Sustained Hover (1)
Lecture 4

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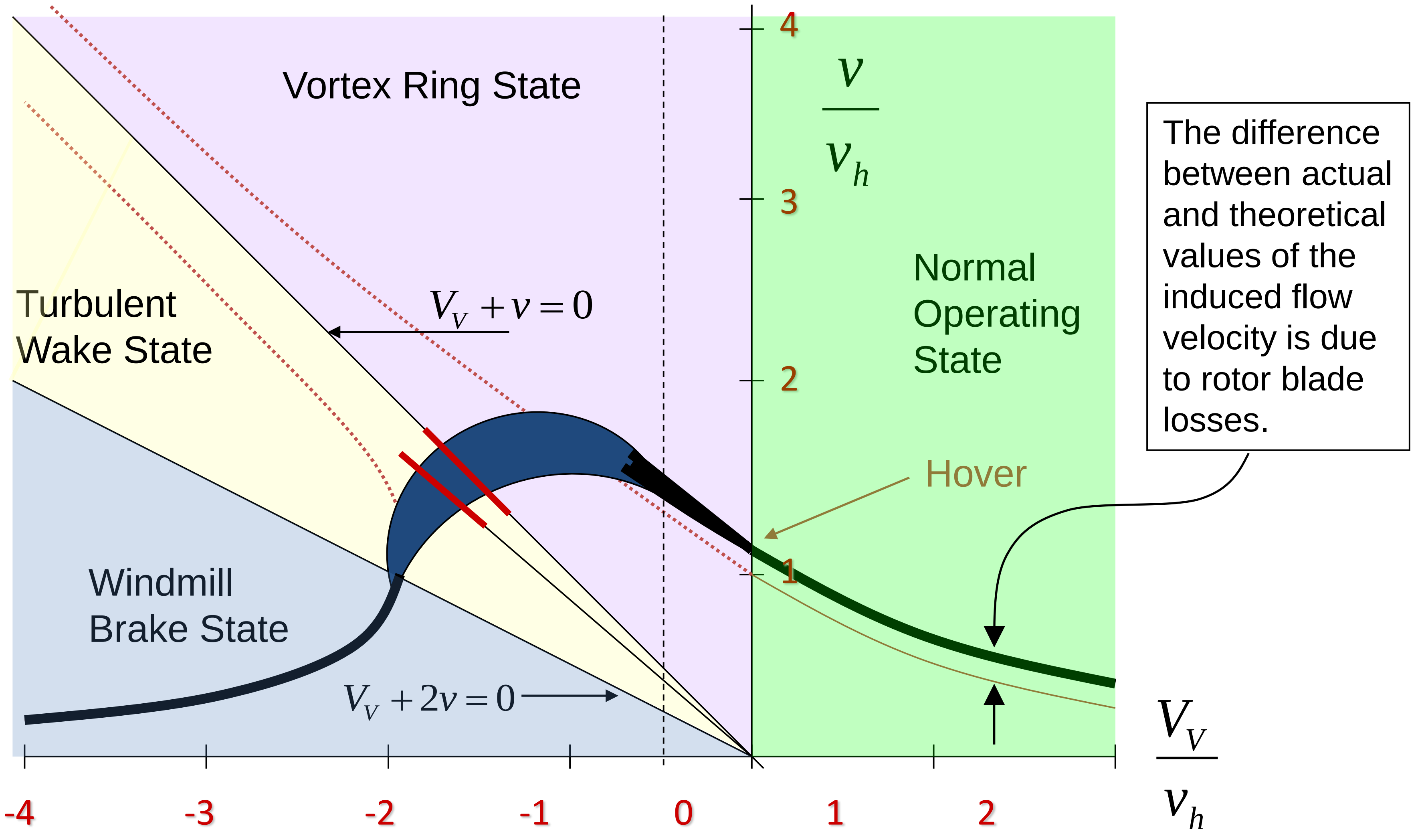
Hover

- Recap on the Universal Induced Velocity Diagram
- Helicopters in Autorotation
- Rotor Efficiency in Hover: Figure of Merit
- Rotor Performance Coefficients
- Maximising the Figure of Merit

The Universal Induced Velocity Diagram



$$\text{Remembering that } P = T(V+v), \text{ then at } (V+v)=0, T \neq 0, P=0$$

HELICOPTER in AUTOROTATION

In general (and this very much depends upon rotor diameter and helicopter weight), helicopters settle down to an autorotational descent rates such that: $\dfrac{V}{v_h} \approx -1.7$

Now in a steady autorotative descent, T = thrust in the hover, so: $T = 2\rho A v_h^2$

The rotor has no net flow through it, so it can be likened to a solid disc of area A.

