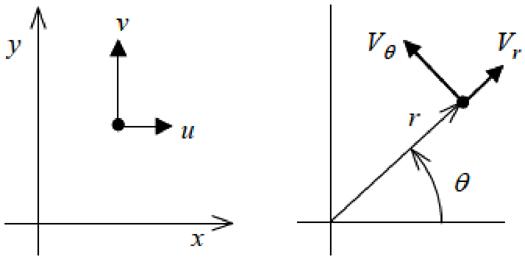
## Useful Equations

*Change between Polar and Cartesian coordinate systems*

$$r = \sqrt{x^2 + y^2}, \quad \theta = \tan^{-1}\left(\frac{y}{x}\right), \quad x = r\cos\theta, \quad y = r\sin\theta,$$

$$u = V_r\cos\theta - V_\theta\sin\theta, \qquad v = V_r\sin\theta + V_\theta\cos\theta$$

$$V_r = u\cos\theta + v\sin\theta, \qquad V_\theta = -u\sin\theta + v\cos\theta$$




*2D Potential Flow*

Velocity components in two-dimensional irrotational flow, in terms of the stream function and potential are given by

$$\underbrace{V_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} \qquad V_\theta = -\frac{\partial\psi}{\partial r} \qquad V_r = \frac{\partial\phi}{\partial r} \qquad V_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta}}_{\text{polar coordinates}}$$

$$\underbrace{u = \frac{\partial\psi}{\partial y} \qquad v = -\frac{\partial\psi}{\partial x} \qquad u = \frac{\partial\phi}{\partial x} \qquad v = \frac{\partial\phi}{\partial y}}_{\text{Cartesian coordinates}}$$

The stream function & velocity potential in Polar coordinates and the velocity distribution for:

i) A uniform flow $U_\infty$ parallel to the $x$ axis:

$$\psi = U_\infty r\sin\theta, \qquad \phi = U_\infty r\cos\theta, \qquad V_r = U_\infty\cos\theta, \quad V_\theta = -U_\infty\sin\theta, \qquad u = U_\infty, \quad v = 0$$

ii) A source, of strength $\Lambda$, at the origin:

$$\psi = \frac{+\Lambda\theta}{2\pi}, \qquad \phi = \frac{+\Lambda}{2\pi}\ln r, \qquad V_r = \frac{+\Lambda}{2\pi r}, \quad V_\theta = 0, \qquad u = \frac{+\Lambda}{2\pi}\frac{x}{(x^2+y^2)}, \quad v = \frac{+\Lambda}{2\pi}\frac{y}{(x^2+y^2)}$$

iii) A doublet, of strength $\kappa$, at the origin:

$$\psi = \frac{-\kappa}{2\pi}\frac{\sin\theta}{r}, \qquad \phi = \frac{+\kappa}{2\pi}\frac{\cos\theta}{r}, \qquad V_r = \frac{-\kappa}{2\pi r^2}\cos\theta, \qquad V_\theta = \frac{-\kappa}{2\pi r^2}\sin\theta,$$

$$u = \frac{-\kappa}{2\pi}\frac{(x^2-y^2)}{(x^2+y^2)^2}, \qquad v = \frac{-\kappa}{2\pi}\frac{2xy}{(x^2+y^2)^2}$$

iv) A vortex, of strength $\Gamma$, at the origin:

$$\psi = \frac{+\Gamma}{2\pi}\ln r, \qquad \phi = \frac{-\Gamma}{2\pi}\theta, \qquad V_r = 0, \quad V_\theta = \frac{-\Gamma}{2\pi r}, \qquad u = \frac{+\Gamma}{2\pi}\frac{y}{(x^2+y^2)}, \quad v = \frac{-\Gamma}{2\pi}\frac{x}{(x^2+y^2)}$$

Using Bernoulli's equation between 1 and 2 for irrotational flow gives: $p_1 - p_2 = \frac{1}{2}\rho\left(V_2^2 - V_1^2\right)$

If point 1 is in the freestream $\quad p_1 - p_\infty = \frac{1}{2}\rho\left(U_\infty^2 - V_1^2\right) \quad \rightarrow \quad c_p = \dfrac{p - p_\infty}{\frac{1}{2}\rho U_\infty^2} = 1 - \dfrac{V^2}{U_\infty^2}$

**Not given in exam: memorise**