

Orbital Mechanics 2:
Reference systems, Keplerian
Elements, types of orbit

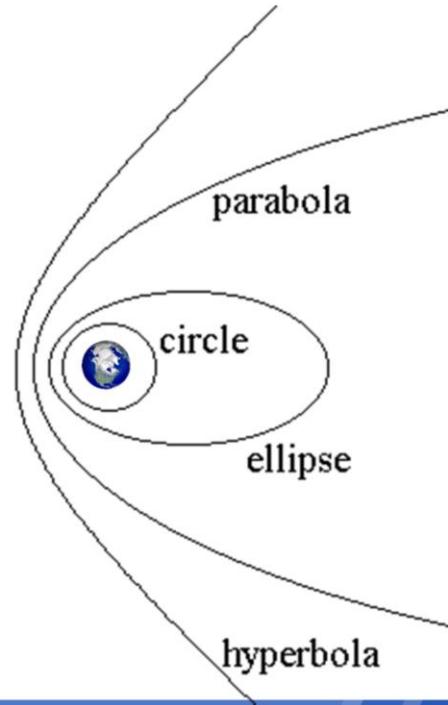
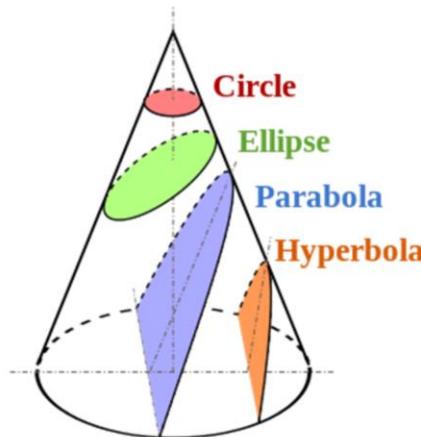
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Remember: Conic Sections



$e=0.00$
circle

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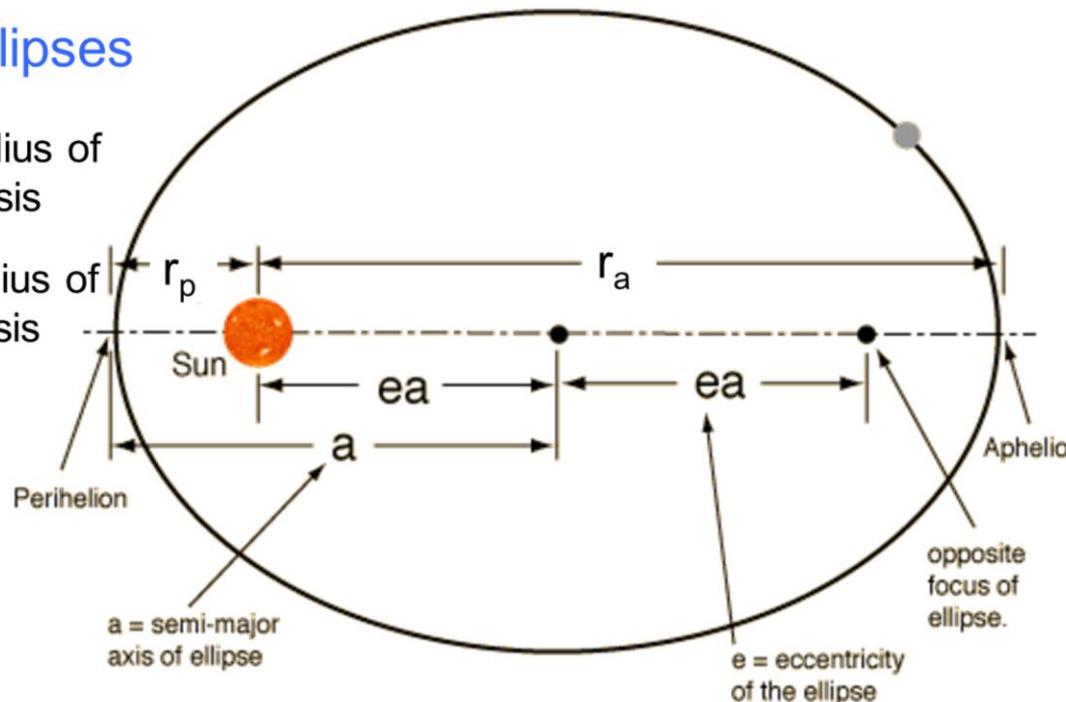
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Remember the influence of eccentricity on shape of the conic section
Circle is a special case of ellipse
Parabola is a special case of hyperbola

Ellipses

r_p : radius of periapsis

r_a : radius of apoapsis



a = semi-major axis of ellipse

e = eccentricity of the ellipse

$$r_a = a(1+e) \quad r_p = a(1- e)$$

Remembering the influence of eccentricity on shape of the conic section.

Vocabulary: peri, ap, semi-major axis.

Learning Outcomes

1. 3 Reference systems
2. 3 planes
3. 6 orbital elements
4. Ground tracks
5. LEO, MEO and GEO
6. Useful types of orbits



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Be able to describe 3 different reference systems ECI, ECEF and TH and the difference between them

Be able to define Vernal equinox, orbital & ecliptic & equatorial planes.

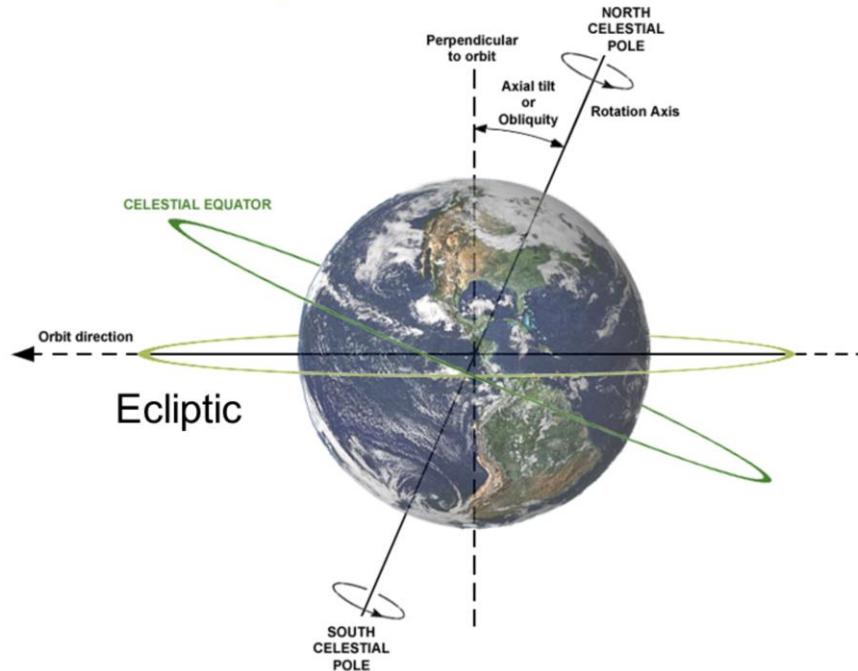
Be able to define Keplerian orbital elements and explain how they are measured

Be able to explain a ground track and how the elements influence it

Be able to explain LEO, MEO and GEO

Be able to define Sun-synchronous and Molniya orbits

Defining a Reference Frame



Working with orbits means we are working in 3D space. Depending on our starting point we might use a different reference system. The Earth tilts on its axis by 23.5deg. The green line is the Equator, which is not the plane of Earth's orbit around the Sun. This (black line) plane around the Sun is called the 'ecliptic'.

