Advanced Bending and Torsion
Shear Stresses in Open Thin-Walled Sections

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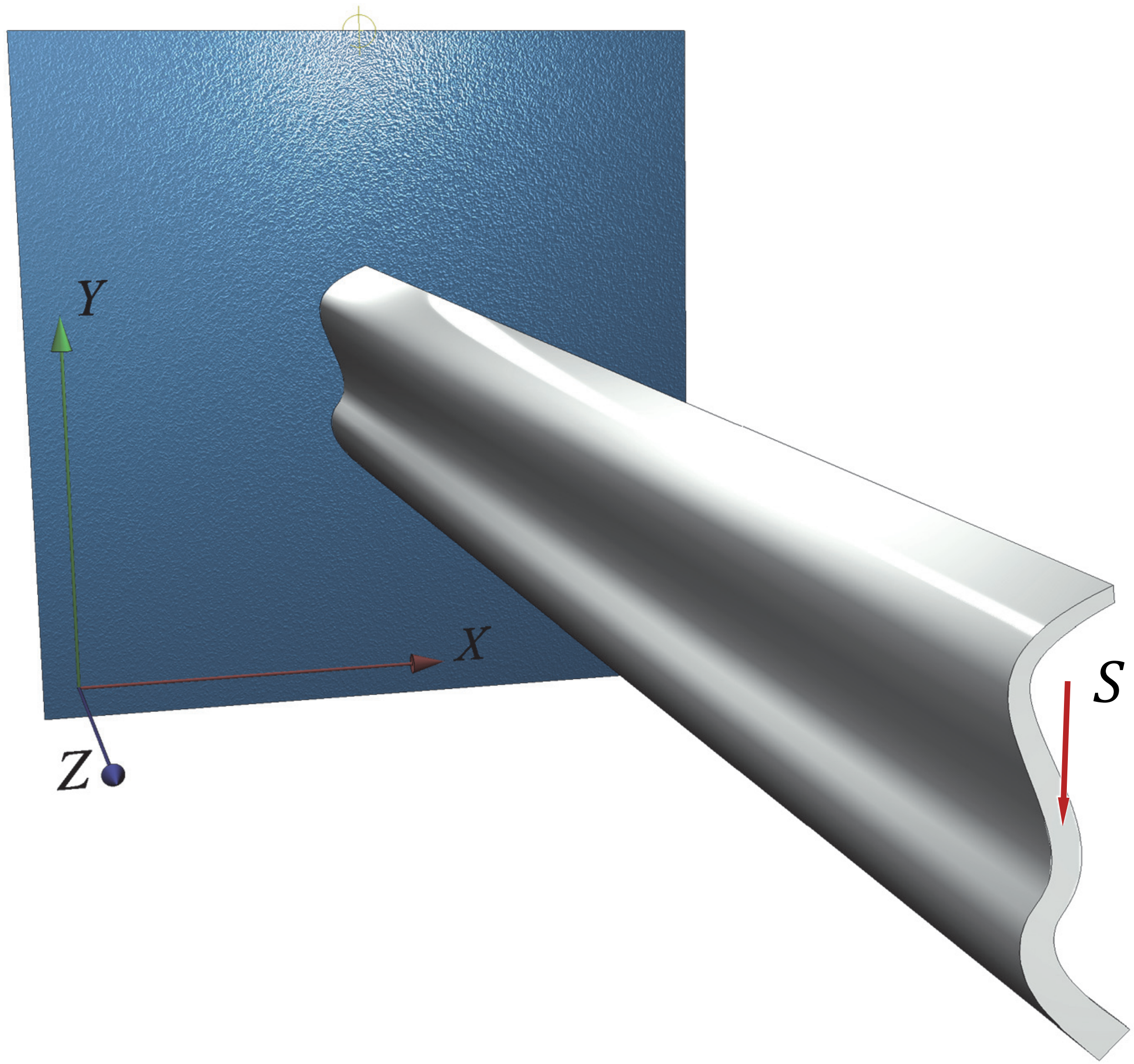
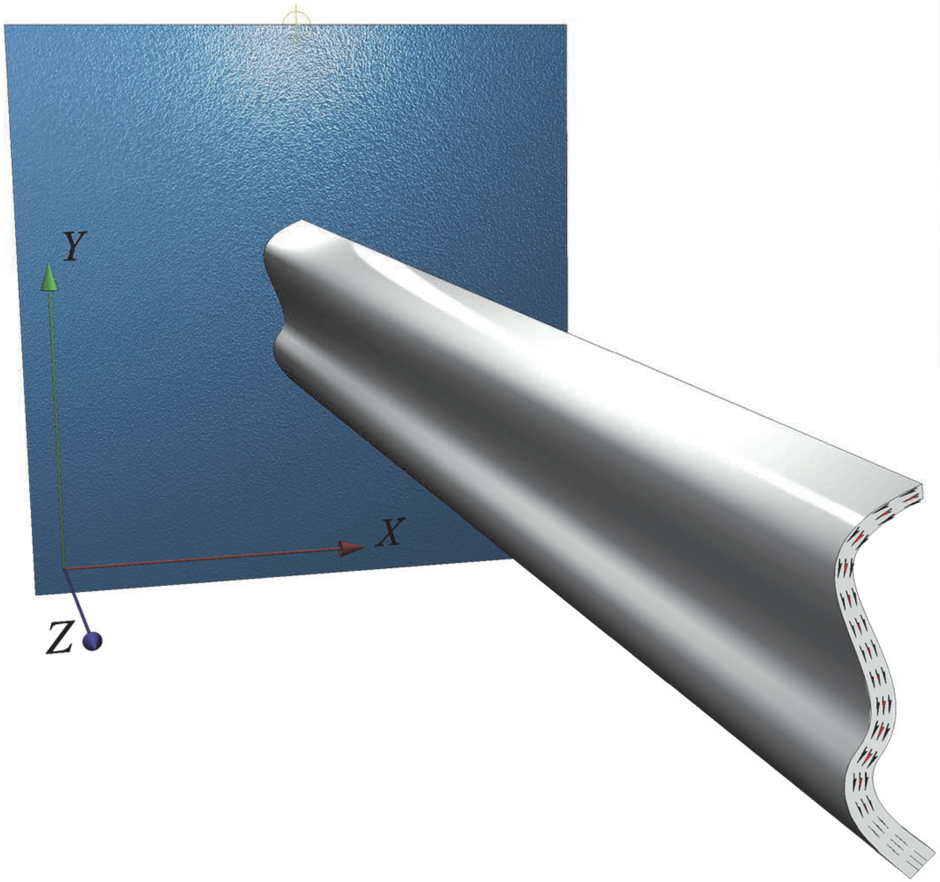
Shearing of Open Thin-Walled Sections - Derivations

