Vibrations 2, Lecture 22
Dynamic Aeroelasticity
Flutter

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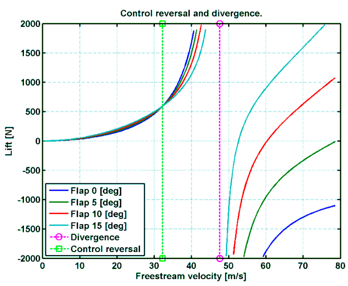
Lecture 21 review

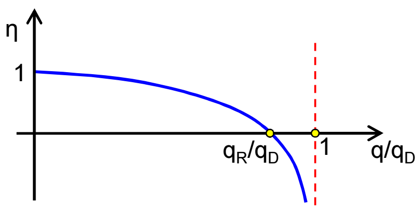
Control reversal:

$$q_R(U_R) = -\frac{k\,C_{L,\beta}}{Sc\,C_{L,\alpha}\,C_{M,\beta}}$$

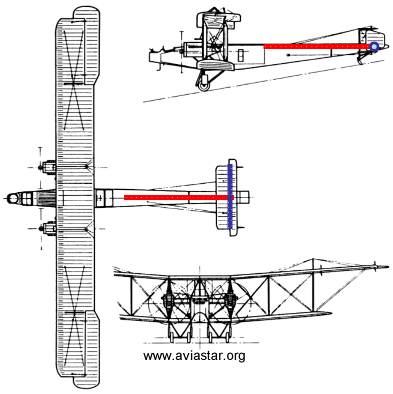


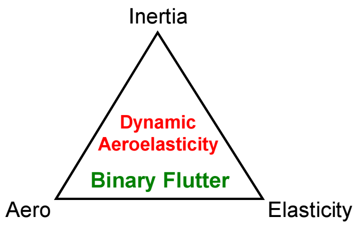
Lift effectiveness:

$$\eta = \frac{\partial L/\partial\beta|_{elastic}}{\partial L/\partial\beta|_{rigid}} = \frac{1-q/q_R}{1-q/q_D}$$



Handley Page O/400

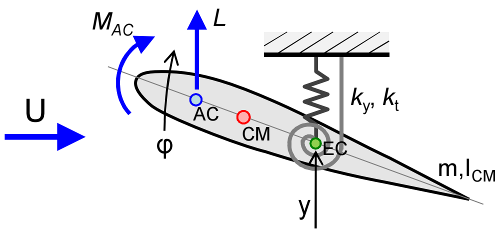




"The first recorded flutter incident was on a Handley Page O/400 twin engine biplane bomber in 1916. The flutter mechanism consisted of a **coupling** of the **fuselage torsion mode** with an **antisymmetric elevator rotation mode**. The elevators on this airplane were independently actuated. The solution to the problem was to interconnect the elevators with a torque tube." M.W. Kehoe, A Historical Overview of Flight Flutter Testing, NASA TM 4720, 1995 (page 3)

2 DOF aeroelastic model

2 DOF model of a rigid wing suspended on the linear and torsional *springs* placed in airstream*:*



Matrix form of the EOM – see lectures 16 and 17:

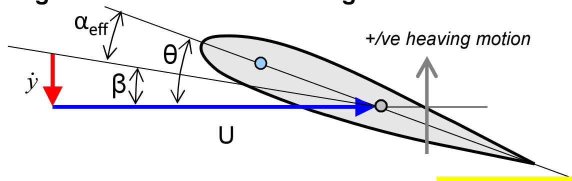
$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}$$

$$\begin{bmatrix} m & mx_{CM} \\ mx_{CM} & I_{CM} + mx_{CM}^2 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} k_y & 0 \\ 0 & k_t \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} L \\ M_{AC} + Lx_{AC} \end{bmatrix}$$

Quasi-steady aerodynamics

Quasi-steady aerodynamics: At any instant in time, the aerodynamic characteristics are assumed to be the same as if the aerofoil were **moving with constant** heave or pitch velocities (c.f. steady aerodynamics = no motion!).

