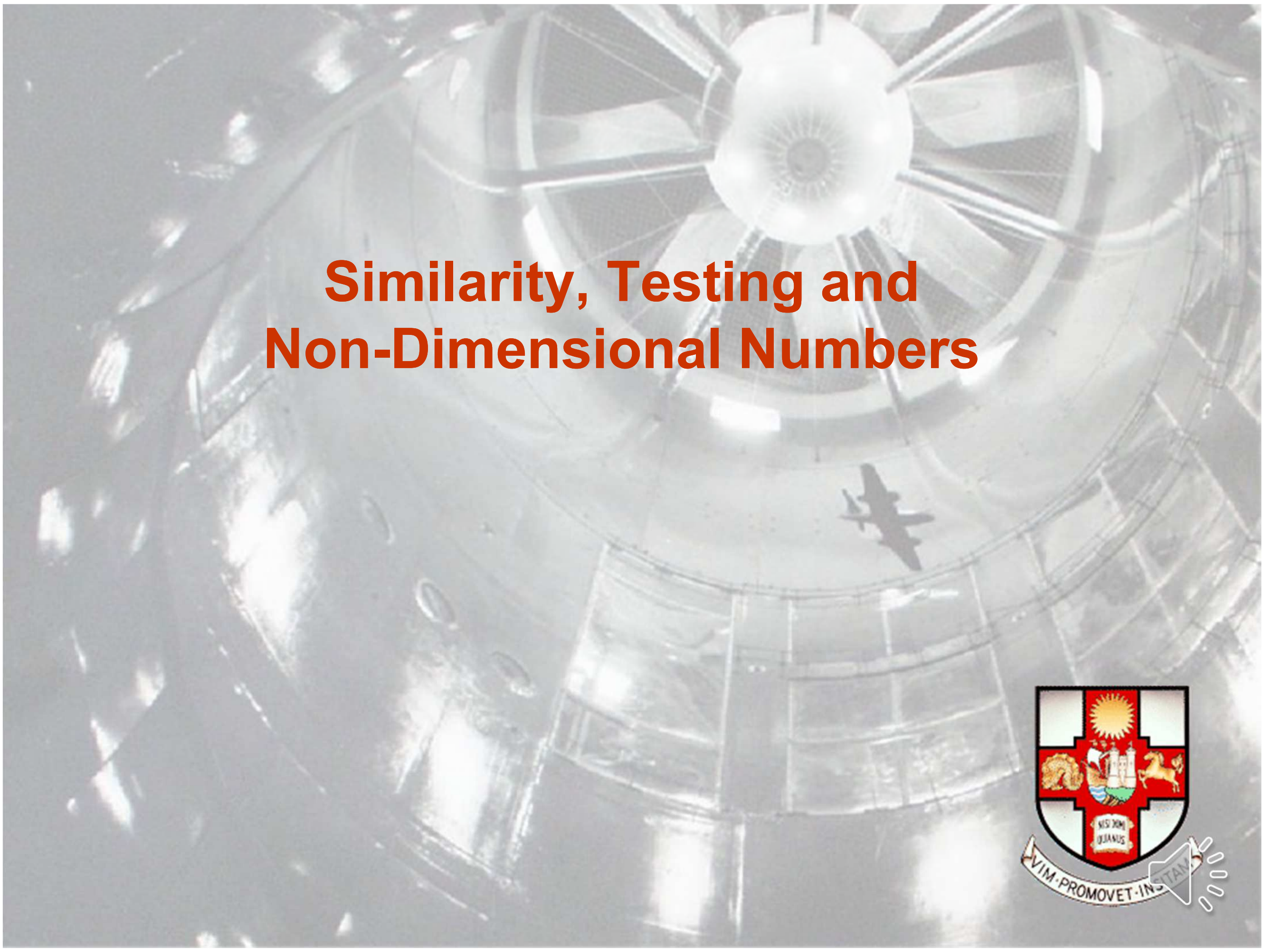
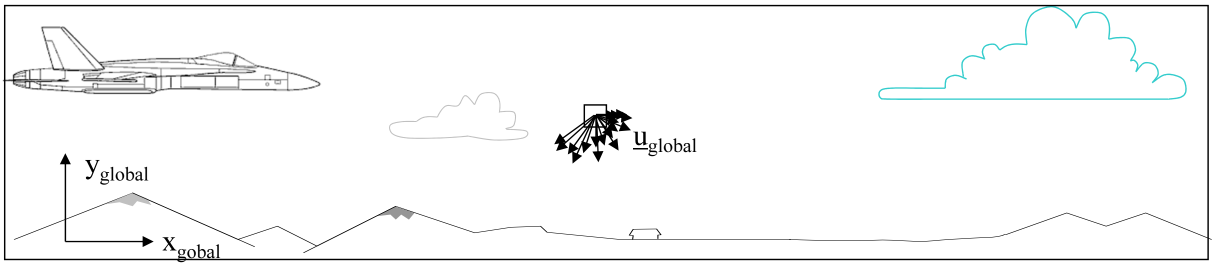


