Revision

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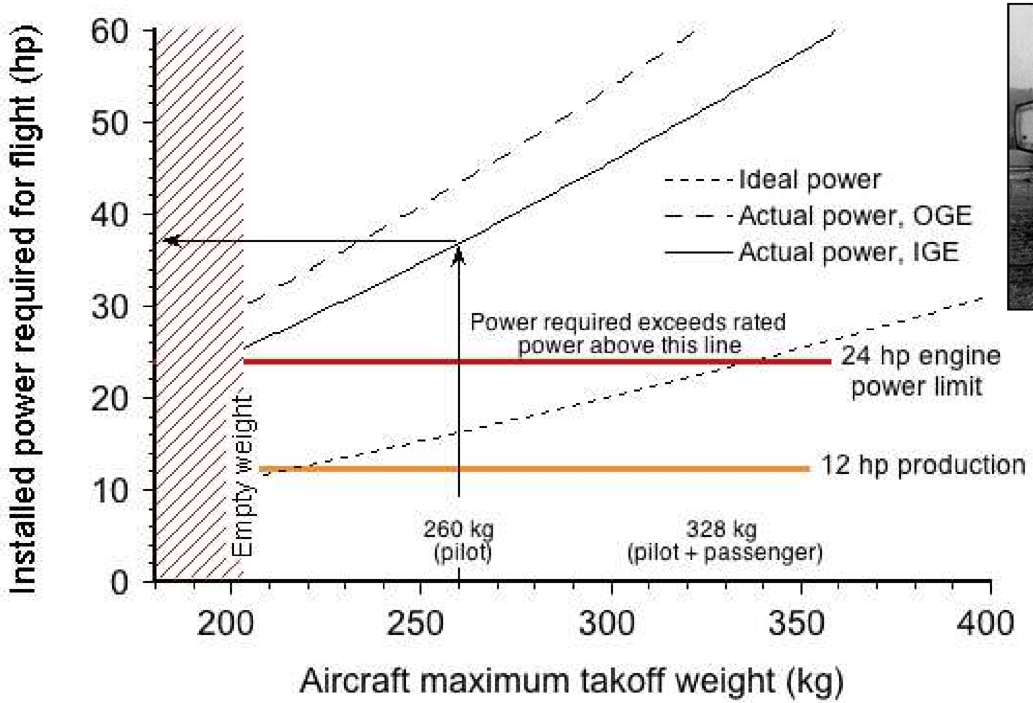
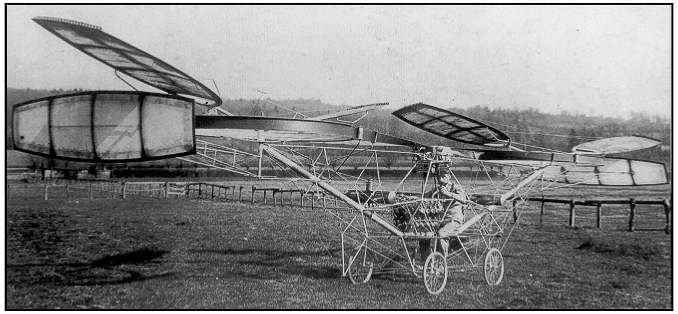
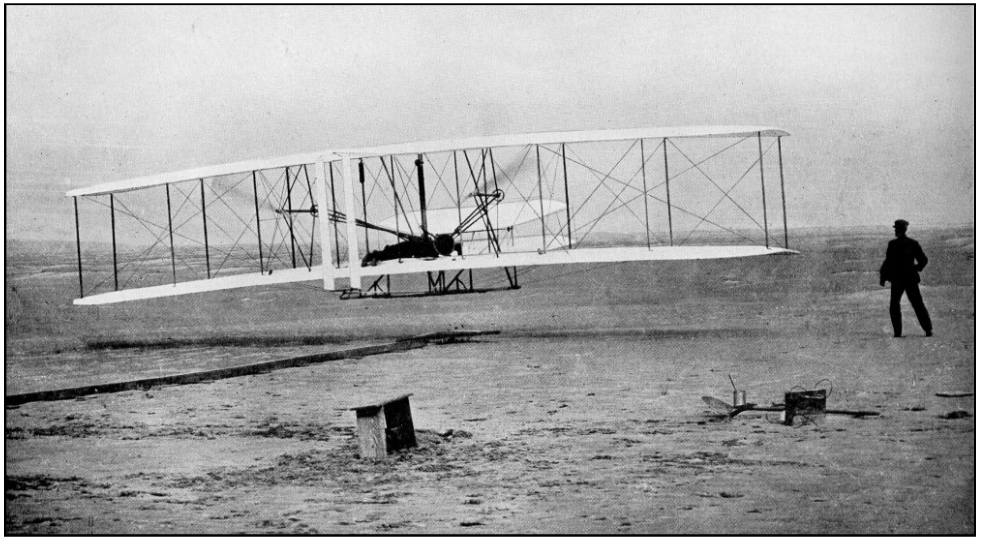
Could It Fly?

The following chart was developed by Dr. Gordon Leishman, a leading helicopter aerodynamicist, to analyze the lifting capability of the Cornu helicopter as described by Cornu himself with the actual power available from his engine.

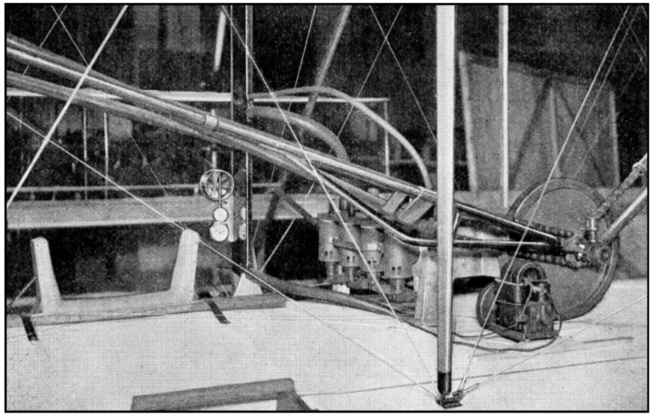




Predictions of power requirements for flight for Cornu's piloted machine, showing that with a 12 to 24 hp Antoinette engine, a free flight, even in ground effect, was a highly improbable event.

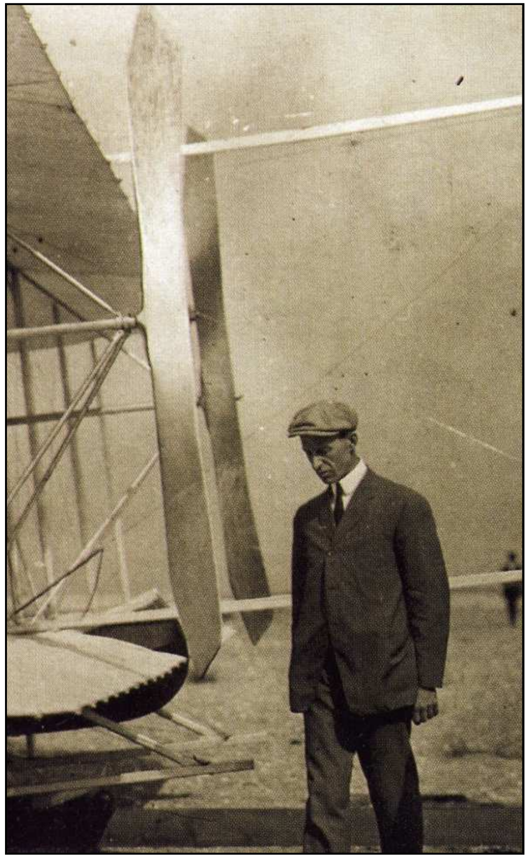


The aircraft weighed 1150 lbs fully laden



24 bhp engine weighs 240 lbs

Wright Flyer



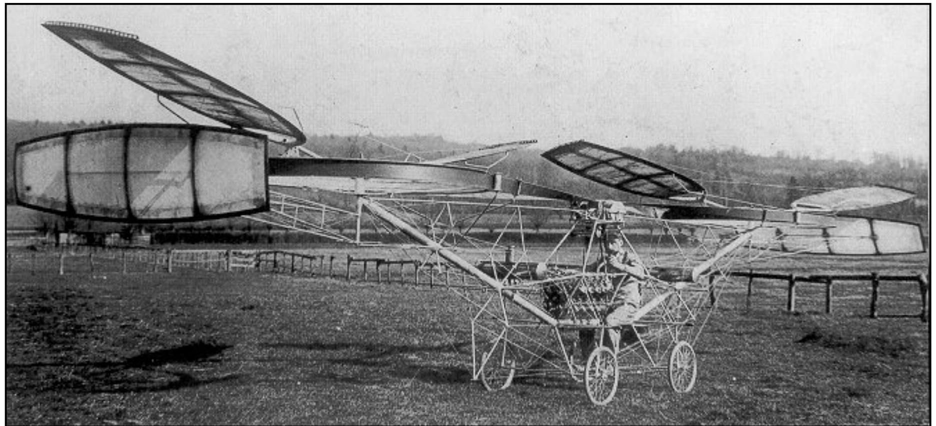
Propellers - 66% efficient

$$\eta_p = \frac{TV}{P}$$ So that at 30knots, L / D = 6

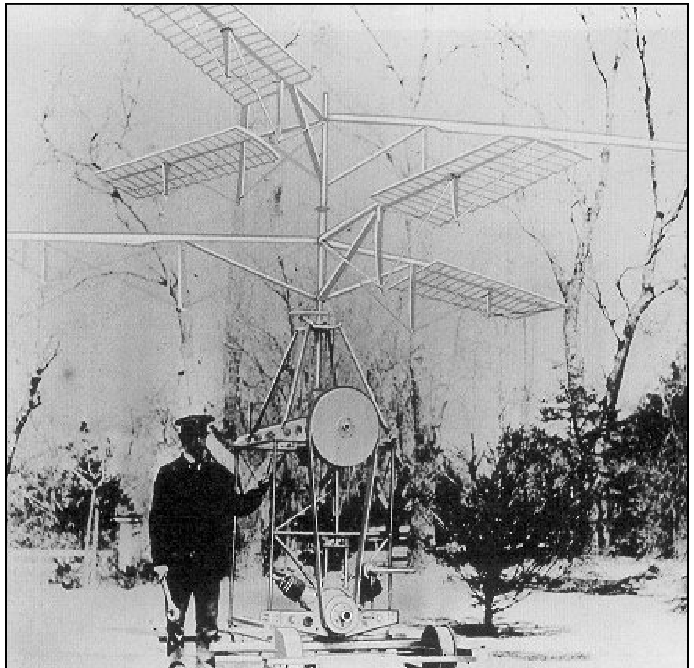
L/D = W/T, so for the Wright Flyer the propeller thrust is 1/6 the aircraft weight.

To take off vertically, Thrust > Weight. This requires thrust > 6 x Wright Flyer's thrust for no extra weight - clearly these early attempts could never succeed.

Some just left the ground but had no control and were always tethered.



Cornu 1907 first manned aircraft to hover



Sikorsky 1909 (unsuccessful)

So the first rotary winged aircraft that flew (in the generally accepted sense of this word) was in 1923 and it did not have a powered rotor !

Cierva C8 Autogiro™ (1926)

Helicopters and Autogyros – A Brief History

Breguet-Dorand Coaxial Helicopter

1936 – Closed circuit flight of 27 mile which took just over 1 hour, flying at 500ft.

Focke-Achgelis Fa 61

1936 – Closed circuit of 50 miles at speed of 76 mph at 8000ft.

Invention of the helicopter is often, wrongly, attributed to Igor Sikorsky. However, he did develop his own machine which set the mould for most designs thereafter.

