# Worked solution

Let $Y, Z$ be the roll of each die and $X = Y + Z$ be their sum, for which we build a table.

| $Z \backslash Y$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 |

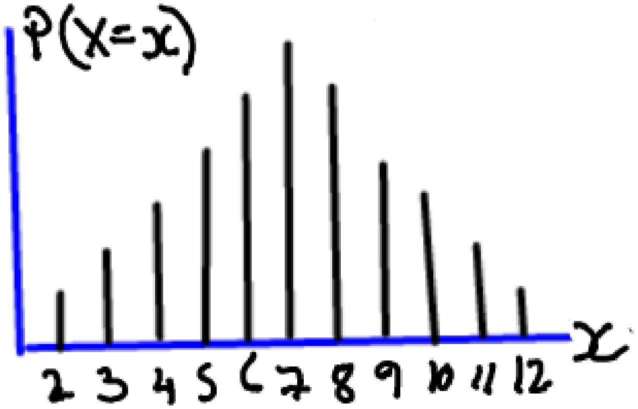
So for example, to find $P(X=4)$, count the number of 4's in the table, each of which has probability $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$, because the dice are independent.

So $P(X=4) = 3/36 = \frac{1}{12}$.

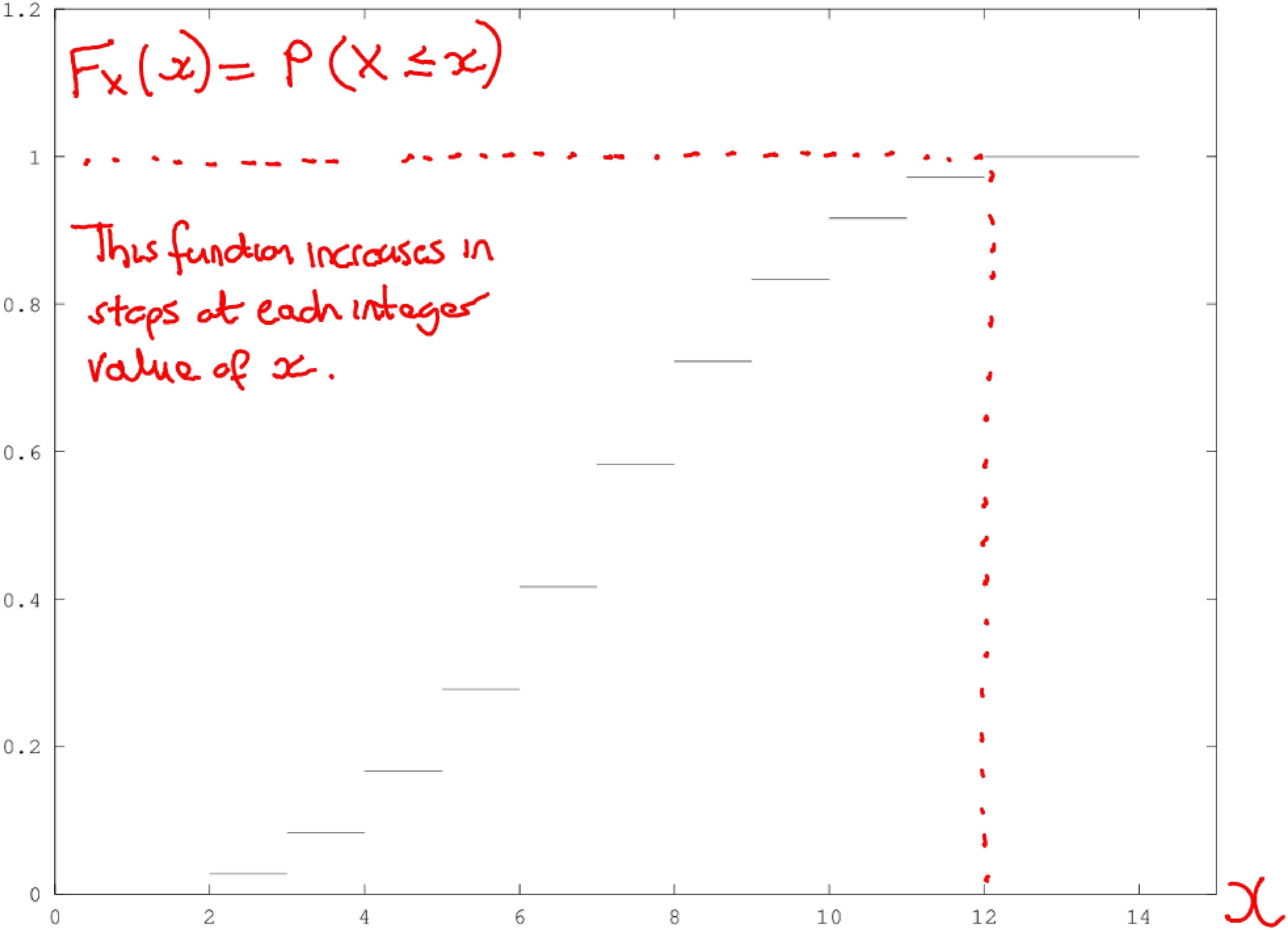
To complete the probability function

$P(X=2) = \frac{1}{36}$, $P(X=3) = \frac{2}{36} = \frac{1}{18}$, $P(X=4) = \frac{1}{12}$, $P(X=5) = \frac{1}{9}$, $P(X=6) = 5/36$, $P(X=7) = \frac{1}{6}$, etc.

Plot of the probability function $P(X=x)$.




For the cumulative distribution, see next slide.



$F_X(x) = P(X \le x)$
This function increases in steps at each integer value of $x$.
1.2
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
10
12
14
$x$

# Exampercise

Compute the mean for

- a roll of one die

$$\mu = E(X) = 1.P(X=1) + 2.P(X=2) + 3.P(X=3) + 4.P(X=4) + 5.P(X=5) + 6.P(X=6)$$
$$= \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6} = \frac{21}{6} = 3\tfrac{1}{2}$$

- a roll of two dice

$$\mu = E(X) = 2.P(X=2) + 3.P(X=3) + 4.P(X=4) + \dots + 12.P(X=12)$$
$$= \frac{1}{36}\left(2.1 + 3.2 + 4.3 + 5.4 + 6.5 + 7.6 + 8.5 + 9.4 + 10.3 + 11.2 + 12.1\right)$$
$$= \frac{252}{36} = 7,$$ perhas not surprsingly!

# Solutions

One die. $E(X) = 3\frac{1}{2}$, already computed.

$$E(X^2) = 1^2.P(X=1) + 2^2.P(X=2) + \ldots + 6^2.P(X=6) = \frac{1}{6}(1^2+2^2+3^2+4^2+5^2+6^2) = 91/6$$

Variance $\sigma^2 = E(X^2) - E(X)^2 = \frac{91}{6} - \left(\frac{7}{2}\right)^2 = \frac{35}{12}$

So the standard deviation $\sigma = \sqrt{35/12} \simeq 1.71$ (3 sf)

Two die. Same idea, $E(X) = 7$ already computed.

$$E(X^2) = 2^2.P(X=2) + 3^2.P(X=3) + \ldots + 12^2.P(X=12)$$
$$= \frac{1}{36}\left(2^2.1 + 3^2.2 + \ldots + 12^2.1\right)$$

Can you fill in the details? — Exercise