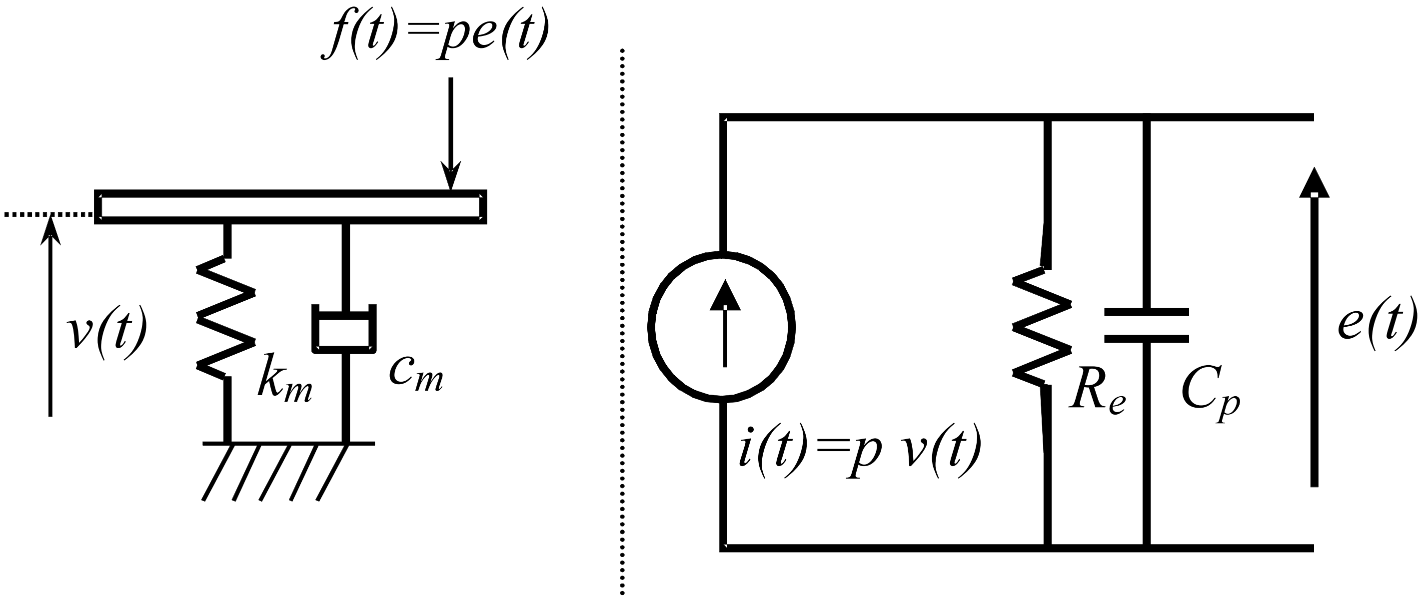
Transducers 2 – Modelling Piezo

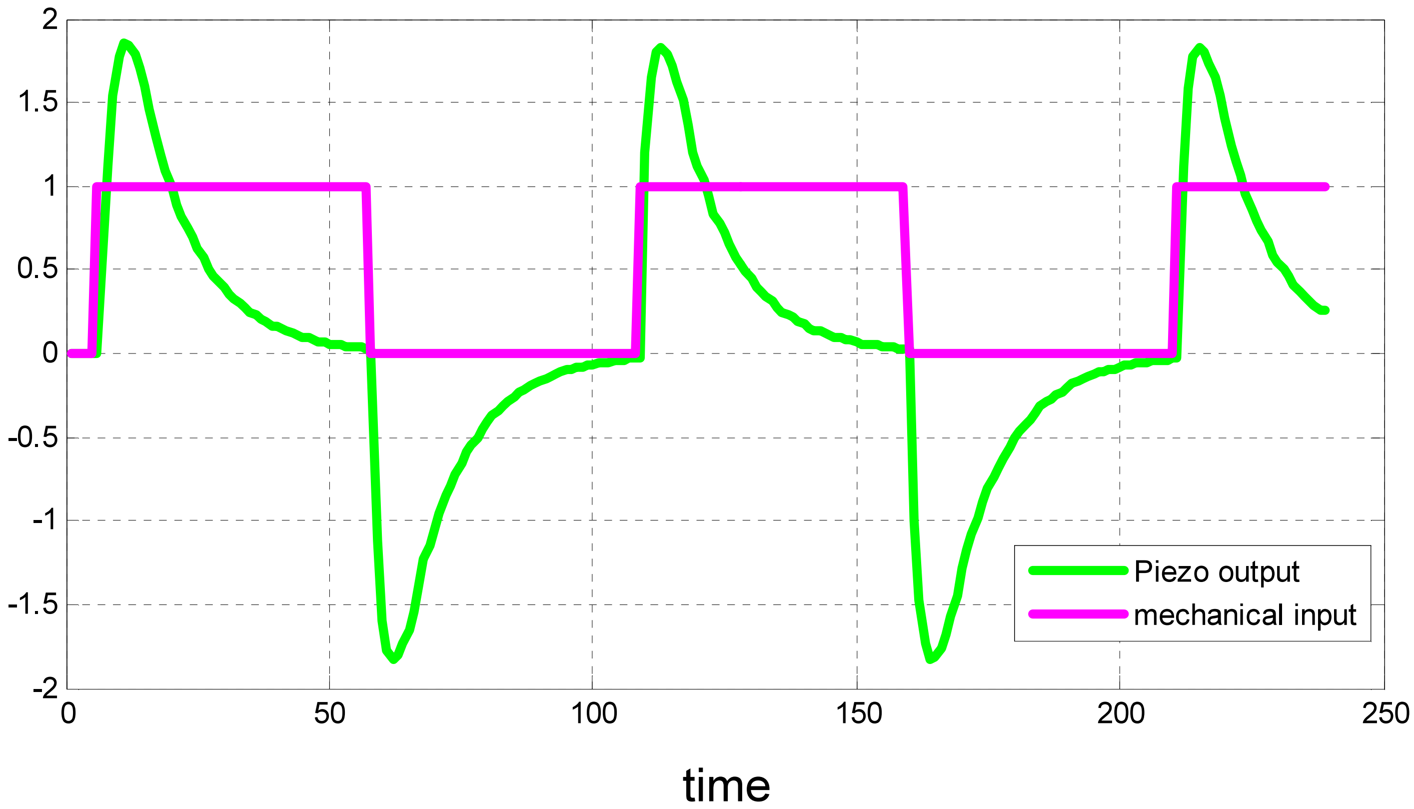
Piezo model

- In the last lecture we derived a model of a piezo electric transducer by considering the underlying physics



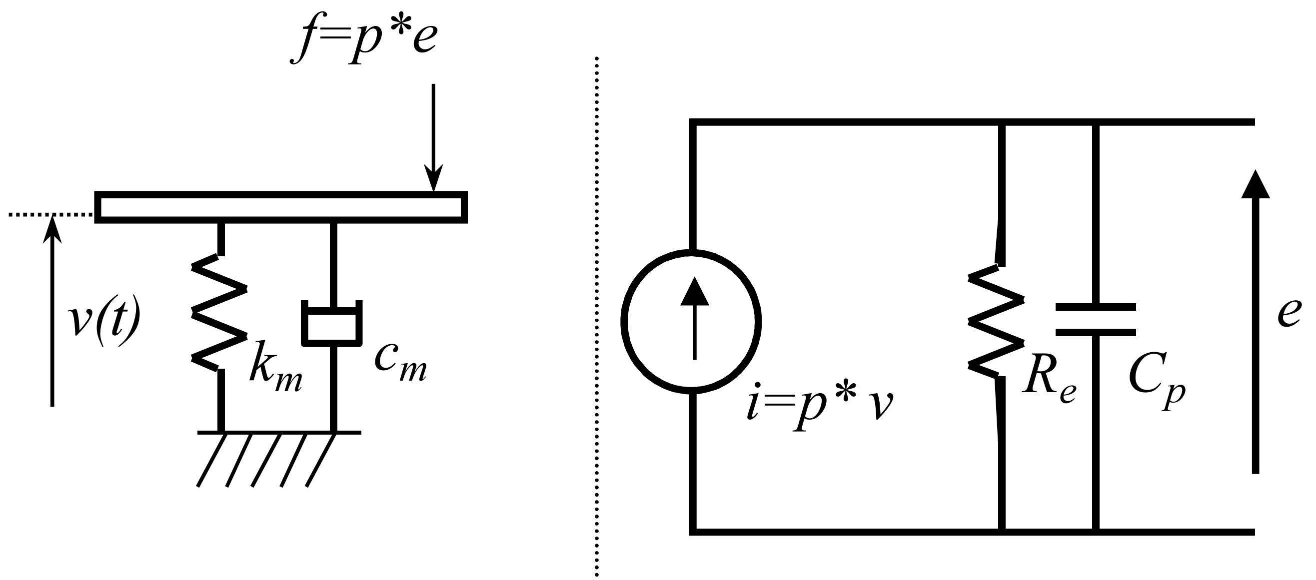
Physical behaviour of the piezo transducer

- We witnessed that if the piezo element was deformed and then held, the electrical output would ‘pulse’ and then decay to zero. The output responded to the differential of the input



