UNIVERSITY OF BRISTOL
FACULTY OF ENGINEERING

First Year Examination for the Degree of Master of Engineering

MAY/JUNE 2010 3 Hours

AENG11101

FLUIDS 1

This paper contains *two* sections

SECTION 1
Answer *all* questions in this section

This section carries *40 marks*.

SECTION 2
This section has *five* questions.

Answer *three* questions.

All questions in this section carry *20 marks* each.

The maximum for this paper is *100 marks.*

Calculators may be used.

For air, assume R = 287 J/kgK. Take 0^oC as 273^oK.
Use a gravitational acceleration of $9.81 m/s^2$

.

Useful Equations

The volume of a sphere is $\frac{4}{3}\pi r^3$ Area of a circle πr^2

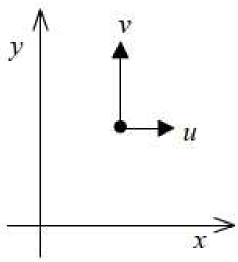
Roots of a quadratic $ax^2 + bx + c = 0 \quad \rightarrow \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

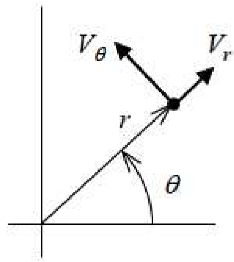
The equation of state for a perfect gas is
$p = \rho RT$

Drag equation Drag = Area $\times C_D \times \frac{1}{2}\rho V^2$

Change between Polar and Cartesian coordinate systems

$$r = \sqrt{x^2 + y^2}, \quad \theta = \tan^{-1}\left(\frac{y}{x}\right), \quad x = r\cos\theta, \quad y = r\sin\theta,$$
$$u = V_r \cos\theta - V_\theta \sin\theta, \qquad v = V_r \sin\theta + V_\theta \cos\theta$$
$$V_r = u\cos\theta + v\sin\theta, \qquad V_\theta = -u\sin\theta + v\cos\theta$$





2D Potential Flow

Velocity components in two-dimensional irrotational flow, in terms of the stream function and potential are given by

$$\underbrace{V_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} \quad V_\theta = -\frac{\partial\psi}{\partial r} \quad V_r = \frac{\partial\phi}{\partial r} \quad V_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta}}_{\text{polar coordinates}} \qquad \underbrace{u = \frac{\partial\psi}{\partial y} \quad v = -\frac{\partial\psi}{\partial x} \quad u = \frac{\partial\phi}{\partial x} \quad v = \frac{\partial\phi}{\partial y}}_{\text{Cartesian coordinates}}$$

The stream function & velocity potential in Polar coordinates and the velocity distribution for:
i) A uniform flow U_∞ parallel to the x axis:

$$\psi = U_\infty r\sin\theta, \quad \phi = U_\infty r\cos\theta, \quad V_r = U_\infty \cos\theta, \quad V_\theta = -U_\infty \sin\theta, \quad u = U_\infty, \quad v = 0$$

ii) A source, of strength Λ, at the origin:

$$\psi = \frac{+\Lambda\theta}{2\pi}, \quad \phi = \frac{+\Lambda}{2\pi}\ln r, \quad V_r = \frac{+\Lambda}{2\pi r}, \quad V_\theta = 0, \quad u = \frac{+\Lambda}{2\pi}\frac{x}{(x^2+y^2)}, \quad v = \frac{+\Lambda}{2\pi}\frac{y}{(x^2+y^2)}$$

iii) A doublet, of strength κ, at the origin:

$$\psi = \frac{-\kappa}{2\pi}\frac{\sin\theta}{r}, \; \phi = \frac{+\kappa}{2\pi}\frac{\cos\theta}{r}, \; V_r = \frac{-\kappa}{2\pi r^2}\cos\theta, \; V_\theta = \frac{-\kappa}{2\pi r^2}\sin\theta, \; u = \frac{-\kappa}{2\pi}\frac{(x^2-y^2)}{(x^2+y^2)^2}, \; v = \frac{-\kappa}{2\pi}\frac{2xy}{(x^2+y^2)^2}$$

iv) A vortex, of strength Γ, at the origin:

$$\psi = \frac{+\Gamma}{2\pi}\ln r, \quad \phi = \frac{-\Gamma}{2\pi}\theta, \quad V_r = 0, \quad V_\theta = \frac{-\Gamma}{2\pi r}, \quad u = \frac{+\Gamma}{2\pi}\frac{y}{(x^2+y^2)}, \quad v = \frac{-\Gamma}{2\pi}\frac{x}{(x^2+y^2)}$$

