Light Aircraft Structures
Skin-Boom Idealisations

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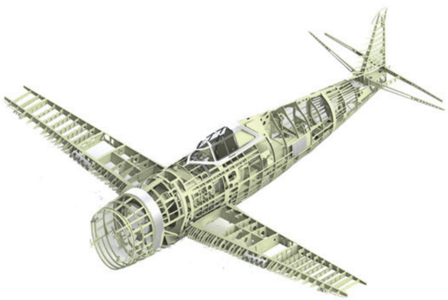
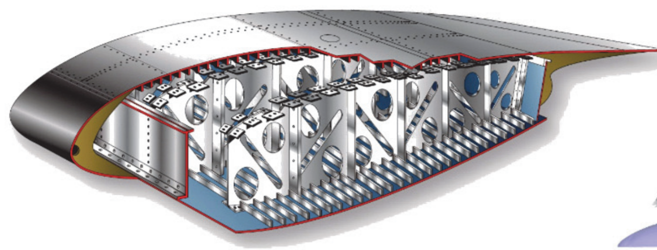
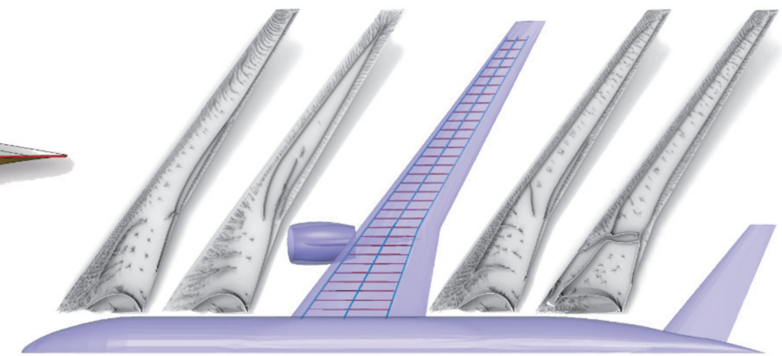
Idealisation of Semi-Monocoque Structures

- Semi-monocoque construction combines both principles:
  - ‘Stressed skin’: outer aerodynamic / hydrodynamic shell resisting loads
  - ‘Spaceframe’: axially loaded members = efficient material usage
- Weight optimisation results in **thin skins** and **large numbers of stringers, webs and frames**





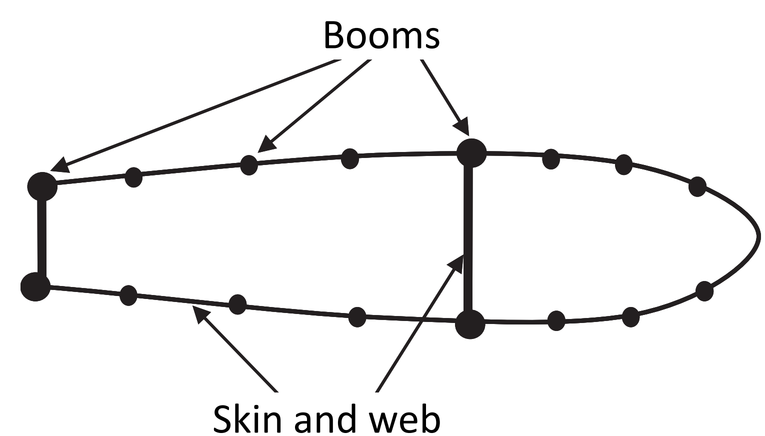


- Skins are good at resisting **in-plane tension and shear**, but offer little resistance to **in-plane compression**
  - Skin may **buckle locally** but stiffeners prevent catastrophic failure, so lightweight designs must account for **post-buckling behaviour**
- Such structures are difficult to analyse, requiring:
  - Structural idealisation (skin-boom approach)
  - Accurate numerical methods (*e.g.* Finite Element Analysis)

Skin-Boom Idealisation

Idealisation:

- 'Booms': axial members that resist direct stresses (i.e. bending)
  - Stringers, spar caps
- 'Skin': thin webs that resist shear stresses (*i.e.* transverse loads & torsion)
  - Outer skin, spar webs



