Sustained Hover (2)
(the unique ability of the rotorcraft!)
Lecture 5

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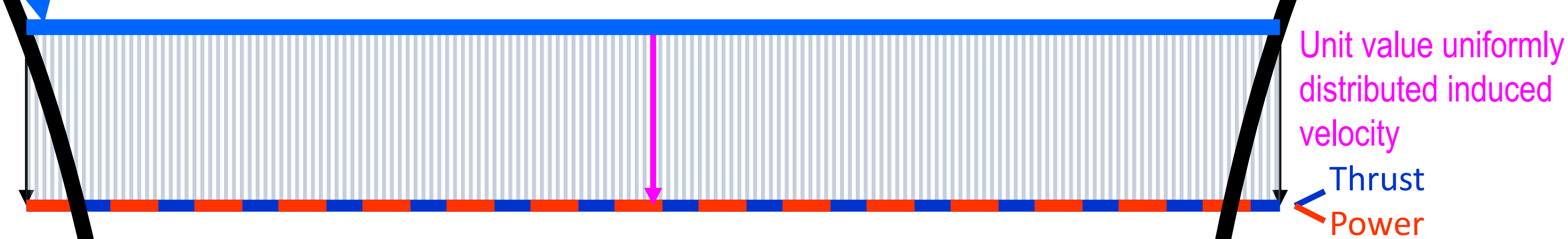
Hover

- Maximising Figure of Merit
  - Induced velocity distribution (recap)
  - Ideal Twist
  - Blade profile drag and rotor solidity
  - Rotor blade tip loss factors
- A note on Coning angle
- A note on Ground Effects

Maximising the Figure of Merit

An ideal rotor (M = unity) requires an actuator disk condition of infinite (zero loss) blades and a zero loss flow state.

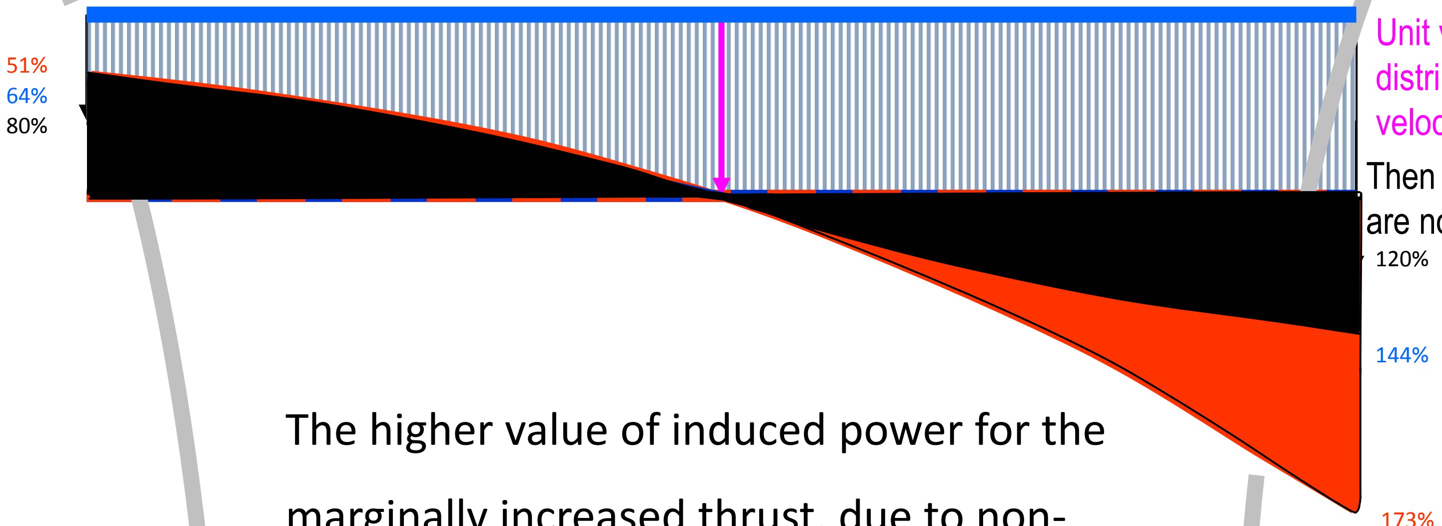


A rotor generates thrust by imparting momentum to the air that flows through it. This can only be efficiently achieved by a uniform distribution of induced velocity as assumed in actuator disk theory. Any local variation in the magnitude of the induced velocity will increase the overall power requirement as:

$$P = Tv = \left(2\rho A v^2\right)v = 2\rho A v^3$$

Maximising the Figure of Merit

Suppose V is 20% less on the LHS, 20% higher on the RHS



Unit value triangularly distributed induced velocity

Then Thrust & Power are no longer constant

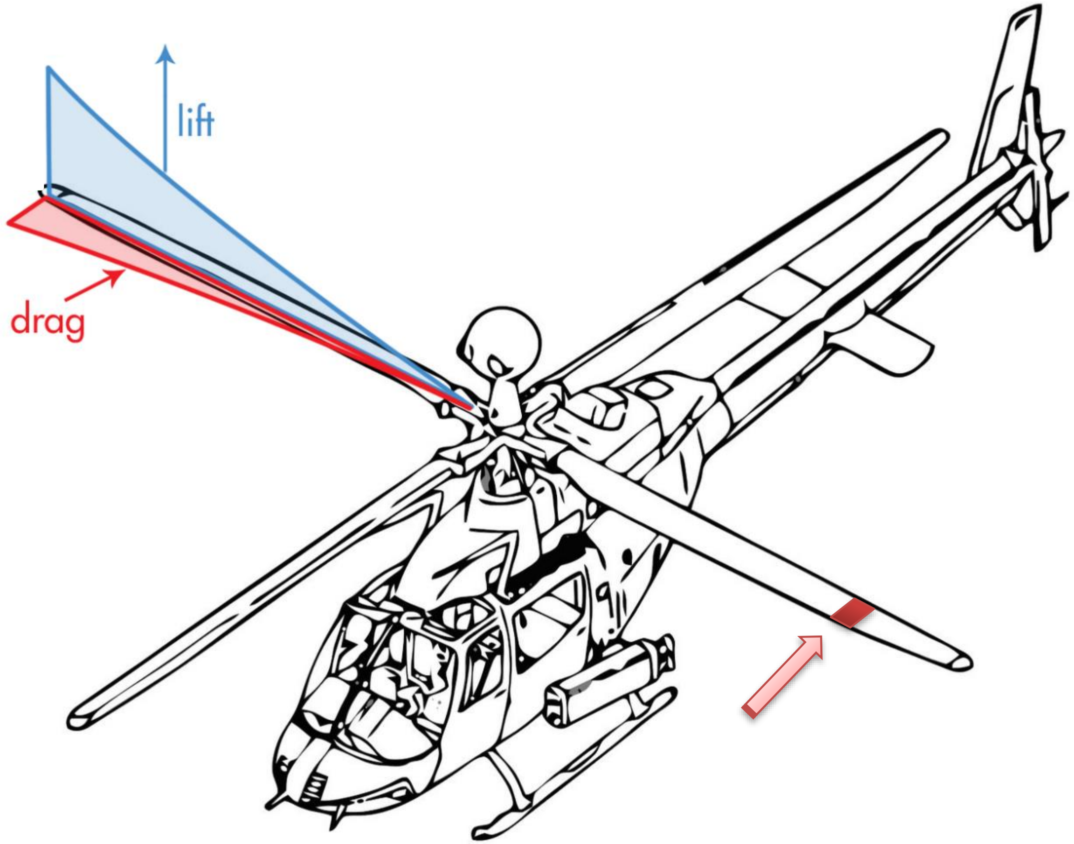
Thrust

Power

The higher value of induced power for the marginally increased thrust, due to non-uniform inflow velocity (v) is seen from the plot of excess power which is effectively a

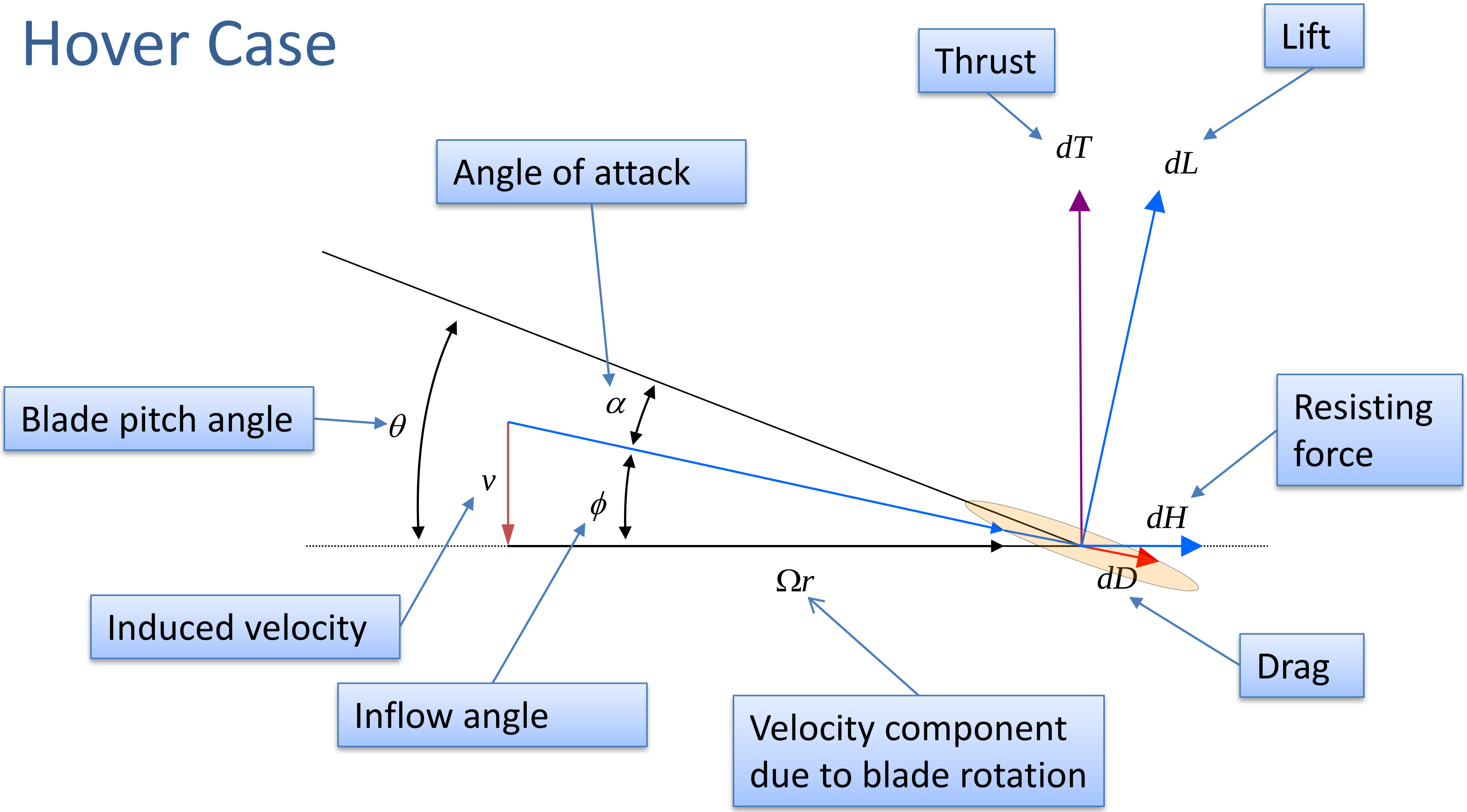
. Power Loss

Forces Acting on the Blade



Forces at a Blade Element

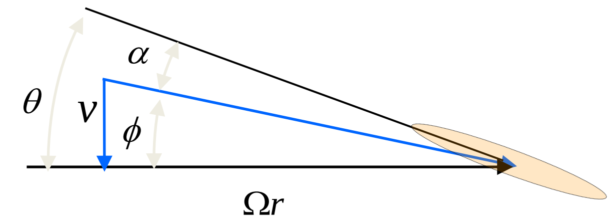
Hover Case



Maximising the Figure of Merit

IDEAL BLADE TWIST results in a constant induced velocity across the rotor disk.