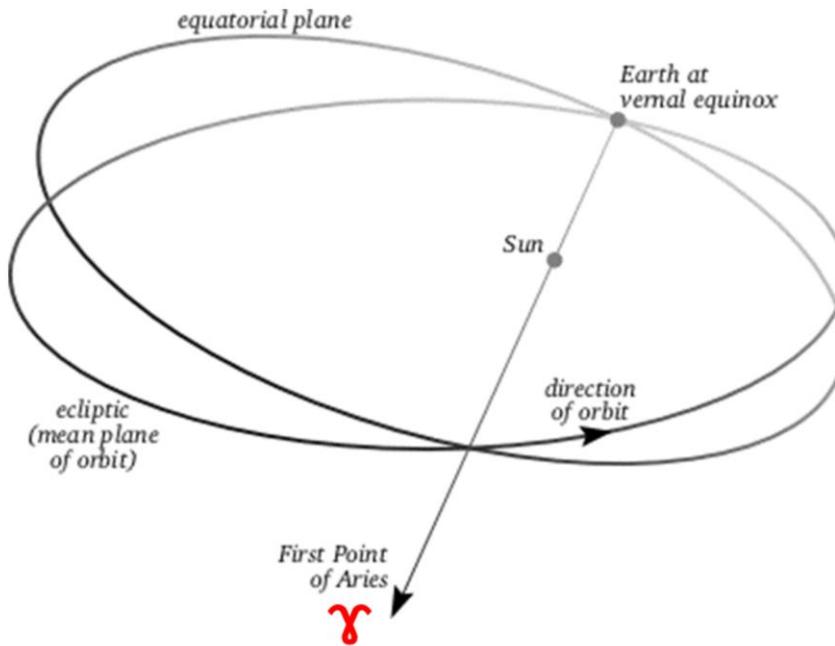
Questions

- Which way round the Sun does the Earth move?
 - a) Clockwise
 - b) Anti-Clockwise
- When would you use the ecliptic and when the equator as your reference plane?

The Earth moves anti-clockwise around the Sun.

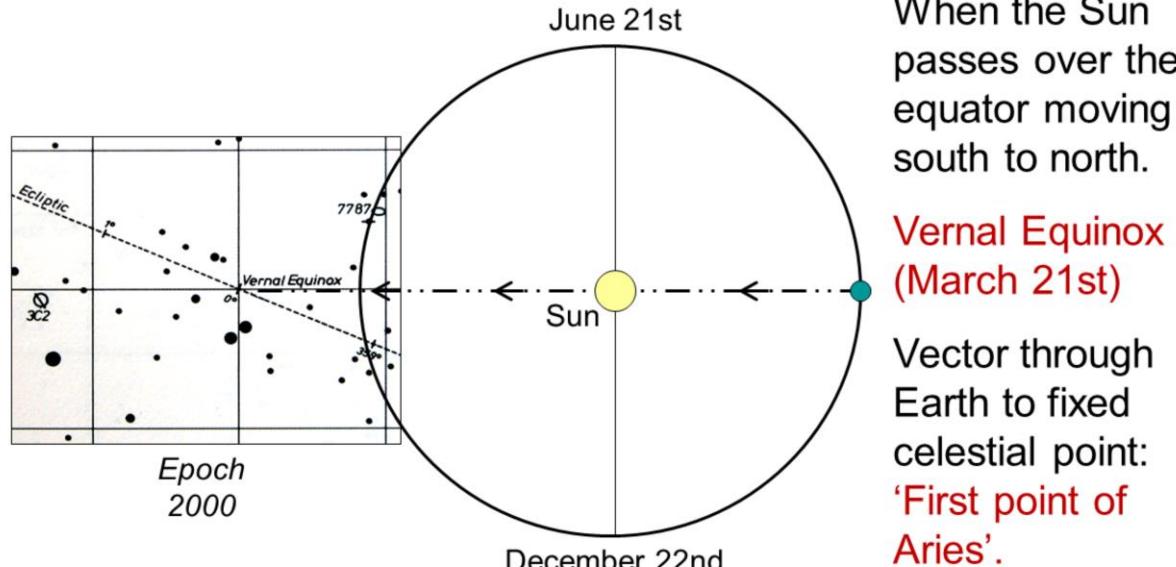
You would use the ecliptic if you were planning an interplanetary journey and the equator if you were in orbit around the Earth.

Defining a Reference Frame



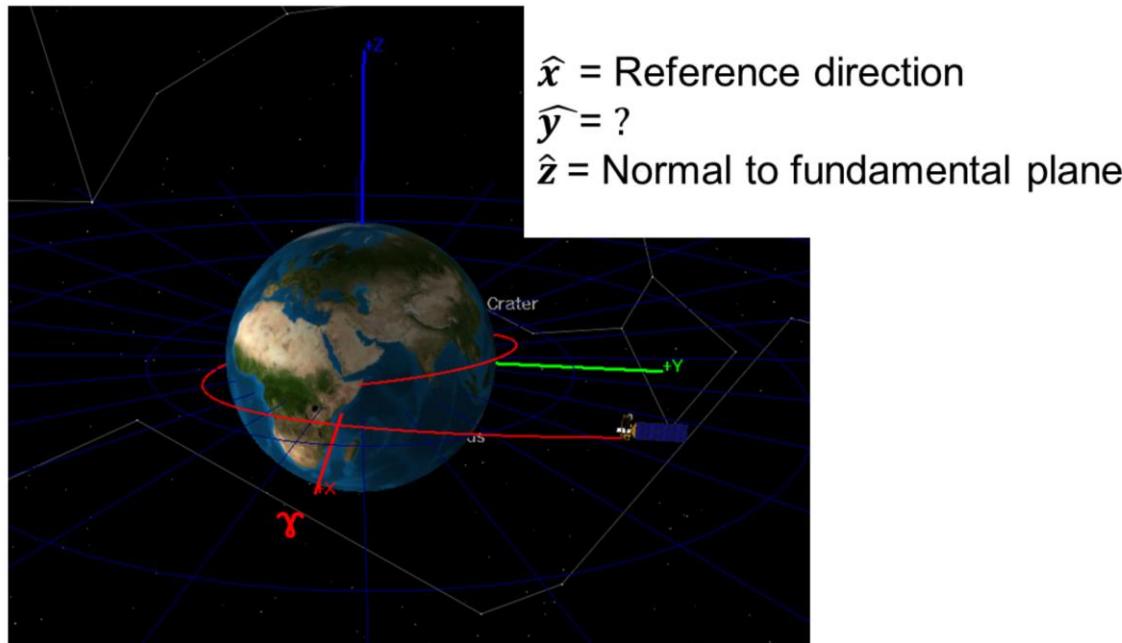
We need to fix a plane wrt to a point, either wrt Earth or to the stars, eg: we can use the prime meridian (Earth) or vernal equinox (stars). The vernal (Spring) equinox is the position of the Sun on March 21st when the Sun crosses the equator going from the Southern to the Northern Hemisphere. The symbol of the ram indicates the First point of Aries (the constellation). This point is where the equatorial plane meets the ecliptic plane. The Vernal equinox and the First point of Aries are in the same direction and are used as a reference direction.

