Flight Project 2

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Contents

1 Flight Project 2

1.1 Task

The objectives are to achieve the:

- Highest value of lift L at any incidence. This is a dimensional result, measured in [N]
- Highest value of lift-drag ratio $\frac{L}{D}$ at any angle of attack. This is a non-dimensional result
- Highest leading edge sweep angle (either forwards or backwards). This is a non-dimensional result
- Lowest volume of foam within the wing

Subject to the constraint that:

- **Your wing must achieve a deflection of less than 4cm at a tip load of 300g. If your wing does not pass this test, you will not be eligible to win the contest! If this happens, consider yourself lucky. In the real world your aeroplane would now be a hole in the ground...**

The method of scoring will be through ranking all entries according to the four objectives above. The rankings will then be summed for each team, and the team with the lowest summed ranking will win (ties are permitted). 20% of marks will be allocated according to your team's final ranking position; the remainder will be for the technical quality of the report.

Notice that the objectives above conflict. A high value of lift requires a large wing area and a high stalling lift coefficient $C_{L_{max}}$. However, a high sweep angle will lower $C_{L_{max}}$, and because of the 7.5cm$\times$35cm$\times$30cm box constraint it will also lower the wing area. A good $\frac{L}{D}$ requires a high aspect ratio, which (with the box constraint) forces a lower area. These are just a couple of examples of interactions and there are many more.

The ranking method is intentionally a sliding scale depending on peer performance; you are competing between groups to achieve a good compromise in the above areas. This is how the real world operates!

1.2 Construction

The CNC foam cutter can only cut prismatic wings. Therefore, the variables you can change are

- Root and tip aerofoils
- Root and tip aerofoil positions; this allows twist, taper, sweep and aspect ratio to change

Remember that a hard constraint is that your wings must fit within the foam block for cutting.

The foamcutter cannot cut thin wings, as these get too hot and warp. If you make any part of your wing (like the tip) too thin, it may no longer be the shape you intended, and we may need to modify the design for you. We suggest that **you do not make the maximum thickness at the tip less than 2cm, and that the tip chord remains above 10cm**.

1.3 Design

You will need to create a full 3D CAD representation of your wings.

In addition, you will need to download the file *gCodeGen.zip* from the Blackboard site. This Matlab software will take the definition of your wing and convert it into G-code when you run *gCodeGen.m*. G-code is the signalling language for the foamcutter. At the moment, it is not possible to create G-code from your CAD files, therefore this step must be done separately to the CAD work.

Once you have designed your wing you must enter the leading edge sweep, taper ratio, twist (positive is tip leading edge down), root chord and dihedral into the file *settings.dat*. Do not change the other inputs. You must also place the coordinates for the root and tip aerofoils in two files named *root.aero* and *tip.aero*, following the formatting convention in the example files. The points should go anti-clockwise around the section.

When you run *gCodeGen.m* it will generate a 3D view of the foamcutting arrangement. If any errors are produced, you must resolve these before sending us the *settings.dat* file by reducing the size of your wing. Errors are normally a result of trying to create a wing that does not fit within the prescribed box constraints.

1.4 Predictions

In order to design your wing you will need to be able to find

- Tip deflection at 300g. The loading position will be adjusted during the test to achieve zero twist at the tip
- $\frac{dC_L}{d\alpha}$ for your root and tip aerofoils (i.e. no 3D effects), and $\frac{dC_L}{d\alpha}$ for your wing (i.e. with 3D effects)
- $C_{L_{max}}$ for your root and tip aerofoils (i.e. no 3D effects), and $C_{L_{max}}$ for your wing (i.e. with 3D effects)
- C_{D_0} for your root and tip aerofoils, and for your wing

In order to make these predictions, you should use simple bending theory and the aerodynamic methods outlined in the following sections.

1.4.1 Finding C_L

http://www.ae.illinois.edu/m-selig/pd.html provides the zero lift incidence of your aerofoil(s) $\alpha_{0_{2D}}$. You may assume that

$$\alpha_{0_{2D}} = \frac{1}{2}\left(\alpha_{0_{root}} + \alpha_{0_{tip}} - \alpha_{tip}\right) \tag{1}$$

where α_{tip} is the nose up twist of the wingtip section. Then take

$$\alpha_{0_{3D}} = \alpha_{0_{2D}} \tag{2}$$

and

$$C_{L_{3D}} = \frac{dC_L}{d\alpha}_{3D}\left(\alpha - \alpha_{0_{3D}}\right) \tag{3}$$

up to stall. We will not predict any post-stall behaviour.