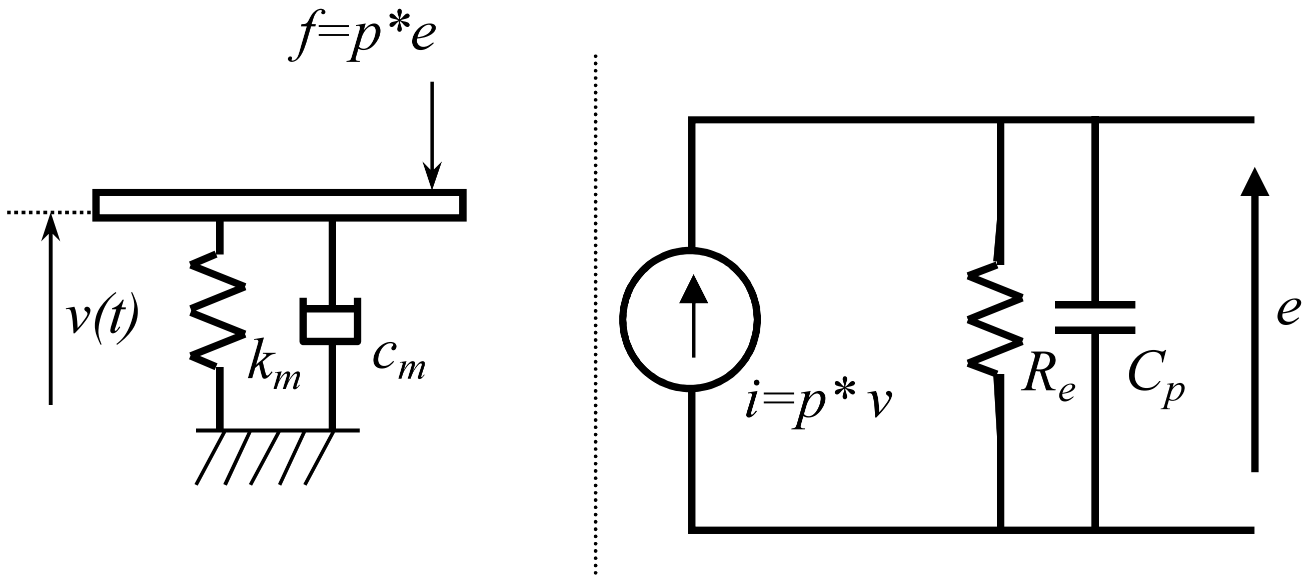
Physical behaviour of the piezo transducer

- Capacitor C_p is positively charged by displacement of the piezo device. When the motion stops at some maxima, charge accumulated on the capacitor slowly drains away through R_e and the output voltage, e, falls to zero. However when we allow the piezo device to return to resting position, charge is moved such that the capacitor is negatively charged, hence we see a negative pulse.



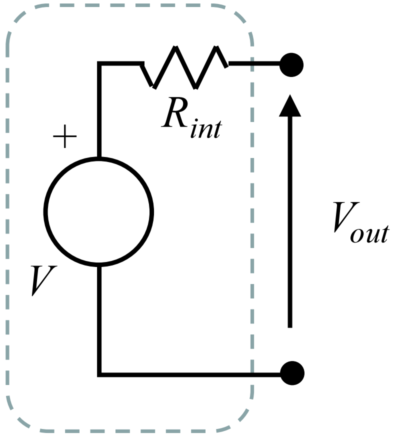
Behaviour – kinematic input

- You will notice that velocity appears on the electrical side of piezo model, hence if our input is *kinematic* i.e. displacement, velocity, acceleration, then the mechanical side components are of no consequence.
- That is not say that the mechanical components will not contribute to the force that an ideal kinematic source must provide, just that if we can provide an *ideal source*, they do not appear in our response.



What is an ideal source?

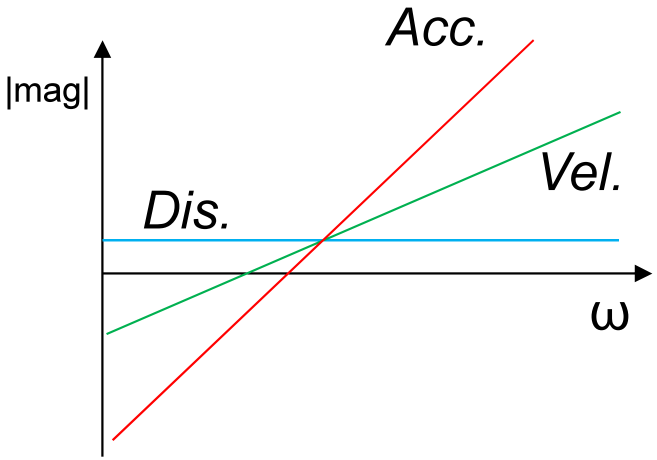
- Any source (input) that has a fixed magnitude – it could be a voltage, force, velocity, current etc. is considered ideal.
- This can be further un-picked:
  - It should have constant magnitude with frequency
  - It should have constant magnitude irrespective of load i.e. a velocity irrespective of the force required to produce that velocity (velocity is 'force invariant') or it could be a force that is constant irrespective of the corresponding velocity created by the force (force is 'velocity invariant')



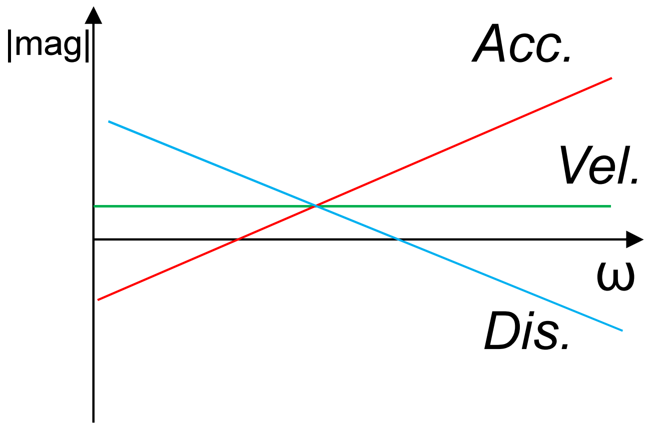
Often real sources are not very ideal but we can represent them with an ideal source and additional components - perhaps the most commonly encountered example is the representation of a battery or power supply as a ideal voltage source in series with a resistive element

Relationship between x, $\dot{x}$ and $\ddot{x}$

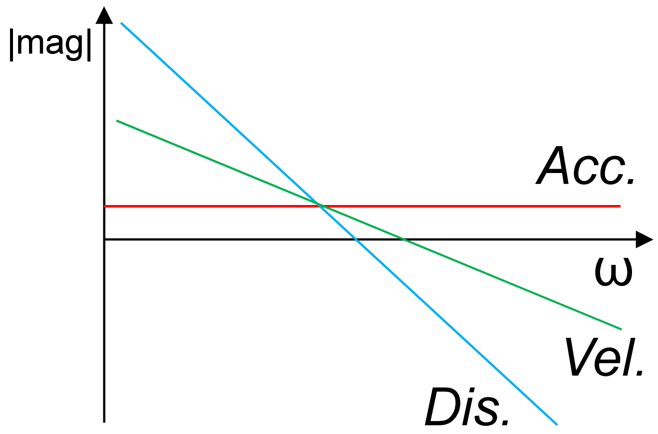
- We may have systems where any of the kinematic relations - displacement, velocity, acceleration – are the variable of interest, and so it is useful to reflect on their relationship with frequency.



If we have constant displacement over all frequencies, this implies the velocity increases with a 20dB/decade slope; acceleration with a 40dB decade slope



If velocity has constant magnitude over all frequencies, this implies the acceleration increases with a 20dB/decade slope; displacement decreases with a -20dB decade slope



If acceleration has constant magnitude over all frequencies, this implies the acceleration decreases with a -20dB/decade slope; displacement decreases with a -40dB decade slope

Relationship between x, $\dot{x}$ and $\ddot{x}$

- We can reason this through in other ways. If x, v and a, describe corresponding displacement, velocity and acceleration:

  - In the time domain:

$$x(t) = sin(\omega t) \qquad v(t) = \frac{dx}{dt} = \omega cos(\omega t) \qquad a(t) = \frac{d^2x}{dt^2} = -\omega^2 sin(\omega t)$$

Magnitude increases with ω

Magnitude increases with ω^2

  - or in the frequency domain;

$$\mathrm{V}(s) = sX(s) \qquad \mathrm{A}(s) = s^2X(s) \qquad \mathrm{X}(s) = \frac{V(s)}{s}$$

etc.

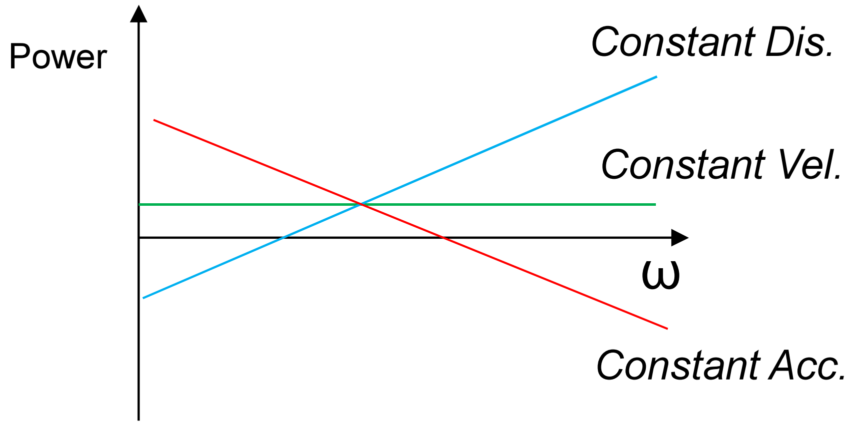
Zero at s = 0

Two zeros at s = 0

Pole at s = 0

Relationship between x, $\dot{x}$ and $\ddot{x}$

- One useful way to consider this is the power that each input implies in a viscous damper:



Only a constant velocity produces a power that is constant with frequency – constant acceleration results in a reducing power; constant displacement with an increasing power.

What are the practical consequences?

