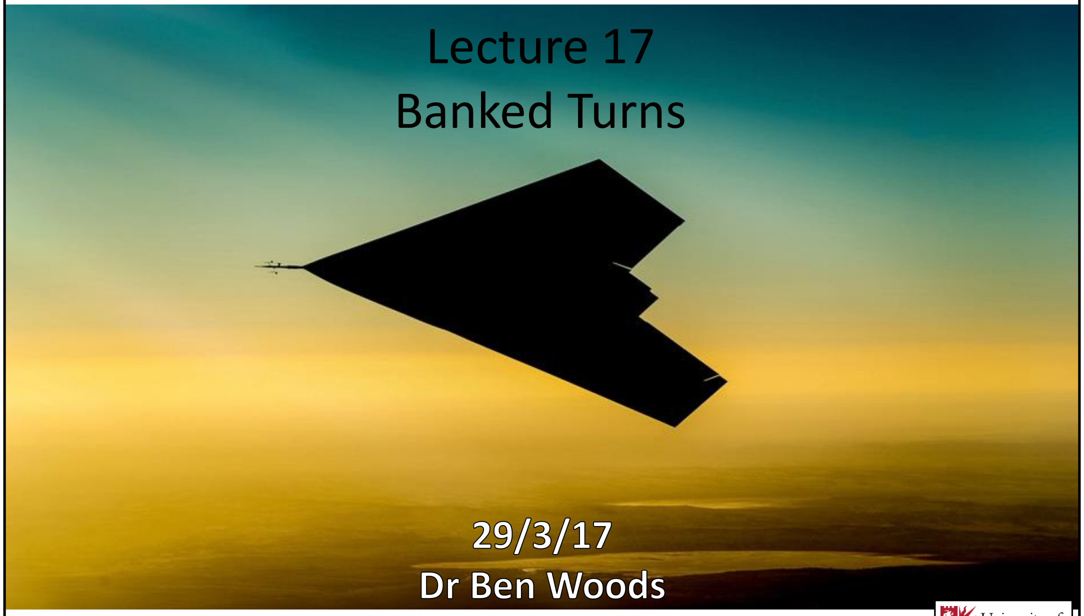
# Aeronautics & Mechanics AENG11301




**Department of Aerospace Engineering**
**University of Bristol**

BAE Taranis

The BAE Systems Taranis is a British demonstrator programme for Unmanned Combat Air Vehicle (UCAV)
A semi-autonomous unmanned warplane, it is designed to fly intercontinental missions, and will carry a variety of weapons, enabling it to attack both aerial and ground targets.
It will utilise stealth technology, giving it a low radar profile,
It will be controllable via satellite link from anywhere on Earth

http://en.wikipedia.org/wiki/BAE_Systems_Taranis

On 5 February 2014, BAE revealed information on Taranis flight tests. The first flight occurred on 10 August 2013 at Woomera Test Range in South Australia. The Taranis is planned to be operational "post 2030" and used in concert with manned aircraft.

http://www.youtube.com/watch?v=uXS1iGx03eg

# Outline for today

- Description of a steady banked turn
- Turn radius
- Turn rate
- Limits on turn performance
  - Stall limited turn
  - Load factor limited turn
  - Thrust limited turn

# Aims for today

- Be able to calculate forces acting in a steady banked turn
- Be able to calculate turn radius
- Be able to calculate turn rate
- Be able to calculate turn radius and rate for:
  - Stall limited turn
  - Load factor limited turn
- Be able to calculate increase in thrust and power required in a steady banked turn