Assumptions:

- Skins only resist shear
- Booms resist only direct stresses
- 'Effective' boom cross-sectional areas depend on type of loading
  - Bending, transverse shear, torsion

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Bending of Idealised Thin-Walled Structures

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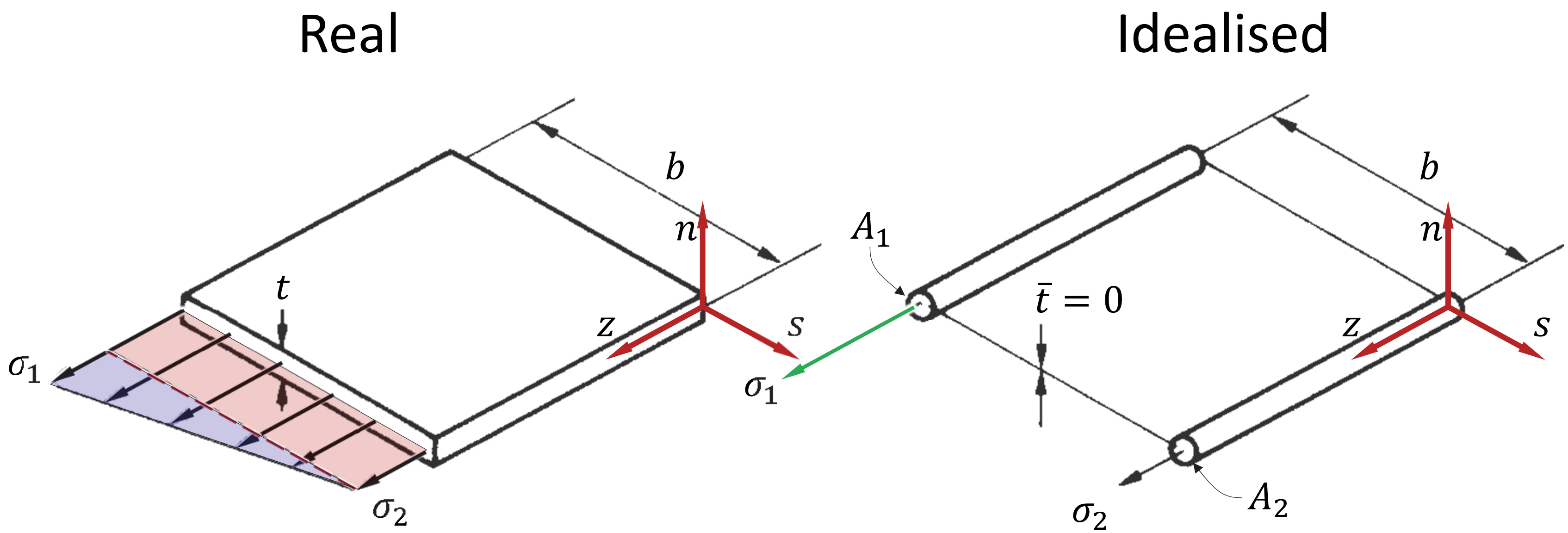
Skin-Boom Bending



- Moments about n:

$$\left[\sigma_2 (t\ b) \left(\frac{b}{2}\right)\right] + \left[\frac{1}{2}(\sigma_1 - \sigma_2)(t\ b)\left(\frac{2}{3} b\right)\right] = \sigma_1\ A_1\ b$$

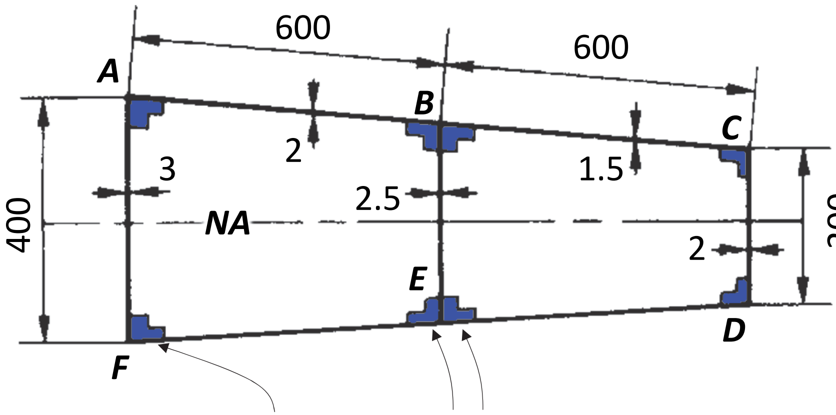
$$A_1 = \frac{t\ b}{6}\left(2 + \frac{\sigma_2}{\sigma_1}\right)$$