and it's Flat Plate Drag, $D = \tfrac{1}{2}\rho V^2 A C_D$

Equating the Thrust and Drag equations gives: $2\rho A v_h^2 = \tfrac{1}{2}\rho \mathrm{V}^2 A C_D$

so, $C_D = \dfrac{4}{\left(V/v_h\right)^2}$

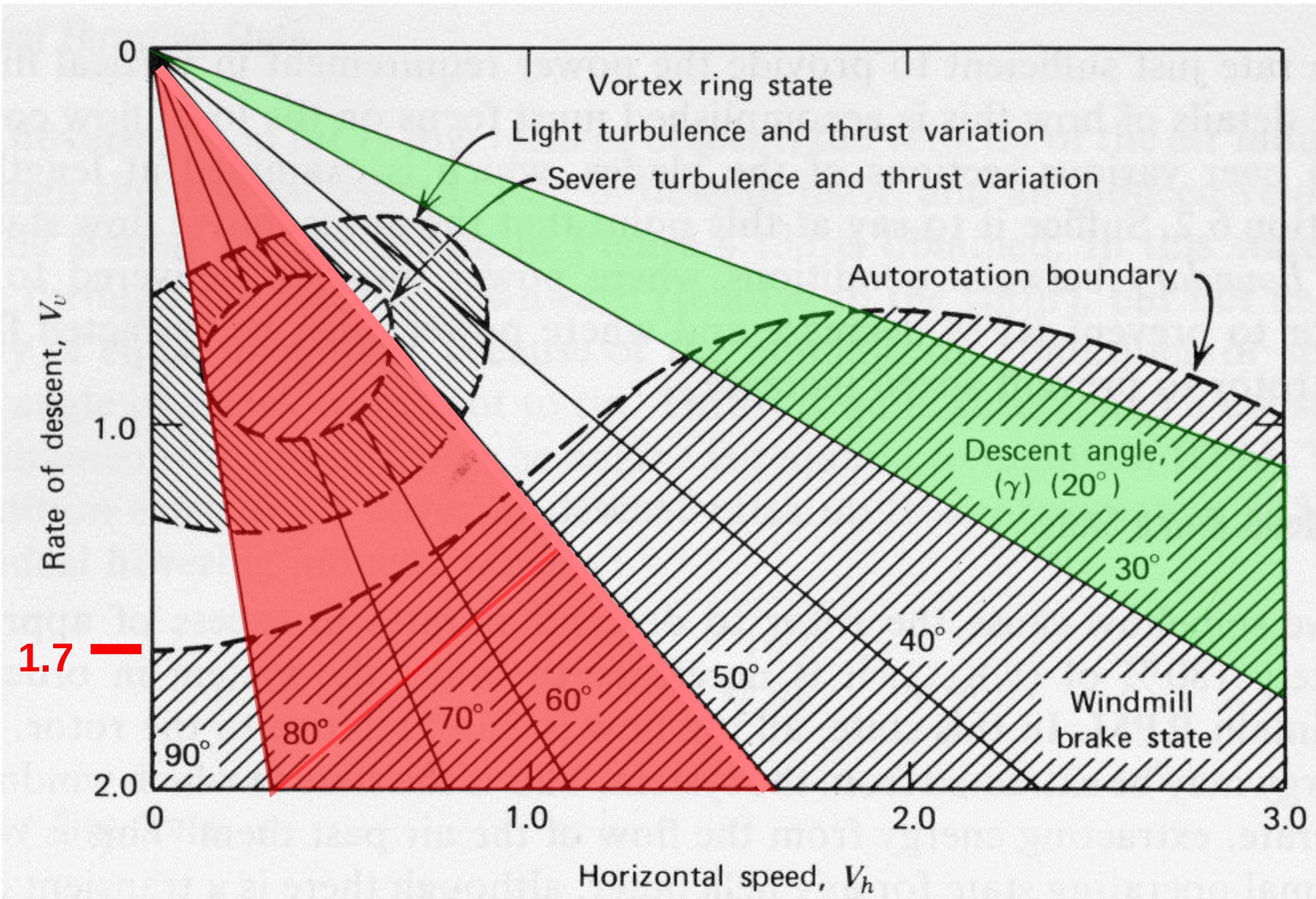
For $V/v_h = -1.7$, $C_D = 1.38$ which is the effective drag coefficient of a parachute.

HELICOPTER in AUTOROTATION

Westland Super Lynx, Mass = 5330kg, Rotor Diameter = 12.8 m

$$v_h = \sqrt{\frac{T}{2\rho A}} = \sqrt{\frac{52287}{315}} = 12.8 \text{ m/s}, \text{ vertical autorotation descent} \approx 22 \text{ m/s}.$$

HELICOPTER in VERTICAL and FORWARD AUTOROTATION



It is important to design for low autorotation rates

HELICOPTER in AUTOROTATION

Rotor Efficiency in the Hover

The measure of propeller efficiency, $\eta_p = TV/P$ is not suitable for the helicopter.

So a "**Figure of Merit**" is used: $\eta_r = FoM = Tv/P$

$$= \frac{T}{P}\sqrt{\frac{T}{2\rho A}}$$

$$= \frac{T}{P}\frac{1}{\sqrt{2}}\sqrt{\frac{T}{\rho \pi R^2}}$$

It should be noted that the FoM is inversely proportional to the rotor diameter and comparative studies therefore should be limited to rotors of the same diameter.

