Energy Methods
Principle of Stationary Potential Energy

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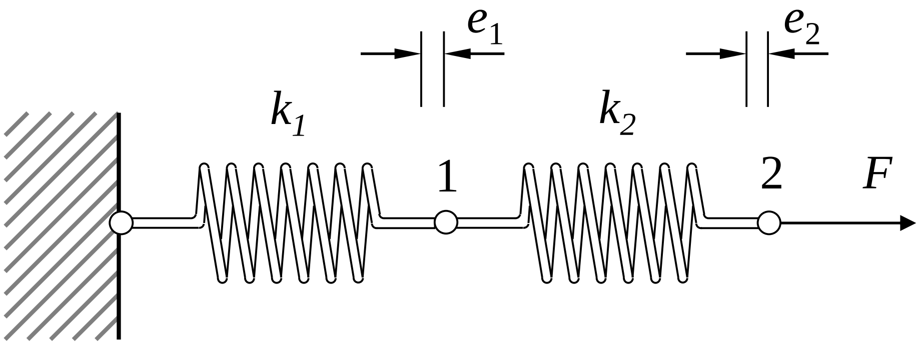
Energy Methods - Contents

Principle of Stationary Potential Energy (PSPE)

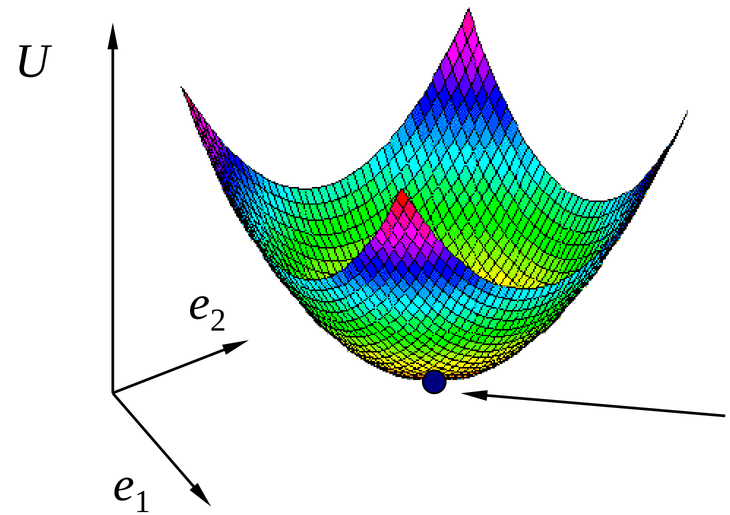
- Assume two axially-loaded members connected 'in series' and subjected to a force F:



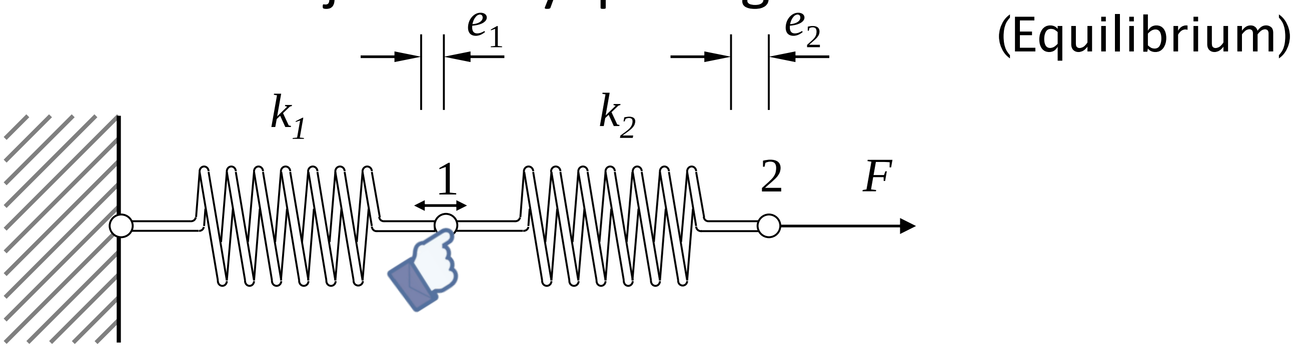
(Equilibrium)

- Joints 1 and 2 will be displaced by e_1 and e_2 respectively
- If the system is under static equilibrium then the total energy must be at a **local minimum**:

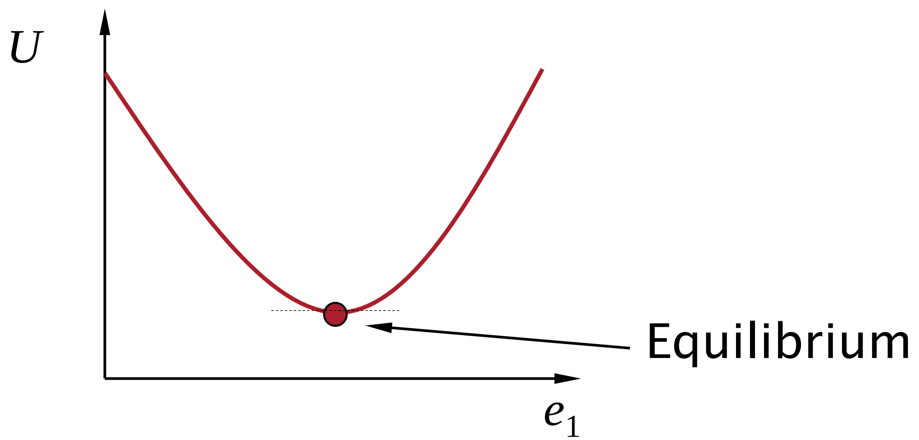
(stationary minimum)



- For a system in equilibrium, imagine that we could introduce a **small arbitrary displacement** in joint 1 by 'poking' it:



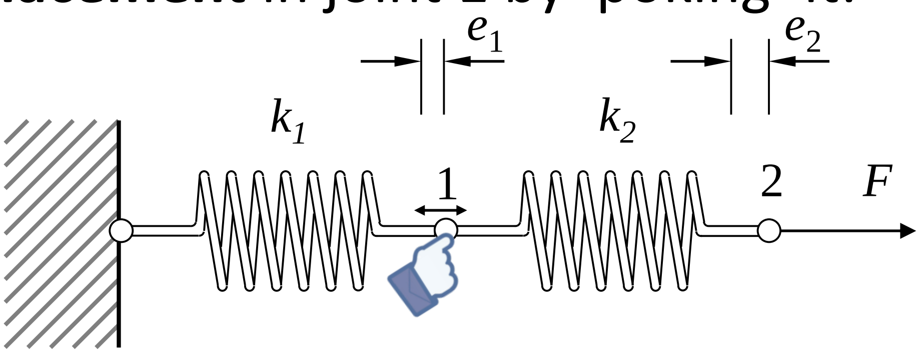
- By doing so we would find that the total system energy does not change when e_1 is varied by a small amount:



$$\frac{\partial U}{\partial e_1} = 0$$

Principle of Stationary Potential Energy (PSPE)

- For a system in equilibrium, imagine that we could introduce a **small arbitrary displacement** in joint 1 by 'poking' it:



(Equilibrium)

- The **PSPE** is related to the **principle of virtual work** used in Mechanics 1. In the latter:
  - Assume a set of **arbitrary but permissible virtual displacements** (i.e. respecting all boundary conditions)
  - Calculate the **virtual work** as the product of **real forces** and **virtual displacements**
  - If the system is in equilibrium then this **virtual work** must be equal to the **virtual change in internal energy**

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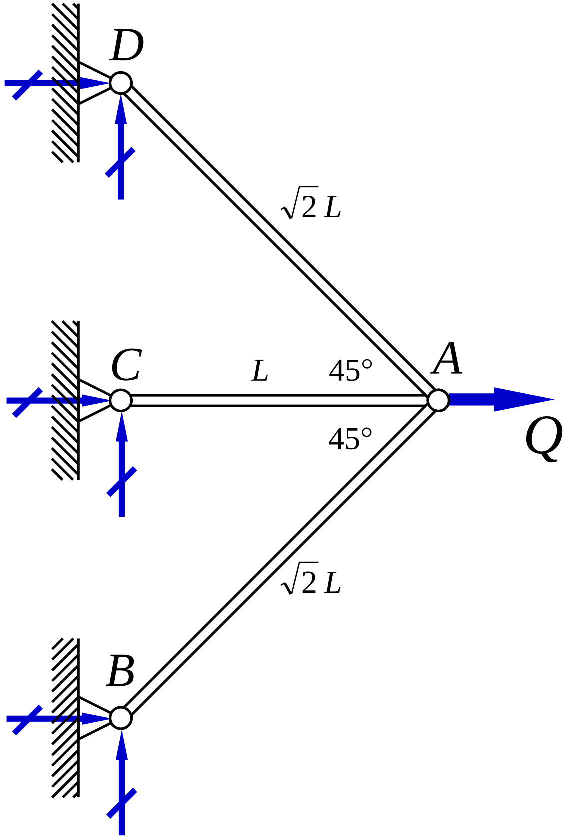
PSPE for Pin-Jointed Trusses

- Consider the following redundant truss structure where all members have the same cross-sectional area A and the same Young's modulus E



Redundancy test:

- Nu = No. unknowns = N°. of reactions + N°. of members

  6 + 3 = 9

- Ne = N°. of equations = N°. joints × N°. DOFs

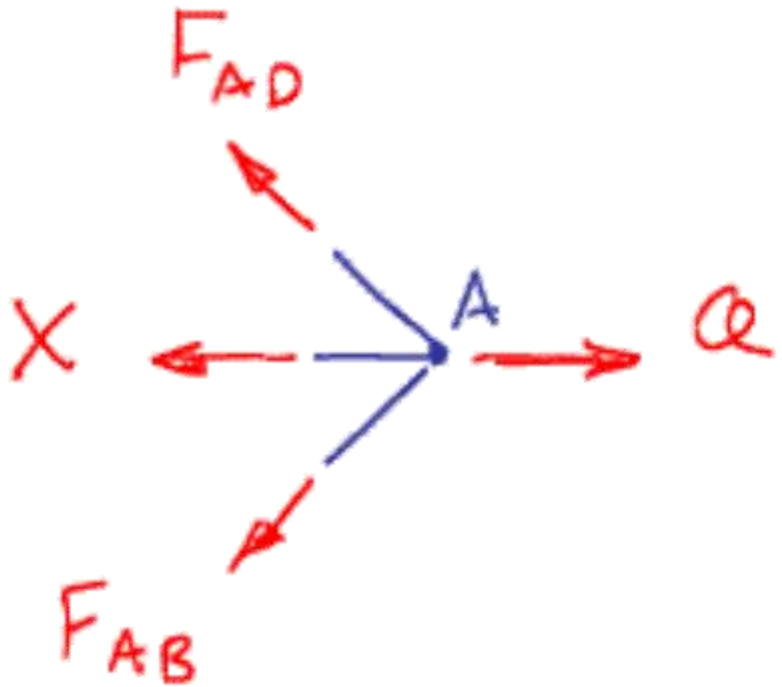
  4 × 2 = 8

- DoR = degree of redundancy = Nu – Ne

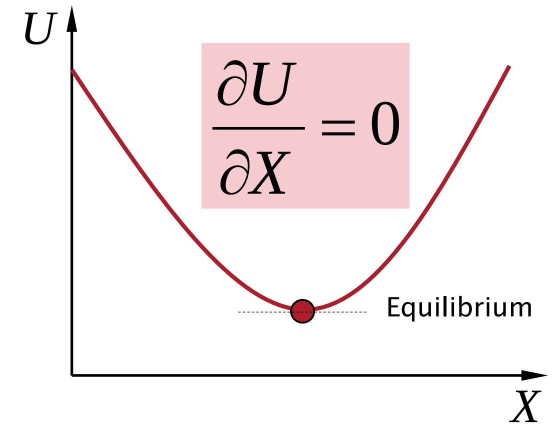
  9 – 8 = 1

We choose one internal force (arbitrarily) as our unknown redundant force X

Let $F_{AC} = X$



We write the PSPE in terms of the unknown X:



The total energy is a function of all internal forces:

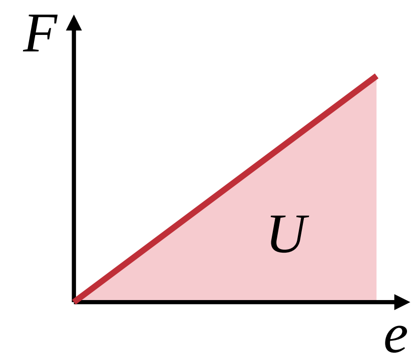
$$U = f\left(F_1, F_2, F_3, \ldots\right)$$

Applying the 'chain rule' for partial differentiation:

$$\frac{\partial U}{\partial X} = \sum\left(\frac{\partial U}{\partial F_i}\cdot\frac{\partial F_i}{\partial X}\right)$$

For axial members:

$$U_i = \int_0^{e_i} F\,\mathrm{d}e = \frac{1}{2}F_i\,e_i = \frac{1}{2}F_i\,\frac{F_i\,L_i}{A_i\,E_i} \quad \therefore \quad U_i = \frac{1}{2}\frac{F_i^{\,2}\,L_i}{A_i\,E_i}$$



$$\boxed{\frac{\partial U_i}{\partial F_i} = \frac{F_i\,L_i}{A_i\,E_i}}$$

Finally, the PSPE statement:

$$\frac{\partial U}{\partial X} = \sum\left(\frac{F_i\,L_i}{A_i\,E_i}\cdot\frac{\partial F_i}{\partial X}\right) = 0$$

→ Find all internal forces in a statically indeterminate pin-joint truss

1. Choose one member to be your 'redundant member', and assume it has an unknown redundant force X

2. Using the method of joints, write expressions for all internal forces F_i in terms of the external load Q and redundant load X

3. Tabulate the expressions for F_i, L_i, A_i, E_i and the derivatives $\frac{\partial F_i}{\partial X}$

4. Perform the summation $\sum\left(\frac{F_i L_i}{A_i E_i}\cdot\frac{\partial F_i}{\partial X}\right)=0$ to find X

5. Once X is known, go back to the table and replace it to find the individual internal forces F_i