

AENG 31200

Structures & Materials 3

Lecture: Creep

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● ● ● | Learning Objectives

The student will be able to;

- Understand the basic phenomenon of creep in metals and polymers and their behaviour under sustained loads or displacements and temperature.
- Undertake a general analysis approach and be able to identify important design considerations.

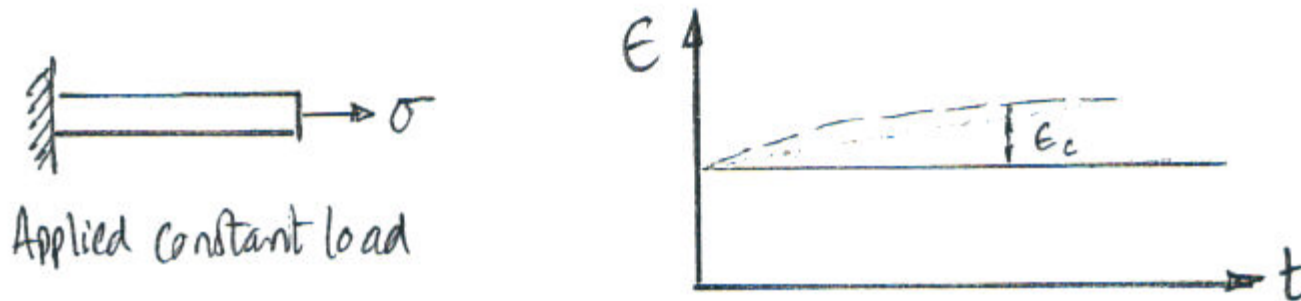
● ● ● | Learning Resources

- WD Callister; “Materials Science & Engineering”, Wiley. TA403
- Penny & Marriot; “Design for Creep” TA418.22
- Bernasconi & Piatti; “Creep of engineering materials and structures” TA418.22
- DWA Rees; “Mechanics of solids and structures” TA4048
- Boresi et al. “Advanced mechanics of materials” TA405

Introduction

Definitions

- Creep: the permanent elongation of a component under static load maintained for a period of time



- Stress Relaxation: the decrease in stress in a component under a constant strain over a period of time



Occurrence

- Phenomenon of metals and some non-metallic materials
e.g. thermoplastics & rubbers
- Under load / time / temperature
- Creep deformation can occur at any temperature but is usually only significant at elevated temperatures for engineering materials:

Metals: Above approximately 40% of T_m (in deg K)

- High σ , High T

Plastics: Above glass transition temperature, T_g (glassy \leftrightarrow rubbery)

- Low σ , Low T

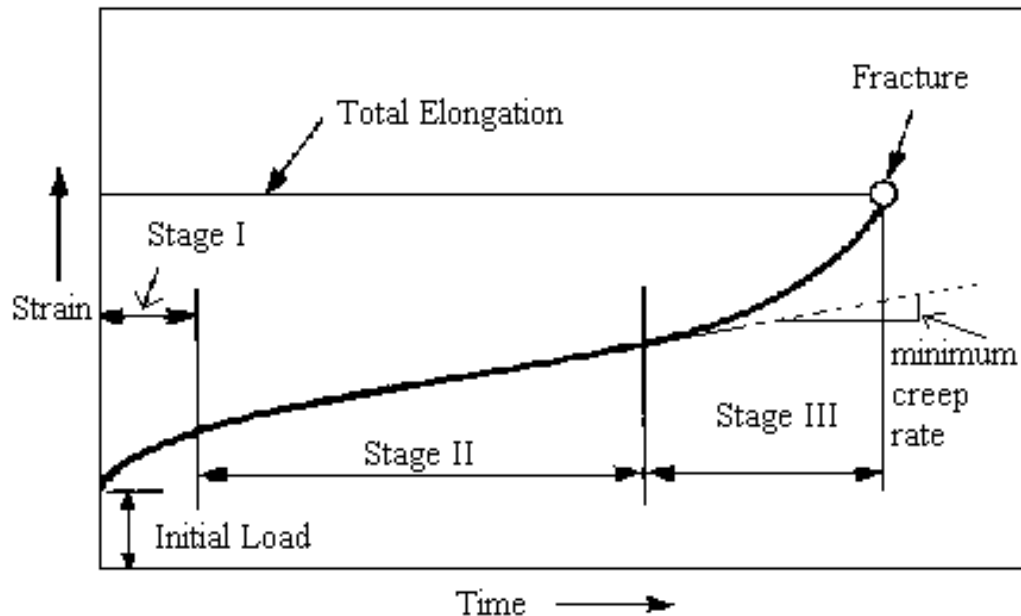


Lead pipes bend due to creep.

