Lecture 10
Elevator Angle to Trim

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Balance vs Stability

To satisfy the requirements for both equilibrium and stability, there is the implication that:

- The horizontal tail must provide a force to ensure a zero pitching moment overall …

Balance

- when the aircraft is disturbed in pitch from steady flight, the change in pitching moment must act to restore equilibrium, i.e. the aircraft must display a positive aerodynamic stiffness in pitch …

Stability

Tail Configurations

Three types of tail configuration can be used to achieve this:

a. fixed tail, plus elevator

- in this case trim is achieved by adjusting the elevator to a datum position, probably different from $\eta = 0$, with only the remaining elevator deflection being available for manoeuvring
- we shall consider later how to establish the fixed position for the horizontal tail (permanently built-in) – the *tail setting angle.*

Tail Configurations

b. trimming tail, plus elevator

- the *whole* horizontal tail can be rotated in pitch to achieve trim, perhaps with $\eta = 0$; the elevator is then used only for additional (more rapidly changing) control (manoeuvring)

c. all-flying tail, no elevator

- the datum position is selected for trim; control is then effected by additional movements of the whole tail away from this datum value.

CG / Balance Diagram

- Prior warning of final year design Project.
- Note the practical limits on C.G. movement, namely about 11%-36%; i.e. must ensure C.G. of A320 lies within ~1 metre!

  Typical C.G. Limits

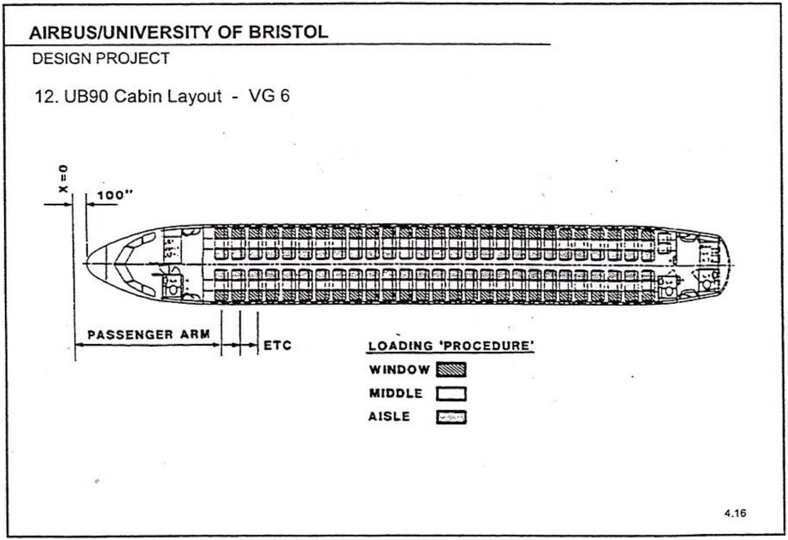
- All of these are based on *aerodynamic* considerations except the rear take-off limit, which is a function of the required load on the nosewheel of the aircraft during take-off, to ensure adequate steering.
- One requirement might be that the load on the nosewheel shall be no less than 2.5% of the total aircraft weight, with full static engine thrust.