Unsteady and Steady Flows

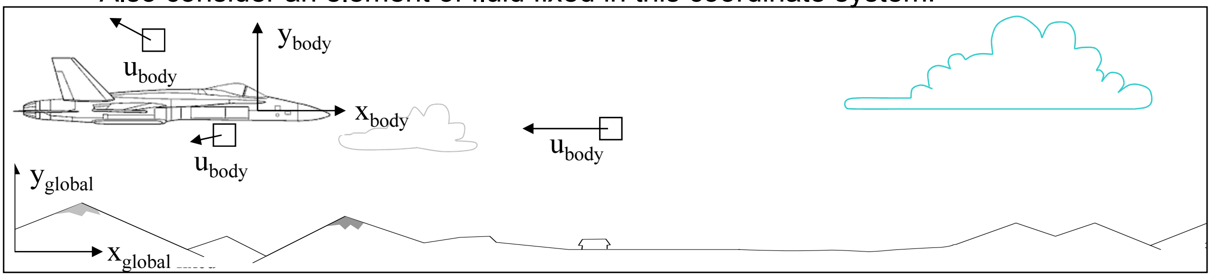
- By considering the motion of the fluid relative to a moving set of reference axes, many familiar unsteady fluid problems can be transformed into equivalent steady situations.
- This is an incredibly simple and powerful piece of analysis, BUT an understanding of the relationship between total and static values (pressure and temperature) is essential. The transformed problem is not identical to the original situation.
- The fundamentals of fluid experiments to model vehicles becomes straight forward once this analysis is carried out.

Unsteady to Steady Flows-Choosing Coordinates

- Consider an aircraft, moving into still air, relative to a global fixed coordinate system.
- Also consider an element of fluid at a constant location in this coordinate system.



- We can see that $\underline{u}_{global}$ varies with time – unsteady flow
- Consider an aircraft moving relative to a body fixed coordinate system.
- Also consider an element of fluid fixed in this coordinate system.



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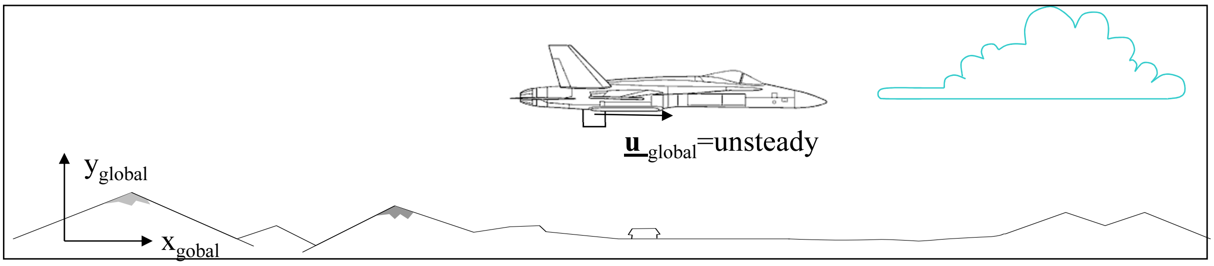

