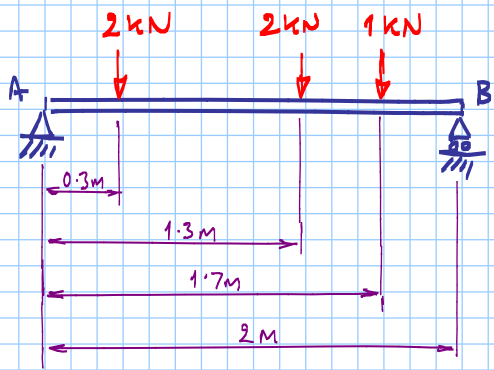
ASD2 Ex1 SS BEAM – PART: Simplified Model

IRF 24.1.2010 ①
22.11.2011

SIMPLY SUPPORTED BEAM:

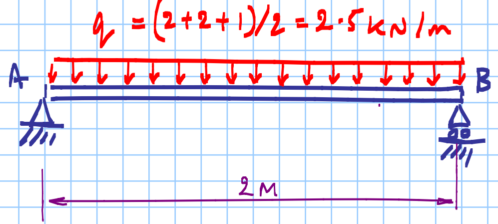


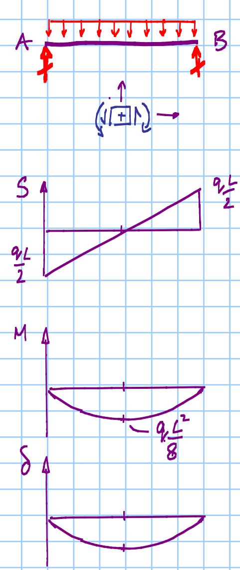
Could solve by M() fn (see STM 1) as multiple point load beam but it would be advisable to start with a simpler model to provide a useful check.

Simplified representation:

②

(i) Uniform Distributed Load model "Standard Beam"





$S_{max} = \pm \frac{qL}{2} = \frac{2.5 \times 2}{2} = \pm 2.5\,kN$ @ ends

$M_{max} = \frac{qL^2}{8} = \frac{2.5 \times 2^2}{8} = 1.25\,kNm$ @ centre

$v_{max} = \frac{5qL^4}{384EI} = \frac{5 \times 2.5 \times 2^4}{384\,EI} = \frac{0.521}{EI}\,m$ @ centre

IDF 7.4.04

26

23.2.09

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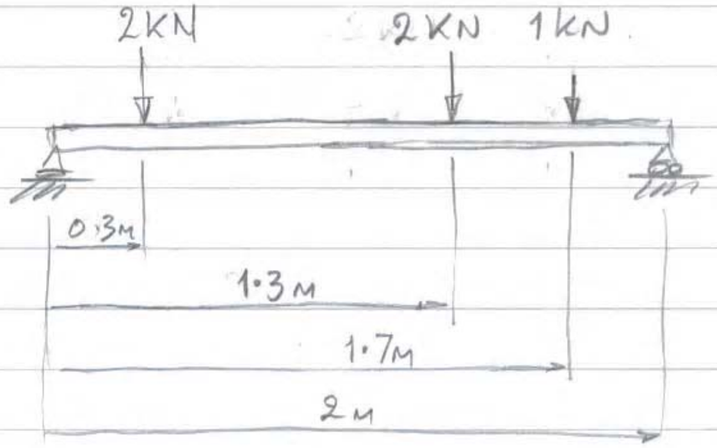
ASD2 Ex1 SS BEAM - PART: Full STM1 Model

(iii)

[STM1 @ 9

WORKED Ex]

Fuselage floor beam: SIMPLY SUPPORTED ENDS

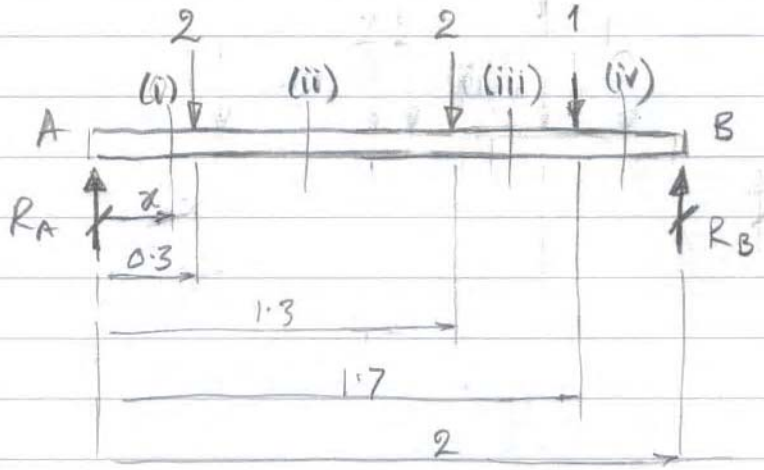


$E = 70'000\ N/mm^2$

$I = 1.5 \times 10^6\ mm^4$

a) SF + BM diagrams. Using $H()$ fn.

First for reactions @ supports
- consider FBD of complete structure:



Equilib'm of forces + moments:

$\Sigma \uparrow^{+} = 0: \quad R_A - 2 - 2 - 1 + R_B = 0$

$\hookrightarrow \quad R_A + R_B = 5$ ——— (1)

$\Sigma \overset{+}{\circlearrowleft}_A = 0: \quad -2 \times 0.3 - 2 \times 1.3 - 1 \times 1.7 + R_B \times 2 = 0$

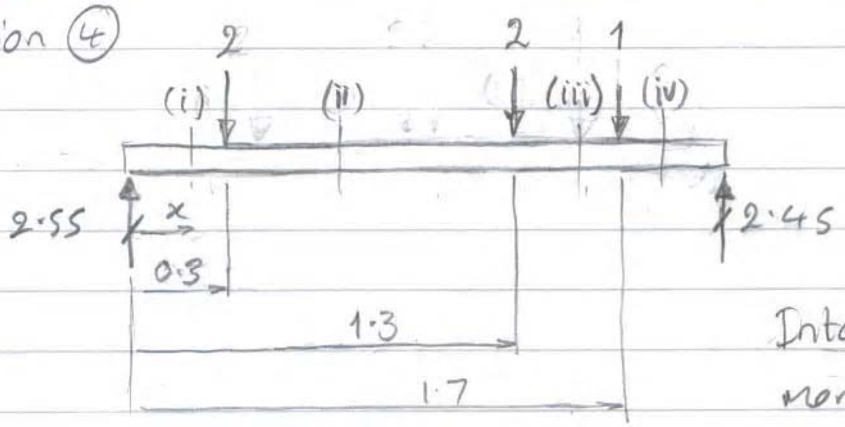
$\hookrightarrow \quad R_B = \dfrac{0.6 + 2.6 + 1.7}{2} = \underline{\underline{2.45\ kN}}$

IRF 6.1.2009

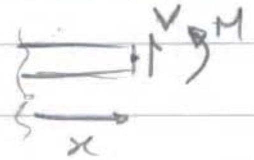
$\therefore$ ① $\rightarrow$ $R_A = \underline{\underline{2{\cdot}55\ kN}}$

Next, to reveal internal forces and moments we need to consider a section in each 'bay' of the beam and equilib^m of the FBD to the LHS (or RHS) of the section.

