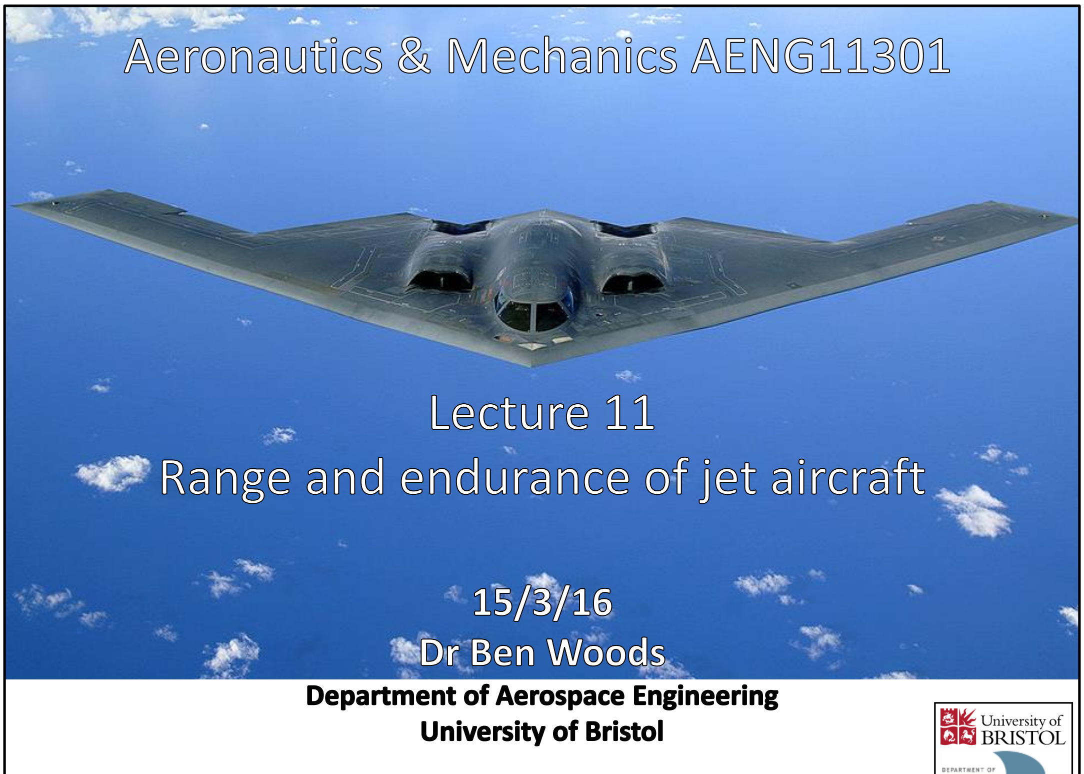


Northrop Grumman B-2 Spirit

with a range of more than 6,000 nautical miles (11,000 km) unrefuelled and over 10,000 nautical miles (19,000 km) with one refueling
B-2s flew non-stop to Kosovo from their home base in Missouri and back.
With aerial refueling support, the B-2 flew one of its longest missions to date from Whiteman Air Force Base, Missouri to Afghanistan and back.
The B-2s flew directly from the U.S. mainland across the Atlantic Ocean to Libya; a B-2 was refueled by allied tanker aircraft four times during each round trip mission.

Mid air refuelling video B2
http://www.youtube.com/watch?v=UuCKO9PKhA4

Tornado refuelling
http://www.youtube.com/watch?v=RtUn2RfV-gY
Better video
http://www.youtube.com/watch?v=Ai8SZQWe5tU

Outline for today

- Definitions of endurance and range
- Breguet Range Equation – Jet aircraft
  - Endurance
  - Cruise climb range
  - Constant altitude range
  - Maximum range for cruise climb
  - Maximum range for constant altitude

Aims for today

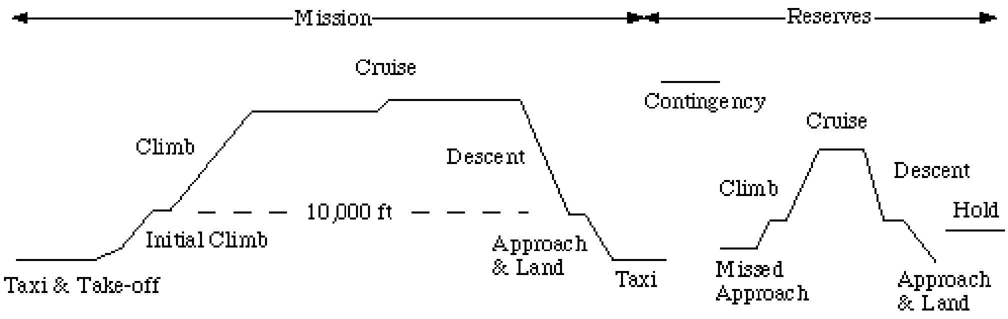
- Be able to define endurance and range
- Be able to define variables held constant in cruise climb vs constant altitude cruise and to be able to comment on the different assumptions involved
- For a jet aircraft be able to calculate:
  - maximum endurance
  - maximum range for cruise climb
  - maximum range for constant altitude flight