SECTION 1   Answer <u>all</u> questions in this section

Q1.1 Calculate the pressure at a depth of 100m below the surface of a lake. Assume that the atmospheric pressure at the lake surface is $1.023 \times 10^5 Nm^{-2}$ and that the density of the water is 1000 kg m^{-3}.

(4 *marks*)

Q1.2 A swimming pool has two square gates (water inlet and exit) each with a height and width of 0.2m. The vertical inlet gate is located in the side of the pool with the bottom edge of the gate at a depth of 1m. The exit gate is located horizontally in the floor of the pool at a depth of 2m. What is the total force acting on each gate, assuming that atmospheric pressure is acting on the outer face of each gate? The density of the water is 1000 kg m^{-3}.

(4 *marks*)

Q1.3 State the assumptions that must be made for Bernoulli's equation to be valid.

(4 marks)

Q1.4 Pressure and velocity are measured at points along a streamline in a slow airflow. At one point the velocity and static pressure are measured at $5ms^{-1}$ and $0.9 \times 10^5 Nm^{-2}$ respectively. Downstream the velocity is measured at $25ms^{-1}$, what is the static pressure? Take the density of the air as 1.2 kg m^{-3}.

(*4 marks*)

Q1.5 A surface with a sharp leading edge moves through still air at a speed of $10ms^{-1}$. Assuming a transition Reynolds number of $Re_x = 5 \times 10^5$, what distance from the leading edge will the flow start to become turbulent?

Take the kinematic viscosity for air as $\nu = 1.47 \times 10^{-5}\, m^2 s^{-1}$

Figure 1.1 below shows two velocity profiles, (a) & (b), non-dimensionalised by the free stream velocity and boundary layer thickness. Identify the turbulent and laminar profiles.

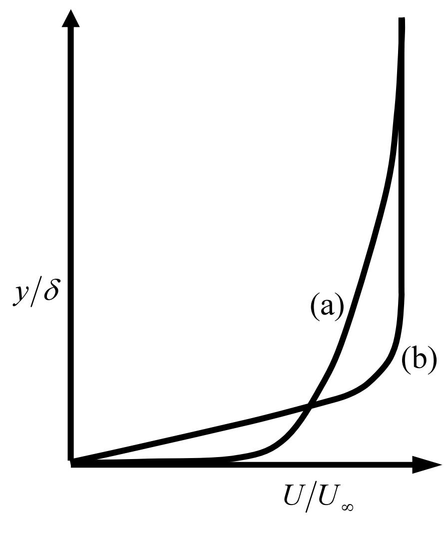


Figure 1.1 non-dimensionalised velocity profiles

(4 marks)

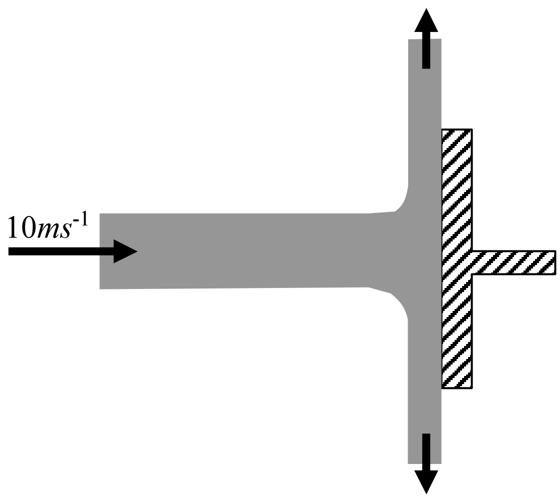
Q1.6 A car drives at $80ms^{-1}$ through air at 20^oC. Using the formulae for the speed of sound in air, $a = \sqrt{\gamma RT}$, calculate the Mach number and comment on the compressibility effects on the flow. For air γ =1.403 and R =287 J/kg K while T is in ^oK.