1.4.2 Finding C_D

Obtain C_{D_0} from *http://www.ae.illinois.edu/m-selig/pd.html* for your chosen aerofoils. If the root and tip are different, use average values of the two.

Then use

$$C_{D_{3D}} = C_{D_o} + \frac{C_{L_{3D}}^2}{e\pi A_R} \tag{4}$$

assuming $e = 0.7$.

1.4.3 Finding $C_{L_{max}}$ and $\frac{dC_L}{d\alpha}$

$C_{L_{max}}$ is the value of lift coefficient at stall; the highest point on the C_L vs. α graph. It is strongly dependent on flow separation, which makes it difficult to predict accurately. Fortunately, you have access to *http://www.ae.illinois.edu/m-selig/pd.html*, which provides experimental data for a range of aerofoils suitable for small aircraft (i.e. low R_e flight).

However, this does mean you must use one of the aerofoils in this database. There are approximately 100 options to choose from. The same data will provide a result for $\frac{dC_L}{d\alpha}$ for each aerofoil.

In order to convert a 2D $\frac{dC_L}{d\alpha}$ into a 3D $\frac{dC_L}{d\alpha}$ for the wing use the Helmbold-Diederich formula

$$\frac{dC_L}{d\alpha}_{3D} = \frac{2\pi A_R}{2 + \sqrt{\frac{A_R^2}{\kappa^2}\left(1 + \tan^2\left(\Lambda_{\frac{1}{2}}\right)\right) + 4}} \tag{5}$$

with

$$\kappa = \frac{\frac{dC_L}{d\alpha}_{2D}}{2\pi} \tag{6}$$

$$\frac{dC_L}{d\alpha}_{2D} = \frac{1}{2}\left(\frac{dC_L}{d\alpha}_{root} + \frac{dC_L}{d\alpha}_{tip}\right) \tag{7}$$

where $\Lambda_{\frac{1}{2}}$ is the half chord sweep angle.

This is most accurate for accurate for larger A_R and R_e, however it will suffice in this case. Make certain to use $\frac{dC_L}{d\alpha}$ values measured in radians *not* degrees.

Although this is not a rigorous approach, you may also use

$$C_{L_{max_{3D}}} = 0.9 C_{L_{max_{2D}}} \cos\left(\Lambda_{\frac{1}{4}}\right) \tag{8}$$

where $\Lambda_{\frac{1}{4}}$ is the quarter chord sweep angle and

$$C_{L_{max_{2D}}} = min(C_{L_{max_{root}}}, C_{L_{max_{tip}}}) \tag{9}$$

To convert between sweep angles use

$$A_R \tan\left(\Lambda_{\frac{1}{2}}\right) = A_R \tan\left(\Lambda_n\right) + 2\left(2n - 1\right)\frac{1-\lambda}{1+\lambda} \tag{10}$$

where $A_R = \frac{b^2}{S}$, b is aircraft wingspan, S is planform area and λ is the taper ratio.

1.4.4 Finding Aerodynamic Centre

The aerodynamic centre is the point about which the pitching moment does not change with angle of attack. Note that this is not the same as the centre of pressure, which is the point about which the aerodynamic loads have no moment.

To find x_{ac} take moments about the hinge point on the balance (this is location where your wing was spiked). Let's say nose down is negative and that x_{ac} is positive behind the hinge, then

$$M = M_0 - L x_{ax} \tag{11}$$

Remember that M_0 is the constant part of the pitching moment about the aerodynamic centre itself, which is independent of α. Non-dimensionalise so that

$$C_M = C_{M_0} - C_L \frac{x_{ax}}{c} \tag{12}$$

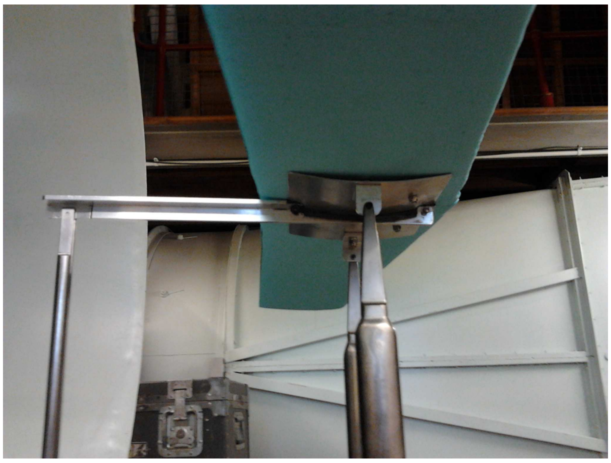
where c is your reference length (the one used to calculate C_M from M in the data your measure). We can differentiate this to get

$$\frac{\partial C_M}{\partial \alpha} = -\frac{\partial C_L}{\partial \alpha}\frac{x_{ax}}{c} \tag{13}$$

