Lecture 10

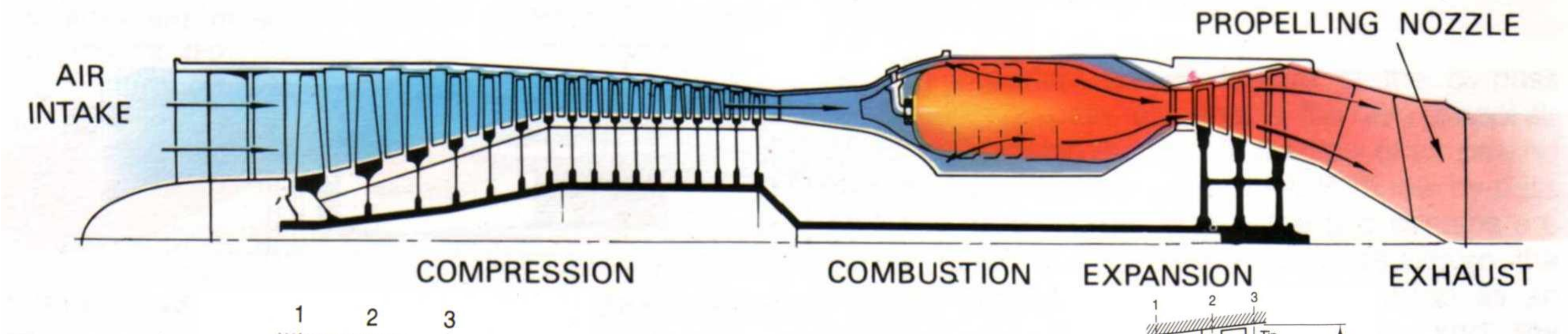
Axial Compressors

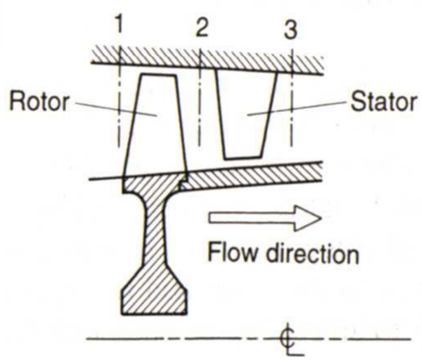
Objective: The "simple" analysis of an Axial Compressor

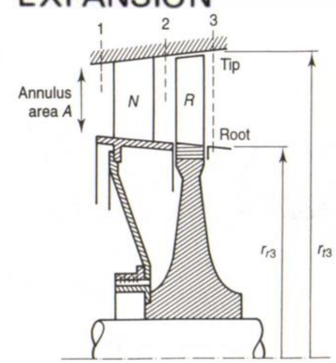
Turbomachinery ~*Introduction*

- The **compressor** raises the pressure of the air before combustion

- The **turbine** extracts work from the hot high pressure combustion products to drive the compressor



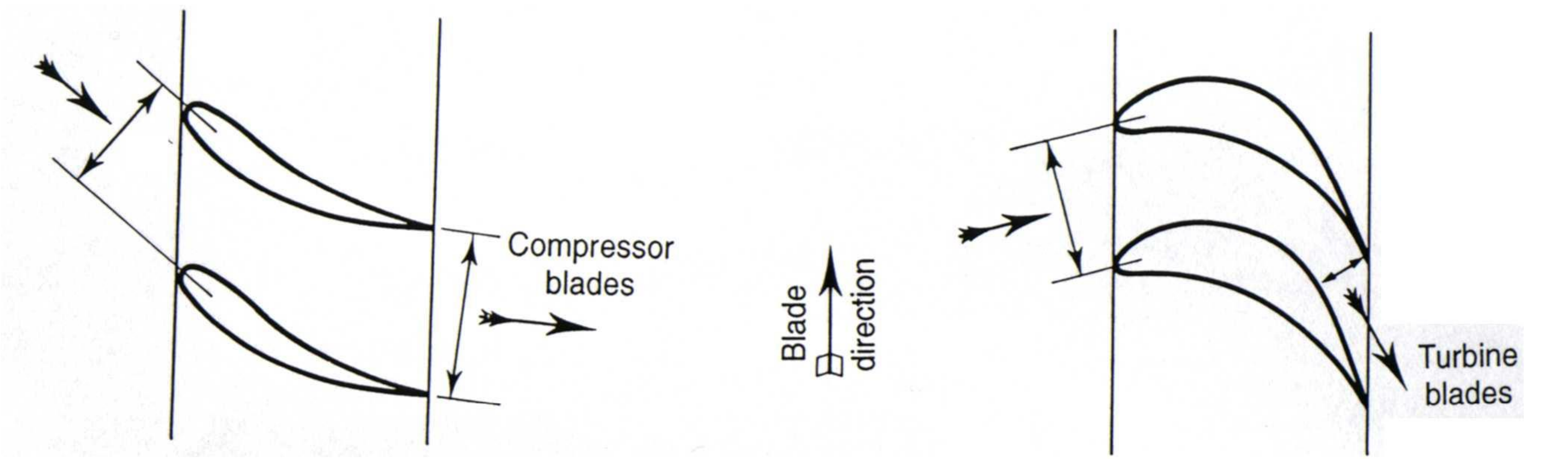




Each stage consists of a row of rotor blades followed by a row of stator blades
Often preceded by an inlet guide vane

Each stage consists of a row of Nozzle Guide Vanes which direct the gas onto the rotor blade

Comparison of compressor & turbine blades



Diverging Passages

Converging Passages

On both stator & rotor blades

Isentropic & Polytropic Efficiency 1

Polytropic or "Small Stage" Efficiency

The isentropic efficiency of an elemental stage, such that it is constant throughout the whole process:

For compression: $\eta_{pol} = \dfrac{dT_O'}{dT_O} = \textbf{Constant}$

For an isentropic process: $\dfrac{T_O}{p_O^{\gamma-1/\gamma}} = \textbf{Constant}$

The Differential form being: $\dfrac{dT_O'}{T_O} = \dfrac{\gamma}{\gamma-1}\dfrac{dp_O}{p_O}$

Isentropic & Polytropic Efficiency 2

Polytropic or "Small Stage" Efficiency

The isentropic efficiency of an elemental stage, such that it is constant throughout the whole process:

The Differential form being: $$\frac{dT_O'}{T_O} = \frac{\gamma}{\gamma - 1}\frac{dp_O}{p_O}$$

Substituting for dT_O' gives $$\eta_{pol}\frac{dT_O}{T_O} = \frac{\gamma}{\gamma - 1}\frac{dp_O}{p_O}$$

Integrating from 1 to 2 gives $$\frac{T_{O2}}{T_{O1}} = \left(\frac{p_{O2}}{p_{O1}}\right)^{\frac{\gamma - 1}{\gamma \eta_{pol}}}$$

Representative of aerodynamic quality and "technology level"

Isentropic & Polytropic Efficiency 3

Isentropic Efficiency $$\eta_{isen} = \frac{T'_{O2} - T_{O1}}{T_{O2} - T_{O1}}$$

Overall Efficiency of compressors of identical aerodynamic quality reduces as overall pressure ratio increases. In a multi stage axial compressor of equal aerodynamic quality i.e. a similar ΔT_0 per stage, the pressure ratio per stage decreases.

Example: Consider a multi stage axial compressor with 5 stages. A stage is added of the same aerodynamic quality i.e. same polytropic efficiency (85%) & ΔT_0 per stage :

Stages	OPR	Overall η_{isen}	Pressure Ratio last stage	Average Stage Pressure Rise
5	4.8	81.4	1.29	1.37
6	6.07	80.9	1.26	1.35