Rotor Efficiency in the Hover

$$\eta_r = FoM = Tv/P \qquad = \frac{T}{P}\sqrt{\frac{T}{2\rho A}}$$

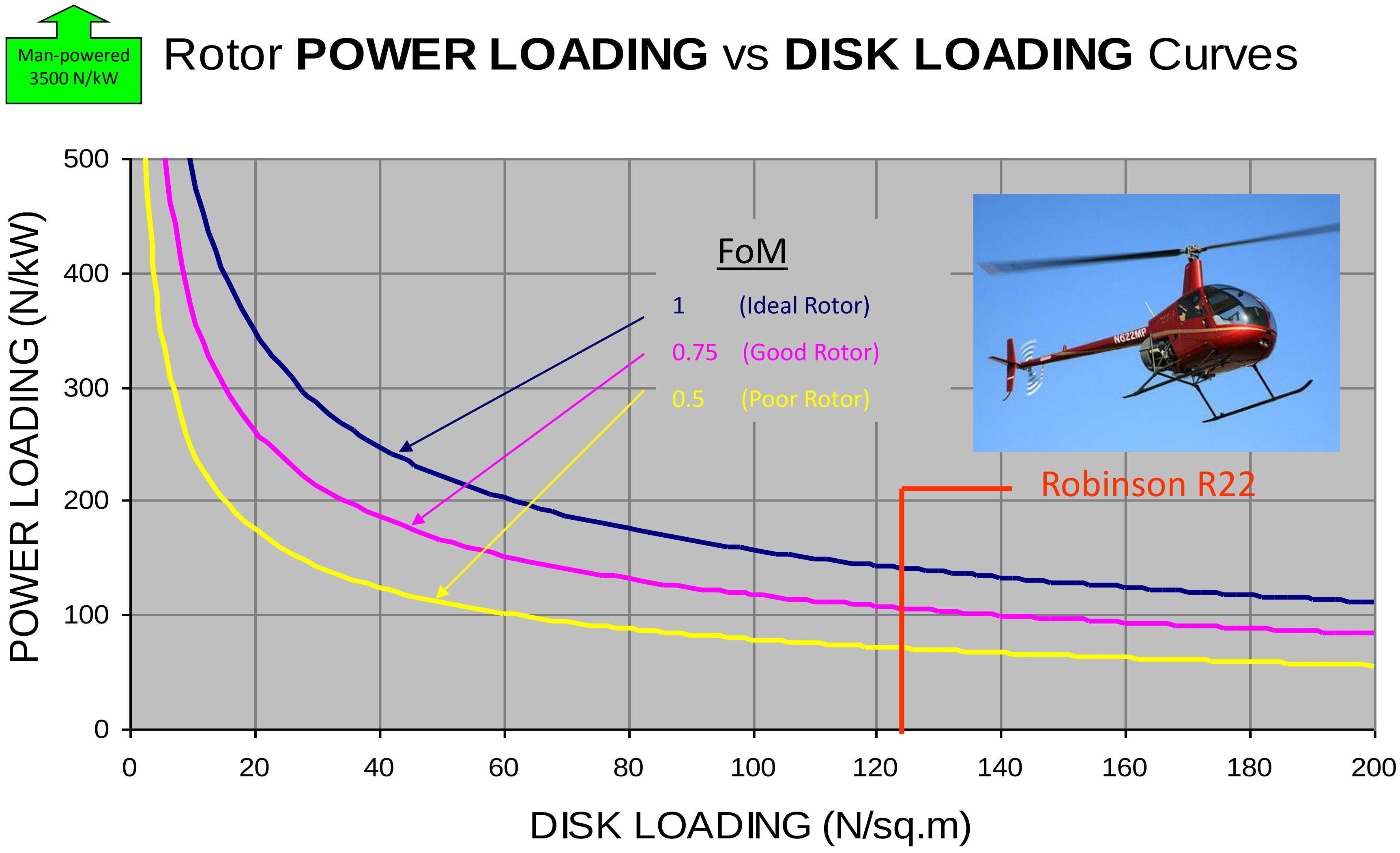
If, $\dfrac{T}{P} = \text{PL}$ (known as Power Loading) and $\dfrac{T}{A} = \text{DL}$ (known as Disk Loading)

then $$\text{PL} = 1.565 FoM \frac{1}{\sqrt{\text{DL}}}$$ This is Dimensional !

