Vibrations 2, Lecture 9
Transmissibility

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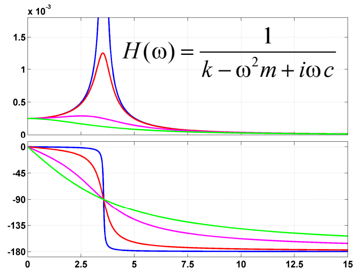
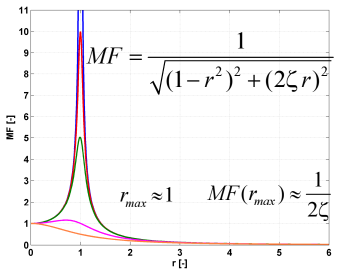
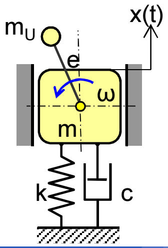
Lecture 8

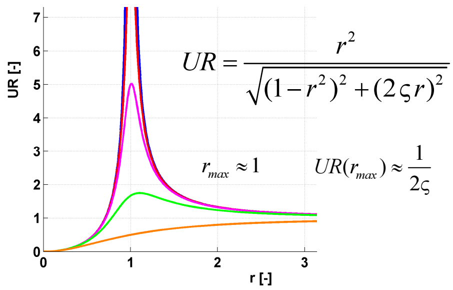
FRF





Unbalance response





Lecture 9

- Base motion excitation
- Displacement transmissibility
- Force transmissibility
- Example

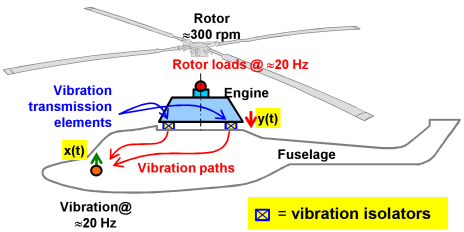
Introduction

In many cases vibrations are caused by the sources other than forces (road surface, pilot's seat vibration due to floor vibrations, etc.). It makes sense to introduce the concept of **vibration transmission**. Consider this problem:

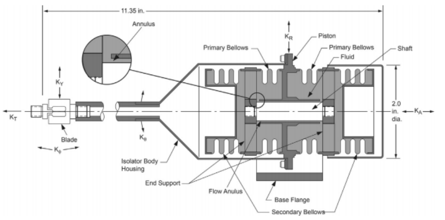
$$\text{Input } Y\exp(i\omega t) \longrightarrow \boxed{T(\omega) = ???} \longrightarrow \text{Response } X\exp(i\omega t)$$

Examples: vibration isolation in helicopters and space applications

Engine-gearbox isolator

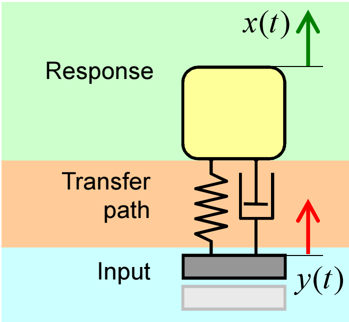


Reaction Wheel Assembly isolator



Davis et al., Advanced 1.5 Hz passive viscous isolation system, 1994

Base motion excitation model



Dynamic equilibrium condition:

$$-f_I + f_D + f_S = 0$$

$$m\ddot{x} - c(\dot{y} - \dot{x}) - k(y - x) = 0$$

$$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$$

Base excitation: y(t)=Y.sin(ωt), complex notation y(t)=Y.exp(iωt). This excitation causes the steady state response x(t)=X.exp(iωt), where X is a complex number. Substituting *y* and *x* into EOM gives:

$$-\omega^2 mXe^{i\omega t} + i\omega cXe^{i\omega t} + kXe^{i\omega t} = i\omega cYe^{i\omega t} + kYe^{i\omega t}$$

$$e^{i\pi}\omega^2 mXe^{i\omega t} + e^{i\pi/2}\omega cXe^{i\omega t} + kXe^{i\omega t} = e^{i\pi/2}\omega cYe^{i\omega t} + kYe^{i\omega t}$$

Base excitation

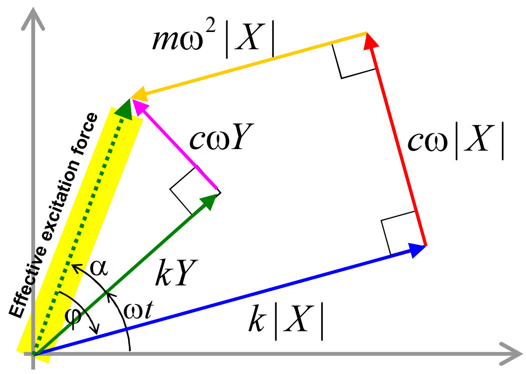
Mathematical and vector representation of the force equilibrium due to harmonic base motion:

$$\omega^2 mXe^{i(\omega t+\pi)} + \omega cXe^{i(\omega t+\pi/2)} + kXe^{i\omega t} = \omega cYe^{i(\omega t+\pi/2)} + kYe^{i\omega t}$$

The magnitude of excitation:

$$F_0 = \sqrt{(\omega cY)^2 + (kY)^2}$$

$$F_0 = Y\sqrt{(\omega c)^2 + k^2}$$



Steady state vibrations due to base motion

From the previous slides:

Input

$F_0 \exp(i\omega t) \rightarrow \boxed{|X| = |H(\omega)| F_0} \rightarrow x(t) = H(\omega) F_0 \exp(i\omega t)$

