Aeronautics & Mechanics
AENG11301

Lecture 6
Standard Atmosphere

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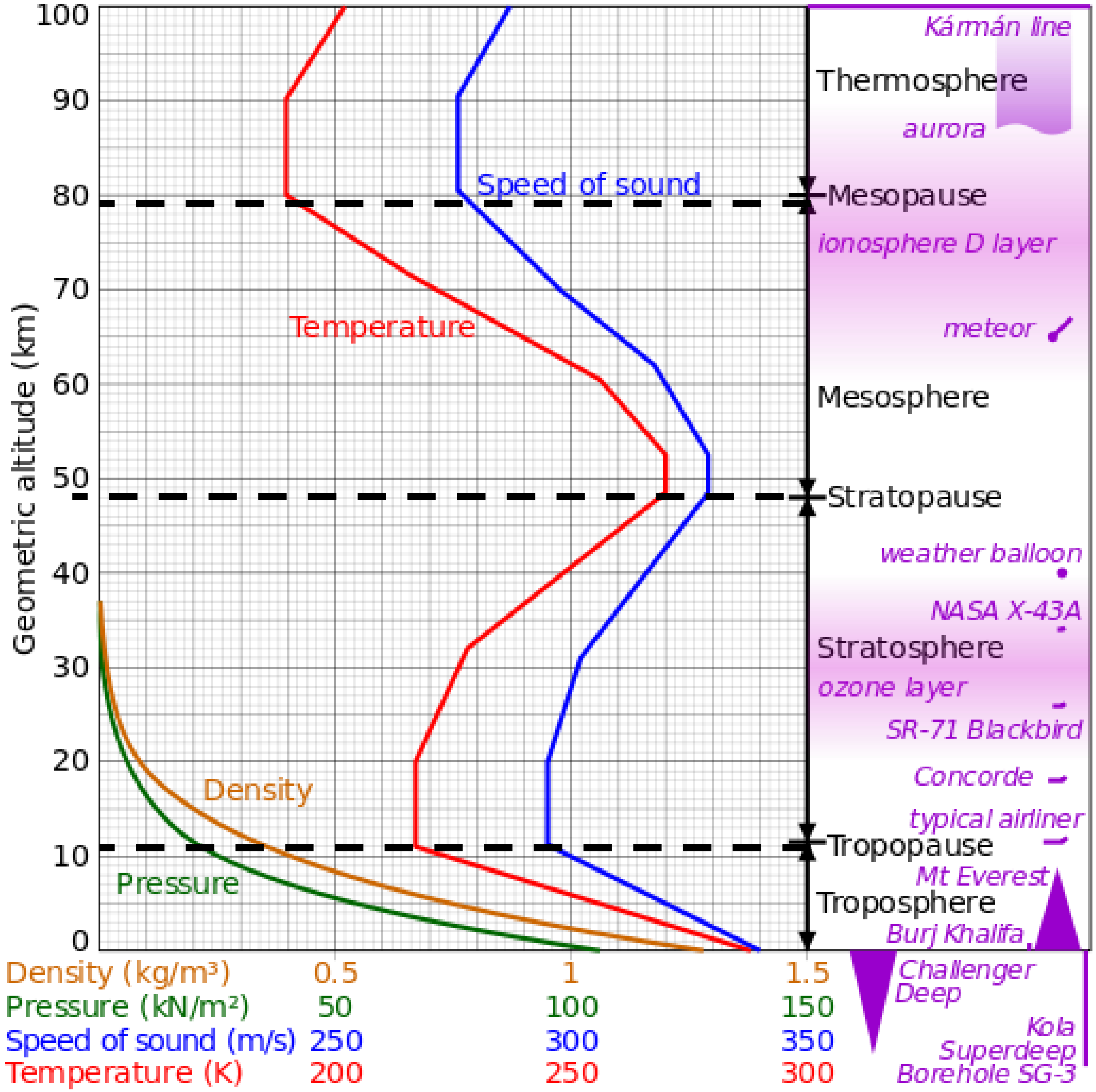
Outline for today

- The atmosphere
- Standard Atmosphere
- Relationship between temperature, pressure, density and other variables
- Examples

Aims for today

- Be able to define different regions of the atmosphere
- Appreciate why we use a Standard Atmosphere
- Calculate relations between temperature, pressure, density, and other variables
- Be able to define standard sea level values

The atmosphere



Zones based on temperature profile:

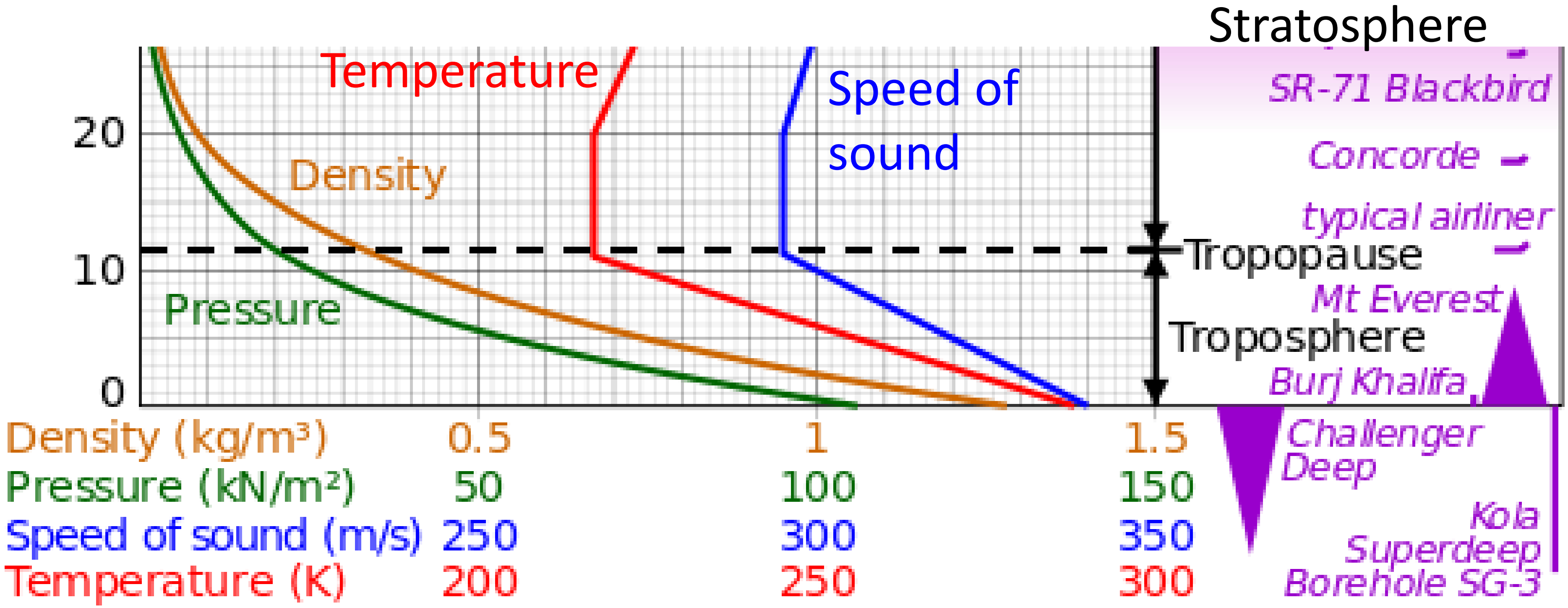
Thermosphere

Mesosphere

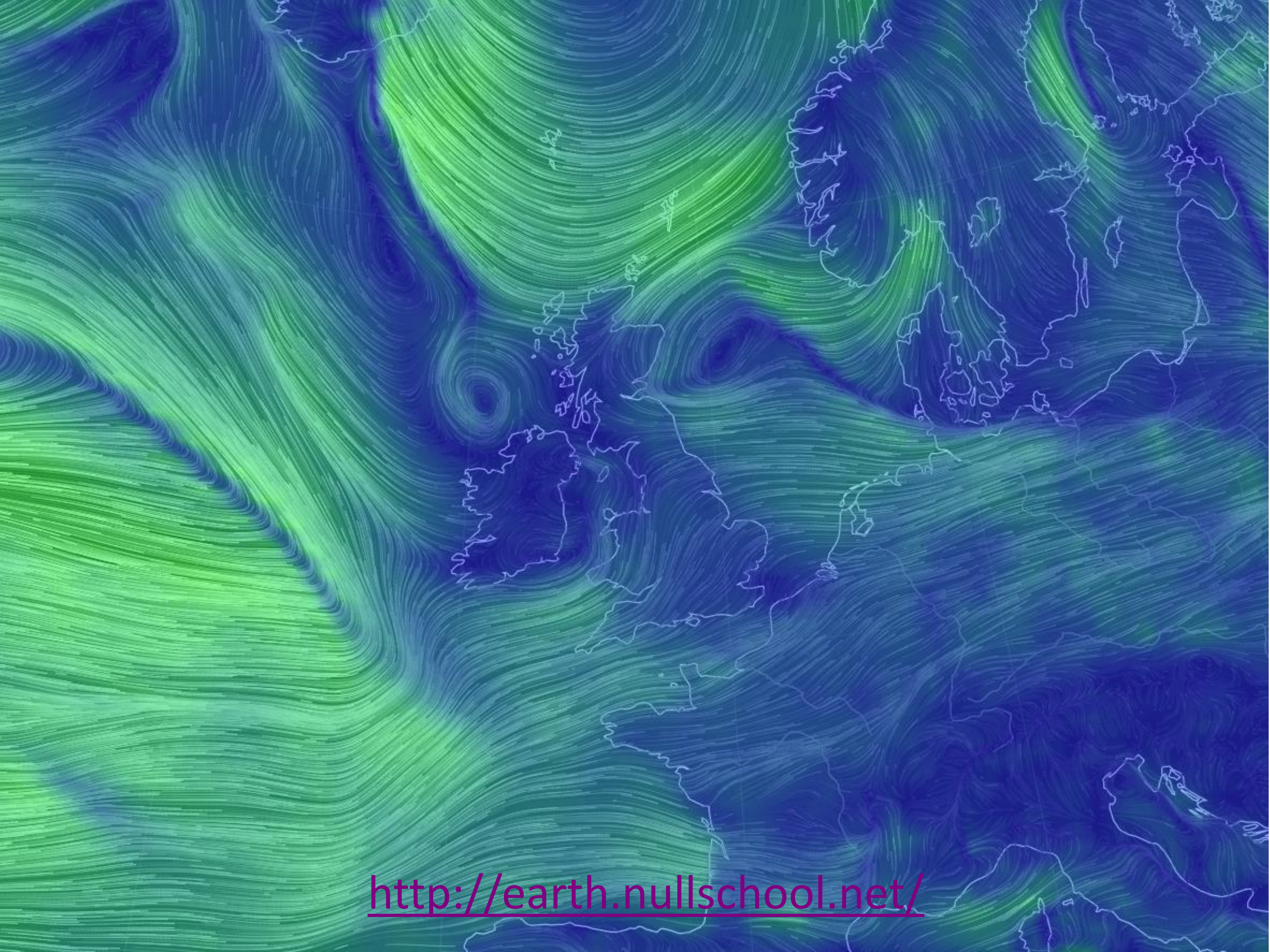
Stratosphere

Troposphere

The atmosphere



- Most aircraft equipped with air-breathing engines operate in the lowest two zones (and <20km)
- In **troposphere** (0-11 km) temperature decreases with altitude
- Lowest part of **stratosphere** (11-20 km) is isothermal
- **Tropopause** is separating boundary between zones