It has been seen that for this to be the case, then $\alpha_r \propto \dfrac{1}{r}$



For this to be the case, $\alpha_r = (\theta - \phi) = \dfrac{R}{r}(\theta_t - \phi_t)$

So, $\phi = \phi_t \dfrac{R}{r}$ where ϕ_t = inflow angle at the tip.

Similarly blade pitch angle $\theta = \theta_t \dfrac{R}{r}$ where θ_t =pitch angle at the tip.

Thus, by careful design, the ideal inflow can be achieved by blade twist, or blade planform taper or a combination of the two.

Unfortunately the other requirements for ideal conditions (zero profile drag, tip losses and swirl in the wake) are not so easily met and must, at best, be minimised.

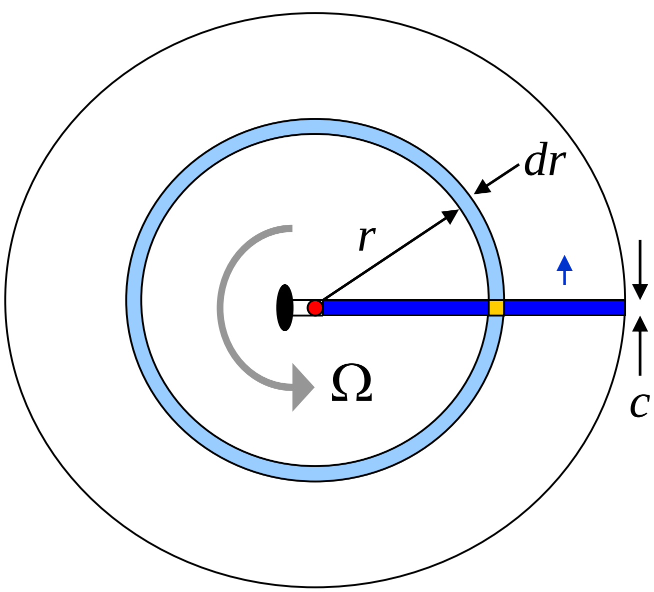
This is best achieved by utilising rotor blade element analysis.

Maximising the Figure of Merit:

Thrust Coefficient for Rotor with Ideally Twisted blades

$$dL = \frac{1}{2}\rho(\Omega r)^2 a \frac{R}{r}(\theta_t - \phi_t)cdr$$

$$L = \int_0^R \frac{N}{2}\rho\Omega^2 rRa(\theta_t - \phi_t)cdr = \frac{N}{4}\rho\Omega^2 R^3 a(\theta_t - \phi_t)c \quad (\approx T)$$



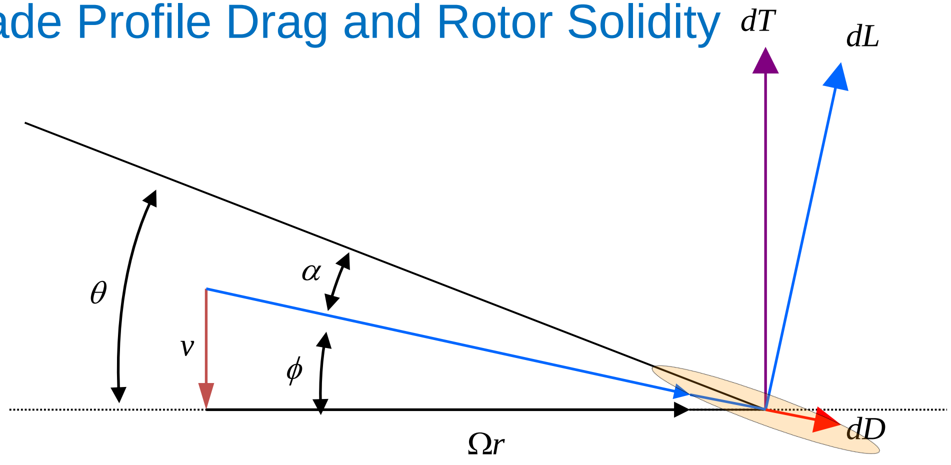
$$C_T = \frac{T}{\rho A(\Omega R)^2}$$

$$= \frac{N}{4}\frac{\rho\Omega^2 aR^3(\theta_t - \phi_t)c}{\rho\pi\Omega^2 R^4}$$

$$= \frac{Na(\theta_t - \phi_t)c}{4\pi R}$$

or, $C_T = \frac{\sigma}{4}a(\theta_t - \phi_t)$, since $\sigma = \frac{Nc}{\pi R}$

Maximising the Figure of Merit: Effect of Blade Profile Drag and Rotor Solidity



The rotor blade element of drag is composed of two components; the profile drag and the induced drag. The resultant drag in the plane of the rotor is:

$$dD\cos\phi + dL\sin\phi$$

Since ϕ is small this can be written:

$dD + dL\phi$, or, in coefficient form as: $C_{d_0} + \phi C_l$

Thus the in-plane drag torque due to this element is:

$$dQ = \frac{N}{2}\rho(\Omega r)^2 c(C_{d_0} + \phi C_l) r dr$$

Assuming that $C_{d_0} = \delta$ is relatively constant over the range of α , then C_{d_0} (a constant).

