Structures Lab

0 Introduction

This set of laboratory experiments is intended to give a brief hands-on introduction to simple structural elements, load conditions and resulting stress, strain and deformation. Some terms are introduced below:

Structural Elements can be categorised according to their loading and function as: bars, beams, plates and shells. In these experiments only bars and beams are considered. Bars are elements which predominantly carry only axial force, *i.e.* tension or compression forces along their length. Bars which carry only tensile force are called 'ties'. Bars which carry only compressive force are called 'struts'. Other general names for bars are 'trusses' or 'rods'. Beam elements can carry transverse loading as well as axial loading and work in bending and shear.

Static Equilibrium requires that the resultant of applied and reacted forces or moments acting on a structure or any part of it must be zero. This means that the sum of resolved forces in a given direction or the sum of moments about a given point must equate to zero, *e.g.* $\sum F_x = 0$, $\sum F_y = 0$, $\sum M = 0$.

Free Body Diagrams (FBD) are diagrams drawn to visualise the structure or part of the structure and the loads acting on it where applied forces, reactions or the influence of another part of the structure are represented by force and moment vectors. Considering a part of a structure separated from the remainder by a 'notional' cut at a section allows us to evaluate the internal forces and moments that must exist at that section for equilibrium.

Engineering Stress, σ 'sigma' or τ 'tau', is a measure of load distribution in a structural element as force per unit area, where the area is the original unloaded cross-sectional area. Units of load per unit area are used, *e.g.* N/mm^2. For 'direct stress' the area is taken as the cross-sectional area perpendicular to the normal force,

$$\sigma = F_N / A_N \tag{0-1}$$

For shear stress the area is taken as the cross-sectional area parallel to the shear force,

$$\tau = F_S / A_S \tag{0-2}$$

Engineering Strain, ε, 'epsilon' is the ratio of the change in length of a structural element under load to the original unloaded length,

$$\varepsilon = \frac{\Delta L}{L_0} \tag{0-3}$$

Strain is dimensionless because it is simply the ratio of two lengths, *e.g.* mm/mm. For engineering applications strains are usually small and the ratio is often expressed as 'micro-strain' $\mu\varepsilon$, *i.e.* $1\,\mu\varepsilon = 10^{-6}$.

Young's Modulus, E, is the constant of proportionality which relates stress to strain according to Hooke's Law, *i.e.*

$$\sigma = E\,\varepsilon \tag{0-4}$$

A similar linear law can be written for shear stress and shear strain, *i.e.*

$$\tau = G\gamma \tag{0-5}$$

Second Moment of Area, I, is a cross-sectional area property of a structural element which provides a measure of the area and its distribution with respect to an axis through the cross-section (usually through the centroid of the cross-section). For a simple rectangular cross-section of width b and depth d,

$$I = \frac{bd^3}{12} \tag{0-6}$$

For other sections the area property can be obtained by the 'parallel axis theorem',

$$I_{XX} = \int y^2 \, \mathrm{d}A \tag{0-7}$$

The Neutral Axis, NA, is a cross-section axis of a beam or strut about which bending occurs. The neutral axis passes through the centre of area (centroid) of the cross-section. Bending stresses vary from tension to compression about this axis. On the neutral axis the bending stress will be zero. Cross-sectional area further from the neutral axis will have a greater moment arm to resist bending (this explains the common use of I-section beams).

0.1 Apparatus

0.1.1 Dial gauges

Analogue and digital dial gauges are used to measure deflection in these experiments. To ensure initial contact and sufficient travel the dial gauges must be positioned to give a non-zero reading (on the digital dial gauges this reading can subsequently be re-zeroed).

0.1.2 Load cells

On some of the experiments load cells are used instead of weights. The load cells consist of loading rings with deflection calibrated in terms of applied load and digital readout in Newtons.

0.1.3 Strain gauges

Strain gauges are fine wire sensors that experience a change in electrical resistance when they are stretched or compressed. Bonded to the surface of a structural element the strain gauges stretch or compress by the same amount as the element under load. Hence, the strain in a member can be determined from the calibrated electrical resistance change of the strain gauge. The strain gauges are connected to digital strain displays where the output is given as micro-strain $\mu\varepsilon$, *i.e.* $1\,\mu\varepsilon = 10^{-6}$.

Note: You should take care not to exceed any of the prescribed loads or displacements to avoid permanent deformation of the structural elements and damage to the transducers.

0.2 Method of working

Working in pairs you will rotate through the different experiments. Each pair will consider a set of conditions at each experiment and record readings and observations on the worksheet at that station. At the end of the laboratory session you should have a full set of measured results for each experiment. You should use any spare time between experiments to process your work sheets. You will need to keep your completed work sheets for further consideration in the Structures lectures and assignments.

0.3 Tasks

Worksheets – each student is required to complete a set of worksheets for the experimental analysis and theoretical predictions of each experiment. You should retain the completed worksheets for use in lectures, examples classes and online tests.

Online Report & Questionnaire – each student is required to complete an online report and a series of online tests on Blackboard. These will be made available towards the end of TB1 – please look out for email announcements.

1 Determinate 2D Pin-Jointed Truss

1.1 Background

Pin-jointed truss structures consist of linear elements connected at their ends through joint fittings which are free to rotate *i.e.* 'pinned'. They create a rigid structural form by 'triangulation' *i.e.* by forming sets of closed triangles.

If there are just enough members to create a triangulated structure it is described as 'determinate'. In this case the loads in each member may be calculated simply from equations of static equilibrium at the joints or at any convenient section through the structure.

If there are more members than needed for basic triangulation or excess supports then the structure is known as 'indeterminate' and in this case the relative stiffness of each member and the relative connectivity must be accounted for as well as the equations of statics in order to calculate the member loads.

If there are too few members or supports then the structure will behave as a mechanism and will translate or rotate as a rigid body, being unable to react the external applied loading.

1.2 Objectives

In this experiment you will measure internal member loads and the structural deflection of a statically determinate truss structure subjected to external loading and compare experimental results with theory.

The force in each member can be calculated from the strains given by the digital strain display using the relationships:

$$\sigma = F / A \tag{1-1}$$

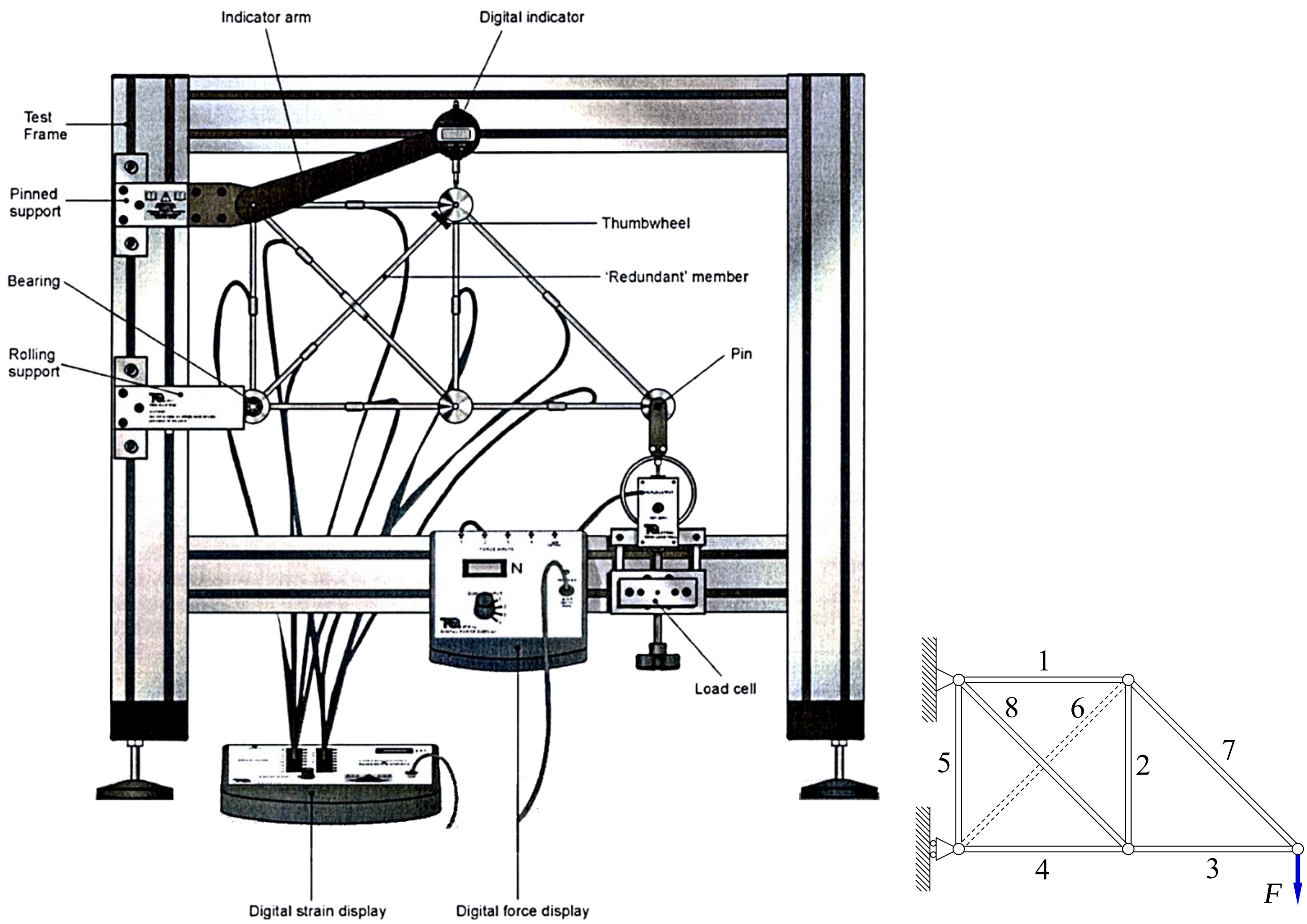
where σ is the direct stress (N/mm^2), F is the force (N) and A is the cross-section area (mm^2) of a member, and

$$\sigma = E\varepsilon \tag{1-2}$$

where E is the Young's Modulus (N/mm^2) and ε is the measured strain (dimensionless) of a member.

Using the above expressions you should evaluate a factor to convert your micro-strain $(\mu\varepsilon)$ readings to obtain force values. Forces should also be calculated from theory of equilibrium using the method of joints or method of sections.

1.3 Apparatus



(a) (b)

Figure 1-1: The truss apparatus, (a) full device and (b) equivalent truss structure.

The experiment consists of a fixed framework of stainless steel rods connected by bosses. The 'redundant' member should be disconnected from the framework. The framework is mounted on two supports, one allowing pivoting only (pinned), the other allowing pivoting and linear translation (roller). An electronic load cell measures the force applied to the truss. The force can be applied by turning the hand-wheel underneath the load cell and the load (in Newtons) read from a digital display. A digital indicator above the truss shows its vertical displacement. Each one of the members in the truss has a 'strain gauge' bonded to its surface, numbered 1 to 8 as shown in Figure 1-1(b).

1.4 Tasks

Complete work sheets, consider experimental and theoretical results and note sources of errors.

Member loads and deflections of a statically determinate 2D pin-jointed truss structure

Tasks

Ensure the redundant member is released to make a statically determinate structure.

