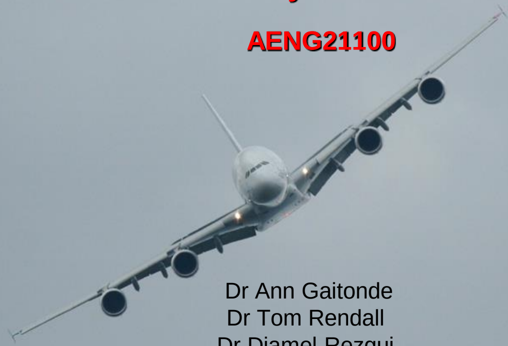
Aerodynamics 2

AENG21100



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Course Aims

- To establish a *basic understanding* of the interactions between fluid flows and aerospace structures
- To provide some *basic tools and fundamental understanding* required for the experimental and theoretical modelling of these flows
- To build on material from *Fluids1*

Course Content & Assessment

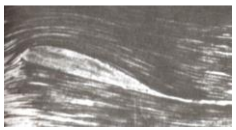
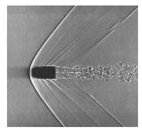
- 35 lectures-introduction, incompressible, compressible and helicopter aerodynamics
- 7 examples classes
- 3 assessed tests
  - short 'progress test' sessions on BB
- 2 laboratory classes and linked MATLAB assignments
  - A1 'pressure distribution on an aerofoil'
  - A2 'compressible flow in a nozzle'
- Exam
  - 3 hours (compulsory section plus 3 questions out of 5)
- Note: No changes from last year's course

Key Points

- Material is a step up from last year. Just because you achieved a good mark in Fluids 1 does not guarantee a good mark in Aero2.
- To do well in Aero 2 you need to develop understanding – memorising alone will not lead to exam success.
- Ways to achieve understanding:
  - Attend lectures, unless a genuine reason not to. Use the Mediasite recording to go over any missed lecture as soon as possible.
  - Make sure you review anything you find difficult in each lecture by using books, online resources and asking questions. As the material builds up, its important to have good foundations.
  - Complete Examples Sheets when set. Ask for help when you get stuck during the examples classes.
- Plagiarism:
  - Make sure you understand exactly what constitutes plagiarism and how to reference and quote appropriately.
  - Better to get 50% for your own work than ending up with 0%!

Unit content

- Introduction (2 lectures)
- Incompressible flow - builds on 2D potential methods from last year. Models for flows past aerofoils and wings (1 lecture per week)
- Compressible flow - isentropic flow, shockwaves and their impact on aerofoils and wings (1 lecture per week)
- Helicopter aerodynamics – builds on from actuator disc theory from last year (will eventually take over both lecture slots as incompressible and compressible sections finish)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REVIEW

Before we can get going we need to review material from Fluids1 and some maths:

- Some basics (covered today)
  - Static, dynamic and total values
  - The relationship between some unsteady flows and their steady equivalents (using the Galilean transformation)
  - Similarity, Euler number, Reynolds number and Mach number
- Derivation of 1D Euler Equation & Laplace's equation (covered today)
- 2D Potential flow (covered in first incompressible flow lecture)
- Maths in handout for you to go through yourselves (solutions are available on the BB site)

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Definitions of Pressure

- **Static pressure:** The actual pressure of the fluid, which is associated not with its motion but with its state. Measured by a barometer placed in the fluid.
- **Total Pressure:** Also known as reservoir stagnation or pitot pressure. The total pressure is the pressure when all the kinetic energy is converted into pressure energy.
- **Dynamic pressure:** The dynamic pressure is equal to the difference between the total pressure and the static pressure.

