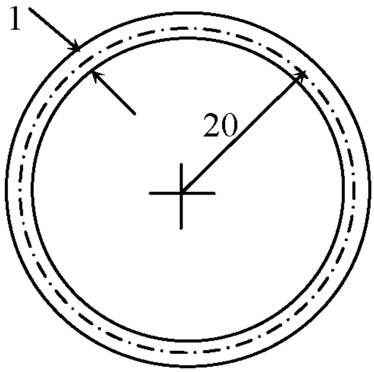
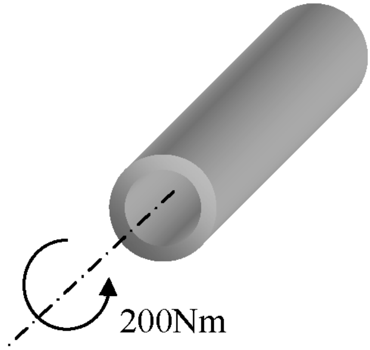
Q1 A thin walled steel circular tube of mean radius = 20mm carries a torque of 200 Nm. Calculate the shear flow in the wall. If the wall thickness is 1mm calculate the internal strain energy and angle of twist per unit mm length of tube. Take G = 70,000 N/mm^2.

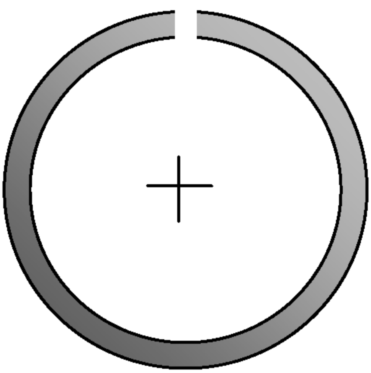


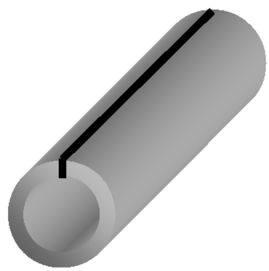


Ans. 79.58 N/mm; 5.68 Nmm; 5.68×10^{-5} rads.

Why is the shear flow constant around a thin walled closed cylinder of arbitrary shape, which is subjected to a pure torque?

Q2 If the thin walled tube of Qn. 3.1.1 is cut to form a longitudinal slit along the whole length of the tube, calculate by how much the torsional stiffness is reduced. Also what is the maximum shear stress in the wall?



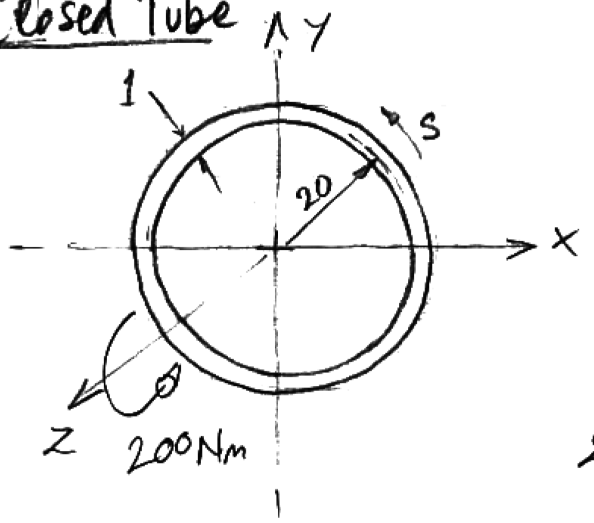


Ans. Reduced from 35.21×10^8 to 0.0293×10^8 Nmm^2/rad. 4778 N/mm^2

3. SHEAR STRESSES UNDER PURE TORSION

3.1 THIN WALLED UNIFORM BEAMS

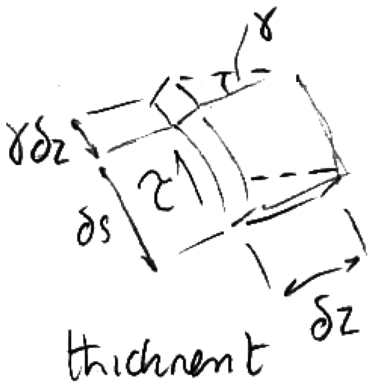
Q 1. Closed Tube



200 Nm = 200'000 Nmm

For strain energy

Considering internal strain energy in element $\delta_s \delta_z$



strain energy $= \frac{1}{2}$ force × distance

$$\delta U = \frac{1}{2}(\tau(\delta s\, t)) \times \gamma\, \delta z$$

subst $\gamma = \frac{\tau}{G}$ for linear stress-strain

$$\rightarrow \quad \delta U = \frac{1}{2}\frac{\tau^2}{G} \underbrace{t\, \delta s\, \delta z}_{= \delta V \text{ ie vol.}}$$

integrating around section

$$\rightarrow \quad U = \left(\oint \frac{\tau^2}{2G} t\, ds\right) dz$$

NOTE, HERE τ, t, G CONSTANT WRT ds, dz

For shear stress τ

First $\quad q = \frac{T}{2A} = \frac{200'000}{2\pi \times 20^2} = \underline{79.6}$ N/mm