CG / Balance Diagram

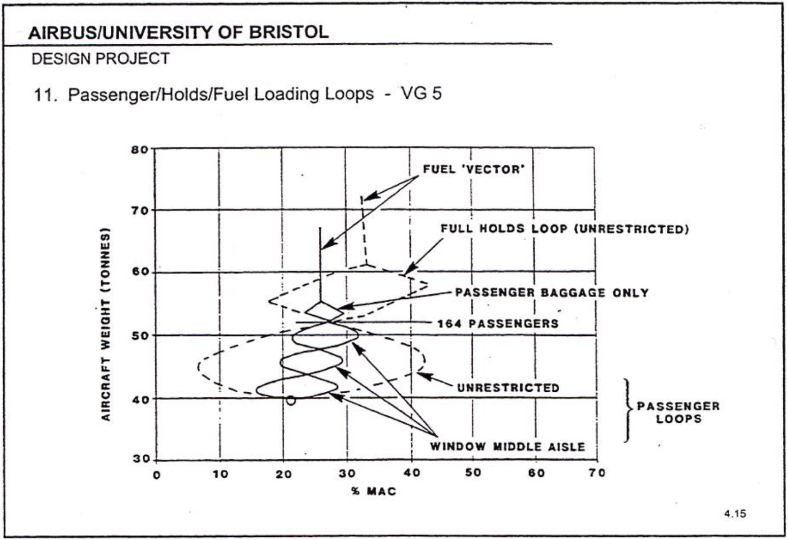
The Loading Loop

- Need to derive the C.G's at other Loaded Configurations.
- Can define a 'reasonable' passenger loading procedure, i.e. how the passengers will be distributed throughout the cabin when they are all on board.
- The effects of the holds and the fuel are shown as added to the case with full passengers but there might be a partly loaded configuration.

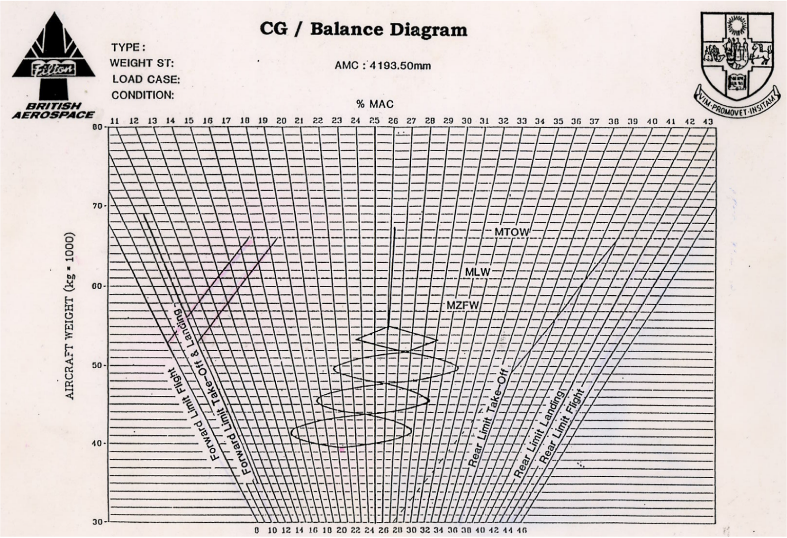
CG / Balance Diagram



CG / Balance Diagram



CG / Balance Diagram



Downwash and Wake Effects at the Tail

Downwash and Wake Effects at the Tail

Introduction

- The local incidence (or angle of attack) of an aircraft wing is not simply the angle between the approaching wind (its direction measured a long way ahead of the wing) and the wing chord from which zero-lift is measured – *there is an effective change in incidence*.

- This is due to the downwash created by vortices shed downstream from the wing: it alters the flow and affects even the wing itself.

Downwash and Wake Effects at the Tail

- Consider the flow field at the horizontal tailplane, where the wing wake is well-developed – we would expect that downwash would be significant.
- The incidences seen by the two aerofoils are not going to be equal and the distribution of incidence across the two aerofoils from tip to tip are not going to be equal. (*More on this in subsequent units* – this year we will consider a representative point.)
- Generally, the forward surface (while it is developing lift) will have a downward influence on the flow downstream so that the rearward surface 'sees' a lesser incidence.

Downwash and Wake Effects at the Tail

- The fuselage and propulsion system, especially propellers, will also have effects on the flow field at the tail – e.g. short take-off video.
- Often a design objective will be to place the horizontal tail where the effect of downwash is limited.

Local Airspeed

- The velocity over the tail will not normally be the same as the free stream. *Note: T-tails are less affected*.
- In general there will be a reduction in airspeed due to wing and fuselage drag.
- In contrast, there will be an increase in airspeed from the engine/propeller slipstream.
- We can account for these effects using the 'efficiency factor' η_T introduced in the previous lecture. This is effectively:

$$U_T^2 = \eta_T U^2 ,$$

but η_T is often simply taken to be 1.0 (assume to be 1.0 in the exam unless told otherwise).