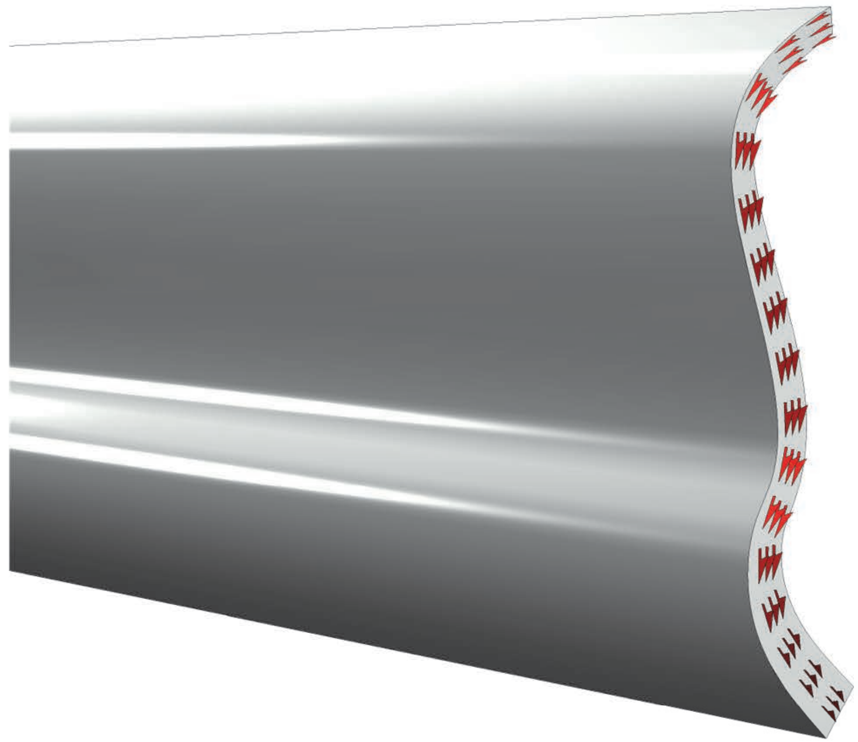
- Assume an arbitrary thin-walled section with variable wall thickness:



