UNIVERSITY OF BRISTOL
FACULTY OF ENGINEERING

First Year Examination for the Degrees of
Bachelor and Master of Engineering

MAY/JUNE 2016 2 Hours

FLUIDS 1

AENG11101

This paper contains *two* sections

SECTION 1
Answer *all* questions in this section
This section carries *20 marks*.

SECTION 2
This section has *three* questions.
Answer *two* questions.
All questions in this section carry *20 marks* each.
The maximum for this paper is *60 marks.*

Calculators may be used.

For air, assume R = 287 J/kgK. Take 0°C as 273 K.
Use a gravitational acceleration of $9.81 m/s^2$
1 bar = 10^5 N/m^2

TURN OVER ONLY WHEN TOLD TO START WRITING
CALCULATORS MUST HAVE THE FACULTY OF ENGINEERING SEAL OF APPROVAL

Useful Equations

The volume of a sphere: $\frac{4}{3}\pi r^3$ Area of a circle: πr^2

Roots of a quadratic: $ax^2 + bx + c = 0 \quad \rightarrow \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The equation of state for a perfect gas is:

$$p = \rho RT$$

Drag equation: $\text{Drag} = \text{Area} \times C_D \times \frac{1}{2}\rho V^2$

Equation of a streamline in 2D flow

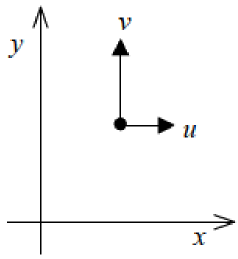
$$\frac{dy}{dx} = \frac{v}{u}$$

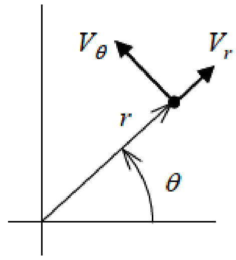
Change between Polar and Cartesian coordinate systems

$$r = \sqrt{x^2 + y^2}, \quad \theta = \tan^{-1}\left(\frac{y}{x}\right), \quad x = r\cos\theta, \quad y = r\sin\theta,$$

$$u = V_r \cos\theta - V_\theta \sin\theta, \qquad v = V_r \sin\theta + V_\theta \cos\theta$$

$$V_r = u\cos\theta + v\sin\theta, \qquad V_\theta = -u\sin\theta + v\cos\theta$$





2D Potential Flow

Velocity components in two-dimensional irrotational flow, in terms of the stream function and potential are given by

$$\underbrace{V_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} \qquad V_\theta = -\frac{\partial\psi}{\partial r} \qquad V_r = \frac{\partial\phi}{\partial r} \qquad V_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta}}_{\text{polar coordinates}}$$

$$\underbrace{u = \frac{\partial\psi}{\partial y} \qquad v = -\frac{\partial\psi}{\partial x} \qquad u = \frac{\partial\phi}{\partial x} \qquad v = \frac{\partial\phi}{\partial y}}_{\text{Cartesian coordinates}}$$

The stream function & velocity potential in Polar coordinates and the velocity distribution for

i) A uniform flow U_∞ parallel to the x axis:

$$\psi = U_\infty r\sin\theta, \qquad \phi = U_\infty r\cos\theta, \qquad V_r = U_\infty \cos\theta, \quad V_\theta = -U_\infty \sin\theta, \qquad u = U_\infty, \quad v = 0$$

ii) A source, of strength Λ at the origin:

$$\psi = \frac{+\Lambda\theta}{2\pi}, \quad \phi = \frac{+\Lambda}{2\pi}\ln r, \qquad V_r = \frac{+\Lambda}{2\pi r}, \quad V_\theta = 0, \qquad u = \frac{+\Lambda}{2\pi}\frac{x}{(x^2 + y^2)}, \quad v = \frac{+\Lambda}{2\pi}\frac{y}{(x^2 + y^2)}$$

iii) A doublet, of strength κ at the origin:

$$\psi = \frac{-\kappa}{2\pi}\frac{\sin\theta}{r}, \qquad \phi = \frac{+\kappa}{2\pi}\frac{\cos\theta}{r}, \qquad V_r = \frac{-\kappa}{2\pi r^2}\cos\theta, \qquad V_\theta = \frac{-\kappa}{2\pi r^2}\sin\theta,$$

$$u = \frac{-\kappa}{2\pi}\frac{\left(x^2 - y^2\right)}{\left(x^2 + y^2\right)^2}, \qquad v = \frac{-\kappa}{2\pi}\frac{2xy}{\left(x^2 + y^2\right)^2}$$

iv) A vortex, of strength Γ, at the origin:

$$\psi = \frac{+\Gamma}{2\pi}\ln r, \quad \phi = \frac{-\Gamma}{2\pi}\theta, \qquad V_r = 0, \quad V_\theta = \frac{-\Gamma}{2\pi r}, \qquad u = \frac{+\Gamma}{2\pi}\frac{y}{\left(x^2 + y^2\right)}, \quad v = \frac{-\Gamma}{2\pi}\frac{x}{\left(x^2 + y^2\right)}$$

Useful integrals

$$\int \sin^2(ax)dx = \frac{x}{2} - \frac{\sin 2ax}{4a} + C$$

$$\int \cos^2(ax)dx = \frac{x}{2} + \frac{\sin 2ax}{4a} + C$$

turn over …

SECTION 1
Answer <u>all</u> questions in this section

Q1 A pressure transducer records a gauge pressure of 2bar when lowered into a lake. How far below the surface is the pressure transducer if the atmospheric pressure at the surface is 1.023bar and the density of the water is 1000 kg m^{-3} ?

