# Structural Loads in Beams
# **Cantilever Beam with a Point Load**

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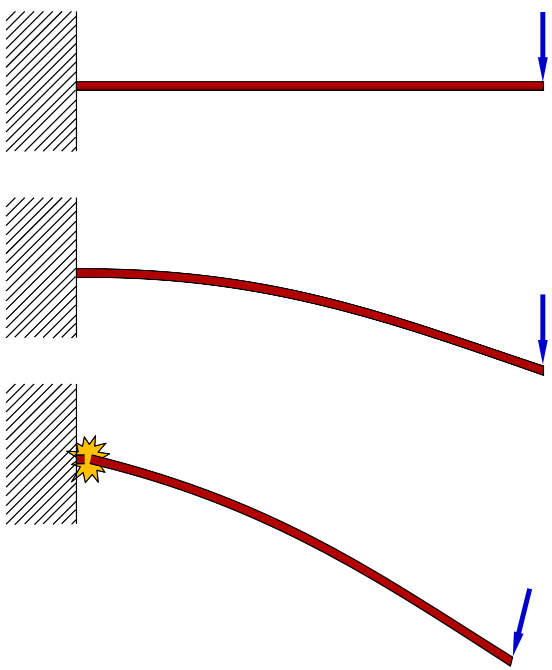
# Shear Force and Bending Moment – Physical Meaning

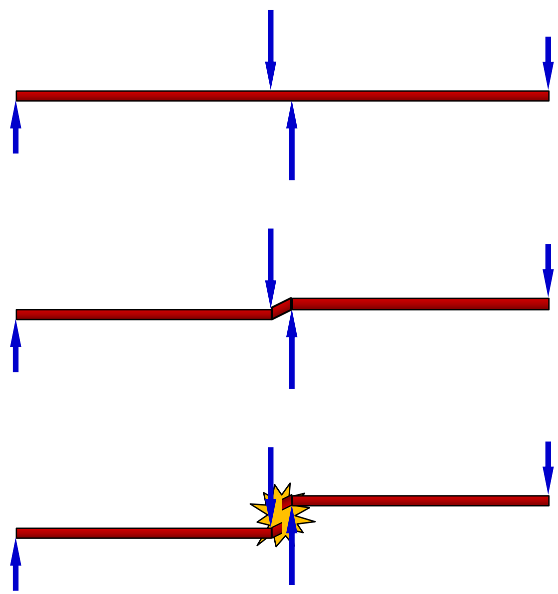
## Bending Moment

- Critical for:
  - Long/thin beams, and/or
  - Transverse forces separated by considerable beam length
- Deformation:
  - Curvature
  - Tension/compression on top/bottom surfaces (depending on sign of $M$)
- Strains: direct strains (tension & compression)
- Failure: 'snapping'



## Shear Force

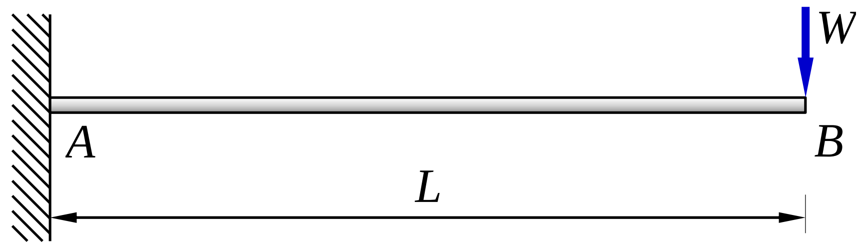
- Critical for:
  - Short/thick beams, and/or
  - Opposing transverse forces close to each other (*e.g.* when cutting with scissors)
- Deformation:
  - 'Angle changes' (or 'diagonals' stretching or shrinking)
- Strains: shear strains
- Failure: 'shearing off'



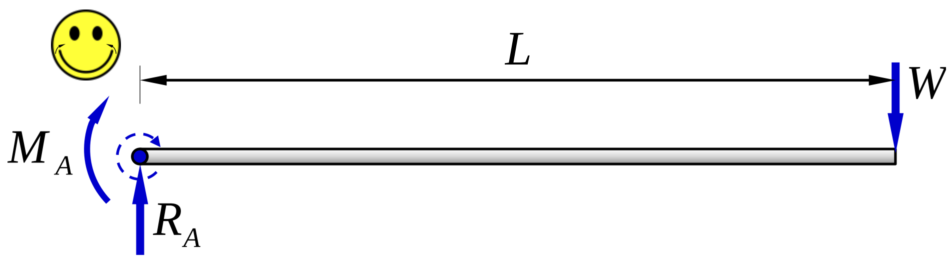
# 2. Structural Loads in Beams - Contents

