Q2 A beam is propped between pinned supports at A and B and is subjected to a vertical tip load of 1 kN at C as shown in Figure Q2(a). The specimen has a Young's modulus of 200 kN/mm^2 and a rectangular cross-section of 30 mm $\times$ 10 mm where the smaller dimension is in the vertical loading direction, as shown in Figure Q2(b).

(a) Calculate the reaction forces at A and B.

[2 marks]

(b) Draw the shear force and bending moment diagrams reasonably to scale and giving principal values.

[8 marks]

(c) Calculate the tip deflection at C.

[10 marks]

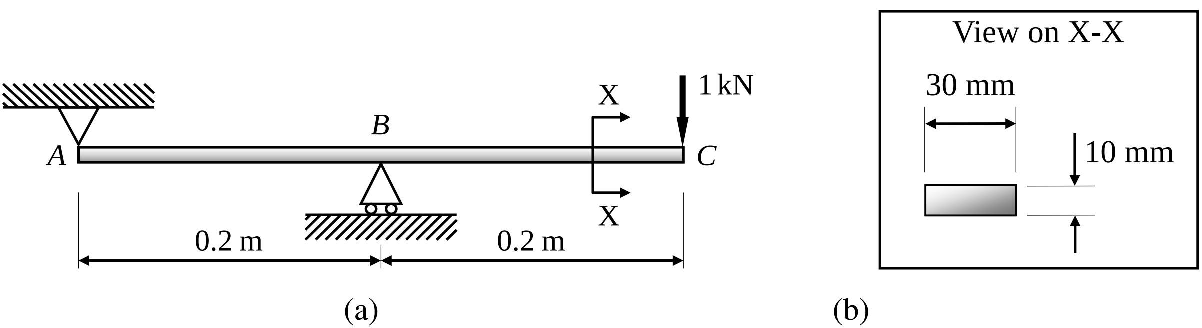
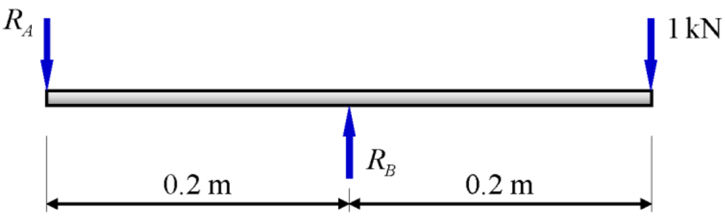


Figure Q2: (a) A beam under load and (b) detail of its cross-section.

Q2

a) The free body diagram is:



And the reactions are:

$$\sum M_{@A}^{CW} = 0$$

$$(1\,\text{kN})(0.4\,\text{m}) - (R_B)(0.2\,\text{m}) = 0$$

$$R_B = 2\,\text{kN}$$

(1 mark)

$$\sum F = 0$$

$$R_B - R_A - 1\text{kN} = 0$$

$$R_A = 1\text{kN}$$

(1 mark)

b) To find the moment at *B* we 'build-in' the beam there and:

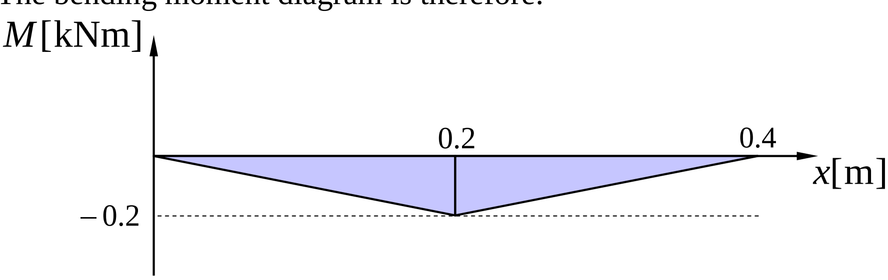
$$M_B = (-1\text{kN})(0.2\,\text{m})$$

$$M_B = -0.2\,\text{kNm}$$

At the ends the moments must be zero (pin-joint and free) therefore:

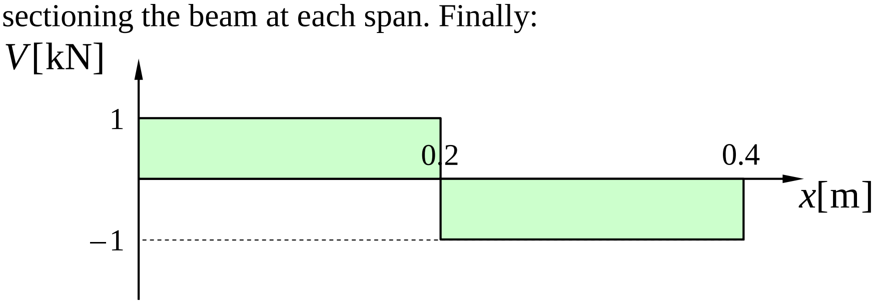
$$M_A = M_C = 0$$

The bending moment diagram is therefore:



(4 marks)

The shear force diagram can be obtained either by analysing the 'slope' of the bending moment diagram, or by sectioning the beam at each span. Finally:



(4 marks)

Q2 (cont.)

c) We can use the *double-integration method* to calculate the deflection.

Balance of moments for a section moving from left to right gives:

$$M_{(x)} = (-R_A)(x) + (R_B)(x - 0.2\text{ m})H(x - 0.2\text{ m})$$

(1 mark)

Therefore the curvature is given by:

$$EI\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = M_{(x)} = (-R_A)(x) + (R_B)(x - 0.2\text{ m})H(x - 0.2\text{ m})$$

$$EI\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -x + 2(x - 0.2\text{ m})H(x - 0.2\text{ m})$$

First integration gives the slope:

$$EI\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{2}x^2 + (x - 0.2\text{ m})^2 H(x - 0.2\text{ m}) + A \qquad (1)$$

(2 marks)

And the second integration gives the deflection:

$$EI\,y_{(x)} = -\frac{1}{6}x^3 + \frac{1}{3}(x - 0.2\text{ m})^3 H(x - 0.2\text{ m}) + Ax + B \qquad (2)$$

(2 marks)

The constants of integration are found via the boundary conditions:

$@\,x = 0,\ y = 0$ and from equation (2) we get $B = 0$

(1 mark)

$@\,x = 0.2,\ y = 0$ and from equation (2):

$$EI\,y_{(x)} = 0 = -\frac{1}{6}0.2^3 + 0 + A(0.2) + 0$$

$A = \frac{1}{6}0.2^2$ and $A = 0.00667\text{ kN}$

(1 mark)

The second moment of area is:

$$I = \left[\frac{1}{12}(30)(10)^3\right]$$

$$I = 2500\,\text{mm}^4 = 2.5\times10^{-9}\text{ m}^4$$

and the flexural modulus is:

$$EI = 200\frac{\text{kN}}{\text{mm}^2}\times 2500\text{ mm}^4$$

$$EI = 5\times10^5\text{ kNmm}^2 = 0.5\,\text{kNm}^2$$

Finally, the deflection is found from equation (2) by substituting $x = 0.4\text{ m}$:

$$EI\,y_C = -\frac{1}{6}(0.4)^3 + \frac{1}{3}(0.4 - 0.2\text{ m})^3 + 0.00667(0.4)$$

$y_C = -0.01067\,\text{m} = -10.67\,\text{mm}$, the negative sign describing a <u>downward</u> deflection.

(3 marks)