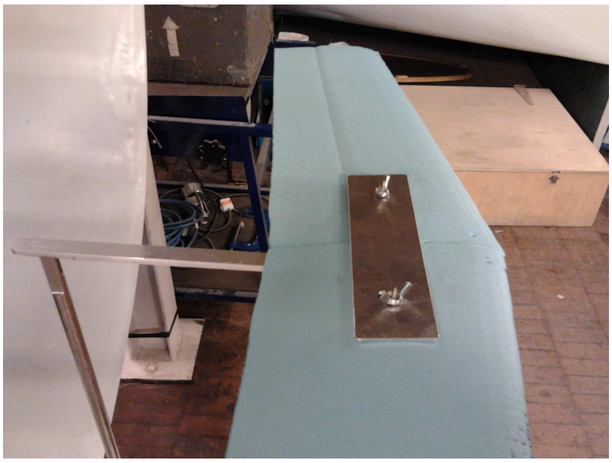
Hence we need the ratio of the two gradients, giving

$$\frac{x_{ax}}{c} = -\frac{\frac{\partial C_M}{\partial \alpha}}{\frac{C_L}{\partial \alpha}} \tag{14}$$

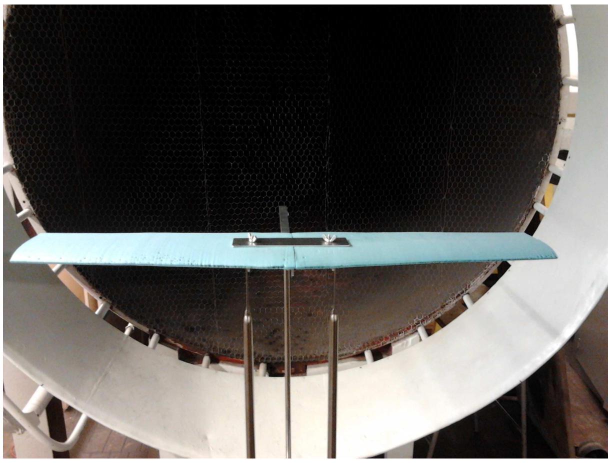
Finally, it is worth remembering that this is all based around small angles of attack. At large angles, $\frac{\partial C_L}{\partial \alpha}$ will change, and the aerodynamic centre will move location. For aircraft stability the value in the linear lift region is what is important.