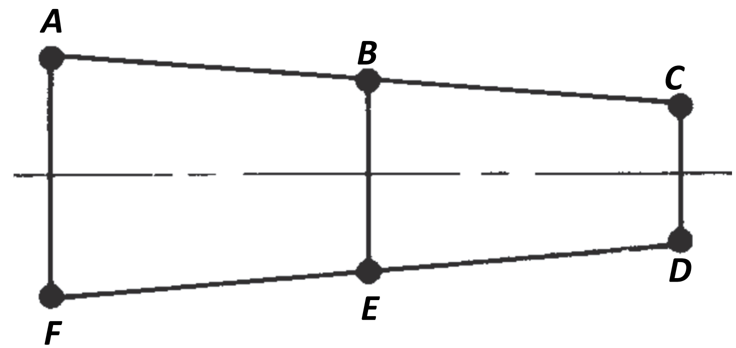
and conversely:

$$A_2 = \frac{t\ b}{6}\left(2 + \frac{\sigma_1}{\sigma_2}\right)$$

Skin-Boom Bending – Example (1/3)

- Wing box subjected to vertical loading (dimension in mm):





$$A_A = 300 + \frac{t_{AF}\, b_{AF}}{6}\left(2 + \frac{\sigma_F}{\sigma_A}\right) + \frac{t_{AB}\, b_{AB}}{6}\left(2 + \frac{\sigma_B}{\sigma_A}\right)$$

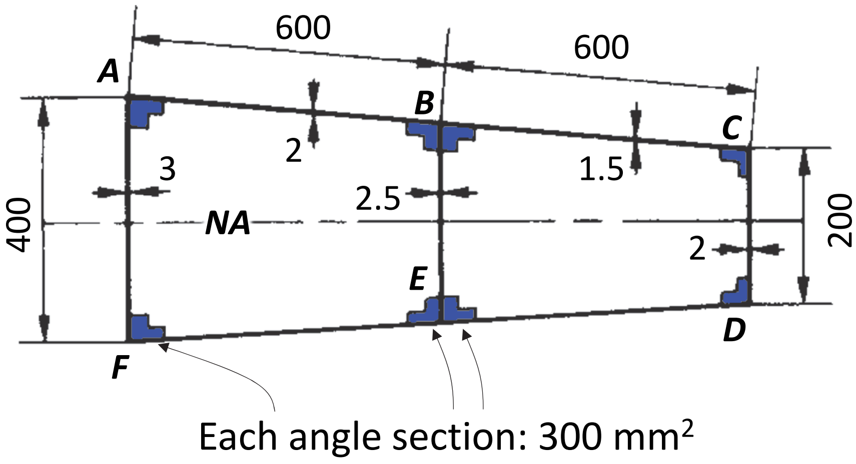
- Bending stresses are proportional to distance from *NA*, therefore:

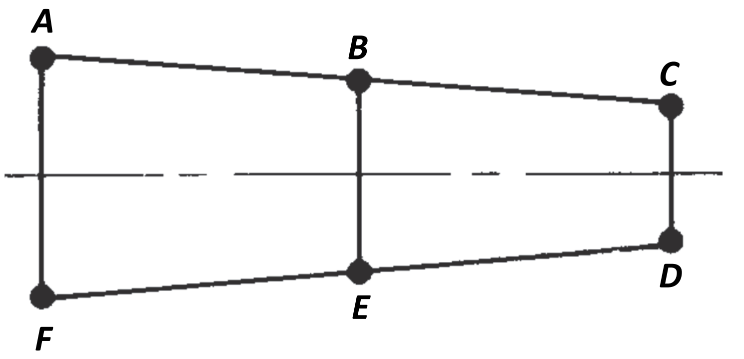
$$A_A = 300 + \frac{(3.0)(400)}{6}\left[2 + \frac{(-200)}{(200)}\right] + \frac{(2.0)(600)}{6}\left[2 + \frac{(150)}{(200)}\right]$$

