R

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DEPARTMENT OF AEROSPACE ENGINEERING

FLIGHT MECHANICS II

Manoeuvre Stability

1.0 Introduction

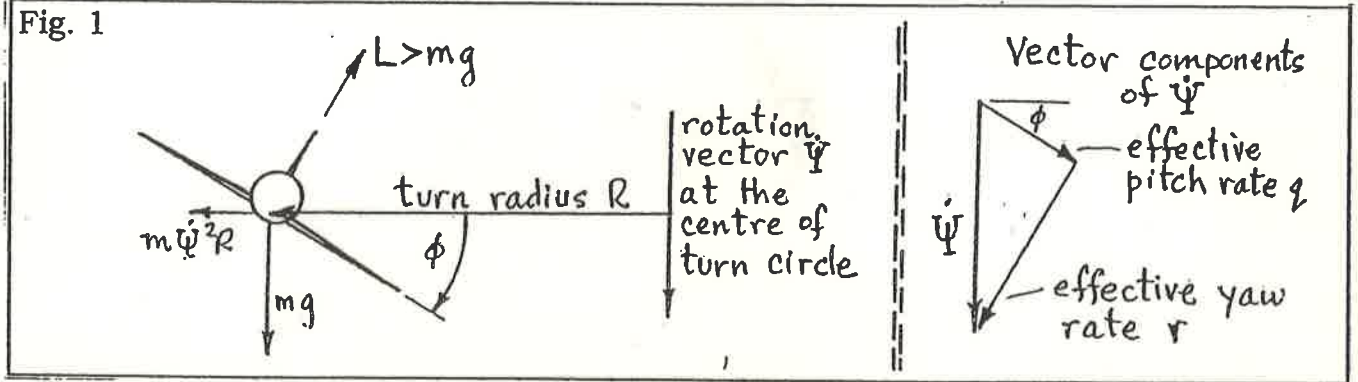
There are two very simple cases which can be used to illustrate a subtle distinction between the static stability of straight-and-level flight and that for "manoeuvring" flight. Whilst we use the simple cases to make the analysis relatively easy, the implication is that the stability for any flight which is "aerobatic" will not necessarily be the same as for the simpler steady cases that have already been considered. Perhaps we should find out if the manoeuvring increases or decreases apparent stability. Does the pilot feel more or less of what we call the "aerodynamic stiffness" in pitch?

The two cases mentioned are as follows, each implying that the aeroplane is subject to "g-loading" that is higher than normal ($n > 1$) and is also subject to a continuous pitch rate q for:

a. flight which continues around a tight, banked, horizontal circle,

or b. pulling out of a dive.

Fig. 1



We shall simplify our analysis by looking at the balance that exists at the bottom of a vertical curve (Fig.2), but we could apply a similar study to the balance which exists for flight in a horizontal circle (Fig.1). The geometry is easily explained for the vertical curve, coming out of a dive.

2.0 Flight in a Vertical Circle

It may be wise to point out that some texts use n to mean "extra-g" whereas n in these notes is "total-g", being normally 1.0 for level flight. The forward speed shown in Fig. 2 is