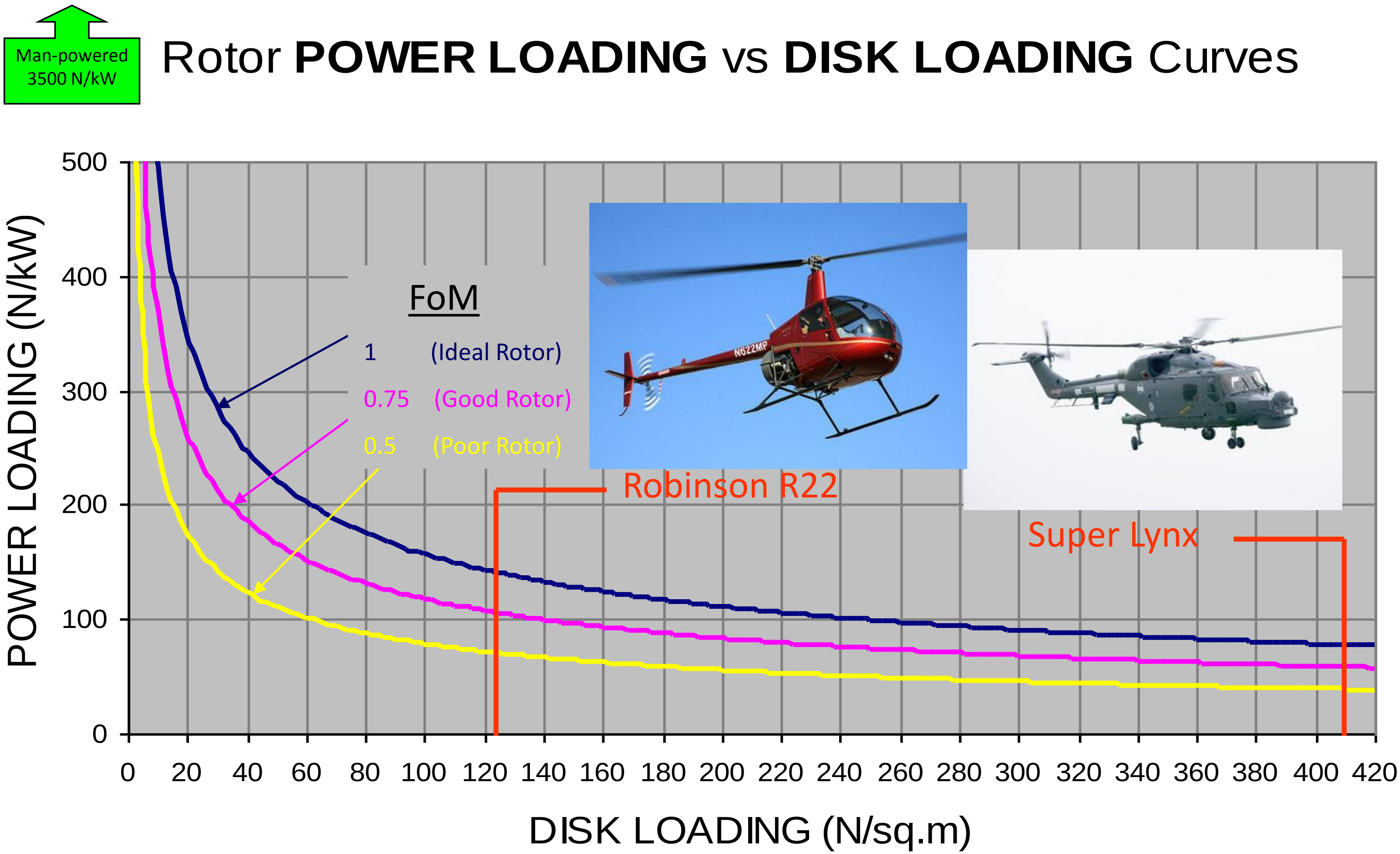
(based upon ρ =1.225 kg/m^3)

This relationship can be plotted and if the Figure of Merit is known, then for a given disk loading the power loading may be found from the graph.

Rotor Efficiency in the Hover



Rotor Efficiency in the Hover



Rotor Performance Coefficients

The general thrust coefficient $T_c = \dfrac{T}{\rho V^2 D^2}$ has limited value for helicopter rotors as it has an infinite value at $V = 0$. A more suitable coefficient is $C_T = \dfrac{T}{\rho A(\Omega R)^2}$ which gives a thrust coefficient based upon rotor tip speed ΩR (m/s) and rotor disk area A .

(*We refer to parameter* ΩR *as the* reference velocity *where rotational speed* Ω *is in* *rads/sec*)

It is also normal to express the forward speed (V) of the helicopter relative to the tip speed parameter ΩR and this is called the **Advance Ratio** μ .

Thus $$\mu = \frac{V}{\Omega R}$$

In a similar way, the flow through the rotor (v in the hover but $V_V + v$ otherwise) is non-dimensionalised by ΩR and this is called the **inflow ratio** λ .

Thus $$\lambda = \frac{V_V + v}{\Omega R}$$

Rotor Performance Coefficients

A note of caution:

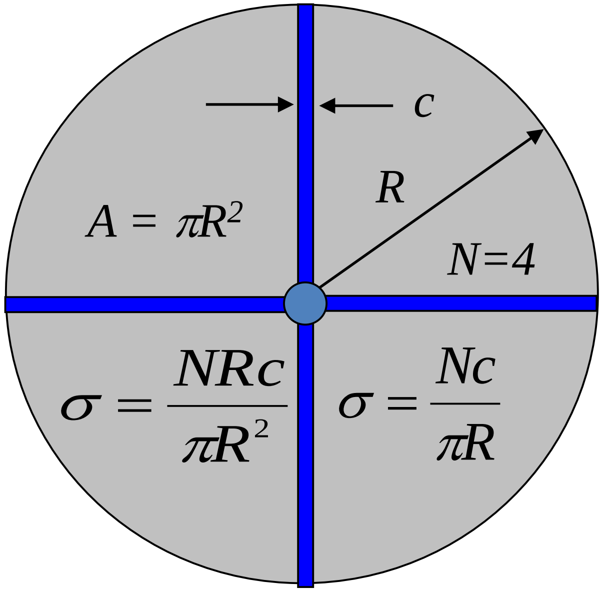
Whilst $C_T = \dfrac{T}{\rho A(\Omega R)^2}$ has become the accepted form,

This one is used in this course

some text books may show:

$$C_T = \frac{T}{1/2\,\rho V_T^2 A} \quad \text{where } V_T = \Omega R \quad \text{(blade tip speed)}$$

Or



$$C_T = \frac{T}{\rho \sigma A(\Omega R)^2}$$

Where $\sigma = \dfrac{Nc}{\pi R}$ (solidity, where N is number of blades)

Rotor Performance Coefficients

Similarly, Torque Coefficient

$$C_Q = \frac{Q}{\rho A R (\Omega R)^2}$$

and Power Coefficient

$$C_P = \frac{P}{\rho A (\Omega R)^3}$$

therefore $C_P = C_Q$

Rotor Performance Coefficients

The Figure of Merit as previously defined can be more conveniently expressed in terms of non-dimensional quantities using the thrust and power coefficients.