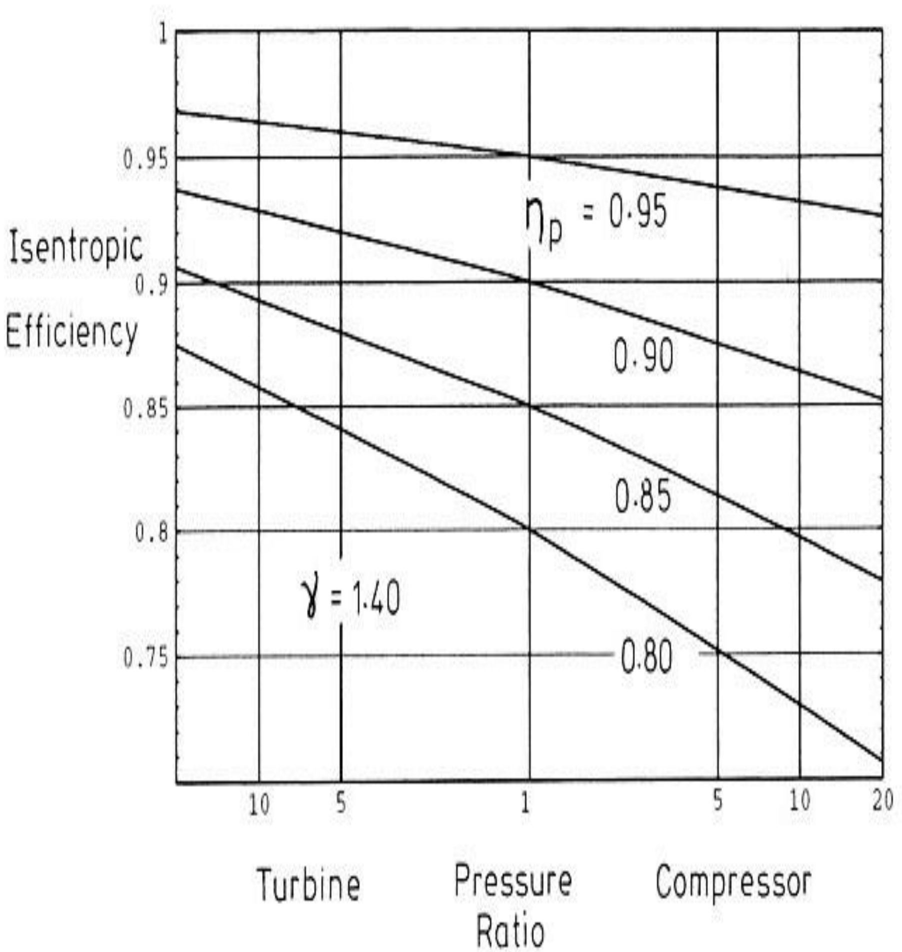
Isentropic & Polytropic Efficiency 4

Polytropic or "Small Stage" Efficiency

This removes the penalty for higher pressure ratios and allows compressors of differing aerodynamic quality and pressure ratio to be compared:

$$\frac{T_{O2}}{T_{O1}} = \left(\frac{p_{O2}}{p_{O1}}\right)^{\frac{\gamma-1}{\gamma\eta_{pol}}}$$

Polytropic Efficiency



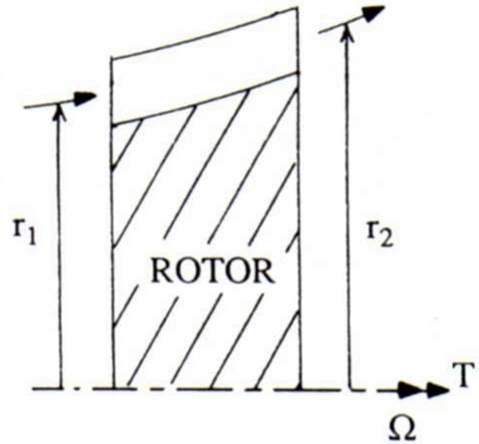
Jet Propulsion
Cumpsty, (p124)

Hence, for a fixed η_{poly} , η_{isen} reduces as pressure ratio increases

Polytropic efficiency is a more fundamental concept than isentropic efficiency and is representative of compressor technology level

Euler Work Equation

Applicable to both compressors & turbines



Consider an elemental mass $\delta\dot{m}$ entering rotor in steady flow conditions.
In time δt an equal mass must leave.

Moment of momentum of entering fluid at r_1 about axis of rotation:

$$\mathbf{M_1} = \delta\dot{m} \cdot r_1 \cdot C_{w1}$$

Corresponding moment of momentum of leaving fluid at r_2:

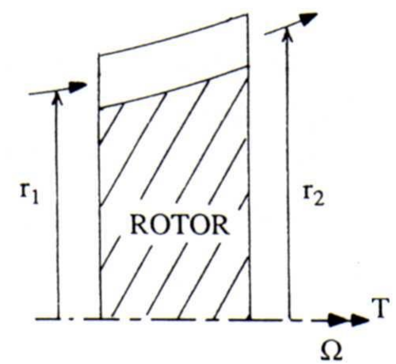
$$\mathbf{M_2} = \delta\dot{m} \cdot r_2 \cdot C_{w2}$$

Since Torque = Rate of change of moment of momentum, then:

$$T = \dot{m}\,(r_2 \cdot C_{w2} - r_2 \cdot C_{w1})$$

Euler Work Equation

Applicable to both compressors & turbines



Power is then given by the Euler Equation:

$$\boldsymbol{P_{ow} = T \cdot \omega} \quad \& \quad \boldsymbol{U = r \cdot \omega}$$

So then:

$$\boldsymbol{P_{ow} = T \cdot \omega = \dot{m}\,(U_2 \cdot C_{w2} - U_1 \cdot C_{1w})}$$

Where U_1 & U_2 are the speed of the blade row at inlet & outlet.

Work Input per unit mass is equal to the change in stagnation enthalpy per unit mass:

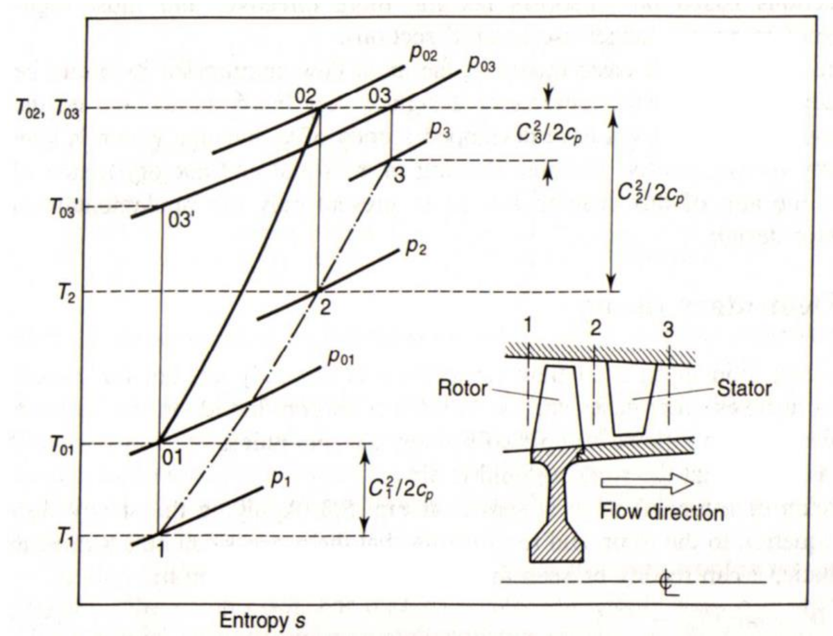
$$\boldsymbol{\Delta h_0 = C_p(T_{02} - T_{01}) = U_2 \cdot C_{w2} - U_1 \cdot C_{1w}}$$

In most cases we can assume that $r_1 = r_2$, hence $U_1 = U_2 = U$:

$$\boldsymbol{\Delta h_0 = U\,(Cw_2 - Cw_1)}$$

T - S Diagram for a Single Stage

Analysis of Single Stage for an Axial Compressor:



POWER ABSORBED

Steady Flow Energy Equation:

For *Rotor* (absorbs all power):

$$P_{ow} = \dot{m} \cdot C_p(T_{02} - T_{01})$$

For *Stator*

$$P_{ow} = 0$$

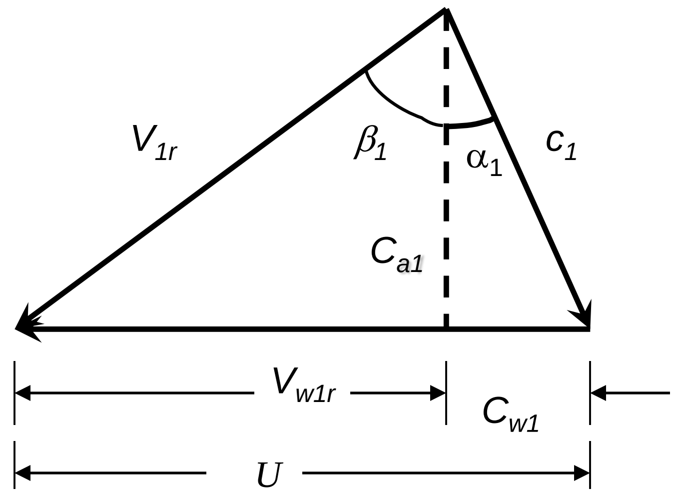
Hence $T_{03} = T_{02}$

Stator transforms KE in air to increase Static Pressure at Constant Stagnation Temperature. However there are losses due to friction.

p_{02} is less than it would have been for an isentropic process and $p_{03} < p_{02}$ due to losses in stator.