$$\boxed{A_A = A_F = 1050\ \text{mm}^2}$$

Skin-Boom Bending – Example (3/3)

- Wing box subjected to vertical loading (dimension in mm):





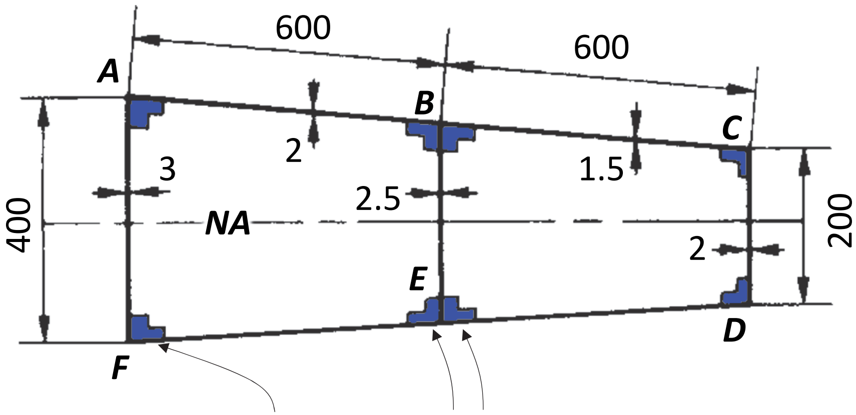
$$A_B = (2)(300) + \frac{t_{AB}\, b_{AB}}{6}\left(2 + \frac{\sigma_A}{\sigma_B}\right) + \frac{t_{BC}\, b_{BC}}{6}\left(2 + \frac{\sigma_C}{\sigma_B}\right) + \frac{t_{BE}\, b_{BE}}{6}\left(2 + \frac{\sigma_E}{\sigma_B}\right)$$

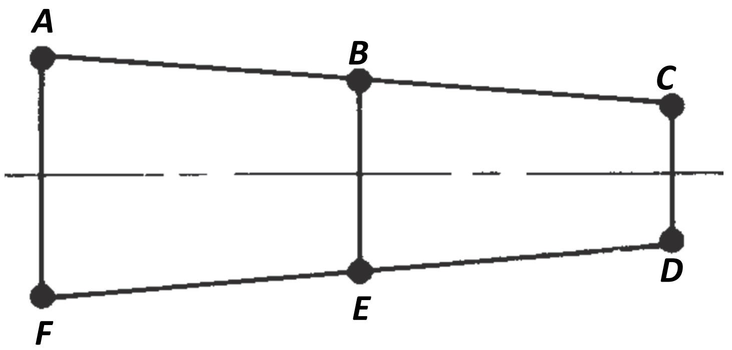
$$A_B = 600 + \frac{(2.0)(600)}{6}\left[2 + \frac{(200)}{(150)}\right] + \frac{(1.5)(600)}{6}\left[2 + \frac{(100)}{(150)}\right] + \frac{(2.5)(300)}{6}\left[2 + \frac{(-150)}{(150)}\right]$$

$$A_B = A_E = 1791.7\ \text{mm}^2$$

Skin-Boom Bending – Example (3/3)

