UNIVERSITY OF BRISTOL
FACULTY OF ENGINEERING

First Year Examination for the Degree of Master of Engineering

MAY/JUNE 2015 2 Hours

AENG11100

FLUIDS I

Solutions

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$$p_G = \rho_{oil}\, g \mathrm{h}_{\mathrm{oil}} + \rho_{water}\, g \mathrm{h}_{\mathrm{water}} = 1000 \times 9.81\left(0.8 * \left(1.5 - \mathrm{h}_{\mathrm{water}}\right) + \mathrm{h}_{\mathrm{water}}\right) = 0.123 \times 10^5\, N/m^2$$

Q 1

$$1.2 + 0.2 * \mathrm{h}_{\mathrm{water}} = \frac{12.3}{9.81} m$$

$$h = 5\left(\frac{12.3}{9.81} - 1.2\right) = 0.269 m$$

(6 *marks*)

Q 2 For vertical component use "weight of water above", in this case that means weight of water in the cylinder and hemisphere directly above the dome and thrust is upwards. For horizontal force use the pressure at the centre of the projected area times the projected area. In both cases use gauge pressure as the atmospheric pressure acts inside the sub.

$$F_v = -\left(\tfrac{2}{3} \times \pi \times 0.5^3 + 12 \times \pi \times 0.5^2\right) \times 1000 \times 9.81 = -\left(\tfrac{1}{3} + 12\right) \times \pi \times 2452.5 = -95025 N$$

$$F_h = 11 \times 1000 \times 9.81 \times \pi \times 1^2 = 339009 N$$

(6 *marks*)

Q 3 Steady, incompressible, inviscid,1D flow

(4 marks)

Q 4 a) is turbulent and b) is laminar. The turbulent profile is "fuller", meaning that it has more momentum near the surface. If the adverse pressure gradient associated with separation is applied throughout the boundary layer then the fuller turbulent profile will remain attached because the resulting momentum change near the surface will not be sufficient for reversed flow.

(*6 marks*)

Q 5 From continuity we know that the velocity upstream of the sudden enlargement must be

$$A_1 V_1 = A_2 V_2 \quad \rightarrow \quad V_2 = \frac{A_1}{A_2} V_1 = \frac{V_1}{4}$$

$$C_{loss} = \left(1 - \frac{A_1}{A_2}\right)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16} = 0.5625$$

Applying Bernoulli (set vertical zero at downstream location)

$$p_1 + \tfrac{1}{2}\rho V_1^2 + \rho g h_1 = p_2 + \tfrac{1}{2}\rho V_2^2 + \rho g h_2 + C_{loss}\tfrac{1}{2}\rho V_1^2$$

$$p_2 = p_1 + \tfrac{1}{2}\left(1 - C_{loss}\right)\rho V_1^2 + \rho g\left(h_1 - h_2\right) - \tfrac{1}{2}\rho \frac{V_1^2}{4^2}$$

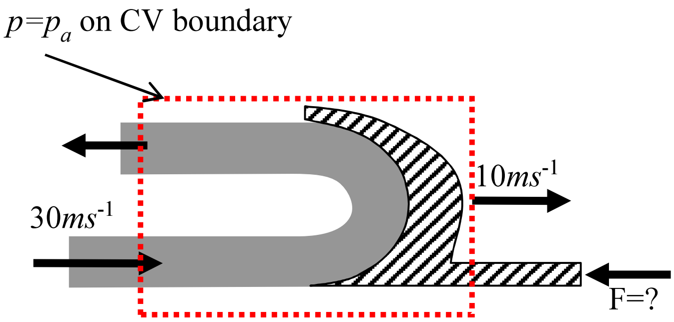
$$p_2 = p_1 + \tfrac{1}{2}\frac{6}{16}\rho V_1^2 + \rho g \times 4$$

$$p_1 = 2 \times 10^5 + \frac{3}{16} \times 1000 \times 4^2 + 1000 \times 9.81 \times 4 = 242240 \text{ N/m}^2 = 2.4224 bar$$

(*6 marks*)