(4 marks)

Q1.7 In the steady flow momentum equation for control volume analysis, the net force can be split into pressure forces, viscous shear forces and body forces. How do these forces act on the control volume boundary and any submerged bodies.

(4 marks)

Q1.8 A horizontal circular water jet of radius 4cm and speed $10ms^{-1}$ hits a flat stationary vertical plate. By using a suitable control volume find the horizontal force on the flat plate if all the water leaving the plate is moving vertically. Assume the water has a density of 1000 kg m^{-3}.



(4 marks)

Q1.9 Explain briefly how a doublet flow is obtained from a combination of elementary potential flows.

(4 marks)

Q1.10 The potential function of a 2D flow is given by

$$\phi = xy + x + y$$

Find expressions for the velocity components and the stream function of this flow.

(4 marks)

SECTION 2 Answer *three* questions in this section

Q2.1 Assuming that the temperature decreases linearly with altitude (at a rate λ) the pressure and temperature at an altitude, z, are given by

$$p = p_{sl}\left(1-\lambda z/T_{sl}\right)^{\frac{g}{R\lambda}} \quad \text{and} \quad T = T_{sl} - \lambda z$$

where the subscript "sl" denotes the value at sea level (z=0), g is the acceleration due to gravity and R is the specific gas constant of air.

(a) A spherical hydrogen balloon of diameter 900mm is used to carry meteorological equipment. Find the altitude it will reach if the total mass of the balloon and instruments is 220g, the instruments have negligible volume and the volume of the balloon remains unchanged with altitude. You may assume that R=287 J/kg K λ=0.0065°K/m, while the temperature and pressure at sea level are 20°C and $1.009\text{x}10^5\text{Nm}^{-2}$ respectively.

(10 marks)

(b) If the density of the air at sea level is 1.2 kg m^{-3}, find the force required to keep the same balloon at sea level before release.
Find the terminal velocity of the balloon once released. You may assume conditions are still effectively at sea level, and the Drag coefficient (C_D) at this speed is 0.45
Use the terminal velocity to find the Reynolds number based on the sphere diameter and a kinematic viscosity for air of $\nu = 1.47\times10^{-5}\,\text{m}^2/\text{s}$.

(7 marks)

(c) Just after the balloon is released and the upward velocity is small, would you expect the drag coefficient to be higher or lower than the value at the terminal velocity? The terminal velocity calculated in (b) means that the flow around the sphere never becomes critical. How would you expect the drag coefficient to change if the velocity of the balloon increased so the flow becomes critical then supercritical?

(3 marks)

Q2.2 Two identical large fuel tanks, A & B, of uniform cross sectional area are joined by a small sharp edged orifice. Fuel is forced from one to the other by pumping air into the top of the tanks. At a particular instant the air pressures in tanks A and B is p_A and p_B respectively, while the height of fluid above the centre of the orifice is h_A and h_B as shown in figure 2.2a below.

(a) Using a quasi steady assumption, show that the mass flow rate of fuel from A to B is given by

$$\dot{m} = CA_o\sqrt{2\rho\left[(p_A - p_B) + \rho g(h_A - h_B)\right]}$$

where ρ is the density of the fuel, A_o is the area of the orifice and C is the ratio of the orifice area to the jet area at the vena-contracta condition. Clearly state all assumptions made during your derivation.

(10 marks)

(b) The sharp edged orifice described in (a) is replaced by a short straight pipe with a smooth inlet (shown in figure 2.2b). The pipe is short enough so that viscous effects can be neglected. The initial pressure and fuel heights at a particular instant are measured as: p_A=2x10^{5}N/m^2, p_B=0.8x10^{5}N/m^2, h_A=1.6m and h_B=0.8m. At the same point in time the fuel in tank A is found to be falling at a rate of 1cm/s. If the specific gravity of the fuel is 0.9, what is the ratio of pipe exit area to the horizontal cross sectional area of the tanks?