Description of a steady banked turn

Turn radius

Turn rate

Limits on turn performance

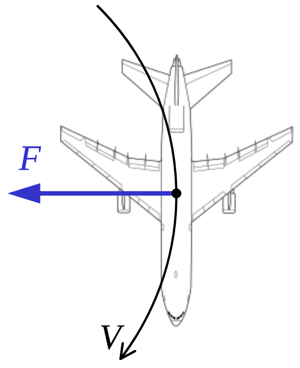
Stall limited turn

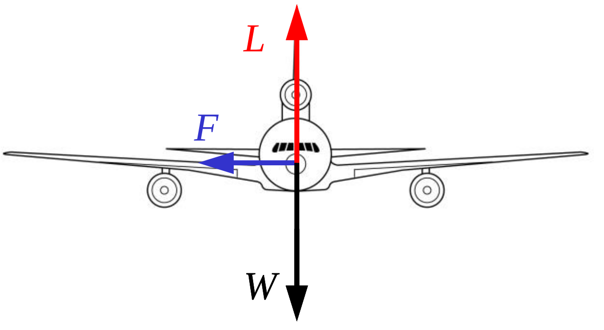
Load factor limited turn

Thrust limited turn

# Turning Flight (1)

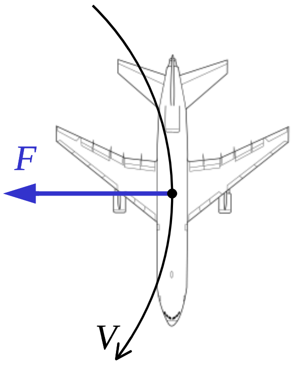
- resultant (side) force needed towards centre of turn
- could use the rudder to provide **sideforce** – with wings held horizontal in a **flat turn**
  - very inefficient – sideforce is small (so turn radius is large), and drag increase due to fuselage **sideslip** is significant

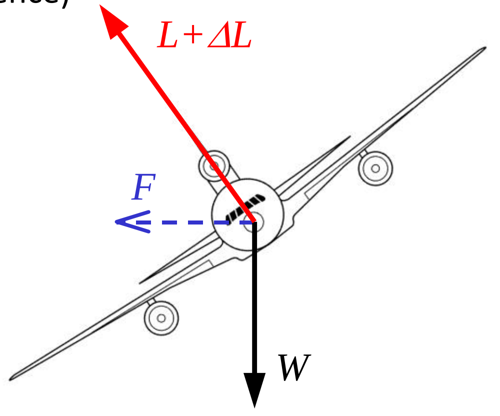


# Turning Flight (2)

- resultant (side) force needed towards centre of turn
- **bank** or **roll** the aircraft so that part of the lift acts in the horizontal plane giving a banked turn
  - large force available so tight turns are possible
  - aerodynamic loads remain symmetrical (left to right in aircraft frame of reference)



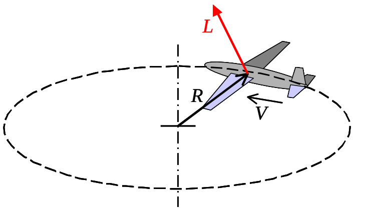



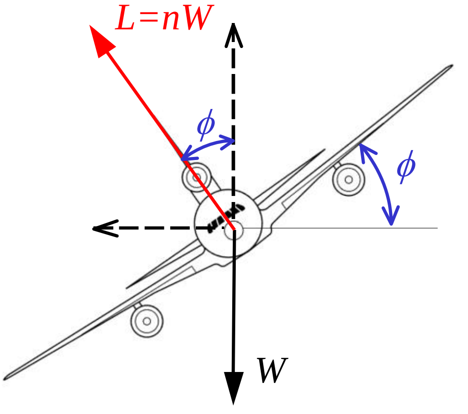
# Load Factor in a Steady Banked Turn

- aircraft is turning with radius $R$ at steady speed $V$
  - thrust and drag are balanced
  - correct **bank angle $\phi$** is set so there is no sideslip
  - note that $V$ is *True Air Speed*
- resolve forces vertically
  - note *no vertical acceleration*

$$L\cos\phi = W \qquad L = nW \quad \Rightarrow \quad \boxed{\cos\phi = \frac{1}{n}}$$

  - high load factors require large bank angle
  - e.g. $n$ =4 $\rightarrow$ $\phi$=75.5°



