(17)

The following notes illustrate basic joint stressing. Items include:

1. Design at Ultimate

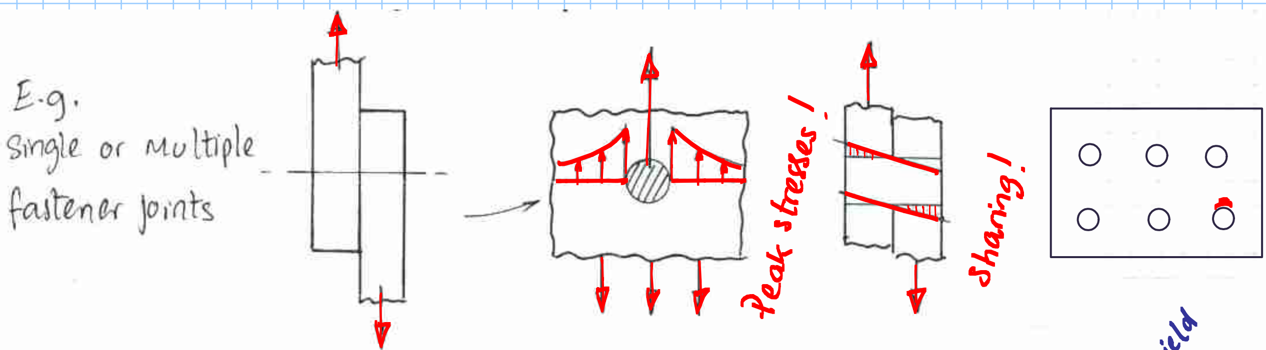
2. Single fastener pinned joints

3. Multiple Fastener fixed joints
   a. ... with concentric loading
   b. ... with concentric and eccentric loading
   c. Fixed joint connection configurations

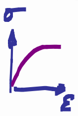
Basic assumptions:
Pins are perfectly rigid (i.e. they do not deform)
Pins have perfect fit (i.e. no tilt)

(18)

1. <u>Design at Ultimate</u> "Net-section average-stress"



Below elastic limit : Unequal load sharing between fasteners
Non uniform stress distribution
– In plane and thru-thickness



Above elastic limit : Plastic yielding
Equal load sharing
Uniform stress distributions

No alleviation below yield

Ductile alleviation of stress conc'n above yield

→ Design using averaged net section stress ok for ductile materials

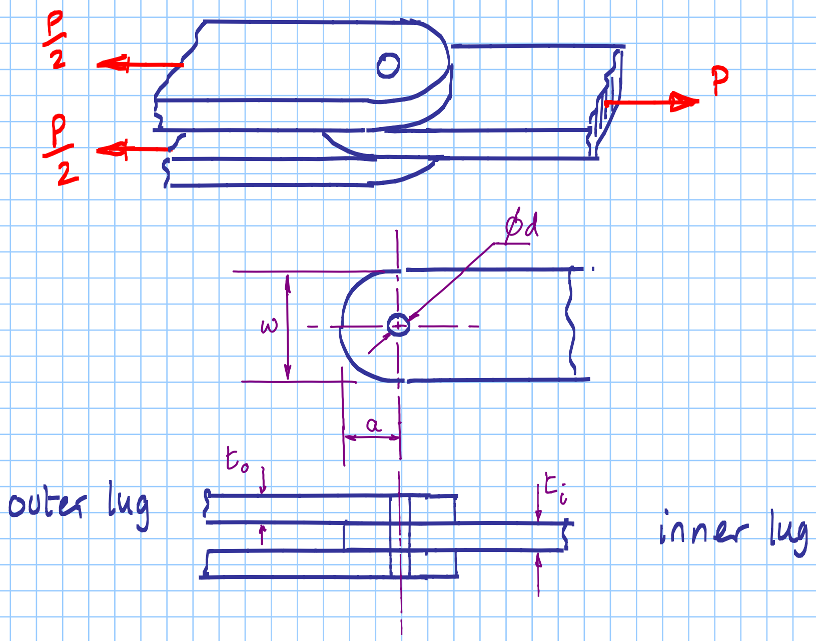
I.e. for an engineering alloy at ultimate loading we assume that yeilding dissipates stress concentrations and promotes even load sharing (but note, below yield this is not the case - with implications for fatigue!)

2. Single-fastener pinned joints

A single pin fastener joint should always be double shear configuration to avoid excessive eccentricity and bending across the joint.

Configuration + Geometry:

E.g.:



Design Statement

Design to ensure that generated stresses do not exceed the allowable strengths of any of the failure modes at ultimate loading.

Design for: pin: $\tau, \sigma_{br}, \sigma_b \leq \tau^*, \sigma_{br}^*, \sigma_b^*$

lug: $\sigma_{br}, \sigma_{tens}, \tau_{so} \leq \sigma_{br}^*, \sigma_{tens}^*, \tau_{to}^*$ etc!