Q 6

Consider a control volume fixed relative to the plate. Effectively apply a Galilean transformation and work with a fixed turning vane and an inflow of 20ms^{-1}. The net horizontal force on the control volume equals the rate of change of momentum in that direction. We also assume that atmospheric pressure acts through the jet diameter so there is no contribution to the net horizontal force from the jet entry into the CV. The force on the control volume in the onset flow direction is therefore



$$F_{CVx} = \dot{m}(V_2 - V_1) = \pi r^2 V\rho(-V - V) = -\pi\times 0.05^2\times 20\times 1000\times(20+20) = -6283.2N$$

$$F = -F_{CVx} = 6283.2N$$

(6marks)

Q 7

From given equations, doublet provides horizontal & vertical velocities:

$$u = \frac{-\kappa}{2\pi}\frac{(x^2 - y^2)}{(x^2+y^2)^2}, \qquad v = \frac{-\kappa}{2\pi}\frac{2xy}{(x^2+y^2)^2}$$

Stagnation point at leading edge, placing origin at the centre of the doublet then

$$0 = U_\infty - \frac{\kappa}{2\pi}\frac{1}{x^2}, \qquad \kappa = 2\pi\times 10\times 0.2^2 = 0.8\pi = 2.5133$$

Velocities at top therefore

$$u = 10 + \frac{-0.8\pi}{2\pi}\frac{(-0.2^2)}{(0.2^2)^2} = 20ms^{-1}, \qquad v = 0$$

Because of viscosity, actual velocity at the top surface of the cylinder is zero.

(6 marks)

Q8

(a) Applying Bernoulli's equation between the reservoir surface (subscript 0) and 2 in the vena contracta:

$$p_0 + 0 + \rho gh = p_a + \tfrac{1}{2}\rho V_2^2 + 0$$

If $p_0 = p_a$ initially and $h = H$ then

$$p_a + \rho gH = p_a + \tfrac{1}{2}\rho V_2^2$$

$$V_2 = \sqrt{2gH} \qquad \text{eqn(1)}$$

The mass flow rate is given by

$$\dot{m} = \rho A_2 V_2 = c_d \rho A \sqrt{2gH} \qquad \text{eqn(2)}$$

(8 marks)

(b)

Once the container is closed the mass of air inside the container does not change. Assuming the air is incompressible

$$p \cdot Vol = Const$$

If the cross sectional area of the container is fixed as A_c, and the pressure of the gas in the closed container (and hence the surface of the water) is given by p_c, then

$$p_a \cdot A_c (H - h_0) = p_c \cdot A_c (H - h)$$

$$p_c = p_a \cdot \frac{(H - h_0)}{(H - h)} \qquad \text{eqn(3)}$$

Again, applying Bernoulli's equation between the reservoir surface and the vena contracta:

$$p_c + 0 + \rho gh = p_a + \tfrac{1}{2}\rho V_2^2 + 0$$

$$p_a \frac{H - h_0}{H - h} - p_a + \rho gh = \tfrac{1}{2}\rho V_2^2$$

$$V_2 = \sqrt{2gh - 2\frac{p_a}{\rho}\frac{(h_0 - h)}{(H - h)}}$$

$$\dot{m} = \rho A_2 V_2 = c_d \rho A \sqrt{2gh - 2\frac{p_a}{\rho}\frac{(h_0 - h)}{(H - h)}} \qquad \text{eqn(4)}$$

To find the height of the jet we can either take Bernoulli's equation from the vena contracta to the top of the jet (*V=0*) or from the top of the reservoir to the top of the jet (which is what we shall do now)

$$p_A + 0 + \rho gh = p_a + 0 + \rho g h_{jet}$$

$$p_a \frac{H - h_0}{H - h} - p_a + \rho gh = \rho g h_{jet}$$

$$\rho g h_{jet} = \rho gh - p_a \frac{h_0 - h}{H - h}$$

$$h_{jet} = h - \frac{p_a}{\rho g}\frac{(h_0 - h)}{(H - h)} \qquad \text{eqn(5)}$$

(12 marks)

