UNIVERSITY OF BRISTOL
FACULTY OF ENGINEERING

First Year Examination for the Degree of
Master of Engineering

MAY/JUNE 2014 3 Hours

FLUIDS 1

AENG11101

This paper contains *two* sections

SECTION 1
Answer *all* questions in this section
This section carries *40 marks*.

SECTION 2
This section has *five* questions.
Answer *three* questions.
All questions in this section carry *20 marks* each.
The maximum for this paper is *100 marks.*

Calculators may be used.

For air, assume R = 287 J/kgK. Take 0^{o}C as 273 K.
Use a gravitational acceleration of 9.81m/s^2
1 bar = 10^5 N/m^2

TURN OVER ONLY WHEN TOLD TO START WRITING

Useful Equations

The volume of a sphere: $\frac{4}{3}\pi r^3$ Area of a circle: πr^2

Roots of a quadratic: $ax^2 + bx + c = 0 \quad \rightarrow \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The equation of state for a perfect gas is:

$$p = \rho RT$$

Drag equation: $\text{Drag} = \text{Area} \times C_D \times \frac{1}{2}\rho V^2$

Equation of a streamline in 2D flow

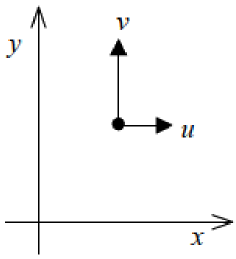
$$\frac{dy}{dx} = \frac{v}{u}$$

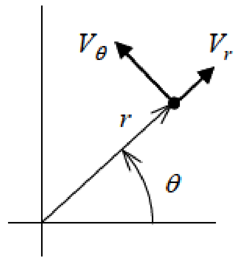
Change between Polar and Cartesian coordinate systems

$$r = \sqrt{x^2 + y^2}, \quad \theta = \tan^{-1}\left(\frac{y}{x}\right), \quad x = r\cos\theta, \quad y = r\sin\theta,$$

$$u = V_r\cos\theta - V_\theta\sin\theta, \qquad v = V_r\sin\theta + V_\theta\cos\theta$$

$$V_r = u\cos\theta + v\sin\theta, \qquad V_\theta = -u\sin\theta + v\cos\theta$$





2D Potential Flow

Velocity components in two-dimensional irrotational flow, in terms of the stream function and potential are given by

$$\underbrace{V_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} \qquad V_\theta = -\frac{\partial\psi}{\partial r} \qquad V_r = \frac{\partial\phi}{\partial r} \qquad V_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta}}_{\text{polar coordinates}}$$

$$\underbrace{u = \frac{\partial\psi}{\partial y} \qquad v = -\frac{\partial\psi}{\partial x} \qquad u = \frac{\partial\phi}{\partial x} \qquad v = \frac{\partial\phi}{\partial y}}_{\text{Cartesian coordinates}}$$

The stream function & velocity potential in Polar coordinates and the velocity distribution for

i) A uniform flow U_∞ parallel to the x axis:

$$\psi = U_\infty r\sin\theta, \quad \phi = U_\infty r\cos\theta, \quad V_r = U_\infty\cos\theta, \quad V_\theta = -U_\infty\sin\theta, \quad u = U_\infty, \quad v = 0$$

ii) A source, of strength Λ at the origin:

$$\psi = \frac{+\Lambda\theta}{2\pi}, \quad \phi = \frac{+\Lambda}{2\pi}\ln r, \quad V_r = \frac{+\Lambda}{2\pi r}, \quad V_\theta = 0, \quad u = \frac{+\Lambda}{2\pi}\frac{x}{(x^2+y^2)}, \quad v = \frac{+\Lambda}{2\pi}\frac{y}{(x^2+y^2)}$$

iii) A doublet, of strength κ at the origin:

$$\psi = \frac{-\kappa}{2\pi}\frac{\sin\theta}{r}, \qquad \phi = \frac{+\kappa}{2\pi}\frac{\cos\theta}{r}, \qquad V_r = \frac{-\kappa}{2\pi r^2}\cos\theta, \qquad V_\theta = \frac{-\kappa}{2\pi r^2}\sin\theta,$$

$$u = \frac{-\kappa}{2\pi}\frac{\left(x^2 - y^2\right)}{\left(x^2 + y^2\right)^2}, \qquad v = \frac{-\kappa}{2\pi}\frac{2xy}{\left(x^2 + y^2\right)^2}$$

iv) A vortex, of strength Γ, at the origin:

$$\psi = \frac{+\Gamma}{2\pi}\ln r, \quad \phi = \frac{-\Gamma}{2\pi}\theta,$$

$$V_r = 0, \quad V_\theta = \frac{-\Gamma}{2\pi r}, \qquad u = \frac{+\Gamma}{2\pi}\frac{y}{\left(x^2 + y^2\right)}, \quad v = \frac{-\Gamma}{2\pi}\frac{x}{\left(x^2 + y^2\right)}$$

