UNIVERSITY OF BRISTOL
FACULTY OF ENGINEERING

First Year Examination for the Degree of Master of Engineering

MAY/JUNE 2010 3 Hours

AENG11100

FLUIDS 1A???

SOLUTIONS

SECTION 1 Answer <u>all</u> questions in this section

Q1.1 $p = p_a + \rho g \mathrm{h} = 1.023 \times 10^5 + 1000 \times 9.81 \times 100 = 1.0833 \times 10^5 N/m^2$ (4 *marks*)

Q1.2 Horizontal force on the inlet gate given by gauge pressure at C.G times the area (use gauge because atmospheric pressure acts on both sides). The vertical force on the exit equals the weight of water supported (again atmosphere acts on both sides)

$$F_{inlet} = (p_{CG} - p_a) \times A_{inlet} = \rho g \mathrm{h} \times A_{inlet}$$

$$F_{inlet} = 1000 \times 9.81 \times 0.9 \times 0.2^2 = 353.16N$$

$$F_{exit} = \rho g (\mathrm{h} \times A_{exit})$$

$$F_{exit} = 1000 \times 9.81 \times 2 \times 0.2^2 = 784.8N$$

(4 *marks*)

Q1.3 Steady, incompressible, inviscid,1D flow

(4 marks)

Q1.4 Density of air is small so we neglect hydrostatic terms & use $p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2 \quad \rightarrow \quad p_2 = p_1 + \frac{1}{2}\rho(V_1^2 - V_2^2)$

$$p_2 = 0.9 \times 10^5 + 0.5 \times 1.2 \times (5^2 - 25^2) = 89640 \mathrm{N/m}^2$$

(*4 marks*)

Q1.5 $\mathrm{Re}_x = \dfrac{Vx}{\nu} \quad \rightarrow \quad x = \dfrac{\mathrm{Re}_x \nu}{V} = \dfrac{5 \times 10^5 \times 1.47 \times 10^{-5}}{10} = 0.735 \text{ m}$

(a)=turbulent (b)=laminar.

(4 marks)

Q1.6 $M = \dfrac{V}{a} = \dfrac{V}{\sqrt{\gamma RT}} = \dfrac{80}{\sqrt{1.403 \times 287 \times 293}} = 0.233$

Compressibility effects will be small and may be neglected in some calculations.

(4 marks)

Q1.7 The pressure and viscous shear forces only have a net effect at the control volume and submerged body boundaries

The pressure acts normal to the boundaries while the viscous shear force acts tangentially.

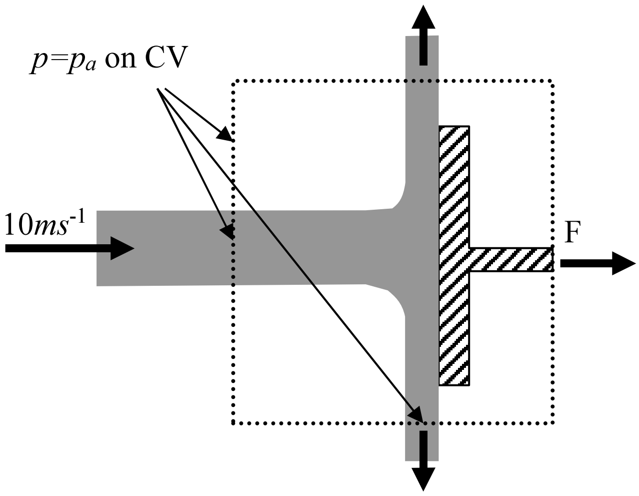
The effects of the body forces must be integrated throughout the control volume.

(4 marks)

Q1.8 Consider a control volume fixed relative to the plate. The net horizontal force on the control volume equals the rate of change of momentum in that direction. We also assume that atmospheric pressure acts through the get diameter so there is no contribution to the net horizontal force from the jet entry into the CV.

$$F_{CVx} = \dot{m}(V_2 - V_1) = \pi r^2 V \rho (0 - V) = -\pi \times 0.04^2 \times 10 \times 1000 \times 10 = -502.65N$$

$$F = -F_{CVx} = 502.65N$$



(4 marks)

Q1.9 A source and sink of equal strength Λ are placed a distance l apart. Then the distance between the source and sink is reduced keeping the term Λl constant. In the limit as the distance tends to zero the flow is a doublet flow.

