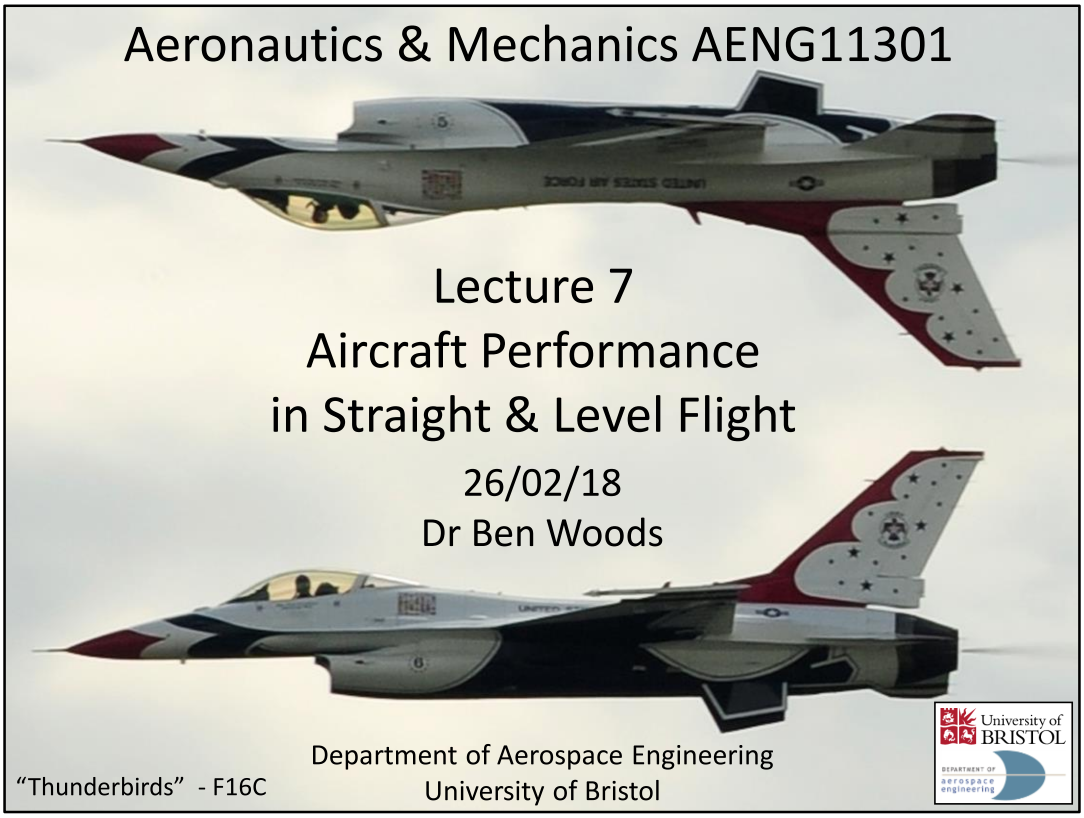


Straight and level… we didn’t say right side up!

Outline for today

- Forces in steady flight
- Equivalent air speed
- Drag equation
- Factors effecting drag
- Examples

Aims for today

- Know force balance in steady flight
- Be able to calculate equivalent airspeed
- Be able to calculate the two components that make up the drag equation
- Be able to calculate minimum drag velocity
- Know the effects of changing altitude and weight on drag

