INTRODUCTION TO POTENTIAL MODELS FOR AEROFOILS

AIMS

- To introduce conformal transformations as a way of obtaining potential solutions about other geometries
- To explain the Kutta condition
- To introduce Kelvin's circulation theorem and its implications for an impulsively started flow about an aerofoil.
- To review some basic facts about aerofoils

1 INTRODUCTION

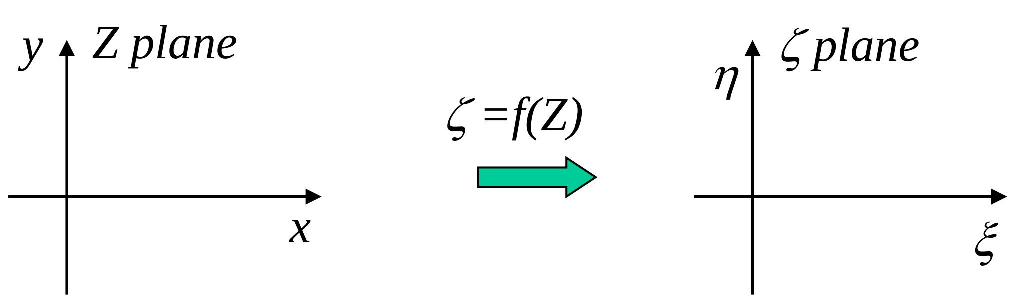
The potential solutions considered so far in Fluids1 (and reviewed in this course) have been for simple geometries. However the material can be extended to model the flow about more relevant geometries. In this introductory handout, the simplest way to extend the theory to model the flow about a particular special type of aerofoil is briefly considered. This will allow us to develop the concept of the Kutta condition which is essential to the modelling of flows past aerofoils. Also Kelvin's circulation theorem is stated and its implications for an impulsively started aerofoil considered.

2 CONFORMAL MAPPING OR TRANSFORMATION

For 2D problems, the stream & potential functions can be combined into one 'complex potential'

$$W(Z) = \phi + i\psi$$

where $Z = x + iy$. Then any *regular* function f applied to the Z ($= x+iy$) domain will map it into the (say) ζ (= $\xi+i\eta$) domain, so that angles between curves are preserved – a *conformal transformation*



A solution for the complex potential in the Z domain is also a solution in the ζ domain *i.e.*

$$W(Z) = W(\zeta)$$

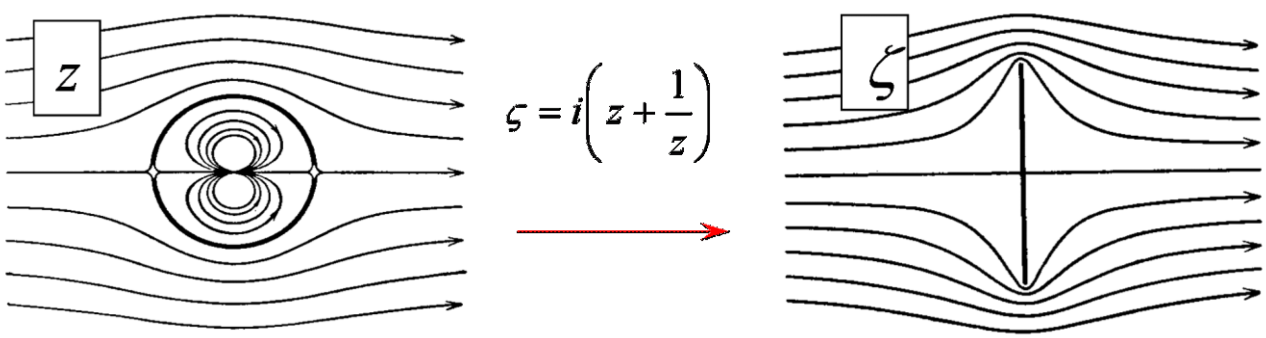
This means that if the potential and stream function is known at a point in the Z plane then the potential and stream function at the mapped point in the ζ plane is the same. Note however that velocities are not the same in the two planes because the velocity components in the Z plane are given by