(4 marks)

Q1.10

$$\phi = xy + x + y$$

$$u = \frac{\partial \phi}{\partial x} = y + 1 \qquad v = \frac{\partial \phi}{\partial y} = x + 1$$

$$\frac{\partial \psi}{\partial x} = -v = -x - 1 \quad \rightarrow \quad \psi = -\frac{x^2}{2} - x + c(y)$$

$$\frac{\partial \psi}{\partial y} = u = y + 1 \quad \rightarrow \quad \frac{dc(y)}{dy} = y + 1 \quad \rightarrow \quad c(y) = \frac{y^2}{2} + y + const$$

$$\psi = \tfrac{1}{2}\left(y^2 - x^2\right) + y - x + const$$

(4 marks)

SECTION 2 Answer *three* questions in this section

Q2.1

(a) The balloon will reach a height where the weight of the displaced air will match the weight of the balloon. At this neutral buoyancy point the average density of the balloon will be the same as the density of the displaced air

So the density of the displaced air is

$$\rho = \frac{mass}{volume} = \frac{0.22}{\frac{4}{3}\pi 0.45^3} = 0.5764 kg/m^3$$

The density of the displaced air is also given by the equation of state for a perfect gas

$$p = \rho R T$$

Applying the given relationships and rearranging to find z in terms of ρ

$$\rho = \frac{p_{sl}\left(1-\lambda z/T_{sl}\right)^{\frac{g}{R\lambda}}}{R\left(T_{sl}-\lambda z\right)} = \frac{p_{sl}}{R\left(T_{sl}\right)^{\frac{g}{R\lambda}}}\frac{\left(T_{sl}-\lambda z\right)^{\frac{g}{R\lambda}}}{\left(T_{sl}-\lambda z\right)} = \frac{p_{sl}\left(T_{sl}-\lambda z\right)^{\frac{g}{R\lambda}-1}}{RT_{sl}\left(T_{sl}\right)^{\frac{g}{R\lambda}-1}} = \frac{p_{sl}}{RT_{sl}}\left(\frac{T_{sl}-\lambda z}{T_{sl}}\right)^{\frac{g}{R\lambda}-1}$$

$$\frac{T_{sl}-\lambda z}{T_{sl}} = \left(\frac{RT_{sl}\rho}{p_{sl}}\right)^{\frac{R\lambda}{g-R\lambda}} \quad \rightarrow \quad z = \frac{1}{\lambda}\left[T_{sl} - T_{sl}\left(\frac{RT_{sl}\rho}{p_{sl}}\right)^{\frac{R\lambda}{g-R\lambda}}\right]$$

$$z = \frac{293}{0.0065}\left[1-\left(\frac{287\times 293\times 0.5764}{1.009\times 10^5}\right)^{\frac{287\times 0.0065}{9.81-287\times 0.0065}}\right] = \frac{293}{0.0065}\left[1-\left(\frac{287\times 293\times 0.5764}{1.009\times 10^5}\right)^{0.23482}\right]$$

$$z = 7129.9m$$

(10 marks)

(b) The resultant force, F, required to keep the balloon at sea level is given by

F=Buoyancy Force – Weight

$$F = \tfrac{4}{3}\pi 0.45^3 \times 1.2\times 9.81 - 0.22\times 9.81 = 2.335N$$

At terminal velocity

$$Drag + Weight = Buoyancy$$

$$Drag = \tfrac{1}{2}\rho V^2 \pi r^2 C_D = \tfrac{1}{2}\times 1.2\times \pi \times 0.45^2 \times 0.45\times V^2 = 0.1718\times V^2$$

Using the previous value for force we have

$$Drag + Weight = Buoyancy$$

$$0.1718\times V^2 = 2.335 \quad \rightarrow \quad V = \sqrt{\frac{2.335}{0.1718}} = 3.687 m/s$$

$$\mathrm{Re}_{crit} = \frac{VD}{\nu} = \frac{3.687\times 0.9}{1.47\times 10^{-5}} = 2.26\times 10^5$$

(7 marks)

(c) C_D initially higher than at terminal velocity.