Stresses at ultimate — Allowable values

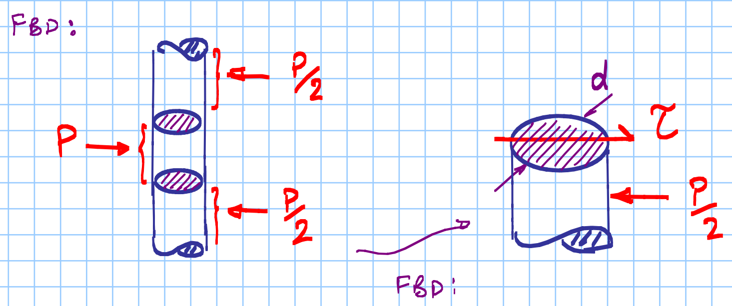
For initial joint design, to cope with the numerous possible modes of failure we often refer to guidelines to help us approach an optimum solution quickly, e.g.:

$$\frac{d}{t} \leq 3.5 \quad , \quad \frac{a}{d} \geq 1.5 \text{ to } 2 \quad , \quad \frac{w}{d} \geq 4$$

Consider each mode in turn:

- Pin Shear

Consider the bolt cross-section being sheared at the interface between the joint plates (lugs)



Shear stress: $\tau = \dfrac{P}{2\pi\left(\frac{d}{2}\right)^2}$

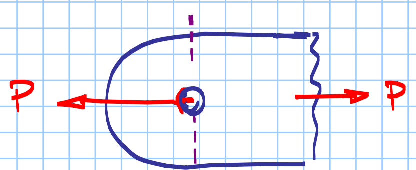
– design for $\underset{\text{actual}}{\tau} < \underset{\text{allowable}}{\tau^*}$

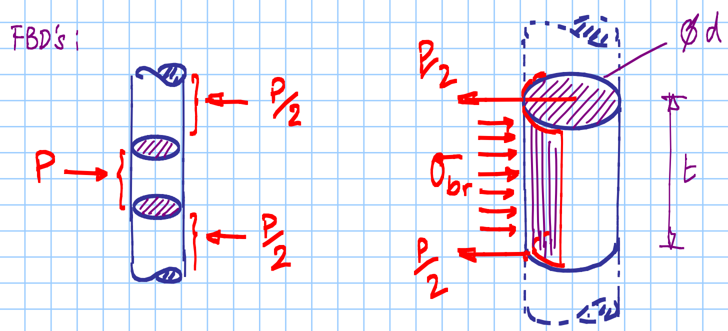
↳ "Reserve factor" $\boxed{RF = \dfrac{\text{Allowable}}{\text{Actual}}}$

RF > 1 required.

- Pin Bearing (usually less critical than lug bearing)

Consider the lug bearing against the side of the bolt:





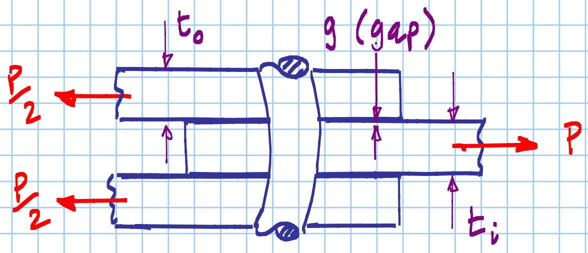
$\sigma_{br} = \dfrac{P}{dt}$ ← projected flat area

Design for $\sigma_{br} < \sigma_{br}^*$

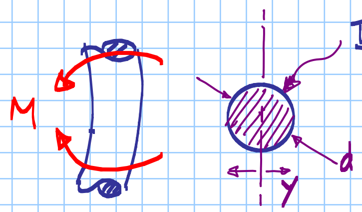
↳ $\boxed{RF = \dfrac{\sigma_{br}^*}{\sigma_{br}}}$

- Pin Bending

Consider the relative dispacement of the lug plates and the effective bending moment on the pin



Can be a significant design driver for thick, single shear or offset joints, e.g. with a filler.



$$I = \frac{\pi R^4}{4} = \frac{\pi d^4}{64}$$

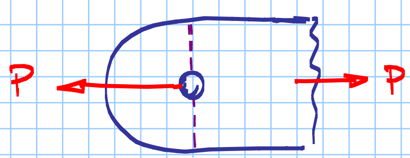
$$\sigma = \frac{M\left(\frac{d}{2}\right)}{I} \qquad M \approx \frac{P}{2}\left(\frac{t_o}{2} + \frac{t_i}{4} + g\right)$$ as a conservative estimate.

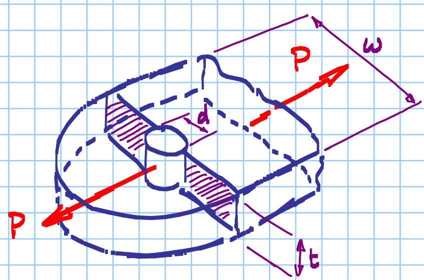
Design for $\sigma < \sigma^*$ $\quad \Rightarrow \quad \boxed{RF = \frac{\sigma^*}{\sigma}}$

- Lug Tension

Consider the direct tension carried by the net lug section at the bolt.

E.g.



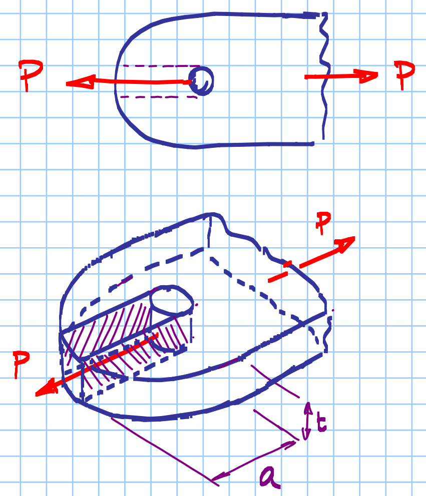


$$\sigma_t = \frac{P}{(w-d)t}$$

Design for $\sigma_t < \sigma^*$ $\quad \Rightarrow \quad \boxed{RF = \frac{\sigma^*}{\sigma_t}}$

- Lug Shear out

Consider the bolt shearing out through the end of the lug.

