Lecture 12
Aircraft Modes

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Typical Longitudinal Responses:

Phugoid and Short Period Modes

Typical Longitudinal Responses

- For a conventional aircraft configuration, the characteristic roots will normally occur in two complex pairs, as shown below.

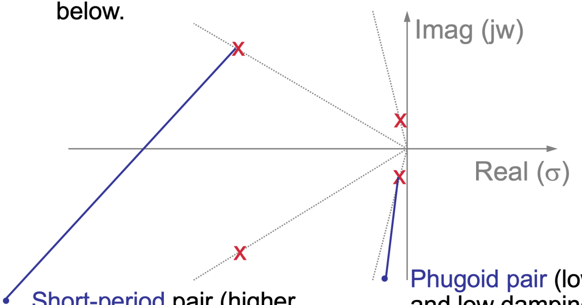


Fig.1

Short-period pair (higher frequency and more heavily damped) σ is much more negative

Phugoid pair (low frequency and low damping ratio) i.e. σ is not large and can even be positive for some flight conditions.

The Short-Period Mode

- The disturbance velocity in the transverse direction, w, helps to define the total incidence during the disturbance because of $\Delta\alpha = \mathrm{w} / \mathrm{U}$.

- Thus, whatever steady flight incidence there had been, a transverse disturbance of w will alter it and the longitudinal balance of forces will be lost.

- Similarly, a disturbance of the pitch balance will lead to pitching action and a pitch rate q. These two disturbances cause the wing and tail lifts to depart from the trimmed values and thus the consequent short-period mode is essentially a seeking of that trimmed state again. It does not take long to achieve and is generally quite heavily damped.

The Short-Period Mode

- The short period motion is thus composed largely of w (or α) and q with virtually no change in forward speed ($u \approx 0$ because $U \approx$ constant);

- the weather-cock action in the vertical plane is over so quickly that no significant changes in drag can occur to alter U.

- A typical period would be 1-4 seconds;

- This motion is important in Handling Quality evaluations because the pilot feels it and can react quickly enough to contribute. There is thus the possibility of a PIO (pilot-induced oscillation).

The Phugoid Mode

- This is primarily u and θ motion with small q and nearly zero w. The u and θ action can be envisaged as follows:

  a. a pitch disturbance ($\Delta\theta$) causes the aircraft to climb slowly,
  b. the forward speed U is caused to drop (u becomes negative)
  c. for a stable aircraft, such a loss of speed will cause a pitching moment (nose down) that tends to restore the trimmed condition, so the positive disturbance ($\Delta\theta$) tends to zero and then becomes negative after the aircraft reaches a peak departure in altitude from its former flight level,

The Phugoid Mode

d. the inevitable overshoot in θ because of the vehicle's pitch inertia, i.e. after $\Delta\theta$ goes negative, causes the aircraft to "start going downhill",

e. the speed slowly picks up (u now becoming obviously positive),

f. again, for the same reason as in c. above, the overall pitching moment becomes positive and begins to bring the nose up, so the negative θ turns to zero and eventually to positive values and the aircraft "starts going uphill",

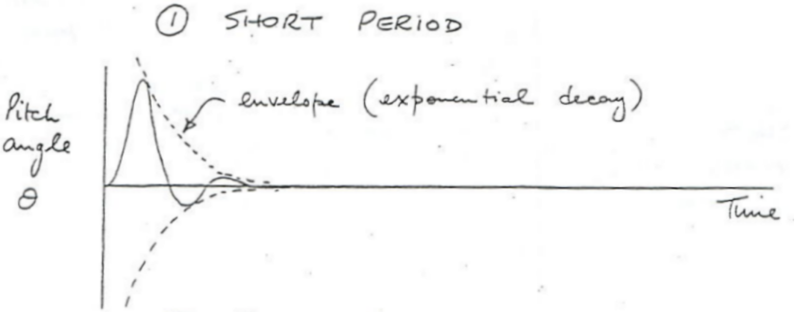
The Phugoid Mode

g. there will have been a noticeable loss of altitude and the climb back to original height begins, at small positive θ.

- The full cycle can involve changes in altitude of some hundreds of feet for a large aircraft, but the pitch action and the changes of forward speed are quite slow and the "roller-coaster" action is not necessarily obvious to a passenger.
- Periods are typically 15-100s and, even if $\sigma > 0$ the unstable motion is controllable by the pilot because of the long period (small frequency); in practice, an autopilot loop often suppresses the motion by raising the damping artificially.
- Kinetic vs Potential!