Shearing of Open Thin-Walled Sections - Derivations

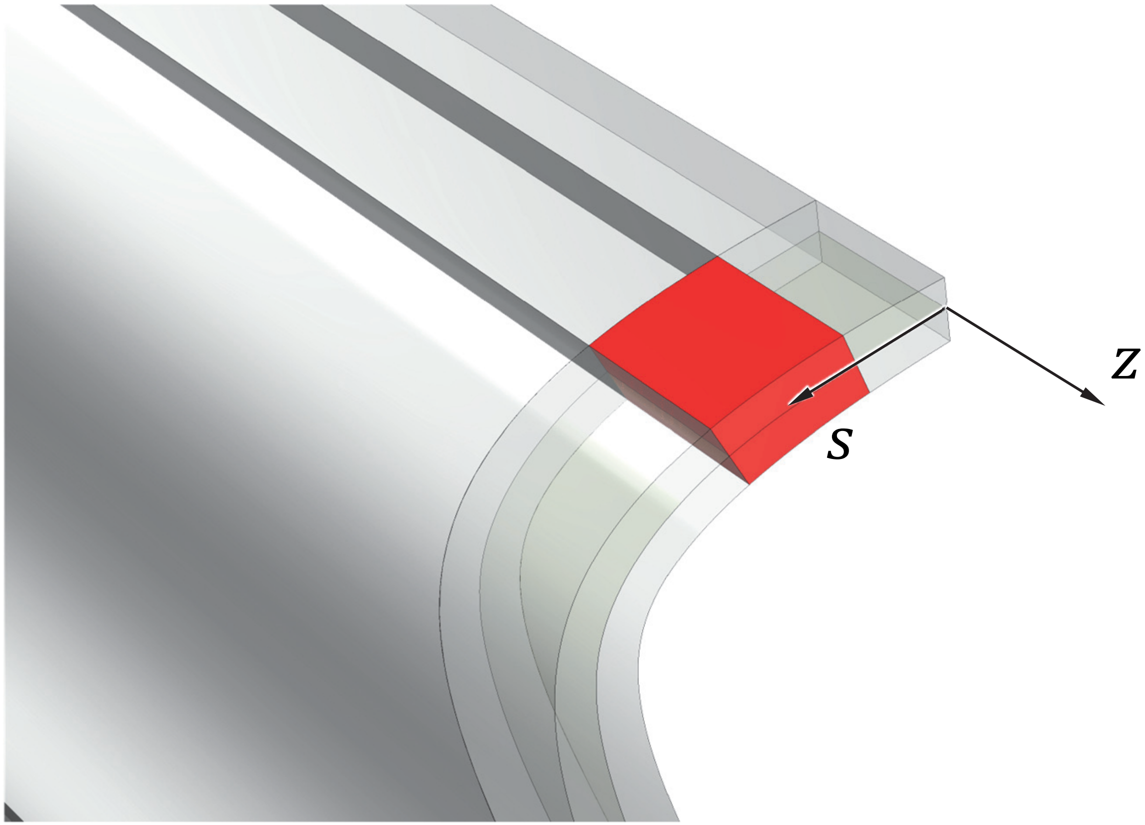
- Assume an arbitrary thin-walled section with variable wall thickness
- Shear stresses now 'flow' along the cross-section:





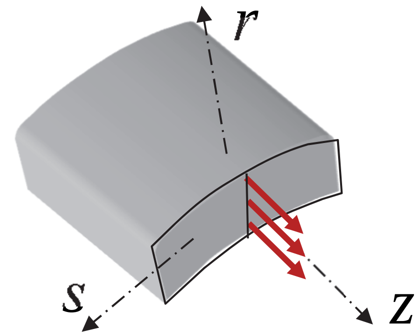
Shearing of Open Thin-Walled Sections - Derivations

- Assume an arbitrary thin-walled section with variable wall thickness
- Shear stresses now 'flow' along the cross-section
- We consider a small element along the arc length s