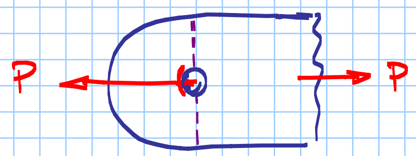


$$\tau = \frac{P}{2at}$$

Design for $\tau < \tau^*$ $\hookrightarrow$ $\boxed{RF = \frac{\tau^*}{\tau}}$

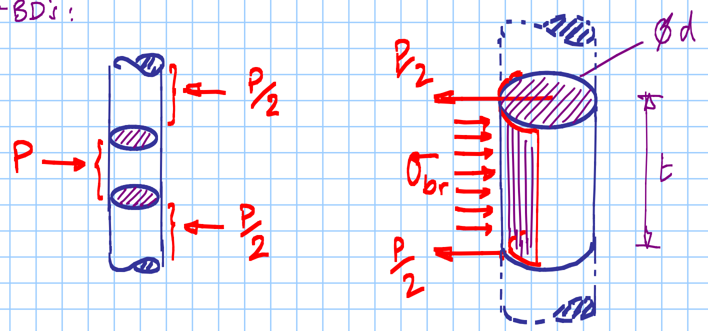
- Lug Bearing*

Consider the bolt bearing on the lug hole surface.



Preferable earliest failure mode since "benign" with warning of loose joint due to local deformation at hole.

FBD's:

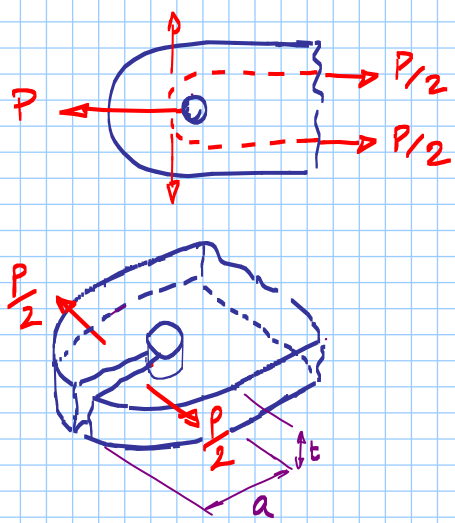


$$\sigma_{br} = \frac{P}{dt}$$ Projected flat area.

Design for $\sigma_{br} < \sigma_{br}^*$ $\hookrightarrow$ $\boxed{RF = \frac{\sigma_{br}^*}{\sigma_{br}}}$

- Lug Bursting (Cleavage)

Consider the effective transverse tension across the lug



$$\sigma_{cL} = {}^{P}\!/_{2} \Big/ at$$

Design for $\sigma_{cL} \leq \sigma^*$ $\quad \hookrightarrow \quad \boxed{RF = \frac{\sigma^*}{\sigma_{cL}}}$

3. <u>Multiple fastener fixed Joints</u>

I.e. no rotation.

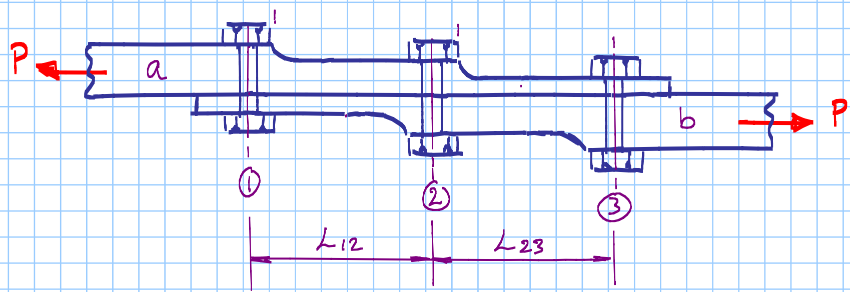
The basic failure modes defined for a simple pin joint also apply to fixed joints with multiple fasteners.

To relate to these modes we need to calculate the load carried by each fastener in a multiple fastener fixed joint.

Methods for estimating individual fastener loads in multiple fastener joints are outlined below.

3a. Multiple fastener fixed joint with Concentric loading

E.g. consider a three pin lap joint:



Note, for highly loaded multi-fastener joints the joint elements must be tailored to promote even load sharing.

Note stiffness of each element: $k_{a_{12}} = \left.\dfrac{AE}{L}\right|_{a_{12}}$ etc

For plates of the same material $E_a = E_b$

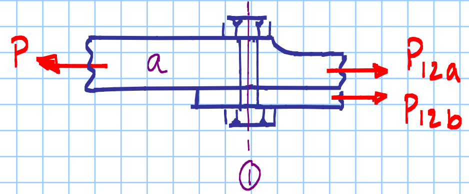
For equal length steps $L_{12} = L_{23}$

For equal width plates $A_{12} = w t_{12}$, $A_{23} = w t_{23}$

where: L = step length
w = step width
t = step thickness

Consider FBD's of sections revealing step loads and pin loads.

FBD1



Note this is a redundant structure, i.e. more than one load path.
So to solve we must consider: equilibrium, constitutive relationships and compatibility.

Equilibrium:

$\sum \overset{+}{\rightarrow} = 0: \quad -P + P_{12a} + P_{12b} = 0 \qquad (1)_{12}$

Constitutive Relations: $P_{12a} = k_{12a} d_{12a} \qquad (2)_{12a}$ where $k_{12a} = \left.\dfrac{AE}{L}\right|_{12a}$

$P_{12b} = k_{12b} d_{12b} \qquad (2)_{12b}$ And $k_{12b} = \left.\dfrac{AE}{L}\right|_{12b}$

Compatibility: $d_{12a} = d_{12b} \qquad (3)_{12}$ I.e. each side of step extends by the same amount.

