Aerodynamics 2- Rotorcraft Aerodynamics

Lecture 6

Translational Flight

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Recapping on Lecture 4 *(figure of merit, performance coeffs, twist & tip loss, blade coning)*

Figure of Merit

The figure of merit was introduced as a measure of rotor efficiency for rotors of the same diameter. The ideal figure of merit, ($M = 1$) assumes no profile losses and an evenly distributed induced flow over the rotor disk.

The figure of merit can be expressed in coefficient form by $M = 0.707 \dfrac{C_T^{3/2}}{C_Q}$

Profile drag losses cannot be avoided but in theory an even distribution of the induced flow across the rotor can be attained. A good approximation to this can be achieved in practice by a suitable twist of the blade from root to tip and this is referred to as the ***Ideal Twist***.

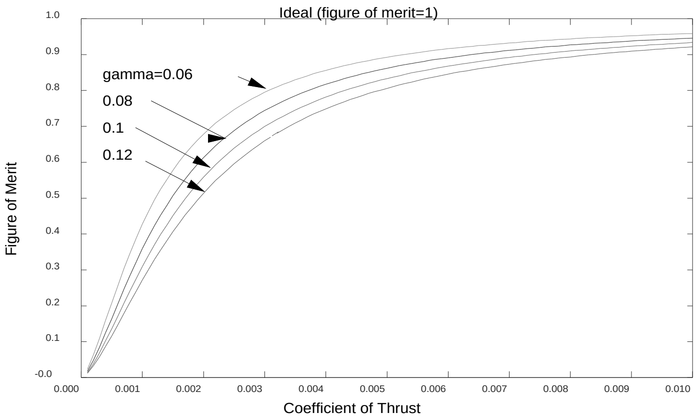
For rotors with ideally twisted blades operating in hover, small angle approximations can be made so that blade element lift is very nearly equal to blade element thrust, ie. $dL \approx dT$

then, $$C_T = \frac{\sigma}{4} a(\theta_t - \phi_t)$$

The inclusion of the profile drag into the expression for the figure of merit gives:

$$\boxed{M = 0.707 \frac{C_T^{3/2}}{\dfrac{C_T^{3/2}}{\sqrt{2}} + \dfrac{\sigma\delta}{8}}}$$

If this expression is plotted for a fixed rotor solidity ($\sigma = 0.04$) then the influence of profile drag can be seen. For zero profile drag, $M = 1$ but as profile drag increases, the figure of merit decreases. At low values of C_T , the figure of merit is also low because δ has a greater influence in the expression. As C_T increases, the relative importance of the δ term decreases and the figure of merit increases.

