Properties of Materials

Theme: Polymers and Composites

Lecture 3: Composites

Dr Chenchen Zhu
chenchen.zhu@bristol.ac.uk
Frank LT PHYS BLDG

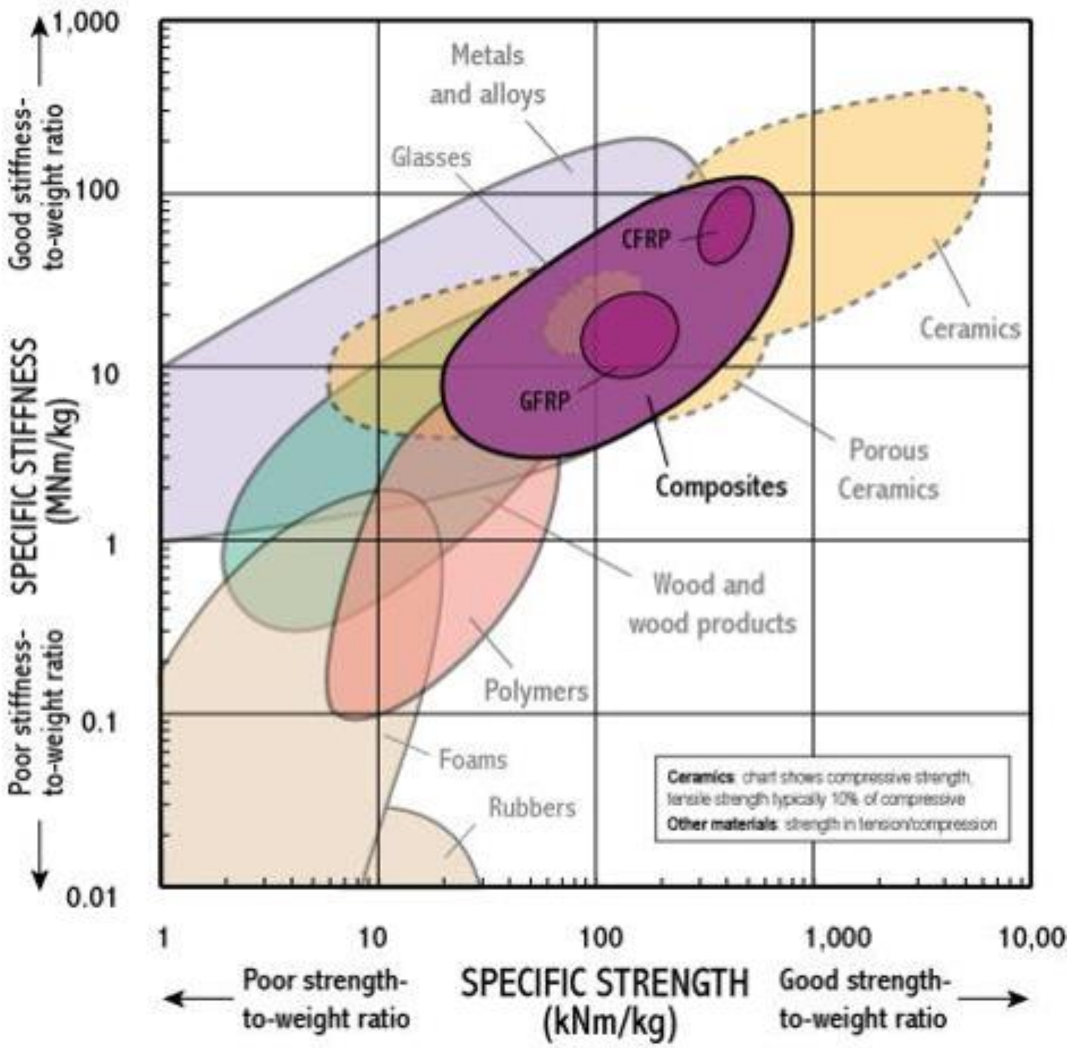
Lecture Contents

- Lecture 1
  - Introduction
  - Basic structure of polymers
- Lecture 2
  - Deformation
  - Chain alignment and viscoelasticity
- Lecture 3
  - Composites
  - Modulus and strength

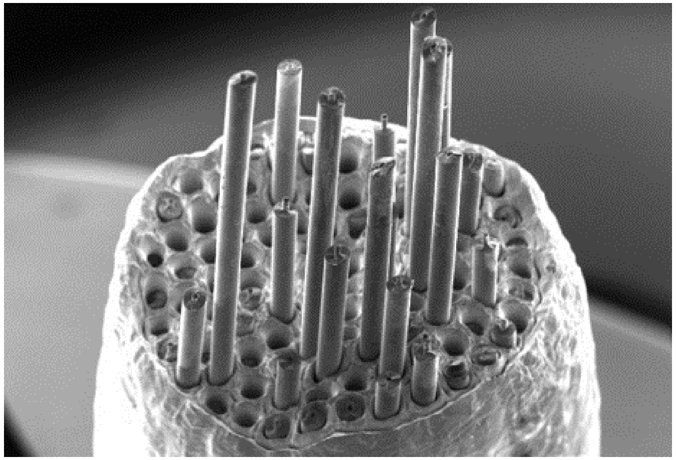
Lecture 3 Composites

1. Composites
2. Carbon fibre
3. Unidirectional carbon fibre/matrix (resin) composites
    - Properties
    - Modulus
    - Strength
    - Anisotropy
    - Toughness

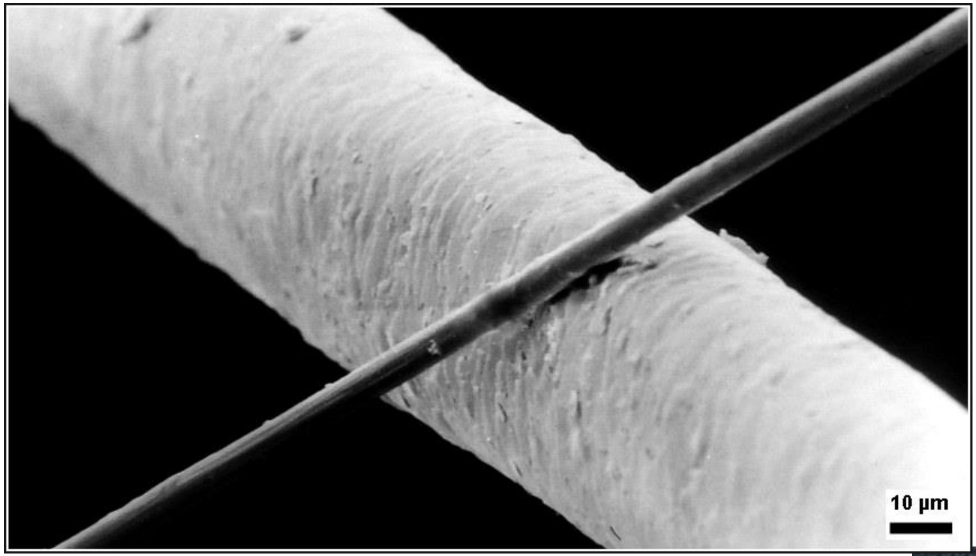
Composites



1,000
100
10
1
0.1
0.01
Good stiffness-to-weight ratio
SPECIFIC STIFFNESS (MNm/kg)
Poor stiffness-to-weight ratio
Metals and alloys
Glasses
CFRP
GFRP
Ceramics
Composites
Porous Ceramics
Wood and wood products
Polymers
Foams
Rubbers
Ceramics: chart shows compressive strength, tensile strength typically 10% of compressive
Other materials: strength in tension/compression
1
10
100
1,000
10,00
Poor strength-to-weight ratio
SPECIFIC STRENGTH (kNm/kg)
Good strength-to-weight ratio