Forces in steady flight

Equivalent air speed

Drag equation

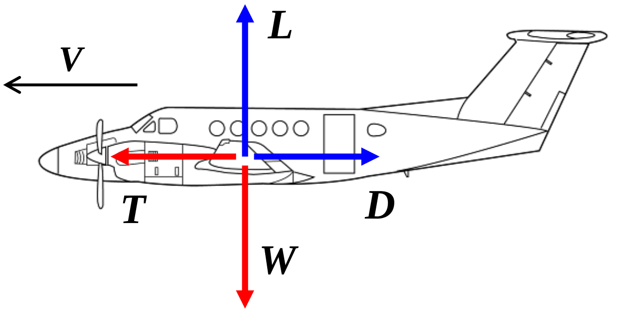
Factors effecting drag

Examples

http://www.youtube.com/watch?v=M-1hZM7v1rQ

Forces Acting on an Aircraft

- straight & level flight at constant speed
  - forces on aircraft in equilibrium
  - treat aircraft as a 'point mass'
- four forces acting on the aircraft
  1. Lift **L** – perpendicular to motion
  2. Drag **D** – opposing the motion
  3. Thrust **T** – assumed to act in flight direction (small α)
  4. Weight **W** – level flight so also perpendicular to the motion



Forces Acting on an Aircraft - Lift

- for an aircraft in equilibrium: $\boxed{L = W}$
- lift coefficient $$\boxed{C_L = \frac{L}{\frac{1}{2}\rho V^2 S} = \frac{W}{\frac{1}{2}\rho V^2 S} = \frac{w}{q}}$$
  - w is the **wing loading** W/S and q is the dynamic pressure $\frac{1}{2}\rho V^2$
- the corresponding flight speed V is then

$$\boxed{V = \sqrt{\frac{W}{\frac{1}{2}\rho S C_L}} = \sqrt{\frac{w}{\frac{1}{2}\rho C_L}}}$$

  - slower flight speed obtained by:
    reducing wing loading w (lighter aircraft or bigger wing) or
    increasing C_L (= increasing incidence α)
  - **minimum possible level flight speed occurs at C_{Lmax} (= stall)**
  - reducing density ρ (= increasing **altitude**) increases flight speed

Equivalent Air Speed

- variation of aerodynamic forces with density (altitude) makes performance calculations awkward

$$\boxed{\sigma = \frac{\rho}{\rho_{SL}} \approx \frac{20 - H}{20 + H}} \quad \rightarrow H = \text{altitude in km}$$

- **True Air Speed** (**TAS**) V or V_T is the speed of the aircraft relative to the air-mass in which it is flying
- for an aircraft flying at altitude H where the air density is ρ ...
- the **Equivalent Air Speed** (**EAS**) V_E is the speed at standard sea level density (ρ_{SL} or $\rho_0 \approx 1.225$ kg/m^3) which will give the same aerodynamic loads
  - for the same aerodynamic configuration, *i.e.* constant C_L etc

$$L = \tfrac{1}{2}\rho V^2 S C_L = \tfrac{1}{2}\rho_0 V_E^2 S C_L \quad \Rightarrow \quad \boxed{V_E = V\sqrt{\frac{\rho}{\rho_0}} = V\sqrt{\sigma}}$$

Equivalent Air Speed (2)

- useful in *calculations* – can often work only in terms of sea level conditions then convert to required altitude
- useful in *flight* – EAS is close to **Indicated Air Speed** (IAS) read by a pilot from an **airspeed indicator** (ASI)
  - ASI uses a **pitot-static tube** to measure dynamic pressure q directly, since from Bernoulli

$$\boxed{p_{pitot} - p_{static} = p_0 - p = \tfrac{1}{2}\rho V^2 = \tfrac{1}{2}\rho_0 V_E^2}$$

- the pilot is most concerned with forces on the aircraft, which depend on q
  - stall will always occur at the same EAS
  - structural limits are defined in terms of EAS
- in most cases $\rho < \rho_0$ so $V_{EAS} < V_{TAS}$

Forces Acting on an Aircraft - Drag

- for an aircraft in equilibrium: $\boxed{T = D}$
- drag coefficient $\boxed{C_D = \dfrac{D}{\frac{1}{2}\rho V^2 S} = \dfrac{T}{\frac{1}{2}\rho V^2 S}}$
- many additional sources of drag on an aircraft
  - profile drag from other major components (fuselage, tail etc)
  - **interference drag** between components (eg wing/fuselage junction)
  - drag due to intake/exhaust of air for internal services
  - **slipstream drag** for propeller–driven aircraft
  - **trim drag** due to aerodynamic load on tailplane for balance
  - **excrescence drag** (aerials, canopies etc)
  - drag due to surface condition (roughness, waviness, rivets, etc)
  - deployment of flaps, airbrakes, spoilers, undercarriage, etc