$(3),(2): \quad \dfrac{P_{12a}}{k_{12a}} = \dfrac{P_{12b}}{k_{12b}}$ ie $\dfrac{P_{12a}}{P_{12b}} = \dfrac{k_{12a}}{k_{12b}} \qquad (4)_{12}$

Note if plates are of same material, E, same step width, w, and step length, L

then: $(4)_{12}: \quad \dfrac{P_{12a}}{P_{12b}} = \dfrac{k_{12a}}{k_{12b}} \rightarrow \dfrac{t_{12a}}{t_{12b}}$ i.e. load in each step proportional to

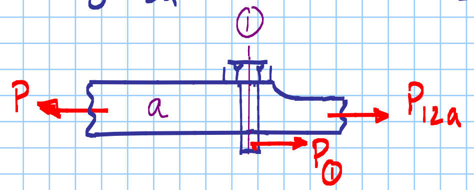
①,④ : Eliminating P_{12b}

$$\hookrightarrow P_{12a} + P_{12a}\frac{k_{12b}}{k_{12a}} = P \quad \rightarrow \quad P_{12a} = P \Big/ \left(1 + \frac{k_{12b}}{k_{12a}}\right) \qquad ⑤_{12a}$$

Similarly eliminating P_{12a} :

$$P_{12b} = P \Big/ \left(1 + \frac{k_{12a}}{k_{12b}}\right) \qquad ⑤_{12b}$$

FBD1a

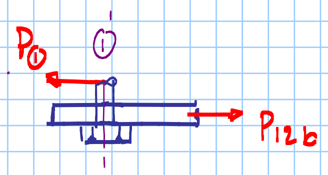


$$\Sigma \xrightarrow{+} = 0 : \quad -P + P_{12a} + P_{①} = 0$$

$$\hookrightarrow P_{①} = P - P_{12a} \quad \text{and } ①: \; P_{12b} = P - P_{12a} \quad \text{so} \quad P_{①} = P_{12b} \qquad ⑥_{12}$$

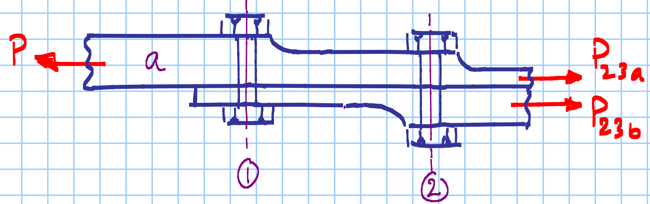
Pin transfer load (↑ $P_{①}$) By-pass load (↑ P_{12a})

FBD1b



$$\Sigma \xrightarrow{+} = 0 : \quad -P_{①} + P_{12b} = 0 \quad : \quad P_{①} = P_{12b}$$ agreeing with previous FBD result.

FBD2



Equilibrium:

$$\Sigma \xrightarrow{+} = 0 : \quad -P + P_{23a} + P_{23b} = 0 \qquad ①_{23}$$

Constitutive Relations:

$$P_{23a} = k_{23a}\, d_{23a} \qquad ②_{23a} \qquad \text{where } k_{23a} = \frac{AE}{L}\Big|_{23a}$$

$$P_{23b} = k_{23b}\, d_{23b} \qquad ②_{23b} \qquad \text{And } k_{23b} = \frac{AE}{L}\Big|_{23b}$$

Compatibility:

$$d_{23a} = d_{23b} \qquad ③_{23}$$

ie. each side of step extends by the same amount.

$$③,② : \quad \frac{P_{23a}}{k_{23a}} = \frac{P_{23b}}{k_{23b}} \qquad ④_{23} \qquad \text{ie} \quad \frac{P_{23a}}{P_{23b}} = \frac{k_{23a}}{k_{23b}}$$

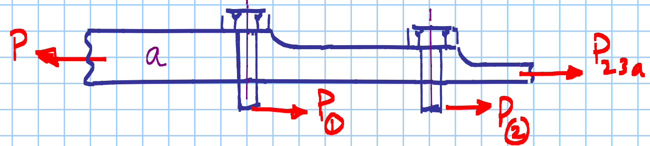
①,④ : Eliminating P_{23b}

$$\hookrightarrow P_{23a} + P_{23a}\frac{k_{23b}}{k_{23a}} = P \quad \rightarrow \quad P_{23a} = P \Big/ \left(1 + \frac{k_{23b}}{k_{23a}}\right) \qquad ⑤_{23a}$$

Similarly eliminating P_{23a} :

$$P_{23b} = P \Big/ \left(1 + \frac{k_{23a}}{k_{23b}}\right) \qquad ⑤_{23b}$$

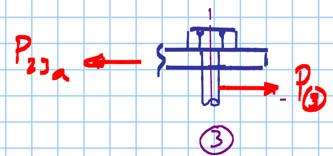
FBD2a



$$\Sigma \xrightarrow{+} = 0: \quad -P + P_1 + P_2 + P_{23a} = 0 \qquad (7)$$

Where $(6)_{12}$, $(5)_{12b}$: $\quad P_1 = P_{12b} = P \Big/ \left(1 + \dfrac{k_{12a}}{k_{12b}}\right)$

And $(5)_{23a}$: $\quad P_{23a} = P \Big/ \left(1 + \dfrac{k_{23b}}{k_{23a}}\right) = P_3$ ie:

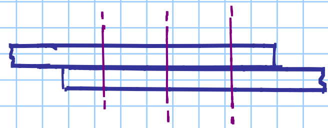


So (7): $\quad -P + P \Big/ \left(1 + \dfrac{k_{12a}}{k_{12b}}\right) + P_2 + P \Big/ \left(1 + \dfrac{k_{23b}}{k_{23a}}\right) = 0$

$$\hookrightarrow \quad P_2 = P - P \Big/ \left(1 + \frac{k_{12a}}{k_{12b}}\right) - P \Big/ \left(1 + \frac{k_{23b}}{k_{23a}}\right)$$

ie. $P_2 = P - P_1 - P_3$

- For untailored joint, ie. no steps:



$k_{12a} = k_{12b} = k_{23b} = k_{23a}$

$$\hookrightarrow \quad P_2 = P - P\big/(1+1) - P\big/(1+1) = P - \frac{P}{2} - \frac{P}{2} = 0 \ !$$

I.e. middle bolt carries no load!