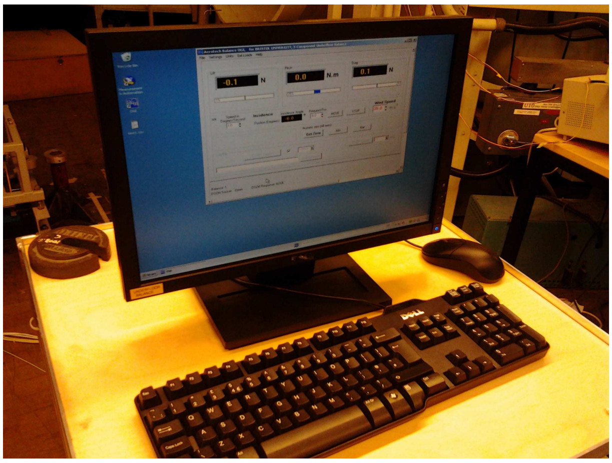
(a) Below rig



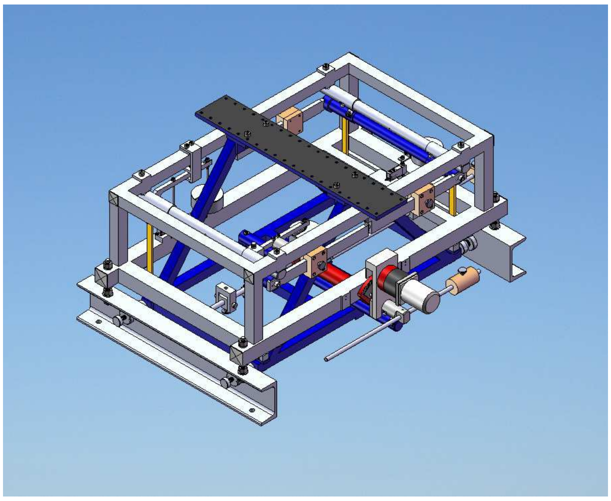
(b) Above rig



(c) Head-on rig



(d) A docile computer



(e) 3-component force balance

Figure 1: Experimental apparatus and the mighty open-jet tunnel

1.4.5 Structural Dynamic and Static Behaviour

Compute as advised during lectures.

2 Aeroelasticity

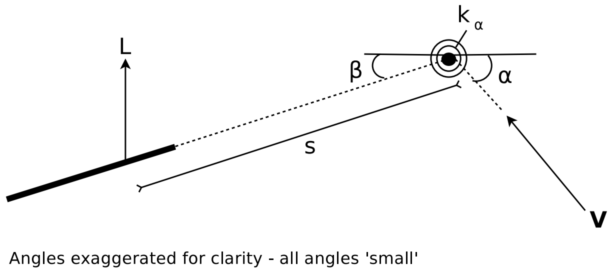


Figure 2: A one degree of freedom pivoting aeroelastic model

The foam wings are visibly flexible. This means that under load they bend and twist, which alters the angle of attack all the way along the span of the wing. In turn, this changes the loads, which of course changes the deflected shape again. The reality is that this is a coupled ‘aeroelastic’ problem (a combination of aerodynamics and elasticity).

Our wings don’t tend to oscillate, mainly due to the high damping in foam, so there are no inertial forces to worry about, which means this is ‘static aeroelasticity’. Dynamic aeroelasticity, where inertial forces are important, is a large field that has many safety implications, and includes subjects such as flutter. Static aeroelastic effects can also be dangerous, but they are more often important for predicting performance.

The wings we have here are swept and tapered. One of the most noticeable effects of sweep is that it couples bending to the aerodynamics loads. To understand why, consider an unswept wing. When this bends, the aerofoil sections do not change their angle relative to the flow vector. However, if the wing is swept, the bending axis lies partly in the flow direction, which means there is a change in angle of attack all the way along the wing.

In our problem the biggest effect of aeroelasticity is to change the lift curve slope of the wing $\frac{dC_L}{d\alpha}$. To explain why this happens, we will first consider a 2D bend-twist model. This isn’t an accurate model and would never be used in design, but it explains the core physics behind the problem.

Adopting the nomenclature of figure 2, then aking moments about the pivot and requiring equilibrium gives

$$Ls\cos(\beta) = k_\alpha \beta \tag{15}$$

However, the angles here are quite small, and we can keep the maths simpler if we say $\cos(\beta) \approx 1$. Dynamic pressure $q_\infty = \frac{1}{2}\rho V^2$, then (where c is chord)

$$Ls = k_\alpha \beta \tag{16}$$

$$\left(\frac{dC_L}{d\alpha}(\beta+\alpha) + C_{L_0}\right) q_\infty cs = k_\alpha \beta \tag{17}$$

$$\frac{dC_L}{d\alpha}(\beta+\alpha) + C_{L_0} = \frac{k_\alpha \beta}{q_\infty cs} \tag{18}$$

$$\frac{dC_L}{d\alpha}\beta\left(1 - \frac{k_\alpha}{\frac{dC_L}{d\alpha}q_\infty cs}\right) = -\left(\frac{dC_L}{d\alpha}\alpha + C_{L_0}\right) \tag{19}$$

$$\beta = -\frac{\alpha + \frac{C_{L_0}}{\frac{dC_L}{d\alpha}}}{1 - \frac{k_\alpha}{\frac{dC_L}{d\alpha}q_\infty cs}} \tag{20}$$

What this means is very simple (ignore C_{L_0} for argument’s sake). If k_α is huge, $\beta = 0$ (the wing is so stiff it is rigid, so there is no deflection). When $k_\alpha = 0$ (there is just a free pivot with no spring), then $\beta = -\alpha$, and the deflection is exactly the opposite of the geometric incidence! This means there is no angle of attack, and no lift. The effect of aeroelasticity has been to remove lift altogether.