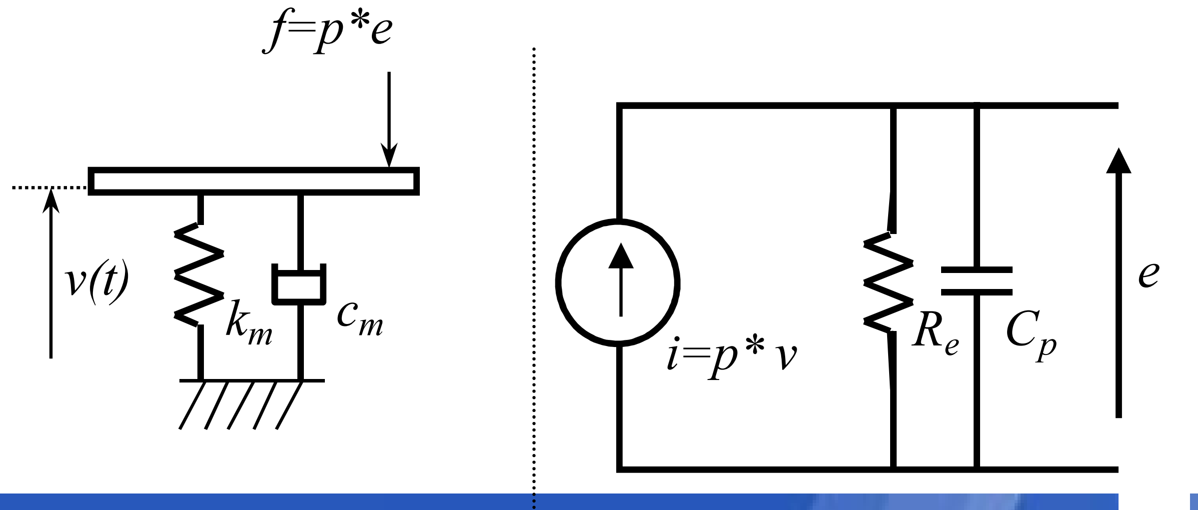
- When we want to describe the frequency response of a system we may need to consider which frequency response

- In real life, it is often considered that power in vibrations decreases with frequency, so that might lead you to characterise your system with constant acceleration input.

- But, since it is not possible to have constant acceleration oscillation at $\omega=0$, the input source could not be ideal…..

Loading and power

- Consider again the piezo transducer. What happens when an input motion deforms the piezo element:
  - A voltage appears across the output
  - This voltage is reflected back as a force in the mechanical domain. This force opposes the input motion.
- Thus the input has to do work.

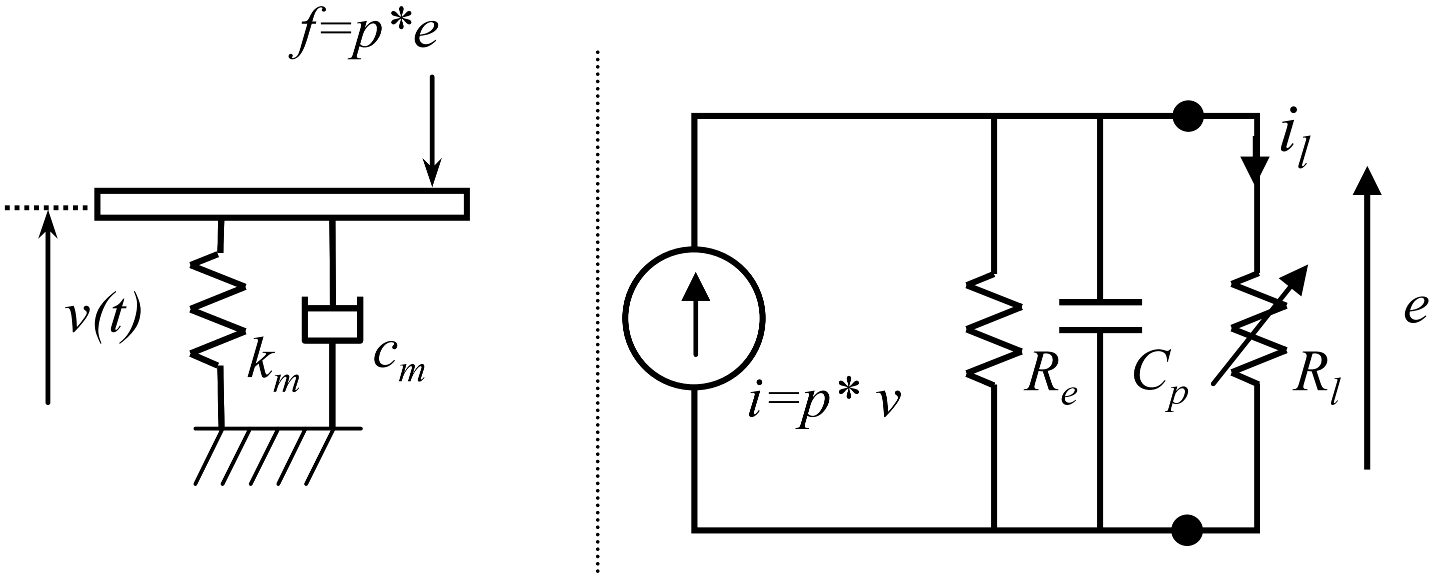


- Important Idea

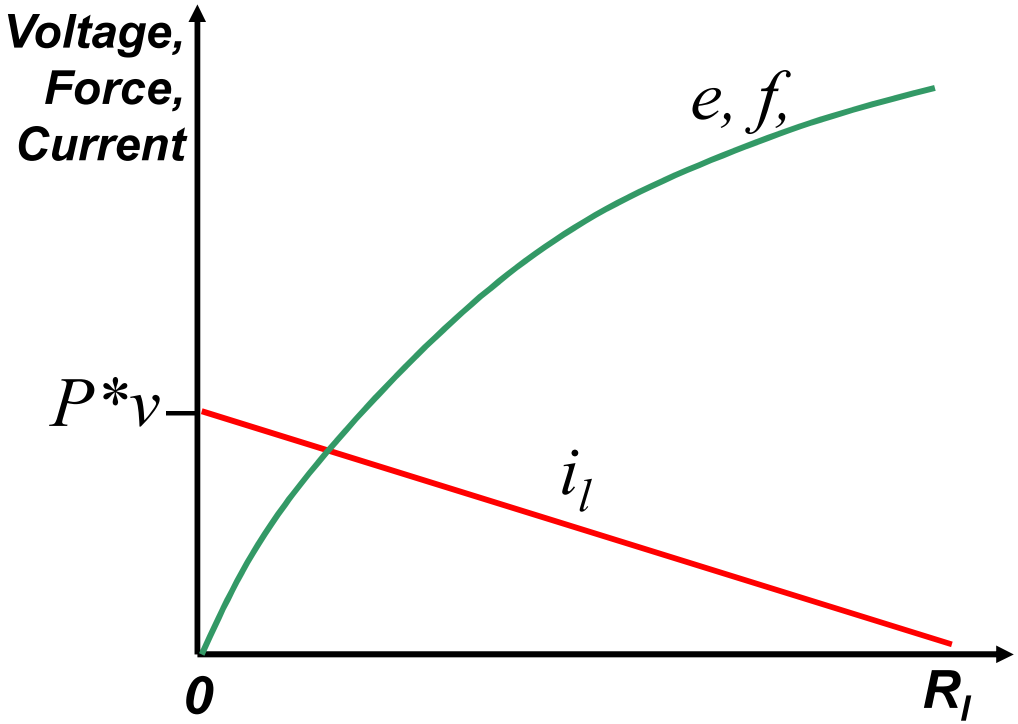
  Measuring something often places a parasitic load on the process being monitored.

Loading and power

- Consider the case where we load our piezo transducer with a variable resistor. The kinematic input has constant amplitude and at a fixed frequency.
- How does the value of R_l effect the voltage developed across it; the current through it?
- How does the value of R_l effect the force reflected back?



Loading and power



- As we increase R_l the output voltage rises, and so does the force reflected back.
- Since we have said the input is fixed, the input power follows the force.
- When $R_l = 0$ then all of displaced charge flows through it. Reflected load is zero.

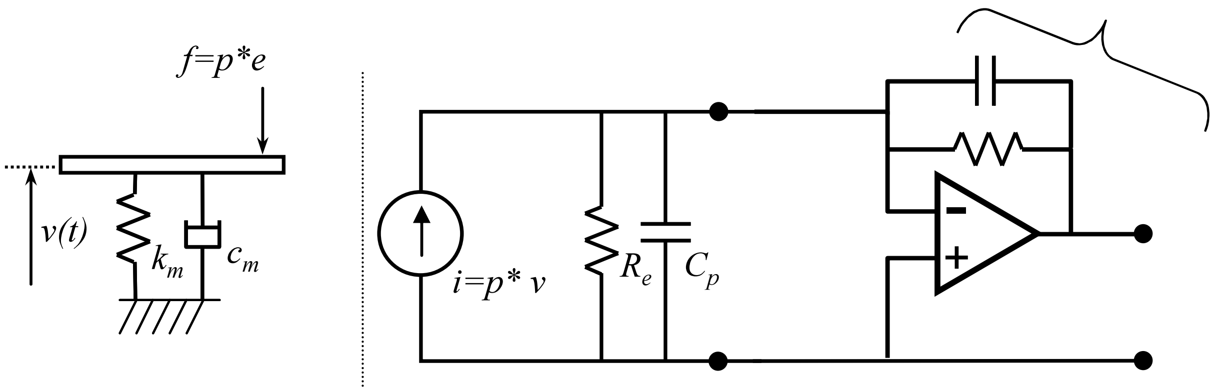
- Important Idea

  To minimise the parasitic power drain of transducers used for sensing we typically try and operate with one of the power producing variables (i.e. voltage/current, force/velocity) as close to zero as possible.

