UNIVERSITY OF BRISTOL
FACULTY OF ENGINEERING

First Year Examination for the Degree of
Master of Engineering

MAY/JUNE 2017 2 Hours

FLUIDS 1

AENG11101

This paper contains *two* sections

SECTION 1
Answer *all* questions in this section
This section carries *20 marks*.

SECTION 2
This section has *three* questions.
Answer *two* questions.
All questions in this section carry *20 marks* each.
The maximum for this paper is *60 marks.*

Calculators may be used.

For air, assume R = 287 J/kgK. Take 0ºC as 273 K.
Use a gravitational acceleration of 9.81m/s^2
1 bar = 10^5 N/m^2

TURN OVER ONLY WHEN TOLD TO START WRITING

Useful Equations

The volume of a sphere: $\frac{4}{3}\pi r^3$ Area of a circle: πr^2

Roots of a quadratic: $ax^2 + bx + c = 0 \quad \rightarrow \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The equation of state for a perfect gas is:

$$p = \rho RT$$

Drag equation: $\text{Drag} = \text{Area} \times C_D \times \frac{1}{2}\rho V^2$

Equation of a streamline in 2D flow

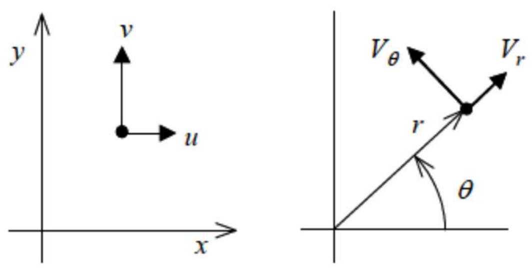
$$\frac{dy}{dx} = \frac{v}{u}$$

Change between Polar and Cartesian coordinate systems

$$r = \sqrt{x^2 + y^2}, \quad \theta = \tan^{-1}\left(\frac{y}{x}\right), \quad x = r\cos\theta, \quad y = r\sin\theta,$$

$$u = V_r \cos\theta - V_\theta \sin\theta, \qquad v = V_r \sin\theta + V_\theta \cos\theta$$

$$V_r = u\cos\theta + v\sin\theta, \qquad V_\theta = -u\sin\theta + v\cos\theta$$



2D Potential Flow

Velocity components in two-dimensional irrotational flow, in terms of the stream function and potential are given by

$$\underbrace{V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \qquad V_\theta = -\frac{\partial \psi}{\partial r} \qquad V_r = \frac{\partial \phi}{\partial r} \qquad V_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta}}_{\text{polar coordinates}}$$

$$\underbrace{u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x} \qquad u = \frac{\partial \phi}{\partial x} \qquad v = \frac{\partial \phi}{\partial y}}_{\text{Cartesian coordinates}}$$

The stream function & velocity potential in Polar coordinates and the velocity distribution for

i) A uniform flow U_∞ parallel to the x axis:

$$\psi = U_\infty r\sin\theta, \quad \phi = U_\infty r\cos\theta, \quad V_r = U_\infty \cos\theta, \quad V_\theta = -U_\infty \sin\theta, \quad u = U_\infty, \quad v = 0$$

ii) A source, of strength Λ at the origin:

$$\psi = \frac{+\Lambda\theta}{2\pi}, \quad \phi = \frac{+\Lambda}{2\pi}\ln r, \quad V_r = \frac{+\Lambda}{2\pi r}, \quad V_\theta = 0, \quad u = \frac{+\Lambda}{2\pi}\frac{x}{\left(x^2 + y^2\right)}, \quad v = \frac{+\Lambda}{2\pi}\frac{y}{\left(x^2 + y^2\right)}$$

iii) A doublet, of strength κ at the origin:

$$\psi = \frac{-\kappa}{2\pi}\frac{\sin\theta}{r}, \qquad \phi = \frac{+\kappa}{2\pi}\frac{\cos\theta}{r}, \qquad V_r = \frac{-\kappa}{2\pi r^2}\cos\theta, \qquad V_\theta = \frac{-\kappa}{2\pi r^2}\sin\theta,$$