- For tailored joint we want equal load sharing:

E.g. for 3-pin example we want:

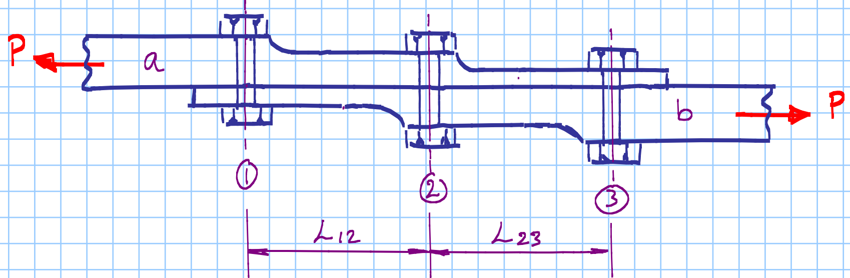
$$P \Big/ \left(1 + \frac{k_{12a}}{k_{12b}}\right) = P \Big/ \left(1 + \frac{k_{23b}}{k_{23a}}\right) = P/3$$

$$\hookrightarrow \quad \left(1 + \frac{k_{12a}}{k_{12b}}\right) = \left(1 + \frac{k_{23b}}{k_{23a}}\right) = 3$$

$$\hookrightarrow \quad \frac{k_{12a}}{k_{12b}} = \frac{k_{23b}}{k_{23a}} = 2$$

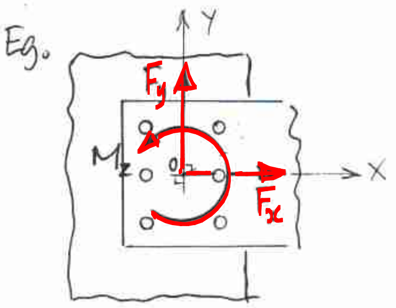
Ie: $\quad k_{12a} = 2k_{12b}$ and $k_{23b} = 2k_{23a}$

etc!



3b. Multiple fastener fixed joints with concentric and eccentric loading

E.g. rivet group loading



Determine most highly loaded fastener

"O" = fastener group centroid

F_x, F_y = "Concentric" loading cpts. thru centroid

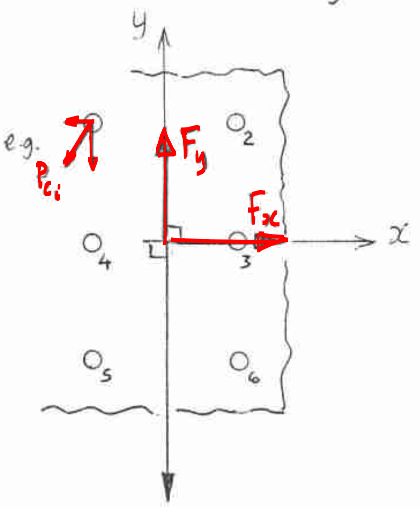
M_z = "Eccentric" loading cpt about centroid.

Obtain from FBD end reactions (or from FE model @ end of discts element)

– Consider separately

I.e. concentric + eccentric components summed by superposition.

Concentric loading



For Equilibrium w.r.t x, y directions:

$$\Sigma \overset{+}{\rightarrow} = 0 \quad : \quad \Sigma P_{c_{x_i}} + F_x = 0$$

$$\Sigma \uparrow^{+} = 0 \quad : \quad \Sigma P_{c_{y_i}} + F_y = 0$$

For n equal fasteners

Assuming uniform shear distribution *

I.e above elastic limit

ok for ultimate design using rivets to join plates with ductile characteristic. →

$$P_{c_{x_i}} = -F_x/n$$

$$P_{c_{y_i}} = -F_y/n$$

Suffix c ⇒ concentric

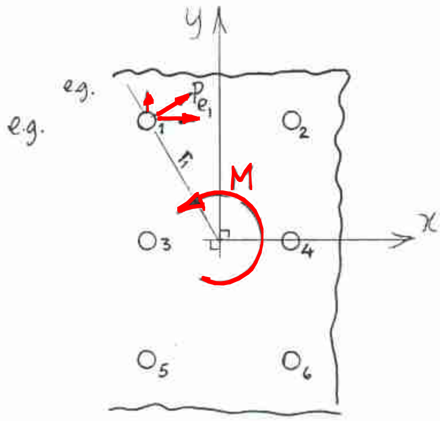
Sign convention:

Applied force cpts: F_y + F_x

Reaction force cpts: $P_{c_{y_i}}$ + $P_{c_{x_i}}$ i

(* Applicable for rivets – but not necessarily for bolts)

Eccentric loading



For Equilibrium :

$$\Sigma \circlearrowleft^{+} = 0 \quad : \quad \Sigma(P_{e_i} \cdot r_i) + M = 0 \qquad ①$$

For n equal fasteners

Assuming fastener load proportional to fastener distance from group centroid

$$\rightarrow \quad P_{e_i} = k \cdot r_i \qquad ②$$

Eliminating k using equilibrium equ^n.