C_D decreases in critical flow region

C_D Increases again (to just below subcritical values) in supercritical region

(3 marks)

Q2.2

(a) For a sharp orifice, there is negligible surface in the jet direction so viscous effects are minimal – so assume inviscid flow. Also assume as flow is water the flow is incompressible. Assuming a quasi-static flow means that at each instant the flow is steady to a good approximation

At vena-contracta: x-sectional area reaches minimum; streamlines parallel; pressure in jet uniform & equal to pressure of "stationary" fluid in B

Apply Bernoulli's equation between the surface of A general & a point in the centre of the jet at the vena contracta (subscript VC)

$$p_A + \tfrac{1}{2}\rho V_{surfaceA}^2 + \rho g h_A = p_{VC} + \tfrac{1}{2}\rho V_{VC}^2$$

If tank large then change in height of surface is negligible. Also the fluid in B is static so the pressure of the flow around the jet is $p_{VC} = p_B + \rho g h_B$

$$p_A + \rho g h_A = p_B + \rho g h_B + \tfrac{1}{2}\rho V_{VC}^2$$

Rearranging $V_{VC} = \sqrt{\dfrac{2[p_A - p_B + \rho g(h_A - h_B)]}{\rho}}$

The area of the jet at the vena-contracta is CA_o

Mass flow rate given by $\dot{m} = CA_o \rho V_{VC} = CA_o\sqrt{2\rho[p_A - p_B + \rho g(h_A - h_B)]}$

(10 marks)

(b) Assuming that the viscosity in the flow is negligible, the jet leaves the pipe with parallel streamlines and the vena-contracta condition is satisfied at the pipe exit but with an area now equal to the orifice. We can therefore use the previous solution where *C =1.*

The mass flow rate is given by

$$\dot{m} = A_o\sqrt{2\rho[p_A - p_B + \rho g(h_A - h_B)]} = A_o\sqrt{2\times 900[1.2\times 10^5 + 900\times 9.81\times(0.8)]}$$

$$\dot{m} = A_o \times 15123.3 kg/s$$

Relating the rate of fall of height to the mass flow rate

$$\dot{m} = \dot{h}_A A_A \rho \qquad \rightarrow \qquad \dot{h}_A = \frac{A_o}{A_A}\frac{15123.3}{900} = 16.80\frac{A_o}{A_A} = 0.01$$

$$\frac{A_o}{A_A} = 5.95\times 10^{-4}$$

(8 marks)

(c) There is a large loss factor associated with the inlet so a flow rates would be slower.

(2 marks)

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Q2.3

(a) Δh is due to the difference in static pressure between the wind tunnel pressure tapping and ambient pressure. Applying Bernoulli's equation between ambient and working section locations and neglecting hydrostatic terms as air density is small

$$p_a = p_w + \tfrac{1}{2}\rho_a V_w^2$$

Applying the hydrostatic equation across the manometer fluid surfaces

$$p_a = p_w + \rho_m g \Delta h$$

Rearranging

$$\rho_m g \Delta h = \tfrac{1}{2}\rho_a V_w^2 \qquad \rightarrow \Delta h = \frac{\rho_a}{2\rho_m g} V_w^2$$

(7 marks)

(b) From continuity and assuming the flow is incompressible, we know that the volume flow rate at the exit must be the same as at the working section.

$$A_e V_e = A_w V_w \qquad \rightarrow \quad V_e = \frac{A_w}{A_e} V_w$$

Similarly the average velocity downstream of the fan must be the same as in the working section (the cross section is constant).