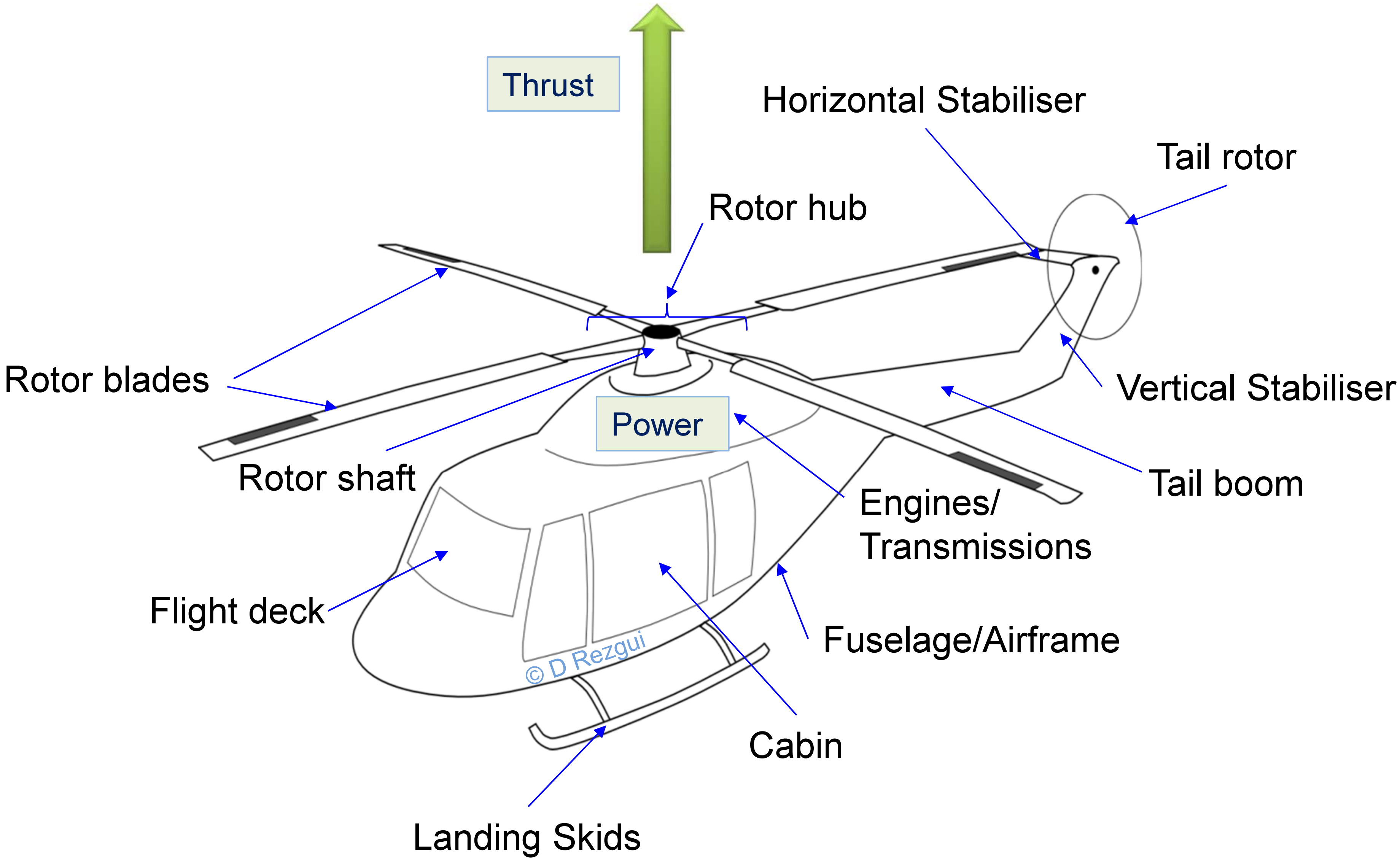
VS-300-C-5a with Single Main Rotor

Sikorsky's developing VS-300-D-1 now becomes the standard "penny farthing" configuration.

XR-4a Collective and Cyclic pitch control by virtue of a swash plate.

Main Helicopter Parts

(Penny Farthing Configuration)



Actuator Disc (Momentum) Theory

The Lifting Rotor in it's most simplistic form is a Propeller.

Mechanical energy (in the form of rotating blades) is used to accelerate (*a*) a mass (*m*) of air.

Newton's law (every action has a reaction), states ***F=ma***, where ***F***, is the rotor thrust (***T***).

Applying Bernoulli's equation:

$$H_0 = P_0 + {}^1/_2\,\rho V^2 = P_1 + {}^1/_2\,\rho(V+v)^2$$
$$H_1 = P_0 + {}^1/_2\,\rho(V+v_1)^2 = P_1 + P' + {}^1/_2\,\rho(V+v)^2$$

Subtracting H_0 from H_1 results in

$$H_1 - H_0 = {}^1/_2\,\rho\,(2Vv_1 + v_1^2) = P'$$

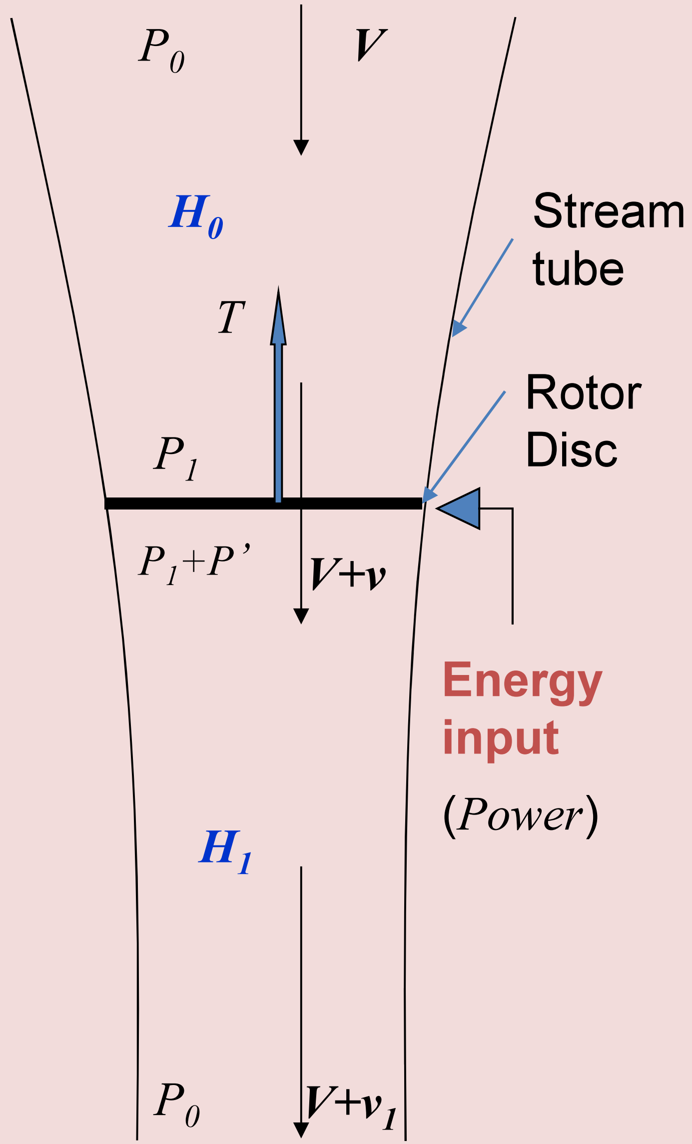
However, the Thrust=change of axial momentum per unit time $\dfrac{T}{A} = P' = \rho(V+v)v_1$

Where ρ is the air density and A is the rotor disc area

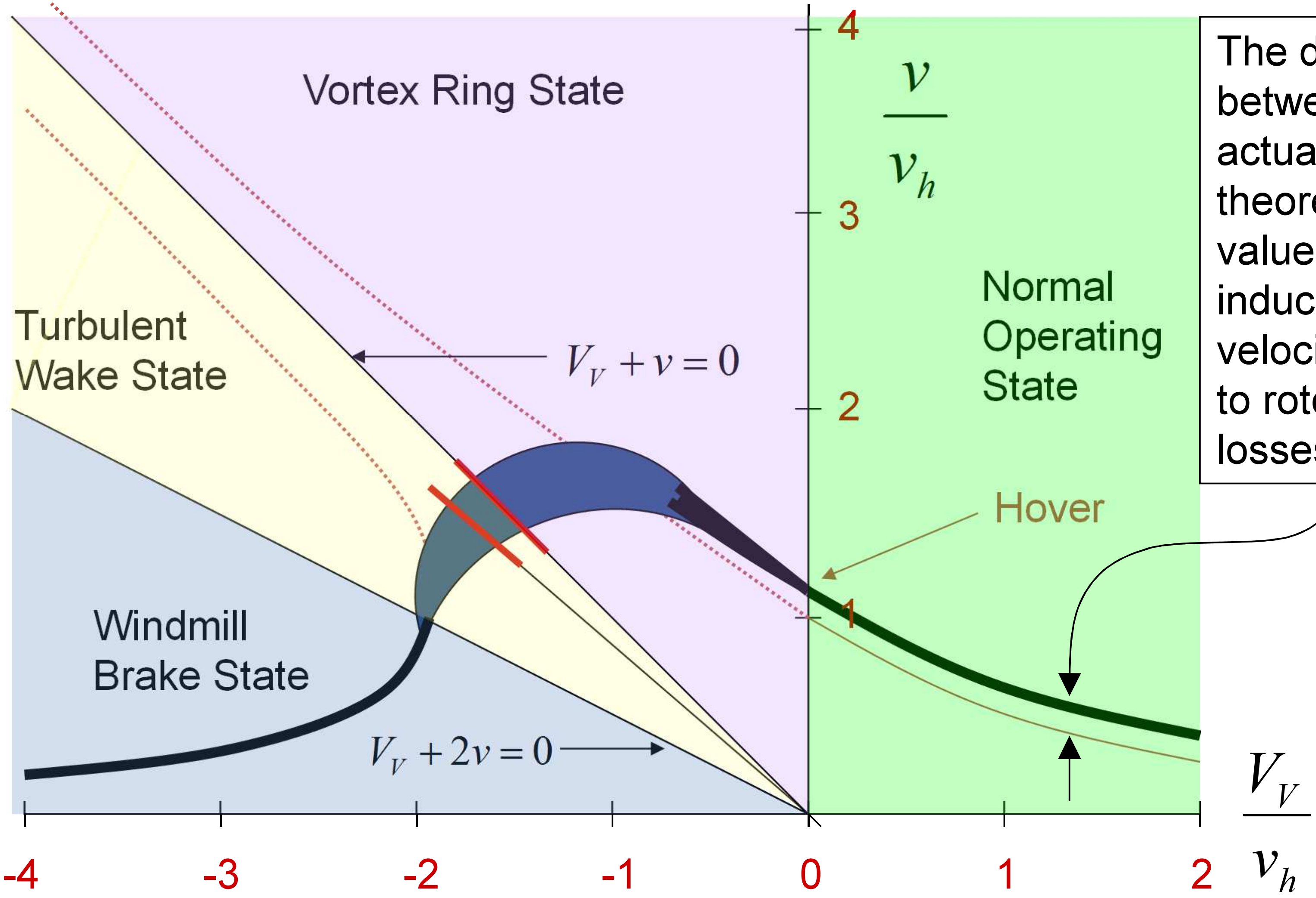
Hence $\dfrac{v_1}{2} = v \quad or \quad v_1 = 2v$

Thrust: $T = 2\rho A(V+v)v$

Power: $P = T(V+v)$



Remembering that $P = T(V + v)$, then at $(V + v) = 0, T \neq 0, P = 0$



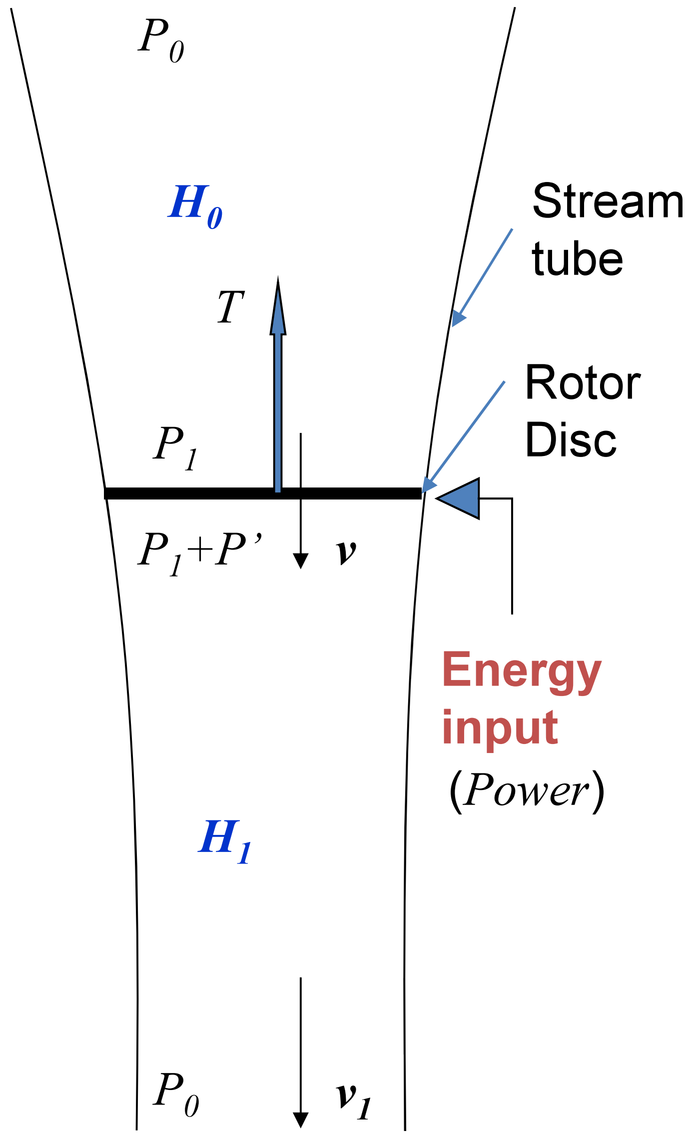
The difference between actual and theoretical values of the induced flow velocity is due to rotor blade losses.

