AVDASI 3: Design Methods.
Design for Aircraft Structures

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1 Introduction

Welcome to **AVDASI 3**: Design Methods - *Design for Aircraft Structures*. This sub-unit is all about applying basic and advanced design methods for the structural analysis of thin-walled structures, obviously very common in aerospace. Although these methods can be used on a broad range of applications, we will focus on the design of an aircraft wing similar, in size, to the one you work with for the performance and aerodynamic coursework.

You are tasked with designing the load-carrying structure of a wing, which will involve a combination of analytical formulas and the development of your own personal numerical solver for preliminary structural analyses of aircraft wings.

By the end of this sub-unit you should be able

1. demonstrate a good understanding of fundamentals from load calculations, to stress/strain and structural failure.
2. to adapt analytical formulas for numerical implementations.
3. to develop preliminary structural analysis software tool, which will prepare you for **AVDASI 4**.

2 Assessment and Working Arrangements

To be clear, there is no coursework assignments in this unit. This sub-unit will be assessed through an individual online test. The test will be open for 24h from 1pm on Thursday the 14$^{\text{th}}$ of May to 1pm on Friday the 15$^{\text{th}}$ (London time). You will be able to start the test anytime during that period, but will only have 4 hours to complete the test once you start. For a <u>prepared</u> student, the test should to take about 2 hours to complete, so you will have plenty of time. You will not know the outcome of the test once you finish it and the test will not provide direct feedback. Marks and feedback will be released once everybody completed and I have post-processed the marks.

The design task I'm giving you will not be directly assessed, but the online test will assess your understanding of structural design. Hence, I strongly recommend you to develop your own code and give your best shot at this wing structural design. The reason is simple, the test will be set with randomised questions to make sure you all have achieved the same level of understanding. The best way for you to get that understanding is to code the physics of the problem yourself. Furthermore, you will have an easier time during the online time if you have a robust, working, code. That said, you can work together and help each other but remember that the online test on BB will be individual.

3 Lectures and Course Materials

Considering the current state of affairs, there will be no lectures, in person or online. Instead, I have written this document to set your task and reference the materials you will need. A document with my answers to your questions will also be made accessible to all students, probably via blackboard. If need be, I will consider setting up an online Q&A session.

You can find all information needed for the course via simple internet searches. Otherwise, you can find most, but not all, the information you need in the following book by Megson. Chapters 16-17-18-19-20 are in particular relevant and exemplify the level of technical details and physical simplifications I expect you to work with.

Figure 1: Recommended textbook, available as e-book on the University website

I have also uploaded older **AVDASI2** design documents to help you, but be careful many formulas you used last year have a lot of underlying simplification (e.g. rectangular symmetric wing box) which do not apply here. However, other formulas, e.g. buckling and displacement, are accurate enough for this exercise.

4 Objectives

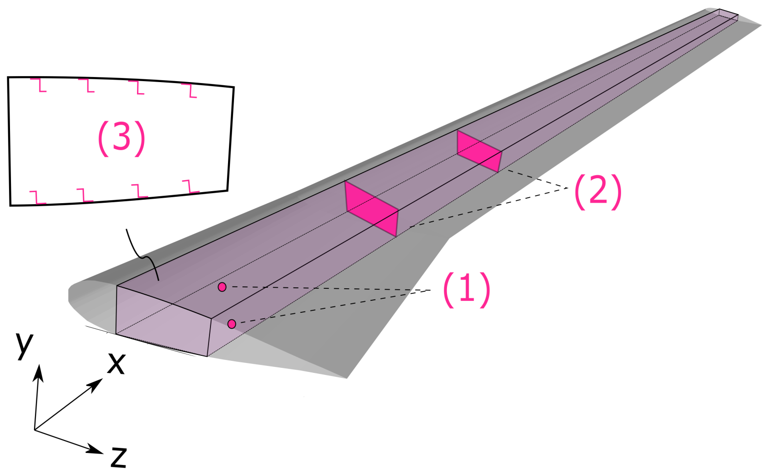
Given a wing geometry, its aerodynamic load and mass distributions, the overarching objective of this subunit is for you to develop a software tool capable of designing the load carrying wing box.