Vernal equinox and first point of aries



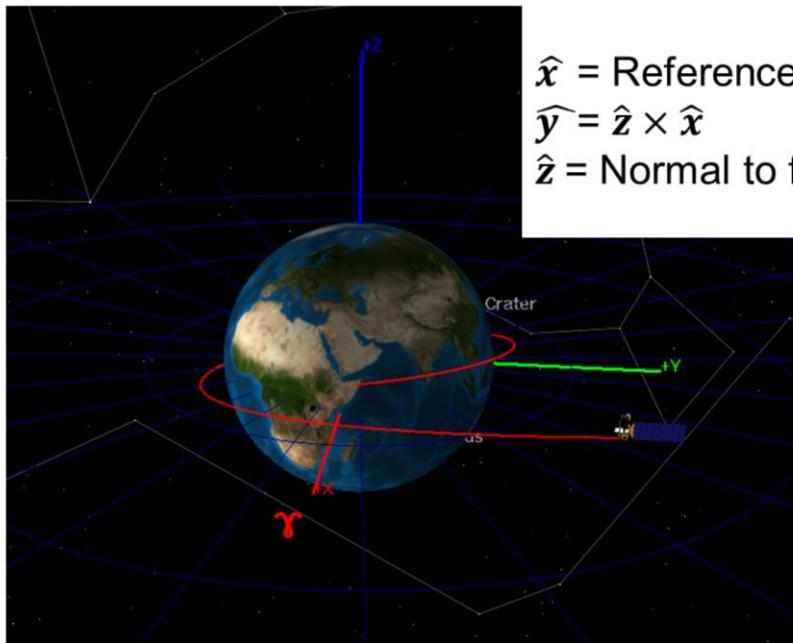
The First Point of Aries is so called because, when Hipparchus defined it in 130 BC, it was located in the eastern extreme of the constellation of Aries. It is currently located within Pisces. The Vernal Equinox drifts $\sim 0.014^\circ$ / year. Orbits are therefore calculated for a specified date and time, (most often Jan 1, 2000, 2050 or today).

Defining a Reference Frame



X is in a reference direction such as first point of aries, z is normal to fundamental plane. The fundamental plane is either the ecliptic or the equatorial plane. Which direction will z be for the equatorial plane? Towards the N pole. Which direction is y? Use RH rule.

Defining a Reference Frame



\hat{x} = Reference direction

$\hat{y} = \hat{z} \times \hat{x}$

\hat{z} = Normal to fundamental plane

Earth Centered Inertial (ECI)

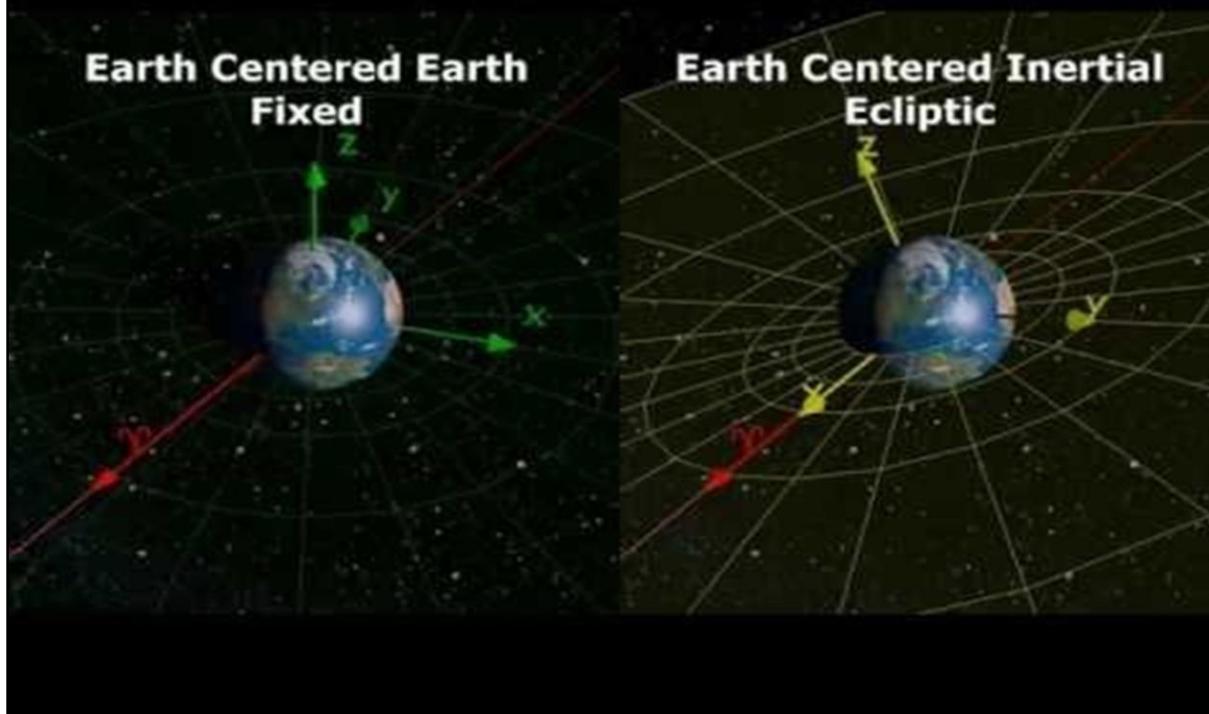
- \hat{x} = Reference direction = first point of Aries
- $\hat{y} = \hat{z} \times \hat{x}$
- \hat{z} = Normal to fundamental plane of equator/ecliptic

Earth Centered Earth Fixed (ECEF)

- \hat{x} = Reference direction = Greenwich meridian
- $\hat{y} = \hat{z} \times \hat{x}$
- \hat{z} = Normal to fundamental plane of equator

Notice that the difference is the x direction.

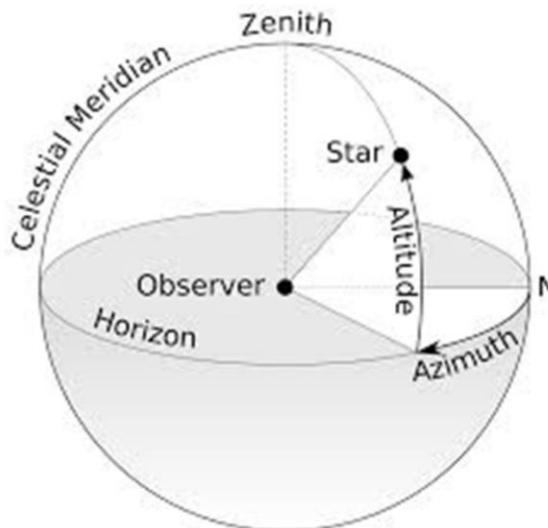
Animation



This video is provided on the Blackboard page.

Topocentric Horizon

- Local definition to observer
- Azimuth and altitude/elevation



The topocentric horizon system is used with telescopes and typically anything Earth-bound as it uses a system which is local to the observer on the Earth. Imagine that observer is on the surface of the Earth, if they look straight above them in the sky they see the zenith, if they look at the horizon then up, then that is the altitude and the direction they look N/S/E/W is the azimuth.

Defining a Reference Frame

Coordinate System	Fundamental Plane	Reference Direction	Coordinates
<i>Topocentric Horizon</i>	Local Horizon	North/South point	Azimuth/ Elevation
<i>Earth-centered, Earth-Fixed (ECEF)</i>	Equatorial plane	Prime Meridian	R, V or Orbital Elements
<i>Earth-centered inertial (ECI)</i>	Equatorial /Ecliptic plane	Vernal Equinox	R, V or Orbital Elements



The coordinates used are the position vector rx , ry , rz and vx , vy , vz or Keplerian (Orbital) Elements which we will now investigate...

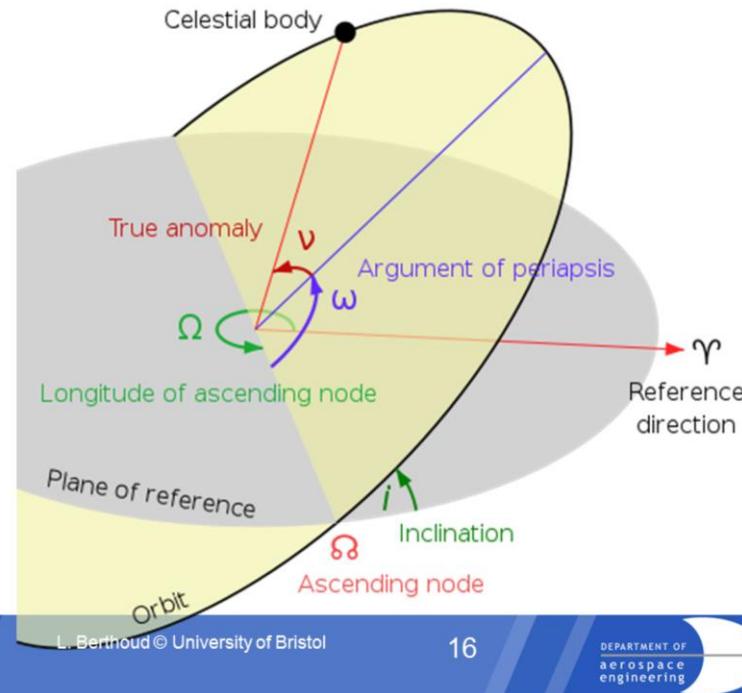
Defining a position can be done in 2 ways...