Carbon fibre



- Hard to make bulk strong carbon
- Easy to make high quality fibre

- Fibre strong in tension
- Weave fibre into fibric for mass use

Unidirectional carbon fibre/matrix (resin) composites

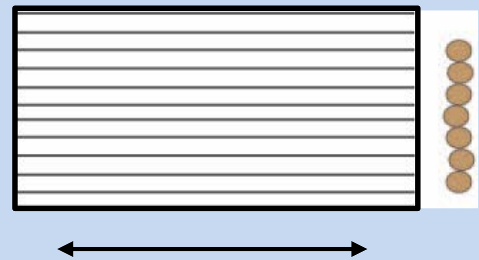
- Fibre provides strength and stiffness
- Resin provides protection (wear, chemical) and holds shape

Volume fraction of composites $V_c = 1$

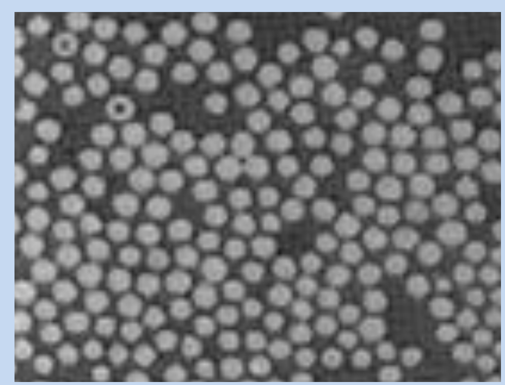
Volume fraction of fibres $V_f = \mathcal{V}_f / \mathcal{V}_c$

Volume fraction of matrix $V_m = 1 - V_f$

volume of composites



Fibre direction



c = composite
f = fibre
m = matrix

Properties

Expect to see volume fraction dependent properties

E.g. Carbon fibre-epoxy

$$\rho_c = V_f \rho_f + (1 - V_f)\rho_m$$

volume fraction of fibres

2.2 GPa

390 GPa

$$\sigma_f = E_f \varepsilon_f$$

0.0056

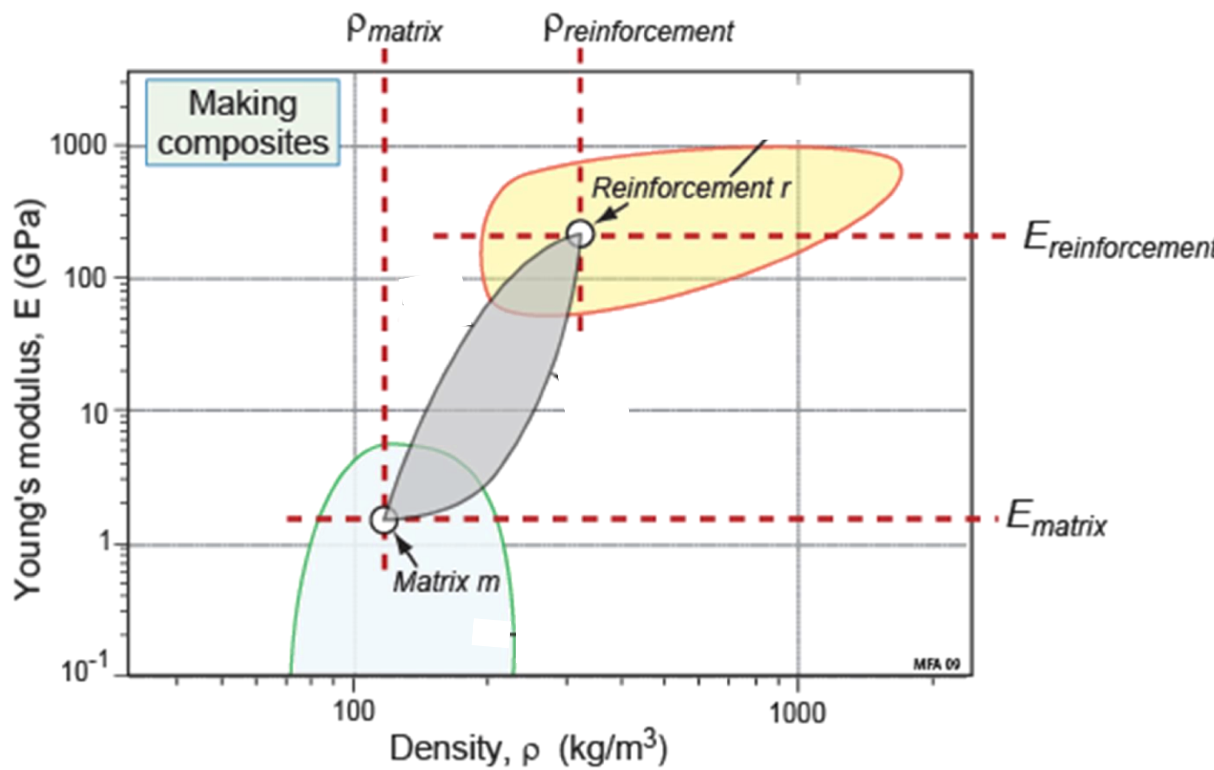
0.06 GPa

3.5 GPa

$$\sigma_m = E_m \varepsilon_m$$

0.017

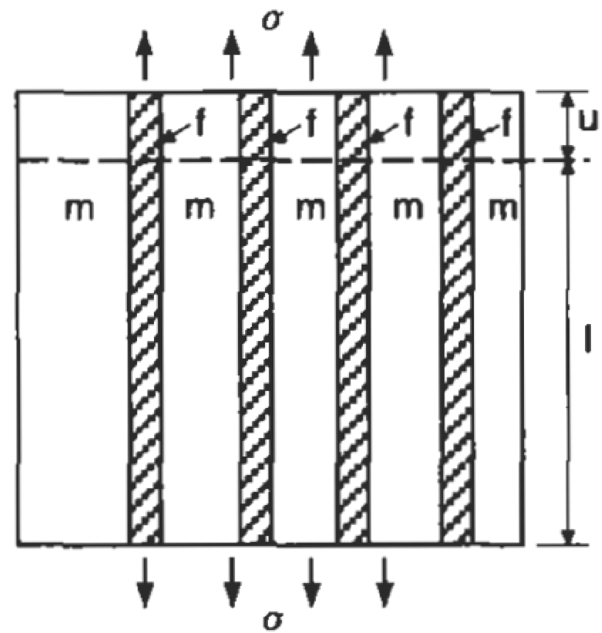
c = composite
f = fibre
m = matrix

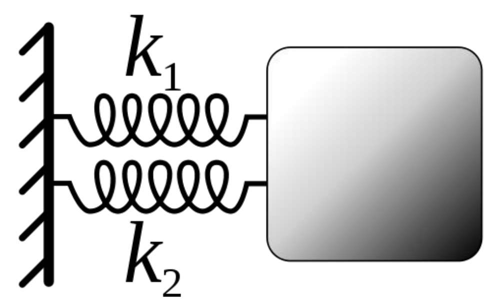


Modulus- parallel with fibres

- Same **strain** in both components
  - $\varepsilon_c = \varepsilon_f = \varepsilon_m$
  - Otherwise continuity breaks
- Fibre higher modulus
  - Same strain, high E = high fibre stress

