Lecture 7/8
ODEs

EXAMPLE

$$\boxed{\begin{gathered}\frac{d^2x}{dt^2} + \frac{dx}{dt} - 2x = 3e^{-t} \\ x(0) = \dot{x}(0) = 0\end{gathered}}$$

1) ~~COMPL~~ SOLUTION OF HOMOG. EQUATION

Char. equation
$$m^2 + m - 2 = 0 \quad \nearrow m_1 = -2, \quad \searrow m_2 = 1$$

$\downarrow$ Complimentary function

$$x_c = Ae^{-2t} + Be^{t}$$

[form solution Ae^{mt}]

2) PARTICULAR ~~SOLUTION~~ INTEGRAL

Since $3e^{-t}$ (exponential multiplied by a constant) we try

$$x_p(t) = Ke^{-t}$$

(SINCE ALL DERIVATIVES of e^{-t} ARE MULTIPLES of e^{-t} !!!)

To check for what values of K x_p is a solution of the non-hom. ODE, SUBSITUTE into equation

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} - 2x = 3e^{-t}$$

$$\frac{dx}{dt} = -Ke^{-t} \; ; \; \frac{d^2x}{dt^2} = Ke^{-t}$$

SUBST

$$\cancel{Ke^{-t}} - \cancel{Ke^{-t}} - 2Ke^{-t} = 3^{-t} \Rightarrow -2K = 3 \Rightarrow K = -3/2$$

$$\Downarrow$$

$$x_p(t) = -\frac{3}{2} e^{-t}$$

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(3) FINALLY

$$x(t) = x_c(t) + x_p(t) =$$

$$= Ae^{-2t} + Be^{t} - \frac{3}{2} e^{-t}$$

(4) I.C.

AT HOME!!!

$x(0) = 0 \Rightarrow$ use sol

$\dot{x}(0) = 0 \Rightarrow$ differentiate solution