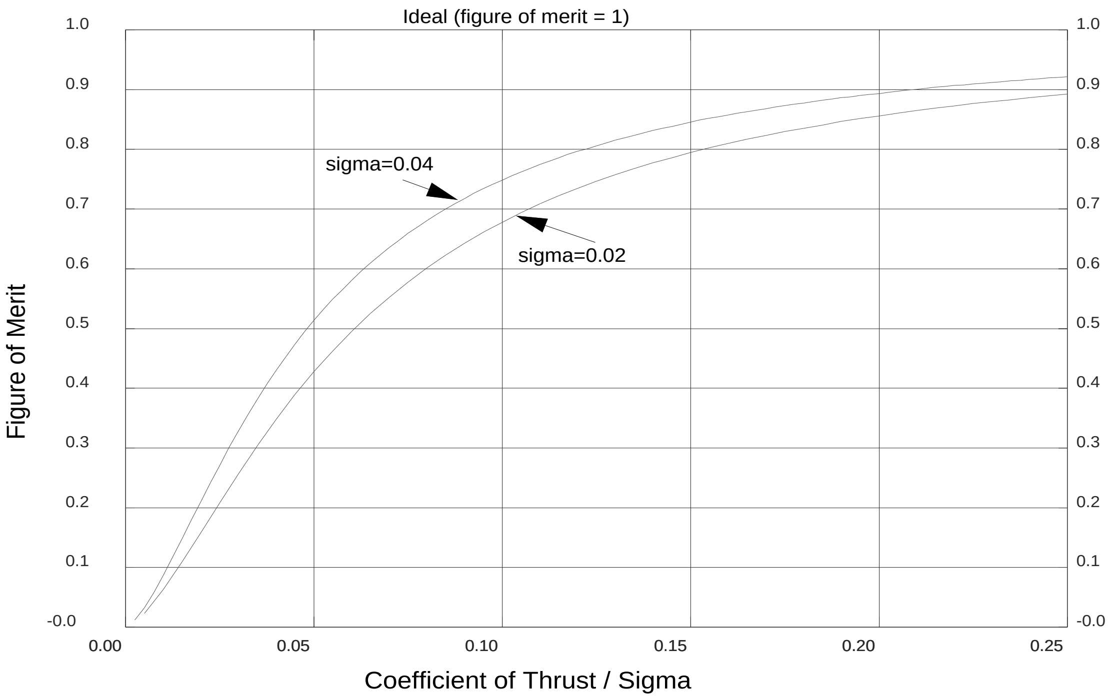


A measure of the lifting performance of the rotor blades is required in order to determine the rotor solidity (blade chord if R and N fixed) for efficient hover. This is chosen so that the blade sections operate at their most efficient lift~drag ratio.

It was shown that the mean lift coefficient of the rotor, $\overline{C_L} = 6 \dfrac{C_T}{\sigma}$

This simple relationship shows that for a rotor of given diameter, the rotor solidity, the mean lift coefficient and the rotational speed are not independent of each other. The term C_T/σ is an important and commonly used parameter of rotor performance, particularly when comparing the hover performance of various rotors operating at the same mean lift coefficient.

The figure of merit against C_T/σ (for $\delta = 0.012$) is shown for solidities $\sigma = 0.04, \sigma = 0.02$.



It shows that if a rotor is operating at a low mean lift coefficient then an increase in C_T/σ by reducing the solidity σ will significantly improve the figure of merit (due to the steepness of the graph in this region). At high values of C_T/σ any improvement in the efficiency can only be obtained <u>from an increase</u> in rotor solidity.

Tip Losses

The Prandtl & Betz approximation for the loss of thrust at the blade tips was discussed. No thrust is assumed to exist outboard of the radius RB, where B is the tip Loss Factor

$$\boxed{B = 1 - \frac{\sqrt{2C_T}}{N}}$$

For a given C_T/σ the tip losses decrease with the number of blades but increasing the number of blades incurs other aerodynamic and structural penalties. Ideally the rotor should be of infinite diameter (to reduce tip losses and induced losses) and have a zero rate of rotation (to avoid profile losses). Blade structural requirements tend to keep the rotor diameter down to reasonable value and since $T \propto \Omega^2$ and $P \propto \Omega^3$ then for efficiency, it pays to keep the rotor speed down.

Unfortunately it is not as simple as that because a high rotational speed is required for:

