Aeronautics & Mechanics AENG11301

Lecture 9
Climb and Glide

8/3/16

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Outline for today

- Gliding flight
  - Force balance
  - Glide angle
  - Sink rate

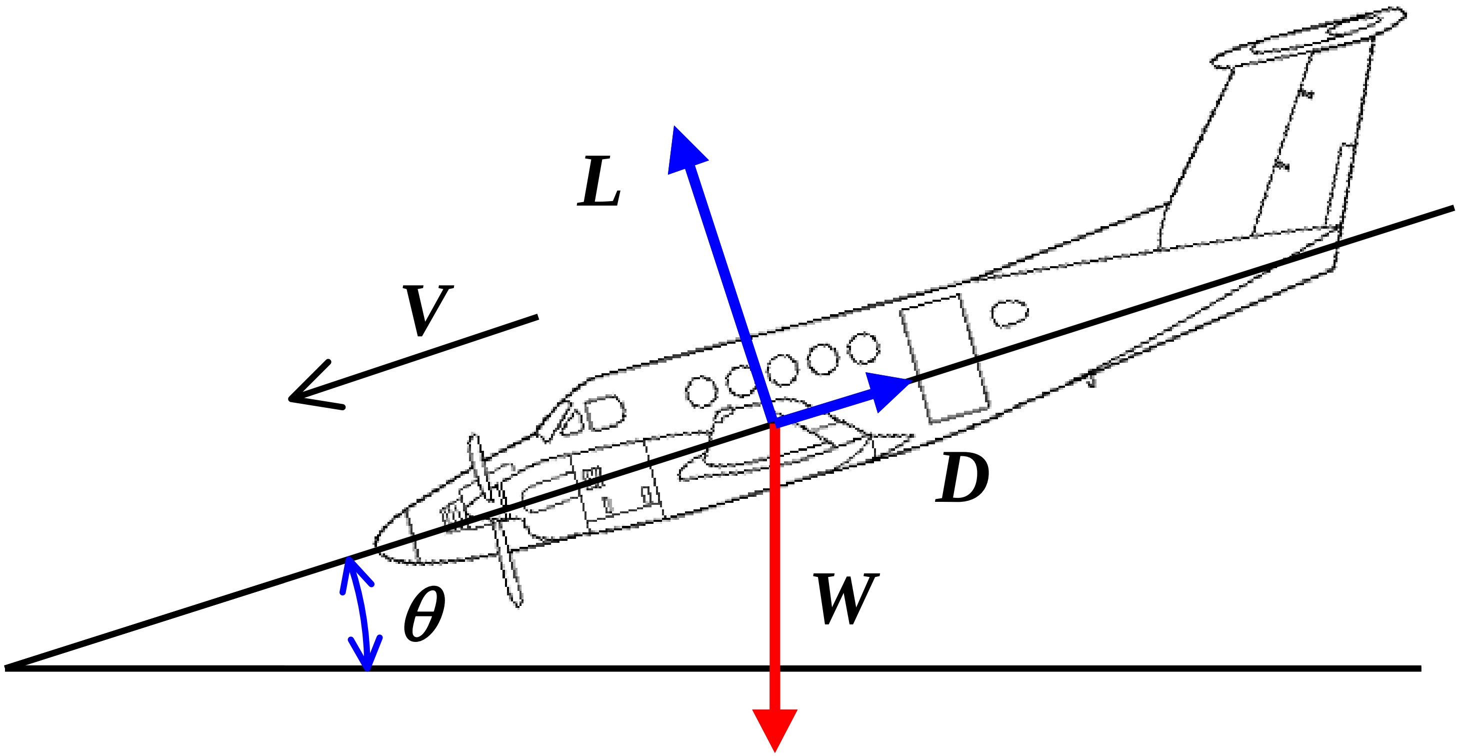
- Climbing flight
  - Force balance
  - Climb angle
  - Climb rate

Aims for today

- Be able to calculate force balance in gliding flight
- Be able to calculate speed for minimum glide angle and speed for minimum sink rate
- Be able to calculate force balance in climbing flight
- Be able to relate maximum climb angle and maximum rate of climb with engine properties

Gliding Flight

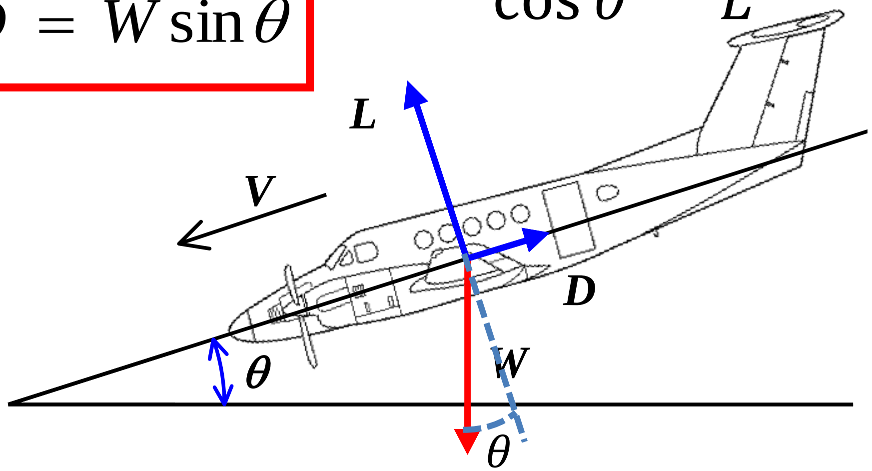
- thrust $T = 0$ but drag $D \neq 0$
  - cannot maintain equilibrium in straight & level flight
  - aircraft descends at **glide angle** θ
  - component of weight along flight path replaces thrust



