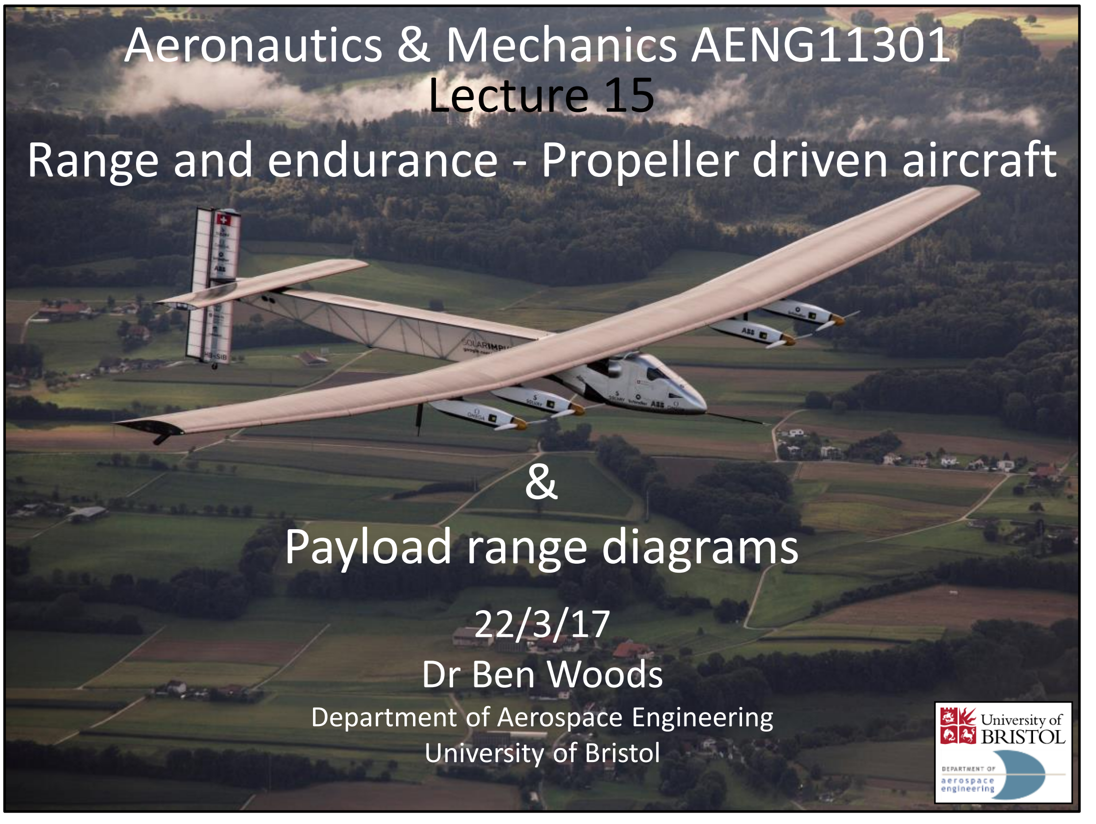


Solar Impulse 2 was built to take up the challenge of achieving the first round-the-world solar flight. This revolutionary airplane will have to do what no one has ever done before: fly through 5 consecutive days and nights without using any fuel, so as to cross oceans from one continent to the next.

This revolutionary single-seater aircraft made of carbon fiber has a 72 meter wingspan (larger than that of the Boeing 747-8I) for a weight of just 2,300 Kg, equivalent to that of a car.

The 17,000 solar cells built into the wing supply four electric motors (17.5 CV each) with renewable energy.
During the day, the solar cells recharge lithium batteries weighing 633 Kg (2077 lbs.) which allow the aircraft to fly at night and therefore to have virtually unlimited autonomy.