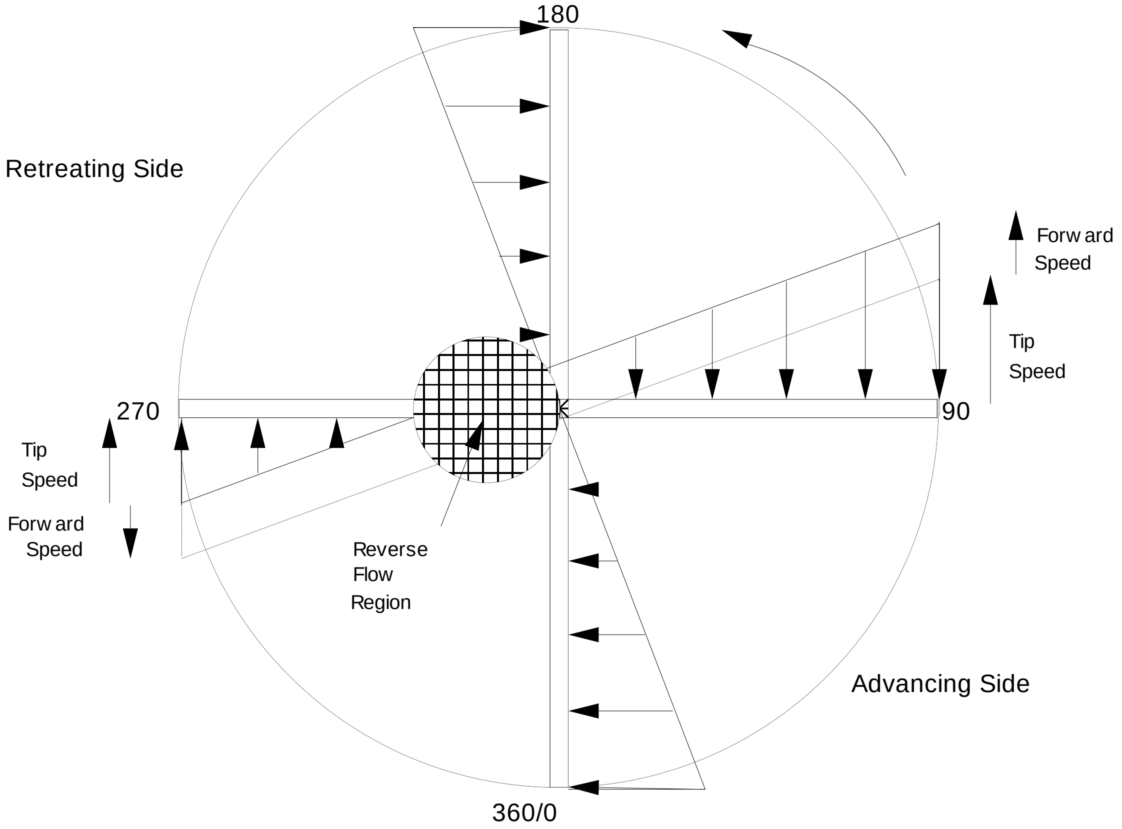
⇒ adequate recovery time to <u>autorotation</u> following engine failure.
⇒ low <u>blade coning</u> angle for structural and aerodynamic reasons
⇒ prevention of <u>retreating blade stall</u> in forward flight

The Rotor in Translational Flight (edgewise flight)

All the previous five lectures on rotary wing aerodynamics have been concerned with axial flow through the rotor. For the helicopter to have any practical use it has to be able to translate and hover over a ground point in windy conditions. The rotor design drivers for translational flight are very different from those for axial flow.

The previous lecture considered the design of a rotor for efficient hovering flight and a high figure of merit was achieved by each blade element operating at its best lift~drag ratio. This could be maintained around the rotor azimuth because the onset velocity at each blade element remained constant.

Consider now the rotor translating forward so that in addition to the blade velocity due to rotation there is a common velocity acting on all elements due to the edgewise flight, as shown below.



The rotor rotates anti-clockwise as viewed from above. The azimuth angle ψ has its datum on the trailing edge of the disk.

The disk comprises of an ***advancing side*** $0^0 \rightarrow 180^0$ and a ***retreating side*** $180^0 \rightarrow 360^0$ At $\psi = 90^0$ the forward velocity adds to the rotational velocity so the tip speed is $\Omega R + V$ and at $\psi = 270^0$ the forward velocity subtracts from the rotational velocity so the tip speed is $\Omega R - V$. At $\psi = 0^0 \,\&\, \psi = 180^0$ the tip speed is ΩR.

Thus edgewise motion of the rotor disk results in an asymmetric disk loading due to the higher velocity over the advancing blades and the lower velocity over the retreating blades. It can be seen from the diagram that there is a point on the retreating blade ($\psi = 270^0$) where the net velocity is zero and inboard of this point, there is a region of reversed flow.

The translational velocity is non-dimensionalised by the rotor tip speed, $\mu = \dfrac{V}{\Omega R}$ where, from basic geometry it can be seen that μ is also the extent of the inboard reverse flow region on the retreating blade, ($\psi = 270^0$).

It was the asymmetric lift that quickly put pay to the early autogyros with their rigid rotor blades. The large rolling moment was powerful enough to roll the aircraft to port often before the take-off speed was reached. Juan de la Cierva had the idea of giving the blades the freedom to flap about their root fitting and so be unable to transfer any rolling moment to the airframe. The advancing blade would flap up and by doing so incur a lower angle of incidence and lift would be reduced. Conversely the retreating blade would flap down, incurring a higher angle of incidence and the lift would be increased. In this way the two sides of the rotor disk produced equal amounts of lift.

