Advanced Bending and Torsion
Shear Centre of Thin-Walled Sections

Dr Luiz Kawashita

Luiz.Kawashita@bristol.ac.uk

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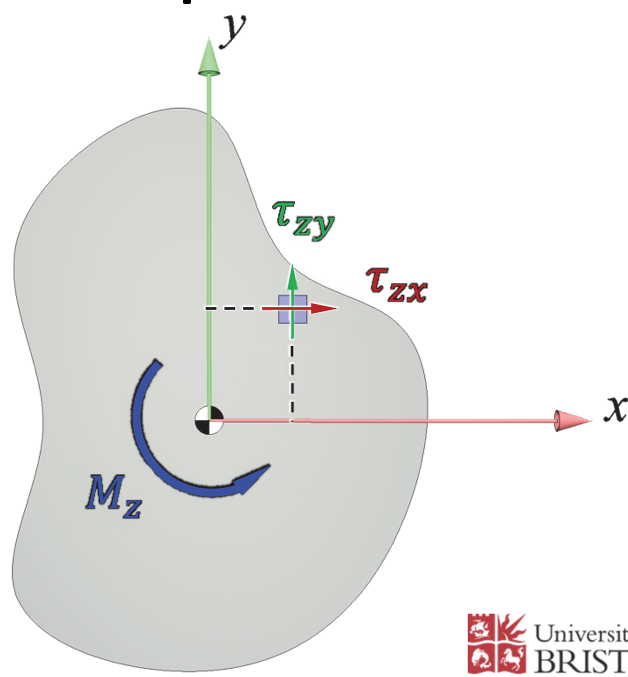
University of BRISTOL

Shear Centre

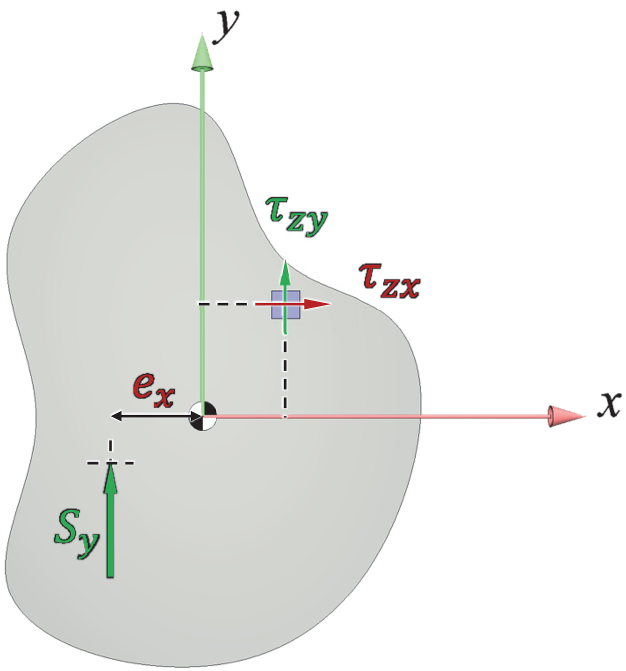
- Is a point somewhere on the x-y plane where transverse shear forces can be applied without causing torsion about the z axis
- A shear centre exists for any cross section, but we will focus only on **convex solid cross-sections** and **thin-walled cross sections**
- As a shear force is applied, shear stresses are generated along the cross-section which can be decomposed into τ_{zx} and τ_{zy}
- These may generate a moment **about a reference point**
  - Important to choose a convenient one
- For a generic cross-section, taking the centroid as reference:

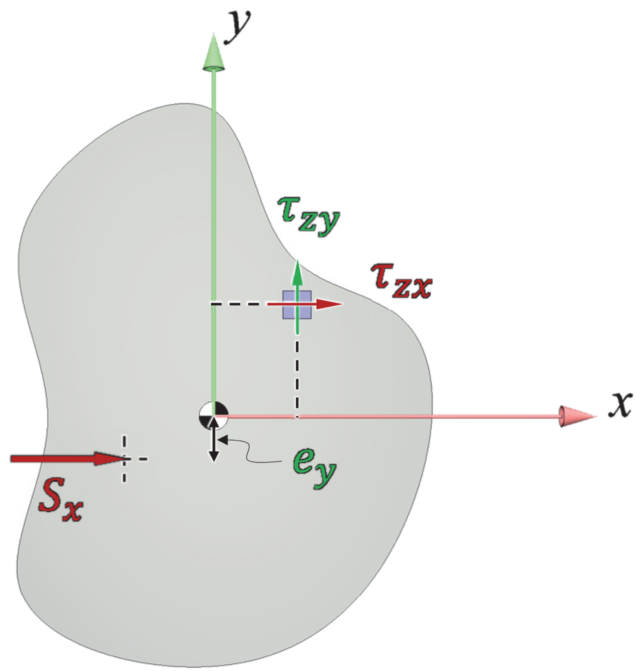
$$M_z = \int (x\,\tau_{zy} - y\,\tau_{zx})\,\mathrm{d}A$$



Shear Centre

- To avoid twisting the beam about z, the shear force must generate a counterbalancing moment about z
- This is done by offsetting the loading point perpendicularly to the loading direction:

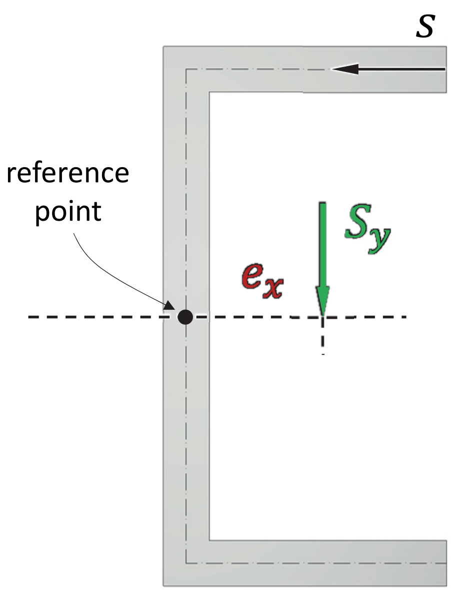




$$S_y\, e_x = \int (y\,\tau_{zx} - x\,\tau_{zy})\, \mathrm{d}A \qquad S_x\, e_y = \int (y\,\tau_{zx} - x\,\tau_{zy})\, \mathrm{d}A$$

Shear Centre – Thin Walled Sections

- For **convex solid sections** the shear centre **is the centroid**
  - Concave solid sections are outside of the scope of StM2
- For thin walled structures the shear centre can easily be found by 'inspection' or by equilibrium of moments
- Thin wall assumptions mean that shear stresses always follow the centreline
- Taking a channel section as example, an picking the origin of X, Y as reference:



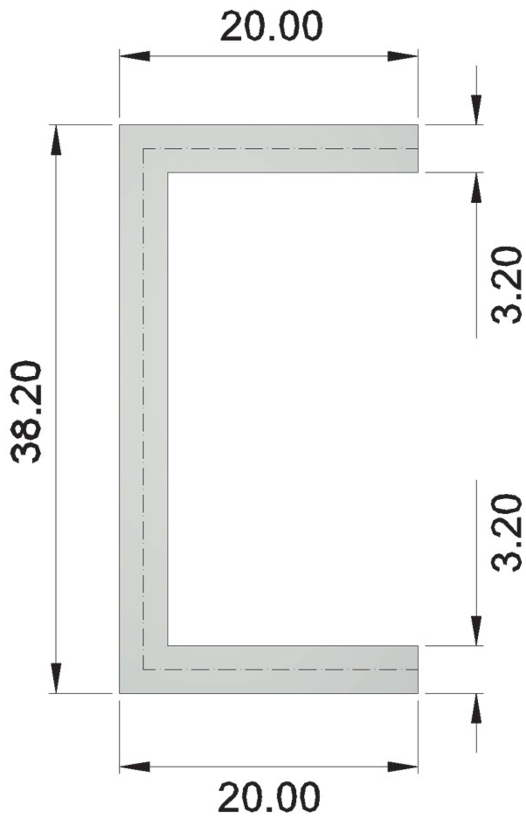
$$S_y\, e_x = \int \left(Y\, \tau_{zx} - X\, \tau_{zy}\right) \mathrm{d}A$$

