UNIVERSITY OF BRISTOL
FACULTY OF ENGINEERING

First Year Examination for the Degree of
Master of Engineering

MAY/JUNE 2012 3 Hours

FLUIDS 1

AENG11101

This paper contains *two* sections

SECTION 1
Answer *all* questions in this section
This section carries *40 marks*.

SECTION 2
This section has *five* questions.
Answer *three* questions.
All questions in this section carry *20 marks* each.
The maximum for this paper is *100 marks.*

Calculators may be used.

For air, assume R = 287 J/kgK. Take 0^{o}C as 273^{o}K.
Use a gravitational acceleration of $9.81 m/s^2$

Useful Equations

The volume of a sphere: $\frac{4}{3}\pi r^3$ Area of a circle: πr^2

Roots of a quadratic: $ax^2 + bx + c = 0 \quad \rightarrow \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The equation of state for a perfect gas is:

$$p = \rho RT$$

Drag equation: $\text{Drag} = \text{Area} \times C_D \times \frac{1}{2}\rho V^2$

Equation of a streamline in 2D flow

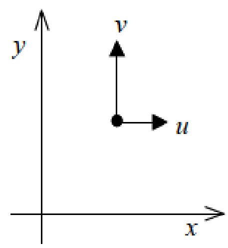
$$\frac{dy}{dx} = \frac{v}{u}$$

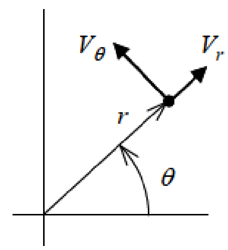
Change between Polar and Cartesian coordinate systems

$$r = \sqrt{x^2 + y^2}, \quad \theta = \tan^{-1}\left(\frac{y}{x}\right), \quad x = r\cos\theta, \quad y = r\sin\theta,$$

$$u = V_r \cos\theta - V_\theta \sin\theta, \qquad v = V_r \sin\theta + V_\theta \cos\theta$$

$$V_r = u\cos\theta + v\sin\theta, \qquad V_\theta = -u\sin\theta + v\cos\theta$$





2D Potential Flow

Velocity components in two-dimensional irrotational flow, in terms of the stream function and potential are given by

$$\underbrace{V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \qquad V_\theta = -\frac{\partial \psi}{\partial r} \qquad V_r = \frac{\partial \phi}{\partial r} \qquad V_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta}}_{\text{polar coordinates}} \qquad \underbrace{u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x} \qquad u = \frac{\partial \phi}{\partial x} \qquad v = \frac{\partial \phi}{\partial y}}_{\text{Cartesian coordinates}}$$

The stream function & velocity potential in Polar coordinates and the velocity distribution for

i) A uniform flow U_∞ parallel to the x axis:

$$\psi = U_\infty r \sin\theta, \qquad \phi = U_\infty r\cos\theta, \qquad V_r = U_\infty \cos\theta, \quad V_\theta = -U_\infty \sin\theta, \qquad u = U_\infty, \quad v = 0$$

ii) A source, of strength Λ at the origin:

$$\psi = \frac{+\Lambda\theta}{2\pi}, \quad \phi = \frac{+\Lambda}{2\pi}\ln r, \qquad V_r = \frac{+\Lambda}{2\pi r}, \quad V_\theta = 0, \qquad u = \frac{+\Lambda}{2\pi}\frac{x}{\left(x^2 + y^2\right)}, \quad v = \frac{+\Lambda}{2\pi}\frac{y}{\left(x^2 + y^2\right)}$$

iii) A doublet, of strength κ at the origin:

$$\psi = \frac{-\kappa}{2\pi}\frac{\sin\theta}{r}, \qquad \phi = \frac{+\kappa}{2\pi}\frac{\cos\theta}{r}, \qquad V_r = \frac{-\kappa}{2\pi r^2}\cos\theta, \qquad V_\theta = \frac{-\kappa}{2\pi r^2}\sin\theta,$$

$$u = \frac{-\kappa}{2\pi}\frac{\left(x^2 - y^2\right)}{\left(x^2 + y^2\right)^2}, \qquad v = \frac{-\kappa}{2\pi}\frac{2xy}{\left(x^2 + y^2\right)^2}$$

iv) A vortex, of strength Γ, at the origin:

$$\psi = \frac{+\Gamma}{2\pi}\ln r, \quad \phi = \frac{-\Gamma}{2\pi}\theta, \qquad V_r = 0, \quad V_\theta = \frac{-\Gamma}{2\pi r}, \qquad u = \frac{+\Gamma}{2\pi}\frac{y}{\left(x^2 + y^2\right)}, \quad v = \frac{-\Gamma}{2\pi}\frac{x}{\left(x^2 + y^2\right)}$$

