Structures and Materials 3 (StM3)
Failure of Materials

Why Study Failure: Case Histories

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Intended Learning Outcomes

Understand material failure, how it relates to engineering design and make simple estimates for strength or life prediction.

Course Content

- Lecture 1: Case Studies
  - Why study failure?
  - Historical Examples
- Lecture 2: Modes of failure
  - Concept of strain energy and toughness
  - Ductile, brittle failure
  - Fractography
  - Factors affecting ductile to brittle transition
- Lecture 3: Introduction to fracture mechanics – Part 1
  - Introduction to fracture mechanics
  - Theoretical stress approach to fracture
  - Stress intensity factor
- Lecture 4: Introduction to fracture mechanics – Part 2
  - Griffith's energy balance approach
  - Irwin's energy balance approach
- Lecture 5: Measuring fracture toughness
  - Fracture process zone and geometrical considerations
  - Measuring toughness
  - Anisotropical materials

Why Study Failure?

Liberty Ships (1941-1945)

Constance Tipper (1894-1995)

- Her major research contribution was to discover why during the Second World War the Liberty Ships were breaking in two.

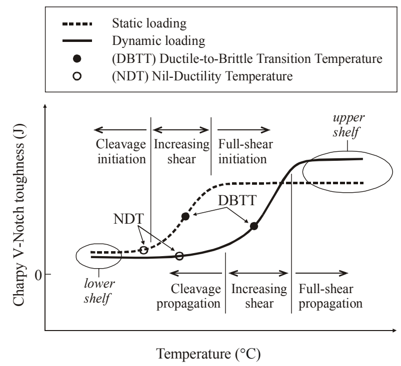
- Liberty Ships were the first all-welded pre-fabricated cargo ships and were mass produced in the United States.

    - 4700 ships were built by 1946;
    - 1250 of these had suffered brittle fractures by 1953;
    - 230 of these fractures were classed as serious;
    - 12 of the ships broke in two.

Liberty Ships: Steel toughness

- The toughness of ship steels is traditionally specified by their Charpy 27 Joule temperature, T27J. The lower the T27J, the tougher the steel.

Constance Tipper (1894-1995)



Lessons Learned

- The very high fracture rate in the Liberty ships was due to the use of steel which was unusually brittle even for its date of manufacture.
  - Tipper demonstrated that the fractures were caused by the steel used rather than the fact that the ships had been welded, as was first thought.
  - She established that there is a critical temperature below which the fracture mode in steel changes from ductile to brittle.
  - Ships in the North Atlantic were subjected to such low temperatures that they would have been susceptible to brittle failure.
- Once steel quality was improved in the immediate post war period there was a rapid reduction in the number of ship brittle fractures.

Historical causes of structural failure

- This lecture will also provide an overview of (some) key causes of structural failure and the lessons learned in the aerospace industry.
  - De Havilland Comet crashes (Mark 1, 1952-1954)
    - Stress concentration of tiny defects and fatigue failure → Fail Safe Philosophy
  - General Dynamics F-111 crash (1969)
    - Large manufacturing flaw growing in fatigue → Damage tolerant philosophy
  - Dan Air Boeing 707 crash (1977)
    - Extensive testing and inspection necessary → Fully tested certification and regular inspections
  - Aloha Airlines Boeing 737 accident (1988)
    - Ageing aircraft will have weakened structures → Ageing structure assessments
  - Lockheed C130A Firefighting Tanker accident (2002)
    - Loading spectrum and sudden loads will affect structural life

De Havilland Comet (Mark 1)(1952-1954)

De Havilland Comet (Mark 1, 1952-54)

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- Jet transportation age began in on May 5 1952 when the De Havilland Comet 1 began scheduled flights from London to Johannesburg.
- The cabin was pressurised to maintain a pressure equivalent to 8 000 feet at an aircraft altitude of 40 000 feet, which was required for efficient operation of the engines. This gave a pressure differential of 8.25 psi (56 kPa) across the fuselage – twice the value previously used.
- It seemed that the future was bright for the British aircraft industry, with orders from France, Canada and the UK. However, a series of 3 accidents occurred where Comet aircraft disintegrated in flight:
  - G-ALYV after leaving Calcutta – May 1953. Violent storms were thought to be involved and some wreckage was recovered. No firm conclusions drawn as to cause.
  - G-ALYP over Elba – January 1954 after 1 286 cabin pressurisation cycles. Little wreckage was recovered and no major problems found in fleet inspection. Fire was assumed the most likely cause and modifications made to improve fire prevention and control. Aircraft returned to service.
  - G-ALYY flying as SA 201 after leaving Rome – April 1954.