- Metal creep is especially important in applications subjected to high σ at elevated T:
 - Gas-turbine blades and similar components in jet engines and rocket motors
 - Supersonic airframes (e.g. Concorde)
 - High temperature steam lines, nuclear-fuel elements
 - Tools, dies during hot working operations such as forging and extrusion
 - Rivets and bolts

Mechanism

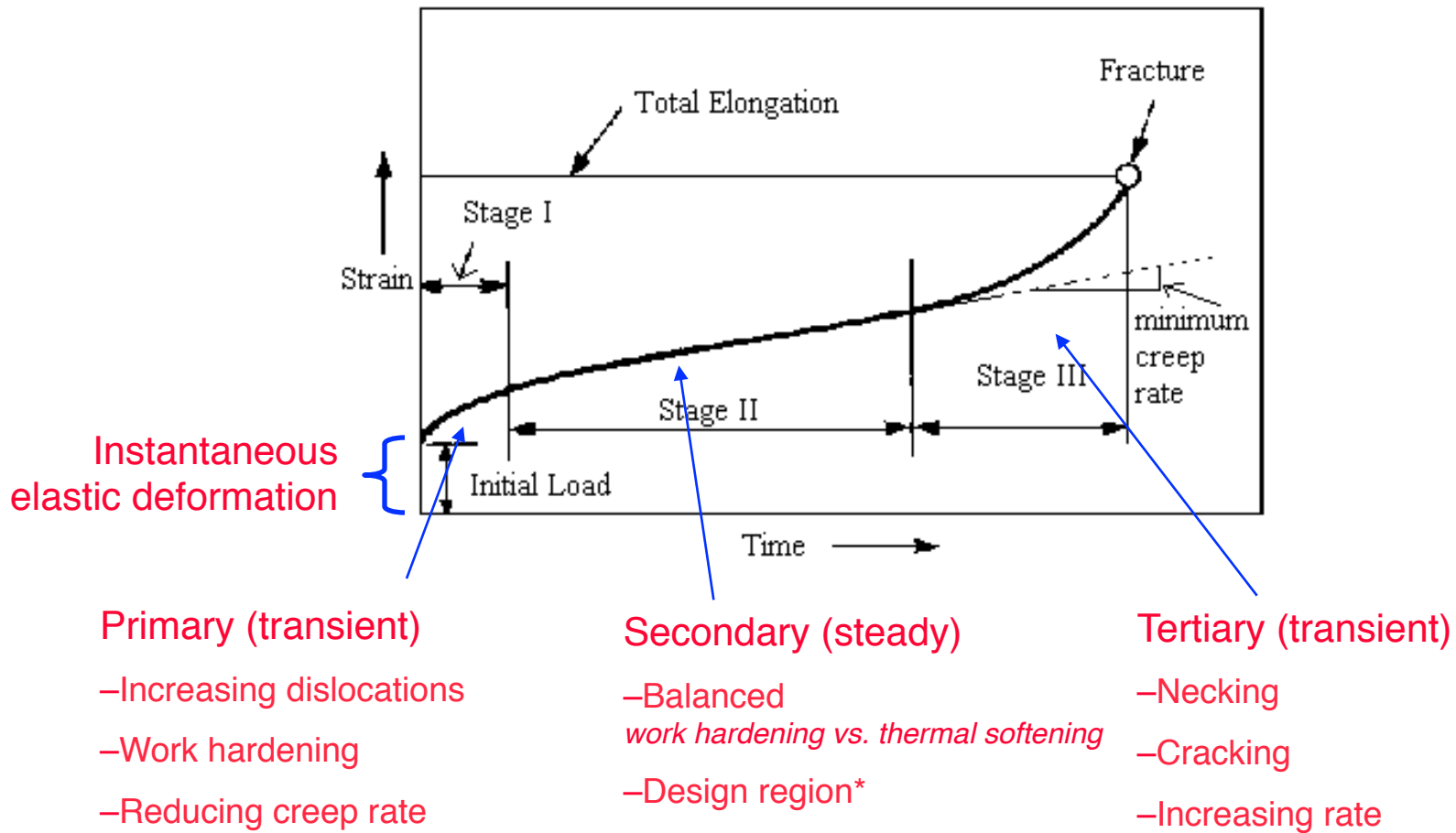
- For creep at elevated temperature;
 - Metals: Grain boundary sliding.
 - Plastics: Slip of polymer chains and alignment parallel to applied stress



