UNIVERSITY OF BRISTOL
FACULTY OF ENGINEERING

First Year Examination for the Degree of Master of Engineering

MAY/JUNE 2013 3 Hours

AENG11100

FLUIDS I

Solutions

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Q 1

$$p_1 - p_a = \rho_1 gh = 1000 \times 9.81 \times 2 = 19620$$
$$p_2 - p_a = \rho_2 gh = SG \times 1000 \times 9.81 \times 2 = SG \times 19620$$
$$p_1 - p_2 = 4000 = 19620(1 - SG) \quad \rightarrow \quad SG = 1 - \frac{4000}{19620} = 0.7961$$

(4 *marks*)

Q 2 For vertical component use “weight of water above”, in this case that means weight of water missing and thrust is upwards. For horizontal force use the pressure at the centre of the projected area times the projected area. In both cases use guage pressure as the atmospheric pressure acts on both sides of the gate.

$$F_v = -\left(\tfrac{1}{4}.\pi.3^2 + 1 \times 3\right) \times 4 \times 1000 \times 9.81 = -395091N$$
$$F_h = 3 \times 4 \times 1000 \times 9.81 \times 2.5 = 294300N$$

(4 *marks*)

Q 3 Along a streamline: Steady, incompressible, inviscid.
Neglecting hydrostatic pressure as the fluid is a gas

$$p_c = p_B + \tfrac{1}{2}\rho V_B^2$$

(4 marks)

Q 4 Consider that the static pressure changes across the streamlines by ρgh, the pitot measures static plus dynamic pressure so that:

$$P_{pitot\,@1} = p_a + \rho gh + \tfrac{1}{2}\rho V_1^2 \quad \rightarrow \quad V_1^2 = 2\frac{(P_0 - p_a)}{\rho} - 2gh = \frac{2 \times 95000}{1000} - 2 \times 9.81 \times 9 = 13.42$$

$$V_1 = 3.663ms^{-1}$$

Use continuity so

$$A_1 V_1 = A_2 V_2 \quad \rightarrow \quad V_2 = \frac{A_1 V_1}{A_2} = \frac{h_1 \times w \times V_1}{h_2 \times w} = \frac{h_1 \times V_1}{h_2} = 3 \times 3.663 = 10.99$$

Again the pitot measures static plus dynamic pressure so that

$$P_{pitot\,@2} = p_a + \rho gh_2 + \tfrac{1}{2}\rho V_2^2 \quad \rightarrow \quad P_{pitot\,@2} - p_a = 1000 \times \left(9.81 \times 3 + \tfrac{1}{2}3.663^2\right) = 36138.78$$

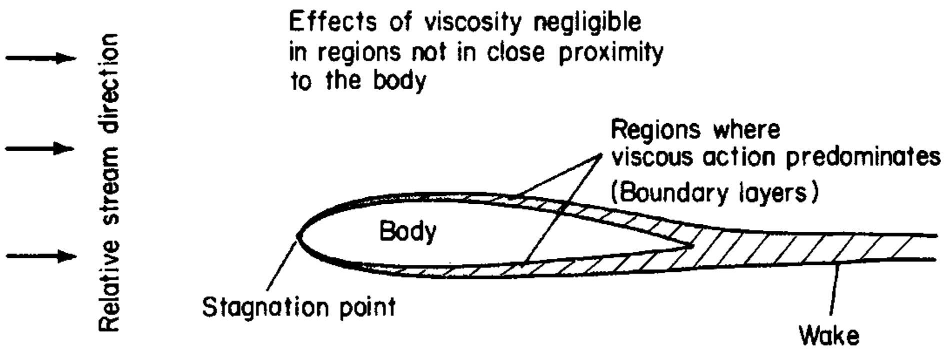
Note: the water surface is not horizontal (part of what drives the flow)

(*4 marks*)

Q 5 (a) The rounded leading edge is the shortest length that displaces the air by the required amount without sharp corners, thereby eliminating separation. Increase in skin friction drag caused by the increased surface area of the elongated trailing edge of the vehicle is outweighed by the reduction in form drag due to the reduction (or elimination) of separated flow.
Also note shrouded wheels
(b) Galilean transformation , static pressure and temperature remain unchanged

