Advanced Bending and Torsion
Torsion of Open Thin-Walled Sections

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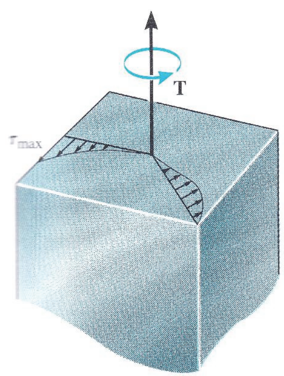
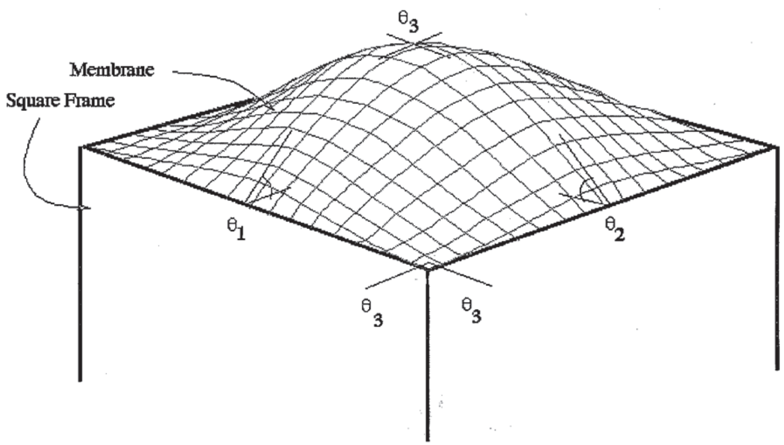
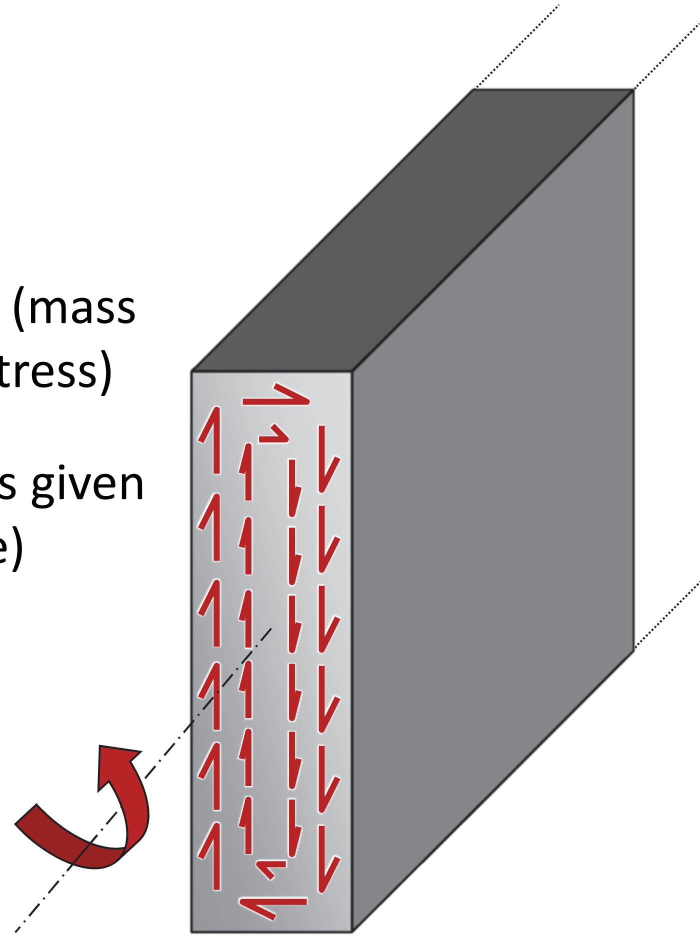
Torsion of open sections

- Open cross-sections resist torsion through the creation of shear flow **within the thickness**
  - Much lower torsion resistance
  - Avoid open sections if torsion is significant
- Using analogies to visualise shear stresses:
  - Closed section: fluid flow within a closed circuit (mass flow = shear flow, local velocity = local shear stress)
  - Open section: 'membrane analogy' (shear stress given by the orthogonal local gradient of the surface)



Shear stress distribution along two radial lines





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Thin Rectangular Beam

- Assume continuous shear flow along 'contour lines'