- Primary and tertiary stages often referred to as transient creep
- Secondary creep also referred to as '*steady state creep*'

Behaviour

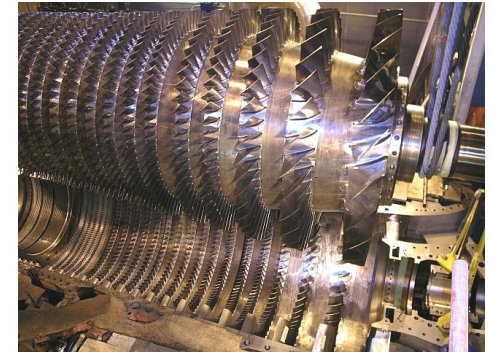
Creep Deformation in metals (general trends, 3 stages)



Behaviour

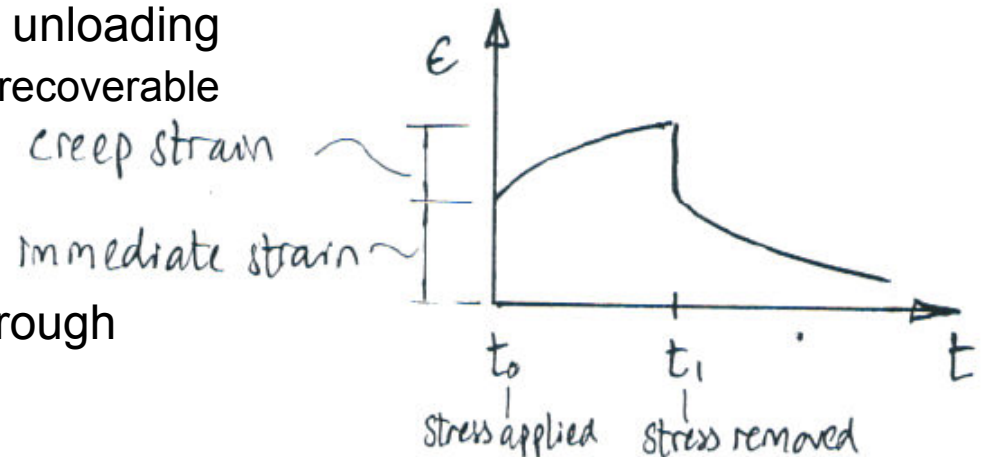
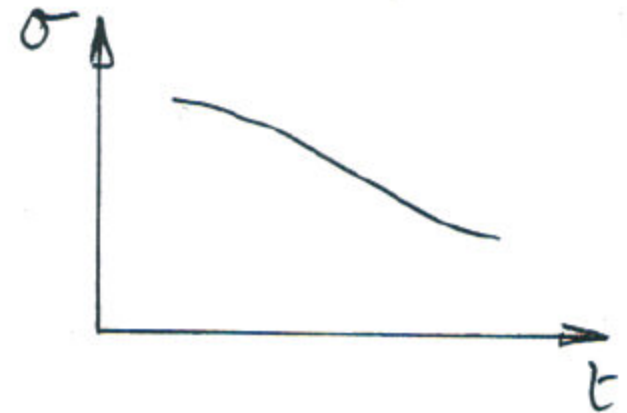
○ Creep Failure

- By excessive deformation, exceeding design allowable
- By excessive deformation, leading to instability
- By rupture i.e. creep fracture



Behaviour

- Stress Relaxation
 - Relaxation of stress through creep at constant strain
- Viscoelasticity (**polymers!**)
 - Combined elastic (immediate)
+ viscous response (time dependent)
- Reverse Creep
 - The reversal of creep strain on unloading
Note: creep in metals is usually unrecoverable
- Recovery
 - The loss of strain hardening through thermal softening



Analysis

Creep strain functions

$$\epsilon_c = f(\sigma, T, t)$$

Note: these parameters are usually interdependent with non-linear relationships

- Empirical functions – to describe one or all 3 creep stages w.r.t. σ , T & t .
 - e.g. considering time dependence only;

1. Low T (i.e. $<0.5T_m$) – **primary** creep dominates

- With low σ , $\epsilon_c = \alpha \ln t$ *i.e. logarithmic trend*
- With high σ , $\epsilon_c = \alpha t^m$ *i.e. power law trend*