Which one depends on the transducer

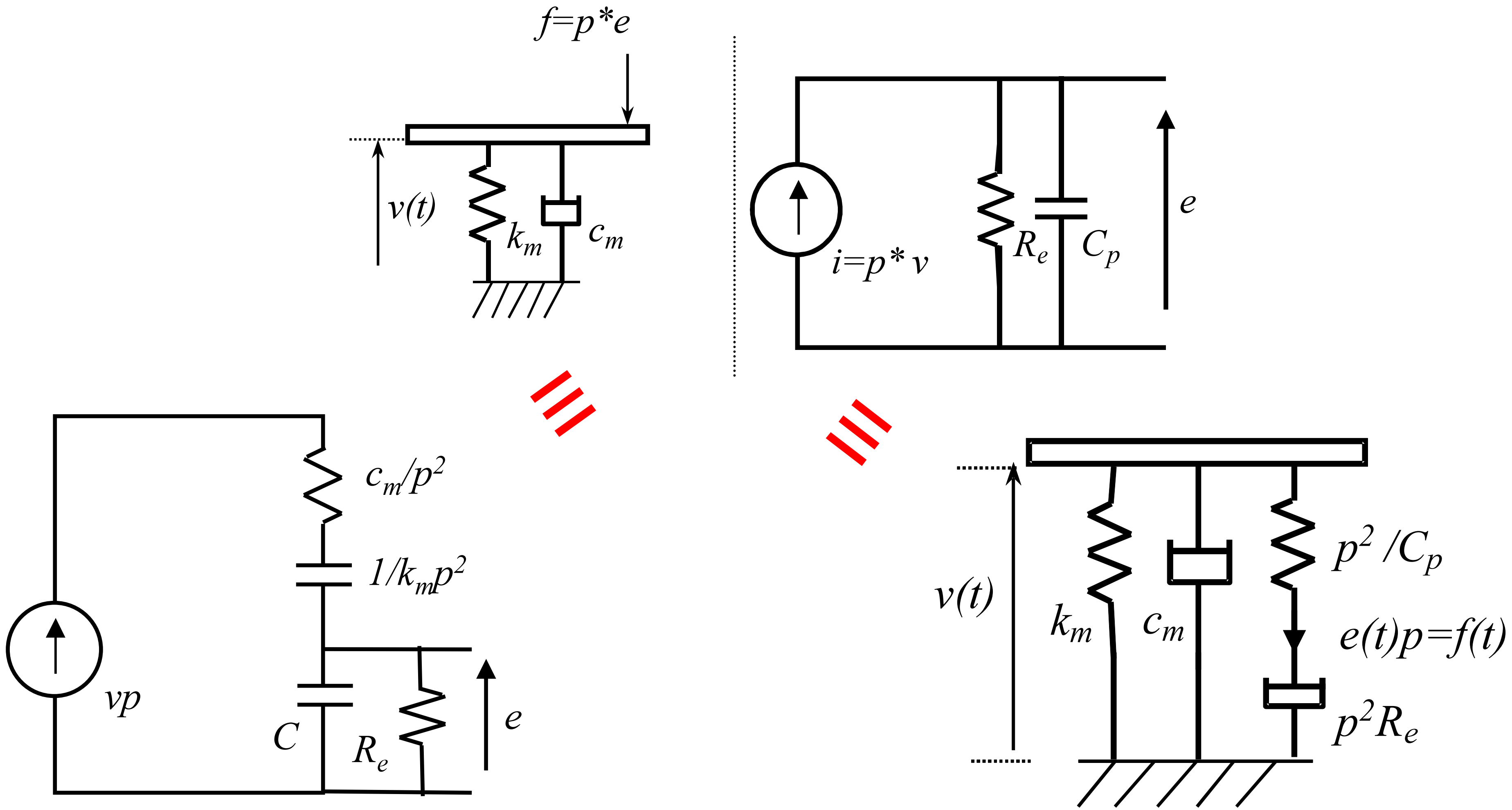
Loading and power

- Piezo transducers are often used with charge amplifiers – converting a input charge to an output voltage.
- The action of the charge amplifier forces the output voltage of the piezo element to remain close to zero (remember the ‘g’ constant?)
  - Since there is no output voltage across the piezo device, its own capacitance or cable capacitance between the piezo device and amplifier does not effect measurement
  - In the ideal case no force is reflected back



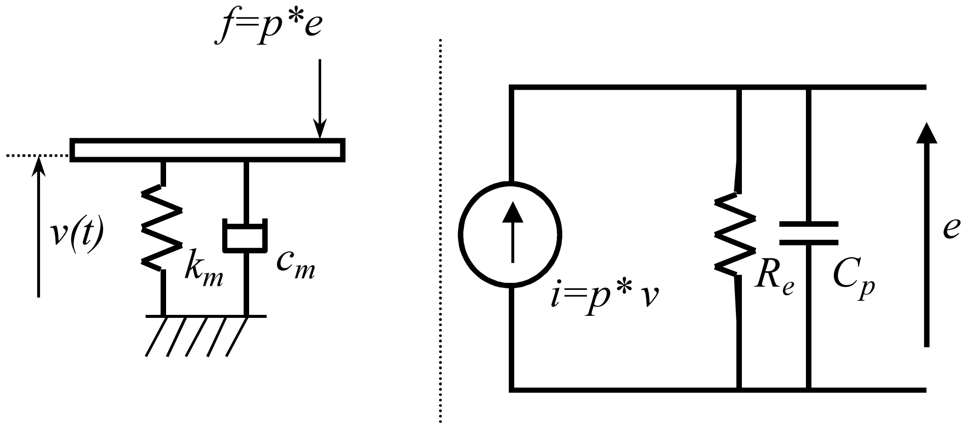
Charge Amplifier – doesn’t allow any voltage to appear across its input

Piezo modelling



The piezo device could be modelled by electrical or mechanical analogies

Piezo modelling – kinematic input (velocity)



Use current balance i.e.
Kirchoff's current law

$$pv = i_{Re} + i_{Cp}$$

$$pv = \frac{e}{R_e} + C_p \frac{de}{dt}$$

$$pV(s) = \frac{1}{R_e} E(s) + C_p s E(s) = E(s)\left(\frac{1}{R_e} + sC_p\right)$$

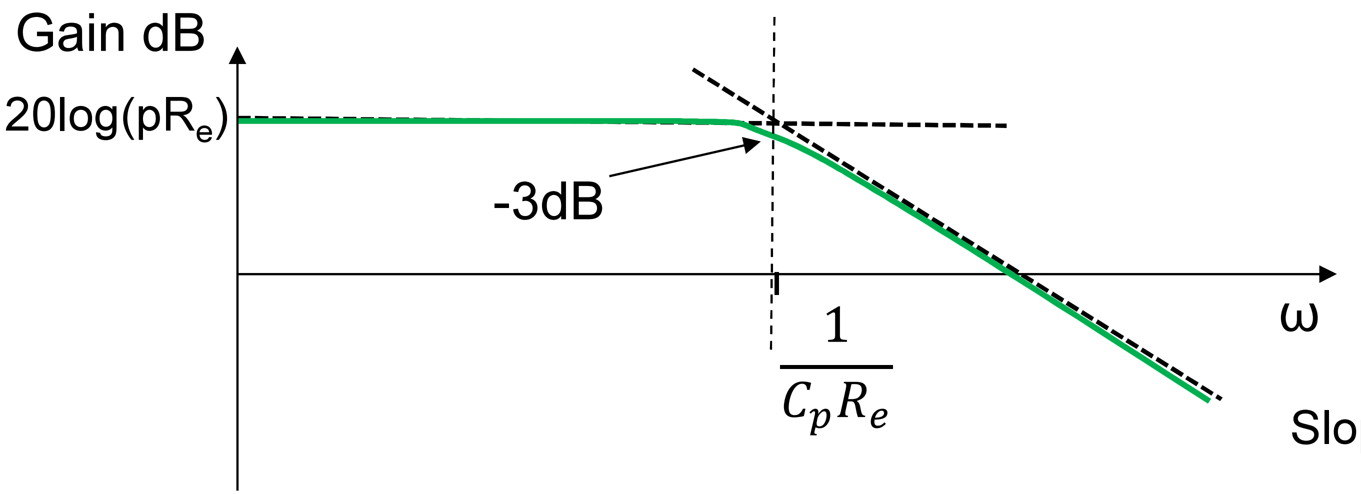
$$\frac{E(s)}{V(s)} = \frac{p}{\frac{1}{R_e} + sC_p} = \frac{p/C_p}{\frac{1}{C_p R_e} + s}$$

Piezo modelling – kinematic input (velocity)

$$\frac{E(s)}{V(s)} = \frac{p/C_p}{\frac{1}{C_p R_e} + s}$$

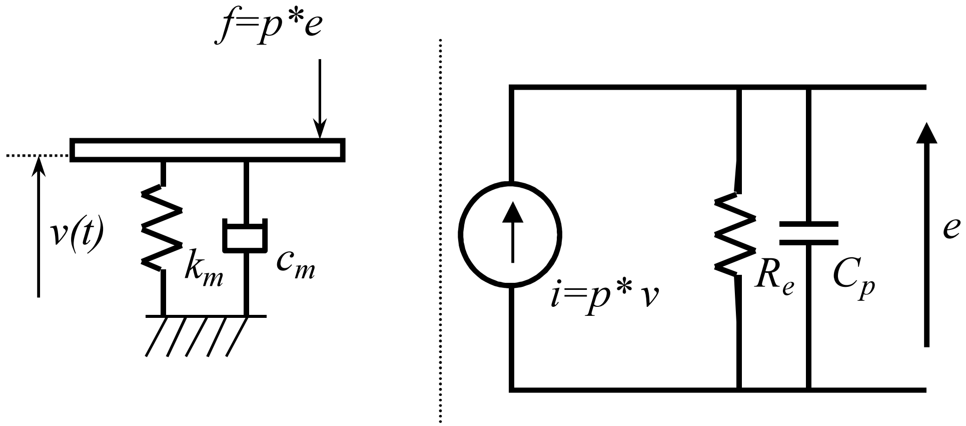
Single pole at $s = -\frac{1}{C_p R_e}$

1^{St} order low pass response with break frequency of $\frac{1}{C_p R_e}$



Slope = - 20 dB / decade
-6 dB / octave

Piezo modelling – kinematic input (disp.)