To be a bit more precise, this software should be able to:

1. Compute the internal shear, bending, and twisting moments at any spanwise location along the wing. This is equivalent to drawing the shear, bending and twisting moment diagrams.
2. Compute cross-sectional properties of a generic, non-isotropic, thin-walled, single-cell closed section. That includes properties like the cross-section centroid, elastic centre, shear centre, the two dimensional second moment of area and bending stiffness matrices.
3. Compute the direct or axial stress due to bending at any cross-sections along the wing span.
4. Compute the shear stress due to shear forces and twisting moments at any cross-section along the wing span.
5. Compute rib and stringer spacing to avoid buckling.
6. Compute the linear wing displacements.
7. (Optional) Evaluate failure indices due direct and shear stress along the wing span and at the joints.

You have already encountered most of the theory behind each of the objectives in STM1, STM2, STM3, and AVDASI 2. The challenge is for you to consolidate your understanding and being able to apply all of these concepts in a consistent and physically meaningful manner, in order to achieve a preliminary structural design. All objectives, except 7, will be evaluated during the online test.

The following chapters will setup the generic design problem. This problem will guide you in developing your code as well as understanding, in more details, what is expected of your software design tool. Note that, you can use any available programming language at your disposal, but I will only able to provide coding help for MATLAB.



Design Variables	Structural measures used for sizing
(1) wing box laminate thicknesses $t(x, y, z)$	direst and shear stresses
(2) ribs spacing R_i	skin and spar panel buckling
(3) stringer thickness S_t and spacing S_i	column buckling and stress

Figure 2: Wing design variables and sizing structural measurements

5 Design Problem and Structural Analyses Software Requirements

This section describe, in details, the design problem prepared for you and the expected capabilities of your software.

5.1 Sign Convention

First of all, I strongly recommend you to use the same sign convention as the one presented in this document. The online test example will also follow that convention. The sign convention is shown in Figure 3.

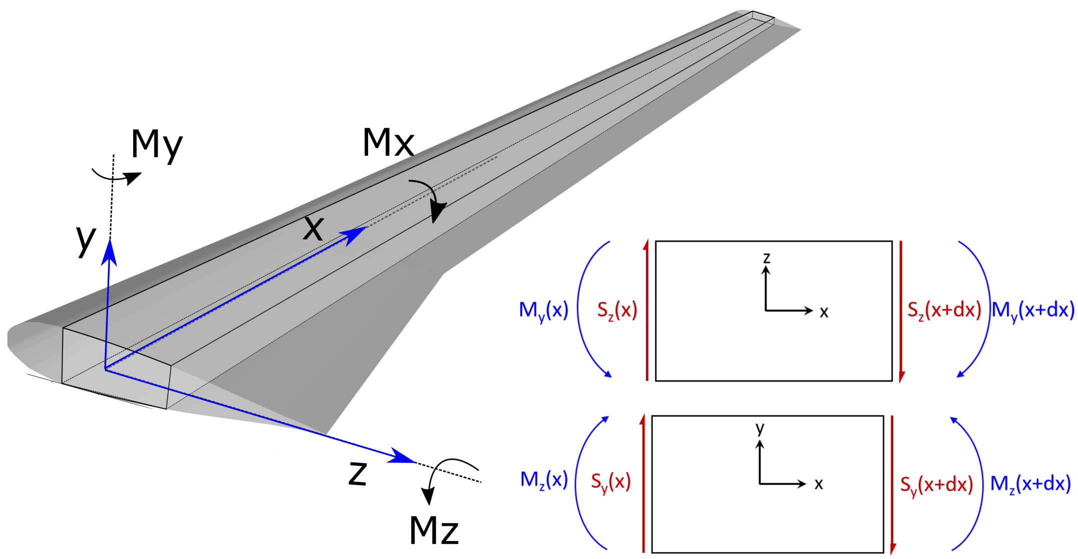


Figure 3: Sign convention recommended. The reference axis (0,0,0) is chosen arbitrarily.

5.2 Wing Geometry

The wing planform geometry and engine location is shown in Figure 4. For simplicity, assume no dihedral angle.