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Key slide!

De Havilland Comet G-ALYP (May 1953)

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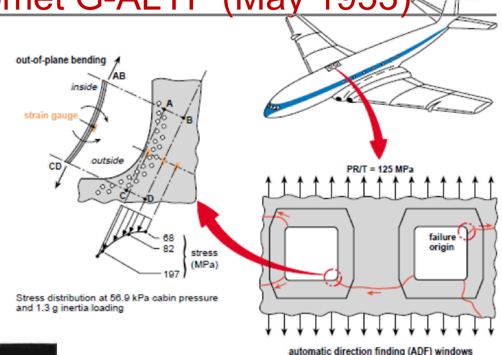
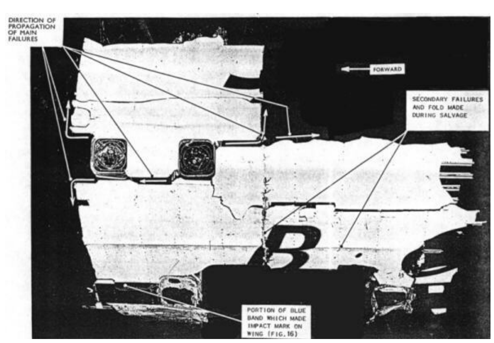
Probable failure origin of service aircraft Comet G-ALYP: stress distribution obtained from test fuselage





Wreckage of first crashed comet, showing the lack of crack-arrest

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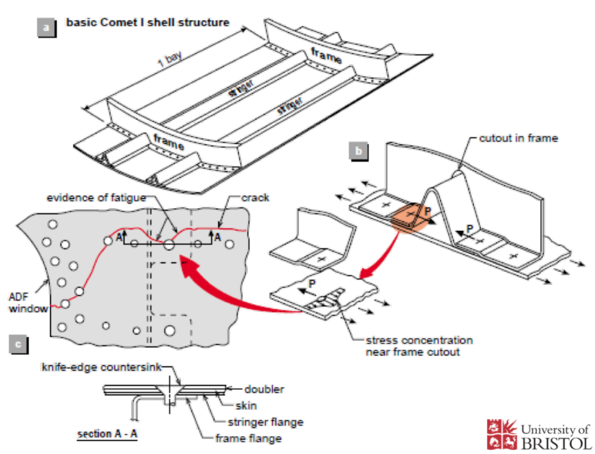
De Havilland Comet G-ALYP (May 1953)

- A cabin pressurization fatigue test was performed under water, with a cabin failure occurring after 3057 pressure cycles.
- This result confirmed that fatigue of the pressurized cabin was the primary cause of failure.
- The origin of the failure (of the first failed aircraft) was found to be cracks that originated at a corner of an ADF window at the top of the cabin. This in itself is not sufficient to explain the catastrophic failure.

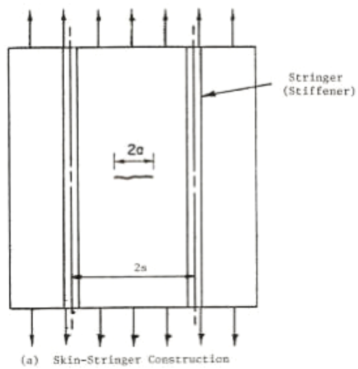
De Havilland Comet G-ALYP (May 1953)

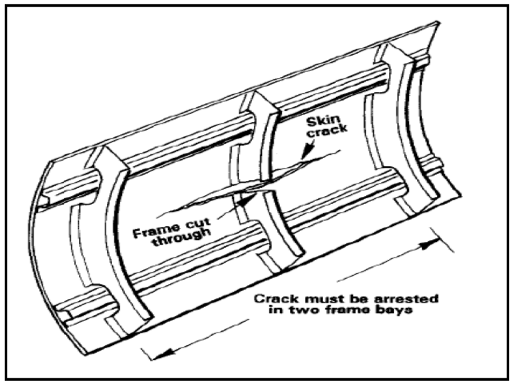
- Failure was catastrophic because the fuselage design did not have features that would allow the pressurisation-driven crack to **arrest** before causing a major failure.
- The Comet fuselage design featured periodic cut-outs in the fuselage frames (to allow the stringers to pass through), which allowed an unstable crack to continue from bay to bay, thereby resulting in a complete failure of the fuselage