- Wing box subjected to vertical loading (dimension in mm):





$$A_C = 300 + \frac{t_{BC}\, b_{BC}}{6}\left(2 + \frac{\sigma_B}{\sigma_C}\right) + \frac{t_{CD}\, b_{CD}}{6}\left(2 + \frac{\sigma_D}{\sigma_C}\right)$$

$$A_C = 200 + \frac{(1.5)(600)}{6}\left[2 + \frac{(150)}{(100)}\right] + \frac{(2.0)(200)}{6}\left[2 + \frac{(-100)}{(100)}\right]$$

$$A_C = A_D = 891.7 \text{ mm}^2$$

- All bending properties are computed based on **booms alone**, *i.e.*:

$$A \cong \sum A_i$$

Idealised boom areas

Idealised boom coordinates

$$Q_{XX} = \sum \bar{Y}_i \, A_i$$

$$\bar{Y} = \frac{Q_{XX}}{A}$$

$$Q_{YY} = \sum \bar{X}_i \, A_i$$

$$\bar{X} = \frac{Q_{YY}}{A}$$

Idealised boom coordinates

$$I_{xx} = \sum (y_i)^2 A_i$$

$$I_{yy} = \sum (x_i)^2 A_i$$

$$I_{xy} = \sum (x_i \, y_i) A_i$$

Light Aircraft Structures

Shear of Idealised Open Sections

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Shear of open sections

- Shear flow in 'real' cross sections:

$$-q_s = \left(\frac{S_x\, I_{xx} + S_y\, I_{xy}}{I_{xy}^2 - I_{xx}\, I_{yy}} \right) \int_0^s x\, t\, \mathrm{ds} + \left(\frac{S_y\, I_{yy} + S_x\, I_{xy}}{I_{xx}\, I_{yy} - I_{xy}^2} \right) \int_0^s y\, t\, \mathrm{ds}$$

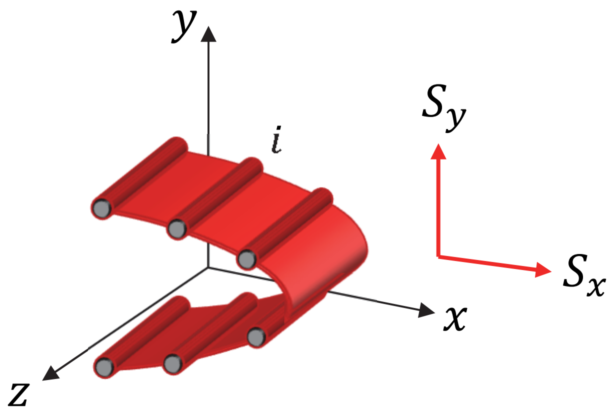
- Shear flow in 'idealised' cross sections:

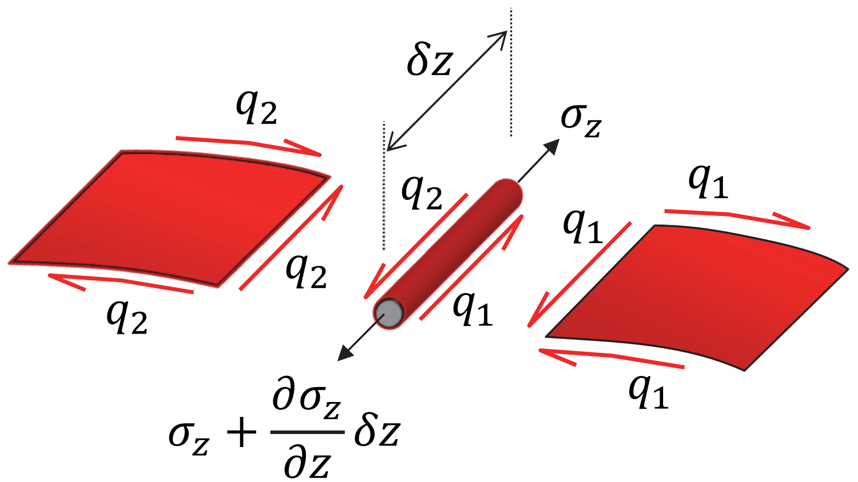
$$-q_s = \left(\frac{S_x\, I_{xx} + S_y\, I_{xy}}{I_{xy}^2 - I_{xx}\, I_{yy}} \right) \sum_{i=1}^{n_s} x_i\, A_i + \left(\frac{S_y\, I_{yy} + S_x\, I_{xy}}{I_{xx}\, I_{yy} - I_{xy}^2} \right) \sum_{i=1}^{n_s} y_i\, A_i$$