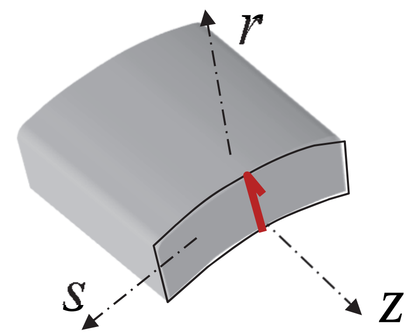


Shearing of Open Thin-Walled Sections - Derivations

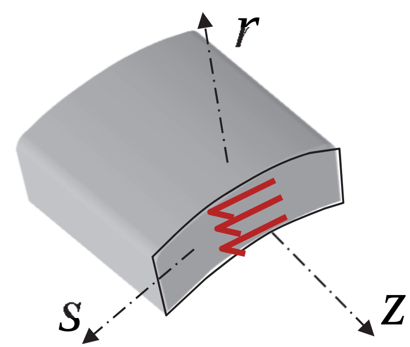
- Thin-wall assumptions
  - Direct stresses are constant through the thickness:



  - Through-thickness direct and shear stresses are negligible:



  - In-plane shear stresses are constant through the thickness:

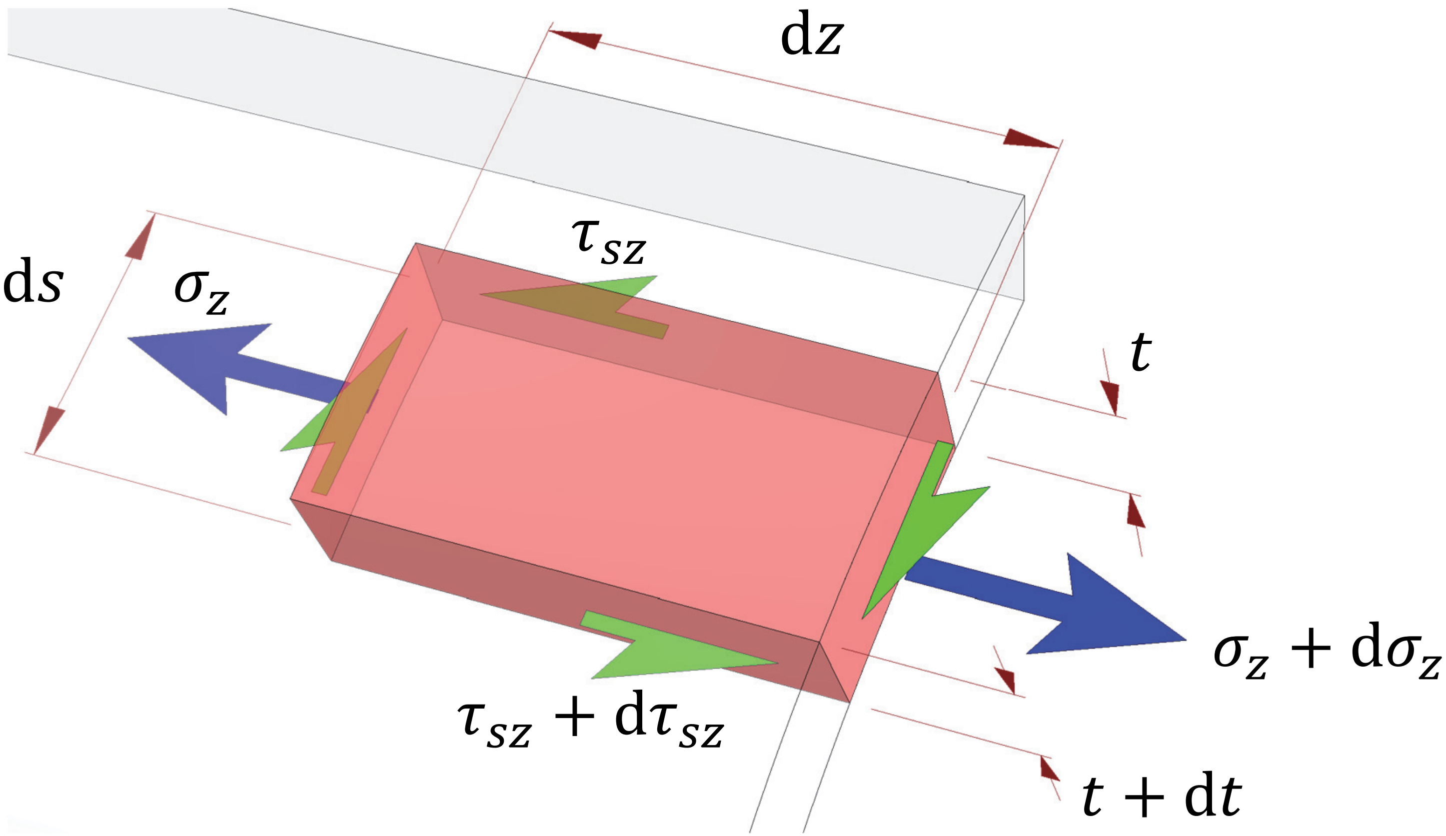


- The **shear flow** (shear force per unit arclength) is defined as:

$$q_{zs} = t\,\tau_{zs}$$

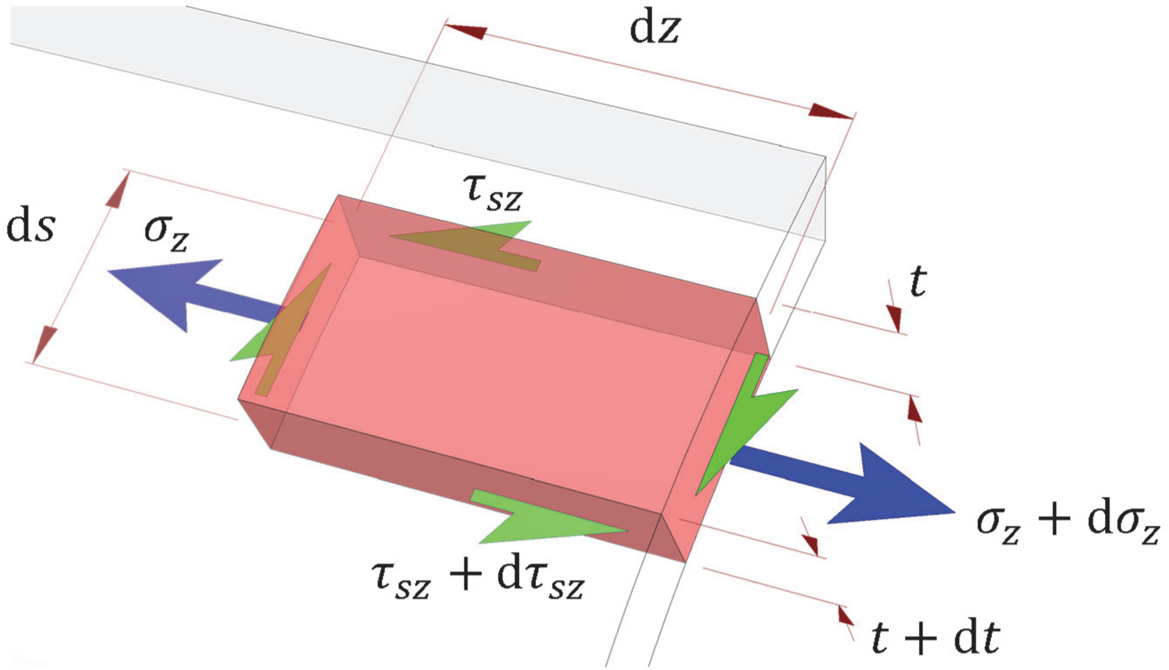
Shearing of Open Thin-Walled Sections - Derivations

- We consider a small element along the arc length s
- There are two major stresses acting: σ_z and τ_{sz}



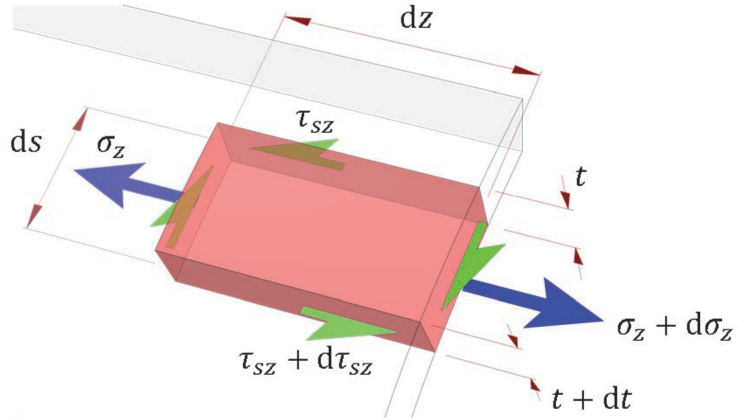
Shearing of Open Thin-Walled Sections - Derivations

