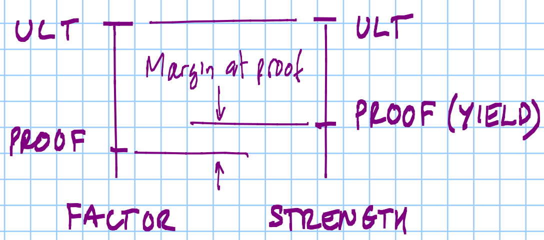
1

Strength Design at Section Level

For strength design we must consider section level and the different components of stress in each section element to ensure that they do not exceed the relevant material strength allowables.

For aircraft structural design we normally design for ultimate then check at proof.

Note, if the ratio of ultimate factor : proof factor exceeds the ratio of ultimate strength: proof or yield strength then the check at proof can be assumed to be covered by the check at ultimate.



2

Initial Checks

17.10.2010

For initial checks consider only the most significant stress in each section element, i.e. direct stress in the flanges and shear stress in the webs:

For area properties use "thin wall assumptions", neglect fillets + corners etc.

Flanges: $\sigma_x = \dfrac{F_x}{A} + \dfrac{M_z y}{I}$ Direct stress in the flanges due to axial loads and bending.

Webs: $\tau_{xy} \approx \dfrac{F_y}{b_w t} + \dfrac{T}{2At}$ Shear stress in the webs due to transverse load and torque.

Considering only single stress types in each element we are effectively using a "non-interactive" single stress failure criteria as an initial check.

I.e. failure occurs when: $\sigma_x \geqslant \sigma_x^*$ in the flanges:

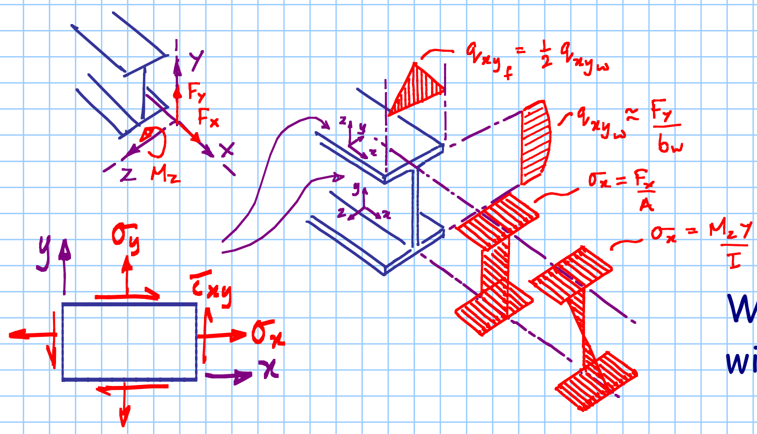
or when: $\tau \geqslant \tau^*$ in the webs:

Where σ^* and τ^* are the yield, proof or ultimate* strengths for direct and shear respectively. (* ultimate strengths are primarily used for aircraft design).

Refined Checks

In the initial checks we only considered the most significant single stresses in each section element but, actually, each section element carries a combination of direct **and** shear stresses. To refine we will must account for the combined stresses.

E.g. a 2D Loading system in the XY plane will result in combined direct and shear stresses in the flanges or webs:



StM2

The question is:
What combinations of σ and τ will cause failure?

To account for the combined stresses, σ_x and τ_{xy} in our failure analysis we need to use an "interactive failure criterion".

Numerous failure criteria are available to deduce the combination of stresses that would cause failure. These failure criteria are usually related to principal stresses and are arranged in the form of "failure index" stress ratios: $R = \frac{\sigma}{\sigma_o}$ or $\frac{\tau}{\sigma_o}$

where σ, τ = applied stresses

and σ_o = stress at failure e.g. yield, proof or ultimate strength* determined by uniaxial test

Failure criteria are expressed as combinations of failure indexes that represent failure, usually involving exponents and factors.

