Vibrations 2, Lecture 18
Energy method for 1DOF systems

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Lecture 18

- Energy method for 1DOF multi-body systems
- Example

Energy method

This lecture looks at the **energy method** for the derivation of the natural frequencies of 1DOF systems with low or no damping.

Assumptions:

- low damping (ζ<0.005) or undamped system,
- harmonic vibration at the frequency ω,
- single degree-of-freedom.

Energy conservation is considered between the two extreme configurations:

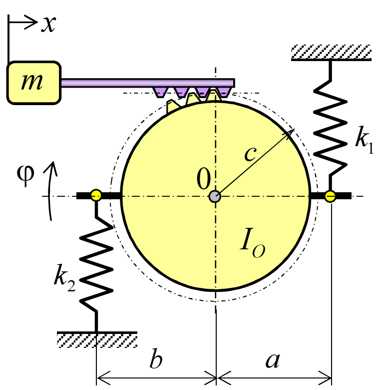
- maximum displacement (zero velocity) $\Rightarrow$ maximum E_P (potential e.)
- maximum velocity (zero displacement) $\Rightarrow$ maximum E_K (kinetic energy)

$$E_{P,tot}(q_{max}) = E_{K,tot}(\dot{q}_{max})$$

$$\sum_{(i)} E_{P,i}(q_{max}) = \sum_{(j)} E_{K,j}(\dot{q}_{max}) \quad \Rightarrow \omega_0$$

Example: energy method

Determine the natural frequency of a part of rudder control mechanism using the energy method.

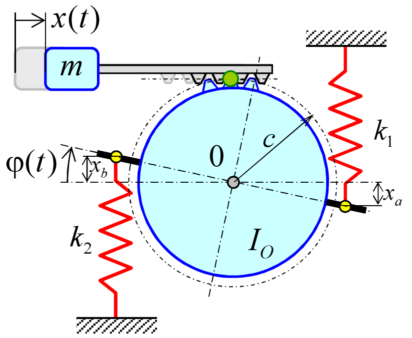


Selected energy formulas:

Energy	x	φ
$2E_K$	$m\dot{x}^2$	$I_0\dot{\varphi}^2$
$2E_P$	$k\,x^2$	$k_t\,\varphi^2$

Example 5: energy method

The following *kinematic* relationship applies: $x = c\varphi \Rightarrow \dot{x} = c\dot{\varphi}$

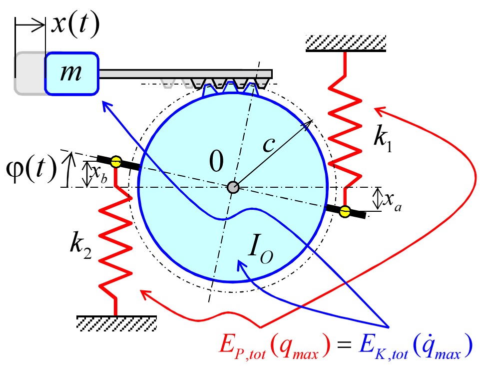


This system consists of two bodies. There exists a (kinematic) relationship between the two involved motions. As a result, this is the 1DOF system. One coordinate (DOF) is chosen for further analysis. Free response of this 1DOF system is then characterized by its (undamped) harmonic response:

$$\varphi = \Phi_0 \sin(\omega t), \dot{\varphi} = \Phi_0 \omega \cos(\omega t) \Rightarrow \varphi_{max} = \Phi_0, \dot{\varphi}_{max} = \Phi_0 \omega$$

Example 5: energy method

Energy analysis and energy balance:



Energy conservation between the two extreme conditions:

$$\underline{E_{P,k1}(\varphi_{max}) + E_{P,k2}(\varphi_{max})} = \underline{E_{K,I0}(\dot{\varphi}_{max}) + E_{K,m}(\dot{\varphi}_{max})}$$

Example 5: using energy method

The total kinetic energy :

$$E_{K,tot} = \frac{1}{2} I_0 \dot{\varphi}^2 + \frac{1}{2} m \dot{x}^2 = \frac{1}{2}\left[I_0 \dot{\varphi}^2 + m (c \dot{\varphi})^2 \right] = \frac{1}{2}\left[I_0 + m c^2 \right]\dot{\varphi}^2 = \frac{1}{2} I_E \dot{\varphi}^2$$

$$E_{K,tot,max} = \frac{1}{2}\left[I_0 + m c^2 \right]\Phi_0^2 \omega^2 = \frac{1}{2} I_E \Phi_0^2 \omega^2$$

The total potential energy :

$$E_{P,tot} = \frac{1}{2} k_1 x_a^2 + \frac{1}{2} k_2 x_b^2 = \frac{1}{2}\left[k_1 (a\varphi)^2 + k_2 (b\varphi)^2 \right] = \frac{1}{2}\left[k_1 a^2 + k_2 b^2 \right]\varphi^2 = \frac{1}{2} k_{E,\varphi} \varphi^2$$

$$E_{P,tot,max} = \frac{1}{2}\left[k_1 a^2 + k_2 b^2 \right]\Phi_0^2 = \frac{1}{2} k_{E,\varphi} \Phi_0^2$$

Energy conservation:

$$E_{P,tot,max} = E_{K,tot,max} \Rightarrow k_{E,\varphi} \Phi_0^2 = I_E \Phi_0^2 \omega^2 \Rightarrow \omega_0^2 = \frac{k_{E,\varphi}}{I_E} = \frac{k_1 a^2 + k_2 b^2}{I_0 + m c^2}$$

Summary

- Energy method can be used with *undamped* 1DOF systems for fast calculation of natural frequencies
- This method is useful in systems with multiple bodies constrained such that the resulting system is a single DOF system
- This method can be extended further to multi-DOF cases in the form of *Rayleigh's method* (assume harmonic vibration and mode shape; energy equation will provide the natural frequency estimate).

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