Outline for today

- Recap of Breguet Range Equation for jet aircraft
- Breguet Range Equation – Propeller driven aircraft
  - Endurance
  - Range
- Payload range diagrams

Aims for today

- For a propeller driven aircraft be able to calculate:
  - maximum endurance
  - maximum range
- Be able to construct and read a payload range diagram

Breguet Range Equation – Propeller-Driven Aircraft

- **“rate at which fuel is burnt**
  = rate at which aircraft weight is reduced”
- define **specific fuel consumption** as
  - ***f*** **= mass of fuel burnt per unit of power per second**
  - consistent units are $kg/W.s$
  - but often given in terms of $kg/kW.hr$ so don’t forget to convert!
- for power P and weight W the basic Breguet equation is

$$\boxed{\frac{dW}{dt} = -fgP}$$

  - a differential equation (in units of N/s)

Endurance - Integration of Breguet Equation

- power delivered by propeller
  - where η = propeller efficiency

$$\boxed{\eta P = DV}$$

$$\Rightarrow \quad \frac{dW}{dt} = -fg\frac{DV}{\eta} \quad \Rightarrow \quad \boxed{dt = -\frac{\eta}{fgV}\frac{C_L}{C_D}\frac{dW}{W}}$$

- assume that C_L/C_D , f and V remain *constant*
  - same as jet cruise-climb
- integrate as before from W_1 to W_2 to obtain the endurance E

$$\boxed{E_{prop} = t_2 - t_1 = t_{12} = \frac{\eta}{fg}\frac{1}{V}\frac{C_L}{C_D}\ln\left(\frac{W_1}{W_2}\right)}$$

Maximum Endurance

- for maximum endurance we require $(C_L/C_D)/V$ to be a maximum
  - or VC_D/C_L to be a minimum

$$\boxed{E_{prop} = \frac{\eta}{fg}\left(\frac{1}{V}\frac{C_L}{C_D}\right)\ln\left(\frac{W_1}{W_2}\right)}$$

- expanding this term we obtain

$$\boxed{V\frac{C_D}{C_L} = \frac{VD}{L} = \frac{VD}{W}}$$

- $D \times V$ is the power required to overcome drag
  $\rightarrow$ **maximum endurance** achieved at **minimum power speed** for propeller-driven aircraft
  - very much slower than for jet aircraft
  - compare with glider performance

Range - Integration of Breguet Equation

- increment in distance dS at velocity V is given by

$$\boxed{dS = Vdt = -\frac{\eta}{fg}\frac{C_L}{C_D}\frac{dW}{W}}$$

- can then integrate from start weight W_1 to end weight W_2 to obtain the range R

$$\boxed{R_{prop} = S_2 - S_1 = \frac{\eta}{fg}\left(\frac{C_L}{C_D}\right)\ln\left(\frac{W_1}{W_2}\right)}$$

- **maximum range** achieved at **minimum drag speed** for propeller-driven aircraft
  - much slower than for jet aircraft

Summary
Range and endurance - Propeller driven aircraft

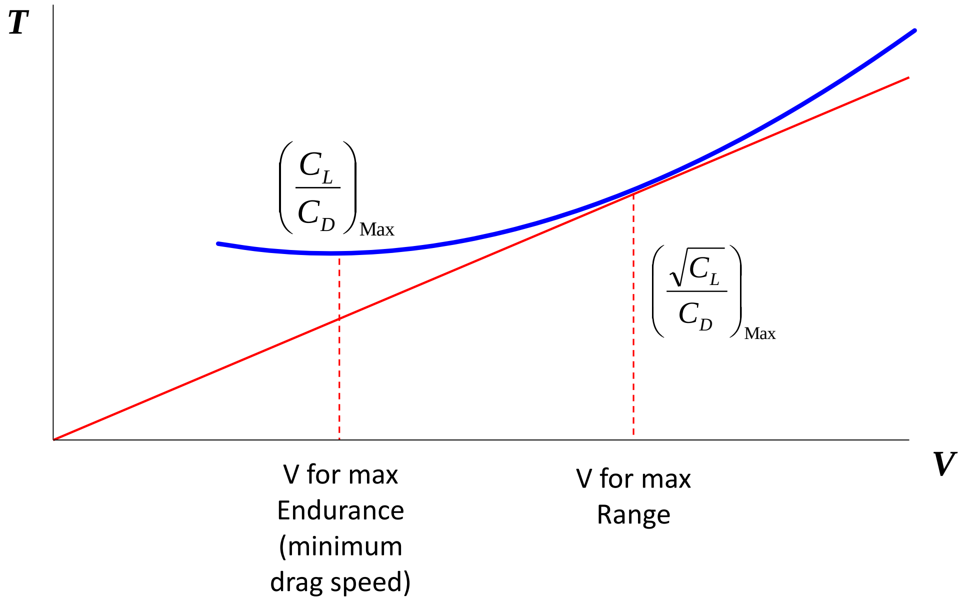
$$\boxed{E_{prop} = \frac{\eta}{fg}\frac{1}{V}\frac{C_L}{C_D}\ln\left(\frac{W_1}{W_2}\right)}$$