(c) Initially $H = 5m,\ p_a = 1bar = 1\times 10^5\ Nm^{-2},\ h_0 = 4.9\,m,\ \rho = 1000\,kg\,m^{-3},\ g = 9.81\,ms^{-2}$

Applying Bernoulli's equation, the initial height of the jet is: $h_{jet} = H = 5m$

From continuity between the vena contracta and the position where the jet has twice the area of the hole (referenced as 3)

$$V_2 A_2 = V_3 A_3$$

$$V_3 = V_2 \frac{c_d A}{A_3} = V_2 \frac{c_d}{2} \qquad \text{eqn (6)}$$

Applying Bernoulli's equation

$$p_a + 0 + \tfrac{1}{2}\rho V_2^2 = p_a + \rho g h_3 + \tfrac{1}{2}\rho V_3^2$$

Rearranging and then using continuity (equation 6) and the previous equation for V_2^2 (equation 1)

$$\rho g h_3 = \tfrac{1}{2}\rho V_2^2 - \tfrac{1}{2}\rho V_2^2 \left(\frac{c_d}{2}\right)^2$$

$$g h_3 = \tfrac{1}{2} V_2^2 \left(1 - \left(\frac{c_d}{2}\right)^2\right) = \tfrac{1}{2} 2gH \left(1 - \frac{c_d^2}{4}\right)$$

$$h_3 = H\left(1 - \frac{c_d^2}{4}\right)^2 = 5 * 0.91 = 4.55m$$

Flow stops when $h_{jet}=0$, so rearranging equation 5

$$p_a(h_0 - h) = \rho g h (H - h)$$

$$(\rho g)h^2 - (p_a + \rho g H)h + p_a h_0 = 0$$

$$h = \frac{(p_a + \rho g H) \pm \sqrt{(p_a + \rho g H)^2 - 4 p_a \rho g h_0}}{2\rho g} = \frac{(149050) \pm \sqrt{(149050)^2 - 192276 \times 10^5}}{19620}$$

$$= \frac{(149050) \pm \sqrt{298830.25 \times 10^4}}{19620} = \begin{cases} 10.38 \\ 4.811 \end{cases}$$

$$\Delta h = 4.9 - 4.811 = 0.089m$$

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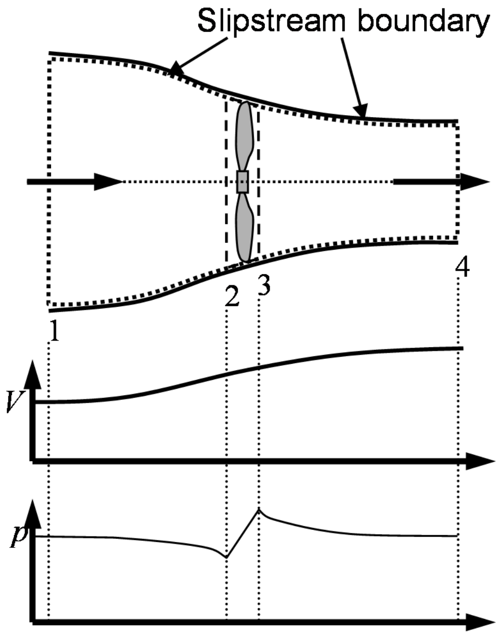
(10 marks)

Q9

(a) Use the actuator disc theory for an ideal propeller, see figure below



Consider the Galilean transformation so that the propeller is fixed. In this case the inflow velocity is now $V_1=V$

Assumptions: Frictionless & incompressible, Steady 1D flow (neglect rotation and variation across the disc radius). Actuator disc is thin so $A_2=A_3=A_d$ & $V_2=V_3=V_d$. $p=p_a$ at all points on slipstream boundary & 1 & 4

Continuity: $Q=V_d A_d$

Bernoulli's equation for CV 1-2 & CV 3-4

$$\begin{aligned} p_1+\tfrac{1}{2}\rho V_1^2 &= p_2+\tfrac{1}{2}\rho V_d^2 \\ p_3+\tfrac{1}{2}\rho V_d^2 &= p_4+\tfrac{1}{2}\rho V_4^2 \end{aligned} \quad \rightarrow \quad p_3-p_2=\tfrac{1}{2}\rho\left(V_4^2-V^2\right)$$