Response

$$F_0 = Y\sqrt{(\omega c)^2 + k^2}$$

$$|X| = |H(\omega)|(Y\sqrt{(\omega c)^2 + k^2}) = \frac{Y\sqrt{(\omega c)^2 + k^2}}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} = \frac{Y\sqrt{1 + (2\zeta r)^2}}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}}$$

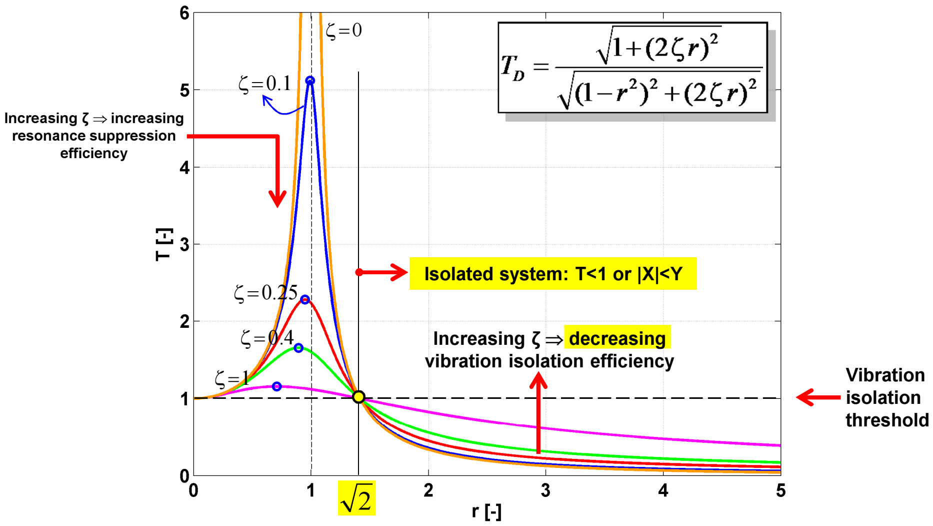
Displacement **Transmissibility**:

$$T_D = \frac{|X|}{Y} = \frac{\sqrt{1 + (2\zeta r)^2}}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}}$$

```
» m=8; k=4000; ze=0.05;
» r=linspace(0,8,1e3);
» Td=sqrt(1+(2*ze*r).^2)./sqrt((1-r.^2).^2+(2*ze*r).^2);
» figure, plot(r,Td), grid
```

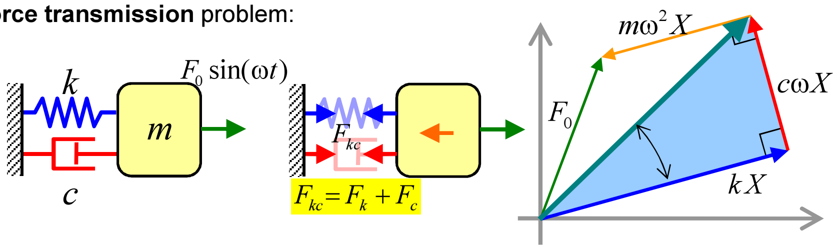
$$T_D(0) = 1,\ T_D(r \to \infty) = 0,\ T_D(r{=}1, \zeta{=}0) \to \infty,\ T_D(r = \sqrt{2}) = 1$$

Transmissibility



Force transmissibility

Consider the harmonic excitation and observe the **force** transmitted to base – the **force transmission** problem:



The magnitude of the force transmitted through the spring and damper is:

$$|F_{kc}| = \sqrt{(\omega c X)^2 + (kX)^2} = \left(\sqrt{(\omega c)^2 + k^2}\right) X = \left(\sqrt{(\omega c)^2 + k^2}\right) |H| F_0$$

Based on this equation we introduce the **force transmissibility**:

$$T_F = \frac{|F_{kc}|}{F_0} = \frac{\sqrt{(\omega c)^2 + k^2}}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} = \frac{\sqrt{1 + (2\zeta r)^2}}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}} = T_D \equiv T$$

This shows that the previous conclusions regarding the displacement transmissibility can be directly applied in the case of the force transmitted to base.

Example 11: Vibration transmission

[SG Kelly; Mechanical vibrations, Problem 3.26]

A 35-kg flow monitoring device is placed on a table in a lab. A pad of stiffness 2×10^5 N/m and damping ratio 0.08 is placed between the apparatus and the table. The table is bolted to the floor. The floor has a steady-state vibration amplitude of 0.5 mm at the frequency of 30 Hz. What is the amplitude of acceleration of the monitoring device?

The natural frequency and frequency ratio are:

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{2 \times 10^5\ N/m}{35\,kg}} = 75.6\,rad/s \quad \Rightarrow r_{op} = \frac{\omega_{op}}{\omega_0} = \frac{2\pi(30\ Hz)}{75.6\,rad/s} = 2.49$$

The amplitude of absolute displacement is:

$$X = T(r,\zeta)Y = (0.0005\,m)\sqrt{\frac{1 + (2(0.08)(2.49))^2}{(1-(2.49)^2)^2 + (2(0.08)(2.49))^2}} = 1.03 \times 10^4\ m.$$

The acceleration amplitude is ($d^2x/dt^2 = \omega^2 x(t)$):

$$A = \omega_{op}^2 X = (2\pi(30\,Hz))^2 (1.03 \times 10^4\ m) = 3.66\,m/s^2.$$

Summary

- Transmissibility:
  - Displacement transmissibility:
    - Output displacement / input displacement
  - Force transmissibility:
    - Output force / input force

- Transmissibility depends on:
  - damping and frequency ratio

- Transmissibility properties:
  - $T<1$ for $r \geq \sqrt{2}$ for all ζ (output < input; vibration isolation)
  - vibration isolation and resonance suppression trade-off
  - see graphs $T=T(r,\zeta)$