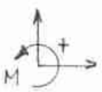
Suffix e ⇒ eccentric

$$①: \Sigma(\overset{\text{CONST}}{k}\, r_i\, r_i) + M = 0, \quad ② \rightarrow \quad P_{e_i} = -\frac{M \cdot r_i}{\Sigma r_i^2}$$

$$\hookrightarrow k = -M/\Sigma r_i^2$$

Sign convention:

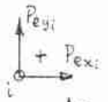
Applied moment:



Wrt x-y co-ords :

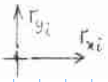
$$P_{e_{x_i}} = \frac{M r_{y_i}}{\Sigma(r_{x_i}^2 + r_{y_i}^2)}$$

Reaction force cpts:



$$P_{e_{y_i}} = -\frac{M r_{x_i}}{\Sigma(r_{x_i}^2 + r_{y_i}^2)}$$

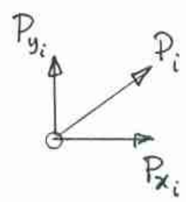
for M +ve a-clock



Summing concentric and eccentric x, y components:

at each fastener
$$P_{x_i} = P_{c_{x_i}} + P_{e_{x_i}}$$

$$P_{y_i} = P_{c_{y_i}} + P_{e_{y_i}}$$



Resultant
$$P_i = \sqrt{P_{x_i}^2 + P_{y_i}^2}$$

Spread sheet !

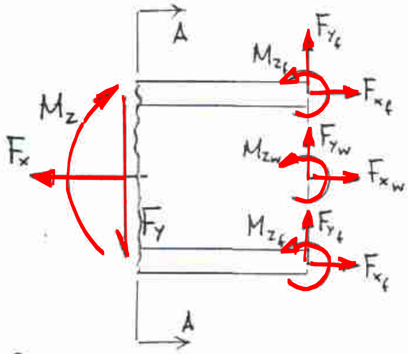
Rivet No.	r_x	r_y	P_{c_x}	P_{e_x}	P_x	P_{c_y}	P_{e_y}	P_y	P
–	–	–	–	–	–	–	–	–	–
–	–	–	–	–	–	–	–	–	–
–	–	–	–	–	–	–	–	–	–

Check joint failure modes @ most highly loaded fastener
(Fastener shear + plate bearing !)

other modes covered by spacing guidelines.

3c. Fixed Joint connection configurations

Transfer of beam loading



E.g. A-A =

Flange and web loads + mmts to be transferred at end of joint.

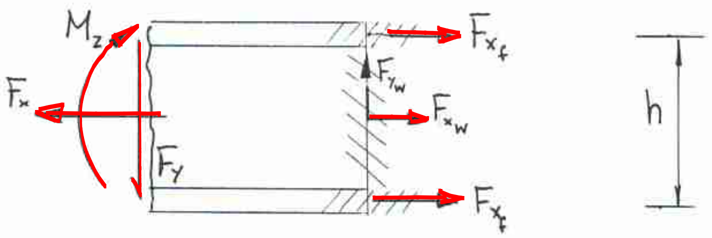
f = flange

w = web

Consider transfer @ flange and web connections

- based on sub-element @ end of joint element

Using flange and web connections



Ie:

Flange joint loading : $F_{x_f} \simeq F_x \dfrac{A_f}{\Sigma(A_f+A_w)} \pm \dfrac{M_z}{h}$ — Bending mont reacted by couple between flange joints

$F_{y_f} \simeq 0$

From equilibrium sums $\Sigma \uparrow^{+} \curvearrowleft^{+} \rightarrow^{+} = 0$

$M_{z_f} \simeq 0$

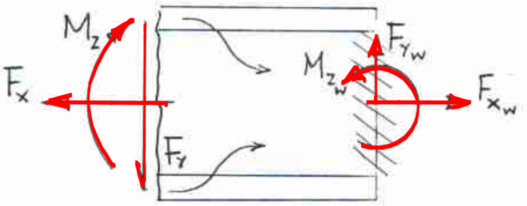
Web joint loading : $F_{x_w} \simeq F_x \dfrac{A_w}{\Sigma(A_f+A_w)}$

Axial load reacted by forces in flange + web joints in proportion to flange+web sections

$F_{y_w} \simeq F_y$ — Shear load reacted by force in web joint

$M_{z_w} \simeq 0$

Using web connection only



Here: flanges off-load into web @ joint
↳ must thicken or reinforce web!

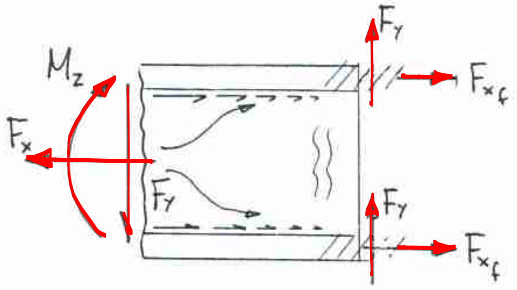
Web joint loading: $F_{x_w} = F_x$

$$F_{Y_w} = F_Y$$

$$M_{z_w} = M_z$$ Web transfers bending moment as eccentric loading.

Note stability of web plate under axial + shear load + bending moment!

Using flange connections only



Here, web off-loads into flanges

Flange joint loading: $F_{x_f} = F_x \dfrac{A_f}{\Sigma A_f} \pm \dfrac{M_z}{h}$ Bending mom't reacted by couple between flanges

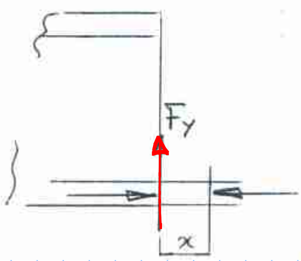
$$F_{Y_f} = F_Y \frac{A_f}{\Sigma A_f}$$ etc.