E.g. failure occurs when: $R_A^{\#} + R_B^{\#} + R_S^{\#} = 1$

Where R_A, R_B and R_S are stress ratios relating to Axial, Bending and Shear stresses and associated failure values. The exponents, #, can be derived or found by testing. Sometimes factors are also included. Eg. $\underline{k}R^{\#}$

Axial and bending stresses can be added by superposition and accounted as a single effective direct stress within the section. Similarly, transverse and torsional shear stresses can also be added as effective single shear stress.

Combined stress failure criteria can be derived from the Principal stresses (which represent the maximum and minimum stresses resulting from a biaxial or triaxial stress system) e.g. for combined axial and shear stresses the following failure index stress ratio sums may be used to define failure:

For brittle materials:

- based on maximum principal stress: $R_A + R_s^2 = 1$ – using $\sigma_0 = \sigma_{ULT}^*$

For ductile materials:

- based on maximum shear $R_A^2 + 4R_s^2 = 1$ – using $\sigma_0 = \sigma_y^*$
- based on energy of distortion:* $R_A^2 + 3R_s^2 = 1$ – using $\sigma_0 = \sigma_{ULT}^*$ or σ_y^*

For this course we will use the "energy of distortion" failure criteria:*

$$\left(\frac{\sigma_x}{\sigma_0}\right)^2 + 3\left(\frac{\tau_{xy}}{\sigma_0}\right)^2 = \text{"FI"}$$

where: $\sigma_0 = \sigma_{ULT}^*$ for a/c design @ ult loads

"FI" = failure index = 1 at failure

$$\frac{1}{\sqrt{FI}} \equiv RF$$

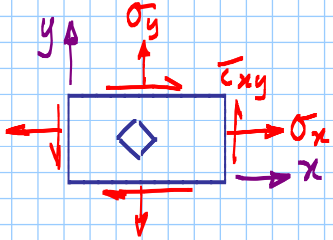
See "Principal Stress Failure Criteria" in StM2 for further background.

Note Title 16/02/2010

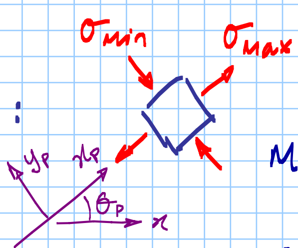
Principal stress Failure Criteria- for a 2D stress system

IJF 15.2.2010

In particular directions an applied 2D stress system $\sigma_x, \sigma_y, \tau_{xy}$ will resolve into maximum or minimum (algebraic) values of pure direct stress. I.e. "Principal stresses" $\sigma_{max}, \sigma_{min}$ in principal axis directions. $\theta_p, \theta_p + 90°$. On these principal planes the shear stress will be zero. At



Applied stresses $\sigma_x, \sigma_y, \tau_{xy}$



Principal stresses $\sigma_{max}, \sigma_{min}$ @ θ_p to x, y axes

since shear is equivalent to diagonal tension and compression.

Max shear stress τ_{max} @ 45° to x_p, y_p axes

$\sigma_{max}, \sigma_{min}, \tau_{max}$ can be deduced by transformation of the applied stresses, $\sigma_x, \sigma_y, \tau_{xy}$ through angle θ then differentiating wrt θ to reveal the maxima and minima.

I.e. $\frac{d\sigma}{d\theta} = 0$: giving the "Principal Stress Equations": (PS Equ^ns)

$$\sigma_{max} = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\sigma_{min} = \frac{1}{2}(\sigma_x + \sigma_y) - \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

Occurring at:

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\tau_{xy}}{(\sigma_x - \sigma_y)}$$

and $\theta_p + 90°$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min})$$ – occurring at $\theta_p \pm 45°$

1) Maximum Principal Stress Failure Criterion

Failure occurs when: $|\sigma_{max} \text{ or } \sigma_{min}| \geq \sigma_0$

principal stresses

Material allowable direct strength obtained from a simple uniaxial tension or compression tests.

$$\sigma_{max,min} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

Writing $\sigma_{max}, \sigma_{min}$ in terms of applied stresses: $\sigma_x, \sigma_y, \tau_{xy}$ from PS equ^ns, putting $\sigma_y = 0$ for simple 2D beam loading, simplifying and rearranging gives:

$$\frac{\sigma_x}{\sigma_0} + \left(\frac{\tau_{xy}}{\sigma_0}\right)^2 = 1 \text{ at failure}$$

"Stress ratio failure sum"

which gives good agreement for brittle materials.

