UNIVERSITY OF BRISTOL
FACULTY OF ENGINEERING

First Year Examination for the Degrees of
Bachelor and Master of Engineering

MAY / JUNE 2017 3 HOURS

STRUCTURES AND MATERIALS 1
AENG 11200

This paper contains *five* questions.

Answer *all* questions.

All questions carry *20 marks* each.

The maximum for this paper is *100 marks*.

CALCULATORS MUST HAVE THE FACULTY OF ENGINEERING SEAL OF APPROVAL

PLEASE DO NOT REMOVE THIS EXAM PAPER FROM THE EXAM ROOM

TURN OVER ONLY WHEN TOLD TO START WRITING

Q1

a) Answer the following questions with no more than one or two sentences.

i) When does a structure become *statically indeterminate*?

ii) When does a structure become a *mechanism*?

iii) What are the maximum and minimum values of the *bending moment* carried by any individual member in a pin-jointed truss?

iv) Write an expression for *axial stiffness* in terms of Young's modulus and geometric properties, following the usual notation.

(4 marks)

b) Figure Q1 shows a plane, pin-jointed truss which is supported at A and B and carries a vertical load of 10 kN at F as shown. All six members have a cross-sectional area of 300 mm^2 and are made of steel with $E = 200$ GPa. Note that the structure is *statically determinate*.

i) Calculate the reactions at A and B.

(3 marks)

ii) Calculate the internal forces in all six members of the truss.

(6 marks)

iii) Using an energy method, calculate the vertical deflection at joint F.

(4 marks)

iv) Using an energy method, calculate the horizontal deflection at joint F.

(3 marks)

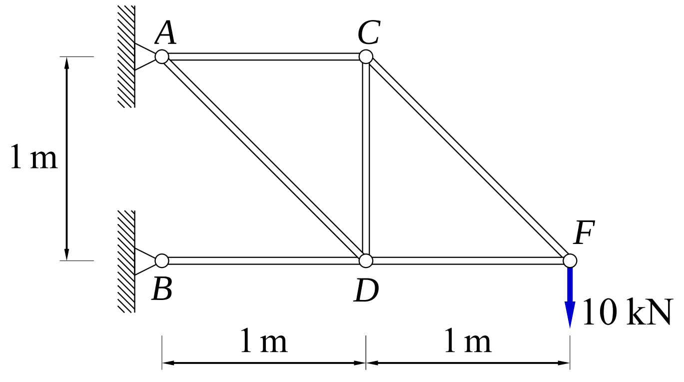


Figure Q1: A plane pin-jointed truss.

Q2

a) Answer the following questions with no more than one or two sentences.

i) Write a formula for the Euler buckling load of a pinned-pinned strut, and explain how you would account for other end conditions.

ii) What is a *Heaviside* function and when would you use it?

iii) Briefly describe what you understand by the term *second moment of area*.

iv) Where would you find the maximum *bending stress* in a beam cross-section?

(4 marks)

b) A beam is propped between pinned supports at A and B and is subjected to a vertical tip load of 1 kN at C as shown in Figure Q2. The beam is made of aluminium alloy with a Young's modulus of 70 GPa and a solid square cross-section measuring 30 mm $\times$ 30 mm.

i) Calculate the reaction forces at A and B.

(2 marks)

ii) Draw the shear force and bending moment diagrams, stating principal values.

(6 marks)

iii) Calculate the deflection at point C.

(8 marks)

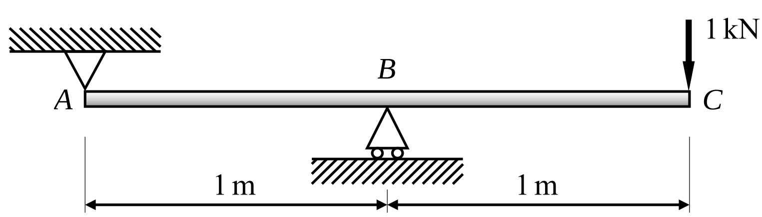


Figure Q2: A beam under load.

Q3

a) Define the following terms:

i) Limit load.
ii) Ultimate load.
iii) Load factor.
iv) Reserve factor.

(4 marks)

b) A long-range transport aircraft has an all up mass of 282,000 kg and a wingspan of 62 m. Two UHB engines are mounted on the rear fuselage. The wing structural mass is 21,000 kg and the wing carries 98,000 kg of fuel. Loads from the fuselage can be assumed to act at the wing-to-fuselage joints which are 3.0 m on either side of the fuselage centreline. The distribution of wing structural mass, lift and fuel mass can be assumed as uniform across the full tip-to-tip wingspan. A wingtip device gives a positive "wing-tip up" point bending moment of 250 kNm at each wingtip but zero nett vertical shear force.