And $\quad \tau = \frac{q}{t} = \frac{79.6}{1} = \underline{\underline{79.6}}$ N/mm²

$$\frac{(N/mm^2)^2 \cdot mm}{N/\cancel{mm^2}}$$

So $\quad U = \dfrac{79.6^2}{2 \times 70'000} \times 1 \times (\pi \times 40) \times 1$

ie/unit length of tube

$U = \underline{\underline{5.69}}$ Nmm per unit length

For rate of twist

Here: t, r, G constant wrt s
and: q, τ constant wrt t. so:

→ Use $\quad \dfrac{\tau}{r} = \dfrac{T}{J} = \dfrac{G\theta}{l}$ $\quad$ * learn!

→ $\quad \dfrac{\theta}{l} = \dfrac{\tau}{Gr}$ or $= \dfrac{T}{GJ}$ $\quad$ ie rate of twist θ'

eg. $\quad \theta' = \dfrac{\tau}{Gr} = \dfrac{79.6}{70'000 \times 20} = \underline{\underline{5.68 \times 10^{-5}}}$ rad/mm

or $\quad \theta' = \dfrac{T}{GJ}$ $\quad$ "Torsional rigidity" ie $\dfrac{T}{\theta'} = GJ$
or "Torsional stiffness"

where $J = 4A^2 \Big/ \oint \dfrac{ds}{t}$ $\quad$ ie torsional constant

$= 4A^2 \Big/ \dfrac{2\pi r}{t}$

$= \cancel{4}(\pi r^2)^2 \Big/ \dfrac{\cancel{2}\pi r}{t}$

$= 2\pi r^3 t$ $\quad$ ie for thin wall tube ie $\theta' = \dfrac{T}{GJ} = \dfrac{T}{G \times 2\pi r^3 t}$

$= \dfrac{200'000}{70'000 \times 2\pi \times 20^3 \cdot 1}$

$\theta' = \underline{\underline{5.68 \times 10^{-5}}}$ rad/mm

- See lecture notes.

<u>Torsional Stiffness</u>:

$\dfrac{T}{\theta'} = GJ =$

<u>For closed tube</u> $J = 4A^2 \oint \dfrac{ds}{t}$

$= 2\pi r^3 t$ - from 3.1.1 above

$= 2\pi \times 20^3 \times 1$

$J = 50'265 \text{ mm}^4$

$\rightarrow \quad GJ = 70'000 \times 50265 = \underline{\underline{3.518 \times 10^9 \text{ Nmm}^2}}$

For open tube

$$J = \Sigma \frac{bt^3}{3}$$

$$= \frac{2\pi r t^3}{3}$$

$$= \frac{2 \times \pi \times 20 \times 1^3}{3}$$

$$J = 41.9 \text{ mm}^4$$

$$\rightarrow \quad GJ = 70'000 \times 41.9 = \underline{\underline{2.932 \times 10^6}} \text{ Nmm}^2$$

$\therefore$ Reduction in torsional stiffness by cutting tube is from 3.518×10^9 to 2.932×10^6 Nmm^2 !

ie reduction of 3 orders of magnitude!

For maximum shear stress in open tube

Note, here q, τ vary with t

and $\frac{\tau}{r} = \frac{T}{J} = \frac{G\theta}{l}$ becomes $\frac{\tau}{t} = \frac{T}{J} = \frac{G\theta}{l}$ ie $r \rightarrow t$

Here $r \equiv t$, $J \equiv \Sigma \frac{bt^3}{3} = \frac{(2\pi r)t^3}{3}$ for circle

$= 41.9$ mm^4 from above

$$\rightarrow \quad \tau_{max} = \frac{Tt}{J} = \frac{200'000 \times 1}{41.9} = \underline{\underline{4773 \text{ N/mm}^2}} \ !$$

Or/ $\tau_{max} = G\frac{\theta}{l} \cdot t$

where $\frac{\theta}{l} = \theta' = \frac{T}{GJ}$

$= \frac{200'000}{70'000 \times 41.9}$

$= 0.0682 \text{ rad/mm}$

$\rightarrow \quad \tau_{max} = 70'000 \times 0.0682 \times 1$

$= \underline{\underline{4773}} \text{ N/mm}^2$!

$\Rightarrow$ Failed

Note, this predicted value of shear stress greatly exceeds the material proof and ultimate strength and so the applied torque can not be carried by the open tube. I.e. the open tube will deform plastically and fail before achieving this stress! (Also note according to the rate of twist a 10 cm length of this tube would have to rotate by 6.82 rad, ie more than one full rotation and still behave elastically!)

But of course this is not valid because the plastic limit would be exceeded long before this!