Structural Loads in Beams
Introduction

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11 October 2017

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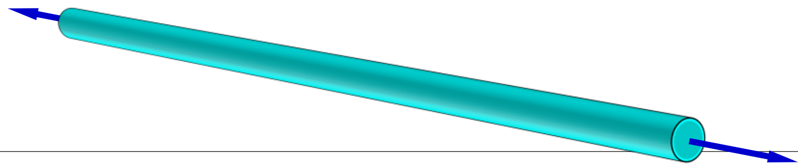
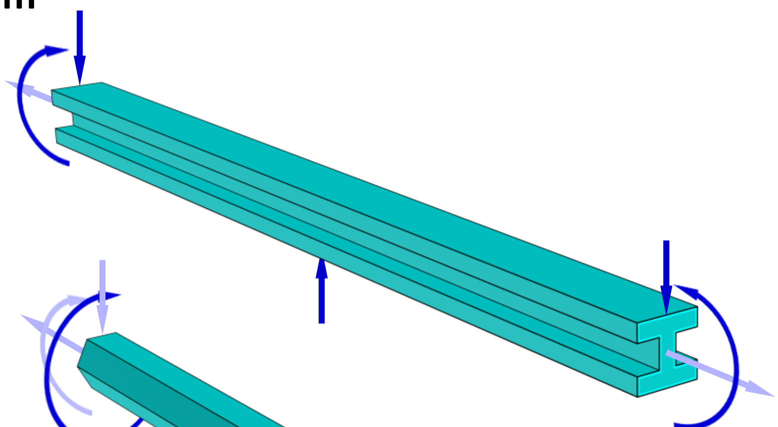
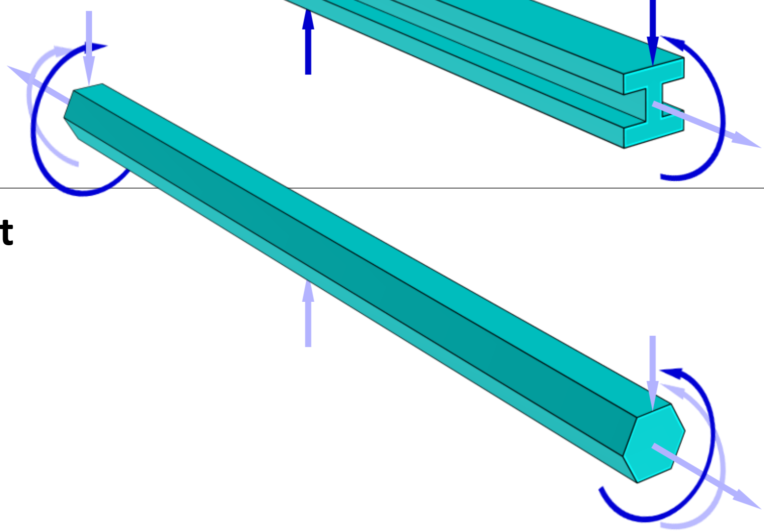
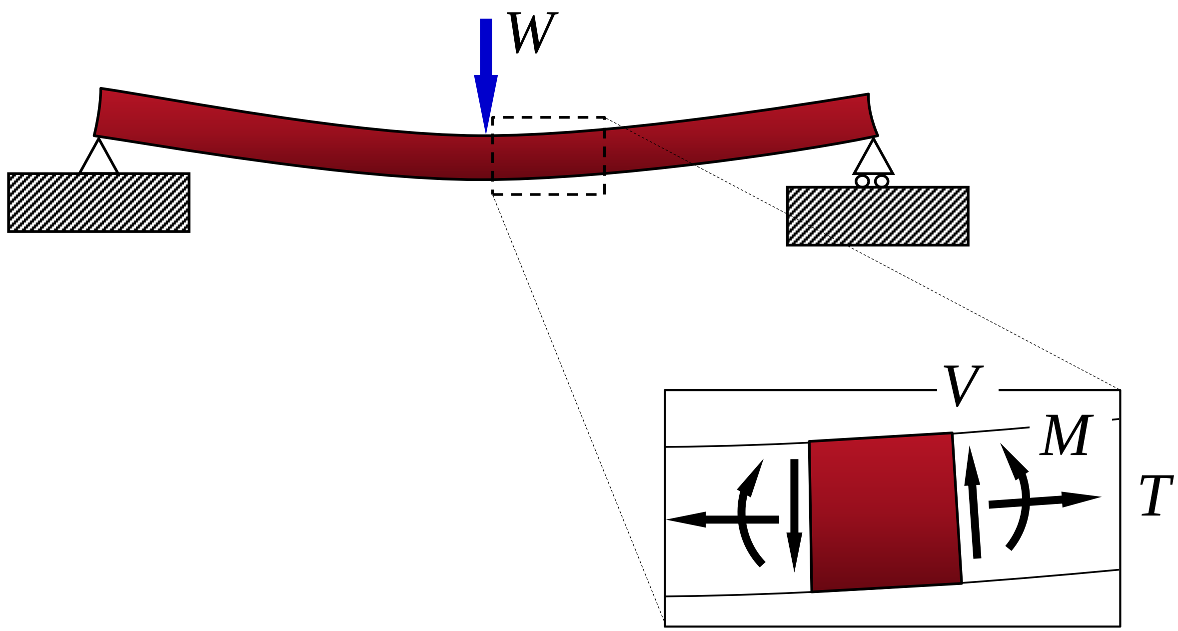
Basic Structural Elements

Names	Supported Loads	Stresses & Strains
Truss (‘axial member’, ‘bar’, ‘rod’, ‘tie’ or ‘strut’) 	• Axial force	• Direct (tensile OR compressive)
Beam 	• Transverse (shear) force • Bending moment • (Axial force) • (Torque)	• Direct • ‘Bending’ (tensile AND compressive) • Shear (through the thickness)
Shaft 	• Torque • (Axial force) • (Transverse shear force) • (Bending moment)	• Shear (torsion) • Direct (tensile OR compressive) • ‘Bending’ (tensile AND compressive) • Shear (through the thickness)

2.1 Beam Element Definition

- A structural member capable of carrying **transverse forces, bending moments** and **axial loads**
  - We will leave axial loads aside for now → 'simple bending'
- A beam subjected to transverse loads will deflect until:
  - the **internal forces** and **moments** generated at **any section** in the beam balance the external loading