└ using $\sigma_0 = \sigma_{ULT}^*$

2) Maximum Shear Stress Failure Criterion

Failure occurs when: $|\tau_{max}| \geq \tau_0$

Max shear stress — Material allowable shear strength obtained from a simple uniaxial tension or compression test.

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min})$$

where $\tau_0 = \frac{1}{2}\sigma_0$

Writing τ_{max} in terms of applied stresses: $\sigma_x, \sigma_y, \tau_{xy}$ from PS eqn's, putting $\sigma_y = 0$ for simple 2D beam loading, simplifying and rearranging gives:

$$\left(\frac{\sigma_x}{\sigma_0}\right)^2 + 4\left(\frac{\tau_{xy}}{\sigma_0}\right)^2 = 1 \text{ at failure}$$

"stress ratio failure sum"

which gives good agreement for ductile

using $\sigma_0 = \sigma_y^*$

3) Energy of Distortion Failure Criterion *

Failure occurs when the energy of distortion per unit volume equals the energy of distortion per unit volume absorbed at failure in a inelastic

Failure occurs when: $\sigma_{max}^2 + \sigma_{min}^2 - \sigma_{max}\sigma_{min} = \sigma_0^2$

Writing $\sigma_{max}, \sigma_{min}$ in terms of applied stresses: $\sigma_x, \sigma_y, \tau_{xy}$ from PS eqn's, putting $\sigma_y = 0$ for simple 2D beam loading, simplifying and rearranging gives:

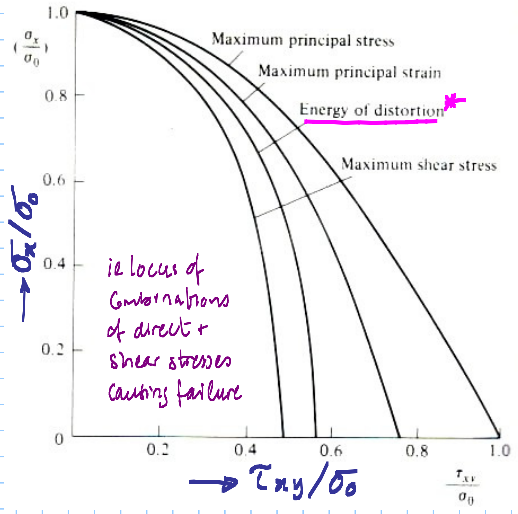
$$\left(\frac{\sigma_x}{\sigma_0}\right)^2 + 3\left(\frac{\tau_{xy}}{\sigma_0}\right)^2 = 1 \text{ at failure}$$

"stress ratio failure sum"

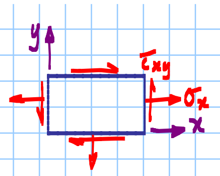
which gives good agreement for ductile materials.

using $\sigma_0 = \sigma_y^*$ or σ_{ULT}^* according to req'd check

Comparison of failure



For:



$\sigma_y = 0$

Note, Max principal stress and max principal strain boundaries would be straight if plotted against principal stress ratios $\frac{\sigma_{max}}{\sigma_0}$, $\frac{\sigma_{min}}{\sigma_0}$

For this course we will use the Energy of Distortion failure criteria.

$$\left(\frac{\sigma_x}{\sigma_0}\right)^2 + 3\left(\frac{\tau_{xy}}{\sigma_0}\right)^2 = \text{"FI"}$$

where:

$\sigma_0 = \sigma_{ULT}$ for a/c design @ ult loads

"FI" = failure index
= 1 at failure

From the resulting failure index "FI" value we will define our reserve factor

$$\text{"RF"} = \frac{1}{\sqrt{FI}}$$

References / Bibliography:

[1] Muvdi it McNabb "Engineering mechanics of Materials"

[2] Peery of Azaar "Aircraft structures"
McGraw-Hill 1976. ISBN 0-07-049196-8