Lecture 11
Static Stability

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Lecture Overview

- Stability – Definitions
- Pitching Moment Equation
- Neutral Point
- Static Margin

Static Stability

Definitions

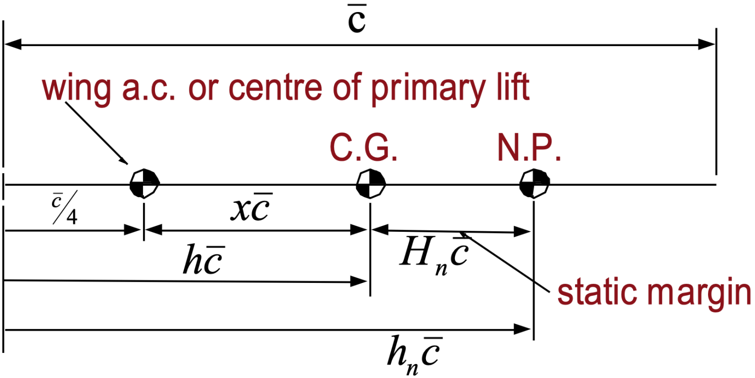
The previous calculations for elevator angle to trim do not guarantee:

- that the necessary angle for η is available
- that the trim condition is *stable*

The Neutral Point and Static Margin

The **neutral point** is the rearmost position of the c.g. before an unstable condition occurs.

The Neutral Point and Static Margin



- Note that our earlier use of c.g. position x (relative to the lift) is replaced here by an alternative measure h from the MAC l.e. or $x=h-\frac{1}{4}$ (MAC - *Mean Aerodynamic Chord*)

Static Stability

The Influence of Aerodynamic Stiffness on Stability

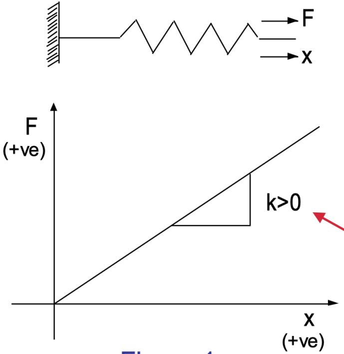


Figure 1

- Consider the simple spring shown in Figure 1
- Relationship between force and displacement?
- Applying force +F results in displacement +x
- Hence positive slope k

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- The force F is associated with extension x and results in a positive slope k. An alternative approach gives:

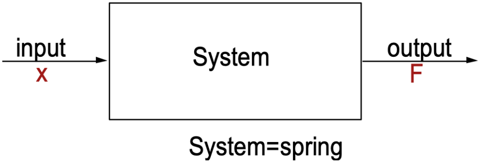


Figure 2

- Using the generalised system-approach of Fig. 2 the force caused by deflection x is actually *negative*.

Static Stability

- This is a restoring force which tries to restore x to zero and the force which we normally plot (Fig. 1) is actually the force required for equilibrium and which counteracts the force produced by the "system".

- Applied displacement x
- results in –ve force
- therefore –ve slope k

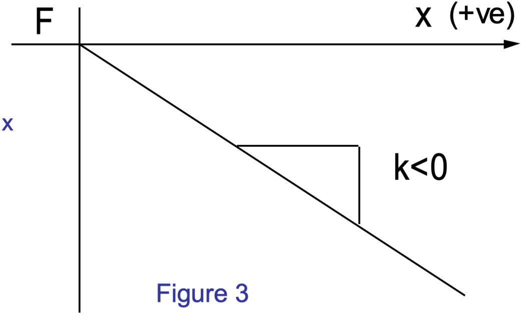
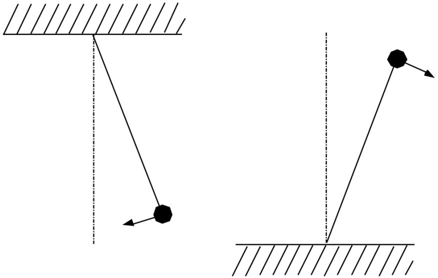


Figure 3

Static Stability



- We normally associate a positive stiffness with a restoring force (the pendulum supported at its top)
- *but* when the displacement and the force caused by it are defined as positive in the same direction a restoring force will be one which displays negative stiffness.

