.



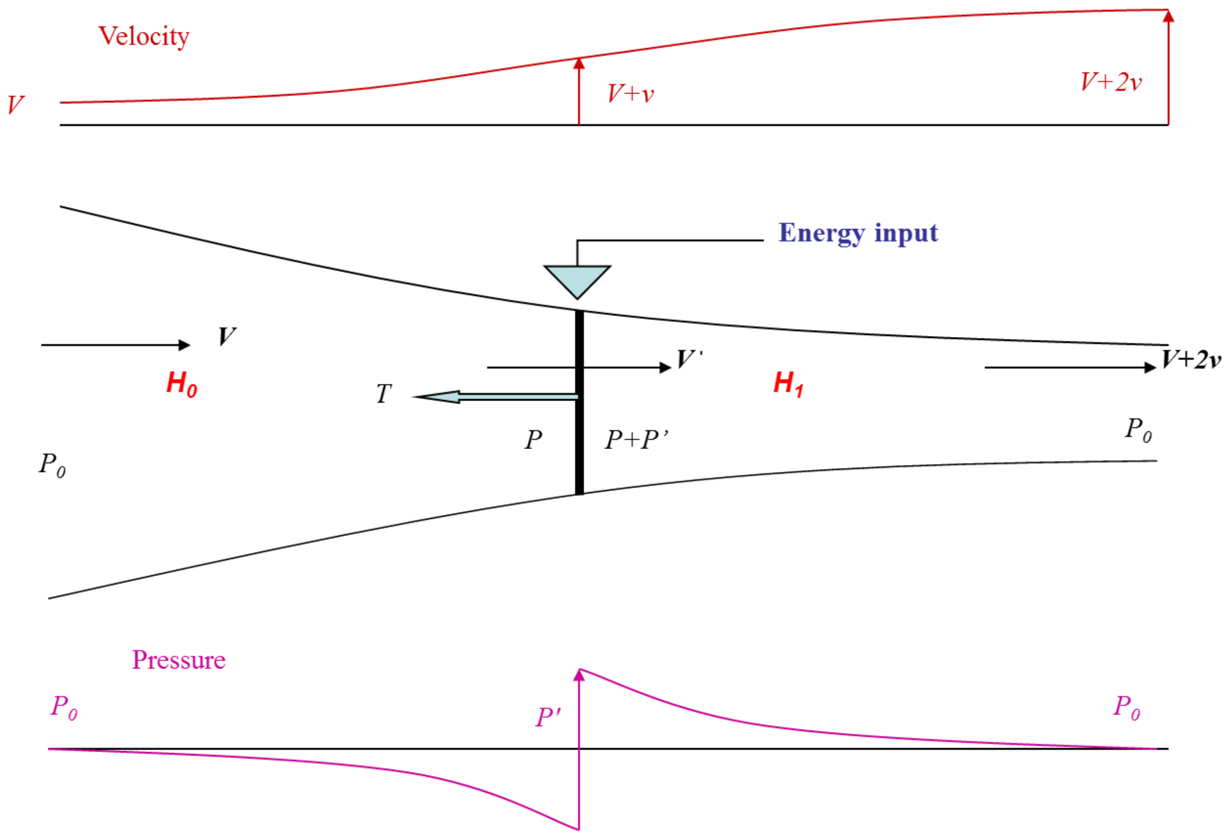
Vortex Theory

This method considers a vortex line bound to the blade and running along it from root to tip. This bound vortex of strength ***K*** cannot terminate abruptly so must be continued as a free vortex emanating from the root and the tip. The former will combine with that from all other blades and will be a new bound vortex running along the propeller axis. The free vortex from the tip will produce a flow field which constitutes the slipstream of the propeller and the motion of the fluid in the slipstream can be calculated as the induced velocity. This is the simplest case which may be developed to include shed vortices at any radial station, thus modifying the induced flow field. A conventional blade element approach can then be used to, as described above.



3. Momentum Analysis (detailed)

Mechanical energy (in the form of rotating blades) is used to accelerate (**a**) a mass (m) of air. Newton's law (every action has a reaction), states **F = ma**, where **F**, is the propeller thrust (**T**).



Appling Bernoulli's equation:

$$H_0 = P_0 + \tfrac{1}{2}\rho V^2 = P + \tfrac{1}{2}\rho (V+v)^2$$
$$H_1 = P_0 + \tfrac{1}{2}\rho (V+v_1)^2 = P + P' + \tfrac{1}{2}\rho (V+v)^2$$

Thrust=force due to pressure differential (P`) exerted at disc area (A) = change of axial momentum per unit time:

$$\frac{T}{A} = \rho (V+v)v_1 = P' = \rho v_1 (V + \frac{v_1}{2})$$

Therefore

$$\frac{v_1}{2} = v$$
$$T = 2\rho A (V+v)v$$

Since we have assumed no losses at the actuator disc, then the energy input (generating the pressure rise at the actuator disc) must be equal to the rise in kinetic energy in the fully developed wake.