$$u - iv \quad = \frac{dW}{dZ}$$

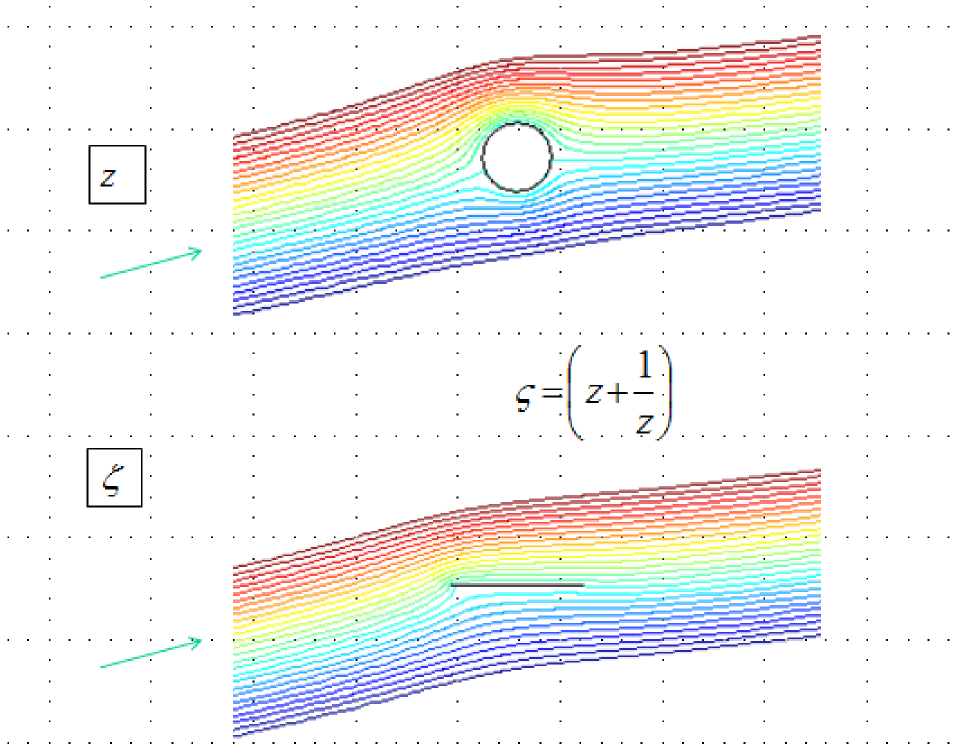
and the velocity components in the ζ plane are given by

$$v_\xi - iv_\eta = \frac{dW}{d\zeta}$$

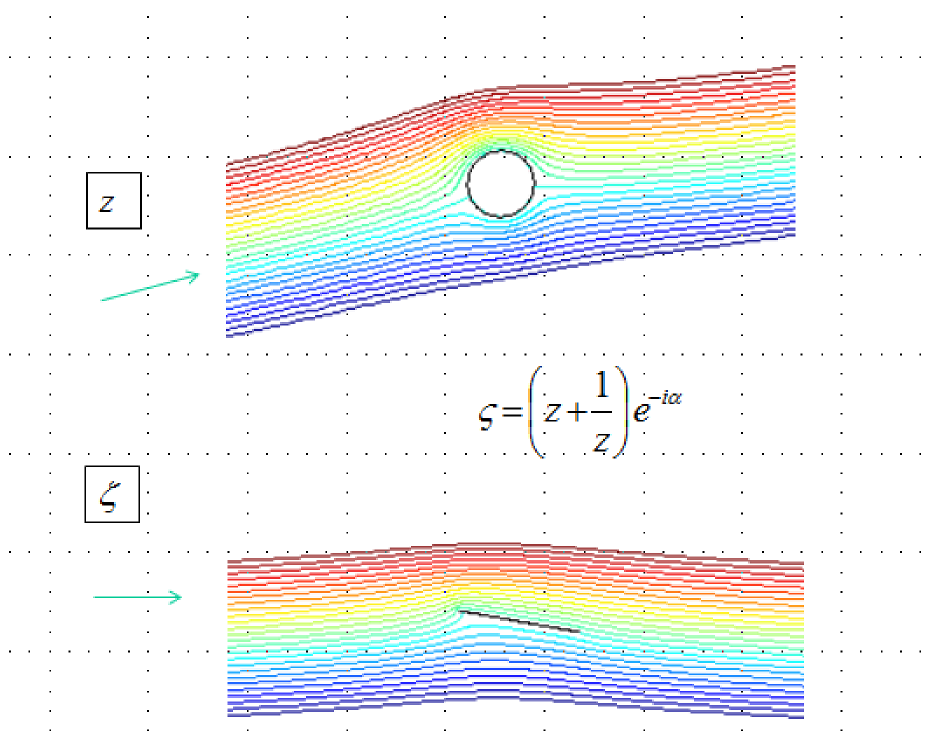
2.1 Example 1 – Unit circle to a flat plate