The Drag Equation

- accurate estimation of drag a long and complex process
  - datasheets, **computational fluid dynamics**, wind tunnel testing
- for purposes of flight performance studies we will assume operation in the linear lift region
- drag can therefore be assumed to consist of:
  1. a part independent of incidence
     – the **zero lift drag coefficient** C_{D0}
  2. a part proportional to C_L^2 – the induced drag plus the part of the form drag that varies with incidence
     \- **drag due to lift**

$$\Rightarrow \quad \boxed{C_D = C_{D_0} + KC_L^2}$$

Oswald efficiency factor

- Drag equation can also be written:

$$\boxed{C_D = C_{D_0} + \frac{C_L^2}{\pi e AR}}$$

where:

$$\boxed{\frac{1}{\pi e AR} = K}$$

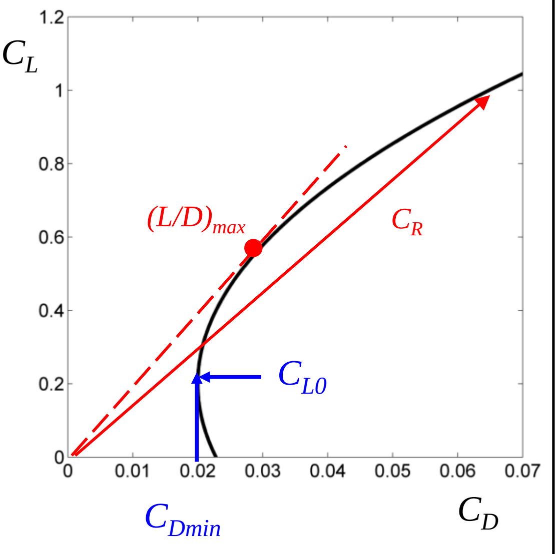
- Redefine ***e***, the span efficiency factor from an isolated wing to include the effects of variation in profile drag with lift for the whole aircraft, now termed the **Oswald efficiency factor**

Drag Polar

- for a general aircraft with a cambered wing, minimum drag no longer necessarily occurs at zero lift
  - in which case the drag equation becomes

$$C_D = C_{D_{min}} + K\left(C_L - C_{L_0}\right)^2$$

  - sometimes plotted as C_L vs C_D
  - line from origin gives total aerodynamic force R and **lift/drag ratio** L/D
  - tangent gives L/D_{max}
- For simplicity we will assume C_{L0} = 0 in this course



The Drag Equation Expanded

- take basic lift and drag equations and rearrange to obtain an equation in velocity V for drag D

$$\boxed{C_D = C_{D_0} + KC_L^2}$$

$$\boxed{C_L = \frac{W}{\frac{1}{2}\rho V^2 S}}$$

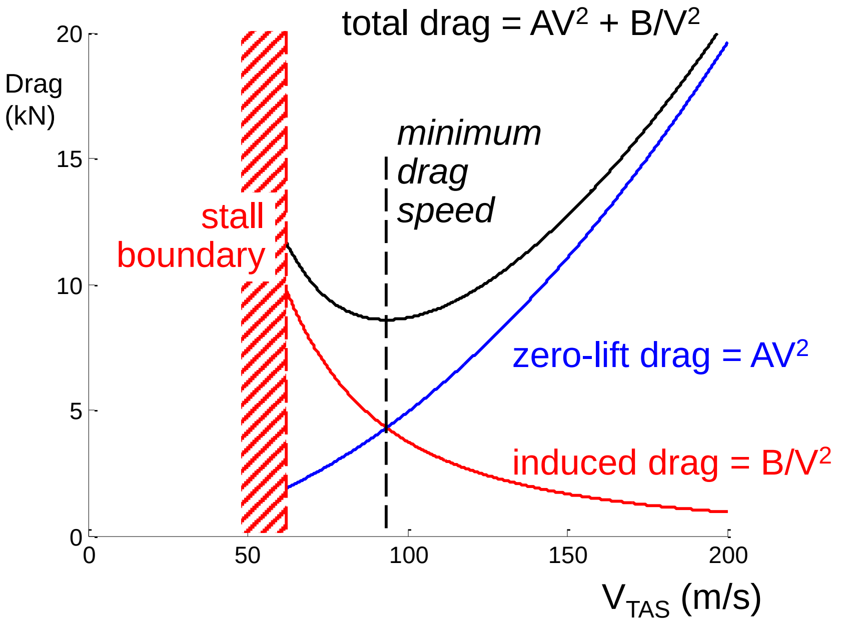
$$\begin{aligned} D &= C_{D_0}\tfrac{1}{2}\rho V^2 S + KC_L^2 \tfrac{1}{2}\rho V^2 S \\ &= C_{D_0}\tfrac{1}{2}\rho V^2 S + \frac{KW^2}{\frac{1}{2}\rho V^2 S} \end{aligned}$$