- **maximum endurance** achieved at **minimum power speed**

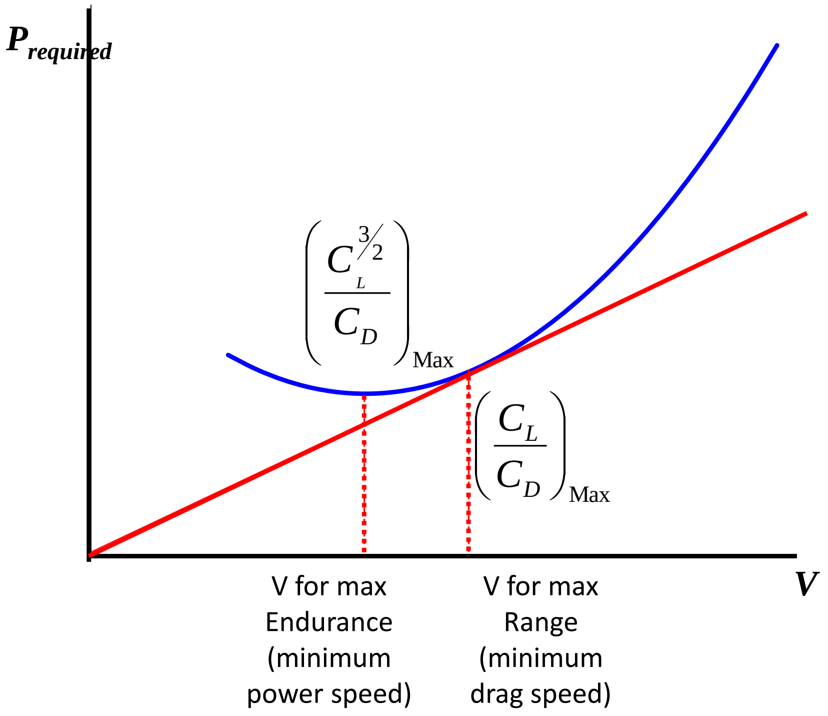
$$\boxed{R_{prop} = \frac{\eta}{fg}\left(\frac{C_L}{C_D}\right)\ln\left(\frac{W_1}{W_2}\right)}$$

- **maximum range** achieved at **minimum drag speed**

Jet Aircraft



Propeller Aircraft



Drag polar relationships

$\frac{C_L}{C_D}$ relation	Maximised when	C_L	C_D	Relates to
$\frac{C_L^{3/2}}{C_D}$	$C_{D0} = {}^1\!/_3\, KC_L^2$	$\sqrt{\frac{3C_{D0}}{K}}$	$4C_{D0}$	Min power required Min sink rate Max prop endurance
$\frac{C_L}{C_D}$	$C_{D0} = KC_L^2$	$\sqrt{\frac{C_{D0}}{K}}$	$2C_{D0}$	Min drag Min glide angle Max prop range Max jet endurance
$\frac{C_L^{1/2}}{C_D}$	$C_{D0} = 3KC_L^2$	$\sqrt{\frac{C_{D0}}{3K}}$	${}^4\!/_3\, C_{D0}$	Max jet range

Payload-Range Diagrams

Need to ID aircraft

What is a Payload-Range Diagram?