Force Balance in a Glide

- resolve forces
  - perpendicular to flight path $\boxed{L = W\cos\theta}$
  - parallel to flight path $\boxed{D = W\sin\theta}$

$$\frac{\sin\theta}{\cos\theta} = \frac{D}{L}$$



- hence

$$\boxed{\tan\theta = \frac{1}{L/D} = \frac{1}{C_L/C_D}}$$

- **glide angle** depends solely on the aerodynamic characteristics
  - weight only affects the speed at which a particular C_L/C_D is achieved
- best (shallowest) glide angle occurs at maximum L/D
  - **minimum glide angle** occurs at minimum drag speed

Glide ratios




	Glide ratio (L/D ratio)
Modern Sailplane	40-60
Lockheed U-2	28
Albatross	20
Boeing 747	17
Herring gull	10
Cessna 150	7
Space Shuttle	4.5
House sparrow	4
Flying squirrel	2



Speed for Minimum Glide Angle

- minimum glide angle occurs at minimum drag condition
  - gives maximum distance from a given start height
  - but corresponding speed depends on glide angle …

$$V = \sqrt{\frac{L}{\frac{1}{2}\rho S C_L}} = \sqrt{\frac{W\cos\theta}{\frac{1}{2}\rho S C_L}} = \sqrt{\frac{W}{\frac{1}{2}\rho S C_L}}\sqrt{\cos\theta}$$

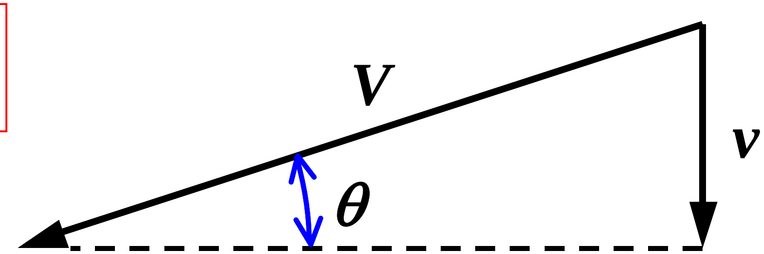
  - **for a given C_L, speed is less than for level flight by the factor $\sqrt{\cos\theta}$**
- for glide angle < 10° error is < 1%
  - *applies for all normal glide performance* … but must check!
- make small angle assumption $\rightarrow \cos\theta = 1$, $\sin\theta = \theta$

$$\Rightarrow \quad L \approx W \quad \Rightarrow \quad \boxed{V_{\min\theta} \approx V_{MD} = \left(\frac{2W}{\rho S}\right)^{\frac{1}{2}}\left(\frac{K}{C_{D0}}\right)^{\frac{1}{4}}}$$

Speed for Minimum Sink Rate

- sinking speed (**rate of sink**) v is vertical component of flight speed V

$$\boxed{v = V\sin\theta}$$



- from force balance and velocity equation
  - making small angle assumption, so that $L \approx W$

$$v = V\frac{D}{W} \approx V\frac{D}{L} = V\frac{C_D}{C_L} \Rightarrow \boxed{v = \sqrt{\frac{2L}{\rho S C_L}}\frac{C_D}{C_L} \approx \sqrt{\frac{2W}{\rho S}}\frac{C_D}{C_L^{3/2}}}$$

