Topic III - Energy Balances

Polytropic Processes (Lecture 2/4)

Contents

5. Polytropic processes

An example (compares processes)

Objectives:

Follows last lectures discussion of constant volume/ temperature/ pressure processes. Feeds into **Non-Flow Energy Eqn**

Applications:

Processes listed here apply to engine cycles (Topic 5)

5 Polytropic Processes

By observation:

$$pV^n = \text{constant} \ldots = p_1 V_1^n = p_2 V_2^n \quad (5)$$

Term n is an **index of expansion or compression**

- Constant pressure, $n = 0$
- Constant volume, $n = \text{infinity}$
- Constant temperature, $n = 1$
- Frictionless and adiabatic, $n \rightarrow \gamma = c_p/c_v$ (Lecture 4.4)

Slope of path

$$\frac{dp}{dV} = -\frac{n\, const\; V^{-n}}{V} = -\frac{n\,p}{V} \quad (6)$$

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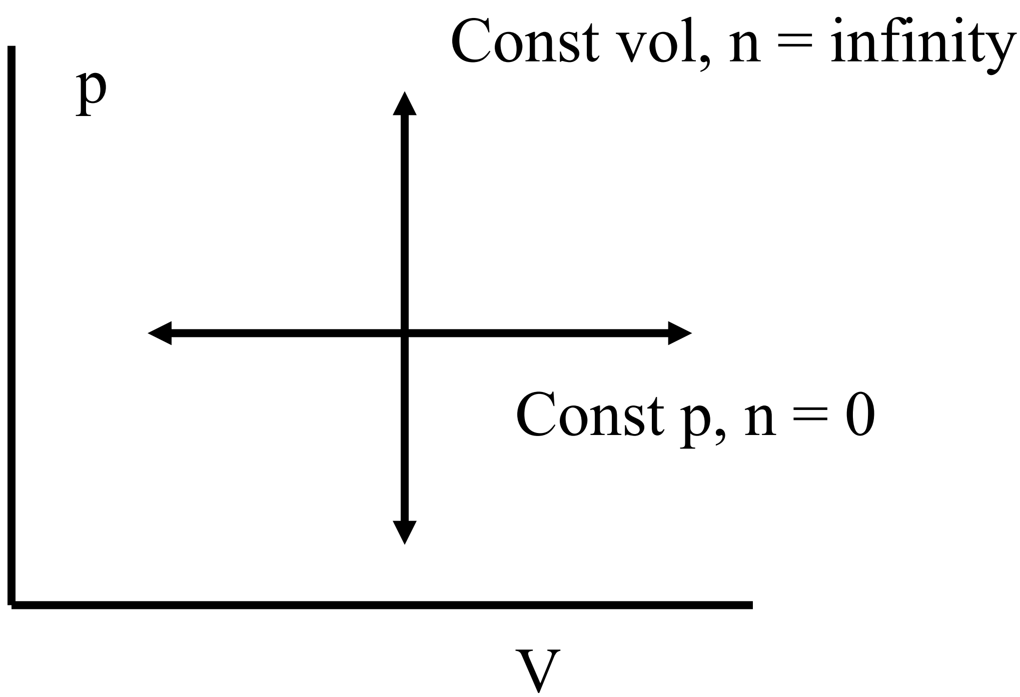
Polytropic Processes (Lecture 2/4)

Plot of p-versus-V



If $p V^n$ = constant, and we add the ideal gas law $pV = m R T$

<u>Also</u>

$p V^n = (pV) V^{n-1} = (mRT) V^{n-1}$ = constant

$T V^{n-1}$ = constant

Also $p = \text{constant}\ T^{n/(n-1)}$

5 Polytropic Processes

Note that in writing

$$pV^n = \text{constant}$$

no gas laws are used. Can integrate p w.r.t. V to get work.

$$W_b = \frac{p_2 V_2 - p_1 V_1}{n-1} = \frac{mR(T_2 - T_1)}{n-1} \quad (7)$$

Isothermal case, n = 1 is a special case: both the denominator and numerator are zero. Redo integration, referring back to last week.