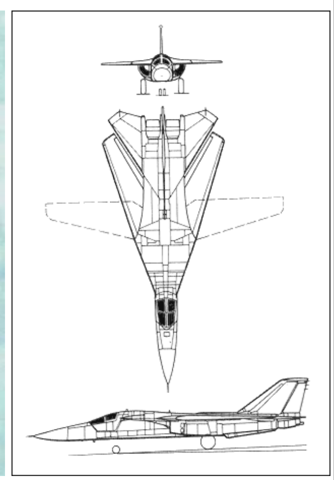
Lessons Learned

- As a result of the Comet accidents, crack-arrest design features and using separate static and fatigue test-articles have become standard practices in the aircraft industry.
- Modern fuselage design has evolved to a configuration of continuous frames that will arrest any crack that may develop between frames.





General Dynamics F-111 Crash - 1969

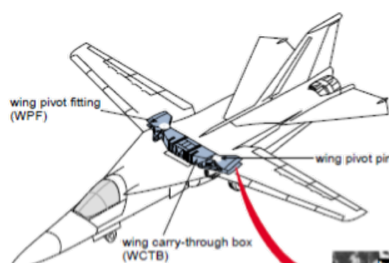


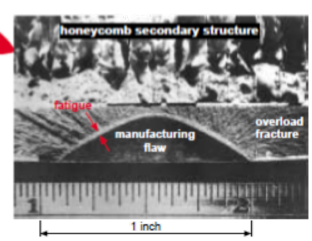
General Dynamics F-111 Crash

- In 1964 the General Dynamics Corporation was awarded a contract for the development and production of the F-111 aircraft, subsequently to be procured by the United States Air Force (USAF) and others.
- The F-111 is an unusual aircraft: it is a variable geometry ‘swing-wing’ (supported by D6AC steel pivot-fittings) fighter-bomber; and it uses high-strength steel in major airframe components, namely the wing carry-through box, wing pivot fittings, some of the centre fuselage longerons and the empennage carry-through structure.
- The F-111 fighter-bomber, which was designed according to the safe-life philosophy, entered service with the USAF in 1968.

General Dynamics F-111 Crash

- On December 22, 1969, just over a year after entering service, F-111 #94 lost the left wing during a low-level training flight. The aircraft had accumulated only 107 airframe flight hours, and the failure occurred while it was pulling about 3.5g, less than half the design limit load factor .

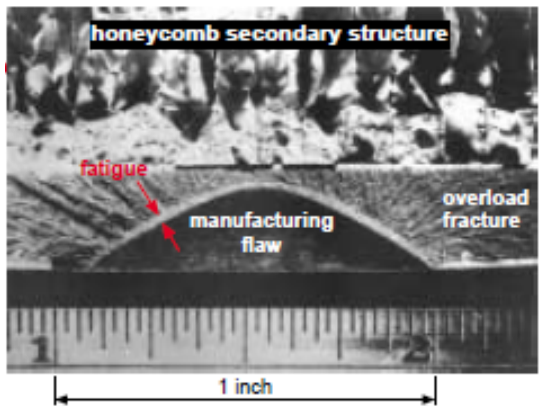




Failure origin: a manufacturing flaw in the high-strength steel lower plate of the left-hand wing pivot fitting

General Dynamics F-111 Crash

- An immediate on-site investigation revealed a flaw in the lower plate of the left-hand wing pivot fitting.
- The cause of the failure was traced to an undetectable surface flaw present in the high strength D6AC steel pivot-fitting. The flaw originated during manufacturing and grew until failure, to a semi-elliptical shape of about 24 mm long by 6 mm deep



Lessons Learned

- This failure directly led to the replacement of the **safe-life philosophy** by the **damage tolerance philosophy** by the USAF in 1974, which was also adopted by the FAA in 1978 (see Boeing 707 Accident at Lusaka, next example).

Boeing 707 Accident at Lusaka (1977)

Boeing 707 Accident at Lusaka (1977)

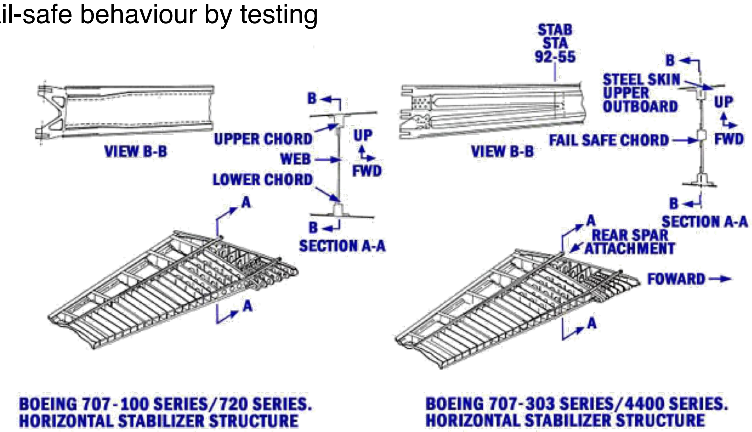
- In 1977, a Dan-Air B707-321C aircraft crashed at Lusaka, Zambia after the entire right-hand horizontal stabilizer separated in flight during the approach to landing. All six passengers were killed.