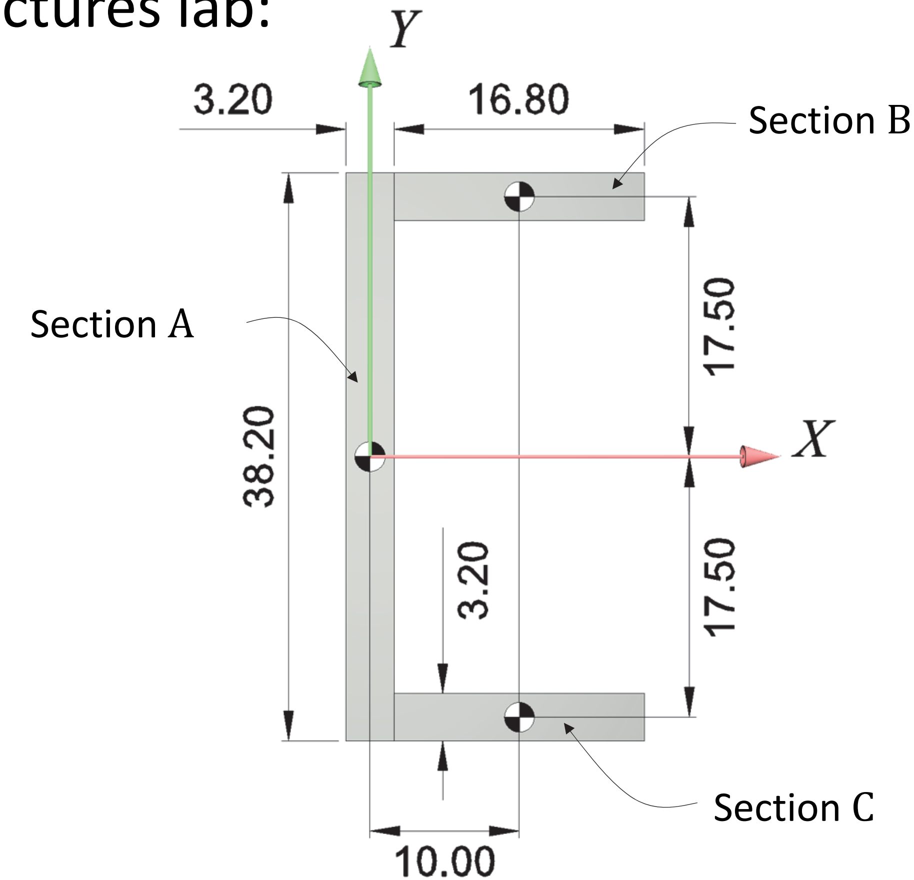
$$S_y\, e_x = \int \left(Y\, q_{s,x} - X\, q_{s,y}\right) \mathrm{d}s$$

Where $q_{s,x}$ and $q_{s,y}$ are the components of q_s along the x and y coordinates, respectively, arising from the application of a shear force S_y

Channel Section Example

- Channel section see in the Structures lab:





$$A_\mathrm{A} = (3.2)(38.2) = 122.24 \text{ mm}^2$$

$$\bar{X}_\mathrm{A} = 0$$

$$\bar{Y}_\mathrm{A} = 0$$

$$A_\mathrm{B} = (16.8)(3.2) = 53.76 \text{ mm}^2$$

$$\bar{X}_\mathrm{B} = 10.0 \text{ mm}$$

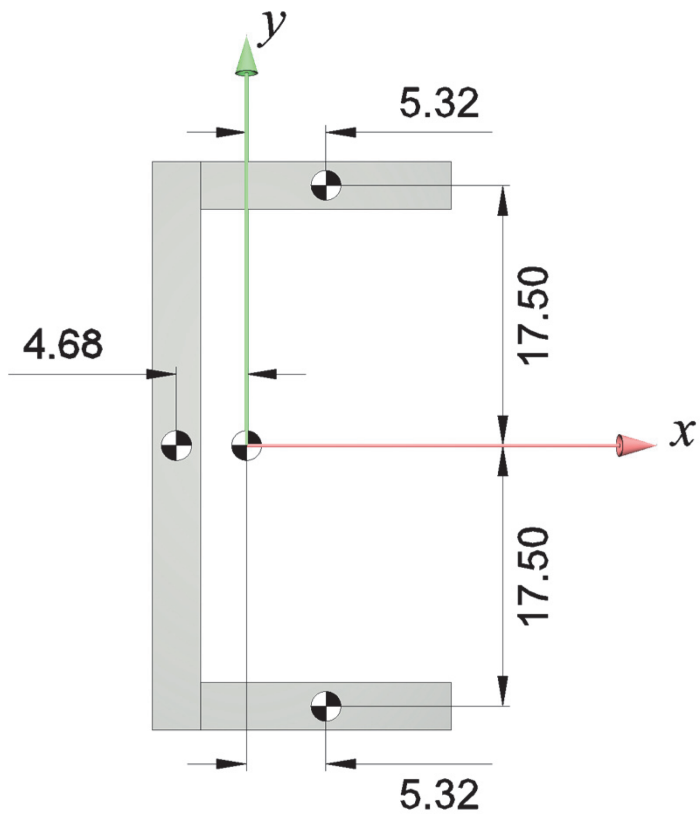
$$\bar{Y}_\mathrm{B} = 17.5 \text{ mm}$$

- We can now find the centroid of the compound section:

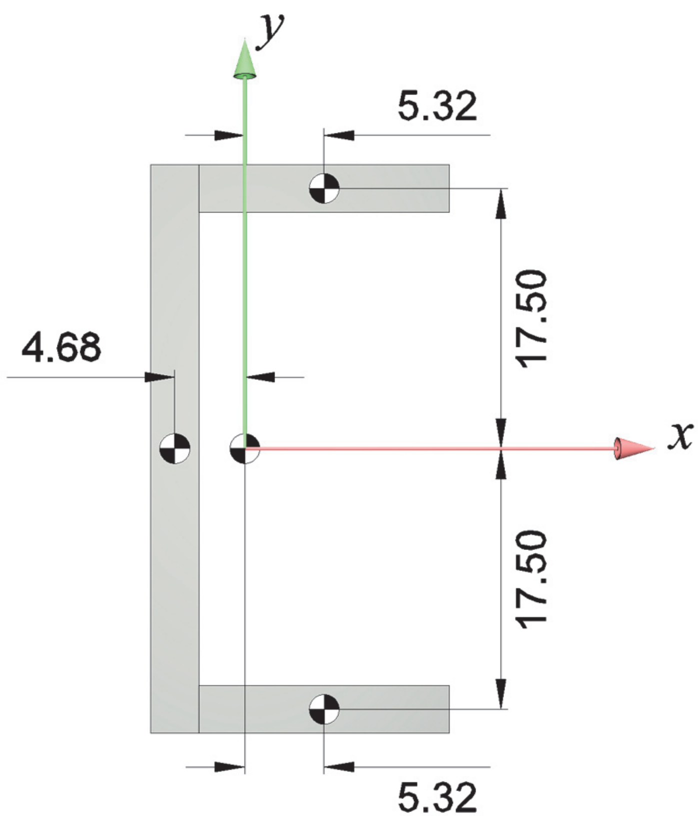
$$\bar{X} = \frac{\sum \bar{X}_i A_i}{\sum A_i} = \frac{\bar{X}_A A_A + \bar{X}_B A_B + \bar{X}_C A_C}{A_A + A_B + A_C} = \frac{2\,(10.0)(53.76)}{(122.24) + 2\,(53.76)}$$

