Advanced Bending and Torsion
Shear Centre Example – Angled Flanges

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07 November 2018

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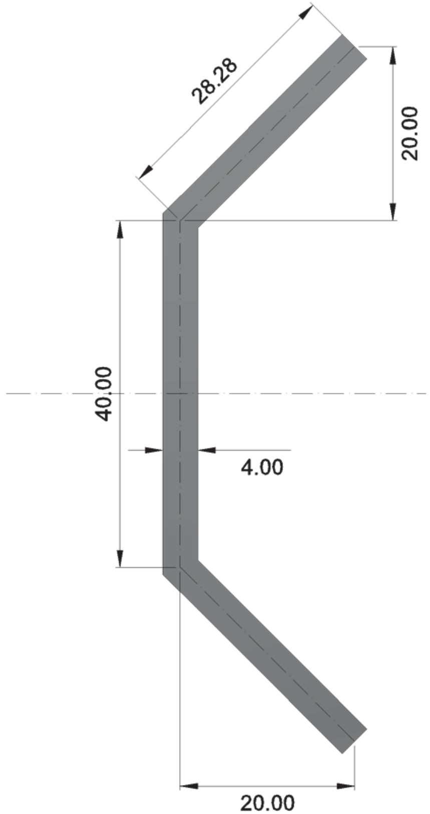
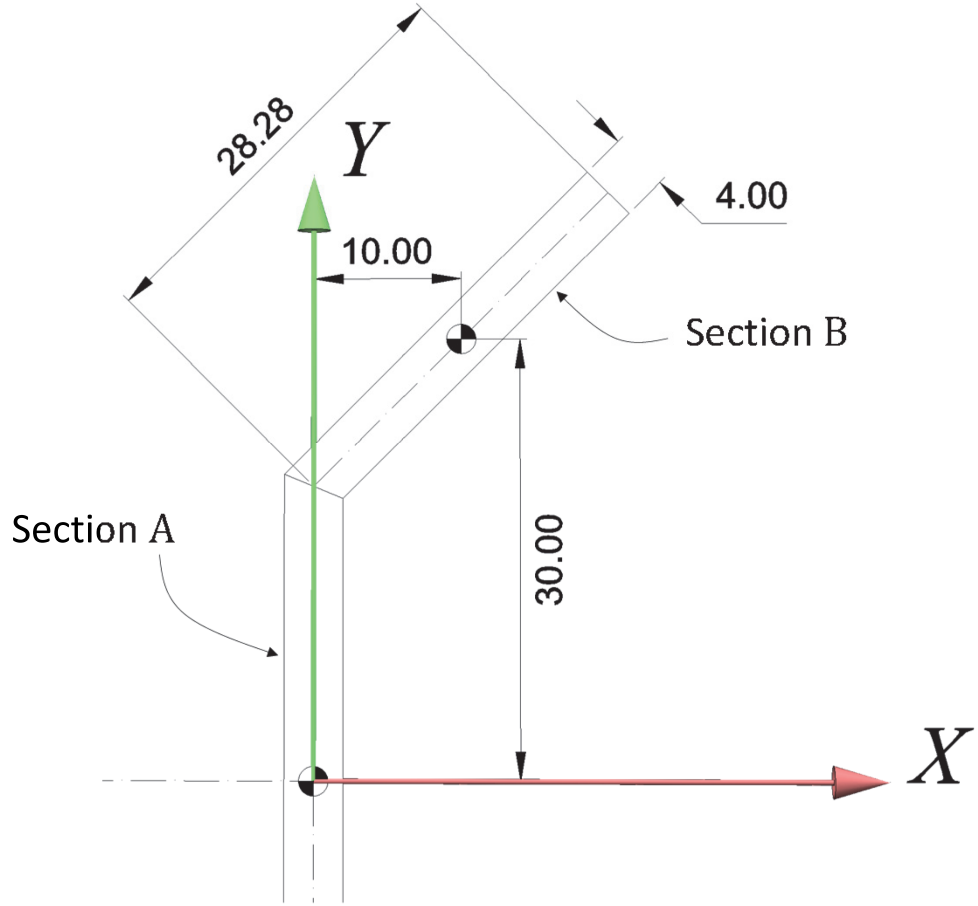
Angled Flange Section