Heaving motion & the effective angle of attack:



$$\tan\beta = \frac{\dot{y}}{U} \approx \beta$$

$$\alpha_{eff} = \theta - \beta \approx \theta - \frac{\dot{y}}{U}$$

Moment change due to pitching velocity: $C_{M_{ac}} = C_{M_{ac},\dot{\theta}} \dfrac{\dot{\theta}c}{U}, \quad C_{M_{ac},\dot{\theta}} < 0$

$C_{M_{ac},\dot{\theta}}$ *is the rate of change of moment w.r.t. (non-dimensionalised) pitching velocity and is typically negative.*

Generalized loads with quasi-steady modifications:

$$Q_1 = L = qSC_L = q\,S\,C_{L,\alpha}(\theta - \dot{y}/U)$$

$$Q_2 = M_{AC} + Lx_{AC} = q\,S\,c\,C_{M_{ac},\dot{\theta}}(\dot{\theta}c/U) + q\,S\,C_{L,\alpha}(\theta - \dot{y}/U)\,ec$$

$S = s\,c$
where *s* is the wing span and *c* is the chord length

Aeroelastic model in matrix form

Matrix form of the EOM:

$$\underbrace{\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q}}_{} = \underbrace{\frac{1}{2}\rho U \begin{bmatrix} -s\,c\,C_{L,\alpha} & 0 \\ -s\,c^2 e C_{L,\alpha} & sc^3 C_{M_{ac},\dot{\theta}} \end{bmatrix} \begin{Bmatrix} \dot{y} \\ \dot{\theta} \end{Bmatrix} + \frac{1}{2}\rho U^2 \begin{bmatrix} 0 & s\,c\,C_{L,\alpha} \\ 0 & s\,c^2 e C_{L,\alpha} \end{bmatrix} \begin{Bmatrix} y \\ \theta \end{Bmatrix}}_{}$$

Structural part
mass stiffness

aerodynamic part of the model.
aerodynamic damping aerodynamic stiffness

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = (1/2)\rho U\,\mathbf{A}_{\dot{q}}\dot{\mathbf{q}} + (1/2)\rho U^2\,\mathbf{A}_q\mathbf{q}$$

$$\mathbf{M}\ddot{\mathbf{q}} + (\mathbf{0} - (1/2)\rho U\,\mathbf{A}_{\dot{q}})\dot{\mathbf{q}} + (\mathbf{K} - (1/2)\rho U^2\,\mathbf{A}_q)\mathbf{q} = \mathbf{0}$$

Thus, the aeroelastic free vibration problem is:

Note the presence of (aerodynamic) damping even though our original problem was UNDAMPED!

$$\tilde{\mathbf{M}}\ddot{\mathbf{q}} + \tilde{\mathbf{D}}\dot{\mathbf{q}} + \tilde{\mathbf{K}}\mathbf{q} = \mathbf{0}$$

New mass and stiffness matrices depend on the aerodynamic parameters and airflow conditions!

Damped vibrations – revision

- **1 DOF problems**
  - Free vibration problem $m\ddot{x}+c\dot{x}+kx=0$
    - Free response $x(t)=Ae^{st},\ s=s_R+is_I$
    - Characteristic equation $ms^2+cs+k=0$
    - Roots=Eigenvalues=damp+freq $s_{1,2}=-\delta\pm i\omega_D,\ \delta=\zeta\omega_0$
    - Damping + natural frequency $x(t)=Ae^{-\delta t}\cos(\omega_D t-\phi)$
- **N DOF problems**
  - Free vibration problem $\mathbf{M}\ddot{\mathbf{x}}+\mathbf{K}\mathbf{x}=\mathbf{0}$
    - **Undamped** free response $\mathbf{x}=\mathbf{a}\sin(\omega t+\varphi)$
    - Eigenvalue problem $(\mathbf{K}-\omega^2\mathbf{M})\mathbf{a}=\mathbf{0}$
    - Characteristic equation $\det(\mathbf{K}-\omega^2\mathbf{M})=0,\ a(\omega^2)^2+b(\omega^2)+c=0$
    - Roots=Eigenvalues=frequency $\omega_{0,j},\ j=1,2,\ldots,N$
  - Forced harmonic vibration $\mathbf{M}\ddot{\mathbf{x}}+\mathbf{C}\dot{\mathbf{x}}+\mathbf{K}\mathbf{x}=\mathbf{f}_0 e^{i\omega t},\ \mathbf{x}(t)=\mathbf{x}_0 e^{i\omega t}$
    - **Damped** steady state vibrations $(-\omega^2\mathbf{M}+i\omega\mathbf{C}+\mathbf{K})\mathbf{x}_0=\mathbf{f}_0,\ \mathbf{x}_0=(\ldots)^{-1}\mathbf{f}_0$