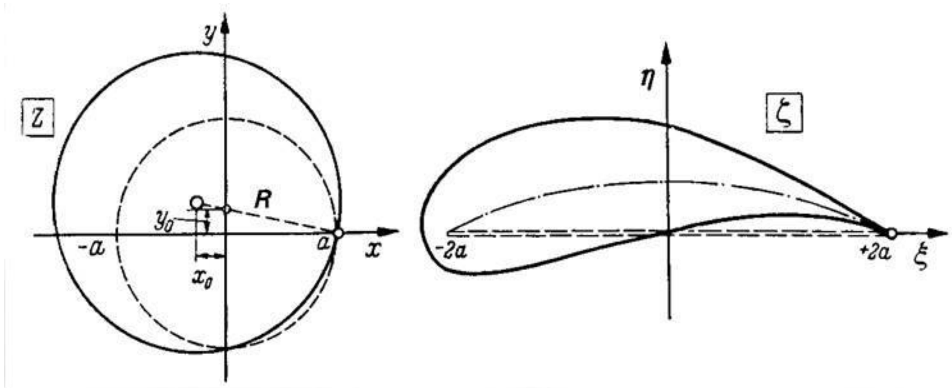
2.2 Example 2a – Unit circle to a flat plate



2.3 Example 2a – Unit circle to a flat plate



2.4 Example 3 – Joukowski Transformation (1910)



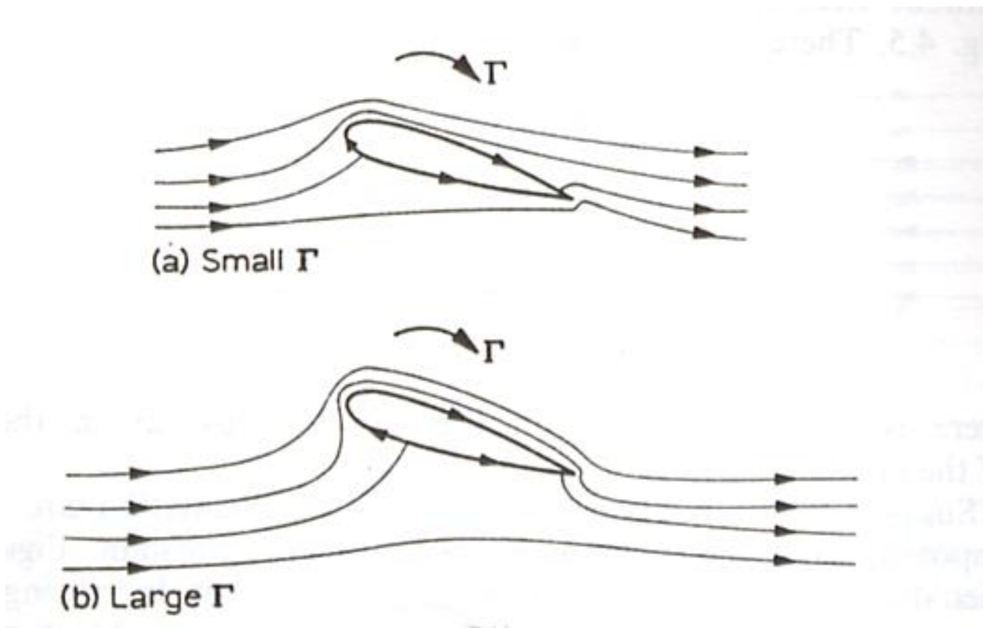
This transformation generates aerofoils with varying thickness & camber. It was the first series of theoretically designed aerofoil sections and with analytical potential solution they were used to validate numerical methods. Aerofoils from the series were tested experimentally in the 1910s –however they were not very good aerofoils due to cusped trailing-edges and circular arc camber-lines which lead to excessive movement of the aerodynamic centre and manufacture difficult. The Joukowski airfoils

The conformal transformation technique has the major disadvantage of being inverse *i.e.* the aerofoil shape is a product of the solution process. Hence the method cannot answer the question 'what forces does this aerofoil/shape produce?', for an arbitrary input aerofoil shape. It is also strictly limited to 2D problems, as conformal mappings do not exist in higher dimensions. However the Jowkowski aerofoils do help to establish a very important condition needed to develop appropriate potential models for arbitrary aerofoils, namely the Kutta condition.