The Downwash Field

- The lifting wing induces a downwash field in its wake.
- This causes the flow direction at the tail to be different from that of the free stream.
- Downwash also changes quite noticeably with wing incidence.

The change of angle (loss of incidence) is referred to as the downwash angle ε (epsilon).

- A wing will normally affect the whole of a tailplane with the wider wake causing downwash over the full span of the tail.
- The effect of a canard surface will vary depending on whether the point considered is within or without the span of the canard.

The Downwash Field

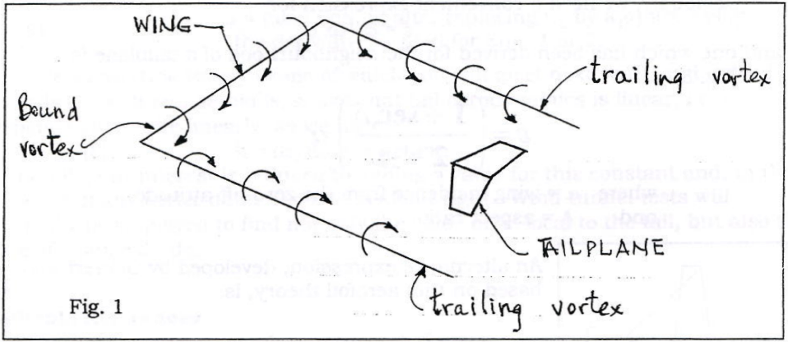


Fig. 1

A wing will produce a downwash field for the tailplane from a (wing) wake that stretches from upstream to downstream of the tail.

The Downwash Field

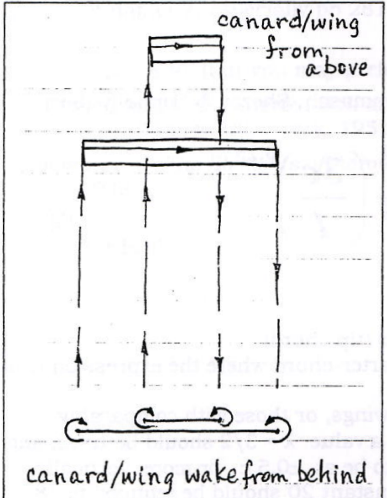


Fig. 2

- A canard will produce additional downwash within the span of the canard
- *but* upwash (leading to increased incidence) outboard of its span.
- *We will consider the conventional configuration.*

Simple Downwash Models for the Tail

- Downwash at the tail is directly proportional to the lift being developed at the wing.
- In order to express the effective tail incidence α_{T} we need to express ε (loss of incidence) at the tail in terms of α_{wing}.
- An approach is to employ a term $\alpha(1-k)$, where:

$$k = \partial\varepsilon / \partial\alpha \qquad (1)$$

- Therefore $$\alpha_{\mathrm{T}} = \alpha - \frac{\partial\varepsilon}{\partial\alpha}\alpha$$

  - where α is actually α_{wing}, from the zero-lift attitude
  - both of these terms become zero when there is no wing lift.

Simple Downwash Models for the Tail

- We need an expression for: $\varepsilon = \dfrac{\partial \varepsilon}{\partial \alpha} \alpha$ (2)

- One model which represents the downwash at the tailplane is:

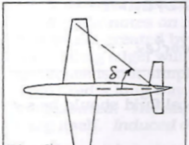


Fig. 3

$$\varepsilon = \left(\frac{1 + \sec \delta}{2 + A} \right) \alpha \qquad (3)$$

(No need to memorise)

where α = wing incidence from the zero-lift attitude

and A = aspect ratio.

Simple Downwash Models for the Tail

- An alternative expression, developed by Glauert and based on thin aerofoil theory, is:

$$\varepsilon = \underbrace{\left(\frac{4}{\pi}\right)^2}_{1.621}\left(\frac{1+\sec\delta}{2+A}\right)\alpha. \qquad (4)$$

- An alternative empirical expression is:

$$\varepsilon = 20\,C_L\,\frac{\lambda^{0.3}}{A^{0.725}}\left(\frac{3c}{l}\right)^{0.25} \qquad (5)$$

where: ε is in degrees

c is the mean wing chord

λ is the wing taper ratio (root chord/tip chord)

l is the distance aft of the wing quarter-chord where the expression is to be valid (see Fig. 4).