Draw the vertical shear force and bending moment diagrams for a positive symmetric manoeuvre case with a load factor of 2.1, giving the principal values across the full wing span.

(8 marks)

c) Figure Q3 shows a double-shear twin-lug joint at the root of a light aircraft wing spar. The 'limit case' internal loading is shown at the end of the beam and these loads are reacted across the joint. The lug allowable strengths are given in Table Q3. Considering only the LHS lug modes of failure and using an ultimate safety factor of 1.5 calculate the lug stresses and reserve factors.

(8 marks)

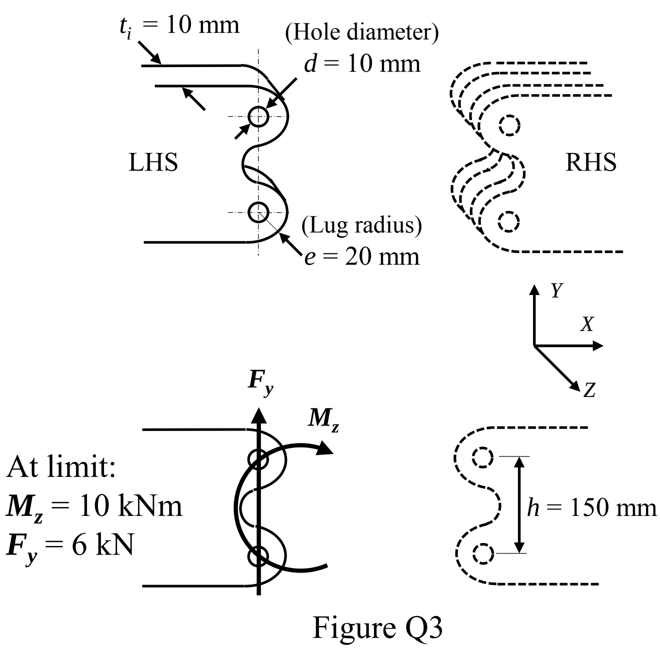


Figure Q3

Table Q3: Lug allowable strengths.

		Units
Tensile strength	700	N/mm^2
Shear strength	400	N/mm^2
Bearing strength	1000	N/mm^2

Q4

a) List the material properties that can be extracted from the following tests:

i) Compression of cylindrical specimens.

ii) Vickers and Rockwell tests.

iii) 3-point bending test.

iv) Charpy V-notch test.

(4 marks)

b) i) Define *resilience* as a material property.

ii) Define the modulus of resilience, U_r, in terms of material stresses and strains (or Young's modulus). Draw the stress-strain curve for an elastic-plastic material, and indicate what the modulus U_r represents in that curve.

(4 marks)

c) Two quantities are commonly used to represent the resistance of materials to fracture and crack growth, namely the 'critical strain energy release rate' G_C, and the 'critical stress intensity factor' K_C.

i) What are other names given to G_C and K_C?

ii) What are the SI units for G_C and K_C?

iii) Write the relationship between G_C and K_C for an isotropic linear elastic material.

(5 marks)

d) The so-called 'Ashby plots' (or selection 'bubble charts') can be used to select materials for a given application very early in the design process.

i) Briefly describe the materials selection process using such charts.

ii) What are the differences between *objectives* and *constraints* in the context of materials selection?

iii) Write the two indices M which must be *maximised* in the selection of materials for a lightweight beam loaded in bending and subject to a *stiffness constraint* and a *strength constraint*, assuming arbitrary cross-sections.

(7 marks)

Q5

a) Write brief notes to describe how covalent, ionic, and metallic bonding differ. Give examples of how each form of bonding influences observed properties.

(5 marks)

b) i) The atomic packing factor (APF) of a primitive cubic structure is 0.52. Show whether the APF of a face centred cubic (FCC) structure packs more or less efficiently, using the dimensions a and r defined in Figure Q5.

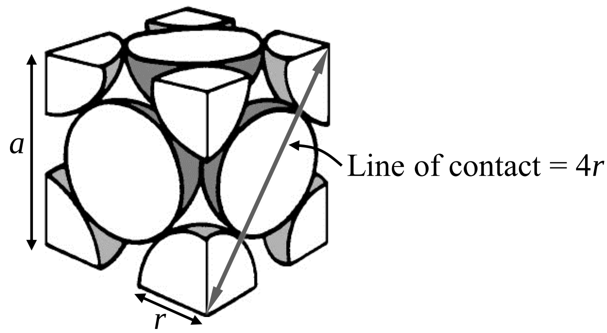


Figure Q5: FCC crystal structure.

ii) Give an example of an element with a FCC structure.