3 THE KUTTA CONDITION

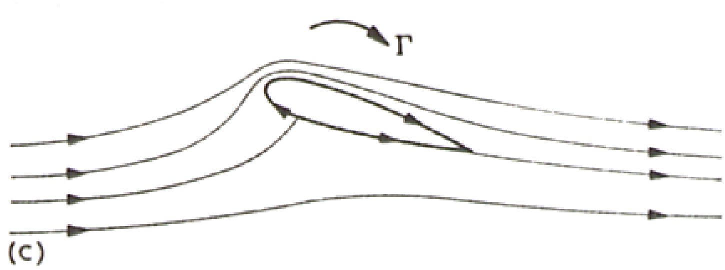
The magnitude of circulation Γ in our potential model of the lifting cylinder is arbitrary. Since solutions for an aerofoil shape can be found by conformal mapping from the circle there will similarly been an infinite number of valid potential models for flows past these aerofoils. In fact this is a general result, that applies to all potential models about aerofoils. However in a real lifting flow over an aerofoil circulation and hence lift are not arbitrary. The question then arises, how can the potential model be made to select the appropriate value for Γ?

To answer this questions consider the potential solutions for an aerofoil with different amounts of circulation.



Consider the case where the circulation is small. In this case the aft stagnation point is on the upper surface of the aerofoil. The flow therefore has to negotiate the sharp trailing-edge. The sharp corner implies infinite local velocity in theoretical potential flow. In a real flow the velocity would be very large; as a result of the large velocity gradients the viscous stresses would tend to move the stagnation point rearwards. When the circulation is large the aft stagnation point is on the lower surface of the aerofoil, and in a similar way the viscous stresses would tend to move the stagnation point rearwards.

The action of viscosity in the real flow is therefore such that it makes the flow leave the trailing-edge *smoothly*, with a *finite* velocity. The effect of viscosity can be modelled in an inviscid flow by adding just enough circulation so that the flow leaves the trailing edge smoothly this is the **Kutta Condition**, which was first described in 1902.

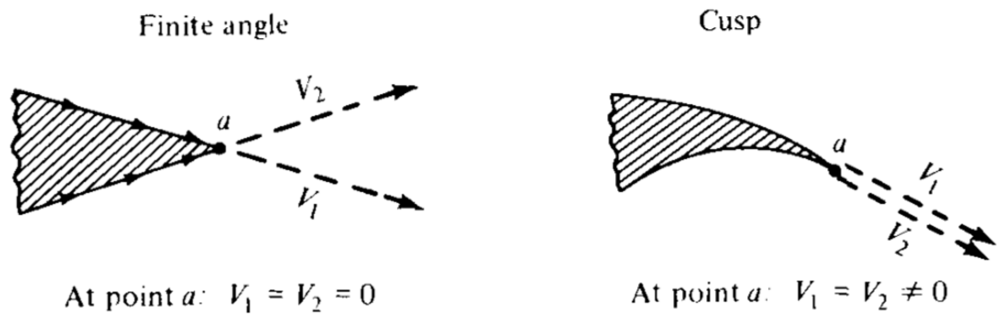


The empirical Kutta condition fixes the value of circulation at any angle of attack. The lift predicted by the model is then uniquely determined via the Kutta-Joukowsky theorem

$$l = \rho_\infty U_\infty \Gamma$$

The implications of the Kutta Condition are that the velocities and hence pressures (by Bernoulli) top and bottom must be equal:

(1) For aerofoils with a finite TE angle there must be a stagnation point at the trailing edge.
(2) For a cusped TE aerofoil, the velocities on the top and bottom surfaces must have same magnitude and direction at trailing edge.



