Vibrations 2, Lecture 2
Stiffness, damping, inertia

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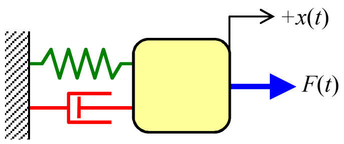
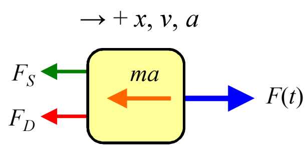
Lecture 1

Vibrating structures



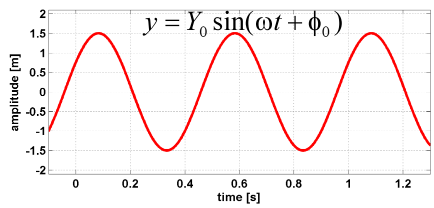
FBD



EOM

$$ma + F_D + F_S = F(t)$$

Harmonic motion



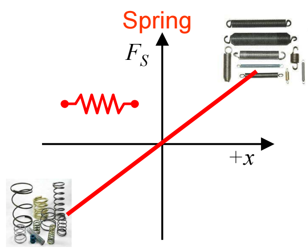
Lecture 2

- Stiffness, damping and inertia
- Solved example
- Measuring vibrations

Springs and stiffness

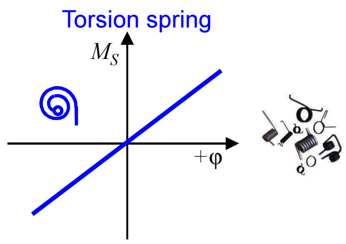
Stiffness is resistance to the displacement from the reference configuration

Definition: $$stiffness = \frac{applied\ load}{deformation}$$



$$F_S = k\,x$$

F_S … restoring force [N]
k … stiffness [N/m]
x … deformation [m]

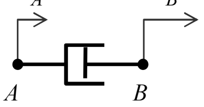


$$M_S = k_r \varphi$$

M_S … restoring moment [Nm]
k_r … torsional stiffness [Nm/rad]
φ … angular deformation [rad]

Dampers and viscous damping

Damping is responsible for the forces which oppose the motion. **Viscous dampers/damping** resists motion with the forces proportional to the *relative* velocity experienced between the ends of the damping element.



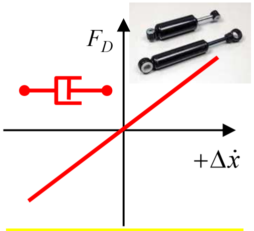
$$F_D = c(v_B - v_A)$$

$$\downarrow$$

$$v_A = \dot{x}_A \qquad v_B = \dot{x}_B$$

Definition: $\longrightarrow$

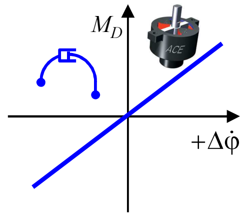
Damper or dashpot



$$F_D = c(\dot{x}_B - \dot{x}_A)$$

F_D … damping force [N]
c … damping constant [Ns/m]
v … relative velocity [m/s]

Torsional damper



$$M_D = c_r(\dot{\varphi}_B - \dot{\varphi}_A)$$

M_D … damping moment [Nm]
c_r … torsional damp. const. [Nsm/rad]
v … relative angular velocity [rad/s]

Rigid bodies and inertia

Newton's method introduces **inertial forces** and **moments.** These loads are in dynamic equilibrium with other applied *and* internal loads.

Inertia properties such as *mass* [kg] and *mass moment of inertia* [kg.m^2] can be seen as a measure of resistance to acceleration.

$$F_I = -m\ddot{x}$$ … translational motion

$$M_{I,A} = -I_A\ddot{\varphi}$$ … rotational motion *about the reference point*

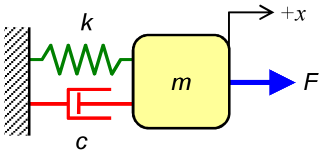
Definition:

$$I_A = \int_{(m)} r_A^2\, dm = I_{A,1} + I_{A,2} + \dots$$

Mass moment of inertia [kg.m^2]

Complete EOM from L1

Using information about the damper and spring forces, the EOM for the problem from Lecture 1 is:



$$ma + F_D + F_S = F(t)$$

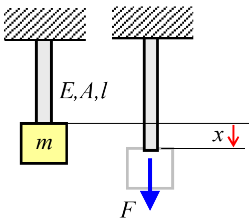
$$ma + cv + kx = F(t)$$

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

Ordinary Differential Equation (ODE) in time with non-zero RHS.

Example: equivalent stiffness

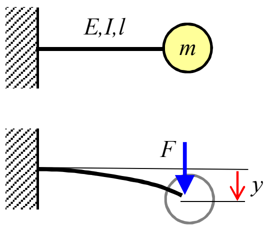
Bars Tensile stiffness	Beams Bending stiffness	Shafts Torsional stiffness



$$x = \frac{F\,l}{E\,A}$$

$$k_{bar} = \frac{F}{x} = \frac{E\,A}{l}$$

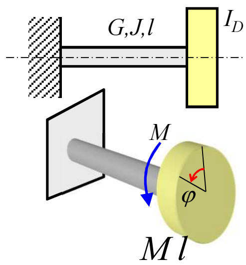
$[N/m]$



$$y = \frac{F\,l^3}{3EI}$$

$$k_{beam} = \frac{F}{y} = \frac{3EI}{l^3}$$

$[N/m]$



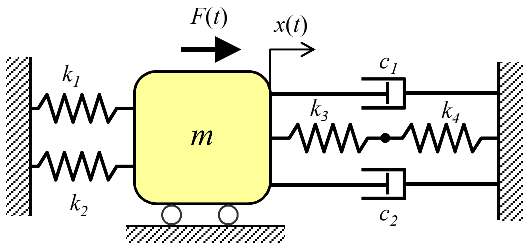
$$\varphi = \frac{M\,l}{G\,J}$$

$$k_{shaft} = \frac{M}{\varphi} = \frac{G\,J}{l}$$

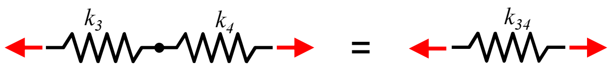
$[Nm/rad]$

Example: 1 DOF spring-mass-damper system

Derive the EOM?