(5 marks)

c) What does the glass transition temperature (T_g) represent and which polymer structural features influence it? Sketch a plot to show the stress-strain behaviour of glassy, plastic, and elastomeric polymers.

(5 marks)

d) i) Define viscoelasticity.

ii) The Maxwell model predicts viscoelastic behaviour in polymers. The Maxwell relaxation time, τ, represents the reduction in stress when a polymer is held at constant strain, ε_0. An elastomer is held at a constant strain of 4% and the empirical data shown in Table Q5 are obtained.

Table Q5: Empirical relaxation data.

Time [s]	Stress [MPa]
0	10.53
120	3.76

Using the equations relating $E(t)$, σ, ε_0, and τ, calculate the relaxation modulus after 250 seconds.

(5 marks)

END OF PAPER

Q1

a)

i. When there are more members or supports than needed to prevent rigid body motion.

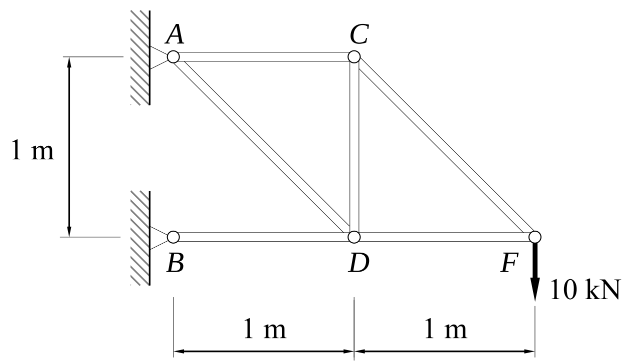
ii. When there are insufficient members or supports to prevent rigid body motion.

iii. Minimum and maximum bending moments are ZERO, since pin-jointed truss elements cannot transfer moments.

iv. $K = \dfrac{AE}{L}$

(4 marks)

b)



i) Support reactions:

A global balance of moments about point *B* gives:

$$
\begin{aligned}
& \sum M_B && = 0 \\
(10000)(2.0) & + R_{A,x}(1.0) && = 0 \\
20000 & + R_{A,x} && = 0 \\
& R_{A,x} && = -20\,\text{kN (i.e. } 20\,\text{kN to the left)}
\end{aligned}
$$

And a global balance of forces gives:

$$R_{B,x} = 20\,\text{kN (to the right)}$$

$$R_{B,y} = 0$$

$$R_{A,y} = 10\,\text{kN (upwards)}$$

(3 marks)

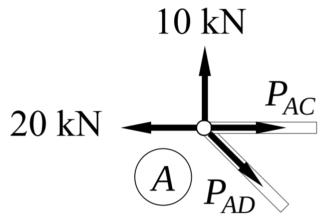
ii) Internal forces

The characteristic angle between members is $\theta = 45^0$.

Joint *B*:

$$P_{BD} = -R_{B,x} = -20\,\text{kN}$$

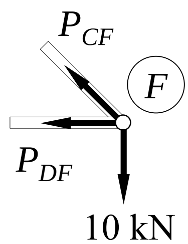
Joint A:



$$
\begin{aligned}
& \sum F_y && = 0 \\
10\ \text{kN} \quad & - P_{AD} \sin\theta && = 0 \\
10\ \text{kN} \quad & - P_{AD} \left(\sqrt{2}/2\right) && = 0 \\
& P_{AD} && = 14142\ \text{N}
\end{aligned}
$$

$$
\begin{aligned}
& & \sum F_x & = 0 \\
20\,\text{kN} \quad & - P_{AD} \cos\theta & - P_{AC} & = 0 \\
& & P_{AC} & = -\left(14142\ \text{N}\right)\cos\theta \quad + 20\,\text{kN} \\
& & P_{AC} & = 10\,\text{kN}
\end{aligned}
$$

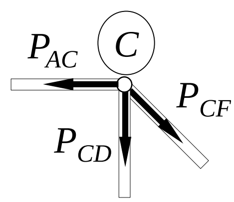
Joint F:



$$
\begin{aligned}
& \sum F_y && = 0 \\
10\ \text{kN} \quad & - P_{CF} \sin\theta && = 0 \\
10\ \text{kN} \quad & - P_{CF} \left(\sqrt{2}/2\right) && = 0 \\
& P_{CF} && = 14142\ \text{N}
\end{aligned}
$$

$$
\begin{aligned}
& \sum F_x && = 0 \\
P_{CF} \cos\theta \quad & + P_{DF} && = 0 \\
& P_{DF} && = -P_{CF} \cos\theta \\
& P_{CF} && = -10\ \text{kN}
\end{aligned}
$$