(*4 marks*)

Q 6 What I was looking for was:



As speed increases the boundary layer thins. Remember that as *Re* increases the viscous forces get smaller relative to the inertia forces. Also note that an increase in thickness is possible if the velocities considered were just before and after transition, however generally this is not the case and you should expect the boundary layer to thin.
As incidence increases flow reaches a point where boundary layer separates, by 20^o we would expect a large separation region (stalled) with viscous forces important far from the upper surface and with a wide wake.

(4marks)

Q 7 We are going through the streamline through the centreline, so we can use continuity because the profiles are the same. From continuity we know that the velocity upstream of the sudden contraction must be

$$A_1V_1 = A_2V_2 \quad \rightarrow \quad \frac{V_2}{V_1} = \frac{A_1}{A_2} = 4$$

Applying Bernoulli

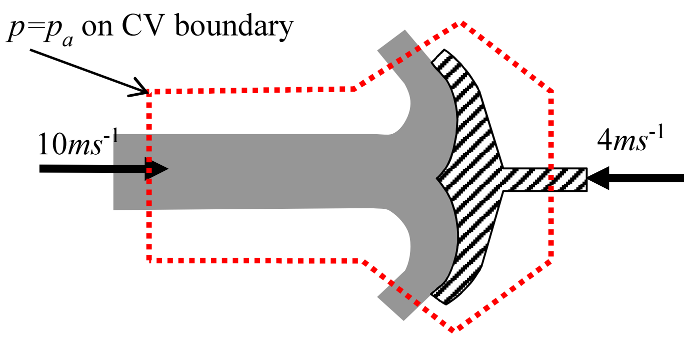
$$p_1 + \tfrac{1}{2}\rho V_1^2 + \rho g h_1 = p_2 + \tfrac{1}{2}\rho V_2^2 + \rho g h_2$$

$$p_1 = p_2 + \tfrac{1}{2}\rho V_2^2\left(1 - \frac{V_1^2}{V_2^2}\right) + \rho g(h_2 - h_1) = p_2 + \tfrac{1}{2}\rho V_2^2\left(1 - \frac{A_2^2}{A_1^2}\right) + \rho g(h_2 - h_1)$$

$$p_1 = 3\times10^5 + \tfrac{1}{2}\times1100\times5^2\left(1 - \frac{1}{16}\right) + 1100\times9.81\times2 = 334472.6 \text{ N/m}^2$$

(4 marks)

Q 8 Consider a control volume fixed relative to the plate. Effectively apply a Galilean transformation and work with a fixed plate and an inflow of 14ms^{-1}. The net horizontal force on the control volume equals the rate of change of momentum in that direction. We also assume that atmospheric pressure acts through the jet diameter so there is no contribution to the net horizontal force from the jet entry into the CV. Note that the force is unaffected by the flow splitting, it is all turned through 130^o .



$$F_{CVx} = \dot{m}(V_2 - V_1) = \pi r^2 V\rho(-V\cos 50^o - V) = -\pi\times0.1^2\times14\times1000\times(14\cos 50^o + 14) = -10115.5N$$

$$F = -F_{CVx} = 10115.5N$$

(4 marks)

Q 9 $u = \partial\psi/\partial y = -\omega y$ Integrating w.r.t $y \Rightarrow \psi = -\frac{1}{2}\omega y^2 + f(x)$

Differentiating w.r.t x $\Rightarrow \partial\psi/\partial x = df(x)/dx = -\omega x$

Integrating w.r.t x $\Rightarrow$ $f(x) = -\frac{1}{2}\omega x^2 + const$ so $\psi = -\frac{1}{2}\omega(y^2 + x^2)$

Eqn of streamline $\psi = const = -\frac{1}{2}\omega(y^2 + x^2)$ $\Rightarrow$ $y^2 + x^2 = const$

(4 marks)