Apply loadings, record strains and deflections and complete the table of values. (Correct strains by subtracting mean offset value at zero load)

(Note: a preload of 50N has already been applied and the load cell zeroed to avoid loading through a range which might include slackness in the joints. The load cell zero should not be altered)

Plot member strains vs structural load for each member on the same graph.

Plot structural load vs deflection.

Calculate member loads at the maximum applied structural loading from the measured strain values using $\sigma = E\,\varepsilon$ and $F = \sigma\,A$

Calculate member loads at the maximum applied structural loading using the method of joints or method of sections.

Compare values of member loads at maximum applied structural loading as derived from strain measurements and equilibrium methods.

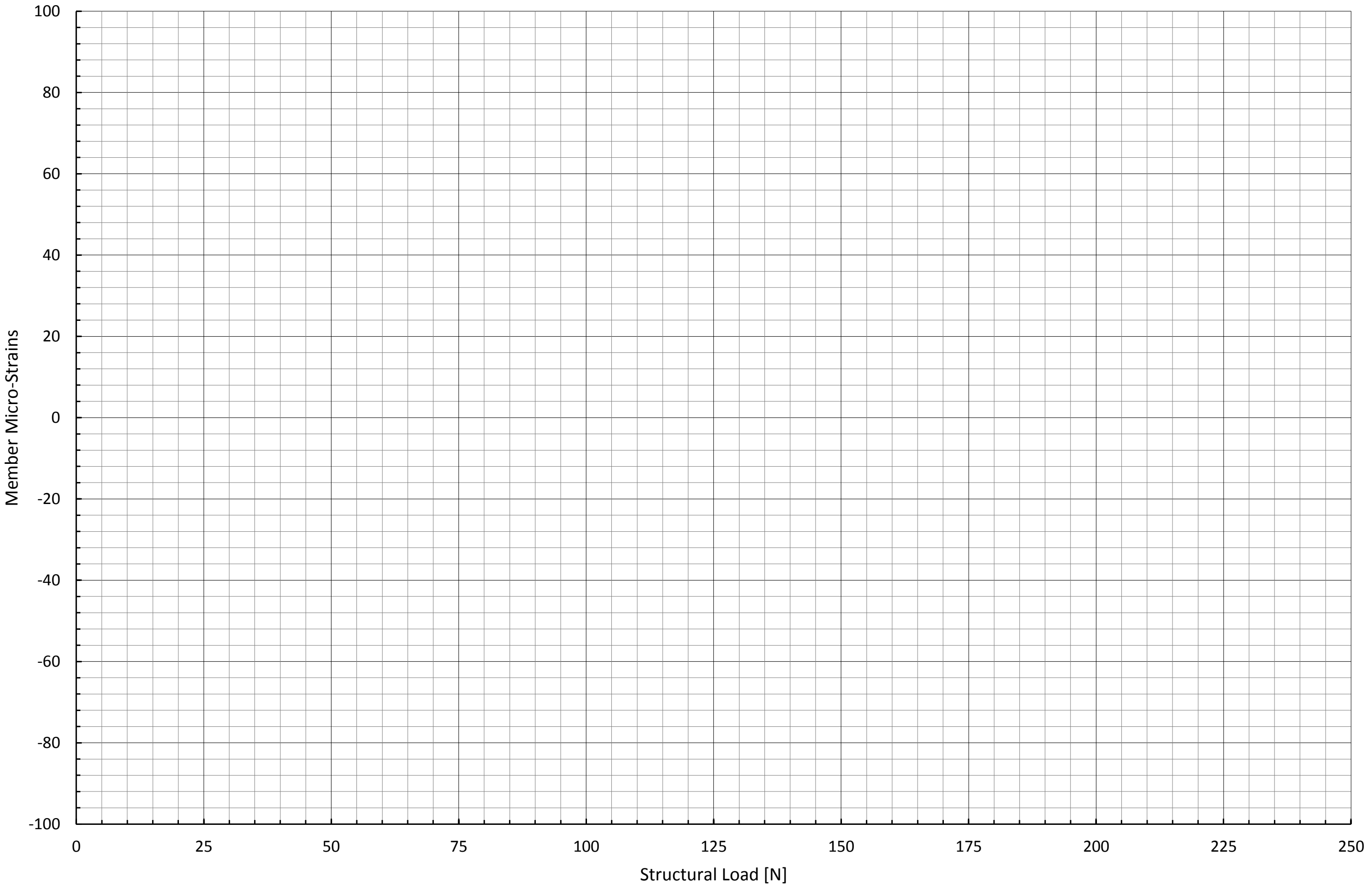
Comment on linearity and differences between values derived from measurement and from theory.

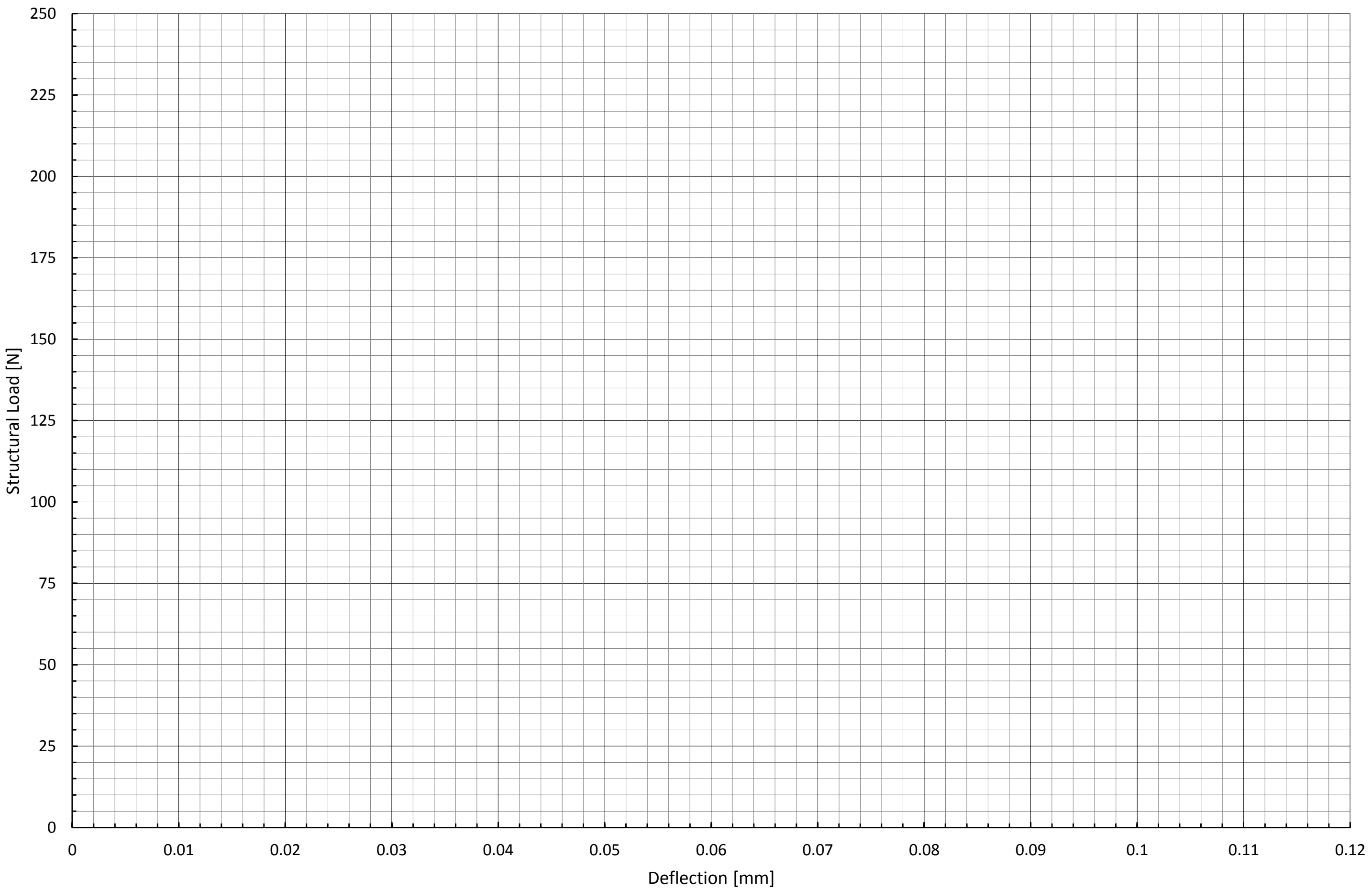
Details

Material =	Stainless steel	Young's Modulus, E =	210 000 N/mm^2
Member diameter D =	5.985 mm	Member cross-sectional area, A =	mm^2
Vertical member length, L_V=	240 mm	Horizontal member length, L_H =	240.00 mm
Diagonal member length, L_D=	339.411 mm		

Applied structural load N		Experimental: Member strains $\mu\varepsilon$, stress and load 1	2	3	4	5	6	7	8	Structure deflection mm
0*	Before loading									
	After loading									
	Mean @ 0 Load									
50	Measured									
	Corrected									
	Stress N/mm ^{2}									
	Load N									
100	Measured									
	Corrected									
	Stress N/mm ^{2}									
	Load N									
150	Measured									
	Corrected									
	Stress N/mm ^{2}									
	Load N									
200	Measured									
	Corrected									
	Stress N/mm ^{2}									
	Load N									
250	Measured									
	Corrected									
	Stress N/mm ^{2}									
	Load N									

	Theoretical							
Member load at 250 N								


Member Micro-Strains
100
80
60
40
20
0
-20
-40
-60
-80
-100
0
25
50
75
100
125
150
175
200
225
250
Structural Load [N]


Structural Load [N]
250
225
200
175
150
125
100
75
50
25
0
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
0.11
0.12
Deflection [mm]

2 Shear Force and Bending Moment in a Beam

2.1 Background

For a transversely loaded beam to be in equilibrium there must exist an internal shear force and bending moment at any section of the beam to balance the external applied loads. Using the concept of structural sections and free body diagrams each free body created by a notional cut at any section must be in equilibrium. In this experiment a notional cut is simulated in a beam where the shear force and bending moment are measured by transducers at the cut.

Static equilibrium requires that:

- 'the **shear force** at a 'cut' is equal to the algebraic sum of the forces acting to the left or right of the cut', or equilibrium of transverse forces, $\sum F_y = 0$.
- 'the **bending moment** at a 'cut' is equal to the algebraic sum of the moments caused by the forces acting to the left or right of the cut', or equilibrium of moments, $\sum M = 0$.

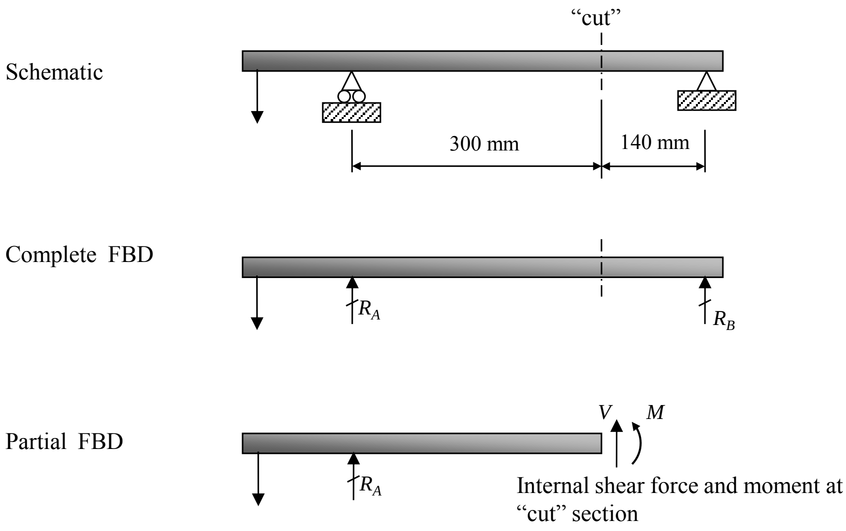


Figure 2-1: Shear forces and bending moments.