$$U = qR \tag{1}$$

and the centripetal acceleration is given by

$$accel. = (n-1)g = \frac{U^2}{R} = q^2 R = qU \tag{2}$$

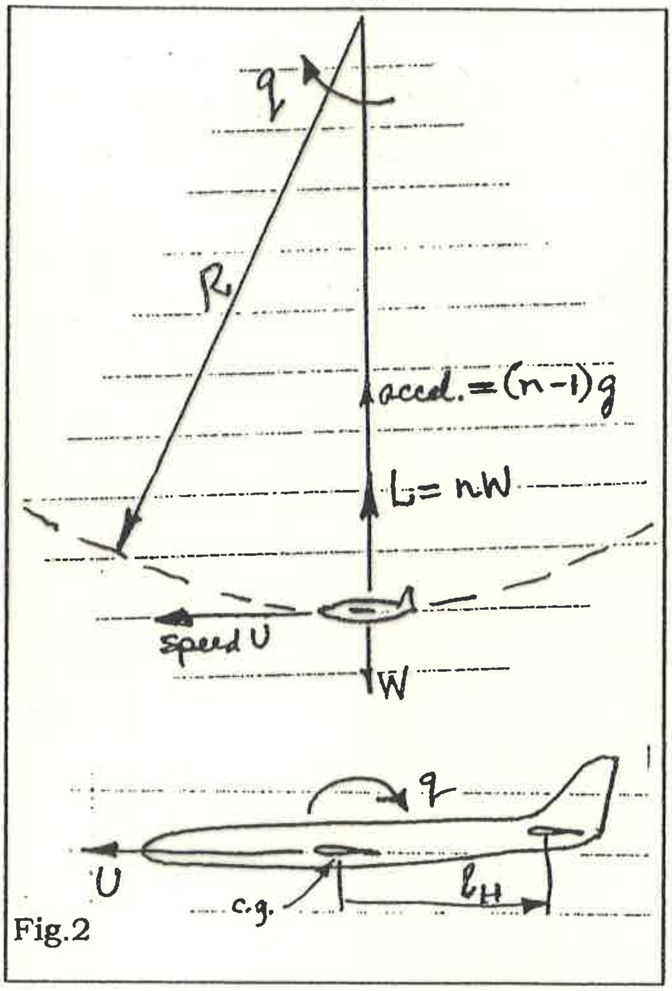


Fig.2

so in terms of the "g being pulled" we can express the consequent pitch rate as

$$q = \frac{(n-1)}{U} g \qquad (3)$$

and clearly for n = 1.0 we have q = 0, as expected.

Because of the continuous pitch rate, there is an additional incidence at the tailplane of

$$\delta \alpha_T = \frac{q\, l_H}{U} \qquad (4)$$

where l_H is the arm from the tailplane to the c.g., about which the aeroplane is pitching at rate q (see Fig.2). Thus the full tail incidence is, using (3) and (4),

$$\begin{aligned} \alpha_T &= i_T + \alpha(1-k) + \frac{q l_H}{U} \\ &= i_T + \alpha(1-k) + \frac{(n-1)}{U^2} g l_H \end{aligned} \qquad (5)$$

It is worth noting that because of the pitch rate q there is additional tail incidence but nearly no additional wing incidence for that reason because the arm akin to l_H is so small. Obviously, there *is* additional wing lift to produce the overall Lift = nW . A change in the *balance* of forces between wing and tail lift components, in order to produce the required lift (under conditions where *both* the wing and tail incidences must have changed), is the reason why the new aerodynamic centre for the whole vehicle (the neutral point) is different for flight in a vertical circle from that for straight-and-level flight.

3.0 In Pursuit of Stability!

We shall continue to use the same criterion for testing stability, i.e. we shall look for the same restoring aerodynamic stiffness by looking at

$$\frac{1}{a_1} \frac{\partial C_M}{\partial \alpha} = \frac{\partial C_M}{\partial C_L}, \qquad (6)$$

requiring this to be negative as before because $\partial C_M / \partial \alpha$ represents the restoring moment that stability implies.

Primarily, we must look at the pitching moment equation while allowing for the new incidence component at the tail and the greater overall lift on the aeroplane.

The lift requirement is no surprise and is given by

$$Lift = nW = \frac{1}{2}\rho U^2 S\left(C_{L_W} + \frac{S_T}{S}C_{L_T}\right) \tag{7}$$

and for later use note that

$$\frac{n}{U^2} = \frac{\rho}{2w}\left(C_{L_W} + \frac{S_T}{S}C_{L_T}\right) \tag{8}$$

where w = wing loading W/S . We shall initially employ the more accurate form of the trim equation, namely

$$C_M = C_{M_0} + x\left(C_{L_W} + \frac{S_T}{S}C_{L_T}\right) - \overline{V}C_{L_T}$$

$$or \qquad C_M = C_{M_0} + xC_{L_W} - \left(\overline{V} - x\frac{S_T}{S}\right)C_{L_T} \tag{9}$$

Now we can repace C_{L_T} by its full form, employing (5) and inserting (8) into it to get

$$C_{L_T} = a_{1_T}\left[i_T + \frac{C_{L_W}}{a_1}(1-k) - \frac{gl_H}{U^2} + \frac{\rho g l_H}{2w}\left(C_{L_W} + \frac{S_T}{S}\underset{\uparrow}{C_{L_T}}\right)\right] + a_{2_T}\,\eta \tag{10}$$

in which we ignore the tab term because the pilot isn't going to be attempting a steady trim while doing the pull-up!

The arrow below Eqn.10 helps us to recognise that we have a problem here; the expression for C_{L_T} depends upon itself. Clearly, we could gather the two terms together such that the left hand side appeared as $C_{L_T}(1-\underset{\uparrow}{.....})$ but this term is very small in practice (of order 0.005) and can be ignored.