Q 10 From given equations, doublet provides horizontal & vertical velocities:

$$u = \frac{-\kappa}{2\pi}\frac{(x^2 - y^2)}{(x^2 + y^2)^2}, \qquad v = \frac{-\kappa}{2\pi}\frac{2xy}{(x^2 + y^2)^2}$$

Stagnation point at leading edge, placing origin at the centre of the doublet then

$$0 = U_\infty - \frac{\kappa}{2\pi}\frac{1}{x^2}, \qquad x^2 = \frac{\kappa}{2\pi}\frac{1}{U_\infty} = \frac{128\pi}{2\pi}\frac{1}{4} = 16 \qquad x = 4$$

Flow is that over a hemisphere so height = 4m

Velocities at top therefore

$$u = 4 + \frac{-128\pi}{2\pi}\frac{(-4^2)}{(4^2)^2} = 8ms^{-1}, \qquad v = 0$$

(4 marks)

Q11 Note: Cannot use *pv=constant* as temperature changes with altitude.

(a) $\rho_H = \dfrac{mass}{volume} = \dfrac{m_H}{\frac{4}{3}\pi r^3}$

As mass of Hydrogen inside the balloon is constant, we can compare initial values (subscript sl) and values during the assent

$$\frac{\rho_H}{\rho_{H_{sl}}} = \frac{m_H}{\frac{4}{3}\pi r^3} \Big/ \frac{m_H}{\frac{4}{3}\pi r_{sl}^3} = \frac{r_{sl}^3}{r^3}$$

If we assume that the temperature and pressure remain equal internal and external to the balloon then using the equation of state for a perfect gas (hydrogen and air):

$$p = \rho_H R_H T = \rho R T$$

Note: We can only use the perfect gas equation for the hydrogen not the balloon as a whole.

$$p_{sl} = \rho_{H_{sl}} R_H T_{sl} = \rho_{sl} R T_{sl}$$

$$\frac{\rho_H}{\rho} = \frac{\rho_{H_{sl}}}{\rho_{sl}} \quad \rightarrow \quad \frac{\rho_H}{\rho_{H_{sl}}} = \frac{\rho}{\rho_{sl}}$$

Combining these relationships gives

$$\frac{\rho}{\rho_{sl}} = \frac{r_{sl}^3}{r^3}$$

If the total mass of the balloon is given by m_B, then the net upwards force F is given by

$$F = B - m_B g$$

$$F = \tfrac{4}{3}\pi r^3 \rho g - m_B g$$

$$F_{sl} = \tfrac{4}{3}\pi r_{sl}^3 \rho_{sl} g - m_B g$$

$$r_{sl}^3 \rho_{sl} = r^3 \rho$$

$$F_{sl} = \tfrac{4}{3}\pi r^3 \rho g - m_B g = F$$

(6 marks)

(b) The density of the displaced air is also given by the perfect gas state equation $p = \rho R T$.
Again note that we cannot use *pv=constant* as temperature changes with altitude
Applying the given relationships

$$\rho_{sl} = \frac{p_{sl}}{R T_{sl}} \qquad \rho = \frac{p_{sl}\left(1 - \lambda z / T_{sl}\right)^{\frac{g}{R\lambda}}}{R\left(T_{sl} - \lambda z\right)}$$

$$\frac{\rho}{\rho_{sl}} = \frac{T_{sl}\left(1 - \lambda z / T_{sl}\right)^{\frac{g}{R\lambda}}}{\left(T_{sl} - \lambda z\right)} = \frac{\left(1 - \lambda z / T_{sl}\right)^{\frac{g}{R\lambda}}}{\left(1 - \lambda z / T_{sl}\right)} = \left(1 - \lambda z / T_{sl}\right)^{\frac{g}{R\lambda} - 1}$$

$$\frac{\rho}{\rho_{sl}} = \left(1 - \lambda z / T_{sl}\right)^{\frac{g}{R\lambda} - 1} = \left(\frac{r_{sl}}{r}\right)^3$$

$$z = \frac{T_{sl}}{\lambda}\left(1 - \left(\frac{r_{sl}}{r}\right)^{\frac{3R\lambda}{g - R\lambda}}\right)$$

$$z = \frac{293}{0.0065}\left[1 - \left(\frac{1}{3}\right)^{\frac{3\times 287\times 0.0065}{9.81 - 287\times 0.0065}}\right] = \frac{293}{0.0065}\left[1 - \left(\frac{1}{3}\right)^{0.70445}\right] = 24287.3$$

$$z = 24287.3m$$

(7 marks)