$$\boxed{\bar{X} = 4.68\ \text{mm}}$$

$$\boxed{\bar{Y} = 0}$$



- Applying the parallel axis theorem for section A:



$$I_{x_A x_A} = \frac{b\,h^3}{12} = \frac{(3.2)(38.2)^3}{12} = 14{,}864.79\ \text{mm}^4$$

$$\bar{y}_A = \bar{Y}_A - \bar{Y} = 0$$

$$I_{xx}^{A} = I_{x_A x_A} + A_A(\bar{y}_A)^2$$

$$I_{xx}^{A} = 14{,}864.79\ \text{mm}^4$$

$$I_{y_A y_A} = \frac{b\,h^3}{12} = \frac{(38.2)(3.2)^3}{12} = 104.31\ \text{mm}^4$$

$$\bar{x}_A = \bar{X}_A - \bar{X} = 0 - 4.68 = -4.68\ \text{mm}$$

$$I_{yy}^{A} = I_{y_A y_A} + A_A(\bar{x}_A)^2$$

$$I_{yy}^{A} = 2{,}781.28\ \text{mm}^4$$

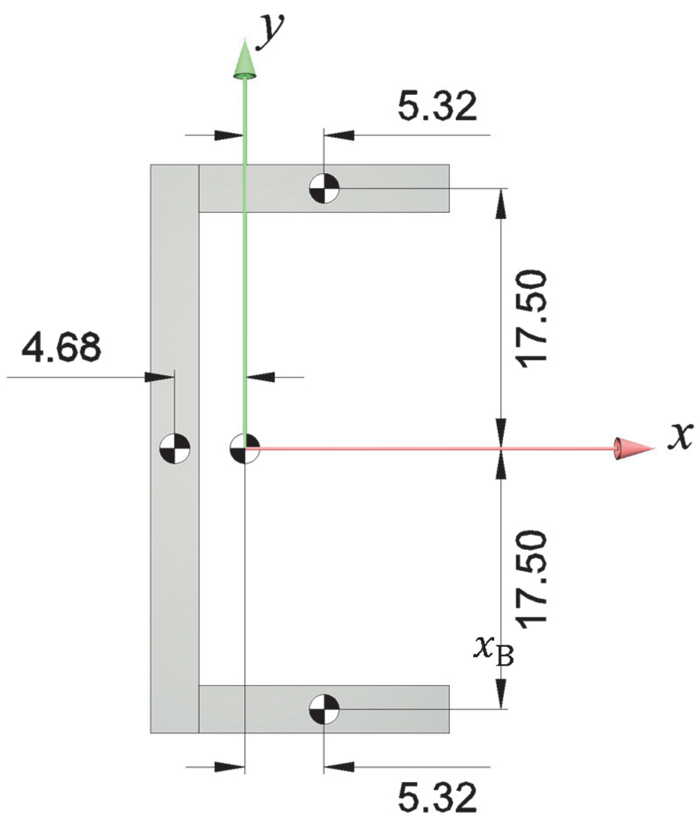
$$I_{x_A\, y_A} = 0\ \text{(symmetric cross-section)}$$

$$I_{xy}^{A} = I_{x_A y_A} + A_A(\bar{x}_A \bar{y}_A)$$

$$I_{xy}^{A} = 0$$

Channel Section Example

- Applying the parallel axis theorem for sections B and C:



$$I_{x_B x_B} = \frac{b\,h^3}{12} = \frac{(19.95)(3.26)^3}{12} = 45.88\ \text{mm}^4$$

$$\bar{y}_B = \bar{Y}_B - \bar{Y} = 17.5 - 0 = 17.5\ \text{mm}$$

$$I_{xx}^B = I_{x_B x_B} + A_B(\bar{y}_B)^2$$

$$I_{xx}^B = I_{xx}^C = 2{,}786.28\ \text{mm}^4$$

$$I_{y_B y_B} = \frac{b\,h^3}{12} = \frac{(3.26)(19.95)^3}{12} = 1{,}264.44\ \text{mm}^4$$