Simple Downwash Models for the Tail

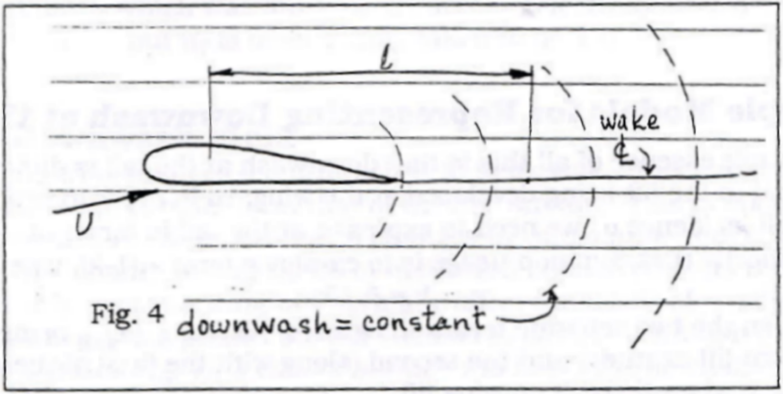


Fig. 4

Simple Downwash Models for the Tail

- Different approximations can be made for different tail positions and wing shapes.
- Differentiate equations (3), (4) (and (5) by replacing C_L by $a_1\alpha$) to find the derivative needed for Eqn. (1) or (2).
- A linear approximation can be used for ε dependency on α, so we have:

$$k = \frac{\partial \varepsilon}{\partial \alpha} = \text{constant}$$

- In the absence of better data, this constant can be approximated with $\boldsymbol{k}$ **= 0.5**.
- Wind tunnel tests are normally used to find not only the value of ε local to the tail but also the rate of change $d\varepsilon/d\alpha$.


Elevator-Angle-to-Trim
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Elevator-Angle-to-Trim

The three equations valid in the plane of symmetry are:

- transverse: $$L_w + L_T = nW \qquad (6)$$
- axial: $$F = D \qquad (7)$$
- pitch: for the moment we shall choose the a.c. of the wing to take the moments about, leading to:

$$M_{ac} = M_0 + nWx\bar{c} - L_T l_T \qquad (8)$$

$$= 0 \quad \text{for equilibirium}$$

Wing, Fuselage & Pod

- For ease of use, we will group together all lift contributions *other* than those due to the tail, and thus refer to the primary lift as L_{WFP} which means lift on the wing, fuselage and pods (engines). *(L_W)*

- We consider tail lift separately.

- There will still be a moment that is independent of lift but it will act about an aerodynamic centre which is not now at the quarter-chord (*contribution of the fuselage*).

Elevator-Angle-to-Trim

Introduction

- To achieve longitudinal balance we need to satisfy equations (6) & (8), and trim the aircraft to meet the following requirements:

  (a) fly fast enough to produce the required lift,

  (b) achieve an incidence, related to the flight speed, to satisfy the conditions in Eqn. (6); since the wing lift is dominant we tend to think of this as a requirement for wing incidence though it is not that simple, i.e. *need to consider the distribution of lift related to* (c).

Elevator-Angle-to-Trim

(c) achieve the pitch balance to satisfy Eqn. (8) by adjusting the tail lift;

- this is also a simplification because any adjustment of tail lift requires an equal and opposite change in wing lift to keep the balance given by Eqn. (6).
- nevertheless, the action to achieve the balance is to move the elevator and thus to change the tail lift.

What we must do is determine η (required elevator deflection) for a chosen speed.

Elevator-Angle-to-Trim

How do you arrange to get the longitudinal balance which is necessary for flight, and what are the variables which you adjust? Assume a fixed and 'sufficient' speed of flight.

- The requirement is $\sum \text{Lift} = \text{Weight}$ and $\sum M = 0$.

- Both wing and tail depend on wing-incidence α so we have to arrange for the correct value of α to provide enough lift to hold us up and have these two components (wing and tail) properly proportioned to produce $\sum M = 0$.

