Vibrations 2, Lecture 20
Static Aeroelasticity
Divergence

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Lecture 19 review

Collar's aeroelastic triangle:

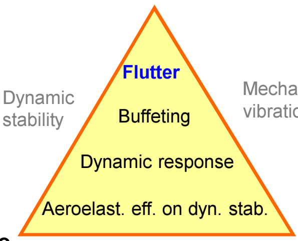
Inertia

Dynamic stability

Mechanical vibrations

Aero

Elasticity



Control system reversal

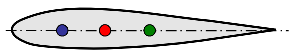
Divergence

Control effectiveness

Load distribution

Aeroelastic effects on static stability

Important sectional points:



a.c. c.m. s.c.

Blade element theory:

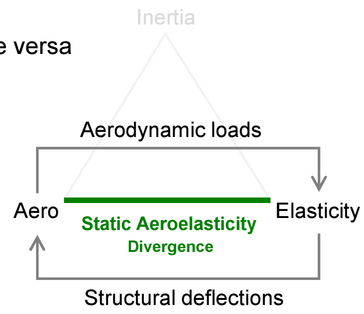
$$C_L = C_{L0} + C_{L,\alpha}\,\alpha + C_{L,\beta}\,\beta \rightarrow L = q\,S\,C_L$$
$$C_{M,a.c.} = C_{M0} + C_{M,\beta}\,\beta \rightarrow M_{a.c.} = q\,S\,c\,C_{M,a.c.}$$
$$q = (1/2)\rho v_\infty^2, S = c\,b$$

Static aeroelasticity studies

Study of flexible aircraft structures under aerodynamic loads.

Assumptions:

- Aero loads influence structural shape and vice versa
- Forces and moments are independent of time
- Only steady aerodynamics is considered



Performance problems studied:

- Jig shape from desired flight shape
- Structural loads in steady flight conditions
- Aero loads (lift, drag/range) loads
- Aircraft trim behaviour
- Static stability and control characteristics

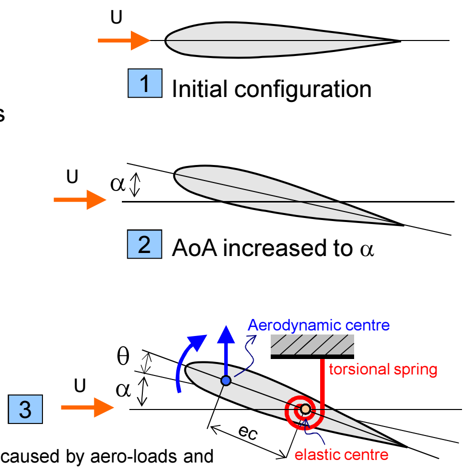
Critical phenomena studied:

- Divergence
- Control reversal

Static aeroelastic response

Physics of torsionally supported rigid wing in airstream – assumptions:

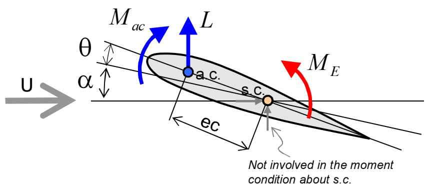
- infinitely long wing
- uniform section
- linear springs
- linear aerodynamics



Deformation θ caused by aero-loads and change in aero-loads due to change in θ.

Equation of moment equilibrium

Static aeroelasticity – an aerofoil or rigid wing is always in the state of static equilibrium (if possible!) between the aerodynamic and elastic (restoring) effects:



Assume:
- small angles

Aerodynamic moment = Elastic moment

Moment equilibrium about shear centre:

$$\sum_{(i)} M_{e.c.,i} = 0$$

$$M_{ac} + L.ec - M_E = 0$$

Lift: $L = qSC_L = qSC_{L,\alpha}(\alpha+\theta)$

Aero moment: $M_{ac} = qScC_{M_{ac}}$

Elastic moment: $M_E = k\theta$

$$qScC_{M_{ac}} + ec.qSC_{L,\alpha}(\alpha+\theta) = k\theta$$

Solve for unknown θ which will satisfy this equilibrium condition!

Aeroelastic response of an aerofoil

Torsional deformation due to finite flexibility:

$$\theta = \frac{qS\left(cC_{M_{ac}} + ec.C_{L,\alpha}\alpha\right)}{k - ec.qSC_{L,\alpha}}$$

Observations:

- θ increases as dynamic pressure q increases
- θ increases to infinity when the denominator approaches 0, or …
- θ → infinity when $k\text{-}ec*q*S*C_{L,\alpha}\rightarrow 0$
- when θ → infinity the airfoil is **divergent**
- $k\text{-}ec*q*S*C_{L,\alpha}\rightarrow 0$ defines q_D and U_D when the divergence occurs
- **divergence** occurs only when e>0 (a.c. forward of e.c.!)
- design aircraft to fly well below **divergence speeds** for all lifting surfaces

$$\boxed{q_D = \frac{k}{ec.SC_{L,\alpha}} \quad\Rightarrow\quad U_D = \sqrt{\frac{2k}{ec.SC_{L,\alpha}\rho}}}$$

Aeroelastic response of an airfoil

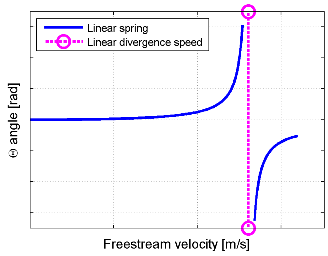
Divergence is a **static aeroelastic** phenomenon, i.e. involves balance between aerodynamic and elastic forces (*no inertial forces, no "rates of change"*).