Aeroelastic eigenvalue analysis

Explore (free) aeroelastic vibrations – using analogy based on the previous slide:

$$\tilde{\mathbf{M}}\ddot{\mathbf{q}}+\tilde{\mathbf{D}}\dot{\mathbf{q}}+\tilde{\mathbf{K}}\mathbf{q}=\mathbf{0} \qquad \mathbf{q}=\mathbf{Q}e^{st}, s=s_R+is_I$$

Q is the vector of amplitudes, s is complex number.

from this the damped or quadratic eigenvalue problem:

$$(s^2\tilde{\mathbf{M}}+s\tilde{\mathbf{D}}+\tilde{\mathbf{K}})\mathbf{Q}=\mathbf{0}$$

nontrivial solutions are obtained if the characteristic equation:

$$\det(s^2\tilde{\mathbf{M}}+s\tilde{\mathbf{D}}+\tilde{\mathbf{K}})=0$$

This polynomial can be solved numerically in Matlab using function **ROOTS**.

For 2 DOF system, this is a quartic polynomial with *U*-dependent coefficients:

$$a_1s^4+a_2s^3+a_3s^2+a_4s+a_5=0,\ a_k=a_k(U)$$

This problem has 4 complex roots (eigenvalues) **AND** they depend on *U*!

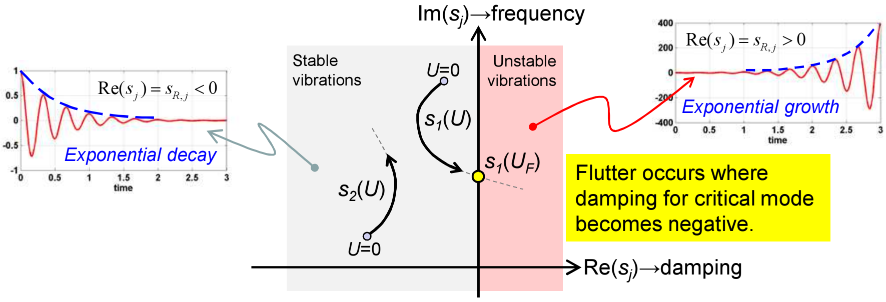
$$s_j=s_j(U),\ \ j=1,2,3,4$$

Eigenvalue analysis

Eigenvalues are often presented in the complex plane. There, they trace the trajectories which depend on one parameter – airstream speed U=[0,U_{max}].

Eigenvalues often come as complex conjugate pairs (symmetric about the horizontal axis) – only one member of each pair is usually shown in graphs.

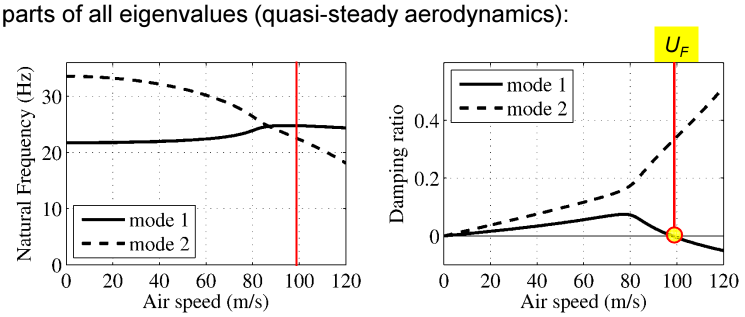
The speed for which an eigenvalue crosses (i.e. the first to cross) from the left (stable) to the right (unstable) half-plane is called the flutter speed U_F.



The stability is assessed by studying the real parts of all eigenvalues for all speeds U.

Flutter diagram and flutter mechanism

Real parts of all eigenvalues (quasi-steady aerodynamics):

