Example 2.1.1

a) A plastic rod of solid circular cross-section with diameter 30 mm and length 0.5 m is subjected to a tensile load of 12 kN as shown in Figure 1. The rod is made of PMMA (polymethyl methacrylate, or 'acrylic') with a Young's modulus of 3.1 GPa. Calculate the elongation e_a of the rod.

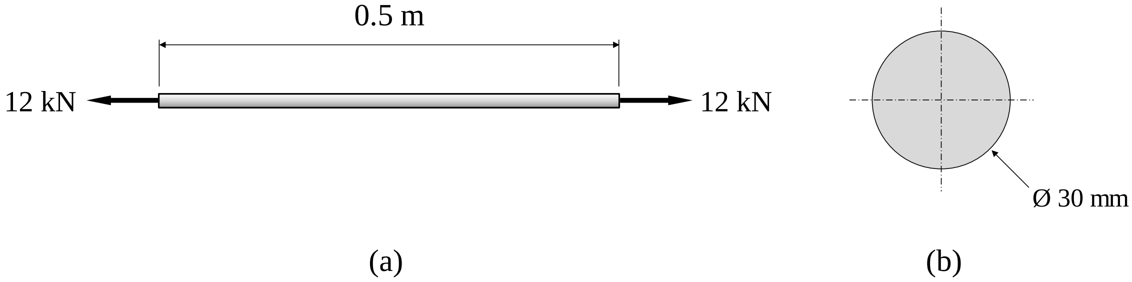

Figure 1: A plastic rod loaded in tension.

b) In order to increase the axial stiffness, engineers decided to adhesively-bond a sleeve on to the rod as shown in Figure 2. The sleeve has an outer diameter of 45 mm, length of 0.3 m and is made of polyamide (or 'Nylon') with a Young's modulus of 2.5 GPa. Calculate the total elongation e_b of the assembly when subjected to the same tensile load of 12 kN. Assume that no slippage can occur between the rod and sleeve.

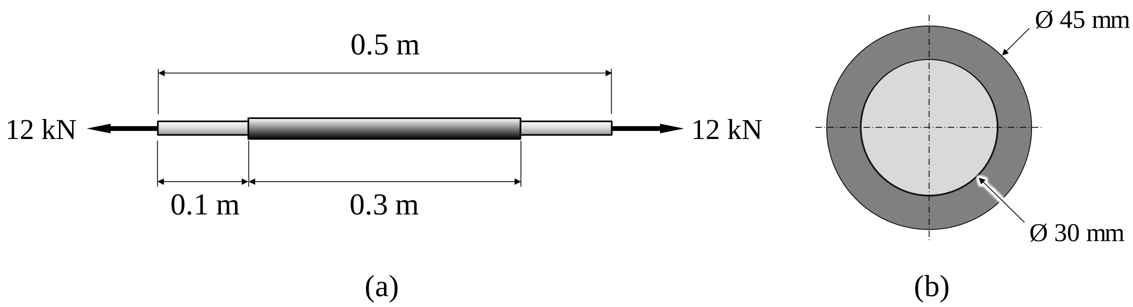

Figure 2: The plastic rod with a bonded sleeve.

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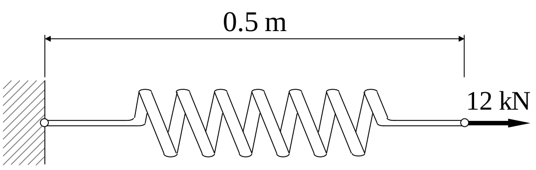
a) This axial member behaves as a single spring of stiffness λ :

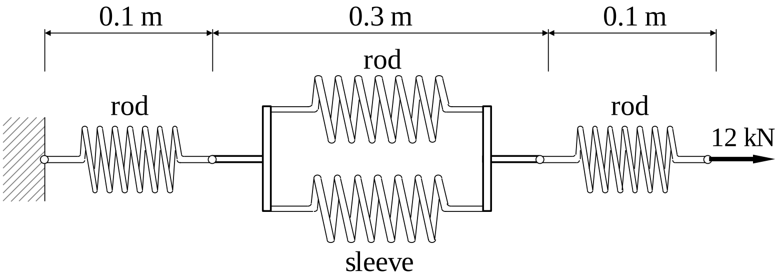


$$F = \lambda\, e \qquad \therefore \qquad e = \frac{F}{\lambda}$$

The stiffness λ is given by: $\lambda_a = \dfrac{E\,A}{L} = \dfrac{E\left(\pi\, r^2\right)}{L} = \left(\dfrac{1}{500\text{ mm}}\right)\left(3100\,\dfrac{\text{N}}{\text{mm}^2}\right)\pi\,(15\text{ mm})^2 \quad \therefore \quad \boxed{\lambda_a = 4382.5\,\dfrac{\text{N}}{\text{mm}}}$

The elongation is therefore: $e_a = \dfrac{F}{\lambda_a} = \dfrac{12\,000\text{ N}}{4382.5\text{ N/mm}} \qquad \therefore \qquad \boxed{e_a = 2.738\text{ mm}}$

b) As the member is now made of two different cross-sections (with or without the sleeve), we can treat it as an assembly of springs:



(Remember the rules for springs in **series** and in **parallel** discussed in lectures.)

Both 'unsleeved' ends behave as springs of length 100 mm:

$$\lambda_1 = \frac{E\left(\pi\, r^2\right)}{L} = \left(\frac{1}{100\text{ mm}}\right)\left(3100\,\frac{\text{N}}{\text{mm}^2}\right)\pi\left(15\text{ mm}\right)^2 \quad \therefore \qquad \lambda_1 = 21912.6\,\frac{\text{N}}{\text{mm}}$$

The sleeved section behaves as two springs in **parallel**:

$$\begin{aligned}\lambda_2 &= \lambda_{\text{rod}} + \lambda_{\text{sleeve}} \\ &= \frac{E_{\text{sleeve}}\left[\pi\left(r_{\text{outer}}^2 - r_{\text{inner}}^2\right)\right]}{L} + \frac{E_{\text{rod}}\left(\pi\, r^2\right)}{L} \\ &= \left(\frac{\pi}{300\text{ mm}}\right)\left(2500\,\frac{\text{N}}{\text{mm}^2}\right)\left[\left(22.5\text{ mm}\right)^2 - \left(15\text{ mm}\right)^2\right] + \left(\frac{\pi}{300\text{ mm}}\right)\left(3100\,\frac{\text{N}}{\text{mm}^2}\right)\pi\left(15\text{ mm}\right)^2 \\ &= \left(7363.1\,\frac{\text{N}}{\text{mm}}\right) + \left(7304.3\,\frac{\text{N}}{\text{mm}}\right)\end{aligned}$$

$$\lambda_2 = 14667.3\,\frac{\text{N}}{\text{mm}}$$

The sleeved and the two unsleeved sections behave as three springs in **series**, therefore:

$$\begin{aligned}e_b &= 2\,e_1 + e_2 \\ &= 2\,\frac{F}{\lambda_1} + \frac{F}{\lambda_2} \\ &= 2\,\frac{12000\text{ N}}{21912.6\text{ N/mm}} + \frac{12000\text{ N}}{14667.3\text{ N/mm}} \\ &= 2\left(0.547\text{ mm}\right) + \left(0.818\text{ mm}\right)\end{aligned}$$

$$e_b = 1.913\text{ mm}$$