Momentum Theory in Hover

Or a lifting rotor
in **hover**, when
the onset
velocity $\boldsymbol{V=0}$

$$\boxed{v_h = \sqrt{\frac{T}{2\rho A}}}$$

$P_h = T v_h$ Hence $\boxed{P_h = \frac{T^{3/2}}{\sqrt{2\rho A}}}$



Rotor Efficiency in the Hover

The measure of propeller efficiency, a "**Figure of Merit**" is used:

$$\boxed{\eta_r = FoM = Tv/P}$$

$$= \frac{T}{P}\sqrt{\frac{T}{2\rho A}}$$

$$= \frac{T}{P}\frac{1}{\sqrt{2}}\sqrt{\frac{T}{\rho \pi R^2}}$$

It should be noted that the FoM is inversely proportional to the rotor diameter and comparative studies therefore should be limited to rotors of the same diameter.

Rotor Efficiency in the Hover

$$\eta_r = FoM = Tv/P = \frac{T}{P}\sqrt{\frac{T}{2\rho A}}$$

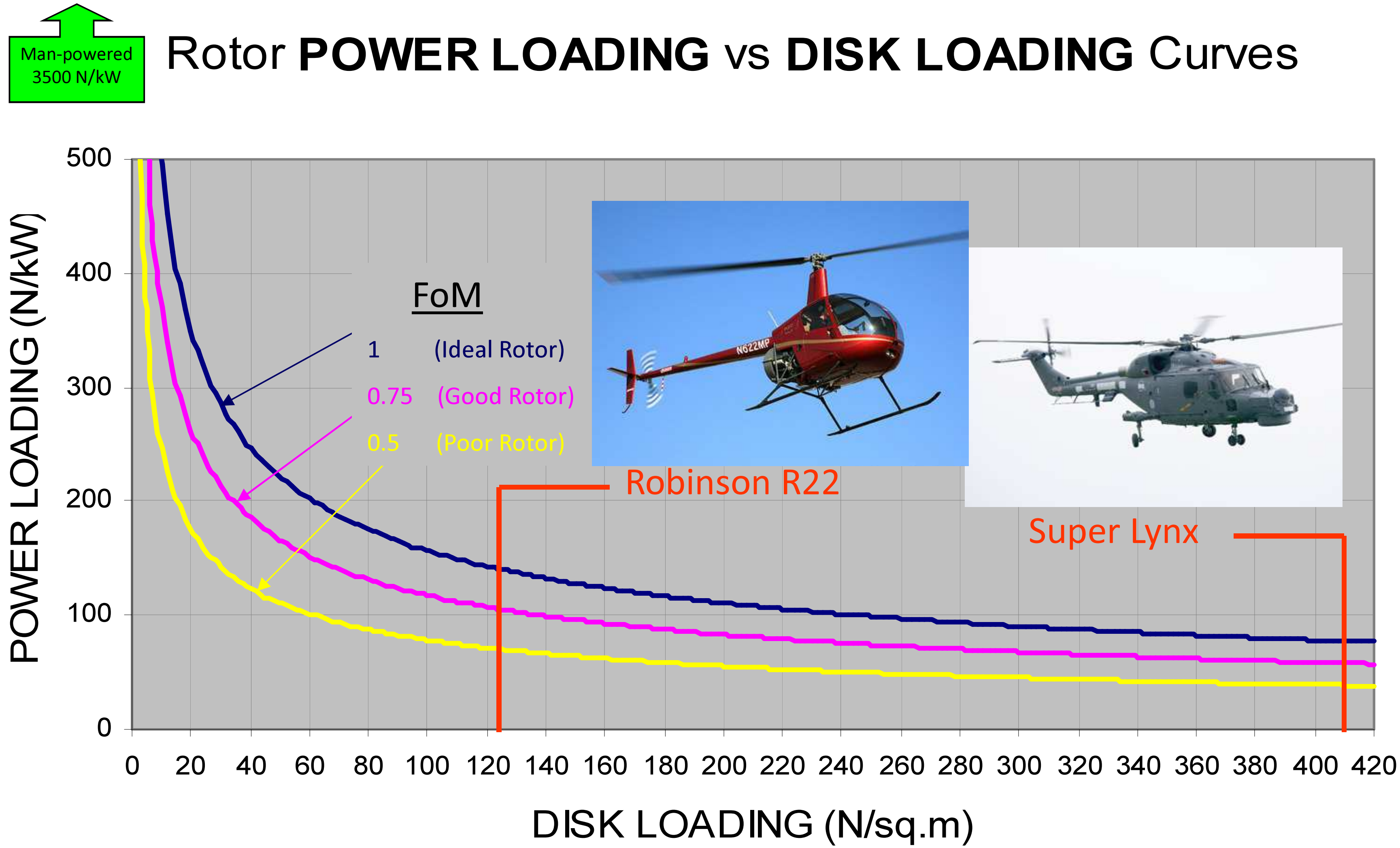
If, $\dfrac{T}{P} = \mathrm{PL}$ (known as Power Loading) and $\dfrac{T}{A} = \mathrm{DL}$ (known as Disk Loading)

then $$\mathrm{PL} = 1.565 FoM \frac{1}{\sqrt{\mathrm{DL}}}$$ This is Dimensional !

(based upon ρ =1.225 kg/m^3)

This relationship can be plotted and if the Figure of Merit is known, then for a given disk loading the power loading may be found from the graph.

Rotor Efficiency in the Hover



HELICOPTER in AUTOROTATION

In general (and this very much depends upon rotor diameter and helicopter weight),

helicopters settle down to an autorotational descent rates such that: $\dfrac{V}{v_h} \approx -1.7$

Now in a steady autorotative descent, T = thrust in the hover, so: $T = 2\rho A v_h^2$

The rotor has no net flow through it, so it can be likened to a solid disc of area A.

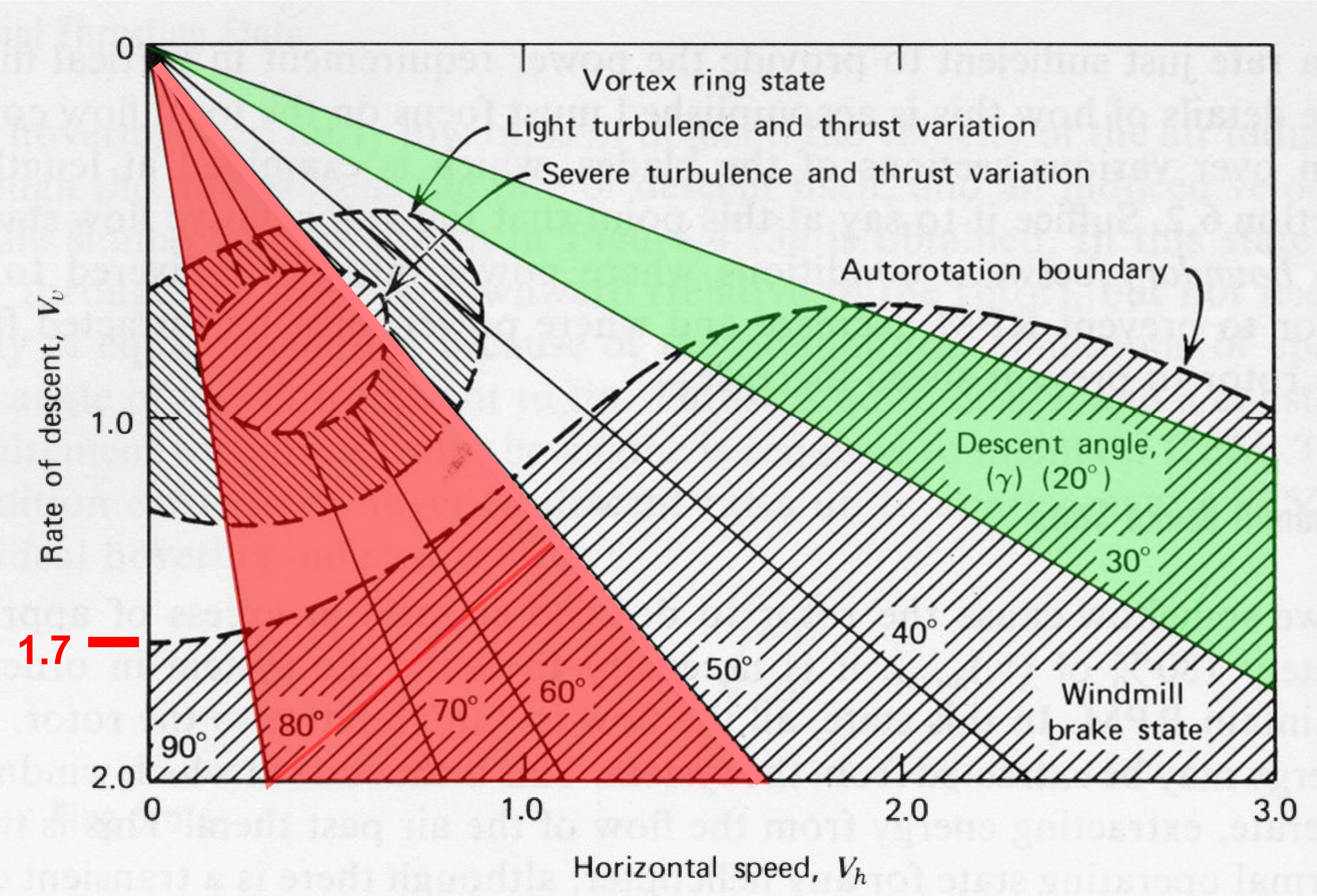
and it's Flat Plate Drag, $D = \tfrac{1}{2}\rho V^2 A C_D$

Equating the Thrust and Drag equations gives: $2\rho A v_h^2 = \tfrac{1}{2}\rho \mathrm{V}^2 A C_D$

so, $C_D = \dfrac{4}{\left(V/v_h\right)^2}$

For $V/v_h = -1.7$, $C_D = 1.38$ which is the effective drag coefficient of a parachute.

HELICOPTER in VERTICAL and FORWARD AUTOROTATION



It is important to design for low autorotation rates

Rotor Performance Coefficients

(*We refer to parameter* ΩR *as the* reference velocity *where rotational speed* Ω *is in* *rads/sec*)

It is also normal to express the forward speed (V) of the helicopter relative to the tip speed parameter ΩR and this is called the **Advance Ratio** μ .

Thus
$$\boxed{\mu = \frac{V}{\Omega R}}$$

In a similar way, the flow through the rotor (v in the hover but $V_V + v$ otherwise) is non-dimensionalised by ΩR and this is called the **inflow ratio** λ .

Thus
$$\boxed{\lambda = \frac{V_V + v}{\Omega R}}$$

Rotor Performance Coefficients

A note of caution:

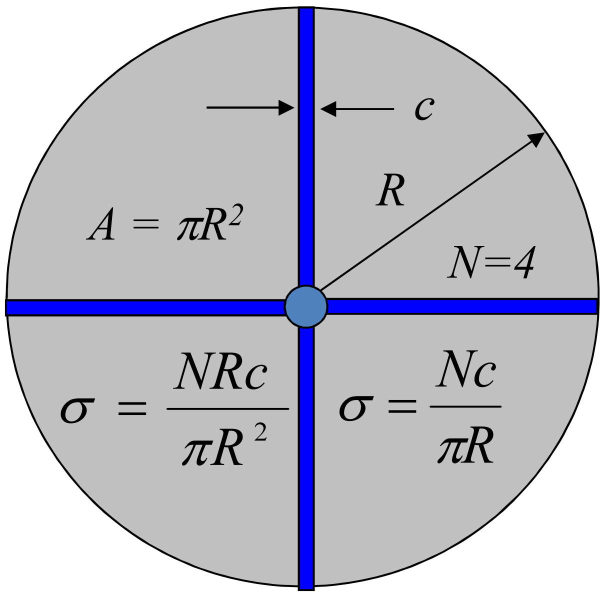
Whilst $\boxed{C_T = \dfrac{T}{\rho A(\Omega R)^2}}$ has become the accepted form,

This one is used in this course

some text books may show:

$$C_T = \frac{T}{1/2\ \rho V_T^2 A}$$ where $V_T = \Omega R$ (blade tip speed)

Or



$$C_T = \frac{T}{\rho\sigma A(\Omega R)^2}$$

Where $\boxed{\sigma = \dfrac{Nc}{\pi R}}$ (solidity, where N is number of blades)

Rotor Performance Coefficients

Similarly, Torque Coefficient

and Power Coefficient

$$\left.\boxed{\begin{aligned} C_Q &= \frac{Q}{\rho A R (\Omega R)^2} \\ C_P &= \frac{P}{\rho A (\Omega R)^3} \end{aligned}}\right\}$$

therefore $C_P = C_Q$

Rotor Performance Coefficients

The Figure of Merit as previously defined can be more conveniently expressed in terms of non-dimensional quantities using the thrust and power coefficients.