In between, it is fairly clear that

$$C_{L_{AE}} = \frac{dC_L}{d\alpha}(\alpha+\beta) + C_{L_0} = \frac{dC_L}{d\alpha}\left(\alpha - \frac{\alpha + \frac{C_{L_0}}{\frac{dC_L}{d\alpha}}}{1 - \frac{k_\alpha}{\frac{dC_L}{d\alpha}q_\infty cs}}\right) + C_{L_0} \tag{21}$$

What we really want is $\frac{dC_{L_{AE}}}{d\alpha}$, which is

$$\frac{dC_{L_{AE}}}{d\alpha} = \frac{dC_L}{d\alpha}\left(1 - \frac{1}{1 - \frac{k_\alpha}{\frac{dC_L}{d\alpha} q_\infty cs}}\right) \tag{22}$$

This tells a similar story. When k_α is huge, $\frac{dC_{L_{AE}}}{d\alpha} = \frac{dC_L}{d\alpha}$. When k_α is zero, $\frac{dC_{L_{AE}}}{d\alpha} = 0$!

A final point: it is not just k_α that matters, but $\frac{k_\alpha}{\frac{dC_L}{d\alpha} q_\infty Cs}$. If you ponder over this non-dimensional number, you will realise that it represents the ratio between stiffness (structural) forces and aerodynamic forces (just like the Reynolds number is the ratio between inertial and viscous forces). The value of $\frac{dC_{L_{AE}}}{d\alpha}$ not only depends on the stiffness of the wing, but also the speed and altitude (i.e. dynamic pressure) at which we fly!

In summary, we can expect the wing deflections to lower $\frac{dC_L}{d\alpha}$. How much will depend on the stiffness and tunnel speed. Unfortunately the analysis above is rather simplistic, and deciding on the value of k_α to use to represent a 3D wing as a 2D model is not easy. Furthermore, the offset pivot model is not accurate (bend-twist coupling is quite complicated and requires beam theories you haven't yet encountered), and the 3D aerodynamics is actually linked to the 3D shape of the wing which our 2D model cannot handle. So, it is tough to use it to make a prediction here. Instead you will carry out two tunnel tests, one with a rigid wing and one with a flexible wing. The comparison will illustrate our aeroelastic effect.

3 Example Results

Defining force and moment coefficients with $q_\infty = \frac{1}{2}\rho V_\infty^2$ as

$$C_L = \frac{L}{q_\infty S} \tag{23}$$

$$C_D = \frac{D}{q_\infty S} \tag{24}$$

$$C_M = \frac{M}{q_\infty S c_{root}} \tag{25}$$

where the total wing area is S and c_{root} is the root chord.

Remember that non-dimensionalisation is arbitrary. These are the definitions we shall use here, but in future always check what the reference lengths and areas are for any data you use; these definitions can even vary between aircraft manufacturers!

Also remember that non-dimensional coefficients are not explanations of anything in their own right. For example, it is futile to argue that C_D drops because S goes up, and therefore D drops. The choice of S and c_{root} is only to get numbers that are 'pleasant' to work with, in that they have values that fit into the context of the rest of the numbers we are using. You could equally well define S to be the area of the palm of your hand, but it would be a little eccentric (and would not represent the physical size of the aircraft in any way...).

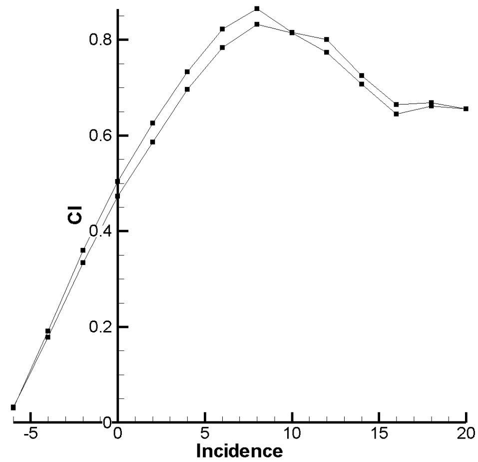
The results for a crude wing are in figure 3, which illustrates stall, and the tendency of the drag to vary quadratically with C_L (this is a 3D effect you will learn about in the future). The curves show $\frac{dC_L}{d\alpha} \approx 3.05$ and $\frac{dC_M}{d\alpha} \approx 0.47$, therefore $x_{ac} \approx \frac{-0.47}{3.05} = -0.15$ (postive moment is nose up). The root chord was 13cm, so x_{ac} lies about $-0.15 \times 13 = 2$cm in front of the hinge location shown in figure 1(a) (positive would be behind).