Steady Flow momentum for CV 2-3:

$$(p_2-p_3)A_d+F_{CV}=\rho Q(V_d-V_d)=0 \quad \rightarrow \quad F_{CV}=(p_3-p_2)A_d$$

Applying results from Bernoulli's equation above

$$F=\tfrac{1}{2}\rho A_d\left(V_4^2-V^2\right)$$

(8 marks)

(b)

Steady Flow momentum for CV 1-4:

$$0+F_{CV}=\rho Q(V_4-V)$$

From momentum & continuity

$$(p_3-p_2)A_d=\rho V_d A_d(V_4-V)$$

Eliminating $(p_3\text{-}p_2)$ using Bernoulli's equation above

$$\rho V_d\left(V_4 - V\right) = \tfrac{1}{2}\rho\left(V_4^2 - V^2\right) = \tfrac{1}{2}\rho\left(V_4 - V\right)\left(V_4 + V\right)$$
$$V_d = \tfrac{1}{2}\left(V_4 + V\right)$$
$$V_4 = 2V_d - V$$

Continuity from 1 to 2

$$A_1 V = A_d V_d$$
$$V_d = \frac{A_1 V}{A_d}$$
$$a = \frac{\left(A_1 - A_d\right)}{A_d} \quad \rightarrow \quad \frac{A_1}{A_d} = a + 1 \quad \rightarrow \quad V_d = \left(a+1\right)V$$
$$V_d = \left(a+1\right)V = \tfrac{1}{2}\left(V_4 + V\right) \quad \rightarrow \quad V_4 = \left(2a+1\right)V$$

Substituting back in gives

$$F = \tfrac{1}{2}\rho A_d\left(V_4^2 - V^2\right) = \tfrac{1}{2}\rho A_d V^2\left(\left(2a+1\right)^2 - 1\right) = \rho A_d V^2 2a\left(a+1\right)$$
$$F = \tfrac{1}{2}\pi\rho d^2 V^2 a\left(a+1\right)$$

The power supplied to the disc is

$$P_{disc} = FV_d$$

Power output

$$P_{out} = FV$$

The efficiency of the rotor is therefore (remember efficiency for turbines and propellers is not the same)

$$\eta = \frac{P_{\text{out}}}{P_{\text{disc}}} = \frac{FV}{FV_d} = \frac{V}{V_d} = \frac{1}{a+1}$$

(14 marks)

(d) From parts a and b we have

$$F = \tfrac{1}{2}\pi\rho d^2 V^2 a\left(a+1\right)$$
$$8000 = \tfrac{1}{2}\pi \times 1.2 \times d^2 \times 85^2 \times 0.2\left(1.2\right) = \pi \times d^2 \times 85^2 \times 0.1\left(1.2\right)^2$$
$$d^2 = \frac{80000}{\pi \times 85^2 \times \left(1.2\right)^2} \quad \rightarrow \quad d = \frac{100}{85 \times 1.2}\sqrt{\frac{8}{\pi}}$$
$$FV_d = \frac{FV}{\eta}$$
$$\eta = \frac{1}{1.2}$$
$$FV_d = FV \times 1.2 = 8000 \times 85 \times 1.2 = 816000W$$

(8 marks)

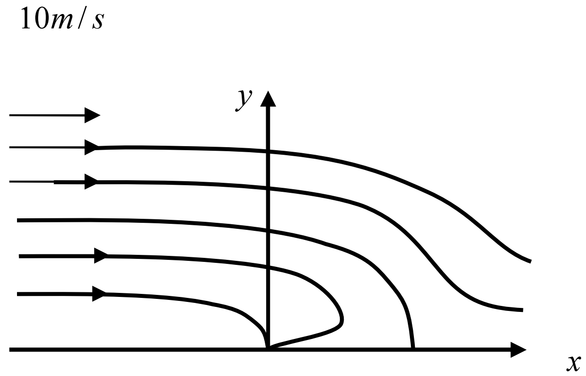
Q10

(a) Streamlines are lines of constant stream function. The change in ψ between streamlines corresponds to the volume flow (per unit depth) between these lines or the mass flow between the lines scaled by the density.