- Represents trade-off between the payload an aircraft carries and the range it can fly
- Defines how an airline can use the airplane
- Provides a better understanding of the utility of the airplane
- Other than fuel economy and cost, probably the most important information for the sales group

Breguet Range Equation

$$\boxed{R = \frac{a}{g * tsfc}\left(M\,\frac{L}{D}\right)\ln\frac{W_{initial}}{W_{final}}}$$

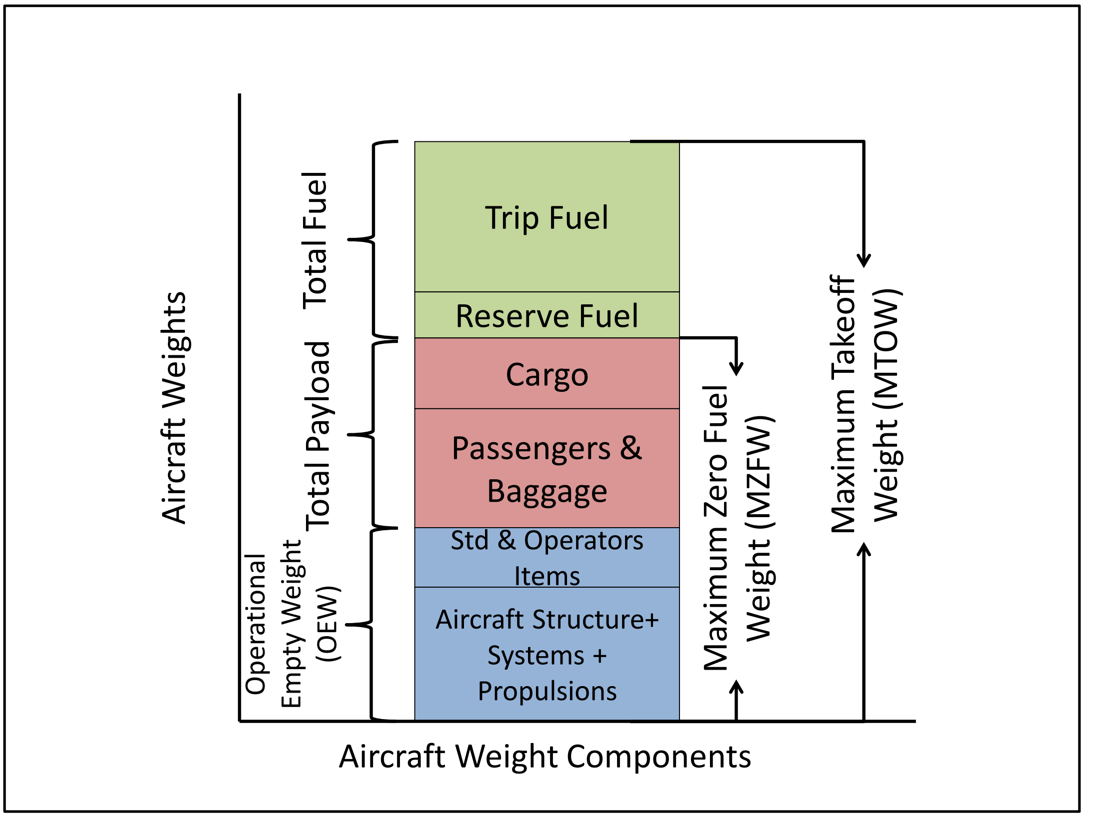
- The Breguet range equation tells us the range of an aircraft as a function of:
  - Altitude through sound speed (a), and $tsfc$
  - Engine efficiency, through $tsfc$ (thrust specific fuel consumption)
  - Aerodynamic efficiency, through $M(L/D)$
  - Structural efficiency through ($W_{initial}/W_{final}$)

Aircraft weight

- **Maximum Takeoff Weight (MTOW)** - maximum authorized weight for takeoff due to strength and airworthiness requirements.
- **Operational Empty Weight (OEW)** – basic weight of aircraft including all equipment and supplies needed for standard operation
- **Maximum Zero Fuel Weight (MZFW)** – maximum weight allowed before fuel is loaded due to strength and airworthiness requirements.