(8 marks)

(c) Comment on the use of the short pipe described in (b) if the fuel flowed from B to A instead of A to B.

(2 marks)

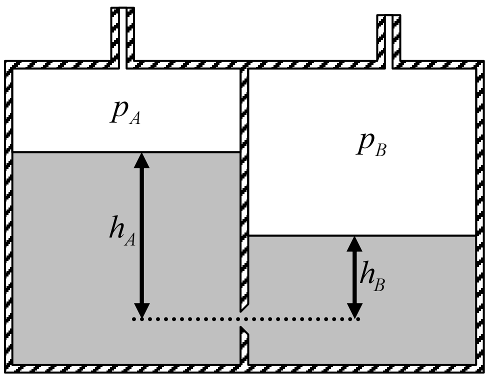


Figure 2.2a: Levels and pressure in two fuels tanks connected by a sharp edged orifice

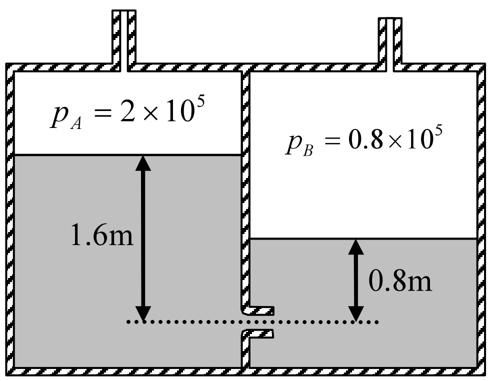


Figure 2.2b: Initial state for fuel tanks connected by short pipe with smooth inlet

Q2.3 Figure 2.3a below shows a low speed, open section wind tunnel. The air is drawn from static atmospheric conditions, through a smooth contraction designed to eliminate total pressure losses, into a parallel working section of area A_w. The air in the working section has a uniform velocity of V_a, The air then passes over the fan, where the area remains fixed before exiting to atmospheric conditions through an expansion and straight section with an exit area of A_e.

(a) Find the differential height of manometer fluid, Δh, in terms of the air velocity V_a, air density ρ_a, the manometer fluid density ρ_m and the acceleration due to gravity g.
State all the assumptions you have made.

(7 marks)

(b) The pressure at the exit, downstream of the fan, is atmospheric ($p{=}p_a$). Derive an expression for the change in static pressure across the fan, Δp_f, in terms of only: A_w, A_e, ρ_a & V_a.

(7 marks)

(c) A model mounted on a vertical strut (see figure 2.3b) is placed in the working section of the wind tunnel. If the total drag on the model and strut, D, is given by

$$D = C_D \times \tfrac{1}{2}\rho_a V_m^2 \times A_m$$

where C_D is the constant drag coefficient, A_m is the frontal area of the model plus strut and V_m is the new working section velocity upstream of the model. If the pressure jump across the fan (Δp_f), is unaltered by the presence of the model show that

$$V_m = \left[1 + C_d \frac{A_m A_e^2}{A_w^3}\right]^{-\frac{1}{2}} V_w$$

(6 marks)