Range and Endurance

- **endurance** - the time an aircraft can remain in flight
- **range** – the horizontal distance that an aircraft can cover

1. **Safe Range** – the maximum distance between two airfields for which an aircraft can fly a safe and reliably regular service with a specified payload
   – rather lengthy calculation of full mission profile (takeoff/climb/cruise/descent/landing, headwinds, diversion allowance etc)
   – therefore simplified measures of range used in project work **Still Air Range** (SAR)
   – take-off with full fuel, climb to cruise altitude, cruise until all fuel expended (!)
2. **Gross Still Air Range** (GSAR)
   – begin at selected altitude with full fuel, cruise until all fuel used
   – approximate factor between GSAR and Safe Range known

Typical Mission Profile



Commercial Transport

Breguet Range Equation – Jet Aircraft

- actually derived by Coffin in the 1920's
  - compact and simple way of calculating GSAR
- **"rate at which fuel is burnt**
  = rate at which aircraft weight is reduced"
- define **thrust specific fuel consumption** (sfc or TSFC) as
  - ***f* = mass of fuel burnt per unit of thrust per second**
  - consistent units are $kg/N.s$
  - but often given in terms of $kg/N.hr$ so don't forget to convert!
- for thrust T and weight W the basic Breguet equation is

$$\boxed{\frac{dW}{dt} = -fgT}$$

  - a differential equation (in units of N/s)

Endurance - Integration of Breguet Equation

- rearranging and substituting $T = D$ and $W = L$

$$dt = -\frac{dW}{fgT}$$

$$T = \frac{D}{L}W = \frac{C_D}{C_L}W$$

$$\boxed{dt = -\frac{1}{fg}\frac{C_L}{C_D}\frac{dW}{W}}$$

- assume that C_L/C_D and f remain *constant*
- can then integrate from start weight W_1 to end weight W_2 to obtain the endurance E

using

$$\int \frac{dx}{x} = \ln x$$

$$\boxed{E = t_2 - t_1 = t_{12} = \frac{1}{fg}\frac{C_L}{C_D}\ln\left(\frac{W_1}{W_2}\right)}$$

- maximum endurance at maximum C_L/C_D

$$\boxed{C_{D_{\max E}} = 2C_{D0}\ , \quad C_{L_{\max E}} = \sqrt{C_{D0}/K}}$$

Range – constant airspeed

- increment in distance dS at velocity V is given by

$$\boxed{dS \;=\; Vdt \;=\; -\frac{V}{fg}\frac{C_L}{C_D}\frac{dW}{W}}$$

- assume that *true air speed* V remains *constant*
- can then integrate from start weight W_1 to end weight W_2 to obtain the range R

$$\boxed{R \;=\; S_2 - S_1 \;=\; \frac{V}{fg}\frac{C_L}{C_D}\ln\left(\frac{W_1}{W_2}\right)}$$

Implications of Assumptions (1)

- is it justifiable to assume C_L/C_D, f and V_{TAS} to be constant?
  - only in particular circumstances…
- as fuel is burnt weight W and hence required lift L reduces
- if V_{TAS} held constant then either C_L and/or σ must reduce
- constant $C_L/C_D \rightarrow$ constant C_L (= constant incidence α)
  $\rightarrow$ therefore σ must decrease $\equiv$ **altitude must increase**
- constant C_L/C_D implies constant C_D
  $\rightarrow$ therefore drag D decreases in proportion to σ
- since $T = D$, thrust must also decrease with altitude while constant sfc f implies **constant throttle setting**
  $\rightarrow$ approximately true for turbojet in **stratosphere** (above ~ 11km), where $\boxed{T \approx kT_0\sigma}$

$$\sigma = \left(\frac{\rho}{\rho_o}\right)$$

$$L = W = \tfrac{1}{2}\rho_0 \sigma V^2 S C_L$$

Cruise-Climb

- this cruise case often referred to as a **cruise-climb**
  - usually precluded by air traffic control restrictions!
- note that below 11km (in the **troposphere**) temperature falls with altitude