- Section with flanges at 45°:





$$A_{\mathrm{A}} = (4.00)(40.0.)\ \mathrm{mm}^2 \qquad A_{\mathrm{B}} = (4.00)(28.28)\ \mathrm{mm}^2$$

$$A_{\mathrm{A}} = 160.00\ \mathrm{mm}^2 \qquad A_{\mathrm{B}} = 113.14\ \mathrm{mm}^2$$

$$\bar{X}_{\mathrm{A}} = 0 \qquad \bar{X}_{\mathrm{B}} = 10.00\ \mathrm{mm}$$

$$\bar{Y}_{\mathrm{A}} = 0 \qquad \bar{Y}_{\mathrm{B}} = 30.00\ \mathrm{mm}$$

Angled Flange Section

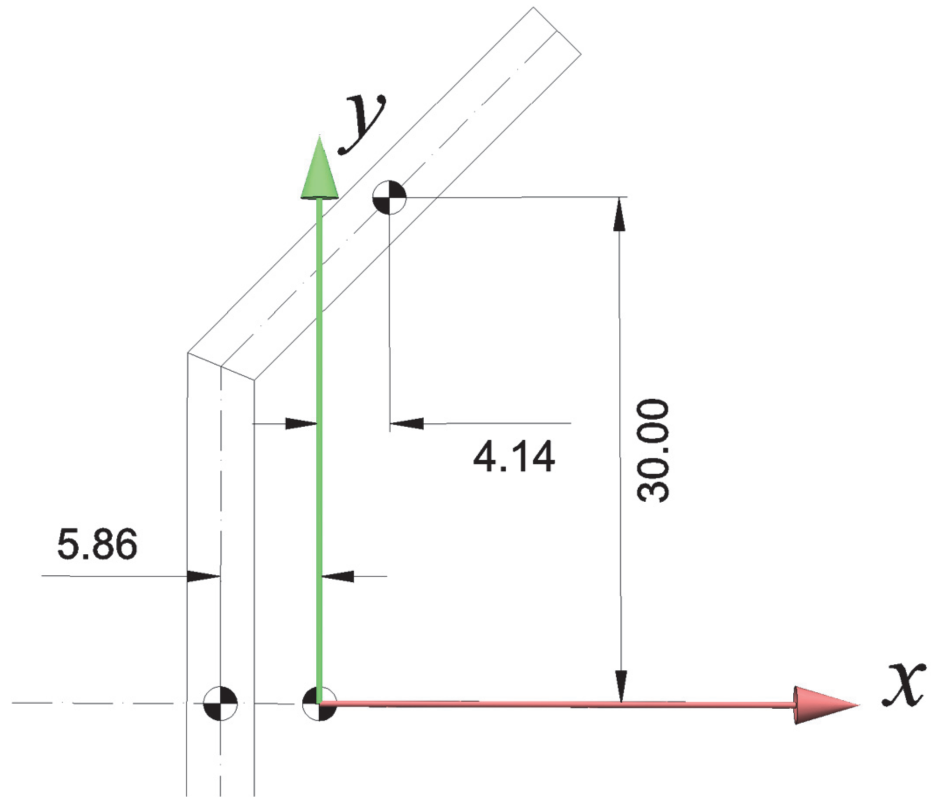
- Centroid of the compound section:

$$\bar{X} = \frac{\sum \bar{X}_i A_i}{\sum A_i} = \frac{\bar{X}_\mathrm{A} A_\mathrm{A} + 2\,\bar{X}_\mathrm{B} A_\mathrm{B}}{A_\mathrm{A} + 2\,A_\mathrm{B}} = \frac{(0)(160.00) + 2\,(10.00)(113.14)}{(160.00) + 2\,(113.14)}$$

