(1)

Stability Design at Part Level - Global Buckling

At part level we consider "global" modes of buckling failure. Examples of global buckling modes include: axial buckling, lateral buckling and torsional buckling.

Here we will consider axial strut/column buckling based on the Euler equation:

$$P_{CRIT} = k\frac{\pi^2 E I}{L^2}$$ where k is an end fixity constant

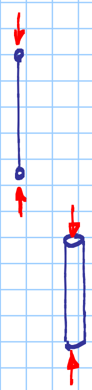
Commonly, we refer to "Slenderness Ratio" L/ρ

e.g. rewriting the Euler equation:

$$\sigma^*_{CRIT} = k\frac{\pi^2 E I}{A L^2} = k\pi^2 E\left(\frac{\rho}{L}\right)^2$$ where $\rho = \sqrt{\frac{I}{A}}$ = "Radius of gyration"

(2)

Plastic Buckling of Columns and Panels

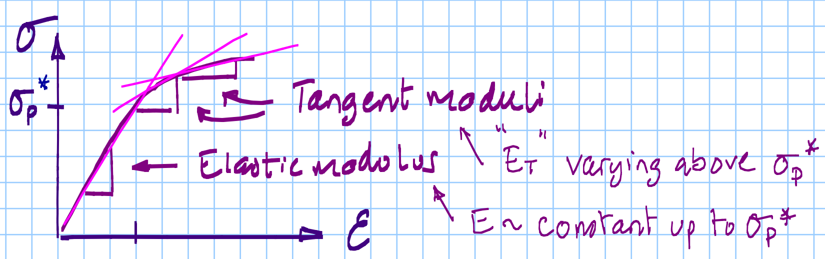


Classic Euler buckling assumes that the material remains elastic and this will be true for slender struts which buckle at stress levels well below their yield or proof strength.

However, if the L/ρ slenderness ratio is low then the stress level necessary for buckling may exceed the elastic limit of the material before the onset of buckling.

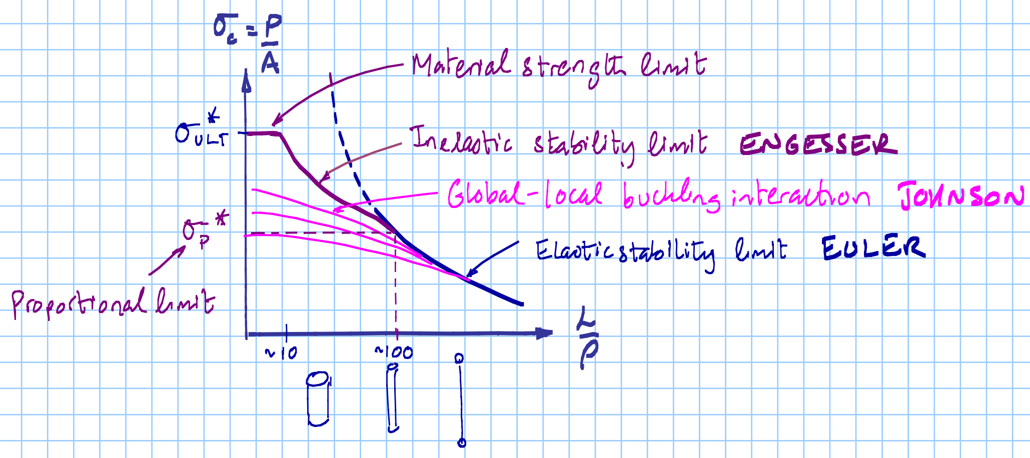
If this occurs then we can no longer refer to the elastic modulus, i.e. Young's modulus, in our analysis and instead we need to refer to the "Tangent modulus."

E.g. Consider a typical aluminium alloy stress strain curve:



The transition between elastic and plastic buckling can be represented on diagrams of compressive stress σ_c vs slenderness ratio L/ρ e.g. Engesser curves which depart from the Euler trend at $L/\rho < 100$

E.g.



