Vibrations 2, Lecture 10
Other forms of excitation

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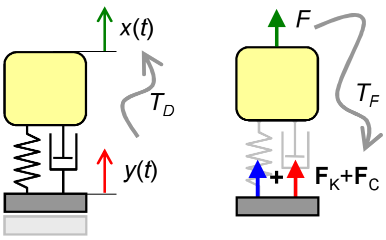
Lecture 9

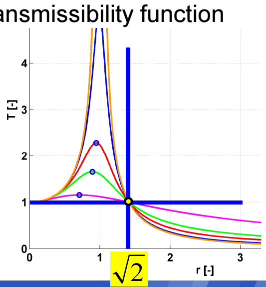
Vibration transmission model



Vibration response due to base motion

$$|X| = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}\,Y$$

Transmissibility function



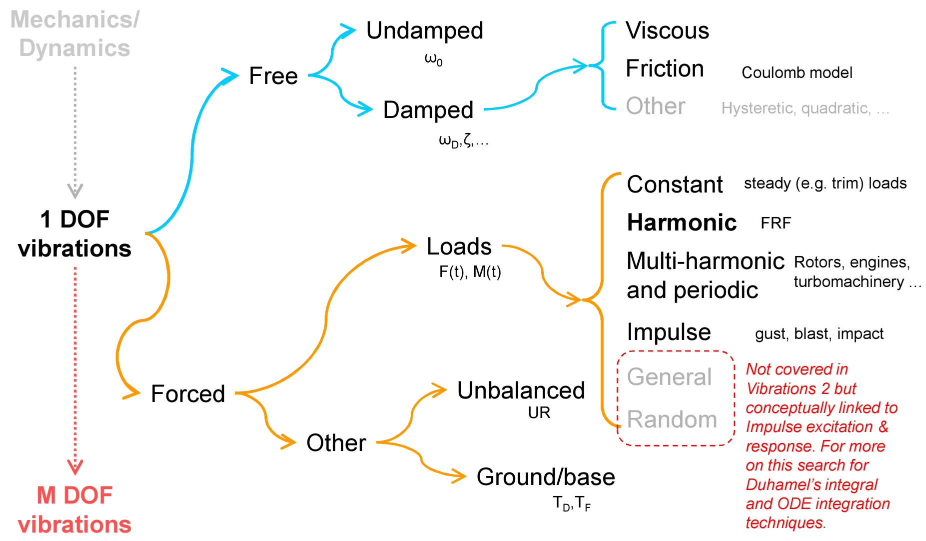
Displacement and force transmissibility

$$T_D = \frac{|X|}{Y} = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} = \frac{|F_{kc}|}{F_0} = T_F$$

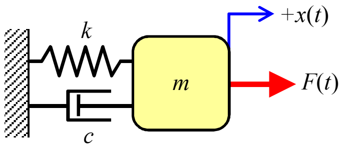
Lecture 10

- Multi-harmonic excitation
- Impulse excitation
- Solved example

Vibrations map (so far) …



Multi-harmonic excitation



Equation of motion:

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

$$F(t) = F_{C,1}\exp(i\omega_1 t) + F_{C,2}\exp(i\omega_2 t)$$

$$F(t) = F_{0,1}\sin(\omega_1 t + \phi_1) + F_{0,2}\sin(\omega_2 t + \phi_2)$$

From mathematics: linear systems can be solved using the **principle of superposition** where the total solution is the sum of all partial solutions resulting from individual components of excitation:

The partial steady state solutions:

$$m\ddot{x}_1 + c\dot{x}_1 + kx_1 = F_{C,1}\,e^{i\omega_1 t} \rightarrow x_1 = (H(\omega_1)F_{C,1})e^{i\omega_1 t}$$