@ section ④



Internal forces + moments revealed @ each section



For SF, V, $\Sigma \uparrow^{+} = 0$:

@ (i) $2{\cdot}55 + V = 0 \rightarrow V = -2{\cdot}55$

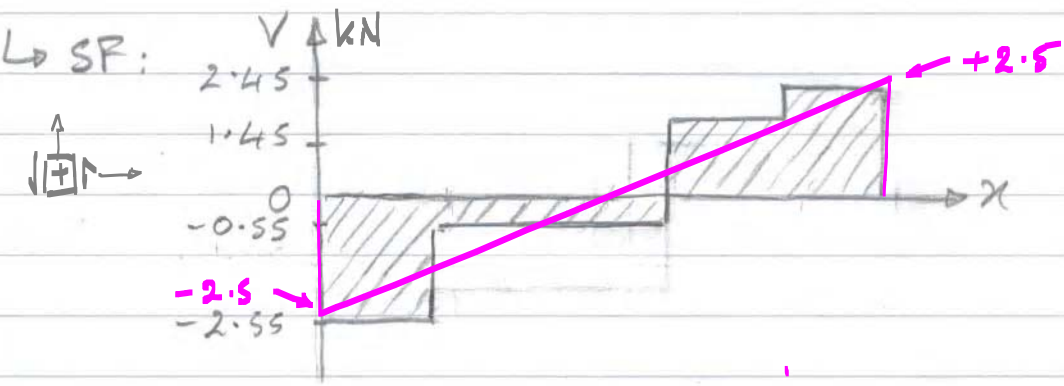
@ (ii) $2{\cdot}55 - 2 + V = 0 \rightarrow V = -0{\cdot}55$

@ (iii) $2{\cdot}55 - 2 - 2 + V = 0 \rightarrow V = 1{\cdot}45$

@ (iv) $2{\cdot}55 - 2 - 2 - 1 + V = 0 \rightarrow V = 2{\cdot}45.$

Could also use Heaviside fn for shear force expressions!

$\hookrightarrow$ SF:



* c/w simple UDL model ✓

For BM M $\Sigma \circlearrowright_{x} = 0$:

We can use Heaviside step functions and proceed to the end bay to develop a single representative expression:

$$-2.55x + 2(x-0.3) + 2(x-1.3) + 1(x-1.7) + M = 0.$$

$$\hookrightarrow M = 2.55x - 2(x-0.3)H(x-0.3) - 2(x-1.3)H(x-1.3) - 1(x-1.7)H(x-1.7)$$

So:

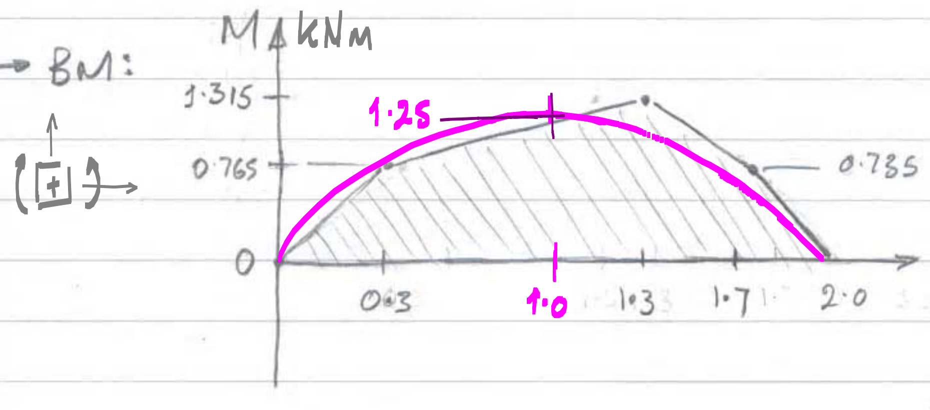
@ (i) $0 < x < 0.3$, $M = 2.55x$ — @ $x=0$, $M = 0$ kNm; $x = 0.3$, $M = 0.765$

@ (ii) $0.3 < x < 1.3$, $M = 2.55x - 2(x-0.3)$
$= 0.55x + 0.6$ linear — @ $x = 0.3$, $M = 0.765$; $x = 1.3$, $M = 1.315$

@ (iii) $1.3 < x < 1.7$, $M = 0.55x + 0.6 - 2(x-1.3)$
$= 0.55x + 0.6 - 2x + 1.6$
$= -1.45x + 3.2$ linear — @ $x = 1.3$, $M = 1.315$; $x = 1.7$, $M = 0.735$

@ (iv) $1.7 < x < 2$, $M = -1.45x + 3.2 - 1(x-1.7)$
$= -1.45x + 3.2 - x + 1.7$
$= -2.45x + 4.9$ linear — @ $x = 1.7$, $M = 0.735$; @ $x = 2$, $M = 0$ check ✓

$\hookrightarrow$ BM:



ie construct each section of BM diagram.

* c/w simple UDL model ✓

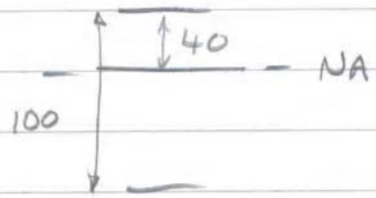
Specific to STM1 Ex. Not relevant here.

b) Position and magnitude of σ_{MAX}

Position = 1·3 m from LHS ie pos^n of max BM.