The induced velocity $$\boldsymbol{v} = \sqrt{\frac{T}{2\rho A}} = \sqrt{\frac{C_T \rho A (\Omega R)^2}{2\rho A}} = \Omega R \sqrt{\frac{C_T}{2}}$$

But since $\lambda = \dfrac{V_V + v}{\Omega R}$ then for $V_V = 0$, $\boxed{\lambda = \sqrt{\dfrac{C_T}{2}}}$

$$FoM = \frac{Tv}{P} = \frac{C_T \rho A (\Omega R)^2 v}{C_P \rho A (\Omega R)^3} = \frac{C_T}{C_P}\frac{v}{\Omega R} = \frac{C_T}{C_P}\lambda$$

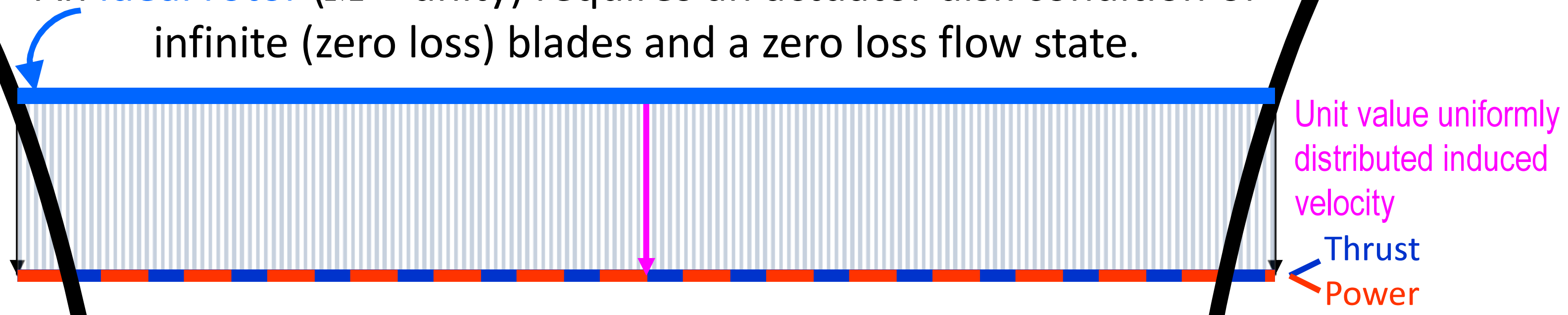
$$= \frac{1}{\sqrt{2}}\frac{C_T^{3/2}}{C_P}$$

Or more commonly……………………………

$$\boxed{FoM = 0.707\,\frac{C_T^{3/2}}{C_Q}}$$

Maximising the Figure of Merit

An ideal rotor (M = unity) requires an actuator disk condition of infinite (zero loss) blades and a zero loss flow state.

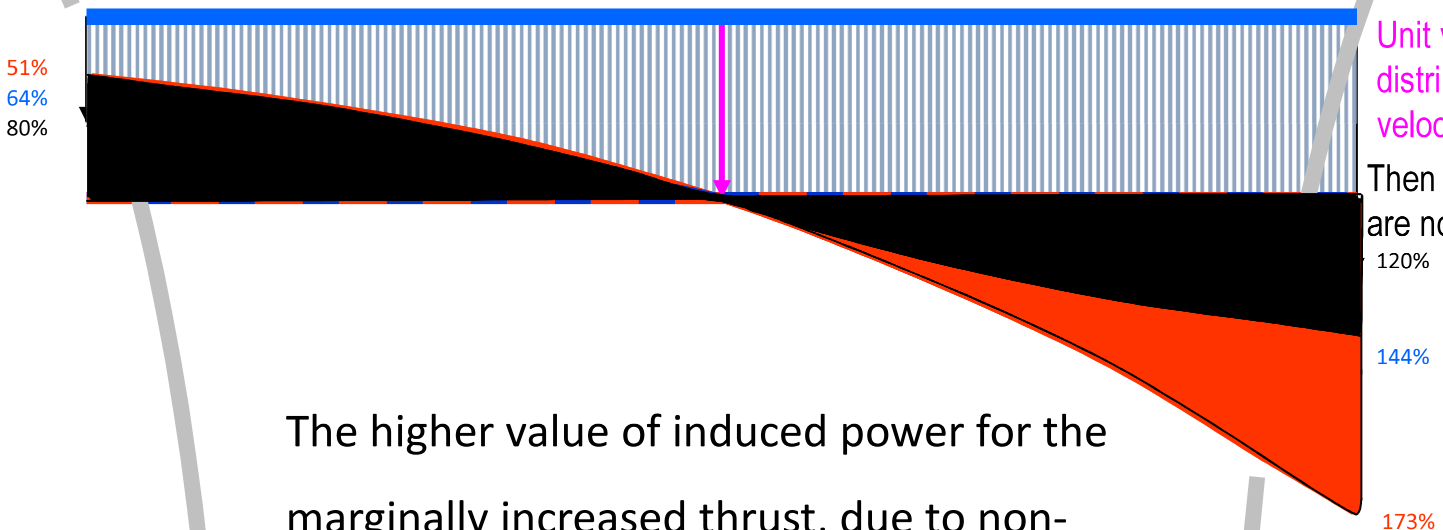


A rotor generates thrust by imparting momentum to the air that flows through it. This can only be efficiently achieved by a uniform distribution of induced velocity as assumed in actuator disk theory. Any local variation in the magnitude of the induced velocity will increase the overall power requirement as:

$$P = Tv = \left(2\rho A v^2\right)v = 2\rho A v^3$$

Maximising the Figure of Merit

Suppose v is 20% less on the LHS, 20% higher on the RHS



Unit value triangularly distributed induced velocity

Then Thrust & Power are no longer constant

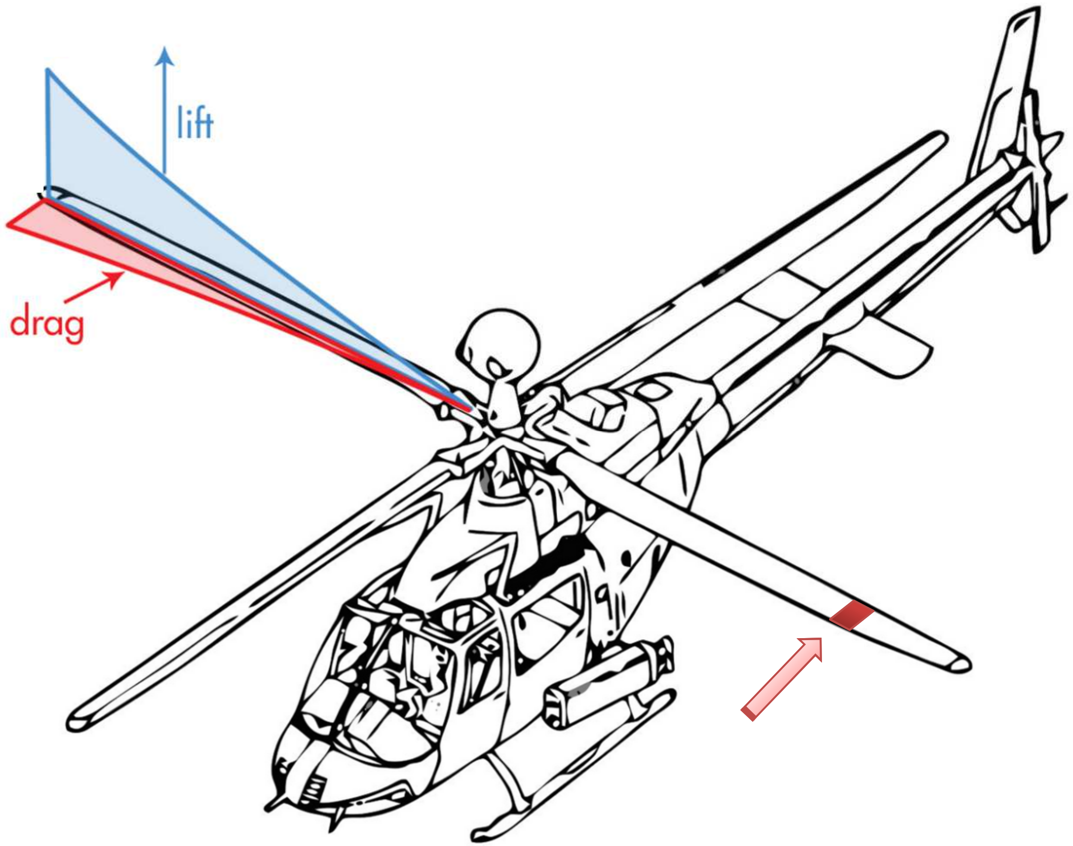
Thrust

Power

The higher value of induced power for the marginally increased thrust, due to non-uniform inflow velocity (v) is seen from the plot of excess power which is effectively a

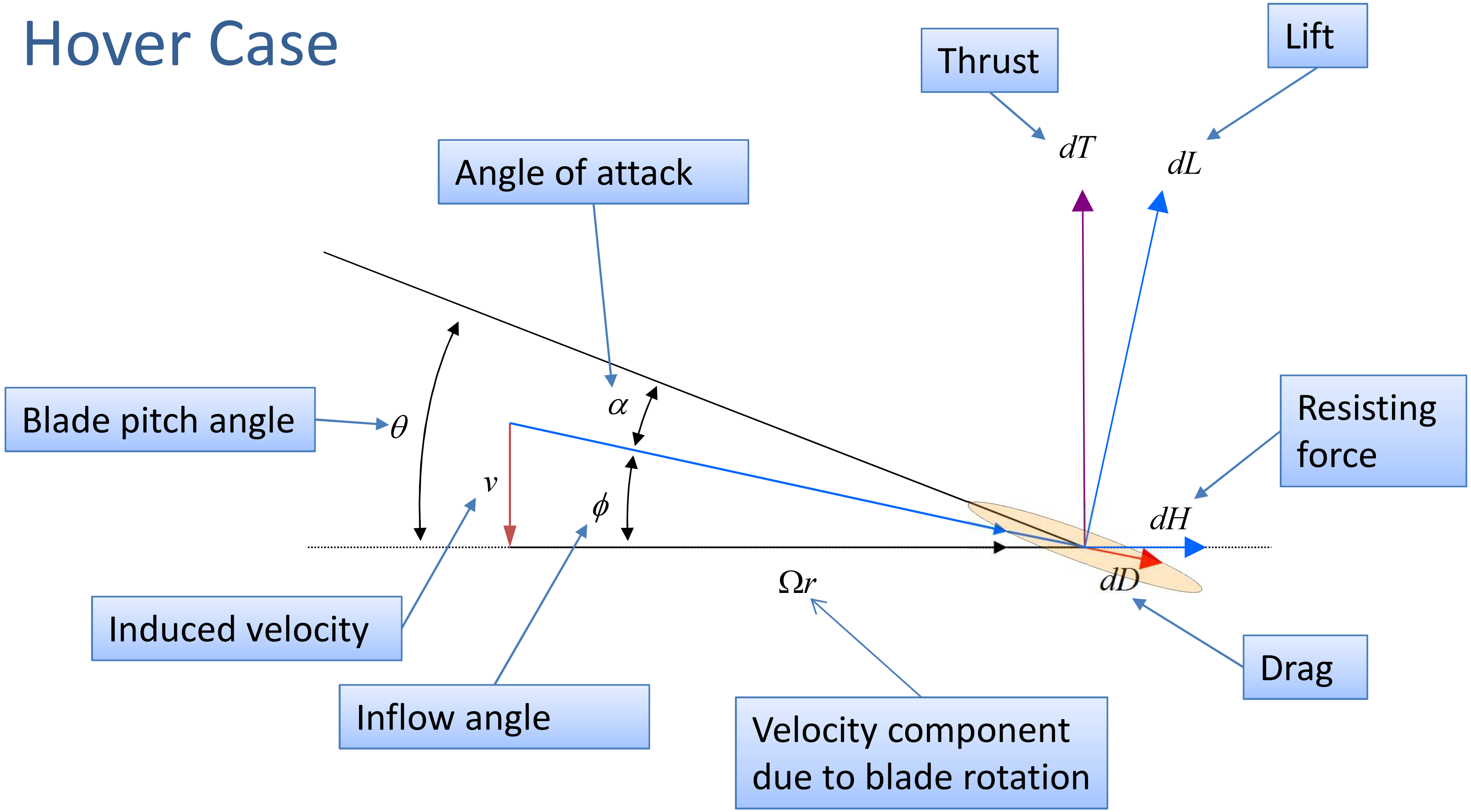
. Power Loss

Forces Acting on the Blade



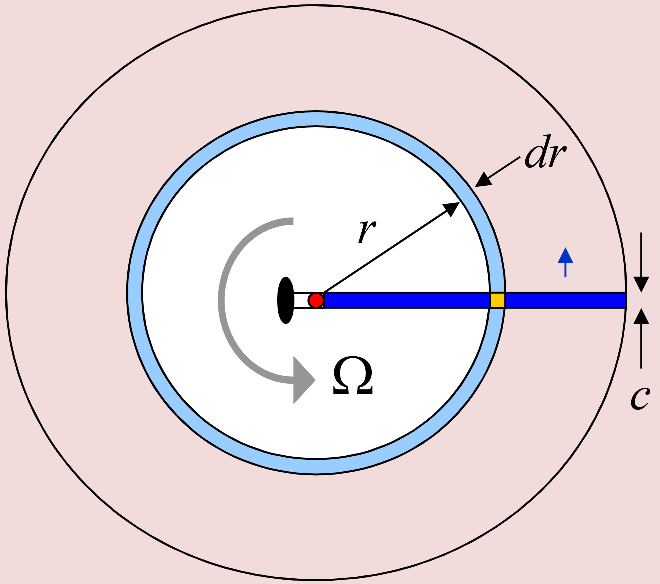
Forces at a Blade Element

Hover Case



Maximising the Figure of Merit

For a helicopter rotor, which has a finite number of blades, the blade element theory can be used to equate the lift on a blade element to the induced velocity in the swept annulus of that element.