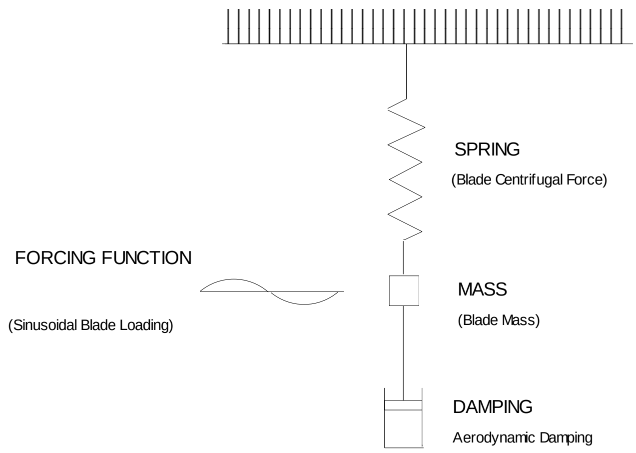
This was the break through that was needed and it was simple and worked perfectly.

It is still employed on the majority of autogyros but for helicopters the blade pitch changes (referred to as “feathering”) are achieved mechanically so the blades do not have to flap. Feathering is an equivalent “means to the same end” as flapping and this is commonly known as the ***flapping-feathering equivalence***.

Whichever method is used, the retreating blade is required to operate at a higher lift coefficient to compensate for the velocity deficit. This could present a problem if the rotor has been designed for maximum efficiency in hover. The demand for increased lift coefficient may well push the retreating blade into stall. The helicopter must not be so designed that its efficiency in one mode of operation is at the expense of its effectiveness in others!

Blade Flapping, Feathering and Equivalence

For simplicity a two-bladed teetering rotor is considered. This allows the blades freedom to flap about a hinge coincident with the rotor shaft. The dynamics and aerodynamics of a rotor are very inter-related. The rotor is considered to be a classical dynamic system where the mass-spring-damper is represented by the blade, the C.F. forces and the flapping induced aerodynamic forces respectively. The system’s forcing function is the sinusoidal blade loading which results from blade velocity variations.



Using the general equation for rotational oscillations, the natural frequency is given by:

$$\omega_n = \sqrt{\frac{K}{I}} \qquad \text{where } K = \text{spring constant } (\boldsymbol{Nm\ /\ rad})$$

$$\text{and } I = \text{moment of inertia } (\boldsymbol{kgm^2})$$

The spring constant of the rotor blade is the restoring force provided by the centrifugal force.

The centrifugal force $(C.F.) = mr\Omega^2$, (for a blade element at radius r)

which results in a moment about the flapping hinge, $(C.F.)r\sin\beta$

which for small angle approximations can be expressed as $(C.F.)r\beta$

Summating for all blade elements, $(C.F.)$ moment = $\int_0^R \Omega^2 r^2 \beta m dr = m\Omega^2\beta\frac{R^3}{3} = M\Omega^2\beta\frac{R^2}{3}$

The flapping moment of inertia of the rotor blade is $I = \frac{MR^2}{3}$.

Thus the natural flapping frequency of the blade is $\omega_n = \sqrt{\Omega^2} = \Omega$ rads/sec

This shows that for a blade that flaps about a hinge on the rotor axis, the natural flapping frequency is the rotational frequency. Thus the rotor is in resonance and the phase angle (that is the force-displacement angle) is 90^0.

The forcing function (in forward flight) has a maximum and minimum value at $\psi = 90^0$ and $\psi = 270^0$ respectively. It follows then that the displacement has a maximum and minimum value at $\psi = 180^0$ and $\psi = 360^0$ respectively. This means that in forward flight the rotor disc will pitch upwards (in the positive sense).

In addition to the lateral lift asymmetry in forward flight, there is a longitudinal asymmetry of lift due to the coning of the rotor disk. This is exacerbated by the rearward tilt of the rotor due to the lateral asymmetry. The forward edge of the rotor disk ($\psi = 90^0 \rightarrow 270^0$) is subject to a vertical component of the onset velocity that effectively increases the blade incidence α. The vertical component is $V\sin(a_1 + a_0)$ where a_1 is the tilt back angle. Conversely the trailing edge of the rotor disk ($\psi = 270^0 \rightarrow 90^0$) has a velocity of $V\sin(a_1 - a_0)$ which by virtue of the coning angle results in a lower blade incidence. This forcing function produces a lateral tilt b_1 (to starboard for anticlockwise rotors in forward flight).