Note this year we are only concerned with flows of air (both incompressible and compressible), so hydrostatic pressures can be neglected

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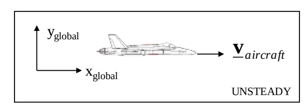
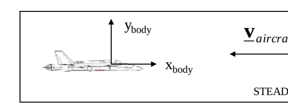
Unsteady and Steady Flows

- Some interesting unsteady flows in aerodynamics can be modelled as steady flows if the fluid motion relative to a moving, rather than fixed coordinate system, is considered using a Galilean transformation.





$$\begin{aligned} \underline{\mathbf{x}}_{body} &= \underline{\mathbf{x}}_{global} + \underline{\mathbf{v}}_{aircraft}\, t \\ \underline{\mathbf{u}}_{body} &= \underline{\mathbf{u}}_{global} + \underline{\mathbf{v}}_{aircraft} \end{aligned} \quad \Rightarrow \quad \begin{aligned} p_{body} &= p_{global} \\ T_{body} &= T_{global} \end{aligned}$$

- Applied to the Navier-Stokes equations, transformation changes the velocity by the *constant* relative motion $\underline{\mathbf{v}}_{aircraft}$ but leaves the **STATIC pressure** & **temperature UNCHANGED**.

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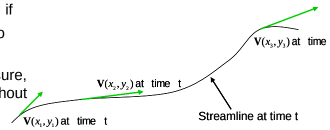
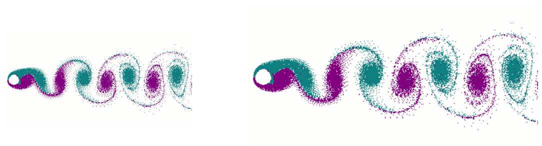
Flow Similarity

- Two flows over two different bodies are similar if
  - Streamlines are the same (lines tangent to the fluid velocity)
  - Plots of flow quantities e.g. velocity, pressure, density, temperature are the same throughout
  - Force coefficients are the same
- This requires that the
  - Bodies are geometrically similar
  - Similarity parameters, that arise from dimensional analysis, are the same for both flows





Pressure Coefficient, Reynolds and Mach Numbers

$$C_p = \frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{pressure \quad force}{inertia \quad force}$$

- pressure coefficient has this form

$$Re = \frac{\rho V L}{\mu} = \frac{VL}{\nu} = \frac{inertia \quad force}{viscous \quad force}$$

- Low Re – viscous forces are important
- High Re – viscous effects confined to thin region near body

$$M = \frac{V}{a} = \sqrt{\frac{\rho V^2}{E_V}} = \frac{inertia \quad force}{elastic \quad force}$$

- Compressibility parameter
- Transitions from *subsonic* to *transonic* to *supersonic* have profound impact on flow phenomena
- Low M – compressibility effects can usually be neglected

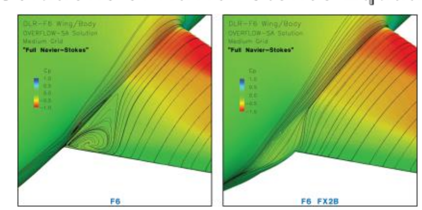
Derivation of the 1D Euler and Laplace Equations From the Navier-Stokes Equations

The Navier-Stokes equations in a fixed Cartesian coordinate system describe unsteady, viscous, compressible flow with body forces. Equations are:

- Continuity, conservation of mass (1 eqn)
- Conservation of Momentum in each coordinate direction (3 eqn)
- Conservation of total Energy
- Two "equations of state"

These equations determine the behaviour of all the flow quantities such as density, the three components of velocity, energy, temperature and pressure.

Solutions of Navier-Stokes Equations

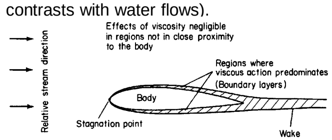


- Numerically resolving all the detail of an NS solution is not possible at Reynolds numbers typically found in aerodynamics
  - because of very small eddies -> turbulence
  - use turbulence modelling -> even more subtle level of approximation!
- Main development period 1990s to present. Research is still ongoing (esp. for drag prediction) and NS solutions are not routinely calculated .