2. High T (i.e. $>0.5T_m$) – **secondary** creep becomes significant

$$\epsilon_c = \alpha t^m + \beta t$$

(1ary) (2ary - linear)

where α & $\beta = f(\sigma, T)$, empirically derived

Useful generalised form;

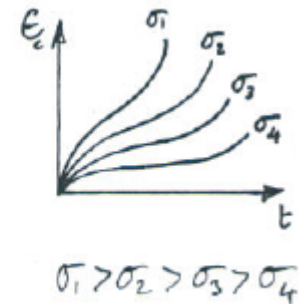
$$\epsilon_c = \alpha t^{1/3} + \beta t + \gamma t^3$$

- good agreement with high temp alloys over wide range

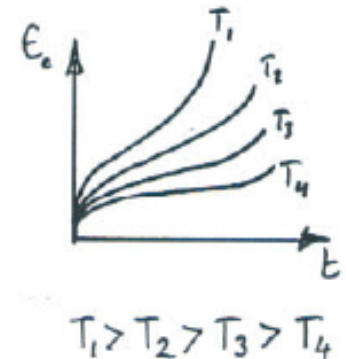
Analysis

- Secondary creep is usual design range.
 - Designs nominally restricted to this region
 - Importance of secondary creep rate $d\epsilon_s/dt$ modelling
- e.g. considering time dependence only;
 1. At low σ , $d\epsilon_s/dt = A \sigma$
 2. At medium σ , $d\epsilon_s/dt = A \sigma^n$ (where $3 \leq n \leq 8$)
 3. At high σ , $d\epsilon_s/dt = A e^{B\sigma}$
- e.g. considering temperature dependence only;
 1. Arrhenius equn. $d\epsilon_s/dt = D e^{-Qc/RT}$
 (where D is constant,
 R is characteristic gas constant,
 Qc is creep activation energy)

Const temp
Curves



Const stress
curves



Analysis

○ Creep strain rate functions

- Assuming that for a given creep time or creep strain; the creep rate is a function of the current σ and T only.

(i.e. implying previous strain history does not influence creep rate)

- Two state equations:

- “Time hardening” $d\epsilon_c/dt = \dot{\epsilon}_c(\sigma, T, t)$

i.e. creep rate for σ & T @ particular accumulated creep **time**, t

- “Strain hardening” $d\epsilon_c/dt = \dot{\epsilon}_c(\sigma, T, \epsilon_c)$

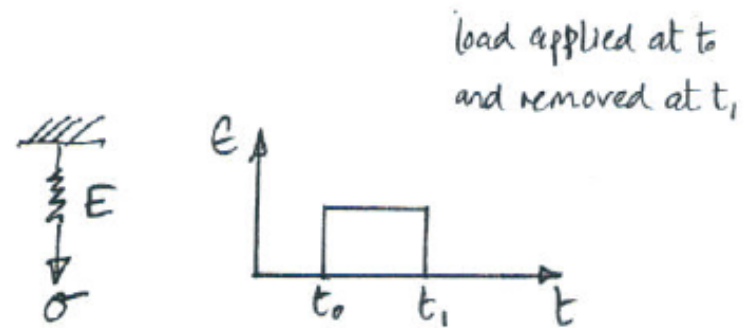
i.e. creep rate for σ & T @ particular accumulated creep **strain**, ϵ_c

Analysis

- Viscoelasticity (**polymers!**)
 - Describes typical behaviour of non-metallic materials e.g. thermoplastics
 - Viscoelastic response is usually described by *rheological* (← relationship between σ and $\dot{\epsilon}$) models:

- For ideal elastic solid;

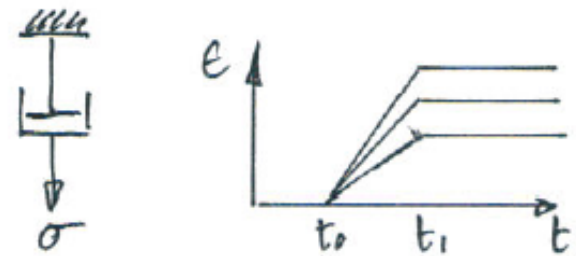
- Hookean *spring* $\sigma = E \epsilon$
 $\Rightarrow \epsilon = \sigma / E$



- For perfectly viscous fluid;