Divergence is experienced when the aerodynamic moment is too large to be balanced by the elastic moment.

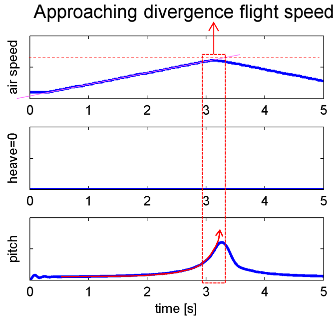
Consider the parameter influences and the ways how to increase U_D!

Solution for a range of air velocities:

$$\theta = \frac{qS\left(cC_{M_{ac}} + ec.a\alpha\right)}{k - ec.qSa}$$



Virtual divergence "flight test":



Effect of wing sweep

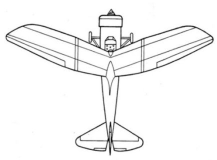
Swept-back wings:

- Increases speed at which shock waves are formed
- Delays onset of associated drag rise
- Reduces effective thickness to chord ratio

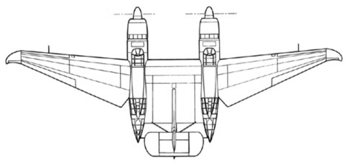
Swept-forward wings:

- Similar drag reduction possible
- Flow separation starts at wing root – better than swept-back where flow separation occurs near tip and diminishing aileron performance
- Potentially useful structural layout (wing-fuselage)
- <u>Static aeroelastic problems - divergence</u>

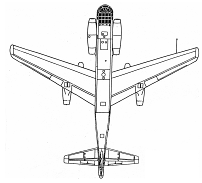
Effect of wing sweep



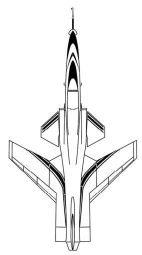
PWS Z-17 "Sęp I", 1932
http://www.samolotypolskie.pl/



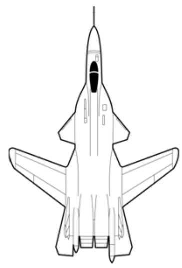
Belyayev DB-LK, 1939
http://xplanes.tumblr.com/



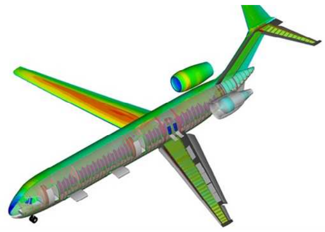
Junkers Ju 287, 1944
https://www.pinterest.com/pin/40187596 6722388804/



Grumman X-29, 1984
www.dfrc.nasa.gov



Sukhoi Su-47, 1997
www.the-blueprints.com

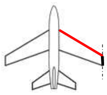


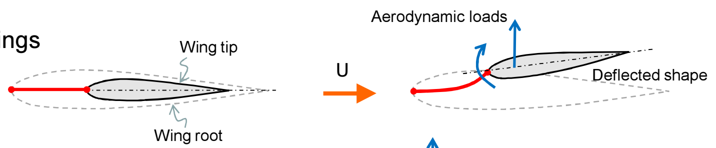
LamAiR project, ongoing
www.dlr.de

Effect of wing sweep

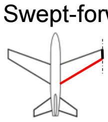
Wing tip deformation patterns:

Swept-back wings

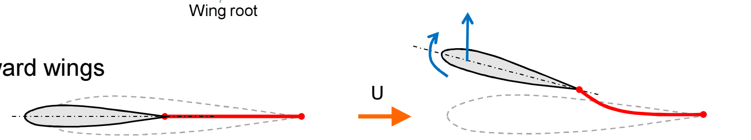




Swept-forward wings



en.wikipedia.org/wiki /Wing_configuration



Forward Swept Aircraft
- Divergence speed becomes limiting case
- Need to use <u>aeroelastic tailoring</u> to counteract effect

Divergence speed

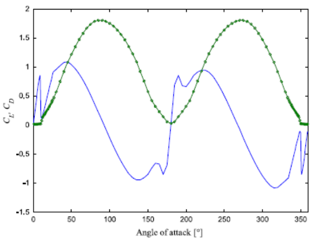
- Increases with sweep back wings
- Decreases with sweep forward wings
- Increasing AR reduces the divergence speed
- Moving the flexural axis away aft of the flexural decreases the divergence speed

Other effects: large angles, …

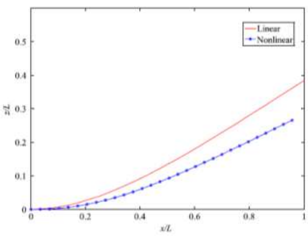
Nonlinear effects of large angles:

- Nonlinear aerodynamics (large AoA): $C_{L,D,Mac}=f_A(\alpha+\theta)$ → Lift, Drag, Moment
- Nonlinear elasticity (e.g. nonlinear springs): $M_E=f_S(\theta)$
- Equations of static equilibrium

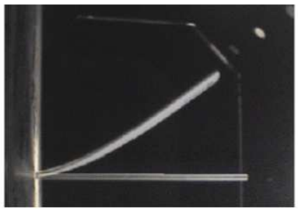
Thwapiah & Campanile, Nonlinear aeroelastic behavior of compliant airfoils, Smart Materials and Structures, 19, 2010



Aerodynamics



Structures



Experiment

Summary

- Static aeroelasticity
  - Assumptions and phenomena
- Physics of torsionally supported rigid wing
  - Equilibrium between aerodynamics and elasticity
  - Divergence and important sectional parameters
- Other effects
  - Wing sweep
  - Nonlinear effects