Vibrations 2, Lecture 15
Tuned Vibration Absorber

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Lecture 14 review

MDOF systems with harmonic excitation:

$$\mathbf{M}\,\ddot{\mathbf{x}} + \mathbf{K}\,\mathbf{x} = \mathbf{f}_0 \cos(\omega t)$$

Response of MDOF systems under harmonic excitation:

$$\mathbf{x} = \mathbf{x}_0 \cos(\omega t)$$

$$\mathbf{x}_0 = (\mathbf{K} - \omega^2 \mathbf{M})^{-1}\,\mathbf{f}_0$$

The amplitudes of the steady-state harmonic response (amplitude-frequency characteristics, 2DOF example) – resonance:

$$x_{0,1}(\omega_i) \to \pm\infty,\;\; x_{0,2}(\omega_i) \to \pm\infty$$

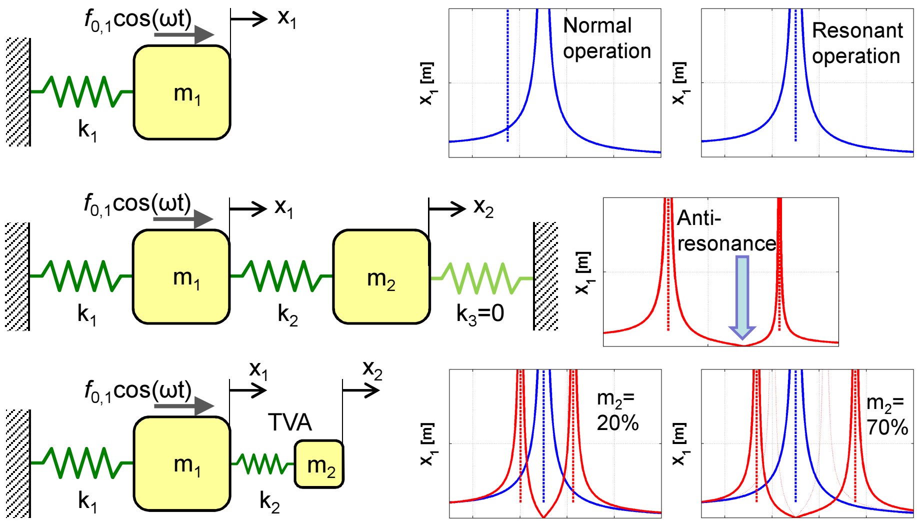
$$\det(\mathbf{K} - \omega_i^2 \mathbf{M}) = 0$$

Lecture 15

- Vibration control
  - By design (see BERP IV blade for AW101)
  - By *passive*, semi-active & active control
- Passive vibration control using TVA
  - Anti-resonance
- Basic TVA principles and design

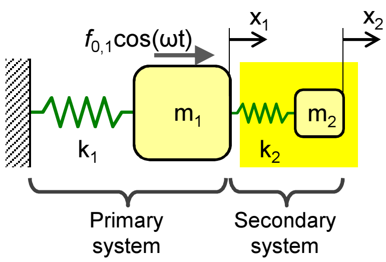
1&2 DOF systems under harmonic excitation

Harmonic response of block 1 (e.g. engine, blade), x_1:



TVA model

2DOF = primary + secondary



Equations of motion:

$$m_1\ddot{x}_1 + k_1 x_1 - k_2(x_2 - x_1) = f_{0,1}\cos(\omega t)$$
$$m_2\ddot{x}_2 + k_2(x_2 - x_1) = 0$$
$$x_i(t) = x_{0,i}\cos(\omega t),\ i = 1,2$$

Harmonic response of 2DOF system with TVA:

$$\begin{bmatrix} k_1 + k_2 - \omega^2 m_1 & -k_2 \\ -k_2 & k_2 - \omega^2 m_2 \end{bmatrix}\begin{bmatrix} x_{0,1} \\ x_{0,2} \end{bmatrix} = \begin{bmatrix} f_{0,1} \\ 0 \end{bmatrix}$$
$$x_{0,1}, x_{0,2} = ?$$

TVA definition

To calculate the amplitudes of the harmonic response use:

- approach described in Lecture 14, where $\mathbf{x}_0=(\ldots)^{-1}\mathbf{f}_0$
- Gauss elimination (e.g. solve for $x_{0,2}$ in eq.2 and substitute to eq.1)

The amplitudes of the harmonic response:

$$x_{0,1} = \frac{(k_2 - \omega^2 m_2) f_{0,1}}{(k_1 + k_2 - \omega^2 m_1)(k_2 - \omega^2 m_2) - k_2^2}$$
$$x_{0,2} = \frac{k_2 f_{0,1}}{(k_1 + k_2 - \omega^2 m_1)(k_2 - \omega^2 m_2) - k_2^2}$$

The amplitude of the harmonic response of block 1 is $x_{0,1}$=0 if:

$$x_{0,1} = 0 \Rightarrow k_2 - \omega^2 m_2 = 0 \Rightarrow \omega = \sqrt{\frac{k_2}{m_2}} = \omega_a$$

If the secondary system is *tuned* (designed) such that its *own* natural frequency is *equal* to the frequency of excitation (which might be the resonant frequency) then the secondary system is called *Tuned Vibration Absorber* (TVA).