- The B707-100 series had been designed to the fail-safe philosophy, which was then permitted by the FAA.

  - Fail-Safe Design: This design concept assumes the possibility of multiple load paths and/or crack arrest features in the structure so that a single component failure does not lead to immediate loss of the entire structure. The load carried by the broken member is immediately picked up by adjacent structure and total fracture is avoided. It is essential; however, that the original failure be detected and promptly repaired, because the extra load they carry will shorten the fatigue lives of the remaining components.

Boeing 707 Accident at Lusaka (1977)

- Boeing assumed that the fail-safe certification for 707-100 Series which was supported by experimentation would also be valid to for the 707-300 Series
- However some key design changes were made. Under the fail-safe philosophy, there was no FAA requirement to specify any periodic structural inspections. Also, there was no FAA requirement to confirm the fail-safe behaviour by testing

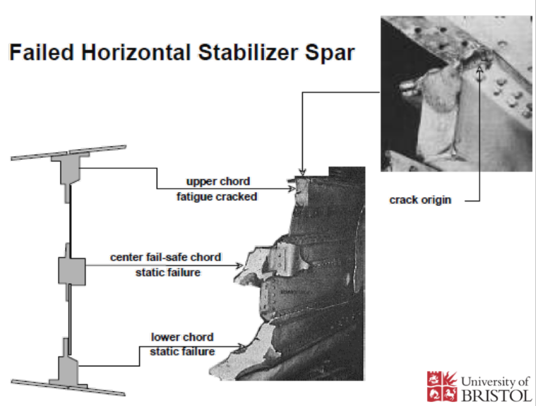


Boeing 707 Accident at Lusaka (1977)

- The aircraft had completed 16,723 flights, compared to a design life of 20,000 flights.
- The single-spar horizontal tail structure was designed to be fail-safe by the addition of an additional chord at the centre of the web. In this way, a failure of either the upper or lower chord would allow the design loads to be carried by the two remaining chords.
- No fatigue or fail-safe testing was ever performed for this design

Post-accident measurements

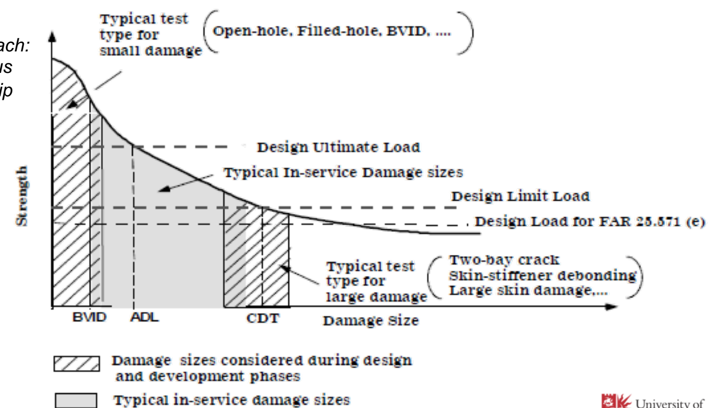
showed significant tail oscillatory loads resulting from airbrake deployment after landing. These loads were unknown during the design of the structure!



Lessons Learned

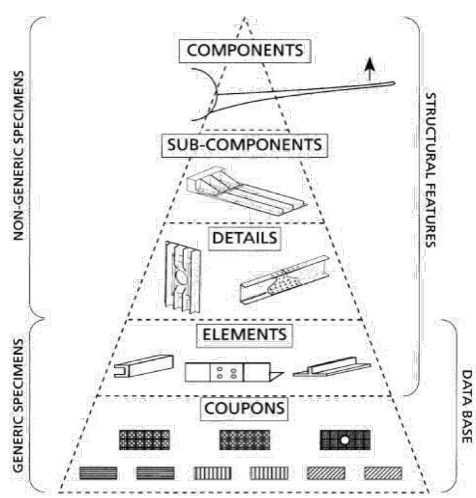
- As a result of this accident, it became generally acknowledged that a fail-safe design is not sufficient to ensure safety, and periodic inspections would also be needed.
- This accident gave the impetus to substitute damage-tolerance requirements (1978) for the fail-safe requirements in the certification of large civilian aircraft.