The longitudinal and lateral tilting of the rotor disk are the two primary considerations for aerodynamic analysis of the rotor in forward flight. There are other aerodynamically generated flapping motions but these are higher harmonic modes ($a_2, b_2, a_3, b_3 ...etc$) which result in blade weaving motions and are of little importance in the study of rotor control and performance. They can have considerable importance in problems of rotor vibration and blade stress however.
The general blade flapping motion can be expressed as:

$$\beta = a_0 - a_1\cos\psi - b_1\sin\psi - a_2\cos 2\psi - b_2\sin 2\psi - ....$$

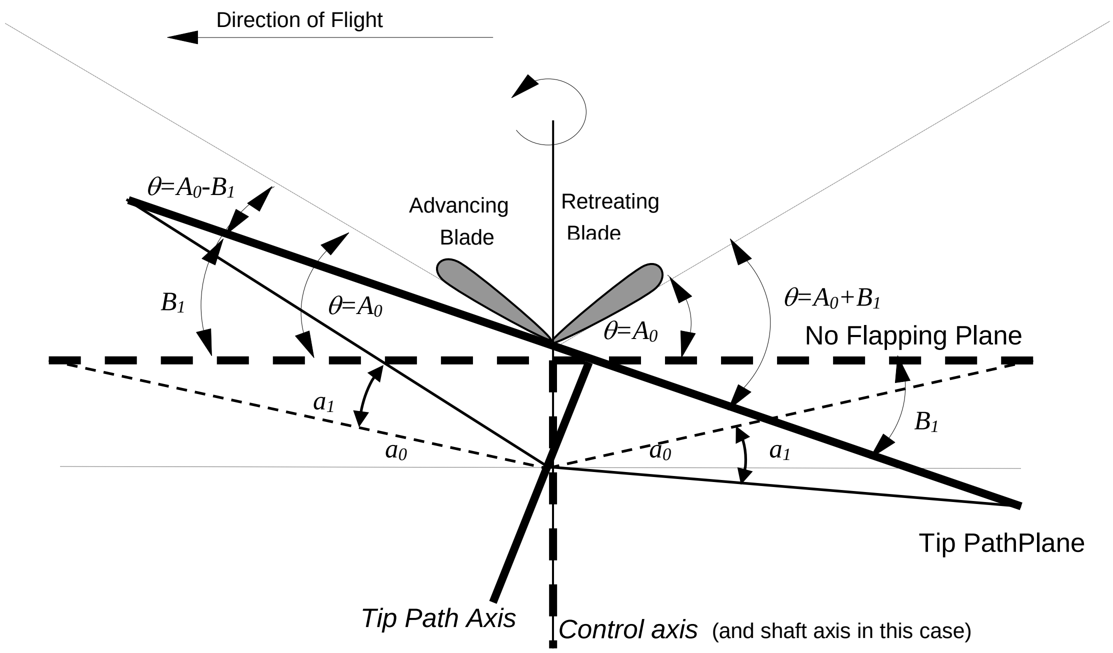
where only the first three terms are generally considered. Relative to the rotor shaft therefore, in translational flight the blades will flap in a manner corresponding to the relationship between β and ψ as given above. The rotor blades can be controlled (by suitable variation in pitch angle θ) so that $\beta = a_0$ for all angles of ψ (ie. no flapping). The variation in pitch angle θ for this to result can be expressed as a similar series of terms as for the flapping angle.

$$\theta = A_0 - A_1 \cos\psi - B_1 \sin\psi - A_2 \cos 2\psi - B_2 \sin 2\psi - .....$$

where again only the first three terms are generally considered unless Higher Harmonic Control is of interest. For this expression,