Kinetic Energy rise in the wake is $\Delta KE = \frac{1}{2}\rho A(V + v)((V + 2v)^2 - V^2)$

$$= 2\rho A(V + v)v(V + v)$$

$$= T(V + v)$$

If P is power input to the actuator disc, $\eta = \dfrac{T(V + v)}{P}$ and can never in practise be unity as it neglects all losses associated with real flows, the most significant of which is the viscous drag of the rotor blades.

For the propeller when $V \neq 0$, then $\eta_p = \dfrac{TV}{P}$, as seen before

and since the ideal power $P = T(V + v)$, then

1 $$\eta_p = \frac{TV}{T(V + v)} = \frac{1}{1 + a} \qquad \text{where } a = \frac{v}{V}$$

This suggests that maximum efficiency equal to unity occurs when $a = 0$

Since $T = 2\rho AV^2 a(1 + a)$ then if $a = 0$, then $T = 0$

This is the trivial case and is of no practical use.

The value of a can be found analytically in terms of the thrust coefficient T_c

where $T_c = \dfrac{T}{\rho V^2 D^2}$,

$$= \frac{2\rho(\pi D^2/4)V^2 a(1 + a)}{\rho V^2 D^2} = \frac{\pi a(1 + a)}{2}$$

Taking only the positive root the quadratic equation $\pi a^2 + \pi a - 2T_c = 0$

$$a = \frac{1}{2}\left\{\sqrt{1 + \frac{8T_c}{\pi}} - 1\right\}$$

Static Thrust

The general thrust coefficient $\boldsymbol{T_c} = \dfrac{\boldsymbol{T}}{\rho \boldsymbol{V}^2 \boldsymbol{D}^2}$ has limited value for propellers as it has an infinite value at $\boldsymbol{V} = \boldsymbol{0}$. A more suitable coefficient is $\boldsymbol{C_T} = \dfrac{\boldsymbol{T}}{\rho \boldsymbol{n}^2 \boldsymbol{D}^4}$ which will give a static thrust coefficient based upon rotational speed $\boldsymbol{n}$ (revs/sec) and the propeller diameter $\boldsymbol{D}$. *(We refer to the parameter* $\boldsymbol{nD}$ *as the* ***reference velocity****)*

It is also normal to express the forward speed (V) of the propeller relative to this same rotational parameter $\boldsymbol{nD}$ and this is called the propeller Advance Ratio $\boldsymbol{J}$

$$\text{Thus} \quad \boldsymbol{J} = \frac{\boldsymbol{V}}{\boldsymbol{nD}}$$

$$\text{Substituting} \quad \boldsymbol{V}^2 = \boldsymbol{n}^2\boldsymbol{D}^2\boldsymbol{J}^2 \quad \text{into} \quad \boldsymbol{T_c} = \frac{\boldsymbol{T}}{\rho \boldsymbol{V}^2 \boldsymbol{D}^2}$$

$$\text{gives} \quad \boldsymbol{T_c} = \frac{\boldsymbol{T}}{\rho \boldsymbol{n}^2 \boldsymbol{D}^4 \boldsymbol{J}^2} = \frac{\boldsymbol{C_T}}{\boldsymbol{J}^2}$$

The inflow ratio $\boldsymbol{a}$ also has limited value in propeller analysis as it too has an infinite value at $\boldsymbol{V} = \boldsymbol{0}$. For the Static Thrust condition the actual induced velocity (v) is required .

$$\boldsymbol{T} = \boldsymbol{2}\rho \boldsymbol{A}(\boldsymbol{V} + \boldsymbol{v})\boldsymbol{v} = \boldsymbol{2}\rho \boldsymbol{A}\boldsymbol{v}^2$$

$$\boldsymbol{v} = \sqrt{\frac{\boldsymbol{T}}{\boldsymbol{2}\rho \boldsymbol{A}}}$$