c = composite
f = fibre
m = matrix

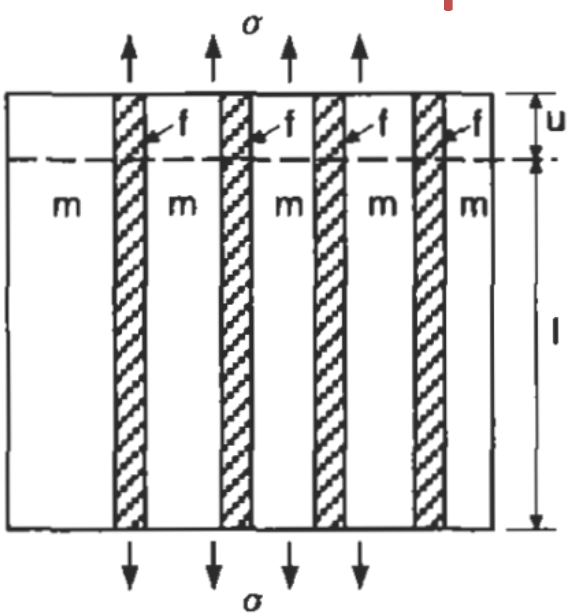




$$\varepsilon_c = \varepsilon_f = \varepsilon_m$$

How to get E_c?

Modulus- parallel with fibres



c = composite
f = fibre
m = matrix

$$F_c = F_f + F_m \quad (1)$$

$$\varepsilon_c = \varepsilon_f = \varepsilon_m \quad (2)$$

$$(1) \xrightarrow{F=\sigma A} \sigma_c A_c = \sigma_f A_f + \sigma_m A_m \quad (3)$$

volume of composites

$$\left.\begin{aligned} \mathcal{V}_c &= lA_c \\ \mathcal{V}_f &= lA_f \\ \mathcal{V}_m &= lA_m \end{aligned}\right\} (5)$$

$$(3) \xrightarrow{\sigma=E\varepsilon\ +(2)} E_c A_c = E_f A_f + E_m A_m \quad (4)$$

$$(4) \xrightarrow{(5)} E_c \mathcal{V}_c = \mathcal{V}_f E_f + \mathcal{V}_m E_m \quad (6)$$

$$(6) \xrightarrow[V_m=\mathcal{V}_m/\mathcal{V}_c]{V_f=\mathcal{V}_f/\mathcal{V}_c} \boxed{E_c = V_f E_f + V_m E_m}$$

volume fraction of fibres

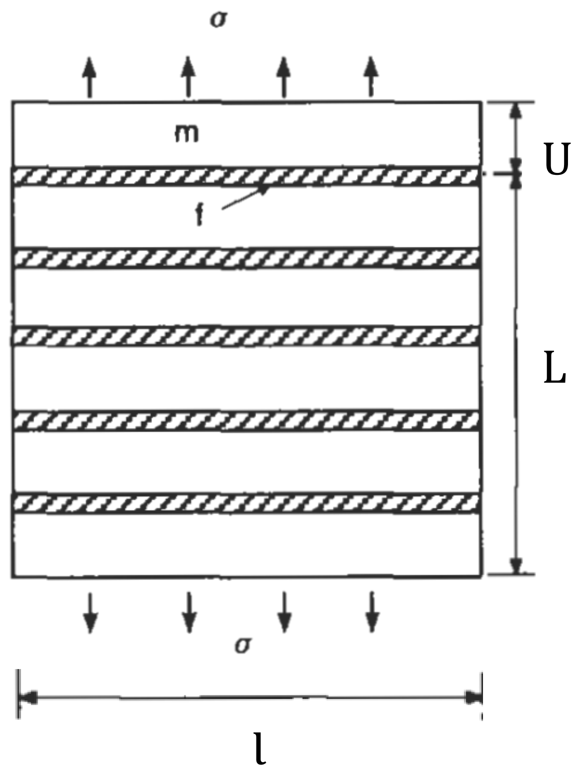
$$\boxed{V_m = 1 - V_f}$$

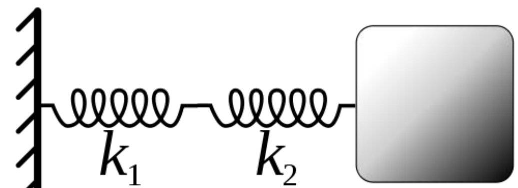
Modulus- perpendicular to fibres

- Same **stress (σ)** in both components
  - $\sigma_c = \sigma_f = \sigma_m$
  - No need for continuity
- Constant length (l) and thickness (t)
- Strain function of E
  - Matrix: low E, high strain
  - Fibre: high E, low strain
- Fibres provide no restraint on matrix strain
  - limited reinforcement

How to get E_c?

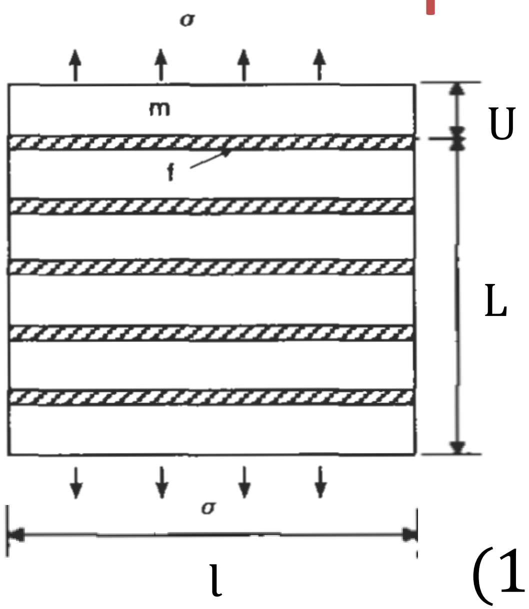
c = composite
f = fibre
m = matrix





$$\varepsilon_c = \varepsilon_f + \varepsilon_m$$

Modulus- perpendicular to fibres



Overall displacement of composites

$$U_c = U_f + U_m \qquad (1)$$

$$\sigma_c = \sigma_f = \sigma_m \qquad (2)$$

$$(1) \xrightarrow{U = L\varepsilon} L_c\varepsilon_c = L_f\varepsilon_f + L_m\varepsilon_m \qquad (3)$$

$$(3) \xrightarrow{\varepsilon=\sigma/E+(2)} L_c/E_c = L_f/E_f + L_m/E_m \qquad (4)$$

volume of composites

$$\left.\begin{aligned} \mathcal{V}_c &= tlL_c \\ \mathcal{V}_f &= tlL_f \\ \mathcal{V}_m &= tlL_m \end{aligned}\right\} (5)$$

$$(4) \xrightarrow{(5)} \mathcal{V}_c/E_c = \mathcal{V}_f/E_f + \mathcal{V}_m/E_m \qquad (6)$$

$$(6) \xrightarrow[V_m = \mathcal{V}_m/\mathcal{V}_c]{V_f = \mathcal{V}_f/\mathcal{V}_c} \boxed{1/E_c = V_f/E_f + V_m/E_m}$$

$$\boxed{V_m = 1 - V_f}$$

Modulus of unidirectional composites

c = composite
f = fibre
m = matrix

Parallel

$$E_c = V_f E_f + (1 - V_f) E_m$$

Conditions:

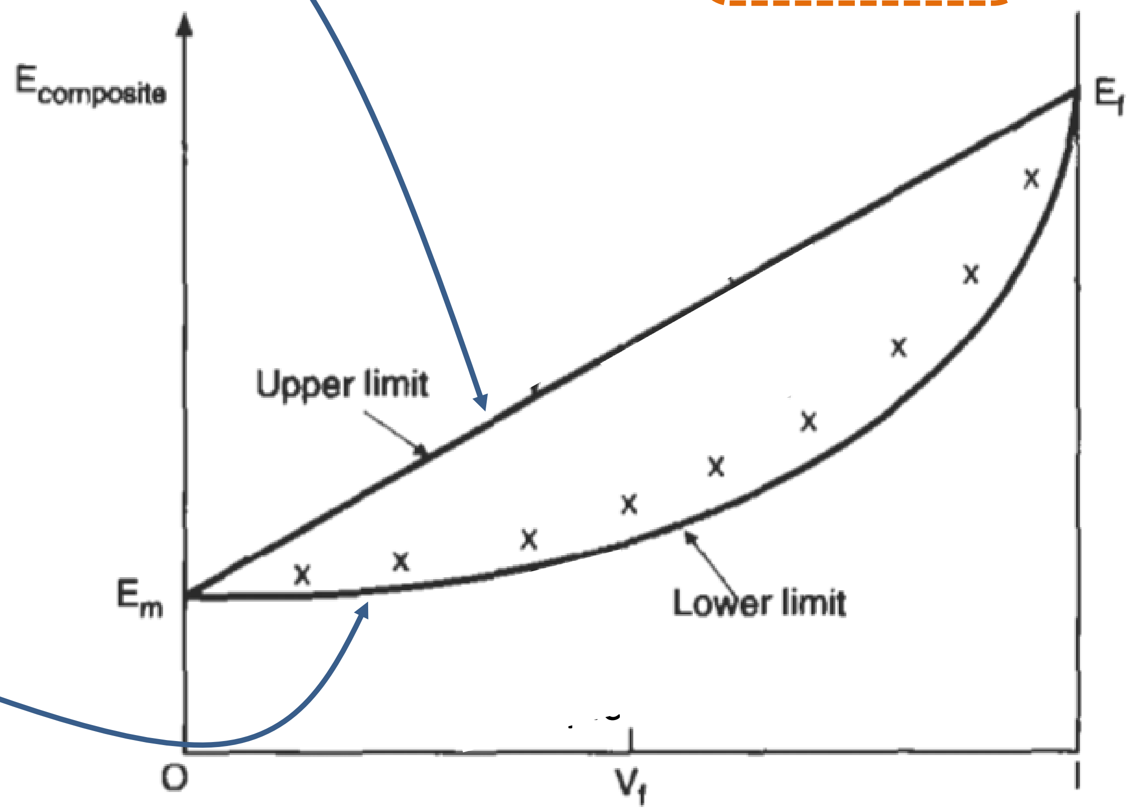
- Same strain ($\varepsilon_c = \varepsilon_f = \varepsilon_m$)

Perpendicular

$$\frac{1}{E_c} = \frac{V_f}{E_f} + \frac{(1 - V_f)}{E_m}$$

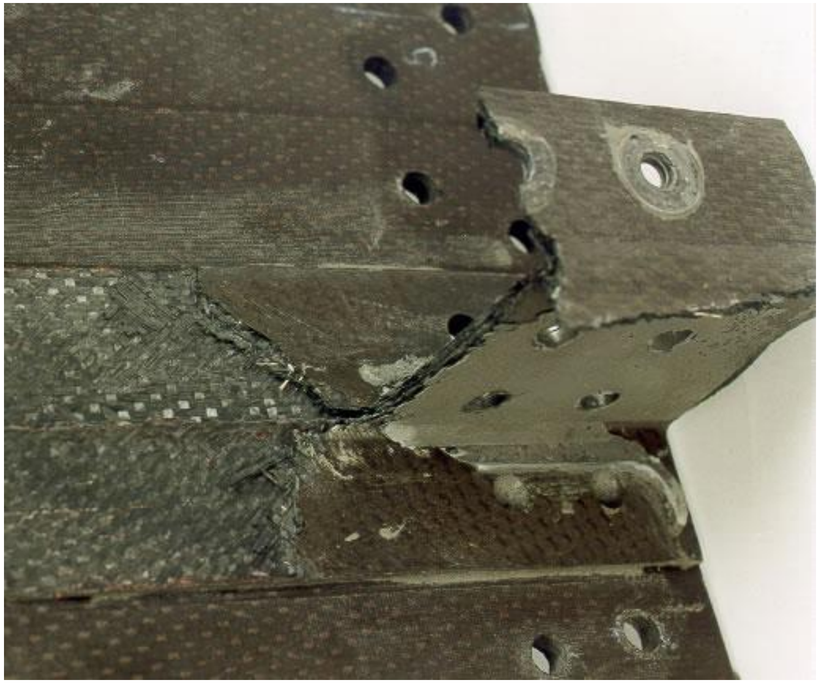
Conditions:

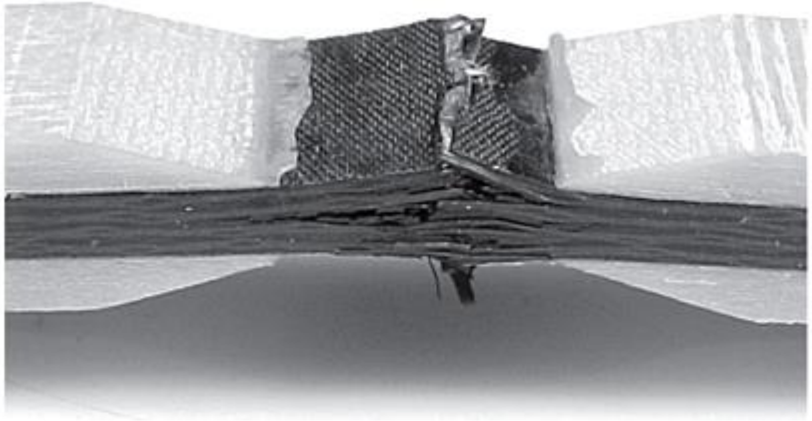
- Same stress ($\sigma_c = \sigma_f = \sigma_m$)



Strength

- Much more complex than modulus
- Multiple failure mechanisms
- Hard to predict compared to metals
  - Major limit on uptake