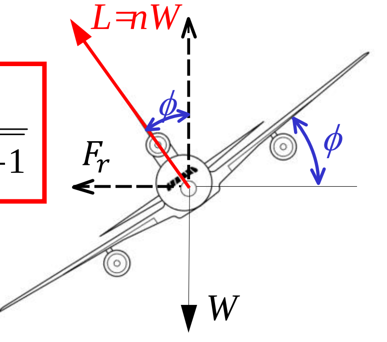
# Turn Radius

- in the horizontal plane the lift component provides the **horizontal resultant force**, $F_r$ to maintain the curved flight path

$$F_r = \sqrt{L^2 - W^2} = W\sqrt{n^2 - 1}$$

$$F_r = m\frac{V^2}{R} = \frac{W}{g}\frac{V^2}{R}$$

$$\boxed{R = \frac{V^2}{g\sqrt{n^2-1}}}$$




- radius $R$ decreases as $n$ (and $\phi$) increase
- radius $R$ increases with $V^2$
  - **minimum turn radius** $R_{min}$ occurs at max load factor $n_{max}$
  - for a given load factor $R_{min}$ occurs at $V_{min}$

# Turn Rate

- rate of change of **heading**

$$\boxed{\omega_t = \frac{V}{R}} \quad \text{and} \quad R = \frac{V^2}{g\sqrt{n^2-1}} \quad \Rightarrow \quad \boxed{\omega_t = \frac{g\sqrt{n^2-1}}{V}}$$

- **turn rate** $\omega_t$ increases as $n$ (and $\phi$) increase
- turn rate $\omega_t$ decreases with $V$
  - **maximum turn rate** $\omega_{max}$ occurs at max load factor $n_{max}$
  - for a given load factor $\omega_{max}$ occurs at $V_{min}$

- time to turn through a given angle also important
  - *e.g.* time to reverse heading

$$\boxed{t_{180} = \frac{\pi}{\omega_t} = \frac{\pi R}{V} = \frac{\pi V}{g\sqrt{n^2-1}}}$$

# Limits on Turn Performance

- sustained (steady) turn performance is a function of $V$ and $n$
- limited by:
  1. stall onset
     - $V_{stall}$ at $C_{Lmax}$
  2. maximum load factor
     - $n_1$
  3. power or thrust available
     - $T_{available} > T_{required}$ & $P_{available} > P_{required}$ for **sustained turn**
     - note higher **instantaneous turn rate** achievable

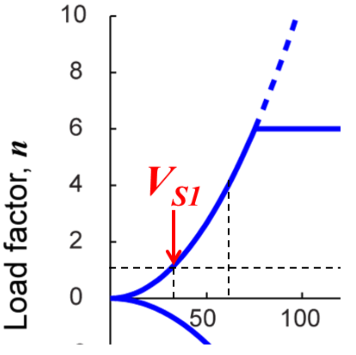
# Stall Limited Turn

- on the stall boundary of the manoeuvre flight envelope $n$ is limited by $C_{Lmax}$
- in straight & level flight

$$W = \tfrac{1}{2}\rho V_{S1}^2 S C_{L\max}$$

- at load factor $n$

$$nW = \tfrac{1}{2}\rho V^2 S C_{L\max} \quad \Rightarrow \quad \boxed{n = \left(\frac{V}{V_{S1}}\right)^2}$$




- substituting in the turn performance equation

$$R = \frac{V^2}{g\sqrt{n^2 - 1}} \quad \Rightarrow \quad R = \frac{V^2}{g\sqrt{\left(\frac{V}{V_{S1}}\right)^4 - 1}}$$

# Performance in a Stall Limited Turn

- rearranging and multiplying through by $(V_{S1}/V)^2$

$$R = \frac{V^2}{g\sqrt{\left(\frac{V}{V_{S1}}\right)^4 - 1}} \Rightarrow \boxed{R = \frac{V_{S1}^2}{g}\left(1 - \left(\frac{V_{S1}}{V}\right)^4\right)^{-\frac{1}{2}}}$$

$$\boxed{t_{180} = \frac{\pi R}{V} = \pi \frac{V_{S1}}{g}\left(\frac{V_{S1}}{V}\right)\left(1 - \left(\frac{V_{S1}}{V}\right)^4\right)^{-\frac{1}{2}}}$$