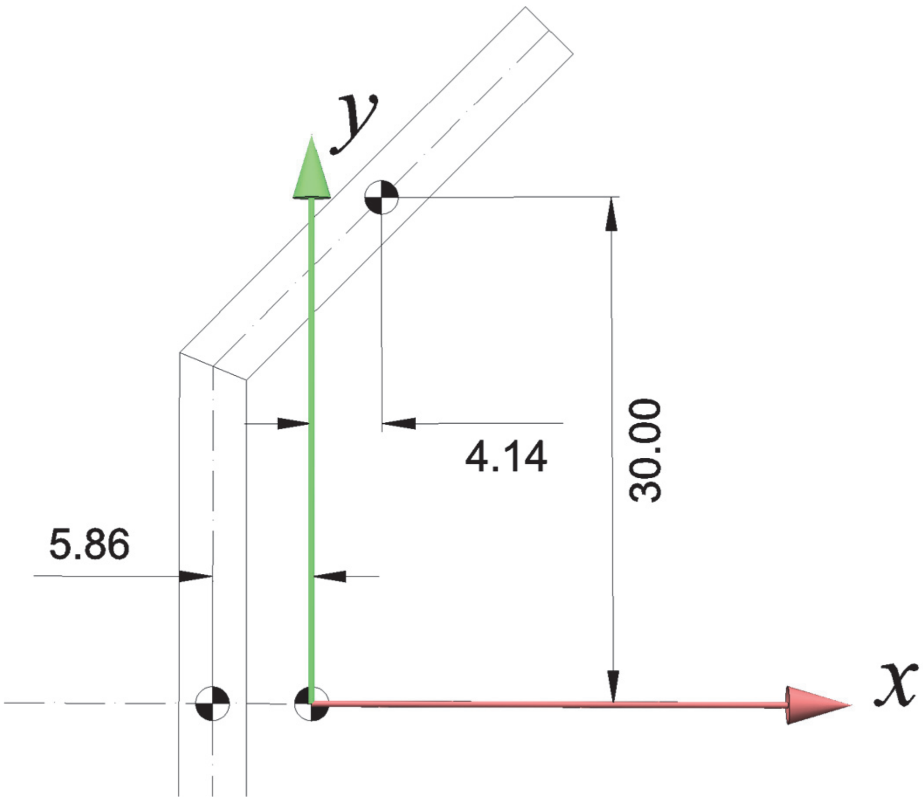
$\bar{X} = 5.86\ \mathrm{mm}$

$\bar{Y} = 0$

- New coordinates:



- Parallel axis theorem for section A:



$$I_{x_A x_A} = \frac{b\,h^3}{12} = \frac{(4.00)(40)^3}{12} = 21{,}333.33\ \text{mm}^4$$

$$\bar{y}_A = \bar{Y}_A - \bar{Y} = 0\ \text{mm}$$

$$I_{xx}^{A} = I_{x_A x_A} + A_A(\bar{y}_A)^2$$

$$I_{xx}^{A} = 21{,}333.33\ \text{mm}^4$$

$$I_{y_A y_A} = \frac{b\,h^3}{12} = \frac{(40.00)(4.00)^3}{12} = 213.33\ \text{mm}^4$$

$$\bar{x}_A = \bar{X}_A - \bar{X} = 0 - 5.86 = 5.86\ \text{mm}$$

$$I_{yy}^{A} = I_{y_A y_A} + A_A(\bar{x}_A)^2$$

$$I_{yy}^{A} = 5{,}703.67\ \text{mm}^4$$

$I_{x_A y_A} = 0$ (symmetric cross-section)