  - minimum sink occurs at minimum $C_D/C_L^{3/2}$

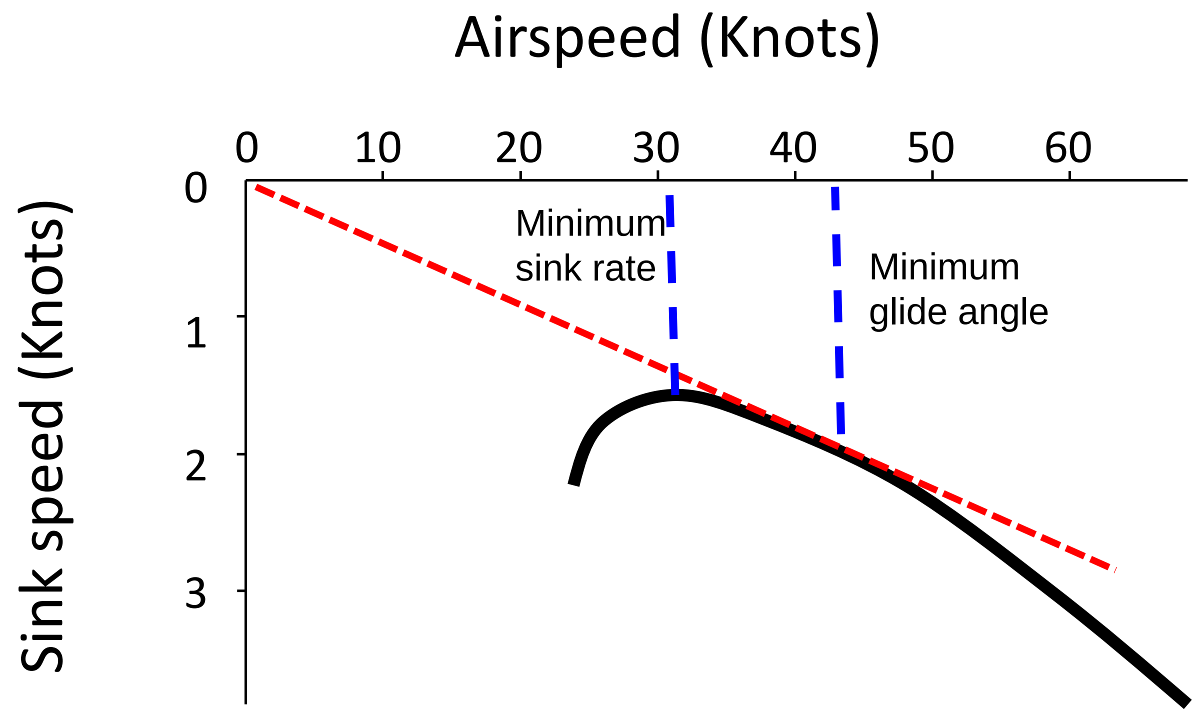
- same as the minimum power condition!

$$\Rightarrow \boxed{V_{min\,sink} \approx V_{MP} = \left(\frac{2W}{\rho S}\right)^{\frac{1}{2}}\left(\frac{K}{3C_{D0}}\right)^{\frac{1}{4}}}$$

Glider Polar Diagram

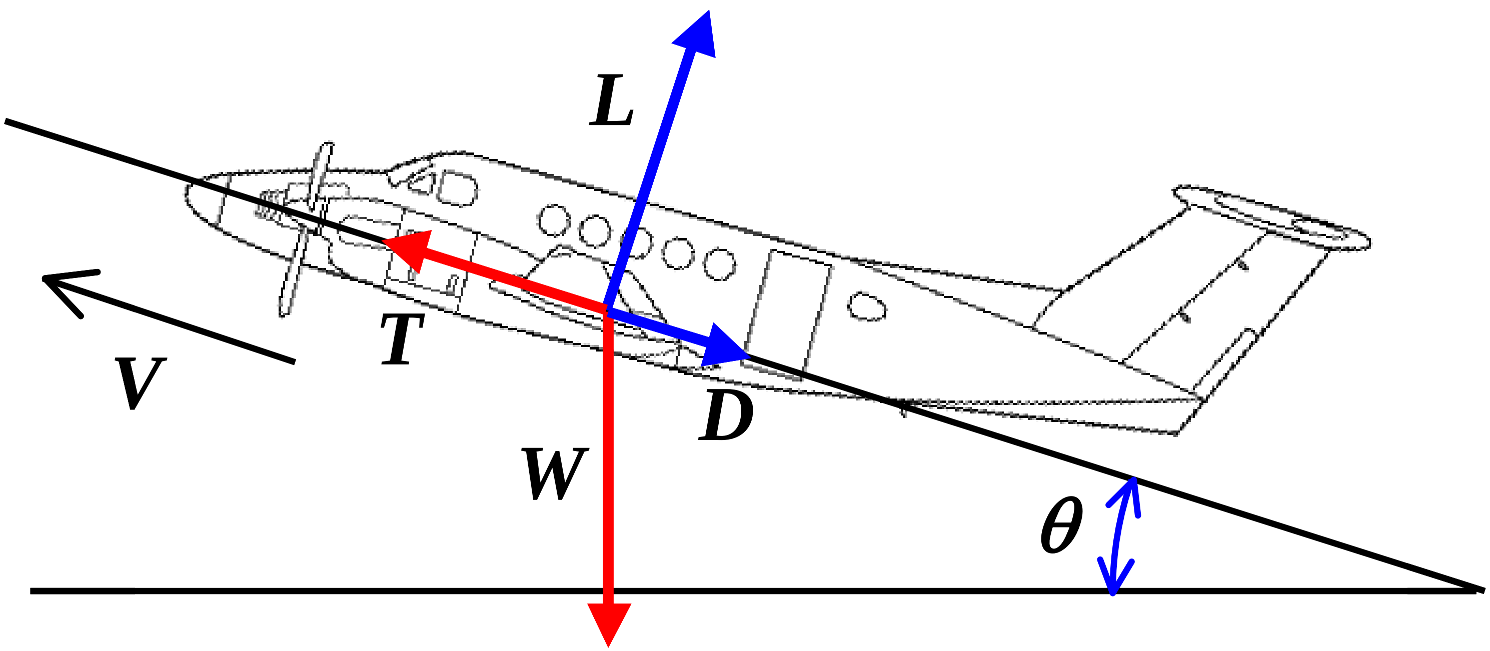
- glider performance usually represented by its ‘polar’
  - plot of sink speed v vs forward speed V
  - indicates variation of D/L with speed
  - readily determined in flight test