$$pV(s) = E(s)\left(\frac{1}{R_e} + sC_p\right)$$

$$V(s) = sX(s)$$

$$psX(s) = E(s)\left(\frac{1}{R_e} + sC_p\right)$$

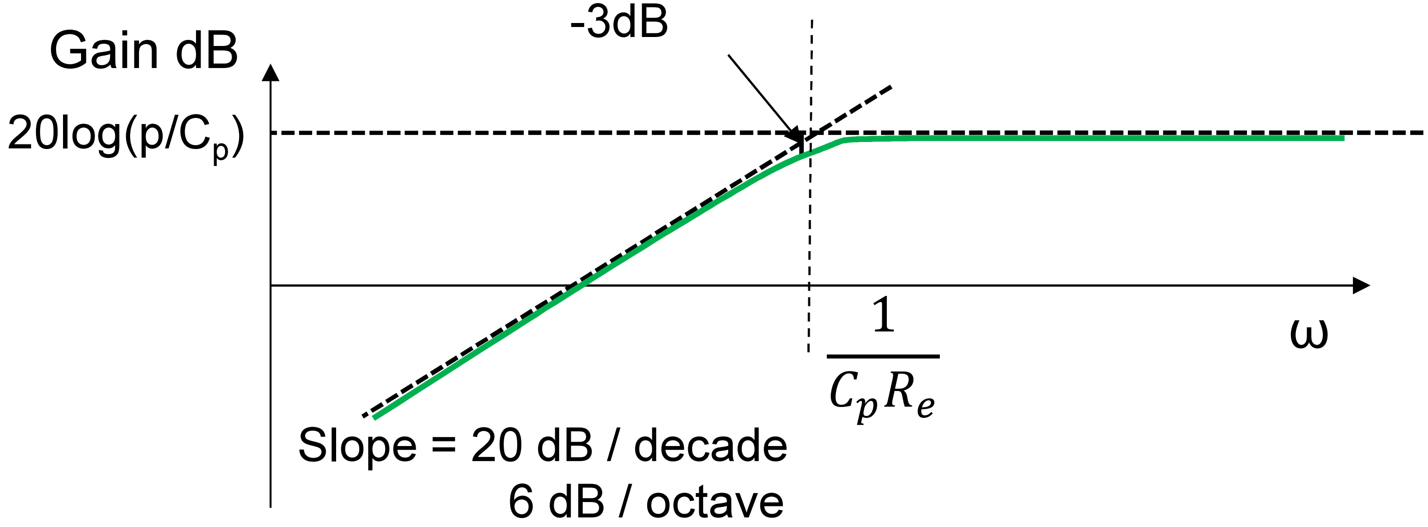
$$\frac{E(s)}{X(s)} = \frac{sp}{\frac{1}{R_e} + sC_p} = \frac{sp/C_p}{\frac{1}{C_pR_e} + s}$$

Piezo modelling – kinematic input (disp.)

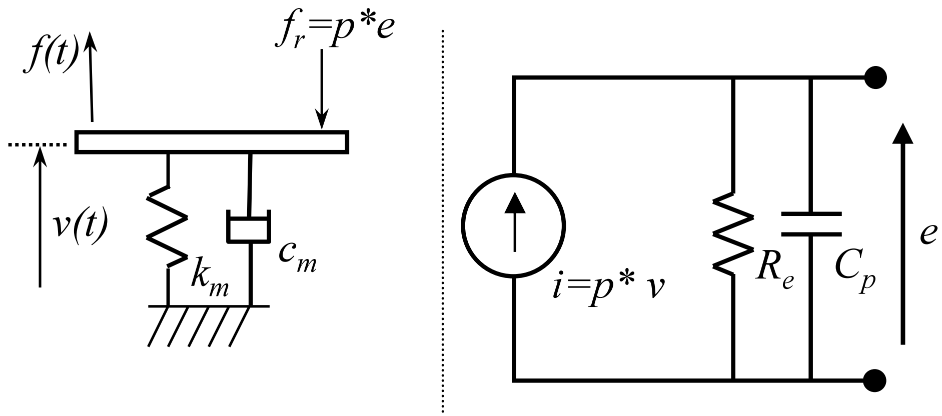
$$\frac{E(s)}{X(s)} = \frac{sp/C_p}{\frac{1}{C_pR_e} + s}$$