- Newtonian *dashpot* $\sigma = \mu d\epsilon/dt$
 $\Rightarrow \epsilon = \int \sigma / \mu \cdot dt$

μ = coefficient of viscosity



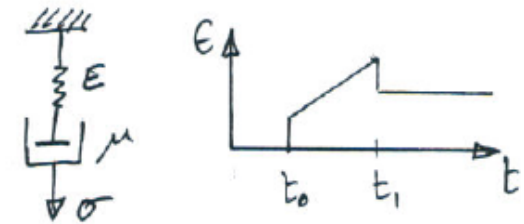
Analysis

- For viscoelastic behaviour;
 - Combined spring & dashpot models

1. Series “Maxwell” model

$$\sigma = \sigma_s = \sigma_d$$

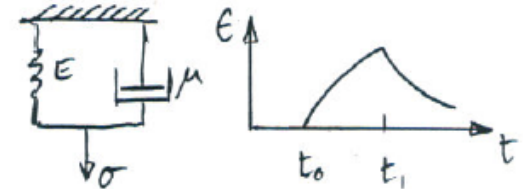
$$\epsilon_m = \epsilon_s + \epsilon_d$$



2. Parallel “Voigt/Kelvin” model

$$\sigma = \sigma_s + \sigma_d$$

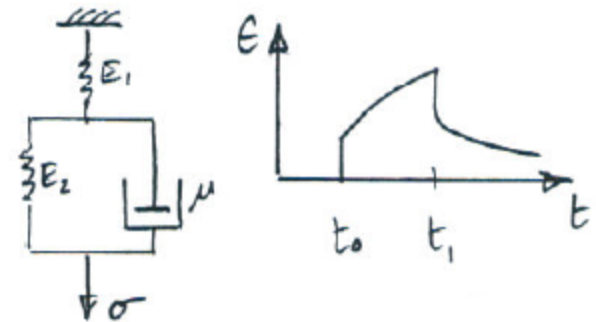
$$\epsilon_v = \sigma/E [1 - \exp(-Et/\mu)]$$



3. Standard Linear Solid model

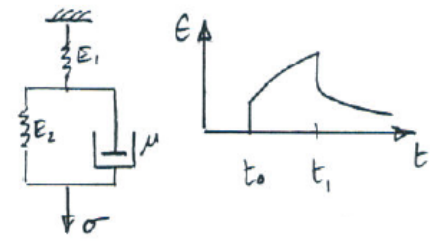
$$\sigma = E_1 \epsilon_1 = E_2 \epsilon_v + \mu d\epsilon_v/dt$$

$$\epsilon = \epsilon_1 + \epsilon_v$$





Analysis



- Note “**SLS**” model = simplest model providing good representation of observed creep + recovery in polymers
- Combined spring & dashpot models
- SLS creep $\Rightarrow \epsilon = \underbrace{\sigma_0/E_1}_{\text{instantaneous } \epsilon} + \underbrace{\sigma_0/E_2 [1 - \exp(-E_2 t_2/\mu)]}_{\text{time dependent creep strain}}$
 $\hookrightarrow \sigma_0/E_2 \text{ as } t \rightarrow \infty$
- SLS recovery after removal of σ_0 at time t_1 , elastic strain σ_0/E_1 is immediately recovered and

$$\epsilon = \sigma_0/E_2 \exp(-E_2 t_2/\mu) [\exp(-E_2 t_2/\mu) - 1]$$

$\epsilon \rightarrow 0 \text{ as } t \rightarrow \infty$

- SLS relaxation

$$\sigma = \epsilon_0 E_1 [E_1 \exp(-\{E_1 + E_2\} t / \mu) + E_2] / (E_1 + E_2)$$

$\sigma \rightarrow (E_1 E_2 \epsilon_0) / (E_1 + E_2) \text{ as } t \rightarrow \infty \text{ i.e. asymptote}$

Note: there are numerous other combinations of springs and dashpots of increasing complexity for modelling creep and relaxation with any required degree of accuracy



Design

- Criteria – design to avoid creep
 - Exceeding allowable deformation
 - Instability due to excessive deformation
 - Creep fracture “rupture”
 - Typical design calcs:
 - σ for given life (t) and T
 - Life (time) for given σ and T
 - σ relaxation time i.e. time for σ to reduce to certain value for given T
 - Relaxed σ for a given t
- For allowable strain
- e.g. bolt relaxation / retightening / creep life