2.2 Objectives

In this experiment you will measure the shear load and bending moment at a simulated notional cut in a beam for different loading conditions and compare with theoretical equilibrium calculations.

2.3 Apparatus

2.3.1 Shear force

The setup consists of a beam which is 'cut' as shown in Figure 2-2. To stop the beam collapsing a mechanism (which allows movement in the shear direction only) bridges the cut on to a load cell thus reacting (and measuring) the shear force. A digital display shows the force from the load cell.

2.3.2 Bending Moment

The setup consists of a beam which is 'cut' by a pivot as shown in Figure 2-3. To stop the beam collapsing a moment arm bridges the cut onto a load cell thus reacting (and measuring) a force that can be used to deduce the bending moment. A digital display shows the force from the load cell.

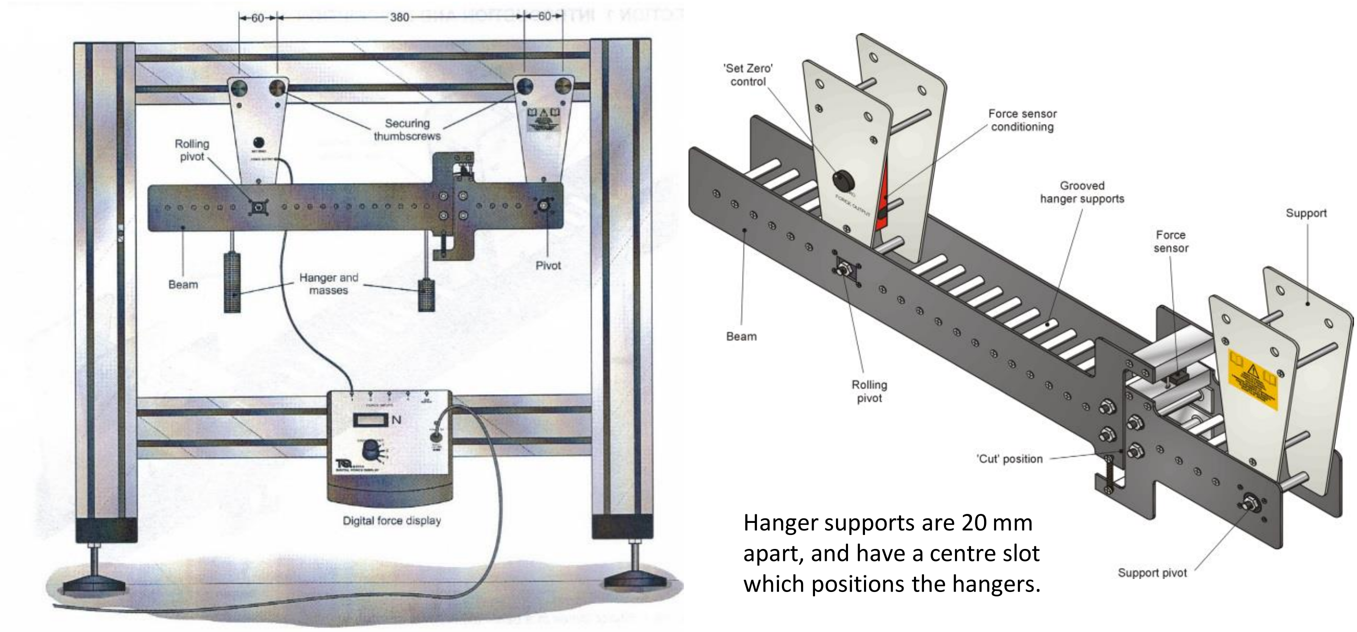


Figure 2-2: Shear force setup.

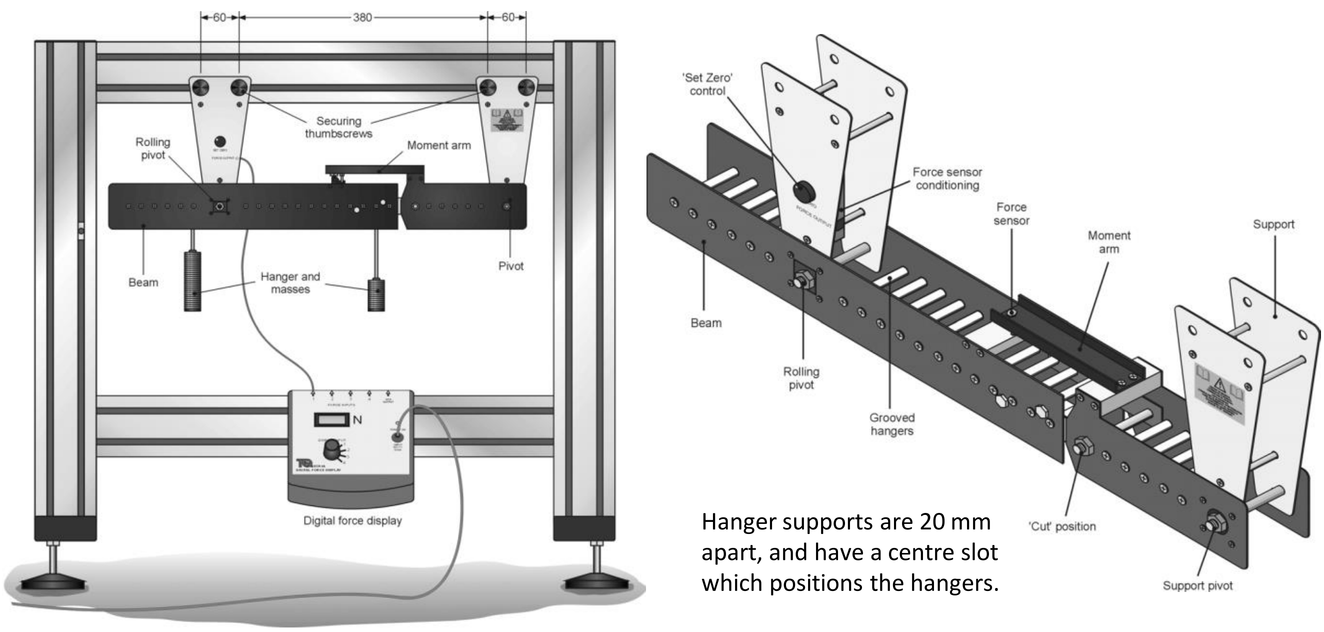


Figure 2-3: Bending moment setup.

2.4 Tasks

Complete work sheets, consider experimental and theoretical results and note sources of errors.

2. Shear force and bending moment in a beam under transverse loading

Tasks

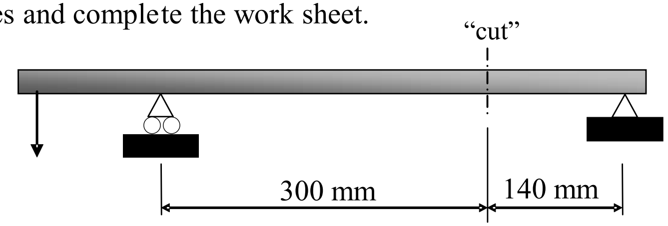
Zero the load cell for the unloaded beam

Apply loading configurations a,b,c to each beam, record the shear force and bending moment transducer force values and complete the work sheet.

Use equilibrium of forces and moments to calculate reactions at supports and forces and moments at the cut section

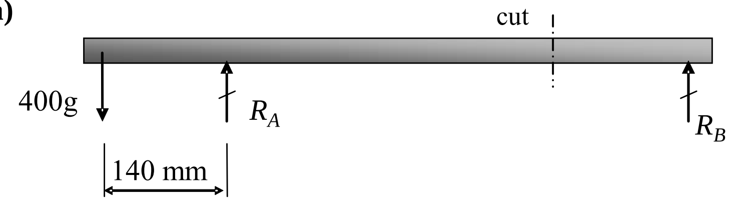
Details

Bending moment NM = BM transducer force 0.125

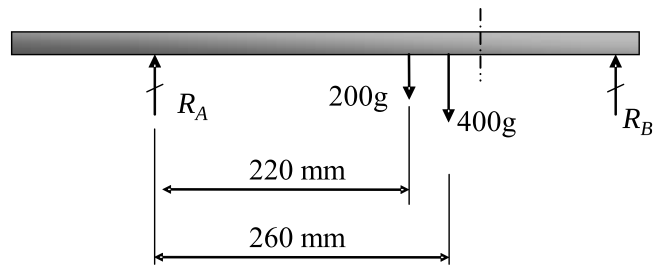


Configuration	Experimental			Theoretical			
	Internal loads at the cut section			Calculated reactions at supports		Internal loads at the cut section	
	SF N	BM transducer force N	BM Nm	R_A N	R_B N	SF N	BM Nm
a							
b							
c							

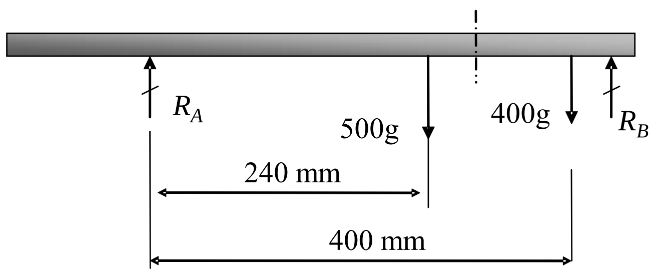
(a)



(b)



(c)



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3 Bending Deflections of Rectangular Section Beams

3.1 Background

The deflection of a beam depends on the cross-sectional area and its distribution about a 'neutral axis'. The neutral axis of a symmetric section loaded along a line of symmetry will pass through the centroid of the cross-sectional area in a direction perpendicular to the plane of loading. For the area to be fully effective then it must be able to resist 'shear' as well as bending, *i.e.* layers of the beam must not be able to slide relative to one another.

The deflection of a beam under transverse load will also be significantly influenced by the end support conditions. The end support conditions can be idealised as free (free to rotate or translate) pinned (free to rotate but not translate) or fixed (prevented from rotating or translating). Perfectly rigid fixed end conditions are in fact difficult to achieve because there will always be some flexibility in the support arrangement.

When there are just enough support conditions to prevent a beam from moving as a rigid body (rotating or translating) the beam is described as determinate and the internal loads and moments and support reactions can be found simply from equations of static equilibrium.

When there are more support conditions than needed to prevent rigid body rotation or translation then the beam is described as indeterminate. In this configuration the stiffness of the beam and some known boundary conditions must be accounted for in order to solve for internal shear force and bending moment and support reactions.

Simple bending analysis which assumes that plane sections of the cross-section remain plane and that the beam deforms as a circular arc provides a theoretical expression for beam bending deflection as,

$$\delta = \frac{kWL^3}{EI} \tag{3-1}$$

where k is a constant depending on the type of loading and support conditions; W is the total applied transverse loading (in Newtons), E is the Young's modulus of the beam material (in N/mm^2) and I is the second moment of area of the beam cross-section (mm^4).