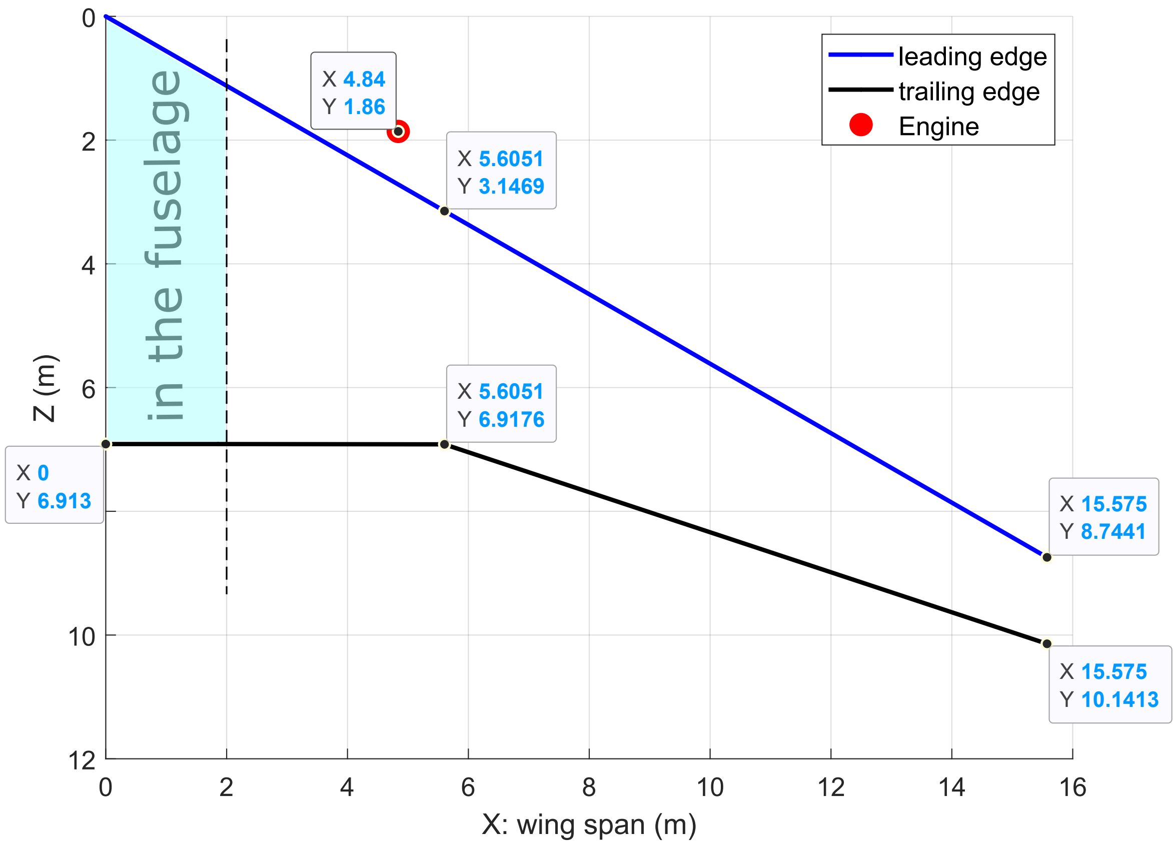


Figure 4: Wing planform geometry

You can freely choose the size of your wing box, but consider leaving space for the trailing and leading edge mechanisms.

The aerofoils profiles for the root, kink, and tip sections are given in Table 1. You can refine the aerofoil profile if you wish but that is not necessary. The profile are twisted with respect to their quarter chord. Twists angles for the root, kink, and tip profiles are $6°, 1.5°, -0.5°$ respectively, following the same sign convention. Finally, assume a linear interpolation of the profiles based on their spanwise locations.