Typical Aircraft Response

- Aircraft response to a disturbance
- Note individual components
- Two sets of roots are well separated in the complex plane



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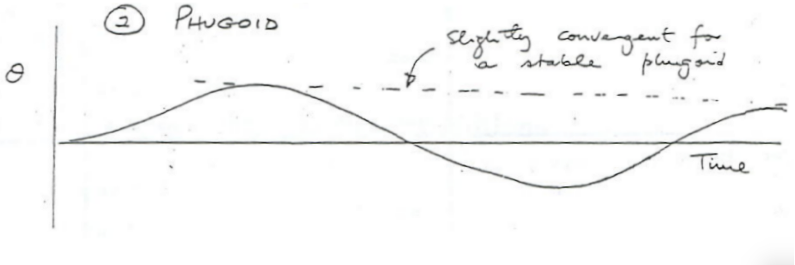

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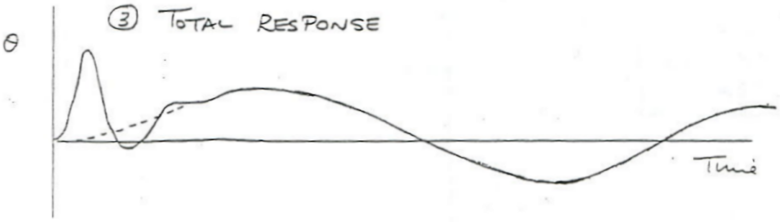


Root locus as CG is shifted aft

- h is the CG position relative to the leading edge; values shown on the loci are for static margin H_n.
- Instability occurs when H_n=0, i.e. starting when the first root crosses into the right half plane.
- The CG position affects the derivative M_w fairly significantly, although other derivatives may also vary.

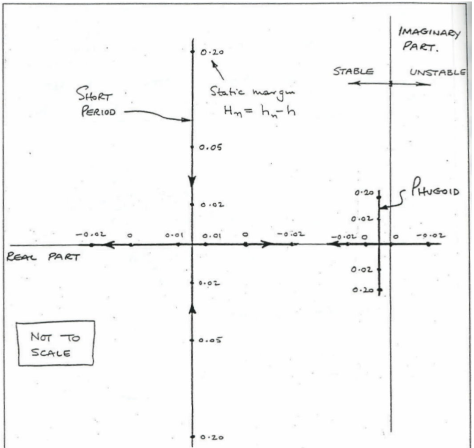


Fig. 3

Lateral Modes

Introduction

- The distinction between lateral freedoms and longitudinal freedoms of motion is based on what happens to the vertical plane of symmetry that passes through the centre of the fuselage.
- For longitudinal motions, the original orientation and disturbed orientation of that plane remain the same.
- For lateral motions, this vertical plane is displaced.

Aircraft Modes

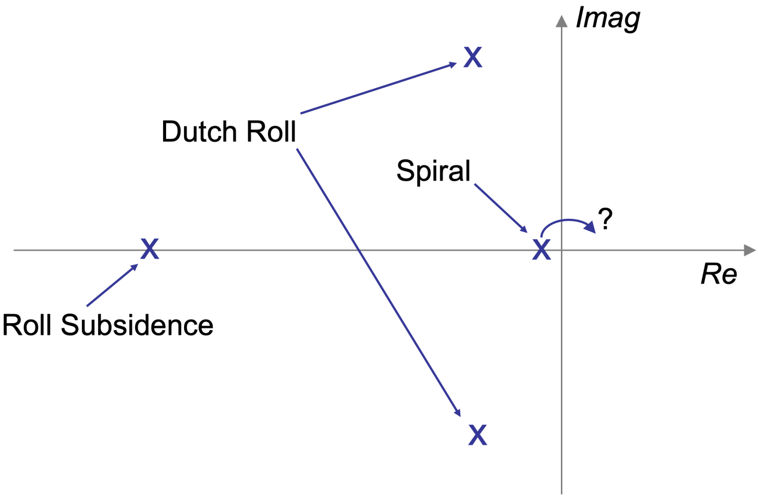
- Short Period
- Phugoid
- Roll Subsidence
- Spiral
- Dutch Roll

Longitudinal

Lateral

Variables?

Lateral Roots



Lateral Modes

1. Large negative real root - the **roll convergence** (***or roll subsidence***).

   This implies almost pure rolling motion and of course cannot last long in reality because after the first 90° you "fall out of the sky"!

   A more realistic attitude suggests that if a roll "pulse" hits the aircraft (e.g. a brief up-gust on one wing), the consequent response in roll would be heavily damped.

Lateral Modes

2. Small real root, of either sign - the **spiral mode**.

   This would be, for an unstable case, a slow divergence in yaw (say nose to starboard) while a roll angle built up (rolling to starboard) and thus also a sideslip would develop.