(c) At terminal velocity

$$Drag = Buoyancy - Weight$$

$$Drag = \tfrac{1}{2}\rho V^2 \pi r^2 C_D = F_{sl} = \tfrac{4}{3}\pi r_{sl}^3 \rho_{sl} g - m_B g$$

$$V^2 = \frac{\tfrac{4}{3}\pi r_{sl}^3 \rho_{sl} g - m_B g}{\tfrac{1}{2}\pi r^2 \rho C_D}$$

Using $r_{sl}^3 \rho_{sl} = r^3 \rho \qquad \rightarrow \qquad r^2 \rho = \frac{r_{sl}^3}{r}\rho_{sl}$

$$V^2 = \frac{r}{r_{sl}} \frac{\tfrac{4}{3}\pi r_{sl}^3 \rho_{sl} g - m_B g}{\tfrac{1}{2}\pi r_{sl}^2 \rho_{sl} C_D}$$

$$V^2 = \frac{1}{3} \frac{\tfrac{4}{3}\pi \times 2^3 \times 1.2 \times 9.81 - 30 \times 9.81}{\tfrac{1}{2}\pi \times 2^2 \times 1.2 \times 0.45} = 9.8424$$

$$V = 3.1373 ms^{-1}$$

(7 marks)

Q12

(a) Applying Bernoulli's equation between the surface and the exit of the siphon, taking the vertical height as zero at the siphon exit. Note that the exit pressure is atmospheric and the surface velocity is assumed zero

$$p_a + \rho g H = p_a + \tfrac{1}{2}\rho V_e^2$$

$$V_e = \sqrt{2gH}$$

$$\dot{m} = \rho A\sqrt{2gH}$$

Applying Bernoulli's equation between the exit and the highest point of the siphon

$$p_h + \rho g(H+h) + \tfrac{1}{2}\rho V_h^2 = p_a + \tfrac{1}{2}\rho V_e^2$$

From continuity

$$AV_h = AV_e \qquad \rightarrow \qquad V_h = V_e$$

Substituting back into Bernoulli's equation

$$p_h + \rho g(H+h) = p_a + \tfrac{1}{2}\rho V_e^2(1-1) = p_a$$

$$h = \frac{p_a - p_h}{\rho g} - H$$

At the maximum point $p_h = p_v$

$$h_{\max} = \frac{p_a - p_v}{\rho g} - H$$

Note: Do not use the static equations to get $h_{\max}$, fluid statics seems to work but that is a coincidence because the area of the pipe doesn't change and the dynamic terms cancel. Consider what would happen if the area at the exit were half or twice the area at $h_{\max}$.

(13 marks)

(b) Applying values to equation for maximum height

$$3 = \frac{1.02\times 10^5 - 2330}{1000\times 9.81} - H \qquad \rightarrow \qquad H = 7.16$$

From previously, velocity at the exit given by

$$V_e = \sqrt{2gH} = \sqrt{2\times 9.81\times 7.16} = 11.8524$$

Applying Bernoulli's equation between the exit and a point 1m below the exit ($p=p_a$ throughout)

$$p_a - \rho g\times 1 + \tfrac{1}{2}\rho V_1^2 = p_a + \tfrac{1}{2}\rho V_e^2$$

$$V_1 = \sqrt{V_e^2 + 2g} = \sqrt{11.8524^2 - 2\times 9.81} = 12.653 ms^{-1}$$

Applying continuity

$$A_1 V = AV_e \qquad \rightarrow \qquad A_1 = \frac{AV_e}{V} = 0.0013\times\frac{11.8424}{12.653} = 0.001218 m^2$$

(7 marks)