If the column is susceptible to local buckling, e.g. panel buckling or wrinkling then an interaction between local and global modes may occur which will result in further reduction of the buckling strength, e.g. as illustrated by Johnson - Euler curves.

Plastic buckling correction factors

If the elastic buckling prediction exceeds the elastic limit of the material, i.e. $\sigma_{c_{CRIT}}^* > \sigma_y^*$ or σ_P^* then the predicted value is not valid and plastic buckling must be considered by referring to the tangent modulus.

However, tangent modulus is not constant and varies with stress level. So, to know what tangent modulus to apply we need to know the stress-level at which buckling occurs! To overcome this we refer to a plastic correction factor "η"

E.g.: $$\sigma_{c_{CRIT}}^* = k\eta\pi^2 E\left(\frac{\rho}{L}\right)^2 \quad \text{for COLUMNS}$$

Similarly: $$\sigma_{c_{CRIT}}^*,\ \sigma_{b_{CRIT}}^*,\ \tau_{CRIT}^* = \underline{\underline{k\eta E\left(\frac{t}{b}\right)^2}} \quad \text{for PANELS}$$

Semi-empirical expressions exist for these correction factors but they are more often accounted for by reference to non-dimensionalised design charts for specific materials which are contrived to give plastic buckling strengths directly without actually evaluating the correction factor itself. Examples for aluminium alloys are given in AVD2 Design Data and further examples can be obtained from references by Bruhn and Niu. E.g.: ⑤

[3]

Compression

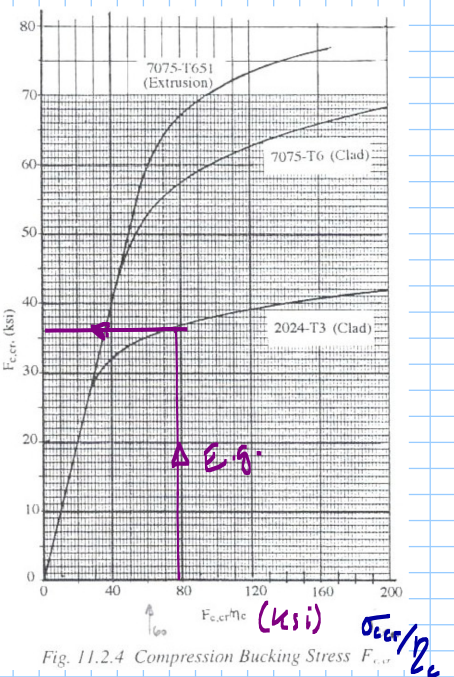


Fig. 11.2.4 Compression Bucking Stress $F_{c,cr}$

Shear

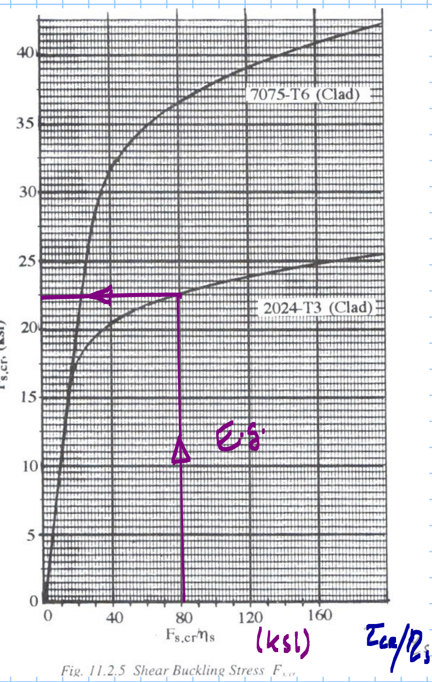
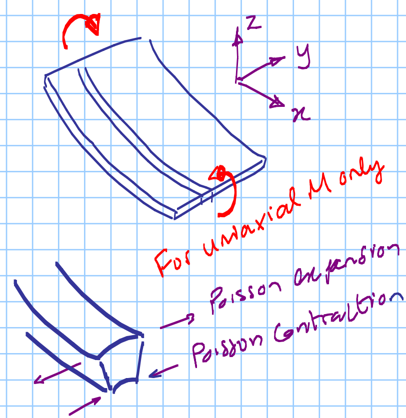


