Example 1.5 – Figure 1 shows a plane, pin-jointed truss structure which is supported at *A* and *B* and carries a vertical load of 40 kN at *F* as shown. All six members have a cross-sectional area of 750 mm^2 and are made of steel with *E*=200 GPa.

a) Calculate the internal forces in all six members.
b) Calculate the horizontal and vertical components of the reactions at *A* and *B*.
c) Find the member with the highest **tensile** stress, state the magnitude of this stress, and calculate the extension (increase in length) of this member due to this stress.
d) Find the member with the highest **compressive** stress, state the magnitude of this stress, and calculate the contraction (reduction in length) of this member due to this stress.

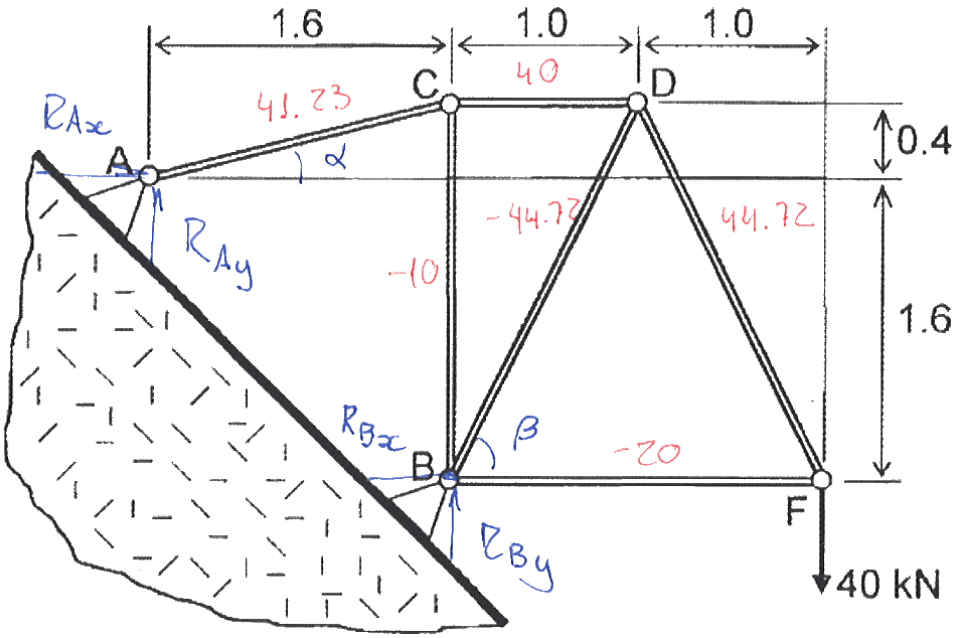
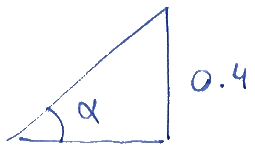


Figure 1: A plane pin-jointed structure (dimensions in metres).

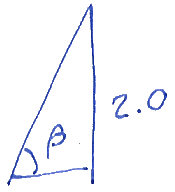
---

a) Internal forces:



$\alpha = \arctan\left(\frac{0.4}{1.6}\right)$ $\quad \cos\alpha \cong 0.970$

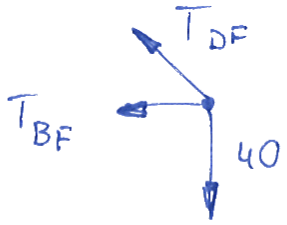
$\alpha \cong 14.04°$ $\quad \sin\alpha \cong 0.242$



$\beta = \arctan\left(\frac{2.0}{1.0}\right)$ $\quad \cos\beta \cong 0.447$

$\beta \cong 63.43°$ $\quad \sin\beta \cong 0.894$

Joint F:



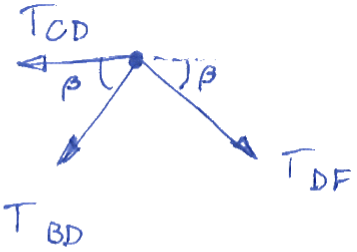
$\Sigma F_y = 0 \therefore \quad (-40) + T_{DF} \sin\beta = 0$

$$T_{DF} = 44.72 \text{ kN}$$

$\Sigma F_x = 0 \therefore \quad -T_{BF} - T_{DF} \cos\beta = 0$

$$T_{BF} = -20 \text{ kN}$$

Joint D:



$\Sigma F_y = 0$

$$(-T_{BD}) \sin\beta - (T_{DF}) \sin\beta = 0$$

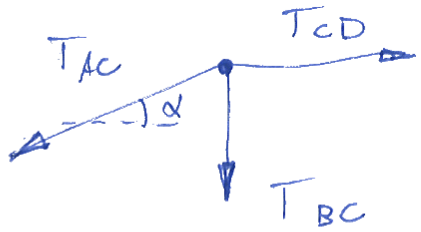
$$T_{BD} = -44.72 \text{ kN}$$

$\Sigma F_x = 0$

$$-T_{CD} - T_{BD} \cos\beta + T_{DF} \cos\beta = 0$$

$$T_{CD} = 40 \text{ kN}$$

Joint C:



$\Sigma F_x = 0 \quad \therefore$

$T_{CD} - T_{AC} \cos\alpha = 0$

$$T_{AC} = 41.23 \text{ kN}$$

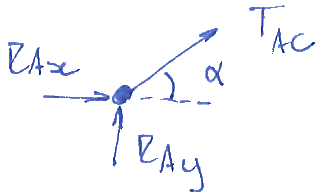
$\Sigma F_y = 0 \quad \therefore$

$- T_{AC} \sin\alpha - T_{BC} = 0$

$$T_{BC} = -10 \text{ kN}$$

b) Reactions:

Joint A:



$\Sigma F_y = 0 \quad \therefore \quad R_{Ay} + T_{AC} \sin\alpha = 0$

$$R_{Ay} = -10 \text{ kN}$$

$\Sigma F_x = 0 \quad \therefore \quad R_{Ax} + T_{AC} \cos\alpha = 0$

$$R_{Ax} = -40 \text{ kN}$$

Global equilibrium:

$$\Sigma F_x = 0 \quad \therefore \quad R_{Ax} + R_{Bx} = 0$$

$$\boxed{R_{Bx} = 40\ kN}$$

$$\Sigma F_y = 0 \quad \therefore \quad R_{Ay} + R_{By} - 40\ kN = 0$$

$$\boxed{R_{By} = 50\ kN}$$

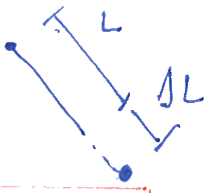
---

c) $T_{DF} = 44.72\ kN$

$$L_{DF} = \sqrt{2^2 + 1^2} = 2.236\ m$$

$$\sigma_{DF} = \frac{44.72 \times 10^3\ N}{750\ mm^2} = \boxed{59.63\ MPa}$$

$$\varepsilon = \frac{\sigma}{E} = \boxed{2.98 \times 10^{-4}}$$



$$\Delta L = \varepsilon . L = 6.67 \times 10^{-4}\ m = \boxed{0.667\ mm}$$

d) $\sigma_{BD} = \boxed{-59.63\ MPa}$

$$\Delta L = \boxed{-0.667\ mm}$$