Static Stability

- So for the system shown in Figure 2, for which the input is the displacement x and the output the resulting force F, we should plot the graph as shown in Figure 3.
- Hence, we need to consider more closely the sign of k expected for a stable system.

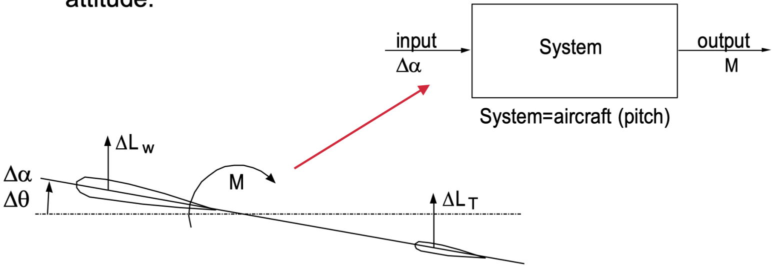
Remember the pendulum example:

restoring force $\longleftrightarrow$ stability

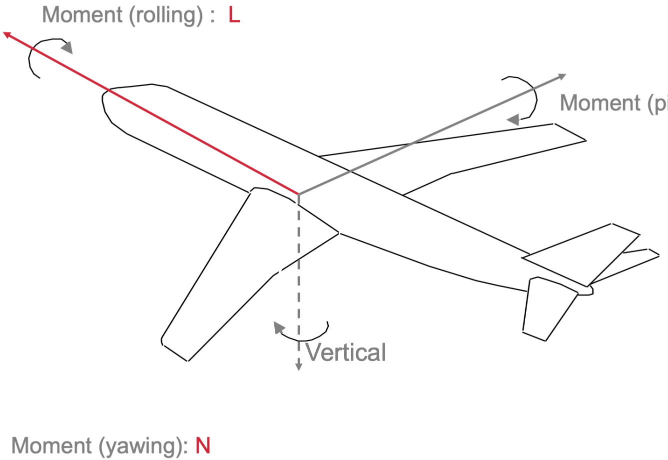
diverging force $\longleftrightarrow$ instability

Static Stability

- The flight mechanics case follows this sign convention and displays the displacement/force characteristics given in Figs. 2 & 3.
- Our study of stability will be directed toward an investigation of the pitching moment ΔM produced when there is a departure ($\Delta\alpha$ or $\Delta\theta$) away from the equilibrium flight attitude.



Body Axes Notation and Sign Conventions



Static Stability

- The "system" is the pitch response of an aircraft; the "input" is a pitch displacement $\Delta\alpha$ and the "output" is the consequent pitching moment ΔM.
- For stable flight we need $\Delta M < 0$ for $\Delta\alpha > 0$ so that there will be a restoring action. This can be seen graphically in Fig. 6.

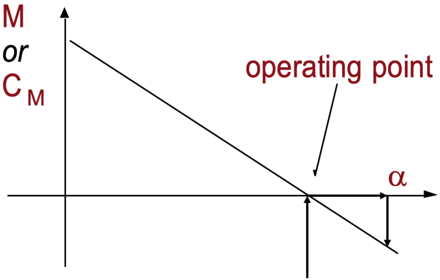


Figure 6

Static Stability

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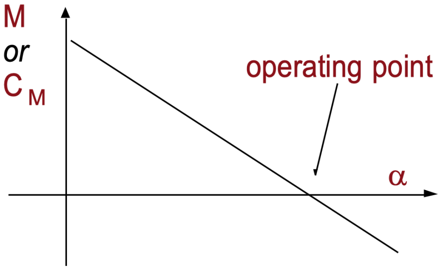

Figure 6

Static Stability

- Consider a perturbation $\Delta\alpha$ away from a trimmed, straight and level operating point? +ve or –ve slope desirable?