3.2 Objectives

In this experiment you will measure the deflection of rectangular cross-section beams of different depths and with different end conditions and compare with simple bending theory.

3.3 Apparatus

The beam deflection experiments consist of sets of aluminium alloy rectangular section beams with different end support conditions, including pinned, fixed or free, as shown in Figure 3-1. Loading is applied by weights and deflections are measured by analogue dial gauges as shown in Figure 3-2.

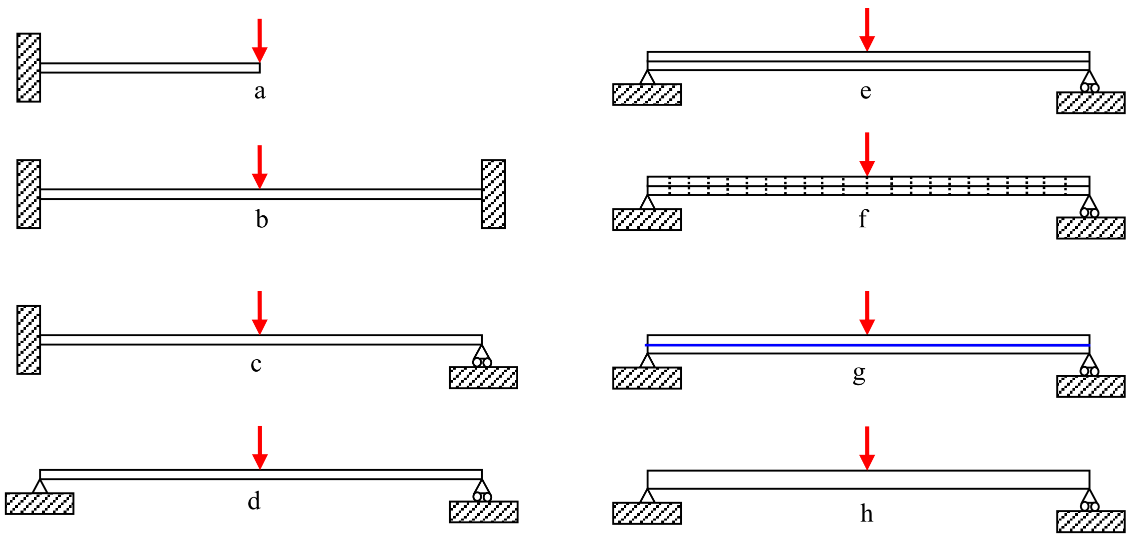


Figure 3-1: Beam supports and configurations.

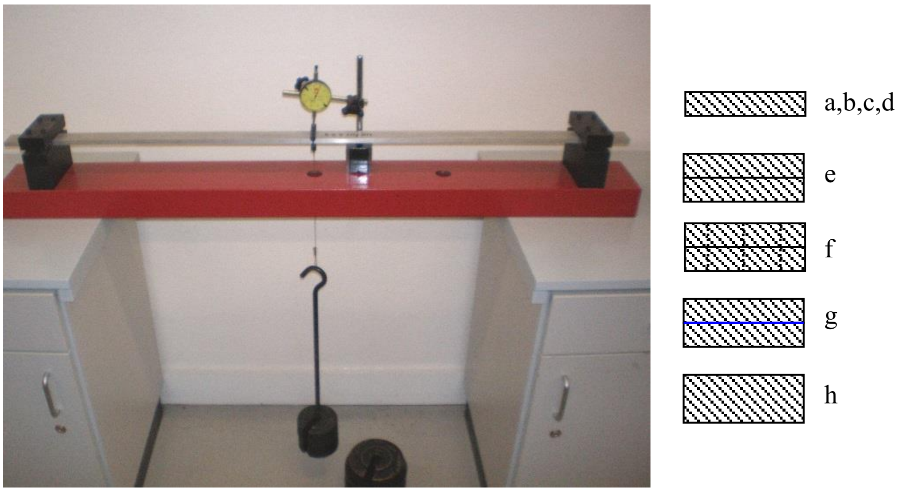


Figure 3-2: Beam bending setup and beam cross sections.

3.4 Tasks

Complete work sheets, consider experimental and theoretical results and note sources of errors.

3. Bending deflections of rectangular section beams of different depth under different support conditions

Tasks

Apply specified loadings to each beam configuration and complete the table of values

Plot mid beam deflections vs structural load on the same graph and comment on linearity and the effect of beam configuartions and supports

Calculate mid beam deflection at the maximum applied structural loading using simple bending theory,

$$\delta = \frac{k W L^3}{EI}$$

Compare measured and calculated values of mid beam deflection at maximum applied structural loading and explain the differences.

Details

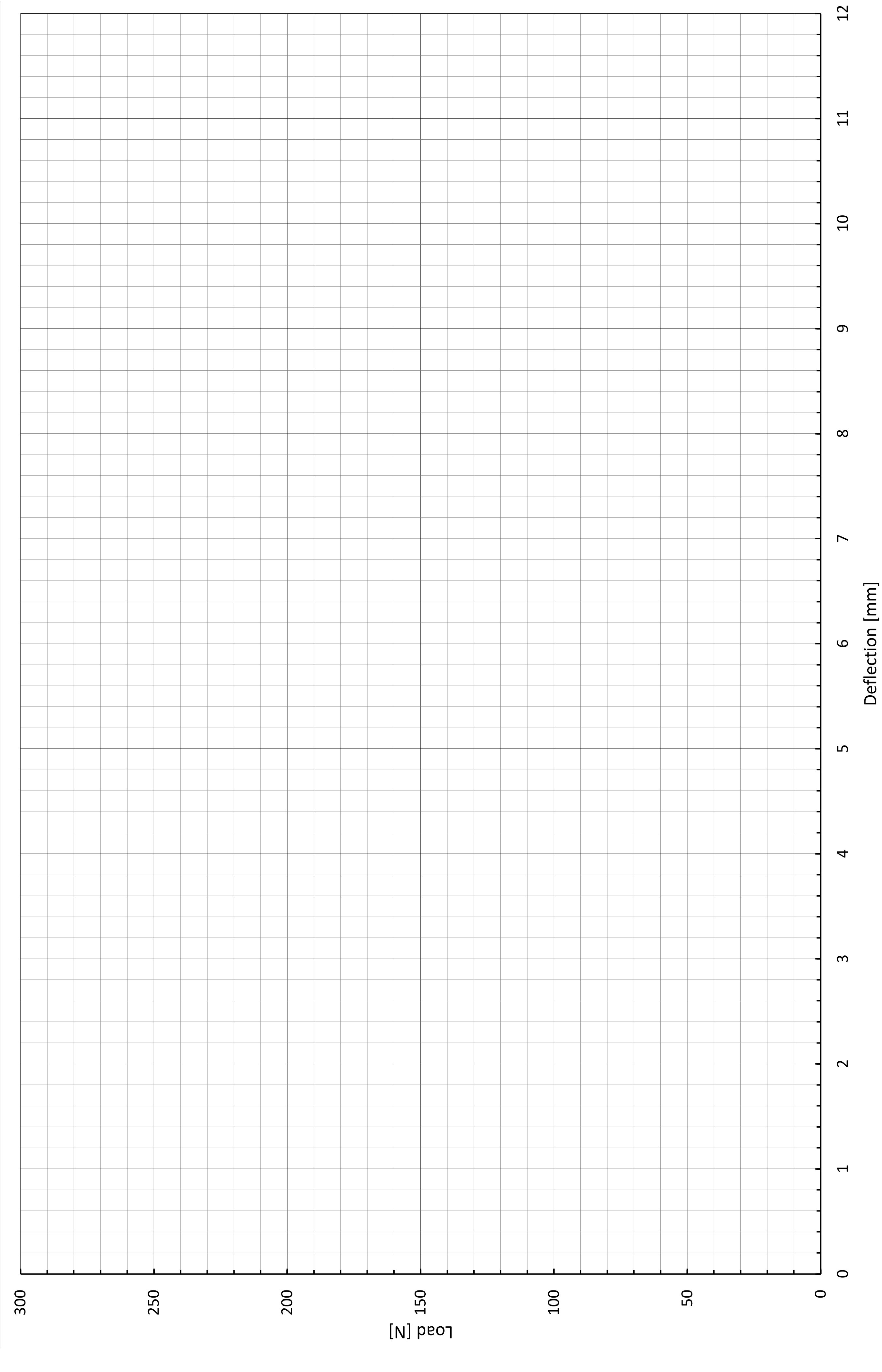
Material = Aluminium alloy Young's Modulus E = 70,000 N/mm^2

Configuration	width b mm	thickness d mm	length l mm	Experimental: Load & Deflection								Theoretical: k	I	EI	Deflection @ max load
a. Single thickness Fixed free (cantilever)	38.10	9.55	500	Mass kg	0.00	1.00	2.00	3.00	4.00	5.00		1/3			
				Load N											
				Deflection mm											
				Corrected mm											
b. Single thickness Fixed fixed (encastre)	38.10	9.55	1000	Mass kg	0.00	5.00	10.00	15.00	20.00			1/192			
				Load N											
				Deflection mm											
				Corrected mm											
c. Single thickness Fixed pinned (propped cantilever)	38.10	9.55	1000	Mass kg	0.00	4.00	8.00	12.00	16.00			7/768			
				Load N											
				Deflection mm											
				Corrected mm											
d. Single thickness Pinned pinned	38.10	9.55	1000	Mass kg	0.00	2.00	4.00	6.00	8.00	10.00		1/48			
				Load N											
				Deflection mm											
				Corrected mm											
e. Double thickness (separate) Pinned pinned	38.10	2 x 9.55	1000	Mass kg	0.00	4.00	8.00	12.00	16.00	20.00		1/48			
				Load N											
				Deflection mm											
				Corrected mm											
f. Double thickness (riveted) Pinned pinned	38.10	19.10	1000	Mass kg	0.00	5.00	10.00	15.00	20.00	25.00		1/48			
				Load N											
				Deflection mm											
				Corrected mm											
g. Double thickness (bonded) Pinned pinned	38.10	19.10	1000	Mass kg	0.00	5.00	10.00	15.00	20.00	25.00		1/48			
				Load N											
				Deflection mm											
				Corrected mm											
h. Double thickness (solid) Pinned pinned	37.98	18.88	1000	Mass kg	0.00	5.00	10.00	15.00	20.00	25.00	30.00	1/48			
				Load N											
				Deflection mm											
				Corrected mm											

Calculations

$I = b d^3 / 12$

Beam deflection $\delta = \left(k W L^3\right)/\left(EI\right)$


Load [N]
0
50
100
150
200
250
300
Deflection [mm]
0
1
2
3
4
5
6
7
8
9
10
11
12

4 Direct Stresses in a T-Section Beam in Bending

4.1 Background

When a beam is loaded in bending a distribution of direct stresses are created along the length and through the depth of the beam to form an internal reacting bending moment at any section (*e.g.* experiment No. 2). To form this internal reacting bending moment the direct stresses must vary from tension to compression across the beam, passing through zero on the 'neutral axis'. The neutral axis of a symmetric section loaded along a line of symmetry will pass through the centroid of the cross-sectional area in a direction perpendicular to the plane of loading. In this experiment the direct stresses acting along the beam which result in this internal bending moment are deduced from strain readings at the mid length of the beam and different heights on the beam section. The magnitude of stress required to generate this internal bending moment will also depend on the cross-sectional area and it's distribution about this neutral axis. The area property used to define the area and it's distribution is known as the 'second moment of area' I .

