Propellers and Ducted Fans

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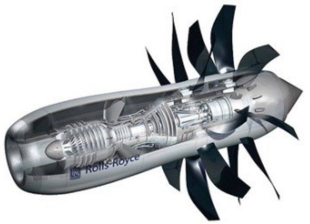
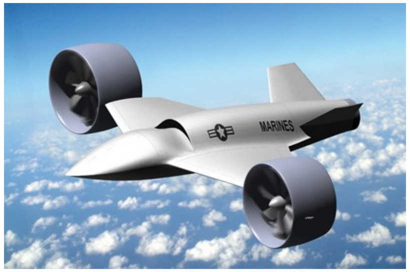
Room 2.40 QB





Revision

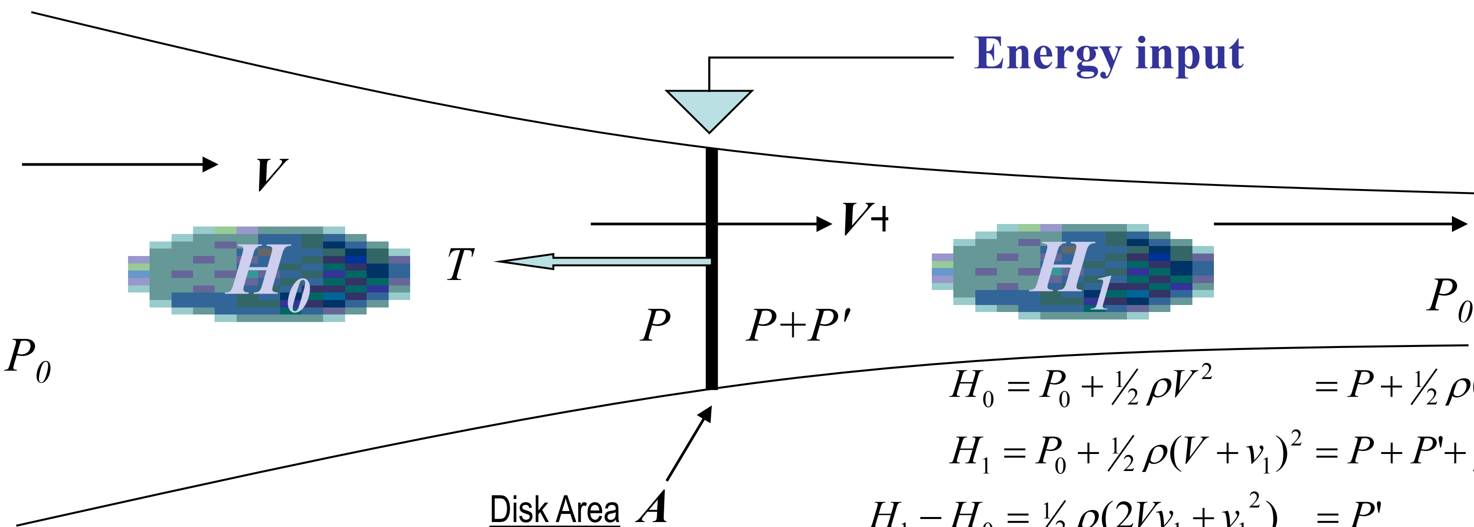
Lecture 5

Notes in Blackboard: https://www.ole.bris.ac.uk

Actuator Disc Theorem

The Propeller or Axial Fan .

Mechanical energy (in the form of rotating blades) is used to accelerate ***(a)*** a mass ***(m)*** of air. Newton's law (every action has a reaction), states ***F = ma***, where ***F***, is the propeller thrust ***(T)***.



$$H_0 = P_0 + \tfrac{1}{2}\rho V^2 \qquad = P + \tfrac{1}{2}\rho(V+v)^2$$

$$H_1 = P_0 + \tfrac{1}{2}\rho(V+v_1)^2 = P + P' + \tfrac{1}{2}\rho(V+v)^2$$

$$H_1 - H_0 = \tfrac{1}{2}\rho(2Vv_1 + v_1^2) \quad = P'$$

$$= \rho v_1\left(V + \frac{v_1}{2}\right) \qquad = P'$$

Momentum Analysis based on Actuator Disk Theorem

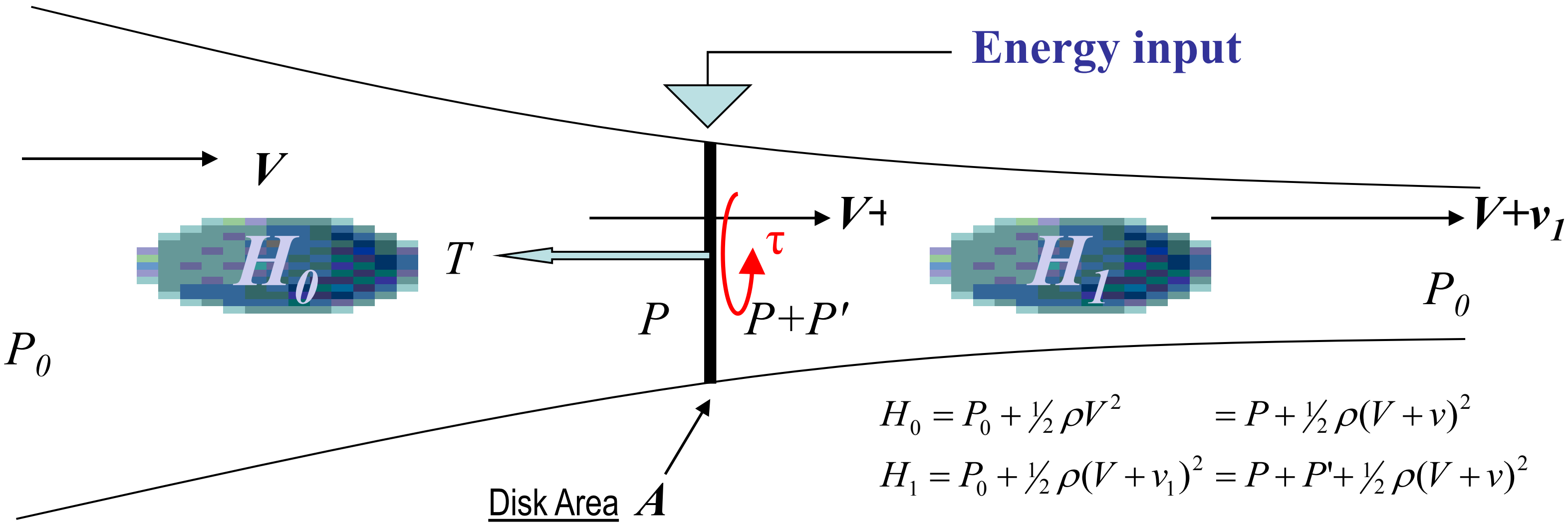
$$\text{Now, } T = \rho A(V+v)v_1, \qquad \frac{T}{A} = \rho v_1(V+v)$$

ρ Air density

A Disc area

$$so\ \frac{v_1}{2} = v \qquad \boxed{\therefore T = 2\rho A(V+v)v}$$

The actuator disk can add only static pressure to the flow and <u>nothing else</u>



$$H_0 = P_0 + \tfrac{1}{2}\rho V^2 \qquad = P + \tfrac{1}{2}\rho(V+v)^2$$

$$H_1 = P_0 + \tfrac{1}{2}\rho(V+v_1)^2 = P + P' + \tfrac{1}{2}\rho(V+v)^2$$

More generally in both H_0 and H_1 states;

$$\overline{P} + \tfrac{1}{2}\rho\overline{V}^2 = \text{contant, thus } \frac{\overline{P}}{\rho} + \tfrac{1}{2}\overline{V}^2 = \text{constant}$$

Momentum Analysis based on Actuator Disk Theorem can be extended to include swirl, though strictly speaking it can no longer be referred to as an Actuator Disk.

$$\text{or } \frac{\overline{P}}{\rho} + \tfrac{1}{2}\overline{V}^2 + \frac{K^2}{A} = \text{constant, where } K = \frac{\tau}{\dot{m}}$$

[“K” *is a* “swirl parameter” based on a free vortex

and τ is fan torque and $\dot{m}$ is the mass flow rate]

Thus, efficiency of a thrust generator is $\eta = \dfrac{T(V+v)}{P}$ where:

T is output thrust
V is onset velocity
v is induced velocity
and P is input (shaft) power

For an aircraft in flight cruise, the **propeller** efficiency is $\eta_p = \dfrac{\boldsymbol{TV}}{\boldsymbol{P}}$

The **IDEAL** propeller is one with **NO LOSSES**.

Thus all the input power (P) is converted to increased kinetic energy (ΔKE),

$\boldsymbol{P = \Delta KE = T\,(V + v)}$,

For the ideal propeller, when $\boldsymbol{V \neq 0}$ then, $\eta_p = \dfrac{TV}{T(V+v)} = \dfrac{1}{1+a}$

Where $\boldsymbol{a} = \dfrac{\boldsymbol{v}}{\boldsymbol{V}}$ and is known as the **axial interference factor (inflow ratio)**.

This suggests that maximum efficiency occurs when $\boldsymbol{a}=0$.

Since $\boldsymbol{T = 2\rho A V^2 a(1 + a)}$ then when $\boldsymbol{a}=0$, $\boldsymbol{T}=0$.

(This is the trivial case and has no practical use)

PROPELLER PERFORMANCE

Recalling the Coefficients used in the analysis of propeller performance

$$J = \frac{V}{nD} \text{ (where } J \text{ in the Advance Ratio)}$$

$$C_P = \frac{P}{\rho n^3 D^5}$$

$$C_T = \frac{T}{\rho n^2 D^4}$$

$$\eta_p = \frac{TV}{P} = \frac{C_T \rho n^2 D^4 V}{C_P \rho n^3 D^5} = \frac{C_T J}{C_P}$$

Where ρ = air density, n = propeller rotational speed (revs/sec), D = propeller diameter

The POWER of an IDEAL propeller.

The Power is non-dimensionalised in a similar manner to thrust but with an additional ***nD*** term as $P \propto TV$ (and ***nD*** is the reference velocity parameter)

Thus **power coefficient** $C_P = \dfrac{P}{\rho n^3 D^5}$

The propeller efficiency $\eta_p = \dfrac{TV}{P}$

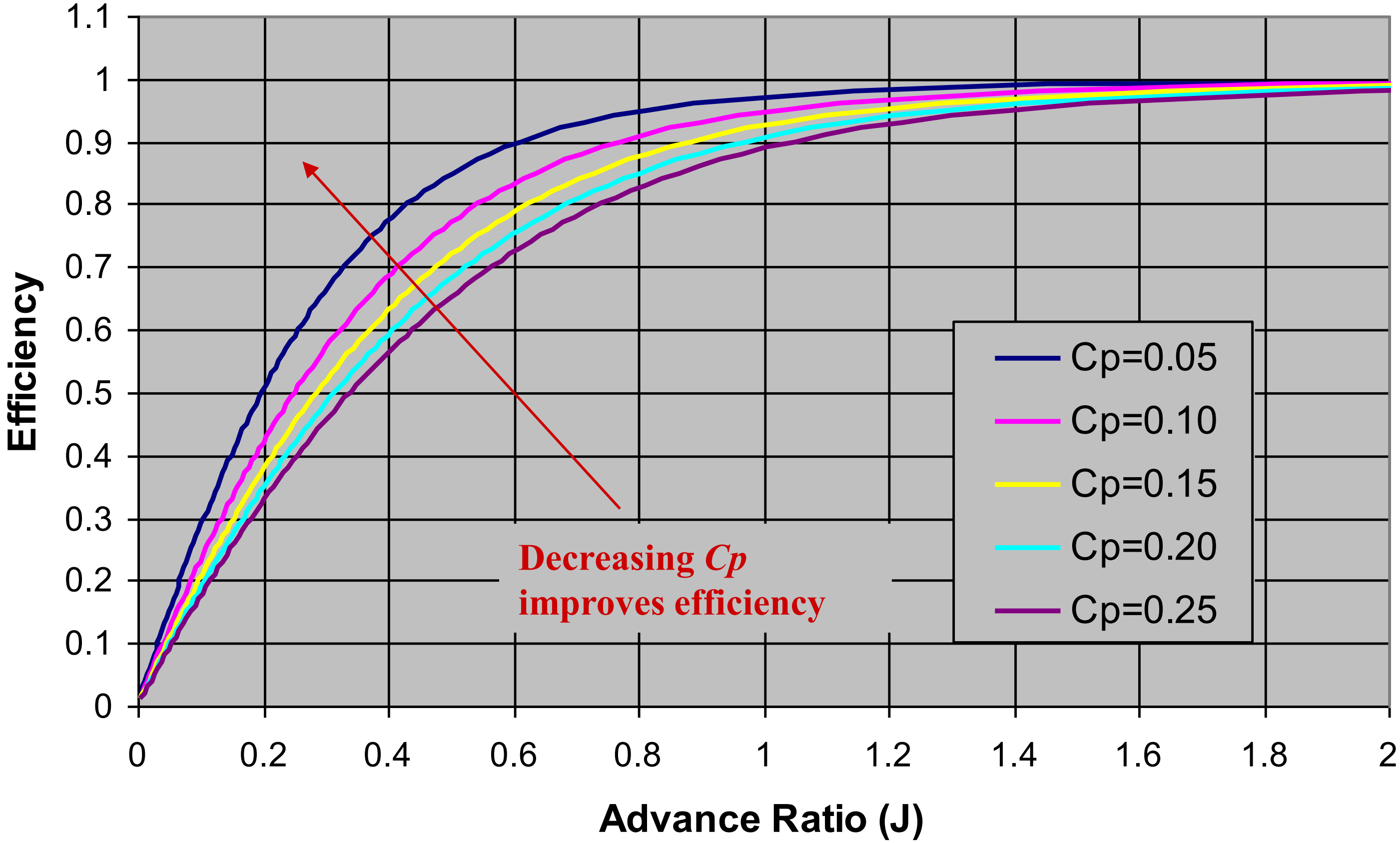
Thus $\eta_p P = \dfrac{\pi}{2} D^2 \rho V^3 (1+a)a = \dfrac{\pi}{2} D^2 \rho V^3 \dfrac{(1-\eta_p)}{\eta_p{}^2}$

and $$\frac{1-\eta_p}{\eta_p{}^3} = \frac{2P}{\pi\, \rho D^2 V^3} = \frac{2C_P}{\pi\, J^3}$$

Of academic interest only (ideal rotor) but gives an indication of parameter sensitivity.