The induced velocity $\boldsymbol{v} = \sqrt{\dfrac{T}{2\rho A}} = \sqrt{\dfrac{C_T \rho A(\Omega R)^2}{2\rho A}} = \Omega R\sqrt{\dfrac{C_T}{2}}$

But since $\lambda = \dfrac{V_V + v}{\Omega R}$ then for $V_V = 0$, $\boxed{\lambda = \sqrt{\dfrac{C_T}{2}}}$

$$FoM = \frac{Tv}{P} = \frac{C_T \rho A(\Omega R)^2 v}{C_P \rho A(\Omega R)^3} = \frac{C_T}{C_P}\frac{v}{\Omega R} = \frac{C_T}{C_P}\lambda$$

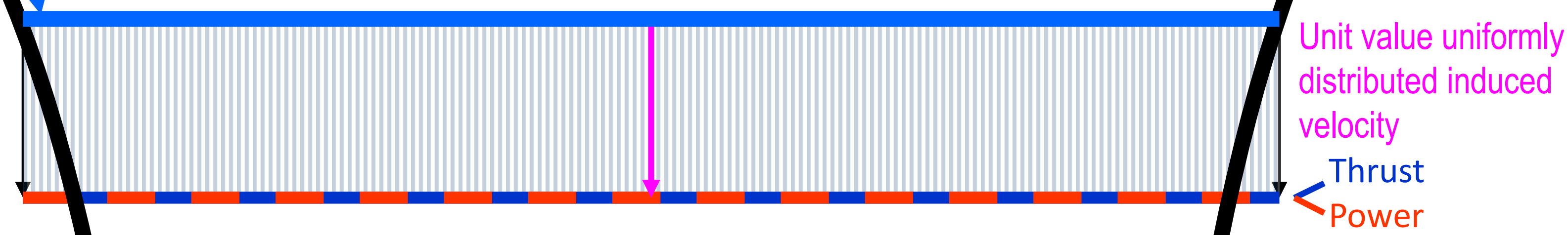
$$= \frac{1}{\sqrt{2}}\frac{C_T^{3/2}}{C_P}$$

Or more commonly……………………………

$$\boxed{FoM = 0.707\frac{C_T^{3/2}}{C_Q}}$$

Maximising the Figure of Merit

An ideal rotor (M = unity) requires an actuator disk condition of infinite (zero loss) blades and a zero loss flow state.