$T =$ Axial force

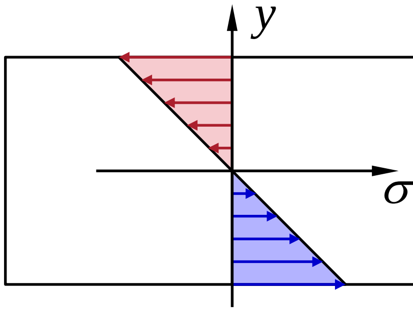
$M =$ Bending moment

$V =$ Shear force

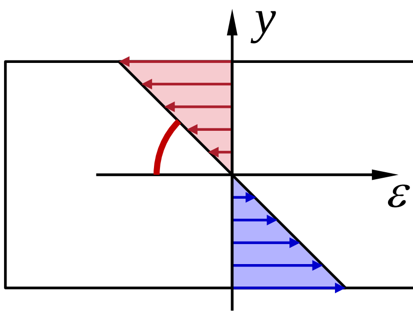
2.1 Beam Element Definition

- Bending moments are related to:

  – Stresses: longitudinal **direct stresses**



  – Strains: longitudinal **direct strains** and **bending curvature**



- Bending stresses will be studied in detail soon

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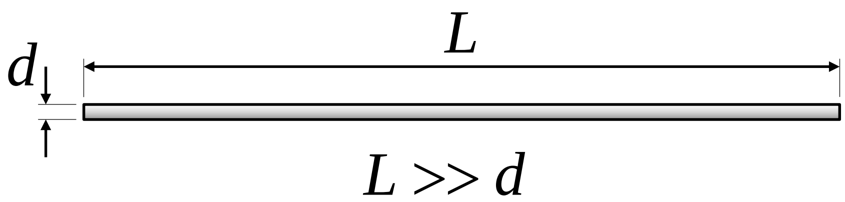
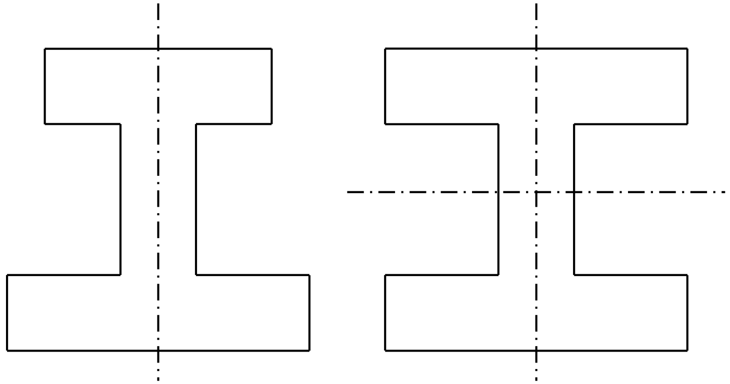
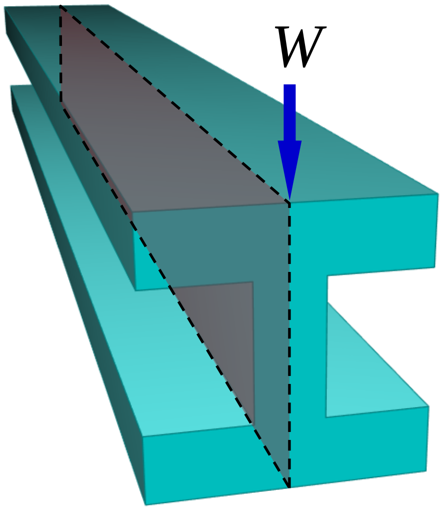
2.2 Idealisations and Assumptions for Beams



- Beams are **straight** and **slender**

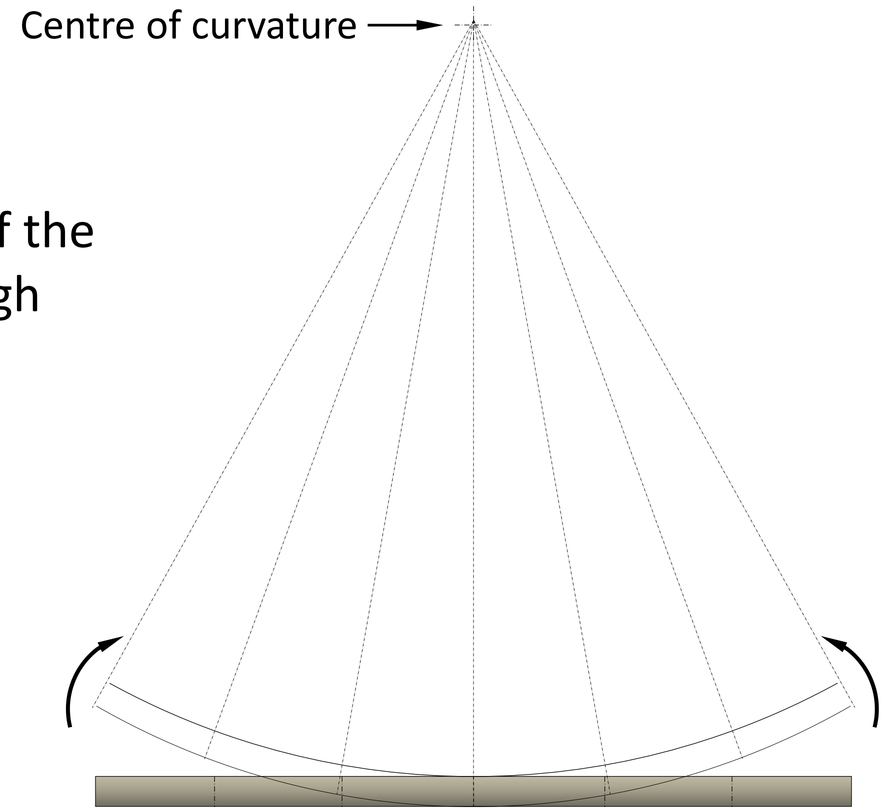


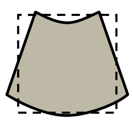
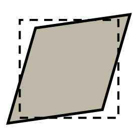
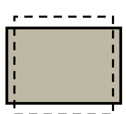
- Cross-sections are ‘singly’ or ‘doubly’ symmetric
- Loads and moments are applied in one symmetric plane of the cross-section