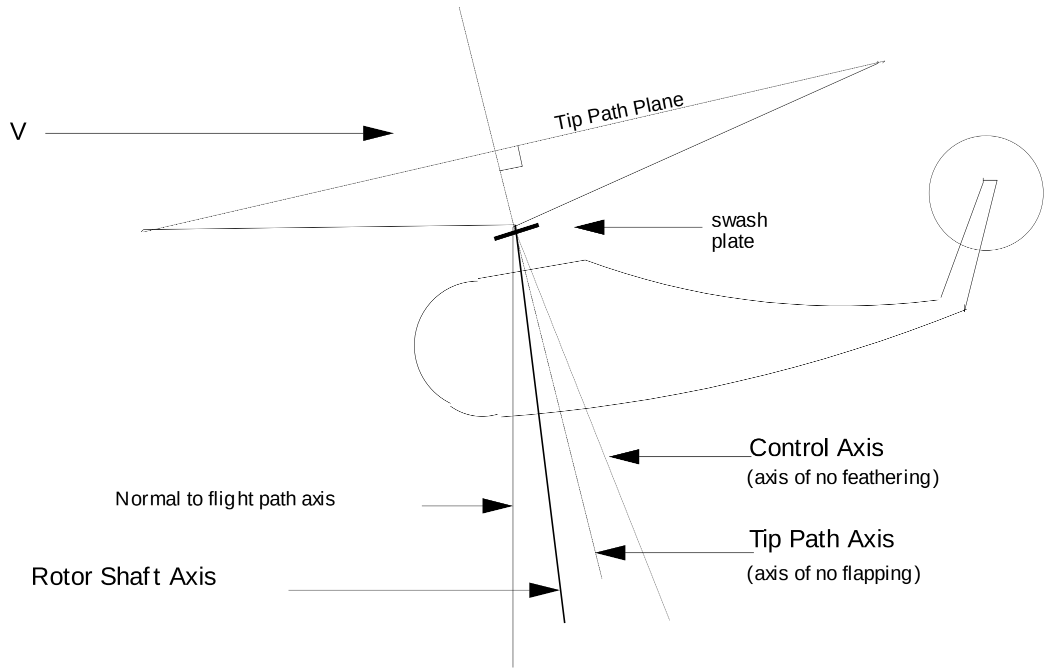
$\theta =$ blade pitch at azimuth position (usually 0.75R)
$A_0 =$ mean blade pitch angle
$A_1 =$ first harmonic (longitudinal pitch input)
and $B_1 =$ first harmonic (lateral pitch input)

The mean blade pitch is effected by the ***collective pitch*** control. All other blade pitch changes are achieved by application of the ***cyclic pitch*** control.



Equivalence of flapping and feathering

The plane prescribed by the blade tips is referred to as the **Tip Path Plane** (TPP) and by definition, this must also be the plane of zero flapping. For the same blade motion, there is another reference plane in which flapping exists but the blade pitch remains constant. This is the **No Feathering Plane** (NFP). The axes associated with the TTP and the NFP are the ***tip path plane axis*** *(tpa)* and the ***no feathering axis*** *(nfa)* and also referred to as ***control axis***) respectively. These axes are never coincident in forward flight and are rarely coincident with the shaft axis or the normal to flight path axis. The diagram below shows a typical arrangement of the axes in forward flight.



In one aspect, the above diagram may appear to contradict what has been said with regard to translational flight - namely that the rotor disk tilts back in forward flight. For an autogyro that remains the case but for the helicopter (as shown here) a forward velocity requires a forward pointing component of the rotor lift force, so the TPP has to be inclined forward. Thus the pilot needs to apply enough cyclic pitch to counter the tilt-back (equivalence by feathering) plus a little more to provide the propulsive component.

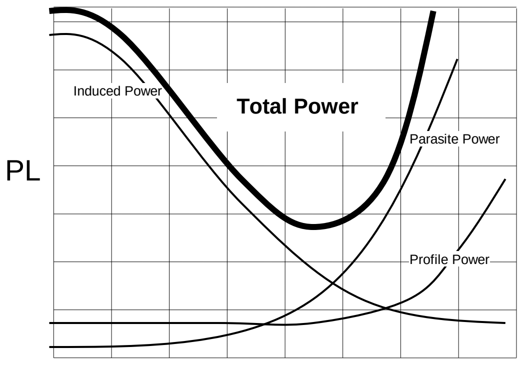
Induced Velocity in Translational Flight

In translational flight, as in the climb, the flow of air through the rotor is greater than that induced for the hover case. As the thrust is due to the rate of change of momentum, the higher mass flow rate requires a lesser increase in velocity for the same thrust.