Q13

(a) From the conservation of mass between station A and station B we have,

$$\int_{r=0}^{R}\int_{\theta=0}^{2\pi}\rho U_A r d\theta\, dr = \int_{r=0}^{R}\int_{\theta=0}^{2\pi}\rho U_B r d\theta\, dr$$

assuming incompressible flow

$$\int_{r=0}^{R}\int_{\theta=0}^{2\pi}U_A r d\theta\, dr = \int_{r=0}^{R}\int_{\theta=0}^{2\pi}U_B r d\theta\, dr$$

Using given equation for U_B

$$\int_{r=0}^{R}\int_{\theta=0}^{2\pi}U_A r d\theta\, dr = \int_{r=0}^{R}\int_{\theta=0}^{2\pi}U_{\max} r\left(\frac{r^3}{R^3}-3\frac{r^2}{R^2}+3\frac{r}{R}\right)d\theta\, dr$$

$$U_A\int_{r=0}^{R}\int_{\theta=0}^{2\pi}r d\theta\, dr = U_{\max}\int_{r=0}^{R}\int_{\theta=0}^{2\pi}\left(\frac{r^4}{R^3}-3\frac{r^3}{R^2}+3\frac{r^2}{R}\right)d\theta\, dr$$

integrating

$$U_A\cdot\pi R^2 = U_{\max}\int_{\theta=0}^{2\pi}\left[\left(\frac{r^5}{5R^3}-3\frac{r^4}{4R^2}+3\frac{r^3}{3R}\right)\right]_{r-0}^{R}d\theta$$

$$U_A\cdot\pi R^2 = U_{\max}\cdot 2\pi\left(9\frac{R^2}{20}\right)$$

$$U_{\max} = U_A\left(\frac{10}{9}\right)$$

$$U_{\max} = U_A\frac{10}{9} = 11.111 ms^{-1}$$

(10 marks)

(b)

Taking a control volume from A to B and just inside wind tunnel walls, consider conservation of momentum in the streamwise direction. Note that the pressure on the CV parallel to the wind tunnel walls has no component in the Drag direction and so is neglected.

$$-D+\pi R^2\times p_A-\pi R^2\times p_B = \int_{\theta=0}^{2\pi}\int_{r=0}^{R}\rho U_B^2 r dr d\theta - \int_{\theta=0}^{2\pi}\int_{r=0}^{R}\rho U_A^2 r dr d\theta$$

$$-D+\pi R^2\times(p_A-p_B) = \rho\int_{\theta=0}^{2\pi}\int_{r=0}^{R}U_{\max}^2 r\left(\frac{r^3}{R^3}-3\frac{r^2}{R^2}+3\frac{r}{R}\right)^2 dr d\theta - \rho U_A^2\pi R^2$$

$$-D+\pi R^2\times(p_A-p_B) = -\rho U_A^2\pi R^2 + \rho U_{\max}^2\int_{\theta=0}^{2\pi}\int_{r=0}^{R}r\left(\frac{r^6}{R^6}-6\frac{r^5}{R^5}+15\frac{r^4}{R^4}-18\frac{r^3}{R^3}+9\frac{r^2}{R^2}\right)dr d\theta$$

$$-D+\pi R^2\times(p_A-p_B) = -\rho U_A^2\pi R^2 + \rho 2\pi U_{\max}^2\left(\frac{R^2}{8}-6\frac{R^2}{7}+15\frac{R^2}{6}-18\frac{R^2}{5}+9\frac{R^2}{4}\right)$$

$$-D+\pi R^2\times(p_A-p_B) = -\rho U_A^2\pi R^2 + \rho\pi U_{\max}^2 R^2\left(\frac{351}{420}\right)$$

The pressure difference measured in the manometer is given by the hydrostatic equation

$$p_A = p_B + \rho_w gh$$

$$p_A - p_B = \rho_w gh$$

Substituting the pressure difference and $U_{\max}$ into the CV equation and rearranging

$$D = \pi R^2\rho_w gh + \pi R^2\rho U_A^2 - \pi\rho U_A^2\left(\frac{10}{9}\right)^2 R^2\left(\frac{351}{420}\right) = \pi R^2\rho_w gh - \pi R^2\rho U_A^2\left(\frac{2}{63}\right) = \pi R^2\left[\rho_w gh - \rho U_A^2\left(\frac{2}{63}\right)\right]$$