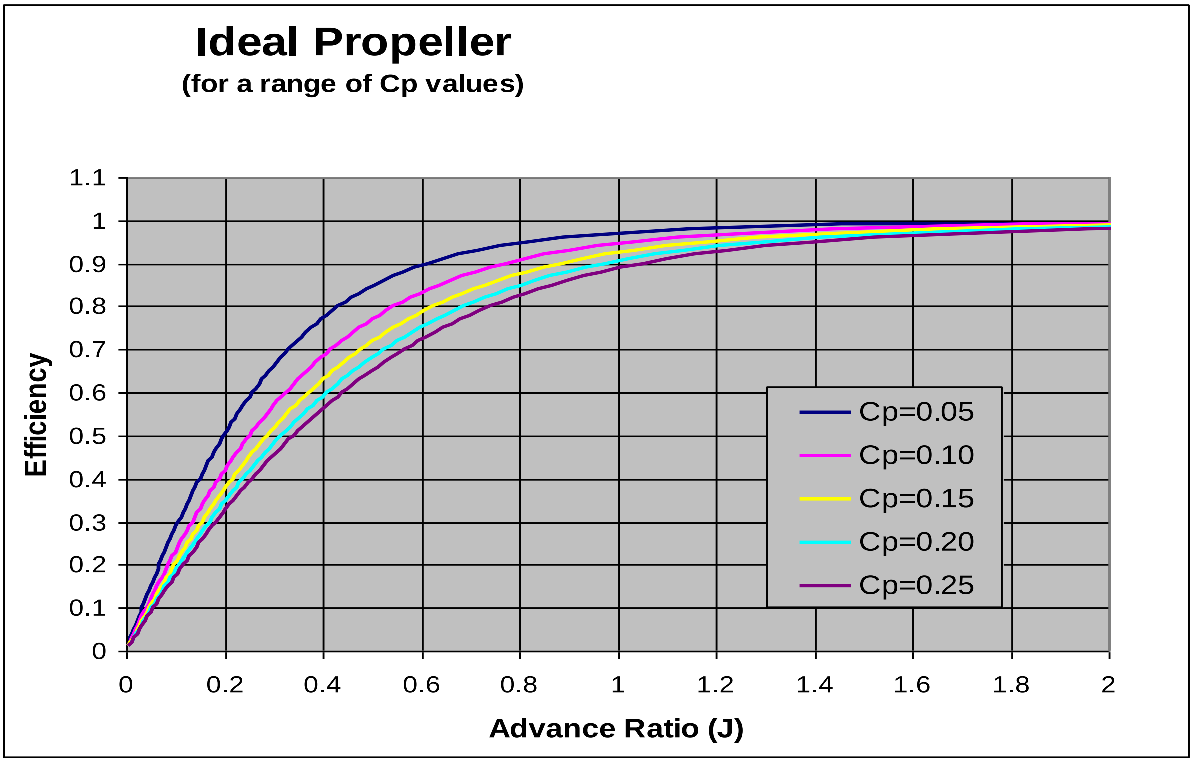
Power Coefficient

The Power is non-dimensionalised in a similar manner to thrust but with an additional nD term as $P \propto TV$ (and nD is the reference velocity parameter)

Thus $C_P = \dfrac{P}{\rho n^3 D^5}$. and propeller efficiency $\eta_p = \dfrac{\boldsymbol{TV}}{\boldsymbol{P}}$

Thus
$$\eta_P P = \frac{\pi}{2} D^2 \rho V^3 (1+a)a = \frac{\pi}{2} D^2 \rho V^3 \frac{(1-\eta_p)}{\eta_p{}^2}$$

and
$$\frac{1-\eta_p}{\eta_p{}^3} = \frac{2P}{\pi \rho D^2 V^3} = \frac{2C_P}{\pi J^3}$$



Frames of Reference

The aircraft in still air (moving forward at velocity ***V*** relative to the ground):

$$T = \rho A(V_{A/C} + v)2v$$

$$\frac{\partial \Delta KE_{air}}{\partial t} = P_{\Delta KE_{air}} = \tfrac{1}{2}\rho A(V_{A/C} + v)(2v)^2$$

$$P = TV_{A/C} + P_{\Delta KE_{air}} = 2\rho A(V_{A/C} + v)vV_{A/C} + \tfrac{1}{2}\rho A(V_{A/c} + v)(2v)^2$$

$$P = 2\rho Av\left[(V_{A/C}^2 + vV_{A/C}) + (V_{A/C}v + v^2)\right] = 2\rho Av(V_{A/C} + v)^2$$

The aircraft in a head wind (of velocity V relative to the ground):

$$T = \rho A(V_A + v)2v$$

$$\frac{\partial \Delta KE_{air}}{\partial t} = P_{\Delta KE_{air}} = \tfrac{1}{2}\rho A(V_A + v)((V_A + 2v)^2 - V_A{}^2))$$

$$P = TV_{A/C} + P_{\Delta KE_{air}} = 0 + \tfrac{1}{2}\rho A(V_A + v)((V_A + 2v)^2 - V_A^2))$$

$$P = \tfrac{1}{2}\rho A(V_A + v)(V_A^2 + 4V_Av + 4v^2 - V_A^2) = 2\rho Av(V_A + v)^2$$

Note:

For an aircraft flying against a headwind equal to its own still air velocity, $V_{A/C}$

then: $\eta = \frac{TV_{A/C}}{P} = 0$ where $V_{A/C}$ is the aircraft velocity relative to the ground.

Clearly this is not acceptable so the convention is to use: $\eta = \frac{TV_A}{P} \neq 0$

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