Joint C:



$$
\begin{aligned}
& \sum F_y && = 0 \\
P_{CD} \quad & + P_{CF} \sin\theta && = 0 \\
& P_{CD} && = -P_{CF} \sin\theta \\
& P_{CD} && = -10\ \text{kN}
\end{aligned}
$$

(6 marks)

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iii) Vertical deflection at F

Member	**Li / m**	**Pi / N**	**Pi'**	**Pi Pi' Li / Nm**
AC	1	10000	1	10000
AD	1.414214	14142.14	1.414214	28284.27125
BD	1	-20000	-2	40000
CD	1	-10000	-1	10000
CF	1.414214	14142.14	1.414214	28284.27125
DF	1	-10000	-1	10000
				126568.5425

Applying Castigliano's theorem:

$$\left(\delta_y\right)_F = \sum_{i=1}^{m} \frac{P_i \, P_i' \, L_i}{A_i \, E_i}$$

$$\left(\delta_y\right)_F = \frac{126{,}568.5}{(300\times10^{-6})(200\times10^{9})}$$

$$\left(\delta_y\right)_F = 2.1 \text{ mm}$$

i.e. 2.1 mm downwards.

(4 marks)

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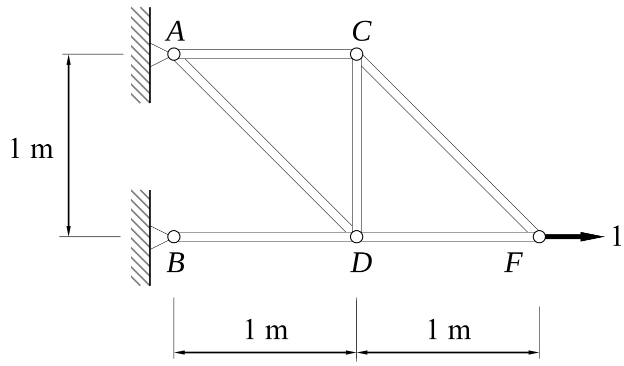
iv) Horizontal deflection at F

Apply a unit load at the joint of interest with the correct orientation:



Now perform a new 'static equilibrium' analysis to obtain P_i' values.

Joint *F*: the non-colinear force must be zero, hence:

$$P'_{CF} = 0$$

$$P'_{DF} = 1$$

There are no vertical loads in the system, therefore $P'_{BD} = P'_{DF} = 1$ and all other P_i' values are zero.

Member	**Li / m**	**Pi / N**	**Pi'**	**Pi Pi' Li / Nm**
AC	1	10000	0	0
AD	1.414214	14142.14	0	0
BD	1	-20000	1	-20000
CD	1	-10000	0	0
CF	1.414214	14142.14	0	0
DF	1	-10000	1	-10000
				-30000

Applying Castigliano's theorem:

$$\left(\delta_x\right)_F = \sum_{i=1}^{m} \frac{P_i \, P_i' \, L_i}{A_i \, E_i}$$

$$\left(\delta_x\right)_F = \frac{-30000}{(300\times10^{-6})(200\times10^{9})}$$

$$\left(\delta_x\right)_F = -0.5 \text{ mm}$$

i.e. 0.5 mm to the left.

(3 marks)

Q2

a)

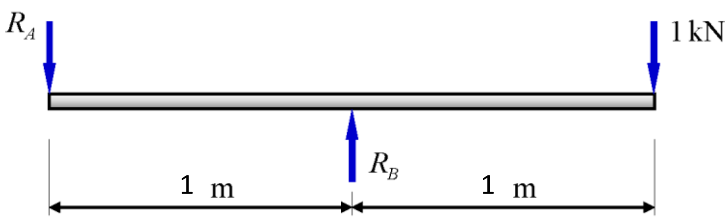
i. $P_{cr} = k\dfrac{\pi^2 EI}{L^2}$, where the constant k must be changed to account for different fixity conditions.

ii. A step function useful for the solution of deflections of muti-bay continuous beam problems.

iii. A measure of cross-section area distribution relevant to bending rigidity. The product of area and the square of its distance from the axis.

iv. The maximum bending stress along the cross-section will occur at the point which is furthest away from the neutral plane.