For this case

$$D = \pi \times 0.5^2 \times \left(1000 \times 9.81 \times 0.003 - 1.2 \times 100 \times \left(\frac{2}{63} \right) \right)$$

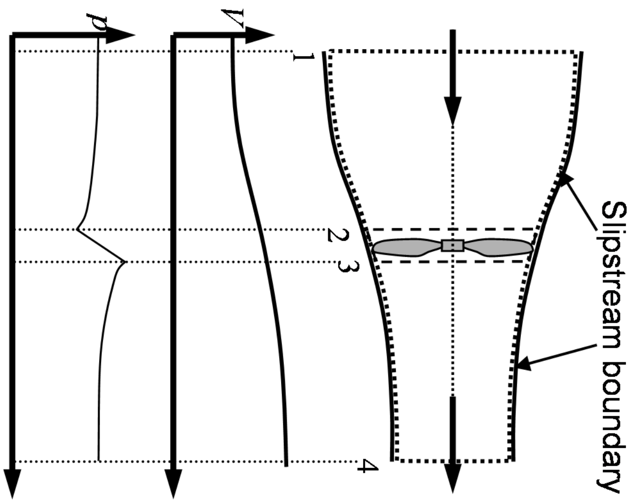
$$D = \pi \times 0.25 \times \left(29.43 - \left(\frac{240}{63} \right) \right)$$

$$D = \pi \times 0.25 \times (25.6205)$$

$$D = 20.1223 N$$

(10 marks)

Q14 **(a)** Use the actuator disc theory for an ideal propeller, see figure below



Consider the Galilean transformation so that the rotor is fixed. In this case the inflow velocity is now $V_1=V$ and the downstream velocity is $V_4=aV$

Assumptions: Frictionless & incompressible, Steady 1D flow (neglect rotation and variation across the disc radius). Actuator disc is thin so $A_2=A_3=A_d$ & $V_2=V_3=V_d$. $p=p_a$ at all points on slipstream boundary & 1 & 4

Continuity: $Q=V_dA_d$

Bernoulli's equation for CV 1-2 & CV 3-4

$$\begin{aligned} p_1+\tfrac{1}{2}\rho V_1^2 &= p_2+\tfrac{1}{2}\rho V_d^2 \\ p_3+\tfrac{1}{2}\rho V_d^2 &= p_4+\tfrac{1}{2}\rho V_4^2 \end{aligned} \quad \rightarrow \quad p_3-p_2=\tfrac{1}{2}\rho\left(a^2V^2-V^2\right)$$

Steady Flow momentum for CV 2-3:

$$(p_2-p_3)A_d+F_{CV}=\rho Q(V_d-V_d)=0 \qquad \rightarrow \quad F_{CV}=(p_3-p_2)A_d$$

Where F is the force on the control volume

Applying results from Bernoulli's equation above

$$F=\tfrac{1}{2}\rho A_dV^2\left(a^2-1\right)=\tfrac{\pi}{8}\rho d^2V^2\left(a^2-1\right)$$

(8 marks)

(b) Steady Flow momentum for CV 1-4:

$$0+F=\rho Q(V_4-V_1) \qquad \rightarrow \quad F=\rho V_dA_dV(a-1)$$

From momentum & continuity

$$(p_3-p_2)A_d=\rho V_dA_d(V_4-V_1)$$

Eliminating (p_3-p_2) using Bernoulli's equation above

$$\rho V_d(aV - V) = \tfrac{1}{2}\rho\left(a^2V^2 - V^2\right)$$
$$V_dV(a-1) = \tfrac{1}{2}V^2(a+1)(a-1)$$
$$V_d = \tfrac{1}{2}V(a+1)$$