↳ transfer of shear load through flange joint needs careful consideration.

$$M_{z_f} = 0$$

Note, stability of flange joint plate under compression and bending due to offset shear load

Eg:

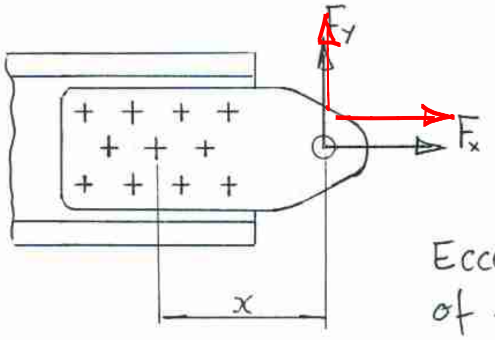


Also, note stability of web plate @ free edge

Further considerations:

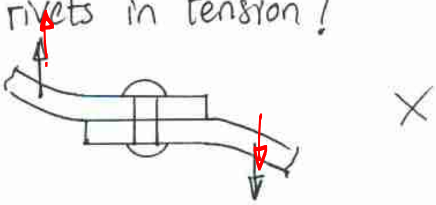
- Eccentric loading in pinned joint fittings

Eg.



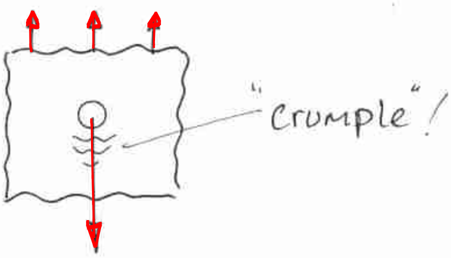
Eccentric loading @ centroid of fitting fastener group.
-due to offset of pin loading
ie. $F_y . x$

- Avoid putting rivets in tension!



ctd.

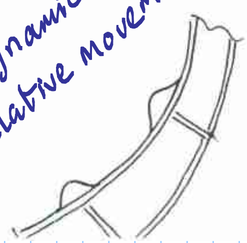
- Beware of local instability @ pin loading in thin plates



Check d/t ratio guidelines
e.g. $d/t \leq 3.5$ etc.

- Must use multifastener joints / fittings in thin plates to disperse concentrated loads
- Consider bushing in pin joint holes to improve bearing
- Avoid using mixed fasteners in same joint eg. rivets + bolts
- For bolted joints check "fitting factors" – Apply to bolted joints especially when subjected to dynamic loading or relative movement.
  ALTERNATIVELY CONSIDER MINIMUM TARGET RF VALUES.
- Use integral fittings where possible
- Use local reinforcement

Eg:



Fitting details

IRF 22.4.09

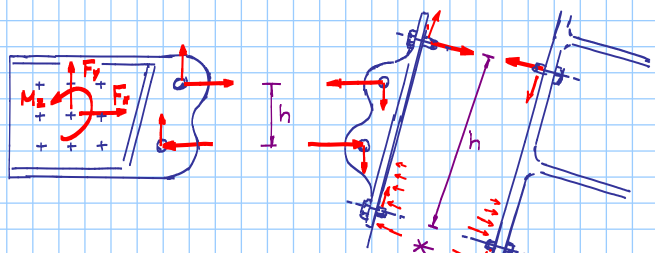
A fixed joint can be created by using multiple fastener connections (a minimum of two to create couple forces).

Fittings can be separate items or integral with the structure to be joined.

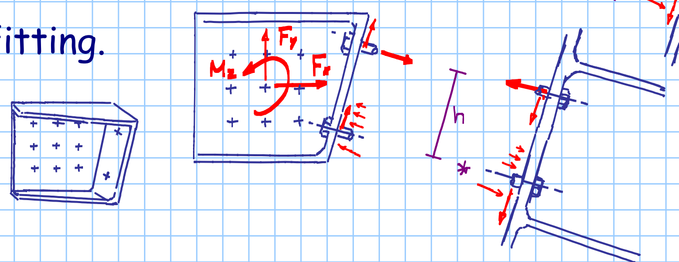
FBD's must be created to understand load transfer as direct and shear loads.

Direct loads will have contributions from F_x and M_z/h couple loads:

E.g. Shear lug fitting
Double shear connection needed



Eg. Tension end plate fitting.



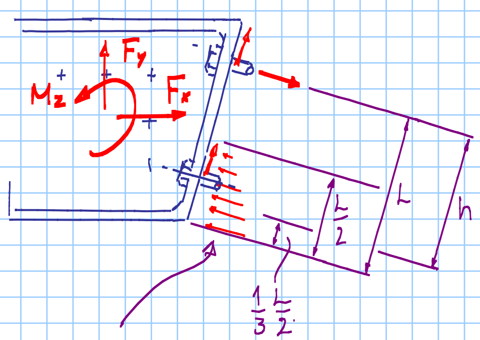
FBD's !

* Note the distribution of the compressive couple load between separate fittings

* Assume the effective centre of the compressive couple load acts at

- either the outer bolt line
- or the centre of an assumed triangular distribution of stress from the edge of the fitting face on the compression side
- whichever is most conservative.

E.g.:



Assumed triangular distribution of compressive stress

Further consideration would be needed for more rows of fasteners.

Joint fittings can

- either be separate items riveted or bolted to the beams to be joined as illustrated (but this requires further parts and fasteners)
- or as integral items within the beam or frame (but this requires significant machining).

Joint analysis includes concentric + eccentric rivet group analysis, lug and pin analysis and fitting analysis.

Depending on your chosen configuration you will need to give some thought to the diffusion of loads into these fittings and their potential failure modes.