- ‘Simple bending’ assumption:
  - Neglecting **shear deformation** (but NOT shear forces!)
  - Neglecting **axial forces** (for now)

Bending deformation:



- **Plane sections remain plane**
  - Sections rotate about a neutral axis of the cross-section subtending a line through the **centre of curvature** of the beam:
- Only **pure bending deformation** is considered
  - Bending 
  - Shearing 
  - Extension 

These **deformation modes** are neglected here

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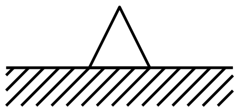
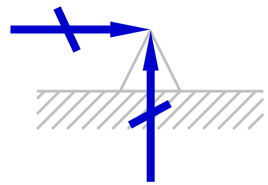
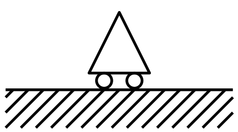
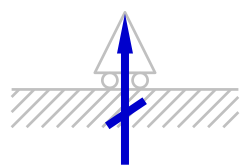
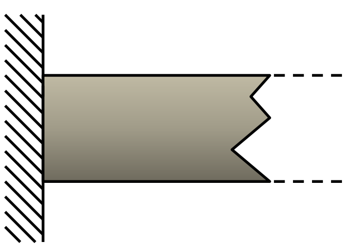
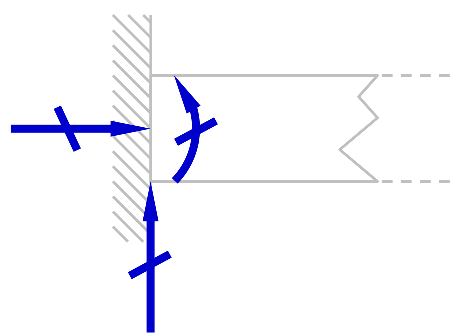
2.3 Support and Loads for Beams

- Similar to trusses, with the addition of the 'built-in' support:

	Support	Possible reactions	Degrees of Freedom
'Pinned' 'Simple'			**Rotational** freedom but **no translational** freedom
'Roller' 'Simple'			**Rotational** freedom + **translational** freedom along the **surface**
'Built-in' 'Encastre' 'Fixed'			**No translational** freedom and **no rotational** freedom

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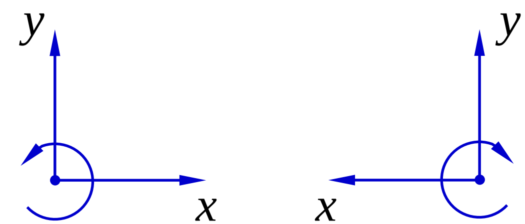
2.4 Sign Convention for Beams

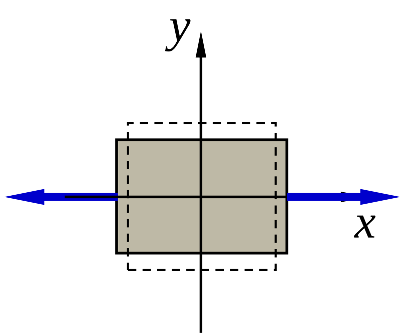
- External sign convention
  - y positive <u>up</u>, but x can be inverted if convenient:



  - For moments, positive sense follows the 'right hand rule'

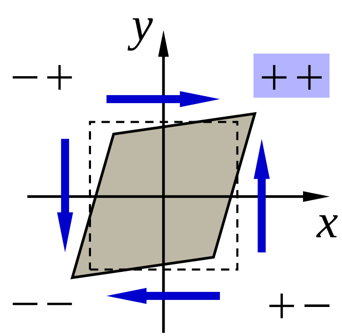
- Internal sign convention

Extension



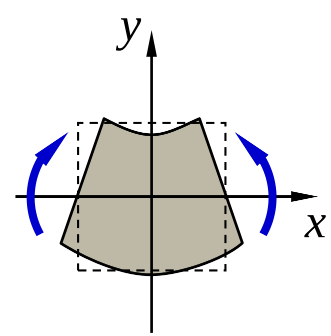
Positive:
'Tension'

Shear



Positive:
'Pinching
the ++ quadrant'

Bending



Positive:
'Sagging'
(or the 'smiley face')

  - Always assume unknown moments and shear forces to be **positive** according to the **internal sign convention** above!!

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2.5 Bending Moment and Shear Force Diagrams

Diagrams are derived using the **method of sections** - similar to the case of a slender truss!

1. Analyse the **global FBD**
   - Resolve reaction forces & moments
2. Choose appropriate origin and sense for the ***x*** **coordinate**
   - Consider loads, boundary conditions, symmetries etc.
3. Section the beam at a given x and **expose internal forces**
   - Always follow the internal sign convention!
   - Write equations of equilibrium and solve unknowns at current x: $\begin{cases} V(x) \\ M(x) \end{cases}$
4. **Move along** the x coordinate and repeat

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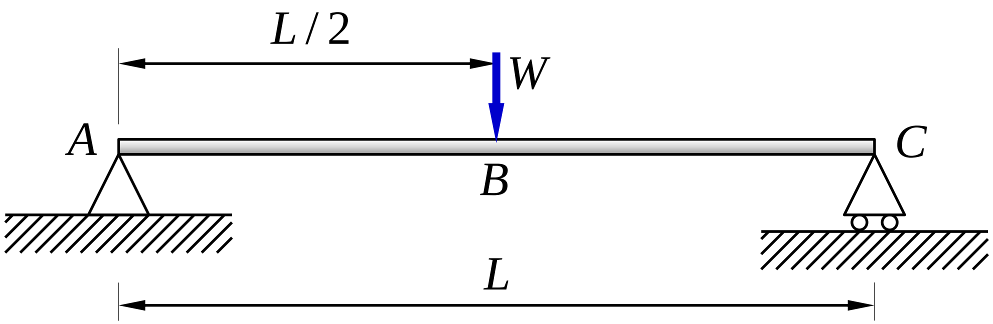
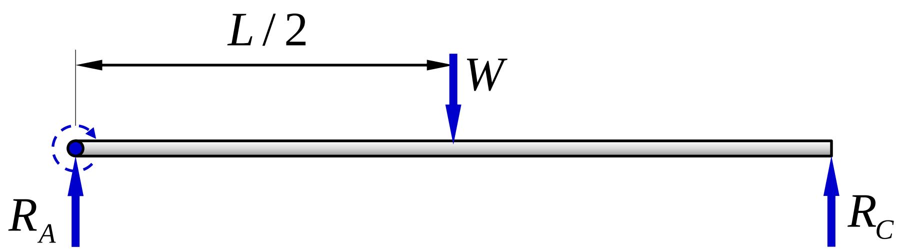
2.5.1 Simply-Supported Beam with Concentrated Load