Magnitude $\sigma_{MAX} = \dfrac{M_{max}\, Y_{MAX}}{I}$

where Y_{max} = furthest distance from NA of cross section



I, Y given in STM1 ex but to be designed here

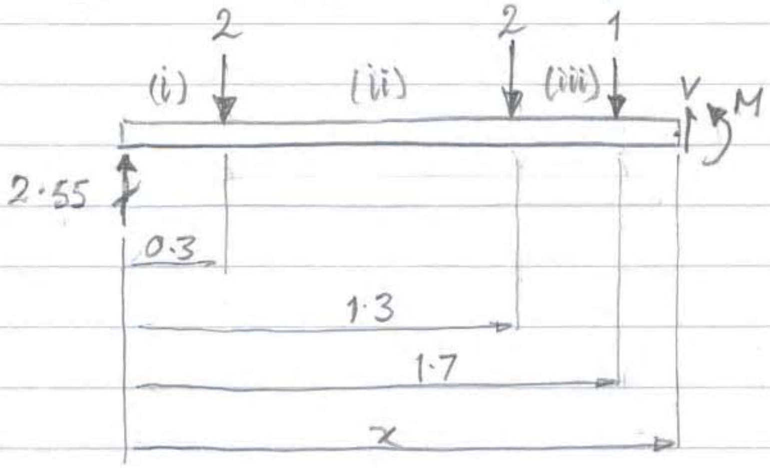
↳ $Y_{max} = 60$ mm, ie @ btm of beam section

$$\therefore \quad \sigma_{MAX} = \frac{\overbrace{1{\cdot}315\times1000\times1000}^{kNm \to Nmm}\times 60}{1{\cdot}5\times10^{6}} = \underline{\underline{52{\cdot}6\ N/mm^2}}$$

Note: using consistent units of N, mm.

c) Maximum deflection, ν_{MAX}

Obtain general expression for bending mom^t's considering end bay and using Heaviside functions



$\Sigma \overset{+}{\circlearrowleft} \textcircled{x} = 0: \quad -2.55.x + 2(x-0.3) + 2(x-1.3) + 1(x-1.7) + M = 0$

Rearranging and applying H fn:

$\hookrightarrow M = 2.55x - \underline{2(x-0.3)H(x-0.3)} - \underline{2(x-1.3)H(x-1.3)} - \underline{1(x-1.7)H(x-1.7)}$

Deflection v @ x can be obtained from diff'l equ^n:

$v'': K \quad EI\frac{d^2v}{dx^2} = M$ — ①

$= 2.55x - \underline{2(x-0.3)H(x-0.3)} - \underline{2(x-1.3)H(x-1.3)} - \underline{1(x-1.7)H(x-1.7)}$

$v': \theta \quad EI\frac{dv}{dx} = 2.55\frac{x^2}{2} - 2\frac{(x-0.3)^2}{2}H(x-0.3) - 2\frac{(x-1.3)^2}{2}H(x-1.3) - 1\frac{(x-1.7)^2}{2}H(x-1.7)$

$+ A$ — ②

$v \quad EIv = 2.55\frac{x^3}{6} - 2\frac{(x-0.3)^3}{6}H(x-0.3) - 2\frac{(x-1.3)^3}{6}H(x-1.3) - 1\frac{(x-1.7)^3}{6}H(x-1.7)$

$+ Ax + B$ — ③

- Applying boundary conditions to determine the constants of integration A, B:

@ $x = 0$, $v = 0$: ③ $\hookrightarrow \underline{\underline{B = 0}}$

@ $x = 2$, $v = 0$:

③ $\hookrightarrow 0 = 2.55 \times \frac{2^3}{6} - 2\frac{(2-0.3)^3}{6} \times 1 - 2\frac{(2-1.3)^3}{6} \times 1 - 1\frac{(2-1.7)^3}{6} \times 1 + A \times 2$

$= 2.55 \times \frac{8}{6} - 2 \times \frac{1.7^3}{6} - 2 \times \frac{0.7^3}{6} - 1 \times \frac{0.3^3}{6} + 2A$

$\hookrightarrow 2A = -3.4 + 1.638 + 0.114 + 0.0045$

$\underline{\underline{A = -0.822}}$ /

Now, position of max deflection occurs when $\frac{dv}{dx} = 0$

ie @ zero slope 

and considering loading on beam the max deflection is likely to occur in bay (ii)

ie $0.3 < x < 1.7$

— Using equ^n ②, setting $\theta = \frac{dv}{dx} = 0$ and setting H fns to 0 or 1 as appropriate, we get:

$$0 = \overset{1.275}{2.55\frac{x^2}{2}} - 2\frac{(x-0.3)^2}{2} \cdot 1 - 0 - 0 - 0.822$$

↳ $$1.275x^2 - (x^2 - 0.6x + 0.09) - 0.822 = 0$$

$$1.275x^2 - x^2 + 0.6x - 0.09 - 0.822 = 0$$

$$0.275x^2 + 0.6x - 0.912 = 0$$

Solving quadratic:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

↳ $$x = \frac{-0.6 \pm \sqrt{0.6^2 - 4 \times 0.275(-0.912)}}{2 \times 0.275}$$

$$= 1.032, \cancel{-3.214}$$

within range ∴ ok. reject '-ve root

↳ $\underline{\underline{x = 1.032\,m}}$ – ie position of max deflection from LHS.

* c/w $x = 1.0$ m assumed in simple UDL model. ✓

Finally for max deflection :

(3) ↳ $EI\,v_{max} = \frac{2.55(1.032)^3}{6} - \frac{2(1.032-0.3)^3}{6} \times 1 - 0 - 0 - 0.822 \times 1.032$

$= -0.512$

↳ $v_{max} = \frac{-0.512}{EI}$ * cw simple UDL model: $= \frac{0.521}{EI}$ ✓

Where $E = 70'000\ N/mm^2$ $= 70 \times 10^6\ kN/m^2$

$I = 1.5 \times 10^6\ mm^4$ $= 1.5 \times 10^{-6}\ m^2$

ie Convert to kN, m to be consistent with previous calcs.

I, Y given in STM1 ex but to be designed here

Specific to STM1 Ex. Not relevant here.

$\therefore\ v_{max} = \frac{-0.512}{70 \times 10^6 \times 1.5 \times 10^{-6}} = -4.875 \times 10^{-3}\ m$

ie $v_{max} \cong 4.88\ mm$ downwards

occuring @ $x = 1.032\ m$ from LHS.

