THE VORTEX LATTICE METHOD

AIMS

- To introduce the vortex lattice method for the modelling of 3D flows

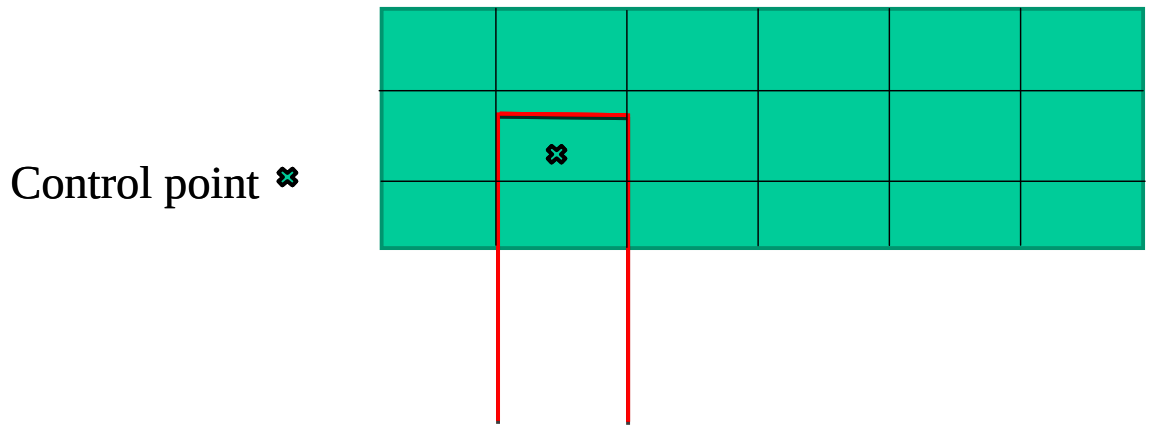
1 INTRODUCTION

The vortex lattice method is in a generalised 3D approach that blends the ideas of lifting line theory and the 2D lumped vortex method in 2D. It uses a panel representation to distribute singularities and so can be thought of as a simple panel method for a thin wing. It is simpler and quicker to implement that a full 3D panel method, but is less accurate. However for many problems where geometries are thin, it gives reasonable results and it is very widely used in aerodynamics, particularly for wing analysis.

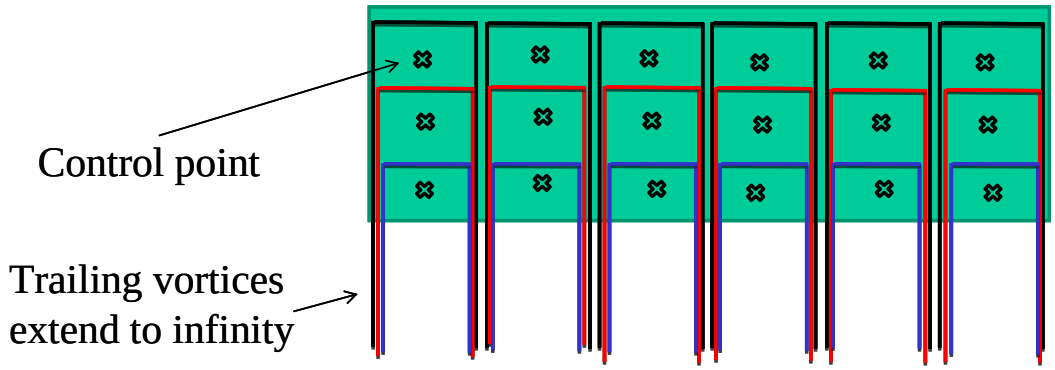
2 BASIC METHOD

Thinking of the lifting line method considered this far, it allows for a spanwise circulation distribution but does not account for the chordwise circulation variation. The 2D lumped vortex method was developed from thin aerofoil theory and gave a discrete representation of the chordwise distribution of circulation. The discrete vortices were distributed using a panel representation of the geometry with a vortex placed at the quarter chord point of each panel and a control point (at which no normal flow is enforced) at the three quarter chord point. This ensures that the Kutta condition is satisfied.

Combining the lumped vortex method and a lifting line method with a finite number of horseshoe vortices leads to the vortex lattice method. Then for an uncambered rectangular wing, the wing is divided into panels and a horseshow vortex is placed on each panel as shown in the next figure

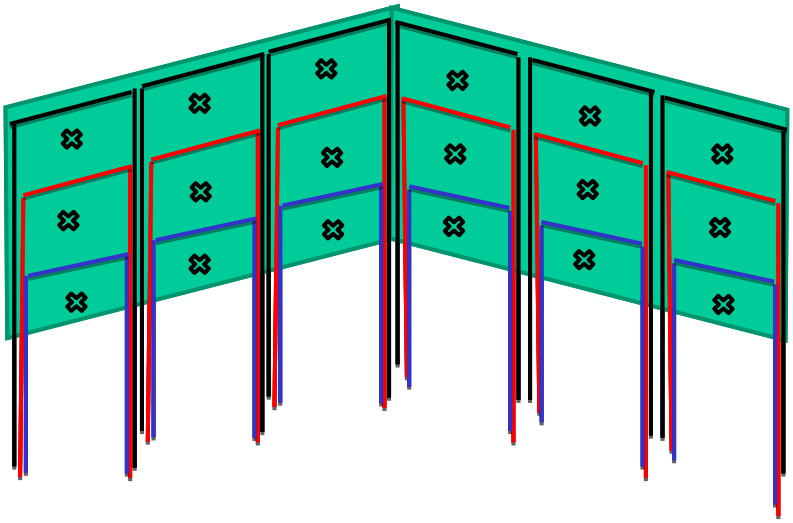


Then when the whole wing is panelled it will appear as follows



Note panels not shown and gaps between vortices added in the drawing only to aid visualisation/understanding

The method can easily be extended to swept wings



but special care must be taken if a control point lies on the extension of any of the bound vortices.

