Cycle Details

$\boldsymbol{T_a} = 223.3\ K$	$\frac{\boldsymbol{P_{o2}}}{\boldsymbol{P_{o1}}} = 1$	$\boldsymbol{\eta_{isen}} = 0.9$ (turbine)
$\boldsymbol{a} = 299.5\ m/s$	$\frac{\boldsymbol{P_{o3}}}{\boldsymbol{P_{o2}}} = 8$	$\frac{\boldsymbol{P_{o4}}}{\boldsymbol{P_{o3}}} = 0.96$
$\boldsymbol{C_a} = 239.6\ m/s$	$\boldsymbol{\eta_{isen}} = 0.87$ (compressor)	$\boldsymbol{\eta_{transmission}} = 0.99$
$\boldsymbol{P_a} = 0.265\ bar$	$\boldsymbol{T_{o4}} = 1200K$	

Step 1: Total Temperature & Pressure at entry

We need to find the stagnation properties at the fan:

$$\boldsymbol{T_{o1}} = T_a\left[1 + \frac{1}{2}(\gamma - 1)M^2\right] = 223.3\left[1 + \frac{1}{2}(1.4 - 1)0.8^2\right] = \boldsymbol{251.9\ K}$$

$$\frac{\mathbf{P_{o1}}}{\mathbf{P_a}} = \left(\frac{\mathrm{T}_{o1}}{\mathrm{T}_a}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{251.9}{223.3}\right)^{\frac{1.4}{0.4}} = 1.525 \rightarrow \mathbf{P_{o1}} = 1.525 \cdot 0.265 = \boldsymbol{0.404\ bar}$$

Step 1-2: Intake

There is no loss, so $\boldsymbol{P_{o2}} = P_{o1} = \boldsymbol{0.404\ bar}$ and $\boldsymbol{T_{o2}} = T_{o1} = \boldsymbol{251.9\ K}$

Step 2-3: Compression

$$\frac{\boldsymbol{P_{o3}}}{\boldsymbol{P_{o2}}} = 8 \rightarrow \boldsymbol{P_{o3}} = 8 \cdot 0.404 = \boldsymbol{3.23\ bar}$$

We calculate the isentropic Temperature rise:

$$\frac{\boldsymbol{T'_{o3}}}{\boldsymbol{T_{o2}}} = \left(\frac{\boldsymbol{P_{o3}}}{\boldsymbol{P_{o2}}}\right)^{\frac{\boldsymbol{\gamma-1}}{\boldsymbol{\gamma}}} \rightarrow \boldsymbol{T'_{o3}} = 251.9 \cdot (8)^{\frac{0.4}{1.4}} = \boldsymbol{456.3\ K}$$

The compressor has an isentropic efficiency of $\eta_{isen} = 87\%$:

$$\boldsymbol{T_{o3}} = \frac{T'_{o3} - T_{o2}}{\eta_{isen}} + T_{o2} = \frac{456.3 - 251.9}{0.87} + 251.9 = \boldsymbol{486.84\ K}$$

Specific power required to drive the compressor:

$$\frac{\boldsymbol{P_{ow_{comp}}}}{\boldsymbol{\dot{m}}} = C_p(T_{o3} - T_{o2}) = 1.005 \cdot (486.84 - 251.9) = \boldsymbol{236.1\ kJ/Kg}$$

Step 3-4: Combustion – Heat addition

$\boldsymbol{T_{o4}} = \boldsymbol{1200\ K}$

There is a pressure loss of 4%

$\boldsymbol{P_{o4}} = 0.96 \cdot P_{o3} = 0.96 \cdot 3.23 = \boldsymbol{3.1\ bar}$ **Ste**