Useful integrals

$$\int \sin^2(ax)dx = \frac{x}{2} - \frac{\sin 2ax}{4a} + C$$

$$\int \cos^2(ax)dx = \frac{x}{2} + \frac{\sin 2ax}{4a} + C$$

turn over …

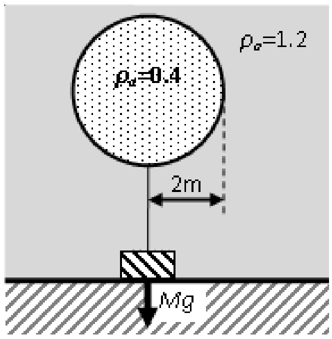
SECTION 1
Answer <u>all</u> questions in this section

Q1 Calculate the pressure and the gauge pressure at a depth of 10*m* below the surface of a lake. Assume that the atmospheric pressure at the lake surface is $1.023\text{x}10^5 Nm^{-2}$ and that the density of the water is 1000 $kg\ m^{-3}$.

(4 marks)

Q2 A weather balloon of radius 2*m* is tied to a mass M with a light inextensible rope. When deflated the balloon has a mass of 10 *kg*, and forms an approximate sphere. The density of the air and gas inside the balloon are 1.2 $kg\ m^{-3}$ and 0.4 $kg\ m^{-3}$ respectively. Find the smallest mass, M, that stops the balloon rising.

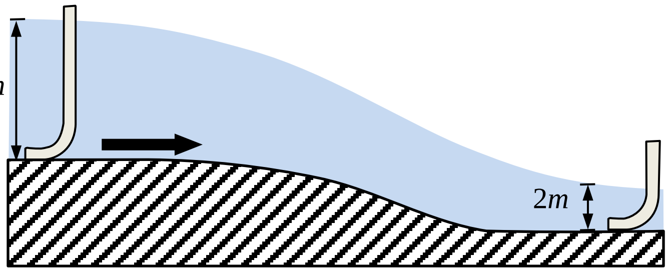
(4 marks)



Q3 State the assumptions that must be made for Bernoulli's equation to be valid.

(4 marks)

Q4 Water flows along a channel with slowly varying depth. The channel cross section is rectangular with a constant width. At a first point where the depth of water is 6*m*, a pitot probe measures a gauge pressure of $0.65\text{x}10^5\ Nm^{-2}$ at the bottom of the flow. At a second point downstream, where the height of the water is 2*m*, find the gauge pitot pressure measured at the bottom of the flow and the velocity of the water assuming 1-D flow and a constant atmospheric pressure throughout. Take the density of the water as 1000 $kg\ m^{-3}$.



(4 marks)

Q5 A light aircraft flies at 70ms^{-1} through air at 20^oC. Using the formulae for the speed of sound in air, $a = \sqrt{\gamma RT}$, calculate the Mach number and comment on the size of any compressibility effects on the flow. For air use γ =1.403.

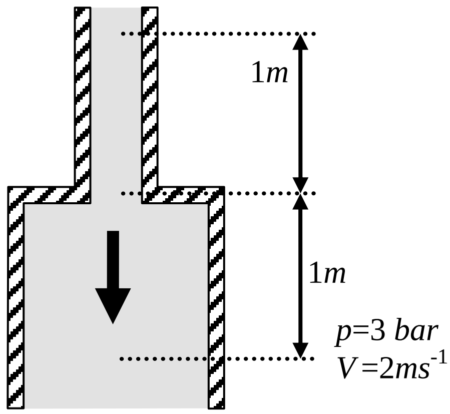
(4 marks)

Q6 A smooth flat surface with a sharp leading edge moves through still water at a speed of $10ms^{-1}$. If the transition Reynolds number is $\text{Re}_x=5.0\text{x}10^5$, what distance from the leading edge will the flow start to be turbulent? Would an increase or decrease in pressure gradient lower the Reynolds number of transition? Give another factor that would lower the transition Reynolds number. Take the kinematic viscosity of water as $1.14\text{x}10^{-6}\ m^2s^{-1}$

(4 marks)