Strength - parallel with fibres

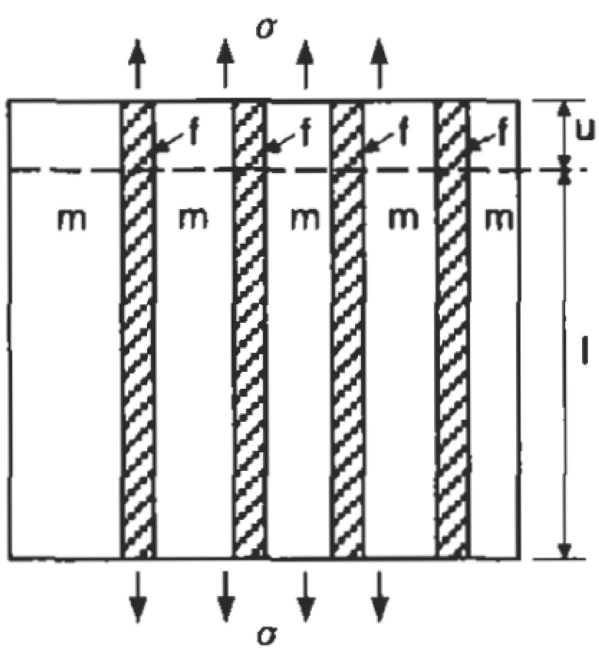
Assume linear elastic fibres and matrix

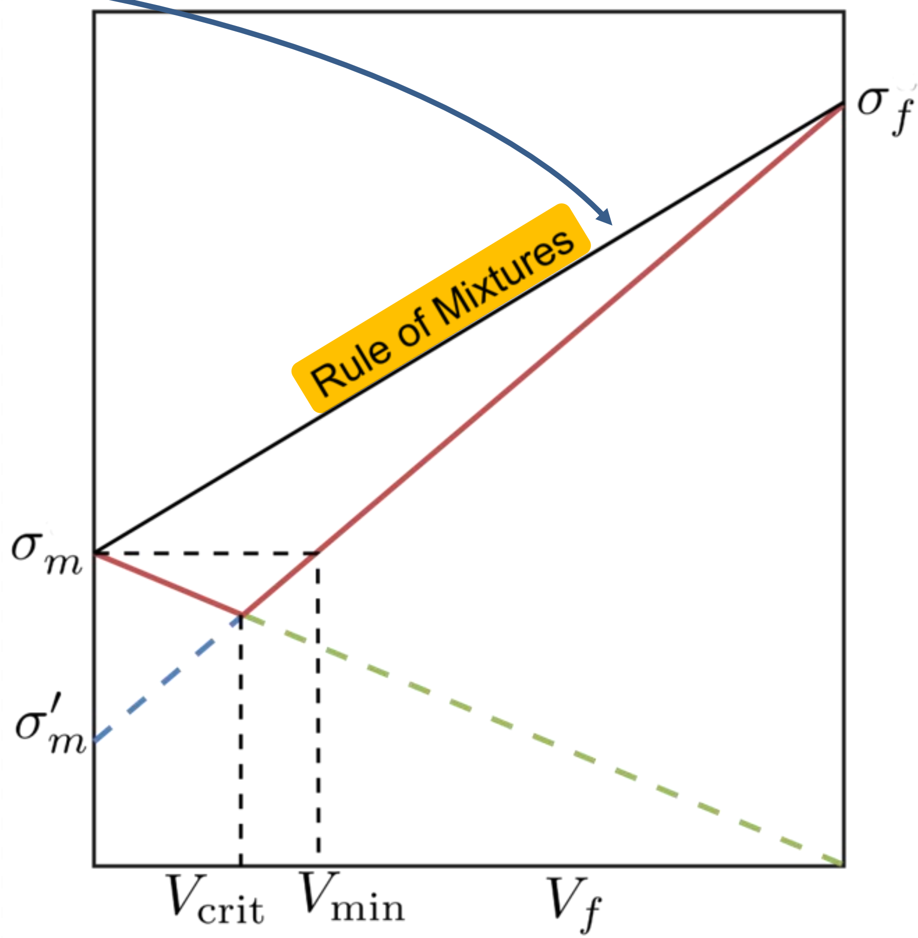
$$\sigma_c = V_f \sigma_f + (1 - V_f)\sigma_m$$

c = composite
f = fibre
m = matrix

Conditions:

- Loads are parallel with fibres
- Same strain ($\varepsilon_c = \varepsilon_f = \varepsilon_m$)





Strength – parallel with fibres

- High fibre fraction
  - Controlled by stiff fibres
  - Fibres fail, matrix fails
  - Reduced matrix contribution

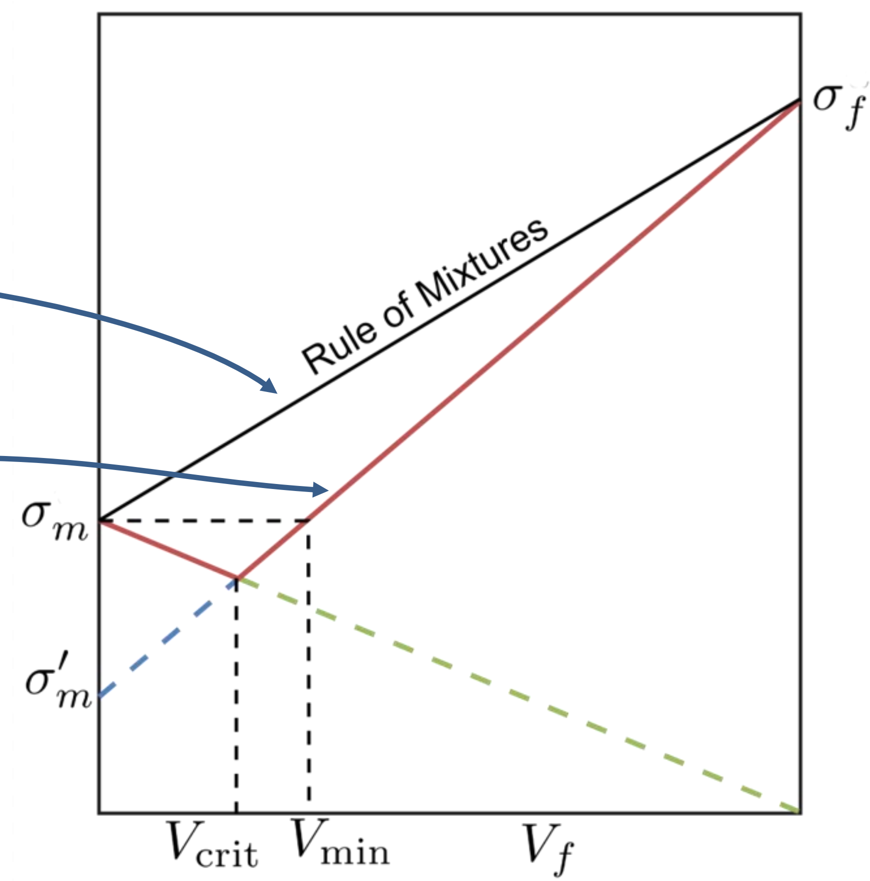
$$\varepsilon'_m = \varepsilon_f \longrightarrow \sigma_m{}' = E_m \varepsilon_f$$

$$\sigma_c = V_f \sigma_f + (1 - V_f)\sigma_m$$

$$\sigma_c = V_f \sigma_f + (1 - V_f)\sigma'_m$$

Conditions:

- Loads are parallel with fibres
- High fibre fraction
- Same strain ($\varepsilon_c = \varepsilon_f = \varepsilon'_m \leq \varepsilon_m$)