Step 4-5: Expansion through turbine

The turbine shaft drives the compressor with a transmission efficiency of 99%:

$$\frac{\boldsymbol{P_{ow_{compressor}}}}{\dot{m}} = \frac{P_{ow_{turbine}}}{\dot{m}} \cdot \eta_{transmission} \rightarrow \frac{\boldsymbol{P_{ow_{turbine}}}}{\dot{\boldsymbol{m}}} = \frac{236.1}{0.99} = \mathbf{238.48}\ \boldsymbol{kJ/kg}$$

$$\frac{\boldsymbol{P_{ow_{turbine}}}}{\dot{m}} = C_{pg} \cdot (-T_{o5} + T_{o4}) = 1.148 \cdot (-T_{o5} + 1200) = \mathbf{238.48}\ \boldsymbol{kJ/kg}$$

So then $\boldsymbol{T_{o5}} = \mathbf{992.26}\ \boldsymbol{K}$

(note: the fuel mass flow $\dot{m}_f$ is neglected)

The turbine has an isentropic efficiency of η_{isen} = 90%:

$$\boldsymbol{T'_{o5}} = -\frac{T_{o4} - T_{o5}}{\eta_{isen}} + T_{o4} = 1200 - \frac{1200 - 992.26}{0.9} = \mathbf{969.18}\ \boldsymbol{K}$$

$$\frac{\mathrm{P}_{o5}}{\mathrm{P}_{o4}} = \left(\frac{T'_{o5}}{\mathrm{T}_{o4}}\right)^{\frac{\gamma g}{\gamma g - 1}} = \left(\frac{969.18}{1200}\right)^{\frac{1.333}{0.333}} = 0.425 \rightarrow \mathbf{P}_{\boldsymbol{o5}} = 0.425 \cdot 3.1 = \mathbf{1.32}\ \boldsymbol{bar}$$

Step 5A: Fully expanded in the ideal Con-Di Nozzle (ideal thrust)

The exhaust gas in this case is fully expanded in an ideal "Convergent/Divergent" Nozzle, such that at some point downstream the static pressure of the jet is the same as that of the ambient air. There are no losses in the duct or nozzle:

$$\boldsymbol{T_{o5}} = \boldsymbol{T_{oN}}$$
$$\boldsymbol{P_{o5}} = \boldsymbol{P_{oN}}$$

The nozzle pressure ratio is: $\frac{\boldsymbol{P_{oN}}}{\boldsymbol{P_a}} = \frac{1.32}{0.265} = \mathbf{4.98}$

So the Temperature of the fully expanded jet will be: $\boldsymbol{T_{FE}} = \frac{T_{oN}}{\left(\frac{P_{on}}{Pa}\right)^{\frac{\gamma g - 1}{\gamma g}}} = \frac{992.26}{4.98^{1/4}} = \mathbf{664.23}\ \boldsymbol{K}$

Using $T_o = T + \frac{C_{FE}^2}{2C_P}$, we know then that

$$\boldsymbol{C_{FE}} = \sqrt{2C_{P_g} \cdot (T_o - T)} = \sqrt{2 \cdot 1148 \cdot (992.26 - 664.23)} = \mathbf{867.85}\ \boldsymbol{m/s}$$

That makes Specific Thrust $\frac{\boldsymbol{F}}{\dot{\boldsymbol{m}}} = (C_{FE} - C_a) = (867.85 - 239.6) = \boxed{\mathbf{628.25}\ \boldsymbol{Ns/kg}}$

(or m/s)

Step 5B: Expansion in a Convergent Nozzle

The exhaust gas in this case is expanded in a convergent nozzle with no losses. We need to find out if the nozzle is choked or not, since that will change the jet velocity.

$\boldsymbol{T_{o5} = T_{oN}}$
$\boldsymbol{P_{o5} = P_{oN}}$

The nozzle pressure ratio is: $\frac{\boldsymbol{P_{oN}}}{\boldsymbol{P_a}} = \frac{1.32}{0.265} = \mathbf{4.98}$