The power supplied to the disc is

$$P_{disc} = FV_d = \rho QV(a-1)V_d$$

Power output

$$P_{out} = FV = \rho QV^2(a-1)$$

The efficiency of the rotor is therefore (remember efficiency for turbines and propellers is not the same)

$$\eta = \frac{P_{\text{out}}}{P_{\text{disc}}} = \frac{FV}{FV_d} = \frac{V}{\frac{1}{2}V(a+1)} = \frac{2}{a+1}$$

(7 marks)

(c) The factor *a* does not make sense here as the inflow velocity is zero, so for a stationary rotor we have

$$V_1 = 0 \quad \rightarrow \quad V_4 = 2V_d$$
$$F = \rho A_d V_d(2V_d - 0) = 2\rho A_d V_d^2$$
$$V_d = \sqrt{F/2\rho A_d}$$
$$V_d = \sqrt{\frac{7000 \times 9.81}{2 \times 1.2 \times \frac{\pi}{4} \times 14^2}} = \sqrt{185.87} = 13.63ms^{-1}$$

Ideal power input is

$$FV_d = F\sqrt{F/2\rho A_d} = F^{\frac{3}{2}}\Big/\sqrt{2\rho A_d}$$

Just use $FV_d = 7000 \times 9.81 \times 13.63 = 936.21kW$

(5 marks)

Q15

(a) The velocity components are given by

$$V_r = \left(1 - \frac{\kappa}{2\pi U_\infty r^2}\right) U_\infty \cos\theta, \quad V_\theta = -\left(1 + \frac{\kappa}{2\pi U_\infty r^2}\right) U_\infty \sin\theta - \frac{\Gamma}{2\pi r}$$

Now the cylinder is a stream line of the flow so there is no flow normal to the cylinder i.e.

$$V_r = 0$$

This means that

$$\left(1 - \frac{\kappa}{2\pi U_\infty r^2}\right) U_\infty \cos\theta = 0$$

for all θ so the doublet strength must be given by

$$\kappa = 2\pi U_\infty R^2$$

The velocity components are then

$$V_r = \left(1 - \frac{R^2}{r^2}\right) U_\infty \cos\theta, \quad V_\theta = -\left(1 + \frac{R^2}{r^2}\right) U_\infty \sin\theta - \frac{\Gamma}{2\pi r}$$

On the cylinder

$$V_r = 0, \quad V_\theta = -2U_\infty \sin\theta - \frac{\Gamma}{2\pi R}$$

The pressure coefficient on the cylinder is given by

$$C_p = 1 - \left(\frac{V}{U_\infty}\right)^2 = 1 - 4\sin^2\theta - \frac{2\Gamma\sin\theta}{\pi R U_\infty} - \left(\frac{\Gamma}{2\pi R U_\infty}\right)^2$$

The pressure distribution on the cylinder is then

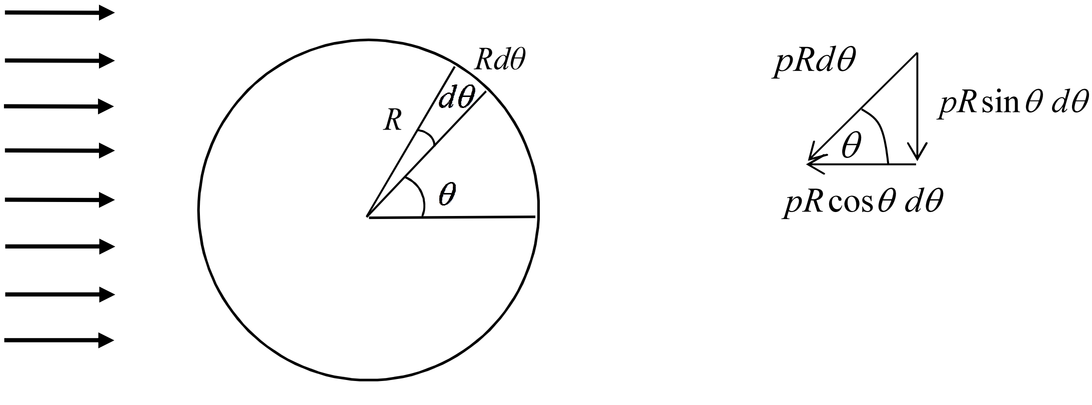
$$p(\theta) = p_\infty + \frac{1}{2}\rho U_\infty^2 \left(1 - 4\sin^2\theta\right) - \left(\frac{\rho U_\infty \Gamma \sin\theta}{\pi R}\right) - \frac{1}{8}\rho\left(\frac{\Gamma}{\pi R}\right)^2$$