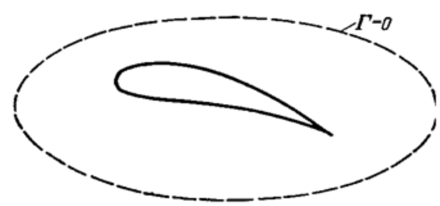
4 GENERATION OF CIRCULATION AND LIFT

The Kutta condition states that the circulation about an aerofoil is the value required to make the flow leave the trailing edge smoothly. In this section consideration is given to how this circulation is generated in a real flow.

4.1 Kelvin's Circulation Theorem

First Kelvin's theorem for an incompressible, inviscid fluid is introduced. This states that :

the time rate of the change of circulation around a closed curve consisting of the same fluid elements is zero.



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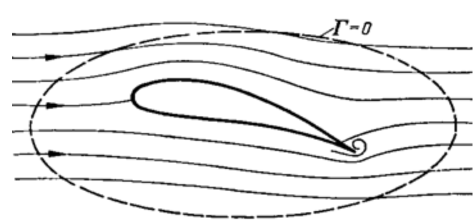
i.e. there is 'conservation of circulation'. This means that circulation (and hence lift) cannot be created in potential flow! In a real flow it is the action of viscosity that leads to the generation of circulation.

4.2 The Starting Vortex



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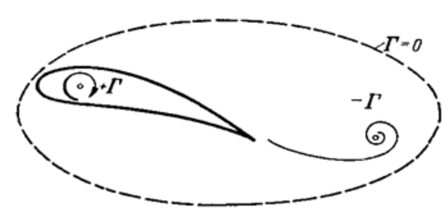
Consider an aerofoil that is initially at rest in a stationary fluid, then the effects of viscosity are negligible. The circulation about the aerofoil (which equals the line integral of velocity around any curve enclosing the aerofoil) is zero.



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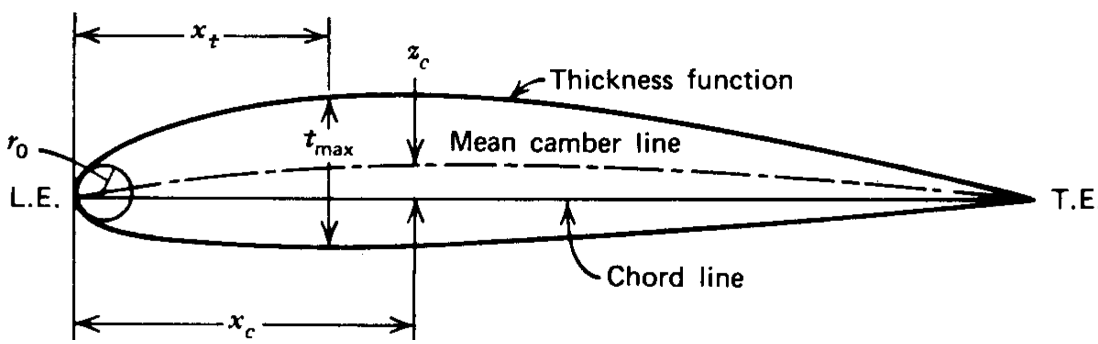
When the aerofoil is impulsively set in motion, the circulation and lift are not produced instantaneously. At the instant of starting the circulation is zero and the rear stagnation point is on the upper surface of the aerofoil and the fluid has to negotiate the sharp trailing edge. In the real flow viscosity acts to damp the large velocity gradients produced and a region of high vorticity forms at the trailing edge. This is shed downstream as the stagnation point moves aft and is accompanied by a progressive increase in circulation and lift around the aerofoil until the Kutta condition is satisfied. The shed vortex rolls up into a point vortex of strength $-\Gamma$ which convects downstream. Kelvin's theorem applies to a curve consisting of the same fluid elements if the flow remains inviscid. Thus if the curve is sufficiently far from the aerofoil to enclose it and the starting vortex, then since the circulation equals zero so the circulation developed about the aerofoil equals to $+\Gamma$ to balance that of the shed starting vortex. When the shed starting vortex is far enough downstream its effects in the vicinity of the aerofoil can be neglected.

Similarly whenever there is a change in aerofoil lift, the local velocity at the TE changes and causes a vorticity to be shed into the flow. The overall circulation remains constant and a starting vortex is shed and convects downstream whose circulation is equal to minus the change in circulation about the aerofoil. After a while when it is far downstream its effects on the aerofoil flow can be ignored. The same theory applies to wings and it should be noted that the shed vortices can interact with (a) tailplanes (affects stability) and (b) other aircraft. Note that when the starting vortex is still close to the wing or aerofoil it must be included in the flow analysis.