$$\frac{v}{V} = \sin\theta = \frac{D}{W} \approx \frac{D}{L}$$



Climbing Flight

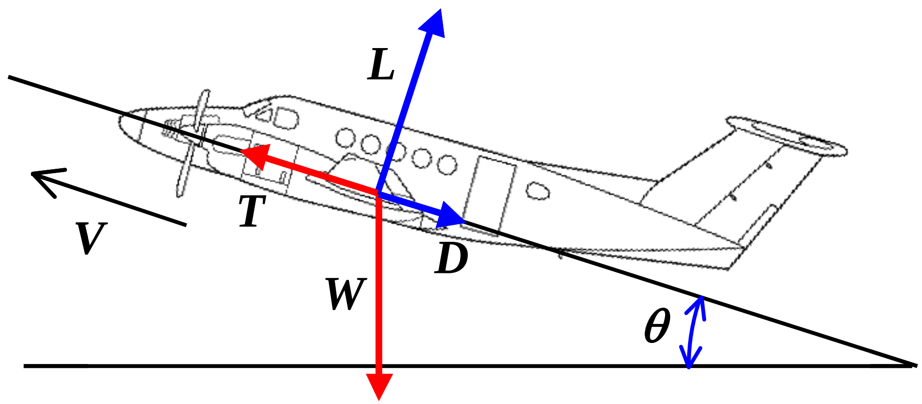
- thrust T > drag D
  - cannot maintain equilibrium in straight & level flight
  - aircraft ascends at **climb angle** θ
  - horizontal component of weight opposes thrust



Force Balance in a Climb

- resolve forces
  - perpendicular to flight path $\boxed{L = W\cos\theta}$
  - parallel to flight path $\boxed{T - D = W\sin\theta}$
- hence **climb angle** is

$$\boxed{\sin\theta = \frac{T-D}{W}}$$



- **rate of climb** v is then

$$\boxed{v = V\sin\theta = \frac{V(T-D)}{W}} \Rightarrow \boxed{Wv = TV - DV}$$

rate of increase of potential energy

power available

power required

excess power

Climb Performance

- maximum angle of climb requires maximum **excess thrust** $(T - D)$

$$\boxed{\sin\theta = \frac{T-D}{W}}$$

- maximum rate of climb requires maximum **excess power** $(TV - DV)$

$$\boxed{v = V\sin\theta = \frac{V(T-D)}{W}}$$

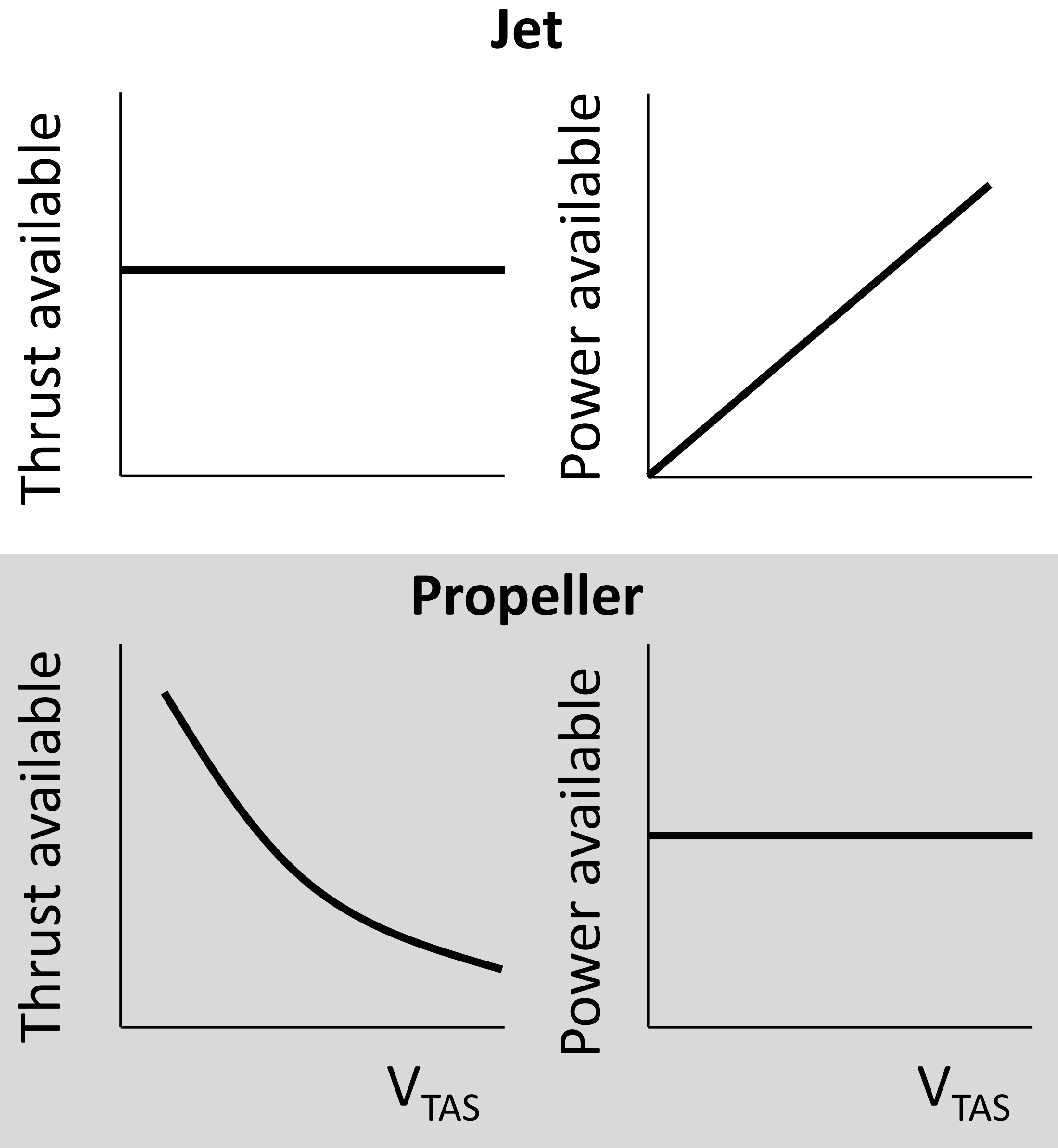
- do not occur together
  - depend on propulsion thrust/power characteristics
  - cannot make small angle assumptions – calculate numerically