$$I_{xy}^{A} = I_{x_A y_A} + A_A(\bar{x}_A \bar{y}_A)$$

$$I_{xy}^{A} = 0$$

Angled Flange Section – I_{xx}

$$x_{(s)} = 10\sqrt{2} - s\frac{\sqrt{2}}{2}$$

$$x_{(s)} = \frac{\sqrt{2}}{2}(20 - s)$$

$$y_{(s)} = 40 - s\frac{\sqrt{2}}{2}$$

$$s_1 = 20\sqrt{2}$$

$$I_{xx}^{\mathrm{B}} = \int_0^{s_1} y^2\, t\, ds$$

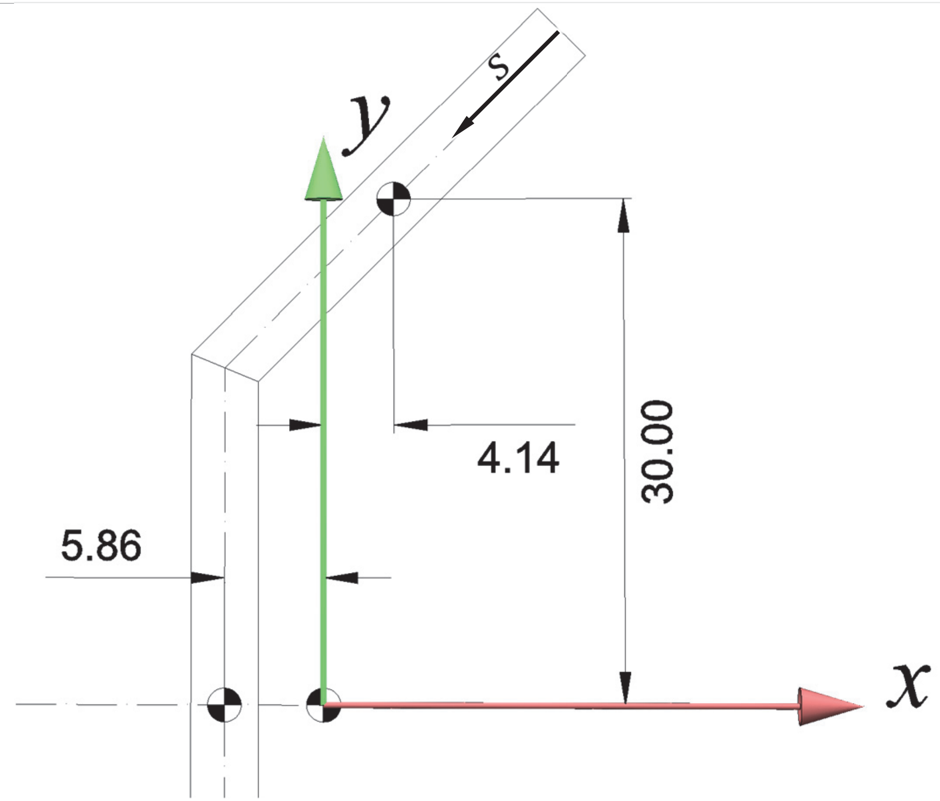
$$I_{xx}^{\mathrm{B}} = \int_0^{s_1} \left(40 - s\frac{\sqrt{2}}{2}\right)^2 t\, ds$$

$$I_{xx}^{\mathrm{B}} = t\int_0^{s_1} \left(40^2 - 40\, s\sqrt{2} + \frac{1}{2}s^2\right) ds$$

$$I_{xx}^{\mathrm{B}} = t\left[40^2 s - 20\, s^2\sqrt{2} + \frac{1}{6}\, s^3\right]_0^{s_1}$$

$$I_{xx}^{\mathrm{B}} = t\left[40^2\, s_1 - 20\, {s_1}^2\sqrt{2} + \frac{1}{6}\, {s_1}^3\right]$$

$$I_{xx}^{\mathrm{B}} = 105{,}595\ \mathrm{mm}^4$$



Angled Flange Section – I_{yy}

$$I_{yy}^{\mathrm{B}} = \int_0^{s_1} x^2\, t\, ds$$

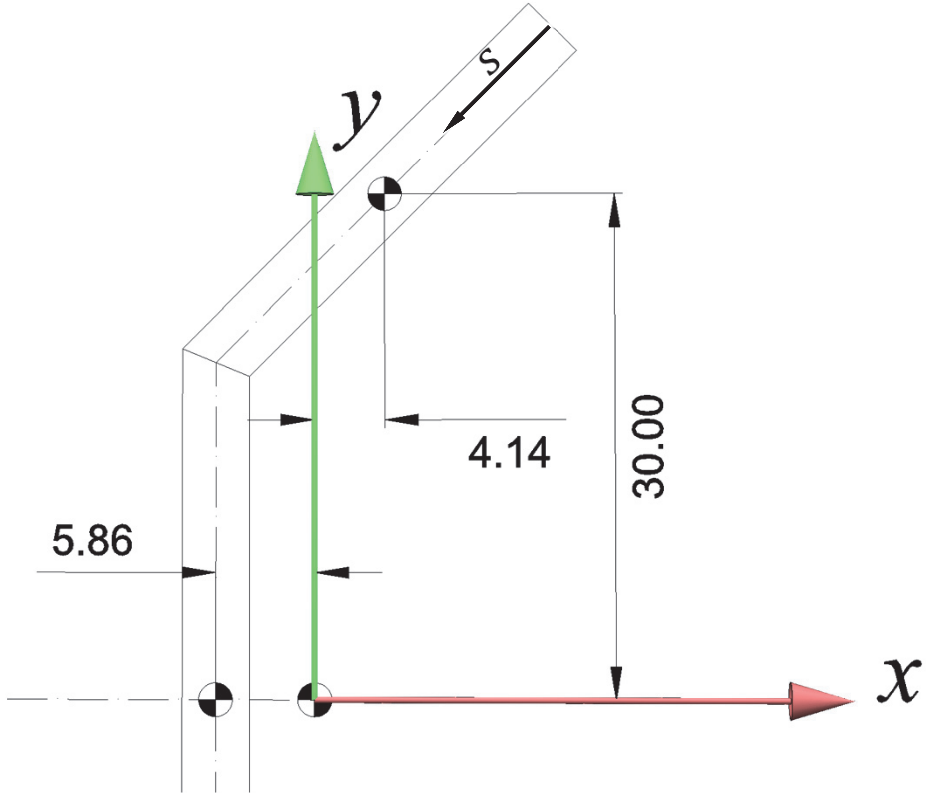
$$I_{yy}^{\mathrm{B}} = \int_0^{s_1} \left[\frac{\sqrt{2}}{2}\,(20 - s)\right]^2 t\, ds$$