5 REVIEW OF AEROFOIL BASICS

A slight change of notation is necessary when moving on to look at aerofoils to be consistent with most other studies. Typically so far 2D flows are assumed to be in the (x,y) plane, however when moving to investigate aerofoils it is usual to use the (x,z) plane. The reason for this is that for a 3D wing or wing body combination it is most usual to use the x coordinate chordwise, y spanwise and z vertically upwards. Thus when taking a slice of the wing, i.e. an aerofoil section, it lies in the (x,z) plane. This notation will be used in the remainder of this course.

5.1 Aerofoil geometry



The standard shape definition was first adopted by NACA in 1929

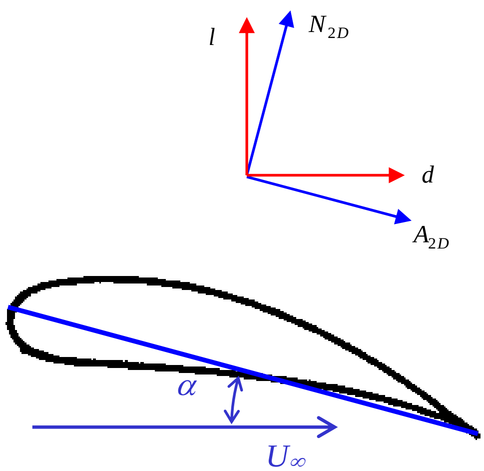
- *thickness envelope* wrapped around a *mean camber line*
- maximum camber z_c at x_c
- maximum thickness $t_{\max}$ at x_t
- leading-edge radius r_0 (+ trailing-edge angle)
- chordline – straight line joining LE and TE (of length c)
  - usual reference line for definition of (geometrical) angle of attack a

5.2 Angle of attack

The angle of attack or incidence, α is the angle between the chord line and free stream velocity (for 2D flow)

5.3 Axis System for Forces

The normal force N_{2D} and axial force A_{2D} are relative to the chord line, whilst the lift l and drag d are defined relative to the free stream direction. No standardised notation so may see alternatives



5.4 Force Coefficients

For aerofoils the lift, drag and moment coefficients are non-dimensionalised as follows

$$c_l = \frac{l}{1/2\rho_\infty V_\infty^2 c}, \quad c_d = \frac{d}{1/2\rho_\infty V_\infty^2 c}, \quad c_m = \frac{m}{1/2\rho_\infty V_\infty^2 c^2}$$

where the moments are usually taken about the leading edge or the quarter chord.

5.5 Centre of Pressure

The forces on the aerofoil are due to distributed loads imposed by pressure and shear stresses. If the loads are specified in terms of the resultant force, where on the aerofoil should the resultant be placed? The answer is that it should be located so that it produces the same effect as the distributed loads. This location is called the ***centre of pressure*** x_{cp} and is the point about which the pitching moment is zero, m_{cp} is zero. An expression can be found by taking moments about the leading edge, then assuming that the angle of attack is small and neglecting the drag contribution

$$\frac{x_{cp}}{c} = \bar{x}_{cp} = \frac{-c_{m_{LE}}}{c_l}$$

see for example Anderson, Fundamentals of Aerodynamics.

For a *symmetric* thin aerofoil x_{cp} = *0.25c, i.e.* is at the quarter chord point.

For a *cambered* aerofoil x_{cp} moves with α. It moves forward from $+\infty$ (!) at zero lift to an asymptote at +0.25*c.*

The movement of x_{cp} for cambered aerofoils is a drawback of using the centre of pressure as a reference point for moment data.

5.6 Aerodynamic Centre

The ***aerodynamic centre****,* x_{ac}, is a *fixed* point where the rate of change of pitching moment m_{ac} with incidence α is zero (ie m_{ac} is *constant*). The following expression can be assuming that the angle of attack is small and neglecting the drag contribution

$$\frac{x_{ac}}{c} = \bar{x}_{ac} = \frac{-dc_{m_{LE}}}{dc_l}$$

m_{ac} is often referred to as m_0, where the zero is used to indicate that this is the 'pitching moment at zero lift'.