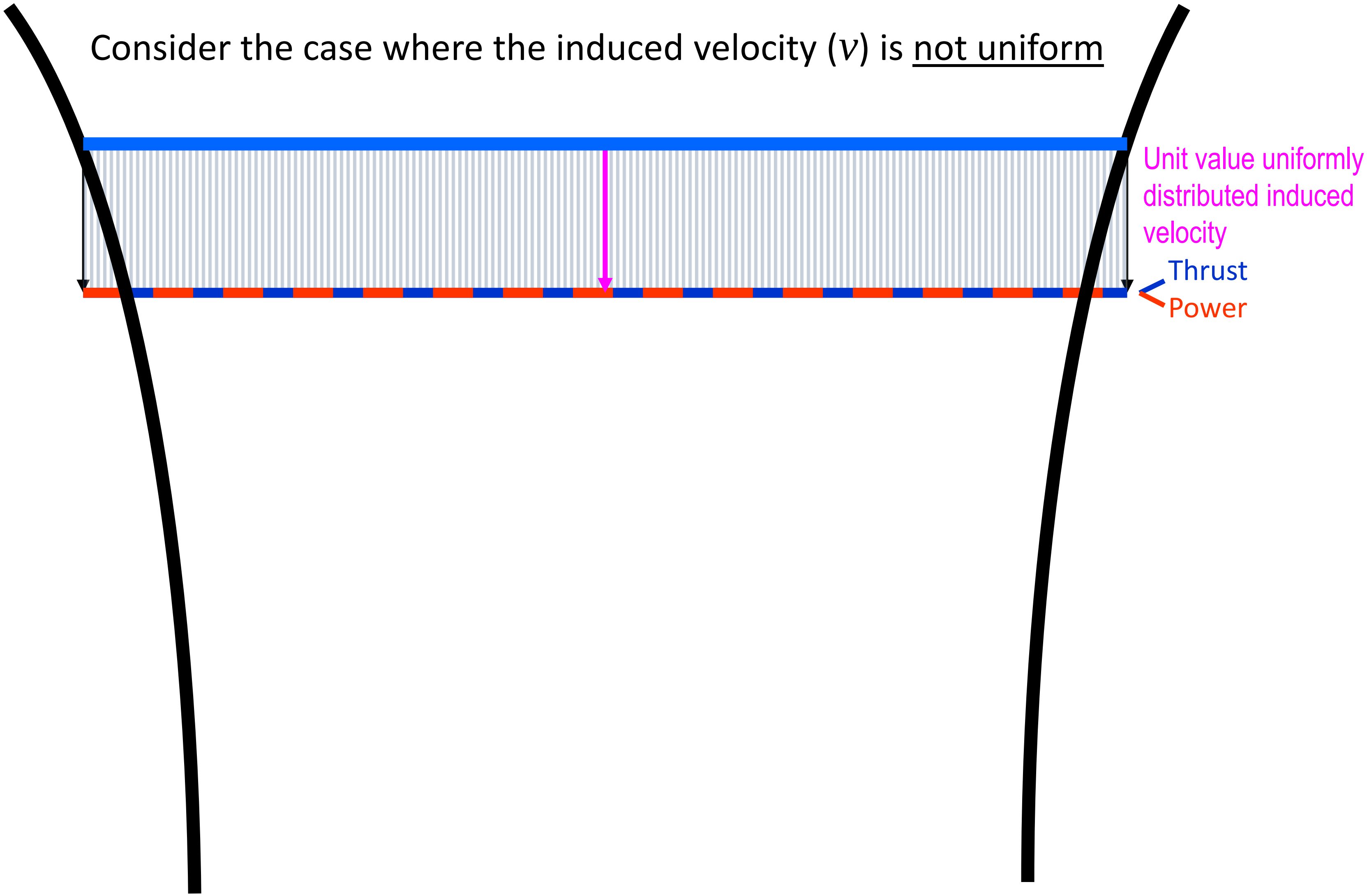


A rotor generates thrust by imparting momentum to the air that flows through it. This can only be efficiently achieved by a uniform distribution of induced velocity as assumed in actuator disk theory. Any local variation in the magnitude of the induced velocity will increase the overall power requirement as:

$$P = Tv = \left(2\rho A v^2\right)v = 2\rho A v^3$$

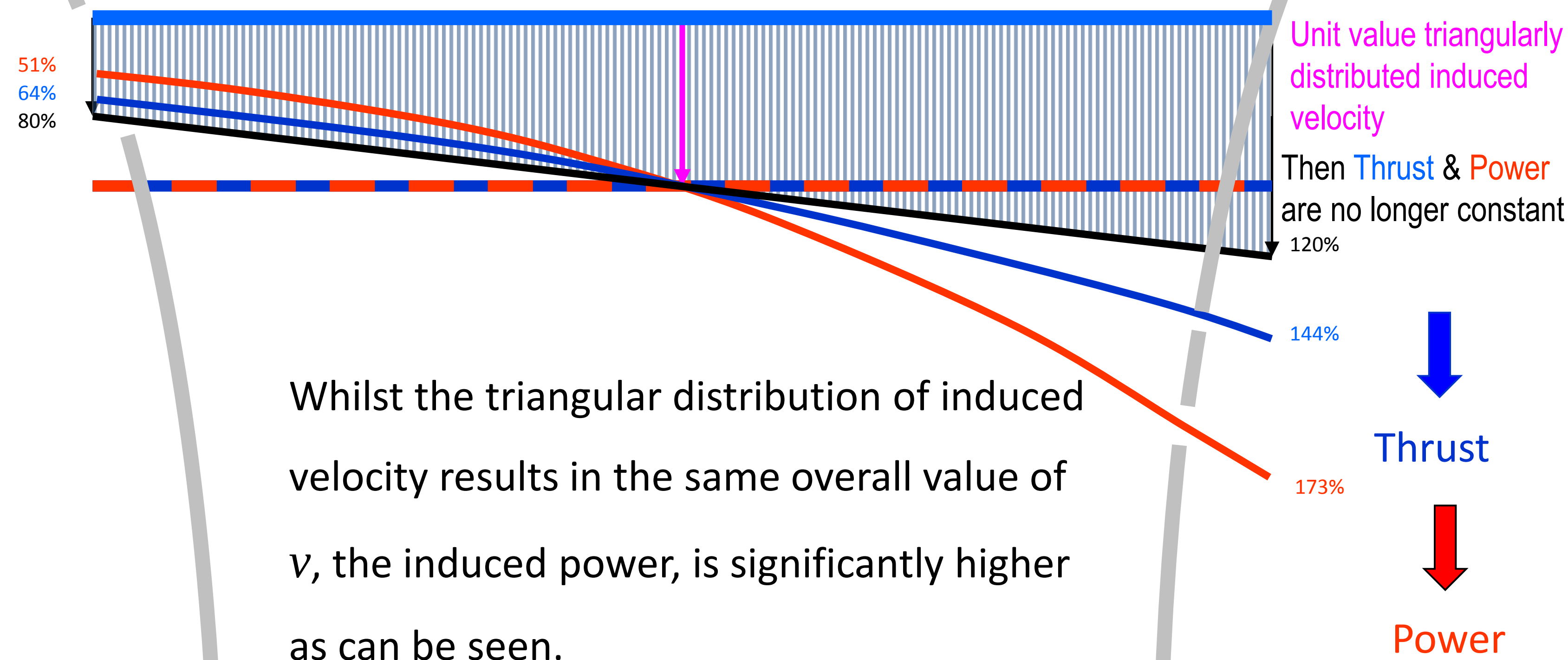
Maximising the Figure of Merit

Consider the case where the induced velocity (v) is not uniform



Maximising the Figure of Merit

Suppose v is 20% less on the LHS, 20% higher on the RHS

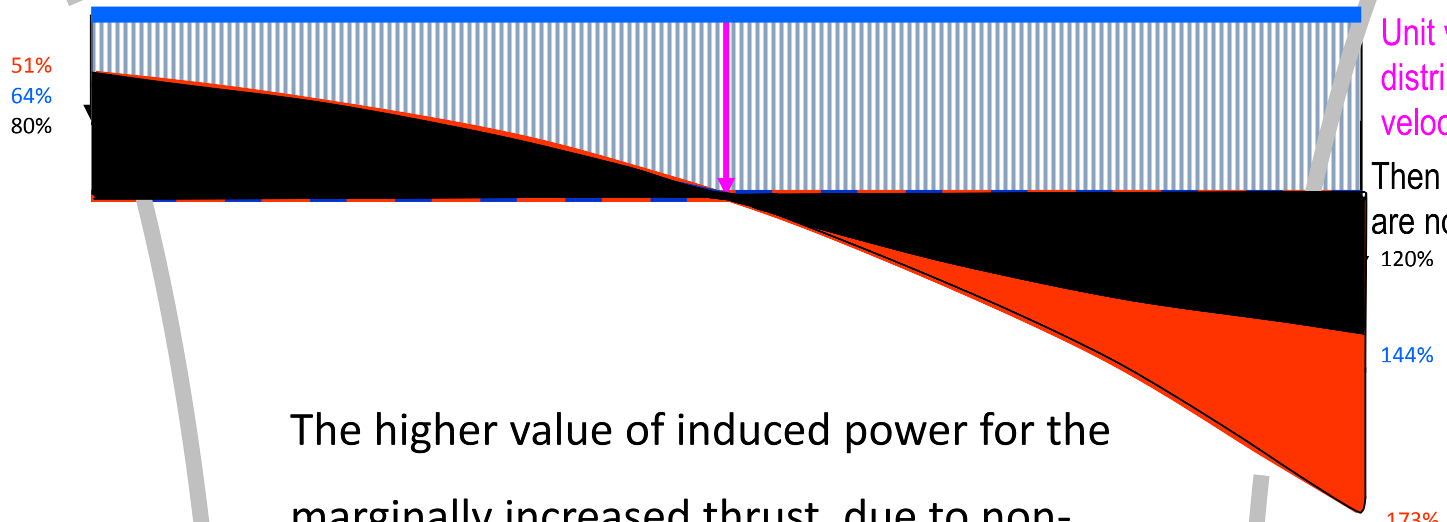


Whilst the triangular distribution of induced velocity results in the same overall value of v, the induced power, is significantly higher as can be seen.

This is Thrust and Power on an actuator disk, <u>NOT</u> the blades of a real helicopter.

Maximising the Figure of Merit

Suppose V is 20% less on the LHS, 20% higher on the RHS



Unit value triangularly distributed induced velocity

Then Thrust & Power are no longer constant

Thrust

Power

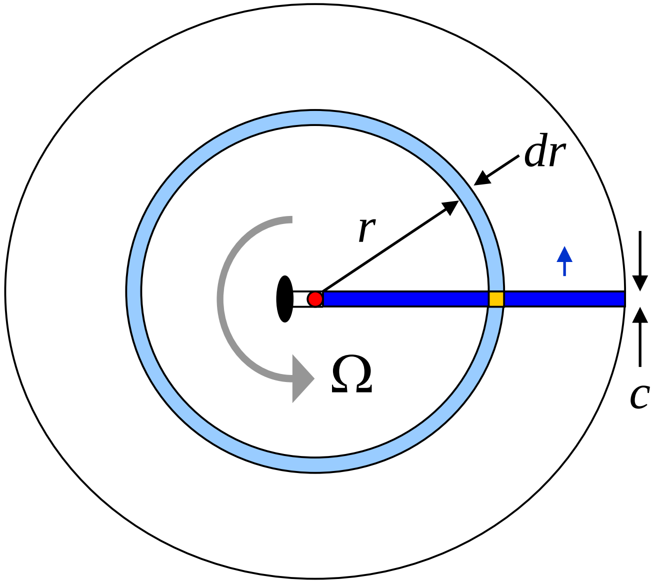
The higher value of induced power for the marginally increased thrust, due to non-uniform inflow velocity (v) is seen from the plot of excess power which is effectively a

. Power Loss

Maximising the Figure of Merit

For a helicopter rotor, which has a finite number of blades, the blade element theory can be used to equate the lift on a blade element to the induced velocity in the swept annulus of that element.