Table 1: Aerofoil normalised profiles

Root Z	Root Y	Kink Z	Kink Y	Tip Z	Tip Y
1	0.0003	1	0.0004	1	0.0008
0.9109	0.0132	0.9021	0.0142	0.9	0.0145
0.8351	0.0224	0.8266	0.0233	0.8	0.0291
0.6488	0.0498	0.7003	0.0388	0.7	0.0437
0.5965	0.0575	0.6001	0.0512	0.6	0.0563
0.5593	0.0626	0.5525	0.0567	0.55	0.0611
0.5034	0.0696	0.5101	0.0614	0.5	0.0646
0.4477	0.0756	0.4547	0.0667	0.45	0.0668
0.3919	0.0804	0.4075	0.0705	0.4	0.0678
0.352	0.0833	0.3602	0.0734	0.35	0.0678
0.3094	0.0858	0.2972	0.0756	0.3	0.0662
0.2504	0.0887	0.25	0.0761	0.25	0.0643
0.1961	0.0905	0.208	0.0756	0.2	0.0606
0.153	0.0907	0.1513	0.0728	0.15	0.0549
0.099	0.0866	0.0996	0.0666	0.1	0.0468
0.074	0.0814	0.0736	0.0613	0.075	0.041
0.0495	0.073	0.053	0.0552	0.05	0.0335
0.0249	0.0582	0.0229	0.0408	0.025	0.0232
0.0143	0.0499	0.0129	0.0332	0.0125	0.016
0.0076	0.0415	0.0075	0.0275	0.0075	0.0123
0.005	0.0372	0.0047	0.0236	0.005	0.01
0.0023	0.0309	0.0026	0.0198	0.0025	0.007
0	0.0177	0	0.0088	0	0
0.0022	0.0038	0.0021	0.0004	0.0025	-0.0051
0.0049	-0.0018	0.0051	-0.0037	0.005	-0.0066
0.0072	-0.0053	0.0078	-0.0062	0.0075	-0.0077
0.0119	-0.0106	0.0139	-0.0103	0.0125	-0.0091
0.0243	-0.0204	0.023	-0.0147	0.025	-0.0116
0.0486	-0.0342	0.0509	-0.0244	0.05	-0.0148
0.0716	-0.0457	0.0725	-0.0301	0.075	-0.0174
0.0979	-0.0516	0.0961	-0.0352	0.1	-0.02
0.1488	-0.0607	0.1513	-0.0432	0.15	-0.0246
0.1953	-0.0632	0.208	-0.0477	0.2	-0.0291
0.2501	-0.0632	0.25	-0.0493	0.25	-0.0331
0.2945	-0.0626	0.3095	-0.05	0.3	-0.0359
0.3579	-0.061	0.3449	-0.0498	0.35	-0.0388
0.3965	-0.0595	0.3981	-0.0486	0.4	-0.0402
0.4543	-0.0563	0.4512	-0.0463	0.45	-0.0404
0.505	-0.0527	0.5066	-0.0428	0.5	-0.0393
0.5556	-0.0482	0.549	-0.0397	0.55	-0.0371
0.6063	-0.0427	0.5966	-0.0357	0.6	-0.0339
0.6485	-0.0375	0.6889	-0.0275	0.7	-0.0257
0.8317	-0.0149	0.8505	-0.0131	0.8	-0.0172
0.941	-0.0053	0.9313	-0.006	0.9	-0.0086
1	-0.0003	1	-0.0004	1	-0.0008

5.3 Load and Mass Distributions

The aircraft maximum take-off weight (MTOW) is 62,820kg, including a fuel capacity of 16,200kg. You are free to choose the fuel mass distribution along the wingspan as long as it is physically meaningful, i.e. your wing box is of appropriate volume. Note that, a well distributed fuel can provide some load relief. The engine location is fixed and the engine mass is approximately $2,500$kg.

In this exercise we will only consider three load cases, cruise 1g, 2.5g and -1g. Instead of using profile polar data, you can simply assume an elliptical distribution of lift forces applied at the aerofoil quarter chord. Furthermore, assume the wings only provide about 87% of the lift required. No need to consider aerodynamic drag or moment coefficient for this exercise, let's focus on the structural design.

Given these load distributions, calculate the shear, bending moment and torsional moment at any point along the span, i.e. equivalent to drawing the diagrams.

5.4 Cross-sectional analysis

The cross-sectional analysis will probably be the most challenging part of this sub-unit. Given the shear, bending and twisting loads you have calculated in the previous section, you must retrieve the direct axial stress and shear stress (or shear flow) in the wing box cross-sections. That is, the code you develop should be able to do that for any cross-section along the wing span. Cross-sections are generally not symmetric, and are simultaneously subject to bending, twisting and shear loads. You also have to consider the influence of stringer on stress distribution. The good news is that cross-sections behave linearly and are thin-walled, single cell closed sections, as illustrated in Figure 5. You can also assume that the effect of taper on the prediction of direct stresses produced by bending is minimal. Finally, you can investigate different stringer shapes, but Z-stringer are sufficient for this sub-unit.