$$\boxed{D = AV^2 + \frac{B}{V^2}}$$

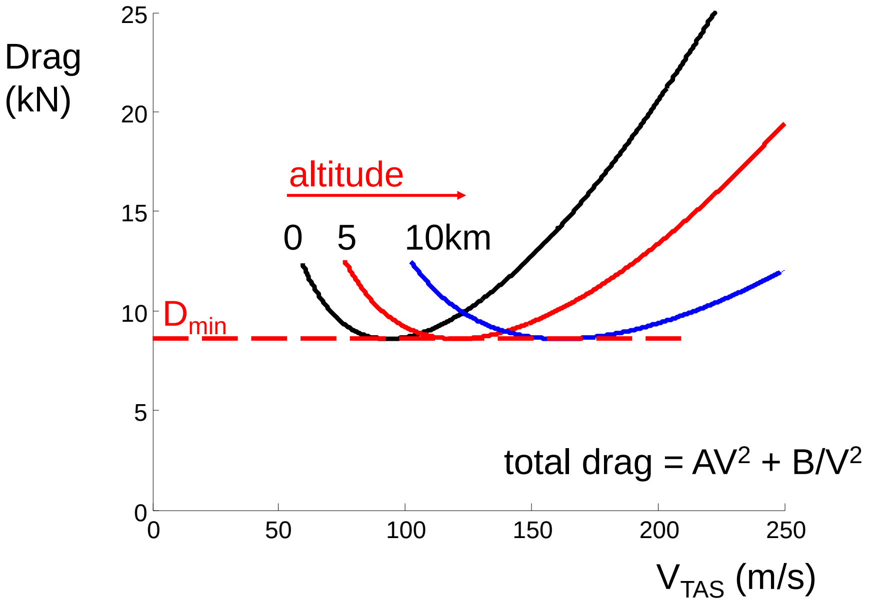
- where A and B are *functions of altitude* / density
  - A is profile drag term
  - B is induced drag term

$$\boxed{A = C_{D_0}\tfrac{1}{2}\rho S \ , \quad B = \frac{KW^2}{\frac{1}{2}\rho S}}$$

Drag at Sea Level



Drag at Altitude



Features of Drag Curves

- combination of V^2 and $1/V^2$ terms gives a minima in the total drag curve at the **minimum drag speed** V_{MD}
  - at low speed the induced drag term B dominates – take-off, landing & air combat
  - at high speed the profile drag term A dominates – cruise conditions
- effect of altitude/density is to shift curves to the right
  - V_{MD} and **stall speed** V_{stall} increase with altitude
  - minimum drag D_{min} remains constant
    $\rightarrow$ weight W and maximum lift/drag ratio L/D are both constant
- if plotted in terms of EAS instead then variation with altitude disappears !