Velocity Triangles for a Compressor Rotor Blade

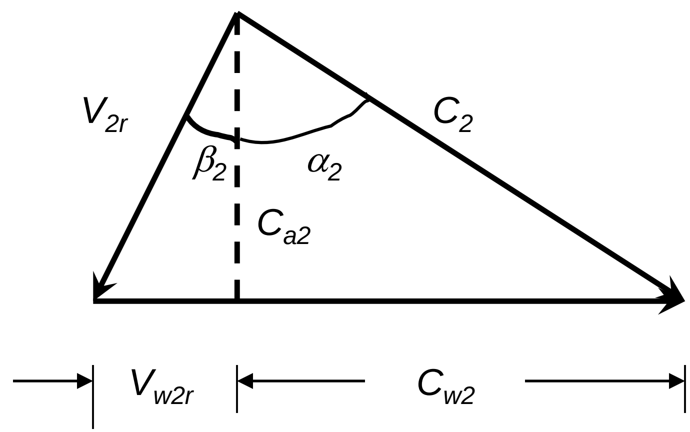
Rotor Inlet



V_{1r} & V_{2r} are the relative velocities of the incoming flow **<u>as seen from the blades</u>**

C_1 & C_2 are the absolute velocities of the flow

Rotor Exit



Thus in the rotor the relative velocity V_{1r} reduces in value to V_{2r} & is deflected by an angle $\beta_1 - \beta_2$

Velocity Triangles for a Single Stage

Rotor Speed = U

Combining the vectors of C_1 and U to give V_1 and α_1

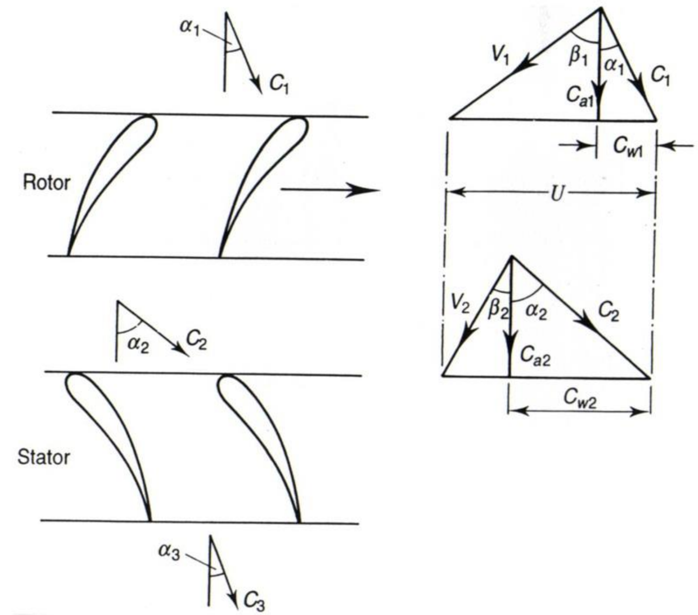
Axial velocity = $\boldsymbol{C_{a1}}$

Whirl velocity = $\boldsymbol{C_{w1}}$

Assume Constant Axial Velocity i.e.

$$C_{a1} = C_{a2} = C_{a3} = C_a$$

Hence at Exit α_3 and C_3 are the same as α_1 and C_1



Simple Blading Analysis

From the velocity triangles:

$$\frac{U}{C_a} = (Tan\,\beta_1 + Tan\,\alpha_1) = (Tan\,\beta_2 + Tan\,\alpha_2\,)$$

By consideration of Angular Momentum (see Euler Equation) it can be shown that:

$$\mathrm{P_{ow}} = \dot{m} \cdot U \cdot (C_{w_2} - C_{w1})$$

In terms of Axial velocity & Air angles:

$$P_{ow} = \dot{m} \cdot U \cdot C_a\,(Tan\,\alpha_2 - Tan\,\alpha_1)$$

or in terms of $\beta_{1:}$

$$P_{ow} = \dot{m} \cdot U \cdot C_a\,(Tan\,\beta_1 - Tan\,\beta_2\,)$$

The input Energy will be absorbed in raising pressure (useful) & overcoming frictional losses (waste).

Regardless of efficiency, Power Input must equal the rise in Stagnation Temperature of the stage.

Simple Blading Analysis

Hence:

$$\dot{m} \cdot C_p \, (T_{03} - T_{01}) = \dot{m} \cdot U \cdot C_a \, (Tan\,\beta_1 - Tan\,\beta_2\,)$$

Stage Temperature rise:

$$T_{03} - T_{01} = U \cdot \frac{C_a}{C_p} \cdot (Tan\,\beta_1 - Tan\,\beta_2\,)$$

$$\downarrow$$

$$\frac{T_{03}}{T_{01}} = 1 + U \cdot \frac{C_a}{C_p \cdot T_{01}} \cdot (Tan\,\beta_1 - Tan\,\beta_2\,)$$

Isentropic Efficiency:

$$\eta_s = \frac{T'_{03} - T_{01}}{T_{03} - T_{01}}$$

Hence Stage Pressure rise:

$$\frac{p_{03}}{p_{01}} = \left(1 + \eta_s \frac{(T_{03} - T_{01})}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} \qquad \frac{p_{03}}{p_{01}} = \left(1 + \frac{\eta_s UCa(\tan\beta_1 - \tan\beta_2)}{C_p T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$$

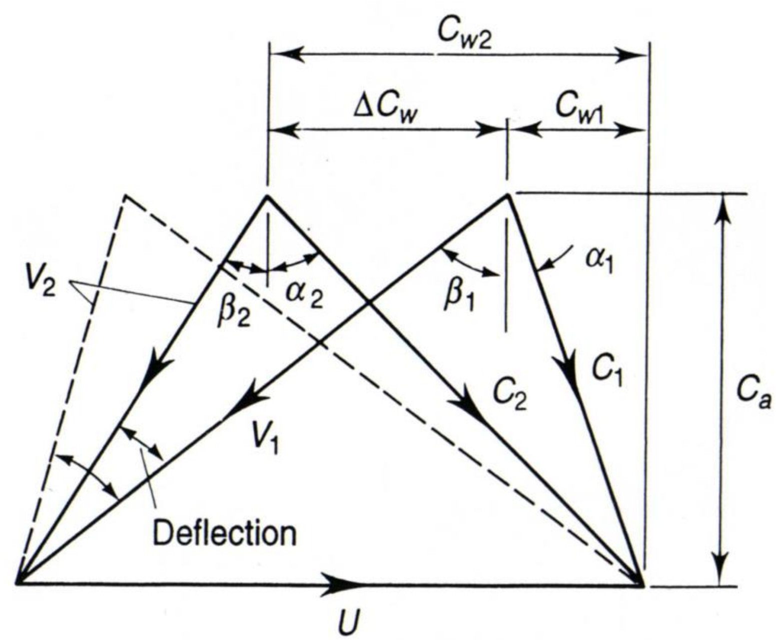
Or using Polytropic Efficiency:

$$\frac{p_{03}}{p_{01}} = \left(1 + \frac{UCa(\tan\beta_1 - \tan\beta_2)}{C_p T_{01}}\right)^{\frac{\eta_p \gamma}{\gamma-1}}$$

Factors effecting Stage Pressure Ratio

$$\frac{p_{03}}{p_{01}} = \left(1 + \frac{UCa(\tan\beta_1 - \tan\beta_2)}{C_p T_{01}}\right)^{\frac{\eta_p \gamma}{\gamma - 1}}$$