It was also shown that: $c_l = a\frac{R}{r}(\theta_t - \phi_t)$ and $\phi = \phi_t \frac{R}{r}$

$$\text{thus, } Q = \int_r^R \frac{N}{2}\rho\Omega^2 r^3 c\left[\delta + \phi_t \frac{R^2}{r^2}(\theta_t - \phi_t)a\right]dr, \text{ so } Q = \frac{N}{4}\rho\Omega^2 R^4 c\left[\frac{\delta}{2} + a\phi_t(\theta_t - \phi_t)\right]$$

$$\text{or, } C_Q = \frac{\sigma\delta}{8} + \phi_t C_T = \frac{C_T^{\frac{3}{2}}}{\sqrt{2}} + \frac{\sigma\delta}{8} \qquad \left[\text{since } \phi_t = \sqrt{\frac{C_t}{2}}\right]$$

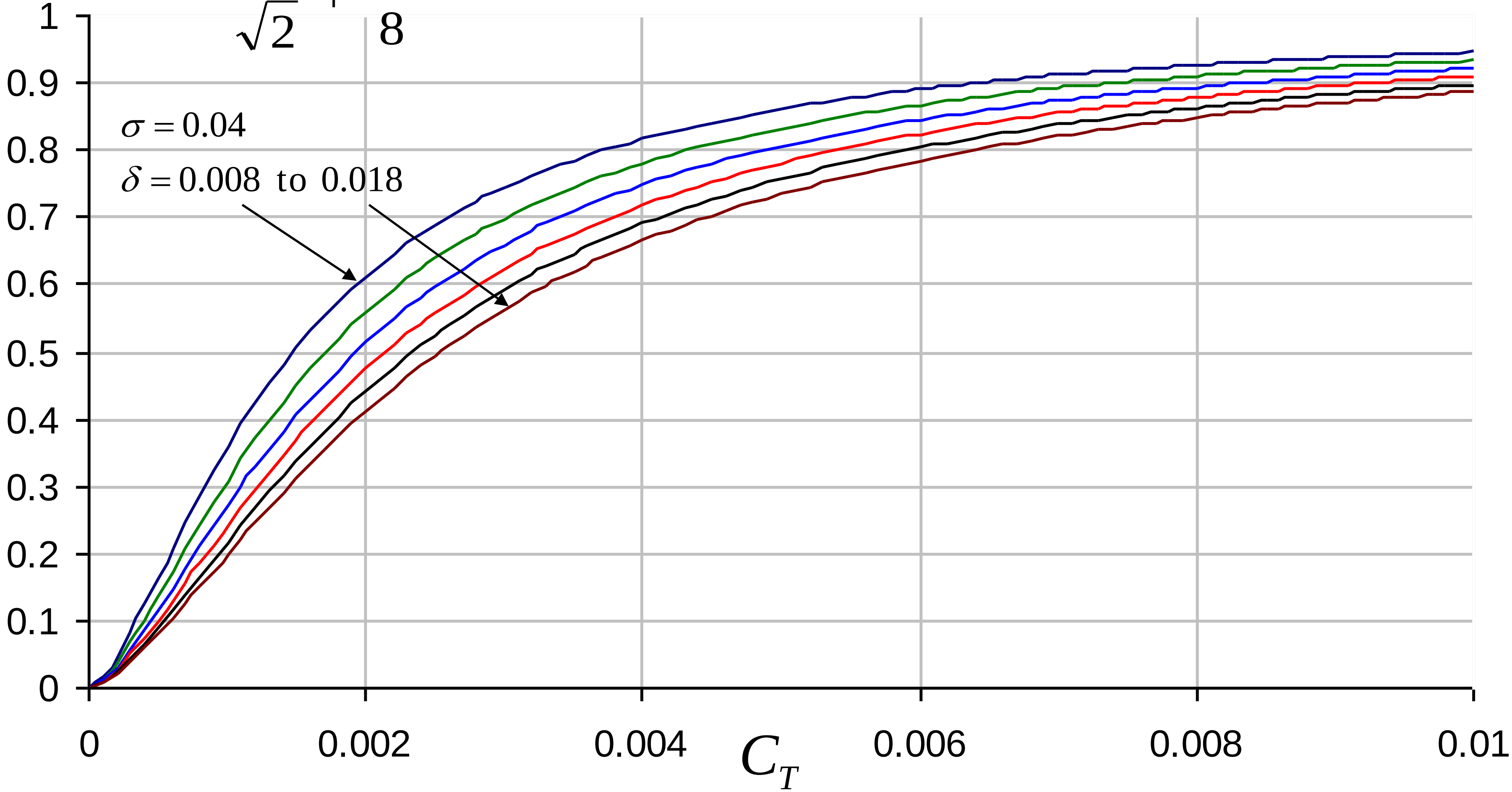
Hence

$$\boxed{M = 0.707\frac{C_T^{\frac{3}{2}}}{\frac{C_T^{\frac{3}{2}}}{\sqrt{2}} + \frac{\sigma\delta}{8}}}$$

Maximising the Figure of Merit

$$M = 0.707 \frac{C_T^{\frac{3}{2}}}{\frac{C_T^{\frac{3}{2}}}{\sqrt{2}} + \frac{\sigma\delta}{8}}$$

Figure of Merit against Coefficient of Thrust for a range of delta's (aerofoil profile drag coefficients) based on a fixed value of rotor solidity ($\sigma = 0.04$).



In the hover, again using small angle approximations, then $L = T = W$ (the weight of the aircraft).