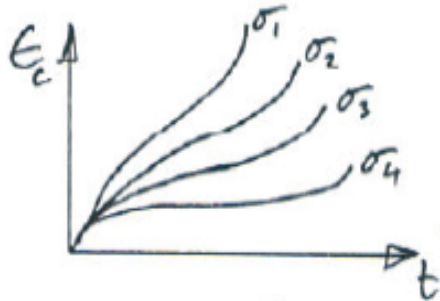
Note: analysis procedures for real problems require computer based numerical techniques because the majority of important parameters have non-linear relationships with each other

Design



- Data – usually based on uniaxial load tests

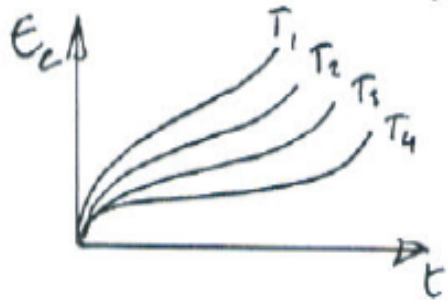
- Constant T creep curves for different σ
- Constant σ creep curves for different T



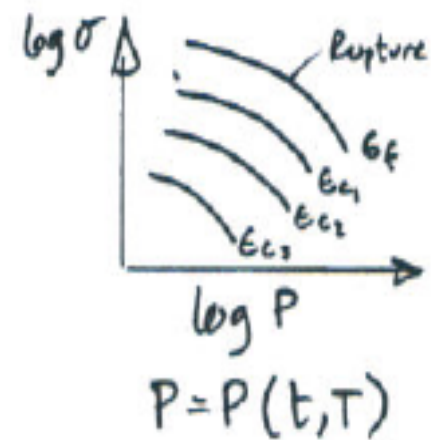
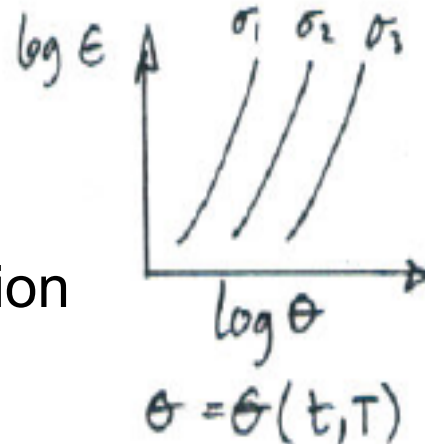
- Secondary creep ϵ rates

i.e. ϵ per unit σ per hour at given T

e.g. $10^{-9} / \text{Nmm}^{-2} / \text{hr} @ 150^\circ\text{C}$ etc.



- State equation parameters correlating creep deformation and rupture life

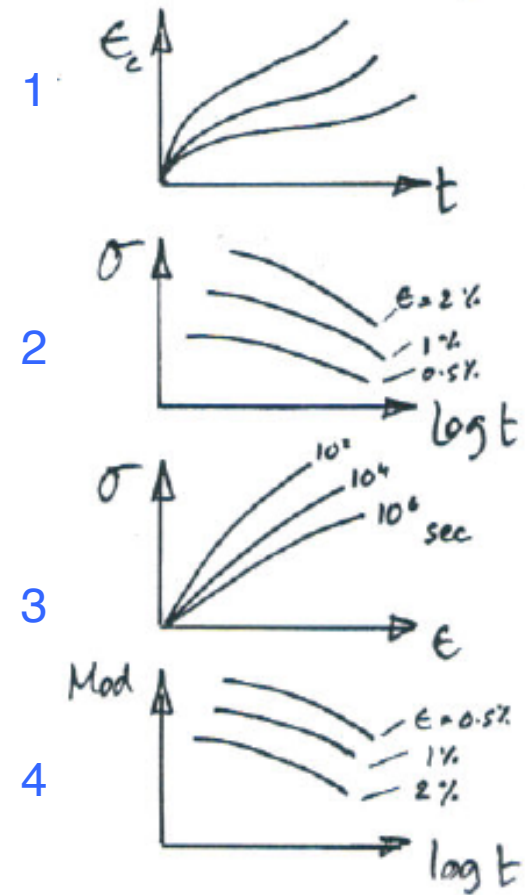


Design

○ Cross-plotted data

(Often used for **thermoplastic** creep design)