- Position Vector: r_x , r_y , r_z in m and v_x , v_y , v_z in m/s
- 6 Orbital/Keplerian Elements:
 - Semi-major axis 'a'
 - Eccentricity 'e'
 - Inclination ' i '
 - Right ascension of the ascending node ' Ω '
 - Argument of Periapsis ' ω '
 - True or mean anomaly ' ν ' or ' θ ' or ' M '

We have a reference system, now we want to define a position.
The top two elements we have already met. The next 4 are new.
The lab will help you understand them in 3D. In the meantime let's look at the definitions...

Orbital or Keplerian elements

- a : Semi-Major Axis (Size)
- e : Eccentricity (Shape)
- i : Inclination
- Ω : Longitude of ascending node
- ω : Argument of Perigee
- v : True Anomaly



This is a summary diagram of all the orbital elements.

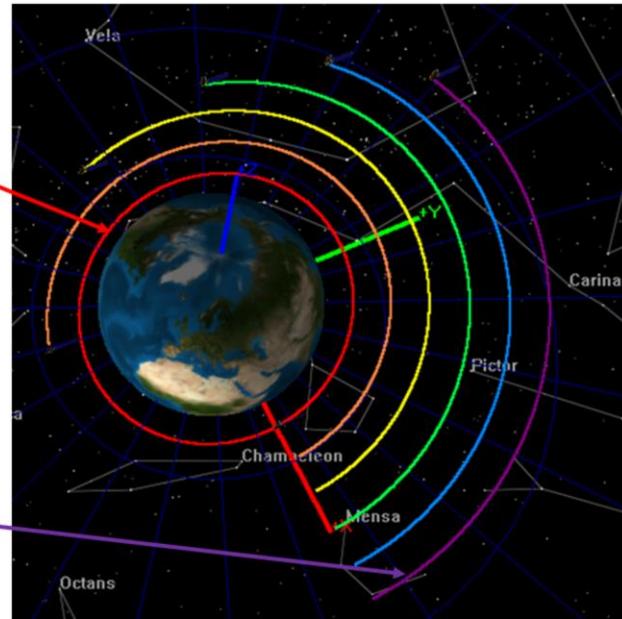
You need to learn the definitions and the symbols for each element.

Which direction is the direction of measurements in each case (clockwise or anticlockwise?)

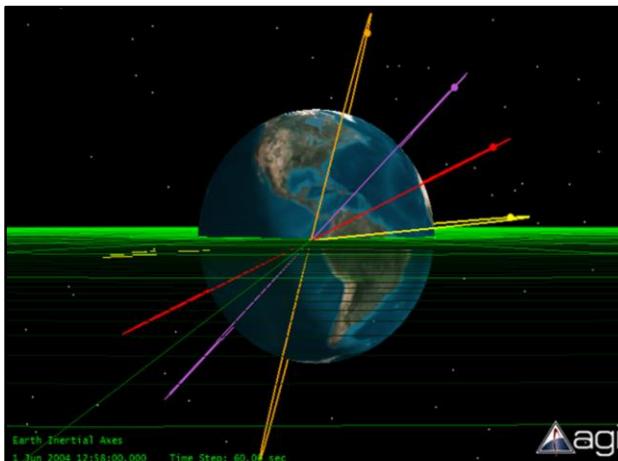
Semi major axis

Smaller
'a'

Larger
'a'



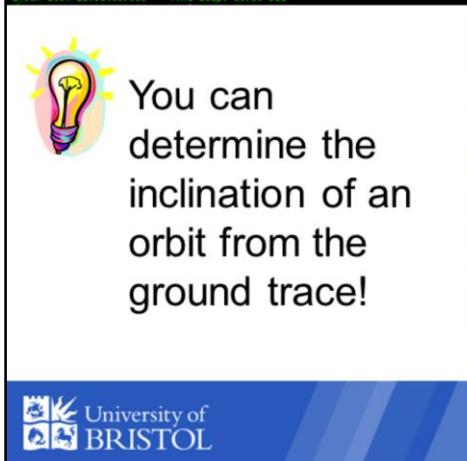
This is a measure of the size of the orbit.



The diagram shows a 3D perspective of Earth with a green horizontal plane representing the equator. Several colored lines (yellow, purple, red) represent different orbital planes tilted at various angles relative to the equatorial plane. A small text box in the bottom left corner reads "Earth Inertial Axes 1 Jul 2004 12:00:00.000 Time Step: 60.00 sec". The AGI logo is in the bottom right.

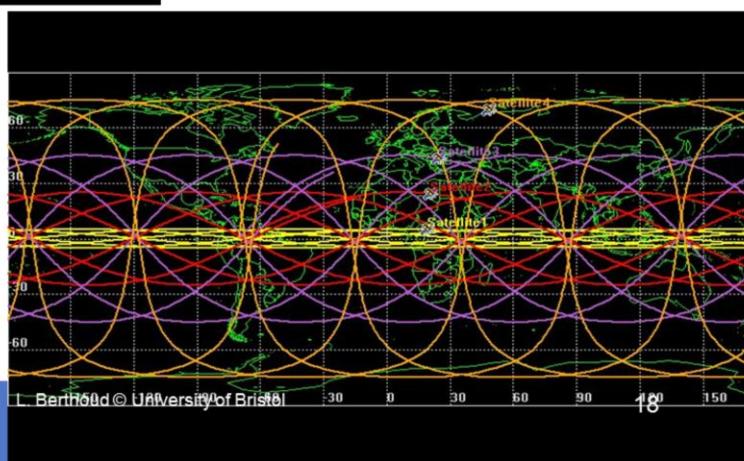
Inclination 'i'

Inclination determines the northern and southern latitude limits.



You can determine the inclination of an orbit from the ground trace!

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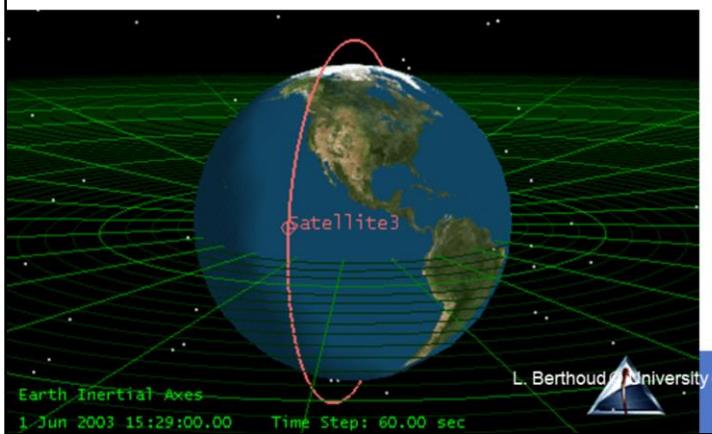
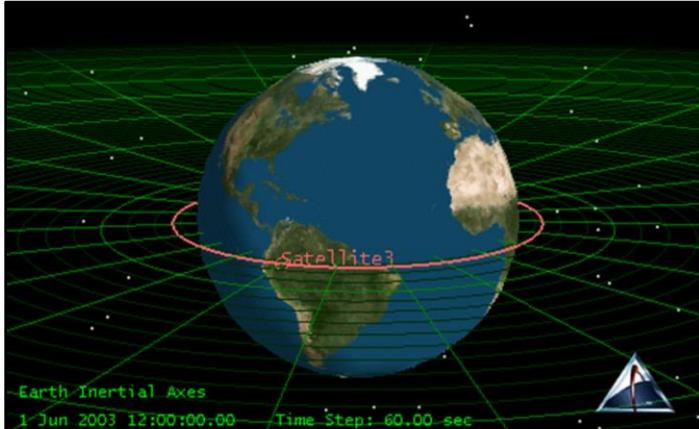


A world map showing the ground traces of multiple satellites. The traces are colored lines (yellow, purple, red) that cross the Earth's surface, forming arcs between the poles. Latitude and longitude lines are overlaid on the map. A small text box at the bottom left says "Berthold © University of Bristol".