$$L = \tfrac{1}{2}\rho V^2 S C_L$$

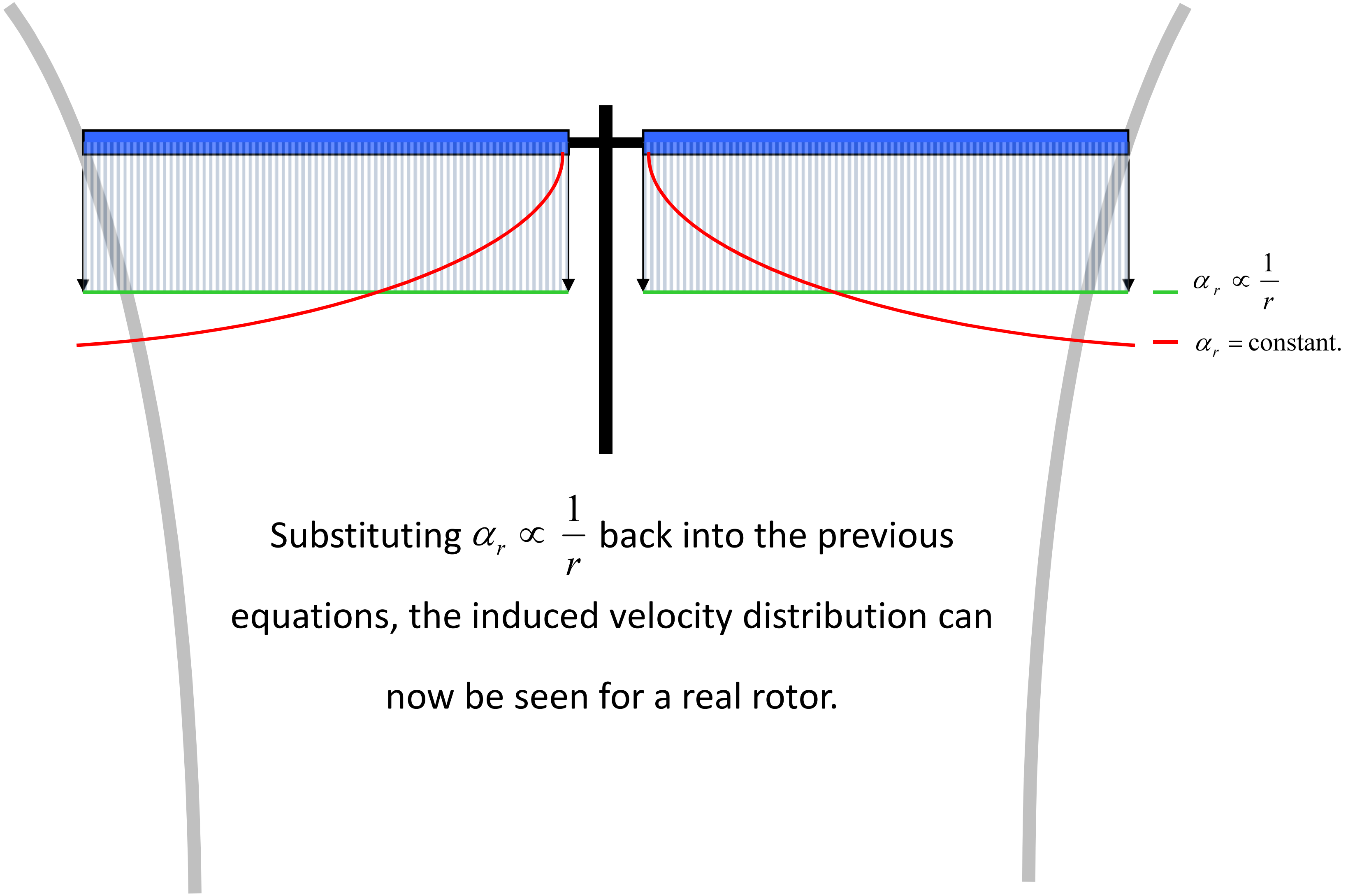
$$dL = \tfrac{1}{2}\rho\,\Omega^2 r^2 c\,dr\,a\,\alpha_r = dT$$

$$v = \sqrt{\frac{T}{2\rho A}},\ dv = \sqrt{\frac{dT}{2\rho 2\pi\,r dr}} = \sqrt{\frac{\rho\,\Omega^2 r^2 c\,dr\,a\,\alpha_r}{8\rho\,\pi\,r\,dr}}$$

$$dv \text{ is proportional to } \sqrt{\frac{r^2\alpha_r}{r}} = \sqrt{r\ \alpha_r}$$

$$\text{so for constant } v,\ \alpha_r \text{ is proportional to } \frac{1}{r}$$

Maximising the Figure of Merit

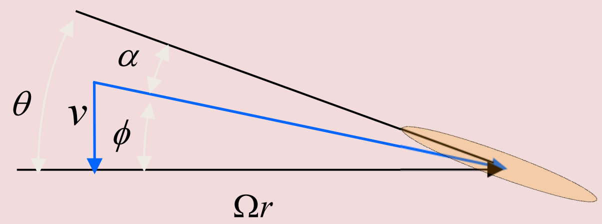


Substituting $\alpha_r \propto \frac{1}{r}$ back into the previous equations, the induced velocity distribution can now be seen for a real rotor.

Maximising the Figure of Merit

IDEAL BLADE TWIST results in a constant induced velocity across the rotor disk.

It has been seen that for this to be the case, then $\alpha_r \propto \frac{1}{r}$



For this to be the case, $\alpha_r = (\theta - \phi) = \frac{R}{r}(\theta_t - \phi_t)$

So, $\phi = \phi_t \frac{R}{r}$ where ϕ_t = inflow angle at the tip.

Similarly blade pitch angle $\theta = \theta_t \frac{R}{r}$ where θ_t = pitch angle at the tip.

Thus, by careful design, the ideal inflow can be achieved by blade twist, or blade planform taper or a combination of the two.

Unfortunately the other requirements for ideal conditions (zero profile drag, tip losses and swirl in the wake) are not so easily met and must, at best, be minimised.

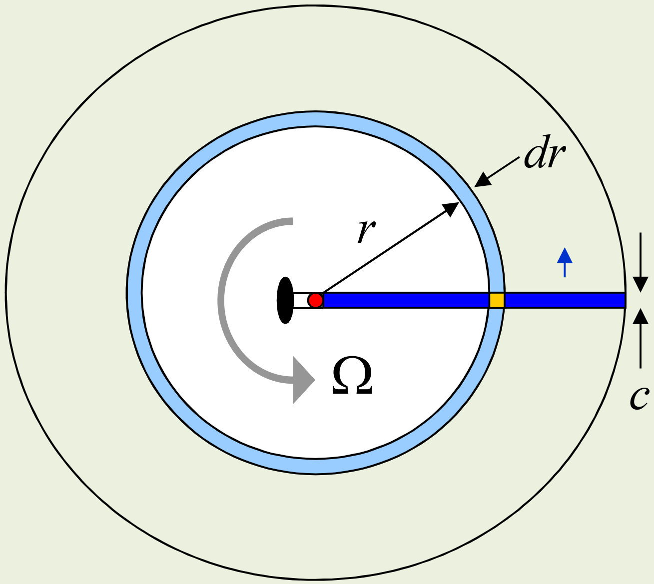
This is best achieved by utilising rotor blade element analysis.

Maximising the Figure of Merit:

Thrust Coefficient for Rotor with Ideally Twisted blades

$$dL = \frac{1}{2}\rho(\Omega r)^2 a \frac{R}{r}(\theta_t - \phi_t)c\,dr$$

$$L = \int_0^R \frac{N}{2}\rho\Omega^2 rRa(\theta_t - \phi_t)c\,dr = \frac{N}{4}\rho\Omega^2 R^3 a(\theta_t - \phi_t)c \quad (\approx T)$$



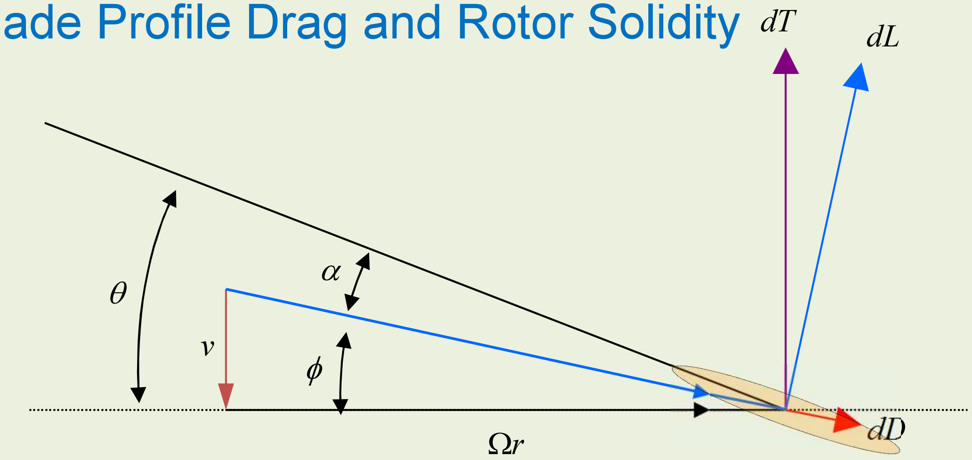
$$C_T = \frac{T}{\rho A(\Omega R)^2}$$

$$= \frac{N}{4}\frac{\rho\Omega^2 aR^3(\theta_t - \phi_t)c}{\rho\pi\Omega^2 R^4}$$

$$= \frac{Na(\theta_t - \phi_t)c}{4\pi R}$$

or, $C_T = \frac{\sigma}{4}a(\theta_t - \phi_t)$, since $\sigma = \frac{Nc}{\pi R}$

Maximising the Figure of Merit: Effect of Blade Profile Drag and Rotor Solidity



The rotor blade element of drag is composed of two components; the profile drag and the induced drag. The resultant drag in the plane of the rotor is:

$$dDCos\phi + dLSin\phi$$

Since ϕ is small this can be written:

$dD + dL\phi$, or, in coefficient form as: $C_{d_0} + \phi C_l$

Thus the in-plane drag torque due to this element is:

$$dQ = \frac{N}{2}\rho(\Omega r)^2 c(C_{d_0} + \phi C_l)rdr$$

Assuming that $C_{d_0} = \delta$ is relatively constant over the range of α , then C_{d_0} (a constant).