Using Hooke's law the stress can be deduced from the measured strains, *i.e.* $\sigma = E\,\varepsilon$.

Using simple beam theory we can also predict this stress for a given bending moment as $\sigma = M\ y/I$ where y is the distance from the neutral axis.

4.2 Objectives

In this experiment you will measure the direct stresses at the mid-section of an inverted T-section beam loaded in bending and compare with simple bending theory prediction.

4.3 Apparatus

This experiment consists of an inverted aluminium T-beam, with strain gauges fixed on the section at the mid beam position. Load is applied load at the top of the beam at two positions each side of the strain gauges. Loading the beam in this way (rather than loading the beam at just one point) allows a gauge to be placed on the top of the beam and provides a constant bending moment area which avoids stress concentration close to the gauge positions.

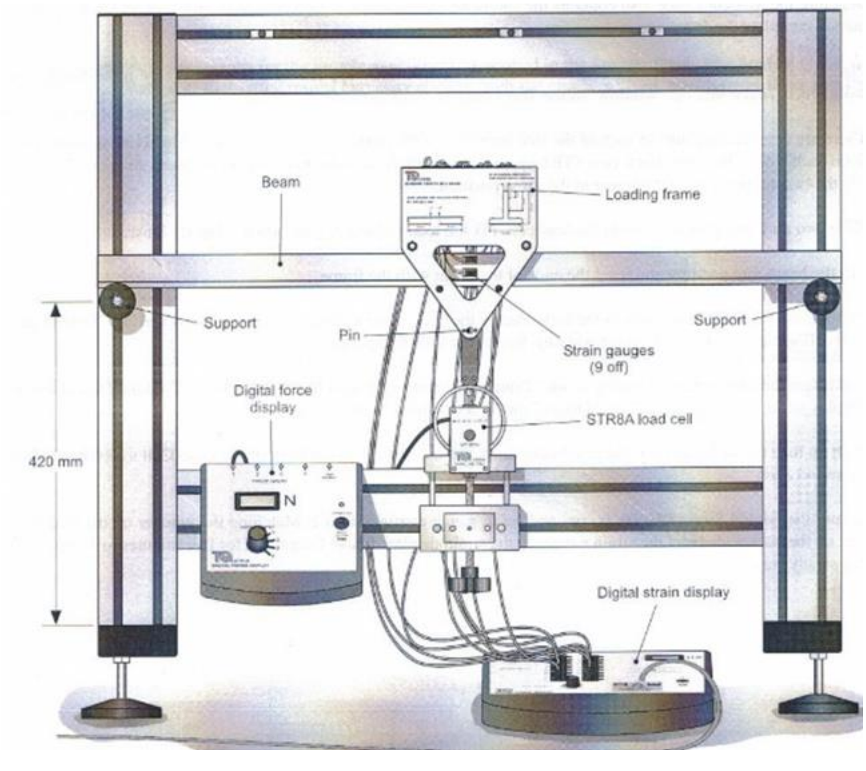


Figure 4-1: T-section beam bending apparatus.

4.4 Tasks

Complete work sheets, consider experimental and theoretical results and note sources of errors.

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Direct stresses in a T-section beam under bending

Tasks

Apply prescribed loading increments, record strains and complete the table of values

(Note a preload of 50N has already been applied and the load cell zeroed to avoid loading through a range which might include slackness in the joints. The load cell zero should not be altered)*

Plot strain vs. bending moment for each strain gauge and comment on linearity.

Plot strain vs strain gauge position for each loading increment and deduce the position of the neutral axis and compare with calculated position of centroid.

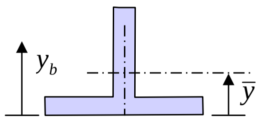
Calculate stress deduced from measured strain and assumed elasticity using $\sigma = E\,\varepsilon$ and explain differences

Deduce the position of the neutral axis (where bending stress is zero) and compare with the calculated position of centroid using 1st moment of area, i.e. $\bar{y} = \frac{\sum A_i y_i}{\sum A_i}$

Calculate stress at the maximum applied structural loading at each strain gauge position using simple bending theory $\sigma = M\ y/I$ and compare with value deduced from measurement

Material =	Aluminium alloy	Young's Modulus E =	69 000 N/mm^2
Flange thickness t_f =	6.38 mm	Flange width b_f =	38.05 mm
Web thickness t_w =	6.36 mm	Web height b_w =	38.08 mm

$I = 60215$ mm^4



Load W N	Bending Moment $M = 350\frac{W}{2}$ N·mm		Experimental Strains $\mu\varepsilon$								
		Gauge No.	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
		Gauge Position y_b	38.1	31.7	31.7	15.1	15.1	6.4	6.4	0.00	0.00
		Before loading									
		After loading									
		Mean @ 0 Load									
100.00		measured									
		corrected									
200.00		measured									
		corrected									
300.00		measured									
		corrected									
400.00		measured									
		corrected									
500.00		measured									
		corrected									

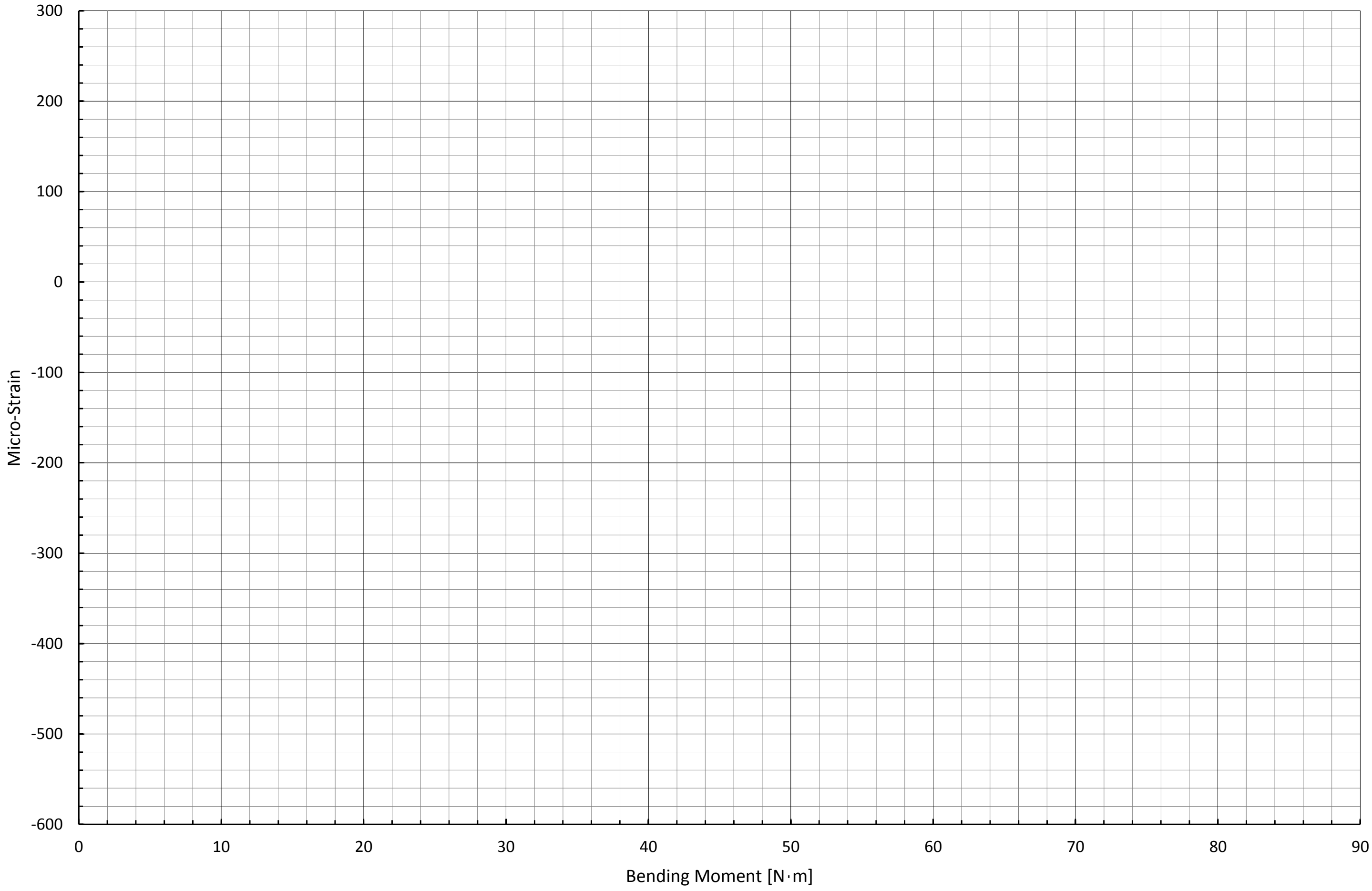
Calculations

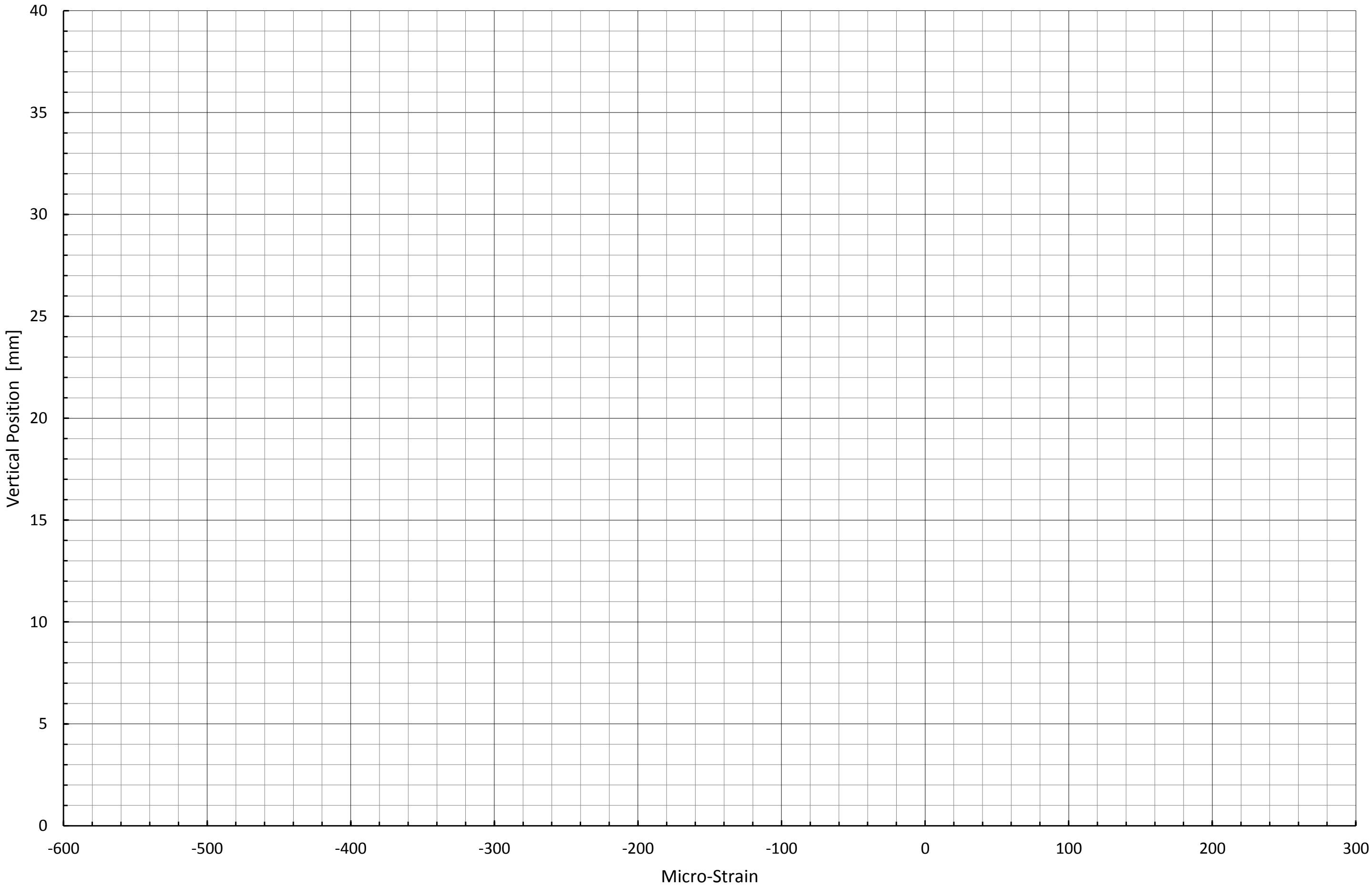
Stress $\sigma = E\,\varepsilon$ @ maximum loading										N/mm^2