The inclination “ i ” is the angle measured from a reference plane to the plane of orbit of the satellite. It is measured at the ascending node, i.e. the location where the satellite transits from the Southern Hemisphere to the Northern Hemisphere, so by definition, its value is between 0° and 180° . You can determine the inclination of an orbit from the ground trace. For example, a satellite with a 45° inclination will have a ground trace ranging from 45° north to 45° south.

Inclination (cont.)

An orbit with an inclination of 0° is called an **equatorial orbit**. An orbit with an inclination of 90° is called a **polar orbit**.



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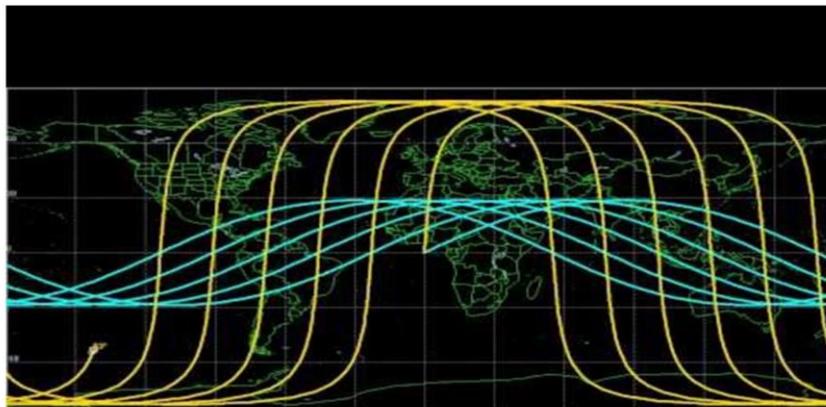
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The terms “equatorial” and “polar” are used to describe orbits with zero and approximately 90 degree inclinations, respectively. Equatorial orbits are very important in the world of communications. However, it should be noted that rarely (if ever) do satellites in equatorial orbits have low altitudes.

Ground traces

We interrupt our regularly scheduled presentation on orbital elements to bring you important information on [ground traces!](#)

A ground trace is a projection of the satellite's orbit onto the earth.



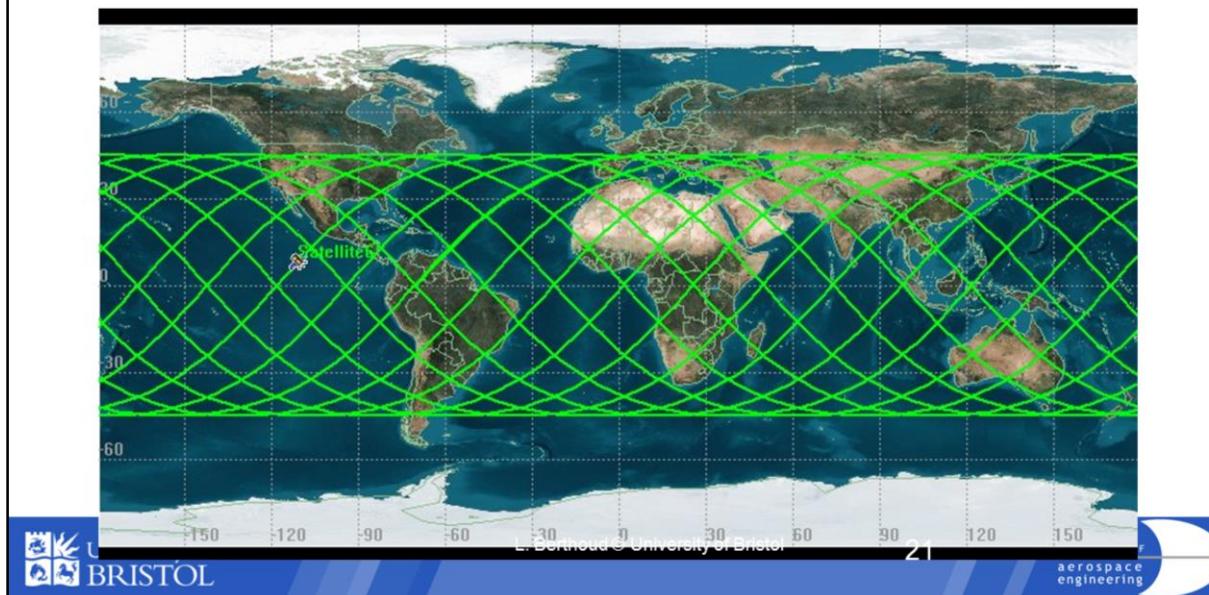
The satellite appears to move westward because Earth is rotating eastward.



If a long string with a magic marker tied to the end of it were hung from a satellite, the path which the magic marker would trace over the ground is the ground trace.

Ground Traces (cont.)

After 1 day the ground trace of a satellite w T= 90 minute:



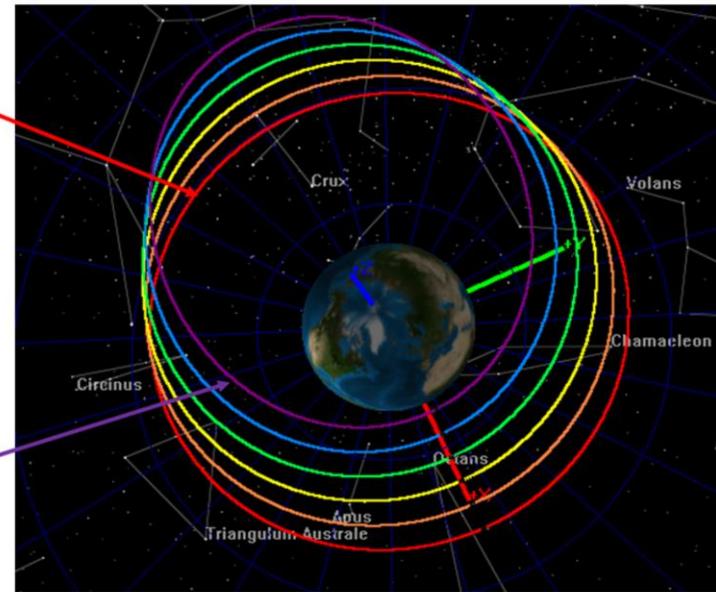
Because the earth continually rotates under the satellite, the ground trace appears to “move” across the Earth over the course of a day.

Why doesn't this ground trace cover the far north and far southern regions of the Earth?