Ideal Propeller
(for a range of Cp values)

$$\frac{1-\eta_p}{\eta_p^{\,3}} = \frac{2P}{\pi \rho D^2 V^3} = \frac{2C_P}{\pi J^3}$$



The REALity is that IDEAL PROPELLERS don't exist, REAL PROPELLERS do!

In analysing and predicting propeller / rotor performance we need to consider:

- Viscosity of the fluid
- Rotor blade tip effects
- Non-uniform inflow
- Blade vortex interaction

$$\eta_p = \frac{TV}{P} \left(= \frac{TV}{T(V+v) + \text{Losses}} \right)$$

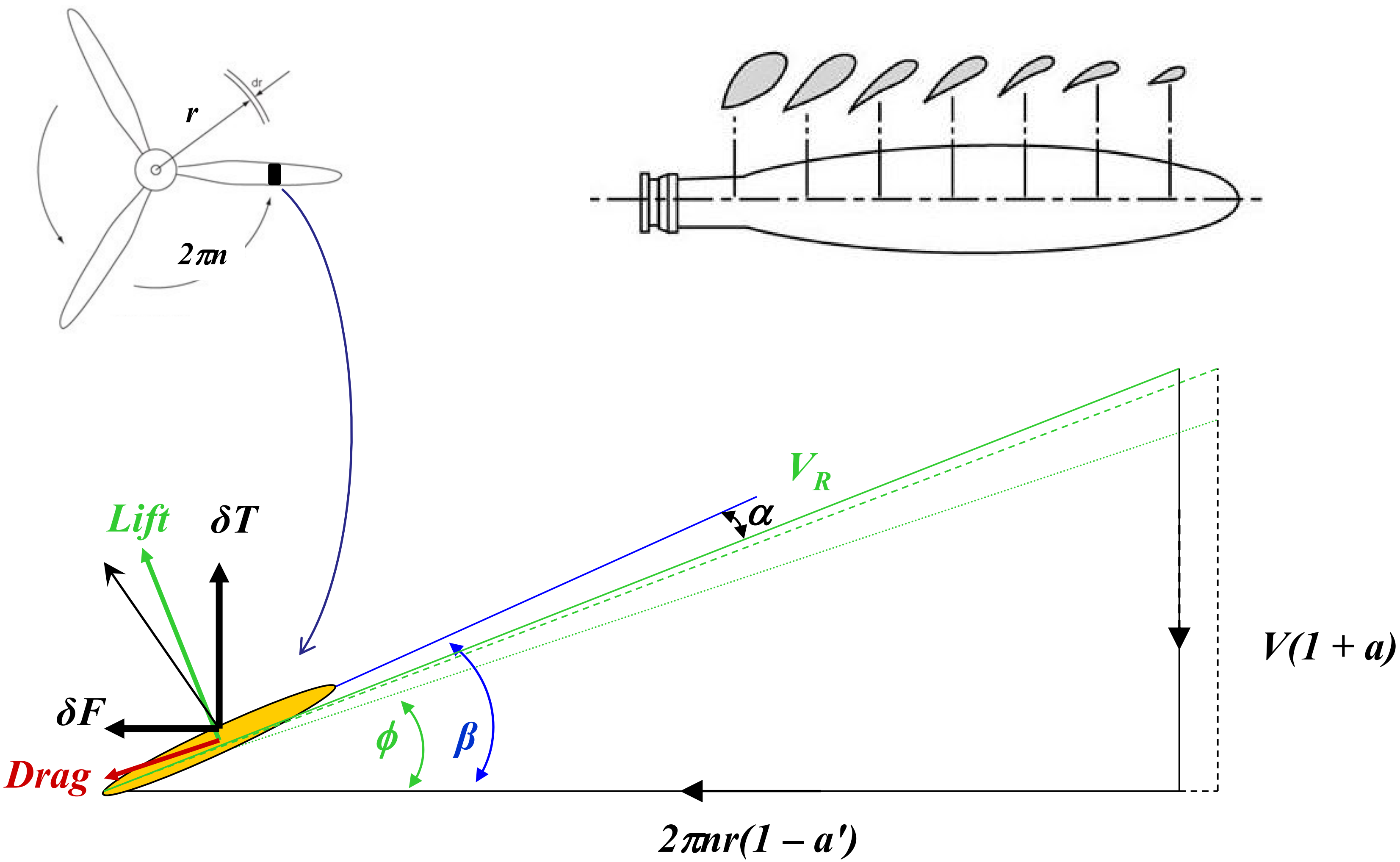
Needless to say, these result in energy loss and therefore reduced efficiency.

In order to obtain a more detailed knowledge of the behaviour of a rotor:

- It is necessary to analyse the forces on the rotor blades.
- The blade has to be considered as a number of separate aerofoil elements.
- Elements are then integrated to represent the characteristics of the whole propeller.

This is achieved by BLADE ELEMENT (often called STRIP ANALYSIS)

Flow Velocity components

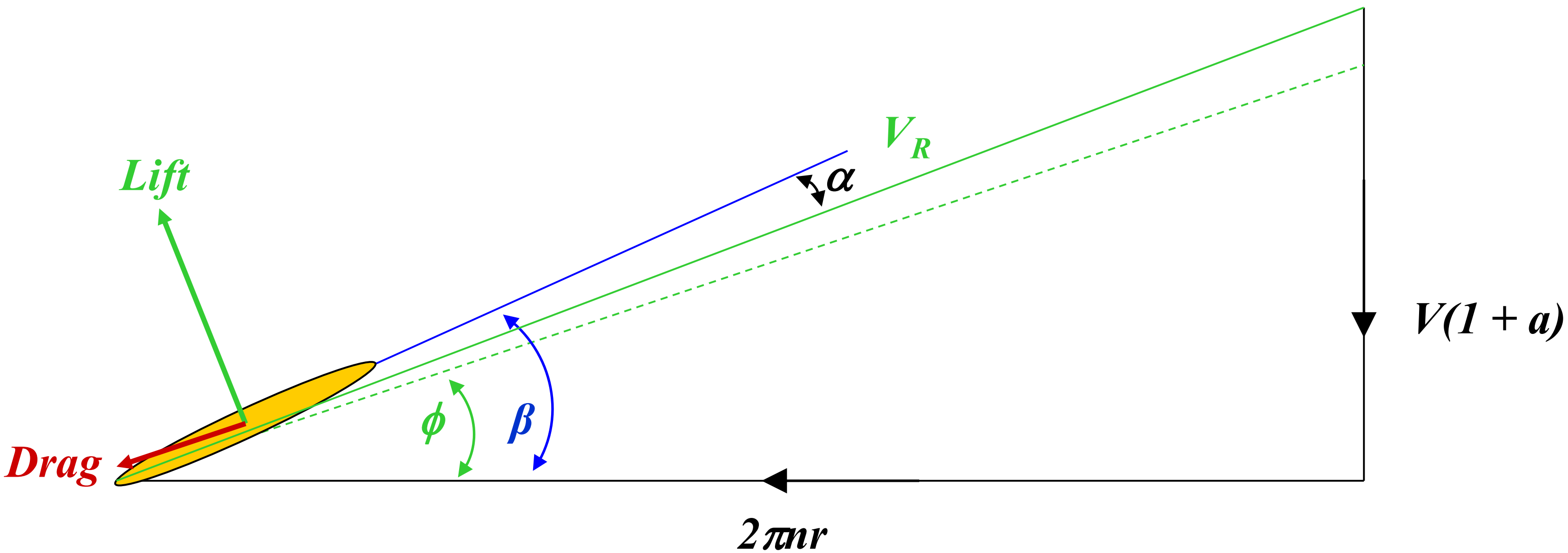


Interference Flows

These are the axial and rotational flows that are in addition to those of the rotor.

Axial

The axial interference flow (also referred to as the "induced" velocity $\boldsymbol{v}$) has been determined previously by momentum analysis. It adds to the onset velocity $\boldsymbol{V}$. This increases the inflow angle, tilts the lift vector backward resulting in induced drag and reduces the angle of incidence given by $\boldsymbol{\alpha = (\beta - \phi)}$.

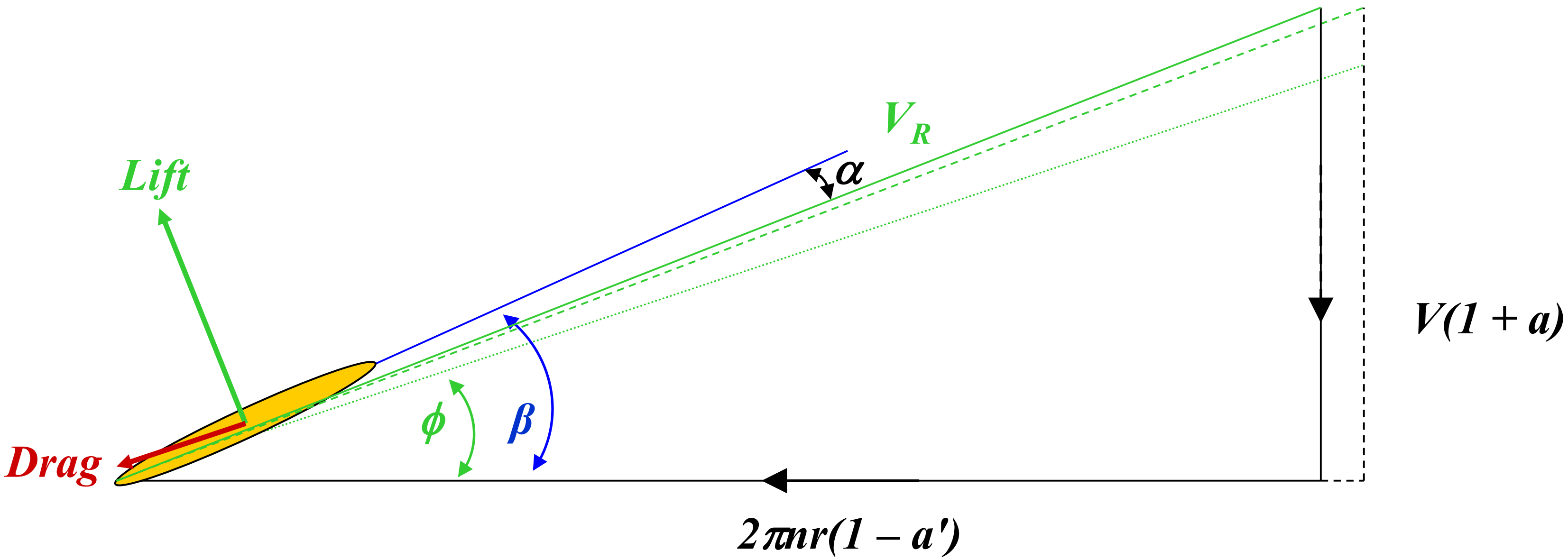


Interference Flows

Radial

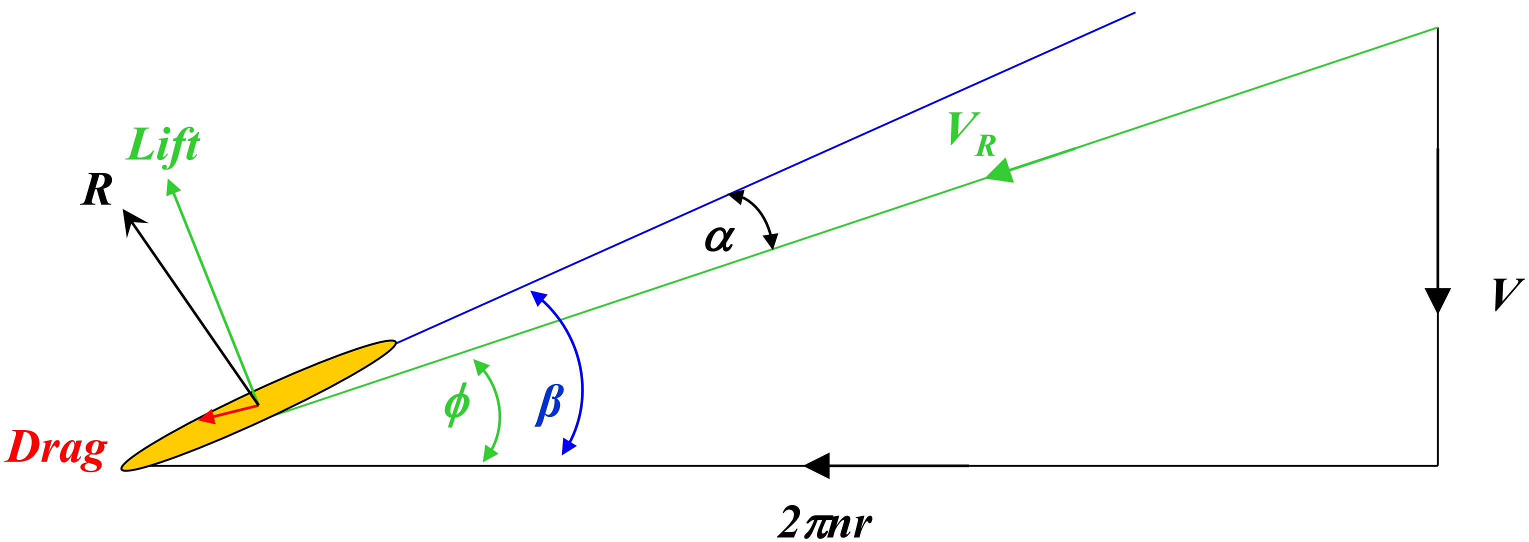
The rotational interference flow (which we shall call ***w***) can be determined from vortex theory and is physically represented by the swirl of the rotor wake. It is suffice to say that it has a value ***w*** at the rotor and ***2w*** in the far wake. Unlike the axial interference flow, it has no value upstream of the rotor and it is not wholly induced, as it contains a contribution from the viscous drag of the rotor blades.

This also increases the inflow angle, tilting the lift vector backward and further increasing induced drag and reducing the angle of incidence given by $\alpha = (\beta - \phi)$.