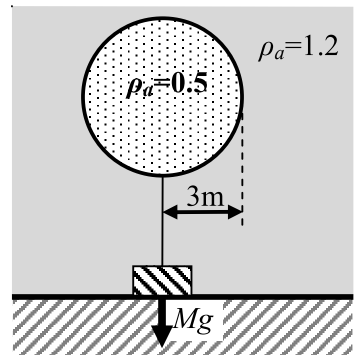
SECTION 1
Answer all questions in this section

Q1 A pressure transducer is lowered into a reservoir and records a gauge pressure of 2 bar. How far below the surface is the pressure transducer if the density of the water is 1000 kg m^{-3}?

(4 marks)

Q2 A balloon near the ground is tethered to a mass M. When deflated the balloon has a mass of 50 kg, and whilst inflated forms an approximate sphere of radius 3 m. Just before release the density of the gas inside the balloon and the density of the surrounding air are 0.5 kg m^{-3} and 1.2 kg m^{-3} respectively. Find the minimum mass M that stops the balloon rising.

(4 marks)



Q3 State the assumptions that must be made for Bernoulli's equation to be valid along a streamline. What additional assumption is required to use Bernoulli's equation for flow inside ducts?

(4 marks)

Q4 Air flows through a duct with a slowly varying cross section. At a certain point where the cross sectional area is 0.1 m^2, a pitot probe measures a total pressure of 1.02×10^5 Nm^{-2} at the centreline of the duct and a static probe measures a pressure of 1.01×10^5 Nm^{-2} at the duct wall. What would be the total and static pressures at a second point in the duct where the cross sectional area is $0.4 m^2$? Take the density of the air as 1.2 kg m^{-3}.

(4 marks)

Q5 (a) The drag of a short cylinder, moving in the direction of its axis, through air is reduced by placing an additional forebody and afterbody at each end of the tube. Draw a rough sketch of how you would expect these shapes to look, including an arrow showing the direction of motion. Explain your reasoning in terms of form drag and skin friction.

(b) In the real flow the body moves through still air. What is the transformation that allows wind tunnel testing of moving air over a static body? Give two flow variables that remain unchanged by this transformation.

(4 marks)

Q6 A surface with a sharp leading edge moves through still water at a speed of 5ms^{-1}. If the transition Reynolds number is $Re_x = 5 \times 10^5$, what distance from the leading edge will the flow start to be turbulent? Take the kinematic viscosity for water as 1.14×10^{-6} $m^2 s^{-1}$ Would accelerating or decelerating flow have a lower Reynolds number of transition? Give two other factors that would also lower the transition Reynolds number.

(4 marks)

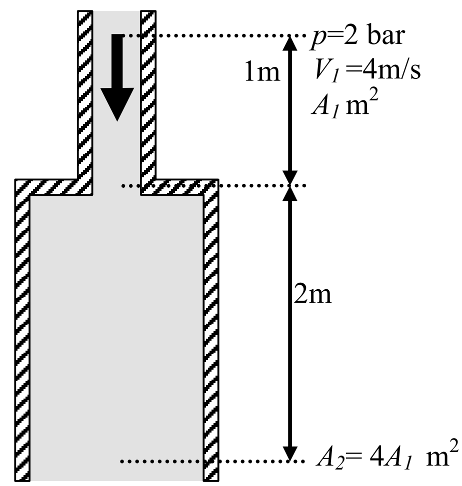
turn over…

Q7 Water flows down a straight smooth vertical pipe with a circular cross-section. At a particular point, the pressure and velocity are 2 bar and 4m/s. After a sudden expansion, the water enters another straight smooth vertical section of the pipe, with an area four times greater than the first section. Using the equation for pressure loss:

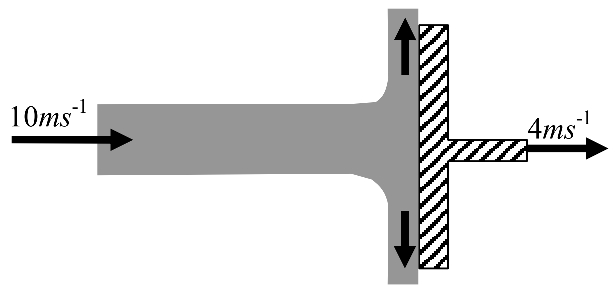
$$\Delta p_{loss} = \tfrac{1}{2}\rho V_1^2 \left(1 - \tfrac{A_1}{A_2}\right)^2$$

Find the pressure 2 m downstream of the contraction. The water density can be taken as 1000 kg m^{-3}.