$$u = \frac{-\kappa}{2\pi}\frac{\left(x^2 - y^2\right)}{\left(x^2 + y^2\right)^2}, \qquad v = \frac{-\kappa}{2\pi}\frac{2xy}{\left(x^2 + y^2\right)^2}$$

iv) A vortex, of strength Γ, at the origin:

$$\psi = \frac{+\Gamma}{2\pi}\ln r, \quad \phi = \frac{-\Gamma}{2\pi}\theta, \qquad V_r = 0, \quad V_\theta = \frac{-\Gamma}{2\pi r}, \qquad u = \frac{+\Gamma}{2\pi}\frac{y}{\left(x^2 + y^2\right)}, \quad v = \frac{-\Gamma}{2\pi}\frac{x}{\left(x^2 + y^2\right)}$$

Useful integrals

$$\int \sin^2(ax)dx = \frac{x}{2} - \frac{\sin 2ax}{4a} + C$$

$$\int \cos^2(ax)dx = \frac{x}{2} + \frac{\sin 2ax}{4a} + C$$

turn over …

SECTION 1
Answer <u>all</u> questions in this section

Q1 A pressure transducer records a gauge pressure of 4bar when lowered into the sea. How far below the surface is the pressure transducer if the atmospheric pressure at the surface is 1.023bar and the density of the water is assumed constant at 1030 kg m^{-3} ?

(2 marks)

Q2 An aquarium has two hemispherical viewing windows, each with a radius of 0.5m. One window is placed on the side wall with its centreline at a depth of 2m, find the horizontal force on this window. The other is placed at the bottom of the aquarium at a depth of 3m, find the vertical force on this window. Assume that atmospheric pressure acts on the surface of the aquarium water and the outside of the viewing windows, also assume that the water (including fish) has a density of 1000 $kg\ m^{-3}$

(3 marks)

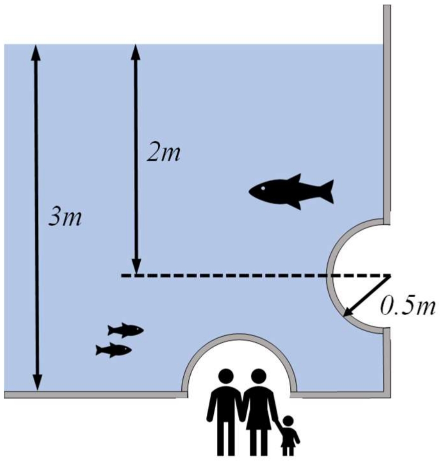


Figure Q2

Q3 State the assumptions that must be made for Bernoulli's equation to be valid.

(3 marks)

Q4 **a)** A 1:4 scale model aircraft placed in a wind tunnel with an airspeed of 30ms^{-1} has a drag of 3N. What would be the drag on the full scale model aircraft flying at twice speed if the air had the same density?

(3 marks)

Q5 Water flows vertically up a straight smooth pipe. At a point 1m below a sudden expansion, the pressure and velocity are 2 bar and 5m/s. After the sudden expansion, the area of the pipe becomes four times greater than the first section. Using the equation for pressure loss:

$$\Delta p_{loss} = \tfrac{1}{2}\rho V_1^2\left(1-\tfrac{A_1}{A_2}\right)^2$$

Find the pressure 2 m above the expansion. The water density can be taken as 1000 kg m^{-3}.