1

Initial Stiffness check – at limit

Beam is likely to be "stiffness critical" ∴ start sizing for deflection

From Part model $v_{MAX} = \dfrac{0.512}{EI}$ m at limit   based on kN, M

Using 2024 T3 Alloy data in Aero h/b :

So, for alloy beam, $E = 72'000\ \text{N/mm}^2 = 72\ \text{kN/mm}^2 = 72\times10^6\ \text{kN/m}^2$

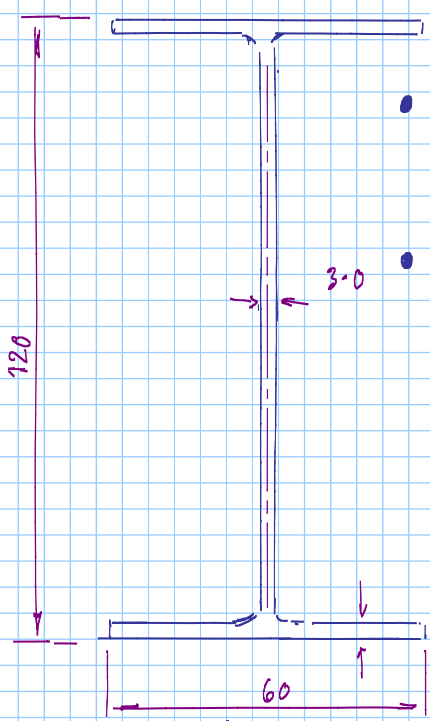
And deflection limit of $0.25\% = \dfrac{0.25}{100}\times 2 = 0.005\ \text{m} = 5\text{mm} = 0.005\text{M}$

$$I = \frac{0.512}{E\,v_{MAX}} = \frac{0.512}{72\times10^6\times 0.005} = 1.422\times10^{-6}\ \text{m}^4$$

$$= 1.422\times10^{6}\ \text{mm}^4$$

2

Trial I-beam (ss ends)



- Try $b_w = 100$, $2b_f = 50$, $t_w = t_f = 3$ mm
  ↳ $I \approx 2\times(50\times3)\times 50^2 = 0.75\times10^6\ \text{mm}^4$
- Try $b_w = 120$, $2b_f = 60$, $t_w = t_f = 3.0$ mm
  ↳ $I \approx 2\times(60\times3.0)\times60^2 = 1.296\times10^6\ \text{mm}^4$ ✓

Accounting for web : $+\ \dfrac{3.0\times120^3}{12} = 0.432\times10^6\ \text{mm}^4$

↳ $I = 1.728\times10^6\ \text{mm}^4$

(slightly high but cladding?) $= 1.728\times10^{-6}\ \text{m}^4$

Deflection check : $v = \dfrac{0.512}{72\times10^6\times1.728\times10^{-6}}$ $= 4.11\times10^{-3}\ \text{m}$
@ limit.   units kN, M $= 4.11\ \text{mm}$

ASD 2 Ex 1 SS BEAM – SECTION: Strength + Stability

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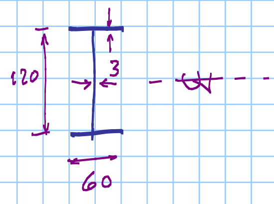
Initial Strength + Stability checks – at ultimate:

3

Simply Supported beam:

Current design, based on stiffness:



Max BM occurs @ $x = 1{\cdot}3\,m$ $M = 1{\cdot}315$ kNm at limit

Max SF " @ $x = 0, 2{\cdot}0\,m$ $S = 2{\cdot}45, -2{\cdot}55$ kN "

STRENGTH:

Flanges : check max direct stress: Int sign Conv'n (StM1)

from StM1 Sol'n.

$$\hookrightarrow \sigma = -\frac{My}{I} = -\frac{1{\cdot}5 \times 1{\cdot}315 \times 1000 \times 1000 \times (\pm 60)}{1{\cdot}728 \times 10^6} = \pm 68{\cdot}5 \text{ N/mm}^2$$

Using 2024 T3 Alloy data in Aero h/b:

$\sigma^*_{ULT} = 440$ N/mm² $\rightarrow RF = \frac{440}{68{\cdot}5}$; $\boxed{RF = 6{\cdot}4}$

assuming $\sigma_c^* = \sigma_t^*$

Webs : check ave shear stress :

$$\hookrightarrow \tau \approx \frac{S_{MAX}}{A_W} = \frac{1.5 \times (\pm 2.55 \times 1000)}{120 \times 3.0} = 10.6 \ \ N/mm^2$$

$$\tau^*_{ULT} = 255 \ \ N/mm^2 \qquad \longrightarrow \ RF = \frac{255}{10.6} : \ \boxed{RF = 24.1}$$

Checks suggest that the flanges and particularly the web are currently overdesigned. However, we need to check the stability before we alter our scheme.

STABILITY :

Flanges : Check max direct compressive stress :

$\sigma_c = 68.5 \ N/mm^2$ (upper flange) Note σ_c value assumed to be -ve

Critical direct compression buckling strength :

$$\sigma^*_{c_{CR}} = K_c \eta_c E \left(\frac{t}{b}\right)^2 \ ① \ = \frac{k_c \pi^2 \eta_c E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2 \ ②$$

Note $\frac{\pi^2}{12(1-0.3^2)} = 0.904$

Edge conditions, ends : ss/ss , sides ss / free and $a/b > 6$: asymptote (ss ≡ hinged)

Using Niu charts :
Niu, p458, Fig 11.3.1 curve ⑩ : $K_c = 0.4$
Bruhn C5.2, Fig C5.2 : $k_c \simeq 0.45$; $K_c = 0.41$
Aero h/b, p19 chart : $K_c = 0.385$

Using $K_c = 0.385$

If ally is clad apply correction factor $\lambda \times t$

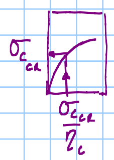
Using cladding correction for t, $3mm = 0.118''$, Niu Fig 11.2.6:

↳ $\lambda = 0.95$: $t = 0.95 \times t = 2.85$

① : $\dfrac{\sigma^*_{c_{CR}}}{\eta_c} = K_c E \left(\dfrac{t}{b}\right)^2 = 0.385 \times 72'000 \left(\dfrac{2.85}{30}\right)^2 = 250.2 \ N/mm^2$

Check Plasticity correction : predicted value close to $\sigma_{0.2\%} = 280 \ N/mm^2$

Using Niu chart, fig 11.2.4, reading $250.2 \ N/mm^2 = 36.3$ ksi on $\dfrac{\sigma_{c_{CR}}}{\eta_c}$ axis



2024 T3 → $\sigma_{c_{CRIT}} = 30.5$ ksi $= 210.3 \ N/mm^2$

↳ $RF = \dfrac{210.3}{68.5}$: $\boxed{RF = 3.1}$

<u>Webs</u> - Check shear stress

$\tau = 10.6 \ N/mm^2$

Critical shear buckling strength

$\tau_{CR} = K_s \eta_s E \left(\dfrac{t}{b}\right)^2$ ① $= \dfrac{k_s \pi^2 \eta E}{12(1-\nu^2)} \left(\dfrac{t}{b}\right)^2$ ②