ψ is only defined for 2D flow whereas ϕ exists for 3D flows. ψ can be used in rotational flows, whereas ϕ implies irrotationality.

(4 marks)

(b) i)



(3 marks)

ii) The stream function for the combined uniform freestream and point source is

$$\psi = 10y - 65\tan^{-1}\left(\frac{y}{x}\right)$$

To find the location of the stagnation point at the nose the first step is to determine the velocity components. The stream function is in terms of Cartesian coordinates so use $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. Thus

$$u = \frac{\partial\psi}{\partial y} = 10 - 65\frac{1}{(1+(y/x)^2)}\left(\frac{1}{x}\right) = 10 - 65\frac{x}{x^2+y^2}$$

$$v = -\frac{\partial\psi}{\partial x} = 65\frac{1}{(1+(y/x)^2)}\left(\frac{-y}{x^2}\right) = -65\frac{y}{x^2+y^2}$$

Then find the stagnation point when $u=v=0$

$v = -65\dfrac{y}{x^2+y^2} = 0$ when $y=0$ i.e. on the centre line.

$u = 10 - 65\dfrac{x}{x^2+y^2} = 10 - 65\dfrac{1}{x}$ when $y=0$

Therefore u=0 when $x = 6.5$. The stagnation point is on the centre line a distance 6.5 m downstream of the source.

(6 marks)

(iii) The stagnation point is located on the dividing streamline that represents the cliff surface so the value of the stream function on the dividing streamline can be evaluated. Substituting y=0, $x = 6.5$ into the equation for the stream function gives

$\psi_{DS} = -65\tan^{-1}(0)$

$\tan^{-1}(0)=0$ or π, but since the stagnation point is downstream of the source (which is at the origin of the Cartesian coordinate system $x=y=0$) so 0 is the correct value to use and thus

$\psi_{DS} = 0$

Then the height of the cliff where it intersects the y- axis is the value of value of y when $x=0$ on the dividing stream line.

$$\psi_{DS} = 0 = 10y - 65\tan^{-1}\left(\frac{y}{0}\right) = 10y - 65\frac{\pi}{2}$$

Hence it can be shown that the height is $y_{cliff} = 65\pi/20\ m = 10.210\,m$

(4 marks)

(iv) To determine the cliff height far downstream as $x \to -\infty$, use the stream function.

$$\psi = 10y - 65\tan^{-1}\left(\frac{y}{x}\right) \to 10y - 65\pi \quad \text{as } x \to -\infty$$

since as $x \to -\infty$ the $\tan^{-1}$ term $\to \pi$.

Then it follows that the dividing streamline is in fact

$$\psi_{DS} = 0 \quad \to \quad 10y - 65\pi \ \text{ as } x \to -\infty$$

$$y_{DS} \to \frac{65\pi}{10} \ \text{ as } x \to -\infty$$

The ultimate cliff height is thus 6.5π m = 20.42 m

(4 marks)

(v) work out the velocities at this location

$$u = 10 - 65\frac{x}{x^2+y^2} = 10m/s$$

$$v = -65\frac{y}{x^2+y^2} = \frac{65}{y}m/s$$

Then the velocity at this point is

$$V = \sqrt{u^2+v^2}$$

Then the height of the cliff at this point is given in (iii) and therefore the velocity is being measured at a height of $y = 3.25\pi + 1$. Then substituting into the above equation gives

$$V = \sqrt{100 + \frac{65^2}{(3.25\pi+1)^2}} = 11.559$$

This is larger than the freestream velocity, hence the percentage difference relative to the free stream is

$$100\left(\frac{V-U_\infty}{U_\infty}\right) = 100\left(\frac{1.559}{10}\right) = 15.59\%$$

(9 marks)