(3 marks)

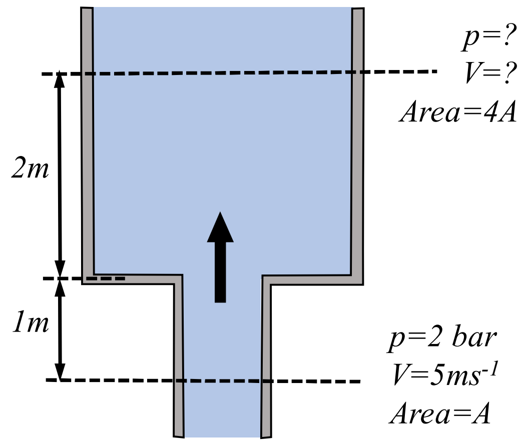


Figure Q5

Q6 A horizontal circular water jet of diameter 10cm and speed 10m/s is turned smoothly through 90º by a stationary vane. By using a suitable control volume, find the horizontal and vertical force on the turning vanes, where the positive direction is as shown in Figure Q6. Assume the water has a density of 1000 kg m^{-3}.

(3 marks)

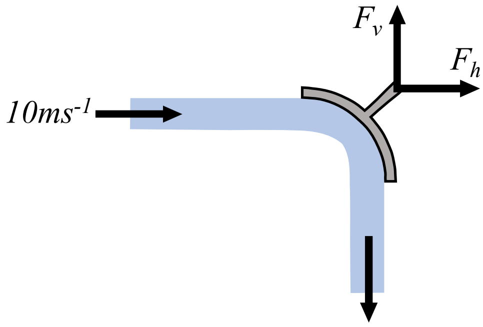


Figure Q6

Q7 An air flow over a long spinning circular cylinder, with a radius of 0.5*m*, can be modelled by a horizontal onset flow of 10 ms^{-1} a doublet of strength 5π m^2s^{-1} and a vortex of strength 5π m^2s^{-1}. Find the streamwise velocity, u, at the top of the cylinder assuming potential flow.

(3 marks)

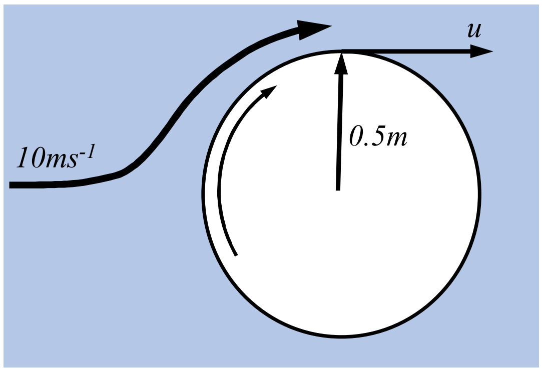


Figure Q7

turn over…

SECTION 2
Answer *two* questions in this section

Q8 Water is siphoned from a large tank (as shown in figure Q8) using a pipe with a constant internal cross sectional area. The highest point of the pipe is h above the water level in the tank, while the exit of the pipe is at a height H below the water level in the tank.

(a) Show that the water exit velocity, v, when the exit of the pipe is H below the water surface is

$$v = \sqrt{2gH}$$

and the maximum height of the siphon is given by

$$h_{\max} = \frac{(p_a - p_v)}{\rho g} - H$$

where p_a and p_v are the atmospheric and water vapour pressure respectively.

(10 marks)

(b) For the siphon system defined above, find the velocity and cross sectional area of the jet $4m$ above the exit of the siphon if the maximum height of siphon above the water surface is (h=h_{max}=$2m$). Assume that the atmospheric pressure is $1.015 \times 10^5 Pa$, the vapour pressure is $3400 Pa$, the siphon exit has a cross sectional area of $0.001 m^2$ and the density of water is $1000\ kg\ m^{-3}$.

(10 marks)