The solution process is exactly the same as seen previously for 2D panel methods i.e. if the horseshow vortex on panel *j* has strength Γ_j then it induces a component of velocity normal to the flat plate wing at control point *i* equal to

$$a_{i,j}\Gamma_j$$

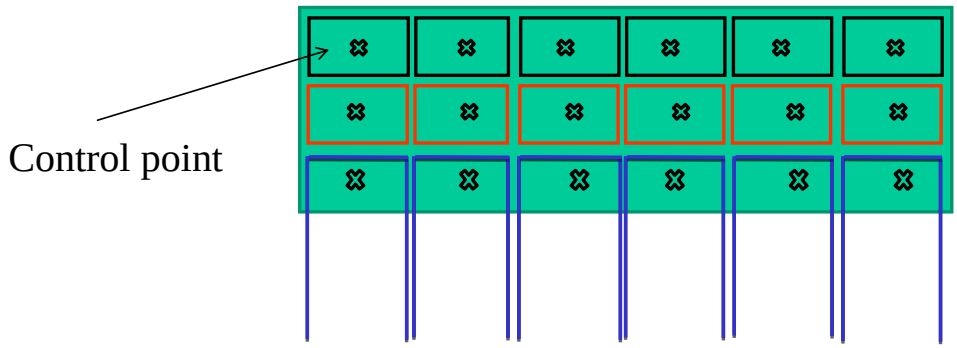
where the term $a_{i,j}$ is the velocity influence coefficient. Then applying the boundary conditions at all control points leads to a linear equation for the vortex strengths

$$\begin{bmatrix} a_{1,1} & a_{1,2} & \ldots & a_{1,N} \\ a_{2,1} & a_{2,2} & . & . \\ : & . & . & . \\ a_{N,1} & . & . & a_{N,N} \end{bmatrix} \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ : \\ \Gamma_N \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \\ : \\ R_N \end{bmatrix}$$

where the right hand side is equal to minus the normal components of the free stream at control points.

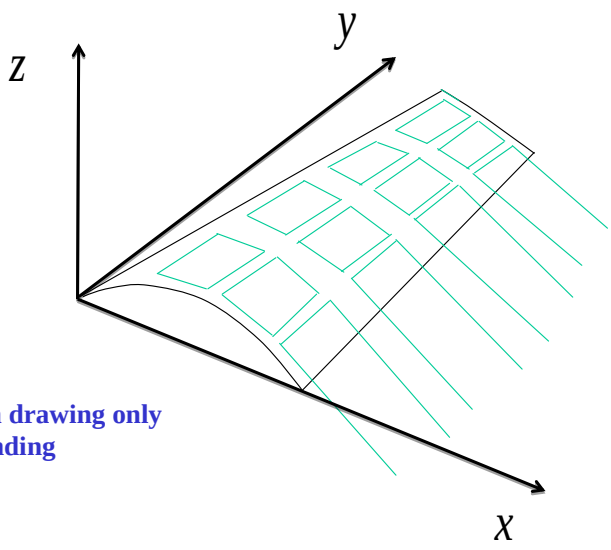
3 GENERALISED METHOD

The method shown so far can be thought of in another way (similar to the two ways of visualising lifting line, see section 1) i.e. rather than thinking of horseshoe vortices think of vortex rings



Note gaps between vortices in drawing only to aid visualisation/understanding

Then this can be generalised to thin cambered wings



Note gaps between vortices in drawing only to aid visualisation/understanding

Now the wake leaves the wing smoothly, but in the far field the wake should be aligned with the freestream (also true for uncambered wings at non-zero incidence). This can lead to inaccuracy and so a more detailed representation of the wake, which allows for a non-planar wake, is desirable. More information will be given on the lecture on 3D panel methods as the methodology is identical.

4 ADVANTAGES AND DISADVANTAGES

- Quicker and simpler than full 3D panel method.
- Easy to do multi-element aerofoil sections (i.e. Flaps), but hard to include nacelles/bodies

- But has similar limitations to thin aerofoil theory
  - thin shapes
  - not very good at stagnation
- Also limitations of a potential method
  - Can extend applicability by using compressibility correction and coupling to a boundary layer code
- These problems are not too important if the vehicle to be analysed is an aircraft dominated (aerodynamically) by the wing.
- Perhaps biggest advantage/widest application is for aeroelastic work:
  - Structural representation of a wing is often as flat panels – can be coupled directly to vortex lattice method
  - Fast solution of linear aerodynamic equations
  - Available in commercial FE solution packages such as NASTRAN and ZAERO

REVISION OBJECTIVES

You should be able to:
describe the stages in the development of the vortex lattice method.