Fig. 11.2.5 Shear Buckling Stress $F_{s,cr}$

Stability Design at Section Level - Panel Buckling

ICF 25. 1. 2010 ⑥

Consider a thin flat plate subjected to bending:



E.g. $\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E}$ ①

and $\varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E}$ ②

In isolation a single strip of the plate would also curve laterally about the neutral line as well as longitudinally about the neutral axis due to opposing "Poisson strains".

E.g. for the sense of bending illustrated there is longitudinal compression of the upper surface and tension of the lower surface and associated opposite transverse strains according to Poisson's ratio.

For an unrestrained isolated strip the "Poisson strains" would not generate a stress.

However, within a plate each strip is restrained by it's neighbour resulting in a transverse "Poisson stress" which increases towards the surface.

But if $\varepsilon_y = 0$ due to constraint ②: $\sigma_y = \nu\sigma_x$ so ①: $\varepsilon_x = \frac{\sigma_x}{E}(1-\nu^2)$

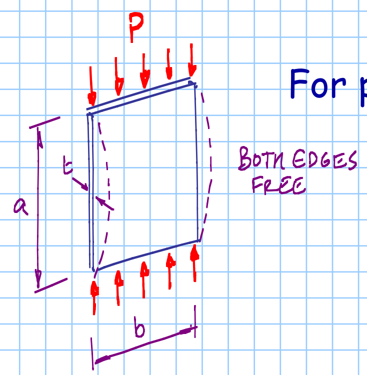
(7)

So compared with a beam, for a given bending moment, a plate will have a smaller strain and curvature by the factor $(1-\nu^2)$. e.g. about 10% less for a Poisson's ratio of 0.3. Essentially, a "plate stiffening effect". Accordingly we can adapt our beam bending equation to apply to plates:

$$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R} \quad : \quad K = \frac{1}{R} = \frac{M}{EI} \rightarrow K = \frac{M}{EI}(1-\nu^2)$$

- For a plate with <u>simply supported ends and free edges</u>, Euler's equation becomes:

$$P_{CR} = \frac{1}{(1-\nu^2)} \frac{\pi^2 EI}{L^2}$$



For plate dimensions $a \times b \times t$: $\sigma_{CR}^* = \dfrac{P_{cr}}{bt}$,

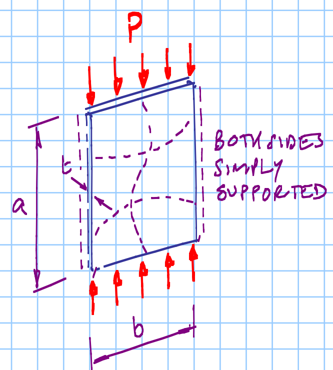
and $I = \dfrac{bt^3}{12}$

$$\hookrightarrow \sigma_{CR}^* = \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{a}\right)^2$$

For plates with other edge conditions the plate width "*b*" becomes the more important dimension because it controls the deflected mode shape of the panel.

(8)

- For a plate with <u>simply supported ends and edges</u> the lengthwise mode shape is dependent on the transverse (widthwise) mode shape. The buckling stress equation then becomes:



$$\sigma_{CR}^* = \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{bm}{a} + \frac{a}{bm}\right)^2 \left(\frac{t}{b}\right)^2$$

where *m* is the number of buckled waves in the sheet depending on the edge restraints and the panel *a/b* aspect ratio.