   The later stage would be a tightening spiral dive with all three motion variables involved.

   In practice, a pilot can control an unstable spiral mode (*slow!*).

Lateral Modes

3. The complex pair - **Dutch Roll**.

   Strictly speaking, all freedoms are active here, in an oscillatory sense, and out of phase with each other.

   This mode can be badly (though positively) damped and will affect Handling Qualities. The frequency is probably lower (the period a bit longer) than the longitudinal short period mode.

   Can be unpleasant if poorly damped! (nausea)

   Often poorly damped on swept wing aircraft

   Excited with the rudder or aileron

   Yaw damper

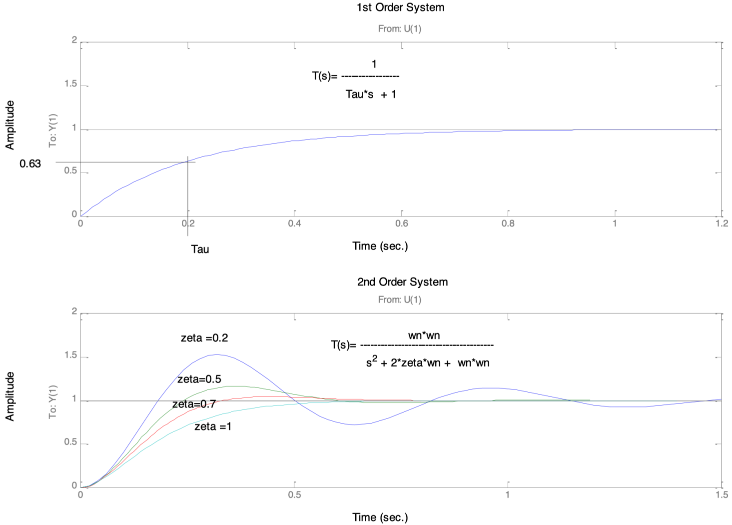
System Stability

System Stability

Introduction

- Measures of transient performance may include steady state errors, overshoot, settling time, etc…
- The dynamic characteristics which govern the response cannot be separated from the stability of the system as implied by its characteristic roots.

System Stability



System Stability

- A system is said to be stable if its impulse response tends to a finite value (often, but not necessarily, zero) as $t \rightarrow \infty$.

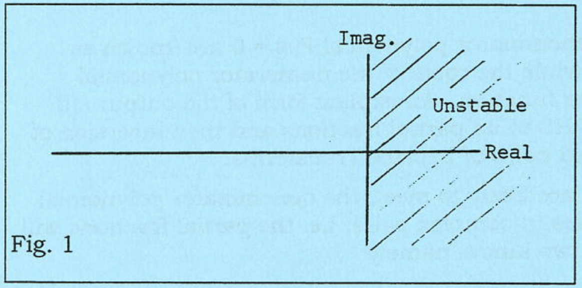


Fig. 1

System Stability

- Stability is ensured if all the characteristic roots have negative real parts, which commits them to a position in the left half of the (complex) root plane and for the general case,

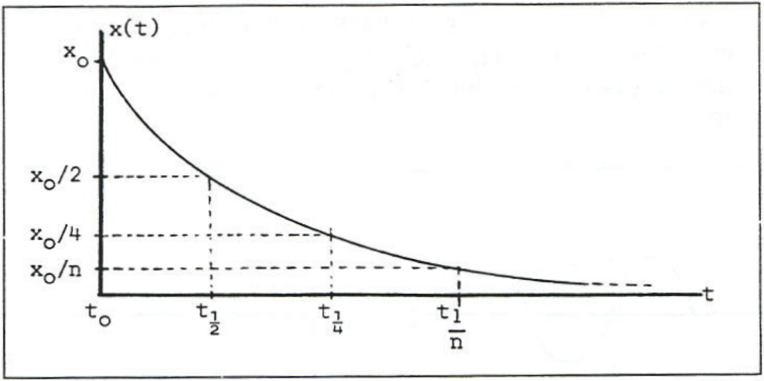
$$\lambda = \sigma \pm j\omega$$

- the **boundary** between stability and instability is $\sigma = 0$ for any root, real or complex (see Fig. 1).