(4 marks)



Q8 A horizontal circular water jet of diameter 10 cm and speed 10 ms^{-1} hits a flat vertical plate moving with a constant velocity of 4 ms^{-1} away from the jet. By using a suitable control volume find the horizontal force on the flat plate if all the water leaving the plate is moving vertically relative to the plate. Assume the water has a density of 1000 kg m^{-3}.

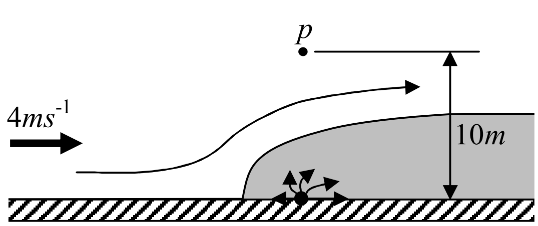


(4 marks)

Q9 For an inviscid incompressible flow derive the equation for the pressure coefficient, c_p, in terms of: p, p_∞, ρ, & U_∞. Define each variable used. From this derive an equation for the pressure coefficient in terms of only the local and far-field velocities.

(4 marks)

Q10 A flow can be modelled as a horizontal onset flow of 4 ms^{-1} and a source of strength 5 m^{2}s^{-1}. Find the horizontal and vertical velocity at the point p, 10 m above the source.



(4 marks)

SECTION 2
Answer *three* questions in this section

Q11 (a) A dam is constructed as a right-angled triangular prism of height H and length L as shown in figure Q11 below. The reservoir side of the dam slopes at an angle α while the downstream water level is at a height h. Assuming the effect of atmospheric pressure is negligible, show that the value of α which just provides a tipping moment about the line A-A is given by

$$\tan(\alpha) = \sqrt{\frac{(\rho_D + 2\rho_w)}{\rho_w}\frac{H^3}{(H^3 - h^3)}}$$

where ρ_D is the density of the dam material and ρ_w is the density of the water. Note that the second moment of area of a rectangle of height y and width b is given by

$$I_{xx} = by^3/12$$

while the centre of area of a triangle is 1/3rd of the height from the base.

(13 marks)

(b) A triangular prism, as described in (a), made of concrete with a specific gravity of 2.5 is used to retain a 10 m width of water to a height of 5 m. Assuming that the prism cannot slip horizontally and that there is no leak of water under the prism, what is the mass of the lightest triangular prism that can be used? The water density can be taken as 1000 kg m^{-3}.

(7 marks)