- The shear flow is assumed to be **constant between booms**

Shear Flow in Idealised Sections – Derivation (1/3)

- We equate direct stresses in booms with shear stresses in the skin:





$$\left(\sigma_z + \frac{\partial \sigma_z}{\partial z}\delta z\right)A_i - (\sigma_z\, A_i) + (q_2\,\delta z) - (q_1\,\delta z) = 0$$

$$\left(\frac{\partial \sigma_z}{\partial z}\delta z\right)A_i + (q_2\,\delta z) - (q_1\,\delta z) = 0$$

$$\left(\frac{\partial \sigma_z}{\partial z}\right)A_i + q_2 - q_1 = 0$$

$$\boxed{q_2 - q_1 = \frac{\partial \sigma_z}{\partial z}A_i}$$

Shear Flow in Idealised Sections – Derivation (2/3)

- Finding the RHS:

$$\sigma_z = \frac{M_y\, I_{xx} + M_x\, I_{xy}}{I_{xy}^2 - I_{xx}\, I_{yy}} x + \frac{M_x\, I_{yy} + M_y\, I_{xy}}{I_{xx}\, I_{yy} - I_{xy}^2} y$$

$$\frac{\partial \sigma_z}{\partial z} = \frac{\frac{M_y}{\partial z}\, I_{xx} + \frac{M_x}{\partial z}\, I_{xy}}{I_{xy}^2 - I_{xx}\, I_{yy}} x + \frac{\frac{M_x}{\partial z}\, I_{yy} + \frac{M_y}{\partial z}\, I_{xy}}{I_{xx}\, I_{yy} - I_{xy}^2} y$$

$$\frac{\partial \sigma_z}{\partial z} = \frac{S_x\, I_{xx} + S_y\, I_{xy}}{I_{xy}^2 - I_{xx}\, I_{yy}} x + \frac{S_y\, I_{yy} + S_x\, I_{xy}}{I_{xx}\, I_{yy} - I_{xy}^2} y$$

- Replacing: $q_2 - q_1 = \frac{\partial \sigma_z}{\partial z} A_i = \frac{S_x\, I_{xx} + S_y\, I_{xy}}{I_{xy}^2 - I_{xx}\, I_{yy}} x_i\, A_i + \frac{S_y\, I_{yy} + S_x\, I_{xy}}{I_{xx}\, I_{yy} - I_{xy}^2} y_i\, A_i$
- Starting from a free edge where $q_s = 0$ and 'integrating' around the section, *i.e.* summing from boom 1 to boom n_s (0 to s) the shear flow becomes:

$$-q_s = \left(\frac{S_x\, I_{xx} + S_y\, I_{xy}}{I_{xy}^2 - I_{xx}\, I_{yy}} \right) \sum_{i=1}^{n_s} x_i\, A_i + \left(\frac{S_y\, I_{yy} + S_x\, I_{xy}}{I_{xx}\, I_{yy} - I_{xy}^2} \right) \sum_{i=1}^{n_s} y_i\, A_i$$

Shear Flow in Idealised Sections – Derivation (3/3)

- Note that if the skin between the stiffeners was also capable of carrying direct stress then we would have:

$$-q_s = \left(\frac{S_x\, I_{xx} + S_y\, I_{xy}}{I_{xy}^2 - I_{xx}\, I_{yy}}\right)\left(\underbrace{\int_0^s x\, \bar{t}\, \mathrm{d}s}_{\text{Skin}} + \underbrace{\sum_{i=1}^{n_s} x_i\, A_i}_{\text{Boom}}\right) + \left(\frac{S_y\, I_{yy} + S_x\, I_{xy}}{I_{xx}\, I_{yy} - I_{xy}^2}\right)\left(\underbrace{\int_0^s y\, \bar{t}\, \mathrm{d}s}_{\text{Skin}} + \underbrace{\sum_{i=1}^{n_s} y_i\, A_i}_{\text{Boom}}\right)$$