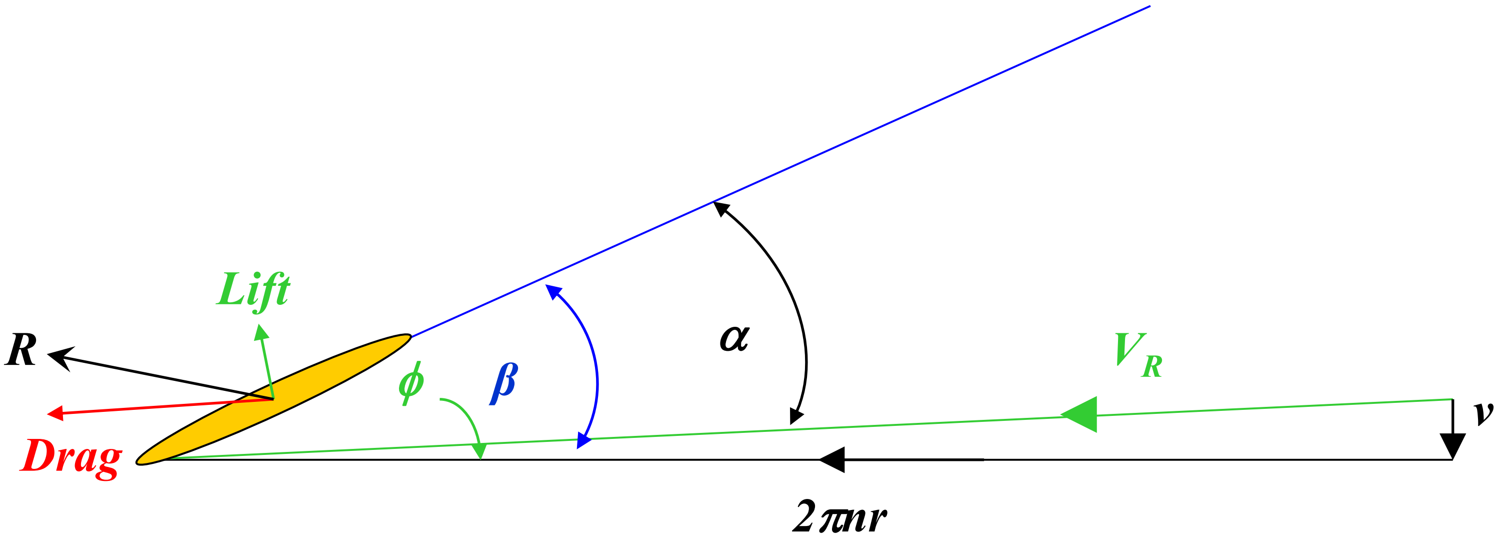
Generally, for propellers in flight, *$V >> v$* and so the axial and rotational interference factors can be neglected. This is not true during the take-off phase of flight and this will be discussed later…..

The velocity diagram below represents propeller inflow for an aircraft in steady flight with the angle of incidence ***"α"*** being a modest angle, most probably close to the angle for <u>best lift/drag</u> of the blade aerofoil section.



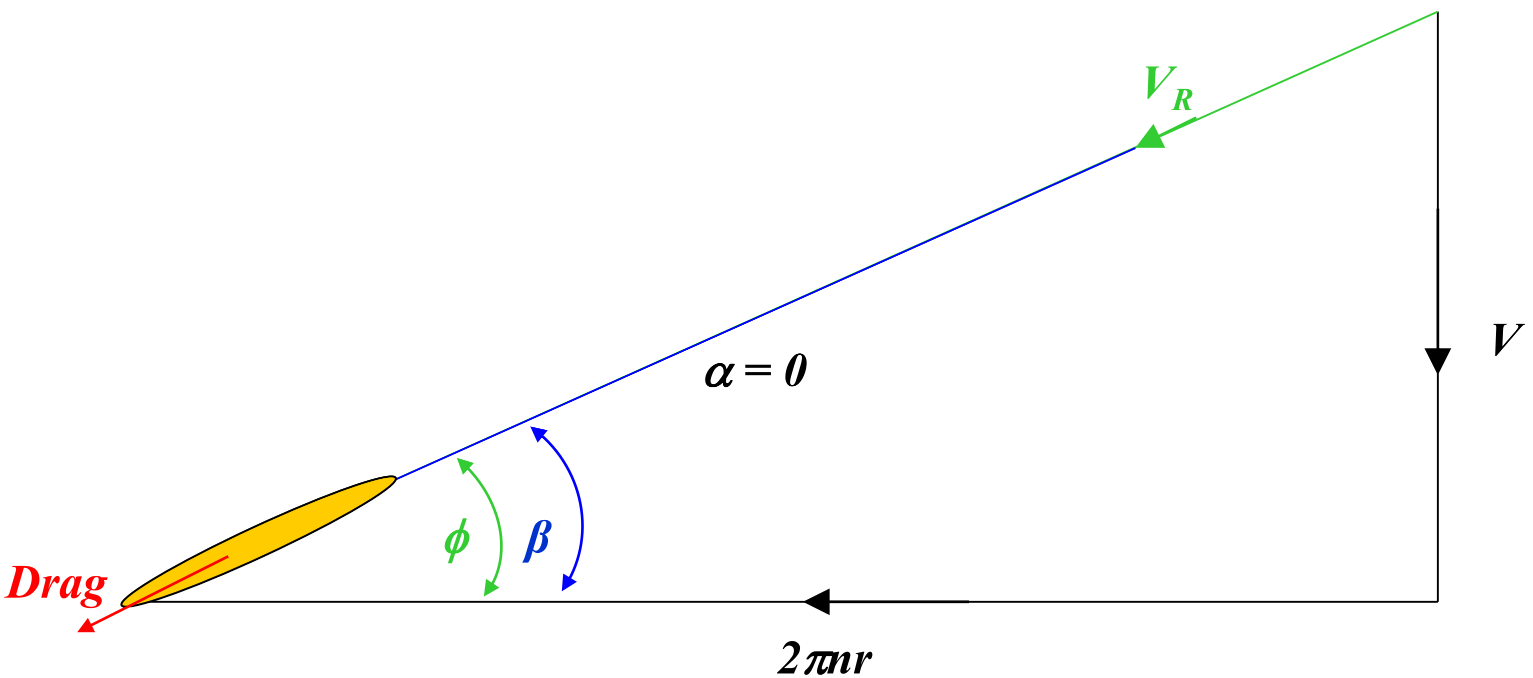
If the blade Pitch Angle β is fixed and the propeller rotational speed is constant, which is normally the case, then the optimum angle of incidence (for best lift/drag) occurs at only one aircraft velocity.

The velocity diagram below represents propeller inflow for an aircraft at the start of the take-off run with the angle of incidence "α" being a very large angle and well beyond the point of stall. Whilst some lift will be produced it will be small compared with the drag resulting in low thrust and high torque.



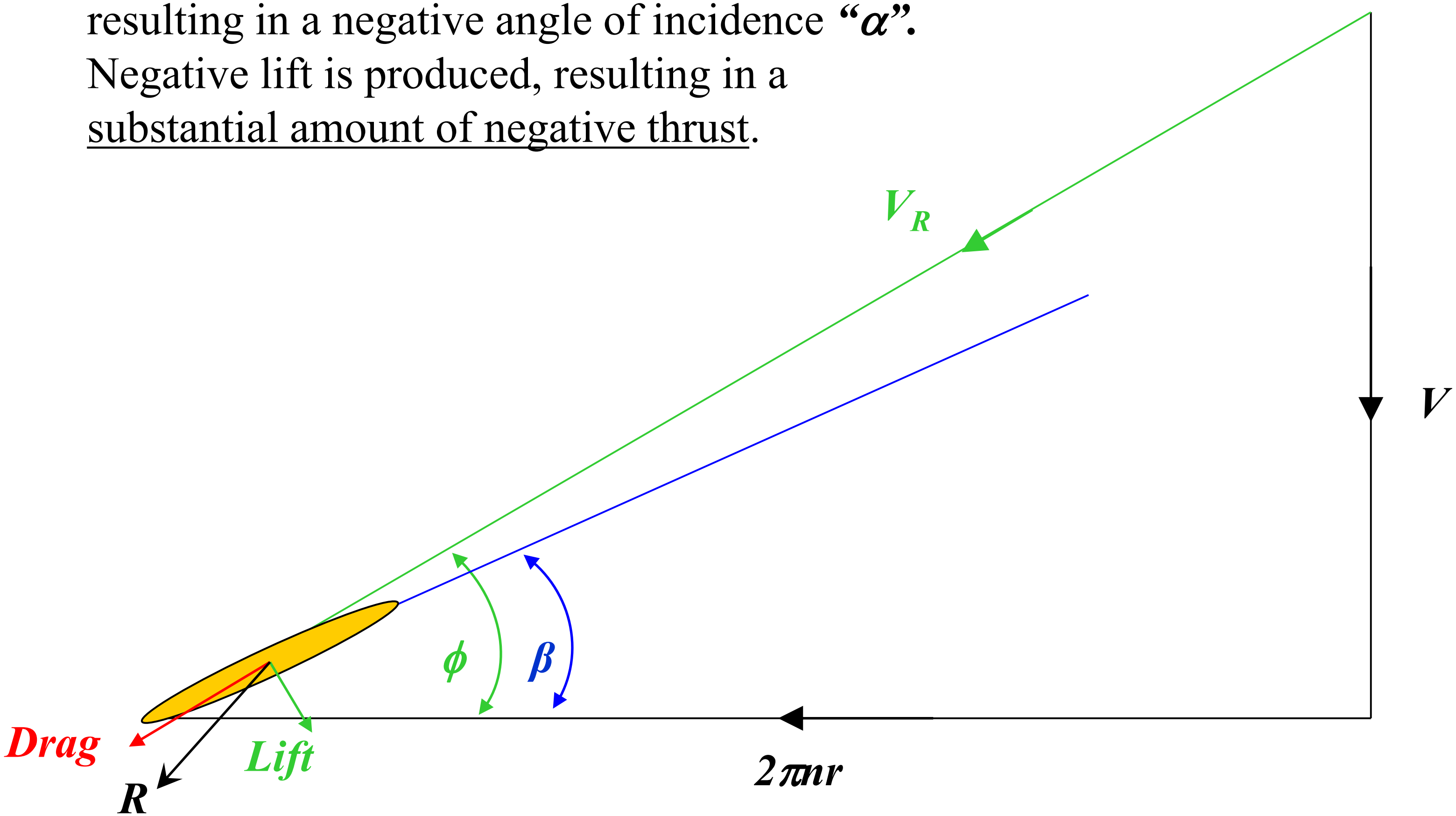
If the blade Pitch Angle (*β)* remains fixed and the propeller rotational speed is constant then there is a limit to the aircraft speed if it relies upon the thrust from the propeller.

The velocity diagram below represents propeller inflow for an aircraft in a shallow dive with the Inflow Angle equal to the Pitch Angle resulting in zero angle of incidence ***"α".*** No lift is produced, resulting in a small amount of negative thrust.



If the blade Pitch Angle β remains fixed and the propeller rotational speed is constant then the propeller can provide a limit to the aircraft speed, a useful feature when diving in cloud, akin to speed limiting brakes.

The velocity diagram below represents propeller inflow for an aircraft in a steep dive with the Inflow Angle exceeding the Pitch Angle resulting in a negative angle of incidence **"α"**. Negative lift is produced, resulting in a <u>substantial amount of negative thrust</u>.

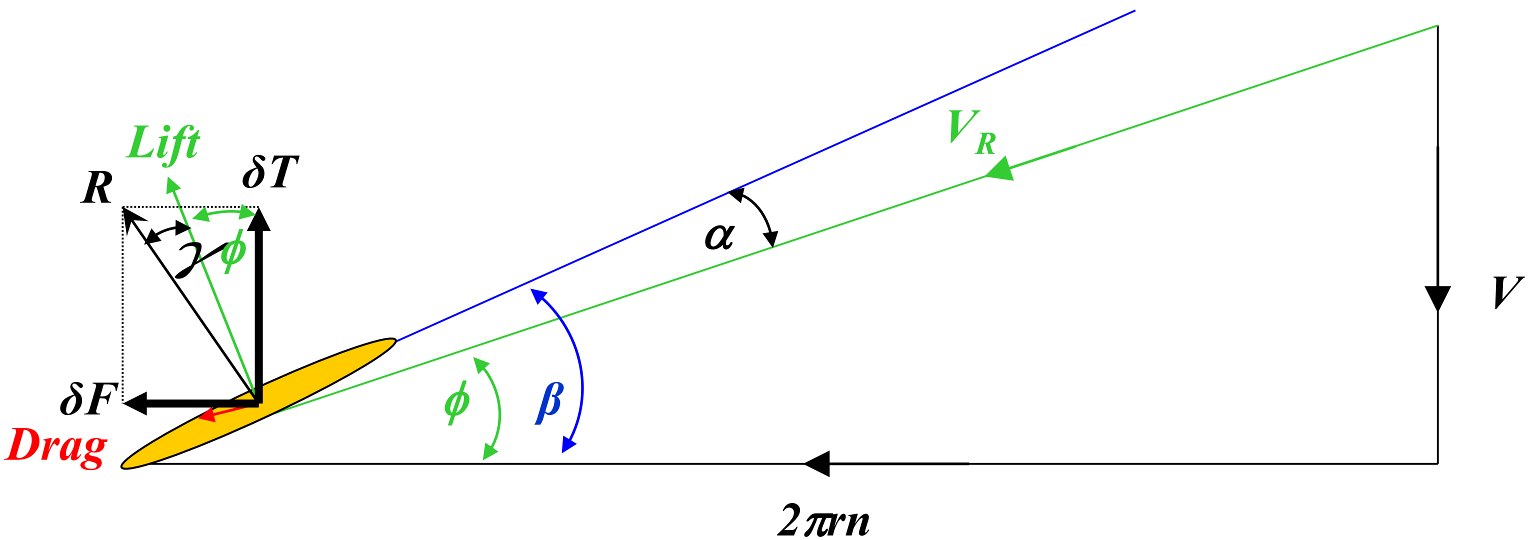


If γ is the angle between the lift component and the resultant force then:

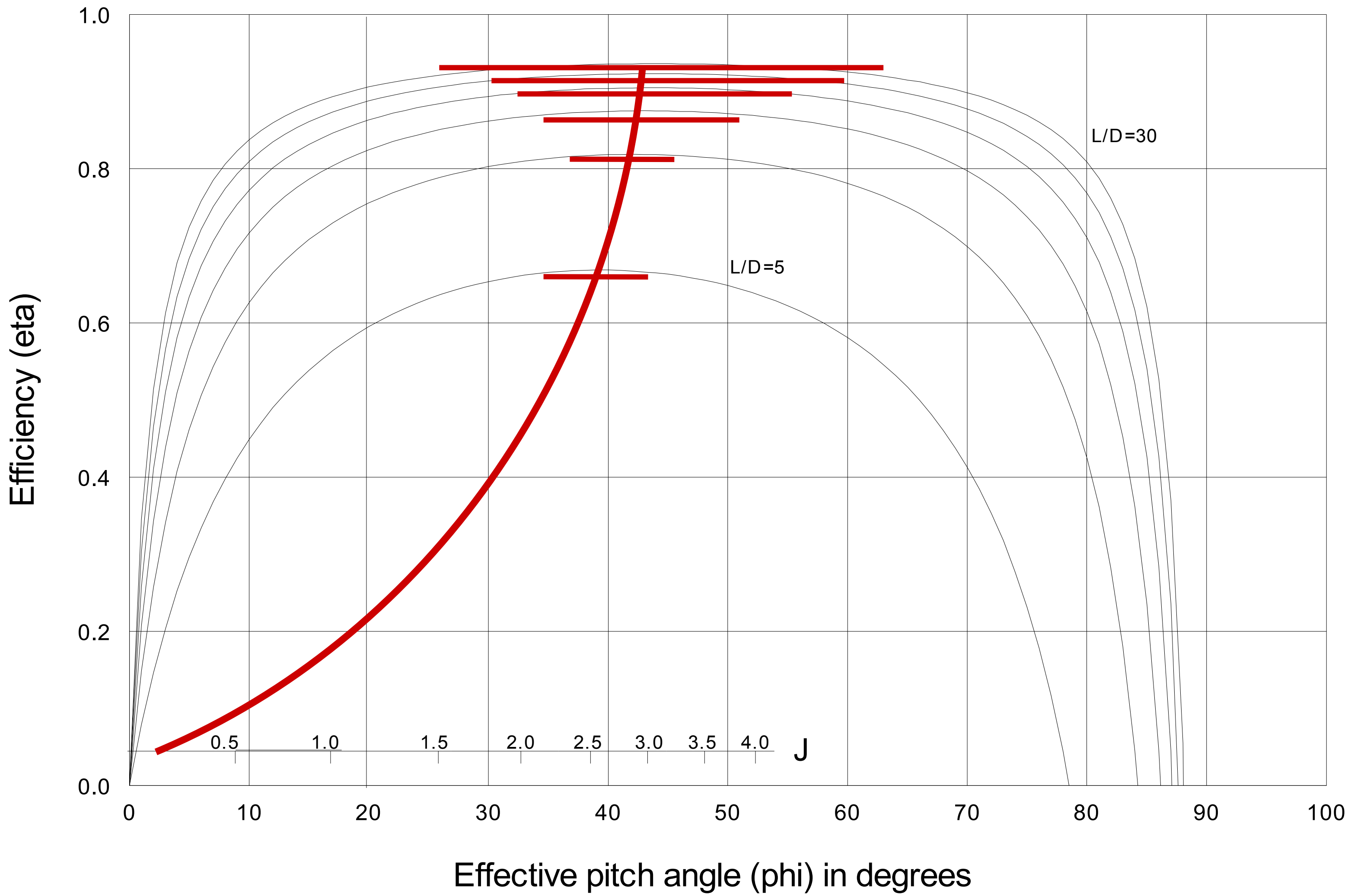
$$\boxed{\tan\gamma = \frac{D}{L} = \frac{C_D}{C_L}}$$

$$\boxed{\eta' = \frac{\delta T V}{\delta F\, 2\pi r n} = \frac{\delta R \cos(\phi + \gamma) V}{\delta R \sin(\phi + \gamma) 2\pi r n} = \frac{\tan\phi}{\tan(\phi + \gamma)}}$$

..from geometry or analytical analysis $\left(\text{as in notes using solidity } \sigma = \frac{Nc}{2\pi r}\right)$



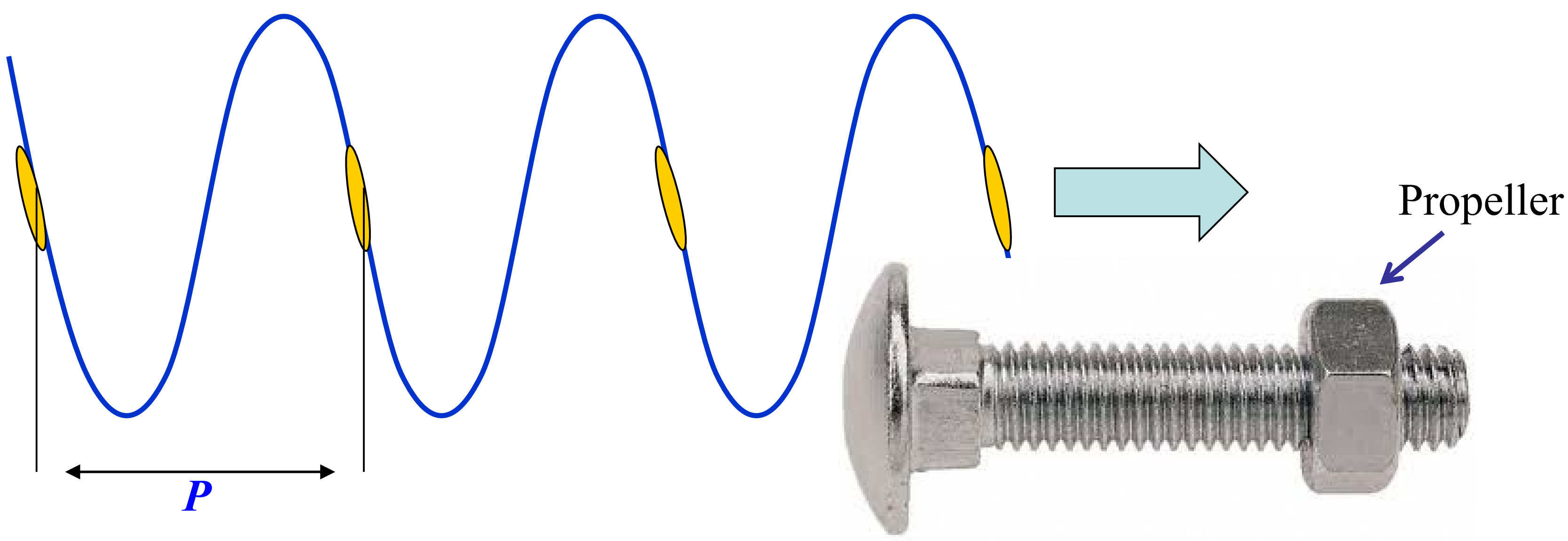
Blade-element efficiency as a function of effective pitch angle



Propeller Pitch

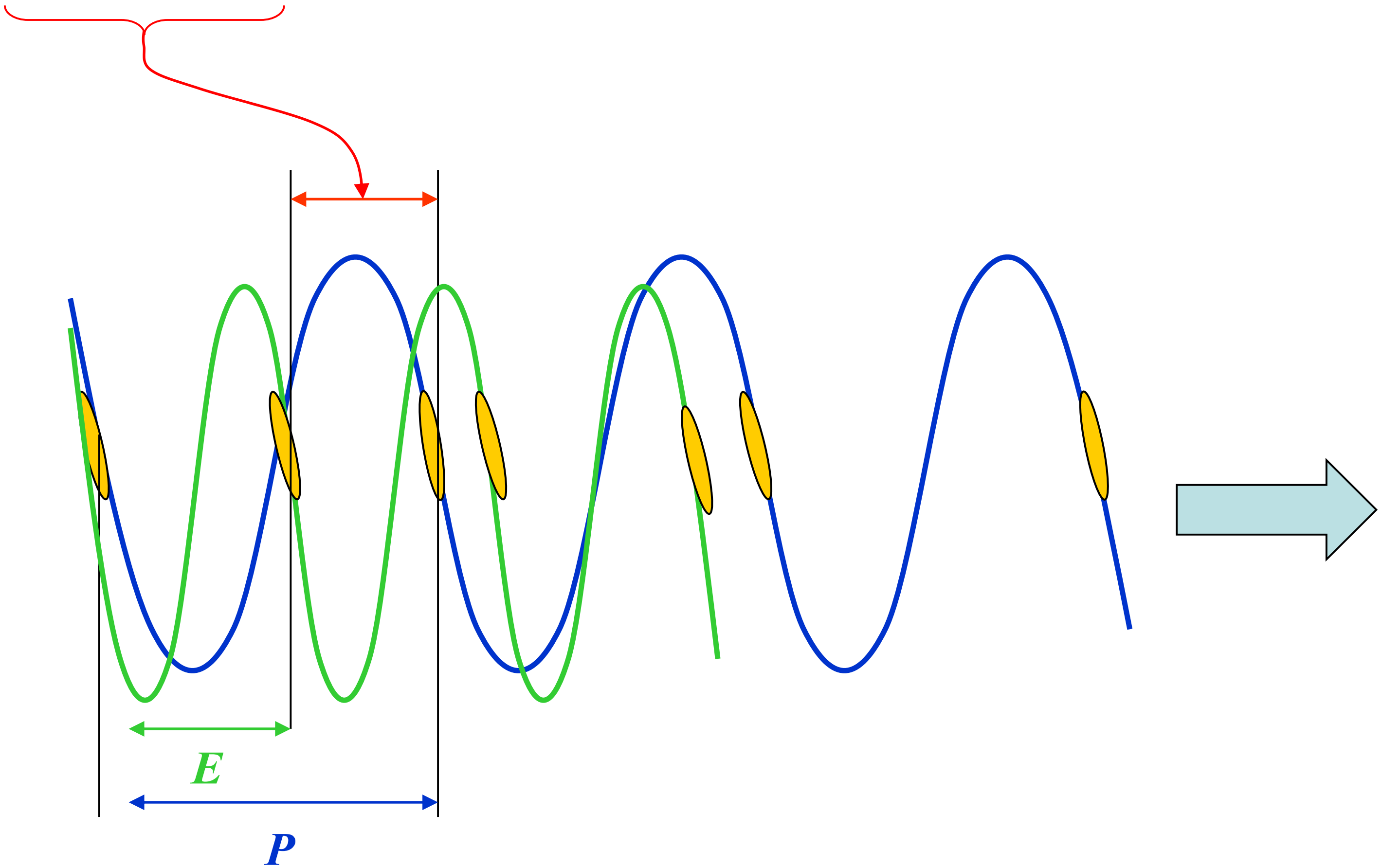
Whilst Propeller Pitch Angle (β) has been referred to in the previous illustrations, Propeller Pitch is a linear distance not an angle and if Propeller Pitch Angle is quoted it should state the radial station at which it is measured (e.g. $\beta_{0.75R}$)

The Geometric Pitch (P) of a propeller is the axial displacement of the propeller prescribed by the geometric chord in one revolution. This is analogous with the mechanical screw thread (which is why propellers were originally called "airscrews").

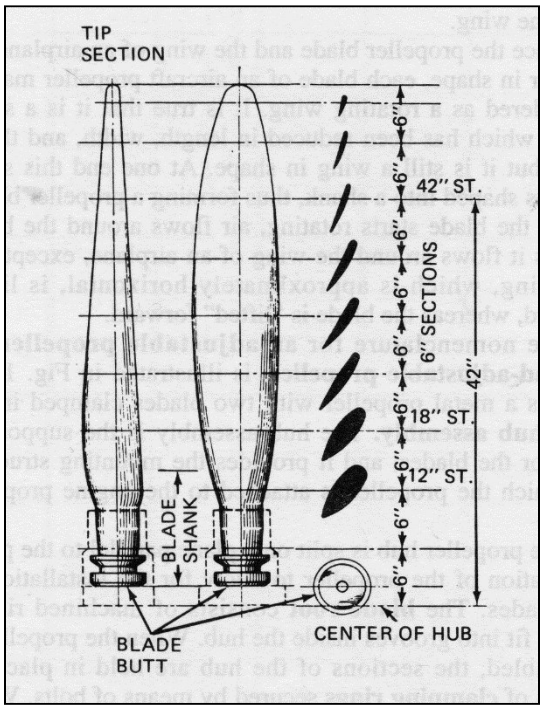


Propeller Pitch

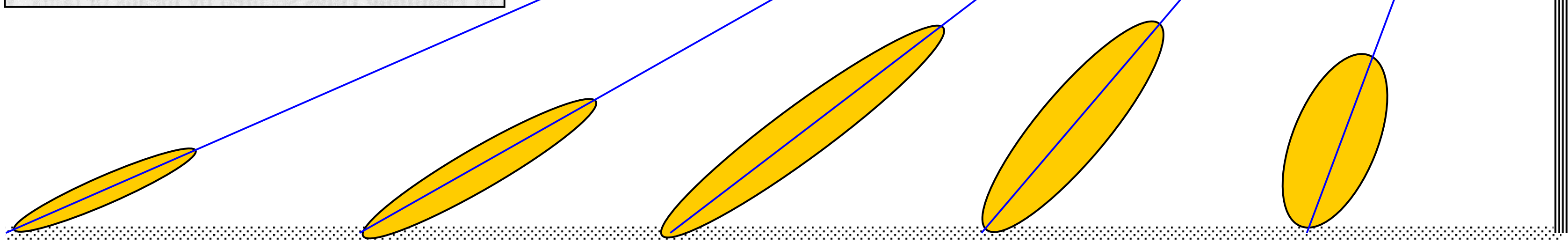
Whilst the Geometric Pitch (***P***) of a propeller is the theoretical axial displacement of the propeller prescribed by the geometric chord in one revolution, the Effective Pitch (***E***) is the actual axial displacement of the propeller. The difference between these two lengths is called the Propeller Slip. If thrust is to be produced there must be a finite value of Propeller Slip.



If the propeller blade has a radial twist such that all stations have the same Geometric Pitch the propeller is described as a Constant Pitch Propeller, as shown below.



It can be seen that the aerofoil section is thicker in the root area to support the high bending stresses at the propeller hub.