Propulsion type

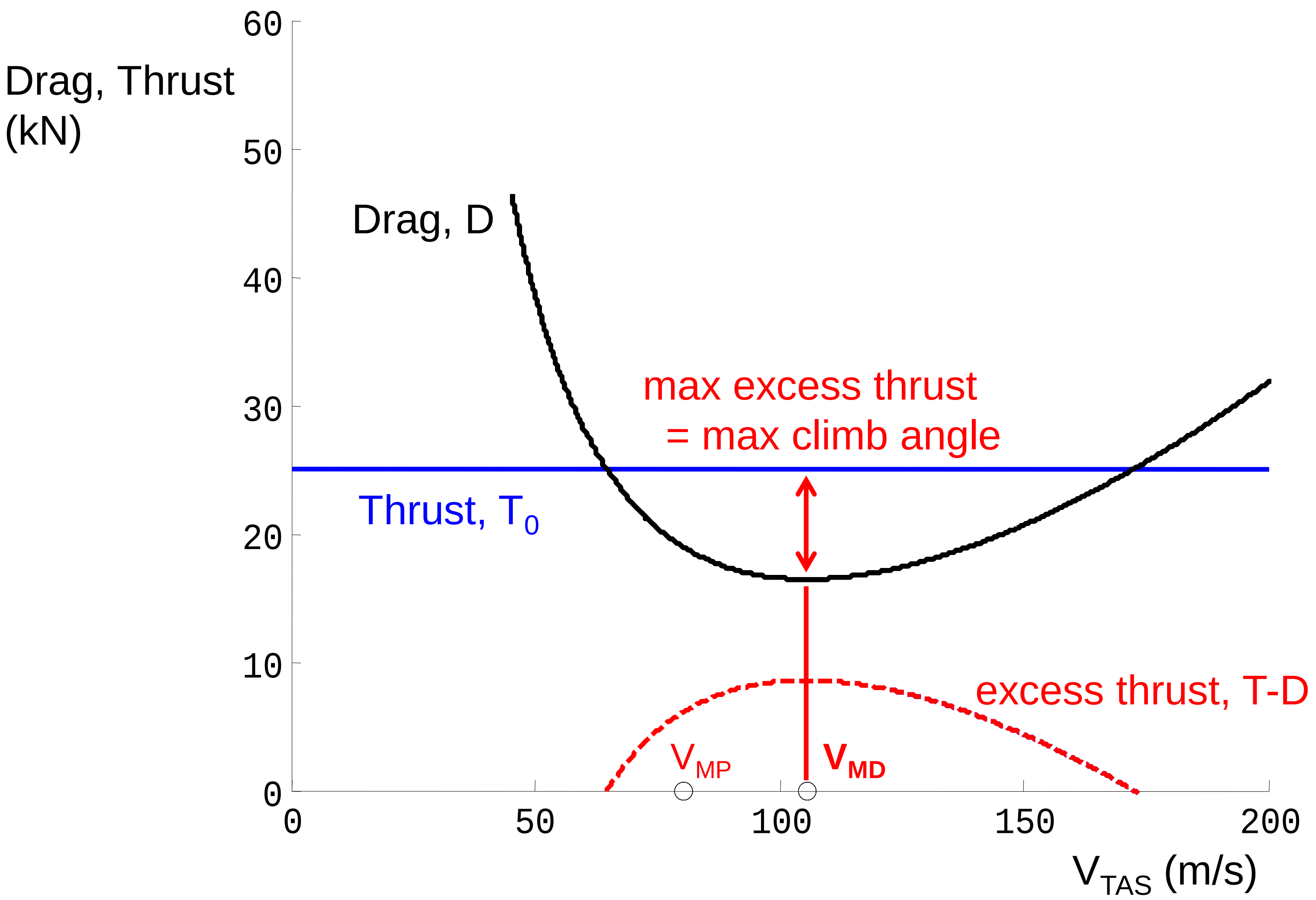
Gross simplification – but useful for preliminary performance work