(6 marks)

(b)

Consider a small arc of the surface, as shown in the sketch, of size $R d\theta$. The force acting on this element is given by

$$p(\theta) R d\theta$$



this acts normal to the surface and must be resolved to get the components (see above)

So the force in the vertical direction over the entire surface is given by

$$l = -\int_0^{2\pi} p(\theta) R \sin\theta d\theta$$

Now using the results from part (a)

$$p(\theta) = p_\infty + \frac{1}{2}\rho U_\infty^2\left(1 - 4\sin^2\theta\right) - \left(\frac{\rho U_\infty \Gamma \sin\theta}{\pi R}\right) - \frac{1}{8}\rho\left(\frac{\Gamma}{\pi R}\right)^2$$

We find that

$$l = -\int_0^{2\pi}\left[p_\infty + \frac{1}{2}\rho U_\infty^2\left(1 - 4\sin^2\theta\right) - \left(\frac{\rho U_\infty \Gamma \sin\theta}{\pi R}\right) - \frac{1}{8}\rho\left(\frac{\Gamma}{\pi R}\right)^2\right] R\sin\theta d\theta$$

$$= R\int_0^{2\pi}\left[\left(-p_\infty - \frac{1}{2}\rho U_\infty^2 + \frac{1}{8}\rho\left(\frac{\Gamma}{\pi R}\right)^2\right)\sin\theta + 2\rho U_\infty^2 \sin^3\theta + \frac{\rho U_\infty \Gamma \sin^2\theta}{\pi R}\right] d\theta$$

$$= R\left(-p_\infty - \frac{1}{2}\rho U_\infty^2 + \frac{1}{8}\rho\left(\frac{\Gamma}{\pi R}\right)^2\right)\int_0^{2\pi}\sin\theta d\theta + 2\rho U_\infty^2 R\int_0^{2\pi}\sin^3\theta d\theta + \frac{\rho U_\infty \Gamma}{\pi}\int_0^{2\pi}\sin^2\theta d\theta$$

$$= \frac{\rho U_\infty \Gamma}{\pi}\int_0^{2\pi}\sin^2\theta d\theta = \frac{\rho U_\infty \Gamma}{\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{2}\right]_0^{2\pi} = \rho U_\infty \Gamma$$

So finally the lift force acting is equal to

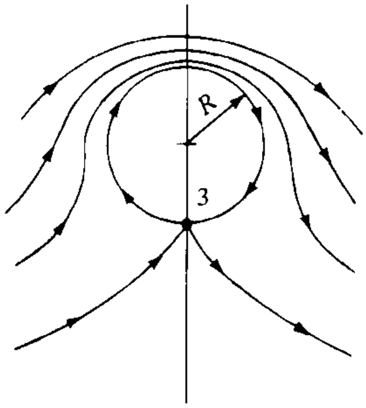
$$l = \rho U_\infty \Gamma$$

(9 *marks)*

The stagnation point at $\theta = 270^o$ so $\sin\theta = -1$, substitute into equation for radial velocity with $r=R$
From previous

$$V_r = 0, \quad V_\theta = -2U_\infty \sin\theta - \frac{\Gamma}{2\pi R} = 0$$

$$2U_\infty = \frac{\Gamma}{2\pi R} \qquad \rightarrow \qquad \Gamma = 4U_\infty \pi R$$



Hence lift per unit length is

$$l = 4\rho U_\infty^2 \pi R$$

$$l = 4 \times 1.2 \times 4^2 \times \pi \times 0.5 = 120.64N$$

(5 *marks)*