Tip -- r -- Root

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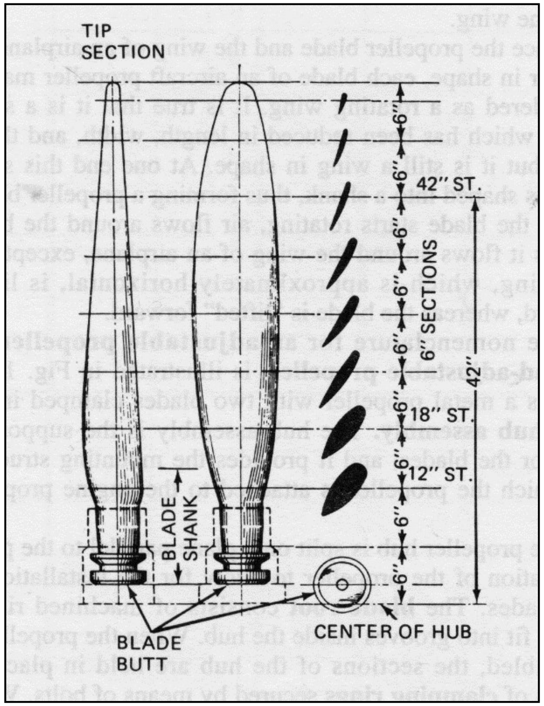

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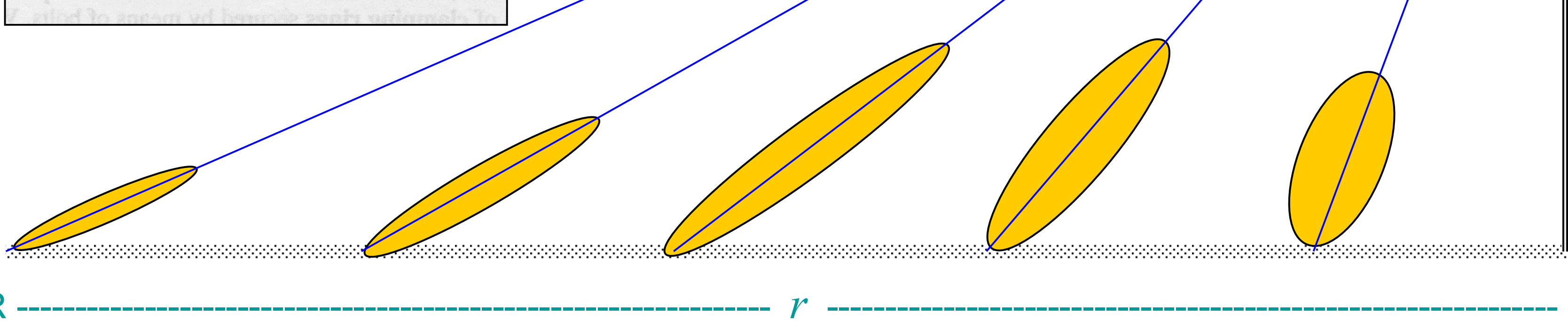
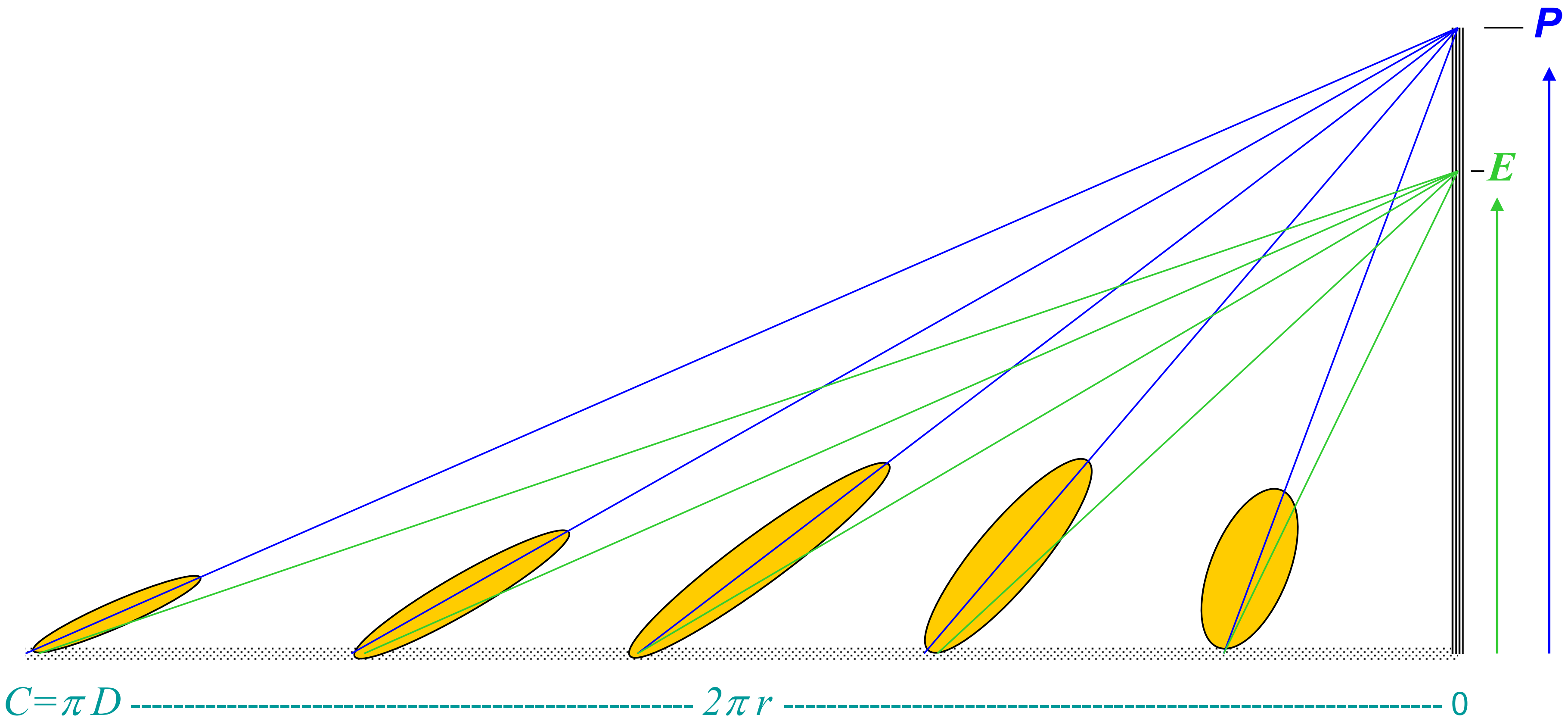


If the propeller blade has a radial twist such that all stations have the same Geometric Pitch the propeller is described as a Constant Pitch Propeller, as shown below.

The radial twist shown is normal for propellers. Whilst the rotational speed "***n***" is constant, the velocity in the plane of rotation increases with radius. Thus the static analysis that indicates Geometric and Effective Pitch



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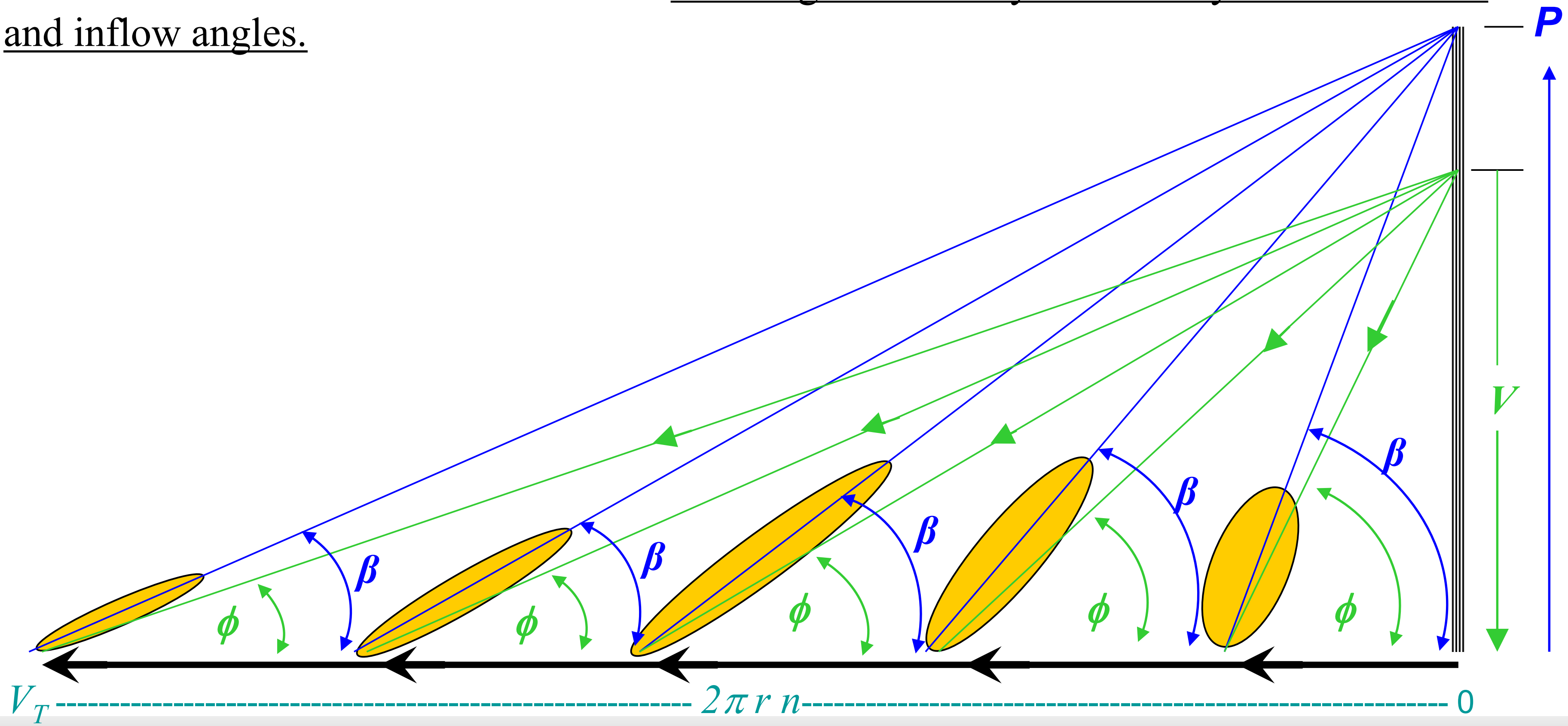
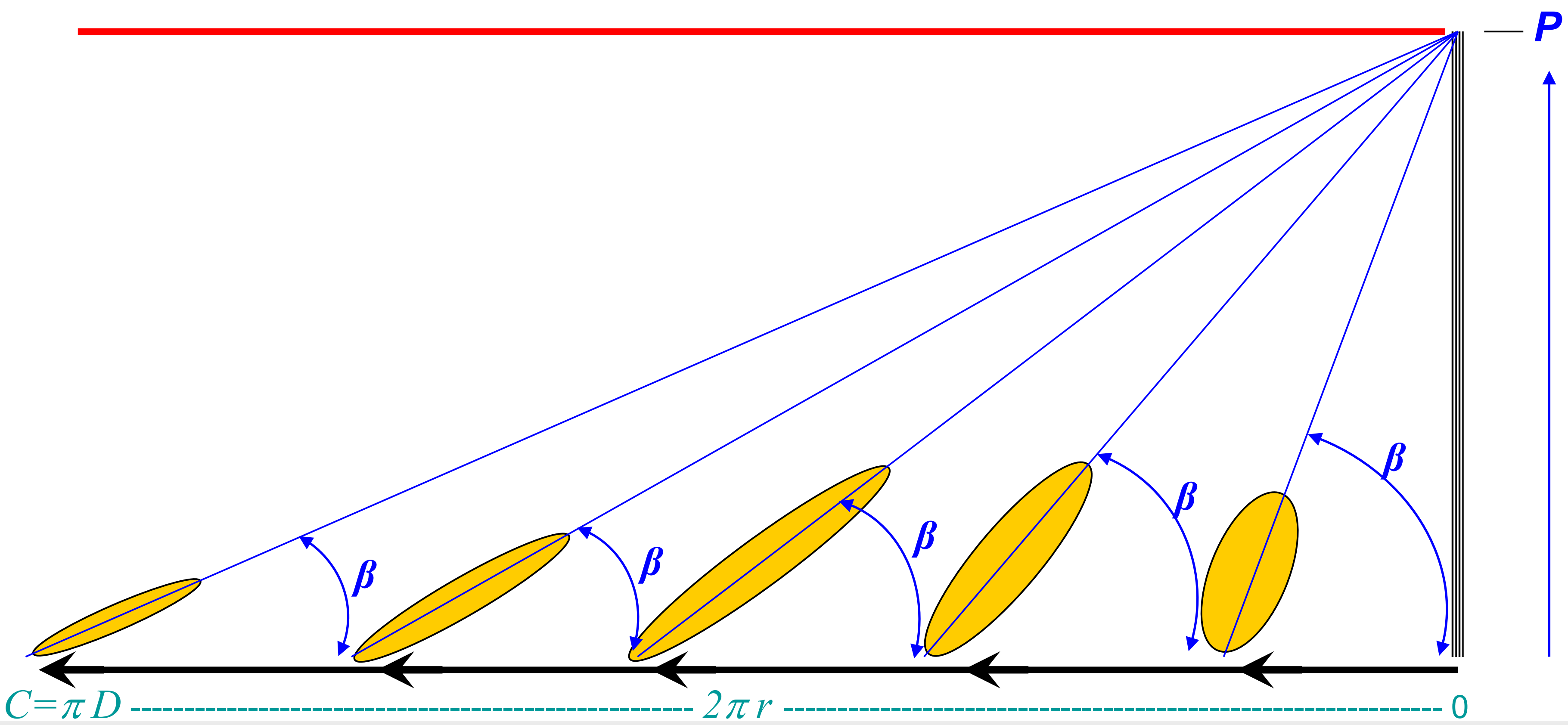
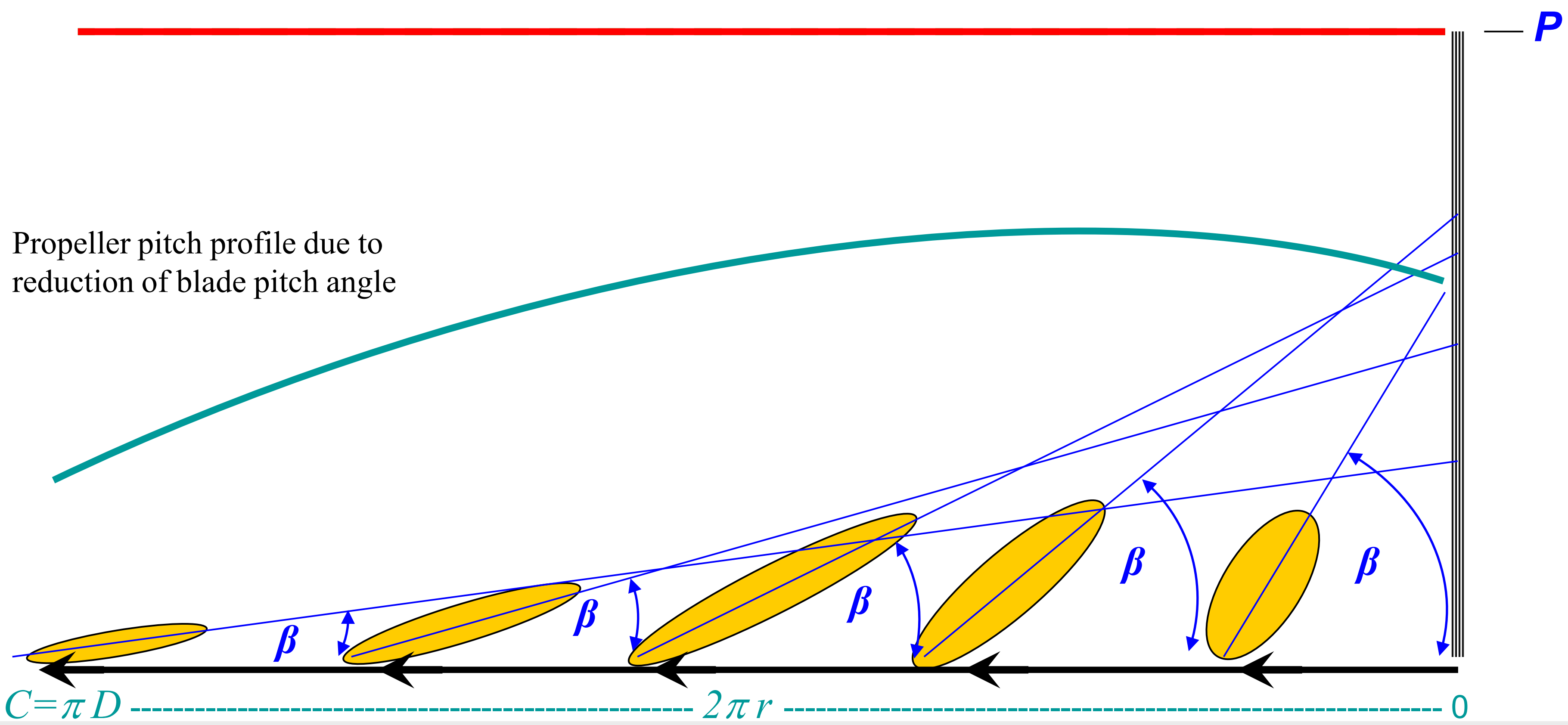
The radial twist shown is normal for propellers. Whilst the rotational speed “***n***” is constant, the velocity in the plane of rotation increases with radius. Thus the static analysis that indicates Geometric and Effective Pitch <u>is analogous to the dynamic analysis of velocities and inflow angles.</u>