b)



i) Support reactions

$$\sum M_{@A}^{CW} = 0$$

$$(1\,\text{kN})(2\,\text{m}) - (R_B)(1\,\text{m}) = 0 \qquad R_B = 2\,\text{kN}$$

$$\sum F = 0$$

$$R_B - R_A - 1\,\text{kN} = 0 \qquad R_A = 1\,\text{kN}$$

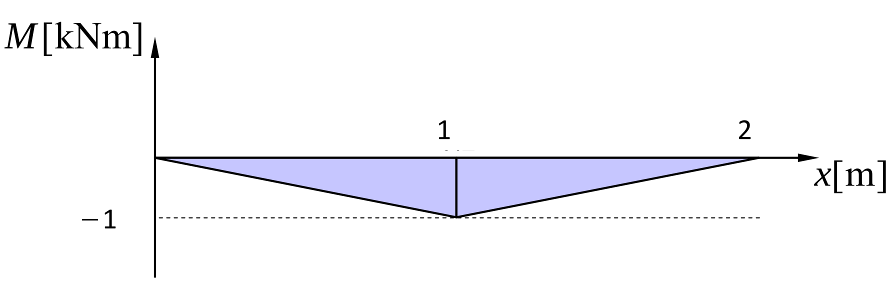
ii) Shear force & bending moment diagrams

$$M_B = (-1\,\text{kN})(1\,\text{m}) \qquad M_B = -1\,\text{kNm}$$

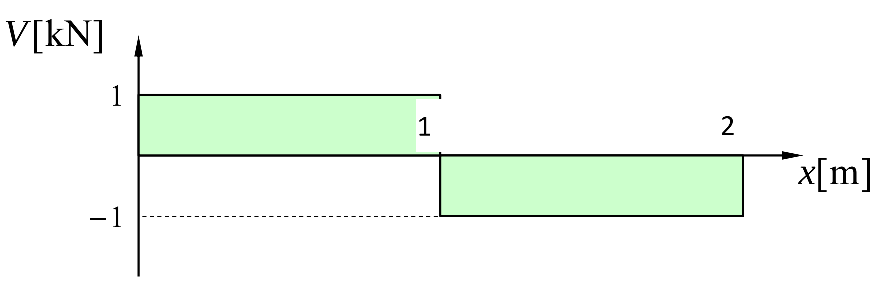
At the ends the moments must be zero (pin-joint and free) therefore:

$$M_A = M_C = 0$$

The bending moment diagram is therefore:



The shear force diagram can be obtained either by analysing the 'slope' of the bending moment diagram, or by sectioning the beam at each span. Finally:



(6 marks)

iii) Deflection at C

Balance of moments for a section moving from left to right gives:

$$M_{(x)} = (-R_A)(x) + (R_B)(x - 1\,\mathrm{m})H(x - 1\,\mathrm{m})$$

Therefore the curvature is given by:

$$EI\,\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = M_{(x)} = (-R_A)(x) + (R_B)(x - 1\,\mathrm{m})H(x - 1\,\mathrm{m})$$

$$EI\,\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -x + 2(x - 1\,\mathrm{m})H(x - 1\,\mathrm{m})$$

First integration gives the slope:

$$EI\,\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{2}x^2 + (x - 1\,\mathrm{m})^2 H(x - 1\,\mathrm{m}) + A \qquad (1)$$

And the second integration gives the deflection:

$$EI\,y_{(x)} = -\frac{1}{6}x^3 + \frac{1}{3}(x - 1\,\mathrm{m})^3 H(x - 1\,\mathrm{m}) + Ax + B \qquad (2)$$

The constants of integration are found from boundary conditions:

$@\,x = 0,\ y = 0$ and from equation (2): $B = 0$

$@\,x = 1,\ y = 0$ and from equation (2):

$$EI\,y_{(x)} = 0 = -\frac{1}{6}(1)^3 + 0 + A(1) + 0 \qquad A = \frac{1}{6}\,\mathrm{kNm}^2$$

The second moment of area is:

$$I = \left[\frac{1}{12}(30)^4\right]$$

$$I = 6.75 \times 10^4\,\mathrm{mm}^4 = 6.75 \times 10^{-8}\,\mathrm{m}^4$$

and the flexural modulus is:

$$EI = 70\,\frac{\mathrm{kN}}{\mathrm{mm}^2} \times 6.75 \times 10^4\,\mathrm{mm}^4$$

$$EI = 4.725 \times 10^6\,\mathrm{kNmm}^2 = 4.725\,\mathrm{kNm}^2$$

Finally, the deflection is found from equation (2) by substituting $x = 2\,\mathrm{m}$:

$$EI\,y_C = -\frac{1}{6}(2)^3 + \frac{1}{3}(2 - 1)^3 + \frac{1}{6}(2)$$

$$y_C = -\frac{2}{3}\frac{1}{EI}$$

$y_C = -0.141\,\mathrm{m} = -141\,\mathrm{mm}$, the negative sign describing a <u>downward</u> deflection.