It was also shown that: $c_l = a\frac{R}{r}(\theta_t - \phi_t)$ and $\phi = \phi_t \frac{R}{r}$

thus, $Q = \int_0^R \frac{N}{2}\rho\Omega^2 r^3 c\left[\delta + \varphi_t \frac{R^2}{r^2}(\theta_t - \varphi_t)a\right]dr,$ so $Q = \frac{N}{4}\rho\Omega^2 R^4 c\left[\frac{\delta}{2} + a\varphi_t(\theta_t - \varphi_t)\right]$

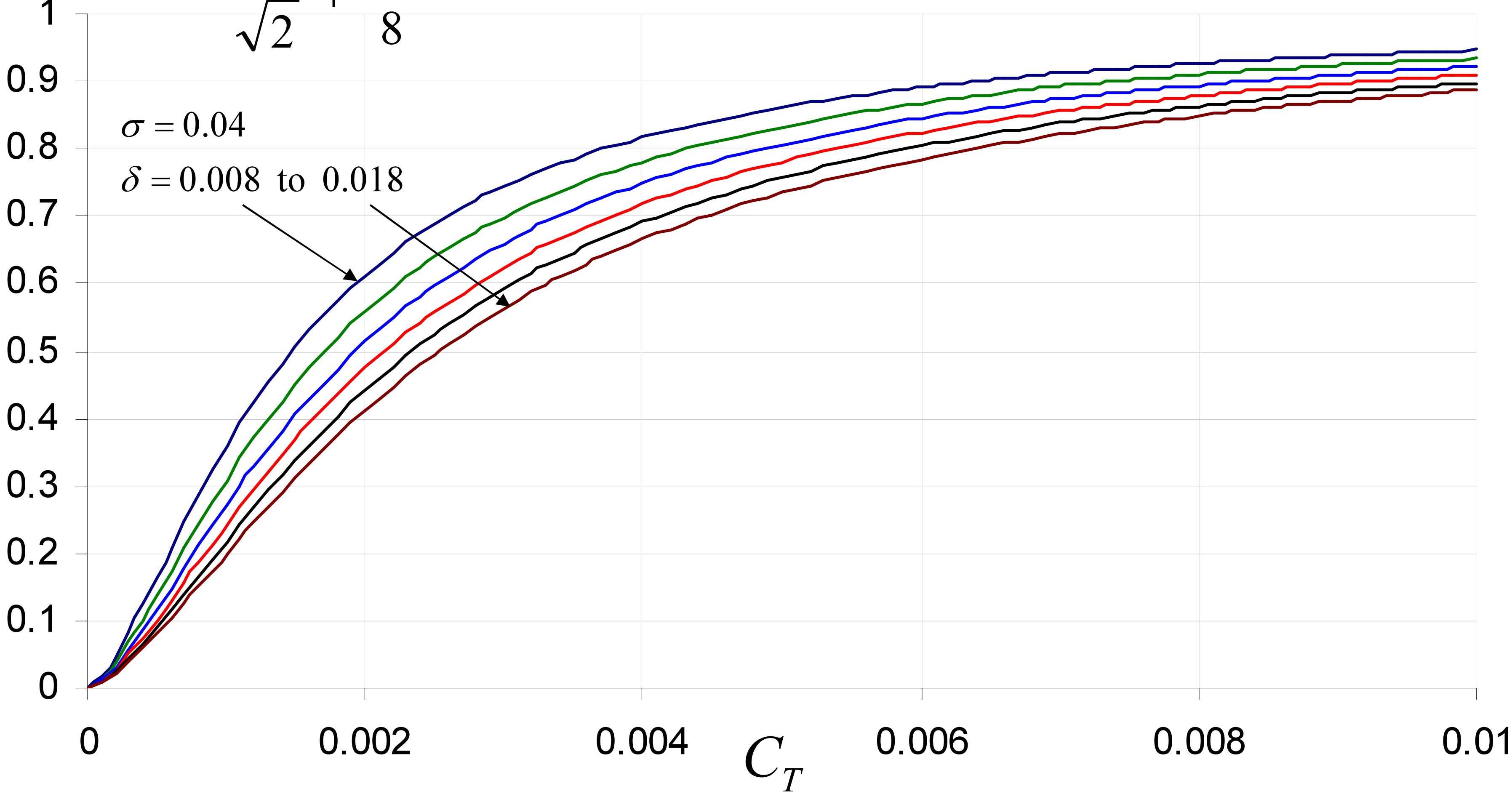
or, $\boxed{C_Q = \frac{\sigma\delta}{8} + \varphi_t C_T = \frac{C_T^{\frac{3}{2}}}{\sqrt{2}} + \frac{\sigma\delta}{8}}$ $\left[\text{since } \varphi_t = \sqrt{\frac{C_t}{2}}\right]$

Hence $\boxed{M = 0.707\frac{C_T^{\frac{3}{2}}}{\frac{C_T^{\frac{3}{2}}}{\sqrt{2}} + \frac{\sigma\delta}{8}}}$

Maximising the Figure of Merit

$$M = 0.707 \frac{C_T^{\frac{3}{2}}}{\frac{C_T^{\frac{3}{2}}}{\sqrt{2}} + \frac{\sigma\delta}{8}}$$

Figure of Merit against Coefficient of Thrust for a range of delta's (aerofoil profile drag coefficients) based on a fixed value of rotor solidity ($\sigma = 0.04$).



In the hover, again using small angle approximations, then $L = T = W$ (the weight of the aircraft).

This is the summation of all the blade elemental lift forces:

$$W = L = \underbrace{\int_0^R \frac{N}{2}\rho(\Omega r)^2 C_l c dr}_{\text{Total lift generated on blades}} = \underbrace{\bar{C}_L \int_0^R \frac{N}{2}\rho(\Omega r)^2 c dr}_{\text{Total lift generated on blades based upon mean value of } C_L} = \underbrace{T = C_T \pi R^2 \rho(\Omega R)^2}_{\text{Total thrust generated by rotor}}$$

(where $\bar{C}_L$ is the mean lift coefficient)

Thus $\frac{1}{6}\bar{C}_L \rho \Omega^2 R^3 N c = C_T \pi R^2 \rho (\Omega R)^2$

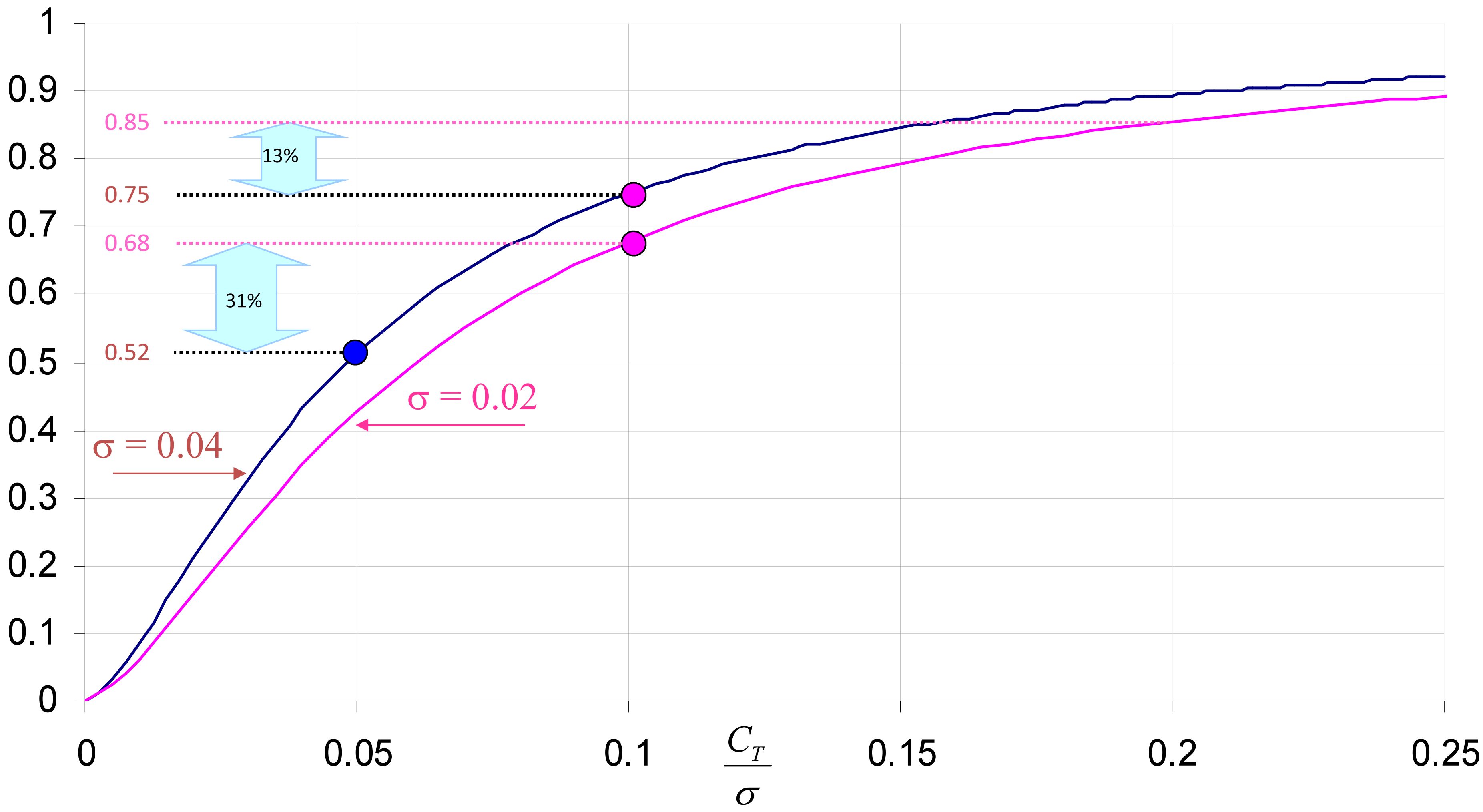
Therefore $\boxed{\bar{C}_L = 6\frac{C_T}{\sigma}}$

This "equivalent" fixed wing lift coefficient gives an indication of *"how hard the rotor blades are working".*

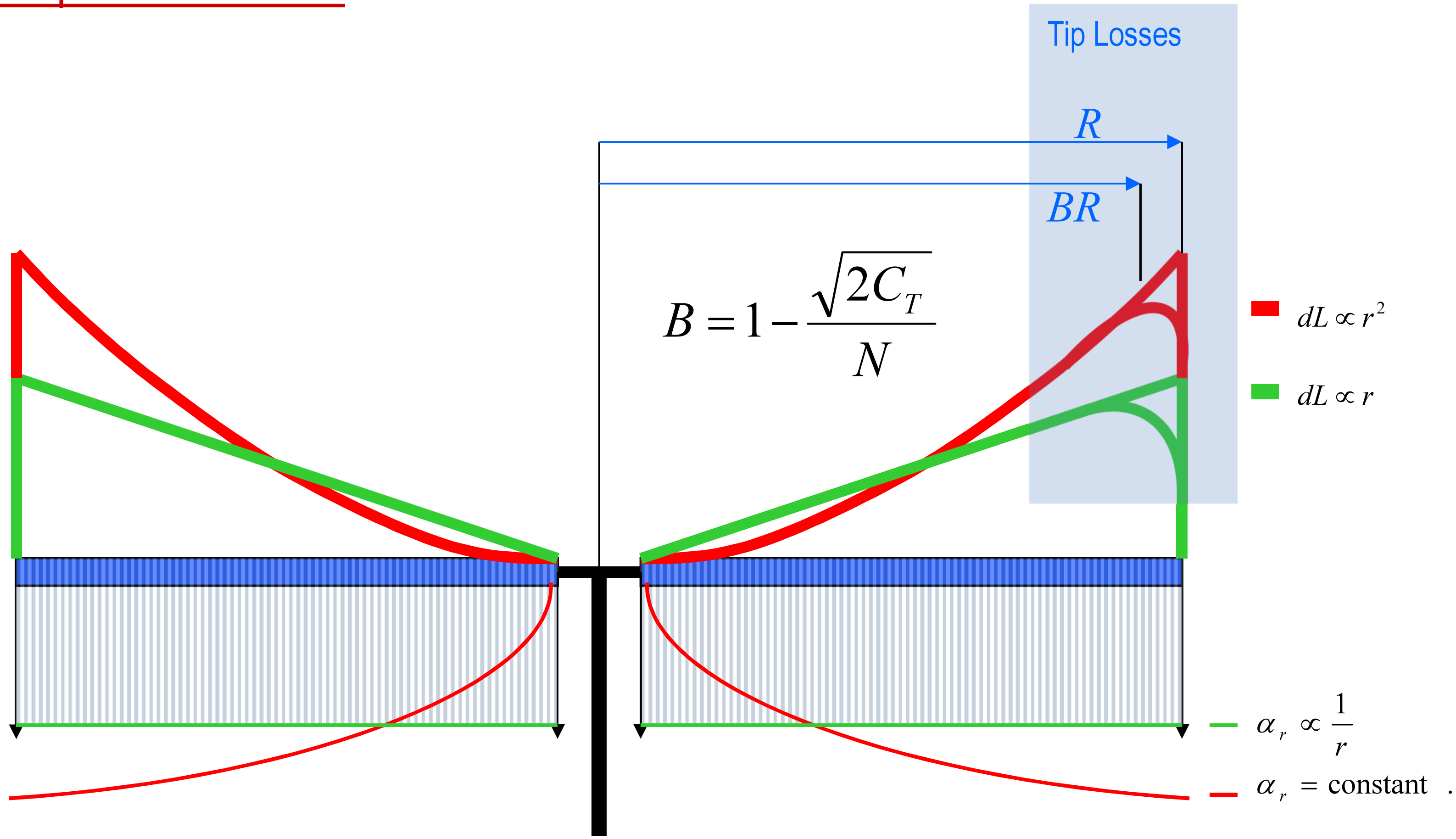
When plotted against the Figure of Merit the effect of solidity becomes apparent.

