Examples Sheet StM2 - 2D Elasticity

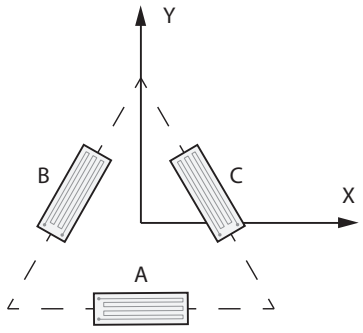
$$\boldsymbol{T} = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$$

Q1: From first principles, derive the stress transformation equations to calculate stresses in an $X'Y'$ coordinate system at a CCW angle θ from the original XY coordinate system.

Q2: For the following stress state, $\sigma_{xx} = 120$ MPa, $\sigma_{yy} = -40$ MPa, $\tau_{xy} = -60$ MPa, calculate (i) the principal stresses and directions, (ii) the maximum/minimum shear stress, and (iii) draw the corresponding Mohr's circle.

Q3: Sketch a Mohr's circle for stress (for an arbitrary stress state), and use it to derive analytical expressions for the principal stresses and directions, and maximum/minimum shear stress.

Q4: The strain on a structure is measured using a delta strain gauge rosette, where strain gauge A is aligned with the structural X-axis; $\varepsilon_A = 50\mu\varepsilon$, $\varepsilon_B = -70\mu\varepsilon$, $\varepsilon_C = 130\mu\varepsilon$. Calculate the strains in the XY coordinate system, and determine the direction of maximum strain.



Q5: Consider a linear-elastic, isotropic material with $E = 70$ GPa and $\nu = 0.3$. For the following stress state, $\sigma_{xx} = -96$ MPa, $\sigma_{yy} = 72$ MPa, $\tau_{xy} = 34$ MPa, calculate the resulting strains. What strain would a strain gauge mounted at 45° to the XY-axes measure?

Q6: From first principles, derive an expression for the bulk modulus K and shear modulus G, in terms of Young's modulus E and Poisson's ratio ν. Discuss what bounds these elastic moduli place on allowable values for Poisson's ratio in linear-elastic, isotropic materials.

Q7: A thin plate is subjected to a uniform strain field of $\varepsilon_{xx} = 1540\mu\varepsilon$, $\varepsilon_{yy} = -320\mu\varepsilon$, $\gamma_{xy} = 632\mu\varepsilon$. The material is Aluminium 6061-T6 with $E = 70$ GPa, $\nu = 0.3$, and a yield stress of $\sigma_Y = 240$ MPa.

i) find the principal stresses and maximum shear stress;

ii) find the orientations of the principal stress and maximum shear stress planes, and verify the results obtained by representing the stress state with the help of Mohr's circle;

iii) using the Tresca and Von Mises failure criteria, verify the strength of the thin plate;

iv) sketch the failure envelopes in principal stress space, indicating the current stress state;