http://earth.nullschool.net/

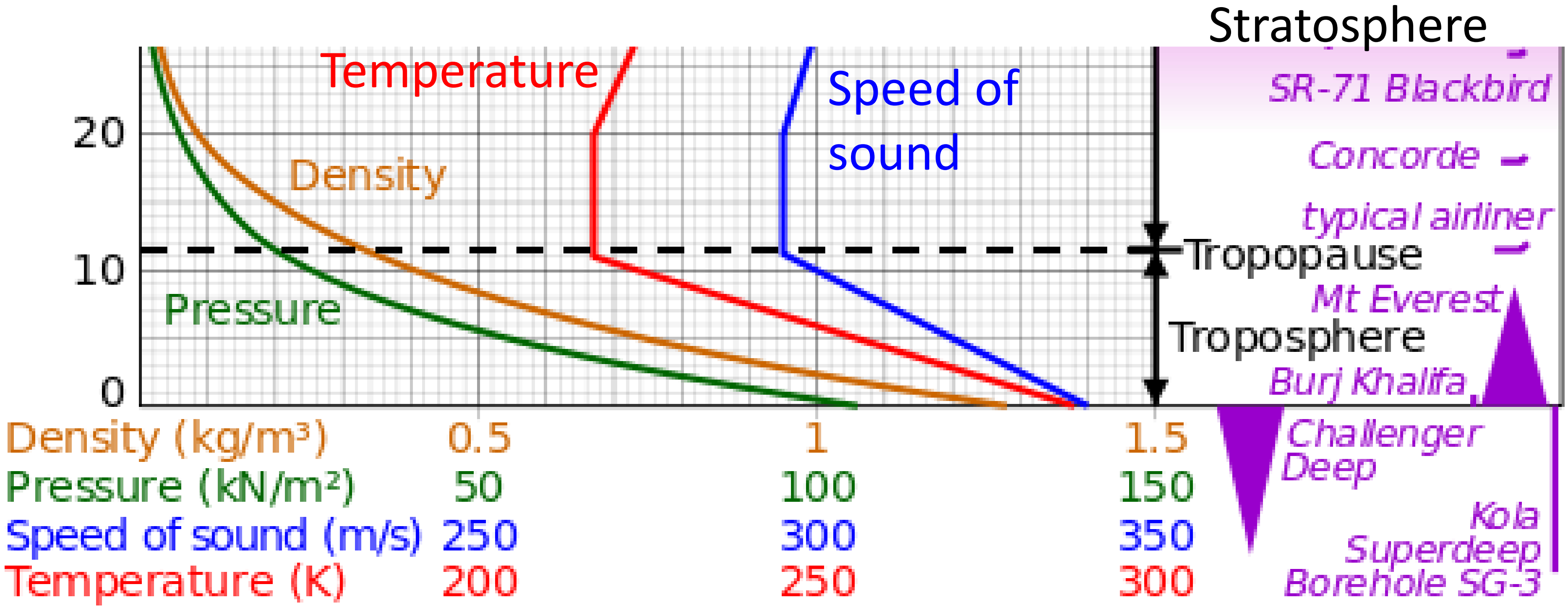
The International Standard Atmosphere

- A standard atmosphere used so all aircraft and engine performance data can be compared on the same basis.
- The International Standard Atmosphere (ISA) is a set of agreed values for how the physical properties of air vary with altitude.
- Established by international agreement through the International Civil Aviation Organization (ICAO)
- The ISA model represents average atmospheric conditions in North America and Europe and it is based on the assumptions that :
  - air is a perfect gas
  - air is dry
  - gravitational acceleration does not vary with altitude
  - hydrostatic equilibrium exists

ISA - sea level values

Variable	Symbol	Value
Pressure	p	101.325 kPa