- Thus, the elevator is used to orientate the aircraft in pitch to satisfy both of these criteria.

Elevator-Angle-to-Trim

$U \rightarrow$ Fixed

$\alpha \rightarrow$ Variable

$\eta \rightarrow$ Variable

$i_T \rightarrow$ Variable (tail setting angle)

An Expression for the Elevator Angle η

- Assume a conventional configuration.
- Note that the tailplane is attached to the fuselage with a different basic incidence, greater than the wing incidence by the setting angle i_T.
- Thus the effective incidence at the tail is given by:

$$\alpha_T = i_T + \alpha - \varepsilon \qquad (9)$$

w subscript implicit
i.e. wing incidence

An Expression for the Elevator Angle η

- The downwash from the wing, which alters the incidence at the tail, is proportional to C_{L_W} and this in turn is proportional to α so we can put

$$\varepsilon = \frac{\partial \varepsilon}{\partial \alpha}\,\alpha \qquad (10)$$

- which implies exactly what was stated in the previous section on downwash/wake, namely that

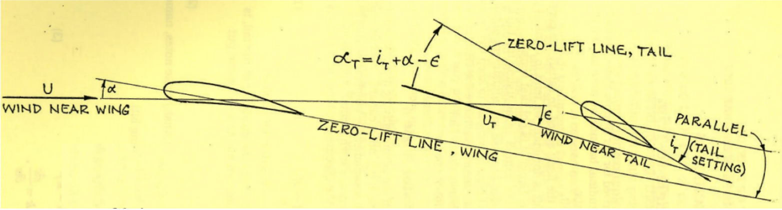
$$\varepsilon = 0 \ \text{for} \ \alpha = 0 \qquad \text{(no wing lift).}$$

An Expression for the Elevator Angle η

Eqn. (9) above then becomes

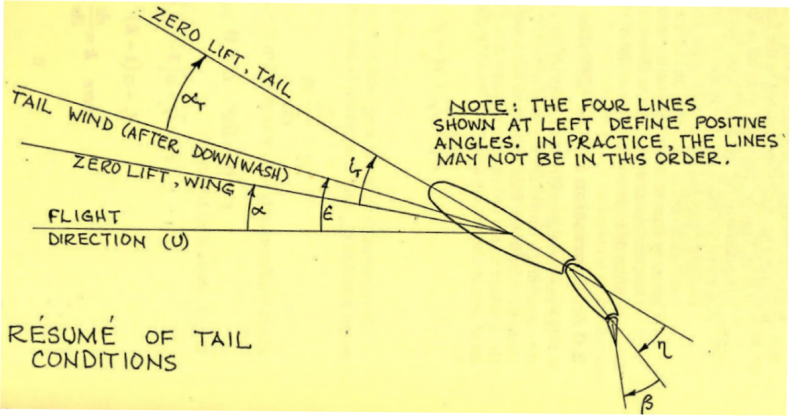
$$\begin{aligned} \alpha_T &= i_T + \alpha\left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right) \\ &= i_T + \alpha\left(1 - k\right) \qquad (11) \\ &\text{where } k = \frac{\partial\varepsilon}{\partial\alpha} \end{aligned}$$

An Expression for the Elevator Angle η



e.g. find i_T, then fix i_T and find η for a new condition.

An Expression for the Elevator Angle η



An Expression for the Elevator Angle η

- Then, to involve the elevator explicitly, we write the tail lift coefficient as:

$$
\begin{aligned}
C_{L_T} &= a_{1_T}\,\alpha_T + a_{2_T}\,\eta \\
&= a_{1_T}\left(i_T + \alpha\,[1-k]\right) + a_{2_T}\,\eta
\end{aligned}
\tag{12}
$$

- which implies a symmetric section for the horizontal tailplane, having $a_{0_T} = 0$.

- We can now use the moment equation Eqn. (12) from Lecture 9 (the moment equation) to solve for the balance but be careful about the choice of reference point – a.c. versus c.g.

