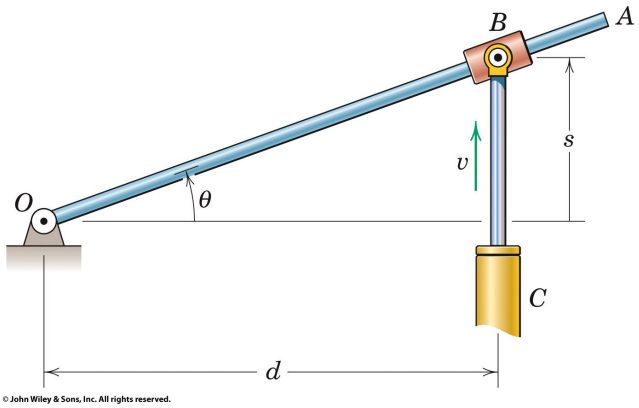
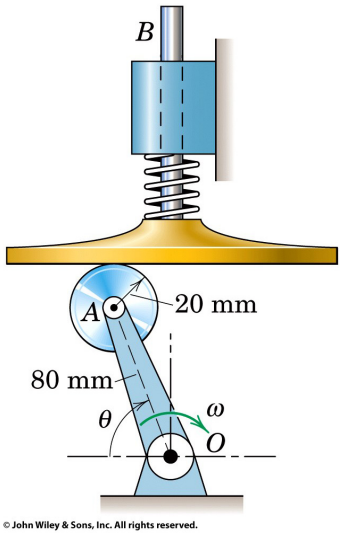
Example Sheet 3 – Kinematics and Dynamics of Rigid Bodies

Mechanisms & Kinematics of Rigid Bodies

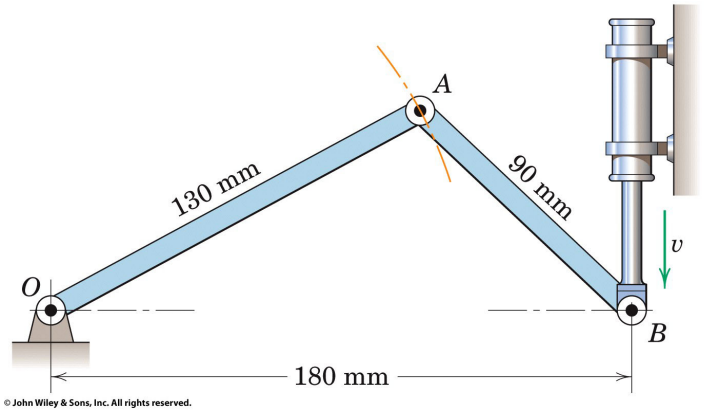
Q1: The fixed hydraulic cylinder C imparts a constant upward velocity v to the collar B, which slides freely on rod OA. Determine the resulting angular velocity ω_{OA} in terms of v, the displacement s of point B, and the fixed distance d.



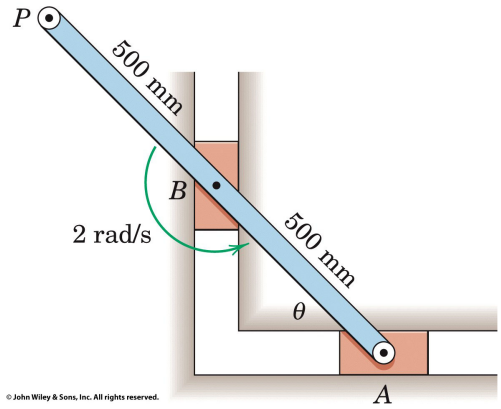
Q2: Determine the vertical acceleration of shaft B for $\theta = 60°$ if crank OA has an angular acceleration $\ddot{\theta} = 8$ rad/s^2 and an angular velocity $\dot{\theta} = 4$ rad/s at this position. The spring maintains contact between the roller and the surface of the plunger.



Q3: The motion of two connected links OA and AB are driven by a vertical piston attached to point B. For the instant represented, point B crosses the horizontal axis through point O with a downward velocity $v = 0.6$ m/s. Determine the corresponding value of the angular velocity ω_{OA} of link OA. Solve this problem using (a) relative motion, and (b) the point of zero velocity.

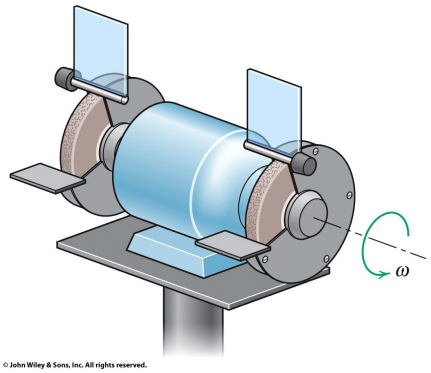


Q4: The motion of the bar AP is controlled by the constrained paths of A and B. If the angular velocity of the bar is 2 rad/s CCW, at $\theta = 45°$, determine the magnitude of the velocities of points A and P. Try to solve this problem using (a) absolute motion, (b) relative motion, and (c) point of zero velocity.