Along each contour line we have:

$$q_{zs} = q_{zn}$$

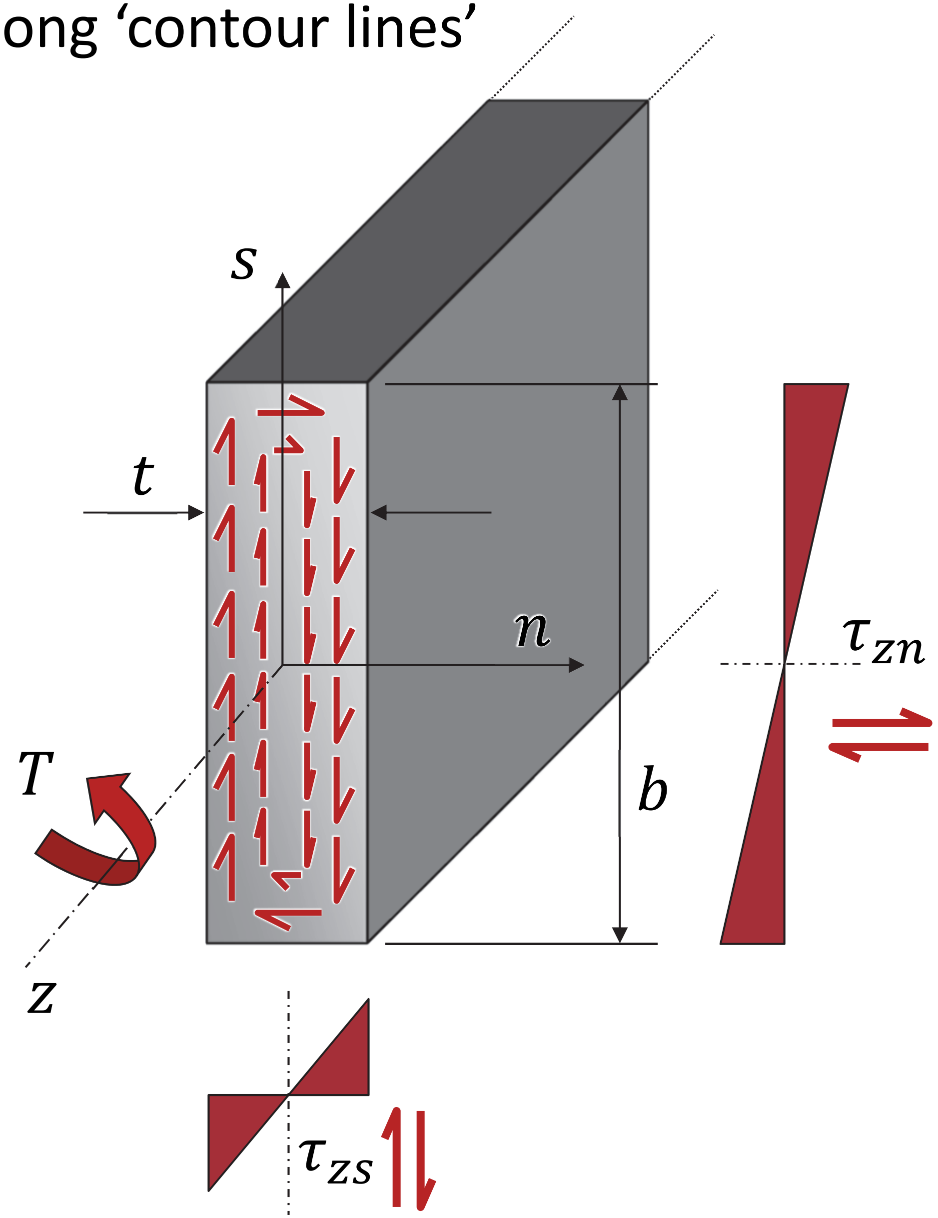
Therefore:

$$\tau_{zs}\, t = \tau_{zn}\, b$$

And:

$$\frac{\tau_{zs}}{\tau_{zn}} = \frac{b}{t}$$

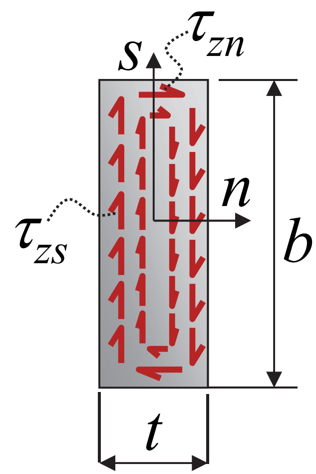
So shear flow is constant along contour lines, but shear stresses are **not** constant