4 Report

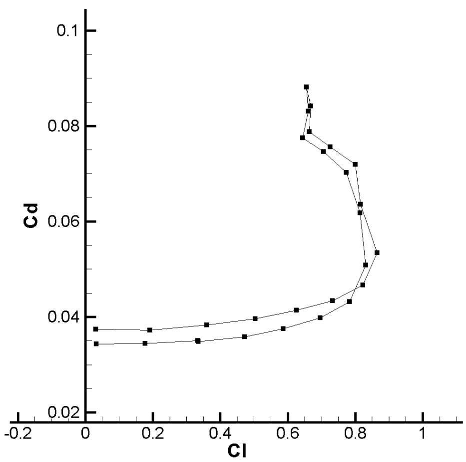
Each group must submit a report not exceeding 2000 words.

A famous man[1] once said 'a scientist discovers that which exists. An engineer creates that which never was'. Creativity embedded in analysis almost defines engineering, so for this reason the specifications for your report are fairly open.

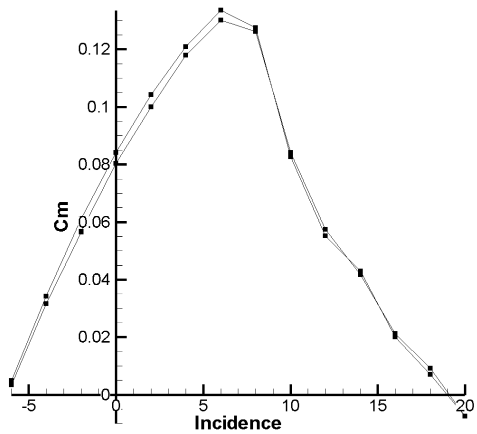
[1] Theodore von Kármán



(a) C_L



(b) C_D against C_L



(c) C_M about hinge

Figure 3: Example results

If you want to get the tone of your report right, aim to justify both the design you came up with, and the analysis of that design (here, through comparison to your experimental data).

You must include the sections numbered 1-6 below. The % marks available for each section are also given. Each section must appear in your report as titled and numbered here, but within each section the bullet points below are suggestions as to what you might include; these are for guidance, it is not necessarily expected that you will cover all of them.

1. Design process - 10%
   - How did you arrive at this configuration - what tradeoff studies did you perform?
   - How did the structural deflection requirement influence the aerodynamic design, and how did the aerodynamic requirements drive the structural shape?
   - Did you prioritise any objectives over others?
2. 3-view CAD Drawings - 5%
3. Structural Behaviour - 25%
   - Predicted tip deflections using each method you applied. There is plenty of scope here for different methods; these could include everything from a 'back-of-envelope' method to an incremental scheme that splits the beam into sections, to a numerical finite difference approach
   - Comparison between predicted and measured structural behaviour. You **must** include your load vs. deflection plot in bending
4. Aerodynamic Behaviour - 25%
   - Lift versus incidence curves for the aerofoil sections you chose
   - Predicted $\frac{dC_L}{d\alpha}$, $C_{L_{max}}$ and $\frac{L}{D}_{max}$ for your wing
   - Comparison between the experimental results and your predictions. You **must** include your C_L vs. incidence, C_L vs. C_D, and C_M vs. incidence plots. Also calculate the location of your wing's aerodynamic centre relative to the hinge point on the force balance (remember, the furthest forward hole on your wing corresponds to the hinge point, so it's easy to remember where it was!)
5. Dynamic Behaviour - 25%
   - Predictions for the 1^{st} bending and torsional frequencies. You can use the stiffness values here that you measured in the static structural tests. If you combine these with estimates for the moment of inertia in bending or torsion, you can arrive at an approximate frequency
   - Comparison between the experimental results and your predictions. Why do the measured results differ from your predictions? Include here any graphs for vibratory response that you may have. What is the approximate value of the damping? What may be generating the damping?
6. Conclusion - 10%
   - Was your design a success or failure? Marks are only available here for honesty and accuracy!
   - What aspects of you wing would you change if you were to repeat this exercise? Was it too stiff, so could you have saved material? Was it too flexible?