Strength - parallel with fibres

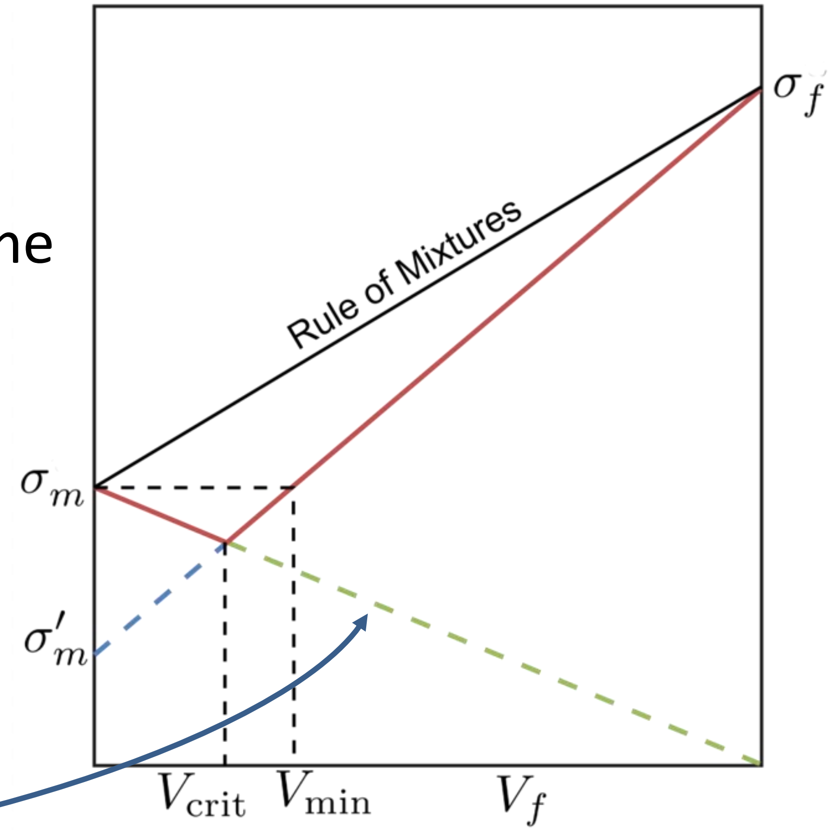
- Low fibre fraction
  - Controlled by matrix
  - Fibres already fractured by the time the matrix reaches failure strain

$$\varepsilon_c \approx \varepsilon_m$$

  - Fibres don't contribute

$$\sigma_c = V_f \sigma_f + (1 - V_f)\sigma_m$$

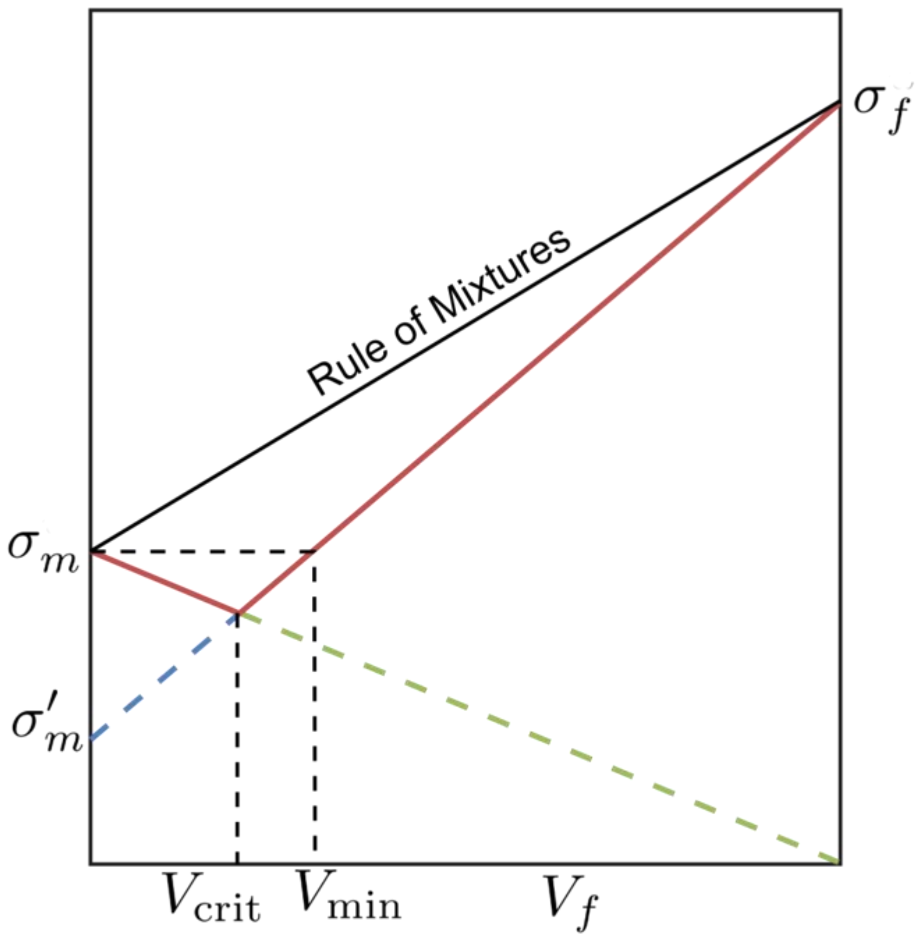
$$\boxed{\sigma_c = (1 - V_f)\sigma_m}$$



Conditions:

- Loads are parallel with fibres
- Low fibre fraction (V_f << 1- V_f)
- Same strain ($\varepsilon_c \approx \varepsilon_m$>> ε_f)

Strength of composites



- Less benefit than expected
- Need minimum V_f to improve compared to matrix
- Actually compromise strength prior to V_{min}
  - Very low for strong fibres/weak matrix
  - Worst strength at V_{crit}

Example

- Assume in a fibre/matrix composites, E_f = 350 GPa, ε_f = 0.006, E_m =12 GPa and ε_m =0.03. Please use the ‘rule of mixtures’ to calculate the ratio of matrix stress to composite stress (σ_m / σ_c) *for* V_f =12 % and V_f =40 %.

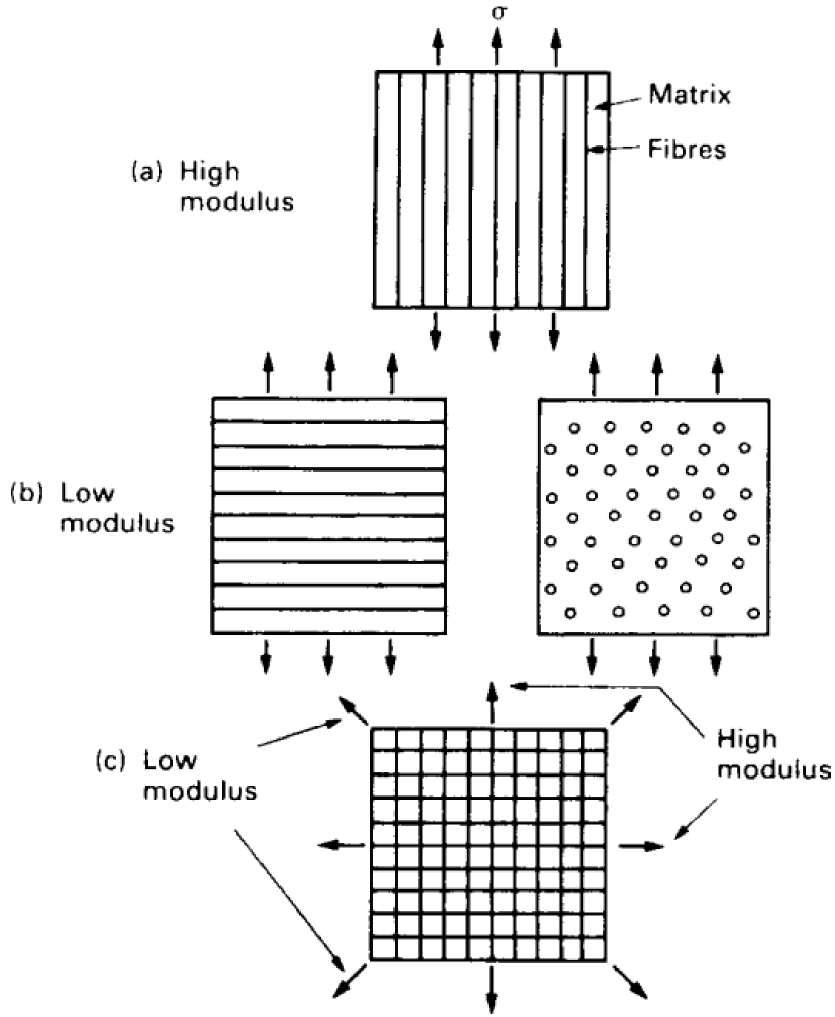
$$\sigma_c = V_f \sigma_f + (1 - V_f)\sigma_m$$