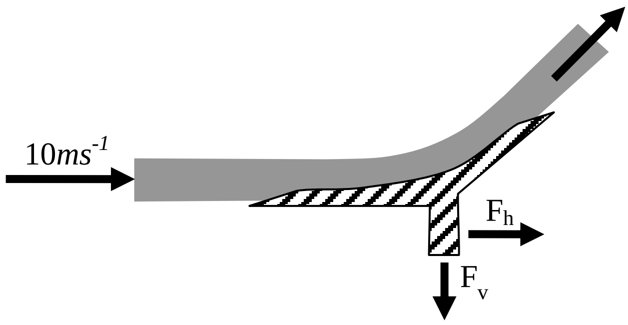
Q7 Oil, with a specific gravity of 0.9, flows down a straight smooth vertical pipe with a circular cross-section. At a particular point, 1m downstream of a sudden enlargement, where the flow may be assumed to have returned to uniform, the pressure and velocity are 3 *bar* and 2ms^{-1} respectively.

If the area ratio across the enlargement is 2 then the loss coefficient is 0.25. Hence, find the static pressure 1m upstream of the expansion. Assume the water has a density of 1000 kg m^{-3}.



(4 marks)

Q8 A horizontal circular water jet of diameter 10*cm* and speed 10ms^{-1} hits a stationary turning vane that smoothly turns the water through 45^o. By using a suitable control volume, find the horizontal and vertical forces on the turning vane. Assume the water has a density of 1000 $kg\ m^{-3}$.



(4 marks)

Q9 State the equation for the incompressible pressure coefficient, c_p, in terms of: p, p_∞, ρ, & U_∞. Define each variable used. From this derive an equation for the pressure coefficient in terms of only the local and far-field velocities.

(4 marks)

Q10 Which three basic flows are combined to model the flow over a lifting cylinder? Similarly, the flow over an oval can be modelled as a combination of which three basic flows?

(4 marks)

turn over…

SECTION 2
Answer *three* questions in this section

Q11 (a) A dam is constructed as an isosceles triangular prism of height H and length L, as shown in figure Q11 below. The sides of the dam slope at an angle α to a maximum height H while the water level is at a height h. Assuming the effect of atmospheric pressure is negligible and that the dam can only move by tipping, show that the value of α which just stops the dam tipping is given by

$$\tan(\alpha) = \sqrt{\left(6SG\frac{H^3}{h^3} + 6\frac{H}{h} - 1\right)}$$

where SG is the specific gravity of the dam material. Note that the centre of area of a triangle is 1/3rd of the height from the base, while the second moment of area of a rectangle of height y and width b is given by
$I_{xx} = by^3/12$

(14 marks)

(b) A dam, as described in (a), made of concrete with a specific gravity of 2.5 is used to retain a 200*m* width of water to a height of 50*m*. Assuming that the dam cannot slip horizontally and that there is no leak of water under the dam, what is the minimum volume of concrete that can be used? Assume water has a density of 1000 $kg\ m^{-3}$.

(6 marks)