- Blade Speed
- Axial velocity
- High Deflection in Rotor Blades
- Efficiency

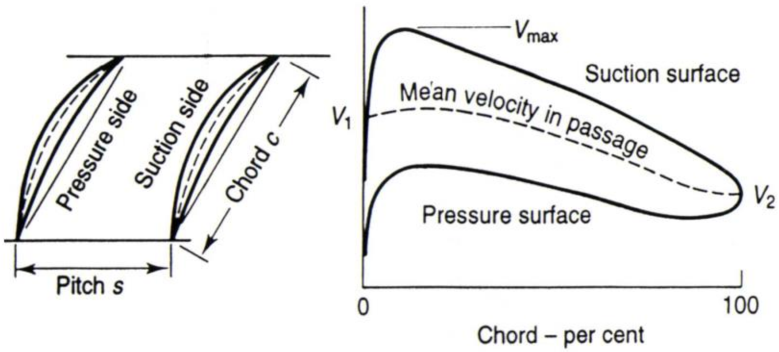


de Haller Number = V_2 / V_1

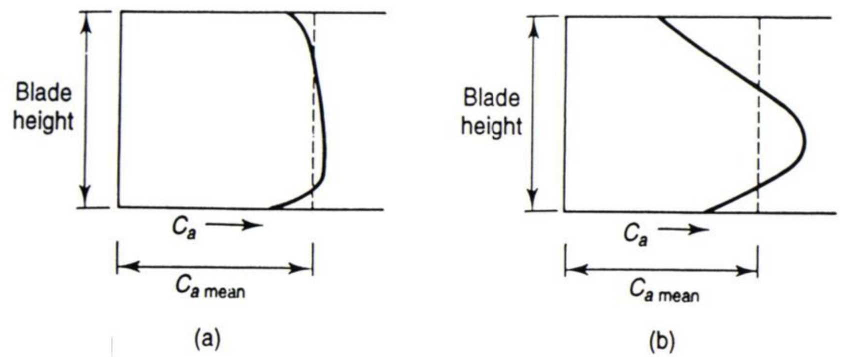
For minimum losses $V_2 / V_1 > 0.72$

Effect of increasing deflection

Blade Spacing & Blockage

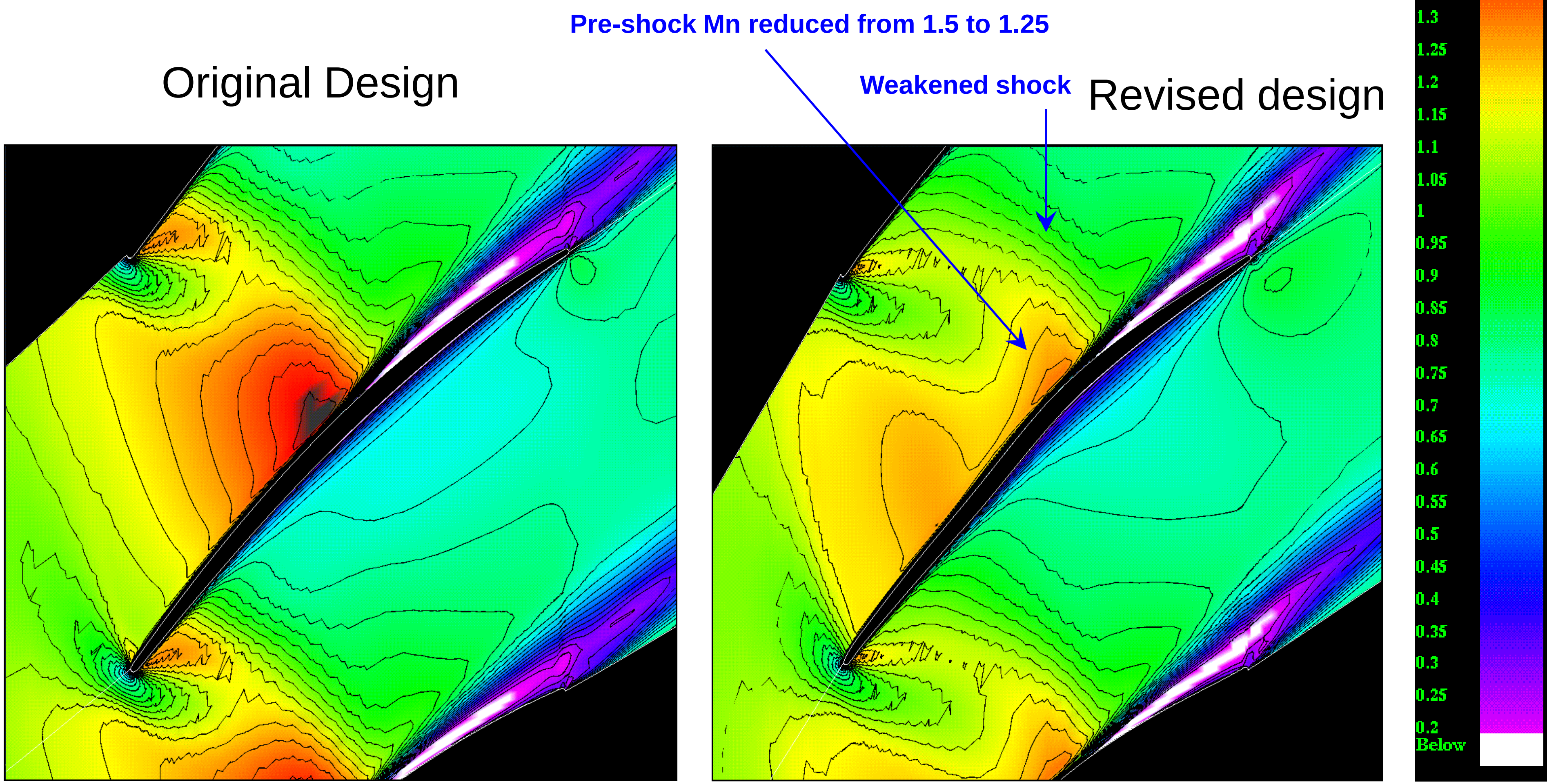


Blade spacing and velocity distribution through passage



Axial velocity distributions: (a) at first stage, (b) at fourth stage

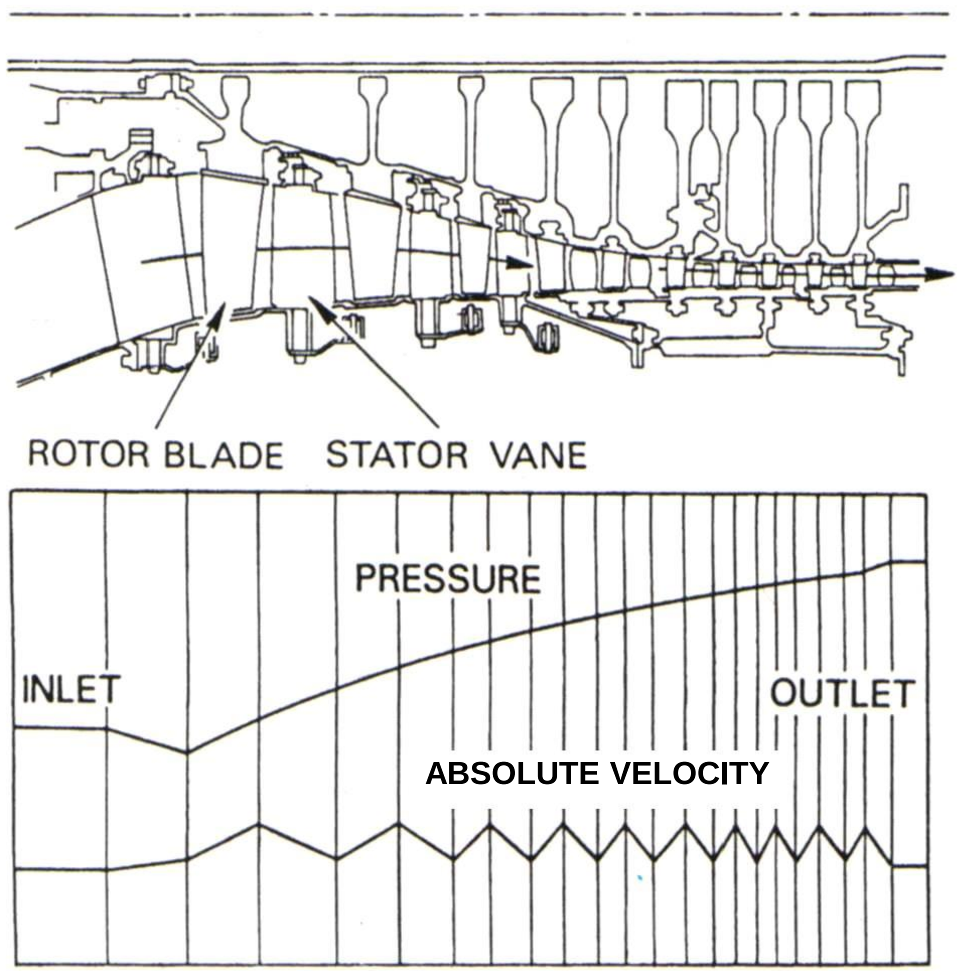
High Mach numbers - aim to control Peak Mach number
Use of CFD to improve Design