Edge conditions : ends + sides SS , $\dfrac{a}{b} > 6$

Niu p460, Fig 11.3.5 curve ④ : $K_s = 5.0$
Bruhn C5.7, Fig C5.11 : $k_s = 5.3$: $K_s = 4.8$
Aero h/b, p20 chart : $K_s = 4.7$

} Using $K_s = 4.7$

① : $\dfrac{\tau_{CR}}{\eta_s} = K_s E \left(\dfrac{t}{b}\right)^2 = 4.7 \times 72'000 \left(\dfrac{2.85}{120}\right)^2 = 190.9 \ N/mm^2$

⑧

Estimate of τ^* from σ^*

Check plasticity correction, c/w $\tau_{0.2\%} \approx \dfrac{\sigma_{0.2\%}}{\sqrt{3}} = \dfrac{280}{\sqrt{3}} \approx 162\ N/mm^2$

– close, so check:

Using Niu chart 11.2.5 p457, $190.9\ N/mm^2 = 27.7\ ksi \equiv \dfrac{\tau_{cr}}{\eta_s}$

2024 T3 : $\hookrightarrow \tau_{cr} = 19\ ksi = 131\ N/mm^2$

$\hookrightarrow RF = \dfrac{131}{10.6}$: $\boxed{RF = 12.3}$

Currently, some oversizing but combined stresses not yet checked, etc.

At this stage we could iterate with with adjusted dimensions for more optimum RF's but only one or two iterations would be worthwhile because we need to refine our analysis.

⑨

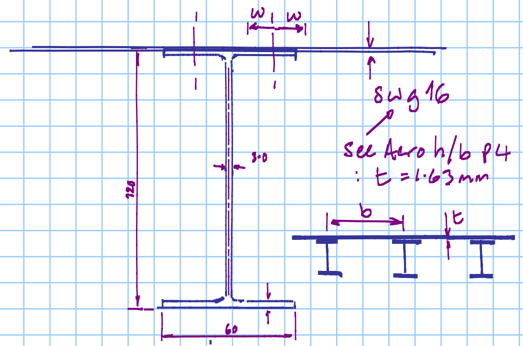
We could then consider new schemes, e.g. new section shapes [, ʃ ⅃⅂ etc accounting for ease of manufacture and connection to other components.

Here, for illustration, we will simply continue with the I-beam from our first initial sizing result, refining our checks to include combined stresses.

Refined Strength + Stability Checks - at ultimate

- Refined Areas: - Attached floor plate

We will assume that the I-beam is attached to a 16 swg 2024-T351 Ally floor plate with similar beams on each side with a b/t ratio > 110.



Assuming riveted attachment at a pitch to avoid inter-rivet buckling we can estimate an effective width of floor plate which can be assumed to carry end load with the floor beam.

From $\sigma_{c_{CR}} = K_c E \left(\frac{t}{b}\right)^2$ we can develop: $2w = b_E = t\sqrt{K_c}\sqrt{\frac{E}{\sigma_c}}$

where $K_c = 6.32$ for $\frac{b}{t} > 110$ ∴ $b_E \simeq 2.5\, t\sqrt{\frac{E}{\sigma_c}}$

w = effective plate width each side of fastener connection

E = Youngs modulus of plate = 72'000 N/mm² (Aero h/b)

σ_c = Limiting stress of stringer, usually the crippling stress

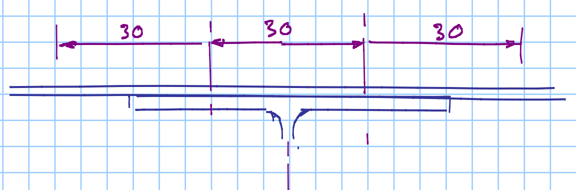
here we will use $\sigma_y = \sigma_{0.002} = 280$ N/mm²

$2w = b_E = 2.5 \times 1.63\sqrt{\frac{72'000}{280}} = 65$mm ∴ $w = 32.5$ mm ∴ Nominally reduce use $w = 30$ mm

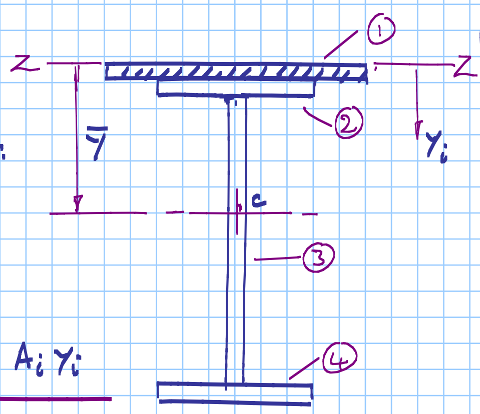
So for the two fasteners across the flanges, for a total flange width of 60 mm, taking edge distance as ~ 15 mm

we can claim $b_E = 2 \times 30 + 30 = 90$ mm

Note the effective width should not be double counted in overlap between fasteners.



Recalculating the 2nd moment of area:



First account for the new centroid of area:

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i}$$

i	A_i mm²		y_i mm		$A_i y_i$
1	90×1.63 =	147	1.63/2	= 0.815	119.8
2	60×3 =	180	1.63 + 3/2	= 3.13	563.4
3	3×114 =	342	1.63 + 120/2	= 61.63	21077.5
4	60×3 =	180	1.63 + 120 + 3/2	= 123.13	22163.4
Σ		849		188.70	43.824×10^3

Tabulate!

$\hookrightarrow \bar{y} = 51.6$ mm

Continuing tabulation for I:

i	A_i	y_i	$A_i y_i^2$	I_{z_i}
1	147	0.815	97.6	$90 \times 1.63^3/12 = 32.5$*
2	180	3.13	1763.4	$60 \times 3^3/12 = 135$*
3	342	61.63	1.299×10^6	$3 \times 114^3/12 = 370'386$
4	180	123.13	2.729×10^6	$60 \times 3^3/12 = 135$*
Σ			4.030×10^6	0.371×10^6

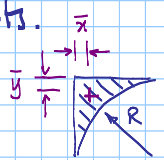
* Note negligible contributions

Σ: $I_{zz} = 4.401 \times 10^6$ mm⁴

// axis: $I_{NA} = I_{z_c} = I_{zz} - A\bar{y}^2 = 4.401 \times 10^6 - 849 \times 51.6^2 = \underline{2.140 \times 10^6}$ mm⁴

- Refined areas: Fillets.