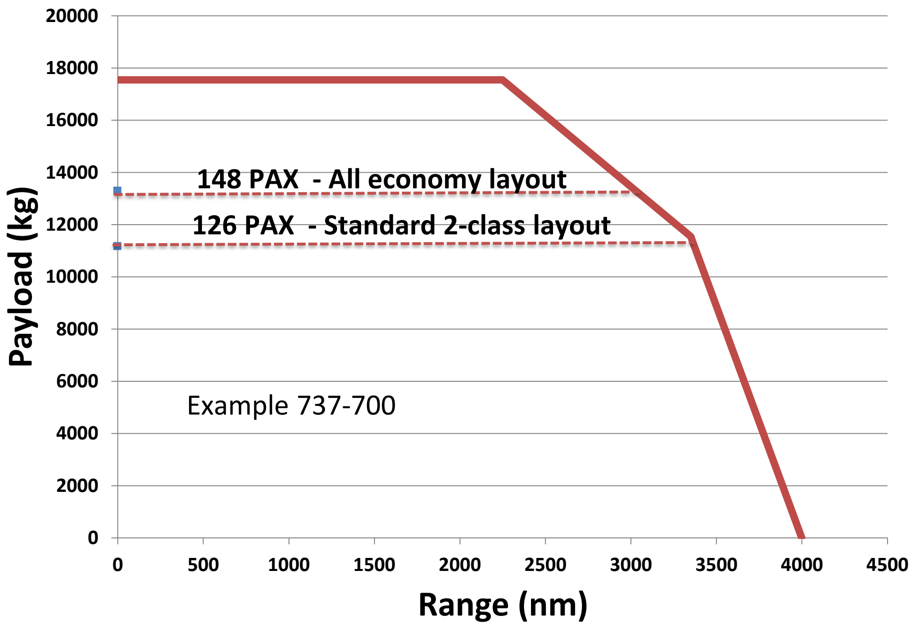
$$\boxed{W_{final} = \text{Operational Empty Weight} + \text{Payload} + \text{Reserve Fuel Weight}}$$

$$\boxed{W_{intial} = W_{final} + \text{Trip Fuel Weight}}$$



Aircraft Weights
Total Fuel
Total Payload
Operational Empty Weight (OEW)
Trip Fuel
Reserve Fuel
Cargo
Passengers & Baggage
Std & Operators Items
Aircraft Structure+ Systems + Propulsions
Maximum Zero Fuel Weight (MZFW)
Maximum Takeoff Weight (MTOW)
Aircraft Weight Components

Sample Payload-Range Diagram



Approx. numbers for Boeing 737-700

- MTOW: 69,400 kg
- OEW + Fuel Reserve: 39,148 kg
- Max Useable Fuel: 18,894 kg
- Maximum Payload: 17,544 kg
- TSFC: 0.017 kg/kN s
- M_{cruise} = 0.79
- L/D_{cruise} = 14.3

Max Range at Max Payload

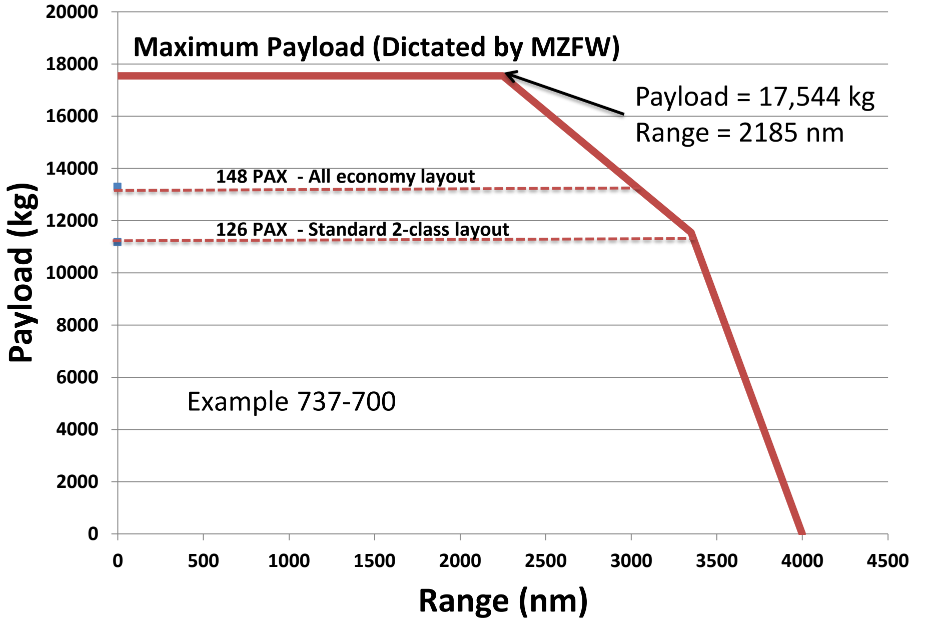
- Wi = MTOW = 69,400 kg
- Wf = OEW + Payload
  = 39,148 + 17,544 = 56,692 kg