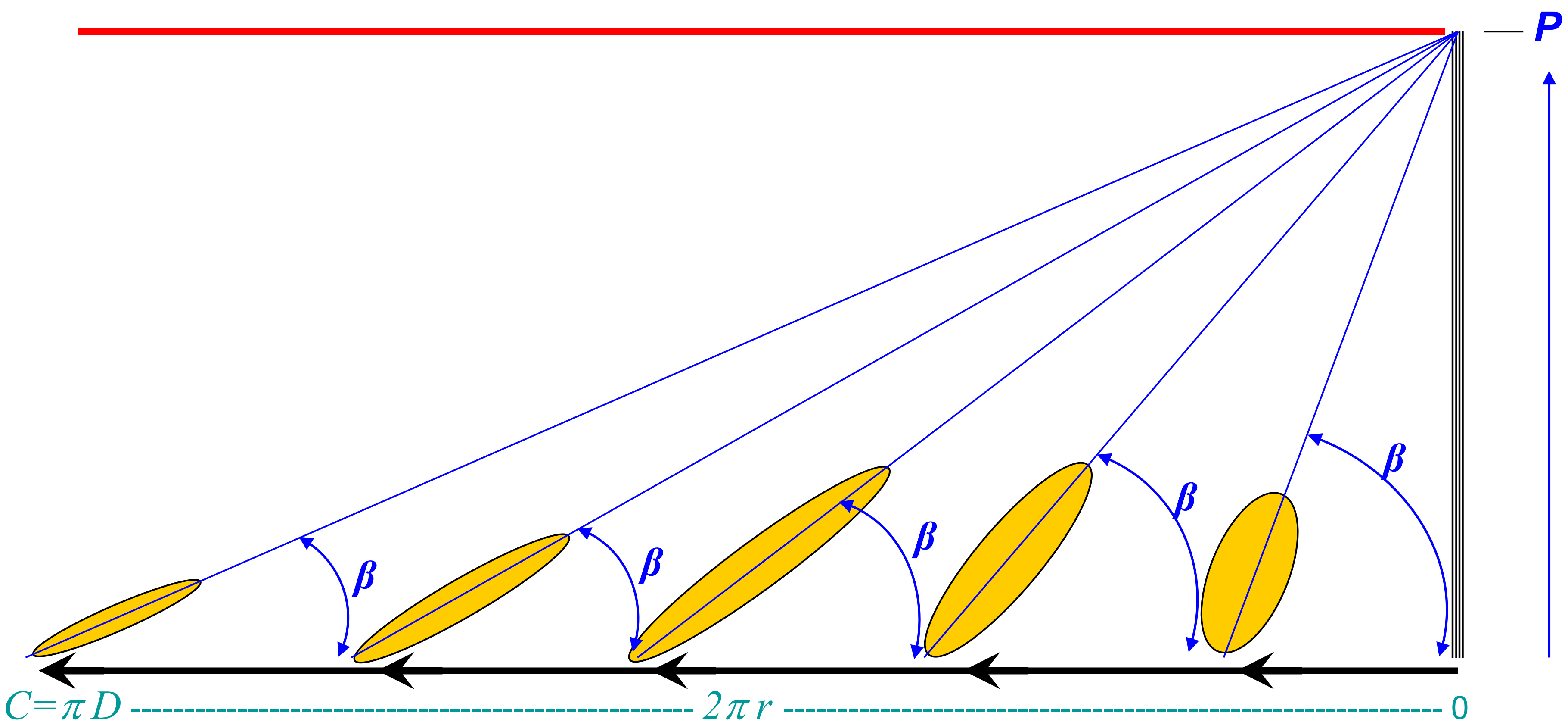


Considering just the Blade Pitch (P) which is **CONSTANT** here,

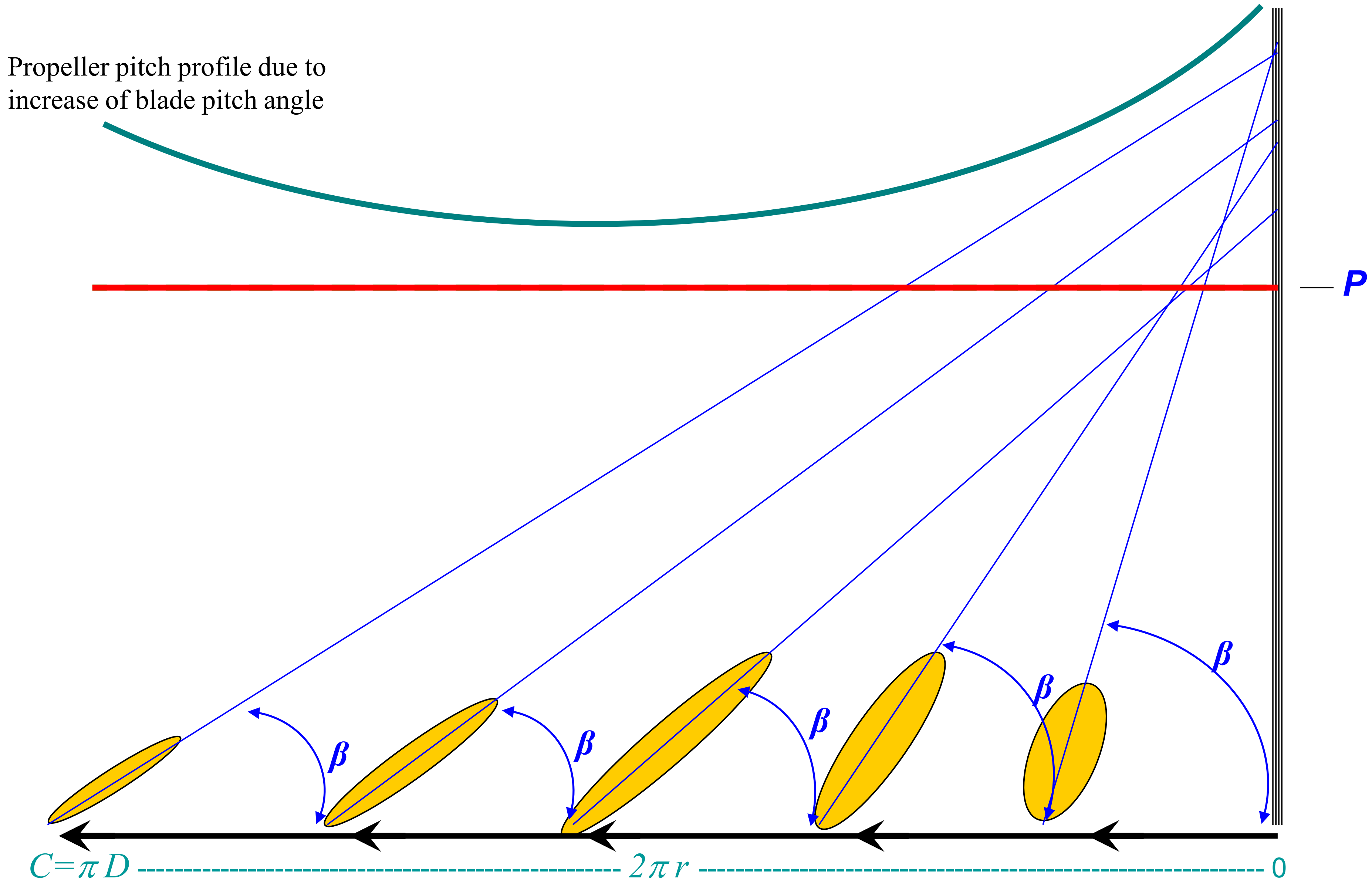


But for a reduction in Blade Pitch Angle (*β*) the Blade Pitch is **NOT CONSTANT**



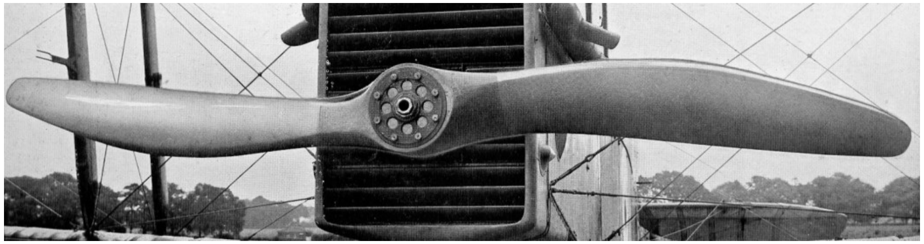

P
β
β
β
β
β
C=π D
2π r
0

and similarly for an increase in Blade Pitch Angle (β)
the Blade Pitch is **NOT CONSTANT**



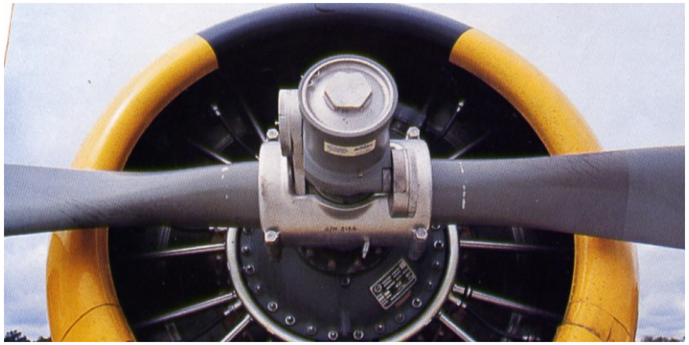
Propeller Types and Designations

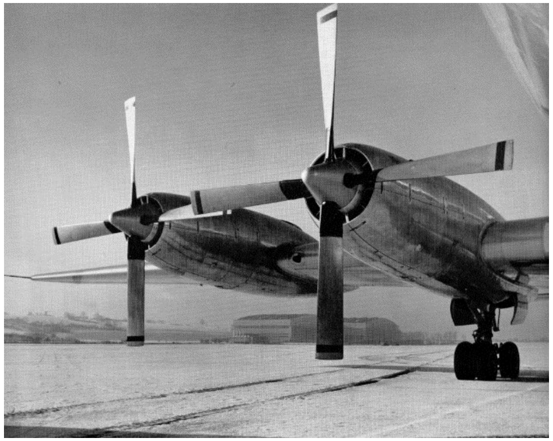
Single piece propellers are Fixed Pitch Propellers



Propellers with a form of automatic blade pitch control are known as Constant Speed Propellers

Adjustable Pitch Propellers have the facility to adjust blade pitch before or during flight





(From Lecture 3)

PROPELLER PERFORMANCE

The methods of propeller analysis described aid the design of propellers and can give reasonable predictions of performance. Lifting line methods and CFD will give better results but at a cost in time and computation.