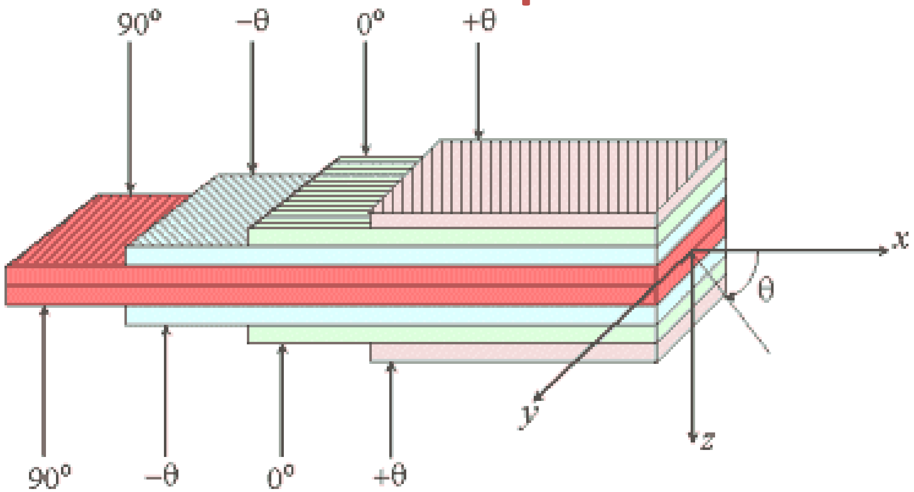
$$\xrightarrow{\sigma = E\varepsilon} \begin{cases} \sigma_c = V_f E_f \varepsilon_f + (1 - V_f) E_m \varepsilon_m & (1) \\ \sigma_m = E_m \varepsilon_m & (2) \end{cases}$$

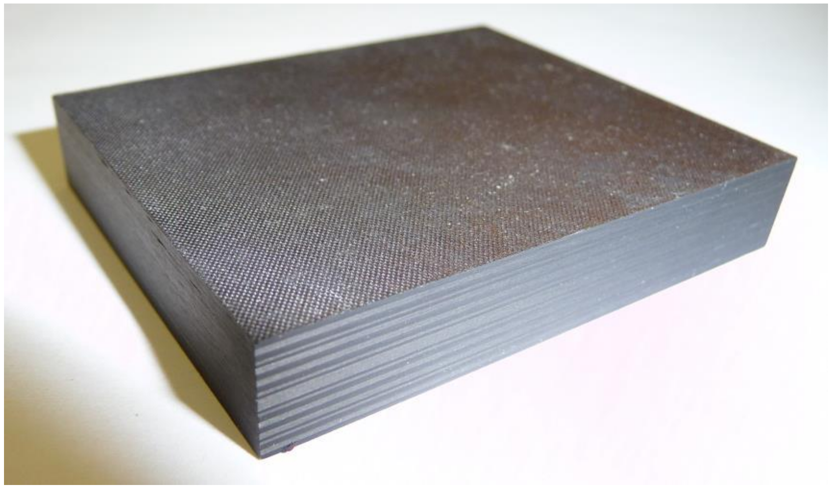
<u>After being used in composites</u>, actually $\varepsilon_c = \varepsilon_m = \varepsilon_f$

$$\xrightarrow[\varepsilon_c = \varepsilon_m = \varepsilon_f]{(2)/(1)} \sigma_m / \sigma_c = \frac{E_m}{V_f E_f + (1 - V_f) E_m} = 0.23\ (12\ \%)\ \text{or}\ 0.08\ (40\ \%)$$

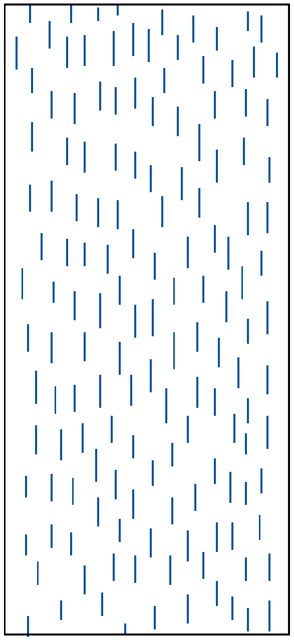
Anisotropy of continuous fibre composites