$$I_{yy}^{\mathrm{B}} = \frac{t}{2}\int_0^{s_1} (400 - 40\,s + s^2)ds$$

$$I_{yy}^{\mathrm{B}} = \frac{t}{2}\left[400\,s - 20\,s^2 + \frac{1}{3}\,s^3\right]_0^{s_1}$$

$$I_{yy}^{\mathrm{B}} = \frac{t}{2}\left(400\,s_1 - 20\,{s_1}^2 + \frac{1}{3}{s_1}^3\right)$$

$$I_{yy}^{\mathrm{B}} = 5{,}712\ \mathrm{mm}^4$$



$$x_{(s)} = \frac{\sqrt{2}}{2}\,(20 - s)$$

$$y_{(s)} = 40 - s\,\frac{\sqrt{2}}{2}$$

$$s_1 = 20\,\sqrt{2}$$

Angled Flange Section – I_{xy}

$$I_{xy}^{\mathrm{B}} = \int_0^{s_1} x\, y\, t\, ds$$

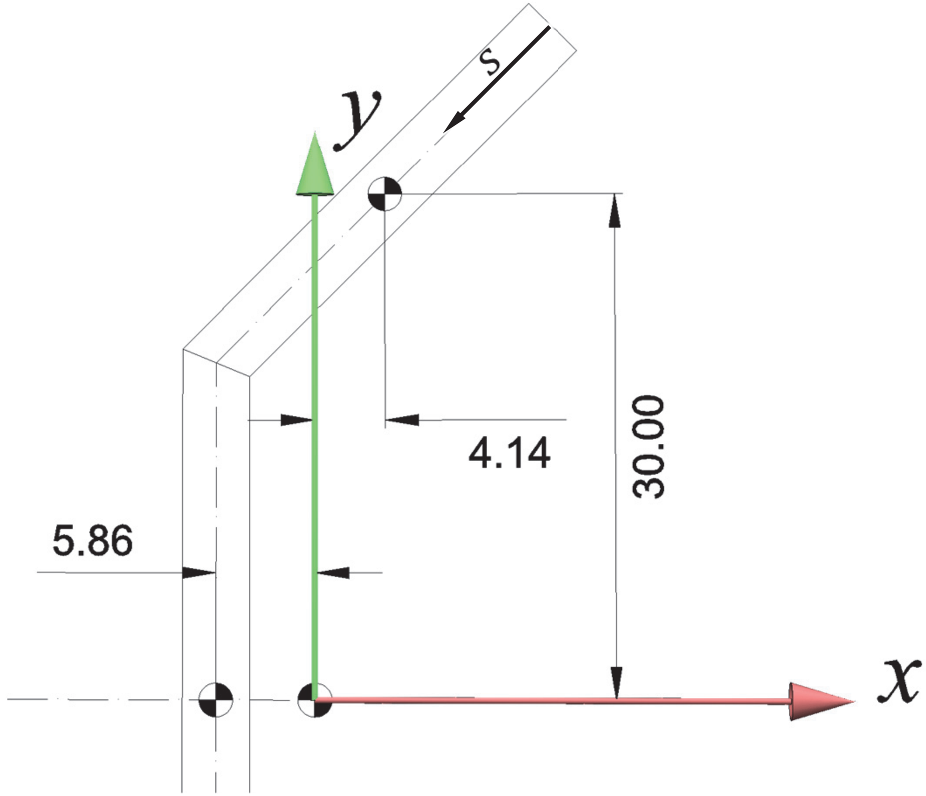
$$I_{xy}^{\mathrm{B}} = \int_0^{s_1} \left[\frac{\sqrt{2}}{2}\,(20 - s)\right]\left[40 - s\frac{\sqrt{2}}{2}\right] t\, ds$$

$$I_{xy}^{\mathrm{B}} = \frac{\sqrt{2}}{2} t \int_0^{s_1} \left(800 - 10\sqrt{2}\, s - 40\, s + \frac{\sqrt{2}}{2} s^2\right) ds$$

$$I_{xy}^{\mathrm{B}} = \frac{\sqrt{2}}{2} t \left(800\, s - 5\sqrt{2}\, s^2 - 20\, s^2 + \frac{\sqrt{2}}{6} s^3\right)_0^{s_1}$$

$$I_{xy}^{\mathrm{B}} = \frac{\sqrt{2}}{2} t \left[800\, s_1 - (5\sqrt{2} + 20)\, {s_1}^2 + \frac{\sqrt{2}}{6} {s_1}^3\right]$$