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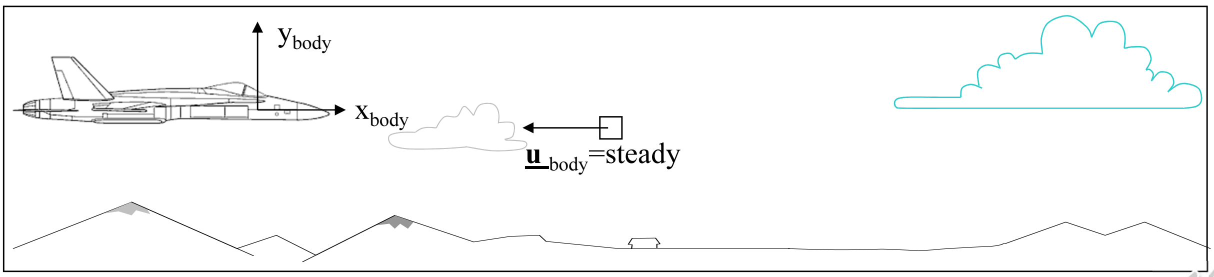


Galilean Transformation

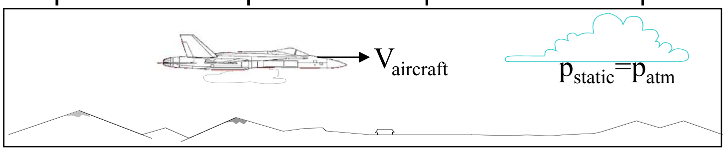
- The Galilean Transformation relates the coordinates of two reference frames which differ only by a constant relative motion.
- The previous example is a Galilean transformation where
- $$\underline{\mathbf{x}}_{body} = \underline{\mathbf{x}}_{global} + \begin{bmatrix} V_{aircraft} \\ 0 \\ 0 \end{bmatrix} t$$
- We need not consider the mathematics. The important points are that:
  – the relative motion of the air and any surfaces remains unchanged
  – the STATIC pressure & temperature remain UNCHANGED.

$$\underline{\mathbf{u}}_{body} = \underline{\mathbf{u}}_{global} + \begin{bmatrix} V_{aircraft} \\ 0 \\ 0 \end{bmatrix} \qquad p_{body} = p_{global} \qquad T_{body} = T_{global}$$

- Now consider a stationary aircraft in a flow of air with an onset velocity of $V_{aircraft}$. The flow is exactly the same as in the previous examples as long as the static pressure and temperature are unchanged. This can be ensured just by making sure that these static values are the same at some location far upstream.

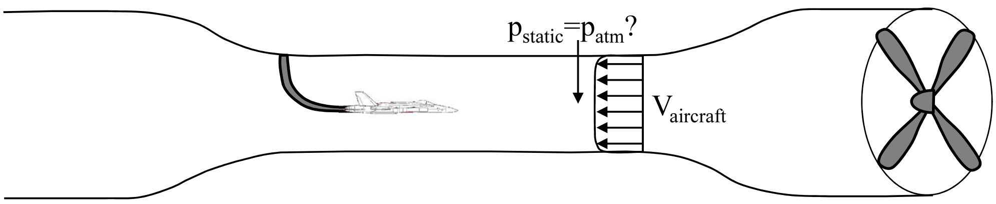
Equivalent Flows

- Again consider an aircraft, moving into still air at a speed $V_{aircraft}$, relative to a global fixed coordinate system.
- The upstream static pressure is equal to the atmospheric pressure



- Consider the same aircraft placed in a large wind-tunnel, with a steady onset flow of $V_{aircraft}$ and the inlet pressure adjusted such that the static pressure of the onset flow is equal to atmospheric. The flow relative to the aircraft motion and the pressure distribution would then be identical in both cases.