Temperature	T	15 °C (288.15 K)
Density	ρ	1.2250 kg/m^3
Speed of sound	a	340.29 m/s
Dynamic viscosity	μ	1.7894 x 10^{-5} kg/(ms)
Gravitational acceleration	g	9.80665 m/s^2
Specific gas constant of air	R	287.05 (J/kg)/K

ISA atmosphere



- Troposphere defined as altitudes 0-11 km, having a linear temperature lapse rate of -6.5 ^{o}C/km
- Tropopause at 11 km, (T=-56 ^{o}C, 216.65 K)
- Lowest part of stratosphere (11-20 km) is isothermal

ISA values

ISA tables can give absolute values or the variables are often stated as ratios of sea level (SL) values:

- Absolute temperature ratio: $\theta = \frac{T}{T_{SL}}$
- Static pressure ratio: $\delta = \frac{p}{p_{SL}}$
- Density ratio: $\sigma = \frac{\rho}{\rho_{SL}}$

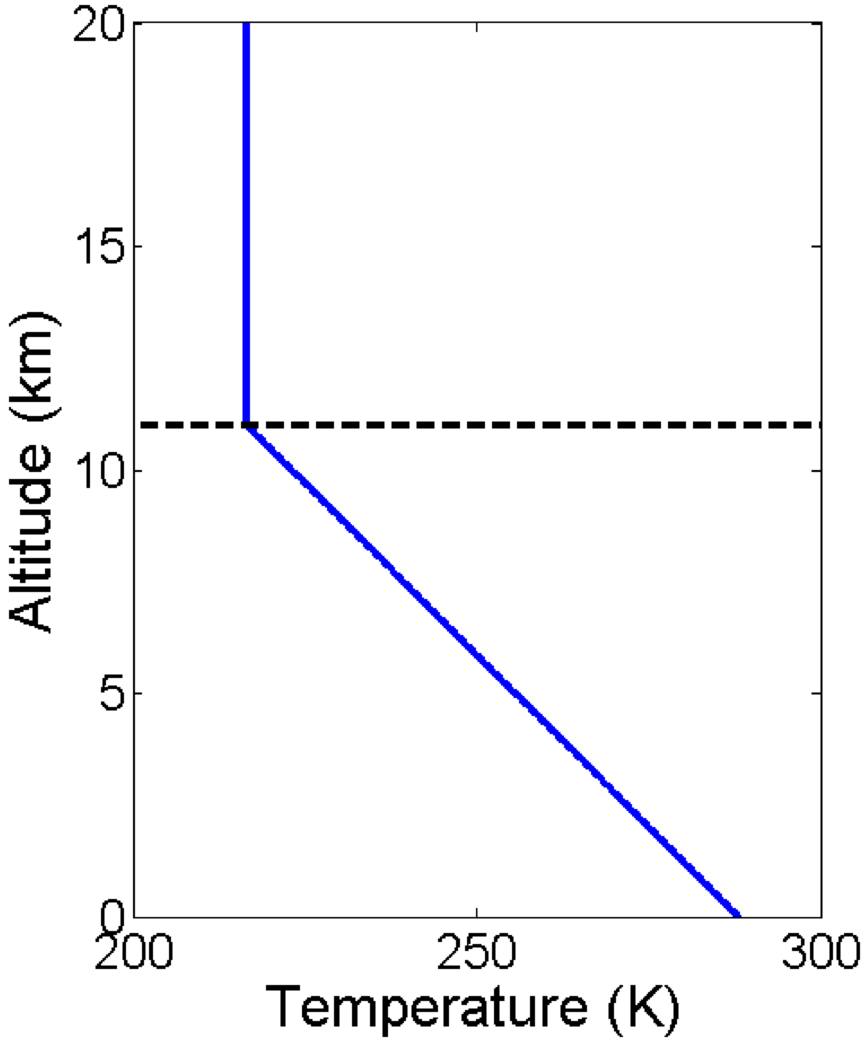
Using the equation of state:

$$\boxed{p = \rho R T}$$

At a given altitude :

$$\frac{p}{p_{SL}} = \frac{\rho}{\rho_{SL}} \frac{T}{T_{SL}} \qquad or \qquad \delta = \sigma\theta$$

ISA temperature



Above the tropopause (11 km):

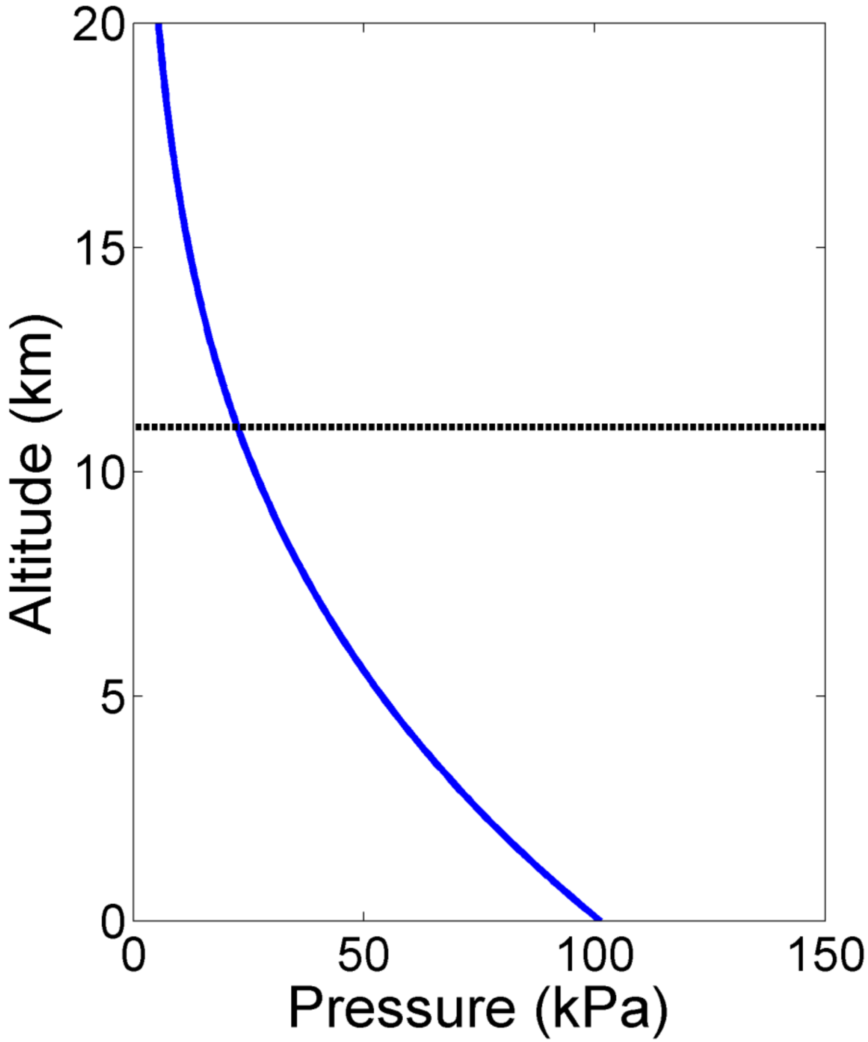
$$T = 216.65 \text{ K}$$

Below the tropopause:

$$T = 288.15 - 6.5 \times 10^{-3} h$$

where h = altitude in m

ISA pressure



Pressure is determined using the Equation of State and from integrating the Hydrostatic Equation:

$$\frac{dp}{dh} = -\rho g$$

Above the tropopause:

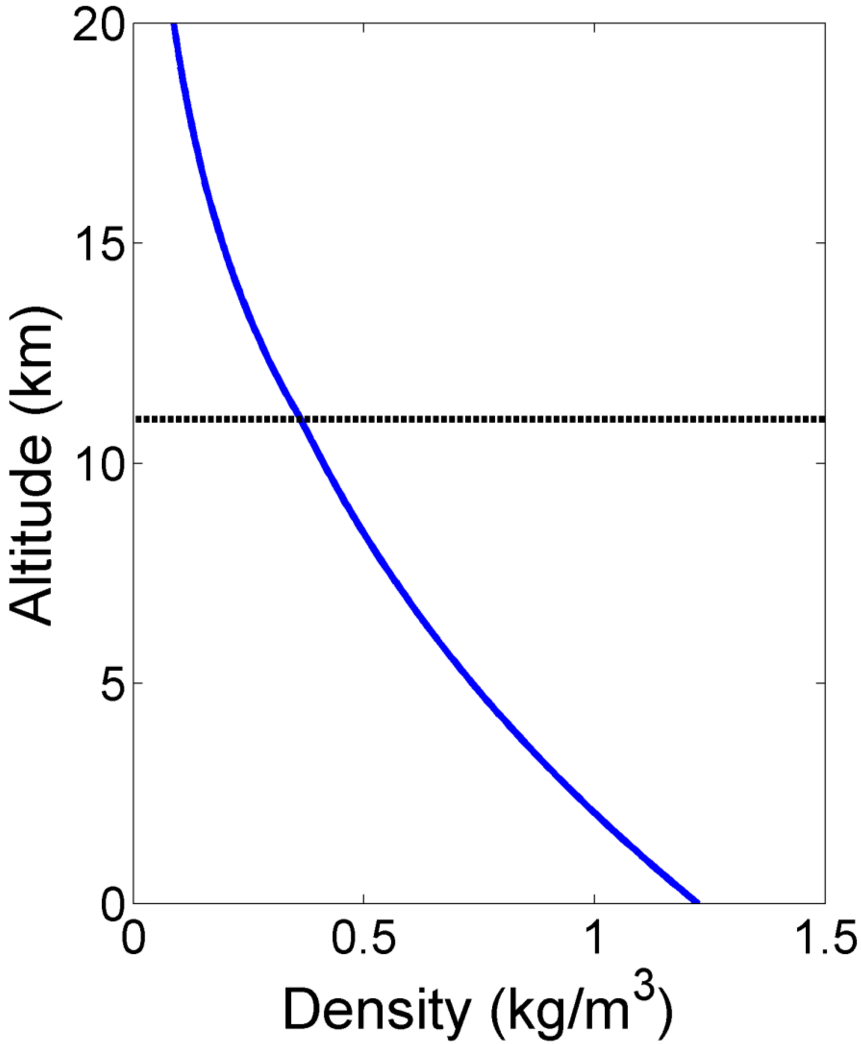
$$\delta = 0.22336e^{(1.7346 - 1.5769\times10^{-4}h)}$$

where e is the base of the natural logarithm

Below the tropopause:

$$\delta = (1 - 2.2558 \times 10^{-5}h)^{5.25588}$$

ISA density



With temperature and pressure known can use equation of state to determine density