E.g. circular fillet:



$A = 0.215 R^2$, $\bar{x} = \bar{y} = 0.223 R$

Ref Bruhn A3.3

Try $R = 5$ mm: $A = 0.215 \times 5^2 = 5.375$ mm², $\bar{x} = 0.223 \times 5 = 1.115$ mm

Consider as point areas for contribution to 2nd mmt of area value I

$\hookrightarrow I_f = 4 \times 5.375 \times (60 - 3 - 1.115)^2 = 0.0671 \times 10^6$ mm⁴

So refined 2nd mmt of area $I_{NA} = 2.140\times10^{6} + 0.0671\times10^{6}$

$= \underline{2.207\times10^{6}\ mm^4} = 2.207\times10^{-6}\ m^4$

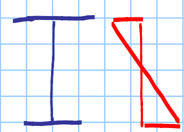
Deflection check: working in kN, M

$$v_{max} = \frac{0.512}{EI}\ m = \frac{0.512}{72\times10^{6}\times 2.207\times10^{-6}} = \underline{3.2\ mm} \text{ at Limit.}$$

– Suggests beam currently overdesigned for required stiffness

Refined Strength + Stability checks – at ultimate:

Check failure under combined direct and shear stress @ ult:



Direct stresses: $\sigma = -\frac{MY}{I}$ $I = I_{NA}$, Y = distance from NA

working in N, mm

Upr

Floor plate: $Y = 51.6$: $\sigma_{fp} = \frac{-1.5\times1.315\times10^{6}\times51.6}{2.207\times10^{6}} = -46.1\ N/mm^2$

Flange : $Y = 51.6 - 1.63 = 50.0$ mm : $\sigma_{fu} =$ " $\times 50.0 = -44.7\ N/mm^2$

Web : $Y = 51.6 - 1.63 - 3 = 47.0$ mm : $\sigma_{wu} =$ " $\times 47.0 = -42.0\ N/mm^2$

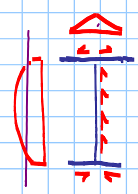
Lwr :

Web : $Y = 51.6 - 1.63 - 120 + 3 = -67.0$ mm : $\sigma_{w_L} =$ " $\times(-67.0) = +59.9\ N/mm^2$

Flange : $Y = 51.6 - 1.63 - 120 = -70.0$ mm : $\sigma_{F_L} =$ " $\times(-70.0) = +62.6\ N/mm^2$

Shear stresses :

#1 Starting from shear flow approximation: $q_w \simeq \frac{S}{b_w}$, and $\tau = \frac{q}{t}$



At ultimate $S = 1.5 \times (-2.55 \times 1000) = -3825\ N$

Accounting for the beam web only as a first estimate:

$q_w \simeq \frac{-3825}{120} = -31.9\ N/mm$ = approx' average shear flow in web

$\hookrightarrow\ \tau_w = \frac{31.9}{3} = -10.6\ N/mm^2$

$q_f \simeq \frac{q_w}{2} \simeq -16.0\ N/mm$ = approx peak shear flow in flange

$\hookrightarrow\ \tau_f = \frac{-16.0}{3} = -5.3\ N/mm^2$

Use these rough values to check refined estimates from shear flow equations:

$$q = \frac{SA\bar{y}}{I} \quad , \quad q = \frac{S}{I}\int t\,y\,ds$$

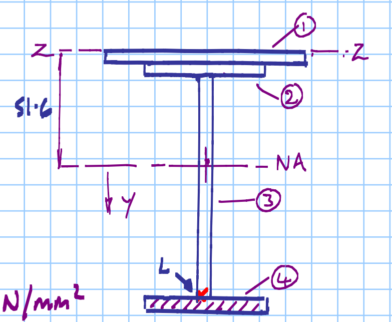
#2 Using $q_{\#} = \frac{SA\bar{y}}{I}$ where $q_{\#}$ = shear flow at position $y_{\#}$ from NA

A = area outside $y_{\#}$

$\bar{y}$ = position of centre of area outside $y_{\#}$ from NA

I = total section 2nd mmt of area.

@ Lwr web-flange intersection : "L"



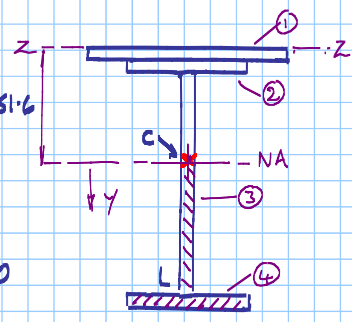
$A = 3 \times 60 = 180\ mm^2$

$\bar{y} = 1.63 + 120 - 51.6 - 3/2 = 68.5\ mm$

$q_{w_L} = \frac{-3825 \times 180 \times 68.5}{2.207 \times 10^6} = -21.4\ N/mm$: $\tau_{w_L} = \frac{21.4}{3} = -7.1\ N/mm^2$

Note: just measure distance y from NA as +ve in shear sum since no reversal occurs about N/A (in contrast to bending stress). I.e sense only determined by sense of transverse load.

@ NA on web : ie centroid "c"



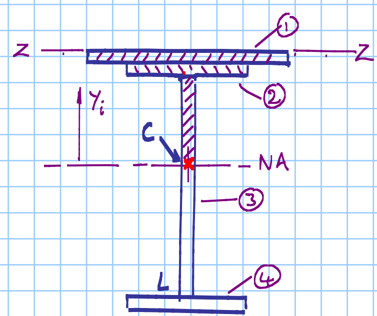
i	A_i		y_i		$A_i y_i$
4	3×60	$= 180$	$1{\cdot}63 + 120 - 51{\cdot}6 - 3/2$	$= 68{\cdot}5$	12'330
3	$3 \times (1{\cdot}63 + 120 - 51{\cdot}6 - 3)$	$= 201$	$(1{\cdot}63 + 120 - 51{\cdot}6 - 3)/2$	$= 33{\cdot}5$	6'733
Σ		381			19'063

$$\hookrightarrow \bar{y} = \frac{19'063}{381} = 50{\cdot}0$$

$$q_{w_c} = \frac{-3825 \times 381 \times 50{\cdot}0}{2{\cdot}207 \times 10^6} = -33{\cdot}0 \text{ N/mm} \quad : \quad \tau_{w_c} = \frac{-33{\cdot}0}{3} = -11{\cdot}0 \text{ N/mm}^2$$

Check considering opposite side.