I (by parallel axis)= mm^4

Centroid position $\bar{y}$ = mm

	Theoretical									
distance of gauges from neutral axis y										
Stress $\sigma = M\ y/I$ @ maximum loading										N/mm^2


300
200
100
0
-100
-200
-300
-400
-500
-600
Micro-Strain
0
10
20
30
40
50
60
70
80
90
Bending Moment [N·m]


40
35
30
25
20
15
10
5
0
Vertical Position [mm]
-600
-500
-400
-300
-200
-100
0
100
200
300
Micro-Strain

5 Buckling of Struts

5.1 Background

Compressive members can be seen in many structures. They can form part of a framework *e.g.* in a truss structure, or they can stand-alone; *e.g.* a support column.

Unlike tension members, which generally only fail by exceeding the material tensile strength, compressive members can fail in two ways; *i.e.*, either by exceeding the material compressive strength or by exceeding the condition of stability 'buckling'. Generally, short wide compressive members 'columns' tend to fail by material 'crushing', whereas long thin compressive members 'struts' tend to fail by 'buckling'. When buckling occurs a strut will no longer be effective in carrying further load and will simply continue to displace *i.e.* it becomes useless as a structural member.

According to theoretical analysis by the famous Swiss mathematician Leonhard Euler (1707-1783) a perfect strut with perfect end conditions will remain stable and straight and return to its stable position under modest lateral disturbances until it reaches a critical compressive load. At this critical load the strut is theoretically in a state of neutral equilibrium and any small lateral disturbance will remain constant. The slightest load increment above this critical load will result in the strut becoming unstable with lateral deflection increasing catastrophically with no further increase in load.

For real struts with real end conditions there will be a natural tendency to deflect in a particular direction from the first increment of load due to imperfections of straightness and alignment of fittings etc. In this experiment you will observe the deflection of struts in their natural direction and also when restrained to buckle in the opposite direction.

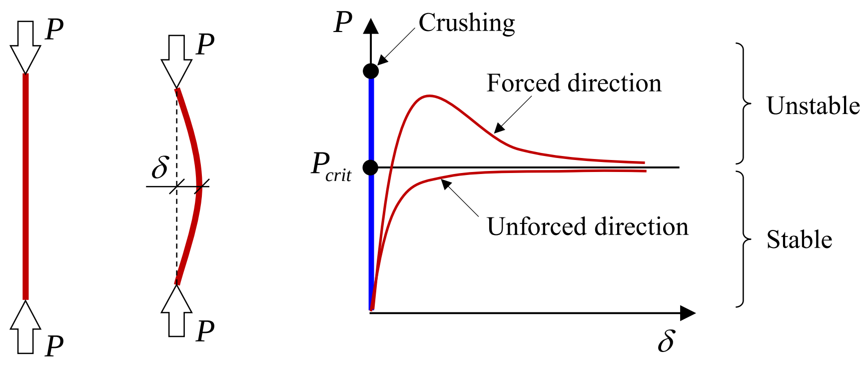


Figure 5-1: Buckling of a strut.

Euler derived formulae to estimate the critical buckling load for struts with different end conditions. The basic formula is

$$P_{crit} = \frac{k\pi^2 EI}{L^2} \qquad (5\text{-}1)$$

where k is a constant depending on end fixity conditions, *e.g.* pinned, fixed or free. For a pinned-pinned strut $k=1$, for a pinned-fixed strut $k \cong 2$ and for a fixed-fixed strut $k=4$. E is the Young's modulus of the beam material (in N/mm^2) and I is the second moment of area of the beam cross-section (mm^4). L is the length of the strut between the supports.

5.2 Objectives

In this experiment you will load various struts until they buckle to investigate the effect of the strut length and the strut end conditions. The Euler buckling formulae should be assessed by comparison with your experimental data.

5.3 Apparatus

The buckling apparatus is shown in Figure 5-2. You are expected to note the features of the apparatus for yourselves and present a succinct description in the appropriate section of your report.

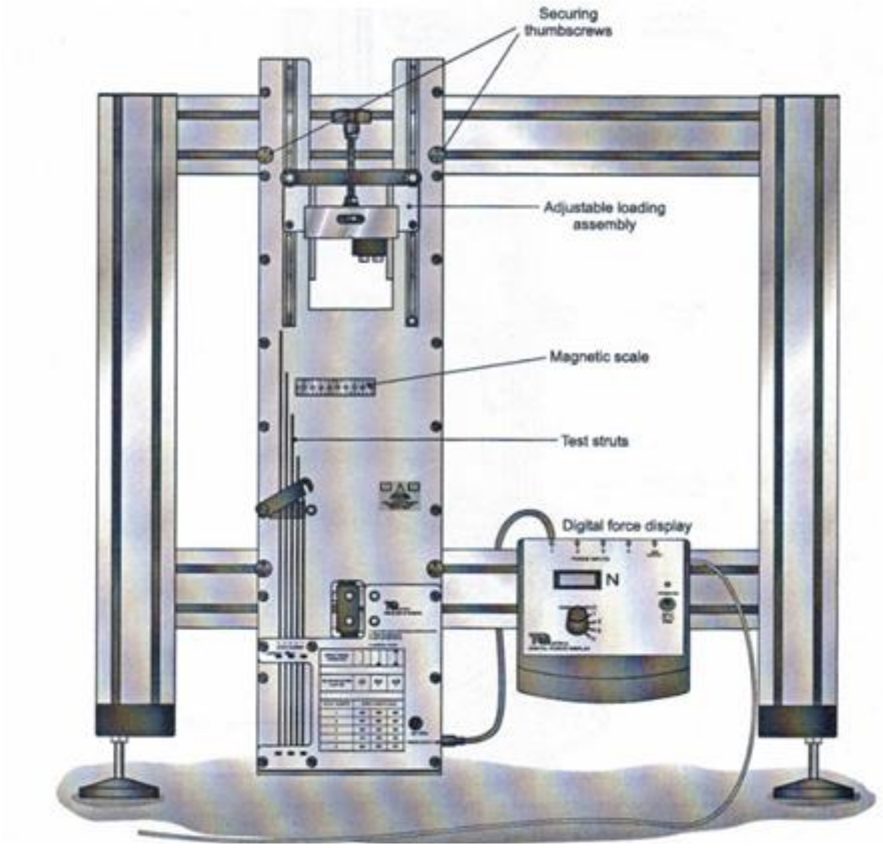


Figure 5-2: Buckling apparatus.

Note: Do not continue to load the struts after the buckling load has been reached, you may permanently deform them!

5.4 Tasks

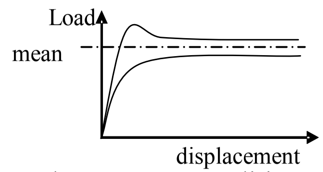
Complete work sheets. Produce a full report as detailed in the Appendix.

Buckling of Struts

Tasks

Apply compressive loading to achieve mid strut deflections at 2mm increments up to a maximum of 20mm. Record strut load at each increment and complete the table.

Plot deflection vs. strut load and deduce the effective buckling load from the average of the natural and forced asymptote values.



Plot buckling load against the reciprocal of squared strut length and comment on the relationship.

Calculate buckling loads for each strut using the Euler formula.

Compare the measured and calculated values of buckling loads and explain the difference.

Sketch the buckled strut shapes for the different end configurations for the longest strut and suggest ‘equivalent pinned-pinned lengths’ for the other support conditions.

Details

Material = Aluminium alloy — Young’s Modulus $E =$ 69000 N/mm²

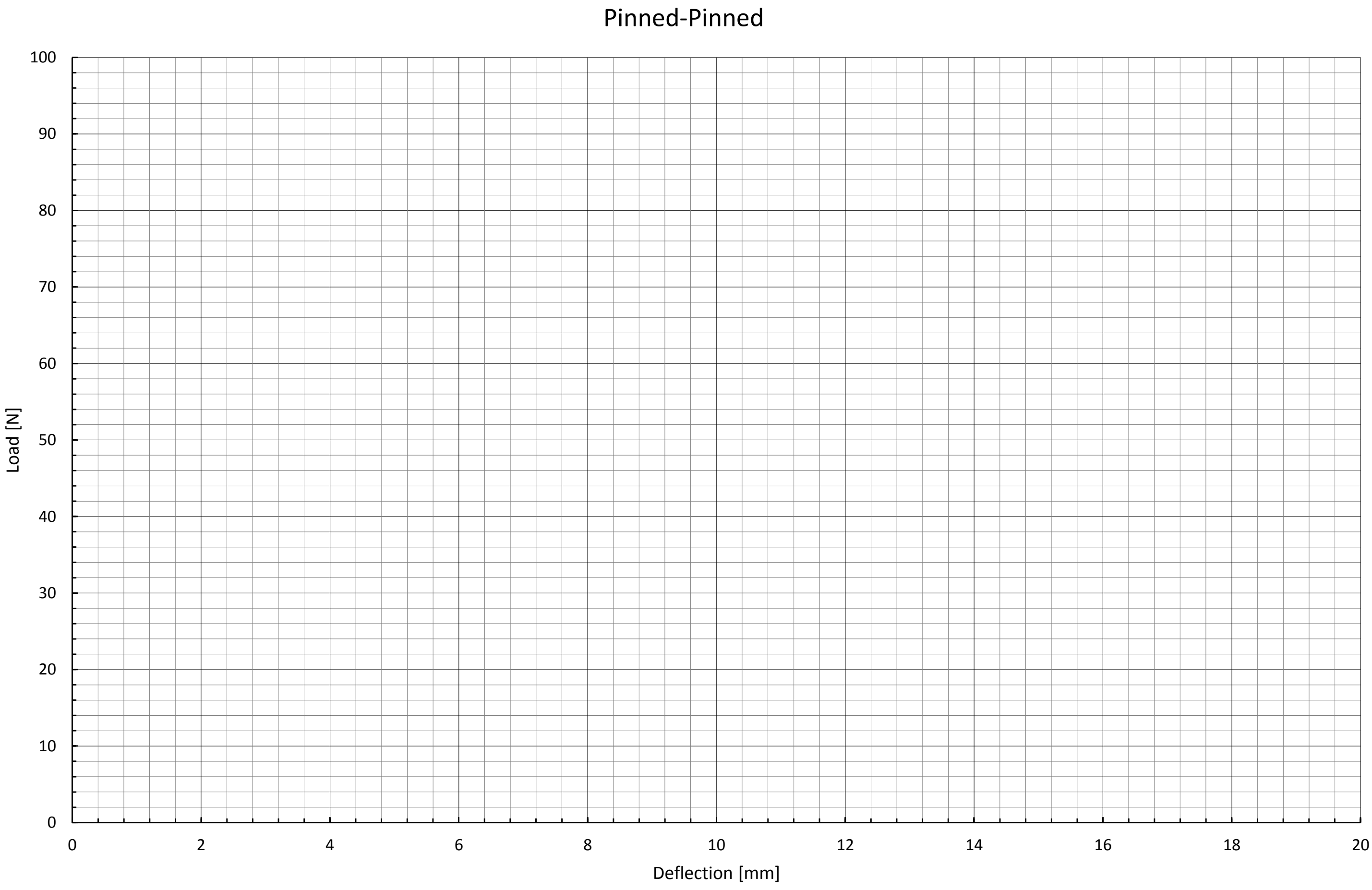
Strut width $b =$ 20 mm — Strut thickness d = 2 mm