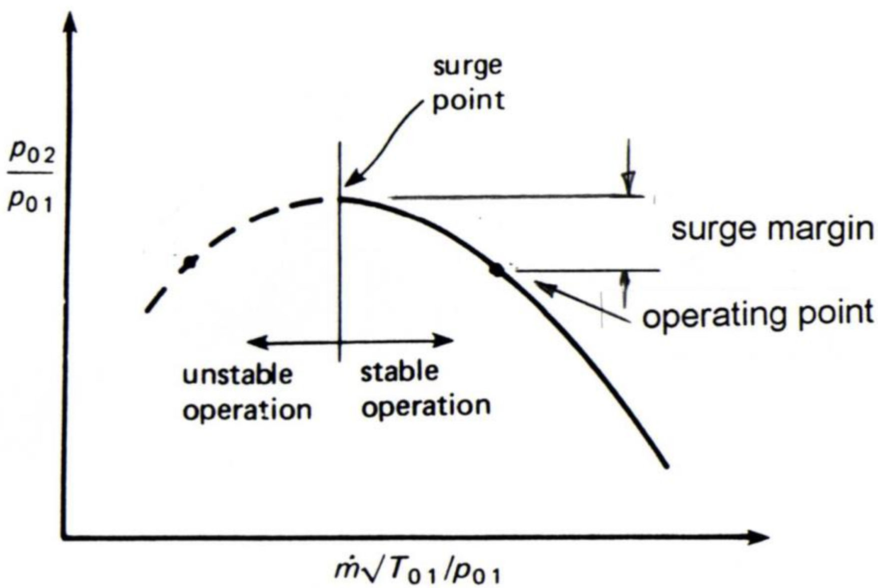
Dimensional Relationships

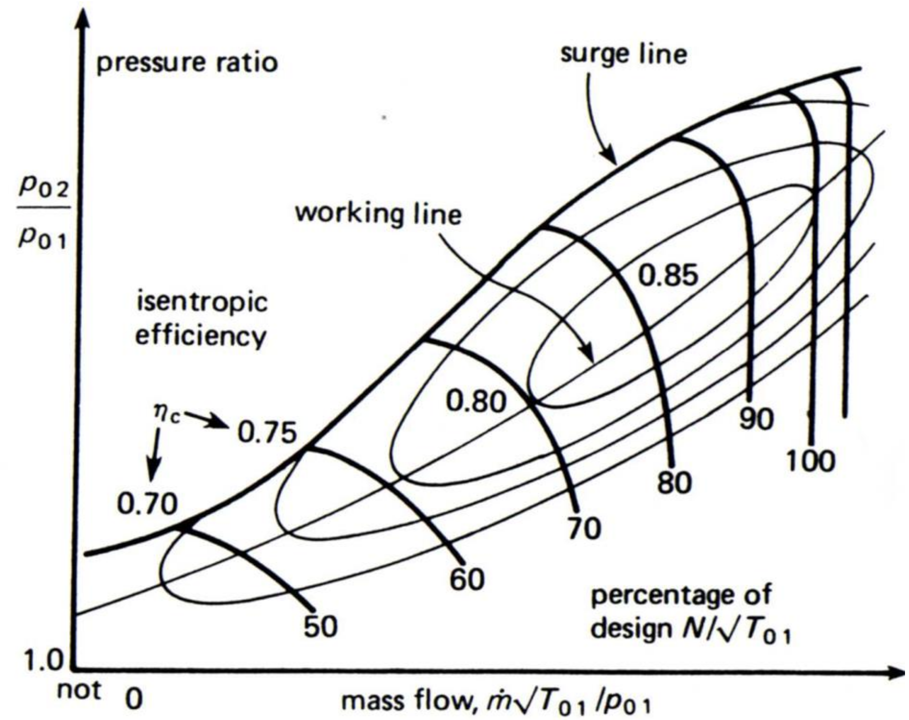
	Non-Dimensional	Quasi-dimensionless Group
1. Flow Velocity	$\dfrac{C}{\sqrt{\gamma RT}}$	$\dfrac{C}{\sqrt{T}}$
2. Rotational Speed	$\dfrac{ND}{\sqrt{RT_O}}$	$\dfrac{N}{\sqrt{T_O}}$
3. Mass Flow	$\dfrac{m\sqrt{RT_O}}{Ap_O}$	$\dfrac{m\sqrt{T_O}}{p_O}$

Pressure & velocity changes through an axial compressor



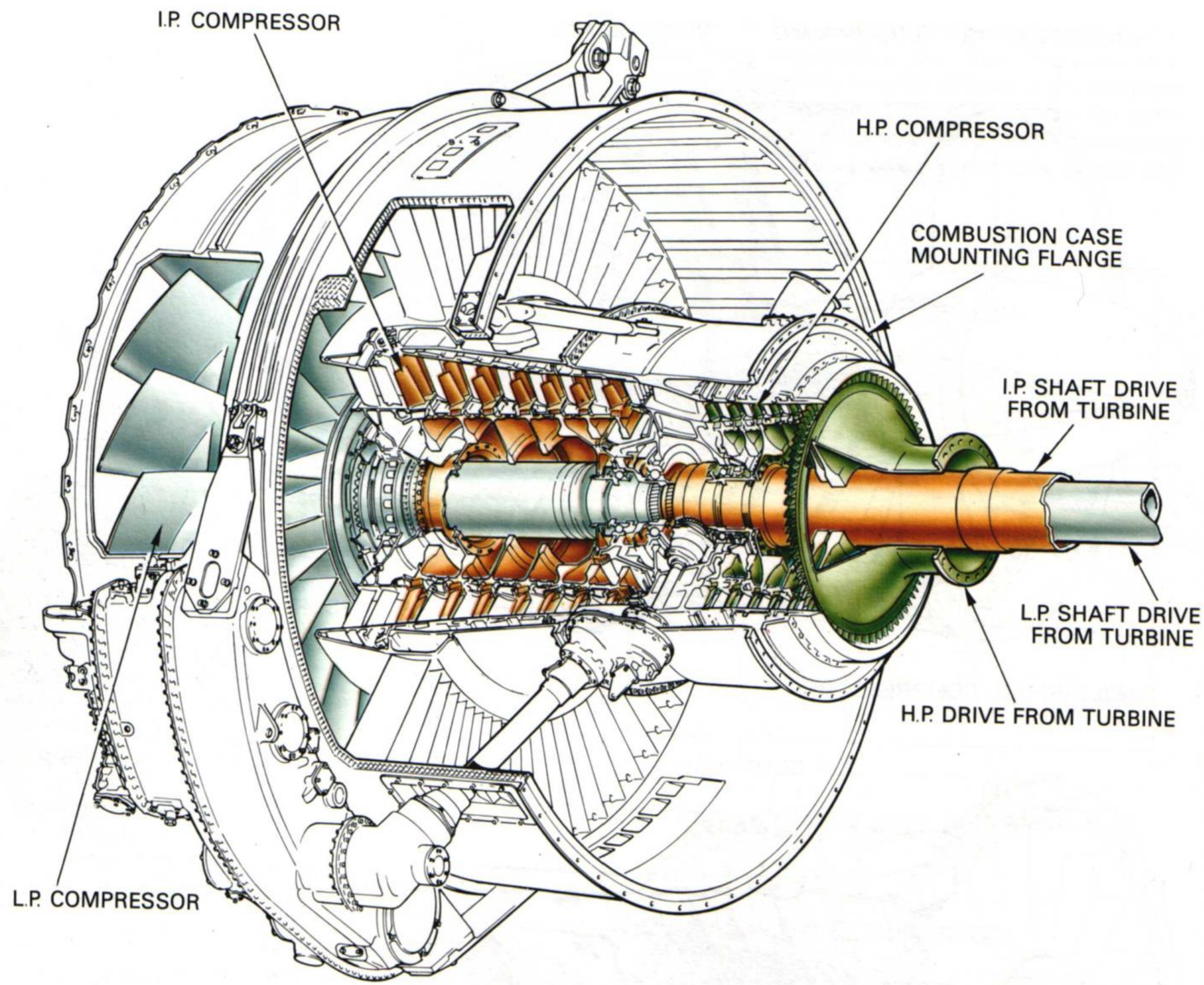
Overall Compressor Characteristics





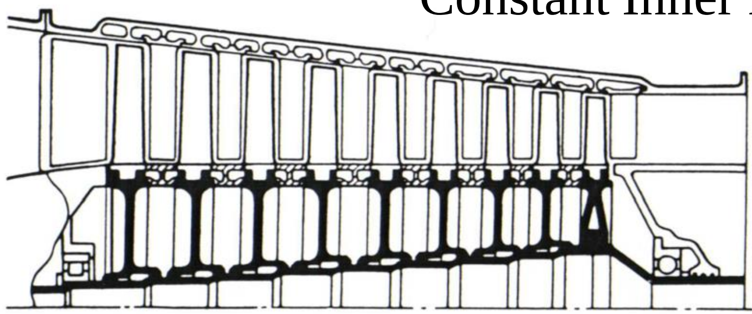
Simplified constant speed characteristic

