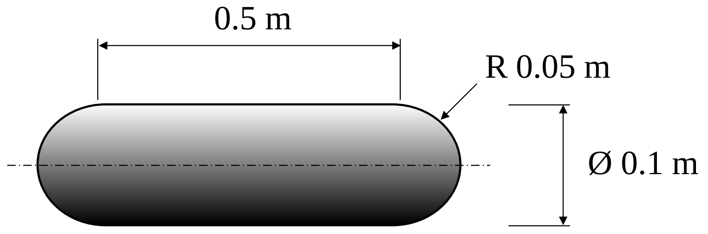
Example 2.1.3

The cylindrical pressure vessel with hemispherical caps shown in Figure 1 is made of stainless steel with E = 200 GPa and ν = 0.33. The wall thickness is 1 mm everywhere. Calculate the longitudinal and hoop strains in the central cylindrical portion (*i.e.* away from the caps) when the vessel is pressurised to 100 kPa.



Wall thickness t = 1 mm

Internal pressure p = 100 kPa

Figure 1: A pressure vessel (not to scale).

Remember: the hoop stress is twice the longitudinal stress for cylindrical vessels:

$$\sigma_H = \frac{p\,R}{t} \quad \text{and} \quad \sigma_L = \frac{1}{2}\frac{p\,R}{t}$$

It is OK to assume that the radius of the neutral plane is 50 mm (instead of 49.5 mm, discounting half the wall thickness).

Therefore the hoop stress is:

$$\sigma_H = \frac{p\,R}{t} = \left(\frac{1}{1\,\text{mm}}\right)\left(0.1\,\frac{\text{N}}{\text{mm}^2}\right)(50\,\text{mm})$$

$$\sigma_H = 5\,\frac{\text{N}}{\text{mm}^2} = 5\,\text{MPa}$$

And the longitudinal stress is:

$$\sigma_L = \frac{\sigma_H}{2}$$

$$\sigma_L = 2.5\,\frac{\text{N}}{\text{mm}^2} = 2.5\,\text{MPa}$$

Now remember Poisson's effect in 2D:

$$\varepsilon_1 = \frac{\sigma_1}{E} - \nu\frac{\sigma_2}{E} \quad \text{and} \quad \varepsilon_2 = \frac{\sigma_2}{E} - \nu\frac{\sigma_1}{E}$$

Therefore:

$$\varepsilon_H = \frac{\sigma_H}{E} - \nu\frac{\sigma_L}{E} = \frac{5\,\text{MPa}}{200\,000\,\text{MPa}} - 0.3\,\frac{2.5\,\text{MPa}}{200\,000\,\text{MPa}}$$

$$\varepsilon_H = 20.87\times10^{-6} = 20.87\,\mu\varepsilon$$

Similarly:

$$\varepsilon_L = \frac{\sigma_L}{E} - \nu\frac{\sigma_H}{E} = \frac{2.5\,\text{MPa}}{200\,000\,\text{MPa}} - 0.3\,\frac{5\,\text{MPa}}{200\,000\,\text{MPa}}$$

$$\varepsilon_L = 4.25\times10^{-6} = 4.25\,\mu\varepsilon$$

(NB. For a radius of 49.5 mm we would get $\varepsilon_H = 20.67\,\mu\varepsilon$ and $\varepsilon_L = 4.21\,\mu\varepsilon$)