First Three Simplifying Assumptions

Starting from the full Navier-Stokes equations a series of assumptions are made to simplify the equations. First 3 assumptions are:

- Assumption 1: Viscous effects are negligible
  - Valid for many real flows where viscous effects are confined to a narrow band near the body surface. Leads to the unsteady Euler equations
- Assumption 2: Steady flow
- Assumption 3: No body forces
  - Valid for aerodynamic flows where gravitational forces etc. are negligible (this contrasts with water flows).



Steady Euler Equations

After the first 3 assumptions arrive at the steady Euler equations for inviscid, compressible flow with no body forces.

- Continuity

$$\nabla\cdot(\rho\mathbf{V}) = 0 \qquad \rightarrow \qquad \frac{\partial\rho u}{\partial x}+\frac{\partial\rho v}{\partial y}+\frac{\partial\rho w}{\partial z} = 0$$

- Conservation of Momentum in x , y & z

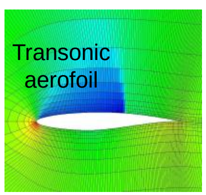
$$\boxed{(\mathbf{V}\cdot\nabla)\mathbf{V} = -\frac{1}{\rho}\nabla p \qquad \rightarrow \qquad \begin{aligned} u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z} &= -\frac{1}{\rho}\frac{\partial p}{\partial x} \\ u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z} &= -\frac{1}{\rho}\frac{\partial p}{\partial y} \\ u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z} &= -\frac{1}{\rho}\frac{\partial p}{\partial z}\end{aligned}}$$

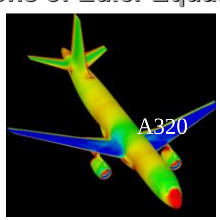
- Conservation of Energy

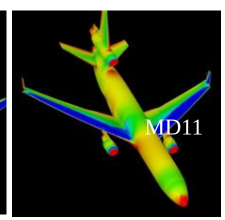
$$(\mathbf{V}\cdot\nabla)e = -\frac{p}{\rho}\nabla\cdot\mathbf{V}+\dot{q} \quad \rightarrow \quad u\frac{\partial e}{\partial x}+v\frac{\partial e}{\partial y}+w\frac{\partial e}{\partial z} = -\frac{p}{\rho}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)+\dot{q}$$

Don't try to memorise these equations you will gain a deeper understanding of these equations as you progress in later years.

Solutions of Euler Equations







- Again no analytic solutions for realistic geometries so need numerical solutions which are pretty good
  - we can still have rotational flow, and nonlinear features such as shocks are modelled (but not separated or turbulent flow).
- Many industrial design CFD codes solve the Euler equations
- Main development period 1980s-present. Drag results obviously exclude viscous effects.

Euler Momentum Equations

Focus only on 3 momentum equations within the steady Euler equations.

- Conservation of Momentum in x , y & z

$$\boxed{(\mathbf{V}\cdot\nabla)\mathbf{V} = -\frac{1}{\rho}\nabla p \qquad \rightarrow \qquad \begin{aligned} u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z} &= -\frac{1}{\rho}\frac{\partial p}{\partial x} \\ u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z} &= -\frac{1}{\rho}\frac{\partial p}{\partial y} \\ u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z} &= -\frac{1}{\rho}\frac{\partial p}{\partial z}\end{aligned}}$$

- The 3 components of the momentum equations can be combined into a single equation if certain assumptions are made

1D Euler Equation

- Either Make Assumption: Flow along a **streamline** only
  - 1D Euler links two points on a streamline

$$dp = -\rho V\,dV$$ 1D Euler equation

- Or Make Assumption: Irrotational flow i.e.

$$\nabla \times \mathbf{V} = 0$$

This means infinitesimally small fluid elements don't rotate. Flow can move in circles.

  - 1D Euler links any two points in the flow field

$$dp = -\rho V\,dV$$ 1D Euler equation

So 1D Euler correct anywhere if flow irrotational, but only for points along a streamline if flow is rotational

Solving 1D Euler

Solution depends on whether flow is compressible or incompressible.