Proof

Rewrite (5) as $p = C V^{-n}$ (6)

$$W_b = -\int_1^2 p\,dV = -\int_1^2 C V^{-n}\,dV \;\ldots$$

$$= -\frac{\left[C\,V^{-n+1}\right]_1^2}{1-n} = \frac{\left[(C V^{-n})V\right]_1^2}{n-1}$$

Term in [ ] is p = C V^{-n} . pV = m RT gives final part of Eqn (6)

$$W_b = \frac{p_2 V_2 - p_1 V_1}{n-1} = \frac{m R (T_2 - T_1)}{n-1}$$

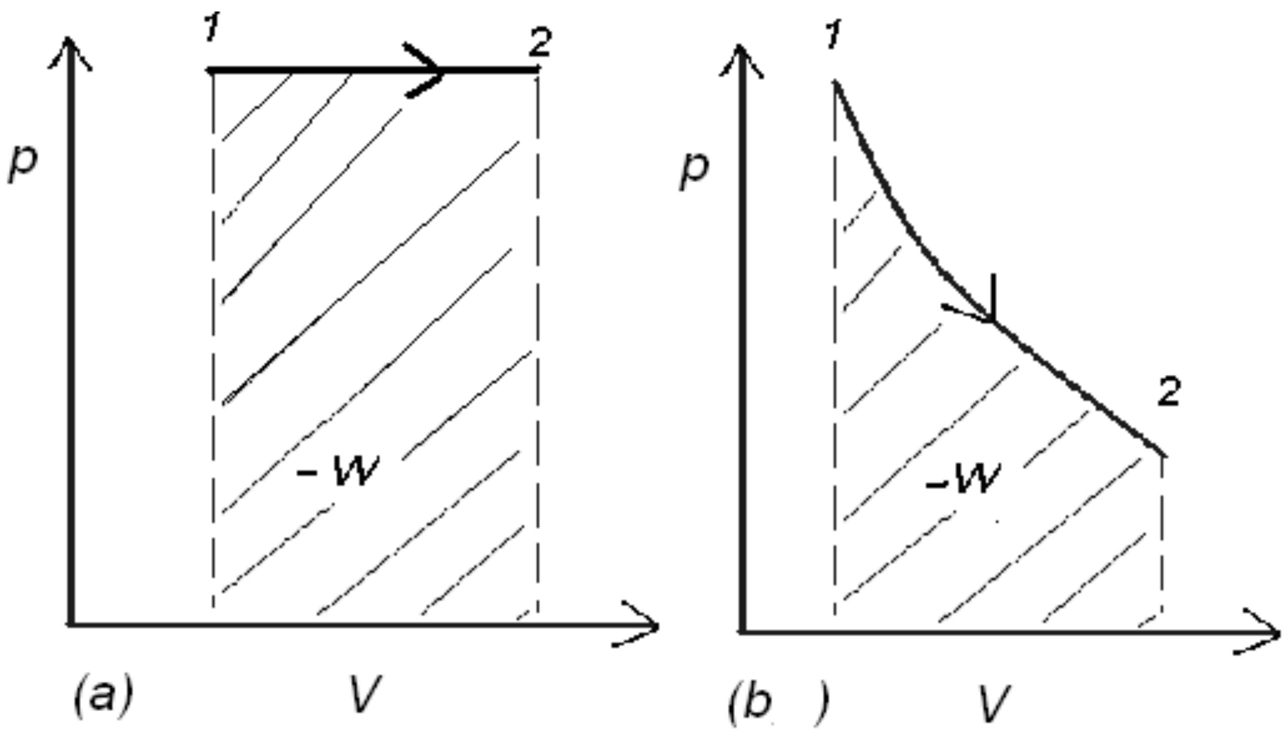
Q.E.D.

5a Isentropic Expansion

… frictionless and adiabatic, one can show

$$p V^{\gamma} = const \qquad ; \qquad \gamma = \frac{c_p}{c_v}$$ (topic notes)

<u>Example</u>: Consider a piston-cylinder for which the start conditions are V_1 = 250 cm^3, p_1 = 6 bar and the end volume is V_2 = 1000 cm^3. What is the boundary work for (a) constant pressure expansion (b) isothermal expansion (c) polytropic expansion (n = 1.3)? (See notes for solution)



Conclusions:

Remember form, $p V^n$ = constant

Can integrate $-p\,dV$ to get work

Thereupon, $Q = \Delta U - W$

Limits

- $n = 0$; constant pressure
- n = infinity : constant volume
- $n = 1$: constant temperature
- $n \to \gamma = c_p/c_v = 1.4$ (air): isentropic

(Lecture 4.4 deals more with isentropic processes)