Typical three spool compressor

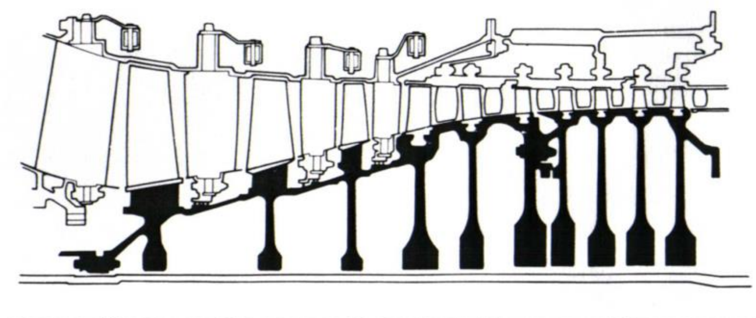


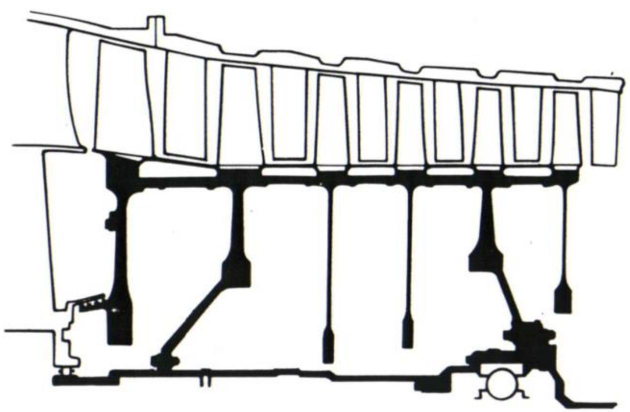
Types of Axial Compressor Construction

Constant Inner Diameter



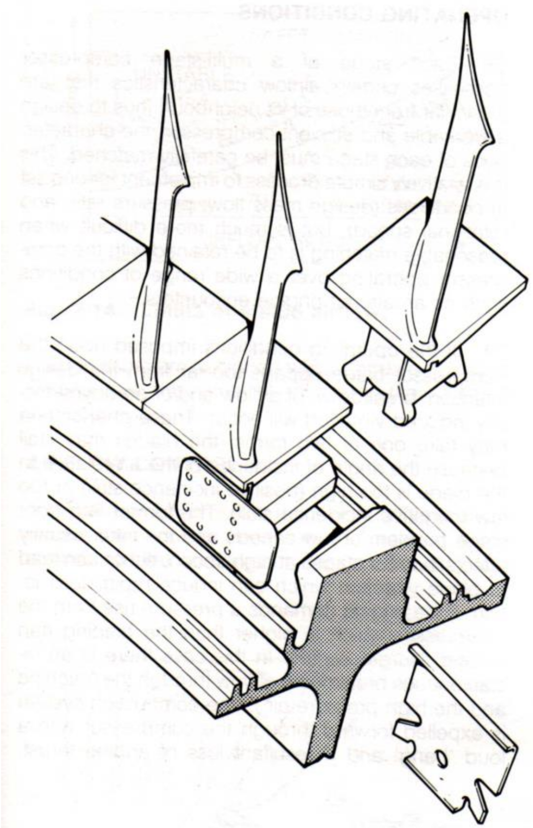
Constant Outer Diameter

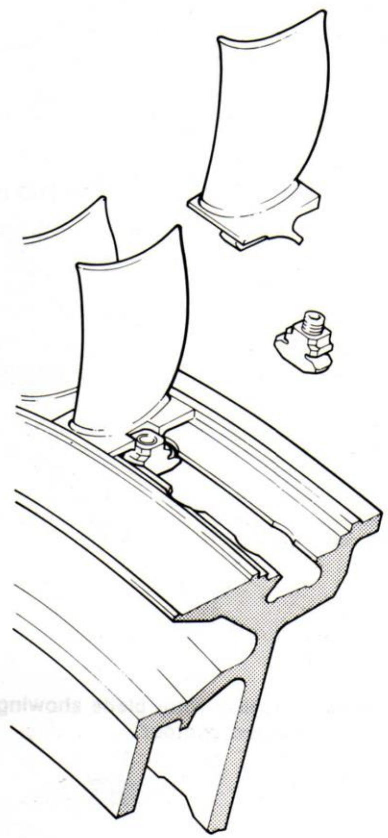


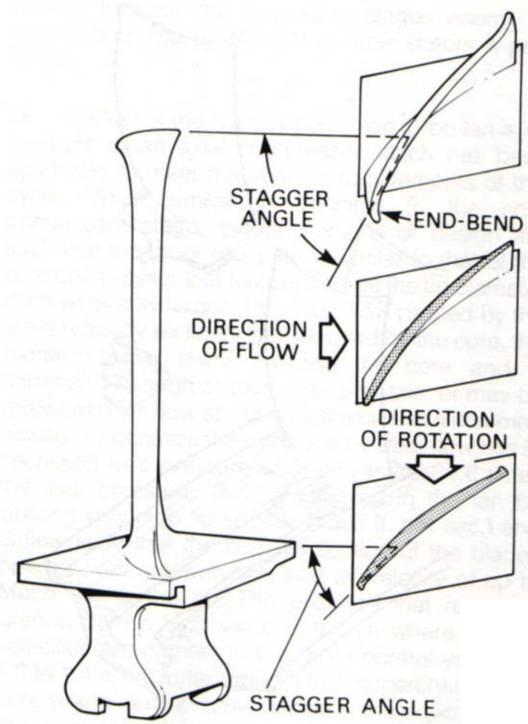


Constant Mean Diameter

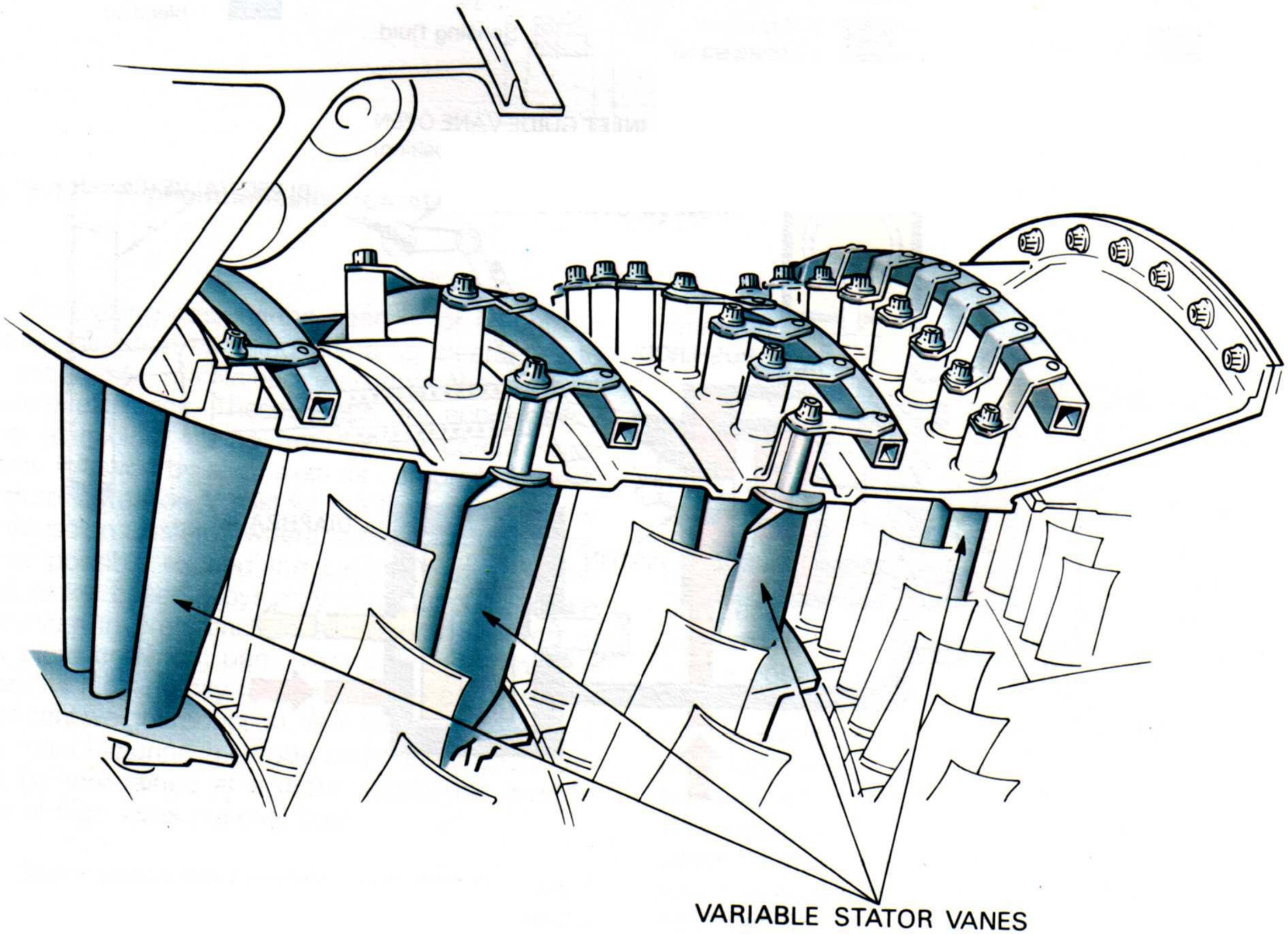
Typical Rotor Blades & Methods of Fixing