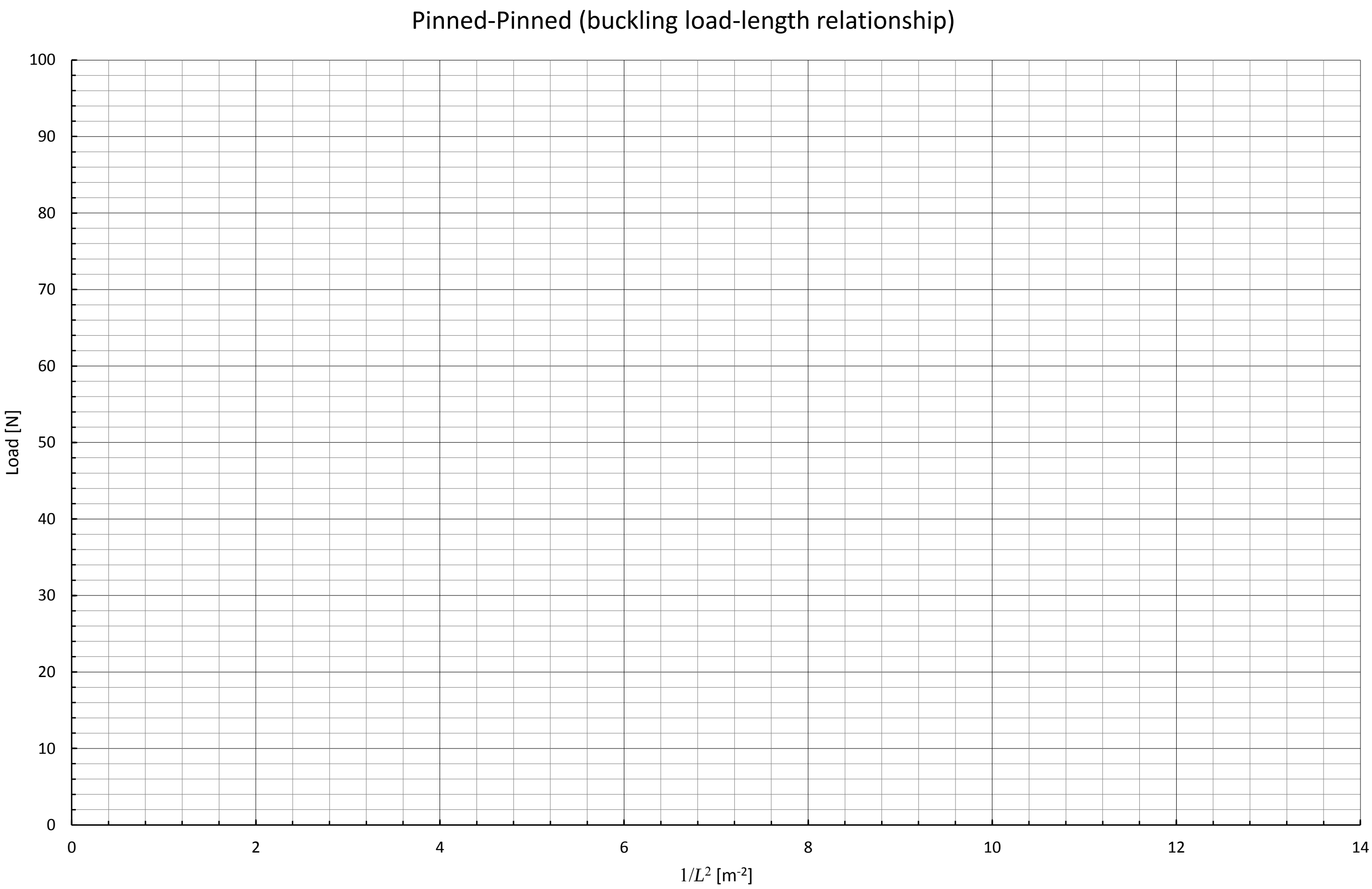
Theoretical (Euler) buckling load: $P_{crit} = \dfrac{k\pi^2 EI}{L^2} \begin{cases} \text{pinned – pinned} & k = 1 \\ \text{pinned – fixed} & k = 2 \\ \text{fixed – fixed} & k = 4 \end{cases}$

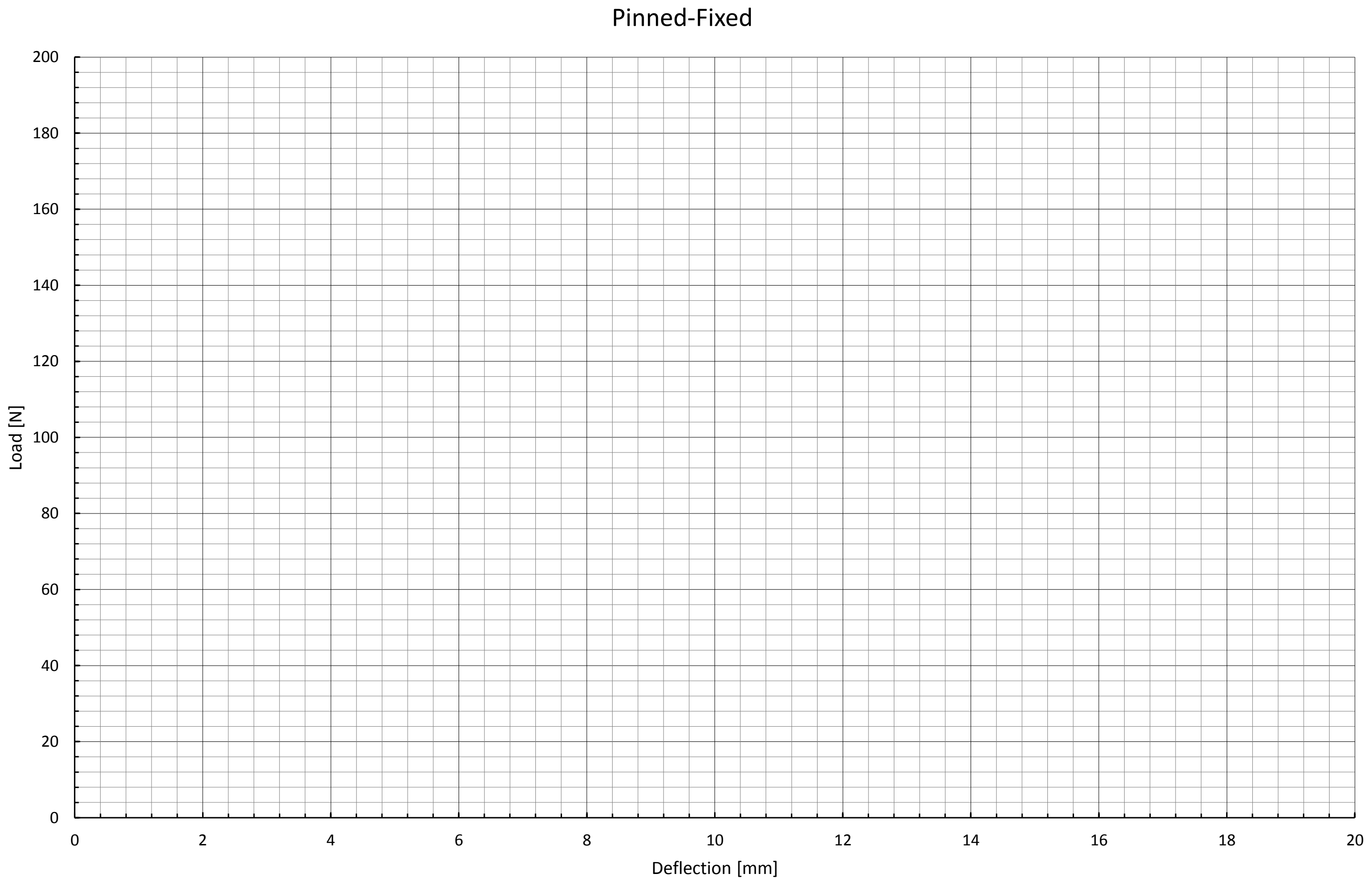
Do not continue to load the struts after the buckling load has been reached, you may permanently deform them.