  $\rightarrow$ thrust falls off less rapidly $\rightarrow$ need to back-off on throttle to maintain V and C_L constant, hence sfc would worsen

maximise cruise speed – ie cruise at high altitude

$$\boxed{R = \frac{V}{fg}\frac{C_L}{C_D}\ln\left(\frac{W_1}{W_2}\right)}$$

a measure of *structural efficiency* – ie minimise fixed weight W_2

a measure of *thermodynamic efficiency* – ie minimise fuel consumption f

a measure of *aerodynamic efficiency* – ie minimise drag C_D

More common notation: commercial aircraft

$$R = \frac{V}{fg}\frac{C_L}{C_D}\ln\left(\frac{W_1}{W_2}\right) = \frac{a}{fg}\left(M\frac{L}{D}\right)\ln\left(\frac{W_1}{W_2}\right)$$

- M(L/D) becomes the important aerodynamic parameter for range
  - Usually, fly at design Mach number
  - Maximum range is at maximum M(L/D)

$$\boxed{R = \frac{a}{g * TSFC}\left(M\frac{L}{D}\right)\ln\frac{W_{initial}}{W_{final}}}$$

a = sound speed
M = Mach number

Range at constant altitude

increment in distance dS at velocity V is given by

$$dS = Vdt = -\frac{V}{fg}\frac{C_L}{C_D}\frac{dW}{W}$$

- assume altitude constant (ρ is constant) and C_L held constant
- substitute velocity equation

$$V = \sqrt{\frac{W}{\frac{1}{2}\rho S C_L}} \Rightarrow dS = -\frac{1}{fg}\sqrt{\frac{2}{\rho S}}\frac{C_L^{1/2}}{C_D}\frac{dW}{W^{1/2}}$$

and integrate

using

$$\int \frac{dx}{x^{1/2}} = 2x^{1/2} \Rightarrow R = S_2 - S_1 = \sqrt{\frac{8}{\rho S}}\frac{1}{fg}\frac{C_L^{1/2}}{C_D}\left(W_1^{1/2} - W_2^{1/2}\right)$$

Implications of Assumptions (2)

- height (hence density ρ) and C_L held constant
- as fuel is burnt weight W and hence required lift L reduces
  $\rightarrow$ V_{TAS} must also reduce
  $\rightarrow$ range is less than for cruise-climb for same C_L/C_D
- since C_D is constant, drag D and hence thrust T also reduce
  $\rightarrow$ therefore throttle setting must be reduced progressively during the cruise
  $\rightarrow$ therefore some variation in sfc f will occur
- need to use average value of f, or treat as a series of shorter steps
  - if each step flown at an increased height an approximation to a cruise-climb profile can be obtained

Maximum Range – Cruise-Climb

- specify start altitude (and hence σ_1) and let thrust vary as required
  $\rightarrow$ sfc f and velocity V constant

$$\boxed{R = \frac{1}{fg}\left(V\frac{C_L}{C_D}\right)\ln\left(\frac{W_1}{W_2}\right)}$$

- for maximum range we still require $V(C_L/C_D)$ to be a maximum
- since σ is not a variable, we can simply substitute the speed equation for V

$$V = \sqrt{\frac{L}{\frac{1}{2}\sigma\rho_0 S C_L}} \quad \Rightarrow \quad \boxed{V\frac{C_L}{C_D} = \sqrt{\frac{W_1}{\frac{1}{2}\sigma_1\rho_0 S}}\,\frac{C_L^{1/2}}{C_D}}$$

$\rightarrow$ therefore need to find minimum $C_D/C_L^{1/2}$

Maximum Range – Cruise-Climb (2)

$$\frac{C_D}{C_L^{1/2}} = \frac{C_{D0} + KC_L^2}{C_L^{1/2}} = \frac{C_{D0}}{C_L^{1/2}} + KC_L^{3/2}$$

$$\Rightarrow \frac{d\left(C_D / C_L^{1/2}\right)}{dC_L} = -\frac{1}{2}\frac{C_{D0}}{C_L^{3/2}} + \frac{3}{2}KC_L^{1/2} =$$

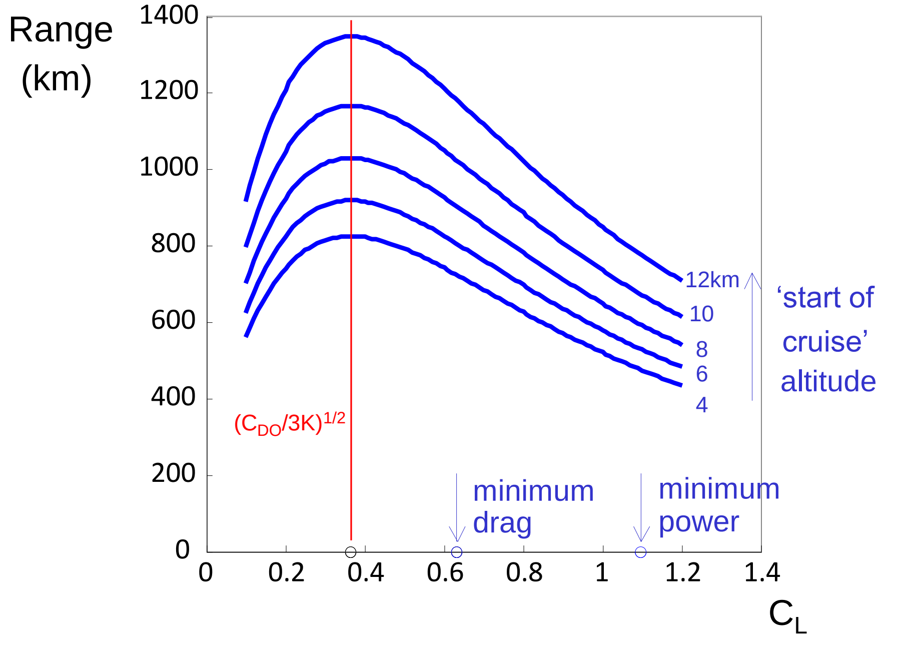
- so at the minimum point $C_{D0} = 3KC_L^2$