Thin Rectangular Beam

- External torque T must be balanced by the torque due to the resultant internal shear stresses, therefore:

$$T = 2\int_0^{t/2} n\,\tau_{zs}\,b\,\mathrm{d}n + 2\int_0^{b/2} s\,\tau_{zn}\,t\,\mathrm{d}s$$



Shear stresses increase linearly from centre:

$$\tau_{zs} = \begin{cases} n = \dfrac{t}{2}, & \tau_{zs}^{\max} \\ n = 0, & 0 \end{cases} \qquad \tau_{zn} = \begin{cases} s = \dfrac{b}{2}, & \tau_{zn}^{\max} \\ s = 0, & 0 \end{cases}$$

So we can write:

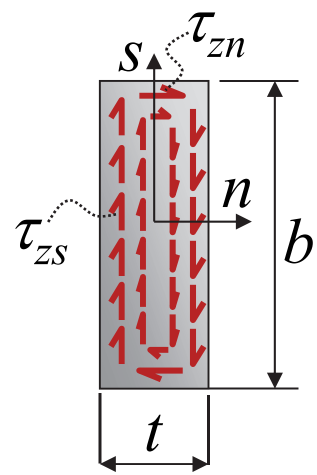
$$\boxed{\tau_{zs} = \left(\frac{2\,\tau_{zs}^{\max}}{t}\right) n} \qquad \tau_{zn} = \left(\frac{2\,\tau_{zn}^{\max}}{b}\right) s$$

Applying the relationship $\dfrac{\tau_{zs}}{\tau_{zn}} = \dfrac{b}{t}$ we get: $\boxed{\tau_{zn} = \left(\dfrac{2t\,\tau_{zs}^{\max}}{b^2}\right) s}$

Thin Rectangular Beam

- Replacing:

$$T = 2\int_0^{t/2} n\left[\left(\frac{2\,\tau_{zs}^{\max}}{t}\right)n\right] b\,\mathrm{d}n + 2\int_0^{b/2} s\left[\left(\frac{2t\,\tau_{zs}^{\max}}{b^2}\right)s\right] t\,\mathrm{d}s$$



$\tau_{zs}^{\max}$ is the overall maximum shear stress $\tau_{\max}$, therefore:

$$T = \frac{4}{3}\frac{b}{t}\,\tau_{\max}\,[n^3]_0^{t/2} + \frac{4}{3}\frac{t^2}{b^2}\tau_{\max}[s^3]_0^{b/2}$$

$\boxed{T = \frac{1}{3} b\,t^2\,\tau_{\max}}$ which can be re-written as: $\boxed{\tau_{\max} = \frac{T}{\left(\frac{b\,t^3}{3}\right)}t}$

Note similarities with the stress in a solid circular cross-section: $\boxed{\tau_{\max} = \frac{T}{J}R}$

Therefore the effective polar 2nd moment of area is: $J = \frac{b\,t^3}{3}$

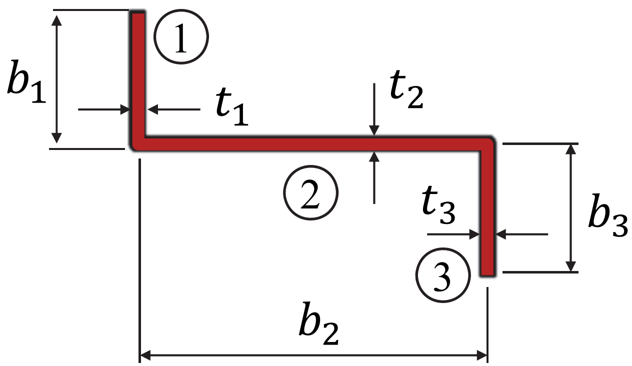
Simple Open Sections

- For open sections consisting of combined narrow rectangular sections, *e.g.*:

- we get:

$$J = \sum_{i=1}^{n} \left(\frac{b_i \, t_i^3}{3} \right)$$

- Example:



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