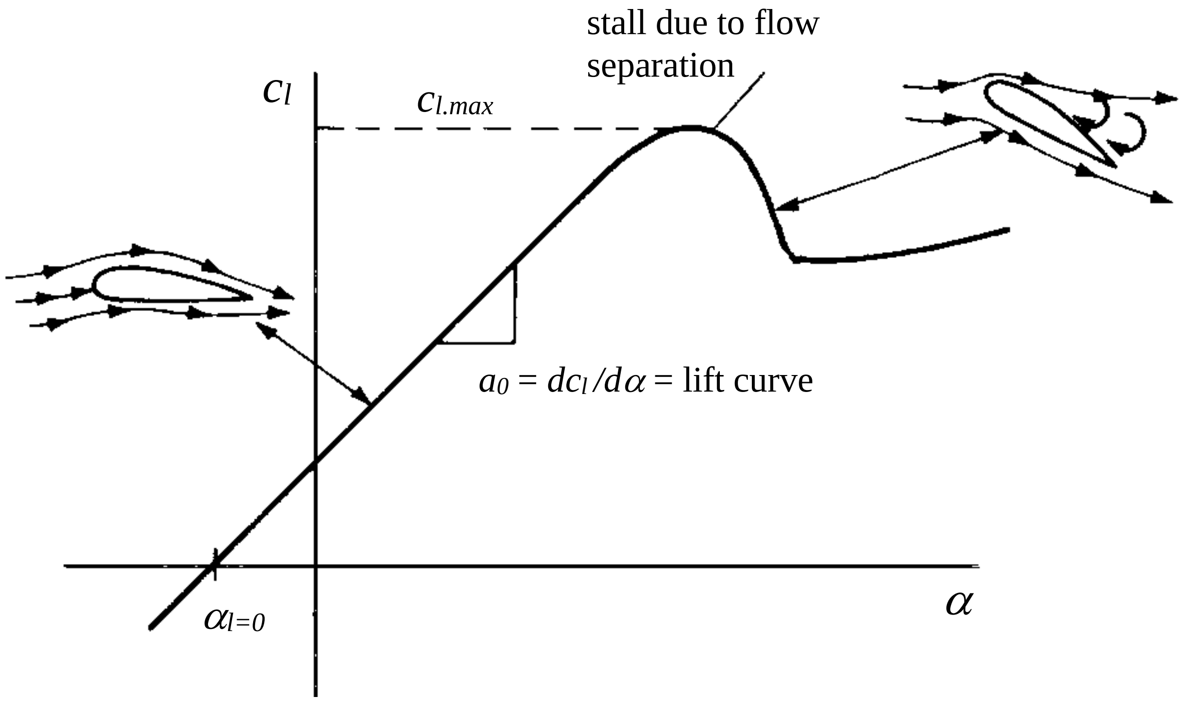
For a *symmetric* uncambered thin aerofoil x_{ac} = *0.25c* and m_0 is zero.

For a cambered aerofoil $x_{ac} \approx$ *0.25c* and m_0 is negative (nose down).

Using the aerodynamic centre is preferable to the centre of pressure as it is a fixed reference point.

5.7 Typical Aerofoil Characteristics

This figure shows the typical variation of the lift coefficient with angle of attack for an aerofoil.



At low to moderate angles of attack, c_l varies linearly with α, and the slope of this line is denoted by a_0 and is called the *lift slope*. In this region the flow is attached over the aerofoil surface. At higher angles of attack, the flow tends to separate from the top surface of the aerofoil due to viscous effects. There is a loss of lift and an increase in drag as the aerofoil stalls. The value of α where the lift equals zero is called the *zero-lift angle of attack* and is denoted by $\alpha_{l=0}$.

REVISION OBJECTIVES

You should be able to:

- Explain how conformal transformations can be used to obtain potential solutions about different geometries
- Explain the Kutta condition and how it determines circulation and lift.
- State Kelvin's circulation theorem
- Explain the development of the flow about an impulsively started aerofoil (i.e. set into motion from rest).
- Identify the chord line and camber line of an aerofoil
- State that angle of attack is measured relative to the chord line
- Draw a diagram showing the direction of forces on an aerofoil
- Give definitions of the centre of pressure and the aerodynamic centre
- Define a_0 and $\alpha_{l=0}$