Taking Bernoulli's equation from just downstream of the fan (subscript fd) to the exit, where the pressure is equal to the ambient pressure

$$p_{fd} + \tfrac{1}{2}\rho_a V_{fd}^2 = p_a + \tfrac{1}{2}\rho_a V_e^2$$

Applying continuity

$$p_{fd} + \tfrac{1}{2}\rho_a V_w^2 = p_a + \tfrac{1}{2}\rho_a \left(\frac{A_w}{A_e}\right)^2 V_w^2 \qquad \rightarrow \quad p_{fd} = p_a + \tfrac{1}{2}\rho_a \left(\frac{A_w}{A_e} - 1\right)^2 V_w^2$$

Therefore

$$p_{fd} - p_w = p_a + \tfrac{1}{2}\rho_a \left(\frac{A_w}{A_e} - 1\right)^2 V_w^2 - p_a + \tfrac{1}{2}\rho_a V_w^2$$

(7 marks)

$$\Delta p_{fan} = \tfrac{1}{2}\rho_a \left(\frac{A_w}{A_e}\right)^2 V_w^2$$

(c) Take a control volume around the model and upstream of the fan. Applying the conservation of linear momentum and neglecting viscous forces (see figure below)

$$p_m A_w - p_{fu} A_w - D = \dot{m} \times (V_m - V_m) = 0$$

$$p_{fu} = p_m - \frac{D}{A_w} = p_m - C_d \times \tfrac{1}{2}\rho_a V_m^2 \times \frac{A_m}{A_w}$$

Applying Bernoulli's equation from ambient conditions to just upstream of the model

$$p_a = p_m + \tfrac{1}{2}\rho_a V_m^2 \qquad p_{fu} = p_a - \tfrac{1}{2}\rho_a V_m^2 - C_d \times \tfrac{1}{2}\rho_a V_m^2 \times \frac{A_m}{A_w}$$

Applying continuity and Bernoulli's equation downstream of the fan

$$p_{fd} + \tfrac{1}{2}\rho_a V_m^2 = p_a + \tfrac{1}{2}\rho_a \left(\frac{A_w}{A_e}\right)^2 V_m^2 \qquad \rightarrow \quad p_{fd} = p_a + \tfrac{1}{2}\rho_a \left[\left(\frac{A_w}{A_e}\right)^2 - 1\right] V_m^2$$

So the pressure change across the fan is now

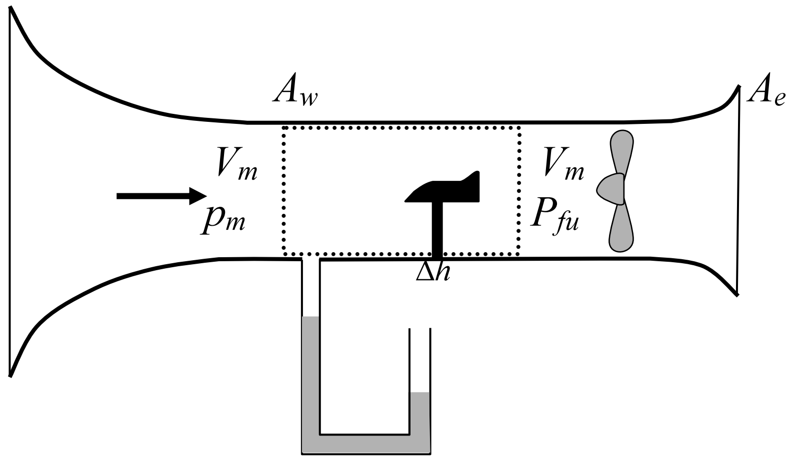
$$\Delta p_f = \tfrac{1}{2}\rho_a\left(\frac{A_w}{A_e}\right)^2 V_m^2 - \tfrac{1}{2}\rho_a V_m^2 + C_d \times \tfrac{1}{2}\rho_a V_m^2 \times \frac{A_m}{A_w} + \tfrac{1}{2}\rho_a V_m^2 = \tfrac{1}{2}\rho_a\left[\left(\frac{A_w}{A_e}\right)^2 + C_d\,\frac{A_m}{A_w}\right]V_m^2$$

