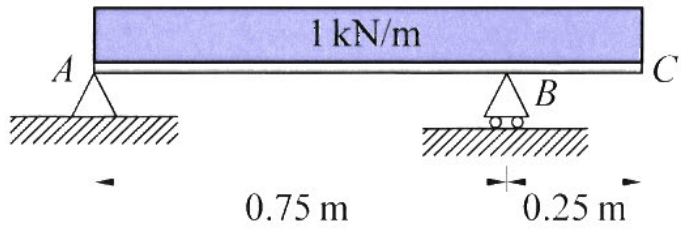
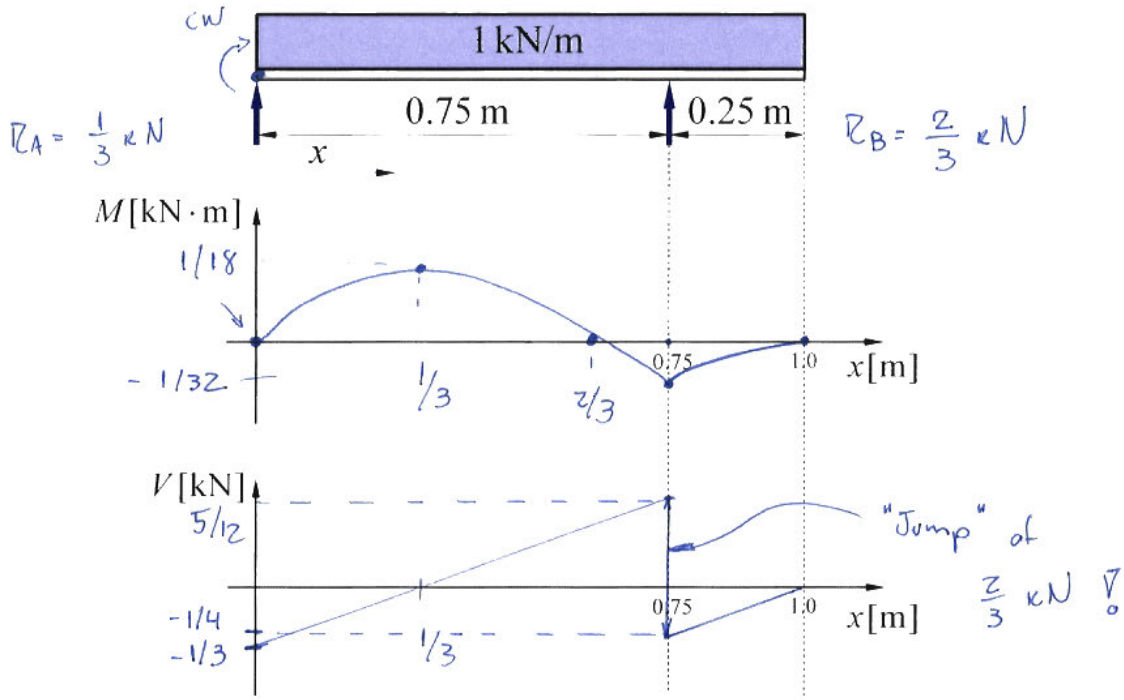
Example 2.3 – Plot the bending moment and shear force diagrams for the following simply-supported beam:





Reactions: $\Sigma F = 0$

$$R_A + R_B - \left(1 \frac{kN}{m}\right)(1\,m) = 0$$

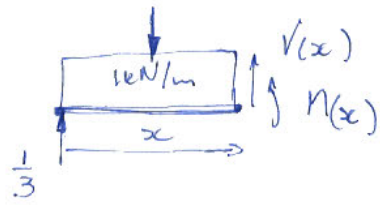
$$\Sigma M^{ccw}_{@A} = 0$$

$$-(0.75\,m)R_B + \left(1 \frac{kN}{m}\right)(1\,m)\left(\frac{1}{2}\,m\right) = 0$$

$$R_B = \frac{2}{3}\,kN$$

$$R_A = \frac{1}{3}\,kN$$

Bending moment : section between A - B :



$$\Sigma M^{ccw}_{@x} = 0$$

$$M(x) + \left(1 \frac{kN}{m}\right)(x)\left(\frac{x}{2}\right) - \frac{1}{3}x = 0$$

$$M(x) = -\frac{1}{2}x^2 + \frac{1}{3}x$$

span AB

at B: $x = \frac{3}{4}$ m

$$M_B = -\frac{1}{2}\left(\frac{3}{4}\right)^2 + \frac{1}{3}\left(\frac{3}{4}\right)$$

$$\boxed{M_B = -\frac{1}{32}\ kN.m} = -0.03125\ kN.m$$

x-axis intersect: root of polynomial

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{cases} a = -\frac{1}{2} \\ b = \frac{1}{3} \\ c = 0 \end{cases}$$

$$\boxed{x_{M=0} = \frac{2}{3}\ m}$$

Peak Moment: $\frac{dM}{dx} = 0$

$$\frac{dM}{dx} = -x + \frac{1}{3} = 0 \quad \therefore \quad \boxed{x = \frac{1}{3}\ m}$$

$$M_{peak} = -\frac{1}{2}\left(\frac{1}{3}\right)^2 + \frac{1}{3}\left(\frac{1}{3}\right)$$

$$\boxed{M_{peak} = \frac{1}{18}\ kN\ m}$$

Shear force: $\Sigma F_y = 0$

$$V_{(x)} + \frac{1}{3} - \left(1 \frac{kN}{m}\right) x = 0$$

$$\boxed{V_{(x)} = x - \frac{1}{3}}$$

$$V_B^- = \frac{3}{4} - \frac{1}{3}$$

$$\boxed{V_B^- = \frac{5}{12} \text{ kN}}$$