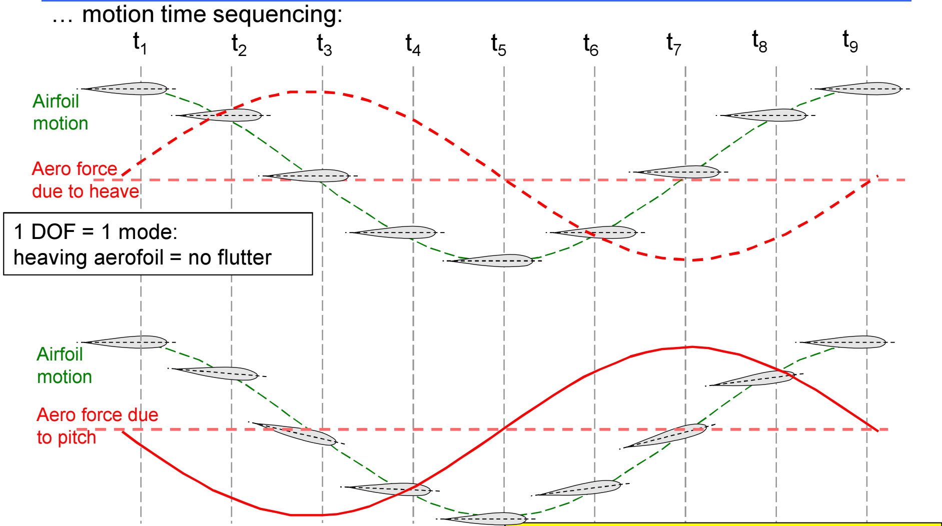


What happens:

- Natural frequencies can move closer together with speed due to aerodynamic stiffness,
- Together with damping, this causes previously uncoupled modes of vibration to couple, i.e. interact,
- Torsion mode can include bending component and vice versa,
- These components can be out of phase with each other *due to damping*,
- Under these conditions, a wing can extract energy from the airflow and become unstable, or lose energy to the airflow rapidly and become highly damped,
- In binary flutter, both of these occur,
- One of the two modes becomes unstable, the other highly damped.

Pitch and heave interaction

… motion time sequencing:

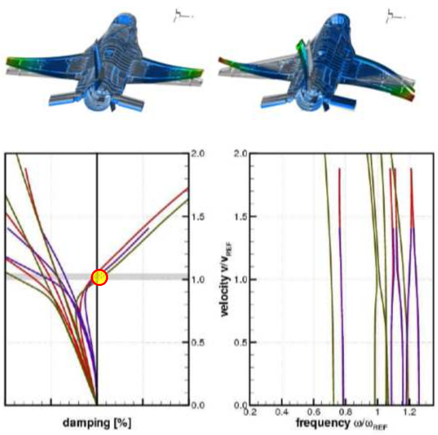


It is possible that the aerodynamic force induced by one mode of vibration does work on the other mode !

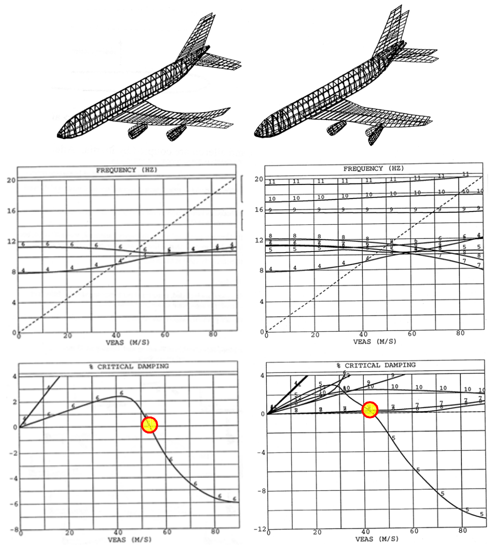
Selected flutter considerations

- Unsteady aerodynamics
  - Aerodynamic coefficients depend on the frequency parameter
  - frequency parameter = $\omega c/U$
- Flutter types
  - Soft (gradual) flutter vs. Hard (sudden) flutter
  - Binary, panel, control surface (balancing), …
- Flutter prevention and design
  - Mass distribution - engines, fuel, tip devices, …
  - c.m. vs. e.c. offset (inertial coupling, see **M** matrix)
- Flutter testing
  - Flight tests vs. Wind Tunnel tests vs. Ground Vibration Tests (GVT)

Examples



B.B. Pprananta, M.H.L. Hounjet and B.J.G. Eussen, Reduction of LCO Sensitivities of the F-16 by Wing Modification, AIAA 2010-2635, 2010



Airbus, Filton, 1996

Summary

- Flutter - most important of aeroelastic phenomena
- Dynamic phenomenon – violent unstable vibration
  - often resulting in structural failure
- Two modes couple to extract energy from the airflow
  - negative damping effect
- Important considerations – the key to flutter prevention is to break up any coupling between modes
  - inertial, aerodynamic, elastic coupling