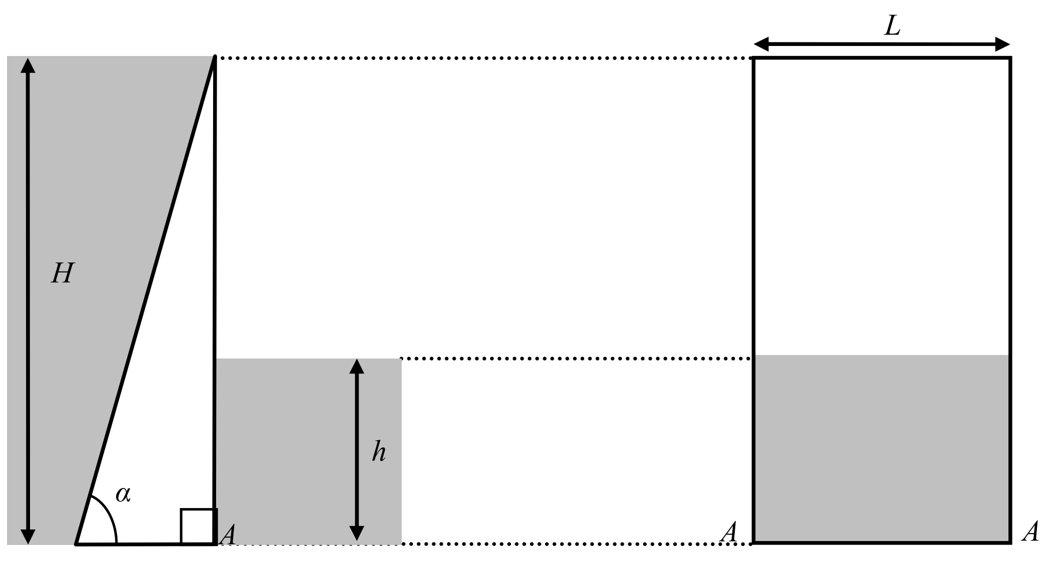


***Figure Q11**: Diagram of dam and reservoirs used in question 11, Side-on view and head-on view from downstream.*

turn over…

Q12 (a) A venturi is placed horizontally, at a height H below the surface of an upper reservoir. Fluid of density ρ_w flows from the upper reservoir and creates low pressure at the throat of the venturi before exiting to the atmosphere. This low pressure draws fluid, of density ρ_v , from a second reservoir whose surface is a height h below the throat of the venturi. If the throat and exit have circular cross sections of diameters D_t & D_e respectively, show that the minimum H that just draws fluid from the lower reservoir into the throat is given by

$$H_{\min} = h\frac{\rho_v}{\rho_w}\frac{D_t^4}{\left(D_e^4 - D_t^4\right)} .$$

(13 marks)

(b) A venturi and reservoirs are arranged as described in (a) where the fluid in the upper reservoir is water (density 1000 *kg* m^{-3}) and the maximum surface height above the venturi is 30 m. The height of the venturi throat above the second reservoir is 1m. If the diameters of the throat and exit are 0.05 m and 0.1 m, find the smallest specific gravity of the fluid in the second reservoir if it is not to enter the venturi throat. Also find the exit velocity of the water at this condition.

(7 marks)

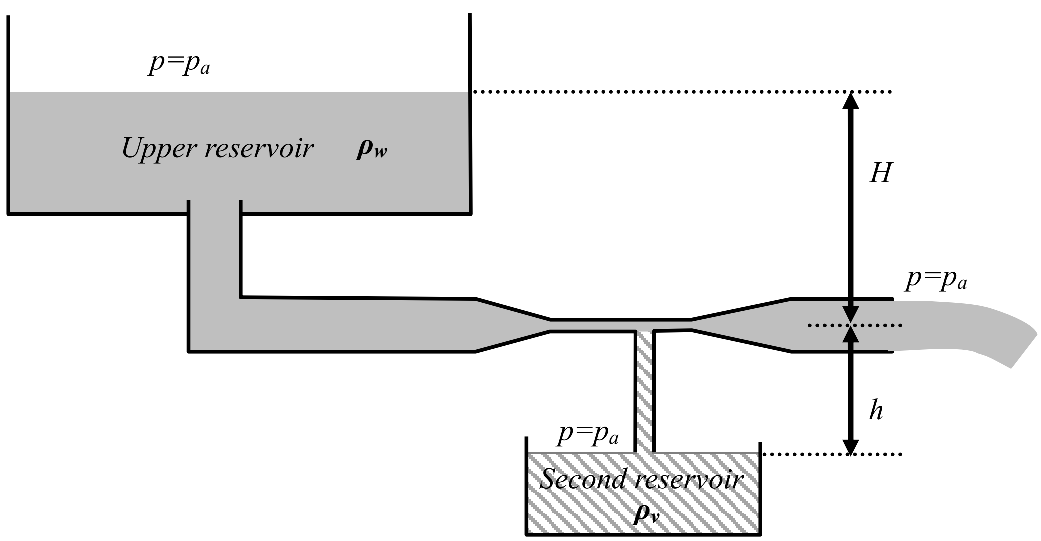


***Figure Q12**: Diagram of reservoirs and venturi used in question 12.*

Q13 Figure Q13 below shows a cylindrical container, of height H and cross sectional area A, with pressurised air above water of height h. At the bottom of the cylinder a small bell mouth opening of exit area A_e is initially blocked. The cylinder is constrained not to move and the opening is unblocked. In the resulting flow you may assume that the jet area is also A_e , the temperature of all fluids remains fixed and that the hydrostatic pressure variation can be neglected for the air.

(a) If the initial height of the water in the cylinder is given as $h=sH$ and the pressure of the air is r times atmospheric, show that the exit velocity of the water can be written as

$$V_e = A\left\{\frac{p_a\left[r\left(\frac{H-sH}{H-h}\right)-1\right]+\rho_w gh}{\frac{1}{2}\rho_w(A^2-A_e^2)}\right\}^{\frac{1}{2}}$$

where p_a is the atmospheric pressure and ρ_w is the density of water.