Connecting to flanges only or web only or web + flanges presents significantly different schemes with conflicting pros and cons in terms of load transfer and ease of manufacture and assembly.

- THIN WALL SECTION JOINTS, e.g. box section

To start we will size according to fastener shear strength in order to estimate the required number of rivets and to provide an initial scheme.

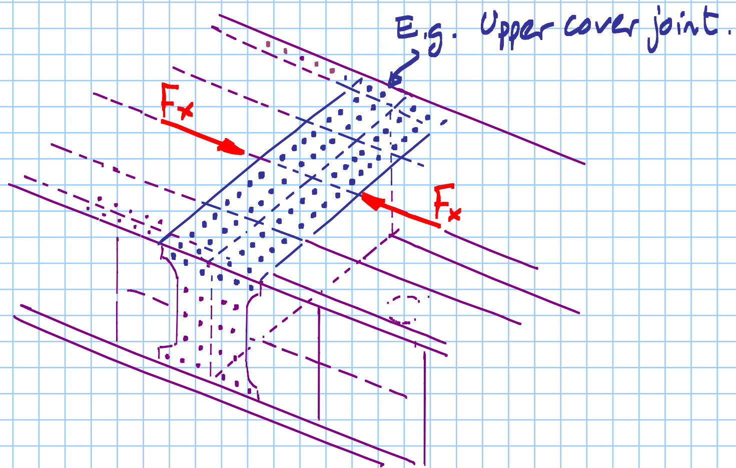
Upper & lower cover lap joints

Design for $\frac{F_x}{n_r} \leq P_r^*$ at ult.

where F_x = End load in cover.

P_r^* = rivet allowable single shear strength

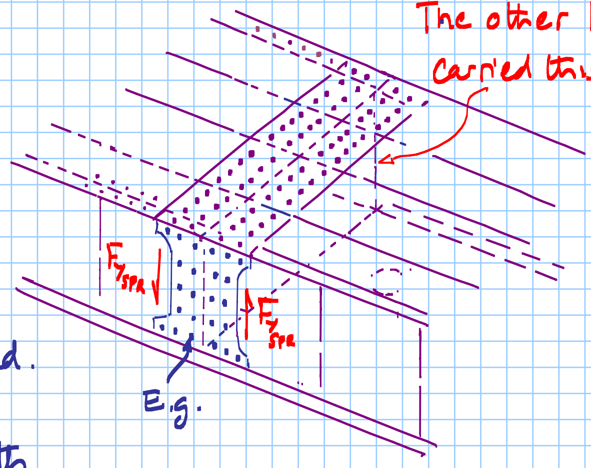
n_r = № of rivets.



Include the "strapping effect" of spar caps and stringers across the joint by considering the share of load according to element cross-section areas.

Assuming uniform load sharing between all fasteners in multiple rows under end load will be unconservative, even at ultimate load, so use a "fitting factor". For max fastener load apply a fitting factor of 1.5 to the average to allow for uneven load sharing between fasteners.

Spar web butt strap joints



Design for $\dfrac{F_{Y_{SPR}}}{n_r} \leq P_r^*$ at ult

where $F_{Y_{SPR}}$ = spar shear load

$= \dfrac{F_Y}{2}$ ie half wing shear load.

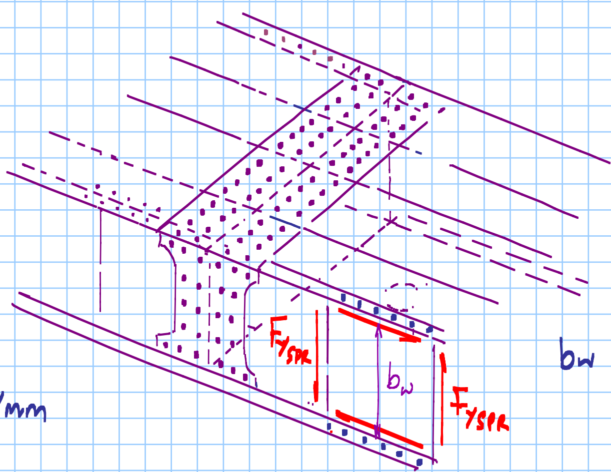
P_r^* = rivet allowable single shear strength

n_r = No of rivets. on one side of butt strap!

Assuming uniform load sharing between all rivets at ultimate.

Also watch out for stability of butt strap.

Spar-skin joint



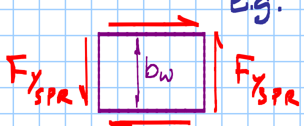
b_w = height of web.

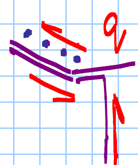
Design for $q.p_r \leq P_r^*$

where q = "shear flow"

$\approx F_{Y_{SPR}}/b_w$ eg. N/mm

E.g.





"shear flow" force per unit length

$F_{Y_{SPR}} = F_Y/2$

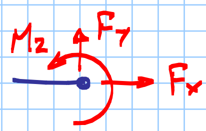
P_r^* = rivet allowable single shear strength

p_r = rivet pitch

Note: "complimentary shear" and "shear flow." See STM2 notes.

For max fastener load apply a fitting factor of 1.25 to average to allow for uneven load sharing between fasteners.

Connection load transfer



Combinations of F_x, F_y, M_z must be transferred at the ends of a beam, depending on the load case and configuration considered,

Values of F_x, F_y, M_z can be obtained from beam FBD and AF, SF, BM diagrams.

The end loads can be interpreted as joint loadings and further consideration of the fittings can be achieved by FBD's of suitable sections to illustrate loading as shown in the examples above.