Aerodynamics 2- Rotorcraft Aerodynamics

Lecture 4 (notes)

Sustained Hover
(the unique ability of the rotorcraft!)

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Ideal Twist

The ideal rotor (Figure of Merit equal to unity) assumes no rotor profile losses and **an even distribution on the induced velocity across the rotor disk**. This of course cannot be achieved in practice. In order to minimise the profile and induced losses across the rotor, the combination of blade element analysis and momentum theory can be used to determine the correct blade planform, in particular, **blade twist**.

The ***ideal twist*** is that which results in **a constant induced velocity across the rotor disk** (as assumed in actuator disk analysis). Hence, the benefit is that the constant inflow distribution produced by the ideally twisted blades reduces the induced power for a given thrust.

To find the twist variation with blade radius for ideally twisted blade, we use a modified momentum theory with a Blade Element Approach:

Assumeing the standard expression of lift:

$L = \frac{1}{2}\rho V^2 S C_L$

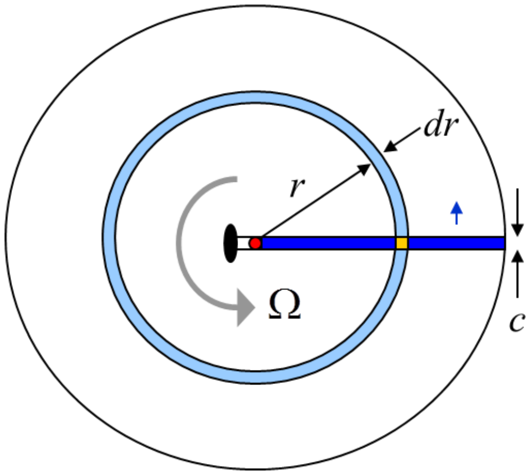
Then, the elemental lift produced by a blade element at a radial position ***r*** is

$dL = \frac{1}{2}\rho\,\Omega^2 r^2 c\,dr\,a\,\alpha_r = dT$

$v = \sqrt{\frac{T}{2\rho A}},\ dv = \sqrt{\frac{dT}{2\rho 2\pi\, r dr}}$

dv is proportional to $\sqrt{\frac{r^2\alpha_r}{r}}$

so for constant v, α_r is proportional to $\frac{1}{r}$



For this to be the case, then $\phi = \phi_t \frac{R}{r}$,

where $\phi_t =$ inflow angle at the tip.

Similarly blade pitch angle $\theta = \theta_t \frac{R}{r}$

The blade incidence at any radial station $r = \alpha_r = (\theta - \phi) = \frac{R}{r}(\theta_t - \phi_t)$

and the elemental lift coefficient $c_l = a\alpha_r$ where $a =$ section lift-curve slope.

Thus the lift on a blade element at radial station r, (assuming flow through the disk to be small compared with the tangential velocities) is given by:

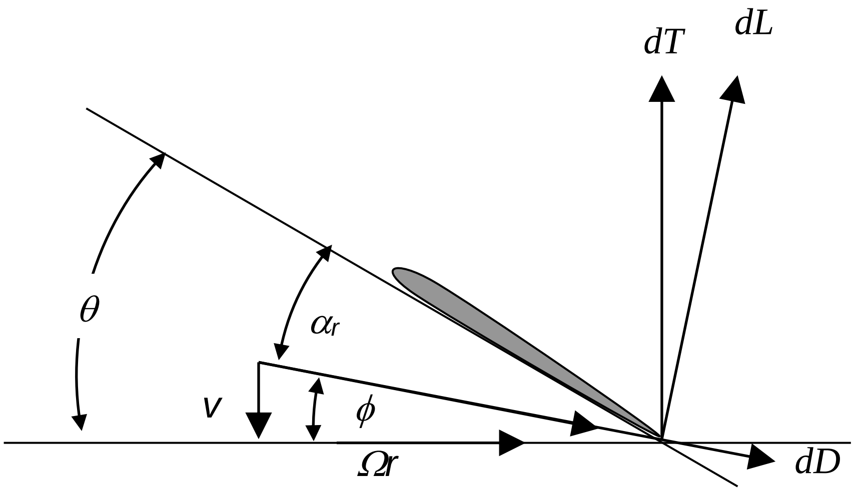
$$dL = \frac{1}{2}\rho(\Omega r)^2 a \frac{R}{r}(\theta_t - \phi_t) c dr \qquad \text{(blade element analysis)}$$

$$L = \int_o^R \frac{N}{2}\rho\Omega^2 r R a(\theta_t - \phi_t) c dr = \frac{N}{4}\rho\Omega^2 R^3 a(\theta_t - \phi_t) c$$

If $L = T$ then $C_T = \dfrac{T}{\rho A(\Omega R)^2} = \dfrac{N}{4}\dfrac{\rho\Omega^2 a R^3(\theta_t - \phi_t)c}{\rho\pi\Omega^2 R^4} = \dfrac{Na(\theta_t - \phi_t)c}{4\pi R}$

but solidity $\sigma = \dfrac{Nc}{\pi R}$, so, $\boxed{C_T = \dfrac{\sigma}{4}a(\theta_t - \phi_t)}$

<u>This is the expression for thrust of an ideally twisted blade</u>



Effects of Blade Profile Drag

The ***rotor drag*** is composed of two elements, the blade profile drag and the blade induced drag. The component of these drag forces in the plane of the rotor for a blade element at radial distance r is $dDCos\phi + dLSin\phi$. Since ϕ is small, small angle approximation may be made so element drag is:

$dD + dL\phi$ or in coefficient form $C_{d_0} + \phi C_l$.