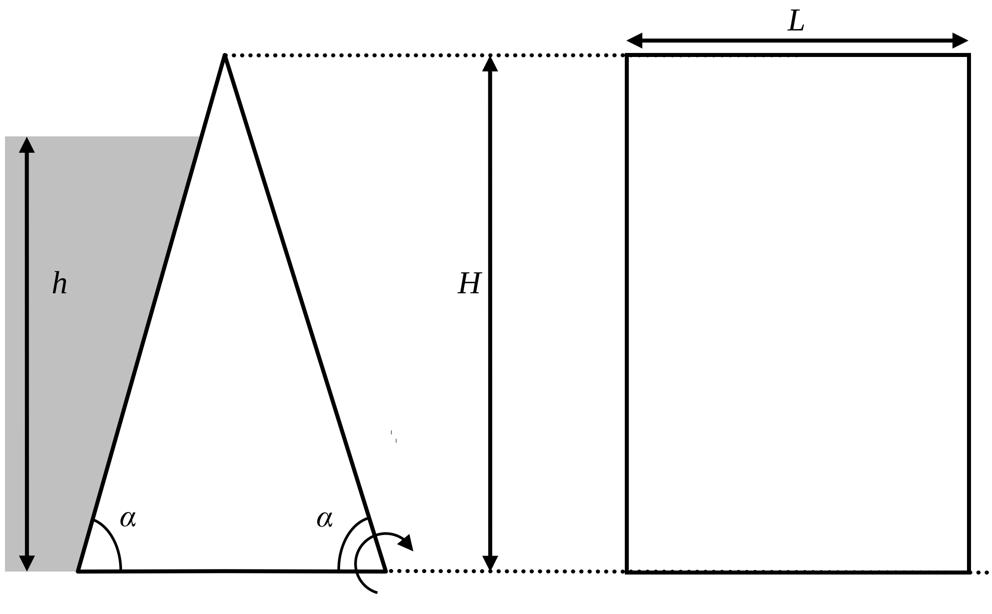


***Figure Q11**: Diagram of dam used in question 11, Side-on view and head-on view from downstream*

Q12 A Venturi Meter can be used to calculate velocity by taking pressures at two points with known cross sectional areas. Such an arrangement is used to calculate the flow rates of water at a hydroelectric power station and is shown in figure Q12 below. Locations 1 and 2 are separated by a vertical distance L and are connected via static pressure tapings and a u-tube manometer

(a) Using subscripts 1 & 2 to identify values at each location, show that the mass flow rate of water is given by

$$\dot{m} = A_1 A_2 \sqrt{2g\Delta h_m \frac{\rho(\rho_m - \rho)}{(A_1^2 - A_2^2)}}$$

where ρ is the density of the water, ρ_m is the density of the manometer fluid, A_1 & A_2 are the cross sectional areas at 1 & 2 respectively and Δh_m is the difference in height of manometer fluid between the arms connected to static tapings at 1 and 2. Clearly state all assumptions made during your derivation. Note that hydrostatic forces must not be neglected in the Venturi or manometer arms.

(10 marks)

(b) Water is drawn from a reservoir, whose surface is at a height 20*m* above location 2, into a straight pipe with a circular cross section of diameter 4*m*. The diameter of the pipe remains unchanged until the contraction at the Venturi where the diameter becomes 0.5*m*. If the manometer fluid has a specific gravity of 9 and the vapour pressure of water is $2.4\text{x}10^3$ Nm^{-2}, what will be the mass flow rate and manometer reading, Δh_m, at the point of onset of cavitation? Take the atmospheric pressure at the reservoir surface to be $1.02\text{x}10^5$ Nm^{-2} and the density of the water as 1000 $kg\ m^{-3}$.

(8 marks)

(c) Reviewing your calculations in (b), what would happen to the maximum mass flow rate if the Venturi Meter were moved closer to the surface of the reservoir.

(2 marks)

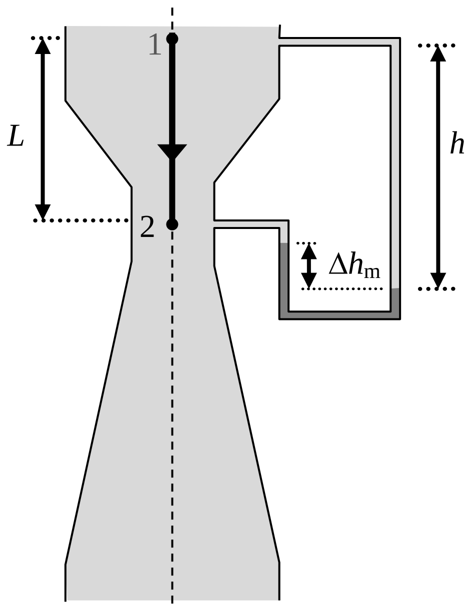


***Figure Q12**: Venturi and u-tube manometer arrangement*

turn over..

Q13 Figure Q13a below shows a low-speed, open-circuit wind tunnel. The air is drawn from static atmospheric conditions, through a smooth contraction designed to eliminate total pressure losses, into a parallel working section of area A_w. The air in the working section has a uniform velocity of V_w, The air then passes through the fan, where the area remains fixed before exiting to atmospheric conditions through an expansion and straight section with an exit area of A_e.

(a) Find the differential height of manometer fluid, Δh, in terms of the air velocity V_w, air density ρ, the manometer fluid density ρ_m and the acceleration due to gravity g. State all the assumptions you have made.

(7 marks)

(b) The pressure at the exit, downstream of the fan, is atmospheric ($p=p_a$). Derive an expression for the change in static pressure across the fan, Δp_f, in terms of only: A_w, A_e, ρ & V_w. *(7 marks)*

(c) A model mounted on a vertical strut (see figure Q13b) is placed in the working section of the wind tunnel. The total drag on the model and strut, D, is given by

$$D = C_D \times \tfrac{1}{2}\rho V_w^2 \times A_m$$

where C_D is the constant drag coefficient, A_m is the frontal area of the model plus strut. If the flow rate through the wind tunnel remains unchanged, show that the force on the fan is given by

$$F_f = \tfrac{1}{2}\rho V_w^2 \left(A_w \left(\frac{A_w}{A_e} \right)^2 + C_d A_m \right)$$