Provided outer streamlines match, so wind tunnel walls match streamlines & viscosity at the walls is negligible

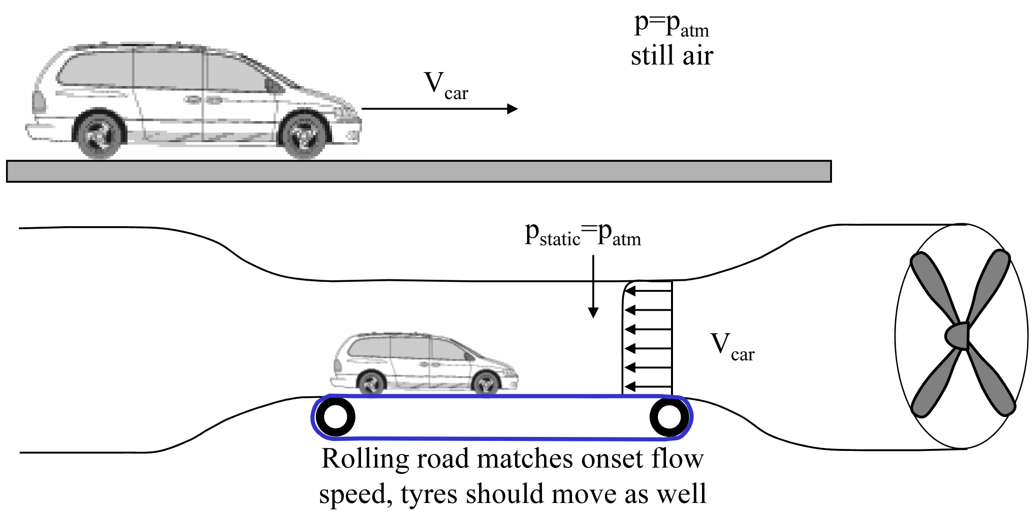


- If fluid starts at atmospheric then when it is accelerated the static pressure will decrease! This is usually OK as it is the pressure gradients that define the flow. The absolute values interact with temperature and density (eqn of state).
- But wind tunnel models are not full scale!

It is important to compare pressure differences rather than absolute values.

Equivalent Flows

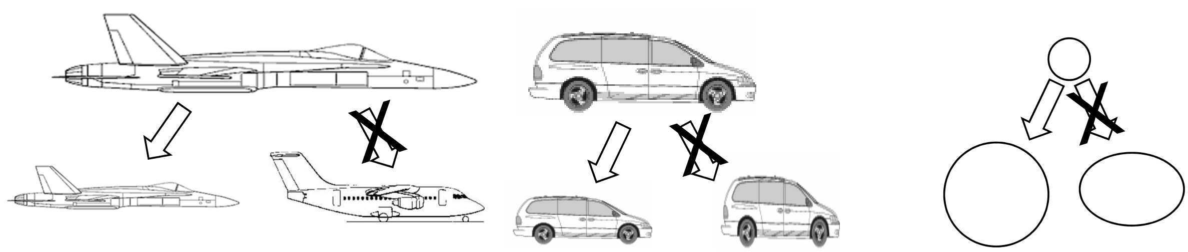
- Similar situation for other vehicles
- Should consider flow around the road and wheels



- Generally, models need to be small enough so outer streamlines are parallel to wind tunnel walls (or use a deformable wind tunnel wall)

Flow Similarity

- Two flows over two different bodies are similar if
  - Streamlines are the same   Remember wind tunnel walls
  - Plots of flow quantities, relative to some reference values, (*V, p, ρ & T*) are the same throughout the flow field
- This requires that the
  - Bodies are geometrically similar (can be different sizes)



  - Similarity parameters are the same for both flows
- This ensures that
  - Force coefficients are the same