Drag Equation in Terms of VEAS

- substitute equivalent air speed V_E and (constant) density ρ_0 into drag equation

$$D = C_{D_0}\tfrac{1}{2}\rho V^2 S + \frac{KW^2}{\tfrac{1}{2}\rho V^2 S}$$

$$\Rightarrow\quad D = C_{D_0}\tfrac{1}{2}\rho_0 V_E^2 S + \frac{KW^2}{\tfrac{1}{2}\rho_0 V_E^2 S}$$

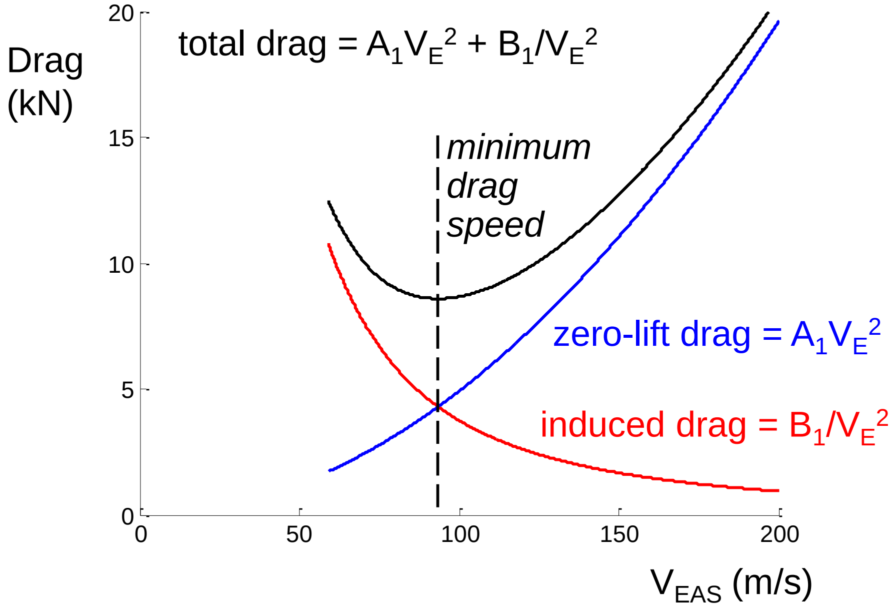
$$\Downarrow$$

$$\boxed{D = A_1 V_E^2 + \frac{B_1}{V_E^2}}$$

- where A_1 and B_1 are now ***constants***

$$\boxed{A_1 = C_{D_0}\tfrac{1}{2}\rho_0 S\ , \quad B_1 = \frac{KW^2}{\tfrac{1}{2}\rho_0 S}}$$

Single Drag Curve



Minimum Drag

- drag given by
  - minimum drag occurs at minimum C_D/C_L

$$D = L \times \left(\frac{D}{L}\right) = W \times \left(\frac{C_D}{C_L}\right)$$

$$\frac{C_D}{C_L} = \frac{C_{D0} + KC_L^2}{C_L} = \frac{C_{D0}}{C_L} + KC_L \quad \Rightarrow \quad \frac{d(C_D/C_L)}{dC_L} = -\frac{C_{D0}}{C_L^2} + K$$

- at minimum point $d(C_D/C_L)/dC_L = 0$
  - *i.e.* when $C_{D0} = KC_L^2$

$$\Rightarrow \quad \boxed{C_{D_{min}} = 2C_{D0} = 2KC_L^2} \quad \Rightarrow \quad C_{L_{MD}} = \sqrt{C_{D0}/K}$$

$$\Rightarrow \quad \left(\frac{C_D}{C_L}\right)_{min} = \frac{2C_{D0}}{\sqrt{C_{D0}/K}} = 2\sqrt{C_{D0}K}$$