$$R = \frac{a}{g * tsfc}\left(M\,\frac{L}{D}\right)\ln\frac{W_{initial}}{W_{final}}$$

$$= \frac{295}{9.8 * 0.017}(0.79 * 14.3) \times \ln\frac{69{,}400}{56{,}692} = 4046\ \text{km}$$

$$= 2185\ \text{nm}\ (\text{km} \times 0.540)$$

Sample Payload-Range Diagram



MTOW Line

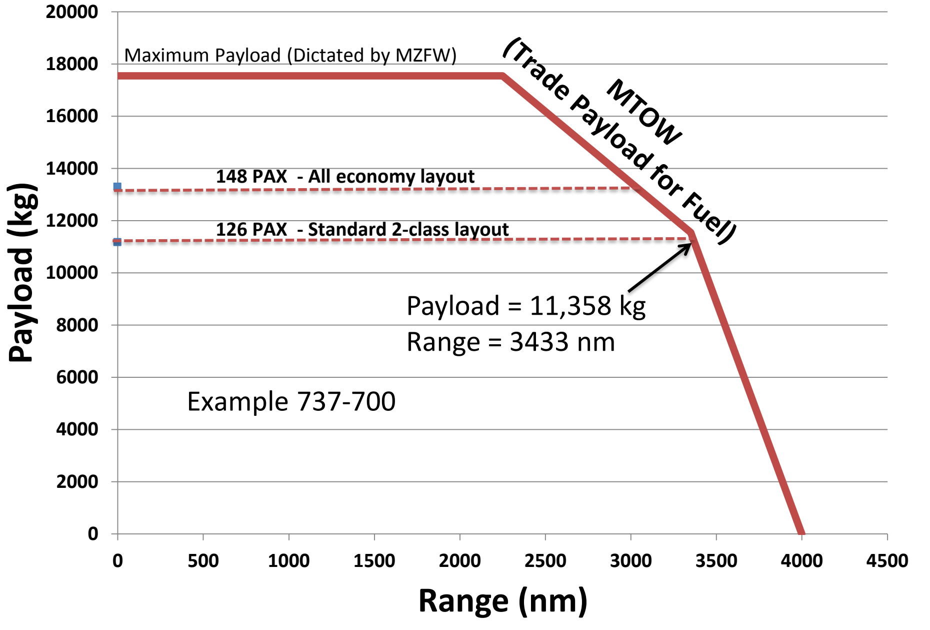
- Wi = MTOW = 69,400 kg
- Payload = MTOW – OEW – Max fuel
  = 69,400 – 39,148 – 18894 = 11,358 kg
- Wf = MTOW - Max fuel = 69,400 – 18894 = 50,506 kg

$$R = \frac{a}{g * tsfc}\left(M\,\frac{L}{D}\right)\ln\frac{W_{initial}}{W_{final}}$$

$$= \frac{295}{9.8 * 0.017}(0.79 * 14.3)\times\ln\frac{69{,}400}{50{,}506} = 6357\ \text{km}$$