Dimensional Analysis

- ‘dimensions’
  - measurable properties used to describe physical state of a system
- SI fundamental dimensions
  - Length [L] Do not confuse with units Meter m
  - Mass [M] Kilogram kg
  - Time [T] Second s
  - Temperature [θ] Kelvin K
- Fourier’s Principle of Dimensional Homogeneity
  - dimensions on each side of an equation must be the same
- basic applications of dimensional analysis
  - use to **determine** relations between parameters
  - use to **check** derivations
- non-dimensional numbers
  - provide a means of **scaling** test results
  - denote regions of **validity** of flow models

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Simple Equation of Motion $V = U + a\,t$

$[L/T] = [L/T] + [L/T^2][T]$

- 1-D Navier Stokes equation

$$\rho\frac{\partial u}{\partial t} + \rho u\frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial x^2}$$

$$\left[\frac{M}{L^3}\frac{LT^{-1}}{T}\right]\left[\frac{M}{L^3}\frac{L}{T}\frac{LT^{-1}}{L}\right] = \left[\frac{ML^{-1}T^{-2}}{L}\right]\left[\frac{M}{LT}\frac{LT^{-1}}{L^2}\right]$$

Dimensional Analysis

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- SI fundamental dimensions

Do not confuse with units

Dimension	Symbol	Unit	Unit symbol
Length	[L]	Meter	m
Mass	[M]	Kilogram	kg
Time	[T]	Second	s
Temperature	[θ]	Kelvin	K

- Fourier’s Principle of Dimensional Homogeneity
  - dimensions on each side of an equation must be the same
- basic applications of dimensional analysis
  - use to **determine** relations between parameters
  - use to **check** derivations
- non-dimensional numbers — Similarity parameters: C_p, Re, M, …
  - provide a means of **scaling** test results
  - denote regions of **validity** of flow models

Euler Number

$$\boxed{C_P = \frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{pressure\ force}{inertia\ force}}$$

- pressure coefficient — Plotted upside-down (suction is positive)
- cavitation coefficient
- factor of ½ for consistency with Bernoulli’s equation
  - gives maximum positive value of +1 at stagnation point
  - ½ omitted in early UK reports
- similar form for force & moment coefficients
  - basic non-dimensionalisation for fluid dynamic loads

Reynolds Number

$$Re = \frac{\rho V L}{\mu} = \frac{VL}{\nu} = \frac{\text{inertia force}}{\text{viscous force}}$$

- *L* is a physically significant reference length
- Low Re – viscous forces are important
  - disturbances damped out – laminar flow
- High Re – viscous effects confined to thin region near body
  - turbulent flow
- Impacts on:
  - friction drag – shear stress at surface
  - flow stability – transition from laminar to turbulent
  - flow separation
  - dynamic similarity in model scaling

Mach Number

$$M = \frac{V}{a} = \sqrt{\frac{\rho V^2}{E_V}} = \frac{\text{inertia force}}{\text{elastic force}}$$

- Compressibility parameter
- Transitions from *subsonic* to *transonic* to *supersonic* have profound impact on flow phenomena
- Low M – compressibility effects can usually be neglected
  - **but** note that ‘low speed’ (ie $V \rightarrow 0$) is not necessarily the same as ‘incompressible’ flow
  - in first case speed of sound *a* is finite, in second *a* is infinite
  - $M<0.3$ in air can usually be considered incompressible
- Very high M – inertia effects dominate
  - Newtonian ‘impact’ theory

Strouhal Number

$$St = \frac{fL}{V}$$ or $$\tau = \frac{tV}{L} = \frac{time}{convective\ time}$$

- again *L* is a physically significant reference length
  - eg chord length *c*
- 'convective time' is the time it takes for a fluid element to convect distance *L* at velocity *V*
- unsteady flows
  - vortex shedding frequency
  - natural frequency of flow instability
- manoeuvring aerodynamics
  - characteristic frequency of aircraft motion
- take care with definition of frequency – *f* or ω ?