$I_{xy}^{\mathrm{B}} = 17{,}830 \text{ mm}^4$ $\quad$ $I_{xy}^{\mathrm{C}} = -17{,}830 \text{mm}^4$



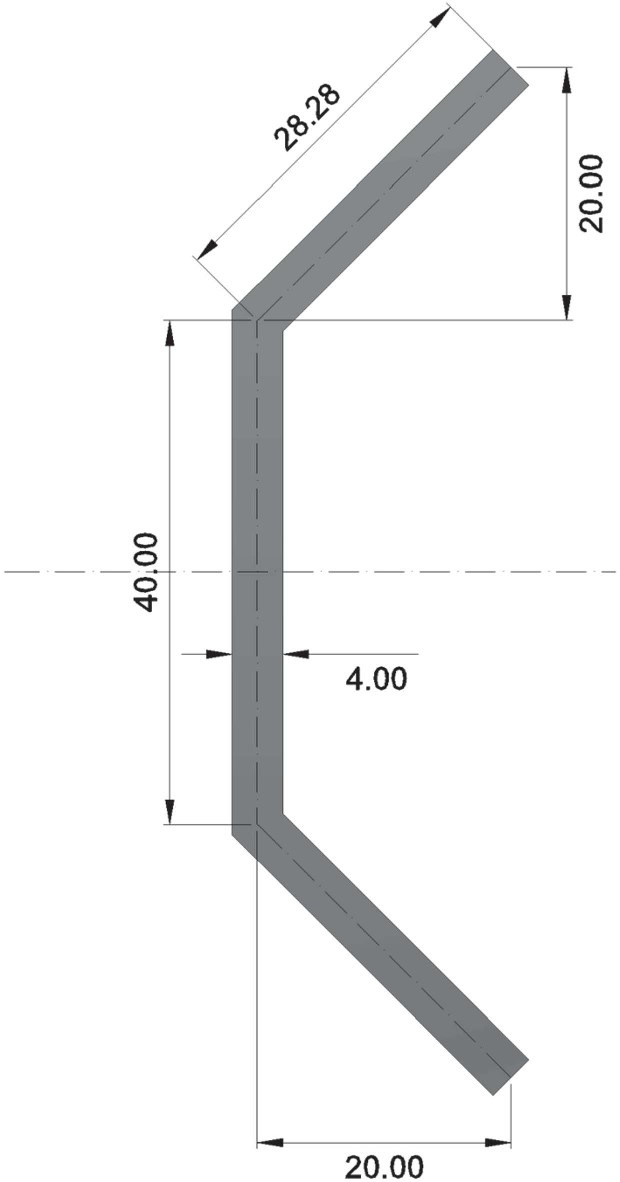
$$x_{(s)} = \frac{\sqrt{2}}{2}\,(20 - s)$$

$$y_{(s)} = 40 - s\frac{\sqrt{2}}{2}$$

$$s_1 = 20\sqrt{2}$$

Angled Flange Section

- Finally, for the compound section:



$$I_{xx} = I_{xx}^{\mathrm{A}} + I_{xx}^{\mathrm{B}} + I_{xx}^{\mathrm{C}}$$

$$I_{xx} = 232{,}523 \text{ mm}^4$$

$$I_{yy} = I_{yy}^{\mathrm{A}} + I_{yy}^{\mathrm{B}} + I_{yy}^{\mathrm{C}}$$

$$I_{yy} = 17{,}128 \text{ mm}^4$$

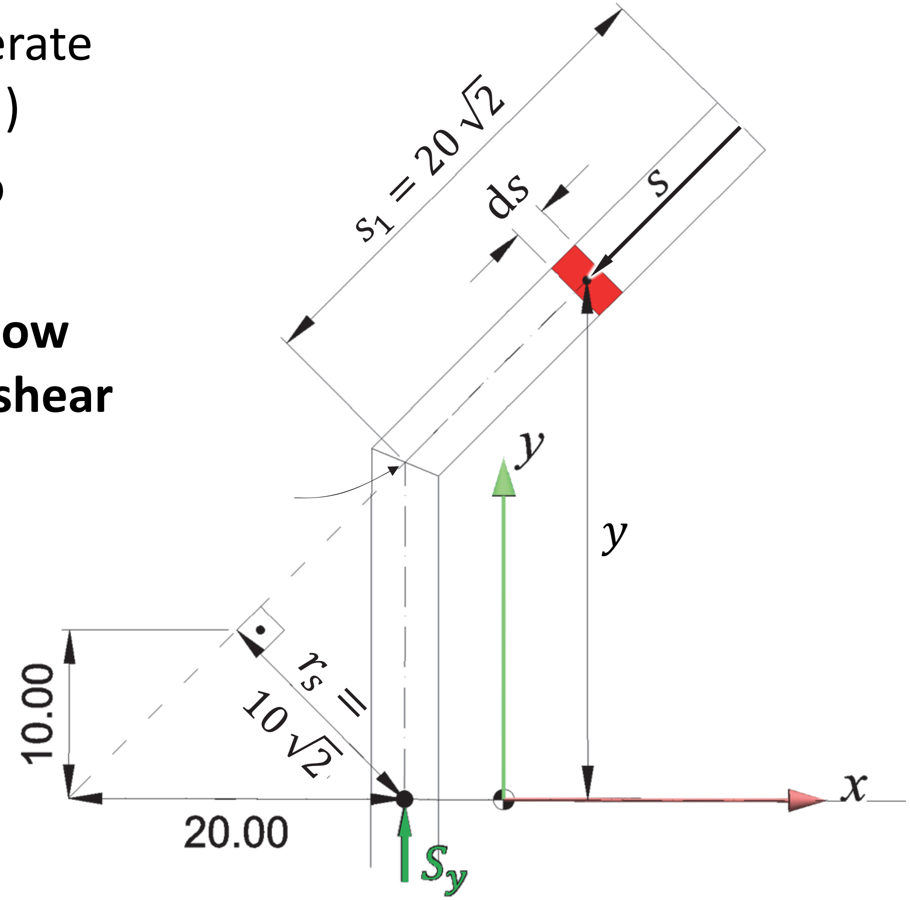
$$I_{xy} = I_{xy}^{\mathrm{A}} + I_{xy}^{\mathrm{B}} + I_{xy}^{\mathrm{C}}$$

$$I_{xy} = 0$$

Angled Flange Section

- **Shear centre**:
  - Most convenient reference point is the bottom left corner (origin of X, Y)
  - Only stresses along section C will generate moments about this reference point (!)
  - We integrate shear flow from $s = 0$ to $s = s_1 = 13.425$ mm
  - **Important**: shear stresses and **shear flow** are defined in terms of $\boldsymbol{x}, \boldsymbol{y}$ while the **shear centre** is defined in terms of $\boldsymbol{X}, \boldsymbol{Y}$

$$(x_0, y_0) = (20 \text{ mm}, 40 \text{ mm})$$
$$r_s = 10\sqrt{2} \text{ mm}$$
$$s_0 = 0$$
$$s_1 = 20\sqrt{2} \text{ mm}$$



Equations:

Shear centre: $$S_y\, e_x = \int r_s\, q_s\, \mathrm{d}s$$

Shear flow: $$-q_s = \left(\frac{S_x I_{xx} + S_y I_{xy}}{I_{xy}^2 - I_{xx} I_{yy}}\right) \int_0^s x\, t\, \mathrm{d}s + \left(\frac{S_y I_{yy} + S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) \int_0^s y\, t\, \mathrm{d}s$$

Angled Flange Section – Shear Centre e_x

$$-q_s = \left(\frac{S_x\, I_{xx} + S_y\, I_{xy}}{I_{xy}^2 - I_{xx}\, I_{yy}}\right)\int_0^s x\, t\, \mathrm{ds} + \left(\frac{S_y\, I_{yy} + S_x\, I_{xy}}{I_{xx}\, I_{yy} - I_{xy}^2}\right)\int_0^s y\, t\, \mathrm{ds}$$

