revision notes
for midsession
2017

12 x 5-mark questions?
covers all the material
in proportion to how long we
spent

15 lectures on algebra

12 lectures on calculus

~~5~~ 5 lectures on complex nos 1
3 lectures on vectors 2
8 lectures on matrices 3

2 functions 4 & curve-sketching
3 differentiation 5 — Taylor series
L'hôpital's rule

4 integration 6 — partial fraction
— improper ∫s
— parts

3 partial differentiation 7

Com-
plex numbers

- basic algebra in cartesian form including $\bar{z} = x + jy$

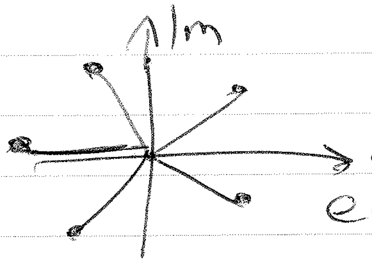
  e.g. $\dfrac{z_1}{z_2} = \dfrac{z_1 \bar{z}_2}{z_2 \bar{z}_2}$

- polar form

  $z = x + jy = re^{j\theta} = r(\cos\theta + j\sin\theta)$

- 3 applications n^{th} roots

(i) $z^m = Re^{j\phi} = R^{\frac{1}{m}} e^{(j\phi + 2n\pi)/m}$



evaluate for each n

equally spaces

(ii) $\cos n\theta$ in terms of $(\sin\theta)^n$, $(\cos\theta)^n$

$\mathrm{Re}(\cos n\theta + j\sin n\theta)$

$= \mathrm{Re}(\cos\theta + j\sin\theta)^n$

$= \mathrm{Re}(\cos^n\theta + nj\cos^{n-1}\theta\sin\theta + \dots)$

↑ expanded using Pascal's triangle

(iii) $\cos^n\theta$ in $\cos m\theta$ $\sin m\theta$

$= \left[\frac{1}{2}(e^{j\theta} + e^{-j\theta})\right]^n$ expand using Pascal's triangle

$$\cosh(x) = \frac{1}{2}(e^{x} + e^{-x})$$
$$\sinh(x) = \frac{1}{2}(e^{x} - e^{-x})$$

$$\cos j\theta = \cosh\theta$$
$$\sin j\theta = j\sinh\theta$$

find all values such that $\cos(z) = 2$.

$$\cos(x+jy) = 2$$

$\cancel{\cos(x)}$ $\cancel{\sin(x)}$

$$\cos(x)\cos(jy) - \sin(x)\sin(jy)$$

$$\ln(z) = \ln(re^{j\theta}) = \ln r + j(\theta + 2n\pi).$$

vectors 2 products dot & cross

component $\underset{\sim}{a}$ in directi[on] $\underset{\sim}{b}$

$$= \frac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{|\underset{\sim}{b}|}$$

lines & planes

geometry

$\underset{\sim}{x} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$ — line

plane

$\underset{\sim}{n}\cdot\underset{\sim}{x} = c$

matrices

i) algebra & transformation
ii) ~~determinatets~~ determinants & inverses
iii) solving equations
iv) eigenvalues & eigenvectors

i) $A + B$ $\quad A \times B = AB$

A^T

$A\underset{\sim}{v} \rightarrow \underset{\sim}{v}$
transformation.

ii) square matrices

det 2x2
$$\begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{matrix}$$
$$= a_{11}a_{22} - a_{12}a_{21}$$

det nxn $\begin{pmatrix} \cdots a_{ij} \cdots \\ \\ \end{pmatrix}$

$+ \; - \; + \; -$
$- \; + \; - \; +$ etc...

determinant ~~of ~~lets~~ whats left.~~ x element a_{ij} x minor inv(A) only exists if $\det(A) \neq 0$.

$$= \frac{1}{\det(A)} \operatorname{adj}(A)$$

adj = transpose of matrix of co-factors

co-factor being $\pm 1 \times$ minor.

alternatively find inverses by row operations

$$\left(\begin{array}{c|cccc} A & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & \ddots 1 \end{array} \right)$$

$$\qquad\qquad I_n$$

$$\sim \left(\begin{array}{c:c} I_n & A^{-1} \end{array} \right)$$

$$AA^{-1} = I_n$$

iii) solving equations

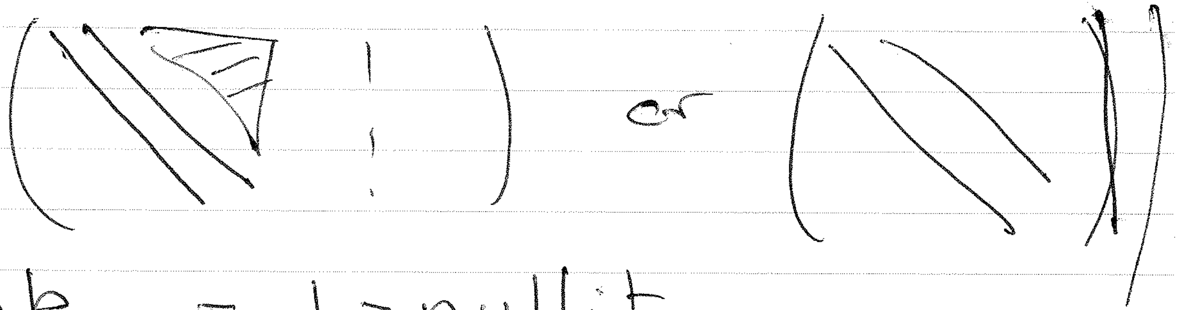
$$A\underline{x} = \underline{b} \qquad \begin{matrix} b_1 \\ b_2 \\ b_3 \end{matrix} \qquad (A|b)$$

$$\begin{matrix} x \\ y \\ z \end{matrix} \text{ or } \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix}$$

6 - or row reduction aka Gaussian elementat.

elementary row operations

method you have to practi—.



or

rank = 1 - nullity

= number of non zero rows after this.

~~fan~~ $\det(A) = 0$ rank$(A|b)$ = ran(A)

underdetermined case

∞-many solut.

rank$(A|b)$ > ran(A)

inconsistent

no soluti.

practice eigenvalues / vectors

$A\underset{\sim}{v} = \lambda \underset{\sim}{v}$ $(A - \lambda I)\underset{\sim}{v} = 0$

characteristic polynomial

nontrivial soluti.

$\det(A - \lambda I) = 0$

roots are eigenvalues — $(A - \lambda I)\underset{\sim}{v} = 0$

Calculus

↳ tips draw a graph

focus on the novel stuff

e.g. implicit differentiation
parametric "

e.g. $y^2 + 3xy + 2y^3 + 4x = 0$

find $\frac{dy}{dx} \Rightarrow 2y\frac{dy}{dx} + 3x\frac{dy}{dx}$
$+ 3y + 2y^2\frac{dy}{dx} + 4 = 0$

solve for $\frac{dy}{dx}$

if = $y = \cos t$
$x = \sin t$ find $\frac{dy}{dx}$

or improper integrals.

→ partial differentiation
↳ chain rule
Hessian matrix $\begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix}$

multi-D Taylor series

$$\Delta f = \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y$$

+ error analysis

abstract definitions

→ continuity
differentiable
domain etc
1-1
onto
inverse function
periodic function.

techniques → for

e.g. Taylor
l'hopital
$\int$ by parts
substitution
partial fractions.