Equating original (no model) and final (model) pressure change

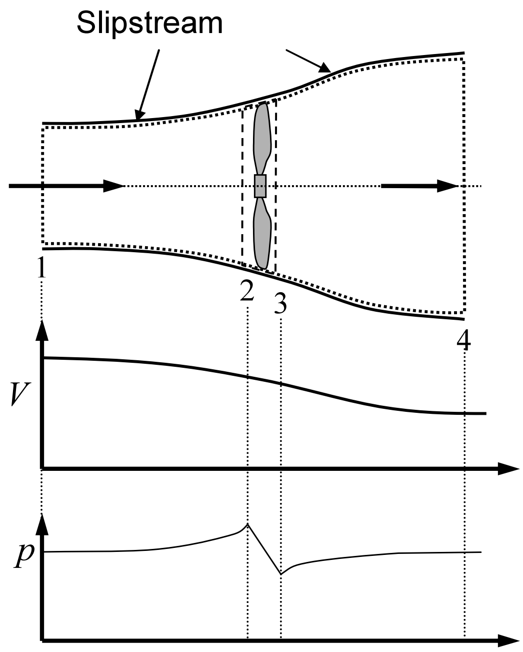
$$\Delta p_f = \tfrac{1}{2}\rho_a\left[\left(\frac{A_w}{A_e}\right)^2 + C_d\,\frac{A_m}{A_w}\right]V_m^2 = \tfrac{1}{2}\rho_a\left(\frac{A_w}{A_e}\right)^2 V_w^2$$

$$V_m = \frac{1}{\sqrt{\left[1 + C_d\,\dfrac{A_m A_e^2}{A_w^3}\right]}}V_w$$

(6 marks)



Q2.4 (a) Use the actuator disc theory for an ideal windmill, see figure below



Assumptions: Frictionless & incompressible, Steady 1D flow (neglect rotation and variation across the disc radius). Actuator disc is thin so $A_2=A_3=A_d$ & $V_2=V_3=V_d$. $p=p_a$ at all points on slipstream boundary & 1 & 4

Continuity: $Q = V_d A_d$

Bernoulli's equation for CV 1-2 & CV 3-4

$$\begin{aligned} p_1 + \tfrac{1}{2}\rho V_1^2 &= p_2 + \tfrac{1}{2}\rho V_d^2 \\ p_3 + \tfrac{1}{2}\rho V_d^2 &= p_4 + \tfrac{1}{2}\rho V_4^2 \end{aligned} \quad \rightarrow \quad p_3 - p_2 = \tfrac{1}{2}\rho\left(a^2V^2 - V^2\right)$$

Steady Flow momentum for CV 2-3:

$$(p_2 - p_3)A_d + F_{CV} = \rho Q(V_d - V_d) = 0 \qquad \rightarrow \quad F_{CV} = (p_3 - p_2)A_d$$

Where F is the force on the control volume

Applying results from Bernoulli's equation above

$$F_{CV} = \tfrac{1}{2}\rho A_d V^2\left(a^2 - 1\right) = \tfrac{\pi}{8}\rho d^2 V^2\left(a^2 - 1\right)$$

Force on the windmill is equal and opposite to the force on the CV so

$$F = \rho \tfrac{\pi}{8} d^2 V^2\left(1 - a^2\right)$$

(8 marks)

(b) Steady Flow momentum for CV 1-4:

$$0 + F_{CV} = \rho Q(V_4 - V_1) \qquad \rightarrow \quad F_{CV} = \rho V_d A_d V(a - 1)$$

From momentum & continuity

$$(p_3 - p_2)A_d = \rho V_d A_d (V_4 - V_1)$$

Eliminating $(p_3$-$p_2)$ using Bernoulli's equation above

$$\rho V_d\left(aV - V\right) = \tfrac{1}{2}\rho\left(a^2V^2 - V^2\right)$$
$$V_d V\left(a-1\right) = \tfrac{1}{2}V^2\left(a+1\right)\left(a-1\right)$$
$$V_d = \tfrac{1}{2}V\left(a+1\right)$$