As we might also want to calculate the appropriate elevator angle for a pull-up, we can choose to put the elevator term on the LHS while inserting the reduced form of (10) into (9). For obvious reasons, it is worth gathering the terms in C_{L_W} prior to the differentiation:

$$C_M + \left(\overline{V} - x\frac{S_T}{S}\right)a_{2_T}\eta = C_{M_0} - \left(\overline{V} - x\frac{S_T}{S}\right)\left[a_{1_T}\left(i_T - \frac{gl_H}{U^2}\right)\right]$$
$$- \left[\left(\overline{V} - x\frac{S_T}{S}\right)\left\{\frac{a_{1_T}}{a_1}(1-k) + \frac{a_{1_T}\rho g l_H}{2w}\right\} - x\right]C_{L_W} \tag{11}$$

The partial differentiation $\partial C_M / \partial C_{L_w}$ just leads to the last line of Eqn. 11 and it will be obvious that when compared with the earlier notes for stick-fixed static margin, there is an additional term now, being

$$\left(\overline{V} - x\frac{S_T}{S}\right)\frac{a_{1_T}\rho g l_H}{2w} \qquad (12)$$

and this represents the shift of the neutral point to the **manoeuvre point**. By reverting to the original less-accurate form, keeping only $\overline{V}$ from the parentheses we have

$$\frac{\partial C_M}{\partial C_{L_W}} = -\left[\underbrace{\overline{V}\frac{a_{1_T}}{a_1}(1-k) - x}_{\uparrow} \quad + \underbrace{\overline{V}\rho g\frac{l_H a_{1_T}}{2w}}_{\Uparrow}\right] = -H_M \qquad (13)$$

and within the brackets the left hand part (↑) is the original static margin whereas the right hand part (⇑) is the additional margin available in a manoeuvre. The **manoeuvre margin is slightly greater than the normal static margin**, typically 5-10% more of the MAC.

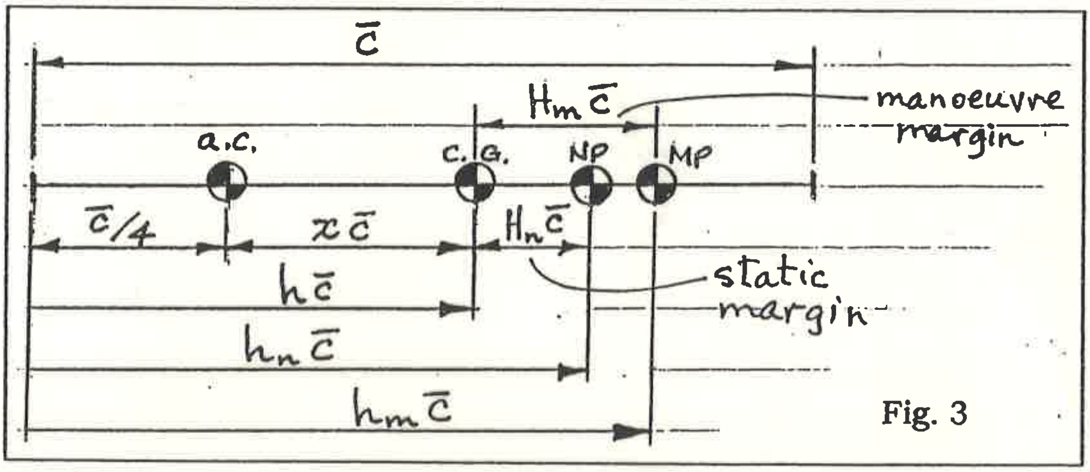


Fig. 3

You will find in the Design Project next year that Eqn.13 is virtually the same as the expression quoted there for studying static stability, but by way of an "elevator angle per g". It is really *additional* elevator angle per *additional* g , beyond the requirements for steady level flight. In fact, this leads to an alternative formulation which offers a different logical justification.

4.0 The Manoeuvre Margin and Elevator-angle-per-g

Another way of approaching the analysis is to try to describe the *changes* from steady level flight to the instantaneous level flight at the bottom of the circular curve as in Fig. 2. In principle, the pitching moment balance given by Eqn. 9 is still valid because q is constant and there is no pitching *acceleration*. As a consequence, if we can determine the *changes* in pitching moments we should be able to set these equal to zero also. The first of these can be gained from Eqn. 9 by inserting Eqn. 10, after deletion of the two terms involving **g** . Then the second can be put into the following form, after a necessary precaution:

We must allow for the fact that the two flight conditions (steady or pitching) must be described with symbols that are not confused when we later do a subtraction. Thus in the following a further subscript n is added to all factors which can change from level flight to the pull-up.