Eccentricity 'e'

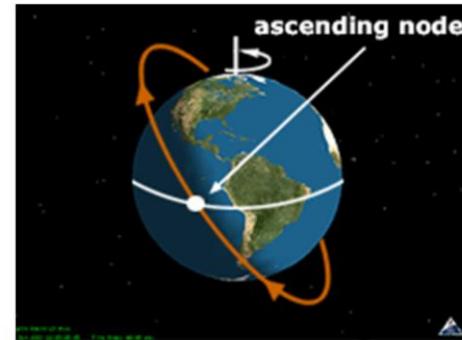
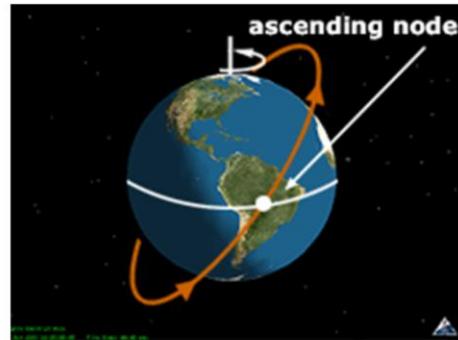
Smaller
'e'

Larger
'e'



Smaller e is circular orbit with red trace.
Larger e is more elliptical with purple trace.

Definition of ascending node

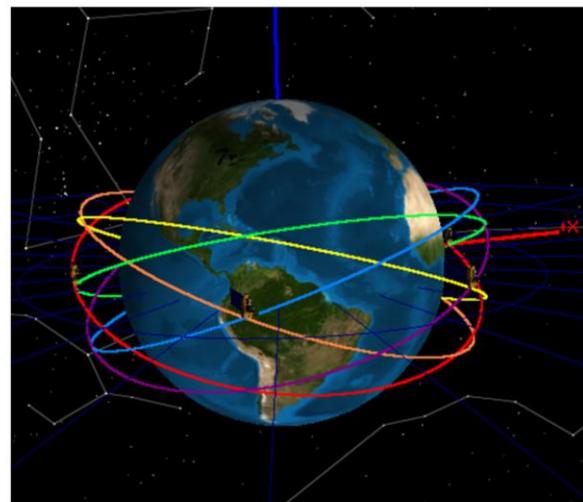


- The ascending node is the point where the spacecraft passes UP (ASCENDS) through a reference plane.
- The descending node is the point where the spacecraft passes DOWN (DESCENDS) through a reference plane.

Orbit passes through a reference plane eg: equator,
180 degrees after the ascending node, the satellite has a descending node.
It is this ascending node which must be located using some non-moving reference point.

Right Ascension of Ascending Node Ω

- Also known as “Longitude of Ascending Node”
- Ω = angle from Vernal equinox to ascending node measured in an anticlockwise direction.



Since we can't use an earth-based reference point, we use a celestial reference point known as the vernal equinox or the first point of Aries. The angular measurement of the right ascension is 0 at the vernal equinox. So, if the ascending node of the orbit is aligned with the vernal equinox, the RAAN is 0. It is measured in the anti-clockwise direction and is an angle measured in degrees.