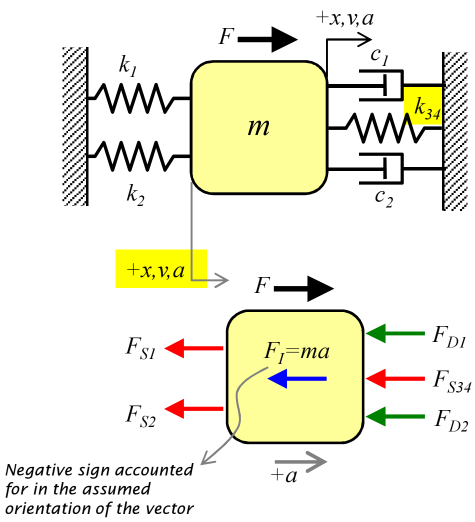
Step 1: Equivalent stiffness for two springs in series



$$x_3 + x_4 = x_{34} \Rightarrow \frac{F}{k_3} + \frac{F}{k_4} = \frac{F}{k_{34}} \qquad \frac{1}{k_{34}} = \frac{1}{k_3} + \frac{1}{k_4}$$

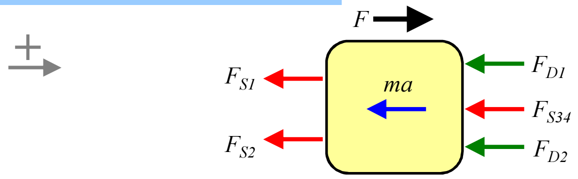
Example 1

Step 2: Free Body Diagram



Example 1

Step 3: Equilibrium (in x-direction)



$$-F_{S1} - F_{S2} - F_{S34} - F_{D1} - F_{D2} - ma + F = 0$$

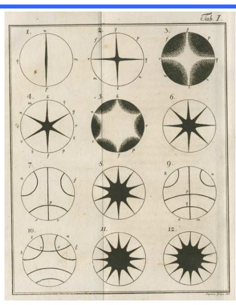
Step 4: Equation of Motion

$$-k_1 x - k_2 x - k_{34} x - c_1 \dot{x} - c_2 \dot{x} - m\ddot{x} + F = 0$$

$$m\ddot{x} + (c_1 + c_2)\dot{x} + (k_1 + k_2 + k_{34})x = F(t)$$

$$m\ddot{x} + c_e \dot{x} + k_e x = F(t)$$ *Compare to slide 7!*

Vibration experiments



Chladni, Entdeckungen über die Theorie des Klanges, 1787

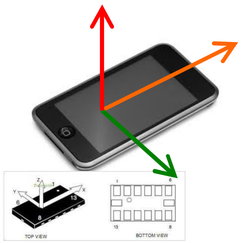
Bakker, 90 years of helicopter design … in the Netherlands, ERF 2010

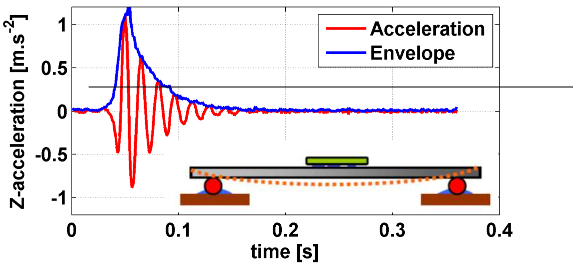
RSL report V-202/17 February 1927:

The tail wheel and nose support strut are replaced by leather footballs with a sac type leather cover. The reason is to facilitate movements of nose and tail in all directions when the helicopter lifts off for short hops. The helicopter controls are tested and the helicopter leaves the ground although secured to the ground with four cables. The control column experiences strong shocks in particular after control inputs to the left or forward. After lift off the nose is much higher than the tail. The longitudinal and lateral oscillations of the fuselage are recorded by a vertical pencil fixed to the fuselage. A piece of paper is held against the pencil during one rotor rotation (fig.7). The numbers in the graph refer to the particular test run. The result helps to make a decision on a next adjustment to the cables or even a modification.

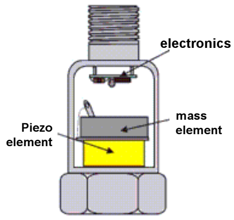
Vibration measurement

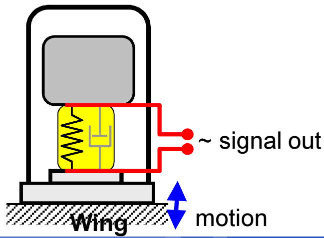
Y1 foam wing design:





Measuring acceleration using single-axis *piezoelectric* accelerometer:





Summary

- Springs, dampers and masses
  - Building blocks of dynamic models
- Practice solved and unsolved problems
  - see example sheets, books, …
- Vibration measurements
  - Sensors, signal processing, identification, …
  - Experimental Methods in Aerospace
    - Experimental Modal Analysis (EMA); Ground Vibration Tests (GVT); Flutter tests; accelerated fatigue tests, …