The equation can be written as:

$$\sigma_{CR}^* = \frac{k\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2 \text{ or } K E \left(\frac{t}{b}\right)^2 \quad \text{where } K = \frac{k\pi^2}{12(1-\nu^2)}$$

Buckling constants, *k* or *K*, can be obtained from charts of *k* or *K* vs *a/b* for different edge conditions and loading configurations. For a Poisson's ratio of 0.3 *K* =0.9*k*. Note, the examples in the Aero Design Handbook and also in the the Bruhn and Niu references - see extracts. Correction factors may also be required, E.g.:

* Plastic buckling correction η

As for columns, plastic buckling correction factors will be required if the predicted elastic plate buckling strength or the applied stress approaches the proof stress or yield strength of the material.

E.g. $\sigma_{ce}^{*} = \eta K_c E \left(\frac{t}{b}\right)^2$ where η = Plastic correction factor

* Cladding correction factors λ

Aluminium alloy sheets are often "clad" with a surface layer of almost pure aluminium for protection "alclad" . However, this layer is significantly "softer" (lower modulus and yield strength). To allow for this we need to "correct" thickness values. Typical clad thickness correction factors range from 0.80 to 0.95 × *t*. See Bruhn and Niu references and data extract.

For the stability design of thin wall sections we consider each element of the section as a panel and then check the possible modes of panel buckling, ensuring that the limit stresses do not exceed the critical panel buckling strength.

For aircraft structural design the buckling stress limit can vary depending on the section element considered and the design philosphy. E.g. for lightweight thin skin semi-monocoque structures some panels can be allowed to buckle below the design limit load (providing that the panel edge support is designed to cope with the off-loading). The effects of aerodynamic profile and passenger confidence may be factors in the setting of panel buckling limit stresses.

17.10.2010

Initial Checks

For initial checks consider only the most significant stress in each section element, i.e. direct stress in the flanges and shear stress in the webs:

For area properties use "thin wall assumptions", neglect fillets + corners etc.

Flanges: $\sigma_x = \frac{F_x}{A} + \frac{M_z y}{I}$ Direct stress in the flanges due to axial loads and bending.

Webs: $\tau_{xy} \approx \frac{F_y}{b_w t} + \frac{T}{2At}$ Shear stress in the webs due to transverse load and torque.

Considering only single stress types in each element we are effectively using a "non-interactive" single stress failure criteria as an initial check.

I.e. failure occurs when: $\sigma_x \geq \sigma^*_{c_{CRIT}}$ in the flanges:

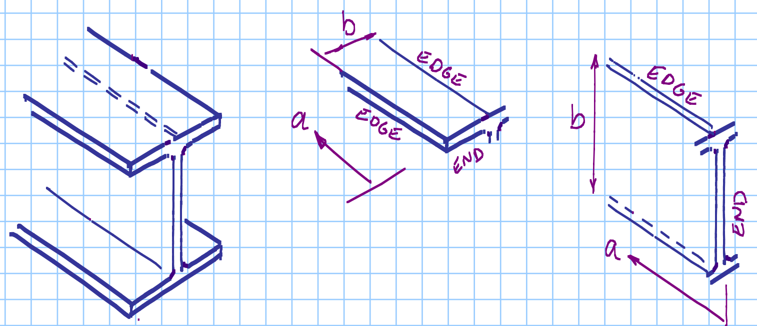
or when: $\tau \geq \tau^*_{CRIT}$ or $\sigma_b \geq \sigma^*_{b_{CRIT}}$ in the webs:

Where σ^*_{CRIT} and τ^*_{CRIT} are the critical panel buckling* strengths for direct and shear respectively.

For a given section we need to identify panels and edge conditions.

E.g. for an I-beam we would consider each half of the flange as a panel and the web as another panel. The ends of the panels at the web/flange intersection could be considered as simply supported or fixed.