pole at $s = -\frac{1}{C_pR_e}$ zero at $s = 0$

1^{St} order high pass response with break frequency of $\frac{1}{C_pR_e}$



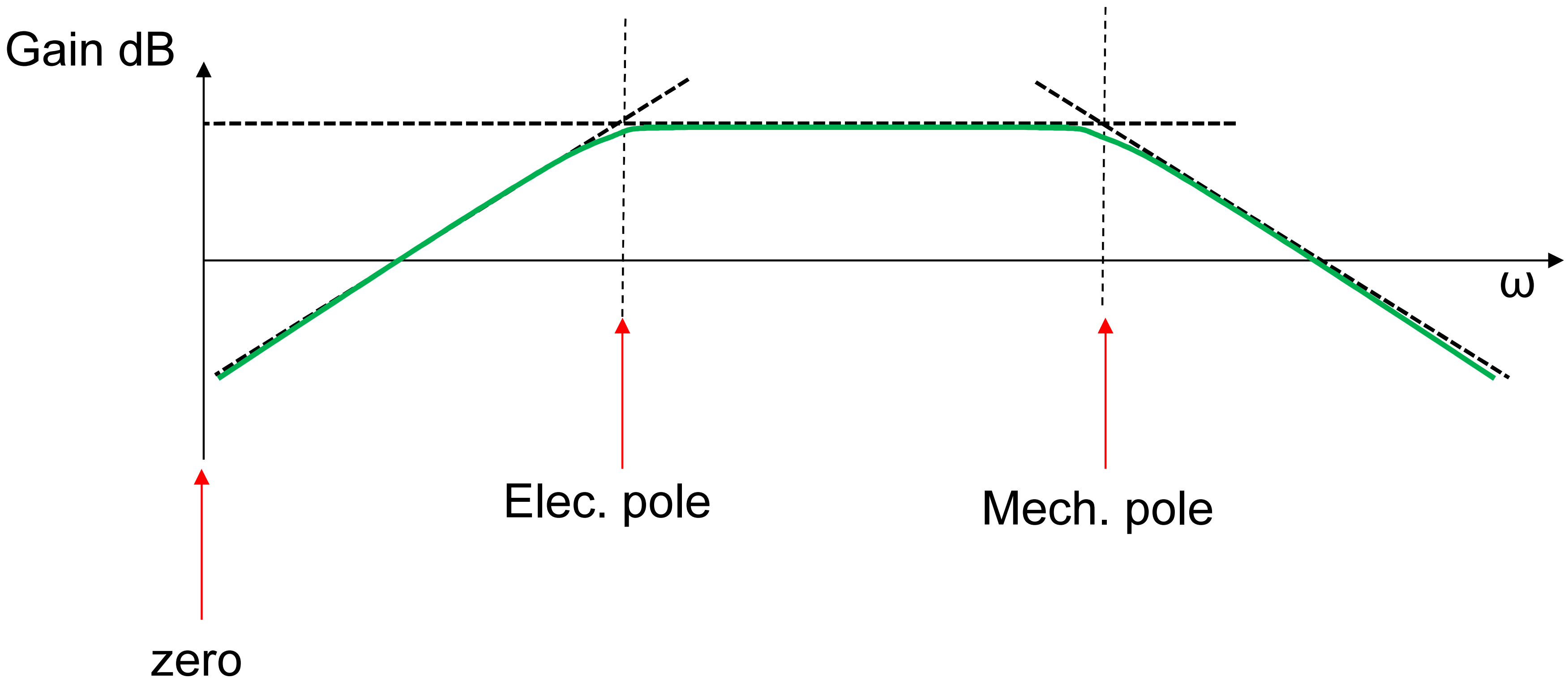
Piezo modelling – force input



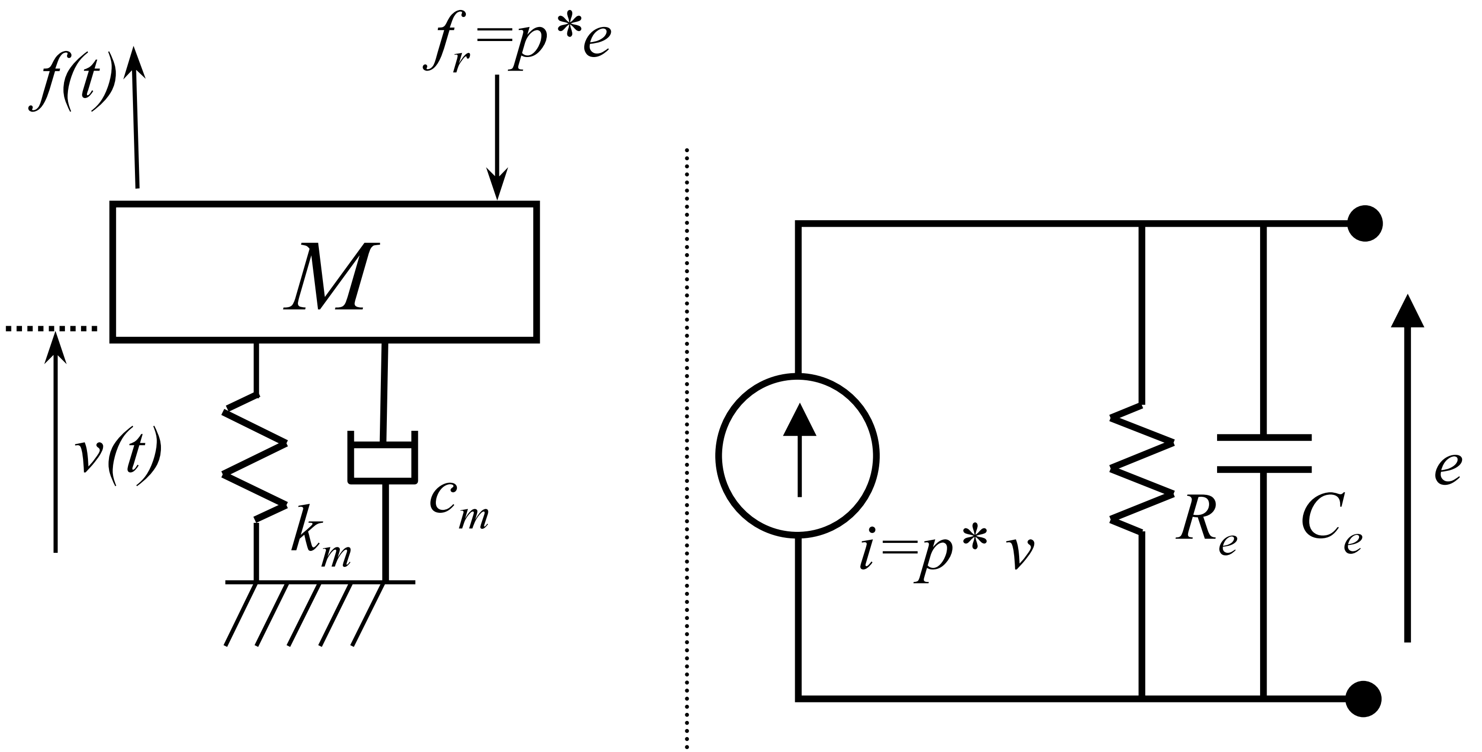
Equate forces $$f = k_m \int v dt + vC_m + pe$$

and currents $$pv = \frac{e}{R_e} + C_e \frac{de}{dt}$$

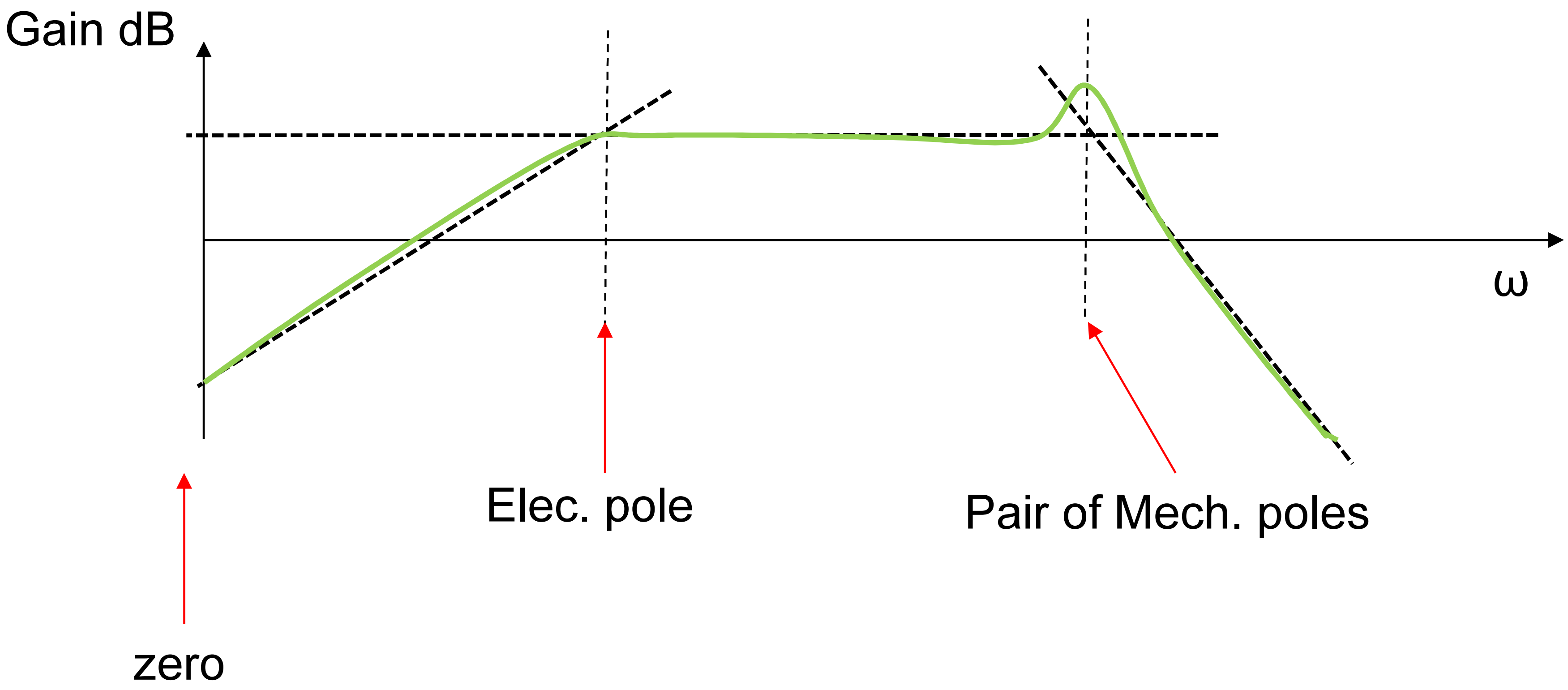
Piezo modelling – force input



Piezo modelling – force input with mass



Piezo modelling – force input with mass ?



Transducers 3 – Electro magnetic and other transducers

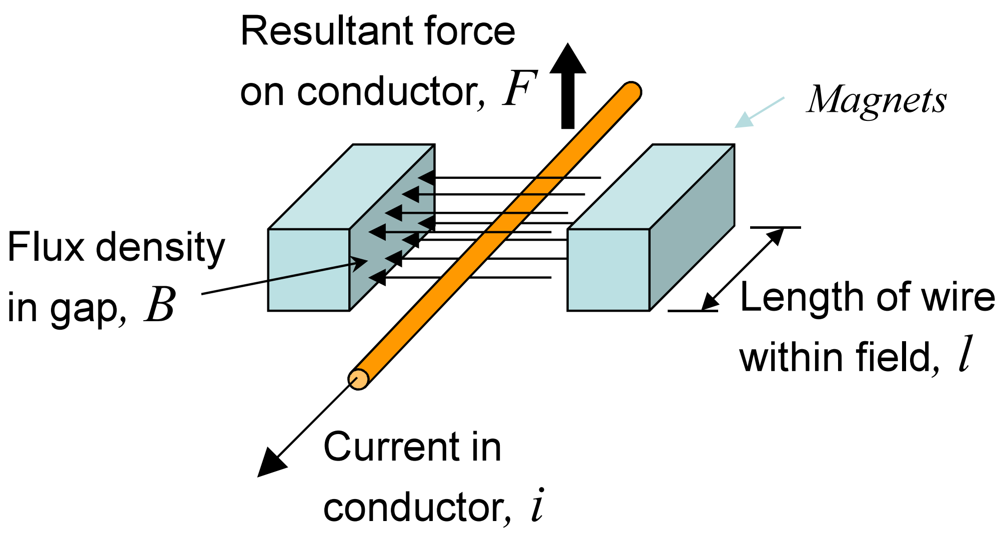
Electro-magnetic transduction

- Piezo electric devices are becoming more prevalent, however the most common form of electro mechanical transducer you will encounter operates using electro-magnetic principles.

- At every scale - from the shaker in your mobile phone to the generator in a power plant – you will find electro-magnetic transducers.

- This is unique – no other transducer technology works well over such a wide range of scales.