The power drawn from the air by the disc is

$$P_{\text{disc}} = -F_{CV}V_d = -\rho Q\left(V_4 - V_1\right)V_d = \tfrac{1}{2}\rho A_d V_d\left(V_1 - V_4\right)V_d = \tfrac{1}{4}\rho A_d\left(aV + V\right)\left(V^2 - a^2V^2\right)$$

Power in the wind passing through the disc area, if the disc were not present.

$$P_{\text{wind}} = \tfrac{1}{2}\dot{m}V_1^2 = \tfrac{1}{2}\rho A_d V_1 V_1^2 = \tfrac{1}{2}\rho A_d V^3$$

The efficiency of the turbine is therefore

$$\eta = \frac{P_{\text{disc}}}{P_{\text{wind}}} = \frac{\tfrac{1}{4}\rho A_d\left(aV+V\right)\left(V^2 - a^2V^2\right)}{\tfrac{1}{2}\rho A_d V^3} = \frac{\left(aV+V\right)\left(V^2 - a^2V^2\right)}{2V^3} = \frac{\left(a+1\right)\left(1-a^2\right)}{2} = \frac{1 + a - a^2 - a^3}{2}$$

(6 marks)

(c) from the definition of efficiency in (b) we have

$$\eta = 0.5 = \frac{1 + a - a^2 - a^3}{2} \quad \rightarrow \quad a - a^2 - a^3 = 0 \quad \rightarrow \quad a\left(a^2 + a - 1\right) = 0$$

$$a = \begin{cases} 0 \\ -\tfrac{1}{2} - \tfrac{\sqrt{5}}{2} \\ \tfrac{\sqrt{5}-1}{2} \end{cases}$$

$a = \tfrac{\sqrt{5}-1}{2}$ is the only feasible solution

$$F = \rho\tfrac{\pi}{8}d^2V^2\left(1-a^2\right) = 1.2\times\tfrac{\pi}{8}\times 20^2\times 12^2\left(1-\left(\tfrac{\sqrt{5}-1}{2}\right)^2\right) = 1.2\times\tfrac{\pi}{8}\times 20^2\times 12^2\left(\tfrac{\sqrt{5}-1}{2}\right) = 16775.5N$$
$$V_d = \tfrac{1}{2}V\left(a+1\right) = \tfrac{1}{4}\times 12\times\left(\sqrt{5}+1\right) = 9.708m/s$$

From Bernoulli's equations in (a)

$$p_2 - p_a = \tfrac{1}{2}\rho\left(V^2 - V_d^2\right) = \tfrac{1}{2}\times 1.2\times\left(12^2 - 9.708^2\right) = 29.85N/m^2$$
$$p_3 - p_a = \tfrac{1}{2}\rho\left(a^2V^2 - V_d^2\right) = \tfrac{1}{2}\times 1.2\times\left(a^2 12^2 - 9.708^2\right) = 23.55N/m^2$$

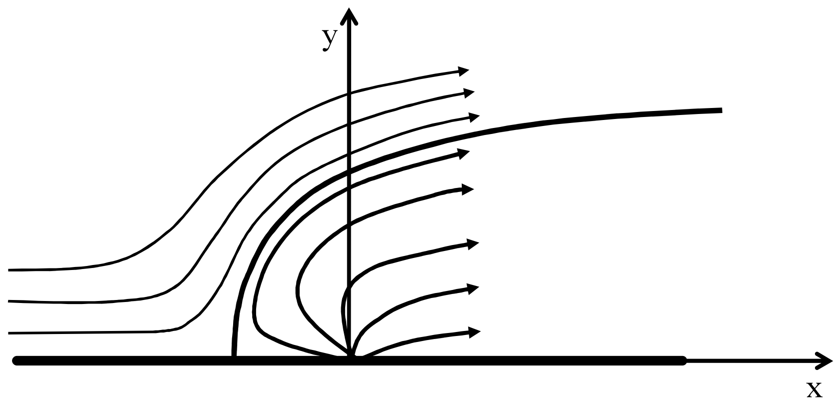
(6 marks)

Q2.5 (a) Streamlines are lines of constant stream function. The change in ψ between streamlines corresponds to the volume flow (per unit depth) between these lines or the mass flow between the lines scaled by the density.