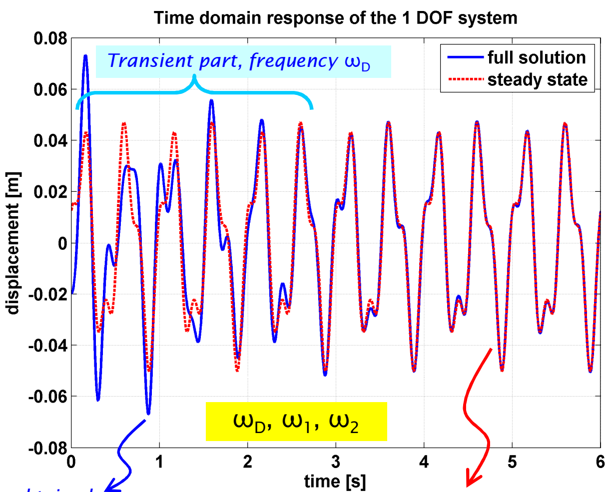
$$m\ddot{x}_2 + c\dot{x}_2 + kx_2 = F_{C,2}\,e^{i\omega_2 t} \rightarrow x_2 = (H(\omega_2)F_{C,2})e^{i\omega_2 t}$$

The total steady-state solution:

$$x(t) = x_1(t) + x_2(t) = H(\omega_1)F_{C,1}\,e^{i\omega_1 t} + H(\omega_2)F_{C,2}\,e^{i\omega_2 t}$$

Multi-harmonic excitation

$$F(t) = \text{Imag}((100+5i)\exp(i2\pi\underline{2}t) + (20-50i)\exp(i2\pi\underline{5}t))$$



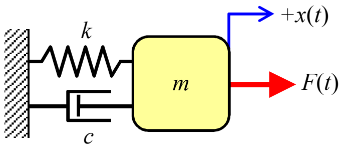
$m = 8\,kg$
$c = 11\,N.s/m$
$k = 4000\,N/m$

$x(0) = -0.02\,m$
$\dot{x}(0) = 0.03\,m/s$

Full solution obtained integrating ODE with F(t)

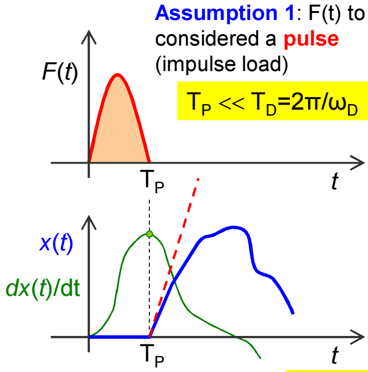
Steady-state solution obtained using approach from slide 5

Impulse excitation



Assumption 1: F(t) to be considered a **pulse** (impulse load)

$T_P << T_D=2\pi/\omega_D$



Assumption 2: $x(T_P)\approx 0$, $dx(T_P)/dt \neq 0$

Equation of motion:

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

We may include assumption 3 (not required): low damping (i.e. $\zeta \to 0$)

To see what happens when $t=T_P$ we try to integrate EOM from 0 to T_P:

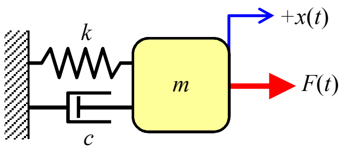
$$\int_0^{T_P} \left(m\frac{d\dot{x}}{dt} + c\frac{dx}{dt} + kx\right)dt = \int_0^{T_P} F(t)\,dt$$

$$\int_0^{T_P} (m\,d\dot{x} + c\,dx + kx\,dt) = I$$

$$\left[m\,\dot{x}(t) + \cancel{c\,x} + k\,\cancel{x\,t}\right]_0^{T_P} = I$$

$$m\,\dot{x}(T_P) - m\,\dot{x}(0) = I \Rightarrow \underline{\dot{x}(T_P) = I/m}$$

Impulse excitation



Assumption 1: F(t) to be considered a **pulse** (impulse load)

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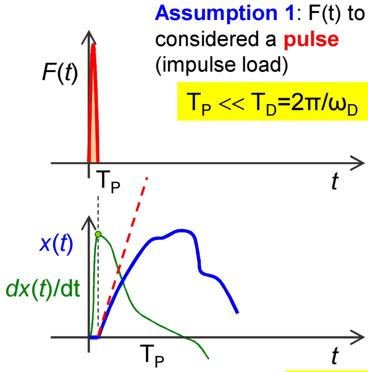


Assumption 2: $x(T_P)\approx 0$, $dx(T_P)/dt=I/m$

Equation of motion:

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

$$x(t) = e^{-\zeta\omega_0 t}\left(A\cos(\omega_D t) + B\sin(\omega_D t)\right)$$

Initial conditions: $T_P \to 0$

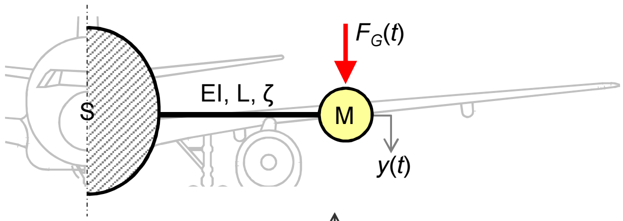
$$x(0)=0, \dot{x}(0)=I/m \qquad I = \int_0^{T_P} F(t)\,dt$$

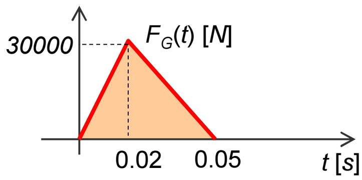
Solution: substitute initial conditions

$$\boxed{x(t) = \frac{I}{m\omega_D} e^{-\zeta\omega_0 t}\sin(\omega_D t)}$$

Example: wing gust response

A simplified equivalent 1 DOF model of aircraft's wing bending vibration is considered. The effective lumped mass due to wing structure and engine is M=4500 kg. A wing is modeled as prismatic beam with bending stiffness EI=4.3×10^7 N.m^2 and length L=6 m. Damping ratio of the system is ζ=12 %. Find the damped natural frequency and the motion of the mass (wing) due to specified gust load.





Example 12

Step 1: Equation of Motion and system parameters

$$M\ddot{y} + c_W\dot{y} + k_W y = F_G(t)$$

$$k_W = \frac{3EI}{L^3} = \frac{3(4.3\times10^7\ N.m^2)}{(6\,m)^3} = 5.972\times10^5\ N/m$$

$$\zeta = \frac{c_W}{2\sqrt{Mk_W}} \Rightarrow c_W = 2\zeta\sqrt{Mk_W} = \ldots = 1.244\times10^4\ N.s/m$$

$$\omega_0 = \left(\frac{k_W}{M}\right)^{1/2} = \ldots = 11.52\,rad/s \Rightarrow f_0 = 1.83\,Hz$$

$$\omega_D = \omega_0\left(1-\zeta^2\right)^{1/2} = \ldots = 11.44\,rad/s \Rightarrow f_D = 1.82\,Hz$$

$$T_D = 1/f_D = 1/(1.82\,Hz) = 0.55\,s$$

Example 12

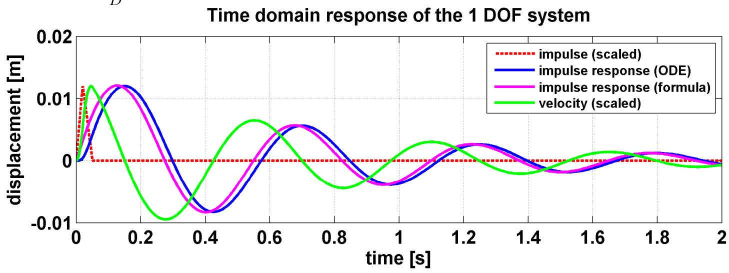
Step 2: Assumption check for impulse duration and impulse calculation

$$T_P / T_D = 0.05 / 0.55 \approx 9.1\% \; < \; 10\%$$ *… threshold value for the input force to be considered as impulse*

$$I = \int_0^{0.05} F(t)\,dt = (0.02\,s)\times(30\,kN)/2 + (0.03\,s)\times(30\,kN)/2 = 750\,N.s$$

Step 3: Wing motion due to impulse

$$x(t) = \frac{I}{M\omega_D} e^{-\zeta\omega_0 t}\sin(\omega_D t) = (0.0146\,m)\,e^{-1.3824t}\sin(11.44\,t)$$



Summary

- Use the *principle of superposition* for cases with multiple excitation forces, e.g. multi-harmonic excitation

- Impulse excitation
  - Short duration (defined relative to T_D)
  - Changes initial velocity (momentum transfer)
  - Impulse response formula