The pitching moment balance for the pull-up then becomes described by:

$$C_M = C_{M_0} + x C_{L_{W_n}} - \left(\bar{V} - x\frac{S_T}{S}\right)\left[a_{1_T}\left(i_T + \alpha_n[1-k_n] + \frac{(n-1)}{U^2} g l_H\right) + a_{2_T}\eta_n\right] \tag{14}$$

The factors identified for change in the pull-up are C_{L_W}, α, k and η, all being reasonable choices even if we cannot quickly find all their new values. A subtraction of the two necessary moment equations then leads to

$$0 = x\left(C_{L_{W_n}} - C_{L_W}\right) - \left(\bar{V} - x\frac{S_T}{S}\right)\begin{bmatrix} a_{1_T}\left(\alpha_n - \alpha - [\alpha_n k_n - \alpha k] + \frac{(n-1)}{U^2} g l_H\right) \\ +a_{2_T}(\eta_n - \eta) \end{bmatrix} \tag{15}$$

and some simplifications are due! If we make the assumptions

a. $x\frac{S_T}{S} \ll \bar{V}$

b. $k_n = k$

then define $\delta C_{L_W} = \left(C_{L_{W_n}} - C_{L_W}\right)$ *and* $\delta\alpha_W = (\alpha_n - \alpha) = \frac{\delta C_{L_W}}{a_1}$ *and* $\delta\eta = \eta_n - \eta$

we can reduce Eqn. 15 to

$$0 = x\,\delta C_{L_W} - \bar{V}\left[a_{1_T}\left(\frac{\delta C_{L_W}}{a_1}(1-k) + \frac{(n-1)}{U^2} g\, l_H\right) + a_{2_T}\,\delta\eta\right]. \tag{16}$$

Two further simplifications will help, namely

c. the recognition that the original static margin was given by

$$H_n = \left[\bar{V}\frac{a_{1_T}}{a_1}(1-k) - x\right] \tag{17}$$

and

d. that $\delta C_{L_W} \approx (n-1) C_{L_W}$.

Then the *additional* elevator angle can be expressed by

$$\delta\eta = -\frac{a_{1_T}}{a_{2_T}}\frac{(n-1)}{U^2} g\, l_H - \frac{1}{\bar{V}a_{2_T}}[H_n](n-1)C_{L_W} \tag{18}$$

and C_{L_W} here is the lift coefficient for straight-and-level one-g flight. From (18) we can deduce that to "pull g" there is a need for a more negative elevator angle. It follows from Eqn. 18 that the desired "elevator-angle-per-g" can be expressed by

$$\frac{\delta\eta}{(n-1)} = -\frac{1}{\bar{V}\,a_{2_T}} H_n C_{L_W} - \frac{a_{1_T}}{a_{2_T}}\frac{g l_H}{U^2}. \tag{19}$$

Now it is worth considering why this elevator-based analysis is useful in studying stability. The argument used a number of times in earlier notes was that a stable aeroplane has positive aerodynamic stiffness offering a restoring moment. As the centre of gravity moves toward the neutral point this aerodynamic stiffness drops, and becomes zero when the c.g. is *at* the N.P. The pilot will feel this because any attempt to pull the nose up will require resistance against the aerodynamic stiffness, and the stronger that stiffness the more the elevator will have to be deflected to counteract it. Or, the more stable the aeroplane the greater will be the elevator angle needed to produce a nose-up vertical turn. When "elevator-angle-per-g" drops to zero, there is a complete loss of the aerodynamic stiffness and the **manoeuvre point** has been reached, i.e. the c.g. has gone back to the point where static stability in the vertical circle is lost.

If we set Eqn. 19 equal to zero and solve for H_n we can show that the result is

$$H_n = -\frac{\bar{V}\, a_{1_T}\, l_H\, \rho\, g}{2\, w} \qquad (20)$$

which indicates that according to the original definition of static margin H_n, a *negative* value is necessary to reach the new **manoeuvre point** M.P., i.e. this point is slightly farther back. This extra distance looks remarkably similar to the extra term (⇑) in Eqn. 13 ! Typical values in (20) would show this to be 5-10% $\bar{c}$ as quoted above.