- However, in StM2 we will always assume $\bar{t} = 0$, *i.e.* the skin cannot carry axial loads
  - 'Idealised' skin-boom scenario

Channel section

- Shear flow:

$$-q_s = \frac{S_y}{I_{xx}} \sum_{i=1}^{n_s} y_i \, A_i$$

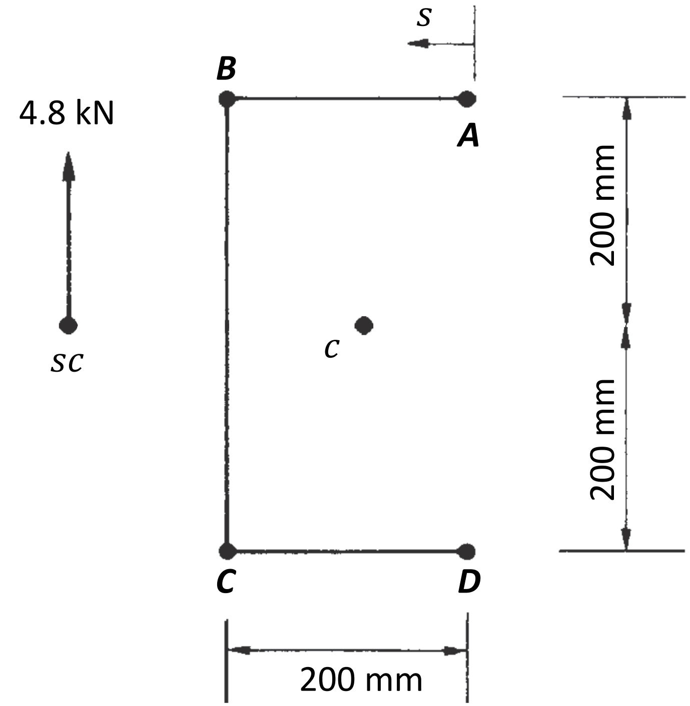
- Second moment of area:

$$I_{xx} = 4 \times (300)(200)^2$$

$$I_{xx} = 48 \cdot 10^6 \text{ mm}^4$$

$$-q_s = \frac{4.8 \times 10^3}{48 \times 10^6} \sum_{i=1}^{n_s} y_i \, A_i$$

$$-q_s = 10^{-4} \sum_{i=1}^{n_s} y_i \, A_i$$



Each boom: 300 mm^2

Shear of an Open Section – Example (2/3)

- Flange AB:

$$-q_s = 10^{-4} \sum_{i=1}^{n_s} y_i \, A_i$$

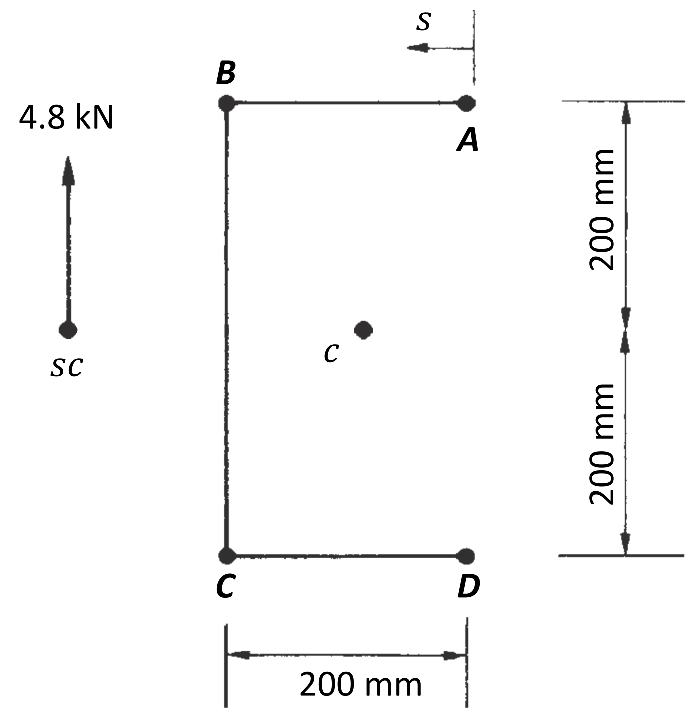
$$-q_{AB} = 10^{-4}(200)(300)$$

$$q_{AB} = -6 \text{ N/mm}$$

- Web BC:

$$-q_{BC} = 6 + (10^{-4})(200)(300)$$

$$q_{BC} = -12 \text{ N/mm}$$



Each boom: 300 mm²

Shear of an Open Section – Example (3/3)