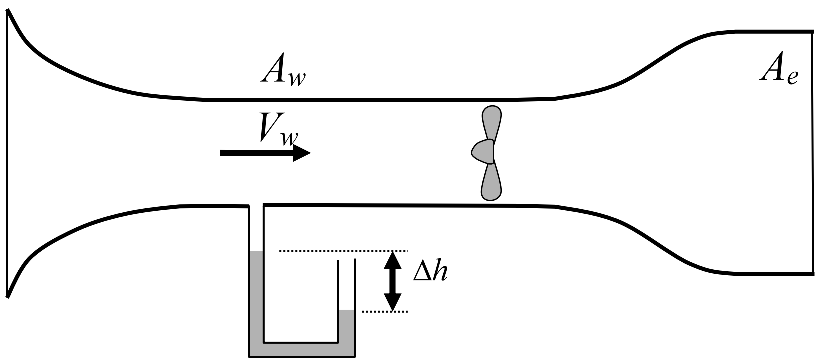


Figure 2.3a: Schematic diagram of empty wind tunnel and manometer

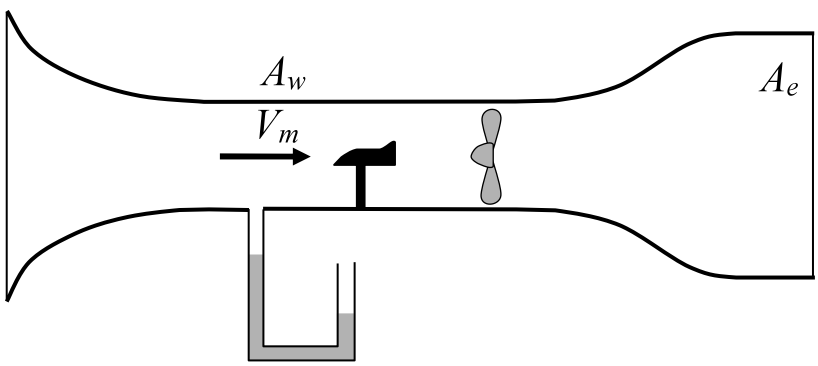


Figure 2.3b: Schematic diagram of the same wind tunnel with model and strut

Q2.4 Air flows at V m/s far upstream of a windmill that sweeps out a circular disc of diameter d m. At a point downstream of the windmill, where the pressure has returned to atmospheric, the velocity of the wind is measured at aV m/s. a is a constant that represents the ratio of downstream to upstream wind speeds.

(a) Use the actuator disc theory for an ideal windmill to show that the force on the windmill can be written as

$$F = \rho \tfrac{\pi}{8} d^2 V^2 \left(1 - a^2\right) \quad ,$$

where ρ is the density of the air. Clearly state all assumptions made during your derivation.

(8 marks)

(b) Continuing the analysis of the windmill defined above, show that the efficiency is given by

$$\eta = \frac{(1+a)\left(1-a^2\right)}{2} = \frac{1+a-a^2-a^3}{2} \quad .$$

(6 marks)

(c) An ideal windmill of diameter 20m, works at an efficiency of $\eta = 0.5$ in a 12ms^{-1} wind. If the air density is 1.2 kg m^{-3}, find the force on the windmill, the air velocity through the disc and the mean gauge pressures just in front of and just behind the disc.

(6 marks)

Q2.5 (a) Briefly describe the physical significance of the stream function ψ in incompressible flow. What are the limits on its application, compared with the potential function ϕ?

(2 marks)

(b) A wind of speed 10 m/s is blowing over a rounded cliff; the situation is modelled as a source of strength $\Lambda = 130\pi\ m^2/s$ at the origin combined with a uniform stream $U_\infty = 10\ m/s$ in the x – direction.

i) Draw a sketch of the streamlines of the flow (internal & external to the cliff).

(2 mark)

ii) Find the location of the stagnation point.

(4 marks)

iii) Find the height of the cliff surface where it intersects the y axis.

(3 marks)

iv) The height of the cliff as $x \to \infty$

(2 marks)

v) At the location where the cliff is half its ultimate downstream size, what is the percentage difference relative to the free stream velocity of the velocity measured by a sensor located 2m above the surface of the cliff?

(7 marks)

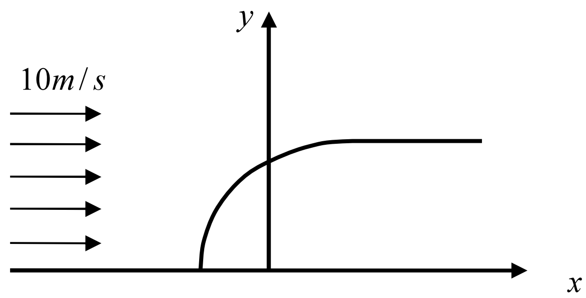


Figure 2.5: Coordinate system for the cliff