# 2.5.2 Cantilever Beam with Concentrated Load

- The cantilever beam is 'built-in' at the 'root' ($A$) and loaded at the 'tip' ($B$):




- Global FBD:



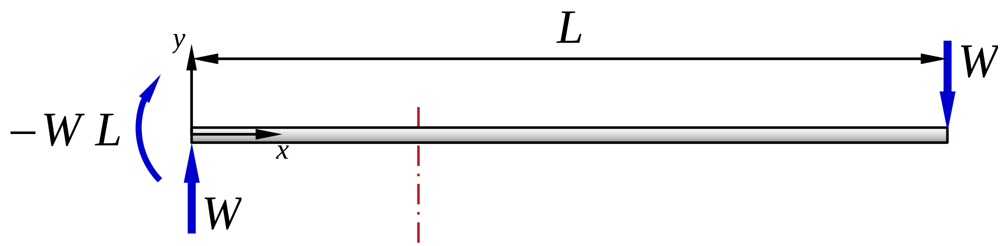

$$\sum M_{@A}^{\text{cw}} = 0 \quad \therefore \quad M_A + (W)(L) = 0 \quad \therefore \quad \boxed{M_A = -W\,L}$$

$$\sum F = 0 \quad \therefore \quad R_A - W = 0 \quad \therefore \quad \boxed{R_A = W}$$

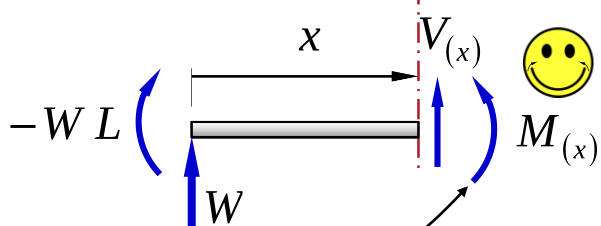
# 2.5.2 Cantilever Beam with Concentrated Load



- Putting the origin of $x$ at point $A$, from left to right:




- Sectioning the beam at an arbitrary point $0 < x < L$ :




$$\sum M_{@x}^{\text{ccw}} = 0 \quad \therefore \quad M_{(x)} - (-WL) - (W)(x) = 0 \quad \therefore \quad \boxed{M_{(x)} = W(x - L)}$$

$$\sum F_{@x} = 0 \quad \therefore \quad W + V_{(x)} = 0 \quad \therefore \quad \boxed{V_{(x)} = -W}$$