- This implies that whichever sign there is for $\Delta\alpha$ away from the equilibrium flight angle α, the pitching moment caused by $\Delta\alpha$ will be of such a sign as to counteract $\Delta\alpha$ and restore the flight to its equilibrium incidence.

The slope given by $\partial M / \partial\alpha$ must therefore be negative for stability.

Static Stability - Definitions

From Fig 6:

- If the coefficient form $\partial C_M / \partial \alpha$ is negative the aircraft will be stable
- Remember that α is α_{wing} and that we can put $\alpha_{wing} = C_{L_W} / a_1$
- Hence there is only a constant factor that would distinguish between $\partial C_M / \partial \alpha$ and $\partial C_M / \partial C_L$.
- Hence if $\partial C_M / \partial C_L$ is negative the aircraft will be stable (restoring moment when in flight)

$$a_1 = \frac{\partial C_L}{\partial \alpha}$$

$$C_L = a_1(\alpha - \alpha_0)$$

zero lift angle α_0

Previous Exam Question

Can you think of the directional equivalent?

Q2 (a) Explain what is meant by longitudinal static stability when related to a classical aircraft configuration. Provide an appropriate diagram.

(7 marks)

(b) You have been presented with the plot shown in Figure Q2.1 for an unknown fighter aircraft showing data across a very wide range of α. Using this plot and with the assumption that the aircraft retains lateral-directional stability throughout the angle of attack range given, describe the likely longitudinal response of the aircraft with α. Within your answer, provide a sketch of α vs time for this aircraft as the elevator is ramped in the direction that generates a positive pitching moment.

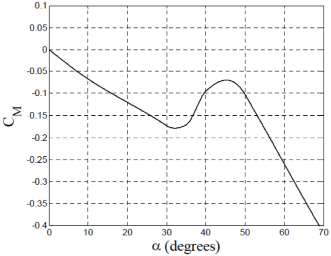

Fig. Q2.1

(13 marks)

(c) For the aircraft described in part (b) suggest how the longitudinal response might be improved for this aircraft when operating at high angles of attack.

(5 marks)


Pitching Moment Equation
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Derivatives of the Pitching Moment Equation

- The pitching moment expression which we obtained was:

$$C_M = C_{M_0} - \bar{V} a_{1_T} i_T - C_{L_{WFP}} \left[\bar{V} \frac{a_{1_T}}{a_1} (1-k) - x \right] - \bar{V} a_{2_T} \eta \qquad (1)$$

$$\boxed{\bar{V} = \frac{S_T l_T}{S \bar{c}}}$$

For trimmed flight $C_M = 0$

- Now consider a disturbance in pitch ($\Delta\alpha$), perhaps caused by an upward gust, and this $\Delta\alpha$ leads to an associated disturbance $\Delta C_L > 0$.

Static Stability

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$$\Delta C_L > 0$$

- On the basis of what we saw above, it should be clear that:

$\Delta C_M > 0$ is adverse $\rightarrow$ unstable aircraft
$\Delta C_M < 0$ is restoring $\rightarrow$ stable aircraft,

Static Stability

and thus in general

$$\frac{1}{a_1}\frac{\partial C_M}{\partial \alpha} \text{ or } \frac{\partial C_M}{\partial C_L} \begin{array}{ll} > 0 & \text{unstable} \\ = 0 & \text{neutrally stable} \\ < 0 & \text{stable} \end{array} \qquad (2)$$

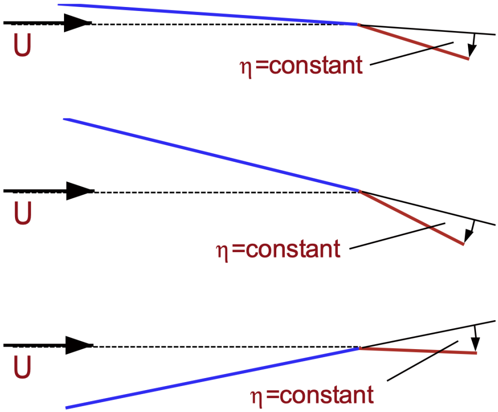
The case for a Fixed Elevator ("Stick-Fixed")

- We will assume that the elevator is held fixed by the pilot or that there is a "control-fixed" condition imposed by the hydraulics when control demands remain constant.