General Aviation (GA) propellers are purchased "off the shelf" to best match the engine and airframe characteristics. Such proprietary propellers are tested in controlled conditions over a range of advance ratios (***J***) and **Performance Charts** issued for:

- single propellers– (fixed pitch)
- family of propellers (as above) of various pitches
- propellers that have 2 or 3 pitch settings – (adjustable)
- propellers with infinite pitch settings (including feathering) – controllable

The performance charts have a number of uses;

- comparing the performance of commercially available propellers
- choosing a propeller to suit a particular engine's power output characteristics
- establishing the best blade pitch setting for a flight mission
- determining the take-off or climb performance of an aircraft
- providing a measure in the development of a new propeller design

PROPELLER PERFORMANCE

Recalling the Coefficients used in the analysis of propeller performance

$$J = \frac{V}{nD} \text{ (where } J \text{ in the Advance Ratio)}$$

$$C_P = \frac{P}{\rho n^3 D^5}$$

$$C_T = \frac{T}{\rho n^2 D^4}$$

$$\eta_p = \frac{TV}{P} = \frac{C_T \rho n^2 D^4 V}{C_P \rho n^3 D^5} = \frac{C_T J}{C_P}$$

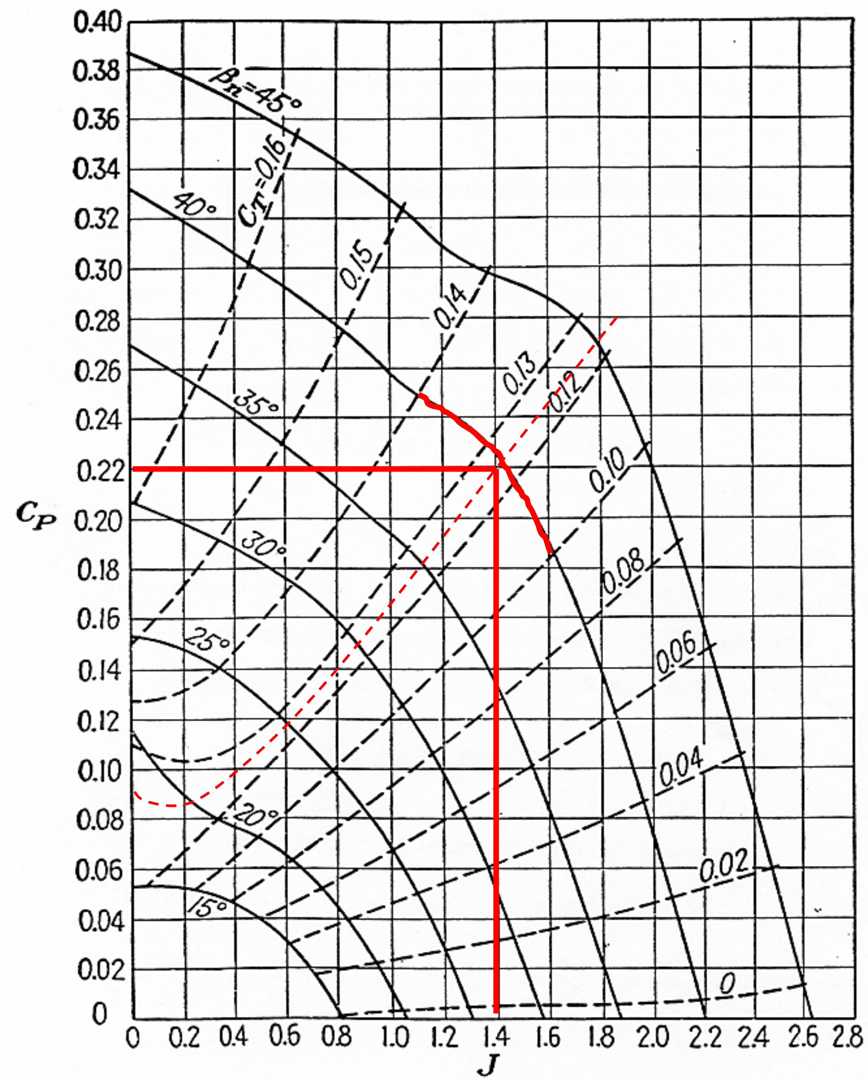
Where ρ = air density, n = propeller rotational speed (revs/sec), D = propeller diameter

PROPELLER PERFORMANCE CHARTS

A typical performance chart for a controllable pitch propeller is shown here.

For example: If the pilot knows the available engine power (*C_P=0.22*) and the speed at which he/she wishes to fly (*J=1.4*), then propeller pitch (*βn*), should be set to 40^0 and the available thrust, from the ***C_T*** value, (*=0.125*) can be determined.

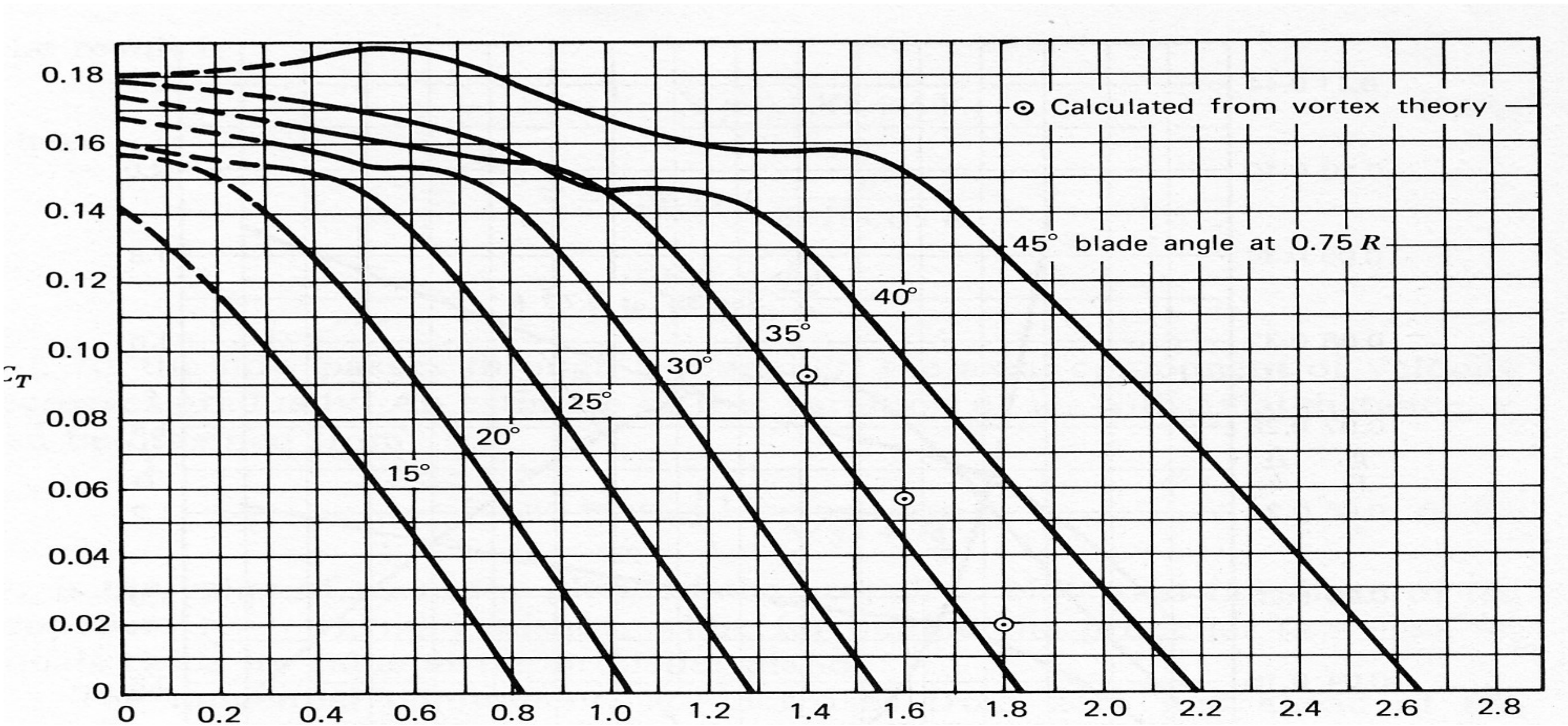
If the aircraft drag at this speed is known, then the rate of climb can also be calculated.



PROPELLER PERFORMANCE CHARTS continued

There are various forms of displaying empirical propeller performance data, often by separate charts for C_P, C_T and η_P

Inspection of the C_T against J chart reminds us of the basic flow vectors, namely that increasing J (and hence V) but holding β constant reduces the angle of incidence α.



J

(From Lecture 3)

PROPELLER PERFORMANCE CHARTS continued

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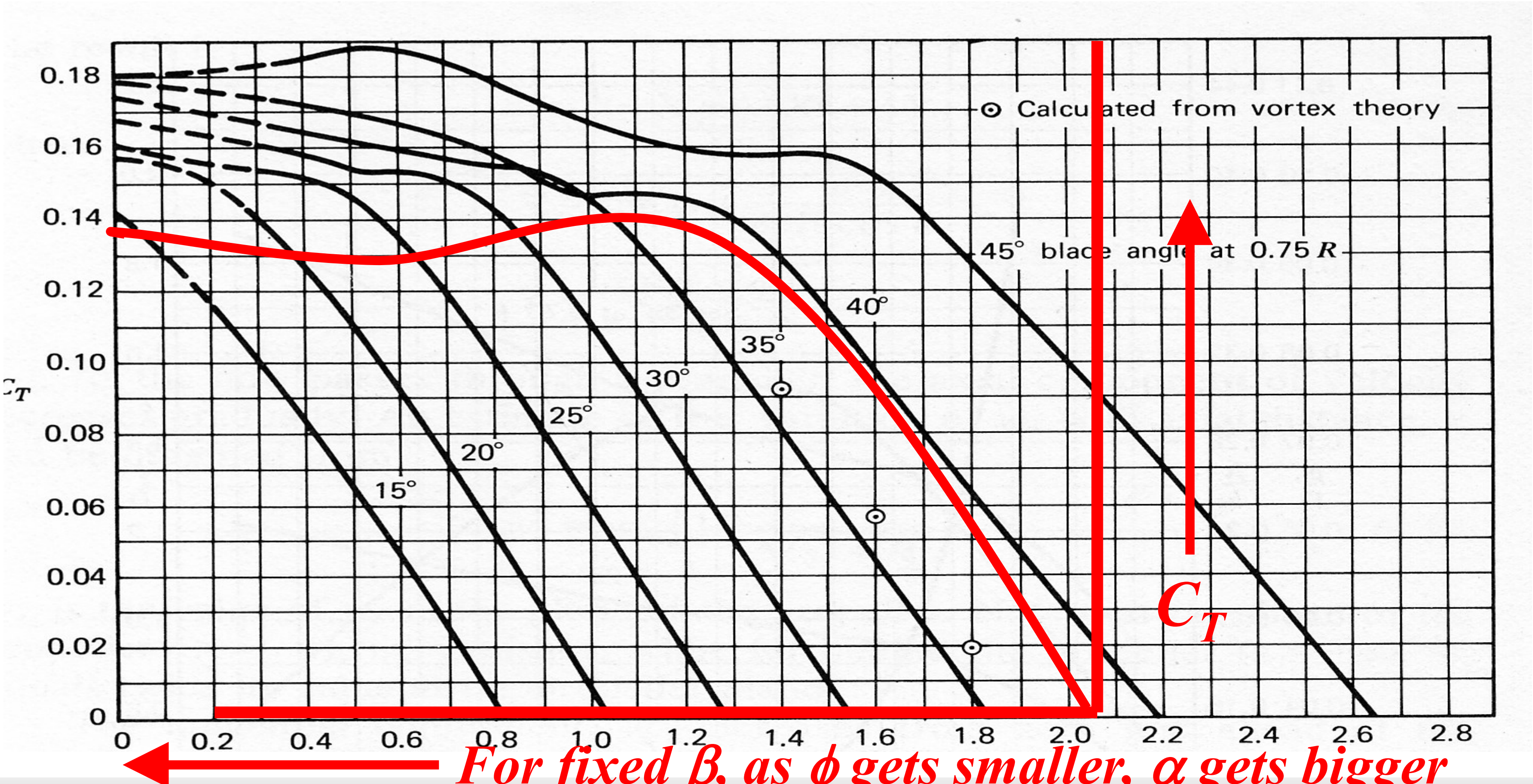
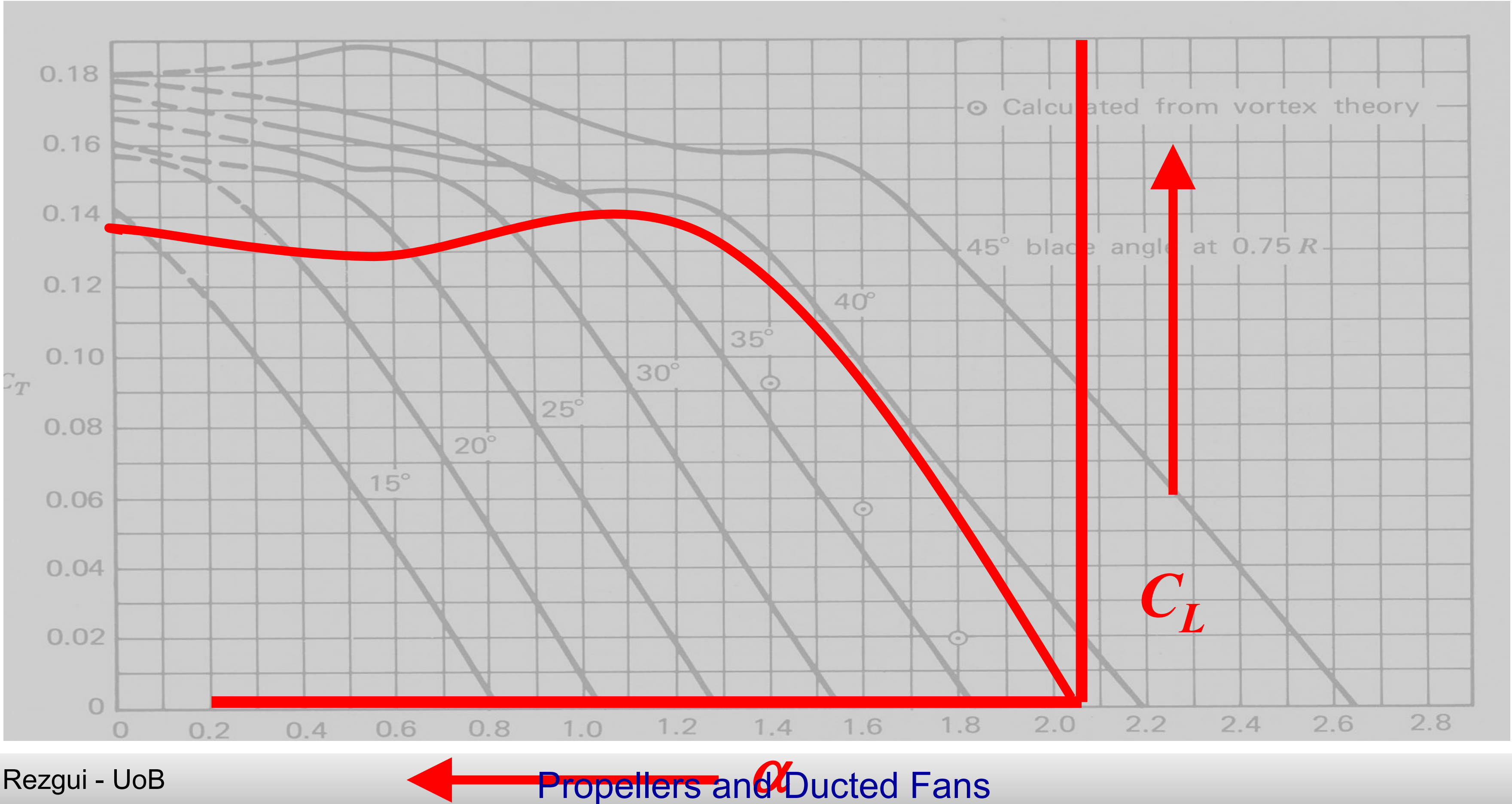


PROPELLER PERFORMANCE CHARTS continued

There are various forms of displaying empirical propeller performance data, often by separate charts for $\boldsymbol{C_P}$, $\boldsymbol{C_T}$ and η_P

Inspection of the $\boldsymbol{C_T}$ against J chart reminds us of the basic flow vectors, namely that increasing J (and hence V) but holding β constant reduces the angle of incidence α.