$$\bar{x}_B = \bar{X}_B - \bar{X} = 10.0 - 4.68 = 5.32\ \text{mm}$$

$$I_{yy}^B = I_{y_B y_B} + A_B(\bar{x}_B)^2$$

$$I_{yy}^B = I_{yy}^C = 16{,}509.88\ \text{mm}^4$$

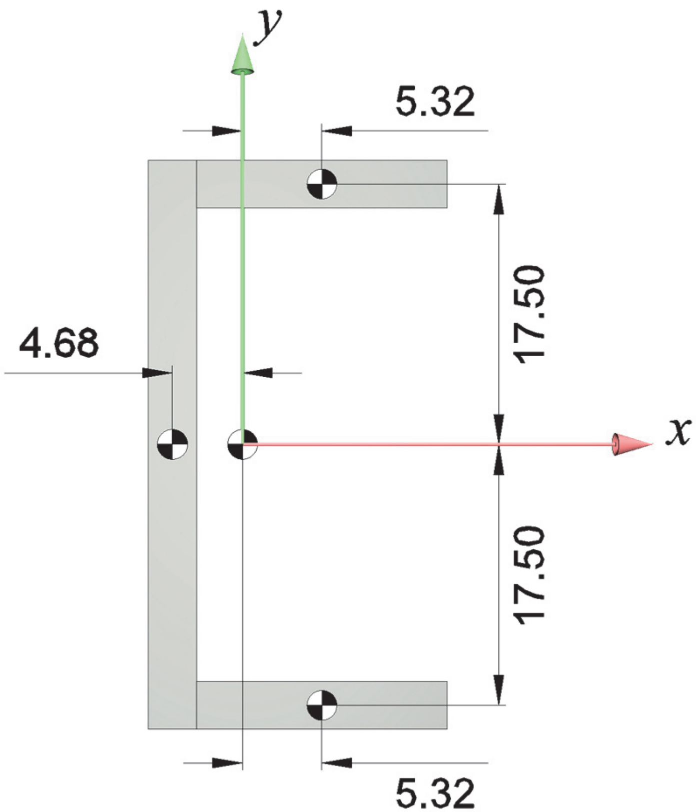
$I_{x_B\,y_B} = 0$ (symmetric cross-section)

$$I_{xy}^B = I_{x_B\,y_B} + A_B(\bar{x}_B \bar{y}_B)$$

$I_{xy}^B = 5{,}005.37\ \text{mm}^4$ and $I_{xy}^C = -5{,}005.37\ \text{mm}^4$

Channel Section Example

- Finally, for the compound section:



$$I_{xx} = I_{xx}^{\mathrm{A}} + I_{xx}^{\mathrm{B}} + I_{xx}^{\mathrm{C}}$$

$$I_{xx} = 47{,}884.54 \text{ mm}^4$$

$$I_{yy} = I_{yy}^{\mathrm{A}} + I_{yy}^{\mathrm{B}} + I_{yy}^{\mathrm{C}}$$

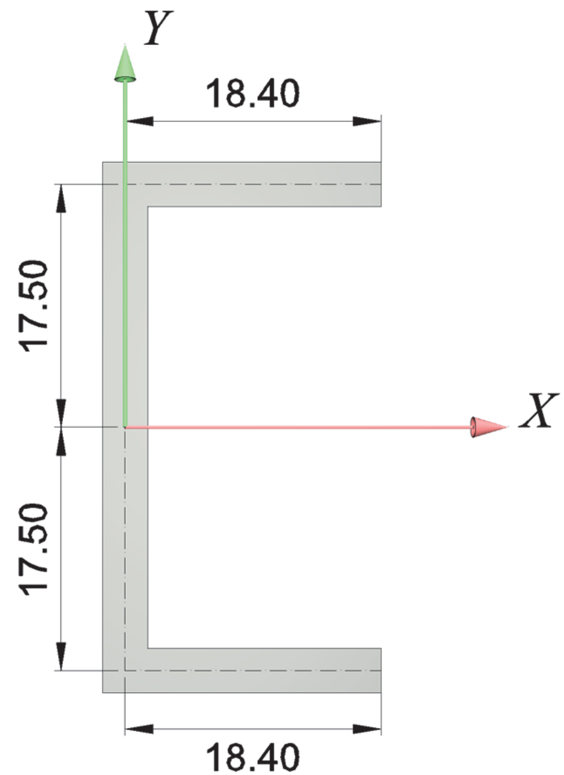
$$I_{yy} = 8{,}353.60 \text{ mm}^4$$

$$I_{xy} = I_{xy}^{\mathrm{A}} + I_{xy}^{\mathrm{B}} + I_{xy}^{\mathrm{C}}$$

$$I_{xy} = 0$$

Channel Section Example

- To calculate the shear centre we apply thin-wall assumptions and consider the centrelines of the cross-section:



Equations:

Shear centre:

$$S_y\, e_x = \int \left(Y\, q_{s,x} - X\, q_{s,y}\right) \mathrm{ds} \qquad S_x\, e_y = \int \left(Y\, q_{s,x} - X\, q_{s,y}\right) \mathrm{ds}$$

Shear flow:

$$-q_s = \left(\frac{S_x\, I_{xx} + S_y\, I_{xy}}{I_{xy}^2 - I_{xx}\, I_{yy}}\right) \int_0^s x\, t\, \mathrm{ds} + \left(\frac{S_y\, I_{yy} + S_x\, I_{xy}}{I_{xx}\, I_{yy} - I_{xy}^2}\right) \int_0^s y\, t\, \mathrm{ds}$$

Channel Section Example

$$-q_s = \left(\frac{S_x\, I_{xx} + S_y\, I_{xy}}{I_{xy}^2 - I_{xx}\, I_{yy}}\right)\int_0^s x\, t\, \mathrm{ds} + \left(\frac{S_y\, I_{yy} + S_x\, I_{xy}}{I_{xx}\, I_{yy} - I_{xy}^2}\right)\int_0^s y\, t\, \mathrm{ds}$$

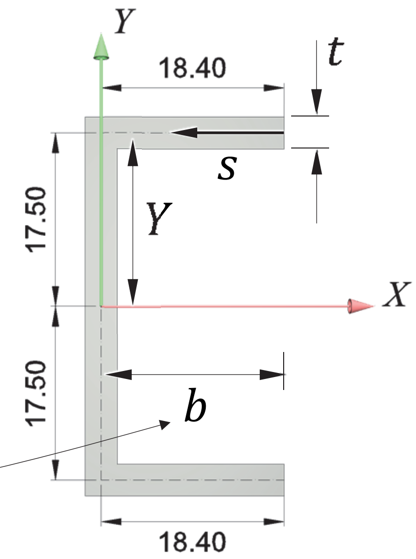
Since $I_{xy} = S_x = 0$:

$$-q_s = \left(\frac{S_y}{I_{xx}}\right)\int_0^s y\, t\, \mathrm{ds} \longrightarrow$$

Integrating:

$$-q_s = \frac{S_y\, y\, t}{I_{xx}}\, s$$

Shear centre formula: $$S_y\, e_x = \int \left(Y\, q_{s,x} - X\, q_{s,y}\right) \mathrm{ds}$$



Combining sections B and C: $$S_y\, e_x = \mathbf{2}\int_0^b \left(Y\, q_s\right) \mathrm{ds}$$

Replacing q_s: $$S_y\, e_x = -2\int_0^b \frac{S_y\, y^2\, t}{I_{xx}}\, s\, \mathrm{ds}$$

Integrating: $$S_y\, e_x = -\frac{S_y\, y^2\, t\, b^2}{I_{xx}}$$

Cancelling out S_y : $$e_x = -\frac{y^2\, t\, b^2}{I_{xx}}$$

$$e_x = -\frac{(17.5)^2\,(3.2)\,(18.4)^2}{(47{,}884.54)}$$

$$e_x = -6.93\ \mathrm{mm}$$

(*i.e.* to the <u>left</u> of the reference point)