Background, not in exam

Froude & Weber Numbers

$$Fr = \frac{V}{\sqrt{gL}} = \sqrt{\frac{inertia\ force}{gravity}}$$

- flows with free surfaces
  - eg wave formation (wave drag) for ships
- dynamic testing of scale models
  - eg spin tunnel testing

$$We = \frac{\rho V^2 L}{\sigma} = \frac{inertia\ force}{surface\ tension}$$

- flows with free surfaces
  - eg droplet formation

Background, not in exam

Prandtl Number

$$Pr = \frac{\mu c_p}{k} = \frac{frictional\ dissipation}{thermal\ conduction}$$

- relates the thickness of the *hydrodynamic* and *thermal* boundary layers
- important in study of compressible, viscous flows

Ratio of Specific Heats

$$\gamma = \frac{c_p}{c_v} = \frac{enthalpy}{internal\ energy}$$

Background, not in exam

Knudsen Number

$$Kn = \frac{\bar{l}}{L} = \frac{mean\ free\ path}{characteristic\ length}$$

- ratio of mean free path of gas molecules to a representative dimension (eg body length or boundary layer depth)
- sets a limit on the fundamental continuum assumption underlying classical aerodynamics
- Kn < 0.01 – '**continuum**' flow, no slip at the surface
- Kn > 0.01 – 'rarified gas' flow, increasing slip at the surface (high Mach Number, or low density)
- Kn > 3 – 'free molecular' flow

Background, not in exam

Ekman Number

$$Ek = \frac{\nu}{\Omega L^2} = \frac{viscous\ \ force}{Coriolis\ \ force}$$

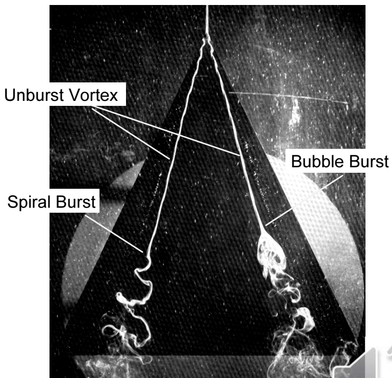
- Ω is the angular velocity of a rotating fluid system
- Coriolis acceleration results from translational motion in a rotating reference frame
- Ekman Number can be thought of as an *inverse* Reynolds Number for rotating flows
- impacts on formation & stability of secondary flows in rotating fluid systems
  - eg spin-up of fluid-filled cylinders
  - relative efficiency of momentum transfer from viscous diffusion (sidewalls) and secondary flows (base)

Background, not in exam

Rossby Number

$$Ro = \frac{U}{\Omega L} = \frac{inertia\ \ force}{Coriolis\ \ force}$$

- inverse of Rossby Number gives the effective **swirl ratio** Ω_0 (or swirl angle) of a rotating flow - ie $1/Ro \sim V_\theta / V_x$
  - when characteristic length = radius
- can be thought of as the ratio of the *convective* time of a fluid particle and the *rotation* time of the fluid system
- impacts on the stability of a rotating fluid system
  - eg vortex breakdown or bursting



Background, not in exam

Impact on Wind Tunnel Testing

- For example, at simplest level

$$C_D = fn(Re, M) = fn\left(\frac{\rho VL}{\mu}, \frac{V}{\sqrt{\gamma RT}}\right)$$

- Very difficult to match both *Re* and *M* at model scale

  a) don't bother
    - 'low' speed – *Re* is dominant effect, *M* can be neglected
    - 'high' speed – *Re* (above a critical level) can be neglected

  b) 'fix' the flow
    - eg transition strips to simulate *Re* effect on boundary layer

  c) test in a pressurised tunnel
    - ie increase ρ to compensate for reduction in *L*

  d) change the working fluid
    - heavy gas, or cryogenic operation

Learning Outcomes: "What you should have learnt so far"

- Galilean transformation
- The idea of flow similarity
- Flow similarity from a) geometrically similar bodies plus b) matching similarity parameters
- Definitions of common similarity parameters: Euler (pressure coefficient) Reynolds and Mach numbers
- The difficulty of matching the similarity numbers important in a flow to provide scale testing (such as wind tunnel testing)