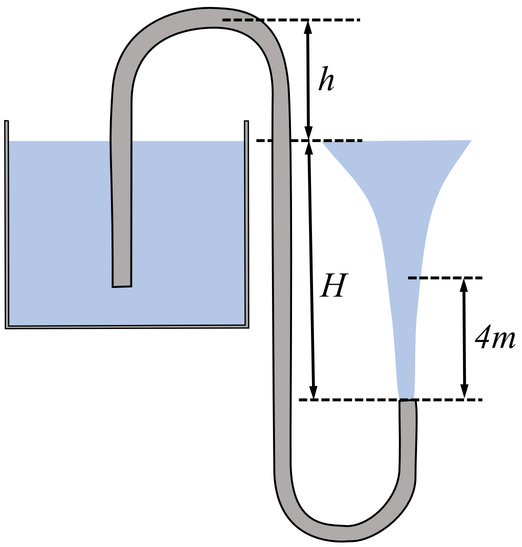


***Figure Q8**: Diagram of siphon system used in question 8.*

Q9 Air flows at V ms^{-1} far upstream of a windmill that sweeps a circular disc of diameter d m. At a point downstream of the windmill, where the pressure has returned to atmospheric, the velocity of the wind is measured at aV ms^{-1}. a is a constant that represents the ratio of downstream to upstream wind speeds.

(a) Use the actuator disc theory for an ideal windmill to show that the force exerted on the windmill can be written as

$$F = \rho \tfrac{\pi}{8} d^2 V^2 \left(a^2 - 1\right) \quad ,$$

where ρ is the density of the air. Clearly state all assumptions made during your derivation.

(8 marks)

(b) Continuing the analysis of the windmill defined above, show that the efficiency is given by

$$\eta = \frac{1 + a - a^2 - a^3}{2} \quad .$$

(6 marks)

(c) The windmill defined above is placed on and provides power to an electric vehicle. The electric motor is assumed 100% efficient and provides a power given by the Drag multiplied by the forward speed. If the drag on the combined windmill and vehicle can be approximated by the force on the windmill, show that the maximum speed v, that the vehicle can maintain into and away from the wind is given by

$$v = \frac{(1+a)}{(1-a)} V \qquad \text{and} \qquad v = \frac{(1+a)}{(3+a)} V$$

respectively.

(6 marks)

turn over…

Q10 (a) Flow over a cylinder is modelled by placing a doublet in a free stream. Find the doublet strength required to produce a cylinder of radius R in a free stream of speed U_∞. Hence find then the velocity and pressure coefficient distributions on the cylinder surface. Then show that the pressure on the cylinder is given by

$$p(\theta) = p_\infty + \frac{1}{2}\rho U_\infty^2 (1 - 4\sin^2\theta)$$

(6 marks)

(b) The pressure around a greenhouse can be modelled using the solution of inviscid incompressible potential flow around a closed semi cylinder, $0 \le \theta \le \pi$, whose radius is R. The greenhouse is fixed to the ground in such a way that the air inside the greenhouse is at rest, with pressure equal to the total pressure of the oncoming flow (see figure Q10). By considering the forces acting on a small element of the upper surface show that the overall lift on the greenhouse per unit length is

$$l = 2\left(p_\infty + \tfrac{1}{2}\rho U_\infty^2\right)R - \int_0^\pi p(\theta) R \sin\theta \, d\theta$$

where U_∞ is the freestream velocity and p_∞ is the freestream static pressure.

(6 marks)

Hence using (a) show that the net lift force per unit length acting on the greenhouse is equal to

$$l = \frac{8}{3}\rho_\infty U_\infty^2 R$$

(8 marks)

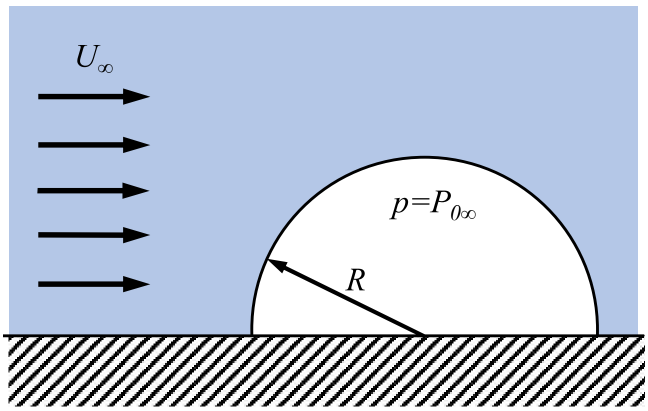


***Figure Q10**: Semi-cylinder greenhouse.*