Checking if the nozzle is chocked (i.e. $\frac{\mathbf{P}_{\boldsymbol{oN}}}{\mathbf{P}_{\boldsymbol{a}}} > \frac{\mathbf{P}_{\boldsymbol{oN}}}{\mathbf{P}_{\boldsymbol{N^*}}}$)
By using $\boldsymbol{T_o = T\left[1 + \frac{1}{2}(\gamma - 1)M^2\right]}$ and setting M = 1, we can find:
$\frac{\mathbf{T}_{\boldsymbol{oN}}}{\mathbf{T}_{\boldsymbol{N^*}}} = 1 + \frac{1}{2}\left(\gamma_g - 1\right) = \frac{2.333}{2} = \mathbf{1.167}$

So then, $\mathbf{T}_{\boldsymbol{N^*}} = \frac{992.26}{1.167} = \mathbf{850.27}\ \boldsymbol{K}$
$\frac{\mathbf{P}_{\boldsymbol{oN}}}{\mathbf{P}_{\boldsymbol{N^*}}} = \left(\frac{\mathrm{T}_{oN}}{\mathrm{T}_{N^*}}\right)^{\frac{\gamma_g}{\gamma_g - 1}} = 1.167^4 = \mathbf{1.85}$
$\mathbf{P}_{\boldsymbol{N^*}} = \frac{\mathrm{P}_{oN}}{1.85} = \mathbf{0.714}\ \boldsymbol{bar}$

$\frac{\mathbf{P}_{\boldsymbol{oN}}}{\mathbf{P}_{\boldsymbol{a}}} > \frac{\mathbf{P}_{\boldsymbol{oN}}}{\mathbf{P}_{\boldsymbol{N^*}}}$ so nozzle is choked.

Speed of sound in the nozzle: $\mathbf{C}_{\boldsymbol{N^*}} = \sqrt{\gamma_g R \mathrm{T}_{N^*}} = \sqrt{1.333 \cdot 287 \cdot 850.27} = \mathbf{570.35}\ \boldsymbol{m/s}$

Using *P = ρRT,* the density at the nozzle is:
$\boldsymbol{\rho_{N^*}} = \frac{\mathrm{P}_{N^*}}{R\mathrm{T}_{N^*}} = \frac{0.714x10^5}{287 \cdot 850.27} = \mathbf{0.293}\ \boldsymbol{kg/m^3}$
(Remember to use pressure SI units for this formula)

$\dot{m} = \rho_{N^*} A_N C_{N^*} \rightarrow \frac{\boldsymbol{A_N}}{\dot{\boldsymbol{m}}} = \frac{1}{0.293 \cdot 570.35} = \mathbf{0.006}\ \boldsymbol{m^2/kg \cdot s^{-1}}$

<u>Specific Thrust calculation</u>:
$\boldsymbol{ST} = \frac{F}{\dot{m}} = (C_{N^*} - C_a) + \frac{A_N}{\dot{m}}(\mathrm{P}_{N^*} - \mathrm{P}_a) = (570.35 - 239.6) + 0.006 \cdot (0.714 - 0.265)x10^5$
$\boldsymbol{ST} = \boxed{\mathbf{600.15}\ \boldsymbol{Ns/kg}}$

Specific Fuel Consumption:

$T_{o3} = 486.84\ K$
$T_{o4} = 1200\ K$
So Combustion Temperature Rise:
$\Delta \boldsymbol{T_o} = 1200 - 486.84 = \mathbf{713.2\ K}$

From the Chart of Combustion Temperature Rise vs Fuel Air Ratio we can find that:
FAR = 0.0194

Knowing that $\dot{m}_f = FAR \cdot \dot{m}$

$$\boldsymbol{SFC} = \frac{\dot{m}_f}{F} = \frac{FAR}{\frac{F}{\dot{m}}} = \frac{0.0194}{600.15} = 3.23x10^{-5} kg/s/N \rightarrow \boxed{\mathbf{0.116\ kg/hr/N}}$$

Overall Efficieny $= \frac{\boldsymbol{C_a}}{\boldsymbol{SFC \cdot Qnet}} = \frac{239.6}{0.116 \cdot 43100} \cdot \frac{3600}{1000} = 0.173 \longrightarrow \boxed{\mathbf{17.3}\%}$