$$= 3433\ \text{nm}$$

Sample Payload-Range Diagram

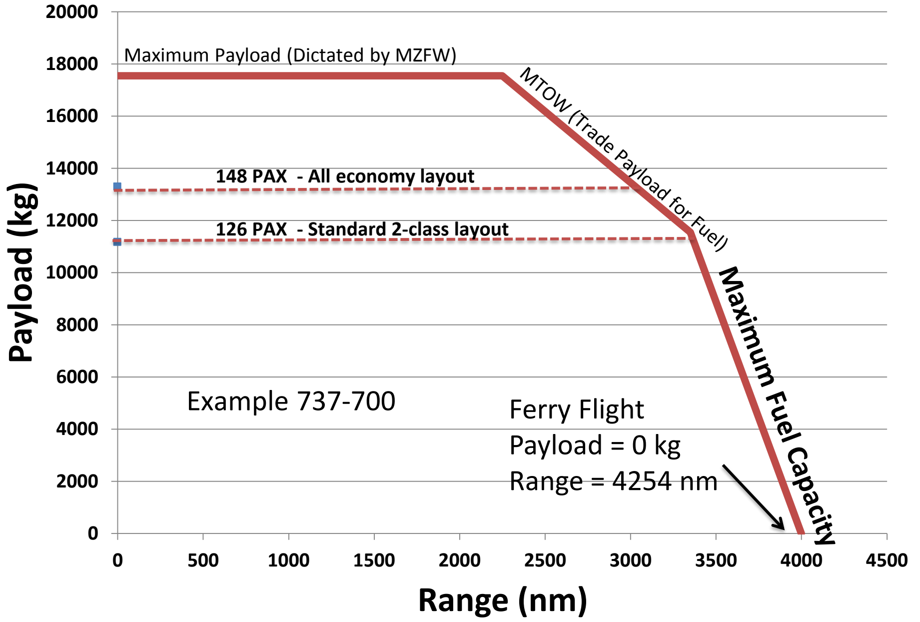


Max Fuel Line

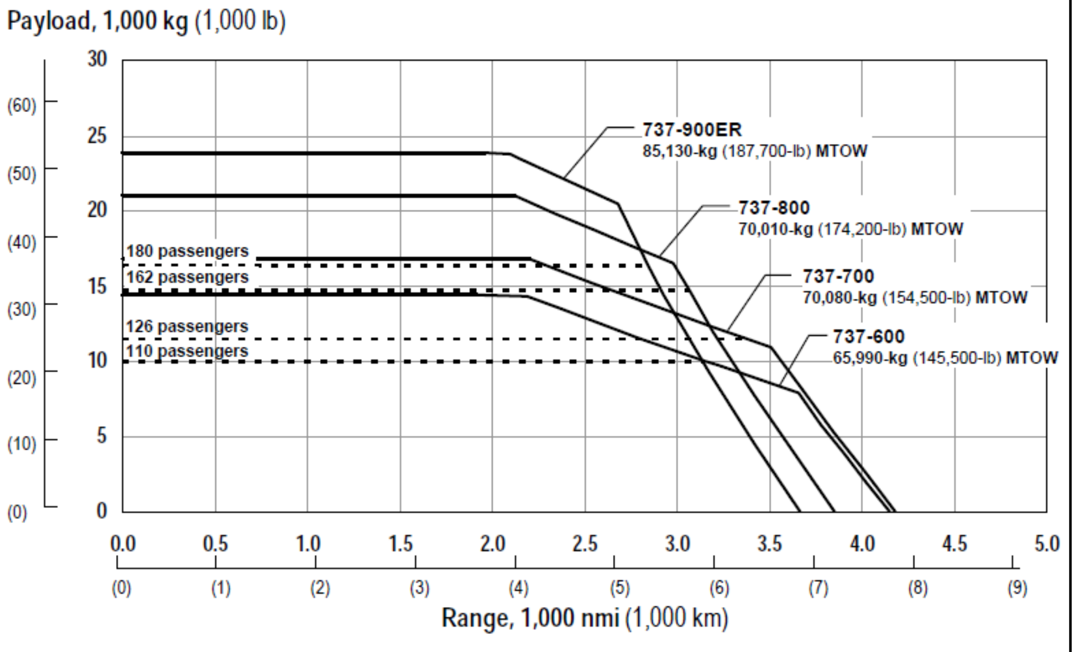
- Wi = OEW + Max fuel = 39,148 + 18,894 = 58,042 kg
- Wf = OEW = 39,148 kg
- Payload = 0 kg

$$
\begin{aligned}
R &= \frac{a}{g * tsfc}\left(M \frac{L}{D}\right)\ln\frac{W_{initial}}{W_{final}} \\
&= \frac{295}{9.8 * 0.017}(0.79 * 14.3) \times \ln\frac{58{,}042}{39{,}148} = 7878\,\text{km} \\
&= 4254\,\text{nm}
\end{aligned}
$$