- Consider the centrally loaded simply-supported beam:



- Global FBD:



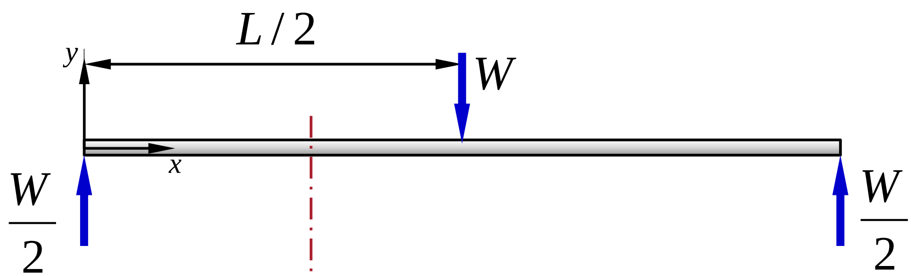
Note: there are **no horizontal forces** or **reactions**, so we omit the subscripts x and y

$$\sum M_{@A}^{\text{cw}} = 0 \quad \therefore \quad W\left(\frac{L}{2}\right) - R_C\left(L\right) = 0 \quad \therefore \quad \boxed{R_C = \frac{W}{2}}$$

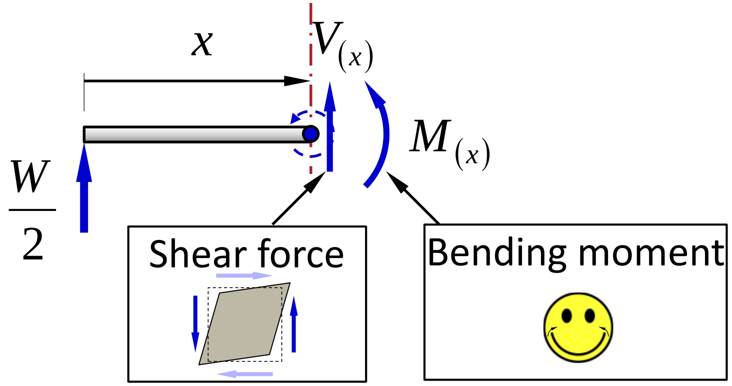
$$\sum F = 0 \quad \therefore \quad R_A + R_C - W = 0 \quad \therefore \quad \boxed{R_A = \frac{W}{2}}$$

2.5.1 Simply-Supported Beam with Concentrated Load

- Putting the origin of x at point A, from left to right:



- Sectioning the beam at an arbitrary point $0 < x < L/2$:



IMPORTANT!
Always assume unknowns to be **positive** following the **internal sign convention**!

$$\sum M_{@x}^{\circlearrowleft\text{ccw}} = 0 \quad \therefore \quad M_{(x)} - \left(\frac{W}{2}\right)x = 0 \quad \therefore \quad \boxed{M_{(x)} = \frac{W}{2}x}$$

$$\sum F_{@x} = 0 \quad \therefore \quad V_{(x)} + \frac{W}{2} = 0 \quad \therefore \quad \boxed{V_{(x)} = \frac{-W}{2}}$$

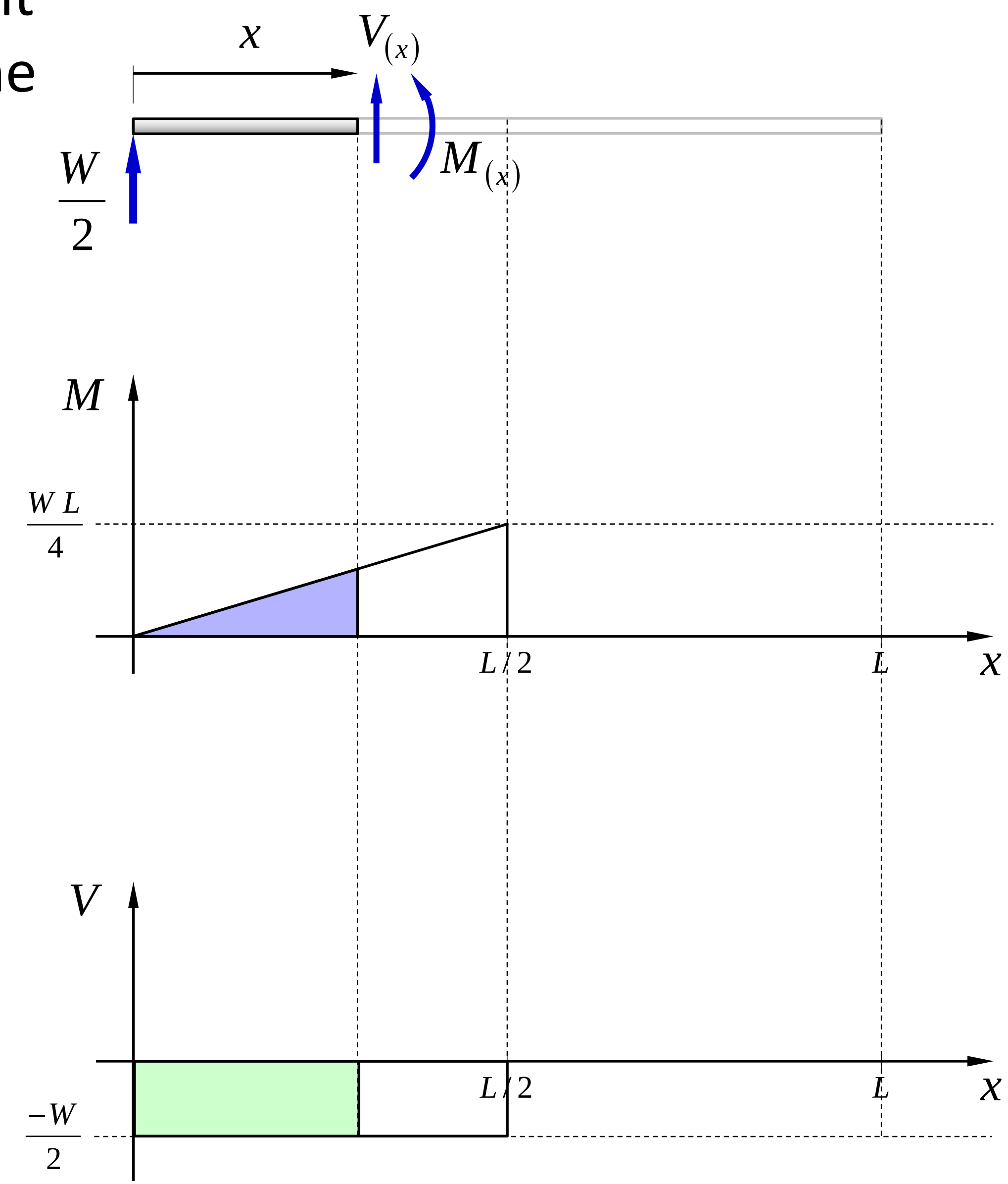
- Note that the bending moment **varies linearly** with x, while the shear force is **a constant**:

$$\boxed{M_{(x)} = \frac{W}{2} x}$$

$$\boxed{V_{(x)} = \frac{-W}{2}}$$

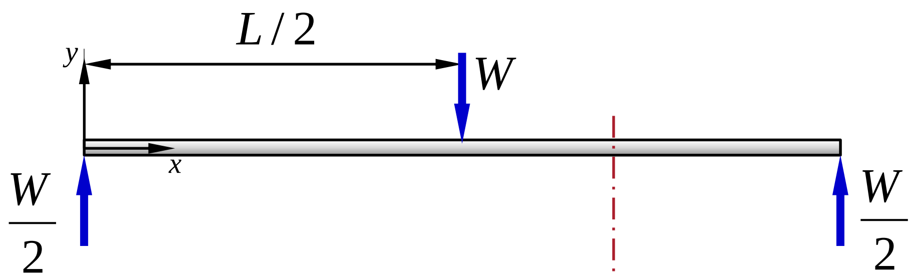
- So we can plot the diagrams:

These solutions are valid for any $0 < x < L/2$.
But what happens after that?

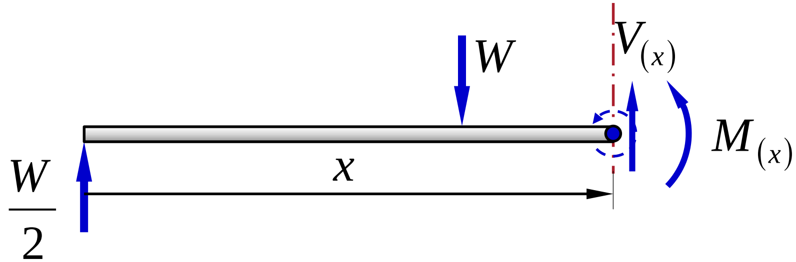


2.5.1 Simply-Supported Beam with Concentrated Load

- Now sectioning the beam at $L/2 < x < L$:



- The load W suddenly 'appears' in our FBD:



$$\sum M_{@x}^{\circlearrowleft\,\mathrm{CCW}} = 0 \quad \therefore \quad M_{(x)} + (W)\left(x - \frac{L}{2}\right) - \left(\frac{W}{2}\right)x = 0 \quad \therefore \quad \boxed{M_{(x)} = \left(\frac{W}{2}\right)(L - x)}$$

$$\sum F = 0 \quad \therefore \quad V_{(x)} - W + \frac{W}{2} = 0 \quad \therefore \quad \boxed{V_{(x)} = \frac{W}{2}}$$

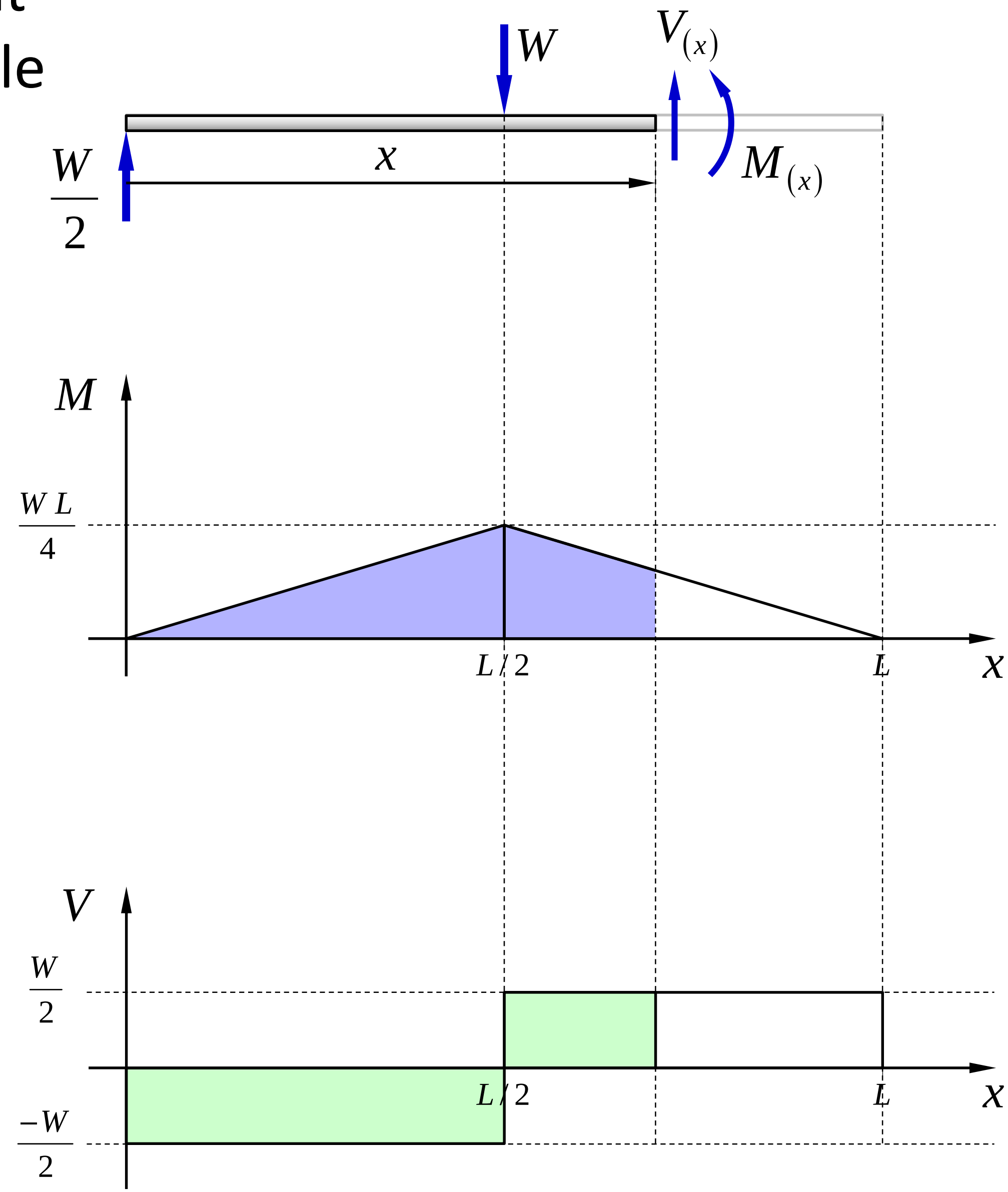
2.5.1 Simply-Supported Beam with Concentrated Load