Maximising the Figure of Merit

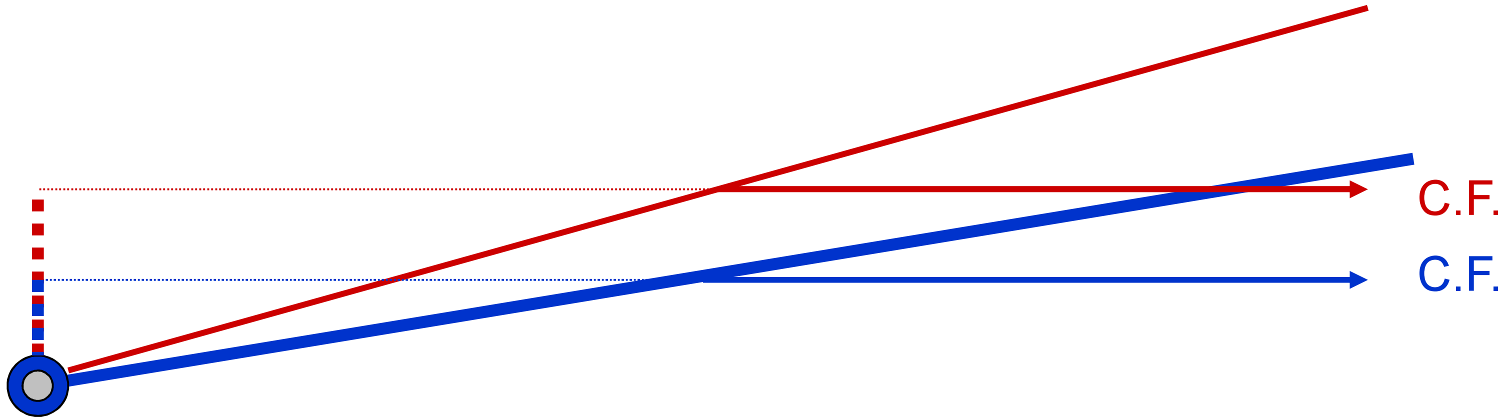
Reducing rotor solidity increases the FoM but with diminishing effect at higher C_T/σ.



Rotor Blade Tip Loss Factors



Unlike the actuator disk, a real rotor blade cannot support lift right out to the blade tip. A tip loss factor B (usually 0.95 -0.97) can be used in analysis whereby it is assumed that blade drag exists over the entire blade length but no lift is generated outboard of radius BR, Blade Flapping Motion



If the blade flaps up, away from it’s steady state, the increased C.F. moment arm will provide a restoring force, acting like a return spring.

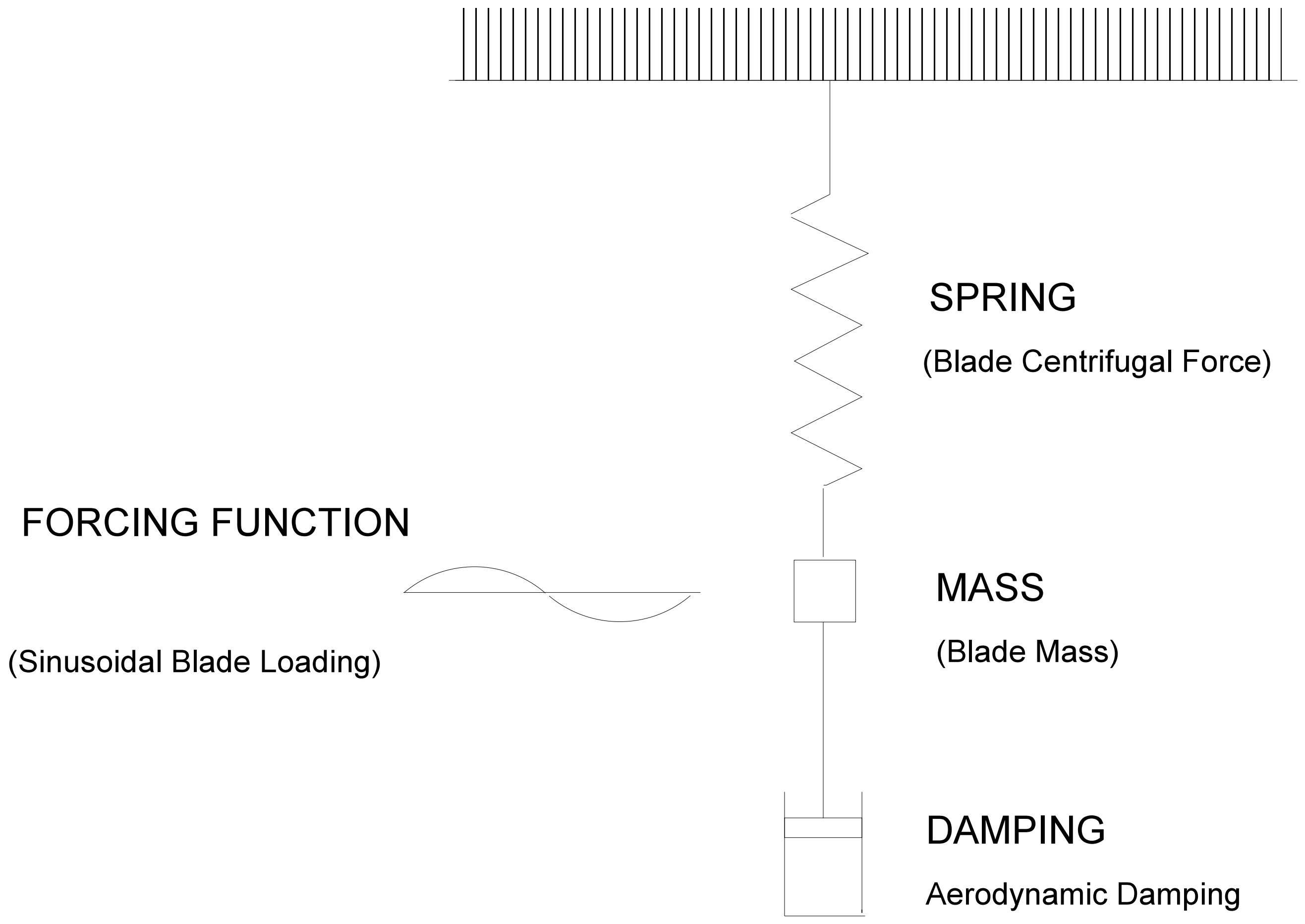
The blade dynamics provide a restoring spring force.

As the blade flaps up, the angle of incidence is reduced and so therefore is the lift which tends to oppose the upward flapping motion.

As the blade flaps down, the angle of incidence is increased and so therefore is the lift which tends to oppose the downward flapping motion.

The blade aerodynamics provide motion damping.

Blade Dynamics



Blade Dynamics

Using the general equation for rotational oscillations, the natural frequency is given by:

$$\omega_n = \sqrt{\frac{K}{I}}$$

where $K =$ spring constant $Nm\ /\ rad$

$I =$ moment of inertia kgm^2

The spring constant of the rotor blade is the restoring force provided by the centrifugal force.

The centrifugal force is $(C.F.) = mr\Omega^2$, (for a blade element at radius r)

which results in a moment about the flapping hinge, $(C.F.)r\sin\beta$

which for small angle approximations can be expressed as $(C.F.)r\beta$

Summating for all blade elements, $(C.F.)$ moment =

$$\int_0^R \Omega^2 r^2 \beta m dr = m\Omega^2\beta\frac{R^3}{3} = \underline{M\Omega^2\beta\frac{R^2}{3}} = K\,\beta$$

The flapping moment of inertia of the rotor blade is $\underline{I = \frac{MR^2}{3}}$

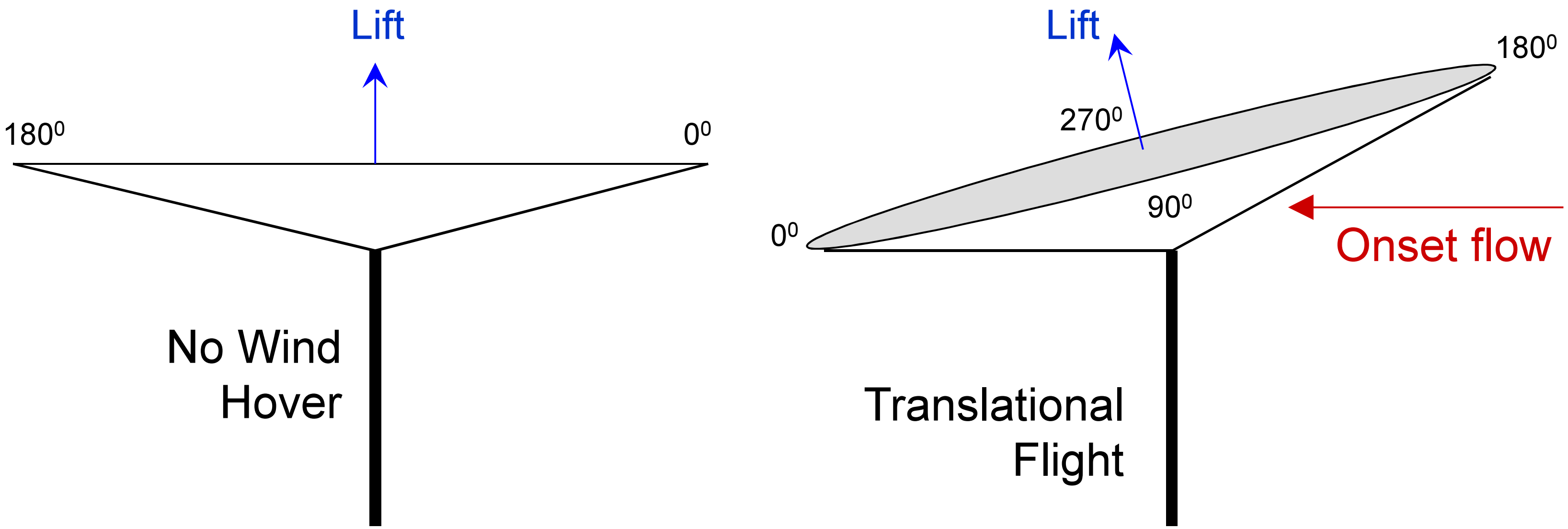
Thus the natural flapping frequency of the blade is $\omega_n = \sqrt{\Omega^2} = \Omega$ rads/sec

The natural frequency is the rotational frequency, so the rotor is in resonance.

Blade Dynamics

This shows that for a blade that flaps about a hinge on the rotor axis, the natural flapping frequency is the rotational frequency. Thus the rotor is in resonance and the phase angle (that is the force-displacement angle) is 90^0

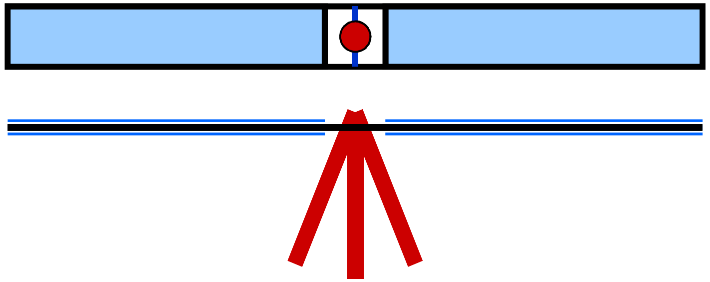
The forcing function (in forward flight) has a maximum and minimum value at $\psi = 90^0$ and $\psi = 270^0$ respectively. It follows then that the displacement has a maximum and minimum value at $\psi = 180^0$ and $\psi = 360^0$ respectively. This means that in forward flight the rotor disc will pitch upwards (in the positive sense).

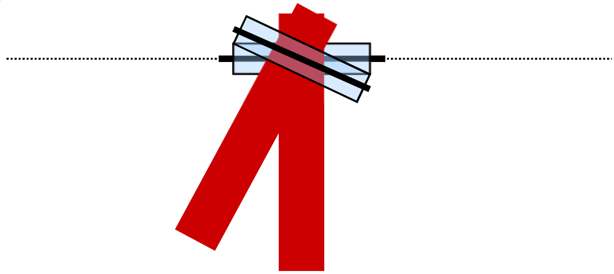


View from Starboard Side ($\psi = 90^0$)

Flapping~Feathering Equivalence

So, the autogyro had a flapping hinge and no feathering hinge but by directly tilting the rotor disk the effective pitch of the blades varied cyclically around the rotor azimuth – this was in effect a cyclic pitch control.





The powered rotor (helicopter) cannot be manually tilted with such ease.

The fully articulated rotor head, controlled across the stationary / rotational interface by a swash plate, can apply collective and cyclic changes of pitch.

Thus the pitch can be cyclically changed to prevent the blades from flapping (in translational flight) and also address the problem of lift asymmetry.