ψ is only defined for 2D flow whereas ϕ exists for 3D flows. ψ can be used in rotational flows, whereas ϕ implies irrotationality.

(2 mark)

(b) i)



(2 mark)

ii) The stream function for the combined uniform freestream and point source is

$$\psi = 10y + 65\tan^{-1}\left(\frac{y}{x}\right)$$

To find the location of the stagnation point at the nose the first step is to determine the velocity components. The stream function is in terms of Cartesian coordinates so use $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. Thus

$$u = \frac{\partial\psi}{\partial y} = 10 + 65\frac{1}{(1+(y/x)^2)}\left(\frac{1}{x}\right) = 10 + 65\frac{x}{x^2+y^2}$$

$$v = -\frac{\partial\psi}{\partial x} = -65\frac{1}{(1+(y/x)^2)}\left(\frac{-y}{x^2}\right) = 65\frac{y}{x^2+y^2}$$

Then find the stagnation point when $u=v=0$

$v = 65\frac{y}{x^2+y^2} = 0$ when $y=0$ i.e. on the centre line.

$u = 10 + 65\frac{x}{x^2+y^2} = 10 + 65\frac{1}{x}$ when $y=0$

Therefore u=0 when $x = -6.5$. The stagnation point is on the centre line a distance $6.5\ m$ forward of the source.

(4 marks)

(iii) The stagnation point is located on the dividing streamline that represents the cliff surface so the value of the stream function on the dividing streamline can be evaluated. Substituting y=0, $x = -6.5$ into the equation for the stream function gives

$\psi_{DS} = 65\tan^{-1}(0)$

$\tan^{-1}(0)=0$ or π, but since the stagnation point is forward of the source (which is at the origin of the Cartesian coordinate system $x=y=0$) so π is the correct value to use and thus

$\psi_{DS} = 65\pi$

Then the height of the cliff where it intersects the *y*- axis is the value of value of *y* when $x=0$ on the dividing stream line.

$$\psi_{DS} = 65\pi = 10y + 65\tan^{-1}\left(\frac{y}{0}\right) = 10y + 65\frac{\pi}{2}$$

Hence it can be shown that the height is $y_{cliff} = 65\pi/20\ m = 10.210\,m$

(3 marks)

(iv) To determine the cliff height far downstream as $x \to \infty$, use the stream function.

$$\psi = 10y + 65\tan^{-1}\left(\frac{y}{x}\right) \to 10y \quad \text{as } x \to \infty$$

since as $x \to \infty$ the $\tan^{-1}$ term $\to 0$.

Then it follows that the dividing streamline is in fact

$\psi_{DS} = 65\pi \to 10y$ as $x \to \infty$

$$y_{DS} \to \frac{65\pi}{10} \quad \text{as } x \to \infty$$

The ultimate cliff height is thus 6.5π m = 20.42 m

(2 marks)

(v) from (c) and (d) the correct *x* location is $x=0$, so next work out the velocities at this location

$$u = 10 + 65\frac{x}{x^2 + y^2} = 10m/s$$

$$v = 65\frac{y}{x^2 + y^2} = \frac{65}{y}m/s$$

Then the velocity at this point is

$V = \sqrt{u^2 + v^2}$

Then the height of the cliff at this point is given in (c) and therefore the velocity is being measured at a height of $y = 3.25\pi + 2$. Then substituting into the above equation gives

$$V = \sqrt{100 + \frac{65^2}{(3.25\pi + 2)^2}} = 11.329$$

This is larger than the freestream velocity, hence the percentage difference relative to the free stream is

$$100\left(\frac{V - U_\infty}{U_\infty}\right) = 100\left(\frac{1.329}{10}\right) = 13.29\%$$

(7 marks)