Choice of Lift Coefficient C_L

- Eqns. (11) & (12) from the Lecture 9 show that we should really use the total C_L, with the consequence that:

$$C_{M_{ac}} = C_{M_0} + x\,C_L - \bar{V}\,C_{L_T} \qquad (13)$$

$$= C_{M_0} + x\left[C_{L_W} + \frac{S_T}{S} C_{L_T}\right] - \bar{V}\,C_{L_T}$$

$$= C_{M_0} + x\,C_{L_W} + \left(x\frac{S_T}{S} - \bar{V}\right) C_{L_T}$$

- and this, rather inconveniently(!), makes both the C_{L_W} term and the C_{L_T} term dependent on x.

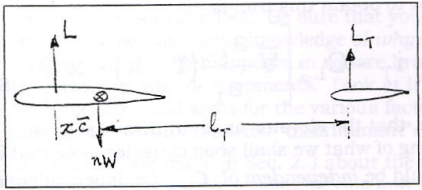
Choice of Lift Coefficient C_L

- Consider the factor: $\left(x\frac{S_T}{S} - \bar{V} \right)$

	Twin Otter	Cessna 402B	Airbus	B-747
$\bar{V}$	1.155	0.958	1.06	1.0
$x\,S_T\,/\,S$	0.013 to 0.02	0.029 to 0.05	0.025 to 0.04	about ± 0.025

- The implication is that relative to the upper line of these figures (i.e. $\bar{V}$), the lower line can be ignored without great loss of accuracy.

Choice of Lift Coefficient C_L



- An alternative model seen in some text-books uses the geometry above, with a different definition of l_T. The comparable moment equation that is derived appears as:

$$C_{M_{CG}} = C_{M_0} + xC_L - \bar{V}C_{L_T} \qquad (14)$$

- in which the C_L is not a total aircraft C_L: C_{L_W} is implied. Also in this equation it is not made obvious that when x changes so must $\bar{V}$ change because of its factor l_T.

Moment Balance Using η

- One of the virtues in simplifying Eqn. (13) to a near copy of the first form shown above, namely:

$$C_{M_{AC}} = C_{M_0} + xC_{L_W} - \overline{V}\,C_{L_T} \qquad (13a)$$

- is that we might want to take partial derivatives of the kind

$$\frac{\partial}{\partial\alpha} \text{ or } \frac{\partial}{\partial C_L}$$

Stability?

- Note: $C_{L_W} = a_1\alpha$

subscript w implied

Elevator-Angle-to-Trim

- Now we can combine Eqn. (12) (lift acting on the tail) & (13) (moment-balance):

$$C_{M_0} + x\,C_{L_{WFP}} - \bar{V} a_{1_T} \left(i_T + \alpha\,[1-k]\right) - \bar{V} a_{2_T} \eta = 0 \qquad (14)$$

- Note that we have terms in $C_{L_{WFP}}$ and in α that could be combined. We replace α by $C_{L_{WFP}} / a_1$ and gather terms in $C_{L_{WFP}}$ to obtain:

$$C_{M_0} - \bar{V} a_{1_T} i_T - C_{L_{WFP}} \left[\bar{V} \frac{a_{1_T}}{a_1} (1-k) - x \right] - \bar{V} a_{2_T} \eta = 0 \qquad (15)$$

Elevator-Angle-to-Trim

- *and thus the elevator angle required to obtain this trim is:*

$$\eta_{trim} = \frac{1}{\overline{V} a_{2_T}} \left\{ C_{M_0} - \overline{V} a_{1_T} i_T - C_{L_{WFP}} \left[\overline{V} \frac{a_{1_T}}{a_1} (1-k) - x \right] \right\} \qquad (16)$$

- which displays, among other things, that the elevator-angle-to-trim is a linear function of $C_{L_{WFP}}$.

Choosing i_T

- Generally, for a fixed tailplane, i_T will be chosen to obtain trim in cruise without having to use the elevator, i.e. η will be nearly zero.

- Obviously, changes in speed and c.g. position alter the need for η so, at best, the value of i_T that requires $\eta = 0$ can be correct at only one (instantaneous) condition.

- For an adjustable horizontal tail, the whole cruise can be flown at $\eta = 0$ while i_T is adjusted continually.

Next Lecture

Static Stability