Advanced Bending and Torsion
Transformation of Axes – Mohr's Circle

Dr Luiz Kawashita

Luiz.Kawashita@bristol.ac.uk

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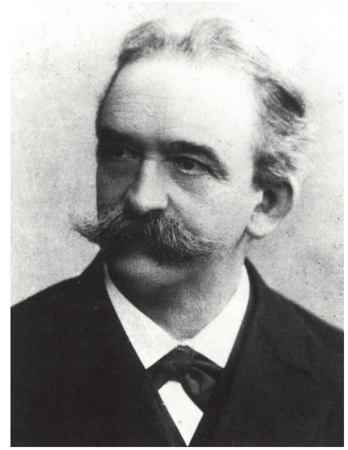
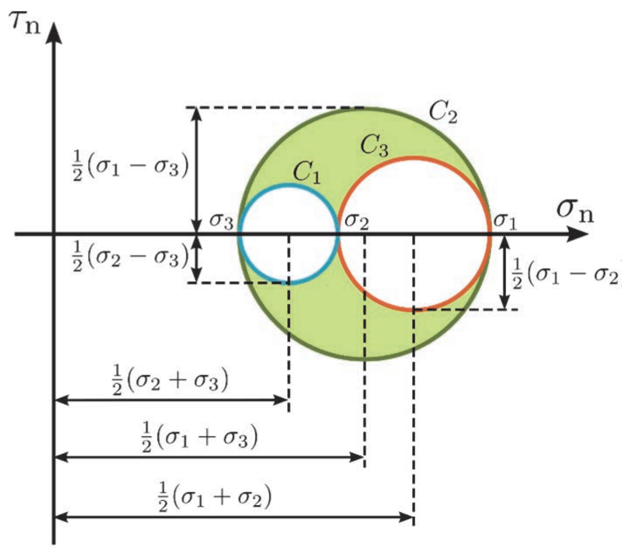
Mohr's Circle

- German engineer Christian Otto Mohr (1835-1918) proposed in 1882 a convenient geometrical representation of tensorial transformations – the **Mohr's Circle**



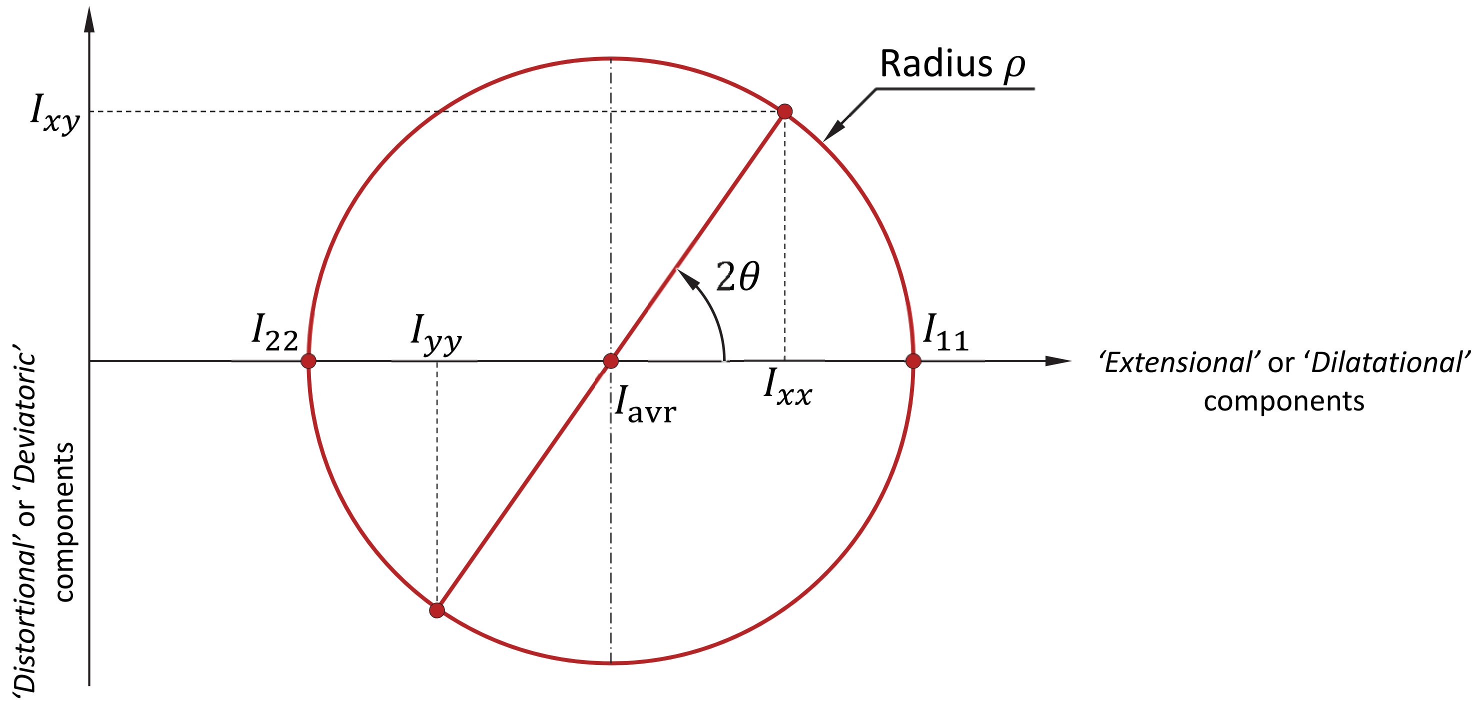
- We will use it twice in StM2:
  - Transformation of 2nd moments of area (I_{xx}, I_{yy}, I_{xy})
  - Transformation of stresses and strains $(\sigma_x, \sigma_y, \tau_{xy})$
- There are two versions: **2D** and **3D**
  - We will use the 2D version only in StM2
  - Essential tool for 2D stress analysis
  - For 2nd moments of area: for infomation only

(*i.e.* not assessed)



Mohr's Circle for 2nd Moments of Area

- In a nutshell: geometrical description of tensorial transformations (rotations) – describing *extensional* and *distortional* components as functions of the rotation angle θ



$$I_{\mathrm{avr}} = \frac{I_{xx} + I_{yy}}{2}$$

$$\rho = \sqrt{\left(\frac{I_{xx} - I_{yy}}{2}\right)^2 + \left(I_{xy}\right)^2}$$

Mohr's Circle for 2nd Moments of Area

- Mohr's circle for the **L-section beam example**:

$$I_{\text{avr}} = \frac{I_{xx} + I_{yy}}{2}$$

$$\rho = \sqrt{\left(\frac{I_{xx} - I_{yy}}{2}\right)^2 + (I_{xy})^2}$$

$$I_{11} = I_{\text{avr}} + \rho$$
$$I_{22} = I_{\text{avr}} - \rho$$

$$\theta_{\text{p}} = \frac{1}{2}\arctan\left(\frac{I_{xy}}{I_{xx} - I_{\text{avr}}}\right)$$

As before:

$I_{xx} = 26{,}554.45\ \text{mm}^4$

$I_{yy} = 5{,}141.10\ \text{mm}^4$

$I_{xy} = -6{,}578.38\ \text{mm}^4$

And now:

$I_{\text{avr}} = 15{,}847.78\ \text{mm}^4$ $\quad I_{11} = 28{,}413.92\ \text{mm}^4$ $\quad \theta_{\text{p}} = -15.78°$

$\rho = 12{,}566.14\ \text{mm}^4$ $\quad I_{22} = 3{,}281.63\ \text{mm}^4$