@ Section NA portion on web : ie centroid "c"



i	A_i		y_i		$A_i y_i$
1	$1{\cdot}63 \times 90$	$= 147$	$51{\cdot}6 - 1{\cdot}63/2$	$= 50{\cdot}8$	7468
2	3×60	$= 180$	$51{\cdot}6 - 1{\cdot}63 - 3/2$	$= 48{\cdot}5$	8730
3	$3 \times (51{\cdot}6 - 1{\cdot}63 - 3)$	$= 141$	$(51{\cdot}6 - 1{\cdot}63 - 3)/2$	$= 23{\cdot}5$	3313
Σ		468			19'511

$$\hookrightarrow \bar{y} = \frac{19'511}{468} = 41{\cdot}7 \text{ mm}$$

$$q_{w_c} = \frac{-3825 \times 468 \times 41{\cdot}7}{2{\cdot}207 \times 10^6} = -33{\cdot}8 \text{ N/mm} \quad : \quad \tau_{w_c} = \frac{-33{\cdot}8}{3} = -11{\cdot}27 \text{ N/mm}$$

ok, some difference due to rounding approximations

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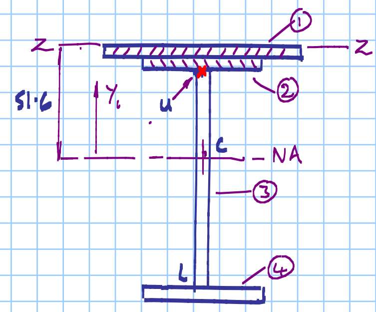
@ Upr web-flange intersection : "u"

i	A_i	y_i	$A_i y_i$
1	$1.63 \times 90 = 147$	$51.6 - 1.63/2 = 50.8$	7468
2	$3 \times 60 = 180$	$51.6 - 1.63 - 3/2 = 48.5$	8730
Σ	327		16'198

$\hookrightarrow \bar{y} = \frac{16'198}{327} = 49.5$ mm



$q_{w_u} = \frac{-3825 \times 327 \times 49.5}{2.207 \times 10^6} = -28.0$ N/mm ; $\tau_{w_u} = \frac{28.0}{3} = -9.3$ N/mm^2

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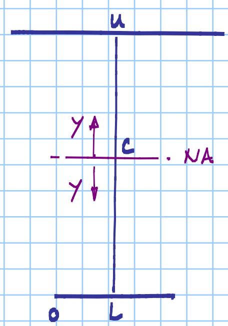
#3 Using $q_s = \frac{S}{I} \int t\, y\, ds$ Assuming thin wall approximations. Representing top flange and floor plate as a single line element. (The relative thicknesses can be accounted for when calculating shear stress $\tau = q/t$)

Note, this analysis illustrates that we are assuming that the floor plate ends at its effective width, suggesting that shear flow is zero at this position but obviously this is not actually true. This limitation also applies to the previous analysis methods. Results obtained are therefore still not rigorous, particularly regarding the top flange / floor panel shear distribution.



$q_s = \frac{-3825}{2.207 \times 10^6} \int t\, y\, ds = -1.733 \times 10^{-3} \int t\, y\, ds$

Using median thickness line as "thin wall" line

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Btm Flange 0-L : $y = 1.63 + 120 - 51.6 - 3/2 = 68.5$ (constant) :

$$q_{f_L} = -1.733\times10^{-3}\int_0^{30} 3.(68.5)\,ds = -0.356\times\left[s\right]_0^{30} = -10.7 \text{ N/mm} \quad : \tau_{f_a} = \frac{-10.7}{3}$$

$\hookrightarrow \tau_{f_L} = -3.6 \text{ N/mm}^2$

Web @ L

$$q_{w_a} = 2\times q_{f_a} = 2\times(-10.7) = -21.4 \text{ N/mm} \quad : \quad \tau_{w_L} = \frac{-21.4}{3} = -7.1 \text{ N/mm}^2$$

Web : @ C : $y = 68.5 - s$

$$q_{w_c} = 2\times(-10.7) - 1.733\times10^{-3}\int_0^{68.5} 3.(68.5 - s)\,ds$$

$$= -21.4 - 5.2\times10^{-3}\left[68.5s - \frac{s^2}{2}\right]_0^{68.5}$$

$$q_{w_c} = -21.4 - 24.4 + 12.2 = -33.6 \text{ N/mm} \quad : \quad \tau_{w_c} = -\frac{33.6}{3} = -11.2 \text{ N/mm}^2$$

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Top of web - as L-C but integrate on to u

$$q_{w_u} = -21.4 - 5.2\times10^{-3}\left[68.5s - \frac{s^2}{2}\right]_0^{117}$$

$120 - 2\times\frac{3}{2} = 117$

$$q_{w_u} = -21.4 - 41.7 + 35.6 = -27.5 \text{ N/mm} \quad : \quad \tau_{w_u} = \frac{-27.5}{3} = -9.2 \text{ N/mm}^2$$

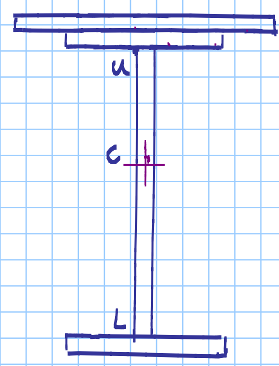
In the flanges at the intersection :

$$q_{f_u} = \frac{-27.5}{2} = -13.75 \text{ N/mm} \qquad : \tau_{f_u} = -\frac{13.75}{3} = -4.6 \text{ N/mm}^2$$

etc. Noting that the distribution along the floor plate assumes a false free edge at the edge of the effective width.

Comparing results from the three methods #1, 2, 3 above:

	#1 "$\frac{S}{b_w}$"		#2 "$\frac{SA\bar{y}}{I}$"		#3 "$\frac{S}{I}\int t\,y\,ds$"	
	q N/mm	τ N/mm²	q	τ	q	τ
f_u	−16·0	−5·3	−14·0	−4·6	−13·7	−4·6
w_u	−31·9	−10·6	−28·0	−9·3	−27·5	−9·2
w_c	−31·9	−10·6	−33·0 −33·8	−11·0 −11·3	−33·6	−11·2
w_L	−31·9	−10·6	−21·4	−7·1	−21·4	−7·1
f_L	−16·0	−5·3	−10·7	−3·6	−10·7	−3·6



f = flange, w = web, L = lwr, u = upr, c = centroid ie @ N/A.

Note, the simplest method gives reasonable results and would be acceptable for preliminary design. Here, having refined our calculations, we will continue with the integration method values.

Checking combined stress failure criteria:

STRENGTH:

Initial design of aircraft structures usually refers to ultimate strength at ultimate loads with checks confirmed by ultimate RF values.

Even so, we still need to check failure by yielding at proof loading. Approximately, we can check this by comparing the "ratio of ultimate to proof loading" with the "ratio of ultimate to proof strength".