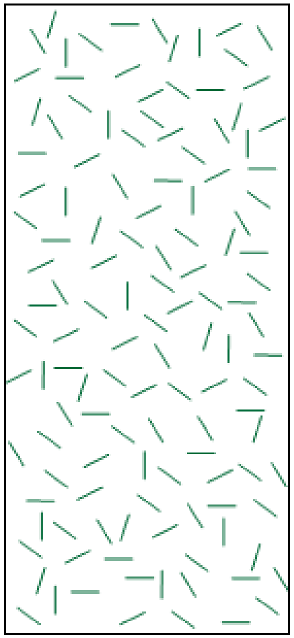




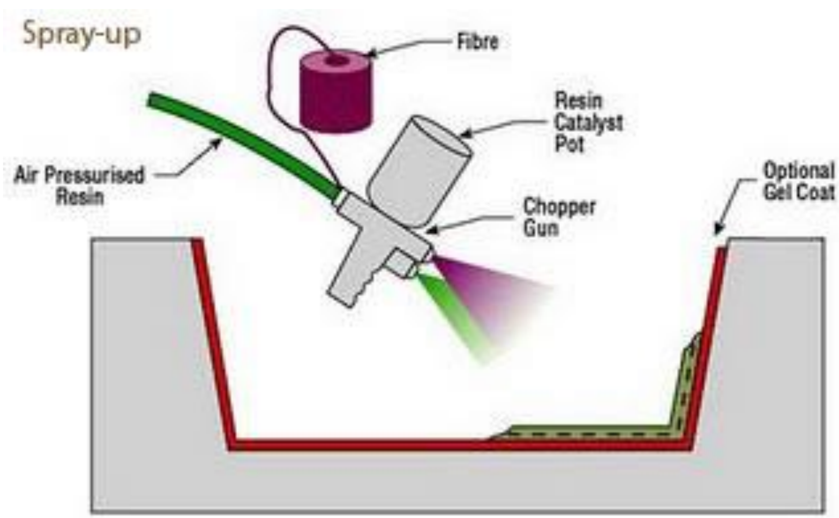
Anisotropy of short fibre composites

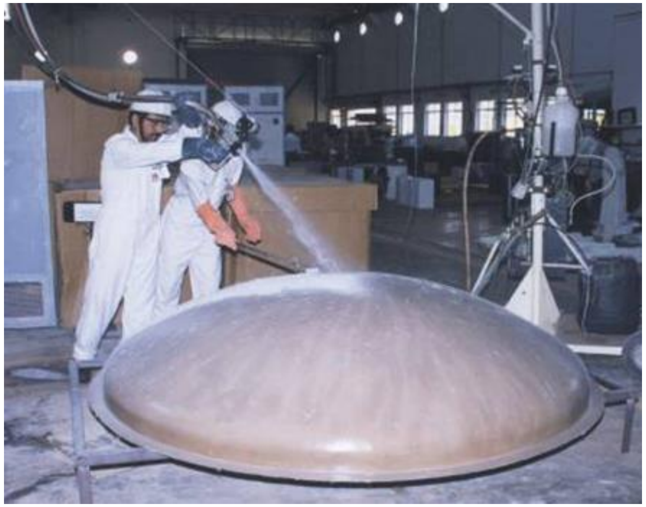


Aligned short fibres
High *E*
High anisotropy



Random short fibres
Low *E*
Low anisotropy



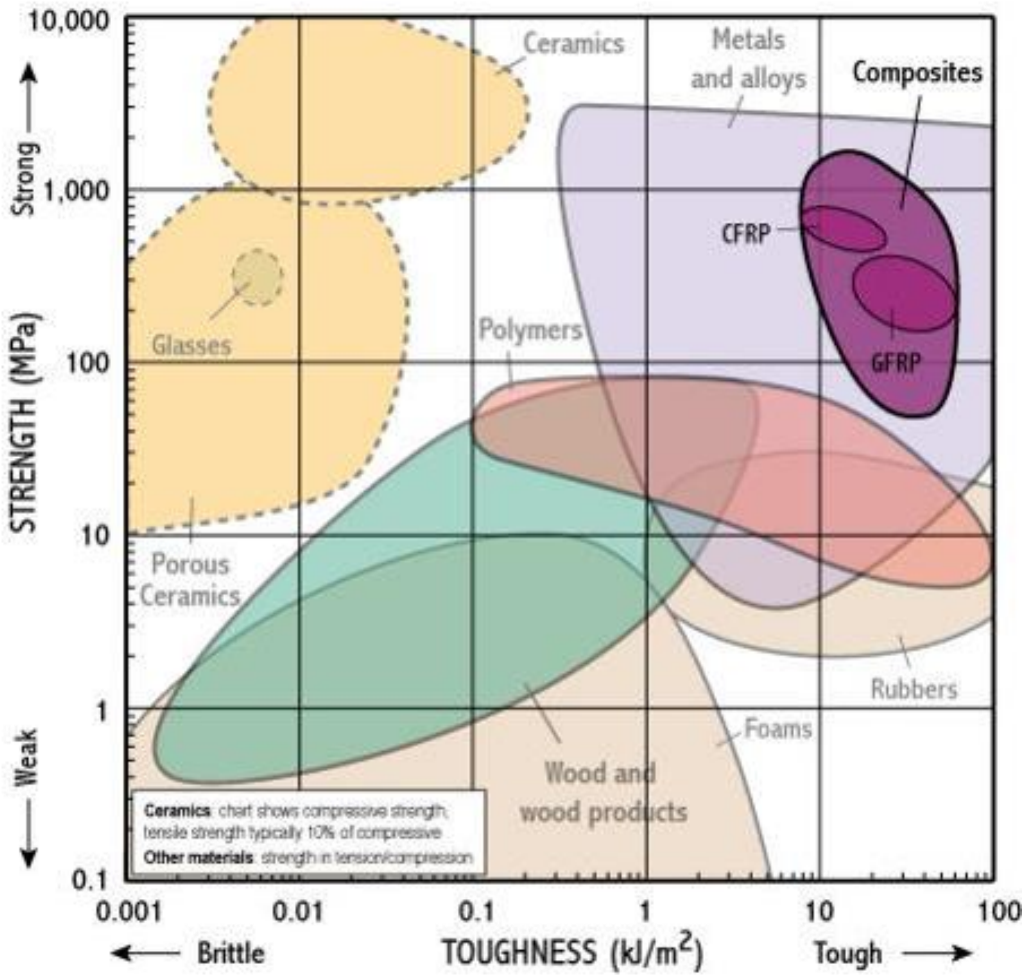


Anisotropy

Opportunity to customise modulus to be high in specified directions

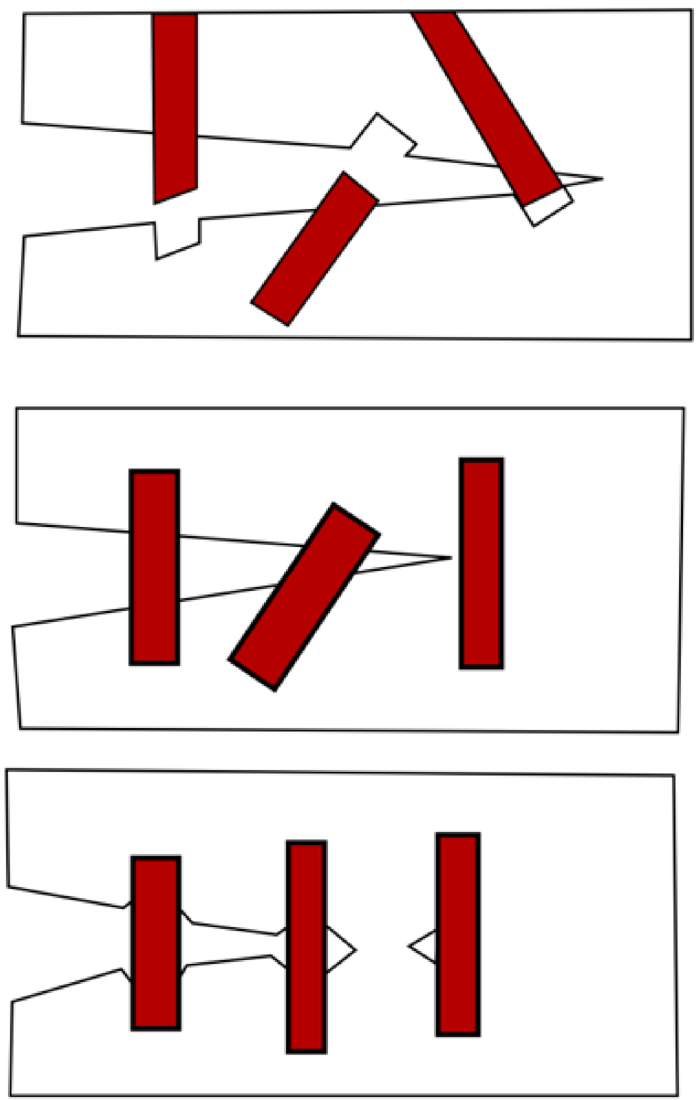
Potential for failure due to unexpected loading!

Toughness



- Composites give E_c and σ_c like ceramic but without the brittleness (not quite)

Toughness of composites



- Fibre pull out
  - Drag fibres from matrix
- Crack bridging
  - Fibres hold crack together and prevent it growing
- Deflection
  - Fibres get in way of crack

Summary

- Composites (and other hybrids) get strengths of both phases and mitigate weaknesses of both
- Potential game changer in design
  - Not properly exploited?
- Introduce new set of complications
  - Either component can fail
  - Multiple failure modes
  - New failure modes
  - Anisotropy in modulus and strength