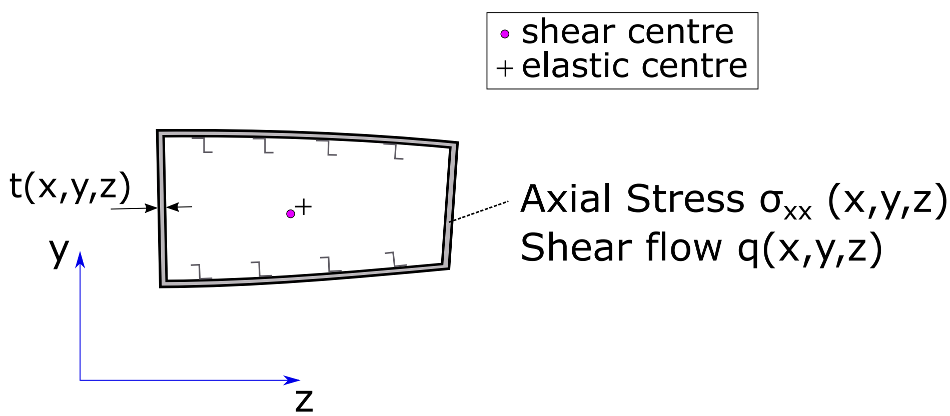


Figure 5: Wing box generic cross-section

Getting Started: You are already familiar with the analytical expression to evaluate simple cross-sectional properties like the centroid. If we find a way to numerically approximate the centroid for any given sections, based on our assumption, then you will find it easier to convert other formulas into their numerical form (e.g. elastic centre, shear stress, bending stiffness). For simplicity, I advice you to start on cross-sections without stringers. The contribution of stringer can be added afterwards. The chapter entitled '*Structural idealization*' in Megson's textbook will guide you in accounting for stringers and showcases simple matlab examples you can use to validate your work.

To get you started, I have decided to show you one way you could use to numerically evaluate cross-section properties. Let us say we want to calculate the z_c position of the centroid, the analytical formula for a continuous geometry is

$$z_c = \frac{1}{A} \int z \, da, \tag{1}$$

where A is the cross-section area. We can find a numerical approximation of the centroid by discretising the cross-section into N rectangular elements, as illustrated in Figure 6. The (x,y) positions, length, and thickness each element are denoted y_i, z_i, L_i, and t_i respectively. Following this notation, the cross-sectional area can be numerically approximated as

$$A = \int da \approx \sum_{i=1}^{N} \Delta A_i = \sum_{i=1}^{N} L_i t_i. \tag{2}$$

This approximation, based on a rectangular discretisation, will only be valid if the approximated geometry closely matches the analytical geometry. You can always increase the number of rectangular elements in order to refine your numerical approximation. The centroid position z_c is evaluated as

$$z_c = \frac{1}{A} \int z \, da \approx \frac{1}{A} \sum_{i=1}^{N} z_i \, \Delta A_i. \tag{3}$$

Many of the formulas you will need can also be approximated using this rectangular discretisation method.

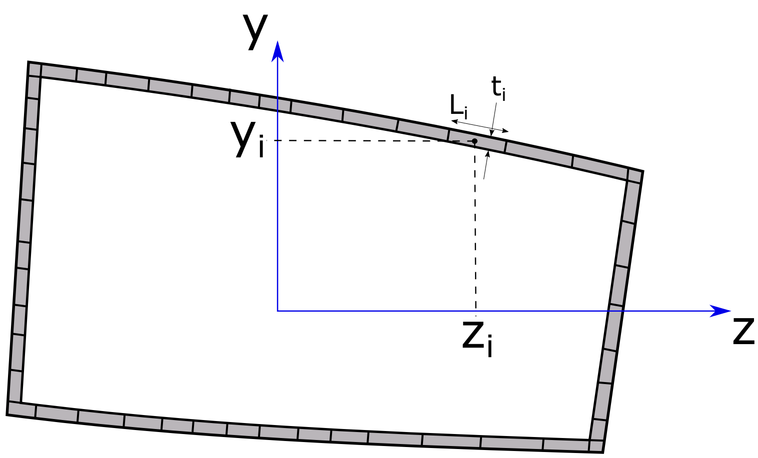


Figure 6: Discretised cross-section for numerical integration

5.5 Wing Box Materials

Composite materials are commonly used in modern wing box design, but this was not always the case. In this exercise, it is important to have an understanding of the relative properties of different materials. Hence, you are encourage to try out various materials to find the best possible design. Best is of course subjective, but you could try to achieve the less expensive design or the lightest design, or a combination of both. At least, I recommend you to design an aluminium and a composite wing box.

Composite laminates should have least have 10% of fibres in each directions. As you may remember, ±45° are good at carrying shear loads, whilst unidirectional laminate are better at carrying load along the fibre direction. Appropriately using composite laminate is key to achieve a lightweight wing box design.

Hint: To account for a composite laminate during your cross-sectional analysis you can use classical laminate analysis (CLA) to first homogenise your laminate properties.

5.6 Wing Linear Deformation

Lastly, it is often important to determine the maximal wing deflection due to bending and twisting. Although you do not yet have experience with finite elements, you can use simple formula introduced in STM1, STM2 and AVDASI2 for prismatic beam elements. Although the wing itself is not prismatic, you can subdivide the wing in many spanwise section and assume each small section to behave like a prismatic section. Once you have computed the deformation of each section, you can compound them to evaluate the wing global transverse deflection.