$$\Rightarrow \boxed{C_{D_{\max R}} = \frac{4}{3}C_{D0}\,, \quad C_{L_{\max R}} = \sqrt{C_{D0}/3K}}$$

- cruise conditions fixed by start altitude and weight
  - C_L and C_D obtained from above
    $\rightarrow$ substitute with W_1 and σ_1 into $V\dfrac{C_L}{C_D} = \sqrt{\dfrac{W_1}{\frac{1}{2}\sigma_1\rho_0 S}}\dfrac{C_L^{1/2}}{C_D}$
  - then substitute into
    Breguet Equation to find range

K missing in second line of handout

Jet Cruise-Climb Range



Maximum Range – Constant Altitude

- specify cruise altitude (and hence σ) and let thrust vary as required
  - altitude (and hence ρ) are constant

$$\boxed{R = \sqrt{\frac{8}{\rho S}}\frac{1}{fg}\frac{C_L^{1/2}}{C_D}\left(W_1^{1/2} - W_2^{1/2}\right)}$$

- for maximum range require $C_D/C_L^{\ 1/2}$ to be a minimum

  $\rightarrow$ *the same as cruise-climb result !*

The reality about C_L and V

- V is driven by the market and Mach 1 drag rise
  - Some customers want more speed
  - Some customers want better fuel consumption
- C_L is driven by many factors
  - Minimum wing area that meets all requirements
  - Buffet boundary
  - Usually between 0.4 and 0.55 for cruise
- Ideal C_L, C_D relationships are only guides but don't determine cruise C_L or M

http://www.youtube.com/watch?v=Ai8SZQWe5tU

Voyager refuelling an RAF Tornado during receiver clearance trials. (c) QinetiQ   Airbus Military - YouTube

Example

Using the given data for a proposed aircraft design calculate:

1. maximum endurance
2. range if M_{cruise} = 0.45, L/D_{cruise} = 15, a_{cruise} = 308 m/s
3. maximum range if cruise climb from initial altitude of 8 km
4. maximum range if cruise at constant altitude of 8 km with unrestricted thrust

W_1 =100 kN
W_2 = 60 kN
S = 50 m^2
C_{D0} = 0.02
K = 0.05
f = 0.0001 kg/Ns

Answers:
1. 137 minutes
2. 1084 km
3. 1028 km
4. 907 km

Summary – Jet aircraft

- Breguet range equation predicts Gross Still Air Range using assumption "rate fuel burnt = rate aircraft weight is reduced"

$$E = \frac{1}{fg}\frac{C_L}{C_D}\ln\left(\frac{W_1}{W_2}\right)$$

$$R = \frac{V}{fg}\frac{C_L}{C_D}\ln\left(\frac{W_1}{W_2}\right)$$

$$R = \frac{a}{g * TSFC}\left(M\frac{L}{D}\right)\ln\frac{W_{initial}}{W_{final}}$$

- maximum endurance at maximum C_L/C_D
- In cruise climb V_{TAS} and C_L/C_D constant and altitude increases
- In constant altitude cruise, height (hence density ρ) and C_L held constant, and V_{TAS} reduced along with throttle
- max range requires minimum $C_D/C_L^{1/2}$

Be able to define endurance and range

Be able to define variables held constant in cruise climb vs constant altitude cruise and to be able to comment on the different assumptions involved

For a jet aircraft be able to calculate:

maximum endurance

maximum range for cruise climb with fixed and varying throttle settings

maximum range for constant altitude flight with fixed and varying throttle settings

Follow-up materials

To help with exam:

- Introduction to Flight – 6.13

To help with exam:
Introduction to Flight – 5.1-5.2

To aid in understanding:
Understanding flight – Chapter 1

For interest:
Introduction to Flight – 5.19 (explanation of lift)