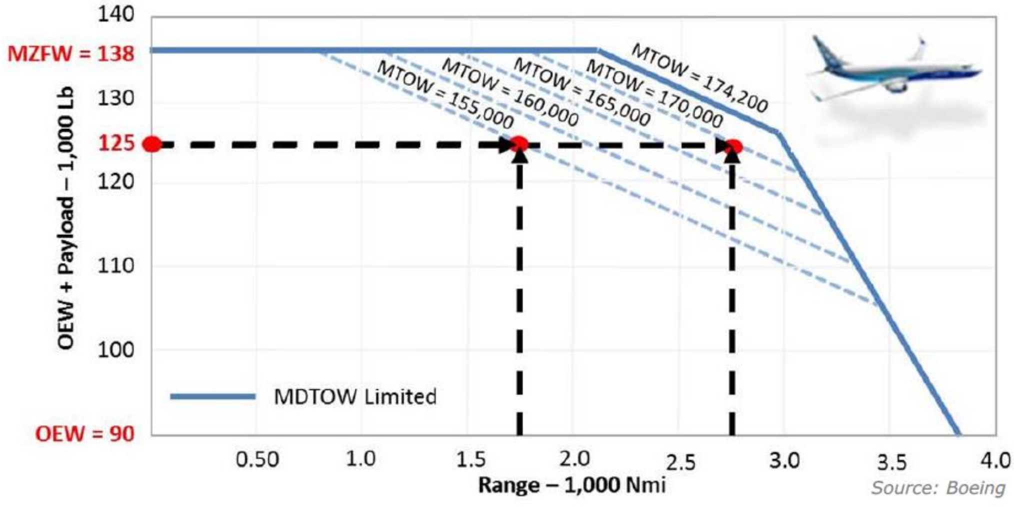
Sample Payload-Range Diagram



737 Family Payload-Range



737-800 different certified MTOW

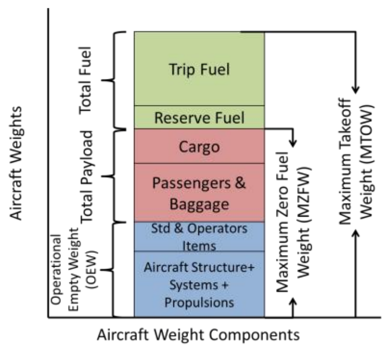


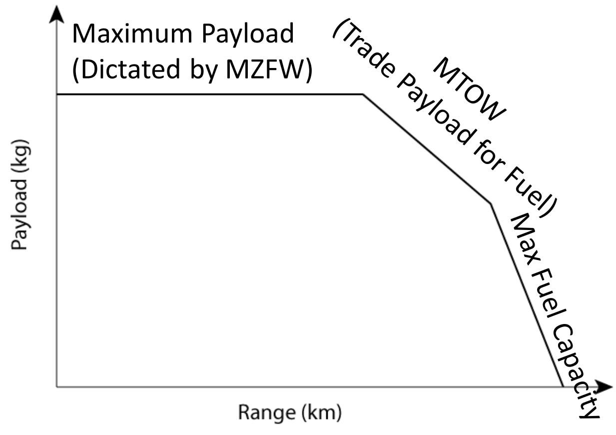
Summary – Payload-Range Diagrams

$$R = \frac{a}{g * tsfc}\left(M\frac{L}{D}\right)\ln\frac{W_{initial}}{W_{final}}$$

W_{intial} = W_{final} + Trip Fuel Weight

W_{final} = OEW+ Payload + Reserve Fuel





Be able to define endurance and range

Be able to define variables held constant in cruise climb vs constant altitude cruise and to be able to comment on the different assumptions involved

For a jet aircraft be able to calculate:

maximum endurance

maximum range for cruise climb with fixed and varying throttle settings

maximum range for constant altitude flight with fixed and varying throttle settings

Follow-up materials

To help with exam:

- Range and endurance of propeller-driven aircraft
  - Introduction to Flight – 6.12
- Drag polar relationships
  - Introduction to Flight – 6.14
- Payload range diagrams
  - Ackert (2013) Aircraft Payload Range Analysis for Financiers [Available on Blackboard]

To help with exam:
Introduction to Flight – 5.1-5.2

To aid in understanding:
Understanding flight – Chapter 1

For interest:
Introduction to Flight – 5.19 (explanation of lift)