Electro-magnetic transduction



The diagram shows the case if we apply a current to a conductor within a field. The resulting force on the conductor is given by;

$$F = Bli$$

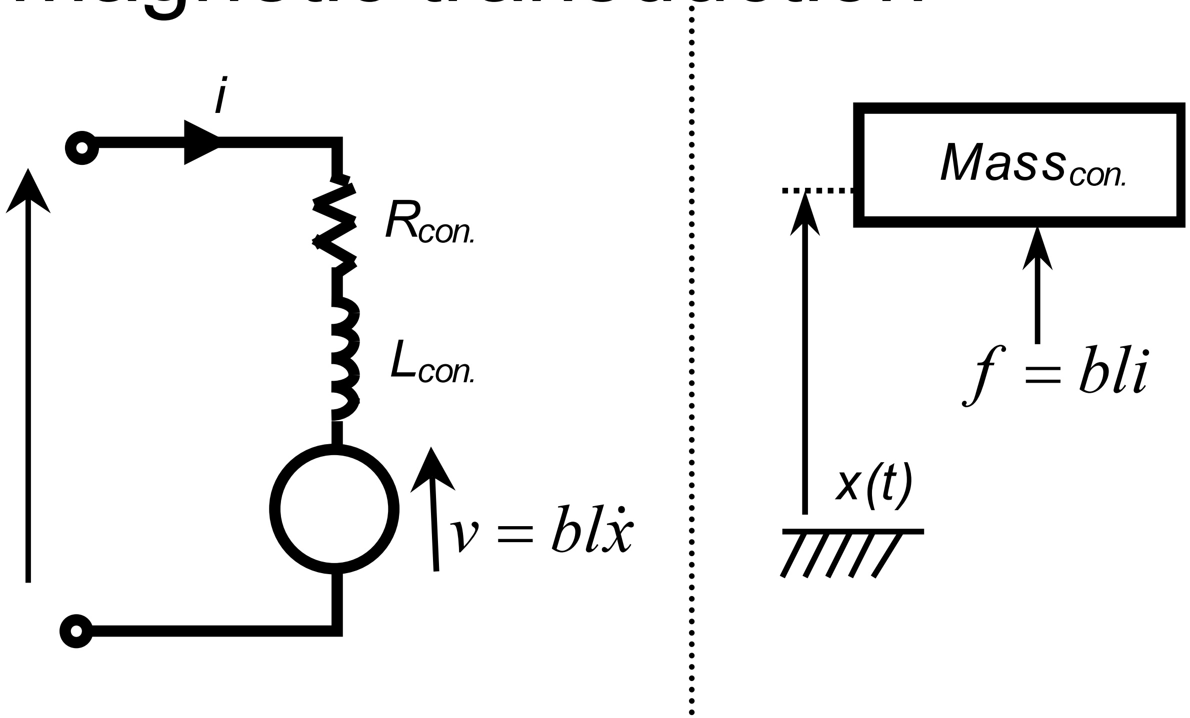
$$\therefore Force \propto i$$

Alternatively if we move the conductor within the field the voltage induced across the active length is given by;

$$E = Blv$$

$$\therefore Voltage \propto velocity$$

Electro-magnetic transduction

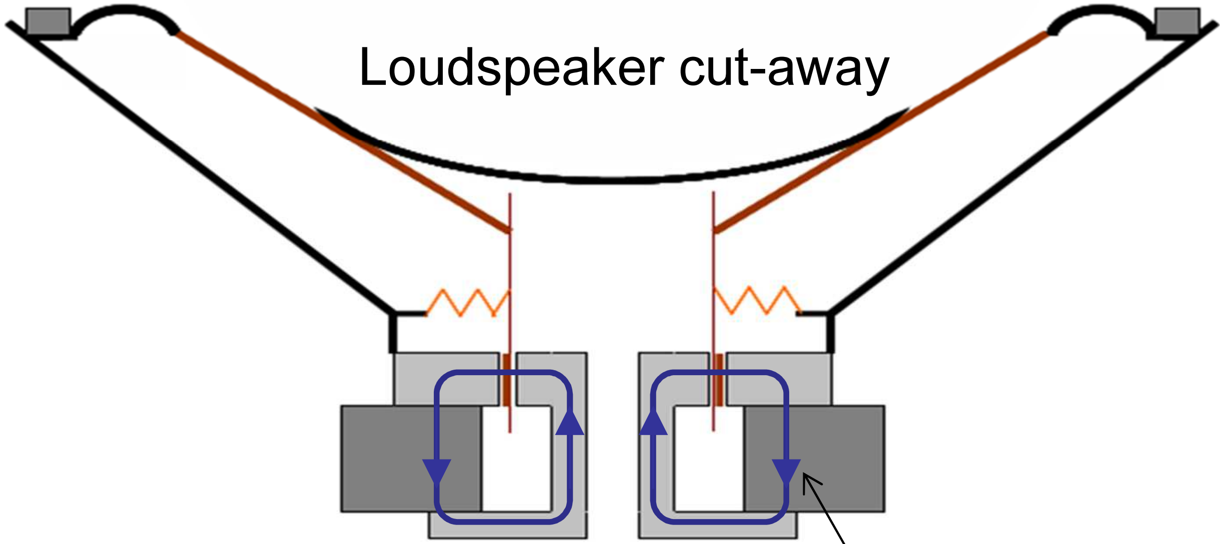


Similar to the way we built up the model for the piezo device, we can add parasitic elements around the transduction forward and converse effects – the conductor has resistance and inductance in the electrical domain and some mass in mechanical domain.

The coupling coefficient between the domains is ‘bl’ i.e. the product of the flux density and the length of the wire. Sometimes it is used as such, typically in voice coils, although often the symbol ‘k’ is used.

Voice coil actuator

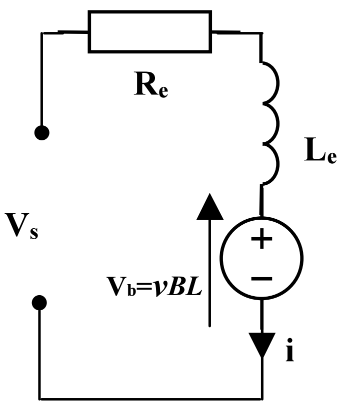
- The voice coil derives its name from use in the loudspeaker. They are used for a range of sensors (e.g. Microphones) and ‘linear’ actuators (i.e. linear travel).
- Since all of the massive parts of the transducer are arranged on the static side of the device, the voice coil can move at very high rates.



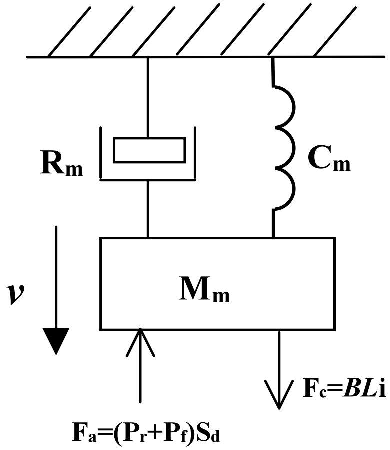
Voice coil

Loudspeaker example

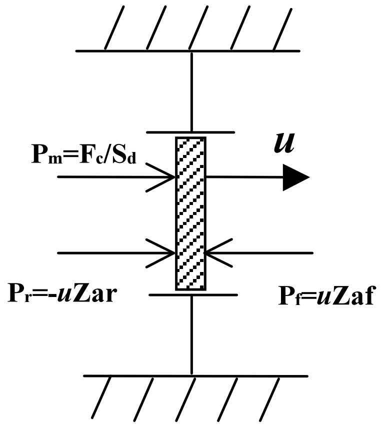
Electrical



Mechanical



Acoustical

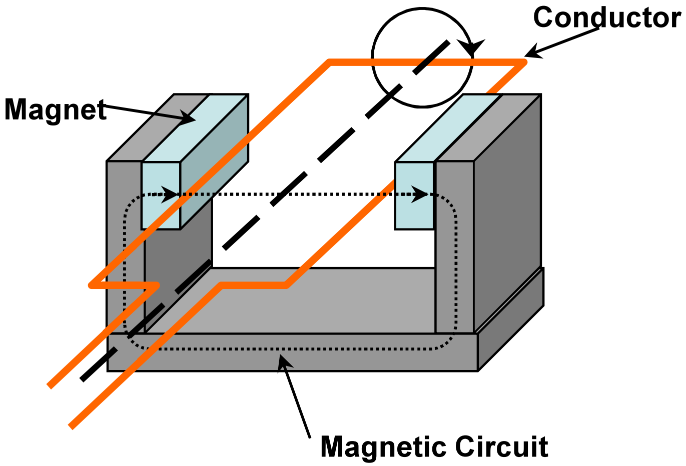


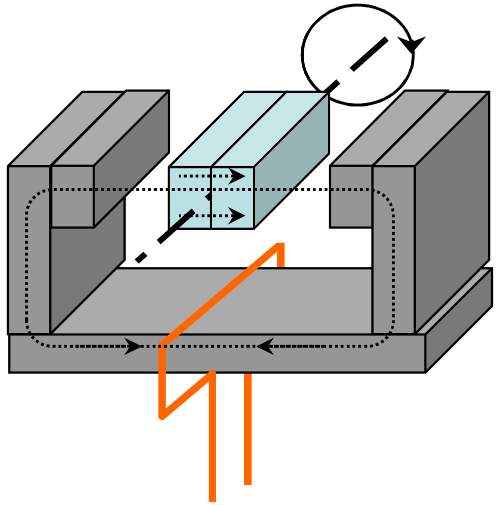
R_e = Resistance.
L_e = Inductance.
V_b = Back emf.
i = Coil current.
V_s = Input signal.
BL = Tesla/metre product.

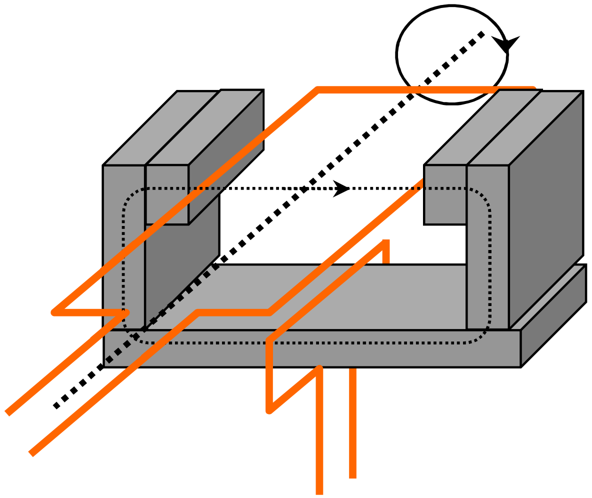
R_m = Mechanical loss.
C_m = Suspension Compliance.
M_m = Moving mass.
v = Cone velocity.
F_c = Force from voice coil.
F_a = Force reflected back through cone.
S_d = Cone surface area.

u = Volume velocity.
P_m = Pressure created by force from voice coil.
Z_{ar} = Radiation impedance acting on rear of cone.
Z_{af} = Radiation impedance acting on front of cone.