In reality this edge condition would be somewhere between simply supported and fixed. For a conservative estimate a simply supported edge condition would be reasonable (particularly for a lightweight thin wall I-beam). Similar arguments could be applied to the panel end conditions.

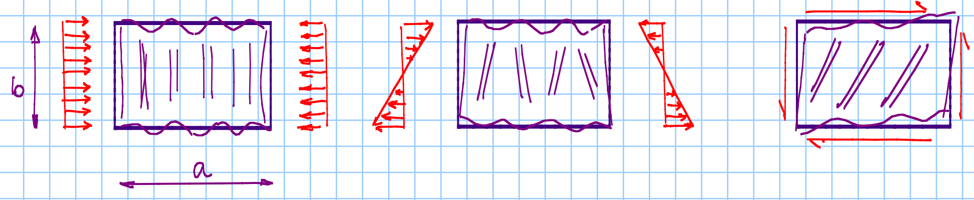


Edge / End conditions:

free
Simply supported (hinged)
Fixed (Clamped)

Each component of stress can result in a particular buckling pattern, e.g.:

Direct stress	Bending stress	Shear stress
σ_c	σ_b	τ



flanges web web

For an initial check considering only the most significant stress in each section panel as a single stress "non-interactive" failure criterion:

Failure occurs in the **flanges** when: $\sigma_c + \sigma_b \geqslant \sigma^*_{c_{CRIT}}$ *

* Note! we consider direct stress and bending stress in the flanges as a single direct stress!

Failure occurs in the **web** when: $\sigma_b \geqslant \sigma^*_{b_{CRIT}}$ or when: $\tau \geqslant \tau_{CRIT}$

So to check for local panel buckling we need to predict the critical panel buckling strengths: $\sigma^*_{c_{CRIT}}$, $\sigma^*_{b_{CRIT}}$, τ^*_{CRIT}

Panel buckling analysis' provides the general relationship:

$$\sigma^*_{c_{CRIT}}\ ,\ \sigma^*_{b_{CRIT}}\ ,\ \tau^*_{CRIT} = K\,\eta\,E\left(\frac{t}{b}\right)^2$$

similar to the Euler expression written in terms of stress and a "slenderness ratio". E.g.:

$$\sigma^*_{CRIT} = \frac{k\pi^2 EI}{AL^2} = k\pi^2 E\left(\frac{\rho}{L}\right)^2$$

where $\rho = \sqrt{\frac{I}{A}}$

Radius of gyration "r"

where:

K = plate buckling constant
depending on: type of stress σ_c, σ_b, τ
panel aspect ratio a/b
panel edge conditions

η = Plastic buckling correction factor

E = Young's modulus

b = plate width

t = plate thickness

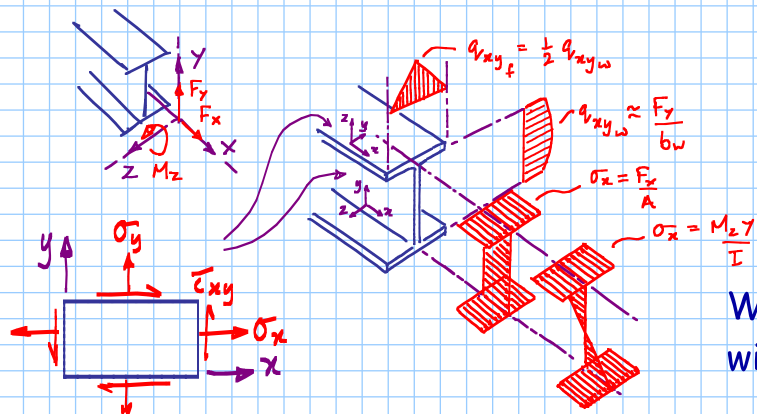
Apply by reference to plastic buckling charts as for columns

ILF
8.12.2013

Refined Checks

To refine we need to account for the combined stresses.