(10 marks)

(b) A cylindrical container, as described in (a), is 0.3 m tall has a 0.05 m radius and a circular bellmouth orifice of radius 0.003 m. While the orifice is blocked the pressure in the air is raised to 5 times the atmospheric pressure and the initial level of water is 0.2 m. Making the assumption that the instantaneous jet velocity is constant, find the maximum thrust produced and the thrust just before all the water is exhausted. Take the atmospheric pressure as 1.013 bar and the density of water as 1000 kg m^{-3}

(10 marks)

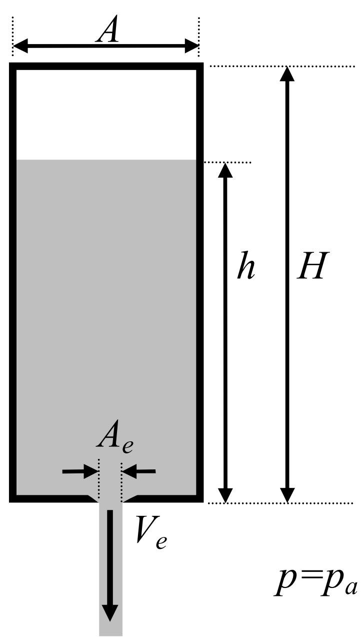


***Figure Q13**: Diagram of cylinder used in question Q13 (a) and (b).*

turn over...

Q14 Air flows at V m/s far upstream of a windmill that sweeps out a circular disc of diameter d m. At a point downstream of the windmill, where the pressure has returned to atmospheric, the velocity of the wind is measured at V/a m/s. a is a constant that represents the ratio of upstream to downstream wind speeds.

(a) Use the actuator disc theory for an ideal windmill to show that the force on the windmill can be written as

$$F = \rho \tfrac{\pi}{8} d^2 V^2 \left(\frac{a^2 - 1}{a^2} \right) \quad ,$$

where ρ is the density of the air. Clearly state all assumptions made during your derivation.

(8 marks)

(b) Continuing the analysis of the windmill defined above, show that the efficiency is given by

$$\eta = \frac{(1+a)(a^2 - 1)}{2a^3} = \frac{a^3 + a^2 - a - 1}{2a^3} \quad .$$

(6 marks)

(c) A windmill of diameter 5 m, works at an efficiency of $\eta = 0.5$ in a 16 ms^{-1} wind. If the air density is 1.2 kg m^{-3}, find the force on the windmill, the air velocity through the disc and the mean gauge pressures just in front of and just behind the disc.

(6 marks)

Q15 (a) A free stream and a doublet can be used to model the non-lifting flow over a cylinder. Find the doublet strength, in terms of the cylinder radius. Hence find then the velocity and pressure coefficient distributions on the cylinder. Then show that the pressure on the cylinder is given by

$$p(\theta) = p_\infty + \frac{1}{2}\rho U_\infty^2 (1 - 4\sin^2\theta)$$

(6 marks)

(b) A greenhouse can be modelled as a closed semi cylinder whose radius is R. It is mounted on tie down blocks as shown in figure Q15. The air under the greenhouse is at rest, with pressure equal to the total pressure of the oncoming flow. By considering the forces acting on a small element of the upper surface and assuming that the flow field over the top of the greenhouse is given by the inviscid incompressible flow over a non-lifting cylinder for $0 \le \theta \le \pi$, show that the overall lift on the greenhouse is

$$l = 2\left(p_\infty + \tfrac{1}{2}\rho U_\infty^2\right)R - \int_0^\pi p(\theta) R \sin\theta \, d\theta$$

(6 marks)

Hence using (a) show that the net lift force acting on the greenhouse is equal to

$$l = \frac{8}{3}\rho_\infty U_\infty^2 R$$

(8 marks)

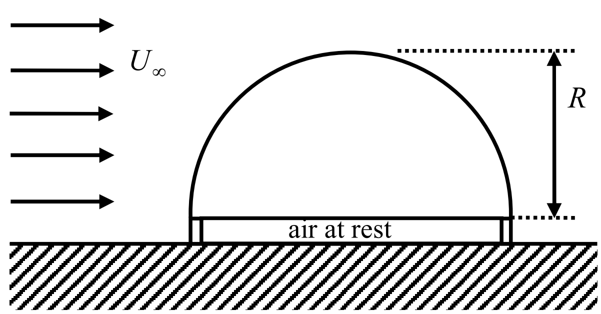


***Figure Q15**: Semi-cylinder greenhouse on blocks.*