(2 marks)

Q2 A dam has a rectangular sluice gate 4m high and 6m wide. The gate is closed and angled at 30^o to the vertical. If the water (shaded on the left in the diagram) just reaches the top of the gate, find the vertical and horizontal components of thrust on the gate.
Assume the water has a density of 1000 $kg\ m^{-3}$

(3 marks)

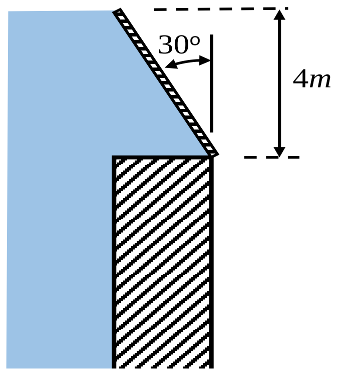


Figure Q2

Q3 State the assumptions that must be made for Bernoulli's equation to be valid.

(3 marks)

Q4 **a)** In wind tunnel testing of steady air flows around bodies such as cars and aircraft, what three non-dimensional parameters must be matched?
b) Which parameter in a) may be neglected in low speed flows & why?

(3 marks)

Q5 Water flows through a smooth pipe that turns from horizontal to vertical. The exit of the pipe is 1m above the inlet. The exit and inlet velocities are 2ms^{-1} and 4ms^{-1} respectively. Given that the inlet pressure is 2 *bar*, find the exit pressure
Assume the water has a density of 1000 $kg\ m^{-3}$.

(3 marks)

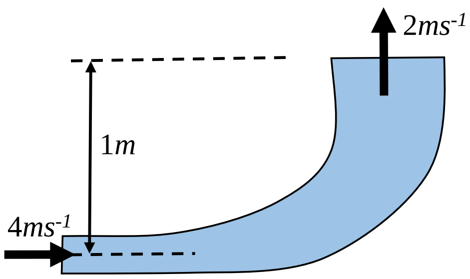


Figure Q5

Q6 A horizontal circular water jet of diameter 20cm and speed 20m/s hits a flat turning vane that smoothly turns the water through 180°. By using a suitable control volume, find the horizontal force on the turning vane if it is moving away from the jet at a constant speed of 3m/s. Assume the water has a density of 1000 kg m^{-3}.

(4 marks)

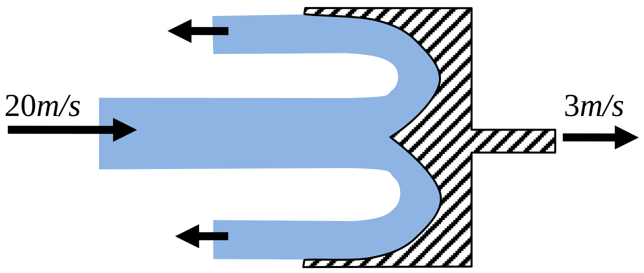


Figure Q6

Q7 How is the potential flow over an oval modelled?

(2 marks)

turn over…

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SECTION 2
Answer *two* questions in this section

Q8 Figure Q8a below shows a low speed, open section wind tunnel. The air is drawn from static atmospheric conditions, through a smooth contraction designed to eliminate total pressure losses, into a parallel working section of area A_w. The air in the working section has a uniform velocity of V_w, The air then passes over the fan, where the area remains fixed before exiting to atmospheric conditions through an expansion and straight section with an exit area of A_e.

(a) Find the differential height of manometer fluid, Δh, in terms of the air velocity V_w, air density ρ_a, the manometer fluid density ρ_m and the acceleration due to gravity g.
State all the assumptions you have made.

(8 marks)

(b) Given that the pressure at the exit, downstream of the fan, is atmospheric ($p=p_a$); derive an expression for the change in static pressure across the fan, Δp_f, in terms of only: A_w, A_e, ρ_a & V_w.