Incompressible – Seen last year

Since density ρ=constant 1D Euler integrates to give Bernoulli's equation

$$p + \frac{1}{2}\rho V^2 = \text{constant}$$

Hydrostatic terms not included in current analysis as we neglected the body forces. OK for air.

This equation can only be applied for flows that are inviscid, constant density, steady **along a streamline *OR* anywhere in irrotational flow. Don't misuse it.**

Compressible – More complicated, not shown

Potential Flow

Solving the 1D Euler equation doesn't provide a complete model of the fluid motion because it involves more than one unknown i.e. the pressure and velocity (and density for compressible flows). This is not surprising as it only comes from momentum equations.

- Consider the special case of irrotational flow, this is a good approximation other than at the surface

  and in the wake for lifting flows

- **Irrotationality** guarantees the existence of a scalar 'velocity potential' function ϕ where

$$u = \frac{\partial \phi}{\partial x},\quad v = \frac{\partial \phi}{\partial y},\quad w = \frac{\partial \phi}{\partial z}$$

Review Topic 5: Potential Flow

Incompressible – using the mass conservation equation applied to the velocity field defined by velocity potential yields

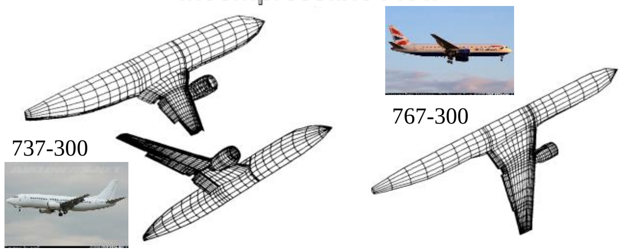
$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0$$ *Laplace equation (1789)*

- **Linear** so solutions can be superimposed
  - flow models built up from 'elementary' flow solutions

Compressible – Not shown, equation is more complex. Beyond scope of this course.

Note ϕ exists in unsteady flow, but for both incompressible and compressible the equation it then satisfies is more complex

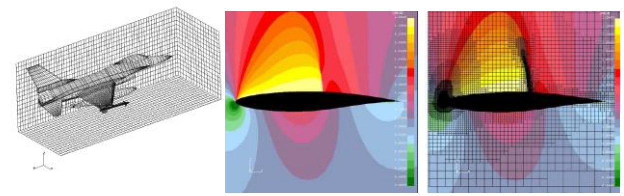
Velocity Potential Solutions for Incompressible Flow



- Models from PANAIR, Boeing's panel method.
  - Converts to a surface problem, but still tricky for even inviscid wakes.
- Concept dates from post WW2.
- Main development period 1970s-present. Developed alongside computers. Widely used.

Velocity Potential Solutions for Compressible Irrotational Flow

These solutions exist and have seen important usage



Transonic aerofoil

- Results from TRANAIR, Boeing's inviscid full potential solver (with viscous-inviscid interaction).
  - Requires a volume mesh.
- Main development period 1980s to present. Used for all recent Boeing aircraft!

For Incompressible Irrotational Flow

- Saw last year that the combination of Bernoulli's equation and the velocity potential calculated from Laplace's equation is sufficient to describe the fluid flow.
- The energy equation doesn't need to be solved as for incompressible flow as the flow is assumed adiabatic so the energy remains constant.

Revision Objectives

- Explain the difference between static, dynamic and total values.
- Explain how some unsteady flows can be transformed to steady flows by changing the coordinate system using the Galilean transformation.
- Define the Euler, Reynolds and Mach numbers.
- State the assumptions needed to reduce the 3 momentum equations of the Euler equations to a 1D equation (note two cases).
- Understand that a velocity potential may only be defined for an irrotational flow which satisfies Laplace's equation for incompressible flow. This means only 1 equation needs to be solved for the three components of velocity.
- Be aware that because Laplace's equation is a linear equation solutions can be found by superposition of other solutions