- **Jet aircraft**
  $\rightarrow$ **thrust** T remains constant with speed
  $\rightarrow$ power $P = TV$ increases with speed

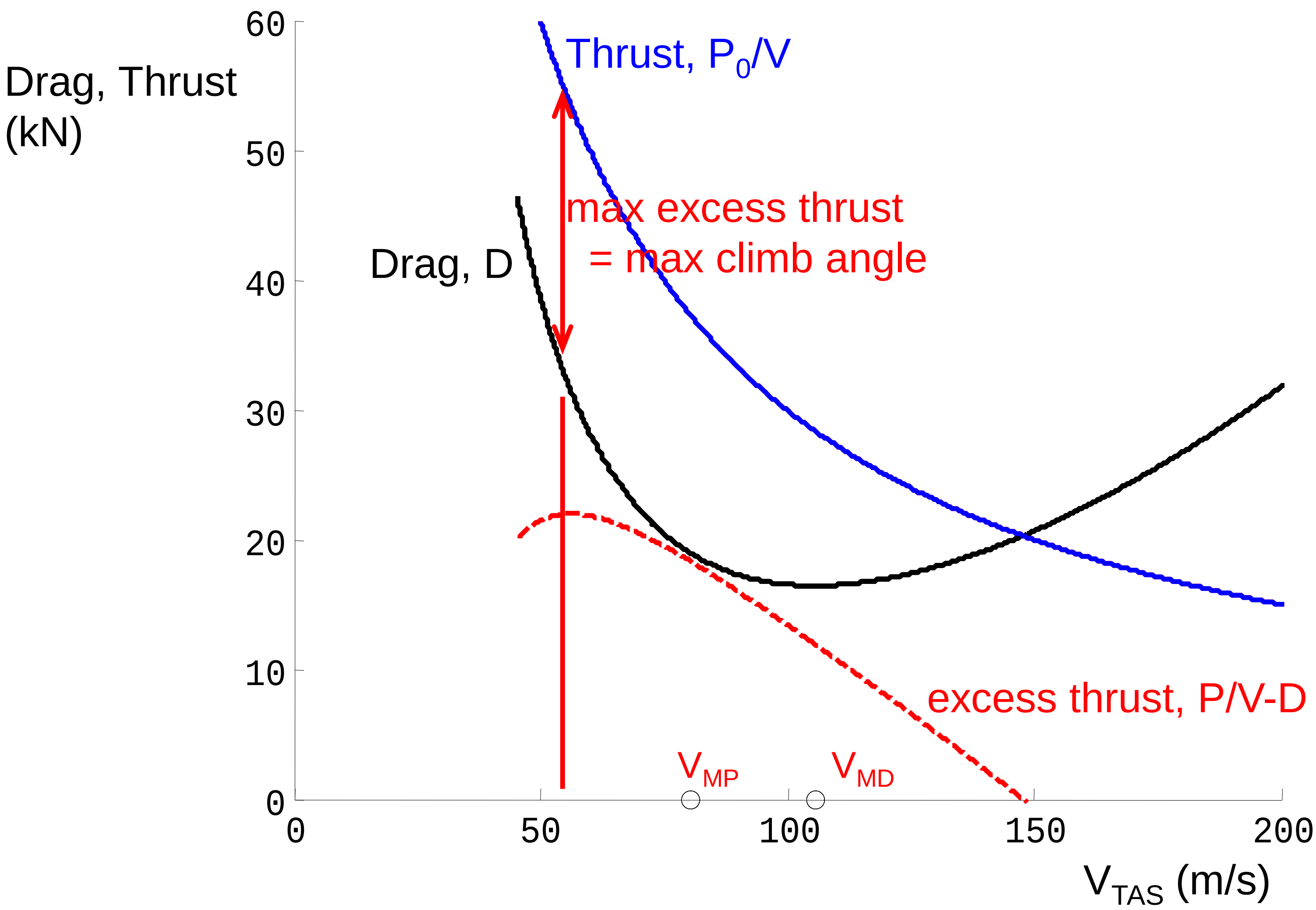
- **Propeller aircraft**
  $\rightarrow$ **power** output P remains constant with speed
  $\rightarrow$ thrust $T = P/V$ reduces with speed