This is the summation of all the blade elemental lift forces:

$$W = L = \underbrace{\int_0^R \frac{N}{2}\rho(\Omega r)^2 C_l c\,dr}_{\text{Total lift generated on blades}} = \underbrace{\bar{C}_L \int_0^R \frac{N}{2}\rho(\Omega r)^2 c\,dr}_{\text{Total lift generated on blades based upon mean value of } C_L} = \underbrace{T = C_T \pi R^2 \rho(\Omega r)^2}_{\text{Total thrust generated by rotor}}$$

(where $\bar{C}_L$ is the mean lift coefficient)

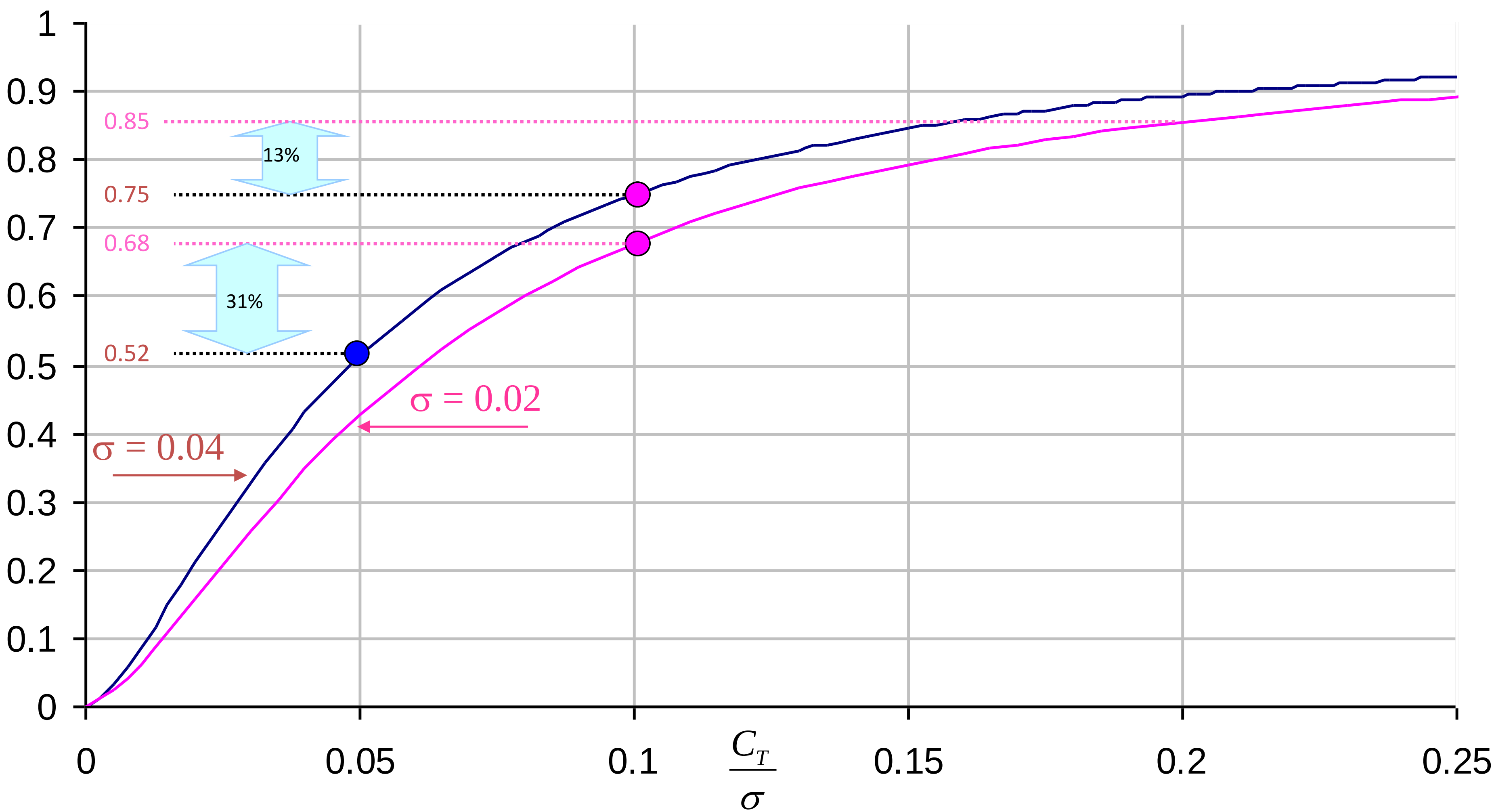
Thus $\frac{1}{6}\bar{C}_L\, \rho\Omega^2 R^3 N c = C_T \pi R^2 \rho(\Omega R)^2$

Therefore $\boxed{\bar{C}_L = 6\frac{C_T}{\sigma}}$ This "equivalent" fixed wing lift coefficient gives an indication of *"how hard the rotor blades are working".*

