Class 1

EMAT20200 Vector Calculus 2016/17

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WARNING: THESE ARE NOT A COMPLETE SET OF NOTES. YOU WILL NEED TO ATTEND THE LECTURES TO COPY FURTHER ANNOTATION TO THE NOTES, AND THE WORKINGS TO EXAMPLES.

0. Overview

7 topics, 7 weeks, 3 slots per week.

1. **Introduction** What is vector calculus? Why is it important? What does it mean to differentiate or integrate $\boldsymbol{v}(x, y, z)$? We need to understand the difference between vector functions, scalar fields, and vectors fields, and what they represent physically and graphically.

2. **Differentiation of scalar fields** What is the gradient of a scalar field? Why is the gradient perpendicular to level contours? What is its relation to 'slope'? How do we take derivatives in other directions? How do we decide which points are maxima, minima or saddles (using the Hessian)?

3. **Differentiation of vector fields** Why are there two notions of differentiation of vector fields (div and curl)? What do they represent physically? Do they obey the rules of calculus I know from 1D? Which second derivatives make sense?

4. **Integration along curves** How do we integrate scalar and vector fields over curves rather than straight lines? Before we do that, we had better find out how to parametrise curves $C(t)$ in 2 and 3 dimensions. Also, what is the arclength $d\boldsymbol{r}$ along a curve in 3 dimensions? How do we use vector calculus to define the work done in moving along a path C. Why does calculating the work done in certain force fields lead to independence of path and in others does not? What is a conservative vector field?

5. **Integration over areas and volumes** How do we integrate when there is more than one variable $\int\int dxdy$ (a double integral)? Does the order matter? What do such integrals represent physically? Can changing co-ordinates help? Is the case of three variables $\int\int\int dxdydz$ really any harder than two variables? Conceptually no, but examples show that the limits can be more tricky.

6. **Integration over surfaces** How do we integrate over a surface $S(x, y, z) = 0$? Before we do that we had better find out how to parametrise the surface, using two parameters. What is the notion of area dS? What is a flux integral and what does it represent physically?

7. **Tying it all together** In 1D we know the relation between integration and differentiation; 'the fundamental theorem of calculus'. How are differential and integral vector calculus related? 1) Integration can help with the physical interpretation of div and curl. 2) The 'fundamental theorem of calculus' grown up = Stokes' theorem + Gauss' theorem.

HINT: Don't panic. 21 lectures/demo classes but only 7 concepts (= 7 types of exam question!): grad, div & curl, line integrals, double integrals, surface integrals and volume integrals. The key is to **visualise**!