(6 marks)

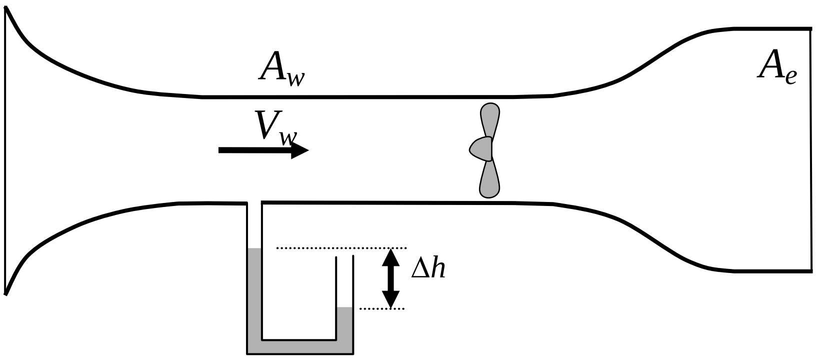


***Figure Q13a**: Schematic diagram of empty wind tunnel and manometer*

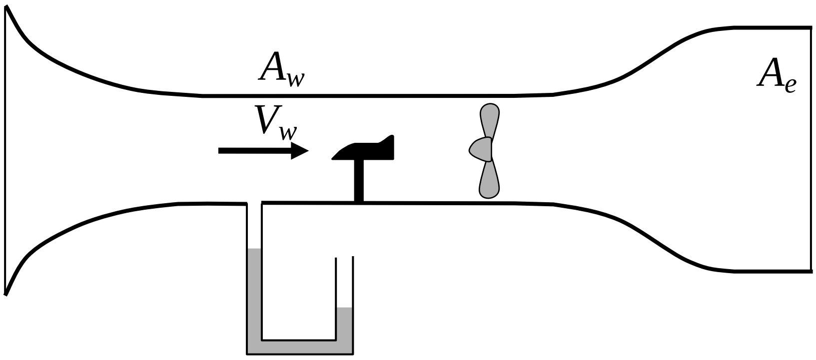


***Figure Q13b**: Schematic diagram of the same wind tunnel with model and strut*

Q14 Air flows at $V\ ms^{-1}$ far upstream of a windmill that sweeps a circular disc of diameter $d\ \ m$. At a point downstream of the windmill, where the pressure has returned to atmospheric, the velocity of the wind is measured at $aV\ ms^{-1}$. a is a constant that represents the ratio of downstream to upstream wind speeds.

(a) Use the actuator disc theory for an ideal windmill to show that the force exerted by the windmill can be written as

$$F = \rho \tfrac{\pi}{8} d^2 V^2 \left(a^2 - 1\right) \quad ,$$

where ρ is the density of the air. Clearly state all assumptions made during your derivation.

(8 marks)

(b) Continuing the analysis of the windmill defined above, show that the efficiency is given by

$$\eta = \frac{1 + a - a^2 - a^3}{2} \quad .$$

(6 marks)

(c) A windmill of radius 10*m*, works at an efficiency of $\eta = 0.5$ in a $10 ms^{-1}$ wind. If the air density is $1.2\ kg\ m^{-3}$, find the force on the windmill, the air velocity through the disc and the mean gauge pressures just in front of and just behind the disc.

(6 marks)

turn over…

Q15 (a) The lifting flow over a rotating cylinder is modelled assuming potential flow. Find the velocity and pressure coefficient distributions on the cylinder surface. Then show that the pressure on the cylinder is given by

$$p(\theta) = p_{\infty} + \frac{1}{2}\rho U_{\infty}^{2}\left(1 - 4\sin^{2}\theta\right) - \left(\frac{\rho U_{\infty}\Gamma\sin\theta}{\pi R}\right) - \frac{1}{8}\rho\left(\frac{\Gamma}{\pi R}\right)^{2}$$

where Γ represents a vortex strength

(6 marks)

(b) Consider a spinning cylinder in an incompressible irrotational flow. Initially write the forces acting on an infinitesimal element of the surface and hence show that the vortex strength required to produce a lift per unit length of, l, is given by

$$\Gamma = \frac{l}{\rho U_{\infty}}$$

(9 marks)

(c) A cylinder, with a diameter of $1m$, was placed in an onset flow of $10ms^{-1}$. Flow visualisation showed that, when spinning, the stagnation point moved 45^{o} around the cylinder compared to when it was stationary. Use this to find the lift per unit length of the spinning cylinder

(5 marks)