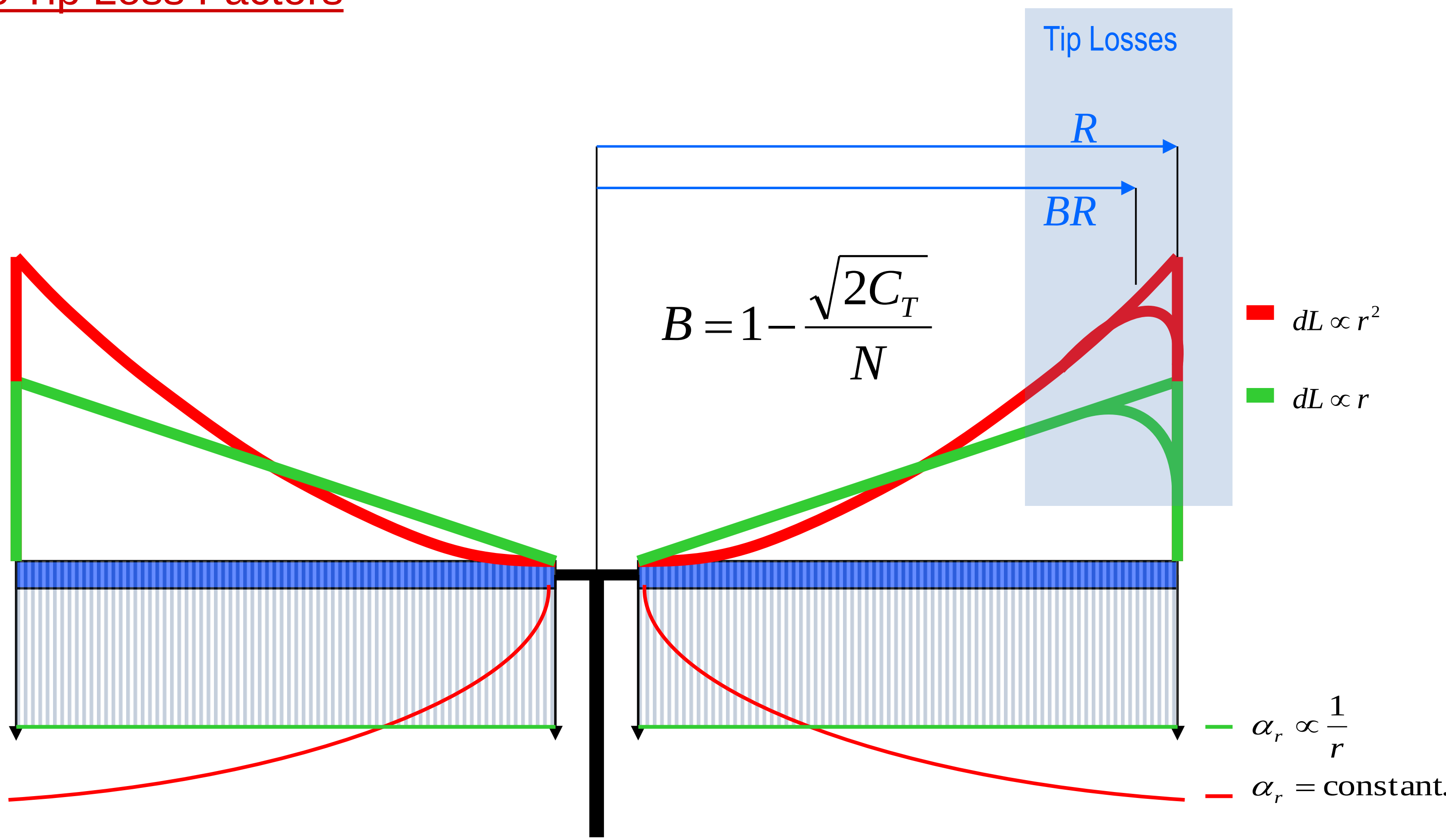
When plotted against the Figure of Merit the effect of solidity becomes apparent.

Maximising the Figure of Merit

Reducing rotor solidity increases the FoM but with diminishing effect at higher C_T/σ.