# 2.5.2 Cantilever Beam with Concentrated Load

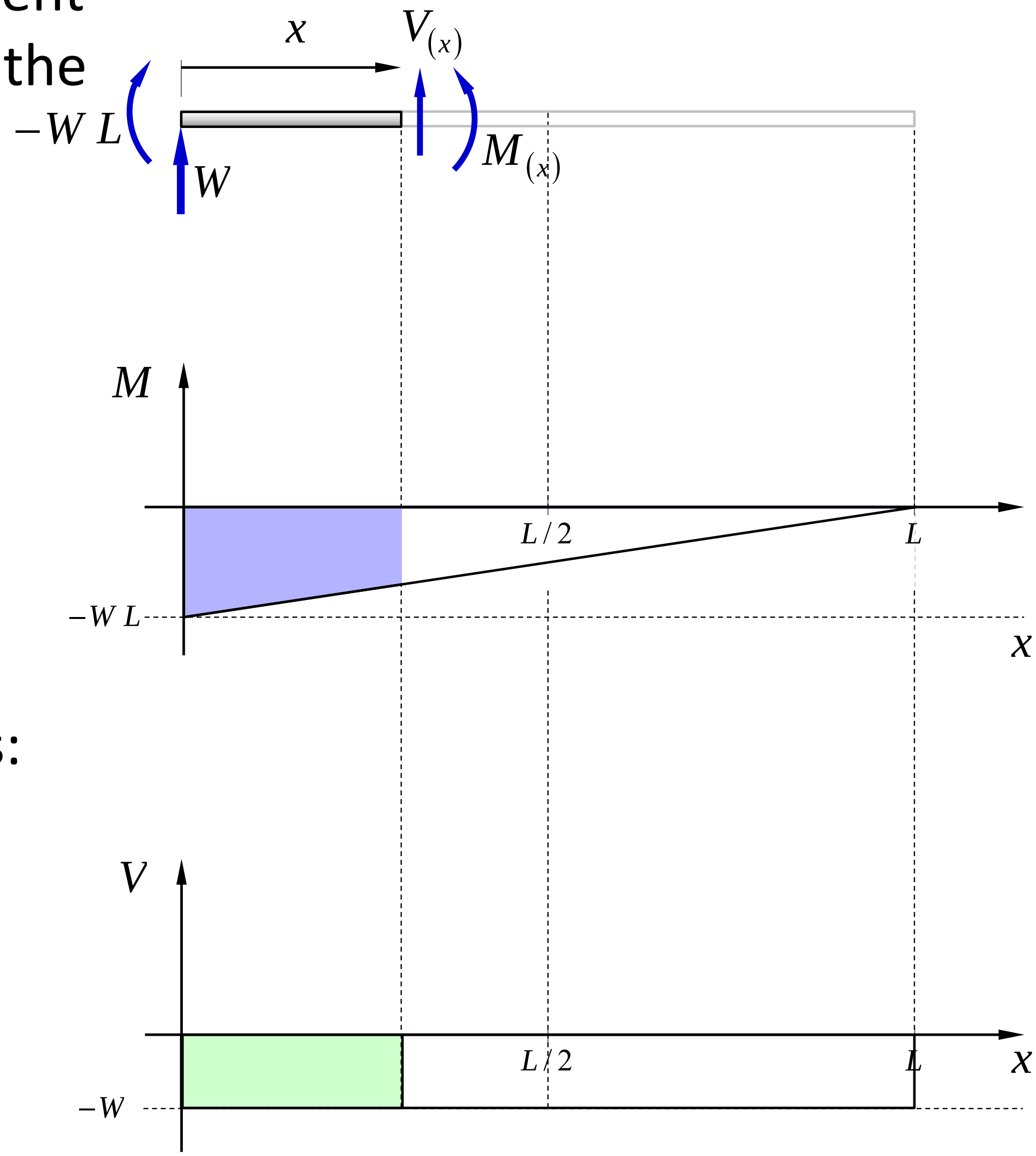
- Note that the bending moment **varies linearly** with $x$, while the shear force is **a constant**:

$$\boxed{M_{(x)} = W\left(x - L\right)}$$

$$\boxed{V_{(x)} = -W}$$

- So we can plot the diagrams:

These solutions are valid for any $0 < x < L$



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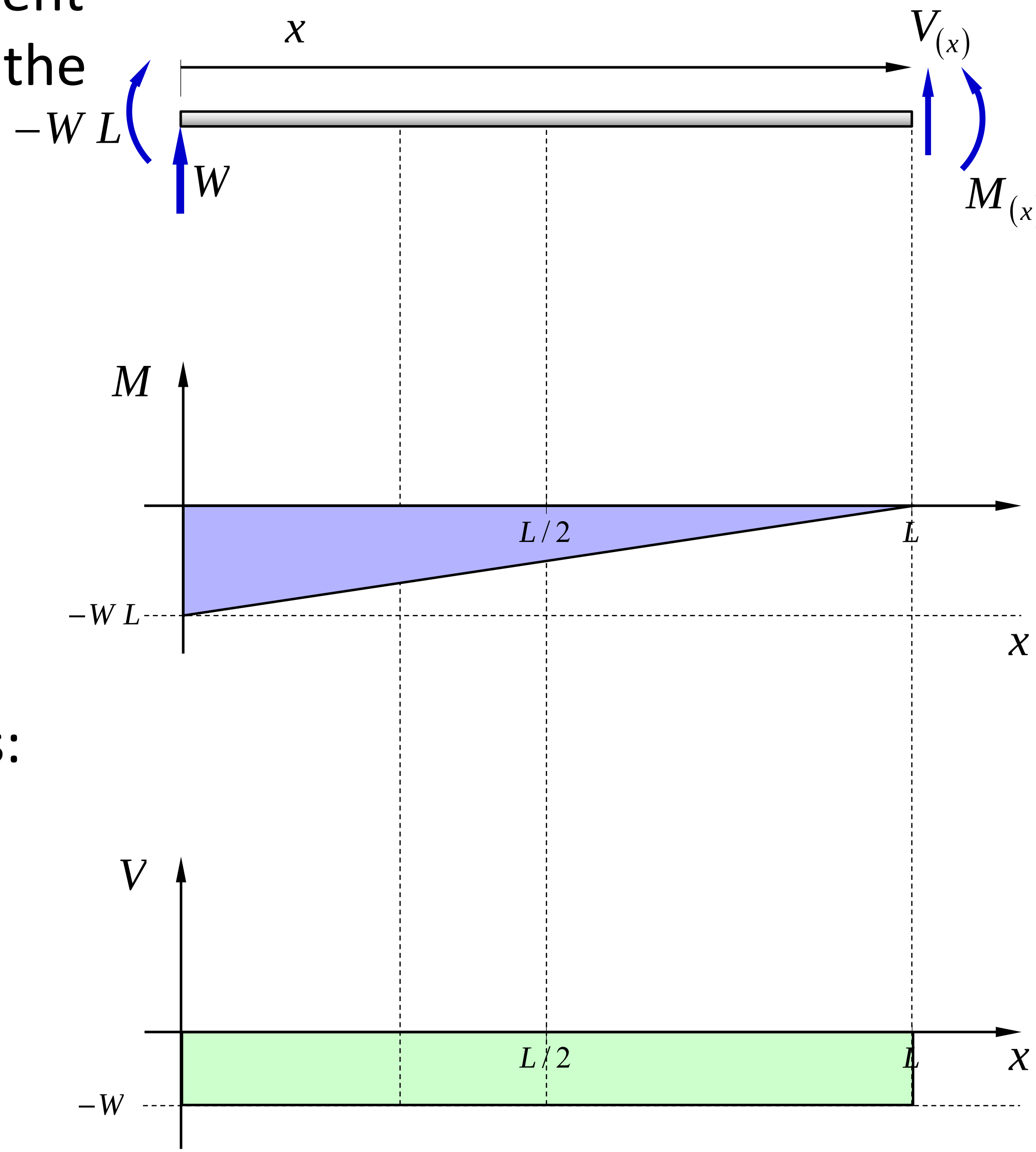


These solutions are valid for any $0 < x < L$

# Structural Loads in Beams
# **Cantilever Beam with a Uniform Load Distribution**

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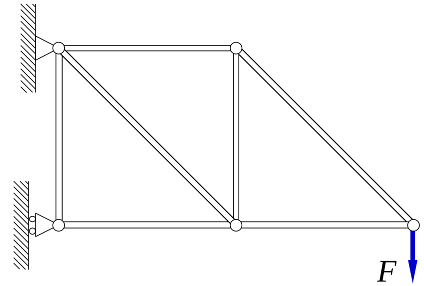
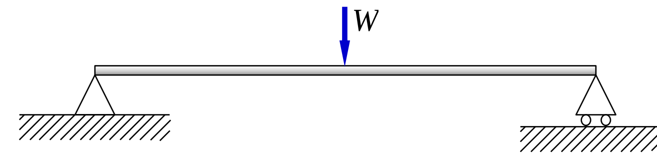
# 2. Structural Loads in Beams - Contents

# Distributed Loads

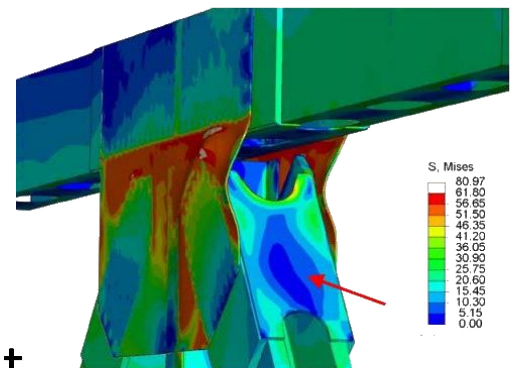


- So far we have been dealing with idealised 'point loads'
  - Forces applied at infinitely small areas → not very realistic