- Note that the bending moment now has **a negative slope**, while the shear force has **changed sign**:

$$M_{(x)} = \left(\frac{W}{2}\right)(L - x)$$

$$V_{(x)} = \frac{W}{2}$$

- And the diagrams become:



2.5.1 Simply-Supported Beam with Concentrated Load

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$$\boxed{M_{(x)} = \left(\frac{W}{2}\right)(L - x)}$$

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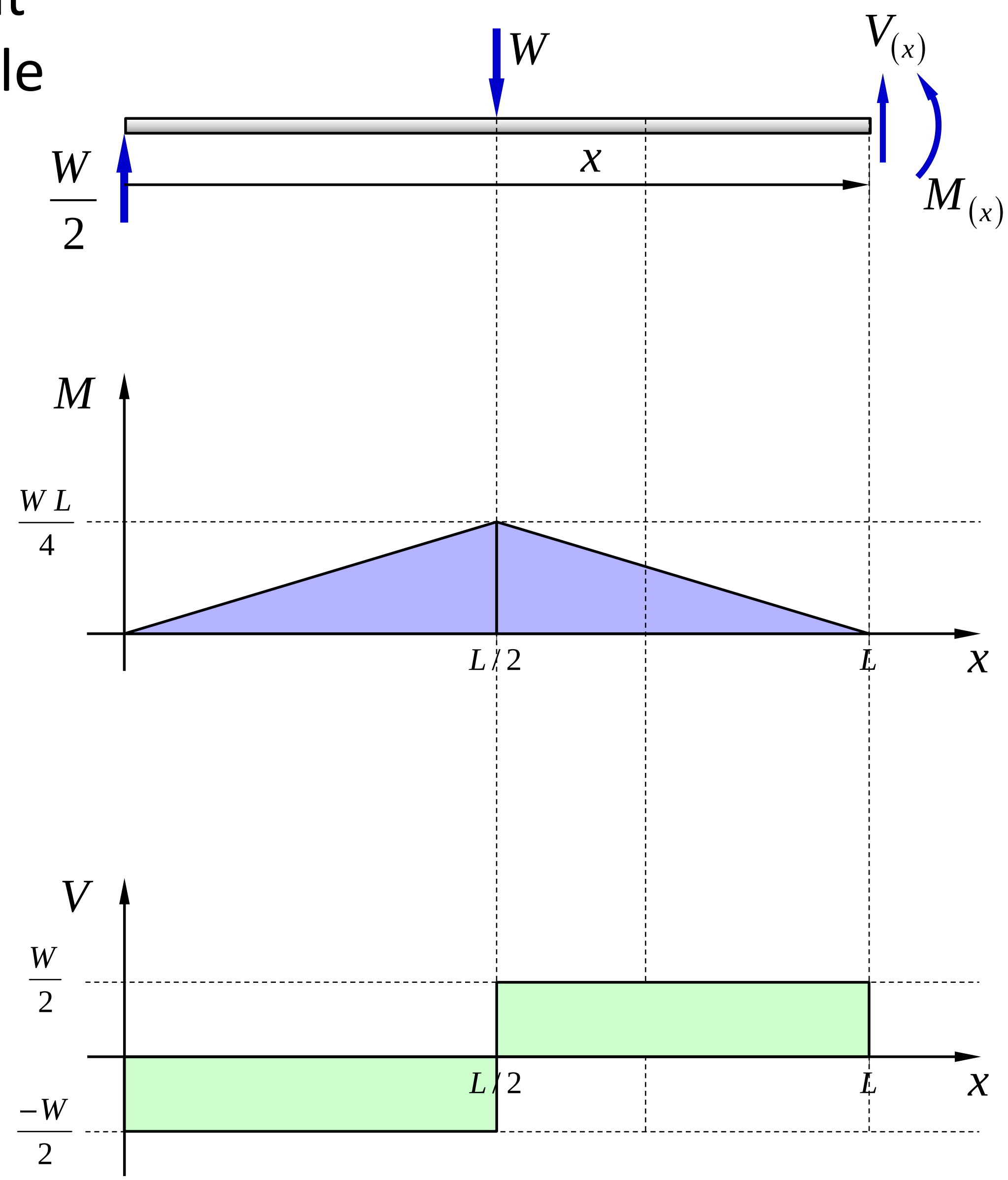
- And the diagrams become:



And at $x = L$ the bending moment finally vanishes

Bending Moment vs. Shear Force

- Shear force is the first derivative of the bending moment:

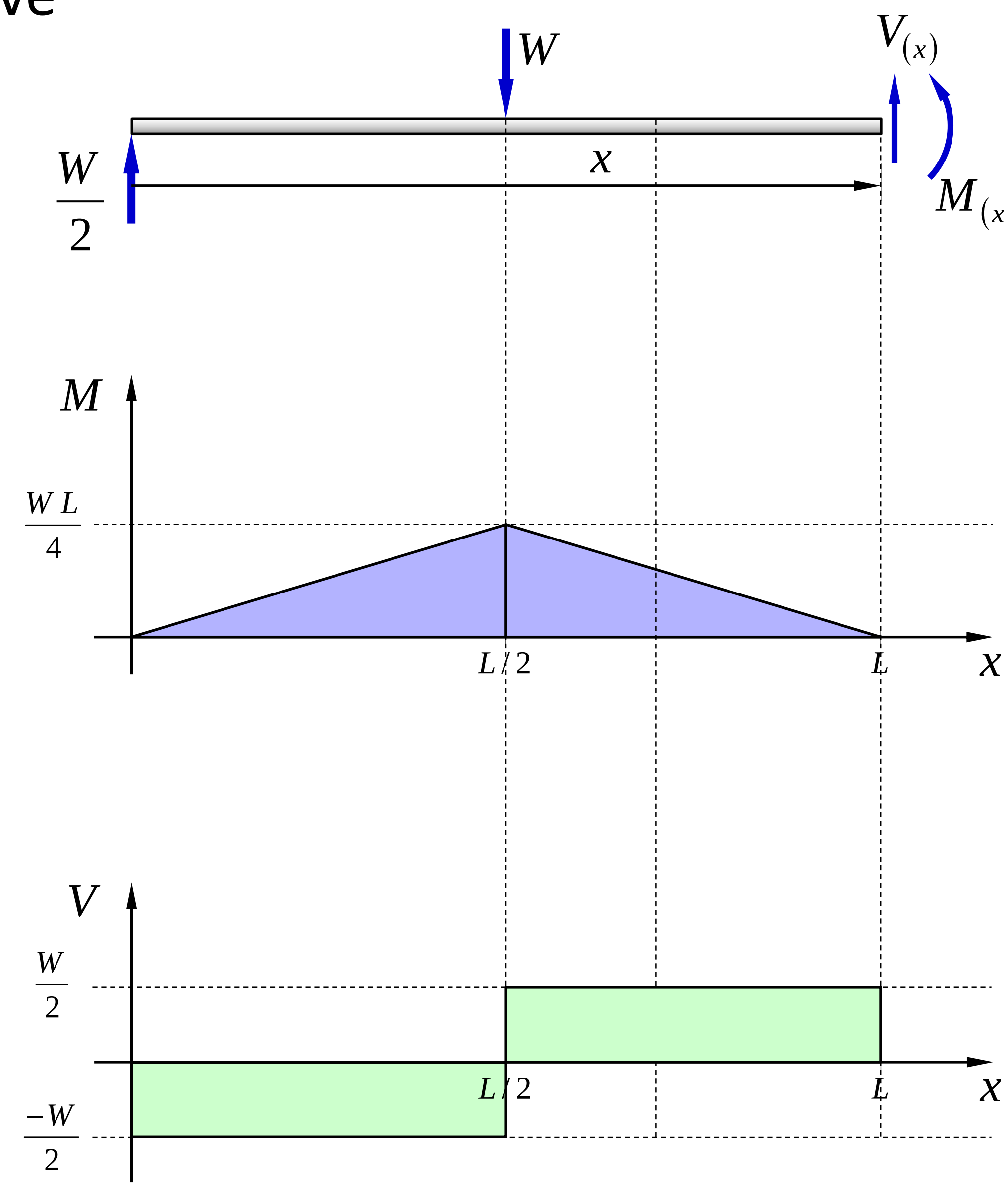
$$\boxed{V_{(x)} = -\frac{\mathrm{d}\,M_{(x)}}{\mathrm{d}\,x}}$$

– i.e. $-V_{(x)}$ is the '**slope**' of $M_{(x)}$

- Conversely, bending moments are the integral of the shear forces:

$$\boxed{M_{(x)} = -\int_0^x V_{(x)}\,\mathrm{d}x}$$

– i.e. $-M_{(x)}$ is the '**cumulative area**' under the graph of $V_{(x)}$



Note: the 'minus' signs in the equations above 'appear/disappear' depending on the direction of the local x-axis !

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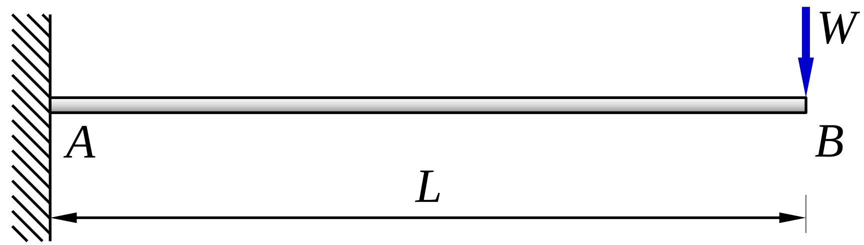
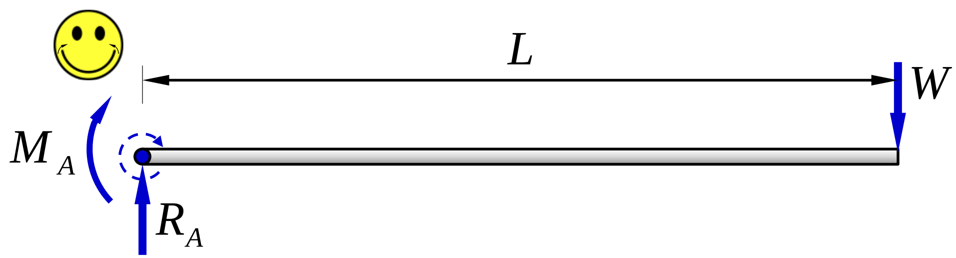
2.5.2 Cantilever Beam with Concentrated Load

- The cantilever beam is 'built-in' at the 'root' (A) and loaded at the 'tip' (B):



- Global FBD:



$$\sum M_{@A}^{\circlearrowright \text{cw}} = 0 \quad \therefore \quad M_A + (W)(L) = 0 \quad \therefore \quad \boxed{M_A = -W\,L}$$

$$\sum F = 0 \quad \therefore \quad R_A - W = 0 \quad \therefore \quad \boxed{R_A = W}$$