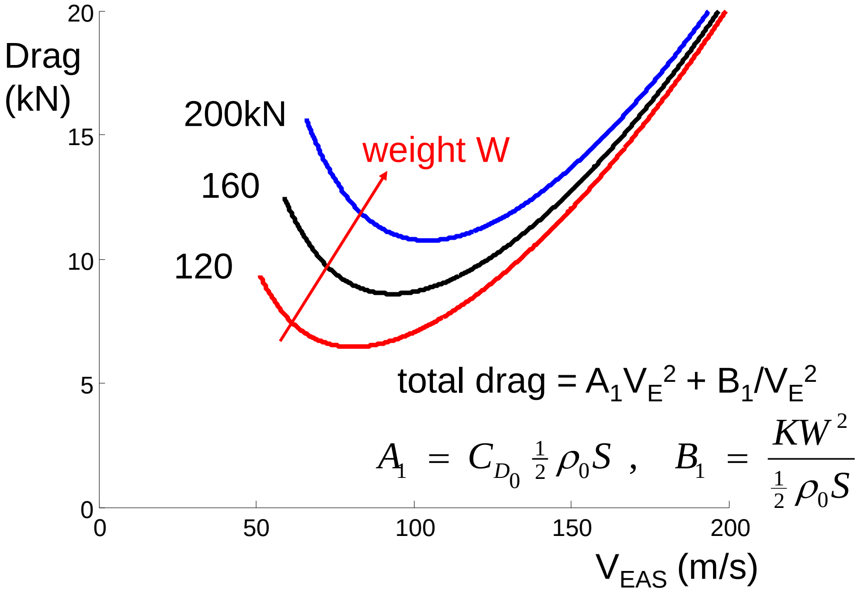
Minimum Drag Speed

- lift at minimum drag is $C_{L_{MD}} = \sqrt{\dfrac{C_{D0}}{K}}$
- substitute into speed equation $V = \sqrt{\dfrac{W}{\frac{1}{2}\rho S C_L}}$

$$\Rightarrow \quad V_{MD} = \sqrt{\frac{W\sqrt{K}}{\frac{1}{2}\rho S\sqrt{C_{D0}}}}$$

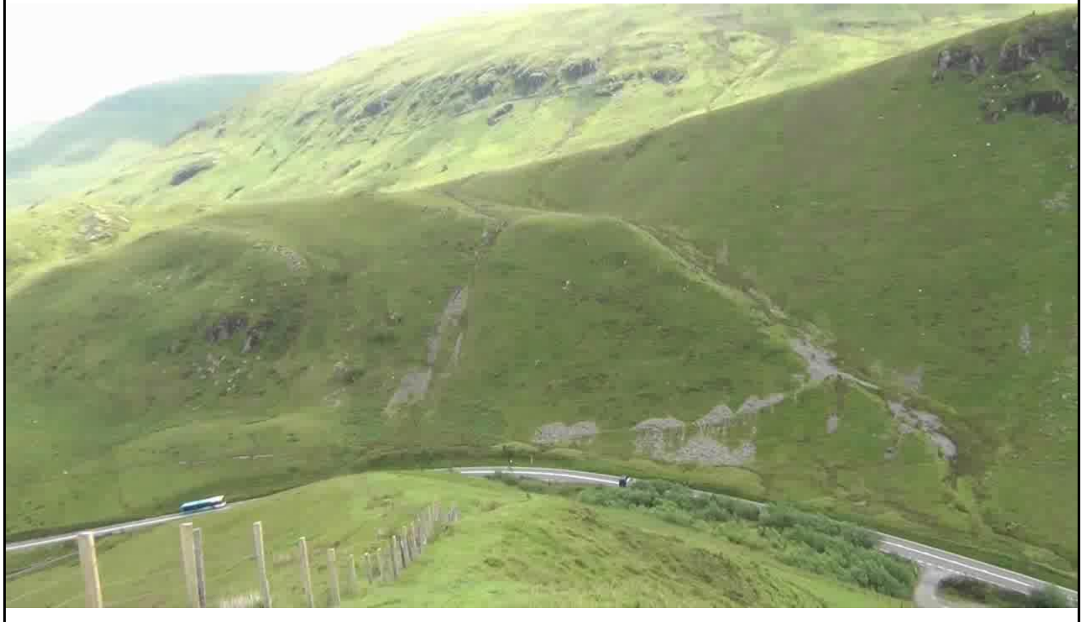
$$\Rightarrow \quad \boxed{V_{MD} = \left(\frac{2W}{\rho S}\right)^{\frac{1}{2}}\left(\frac{K}{C_{D0}}\right)^{\frac{1}{4}}}$$