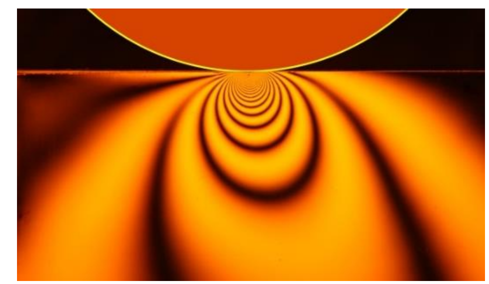



- In reality, loads are applied over a **finite area**, *e.g.*:
  - Stresses (N/m$^2$) in real 3D joints
  - Pressure (N/m$^2$) due to contact

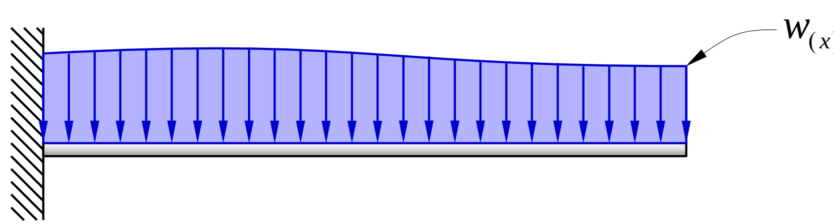



A real truss joint



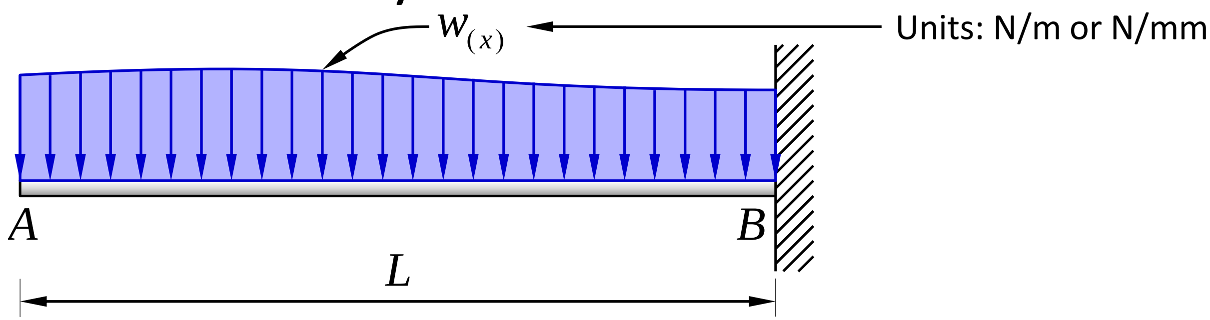
Real contact stresses (photoelastic effect of a cylinder on a flat surface)

- In 2D problems we assume 'unit width', and define distributed loads as <u>**force per unit length**</u> (N/m)

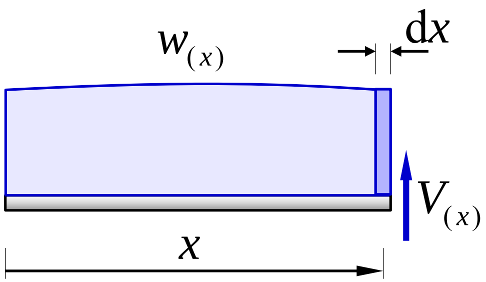


# Distributed Loads

- Cantilever beam with an arbitrary load distribution:




## Shear Force

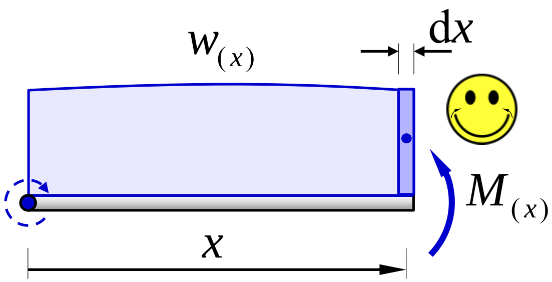



$$V_{(x)} = + \int_0^x w_{(x)} \, dx$$

- Shear force is the **area** under the curve $w(x)$

Signs depend on directions of $x$ and $w(x)$

## Bending Moment

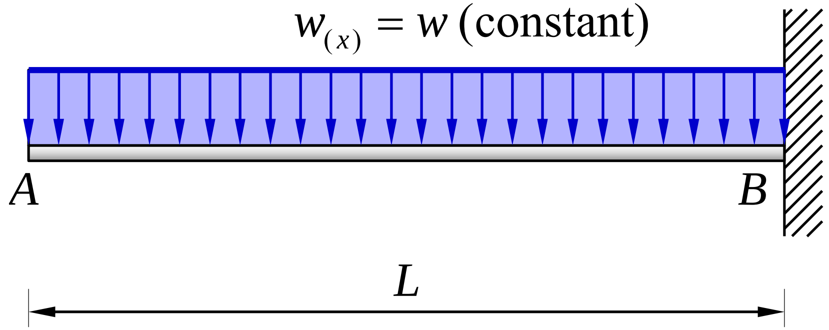



$$M_{(x)} = - \int_0^x w_{(x)} \, x \, dx$$

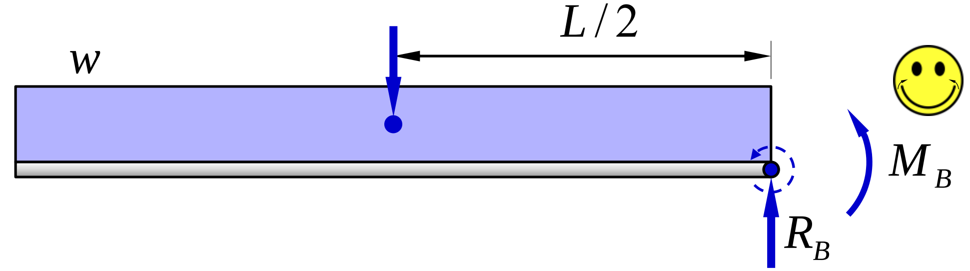
- Bending moment is the **moment of area** of the curve $w(x)$

# 2.5.3 Cantilever Beam with a Distributed Load

- Cantilever beam with constant load distribution:




- Global FBD:




$$\sum F = 0 \qquad \therefore \qquad R_B - (w)(L) = 0 \qquad \therefore \qquad \boxed{R_B = wL}$$

$$\sum M_{@B}^{\circlearrowleft \text{ ccw}} = 0 \qquad \therefore \qquad M_B + (wL)\left(\frac{L}{2}\right) = 0 \qquad \therefore \qquad \boxed{M_B = -\frac{wL^2}{2}}$$

# 2.5.3 Cantilever Beam with a Distributed Load

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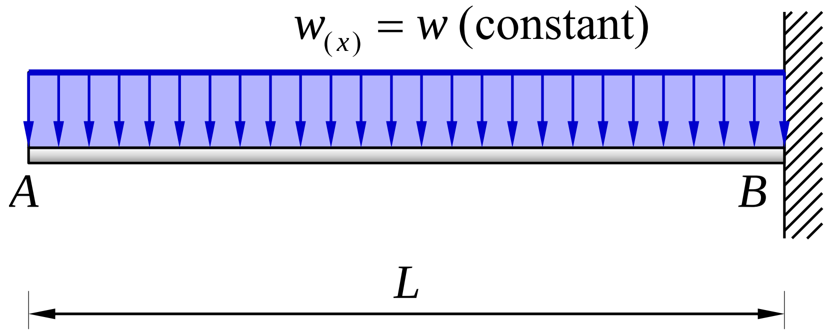
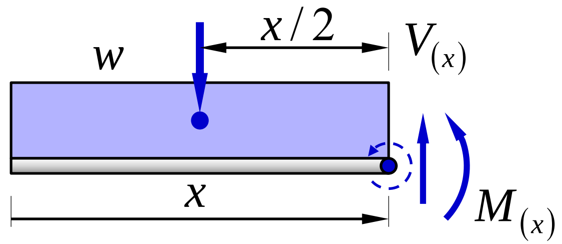



- Section FBD:




$$\sum F = 0 \quad \therefore \quad V_{(x)} - (w\,x) = 0 \quad \therefore \quad \boxed{V_{(x)} = w\,x}$$

$$\sum M_{@x}^{\text{ccw}} = 0 \quad \therefore \quad M_{(x)} + (w\,x)\left(\frac{x}{2}\right) = 0 \quad \therefore \quad \boxed{M_{(x)} = -\frac{w\,x^2}{2}}$$

# Bending Moment *vs.* Shear Force



- Finally we can plot the shear force and bending moment diagrams

$$\boxed{R_B = wL}$$

$$\boxed{V_{(x)} = wx}$$

$$\boxed{M_B = -\frac{wL^2}{2}}$$

$$\boxed{M_{(x)} = -\frac{wx^2}{2}}$$