Quantitative Measures of Stability

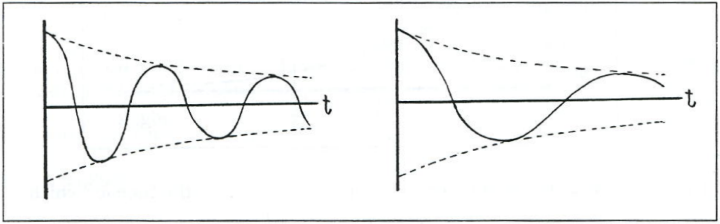
- The definition above for a stability boundary provides only a positive/negative judgement on stability
- though it does suggest that the (non-zero) value of σ could be a useful quantitative measure.
- In fact, σ gives the rate of decay of the exponential envelope whether the response component displays first- or second-order dynamics.
- Thus for constant σ we can calculate the time for a response component to decay to any required fraction of its "current" value.

Time-to-Fractional Amplitude



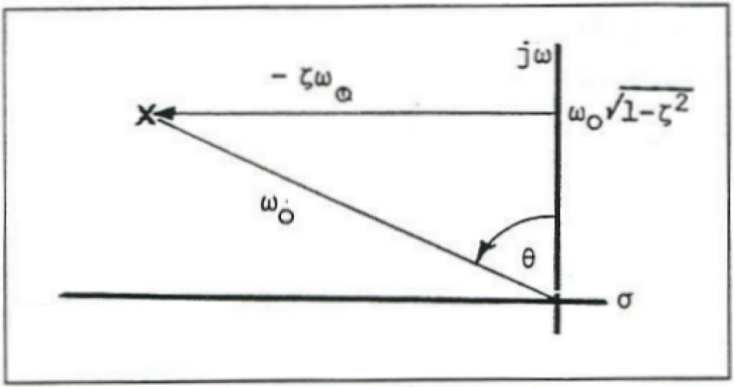
Logarithmic Decrement

- The criterion, $t_{1/2}$, provides a decay rate in time for real or complex roots and is thus independent of the frequency displayed by a complex pair.



Constant Damping Ratio ζ

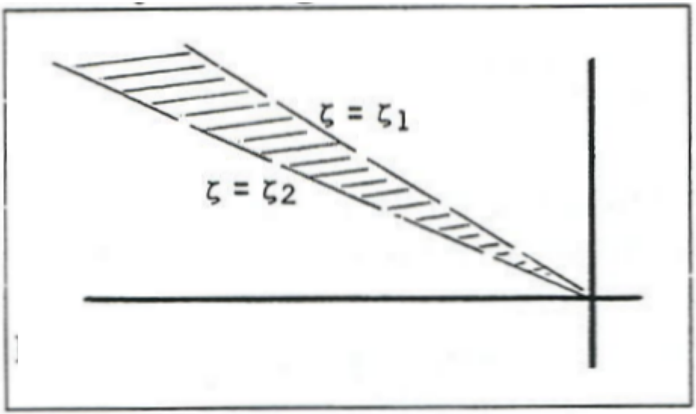
- Where $\lambda = \sigma \pm j\omega$



Constant Damping Ratio ζ

- We can show that:

$$\zeta = \sin\theta$$



Constant Damping Ratio ζ

- For ζ within a required range we must expect the poles to fall within a 'fan-shaped' band emanating from the origin.

- Note, the so-called "optimum damping" of an oscillatory component requires a pair of poles on the lines which are at 45° above and below the negative real axis, because

$$\zeta_{opt} = \frac{1}{\sqrt{2}}$$

Constant Undamped Natural Frequency

- The general expression for a pair of complex roots i.e.

$$\lambda = \sigma \pm j\omega = -\zeta\omega_o \pm j\omega_o\sqrt{1-\zeta^2}$$

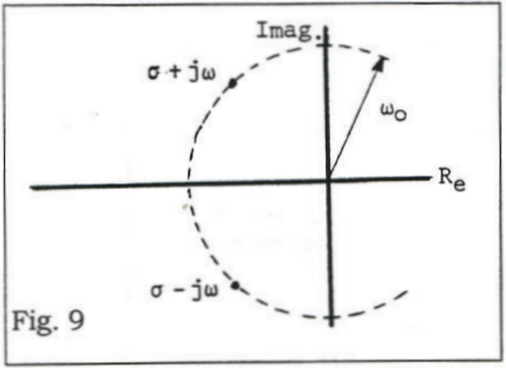
- leads naturally to:

$$\sigma^2 + \omega^2 = \zeta^2\omega_o{}^2 + \omega_o{}^2\left(1-\zeta^2\right) = \omega_o{}^2$$

- which is the equation of a circle in the root plane, centred at the origin and having a radius of ω_0.

Constant Undamped Natural Frequency

- Thus the locus of root positions for a complex pair, when only their damping ratio ζ is varied, is a (semi)circular arc of radius ω_0.



Desired region?