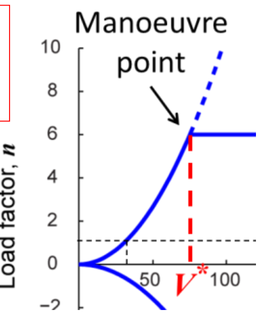
- relatively little effect of $V$ on turn radius $R$
  - ratio $(V_{S1}/V)^4$ becomes very small
  - except near $n = 1$ (*i.e.* near $V = V_{S1}$)
- time to turn $t_{180}$ reduces significantly as speed increases

# Load Factor Limited Turn

- the maximum permissible load factor is $n_{max}$
- the turn performance equations are then

$$R = \frac{V^2}{g\sqrt{n_{\max}{}^2 - 1}}$$

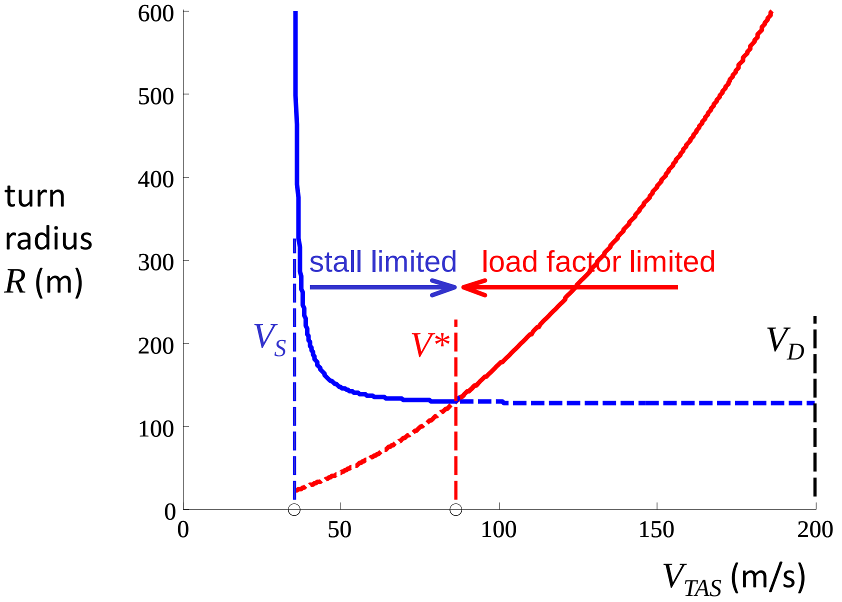
$$t_{180} = \frac{\pi R}{V} = \frac{\pi V}{g\sqrt{n_{\max}^2 - 1}}$$