Thus the in-plane drag torque due to this element is given by:

$$dQ = \frac{N}{2}\rho(\Omega r)^2 c(C_{d_0} + \phi C_l)rdr$$

Assuming C_{d_0} has only a small variation over the range of α then $C_{d_0} = \delta$. It has already been shown that $C_l = a\dfrac{R}{r}(\theta_t - \phi_t)$ and $\phi = \phi_t\dfrac{R}{r}$,

thus $$Q = \int_0^R \frac{N}{2}\rho\Omega^2 r^3 c\left[\delta + \phi_t\frac{R^2}{r^2}(\theta_t - \phi_t)a\right]dr$$

i.e. $$Q = \frac{N}{4}\rho\Omega^2 R^4 c\left[\frac{\delta}{2} + a\phi_t(\theta_t - \phi_t)\right]$$

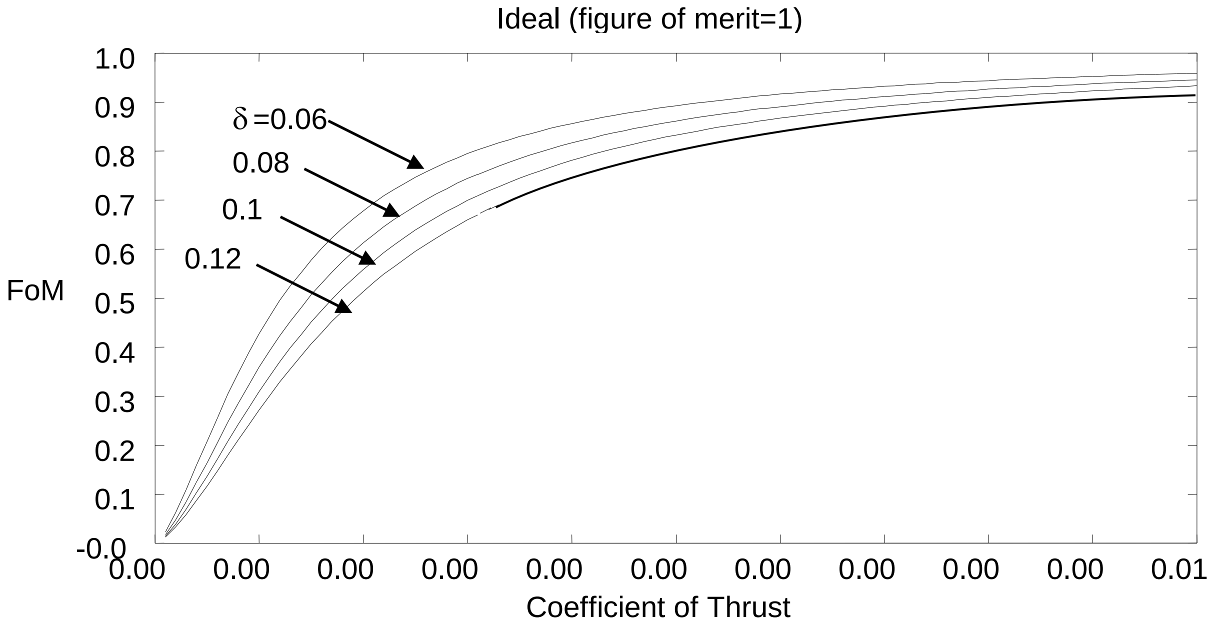
In coefficient form, $C_Q = \dfrac{\sigma\delta}{8} + \phi_t C_T$

but $\phi_t = \sqrt{\dfrac{C_T}{2}}$, so $\boxed{C_Q = \dfrac{C_T^{3/2}}{\sqrt{2}} + \dfrac{\sigma\delta}{8}}$

and thus the Figure of Merit, $\boxed{M = 0.707 \dfrac{C_T^{3/2}}{\dfrac{C_T^{3/2}}{\sqrt{2}} + \dfrac{\sigma\delta}{8}}}$

It can be seen that if $\delta = 0$, then for the ideally twisted blade the Figure of Merit is unity as shown previously. It can also be seen that the Figure of Merit is independent of operating conditions such as disk loading or tip speed.

The variation of the Figure of Merit with Thrust Coefficient is shown here for typical values of profile drag δ and for a rotor solidity $\sigma = 0.04$



In the hover, again using small angle approximations, then $L = T = W$ (the weight of the aircraft). This is the summation of all the blade elemental lift forces:

$$W = L = \int_0^R \frac{N}{2}\rho(\Omega r)^2 C_l c dr = \overline{C_L}\int_0^R \frac{N}{2}\rho(\Omega r)^2 c dr = T = C_T \pi R^2 \rho(\Omega R)^2$$

where $\overline{C_L}$ is the mean lift coefficient

Thus $$\frac{1}{6}\overline{C_L}\rho\Omega^2 R^3 N c = C_T \pi R^2 \rho(\Omega R)^2$$

Therefore $$\boxed{\overline{C_L} = 6\frac{C_T}{\sigma}}$$