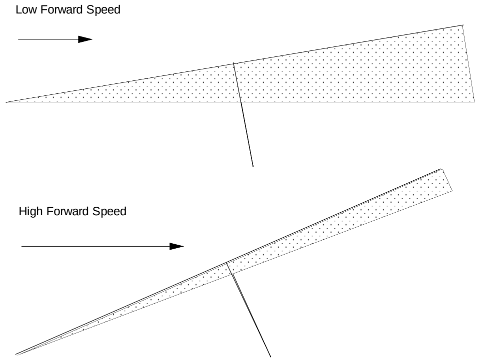
The induced power reduces with increased translational speed, thus the Total Power~Velocity graph for the helicopter is similar to the fixed wing aircraft. A decrease in induced power and an increase in profile and parasitic power with increasing speed.

All the endeavours of the rotor designer to attain a constant level of induced velocity over the rotor disk are literally swept away when the rotor has translational flight.
As the translational speed increases the situation gets a lot worse before it gets better again.

Breakdown of Helicopter Power Losses



At relatively low forward speed, the vorticity in the wake is convected back and will therefore induce more flow through the trailing edge of the rotor than at the leading edge. At very high forward speed, the disk is tilted further forward and a generally lower value of induced velocity will exist.



The reduced induced velocity over the leading edge of the rotor at low to moderate forward speed will increase the local angle of attack so that a positive rolling moment is generated (for anti-clockwise rotors).

It can be seen that at high translational speeds the flow through the rotor is predominantly the component of the onset flow and a more even distribution of induced velocity results.

Thrust $\boldsymbol{T} = (\rho \boldsymbol{A v}) \mathbf{2} \boldsymbol{v}$ for the hovering rotor and this gives $\boldsymbol{v} = \sqrt{\dfrac{\boldsymbol{T}}{\mathbf{2} \rho \boldsymbol{A}}}$.

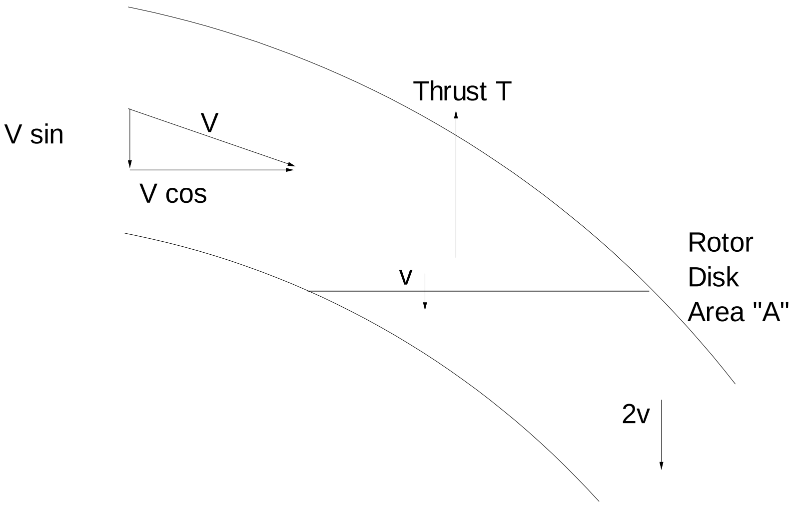
Clearly for translational flight, the unit mass flow $(\rho \boldsymbol{A v})$ has increased as it now includes the translational component of flow through the rotor. So unit mass flow is $\rho A V'$,

$$\text{where } V' = \sqrt{(V\cos\alpha)^2 + (V\sin\alpha + v)^2} = \sqrt{V^2 + 2Vv\sin\alpha + v^2}$$

Thus V' is the vector sum of the translational and induced velocities so in the original thrust equation,

$$\boldsymbol{T} = (\rho \boldsymbol{A V'}) \mathbf{2} \boldsymbol{v}$$

so that $v = \dfrac{T}{2\rho A V'}$.



It has already been shown that :

$$\lambda = \frac{\boldsymbol{V} \ \mathbf{sin}\ \alpha + \boldsymbol{v}}{\Omega \boldsymbol{R}}$$

$$\mu = \frac{\boldsymbol{V}\ \mathbf{cos}\ \alpha}{\Omega \boldsymbol{R}}$$

Thus $V' = \Omega R \sqrt{\lambda^2 + \mu^2}$ and therefore $\boxed{v = \dfrac{\frac{1}{2} C_T \Omega R}{\sqrt{\lambda^2 + \mu^2}}}$

If $V = 0$, then $\mu = 0, \lambda = \dfrac{v}{\Omega R}$ and $v = \Omega R \sqrt{\dfrac{C_T}{2}}$ which is the same as for v in the hover.