1. Linkages

Design 2
AENG21350

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1.1 History

Weaving of clothes gave rise to the need of complex machines to convert rotary motion from waterwheels to complex motions

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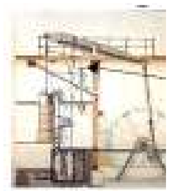
Invention of the steam engine → long linear motion travel needed to harness steam power, and machine tools (i.e., planar mills) did not exist



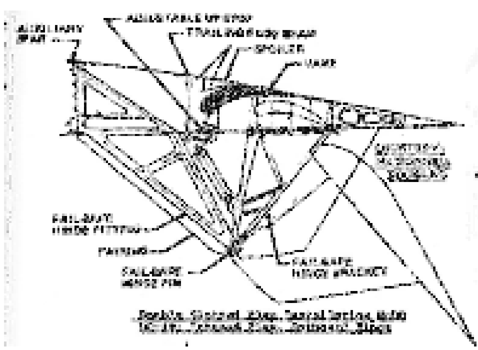
- James Watt (1736 - 1819) used thermodynamics, rotary joints and long links to create straight line motion
- Watt also created the *flyball governor* (first servomechanism) which made steam engines safe and more useful
- Leonard Euler (1707 - 1783) was one of the first mathematicians to study the mathematics of linkage design

Most linkage are planar → motion confined to plane

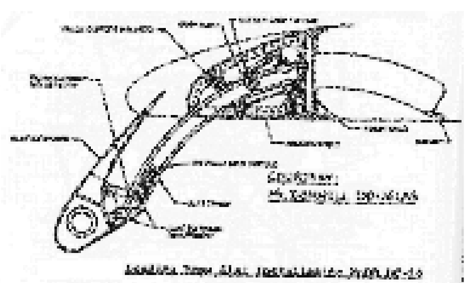
The generic study of linkage motions is called **screw theory**

Sir Robert Stawell Ball (1840 - 1913) is considered the father of screw theory

1.1 Aero examples

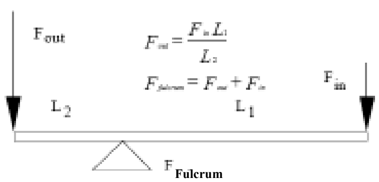


MDD DC-10 – Double slotted flap



MDD DC-10 – Leading edge slat

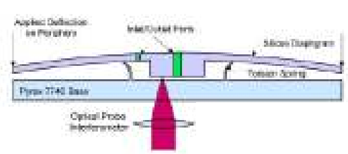
1.2 The Lever (first mechanism!)



$$F_{out} = \frac{F_{in} L_1}{L_2}$$

$$F_{fulcrum} = F_{out} + F_{in}$$

- A lever can be used with a fulcrum (pivot) to allow a small force moving over a large distance to create a large force moving over a short distance.
- The forces are applied through pivots, and thus they may not be perpendicular to the lever



Nanogate – MEMS diaphragm-type lever to filter nanoparticles

1.3 Definitions - 1

Linkage: A system of ***links*** connected at ***joints*** with rotary or linear bearings

- ***Joint (kinematic pairs):*** Connection between two or more links at their nodes, which allows motion to occur between the links
- ***Link:*** A rigid body that possess at least 2 nodes, which are the attachment points to other links

Degrees of Freedom (DOFs):

The number of input motions that must be provided in order to provide the desired output,
OR
The number of independent coordinates required to define the position & orientation of an object

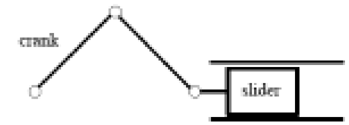
For a planar mechanism, the degree of freedom (mobility) is given by ***Gruebler's Equation***:

$$\boxed{F = 3(n-1) - 2f_1}$$

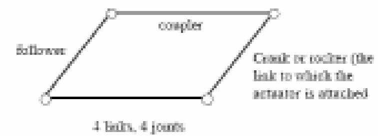
n = Total number of links (including a fixed or single ground link)
$f1$ = Total number of joints (some joints count as f = ½, 1, 2, or 3)

1.3 Definitions - 2

Example: Slider-crank n = 4, $f1$ = 4, F = 1



Example: 4-Bar linkage n = 4, $f1$ = 4, F = 1



The simplest linkage with at least one degree of freedom (motion) is a 4-bar linkage!