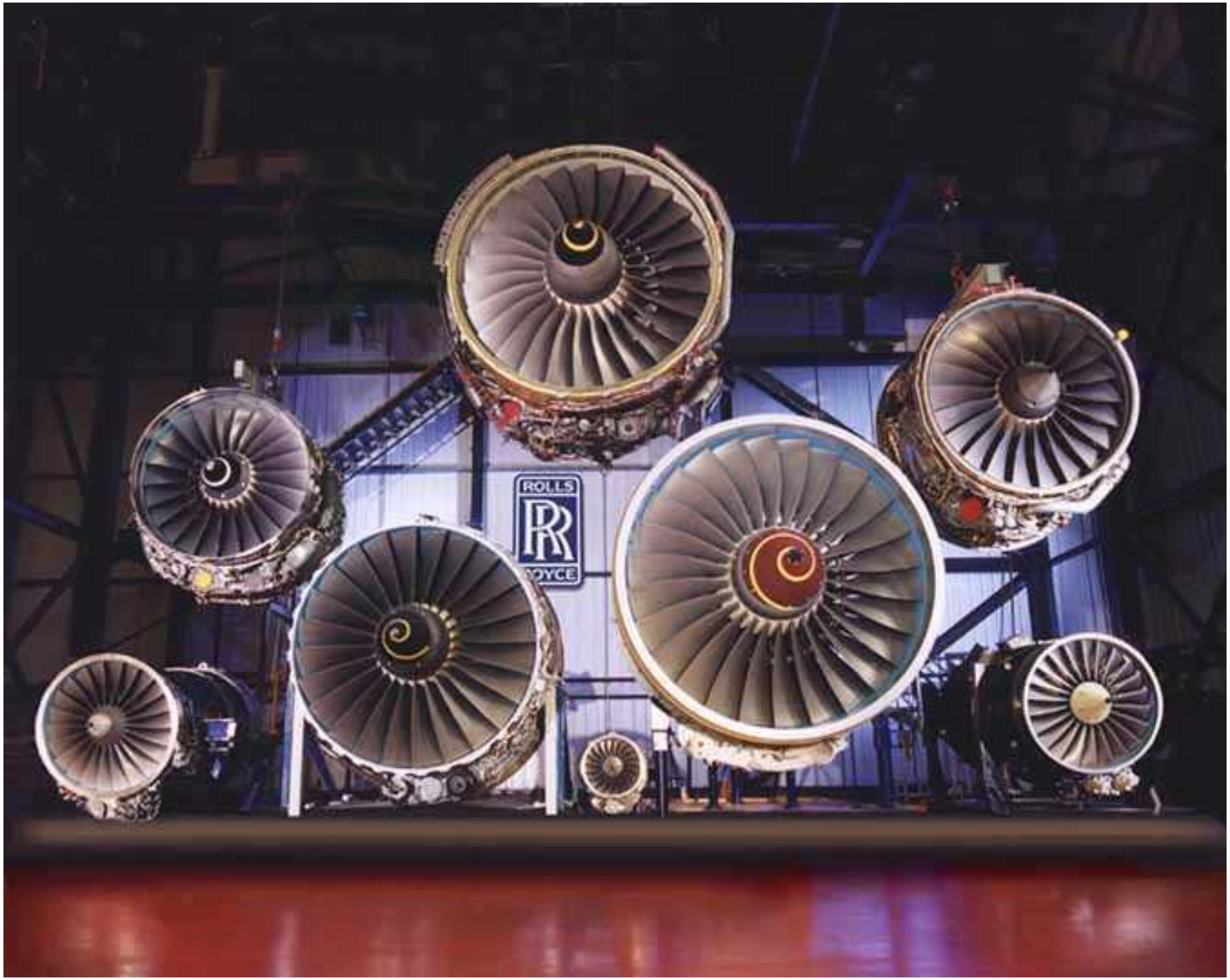




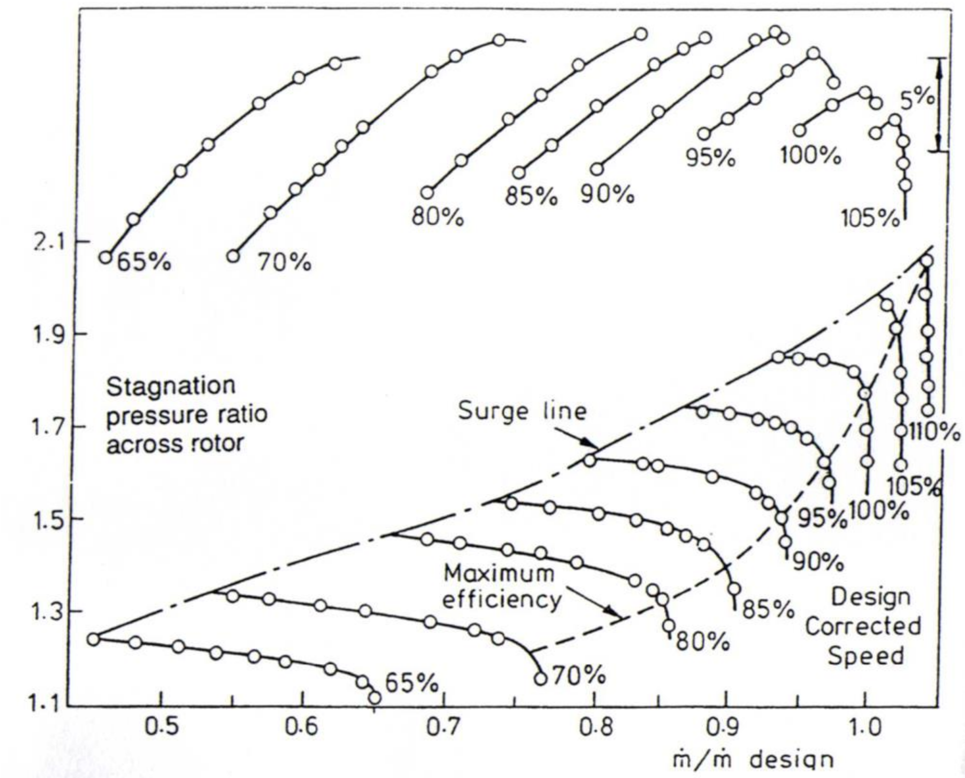
Variable Stator Vanes



FANS



Single Stage Fans

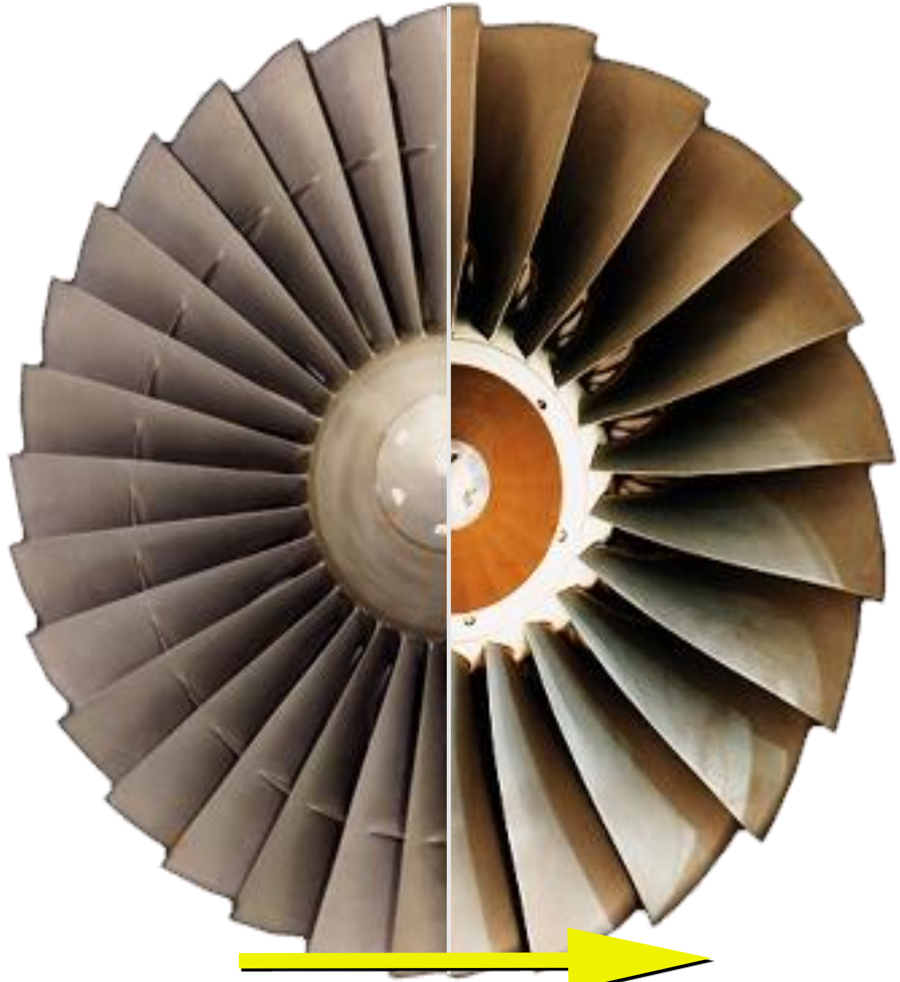


Typical fan characteristic

Fan blades

Fan Blade technology

Clappered



Improved efficiency
reduced weight
reduced noise

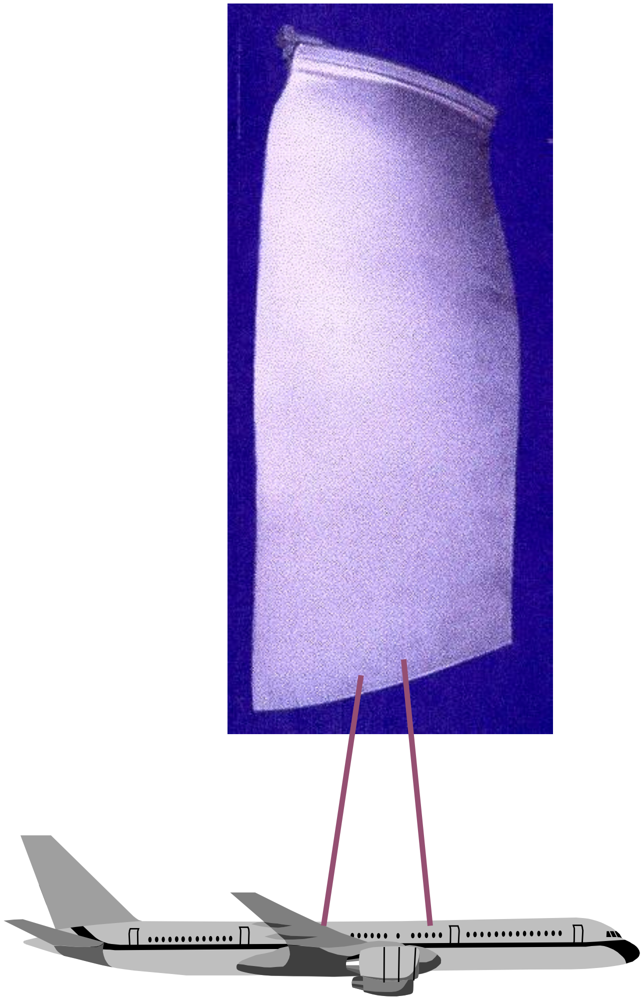
Wide-chord fan

Fans - Some Numbers

Large Civil Turbofan Engine

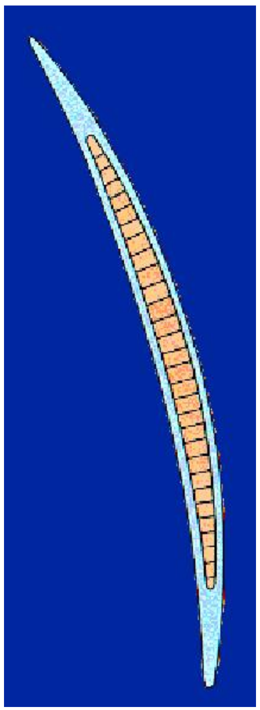
• Thrust	355kN
• Mass flow	1125kg/sec
• Volume flow	$990 m^3$/sec
• Shaft speed	3200 rpm
• Fan Diameter	2.8m