$$\rho = \frac{p}{RT}$$

Approximation for density

$$\sigma = \frac{20 - H}{20 + H}$$

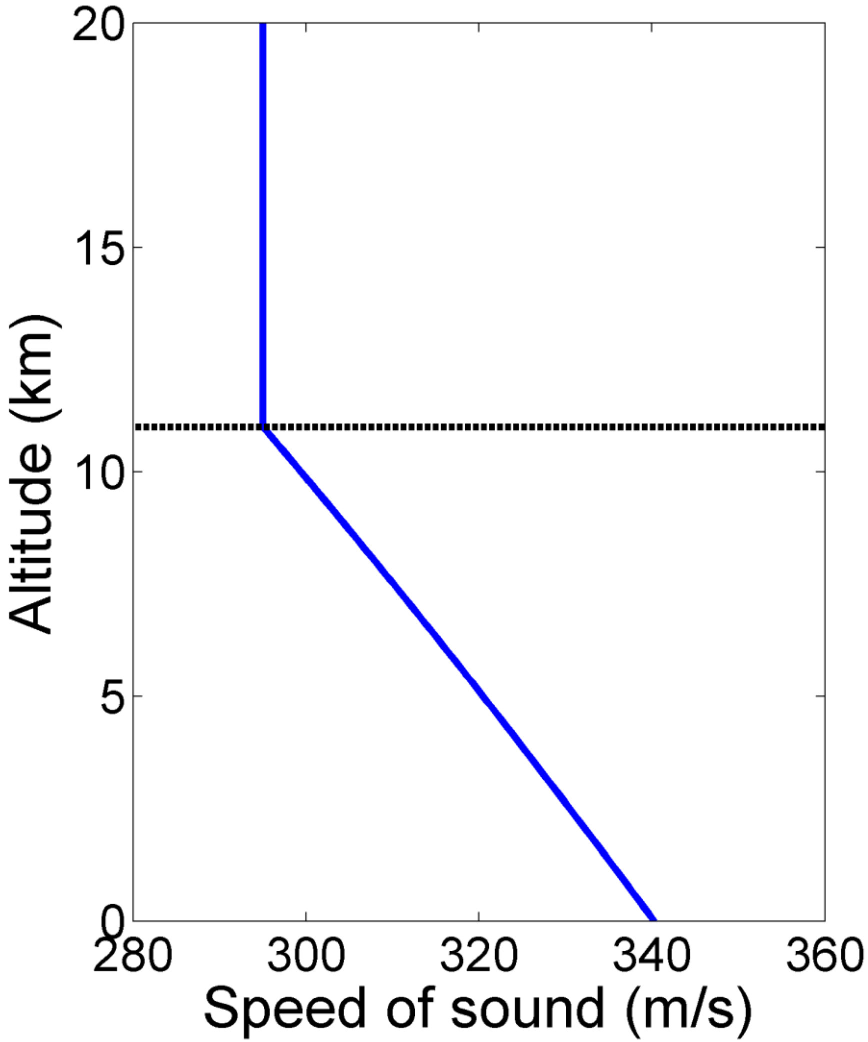
- Where H is the altitude in km

- E.g. At 5 km altitude,

$$\sigma = \frac{20-5}{20+5} = 0.6$$

$$\rho = 0.6\rho_{SL} = 0.6 \times 1.225 = 0.735$$

ISA speed of sound



Speed of sound (a) is related to temperature:

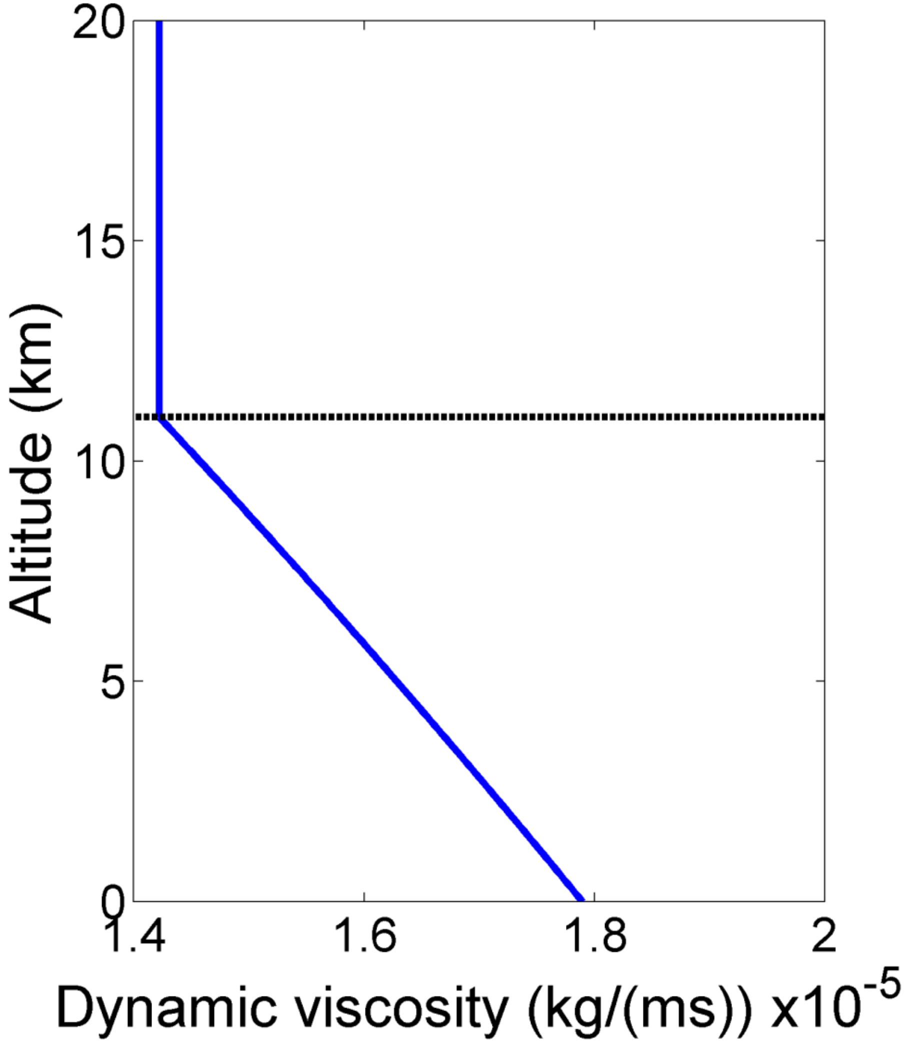
$$\boxed{a = \sqrt{\gamma R T}}$$

where γ is the Ratio of Specific Heats for air and is equal to 1.4

Can also calculate in terms of temperature ratio:

$$a = a_{SL}\theta^{0.5}$$

ISA viscosity



Dynamic viscosity (μ) can be estimated from temperature using Sutherland's Equation:

$$\frac{\mu}{\mu_{SL}} = \left(\frac{T}{T_{SL}}\right)^{3/2} \frac{T_{SL} + T_S}{T + T_S}$$

with the Sutherland constant, $T_S = 110$ K

Or alternatively:

$$\frac{\mu}{\mu_{SL}} = \frac{1.383}{\theta + 0.383}\,\theta^{3/2}$$

Example Question No. 1

Find the Mach number of a B777-300ER flying at 10,000 m with an air speed of 900 km/h

$$a = \sqrt{\gamma R T} = \sqrt{1.4 * 287.05 * T}$$

$$T = 288.15 - 6.5 \times 10^{-3} h$$

$m/s = kph$*0.278

Example Question No. 2

Find the air speed of Concorde flying at Mach 2.0 at an altitude of 18,300 m

$$a = \sqrt{\gamma R T} = \sqrt{1.4 * 287.05 * T}$$

$\mathrm{m}/s =$ kph*0.278 = mph*0.447

$$T = 288.15 - 6.5 \times 10^{-3} h$$

Bonus question: how much faster would it fly if burning highly-caffeinated Pepsi instead of A1 Jet Fuel?

Summary

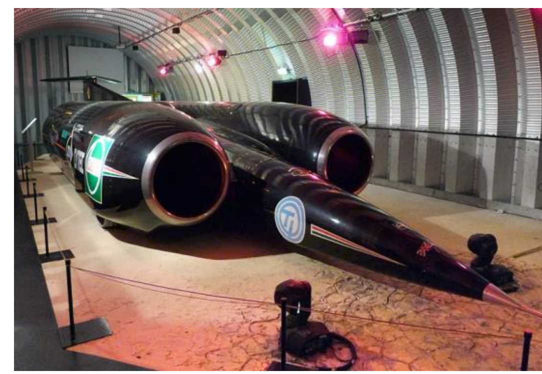
- Troposphere is the region above sea level where temperature decreases with altitude (0 to 11 km)
- Lowest part of stratosphere (11-20 km) is isothermal
- Tropopause is separating boundary between zones
- Standard atmosphere used so all aircraft performance data can be compared on the same basis
- ISA values given in tables and can be calculated from equations

Example Question No. 3

Calculate the Reynolds number of the following in sea level conditions

- Eagle at 10 m/s, Mean Aerodynamic Chord (MAC) = 0.10 m
- Cessna 172 at 100 km/h, MAC = 1.50 m
- ThrustSSC at M1.02, body length of 16.5 m

• A380 at 265 km/h, MAC 17.4 m, at 8,000 m altitude



Follow-up materials

To help with exam:

- Introduction to Flight – Chapter 3

For interest about Meteorology and the atmosphere:

- Meteorology Today – An introduction to weather, climate, and the environment. Donald Ahrens
  13 copies in Geographical Sciences Library