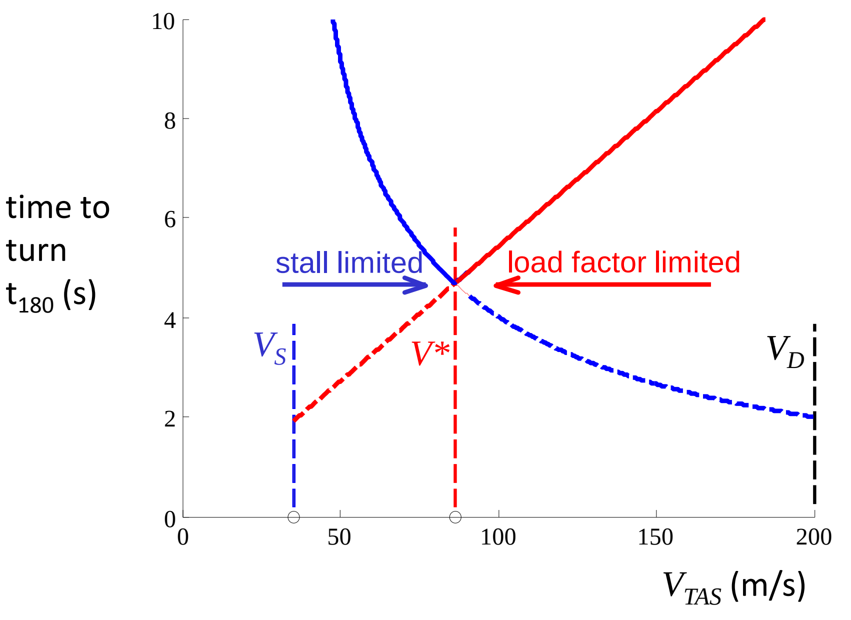

- radius $R$ increases (rapidly) with $V^2$
- time to turn $t_{180}$ increases with $V$
- best turn performance
  - at the **manoeuvre point** on the manoeuvre flight envelope
  - Which occurs at the **corner speed**, $V^*$
- maximum bank angle is constant

$$\cos\phi_{max} = \frac{1}{n_{\max}}$$

# Example of Turning Flight - Radius



# Example of Turning Flight – Time to Turn



# Thrust Limited Turn

- increase in load factor leads to increased induced drag
- at load factor $n$, drag increment $\Delta D$ is

$$D = \tfrac{1}{2}\rho V^2 S C_{D0} + \frac{K(nW)^2}{\tfrac{1}{2}\rho V^2 S}$$

independent of $n$

scales with $n^2$

$$\boxed{\Delta D = \frac{KW^2}{\tfrac{1}{2}\rho V^2 S}\left(n^2 - 1\right) = \Delta T}$$

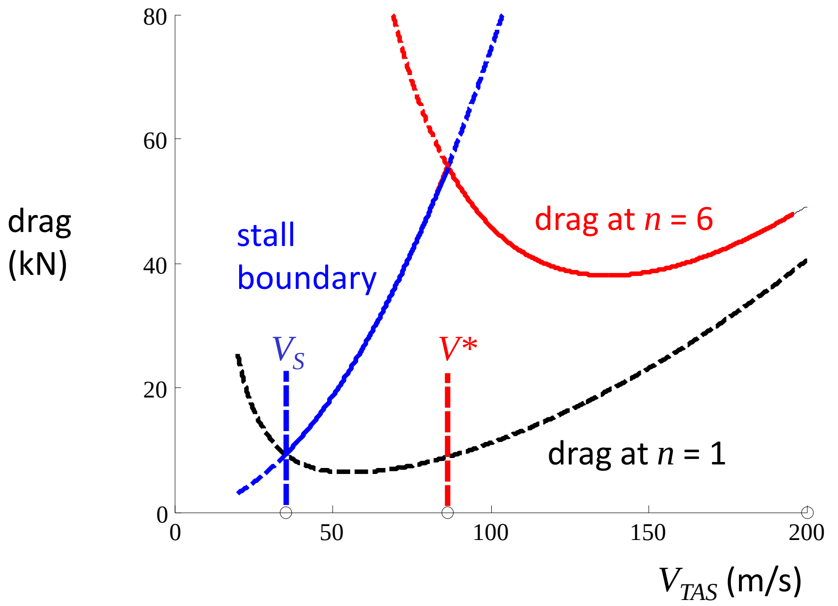
Change in drag will be drag for current $n$ minus drag during steady level flight ($n$ = 1)

- substituting for $n^2$–1 from the turn equation: $R = \dfrac{V^2}{g\sqrt{n^2-1}} \rightarrow n^2 - 1 = \dfrac{V^4}{R^2 g^2}$