Typical Exam Question…..

A twin-engined turbo-prop airliner has a flight cruise speed of **324 km/hr**. The continuous power of it's engines (each rated **700 kW** at mean sea level) reduces linearly with altitude by **50%** to **36,000 ft**. The constant speed propellers are **3.2 m** diameter and have a rotational speed of **1800 rpm**

The aircraft has a mass of **14000 kg**, a wing area of **80m** and the drag equation of the aircraft is:

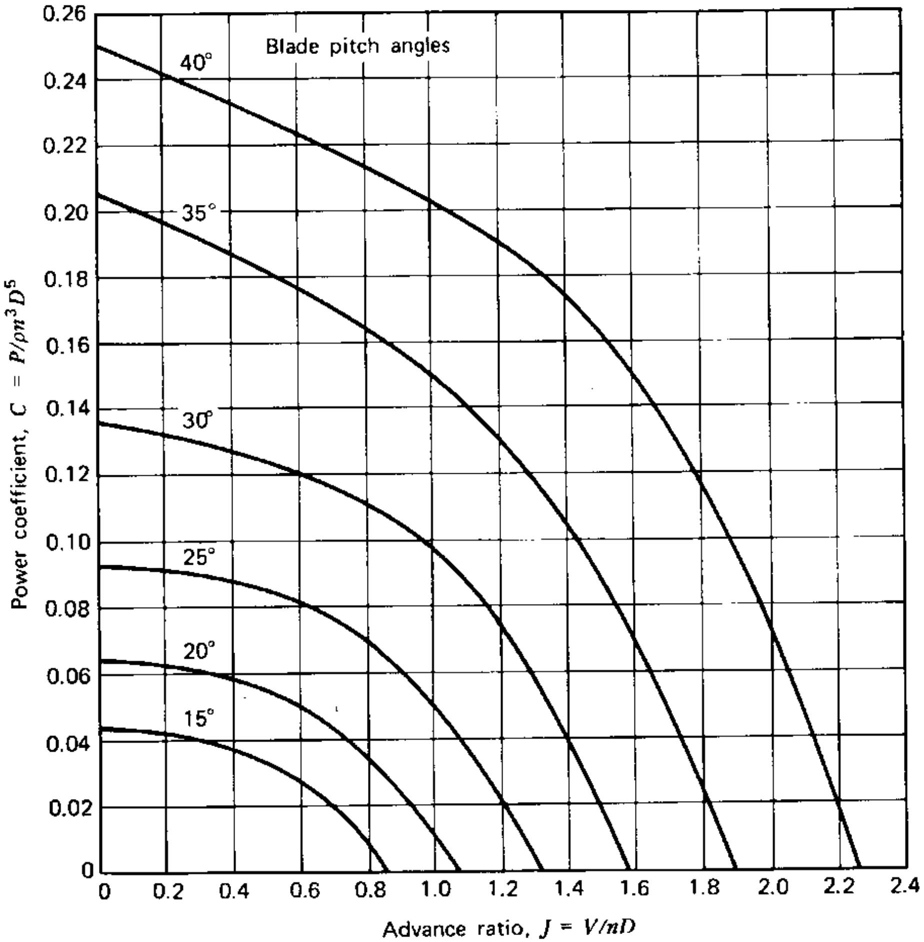
$$C_D = 0.016 + 0.04 C_L{}^2$$

The aircraft is allowed to enter controlled airspace at **FL 170 (17000ft)** on condition that it's rate of climb to **FL 190** is never less than **300 ft/min.**

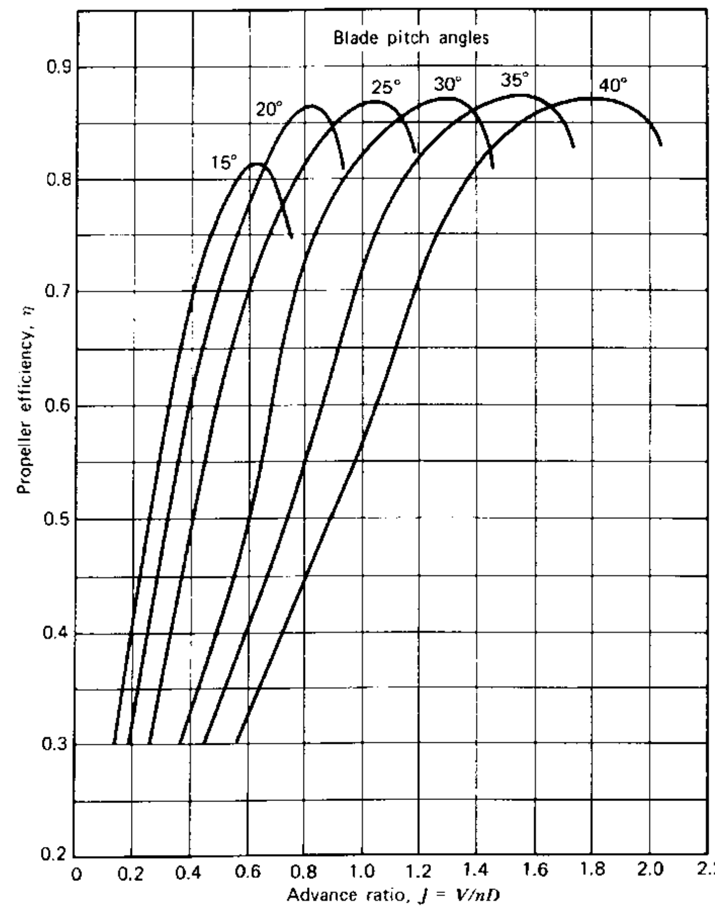
Using the propeller data sheets provided and showing all calculations, determine:

a) whether the aircraft can enter the controlled airspace

b) how long it would take to climb to the higher flight level, (neglecting fuel burn).



Blade pitch angles
40°
35°
30°
25°
20°
15°
Power coefficient, $C = P/\rho n^3 D^5$
0.26
0.24
0.22
0.20
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0
0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0 2.2 2.4
Advance ratio, $J = V/nD$



Blade pitch angles
15°
20°
25°
30°
35°
40°
Propeller efficiency, η
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0 2.2
Advance ratio, $J = V/nD$

$$Rate\ \text{of Climb (ROC)}\ \frac{\Delta \mathrm{P}}{\mathrm{W}} = \frac{Power\_avialable - Power\_required}{\mathrm{W}}$$

Power_available = Propulsive power of the propeller
= Engine power x propeller efficiency

Power_required = Power needed to overcome A/C drag at the required speed
= Drag x V

$Start\ by\ listing\ the\ parameters\ \ required\ :$

$D = propeller\ diameter\ (m) = 3.2$

$n = propeller\ \ speed\ (revs/sec) = 30$

$\rho = air\ density\ (kg/m^3\ )\ at\ altitude \left(\rho = \dfrac{20 - H}{20 + H} \right) where\ H\ is\ in\ km$

$In\ this\ question, the\ available\ power\ (P_a\ )\ is\ given:$

$P_a = 700kw\ at\ SL, reducing\ to\ 535kw\ at\ 17{,}000ft$

$\rho = 1.225\ kg/m^3\ \ at\ SL, reducing\ to\ 0.721kg/m^3\ \ at\ 17{,}000ft$

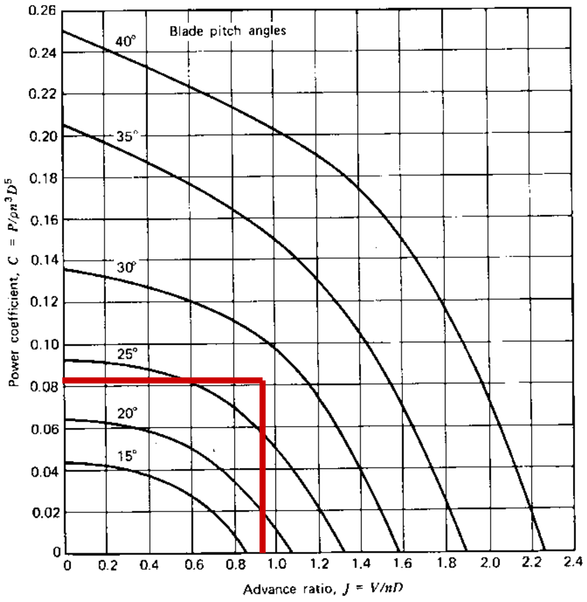
$So\ \ if\ \ D = 3.2, N = 30\ and\ P_a = 535kW,$

$$then\ C_P = \frac{P_a}{\rho n^3 D^5} = \frac{535.E3}{0.721x27.E3x335} = 0.082$$

$$Now\ \ for\ \ V = 324\ km/h, which\ is\ 90\ m/s, then\ J = \frac{V}{nd} = 0.937$$

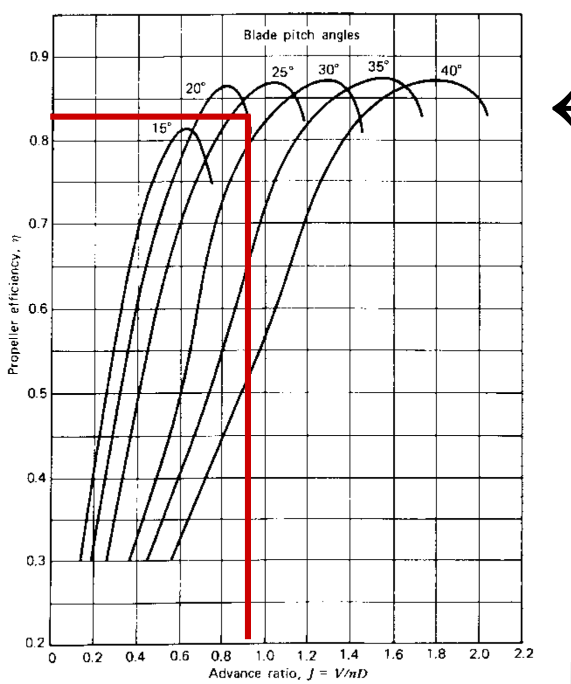
Using the graph for

$$J = 0.937 \; and \; C_P = 0.082$$



← **Graph of C_P against J for various blade pitch angles as measured at 0.75R**

For $C_P = 0.082$ and $J = 0.937$, $\beta = 27.5^o$



← **Graph of C_P against propeller efficiency for various values of pitch angles**

For $J = 0.937$ and $\beta = 27.5^o$, $\zeta = 0.83$

$$\textit{From} \text{ the Graph, for J} = 0.937 \text{ and } \mathrm{C_P} = 0.082$$

$$\textit{then } \beta = 27.5^o \textit{ and } \varsigma = 0.83$$

Thrust Power available is 0.83 x 535 = 444 kw per propeller

Thus total propulsive force is 888 kw

Consider now the aircraft drag :

$$\mathrm{C_L} = \frac{2W}{\rho V^2 S} = \frac{2x14000x9.81}{0.721x(90)^2 x80} = 0.588, \text{ so } \mathrm{C_D} = 0.0298$$

$$\textit{Thus} \text{ D} = 6966\text{N, so } \mathrm{P_{required}} = 626983w$$

$$\textit{Rate} \text{ of Climb (ROC) } \frac{\Delta \mathrm{P}}{\mathrm{W}} = \frac{(888-627)E3}{14000x9.81} = 1.89\ m/s$$

$$1.89\ m/s = 374 \text{ ft/min (which is} > 300 \text{ ft/min required)}$$

Perfect Answer = full marks

$$D = 3.2, n = 1800/60 = 30, Pow. = 700kw$$

$$C_P = \frac{Pow.}{\rho n^3 D^5}$$

$$17000\,ft = 5182m, \rho = \frac{14.818}{25.182}(1.225) = 0.721$$

$$Thrust\ \ power\ available\,at\,17000\,ft = 534.7kw$$

$$C_P = \frac{534700}{6532041} = 0.082, V = 90, J = 0.937$$

$$From\ graph,$$

$$\beta = 27.5^o, \varsigma = 0.83$$

$$Thrust\ \ power\ available = 443.8kw(x2)$$

$$C_L = \frac{2W}{\rho V^2 S} = 0.588, C_D = 0.0298$$

$$D = 6966N, P_{req.} = 626983w$$

$$ROC = \frac{\Delta P}{W} = \frac{887600 - 626983}{137340} = 1.89m/s = 374\,ft/\mathbf{min}\ and\ REPEAT\ for 19000\,ft$$

Notice that both power required and power available depends on air density and hence height

Extra Questions:

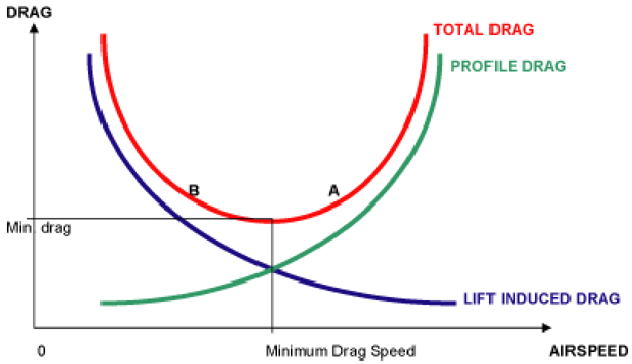
- *b) how long it would take to climb to the higher flight level, (neglecting fuel burn).*
  - *See Matlab file*

- *c) What is the maximum altitude the aircraft can get to?*
  - *See Matlab file*

- *d) At height of 16000ft, what is the maximum straight and level speed of the aircraft?*
  - *Similar to Question “c” but now vary the speed for constant height.*

Minimum Drag Speed

$$C_D = 0.0154 + 0.0192{C_L}^2$$



The minimum drag can be obtained when C_d /C_L is minimum $(Drag = W\left(\frac{Drag}{Lift}\right) = W\left(\frac{C_D}{C_L}\right))$

Hence for a given Weight

0.0154=0.0192 CL^2. ie when CL=0.9 (see below another method for obtaining the drag speed)

$$V_m = \sqrt{\frac{2Mg}{0.9\rho S}} = \dots$$

Alternatively, the minimum drag speed V_m can be directly obtained by differentiating the drag equation (as a function of V) with respect to V and equating the differential to zero (for a give weight W), i.e.

$$\frac{dDrag}{dV} = \frac{d}{dV}\left(\frac{1}{2}\rho SV^2 \times 0.0154 + \frac{0.0192 \times W^2}{\frac{1}{2}\rho SV^2}\right) = 0$$

This gives $V_m = \left(\frac{0.0192}{0.0154}\right)^{0.25}\sqrt{\frac{2W}{\rho S}}$