(8 marks)

Q3

a)

i. Limit load - the maximum load anticipated in operation
ii. Ultimate load – the maximum load a component must withstand without failure
iii. Load factor – the ratio between the inertia loads an aircraft experiences in a particular flight loading case to the 1g steady level flight case.
iv. Reserve factor - the ratio of allowable stress / stress at ultimate load

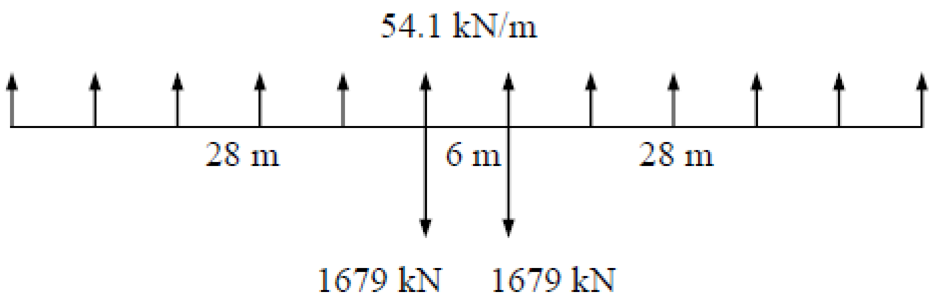
b)

Aerodynamic load on wing = 282,000 x 9.81 x 2.1 = 5809 kN

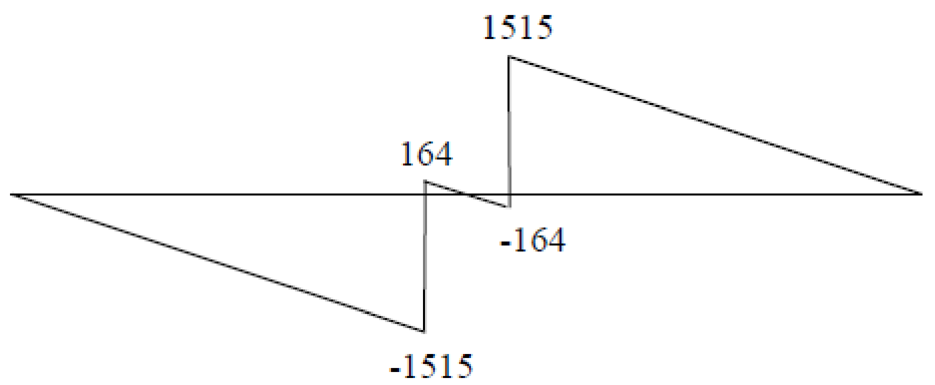
Wing fuel and structure inertia load = (21,000+98,000) x 9.81 x 2.1 = 2452 kN

Nett load from fuselage = 3357 kN, i.e. 1679 kN each side

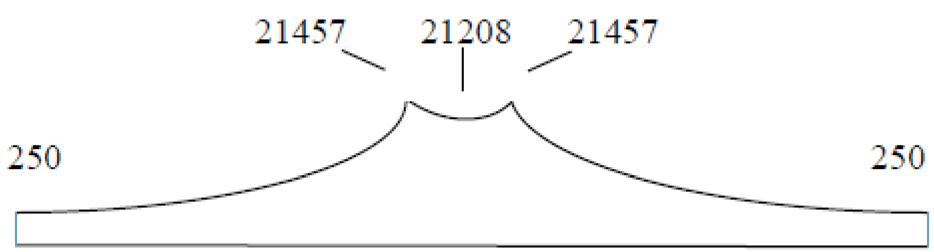
Nett distributed wing loading = 3357/62 = 54.1 kN/m



Shear force diagram with principal values (kN):



Bending moment diagram with principal values (kNm)



c)

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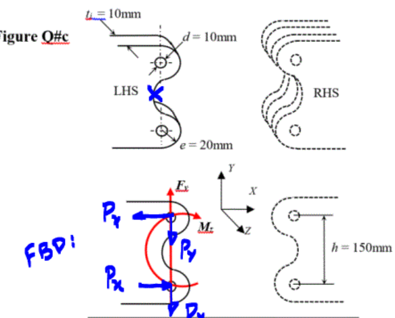
Q

Figure Q#c



At limit:
M_z = 10kNm
F_y = 6kN

Table Q#c

Lug properties	Strength	Units
Tensile	700	N/mm^2
Shear	400	N/mm^2
Bearing	1000	N/mm^2

LHS lug checks only here

Loads transferred across pins:

Referring to FBD: @ULT, working in kN, mm:

$\Sigma \uparrow^{+} = 0: \quad F_y - 2P_y = 0 \quad : P_y = F_y/2 = \frac{1.5\times 6}{2}$

$\hookrightarrow \quad P_y = 4.5\text{kN}$

$\Sigma \circlearrowright^{+} = 0: \quad P_x.h - M_z = 0 \quad : P_x = \frac{M_z}{h} = \frac{1.5\times 10\times 1000}{150}$

$P_x = 100\text{ kN}$

Resultant $P = \sqrt{100^2 + 4.5^2} = 100.1\text{ kN}$
~ approximately in x direction!

