Example 2.2.6

Figure 1a shows the cross-section of a heavy structural beam which is fabricated by joining together three identical I-sections shown in Figure 1b. Determine the second moments of area of the compound cross-section about the axes of symmetry A-A and B-B.

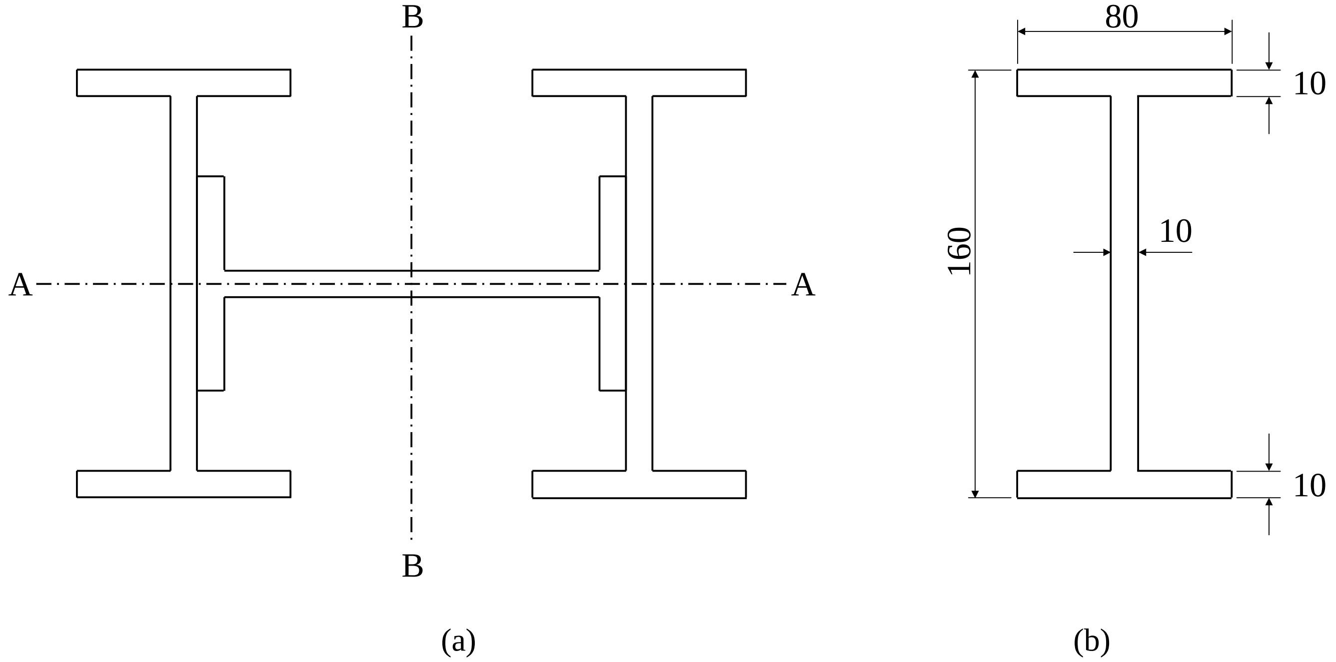
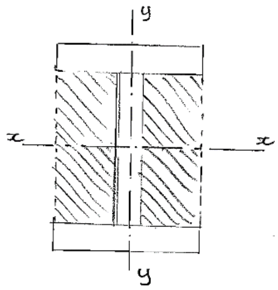


Figure 1: (a) Assembled compound cross-section and (b) dimensions of each component I-section (in millimetres).

First, define local axes *x* and *y* for the single I-beam, and use the additive property of second moments of area to 'add' or 'subtract' areas.



The second moments of area about *x*- *x* and *y*-*y* are therefore:

$$I_{xx} = \frac{1}{12}\left[(80)(160)^3 - (70)(140)^3\right] \text{mm}^4$$

$$I_{xx} = 11.3 \times 10^6 \text{ mm}^4$$

$$I_{yy} = \frac{1}{12}(140)(10)^3 + 2\left[\frac{1}{12}(10)(80)^3\right]$$

$$I_{yy} = 8.65 \times 10^5 \text{ mm}^4$$

The cross-section area for a single I-beam is:

$$A = (80)(160) - (70)(140)\ mm^2$$

$$A = 3000\ mm^2$$

Now consider the composite cross-section. About axis A-A all centroids fall on the axis itself, so the contributions of the three I-beams can simply be added:

$$I_{AA} = 2\,I_{xx} + I_{yy}$$

$$= 2(11.3\times10^6) + (8.65\times10^5)\ mm^4$$

$$\boxed{I_{AA} = 23.465\times10^6\ mm^4}$$

About the B-B axis the 'parallel axis theorem' must be used, as the centroids of two of the I-beams are away from that axis. The second moment of area is therefore:

$$I_{BB} = I_{xx} + 2\,I_{yy} + 2A(85)^2$$

$$= (11.3\times10^6) + 2(8.65\times10^5) + 2(3000)(85)^2\ mm^4$$

$$\boxed{I_{BB} = 56.380\times10^6\ mm^4}$$

Interpretation of results

Since $I_{AA} < I_{BB}$, a strut made of this cross-section would tend to buckle about axis A-A if loaded under axial compression.

If such strut were in a fixed-fixed condition, the critical buckling load would be:

$$P_{cr} = \frac{4\pi^2 EI}{L^2} = \frac{4\pi^2(200.10^3\ N\,mm^{-2})(23.465\cdot10^6\ mm^4)}{(7000\ mm)^2}$$

$$\boxed{P_{cr} = 3.78\ MN}$$

And for a pinned-pinned condition we would have one quarter of that, *i.e.* P_{cr} = 945 kN.