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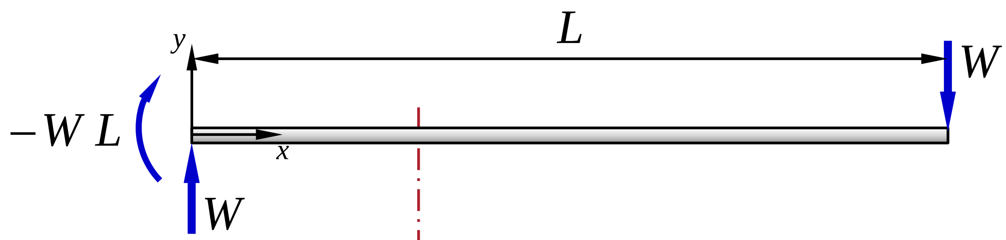
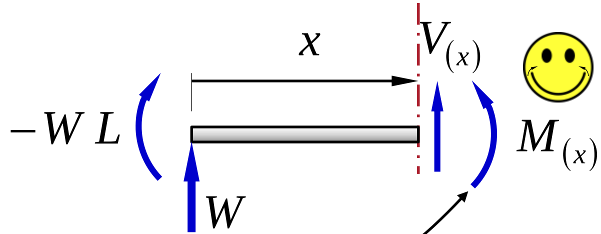
2.5.2 Cantilever Beam with Concentrated Load

- Putting the origin of x at point A, from left to right:



- Sectioning the beam at an arbitrary point $0 < x < L$:



$$\sum M_{@x}^{\text{ccw}} = 0 \quad \therefore \quad M_{(x)} - (-WL) - (W)(x) = 0 \quad \therefore \quad \boxed{M_{(x)} = W(x - L)}$$

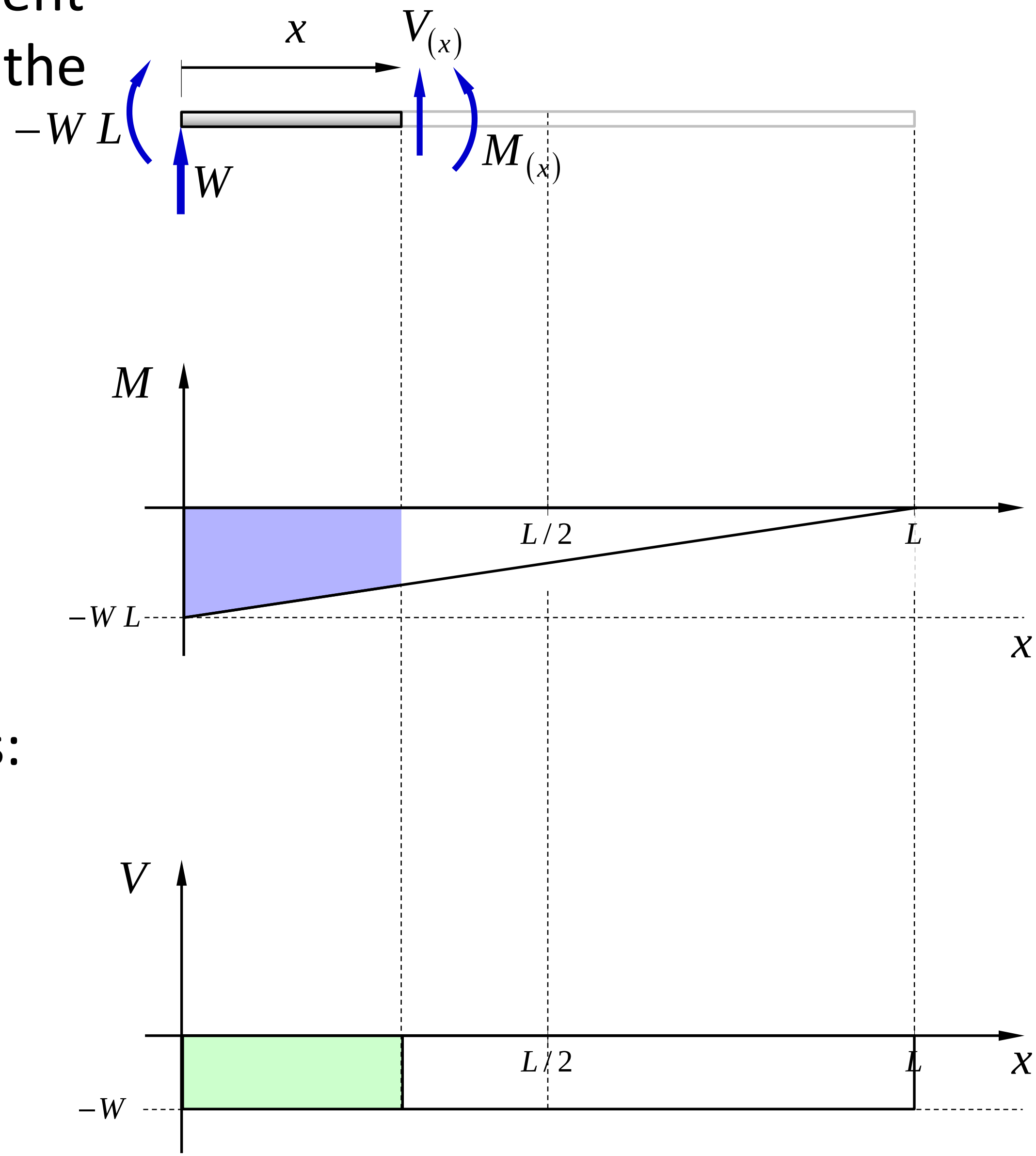
$$\sum F_{@x} = 0 \quad \therefore \quad W + V_{(x)} = 0 \quad \therefore \quad \boxed{V_{(x)} = -W}$$

- Note that the bending moment **varies linearly** with x, while the shear force is **a constant**:

$$\boxed{M_{(x)} = W(x - L)}$$

$$\boxed{V_{(x)} = -W}$$

- So we can plot the diagrams:



These solutions are valid for any $0 < x < L$

2.5.2 Cantilever Beam with Concentrated Load

- Note that the bending moment **varies linearly** with x, while the shear force is **a constant**:

$$\boxed{M_{(x)} = W(x - L)}$$

$$\boxed{V_{(x)} = -W}$$

- So we can plot the diagrams:

These solutions are valid for any $0 < x < L$