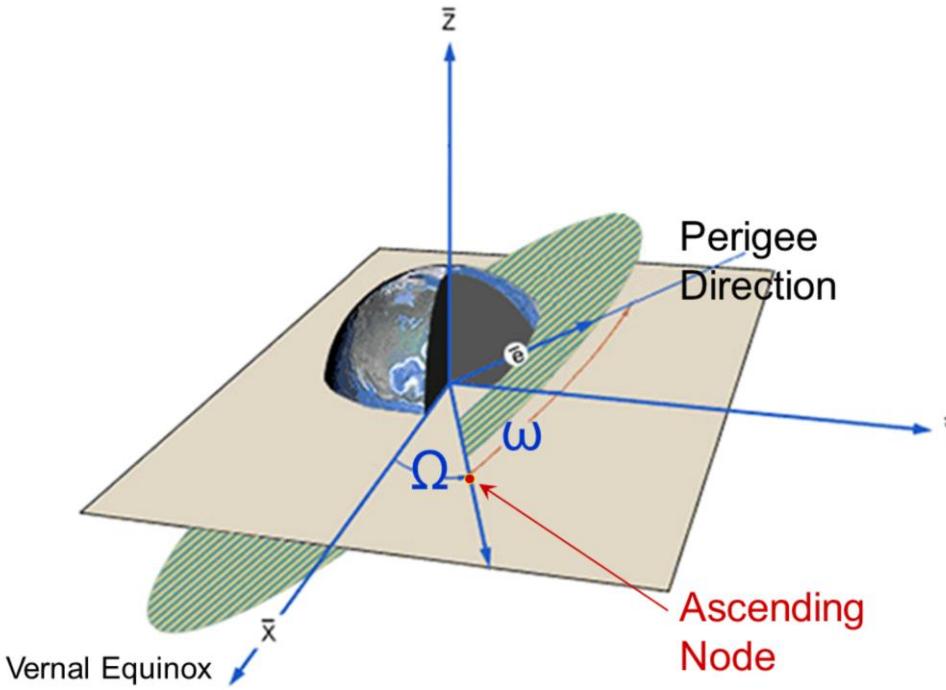
Demo

- Hula hoop and ball

Argument of perigee ω

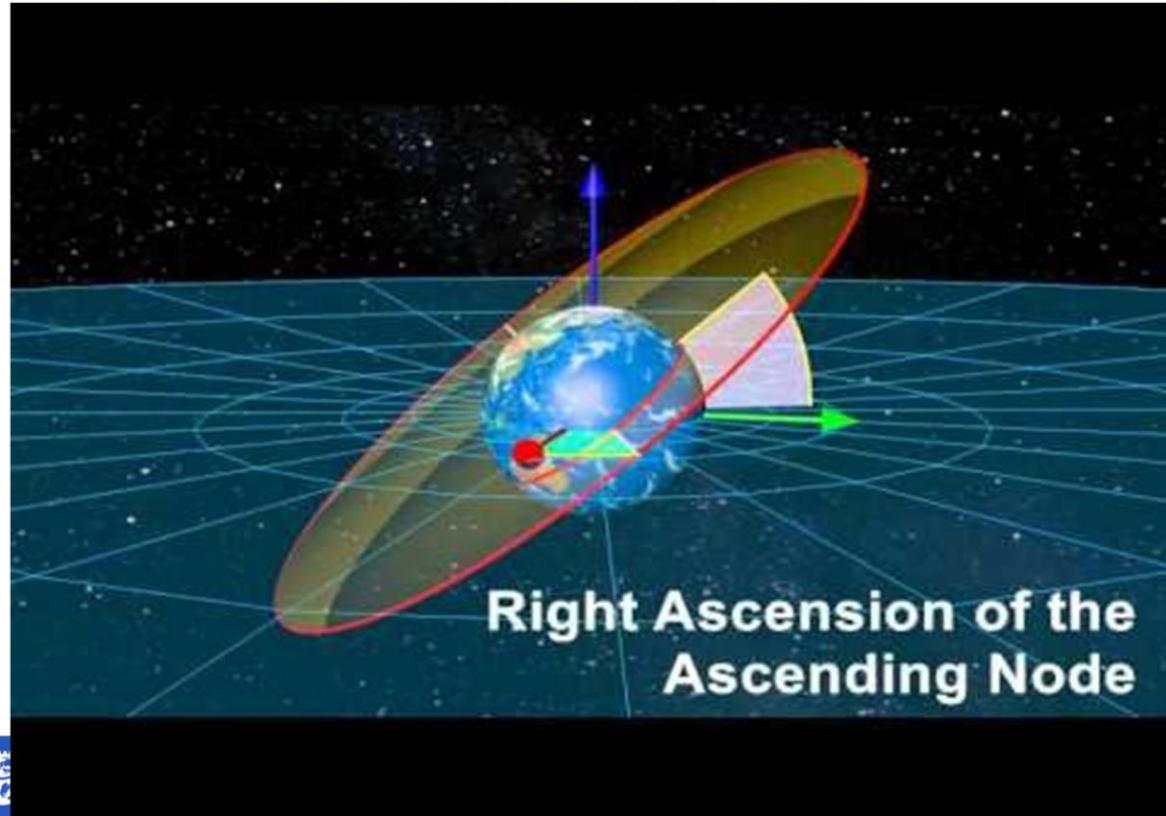
$\Omega = \text{RAAN}$

$\omega = \text{angle}$
from
ascending
node to
perigee on
the orbital
plane



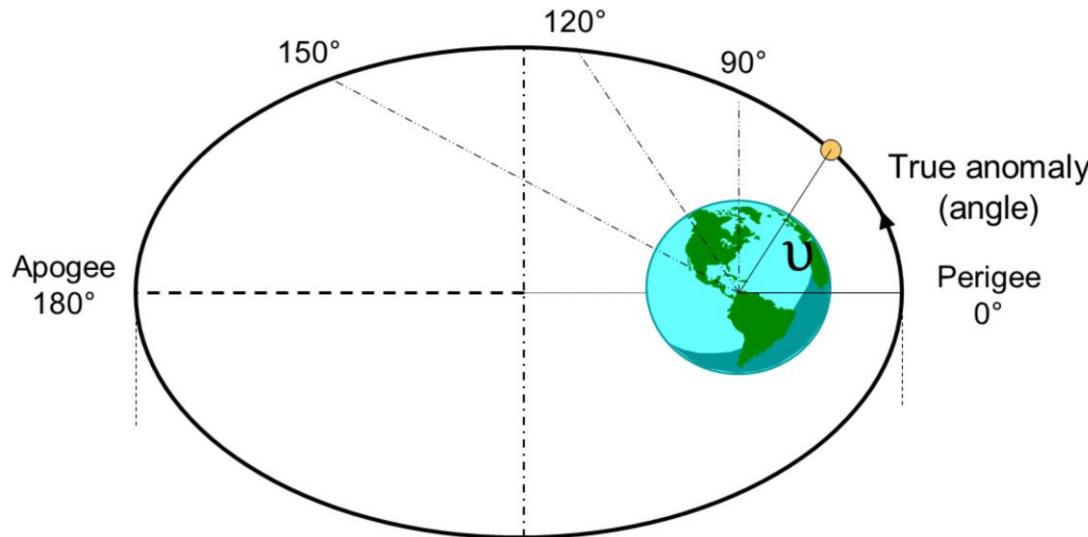
Argument of perigee is measured anti-clockwise from the Ascending node to the perigee (in the orbital plane). It is an angle measured in degrees.

Orbital elements video



This video is provided on the Blackboard page, you can also google it on Youtube with “Orbital Elements Space Universities Network”.

True Anomaly ' ν ' (also ' θ ')



ν defines satellite angle from perigee

Now we use perigee to tell us where in the orbit we are. The true anomaly gives the position of the satellite at that moment in time, it is measured from perigee anti-clockwise round the orbit to the line joining the satellite to the centre of the Earth (in degrees).

Two Line Elements (ISS)

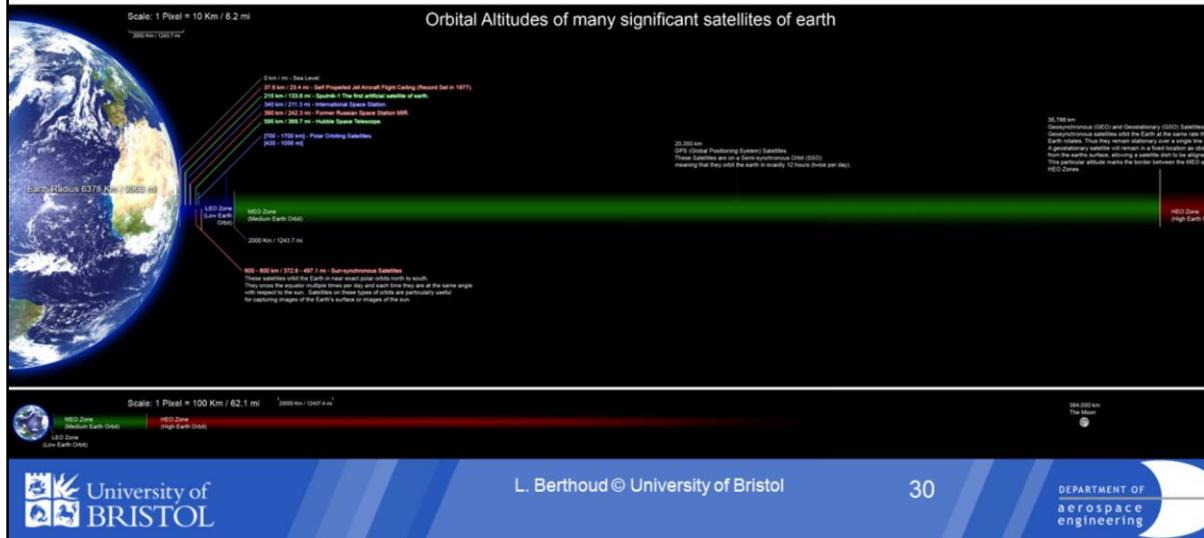
- TWO LINE MEAN ELEMENT SET - ISS
- 1 25544U 98067A 09061.52440963 .00010596 00000-0
82463-4 0 9009
- 2 25544 51.6398 133.2909 0009235 79.9705 280.2498
15.71202711 29176
- Satellite: ISS
- Catalog Number: 25544
- Epoch time: 09061.52440963 = yrday.fracday
- Element set: 900
- **Inclination: 51.6398 deg**
- **RA of ascending node: 133.2909 deg**
- **Eccentricity: .0009235**
- **Arg of perigee: 79.9705 deg**
- **Mean anomaly: 280.2498 deg**
- **Mean motion: 15.71202711 rev/day (*semi-major axis derivable from this*)**
- Decay rate: 1.05960E-04 rev/day²
- Epoch rev: 2917
- Checksum: 315



Two Line Elements are used by NASA as data for satellites, we will see later what mean motion and mean anomaly are. Solving for t given the orbital elements is easy, the other way round was Everest of maths for 300yrs.

Orbital altitude classifications

'LEO' < ~1,000km 'MEO' = 1,000-36,000km 'GEO' = 36,000km



In orbit >100km.

LEO-Low Earth Orbit (<1000km) is most common, but due to Earth's atmosphere and consequent have the shortest life.

MEO-Medium Earth Orbit 1000-3600km is difficult due to radiation belts but is used by eg: GPS constellation. GEO – Geostationary orbit is very useful, as this particular altitude means the satellite appears to hover over a point on the Earth. We will prove this later.

Deep space is anything beyond the Moon.

Homework

- Read the rest of these slides
- Use online notes if you want extra explanation
- Ask questions through discussion board - no such thing as a stupid question!

Summary

1. Reference Systems
 - Earth Centered Inertial (fixed wrt stars),
 - Earth Centered Earth Fixed (fixed wrt Earth)
 - Topocentric horizon (azimuth/elevation)
2. Vernal equinox : points at first point of Aries/Sun on Mar 21
3. Keplerian Orbital Elements: a , e , i , ω , Ω , ν
4. Ground track of a satellite is determined by orbit
5. Types of orbits:
 - LEO-Low Earth Orbit <1000km
 - MEO-Medium Earth Orbit 1000-3600km.
 - GEO – Geostationary altitude 36000km (hovers over point)
 - Sun-synchronous –nodal regression used to stay in Sunlight
 - Molniya – highly elliptical, used as GEO for northern latitudes

Orbital Perturbations

"All orbits evolve"