- Force equilibrium in the z direction:



$$(\sigma_z + \mathrm{d}\sigma_z)\left(t + \frac{\mathrm{d}t}{2}\right)(\mathrm{d}s) - (\sigma_z)\left(t + \frac{\mathrm{d}t}{2}\right)(\mathrm{d}s)$$

$$+ (\tau_{sz} + \mathrm{d}\tau_{sz})(t + \mathrm{d}t)(\mathrm{d}z) - (\tau_{sz})(t)(\mathrm{d}z) = 0$$



- Complementarity of shear: $\tau_{sz} = \tau_{zs}$
- Therefore:

$$(\sigma_z + \mathrm{d}\sigma_z)\left(t + \frac{\mathrm{d}t}{2}\right)(\mathrm{d}s) - (\sigma_z)\left(t + \frac{\mathrm{d}t}{2}\right)(\mathrm{d}s)$$
$$+ (\tau_{zs} + \mathrm{d}\tau_{zs})(t + \mathrm{d}t)(\mathrm{d}z) - (\tau_{zs})(t)(\mathrm{d}z) = 0$$

- Neglecting higher order terms (*e.g.* containing $\mathrm{d}\sigma_z \times \mathrm{d}\tau_{zs} \times \mathrm{d}s$)

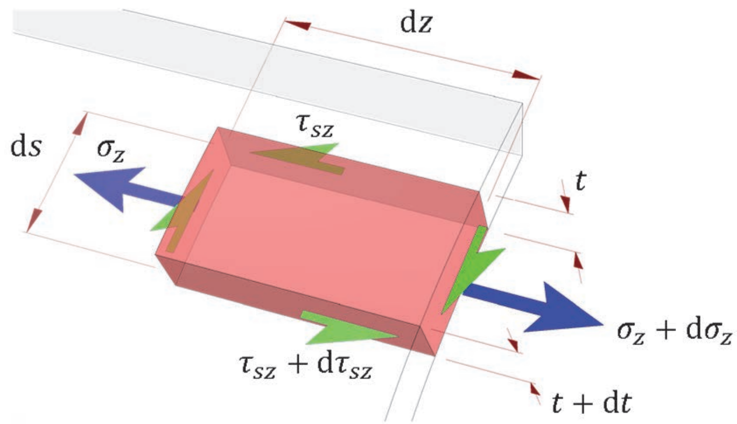
$$t\,\mathrm{d}\sigma_z\,\mathrm{d}s + \tau_{zs}\,\mathrm{d}t\,\mathrm{d}z + t\,\mathrm{d}\tau_{zs}\,\mathrm{d}z = 0$$

- Dividing by $(\mathrm{d}s\,\mathrm{d}z)$

$$t\frac{\mathrm{d}\sigma_z}{\mathrm{d}z} + \tau_{zs}\frac{\mathrm{d}t}{\mathrm{d}s} + t\frac{\mathrm{d}\tau_{zs}}{\mathrm{d}s} = 0$$

$$\boxed{t\frac{\mathrm{d}\sigma_z}{\mathrm{d}z} + \frac{\mathrm{d}(t\,\tau_{zs})}{\mathrm{d}s} = 0}$$

Shearing of Open Thin-Walled Sections - Derivations



- For constant t:

$$t\frac{\mathrm{d}\sigma_z}{\mathrm{d}z} + \frac{\mathrm{d}(\tau_{zs})}{\mathrm{d}s} = 0$$

- Making $q_s = q_{zs} = t\,\tau_{zs}$:

$$t\frac{\mathrm{d}\sigma_z}{\mathrm{d}z} + \frac{\mathrm{d}q_s}{\mathrm{d}s} = 0$$

- Replacing with our stress equation:

$$\sigma_z = \frac{M_y\, I_{xx} + M_x\, I_{xy}}{I_{xy}^2 - I_{xx}\, I_{yy}}x + \frac{M_x\, I_{yy} + M_y\, I_{xy}}{I_{xx}\, I_{yy} - I_{xy}^2}y$$

- Gives finally:

$$-q_s = \left(\frac{S_x\, I_{xx} + S_y\, I_{xy}}{I_{xy}^2 - I_{xx}\, I_{yy}}\right)\int_0^s x\, t\, \mathrm{d}s + \left(\frac{S_y\, I_{yy} + S_x\, I_{xy}}{I_{xx}\, I_{yy} - I_{xy}^2}\right)\int_0^s y\, t\, \mathrm{d}s$$