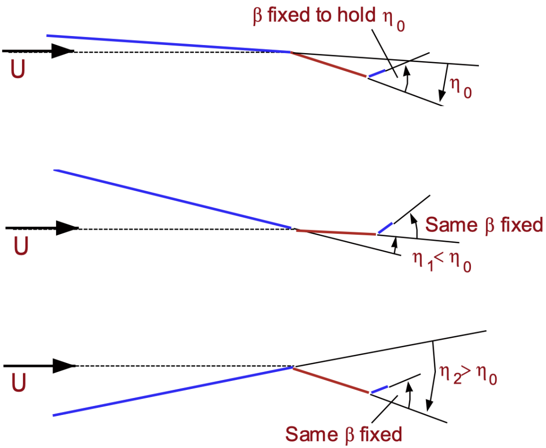
- With the "stick-free" case, there can be a small difference in the pitch stability if the elevator position is allowed to drift. Under such conditions, the nominal value $\eta = \eta_{trim}$ is not retained when the pitch attitude is disturbed.

- We will only consider the "stick-fixed" case.

"Stick-Fixed"



"Stick-Free"



Static Stability

- Thus for the case where the elevator is locked and η = constant (but not necessarily zero), we have from (1) above, for static stability

$$\frac{1}{a_1}\frac{\partial C_M}{\partial \alpha} = \frac{\partial C_M}{\partial C_L} = -\left[\bar{V}\frac{a_{1_T}}{a_1}(1-k) - x\right] < 0 \qquad (3)$$

and the factor in the square brackets must therefore be positive for a stable aircraft.

Static Stability

- For k of order 1/2 and with the c.g. behind the primary lift by a *small* distance (i.e. x is small and positive), the terms in the [   ] lead to a positive value.

- This will remain so, as the c.g. moves forward and passes the primary lift to make $x < 0$.

- However, clearly x *could* become sufficiently large and positive for [   ] to become negative and the aircraft would be unstable.

An Alternative Criterion for Pitch Stability?

- Another way of looking at the longitudinal stability is to use a logical argument which relates C_L and η_{trim} as follows for balanced flight:

1. If flight is to be at a faster speed, it must also be at a lower C_L, i.e. $\Delta C_L < 0$

2. To obtain a lower C_L we must push the nose down to obtain lower incidence α.

3. This lower incidence is obtained by a larger upward force at the tail, *via* a more positive η, i.e. $\Delta\eta > 0$, even if $\eta < 0$.

For Reference

- *and thus the elevator angle required to obtain this trim is:*

$$\eta_{trim} = \frac{1}{\bar{V} a_{2_T}} \left\{ C_{M_0} - \bar{V} a_{1_T} i_T - C_{L_{WFP}} \left[\bar{V} \frac{a_{1_T}}{a_1} (1-k) - x \right] \right\}$$

Static Stability

- Thus in flight we should expect to have $\Delta\eta_{trim}$ positive as speed is increased while ΔC_L becomes negative, and thus:

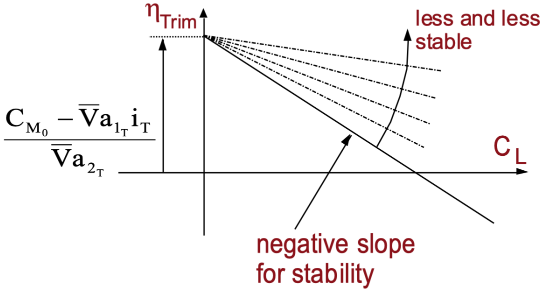
$$\frac{\partial\eta_{trim}}{\partial C_L} < 0$$

- Indeed a differentiation of the expression for η_{trim} will show that this leads to

$$\frac{\partial\eta_{trim}}{\partial C_L} = -\frac{1}{\bar{V} a_{2_T}}\left[\text{same factor in brackets}\right] \quad (4)$$