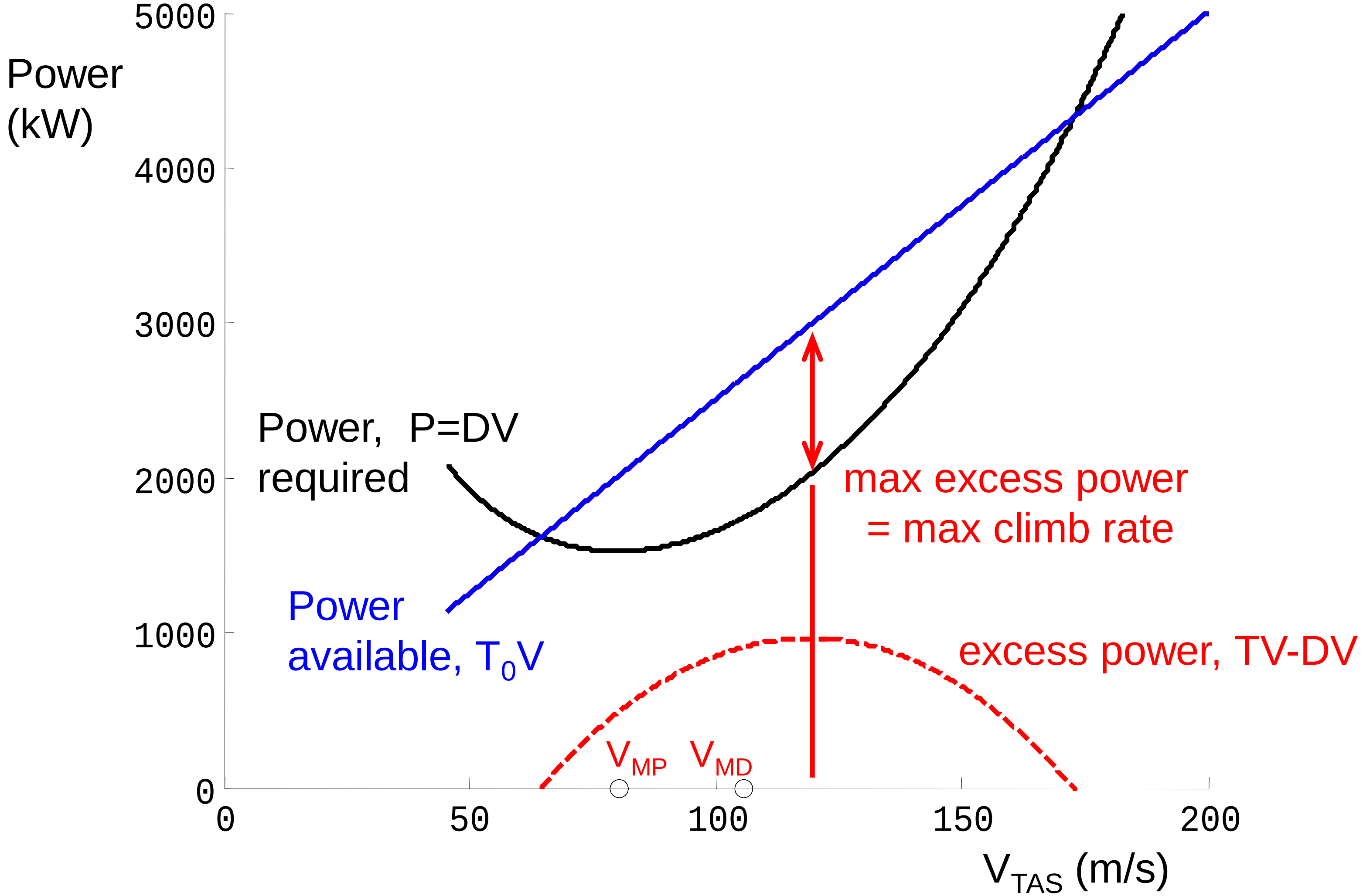
Climb Angle - Jet



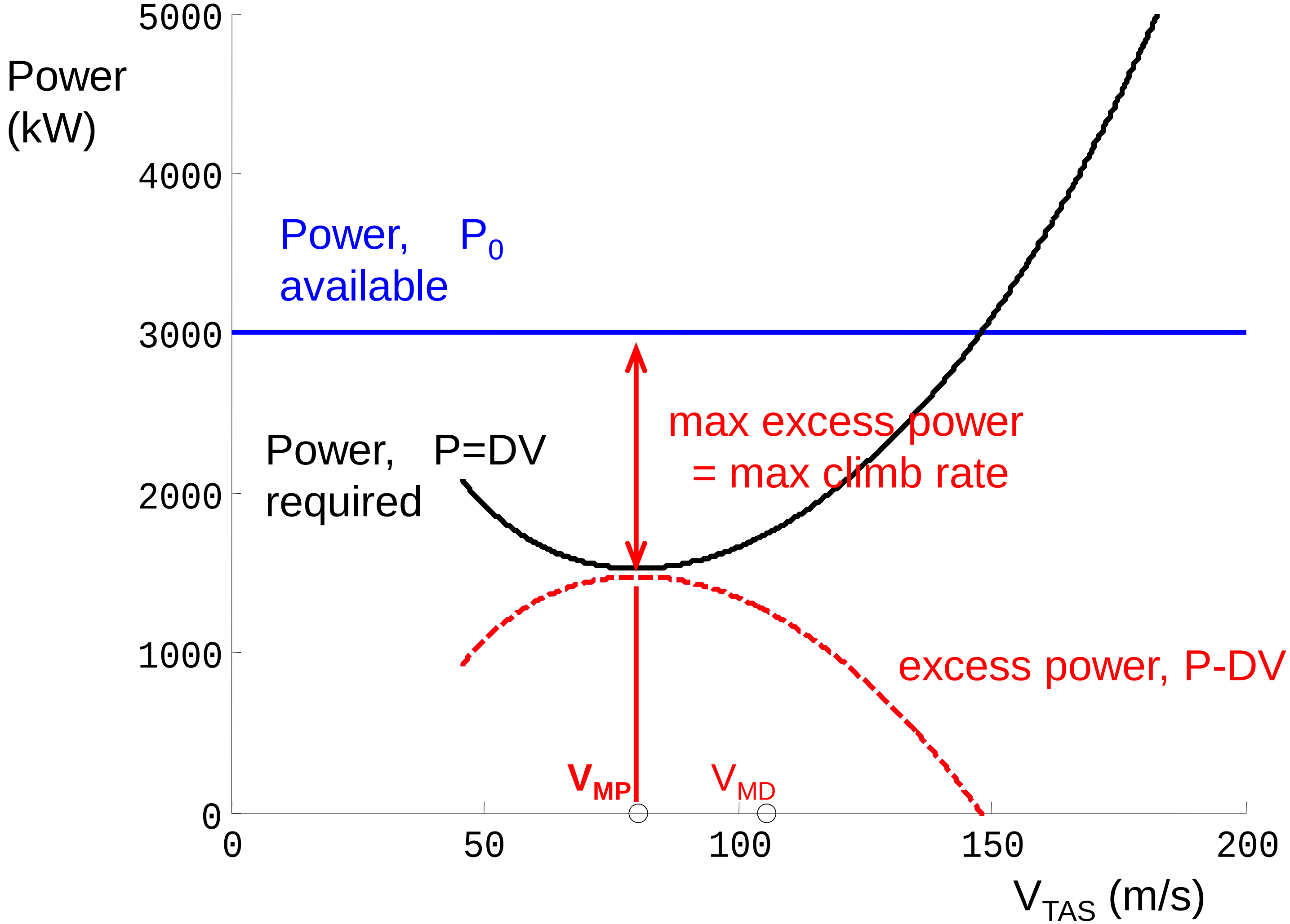
Climb Angle - Propeller



Rate of Climb - Jet



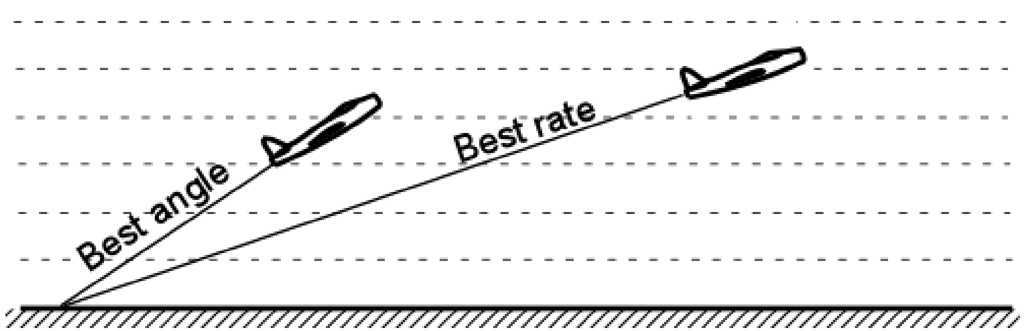
Rate of Climb - Propeller



Climb Performance Summary

- maximum excess **power** = maximum **rate of climb**
- maximum excess **thrust** = maximum **climb angle**

	Propeller Aircraft	Jet Aircraft
Max Rate of Climb	at minimum power speed V_{MP}	$> V_{MP}$
Max Climb Angle	$< V_{MD}$	at minimum drag speed V_{MD}



Example – Glider

Mass = 350 kg

Wing area, S = 11 m^2

Max lift to drag, $(L/D)_{max}$ = 42

Aspect ratio, AR = 21

Oswald efficiency, e = 0.95

Altitude, h = 5000 m
(where $\rho \approx 0.74\ \mathrm{kg/m^3}$)

Calculate:

1) the minimum glide angle
2) horizontal distance glider will cover in still air
3) C_L at minimum glide angle
4) minimum glide angle airspeed
5) how this changes if add 150 kg of water ballast

Summary

Gliding

$$\boxed{L = W\cos\theta} \quad \boxed{D = W\sin\theta} \quad \boxed{\tan\theta = \frac{1}{L/D} = \frac{1}{C_L/C_D}}$$

- Minimum glide angle occurs at minimum drag speed
- Minimum sink rate occurs at minimum power required speed

Climbing

$$\boxed{L = W\cos\theta} \quad \boxed{T - D = W\sin\theta}$$

- Maximum climb rate is at maximum excess Power
- Maximum climb angle is at maximum excess Thrust

	Propeller Aircraft	Jet Aircraft
Max Rate of Climb	at minimum power speed V_{MP}	$> V_{MP}$
Max Climb Angle	$< V_{MD}$	at minimum drag speed V_{MD}

Follow-up materials

To help with exam:

- Introduction to Flight – 6.8, 6.9

To aid in understanding:

- Introduction to Flight – 6.4, 6.6

For interest:
How to use glide polars as a glider pilot

- http://avia.tion.ca/documentation/polar/