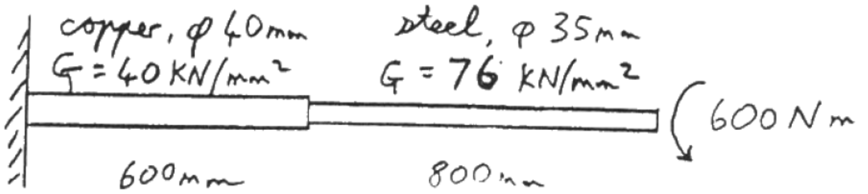
Example 2.4.1

a) A shaft is held rigidly at one end and has a twisting couple of 600 N·m applied at the other end. It can be idealised as being of length 1400 mm, composed of a 40 mm diameter copper bar 600 mm long joined to a 35 mm diameter steel bar 800 mm long. Ca1culale the maximum shear stresses in the two materials, and the angle of twist of the free end. Take the shear modulus of copper to be 40 GPa and that of steel to be 76 GPa.

(Ans: 47.7 MPa, 71.3 MPa, 4.51º)




Study lecture handout on torsion. Take care about surfaces on which shear stresses act.

Statics: torque is constant along the whole length.

$$\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{r} \qquad \text{For circular cross-section, } J = \frac{\pi R^4}{2}$$

$\tau_{max}$ is at $r_{max}$, i.e. at $R$

For copper, $\tau_{max} = \frac{T r_{max}}{J} = \frac{600\times10^3\times20}{\pi\times20^4/2}$ N/mm² $= 47.7$ N/mm²

For steel, similarly, $\tau_{max} = 71.3$ N/mm²

Angle of twist of copper $= \frac{TL}{GJ} = \frac{600\times10^3\times600}{40\times10^3\times\pi\times20^4/2}$ radians $= 0.0358$ radians

Similarly, angle of twist of steel $= 0.0429$ radians.

Hence total angle of twist $= 0.0358 + 0.0429 = 0.0787$ radians

$= 4.51°$

b) A 40 kW motor is to drive the propeller shaft of a boat at 200 r.p.m. If the shaft is to be of solid circular section, and the shear stress in it is to be limited to 70 MPa, calculate the diameter of shaft required. If a hollow shaft of outside diameter 20% greater is used, subject to the same maximum stress, calculate the percentage saving in weight of the shaft, and the percentage change in the twist of the shaft (power = torque × angular velocity).

(Ans: 51.8 mm, 49.5%, -16.7%)

Power is 40 kW at 200 rpm.

Torque $= \frac{\text{power}}{\text{angular velocity}} = \frac{40 \times 10^3}{200 \times 2\pi/60}$ Nm $= 1910$ Nm

Maximum shear stress is at maximum radius, i.e. R

$\tau_{max} = \frac{TR}{J}$, hence $70 = \frac{1910 \times 10^3 \times R}{\pi R^4/2}$ where R is in mm

Hence $R = 25.90$ mm, and diameter of shaft is 51.8 mm.

Consider using hollow shaft of outside diameter 20% greater.

i.e. $R_1 = 25.90 \times 1.2 = 31.08$ mm

In this case, max stress is at $R_1$, and $J = \frac{\pi}{2}(R_1^4 - R_2^4)$

Hence using $\tau_{max} = \frac{TR_1}{J}$, then $R_2 = 25.04$ mm

Cross-sectional area of hollow shaft $= \pi(R_1^2 - R_2^2) = 1065$ mm$^2$

Cross-sectional area of solid shaft $= \pi R^2 = 2107$ mm$^2$

Thus saving in weight = reduction in cross-sectional area

$$= \frac{2107 - 1065}{2107} = 49.5\%$$

For angle of twist, then using $\frac{G\theta}{L} = \frac{\tau_{max}}{r_{max}}$,

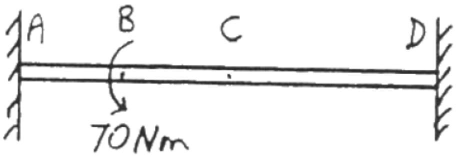
then in this case $\theta \propto \frac{1}{r_{max}}$

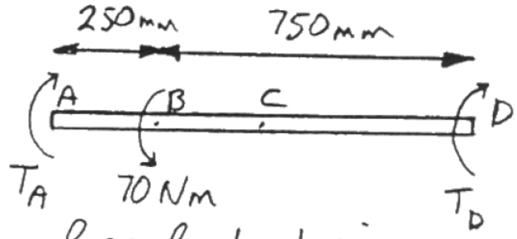
Hence change in angle of twist = change in $\frac{1}{r_{max}}$

$$= \frac{\frac{1}{31.08} - \frac{1}{25.90}}{\frac{1}{25.90}} = -16.7\%$$

c) A steel rod 20 mm diameter, 1000 mm long is rigidly held at both ends. A torque of 70 N·m is applied 250 mm from one end. Calculate the angle of twist of the mid-point of the rod. Shear modulus is 76 GPa.

(Ans: -0.42º)




free body diagram

This is a statically indeterminate problem.
There will be reaction torques at each end,
and equations of statics will just give $T_A + T_D = 70$

We need a geometrical condition. With both ends held rigid, i.e. with no rotation, then geometrical condition is that angle of twist of B considering AB = angle of twist of B considering BD

$\frac{T}{J} = \frac{G\theta}{L}$. Rod is uniform, so G and J are constant.

Hence $\theta \propto TL$. Hence $T_A L_{AB} = T_D L_{BD}$

Thus $T_A \times 250 = T_D \times 750$, i.e. $T_A = 3T_D$

Using this with $T_A + T_D = 70$ gives $T_A = 17.5$ Nm and $T_D = 52.5$ Nm

(Note that this result of $T_A L_{AB} = T_D L_{BD}$ looks like "taking moments about B for the torque". However, this is for the special case of G and J being constant along the length. If either varied, say by changes in diameter, then result would be quite different)

We can now find $\theta_C$ by considering the torque $T_D$ acting over the length CD.

$$J = \frac{\pi}{2}\, r^4 = \frac{\pi}{2}\,(10\text{ mm})^4 \cong 15{,}708\text{ mm}^4$$

Note that length CD is under a **negative torque** (following the right-hand rule, with *x* from left to right), therefore:

$$\theta_C = \frac{T\,L}{G\,J} = \frac{(-17{,}500\text{ N}\cdot\text{mm})\,(500\text{ mm})}{\left(76{,}000\,\frac{\text{N}}{\text{mm}^2}\right)(15{,}708\text{ mm}^4)} = -7.33\;10^{-3}\text{ rad} = -0.42^\circ$$