Joint stressing: working in N, mm

LUGS: Tension: $\sigma_t = \frac{P}{(w-d)t} = \frac{100.1\times 1000}{(40-10)10} = 333.7\text{ N/mm}^2$

c/w $\sigma_t^* = 700$: $\boxed{RF = \frac{700}{333.7} = 2.1}$

Shear-out: $\tau = \frac{P}{2et} = \frac{100.1\times 1000}{2\times 20\times 10} = 250.3\text{ N/mm}^2$

c/w $\tau^* = 400$: $\boxed{RF = \frac{400}{250.3} = 1.6}$

Bearing: $\sigma_{br} = \frac{P}{dt} = \frac{100.1\times 1000}{10\times 10} = 1001\text{ N/mm}^2$

c/w $\sigma_{br}^* = 1000$: $\boxed{RF = \frac{1000}{1001} \approx 1.0}$

Q4

a)

i. Young's modulus, compressive strength
ii. Hardness
iii. Flexural modulus, flexural strength
iv. Impact toughness

(4 marks)

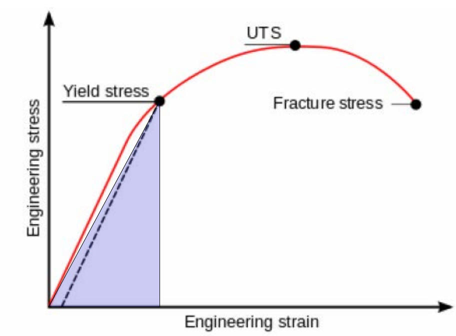
b)

i)

Resilience is the capacity of a material to store <u>elastic energy</u> before yield or irreversible damage mechanisms occur.

(1 mark)

ii)



$$U_r = \frac{1}{2}\sigma_y \varepsilon_y = \frac{1}{2}\sigma_y\left(\frac{\sigma_y}{E}\right) = \frac{\sigma_y^2}{2E}$$

Graphically, U_r is the area of the blue triangle.

(3 marks)

c)

i)

G_C - a.k.a. 'toughness' or 'fracture energy' (both of which are not strictly correct).

K_C - a.k.a. 'fracture toughness'.

(1 mark)

ii)

G_C - $\mathrm{J \cdot m^{-2}}$ or $\mathrm{J/m^2}$ or $\mathrm{N \cdot m^{-1}}$ or $\mathrm{N/m}$ (all are acceptable)

K_C - $\mathrm{Pa\sqrt{m}}$ or $\mathrm{Pa \cdot m^{1/2}}$ or $\mathrm{N \cdot m^{-3/2}}$ (all are acceptable)

(2 marks)

iii)

For linear elastic materials: $G_C = \dfrac{{K_C}^2}{E}$, assuming plane stress condition.

(2 marks)

d)

i)

Ashby plots can be used to compare, rank and select materials if a relevant material index can be derived, and then converted into a selection line (or ranking line). This requires:

- *Translation* of the problem, i.e. identifying the predominant mode of loading, e.g. axial tension, axial compression (buckling), bending, torsion, crack growth etc.
- Based on this translation, choice of objectives and constraints, and derivation of a material index.
- Plotting of a selection line based on this material index.
- Ranking material candidates according to their 'distance' to this selection line.

In this way optimal materials can be identified before the actual structural element is even dimensioned.

(3 marks)

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ii)

Objectives are measures (related to material properties) which we wish to minimise or maximise in order to find the optimal material choice. These measures can be 'better' or 'worse' between candidates, and can be used for ranking (e.g. weight → minimise).

Constraints are requirements which must simply be satisfied – i.e. 'pass/fail' criteria (e.g. price limit → pass or fail).

(2 marks)

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iii)

Minimum mass, stiffness constraint: $M = \frac{E^{1/2}}{\rho}$, where E is the Young's modulus and ρ is the mass density.

Minimum mass, strength constraint: $M = \frac{\sigma_f^{2/3}}{\rho}$, where σ_f is the failure strength.

Q5

a)

Covalent bonding occurs between two non metals, share valence electrons, it is a strong, directional bond between two atoms. Breaking bond lead to fracture of the material (i.e. brittleness). As electrons in e.g. single bond consumed by bonding, electrical conductivity poor (unlike double or triple bonds).