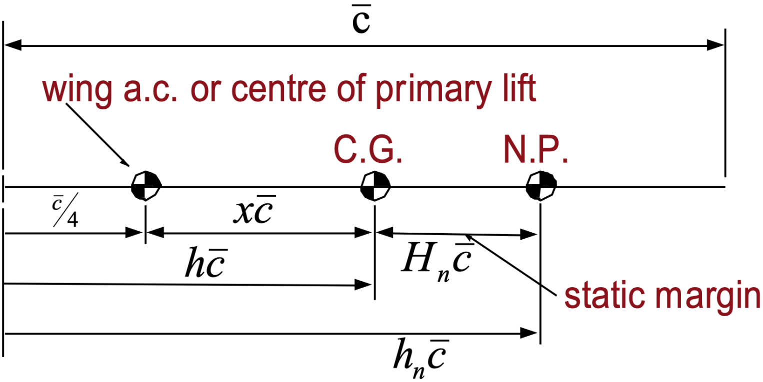
- Clearly, the RHS of Eqn. (4) will be negative if the bracketed expression is again positive; the logical requirement for retaining balanced flight rests on the stability criterion.

Static Stability



- Because the bracketed terms in Eqn. (3) & (4) are the same, the *boundary* between stable and unstable flight is shown by a zero value for that term and thus a horizontal line for the η_{trim} vs C_L graph.
- C.G. change = data for another line (moving significant masses!)

The Neutral Point and Static Margin



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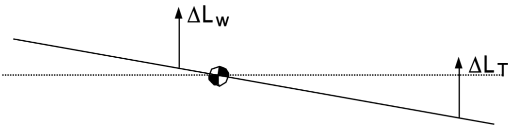
The Neutral Point and Static Margin



- When perturbed in pitch the new (additional) forces define its stability
  - wing force with a small moment arm trying to tip it farther out of alignment
  - while the (smaller) tail force with a long moment arm tries to restore the earlier alignment.
- Rearward movement of the c.g. reduces static stability

The Neutral Point

- Stability becomes neutral when the c.g. moves just back to the neutral point (N.P.), where:

$$h = h_n \qquad\qquad x = h_n - \tfrac{1}{4}$$

- The stick-fixed neutral point can be found by setting to zero the derivative that we saw earlier:

$$\frac{1}{a_1}\frac{\partial C_M}{\partial \alpha} = \frac{\partial C_M}{\partial C_L} = -\left[\bar{V}\frac{a_{1_T}}{a_1}(1-k) - x\right] < 0 \qquad (5)$$

The Neutral Point

- Alternatively: $$\bar{V}\frac{a_{1_T}}{a_1}(1-k)-h_n+\frac{1}{4}=0 \qquad (6)$$
- or, to define the length h_n: $$h_n=\frac{1}{4}+\bar{V}\frac{a_{1_T}}{a_1}(1-k) \qquad (7)$$
- and this defines the stick fixed neutral point relative to the MAC I.e. …

Static Margin

- The static margin is a measure of the distance remaining through which the c.g. could be moved rearward before the aircraft displays neutral static stability.
- It is defined as the remaining distance divided by $\bar{c}$, so it is a chord-fraction and is given by

$$(h_n - h) = H_n \tag{8}$$

- Static Margin is positive if the aircraft is still stable. Note that if in (5) we insert $x = h - 1/4$ we have:

$$\frac{\partial C_M}{\partial C_L} = h - \frac{1}{4} - \bar{V}\,\frac{a_{1_T}}{a_1}\left(1 - k\right)$$

The Neutral Point and Static Margin

- but (7) shows that this is just:

$$\frac{\partial C_M}{\partial C_L} = -(h_n - h) = -H_n. \qquad (9)$$

- Thus (5) is really just:

$$\frac{\partial C_M}{\partial C_L} = -H_n, \qquad (10)$$

- which shows that:

the static margin must be positive for positive static stability.