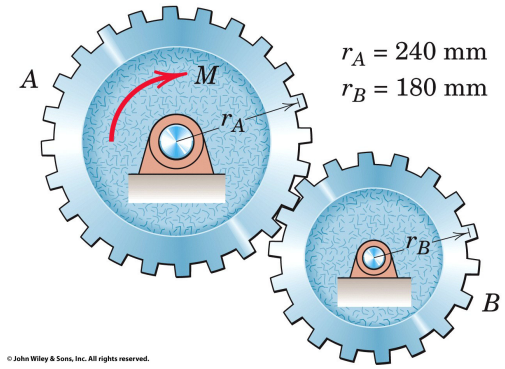


Dynamics of Rigid Bodies

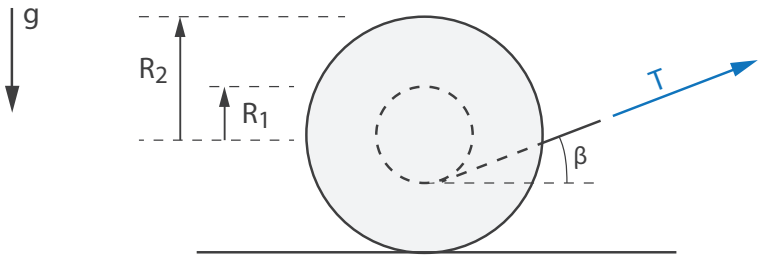
Q5: Each of the two grinding wheels has a diameter of 150 mm, a thickness of 18 mm, and a density of 6800 kg/m^3. The machine accelerates from rest to its operating speed of 3450 rpm in 5 s. When switched off, it comes to rest in 35 s. Determine (a) the motor torque, and (b) the frictional moment. Assume each is constant, and neglect inertia effects of the rotating motor.



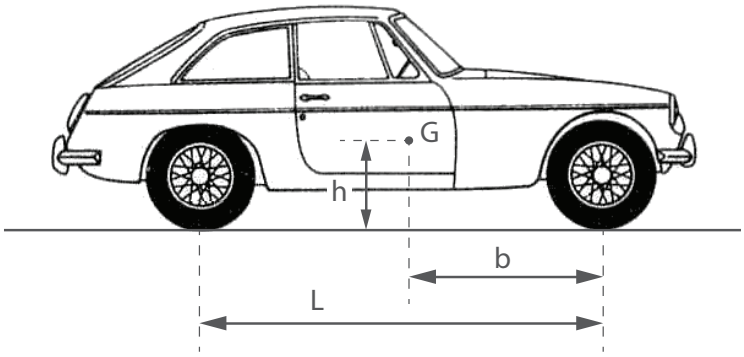
Q6: Two connecting gears are driven by a motor torque $M = 12$ Nm. Gear A has mass 20kg, and radius of gyration 150 mm; the mass of gear B is 10 kg, and radius of gyration of 100 mm. Calculate the angular acceleration of gear B as a result of torque M at shaft A, neglecting fricton.



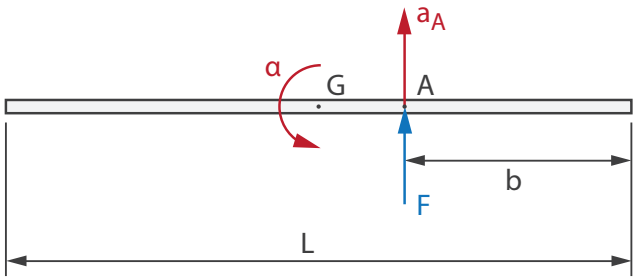
Q7: A yo-yo is pulled by its string at an angle β to the horizontal. Assume that the friction on the floor is sufficient to ensure rolling without slipping. (a) What is the maximum angle β that will allow the spool to be pulled forward? (b) What is the geometric interpretation of this result?



Q8: A classic racing car brakes suddenly. The maximum braking effect is obtained when the tyres are on the verge of slipping, but do not actually slip. The coefficient of friction between the tires and the road is μ, the length of the wheel base is L, and the position of the centre of mass is a distance b behind the front wheels and height h above the road surface. Determine the maximum acceleration of the car, if (a) the wheel front brakes only are used, and (b) if rear wheel brakes only are used. Neglect the inertia of the rotating parts, and assume the suspension of the car is infinitely stiff.



Q9: A uniform slender rod with length $L = 1.6$ m rests on a smooth horizontal surface. A force F is applied at point A, which results in an angular acceleration $\alpha = 18$ m/s^2 and acceleration $a_A = 20$ m/s^2. Determine distance b to point A.



Q10: A uniform beam of length L and mass per unit length ρ is simply supported at each end. The maximum bending moment due to static selfweight has a magnitude of $M = \rho g L^2/8$ midway between the two supports. The support at one end of the beam is then suddenly removed and the beam accelerates under gravity. Just after the support is removed, find the new magnitude and location of the maximum bending moment.

Hints:

Q1: -

Q2: -

Q3: (a) Find an expressions for the velocity of point A with respect to point B and with respect to point O, and equate the two to find ω_{OA}.

Q4: -

Q5: Calculate friction torque first, before calculating motor torque.

Q6: At the point of contact between the two gears, their tangential acceleration is equal; this links the angular acceleration of both gears.

Q7: -

Q8: -

Q9: Calculate the relative acceleration of point A with respect to G, and thereby the acceleration of the centre of mass.

Q10: First, treat the beam as a rigid body and calculate its angular acceleration after the support is removed. Next, cut the beam at a distance λ from the pivot point, and write the equation of motion for this section to find the internal bending moment.

Answers:

Q1: $\omega_{OA} = \frac{vd}{s^2+d^2}$

Q2: $a = -789$ mm/s^2

Q3: $\omega_{OA} = -3.33\boldsymbol{k}$

Q4: $v_P = 1.58$ m/s, $v_A = 0.71$ m/s

Q5: (a) $M_m = 1.005$ Nm, (b) $M_f = 0.126$ Nm

Q6: $\alpha_B = 25.5$ rad/s^2

Q7: (a) $\cos\beta \leq (R_1/R_2)$, (b) applied force passes through contact point between yoyo and ground

Q8: (a) $a = (\mu g\,(L-b))\,/\,(L-\mu h)$, (b) $a = (b\mu g)\,/\,(L+\mu h)$

Q9: $b = 0.533$ m

Q10: $M = \rho g L^2/27$ at $L/3$ from the fixed end