5.0 Stick Forces and Manoeuvres

To the pilot, stick force is a better indicator than elevator angle and indeed the "stick-force-per-g" has often been a feature of flight testing to define performance in terms that a pilot can easily imagine. Since the elevator angle

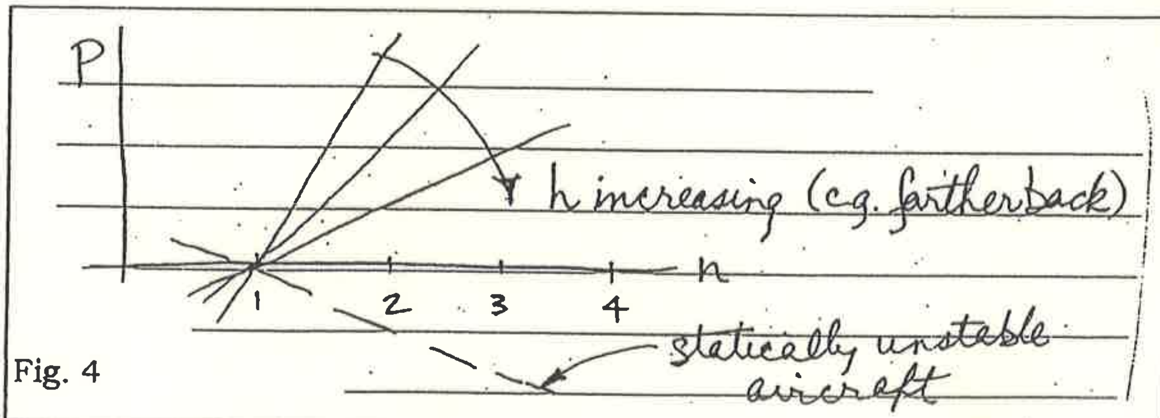


Fig. 4

and the consequent stick force (hinge moment) are closely related it should be no surprise that logical arguments akin to those at the top of this page can be used to verify the following variations, e.g. that positive P is required for increases in n beyond 1.0, for a stable aeroplane.

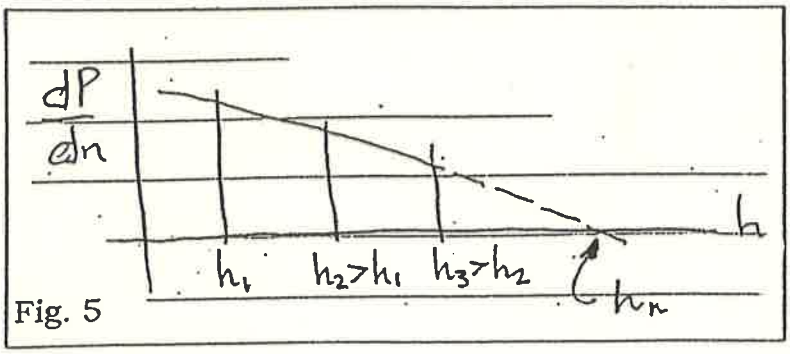


Fig. 5

Fig. 4&5 should also make it clear that a flight test which measured the *slope* for a set of c.g. positions, could allow an extrapolation to find the neutral point, i.e. the value of h for which there would be *no* slope, though clearly *that* flight test would never be done!

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Learning Objectives

(Manoeuvre Stability)

Related to this set of notes, you should be aware that the following are primary objectives which should be borne in mind, though there will be other lesser objectives too, depending on your prior knowledge of specific nomenclature, for example.

1. When we declared a simple model for our analysis of Longitudinal Balance and then determined static stability from the equations that followed, we did not allow for anything but steady level flight. That is too simple for real flight conditions; there will always be a small amount of "manoeuvring" in the vertical plane; i.e. curved flight with transverse acceleration due to the curved flight path, whether for the flight profile from take-off to landing or for some turning. We must examine the possibility that this flight condition could alter the static stability margin and thus, perhaps, lead to less stable flight (although obviously perhaps *more* stable flight too). You must be able to show that you understand this change of flight condition and state what factors may influence the stability.

2. The analysis brings in only one new factor, a change of incidence at the tail, and you must be able to reproduce this change in the appropriate equations. Whilst you will not be required to reproduce the full analysis showing the changes (Eqn. 10-13), you should know how to evaluate the position of the new neutral point, i.e. the change from the original value (Eqn. 12&13).

3. The issue of pilot stick forces is raised and related to the flight testing that might be used to determine real margins. You should be able to describe such tests and show how the process of establishing a margin might be arranged (Sec.5).