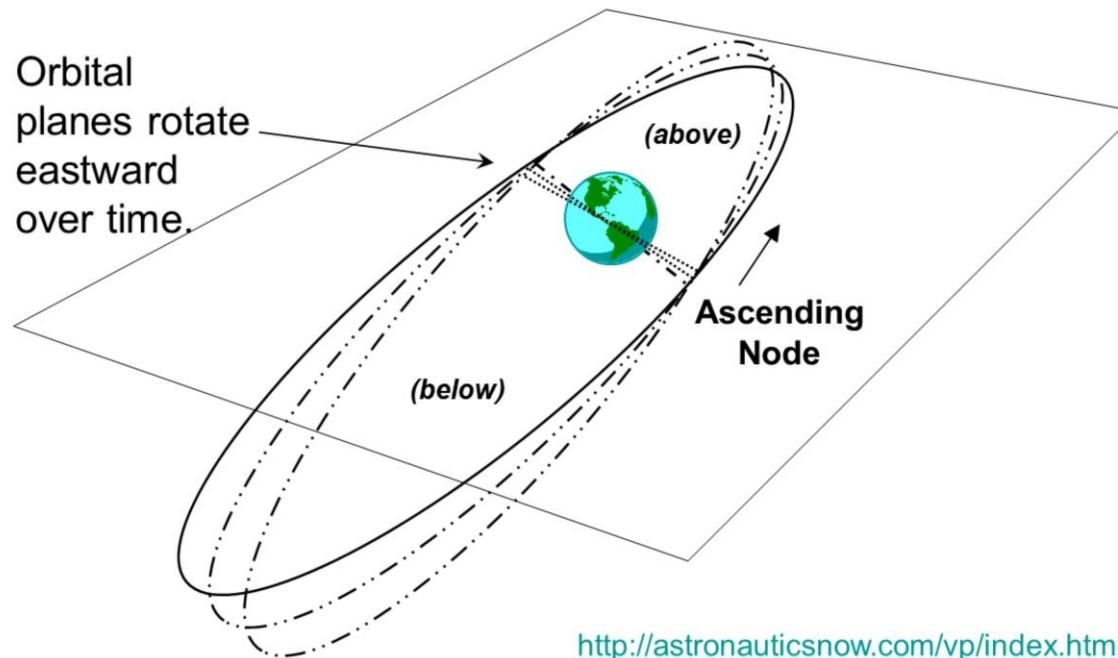
Atmospheric Drag (at LEO altitudes, only)

- Worse during increased solar activity. Insignificant above 800km.

Non-sphericity of the Earth – The Earth is an oblate spheroid. This adds extra “pull” when a satellite passes over the equator – rotating the position of the ascending node of the orbit. This is called **Nodal regression**. The perigee also moves: this is called ‘precession of the apsides’.

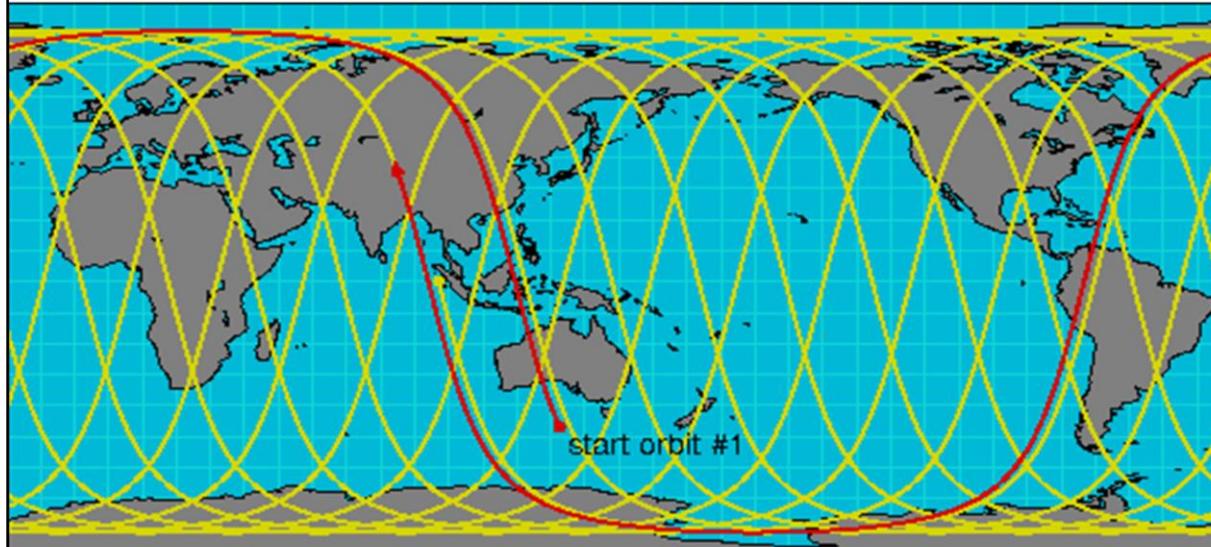
Other Factors – Gravitational irregularities – such as Earth-axis wobbles, Moon, Sun, Jupiter gravity, Solar photon pressure.

Nodal Regression



Nodal regression caused by Earth not being a perfect sphere. It causes the ascending node to move eastwards or westwards depending whether the inclination is more or less than 90deg. Mike Gruntmann's site : <http://astronauticsnow.com/vp/index.html> has several interesting videos, including a very clear one on nodal regression. Nodal regression can be really useful. If we select exactly the right inclination, we can get the node to move at the same rate as the Sun.

Sun-synchronous orbits



- Nodal regression shifts the ascending node by $\sim 1^\circ/\text{day}$
- Satellite sees path under the same lighting conditions

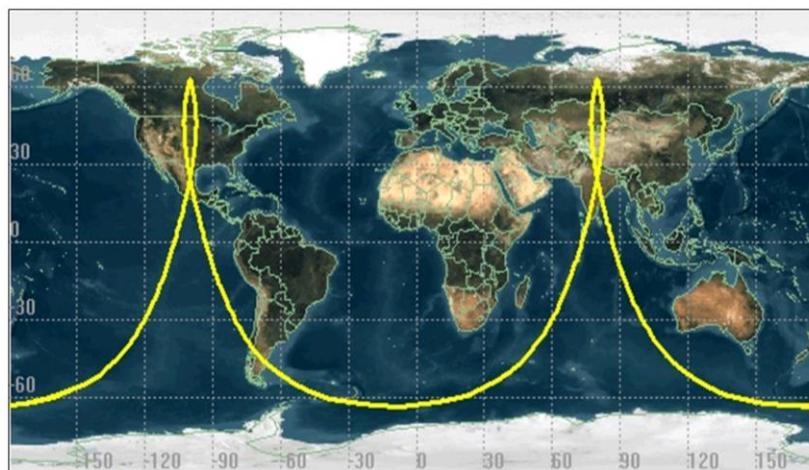
It is possible to choose the parameters of a spacecraft's orbit to take advantage of some or all of these gravitational influences to induce precession, which causes a useful motion of the orbital plane. The result is called a walking or precessing orbit. Nodal regression shifts the ascending node by $\sim 1^\circ/\text{day}$

Satellite then scans the same path under the same lighting conditions ie: same angle of sunlight. This requires a slightly retrograde orbit ($i = 97.56^\circ$ for a 550km).

Used for Earth Observation/reconnaissance.

Molniya orbit - 12hr Period, $i=63.4^\circ$

'Long loitering' high latitude apogee. Used for early warning by USA and Russian Fed.



A satellite in a highly eccentric orbit spends most of its time in the neighborhood of [apogee](#) which for a Molniya orbit is over the northern hemisphere, the sub-satellite point at apogee having a latitude of 63.4 degrees North. As the apogee altitude is high (40,000 km), it will therefore, for a considerable period, have an excellent visibility of the Russian Federation but also of USA/Canada.

Test yourself! (Feedback)

1. ‘The ascending node can be defined as where an orbiting spacecraft crosses the semi-major axis going north.’ True or False?
2. ‘A sun-synchronous orbit is never a precessing orbit’. True or False?
3. What is the angle between the equator and the ecliptic?
4. Does inclination change with increase in altitude?
5. How is RAAN measured?
6. Is argument of perigee measured clockwise or anti-clockwise?
7. What is the approximate inclination of Molniya orbits from the ground track on slide 34?