Free vibration decay in 1 DOF systems with Coulomb friction

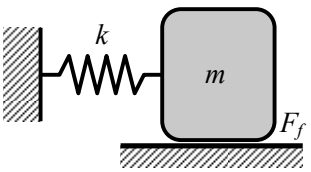

Figure 1 System with 1 DOF and Coulomb friction.

The problem of free vibration decay in 1 DOF system with Coulomb friction, Figure 1, is studied in this note. Figure 2 shows energy losses during vibration with Coulomb friction. Energy losses are shown in the displacement-force diagrams as the shaded areas enclosed by the damping force vs. displacement loops. Time domain traces complement the energy loss diagrams. Free vibration case, Figure 2a), is used for the derivation of the free vibration decay formula, while the forced case, Figure 2b), is provided only for completeness and it is not used for derivations.

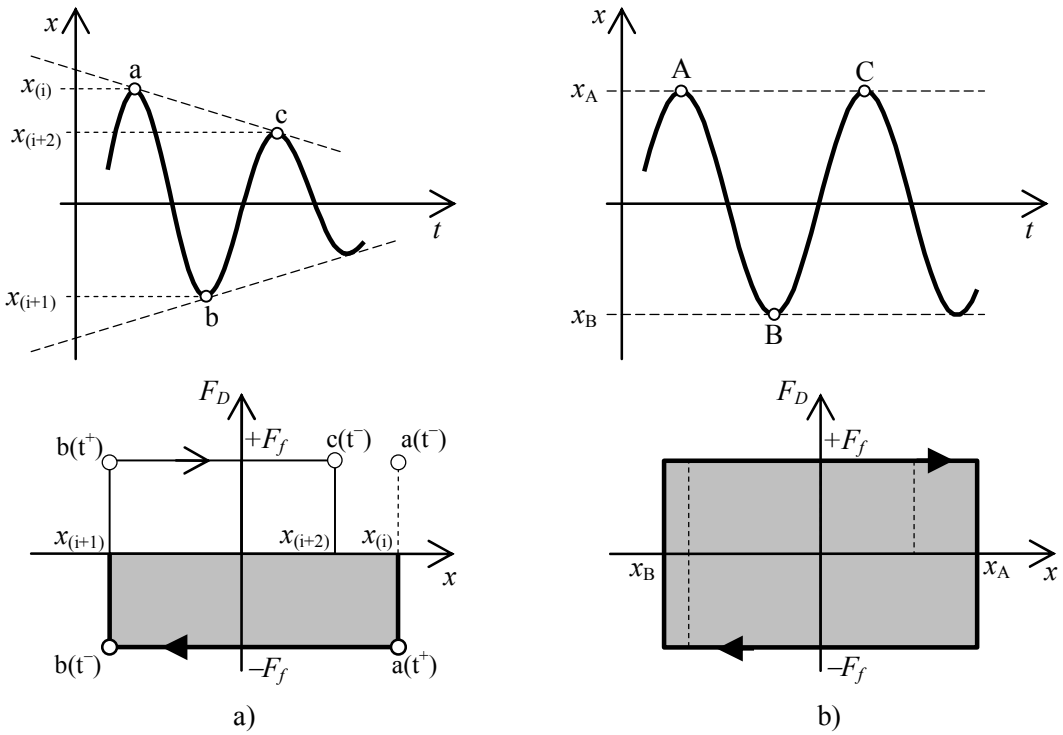

Figure 2 Energy losses during vibration with Coulomb friction: a) free vibration with linear amplitude decay, b) vibration with prescribed harmonic excitation.

Derivation is based on Figure 2a) by considering the energy balance between points **a** and **b** and then extending this result to the point **c**, covering thus one full vibration cycle. The system in Figure 1 has zero kinetic energy and maximum potential energy in points **a**, **b** and **c**. The energy "conservation" condition between points **a** and **b** can be formulated when considering the energy losses due to friction:

$$E_{P,(i)} = E_{P,(i+1)} + E_{D,Coul(i,i+1)}$$

where:

- $E_{P,(i)}$ is the potential energy in point **a**,
- $E_{P,(i+1)}$ is the potential energy in point **b**,
- $E_{D,Coul(i,i+1)}$ is the energy lost due to Coulomb friction between points **a** and **b**.

Potential energies accumulated in the spring:

$$E_{P,(i)} = \frac{1}{2} k\, x_{(i)}^2,\ E_{P,(i+1)} = \frac{1}{2} k\, x_{(i+1)}^2.$$

Energy dissipated during vibration between **a** and **b**, Figure 2a) (indicated by the shaded area)

$$E_{D,Coul(i,i+1)} = F_f\, x_{(i)} + F_f\, x_{(i+1)} = F_f (x_{(i)} + x_{(i+1)}).$$

Resulting energy equation gives:

$$\frac{1}{2} k\, x_{(i)}^2 = \frac{1}{2} k\, x_{(i+1)}^2 + F_f (x_{(i)} + x_{(i+1)}),$$

$$\frac{1}{2} k (x_{(i)}^2 - x_{(i+1)}^2) = F_f (x_{(i)} + x_{(i+1)}),$$

$$\frac{1}{2} k (x_{(i)} - x_{(i+1)})(x_{(i)} + x_{(i+1)}) = F_f (x_{(i)} + x_{(i+1)}),$$

$$x_{(i)} - x_{(i+1)} = \frac{2 F_f}{k}.$$

The last equation from above can be written for the vibration cycle between points **b** and **c**. These two equations are as follows

$$x_{(i)} - x_{(i+1)} = \frac{2 F_f}{k},\ x_{(i+1)} - x_{(i+2)} = \frac{2 F_f}{k}$$

and the resulting formula for free vibration decay in systems with Coulomb friction is

$$\boxed{x_{(i)} - x_{(i+2)} = \frac{4 F_f}{k}.}$$

■