Rotating Electromagnetic transducers



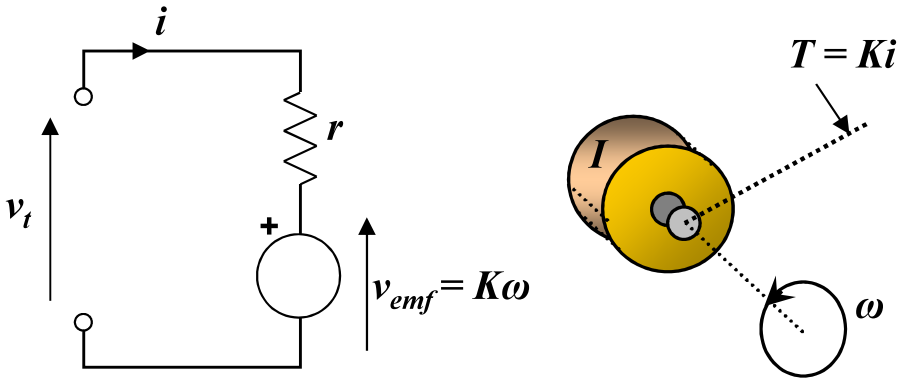


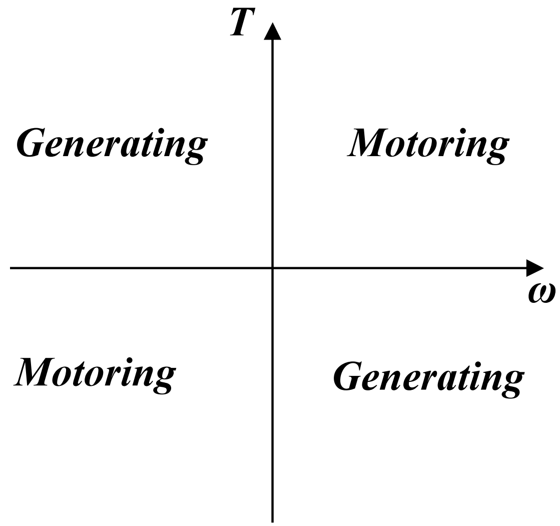


Rotating electromagnetic transducers are found everywhere. It is common to refer to them as ‘machines’ capturing both motoring and generating operation. There are three main topologies – 1) coils rotating in a static magnetic field; 2) magnet rotating in changing field; 3) Coils rotating in a field created by a second coil.

A fourth topology working on the same principle as the gear tooth sensor also exists – the reluctance machine.

Generic machine model





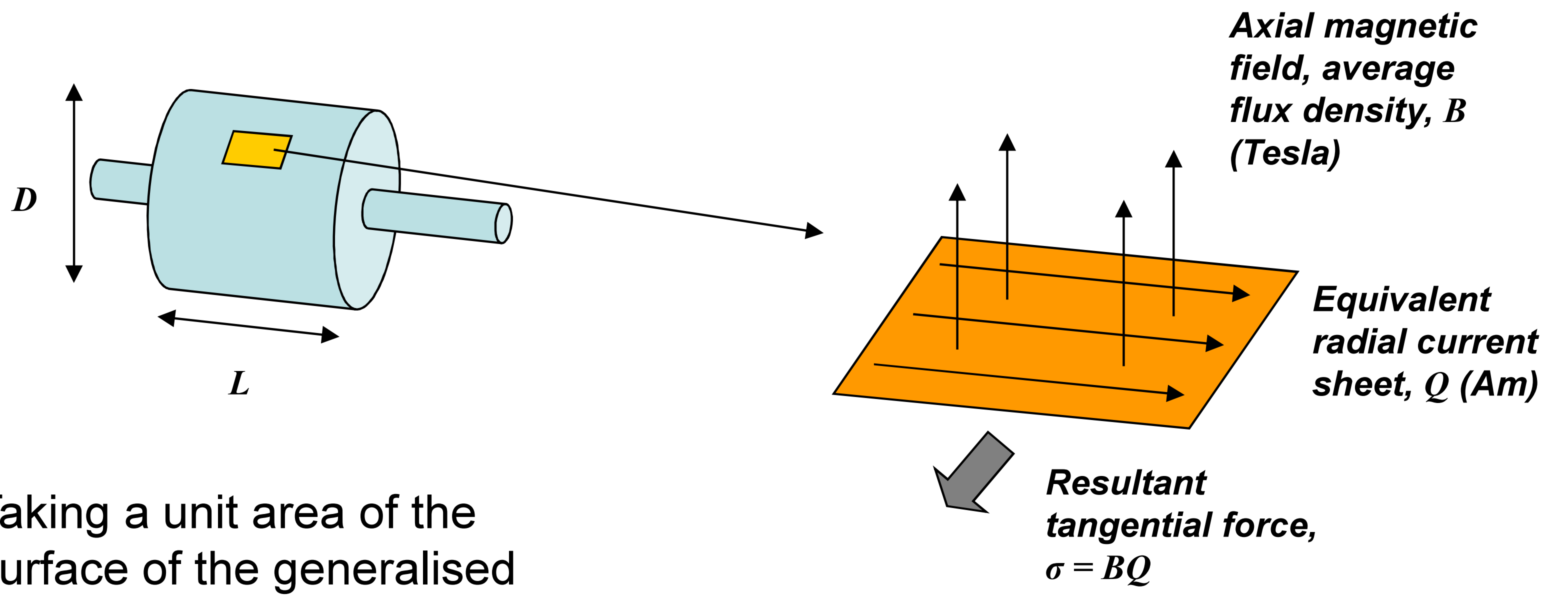
The previous model for electromagnetic transduction can be adapted to form a generic machine model. The core difference is the mechanical domain is rotating – we have angular velocity and torque (rather than velocity and force); instead of mass of the coil, we have moment of inertia of the rotor. Here, coil inductance has been omitted.

The voltage, V_{emf}, know as the 'back emf' is present whenever the machine is turning. When the back emf is less than the terminal voltage when in the motoring region and greater than the terminal voltage when generating.

It is common to use 'k' to describe the electromagnetic coupling coefficient, which might also be described as the '*back-emf constant*' or the '*torque constant*'

Generalised analysis - specific loading

Coils and magnetic of a rotating machine are often buried within the structure, and it can be hard to see where torque is produced. The surface of the rotor is considered as the point at which an virtual current 'sheet' and the magnetic field interact to produce torque.



Taking a unit area of the surface of the generalised machine rotor we can determine the resultant force on the rotor surface and hence the torque

$$Force = \pi DLBQ$$

$$Torque = \frac{1}{2}\pi D^2 LBQ$$

Machine sizing question:

Consider:

$$T \propto BQD^2L$$

$$P = T\omega$$

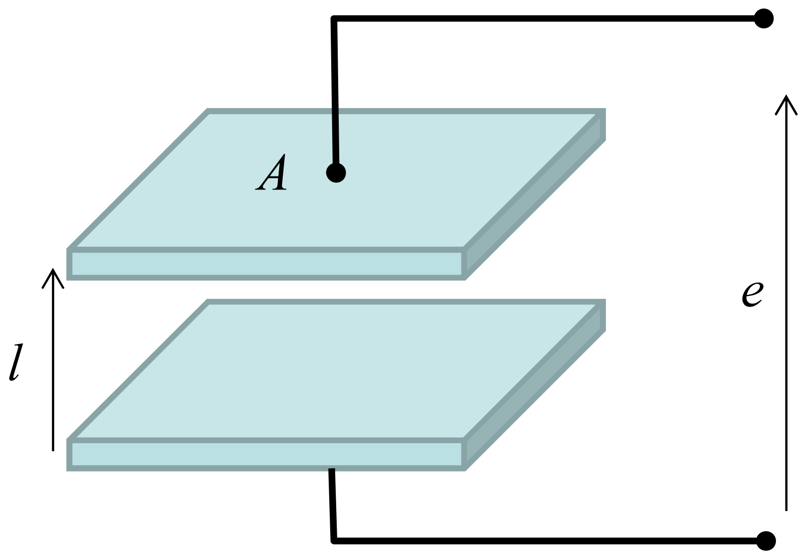
$$P \propto BQD^2L\omega$$

Question: what determines the size of a machine?

Machine sizing question:

- B and Q are limited by properties of the materials used to make the machine:
    - B – the 'magnetic loading' is limited by the magnetic saturation of materials – normally to less then 2 Tesla
    - Q – the 'electric loading' is limited by thermal losses – as we put more current through a conductor we get more energy dissipated as heat
- For a given B and Q, torque is proportional to rotor volume.
- For a given maximum torque, peak power is limited by maximum rotation speed.

Quick look: Electro-static transduction



$$Force = \frac{\varepsilon A V^2}{2l^2}$$

- Electrostatic transducers exploit the attractive and repulsive forces of electric charges on plates.
- Analysis is more complex as displacement changes electrical parameter capacitance, and they need to be excited with charge.
- They have found application for MEMS transducers as the forces scale with the square of physical scale (rather than cubic as with magnetic devices)

Quick look: Magneto-strictive transduction

- Magneto-strictive materials deform when a magnetic field is applied.
- Conversely deforming them will produce a magnetic field.
- An electric coil wound around the magneto-strictive material interfaces with the magnetic filed to complete the transducer

Electro-mechanical Transduction

	Discrete moving parts; Long travel, complex to manufacture	*Solid state no moving parts. Short travel*
Magnetic forces	***Electromagnetic***	***Magneto-strictive***
Electrostatic forces	***Electrostatic***	***Piezo-electric***