Applied Statistics

Lectures 16

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Outline

- Monte Carlo methods

Monte-Carlo methods

If you are struggling to understand statistics, generate a few random numbers and calculate the resulting statistics — with MATLAB (or similar) not by hand. . .

Monte-Carlo methods are easy to use.

- Generate a some random numbers according to the distribution you want.
- Put them into your model.
- Calculate the desired statistics.
- (Repeat several million times!)

Monte Carlo

In its modern form it originated in the 1940s from the nuclear weapons programme — inspired by calculating how often you are likely to win at Canfield Solitaire!

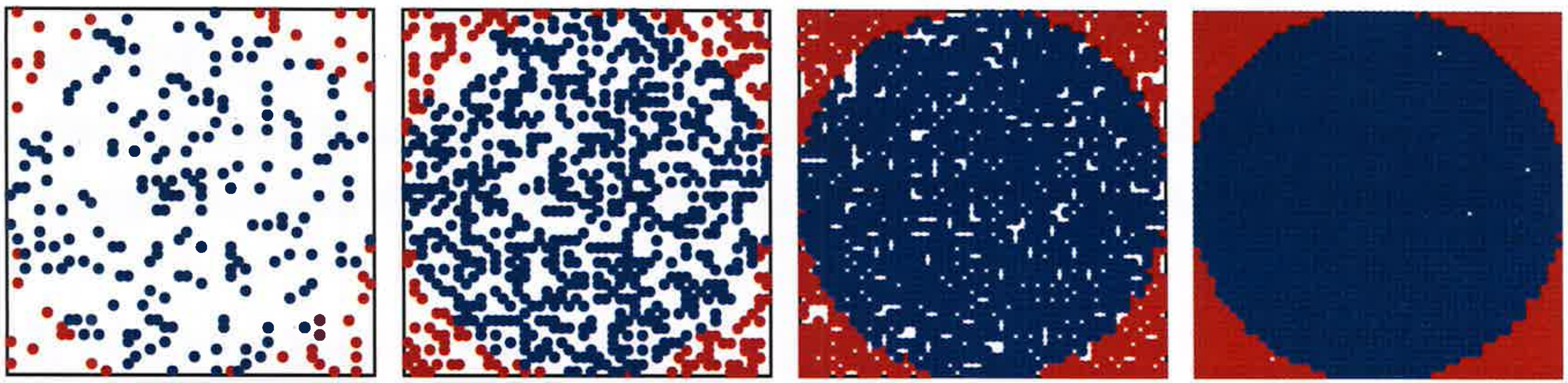
Monaco

Monte Carlo Casino
[Wikipedia]

A simple illustration

x, y $x^2 + y^2 \le 1$

$$\frac{\text{Area of a circle}}{\text{Area of a square}} = \frac{\pi r^2}{(2r)(2r)} = \frac{\pi}{4}$$



$n = 250$ $n = 1000$ $n = 4000$ $n = 16000$

Law of large numbers

While nothing is more uncertain than the duration of a single life, nothing is more certain than the average duration of a thousand lives.

Elizur Wright (1804–1885)
"father of life insurance"

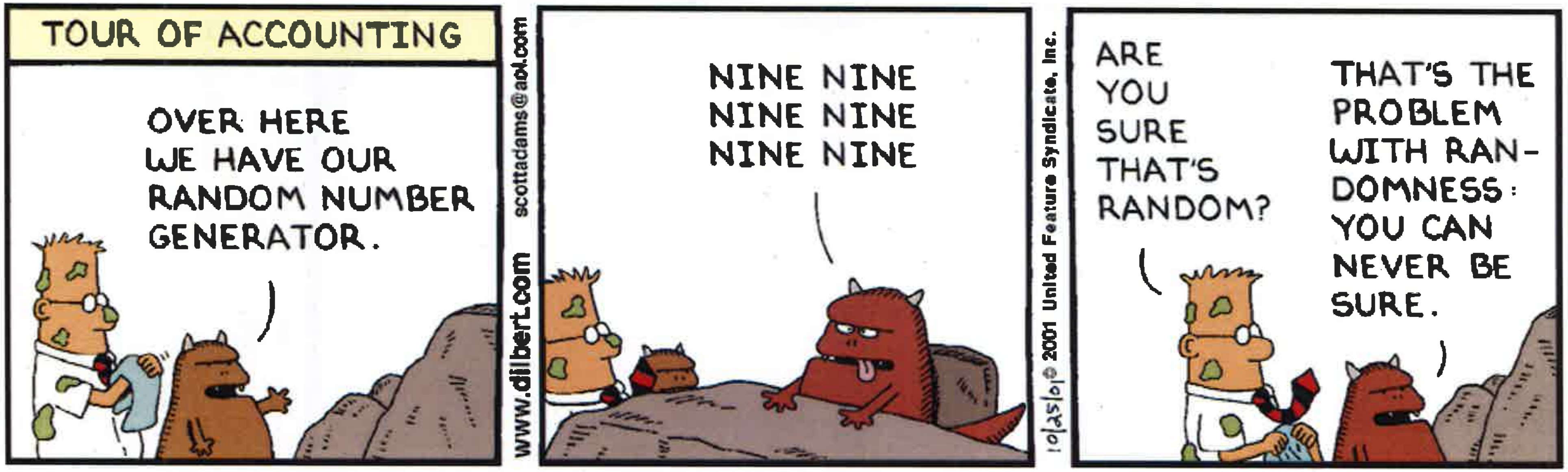
The weak *law of large numbers* states

Theorem (Law of Large Numbers)