Effect of Weight on Drag



Example : C-130 Hercules

The Lockheed C-130 Hercules is a four-engine turboprop military transport aircraft
Capable of using unprepared runways for takeoffs and landings,
Designed as a troop, medical evacuation, and cargo transport aircraft.
Also used as a gunship (AC-130), for airborne assault, search and rescue, scientific research support, weather reconnaissance, aerial refueling, maritime patrol, and aerial firefighting.
It is now the main tactical airlifter for many military forces worldwide
The C-130 entered service with U.S. in the 1950s
The family has the longest continuous production run of any military aircraft in history.
The C-130 is one of the only military aircraft to remain in continuous production for over 50 years with its original customer, as the updated C-130J Super Hercules.



The Mach Loop (also known as the Machynlleth Loop) refers to a series of valleys in the United Kingdom in west-central Wales, notable for their use as low-level training areas for fast jet aircraft. The system of valleys lies 8 miles east of Barmouth and is nestled between the towns of Dolgellau to the north and Machynlleth to the south, the latter of which it takes its name from. The training area is within the Low Flying Area (LFA) LFA7, which covers most of Wales.[1]
Aircraft which use the training area include Royal Air Force Tornado, Typhoon, and Hawk jets, as well as U.S. Air Force F-15E Strike Eagles, which are based at RAF Lakenheath in eastern England.[2]

http://www.youtube.com/watch?v=97rSobuKBxI
http://www.youtube.com/watch?v=W161wBS3XWw

Example – C-130J

Mass = 70,300 kg

Wing area, S = 162 m^2

Wing span, b = 40.4 m

Cruise altitude, h = 8,500 m

(where speed of sound
a = 306 m/s)

Oswald efficiency, e = 0.90

Zero lift drag coefficient,
C_{D0} = 0.028

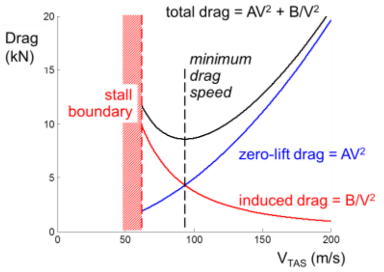
Cruise velocity, V = 0.57 M

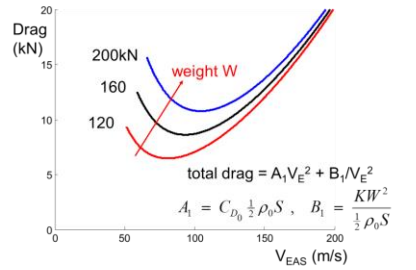
Calculate:

- the equivalent air speed at cruise conditions
- thrust required at cruise
- minimum drag velocity at cruise altitude
- minimum drag velocity at sea level having burnt 17,500 kg of fuel

Summary

- In steady level flight $\boxed{L = W}$ $\boxed{T = D}$
- Equivalent airspeed
$$\boxed{V_E = V\sqrt{\frac{\rho}{\rho_0}} = V\sqrt{\sigma}}$$
- Drag polar $\boxed{C_D = C_{D_0} + KC_L^2}$
- Minimum drag condition $\boxed{C_{D_{min}} = 2C_{D0} = 2KC_L^2}$





Know force balance in steady flight

Be able to calculate equivalent airspeed

Be able to calculate the two components that make up the drag equation

Be able to calculate minimum drag velocity

Know the effects of changing altitude and weight on drag

Follow-up materials

To help with exam:

- Introduction to Flight
  - Equivalent airspeed 4.12.2
  - Steady level flight 6.1 to 6.3

To help with exam:
Introduction to Flight – 5.1-5.2

To aid in understanding:
Understanding flight – Chapter 1

For interest:
Introduction to Flight – 5.19 (explanation of lift)