(8 marks)

(c) If the exit area is reduced to A_{e2} (as shown in figure Q8b below), but the volume flow rate through the fan is unchanged, show that the original pressure difference across the fan divided by the new pressure difference across the fan is given by

$$\left(\frac{A_{e2}}{A_e}\right)^2$$

(4 marks)

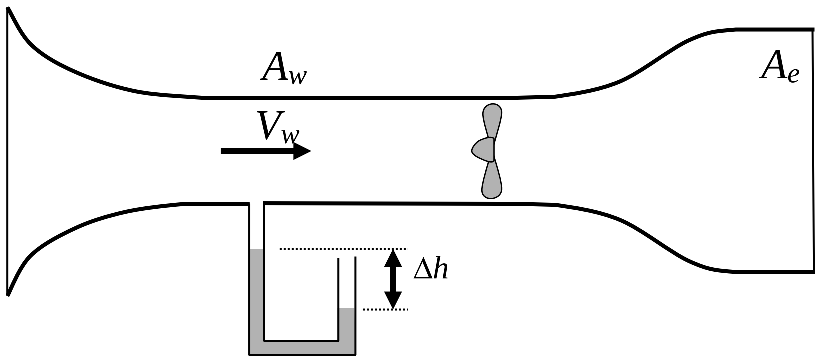

Figure Q8a: Schematic diagram of wind tunnel

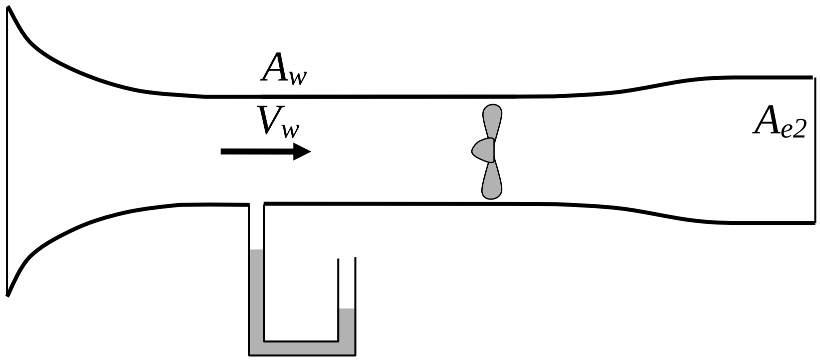

Figure Q8b: Schematic diagram of the same wind tunnel with reduced exit area

Q9 A propeller-driven aircraft flies horizontally, at a speed $V\ ms^{-1}$ relative to the ground, into a headwind of speed $v\ ms^{-1}$. The propeller sweeps out a circular disc of area A while far upstream the streamtube that just encloses the propeller disc has an area of $(a+1)A$ (where the constant a is an inflow factor).

(a) Use the actuator disc theory for an ideal propeller to show from first principles that the force supplied by the propeller can be written as.

$$F = \tfrac{1}{2}\rho A\left(V_4^2 - (V+v)^2\right)$$

where ρ is the density of the air and V_4 is the downstream velocity of the air relative to the disc. Clearly state all assumptions made during your derivation.

(6 marks)

(b) Further, show that the force can be rewritten as

$$F = 2\rho A(V+v)^2 a(a+1)$$

and that the efficiency is given by

$$\eta = \frac{1}{(1+a)}$$

(9 marks)

(c) A light aircraft is being designed to fly at 324 *km/hr*. This requires the propeller to generate a force of 9000N. Find the diameter for the ideal propeller required for this aircraft assuming the air density is given by 1.21 $kg\ m^{-3}$ and the inflow factor is 0.21. Further, find the power required to drive the propeller.

(5 marks)

turn over…

Q10 (a) Using Bernoulli's equation for potential flow, derive an expression for the pressure coefficient in terms of the velocity.

(2 marks)

(b) The flow over a cylinder moving at a speed U_∞ into still air can be modelled using a free stream and a doublet via which transformation? Further, determine the strength of the doublet in terms of the cylinder radius, *R*, and velocity U_∞. Hence show that the pressure variation (with angle θ as shown in figure Q10) on the cylinder is given by

$$p(\theta) = p_\infty + \frac{1}{2}\rho U_\infty^2 (1 - 4\sin^2\theta)$$

(9 marks)

(c) The cylinder, modelled with the doublet strength found from the free stream analysis in (b), moves so that its centre is always a distance *h* above a solid surface. See figure Q10 below. Assuming inviscid potential flow, show that the pressure coefficient at point A (where A is the point on the surface nearest the cylinder as shown in figure Q10) is given by

$$C_p = -4\frac{R^2}{h^2}\left(1 + \frac{R^2}{h^2}\right)$$

(9 marks)

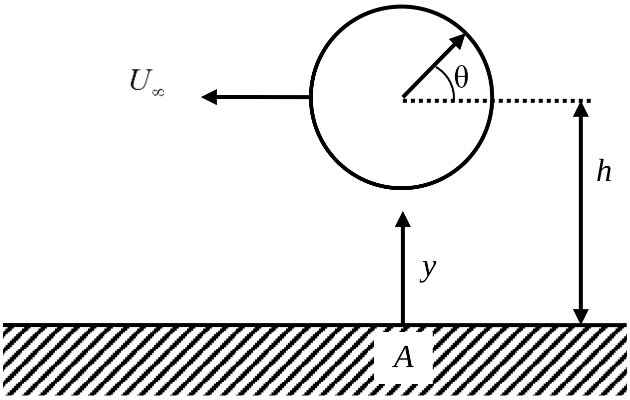

Figure Q10: Cylinder in a freestream with the centre a distance h from a solid surface.

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