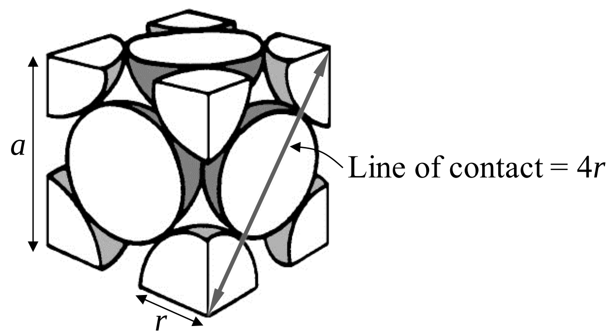
Ionic occurs between metal + non-metal. Exchange of valence electrons takes place (leading to unbalanced charges, an ordered structure with mutual attraction between opposite ions). Non directional bonds. The ions are immobile charge carriers, therefore poor electrical conductivity

Metallic bonding - relatively electropositive atoms give up valance electrons to form a sea of non-localised electrons (positive ions attracted to negative sea). Electrons are charge/heat carriers (excellent conductivity).

[5 marks]

b)

APF of FCC can be derived:



There are **4** atoms per unit cell, **8** corner atoms = 1, **6** face atoms = 3.

Line of contact = 4r, so $(4r)^2 = a^2 + a^2$, $a = 2r\sqrt{2}$

$V_{cell} = a^3 = (2r\sqrt{2})^3 = 16r^3\sqrt{2}$

APF = $(4\text{x}\ 4/3\pi r^3)/a^3 = (16/3\ \pi r^3)/(16r^3\sqrt{2}) = \pi/(3\sqrt{2}) = 0.74$ (more efficient)

[4 marks]

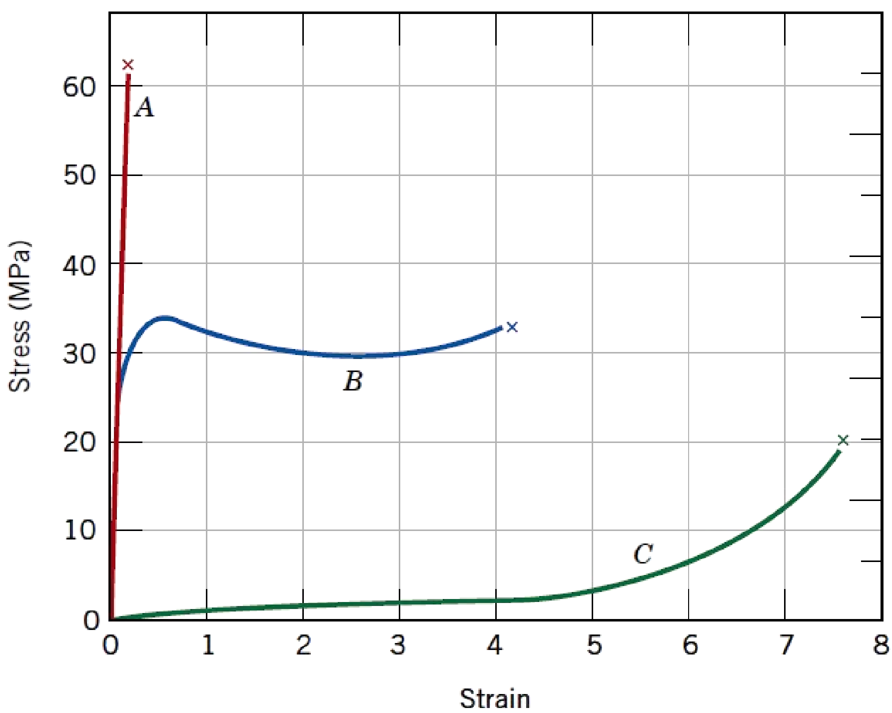
{Aluminium, copper, gold, lead}

[1 mark]

c)

Glass transition temperature (T_g) represents temperature at which change from glassy (brittle) to rubbery behaviour occurs. At the molecular level, represents large scale polymer chain movement/rotation (hindered by entanglement, chain polarity, crosslinking, etc.).

[3 marks]



[2 marks]

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d)

i)

Viscoelasticity – polymers show time-dependent, recoverable strain.

[1 mark]

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ii)

Using σ_0 = 10.52 MPa, σ = 3.76 MPa, t = 250 s

$E(t) = \sigma(t)/\varepsilon_0$

$\sigma = \sigma_0 \exp(-t/\tau)$, rearrange $\tau = -t/[\ln(\sigma/\sigma_0)]$

$\tau = -120/[\ln(3.76/10.52)]$

$\tau = -120/\ln 0.357$

$\tau = -120/-1.028$

τ = 116.73 s

$\sigma = 10.52 \exp(-250/116.7) = 1.23$ MPa

$E(t) = 1.23/0.04 = 30.87$ MPa

[4 marks]