E.g. a 2D Loading system in the XY plane will result in combined direct and shear stresses in the flanges or webs:

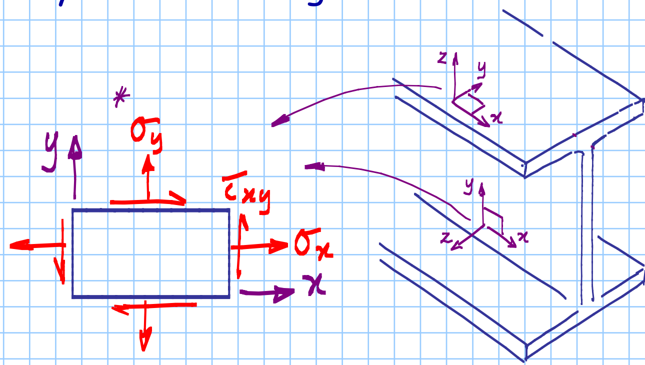


StM2

The question is:
What combinations of σ and τ will cause failure?

To account for the combined stresses, σ_x and τ_{xy} in our failure analysis we need to use an "interactive failure criterion".

Consider the resulting 2D stress system in a flange or web element:



Note, for a simple beam subjected to transverse loading there is no transverse direct stress in the flange or web elements.

Neglecting Brazier loading

*I.e. $\sigma_y = 0$ because a simple beam reacts transverse loads by the generation of longitudinal direct stress σ_x and in plane shear stress τ_{xy}

But this still leaves σ_x and τ_{xy} to consider as combined stresses.

To account for these combined stresses in our buckling failure analysis we need to use an "interactive failure criterion".

As for strength, these failure criteria are arranged in the form of "failure index" stress ratios:

$$R = \frac{\sigma}{\sigma^*_{CRIT}} \text{ or } \frac{\tau}{\tau^*_{CRIT}}$$

where σ, τ = = applied stresses

and $\sigma^*_{CRIT}, \tau^*_{CRIT}$ = critical buckling stresses derived our panel buckling expressions

Failure criteria are expressed as combinations of failure indexes that represent failure, usually involving exponents and factors.

E.g. failure occurs when: $R_A^{\#} + R_B^{\#} + R_S^{\#} = 1$

Where R_A, R_B and R_S are stress ratios relating to Axial, Bending and Shear stresses and associated failure values. The exponents, #, can be derived or found by testing. Sometimes factors are also included. Eg. $\underline{k}R^{\#}$

Axial and bending stresses can be added by superposition and accounted as a single effective direct stress within flange elements but must be considered separately for web elements for panel buckling. Transverse and torsional shear stresses can also be added as effective single shear stress.

For combined shear and compression: $R_s^2 + R_c = 1.0$

.. .. shear and bending: $R_s^2 + R_b^2 = 1.0$

.. .. bending and compression: $R_b^{1.75} + R_c = 1.0$

where $R_c = \frac{\sigma_c}{\sigma^*_{c_{CR}}}$, $R_b = \frac{\sigma_b}{\sigma^*_{b_{CR}}}$, $R_s = \frac{\tau}{\tau^*_{CR}}$

In this course we will use: $R_c + R_b^2 + R_s^2 = 1.0$

$$\left(\frac{\sigma_c}{\sigma^*_{c_{CR}}}\right) + \left(\frac{\sigma_b}{\sigma^*_{b_{CR}}}\right)^2 + \left(\frac{\tau}{\tau^*_{CR}}\right)^2 = \text{"FI"}$$

where we will define our RF as: $RF = \frac{1}{\sqrt{FI}}$

References:

[1] Peery of Azaar "Aircraft structures"
McGraw-Hill 1976. ISBN 0-07-049196-8

[2] Bruhn "Analysis of Design of Flight vehicle structures"
Jacobs publishing. Inc. 1973

[3] Niu "Airframe Stress Analysis and Sizing"
Hongkong Conmilit Press Ltd 1997. ISBN 962-7128-07-4