1.4 Links

Binary Link: Two nodes:

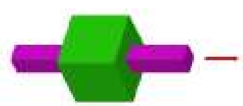
Ternary Link: Three nodes:

Quaternary Link: Four nodes:

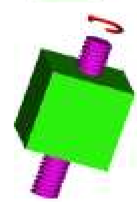
1.5 1 DOF Joints

Lower pairs (first order joints) or *full-joints* (counts as f = 1 in Gruebler's Equation) have one degree of freedom (only one motion can occur):

Revolute (R)
- Also called a **pin joint** or a **pivot.** Ensure that the axle member is firmly anchored in one link, and bearing clearance is present in the other link
- Washers make great thrust bearings
- Snap rings keep it all together
- A *rolling contact* joint also counts as a one-degree-of-freedom revolute joint



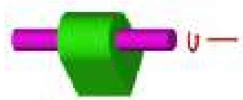
Prismatic (P)
- Also called a slider or sliding joint



Helical (H)
- Also called a **screw. Drawbacks:** thread strength, friction and efficiency

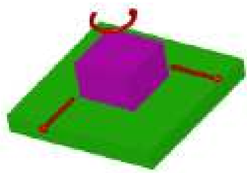
1.6 MDOF Joints

Lower Pair joints with multiple degrees of freedom:



Cylindrical (C) 2 DOF
• A *multiple-joint* ($f = 2$ in Gruebler's Equation)

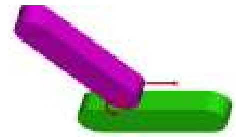
Spherical (S) 3 DOF
• A *multiple-joint* not used in planar mechanisms ($f = 3$ in Gruebler's Equation)



Planar (F) 3 DOF
• A *multiple-joint* ($f = 3$ in Gruebler's Equation)

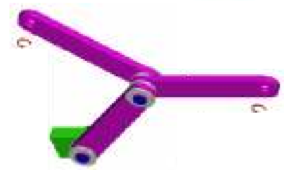
1.7 Higher pair MDOF Joints

Higher Pair joints with multiple degrees of freedom:



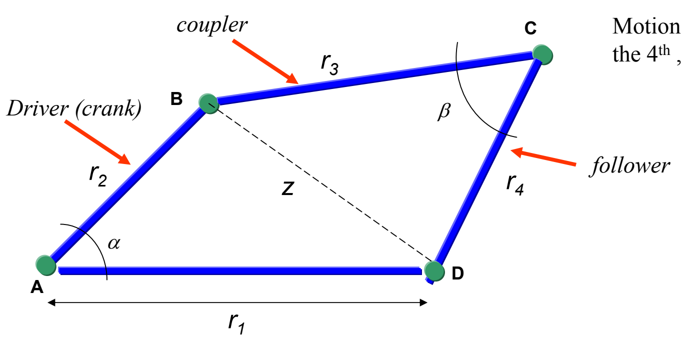
Link against a plane
• A force is required to keep the joint closed (force closed)
– A *half-joint* ($f = 0.5$ in Gruebler's Equation)

Pin-in-slot
• Geometry keeps the joint closed (form closed)
– A *multiple-joint* ($f = 2$ in Gruebler's Equation)



Second order pin joint, 3 links joined, 2-DOF
• A *multiple-joint* ($f = 2$ in Gruebler's Equation)

1.7 4 bar linkage - 1



Motion of 3 links relative to the 4th , which is fixed

Kinematics:

$$z^2 = r_1^2 + r_2^2 - 2 r_r r_2 \cos\alpha$$

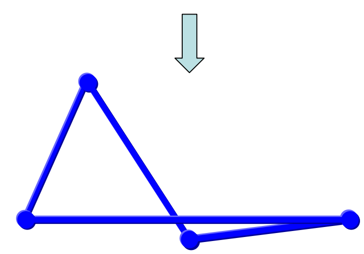
$$z^2 = r_3^2 + r_4^2 - 2 r_3 r_4 \cos\beta$$

$$\Rightarrow \quad r_1^2 + r_2^2 - 2 r_r r_2 \cos\alpha = r_3^2 + r_4^2 - 2 r_3 r_4 \cos\beta$$

1.7 4 bar linkage - 2

$$\beta = \cos^{-1}\left[\frac{r_1^2 + r_2^2 - r_3^2 - r_4^2 - 2 r_1 r_2 \cos\alpha}{-2 r_3 r_4}\right]$$