1. Obtained from basic constant T creep data
2. Isometric σ / t plots
i.e. σ relaxation at given ϵ
3. Isochronous σ / ϵ plots
i.e. σ / ϵ behaviour at given times
4. Tensile creep E / t plots
i.e. creep modulus reduction at given ϵ

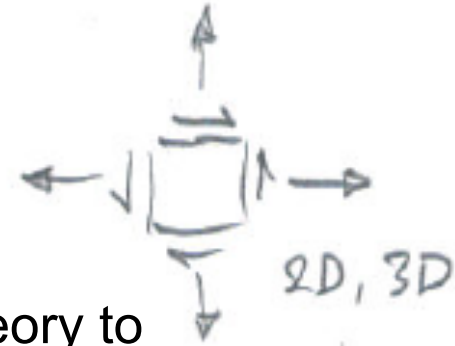


- Isothermal creep data for polymers is often presented in “Isochronous” form –
 - i.e. σ / ϵ curves associated with a particular creep time obtained from a family of creep curves
- For viscoelastic materials, a single modulus for each time may be identified with the slope of the σ / ϵ curve (from Fig. 3 above)

Design of Structures

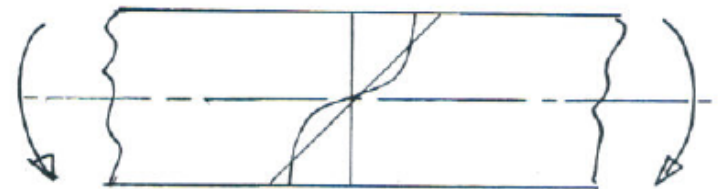
Multi-axial stress states (most structural members)

- Creep is a shear deformation
- Apply Von Mises shear (distortion) strain energy theory to obtain equivalent stress for comparison with uniaxial data
 - e.g. $\sigma_{vm} = f(\sigma_x, \sigma_y, \tau_{xy})$ as a scalar quantity



Varying stress distribution (most structural members)

- e.g. members in bending
- Relaxation \rightarrow redistribution of elastic stress $[My/I]$ to a more uniform condition
- Similarly for members in torsion, discontinuities, etc...



Analysis

○ Creep strain rate functions

- Assuming that for a given creep time or creep strain; the creep rate is a function of the current σ and T only.

(i.e. implying previous strain history does not influence creep rate)

- Two state equations:

- “Time hardening” $d\epsilon_c/dt = \dot{\epsilon}_c (\sigma, T, t)$

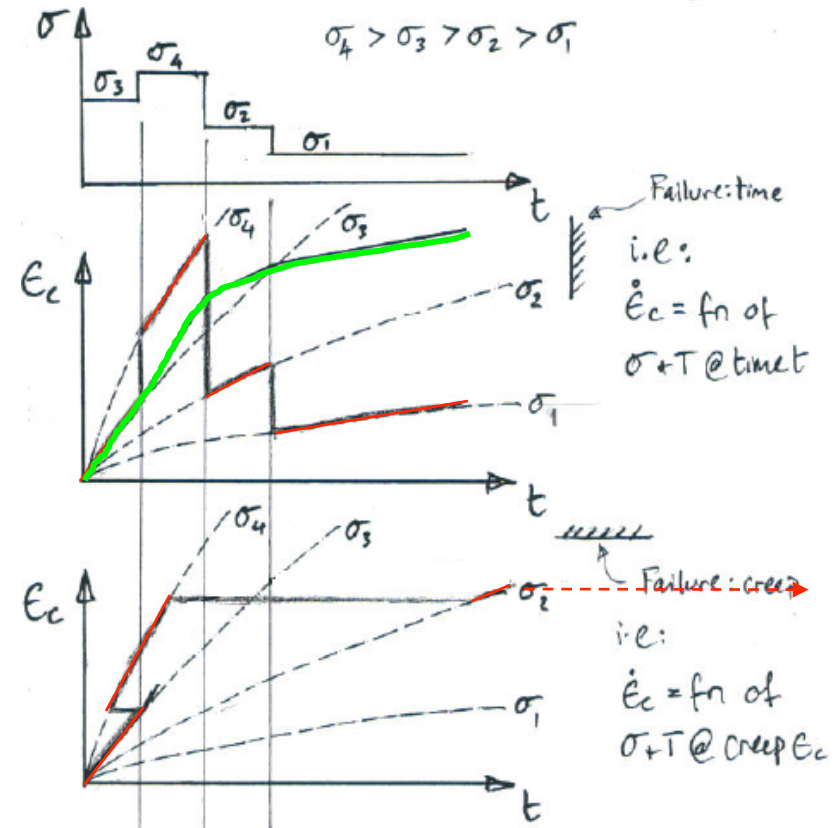
i.e. creep rate for σ & T @ particular accumulated creep **time**, t