E.g. here, assuming a proof factor of 1·0 (typical for civil aircraft) and referring to 2024 T351 alloy:

$$\frac{\sigma_{ULT}}{\sigma_{PROOF}} = \frac{1\cdot5}{1\cdot0} = 1\cdot5 \quad c/w \quad \frac{\sigma^*_{ULT}}{\sigma^*_{0\cdot002}} = \frac{440}{280} = 1\cdot57$$

So, our checks at ultimate should cover proof or yielding failure.

[In more detail we could also evaluate proof reserve factors i.e. Allowable proof strength / Applied proof stress]

Using "energy of distortion failure criterion":

$$\left(\frac{\sigma_x}{\sigma_0}\right)^2 + 3\left(\frac{\tau_{xy}}{\sigma_0}\right)^2 = FI \quad , \quad RF = \frac{1}{\sqrt{FI}}$$

- Upr flange @ ULT: $\sigma_{f_u} = -44\cdot7\ N/mm^2$, $\tau_{f_u} = -4\cdot6\ N/mm^2$ IRF 18.3.2006

"EoD" check: $\left(\frac{-44\cdot7}{-440}\right)^2 + 3\left(\frac{-4\cdot6}{-440}\right)^2 = 0\cdot01065$: $\boxed{RF = 9\cdot7}$

Note, ratios should always be +ve, ie always comparing t:t or c:c.
It is usual to assume tension and compression strengths are equal if only tension value is known.

- Lwr Web @ ULT: $\sigma_{w_L} = 59\cdot9$, $\tau_{w_L} = -7\cdot1$ Considering lower web since higher loading combination

"EoD" check: $\left(\frac{59\cdot9}{440}\right)^2 + 3\left(\frac{-7\cdot1}{-440}\right)^2 = 0\cdot0193$: $\boxed{RF = 7\cdot2}$

- Lwr flange @ ULT: $\sigma_{f_L} = 62\cdot6\ N/mm^2$, $\tau_{f_L} = -3\cdot6\ N/mm^2$

↳ $\left(\frac{62\cdot6}{440}\right)^2 + 3\left(\frac{-3\cdot6}{-440}\right)^2 = 0\cdot02044$: $\boxed{RF = 7\cdot0}$

STABILITY :

Using the combined stress failure criterion:

$$\left(\frac{\sigma_c}{\sigma_{c_{cr}}}\right) + \left(\frac{\sigma_b}{\sigma_{b_{cr}}}\right)^2 + \left(\frac{\tau}{\tau_{cr}}\right)^2 = FI \quad \text{and } RF = \frac{1}{\sqrt{FI}}$$

First calculate the critical buckling strengths.

- Flanges – Here, for the SS beam configuration and the single load case only the top flange is in compression. For other load cases the bottom flange may also be subjected to compression. For a FF beam configuration, even for the present load case the bottom flanges will be in compression at the ends due to reversed bending near the fixed ends.

$\sigma_{c_{cr}}$: In this illustration the dimensions are unchanged from the initial check so we already have $\sigma_{c_{cr}} = 210.3\ N/mm^2$ neglecting restraint from the floor plate.

$\sigma_{b_{cr}}$: Here, $\sigma_{b_{cr}}$ is not relevant to the flanges since stress due to bending is acting effectively as uniform end loading stress, ie as σ_c and there is no in-plane bending stress σ_b in the flanges.

τ_{cr} : We note from the buckling constant curves that τ_{cr} is not usually defined for a panel with a free edge. For our free edge flange panels we will use the condition of SS on all edges (which is unconservative regarding the free edge) and compare against the peak shear stress for the flange (which is conservative since it diminishes to zero at the free edge). The result is obviously not rigorous but should at least provide some account of shear buckling effect in the flanges.

τ_{cr}: For $a/b > 6$, Niu p460, Fig 11.3.5, curve 4 gives $K_s \simeq 5.0$

$$\hookrightarrow \quad \frac{\tau_{cr}}{\eta_s} = K_s E\left(\frac{t}{b}\right)^2 = 5.0 \times 72'000\left(\frac{2.85}{30}\right)^2$$

$$= 3'249\ N/mm^2 = 471\ ksi$$

2.85 = thickness with cladding correction.

This value is obviously greater than the proof strength so, referring to the plasticity correction curves for shear Niu p457, fig 11.2.5 for 2024 T3: using curve end value:

$$\tau_{cr} = 25.5\ ksi = 175.8\ N/mm^2$$

So combined stress check: $\left(\frac{44.7}{210.3}\right) + \left(\frac{4.6}{175.8}\right)^2 = 0.213$: $\boxed{RF = 2.16}$

Note values assumed to be compressive or like sence in failure index ratios.

- Web

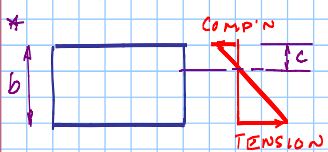
Again, since dimensions are unchanged from the initial check, here we already have $\tau_{cr} = 131\ N/mm^2$ for the web

For the loading case considered we note that we only have in-plane bending stress for the webs, ie σ_b and no σ_c. Note, that this may not be so for other loading cases, particularly when interaction with the fuselage frame is considered.

Applying our buckling formula:

$$\frac{\sigma_{b_{cr}}}{\eta_c} = K_b E\left(\frac{t}{b}\right)^2$$, read K_b from buckling chart Niu p461 fig.11.3.6

for curve 1, ss on all sides and $\frac{b^*}{c} = \frac{114}{(51.6-1.63-3)} = 2.43$



$\hookrightarrow K_b = 21$ using end value of curve (agrees with Bruhn and Also h/b curves)

$$\hookrightarrow \frac{\sigma_{b_{cr}}}{\eta_c} = 21 \times 72'000 \left(\frac{2.85}{114}\right)^2 = 945 \ N/mm^2 = 137 \ ksi$$

Checking plasticity correction - note η_c correction same as for $\sigma_{c_{cr}}$

so using Niu p 456, fig 11.2.4

$$\hookrightarrow \sigma_{b_{cr}} = 40 \ ksi = 276 \ N/mm^2$$

Finally, combined stress check: $\left(\frac{42}{276}\right)^2 + \left(\frac{9.3}{131}\right)^2 = 0.0282$: $\boxed{RF = 5.9}$

Review:

So far the lowest RF is 2.16 for upr flange buckling and the deflection has significant margin so it would be advisable to perform new trials with reduced sizing.

Also, we are combining the maximum stresses occuring at different positions and this will be conservative.