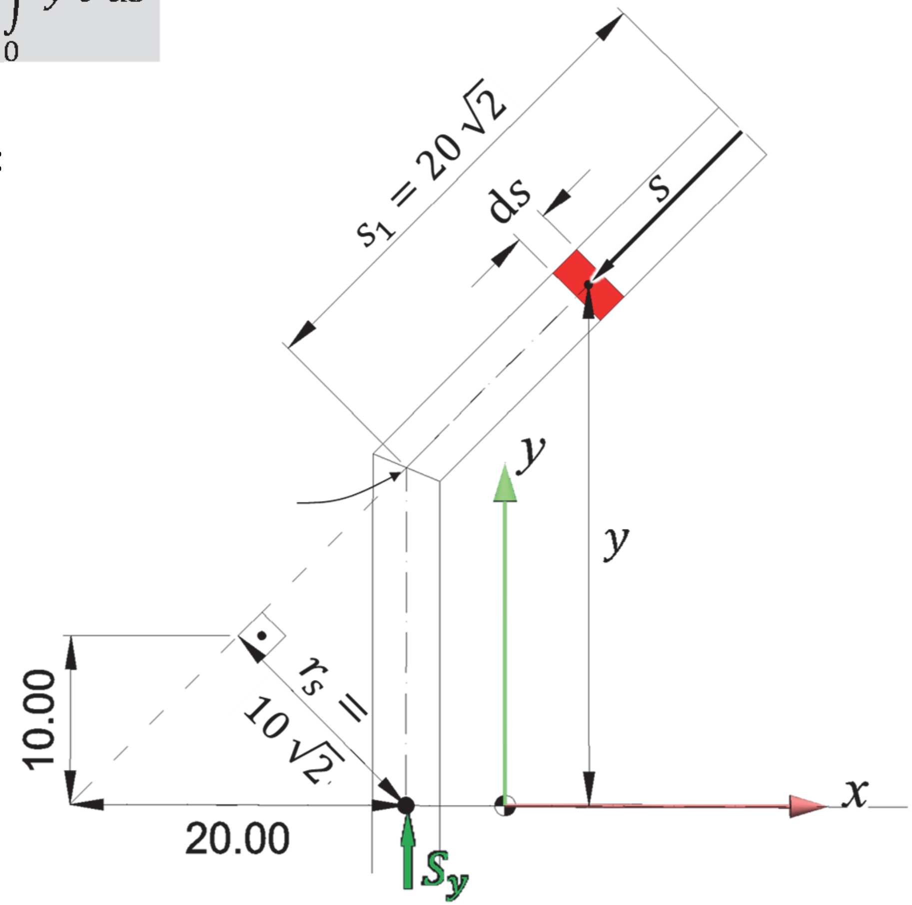
To find e_x we apply S_y, but $S_x = I_{xy} = 0$ and therefore:

$$-q_s = \left(\frac{S_y}{I_{xx}}\right)\int_0^s y\, t\, \mathrm{ds}$$

$$y_{(s)} = 40 - s\frac{\sqrt{2}}{2}$$

$$-q_s = \left(\frac{S_y}{I_{xx}}\right)\int_0^s \left(40 - s\frac{\sqrt{2}}{2}\right) t\, \mathrm{ds}$$

$$-q_s = t\left(\frac{S_y}{I_{xx}}\right)\left(40\, s - \frac{\sqrt{2}}{4}\, s^2\right)$$



Angled Flange Section – Shear Centre e_x

$$S_y\, e_x = \int r_s\, q_s\, \mathrm{d}s$$

Note that there are **two** angled flanges, therefore:

$$S_y\, e_x = \mathbf{2} \int_0^{s_1} \left(10\sqrt{2}\, q_s\right) \mathrm{d}s$$

$$-q_s = t \left(\frac{S_y}{I_{xx}}\right)\left(40\, s - \frac{\sqrt{2}}{4}\, s^2\right)$$

$$S_y\, e_x = 20\sqrt{2} \int_0^{s_1} -\left[t\left(\frac{S_y}{I_{xx}}\right)\left(40\, s - \frac{\sqrt{2}}{4}\, s^2\right)\right] \mathrm{d}s$$

$$e_x = -\frac{20\, t\sqrt{2}}{I_{xx}} \int_0^{s_1} \left(40\, s - \frac{\sqrt{2}}{4}\, s^2\right) \mathrm{d}s$$

$$e_x = -\frac{20\, t\sqrt{2}}{I_{xx}} \left(20\, s^2 - \frac{\sqrt{2}}{12}\, s^3\right)$$

$$e_x = -6.49\ \mathrm{mm}$$