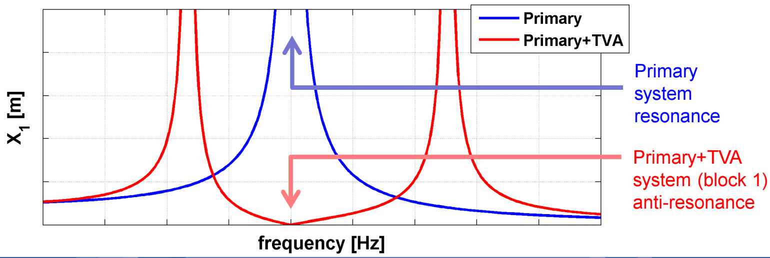
Controlling resonant vibrations

TVAs are useful for *passive control* of *resonant vibrations*. Assuming undamped problem, the harmonic excitation at the undamped natural frequency, $\omega=\omega_0$, causes *resonance*. TVA can be designed such that $\omega_a=\omega=\omega_0$ and the steady-state amplitude $x_{0,1}$ will change from $x_{0,1}\rightarrow$Inf to $x_{0,1}$=0.

The TVA *tuning condition* applied to the system with resonant excitation:

$$\omega_0^2 = \frac{k_1}{m_1} = \frac{k_2}{m_2} = \omega_a^2$$

This situation is illustrated in the following graph:



Further TVA considerations

Properties of 2DOF systems with TVA:

- 1DOF primary system changes to 2DOF system (primary + TVA)
- The primary system (block 1) has one anti-resonance frequency (no motion)
- The *tuning condition* represents one equation with two unknowns k_2 and m_2
  - We need *second* condition (equation), or
  - We *choose* one unknown parameter (k_2 or m_2) and calculate the other
- New tuned 2DOF system has two (undamped) natural frequencies ω_1 and ω_2
  - These natural frequencies can cause additional problems

$$\det(\mathbf{K} - \omega_i^2\mathbf{M}) = 0$$

To understand the influence of k_2 and m_2 on ω_1 and ω_2 divide det(…) by k_1k_2 and use the relationships from the tuning condition:

$$(1 + k_2/k_1 - \omega^2(m_1/k_1))(1 - \omega^2(m_2/k_2)) - k_2/k_1 = 0$$

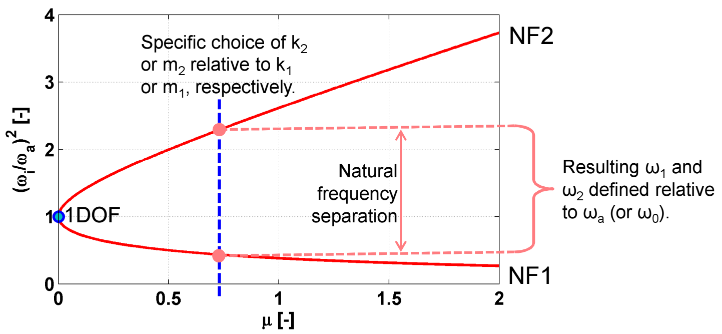
$$(1 + \mu - \omega^2/\omega_a^2)(1 - \omega^2/\omega_a^2) - \mu = 0,\ \ \mu = k_2/k_1 = m_2/m_1$$

Further TVA considerations

Rearrange the previous equation to obtain the equation quadratic in $(\omega/\omega_a)^2$:

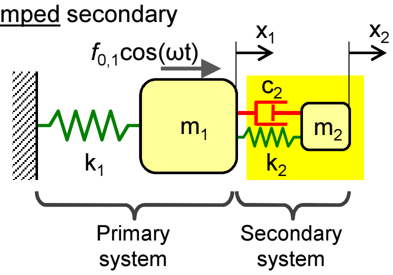
$$\left(\frac{\omega}{\omega_a}\right)^4 - (2+\mu)\left(\frac{\omega}{\omega_a}\right)^2 + 1 = 0$$

The roots of this equation (ratios of the natural frequencies to ω_a squared) depend on μ which, based on its definition, is called the *stiffness* or *mass ratio*.



Initial damped TVA considerations

2DOF = primary + damped secondary



Equations of motion:

$$m_1\ddot{x}_1 + k_1x_1 - k_2(x_2 - x_1) - c_2(\dot{x}_2 - \dot{x}_1) = f_{0,1}\cos(\omega t)$$

$$m_2\ddot{x}_2 + k_2(x_2 - x_1) + c_2(\dot{x}_2 - \dot{x}_1) = 0$$

EOM in matrix form:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f_{0,1} \\ 0 \end{bmatrix}\cos(\omega t)$$

... continued in L16

Summary

- TVAs are used for passive vibration control in systems with harmonic (usually resonant) excitation
- The tuning conditions specifies that the natural frequency of TVA alone should be equal to the natural frequency of the primary system
- Additional conditions are required to ensure correct placement of newly created natural frequencies