• By-pass ratio	Approx 8:1

- Centrifugal force on each blade equivalent to the weight of a fully laden B757 aircraft



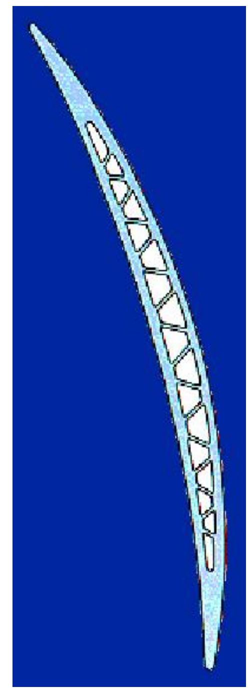
Wide Chord Fan - Hollow Construction

1st generation:
-535E4, -524G,
V2500
EIS 1984



Honeycomb
construction

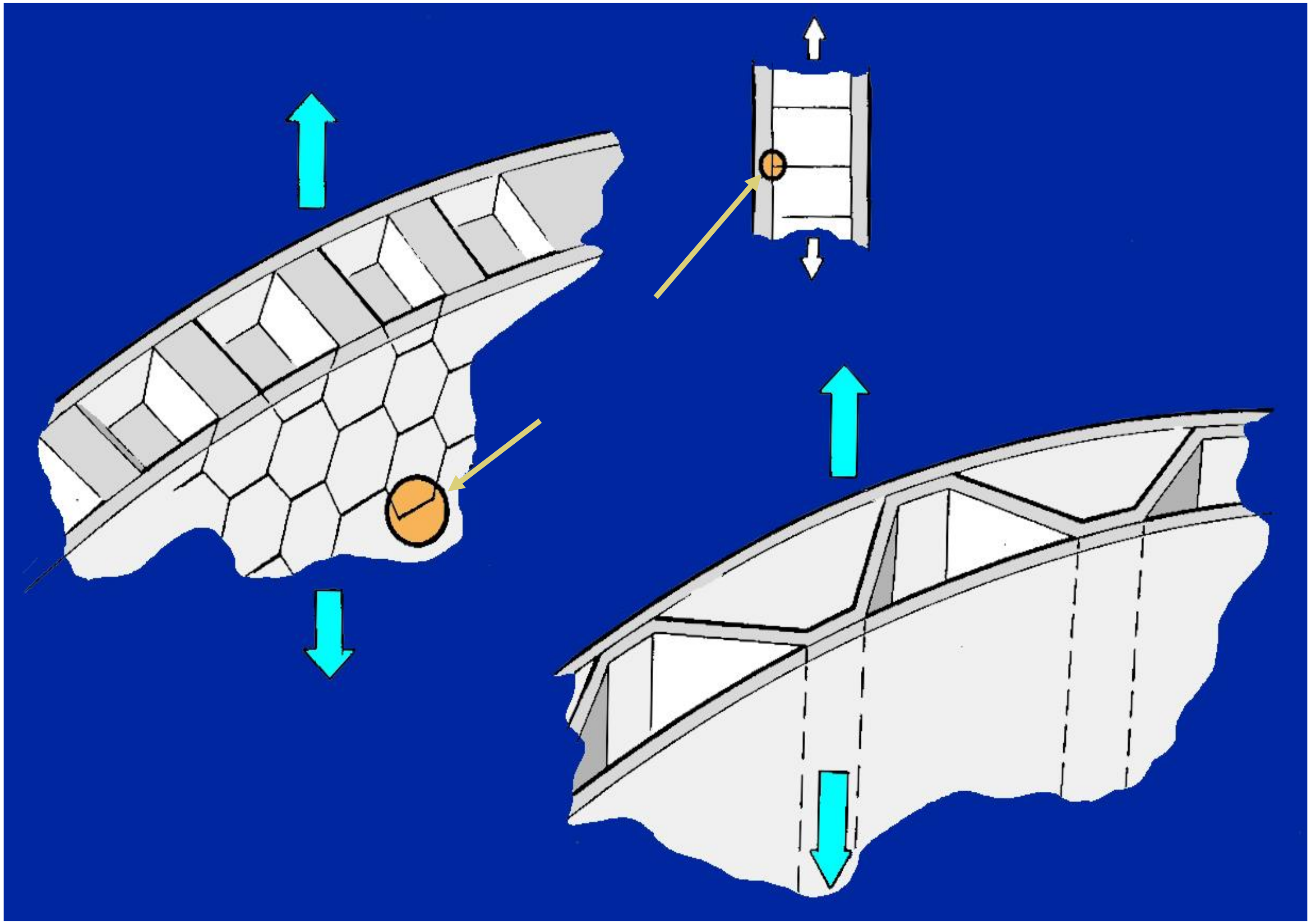
2nd generation
Trent
EIS 1995



DB/SPF
construction

Honeycomb and DB/SPF core comparison

Stress concentration



Radial stress

Honeycomb core

DB/SPF line core

The Next Step

Objective Lecture 6: ***To describe the workings of a Centrifugal Compressor***