Given $X_1, X_2, \ldots,$ *is an infinite sequence of random variables with the same mean* $E[X_1] = E[X_2] = \cdots = \mu$ *then, for any* $\varepsilon > 0$, *we have*

$$P(|\bar{X}_n - \mu| > \varepsilon) \to 0 \text{ as } n \to \infty$$

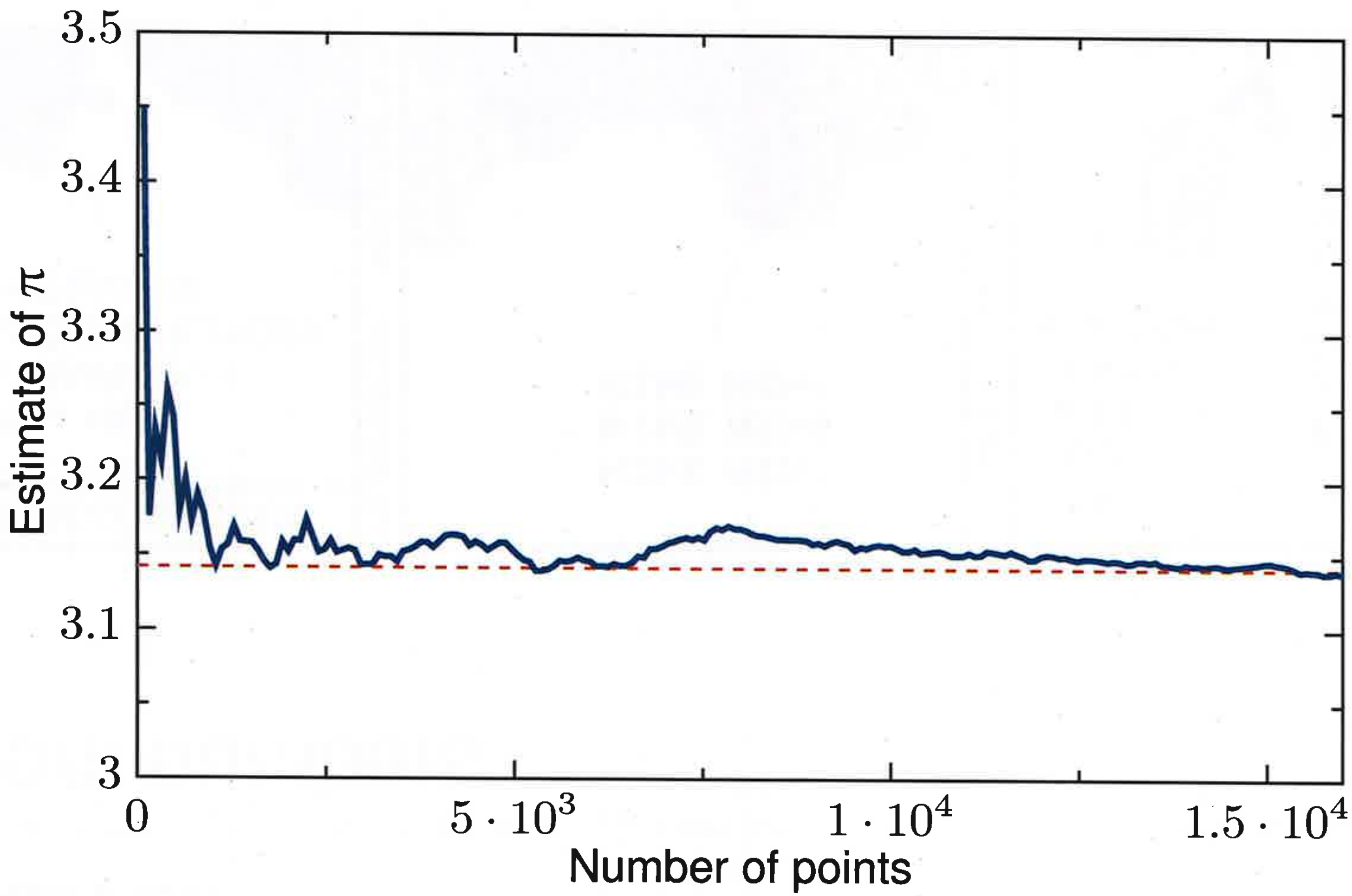
Random numbers



Always be wary of random numbers —

you don't always get the results you expect!

Convergence



- Monte-Carlo methods have slow convergence!

Variance reduction

- Though by the law of large numbers Monte-Carlo methods will converge, they are slow.
- Speed up convergence with *variance reduction methods*
  - Antithetic variates (easiest and can work well)
  - Control variates
  - Low-discrepancy numbers
  - Latin hypercube
  - Importance sampling
- Deterministic (non-random) methods for investigating uncertainty
  - Polynomial chaos
  - Stochastic collocation
- Deterministic methods often suffer from the curse of dimensionality
  - (Smolyak) sparse grids help a lot