BO 102 – Single Bladed Helicopter



$$L = \tfrac{1}{2}\rho V^2 S C_L$$

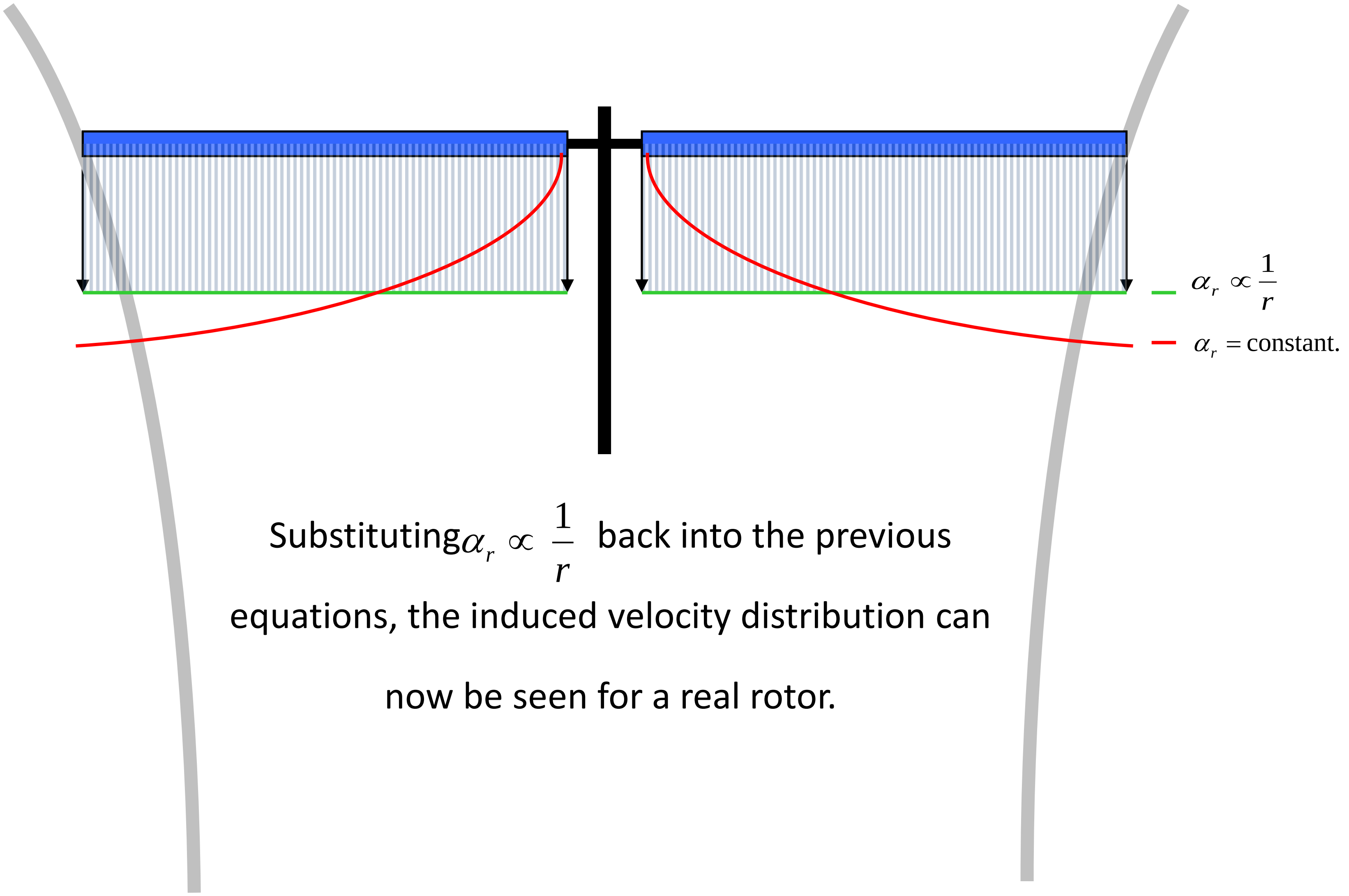
$$\boxed{dL = \tfrac{1}{2}\rho\,\Omega^2 r^2 c\,dr\,a\,\alpha_r = dT}$$

$$\boxed{v =}\sqrt{\frac{T}{2\rho A}},\ dv = \sqrt{\frac{\boxed{dT}}{\boxed{2\rho 2\pi\,rdr}}} = \sqrt{\frac{\rho\,\Omega^2 r^2 c\,dr\,a\,\alpha_r}{8\rho\,\pi\,r\,dr}}$$

dv is proportional to $\sqrt{\dfrac{r^2\alpha_r}{r}} = \sqrt{r\ \alpha_r}$

so for constant v, α_r is proportional to $\dfrac{1}{r}$

Maximising the Figure of Merit

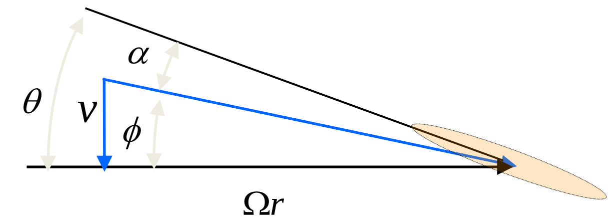


Substituting $\alpha_r \propto \dfrac{1}{r}$ back into the previous equations, the induced velocity distribution can now be seen for a real rotor.

Maximising the Figure of Merit

IDEAL BLADE TWIST results in a constant induced velocity across the rotor disk.

It has been seen that for this to be the case, then $\alpha_r \propto \dfrac{1}{r}$



For this to be the case, $\alpha_r = (\theta - \phi) = \dfrac{R}{r}(\theta_t - \phi_t)$

So, $\phi = \phi_t \dfrac{R}{r}$ where $\phi_t =$ inflow angle at the tip.

Similarly blade pitch angle $\theta = \theta_t \dfrac{R}{r}$ where $\theta_t =$pitch angle at the tip.

Thus, by careful design, the ideal inflow can be achieved by blade twist, or blade planform taper or a combination of the two.

Unfortunately the other requirements for ideal conditions (zero profile drag, tip losses and swirl in the wake) are not so easily met and must, at best, be minimised.

This is best achieved by utilising rotor blade element analysis.