Coning angle ▼ **▼ Rotor flap-back (longitudinal) angle**

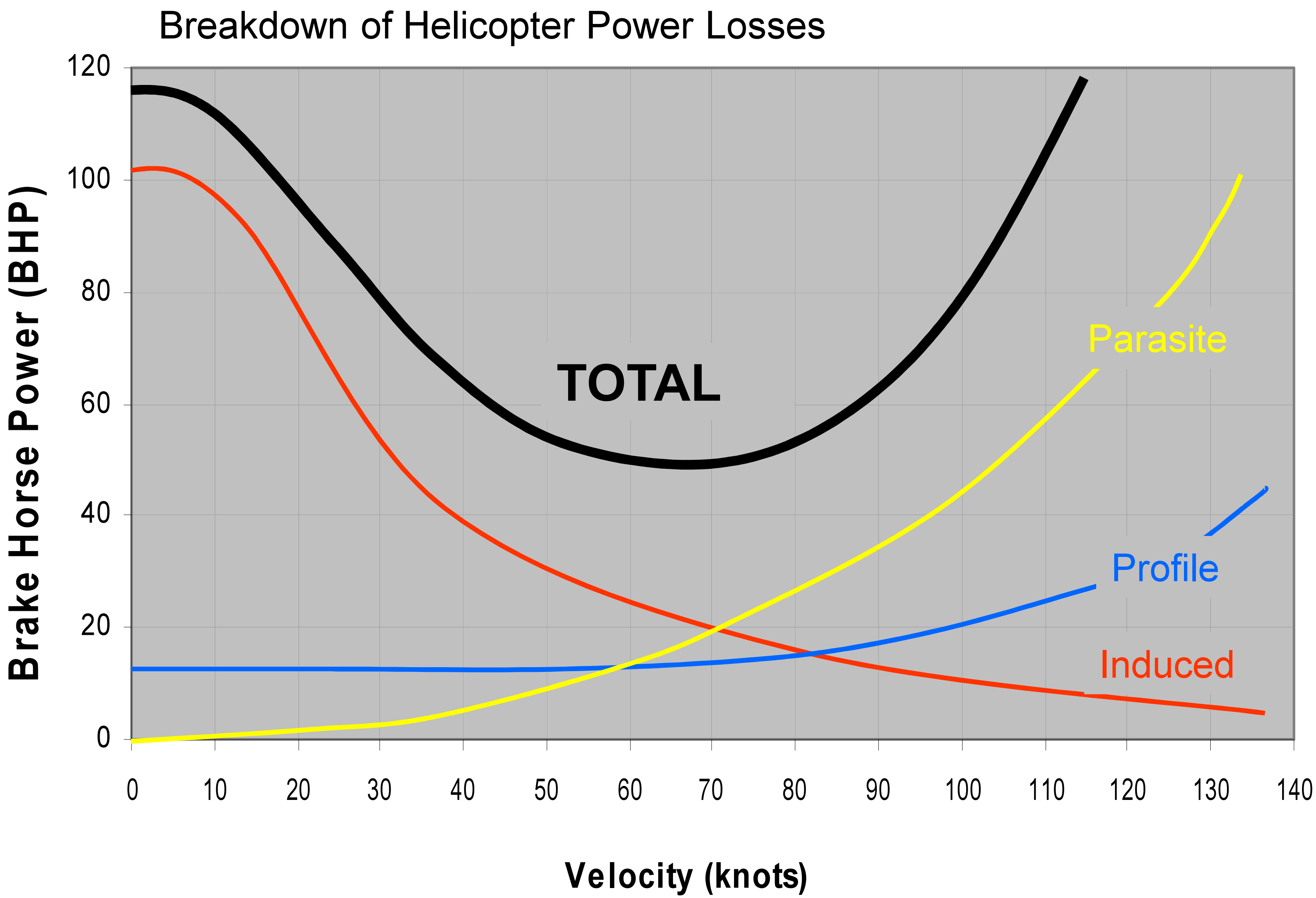
Blade flap angle $\beta = a_0 - a_1 \cos\psi - b_1 \sin\psi - a_2 \cos 2\psi - b_2 \sin 2\psi - \ldots\ldots$

Blade pitch angle $\theta = A_0 - A_1 \cos\psi - B_1 \sin\psi - A_2 \cos 2\psi - B_2 \sin 2\psi - \ldots\ldots$

Collective Pitch angle input ▲ ▲ **▲ Lateral Cyclic Pitch angle input**

Longitudinal Cyclic Pitch angle input will counter lateral rotor tilt

Induced Velocity in Translational Flight



Example Question

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Typical numerical exam question

Before you start, just remember that the induced rotor power is the product of the rotor thrust and the mean induced velocity. If you can't remember the equation for the induced velocity of an ideal rotor in hover, then you should derive it from the momentum theory / actuator disk analysis.

Typical numerical exam question

The helicopter described below has ideally twisted blades that provide a constant value of induced velocity across the rotor disk.

Main Rotor Diameter = 12.8m (42 ft) and All Up Mass *(AUM)* of 4536 kg (10,000 lbs). The main rotor profile power is 30% of the main rotor induced power and the main rotor tip losses are 5% of the main rotor induced power. The tail rotor, all transmission losses and the ancillary drives account for 35% of the total Installed Power *(IP)*.

Determine the Main Rotor Figure of Merit *(FoM)* and the installed power *(IP)* requirement if the helicopter is to hover *(OGE)* at sea level conditions.

Solution:

Determine the Main Rotor Figure of Merit (FoM) and the installed power (IP) at sea level conditions.

Main Rotor

Induced velocity: $v = \sqrt{\frac{T}{2\rho A}} = \sqrt{\frac{AUM \text{ x } 9.81}{2 \text{ x } 1.225 \text{ x } \pi \text{ x } 6.4^2}} = \sqrt{\frac{44498}{315}} = 11.88\ m/s$

Induced power: $P_i = Tv_i = 44498 \text{ x } 11.88 = 528.66\ kW$

Profile power: $P_p = 0.3 \text{ x } P_i = 158.6\ kW$

Tip losses: $P_{TL} = 0.05 \text{ x } P_i = 26.4\ kW$

Total Main Rotor Power: $P_{Total} = 528.66 + 158.6 + 26.4 = 713.7kW$

Figure of Merit: $FoM = \frac{P_i}{P_{Total}} = \frac{528.66}{713.7} = 0.74 = 74\%$

Therefore, Installed Power: $IP = \frac{P_{Total}}{(1-0.35)} = \frac{713.7}{0.65} = 1.1MW$

The tail rotor, all transmission losses and the ancillary drives account for 35% of the total Installed Power (IP)

- If the main rotor has a solidity of 5%, find the collective pitch angle at the 75% radius assuming the rotor has constant chord and ideally twisted blades and which give a uniform induced velocity distribution across the rotor disc.

For Ideally twisted blade, we have

$$C_T = \frac{\sigma}{4} a(\theta_t - \varphi_t) \qquad \dots\dots(1)$$

Where $\theta = \theta_t \frac{R}{r}$ So $\theta_{0.75} = \frac{\theta_t}{0.75}$

From (1)

$$\theta_t = \frac{4C_T}{\sigma a} + \varphi_t = \frac{4C_T}{\sigma a} + \lambda$$

Since $\lambda = \sqrt{\frac{C_T}{2}}$ (given for hover)

Now

$$C_T = \frac{T}{\rho A (\Omega R)^2}$$

Therefore

$$\boxed{\theta_{0.75} = \frac{735.13}{a\Omega^2} + \frac{2.5}{\Omega}}$$

Check for example $\Omega = 35\, rad/s, a = 6.28$ then $\theta_{0.75} = 0.167 rad = 9.56\text{deg}$

- Find the profile drag coefficient of the blades.

Rotor Performance Coefficients - Recap

The Figure of Merit as previously defined can be more conveniently expressed in terms of non-dimensional quantities using the thrust and power coefficients.

The induced velocity $\boldsymbol{v} = \sqrt{\dfrac{\boldsymbol{T}}{2\rho \boldsymbol{A}}} = \sqrt{\dfrac{\boldsymbol{C_T}\rho \boldsymbol{A}(\Omega \boldsymbol{R})^2}{2\rho \boldsymbol{A}}} = \Omega \boldsymbol{R}\sqrt{\dfrac{\boldsymbol{C_T}}{2}}$

But since $\lambda = \dfrac{\boldsymbol{V_V} + \boldsymbol{v}}{\Omega \boldsymbol{R}}$ then for $\boldsymbol{V_V} = \boldsymbol{0}$, $\lambda = \sqrt{\dfrac{C_T}{2}}$

$FoM \Rightarrow M$

$$M = \frac{Tv}{P} = \frac{C_T\rho A(\Omega R)^2 v}{C_P\rho A(\Omega R)^3} = \frac{C_T}{C_P}\frac{v}{\Omega R} = \frac{C_T}{C_P}\lambda$$

$$= \frac{1}{\sqrt{2}}\frac{C_T^{3/2}}{C_P}$$

Or more commonly..............................

$$\boxed{M = 0.707\frac{C_T^{3/2}}{C_Q}}$$

Recap

$$\boldsymbol{C_Q} = \frac{\boldsymbol{Q}}{\rho \boldsymbol{A}\boldsymbol{R}(\Omega \boldsymbol{R})^2} \qquad C_P = \frac{P}{\rho A(\Omega R)^3} \qquad \boldsymbol{C_P} = \boldsymbol{C_Q}$$

$$C_{Pi} = \frac{P_i}{\rho A(\Omega R)^3} = \frac{Tv}{\rho A(\Omega R)^3} = \frac{C_T \rho A(\Omega R)^2 v}{\rho A(\Omega R)^3} = C_T \frac{v}{\Omega R} = C_T \lambda$$

$$\lambda = \sqrt{\frac{C_T}{2}} \qquad\qquad C_{Pi} = \frac{C_T^{\frac{3}{2}}}{\sqrt{2}}$$

$$C_Q = \frac{\sigma\delta}{8} + \phi_t C_T \;\; = \frac{C_T^{\frac{3}{2}}}{\sqrt{2}} + \frac{\sigma\delta}{8} \qquad \left[\text{since } \phi_t = \sqrt{\frac{C_t}{2}}\right]$$

$$C_Q = \frac{C_T^{\frac{3}{2}}}{\sqrt{2}} + \frac{\sigma\delta}{8} \qquad = C_{Pi} + C_{p\_profile}$$

where

$$C_{p\_profile} = \frac{\sigma\delta}{8}$$

$$P_p = 0.3 \times P_i$$

Let's also include the tip losses $P_p = 0.35 \times P_i$

So $C_{P\_profile} = 0.35 \times C_{Pi} = \dfrac{\sigma\delta}{8}$

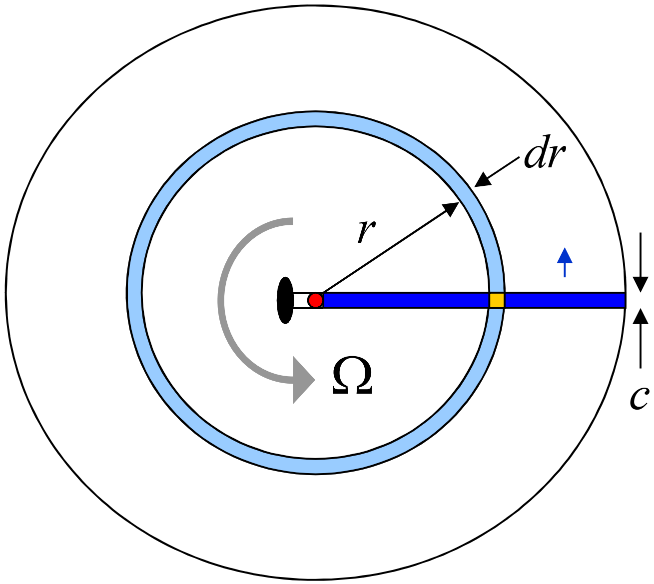
$$\delta = \frac{0.35 \times 8 \times C_{Pi}}{\sigma} = \frac{0.35 \times 8 \times C_{Pi}}{0.05} = \frac{0.35 \times 8}{0.05} \times \frac{P_i}{\rho A(\Omega R)^3} = \frac{0.35 \times 8}{0.05} \times \frac{528660}{1.225 \times 3.14 \times 6.4^5 \times (\Omega)^3}$$

Therefore $\boxed{\delta = \dfrac{716.8}{\Omega^3}}$ $\quad Check\ \ for\ \Omega = 35 rad/s, \qquad \delta = 0.017$

- If the blade has a constant twist, how should the chord of the blades change to provide a constant value of induced velocity across the rotor disc?

Maximising the Figure of Merit

For a helicopter rotor, which has a finite number of blades, the blade element theory can be used to equate the lift on a blade element to the induced velocity in the swept annulus of that element.



$$L = \tfrac{1}{2}\rho V^2 S C_L$$

$$dL = \tfrac{1}{2}\rho\,\Omega^2 r^2 c\,dr\,a\,\alpha_r = dT$$

$$v = \sqrt{\frac{T}{2\rho A}},\ dv = \sqrt{\frac{dT}{2\rho 2\pi\,rdr}} = \sqrt{\frac{\rho\,\Omega^2 r^2 c\,dr\,a\,\alpha_r}{8\rho\,\pi\,r\,dr}}$$

$$dv \text{ is proportional to } \sqrt{r\ c\alpha_r} = \sqrt{r\ c(\theta-\phi\ )}$$

$$dv \text{ is proportional to } \sqrt{r\ c(\theta\ -\phi_t\,\frac{R}{r})} = \sqrt{c(\theta\,r-\phi_t R)}$$

$$\text{so for constant } v,\ c \text{ is proportional to } \frac{1}{\theta\,r-\phi_t R}$$