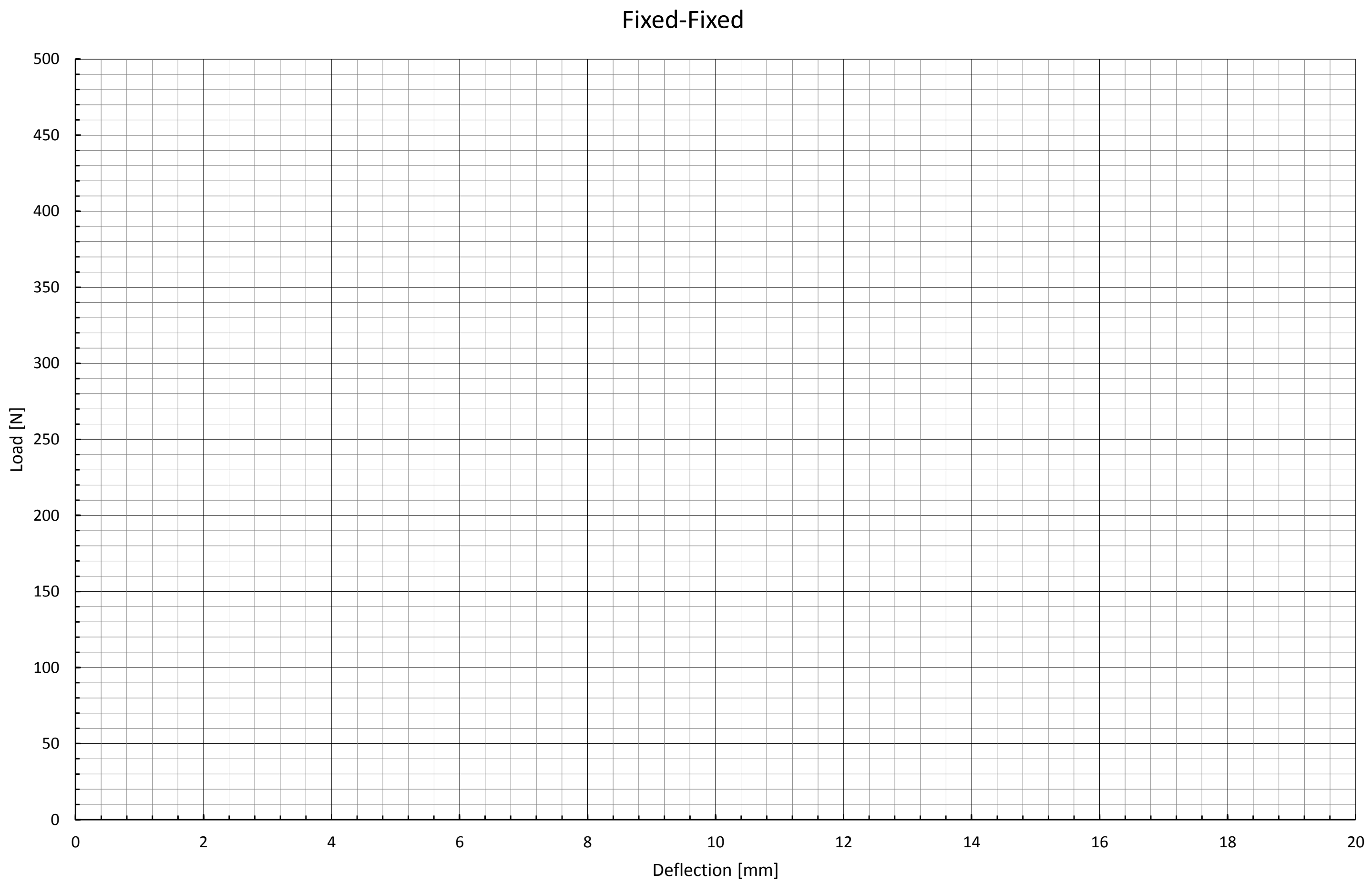
Configuration and length	Length L [mm]		Experimental: Mid-strut deflection [mm], Resulting strut load N: 0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00	16.00	18.00	20.00	Experimental: Buckling load N	Theoretical: Buckling load N
Pinned-pinned Length 1	320	Natural													
		Forced													
Pinned-pinned Length 2	370	Natural													
		Forced													
Pinned-pinned Length 3	420	Natural													
		Forced													
Pinned-pinned Length 4	470	Natural													
		Forced													
Pinned-pinned Length 5	520	Natural													
		Forced													
Pinned-fixed Length 5	500	Natural													
		Forced													
Fixed-fixed Length 5	480	Natural													
		Forced													

Measured buckling load = average of natural and forced asymptotic values


Pinned-Pinned
Load [N]
0
10
20
30
40
50
60
70
80
90
100
0
2
4
6
8
10
12
14
16
18
20
Deflection [mm]


Pinned-Pinned (buckling load-length relationship)
Load [N]
0
10
20
30
40
50
60
70
80
90
100
0
2
4
6
8
10
12
14
$1/L^2$ [m^{-2}]


Pinned-Fixed
Load [N]
0
20
40
60
80
100
120
140
160
180
200
0
2
4
6
8
10
12
14
16
18
20
Deflection [mm]


Fixed-Fixed
Load [N]
0
50
100
150
200
250
300
350
400
450
500
0
2
4
6
8
10
12
14
16
18
20
Deflection [mm]

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6 Appendix

Buckling of Struts – Online Report

This appendix contains further information on the buckling experiment, which will form part of the Structures Lab online assessment.

6.1 Objectives of the Experiment

- Demonstrate the critical nature of compressive loads in slender structural members;
- Investigate the relationship between critical buckling load and strut length;
- Investigate how end constraints affect the critical load of a strut;
- Consider other constraint conditions and the relevance of theoretical predictions.

6.2 Background and Theory

If a slender structural member is subjected to a gradually increasing <u>tensile force</u> 'failure' will eventually occur at a value of the force when the stress causes unacceptable behaviour, usually either yielding or fracture of the material. The value of this stress therefore depends on the strength of the material.

If a slender structural member is subjected to a gradually increasing <u>compressive force</u> it is likely to eventually 'fail' at a value of force much less than that which would produce compressive yielding or fracture of the material. Failure in this case is due to buckling, when the member rapidly develops a transverse deflection and is no longer able to 'support' the compressive load. If the load is not relieved, then the member would finally fail due to bending at only a slightly greater value of the load.

The critical buckling load of a strut is therefore related to the stiffness of the strut in bending and not to its material strength in compression. Stiffness in bending depends on the Young's modulus of the material, and the shape of the cross-section as well as its cross-sectional area. Thus structural members of the same size and shape but made of differing alloys of a given metal may have widely differing failure loads in tension, due to the widely differing strengths possible for different alloys, but would have very similar buckling loads in compression, because the values of Young's modulus of an alloy family does not vary very much. On the other hand, structural members of the same material and cross-sectional area, but with differing cross-sectional shapes, would have the same failure load in tension, but would have significantly different buckling loads.

Because buckling results from the development of transverse bending deflections, constraints at the ends and along the length of the strut will also influence the buckling load considerably. The simplest type of end condition is one which allows the strut to freely pivot at each end, *i.e.* 'pinned'. This causes no bending restriction at the ends, and leads to the lowest possible buckling load (apart from the rather unusual fixed-free condition).

If an end is completely 'built-in' or fixed so that no rotation at the end is possible the effective stability of the member will be higher because the bending deflection will be restricted at the fixing (higher still if both ends are fixed). Also, if transverse movement is restricted or prevented at one or more positions along the length, the buckling load can again be higher, sometimes very much so.

Next, consider the theoretical determination of the buckling force for an idealised situation where the strut is initially perfectly straight with a uniform cross-section, axially loaded in compression by a force acting through the centroid of the cross-section.

For a modest value of the force, the strut will remain perfectly straight; if the middle of the strut is moved transversely and then released, the strut will return to its original straight shape. However, it can be shown that at a certain critical value of the axial force if the middle of the strut is disturbed transversely and then released, the strut will remain in that bent shape. At any higher value of the applied force any transverse disturbance would increase ad infinitum within the terms of the analysis. This is a classic case of stability where the critical condition is neutral stability. The applied force at this critical condition is described as the buckling force. Only an initial small disturbance is considered in the classical analysis but even if the analysis is developed to account for large deflections it can be shown that the force required to continue to buckle the strut only increases slightly until the strut finally fails by yielding or breaking in bending, as indicated in Figure 6-1.

If the strut is not initially perfectly straight, and/or if the force is not acting exactly at the centre of each end, *i.e.* if the strut is initially curved and/or is loaded eccentrically, then the strut will start to bend transversely as soon as the force is started to be applied. In this case, a true instability is not reached, but the rate of increase of the deflection increases more and more rapidly as the buckling force is approached, and again failure finally occurs due to yielding or breaking in bending.

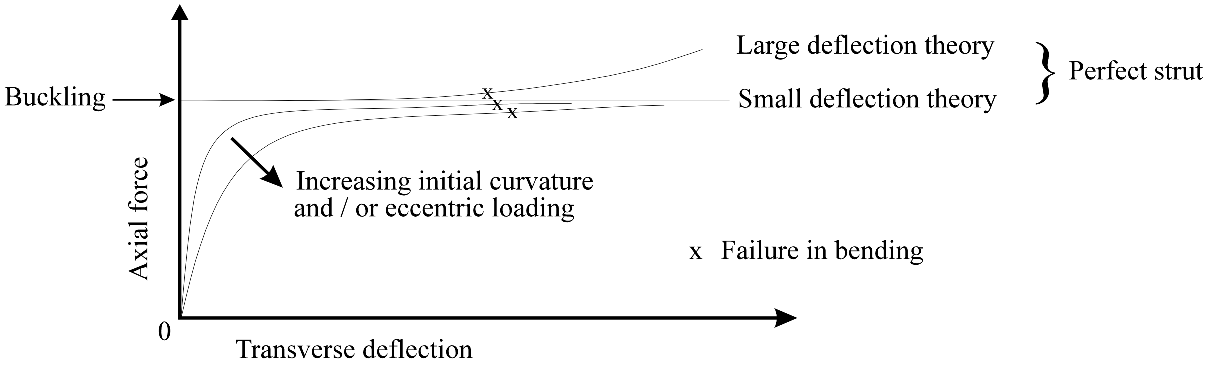

Figure 6-1: Buckling theory.

The theoretical buckling force (small deflection theory) can be expressed as:

$$P_{crit} = \frac{k\pi^2 EI}{L^2} \tag{6-1}$$

where:

- k is a constant which depends on the constraints at the ends and along the length of the strut.
- E is the Young's modulus
- I is the minimum second moment of area of the cross-sectional area about the neutral axis of bending. For a rectangular cross-section where $b > d$, as shown in Figure 6-2, we have $I = I_{zz} = \frac{bd^3}{12}$.
- L is the length of the strut.

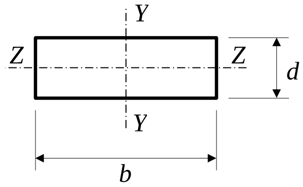

Figure 6-2: Rectangular cross-section.

Formulae for various idealised constraints, together with sketches of the corresponding neutrally stable deflected shapes are shown in Table 6-1.

Table 6-1: Buckling loads and deflected shapes for different end constraints.

Constraints	Buckling load	Deflected shape
Pinned-pinned	$P_{crit} = \frac{\pi^2 EI}{L^2}$	
Pinned-fixed	$P_{crit} = \frac{2.04\pi^2 EI}{L^2}$	
Fixed-fixed	$P_{crit} = \frac{4\pi^2 EI}{L^2}$	

An ideal pinned-end implies zero bending moment at the end, and an ideal fixed (or built-in) end implies a zero slope at the end.

6.3 Results

For the different length struts and for each end condition of the longest strut, plot curves of load against transverse deflection for natural and forced buckling directions, and estimate the buckling load. Also present sketches of the deflected shapes. Comment on any differences between the buckling of the strut in its preferred direction *vs.* the opposite direction.

For the different length struts under pinned-pinned loading conditions, plot load against deflection and estimate the buckling loads. Plot a graph of buckling load against $1/L^2$.

For each strut, give theoretical values of the buckling force for the idealised constraints most appropriate to the situation tested using the actual strut dimensions in each case. Note the possible sources of errors in your experimental and theoretical results and the limitations of the various theoretical results in predicting what happened in the real situations tested. Comment on the overall accuracy of the Euler analysis using a suitable bar chart to illustrate.

Consider other support conditions such as fixed-free (*e.g.* like a mast) and pinned-pinned-pinned (*i.e.* where there is a pinned restraint in the middle). Sketch the likely buckling shapes and suggest the appropriate end fixity k factors which would apply by relating the end fixity factors to the effective pinned-pinned lengths of the struts.