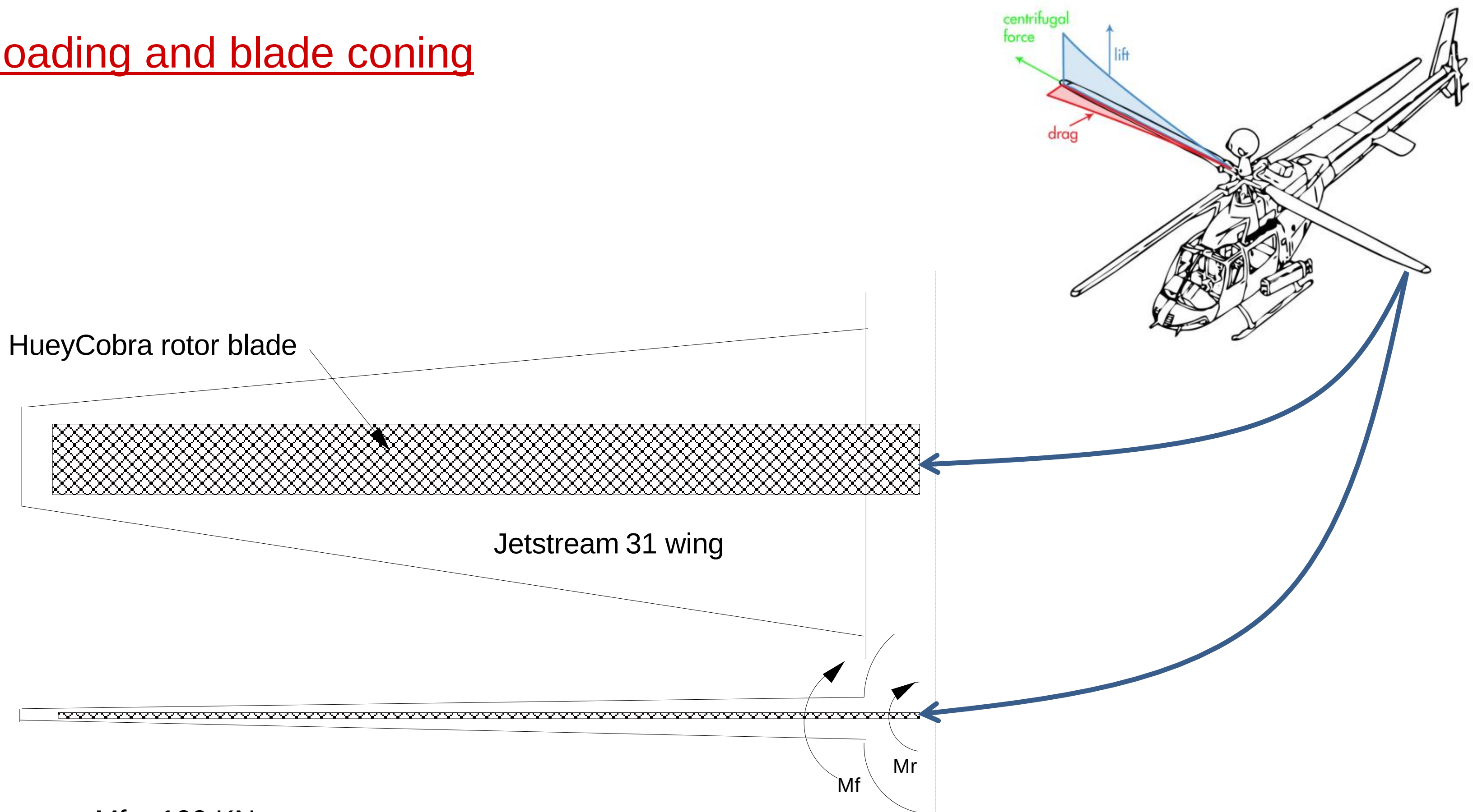


Rotor Blade Tip Loss Factors



Unlike the actuator disk, a real rotor blade cannot support lift right out to the blade tip. A tip loss factor B (usually 0.95 -0.97) can be used in analysis whereby it is assumed that blade drag exists over the entire blade length but no lift is generated outboard of radius BR,

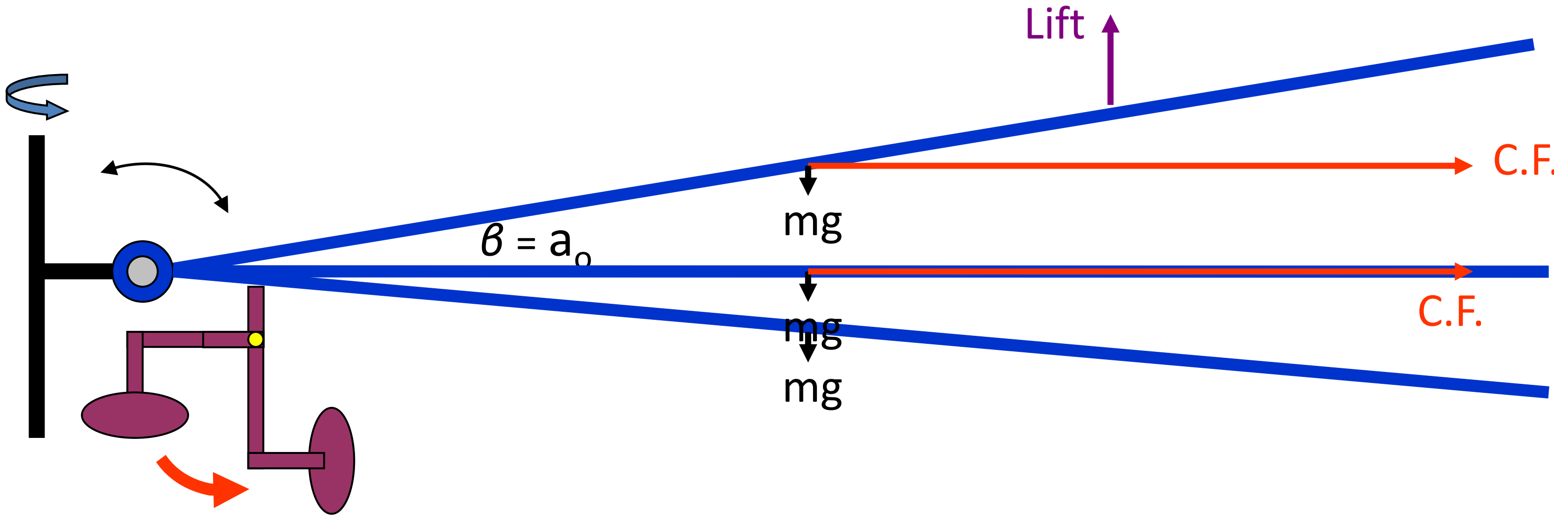
Blade loading and blade coning



Mf = 100 KNm

Mr = 0 KNm

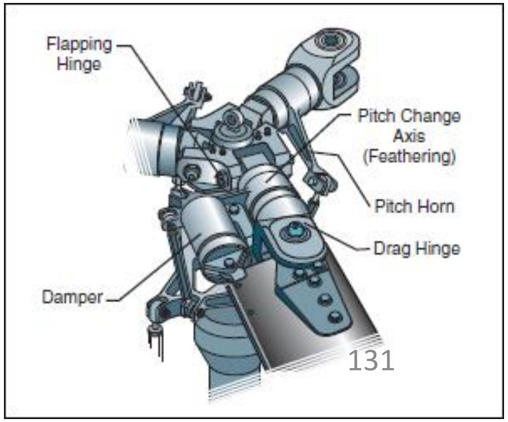
Generally, the rotor blade has the facility to freely flap about a hinge at the rotor hub. To prevent the blade drooping too much in the static case, droop stops are usually employed.



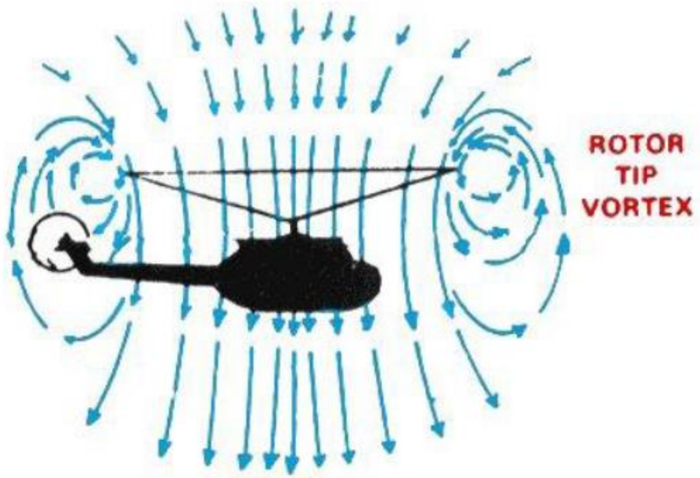
As the rotor starts to turn, the C.F. lifts the blades off the droop stop.

With increasing rotor speed the droop stops are automatically withdrawn.