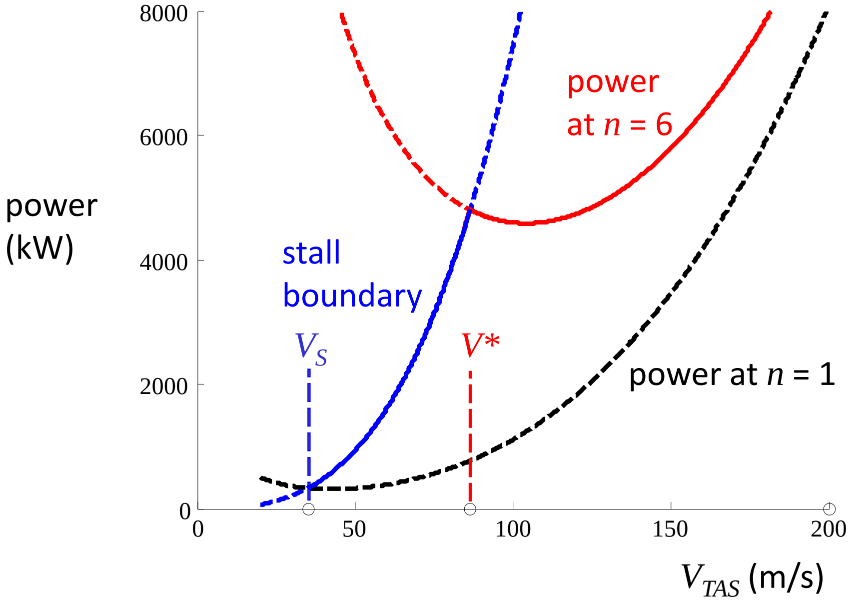
$$\Delta D = \frac{KW^2}{\tfrac{1}{2}\rho S g^2}\frac{V^2}{R^2} \Rightarrow \boxed{\Delta T = \frac{KW^2}{\tfrac{1}{2}\rho S g^2}\frac{V^2}{R^2}} \quad \boxed{\Delta P = \frac{KW^2}{\tfrac{1}{2}\rho S g^2}\frac{V^3}{R^2}}$$

- thrust $\Delta D$ or power $\Delta P$ required rises rapidly with increasing speed $V$ and/or reducing turn radius $R$

# Example of Turning Flight - Thrust



# Example of Turning Flight - Power



# Example

1. For the given aircraft in a steady banked turn at 150 m/s with a load factor of 3 calculate:
   a) horizontal resultant force
   b) bank angle
   c) turn radius
   d) time to turn 90 deg
2. Calculate the aircraft's minimum turn radius and maximum turn rate at sea level
3. Calculate the increase in thrust above straight and level flight conditions required to sustain a steady banked turn at the manoeuvre point

Aircraft parameters

- mass = 2,300 kg
- wing area, $S$ = 19.3 m$^2$
- $C_{Lmax}$ = 2.0
- Max positive load factor, $n$ = 6
- $C_D = 0.02 + 0.06 C_L{}^2$

Hint: $V^* = \sqrt{\dfrac{2 n_{\max} W}{\rho S C_{L\max}}}$

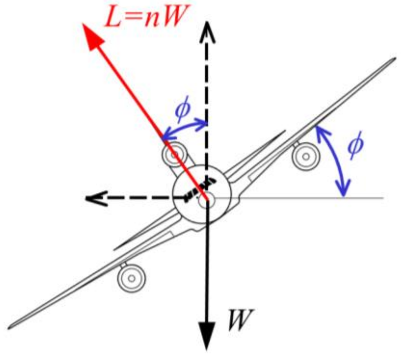
# Answers

1a) 64 kN
b) 70.5 deg
c) 812 m
d) 8.5 s

2) $R_{min}$ = 98.7 m
$\omega_{max}$ = 0.767 rad/s

3) 15.7 kN

## Summary




$$\cos\phi = \frac{1}{n}$$

$$R = \frac{V^2}{g\sqrt{n^2-1}}$$

$$\omega_t = \frac{g\sqrt{n^2-1}}{V}$$

- Turn performance limited by:
  1. stall onset
  2. maximum load factor
  3. power or thrust available

Be able to calculate forces acting in a steady banked turn

Be able to calculate turn radius

Be able to calculate turn rate

Be able to calculate turn radius and rate for:

Stall limited turn

Load factor limited turn

Be able to calculate increase in thrust and power required in a steady banked turn

# Follow-up materials

To help with exam:

- Introduction to Flight – 6.17

To help with exam:
Introduction to Flight – 5.1-5.2

To aid in understanding:
Understanding flight – Chapter 1

For interest:
Introduction to Flight – 5.19 (explanation of lift)