- Flange CD:

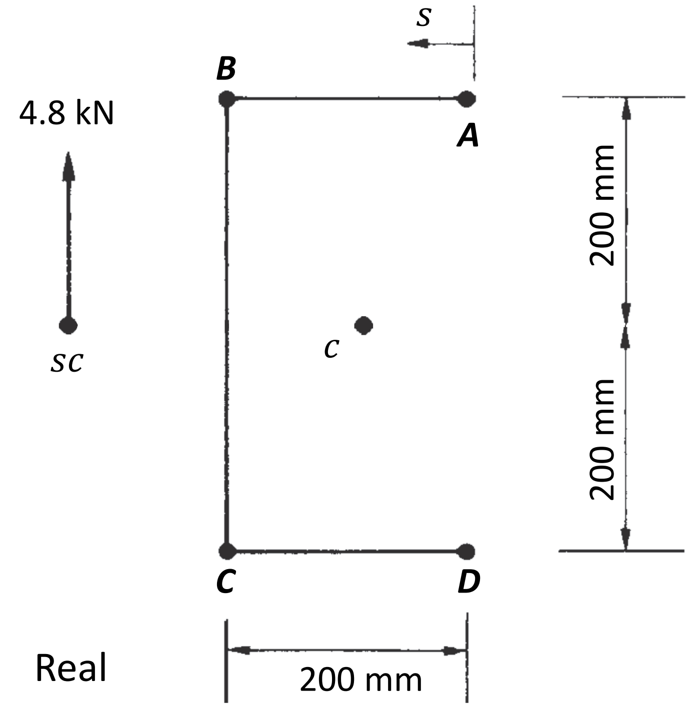
$$-q_{BC} = 12 + (10^{-4})(-200)(300)$$

$$q_{BC} = -6\ \mathrm{N/mm}$$

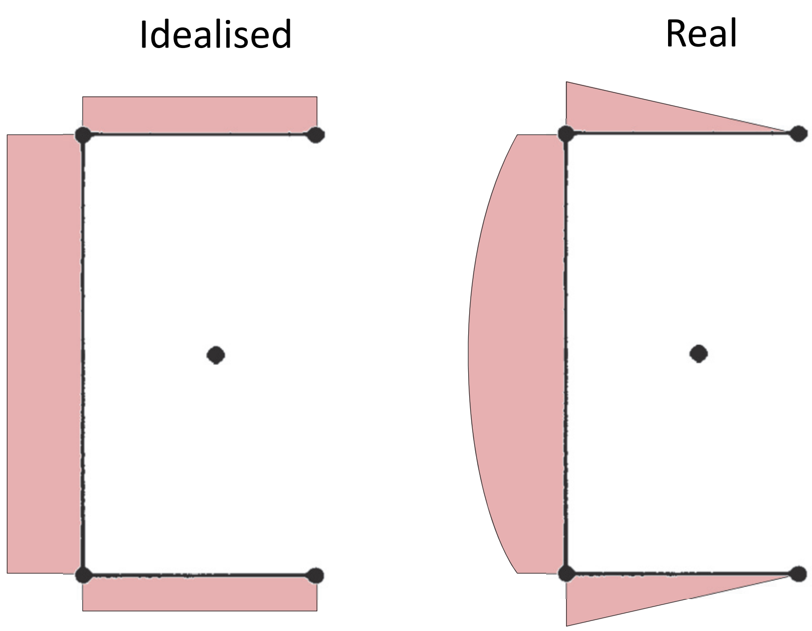
- After point D:

$$6 + (10^{-4})(-200)(300) = 0$$

- Plotting:



Each boom: 300 mm²



Different 'shapes' but same areas