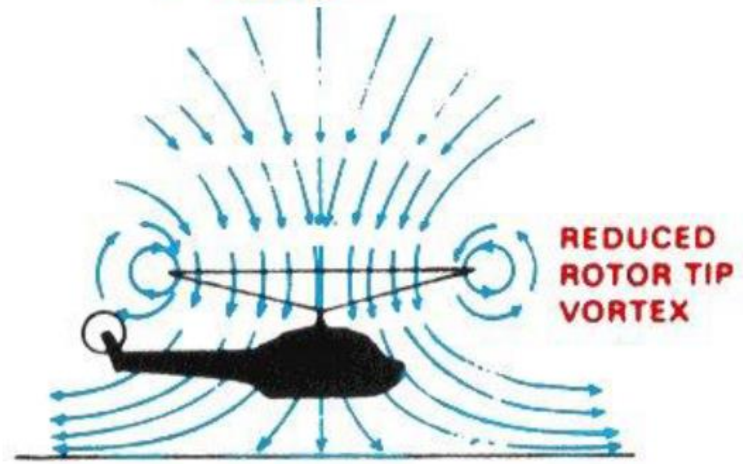
Application of collective pitch causes a lift force to be generated in addition to the existing C.F. force and blade weight. The equalisation of these three blade forces results in a shallow cone which subtends the plane of rotation at an angle (a_0). For a steady state condition (such as hover in zero wind) this is the only component of the blade flapping angle (β).



Ground Effects

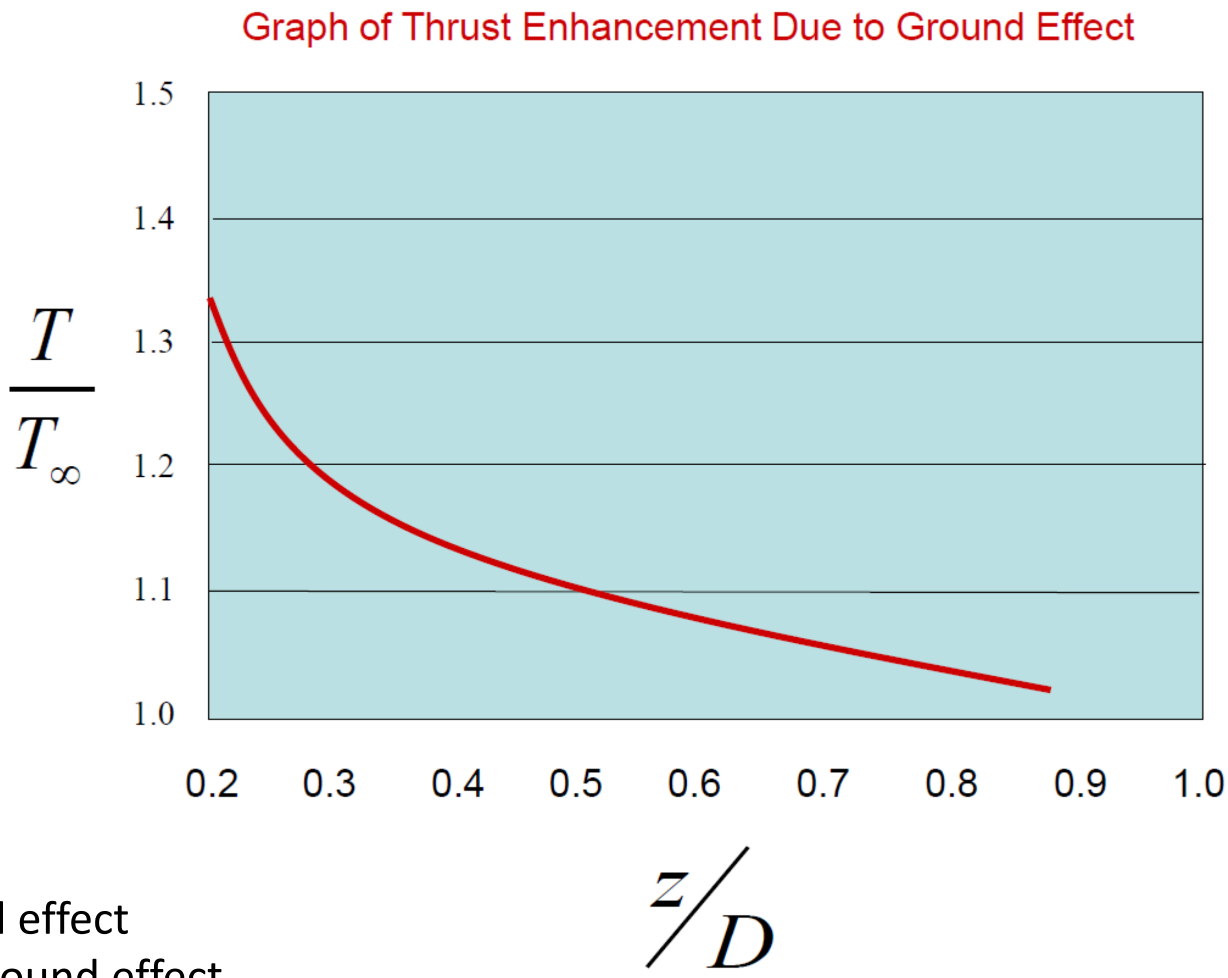


Schematic of a helicopter hovering as described in lectures to date. This is referred to as "operating Out-of-Ground Effect (OGE)"



Schematic of a helicopter hovering close to the ground and benefiting from the reduced induced power requirement. This is referred to as "operating In-Ground Effect (IGE)"

Ground Effects



T: Thrust with ground effect

***T_∞*:** Thrust without ground effect

z: height of rotor from ground

D: rotor diameter

Exercise (you can get the answers in most classical helicopter text books)

Find out what is meant by the “induced power correction factor” and how it is used the in the FoM expression.