- “Strain hardening” $d\epsilon_c/dt = \dot{\epsilon}_c (\sigma, T, \epsilon_c)$

i.e. creep rate for σ & T @ particular accumulated creep **strain**, ϵ_c

Design of Structures

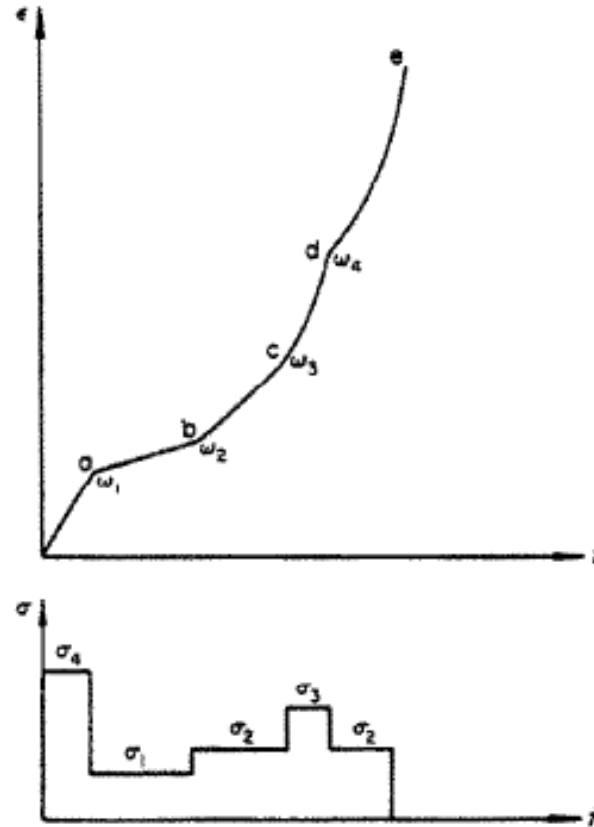
- Changing stress states;
- “Time hardening”
 - $d\epsilon_c/dt = f(\sigma, T, t)$
 - ‘Creep-time fraction rule’ – defining creep w.r.t. accumulated creep time
 - i.e. creep rupture when $\sum_i (t_{ci} / t_{cfi}) = 1$
- “Strain hardening”
 - $d\epsilon_c/dt = f(\sigma, T, \epsilon_c)$
 - ‘Creep-strain fraction rule’ – defining creep w.r.t. accumulated creep strain
 - i.e. creep rupture when $\sum_i (\epsilon_{ci} / \epsilon_{cfi}) = 1$



Note: for most practical applications, predictive methods must be capable of accounting for complex loadings which vary with time



Design of Structures



F.A. Leckie, D.R. Hayhurst Constitutive equations for creep rupture. *Acta Metallurgica*, 25(9), 1977, pp.1059-1070.

Note: for most practical applications, predictive methods must be capable of accounting for complex loadings which vary with time

Material Specification

- Generally, creep resistance increases with T_m (metals) or T_g (polymers)
 - Use stainless steels, superalloys, refractory metals for creep resistance under high σ , T
 - “Creep onset temperatures”

Metals

Al alloys	>200°C
Ti alloys	>315°C
Low alloy steels	>370°C
Stainless steels	>650°C
Super alloys	>1000°C
Refractories	>1500°C

Polymers

(thermoplastics)

- Ambient or slightly above, typically >50°C

(thermosets)

- >50% T_g (Glass Transition Temperature)

Design Examples

- Aero gas turbine blades:
 - elongation + twist $\ll 0.1\%$ total creep
- Airframe subject to kinetic heating
 - Design for $< 0.1\%$ total creep in a/c life
 - e.g. W.M. Doyle, "The Development of Hiduminium-RR.58 Aluminium Alloy: The background to the choice of the main structural material for Concorde", Aircraft Engineering & Aerospace Technology, 41(11),1969, (pp. 11-14)]
- Moving parts generally e.g. steam turbines
 - Design for $< 1\%$ total creep component life
[10,000 hrs \sim 1 year, 100,000 hrs \sim 10 years]
- Avoid creep by:
 - Correct material selection
 - Lowering operational T and / or σ



Case Study