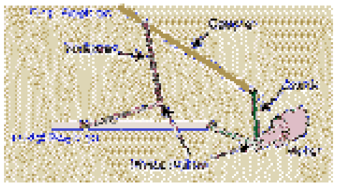
With the dimensions of the linkage given, 2 possible solutions for β. 2nd value corresponds to 2nd mode of assembly of the linkage (**second closure**)



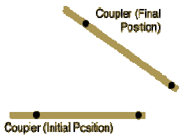
General guidelines:

•$40° < \beta < 140°$ to transmit high forces
•If $\beta < 40°$ linkage tends to bind because of friction in the joints; coupler and follower tend to align and lock

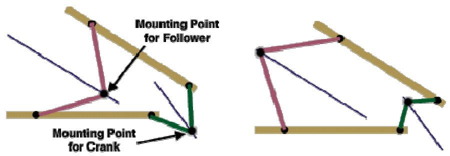
1.8 Design of 4 bar linkage



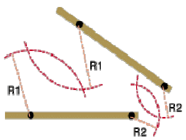
4 steps to design a 4-bar linkage



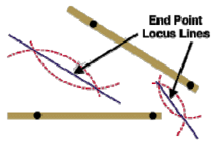
Step 1: Draw Coupler in its Initial and Final Positions



Step 4: Draw Connecting Bars
Draw in the connecting bars. There are many possible location for the mounting point on connecting bars. The two figures show equally valid possibilities for the location of the connecting bars

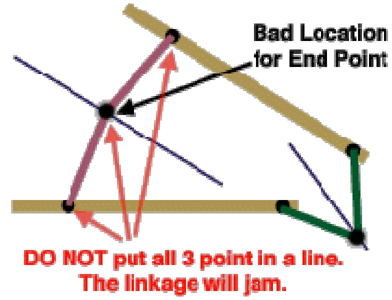


Step 2: Draw Arcs
Draw arcs from each mounting point on the output bar. The radius of the arcs should be the same for each mounting hole



Step 3: Draw Locus Lines
Draw a pair of lines, one connecting the intersections of each pair of arcs. These lines represent the locus of possible end points for the connecting links

1.8 Jamming of a 4 bar linkage!



To prevent the linkage from jamming, the mounting point for the connecting bars should not be placed in line with the output bar's mounting hole's initial and final positions

1.9 Kinematics synthesis of 4-bar linkage

Types of motion can be anticipated using the **Grashof criteria (1883)**:

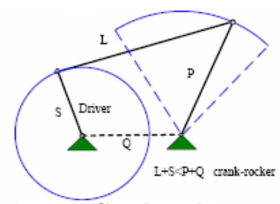
The sum of the shortest (S) and longest (L) links of a planar four-bar linkage cannot be greater than the sum of the remaining two links (P, Q) if there is to be continuous relative motion between two links

- If $L + S < P + Q$, four Grashof mechanisms exist: crank-rocker, double-crank, rocker-crank, double-rocker
- If $L + S = P + Q$, the same four mechanisms exist, but, change-point condition occurs where the centerlines of all links become collinear and the mechanism can **toggle**
- If $L + S > P + Q$, **non-Grashof** triple-rocker mechanisms exist, depending on which is the ground link, but **continuous rotation is not possible**

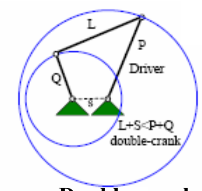
Geometric inversions occur when different pivots are made the ground pivots

1.9 Kinematics synthesis of 4-bar linkage

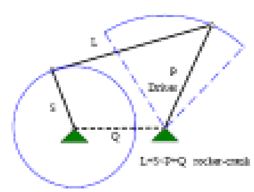
If $L + S < P + Q$, four Grashof mechanisms exist:



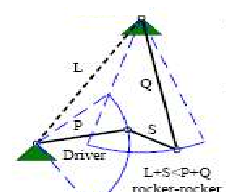
Crank-rocker



Double-crank



Rocker-crank



Rocker-rocker

List of concepts to know

- Definitions of linkage, links and joints
- How to determine the Degrees of Freedom of a linkage (Gruebler's formula)
- 1 DOFs and MDOFs joints
- Kinematics and design of a 4-bar linkage
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- Grashof criteria