Damage tolerance approach: Residual strength versus damage size relationship



Lessons Learned

- This accident also demonstrated the need to **measure flight loads** for new designs of aircraft and to perform **full-scale fatigue testing**.

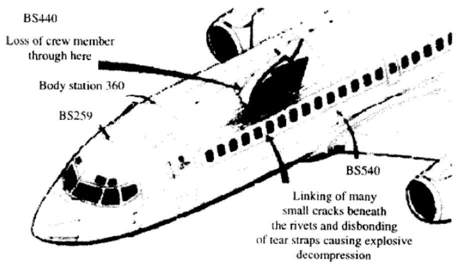


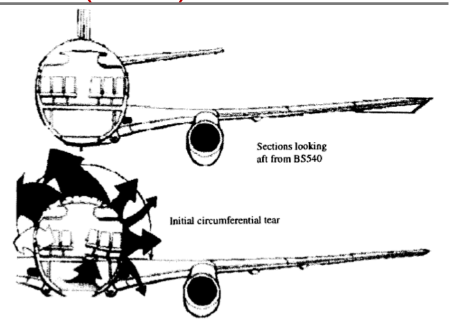
Aloha Airlines Accident (1988)

Aloha Airlines Accident (1998)

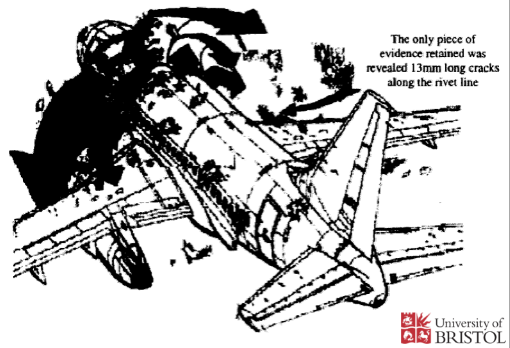
- In 1988, a Boeing 737-200 aircraft, operated by Aloha Airlines, had a sudden failure of the upper portion of the fuselage at a 24,000 ft. altitude over the Hawaiian Islands.
- The aircraft had flown for 19 years and had accumulated 89,680 flights, (approximately 13 flights a day). The aircraft had been designed for only 75,000 flights.
- The Boeing 737-200 was also designed to the fail-safe concept that was then permitted by the FAA.

Aloha Airlines Accident (1998)





Ref: Multiple-site And Widespread Fatigue Damage In aging aircraft, S. Pitt and R. Jones, *Engineering Failure Analysis, Vol. 4~ No. 4, pp. 237 257, 1997*



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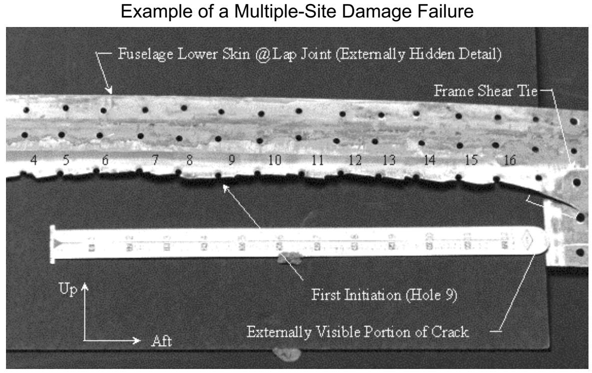
Key slide! Aloha Airlines Accident (1998)

- The failure was attributed to several causes:
  - disbonding of a cold-bonded lap-joint,
  - corrosion at the joint,
  - multiple-site damage, and
  - inadequate inspections.

Example of a Multiple-Site Damage Failure



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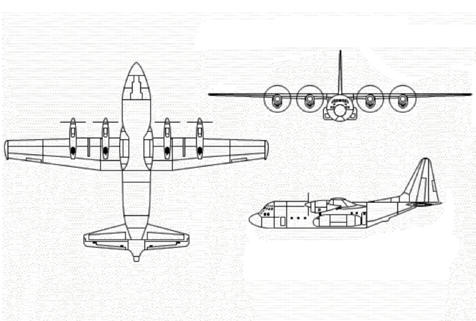
Lessons Learned

- The Aloha incident led to increased awareness of the aging aircraft phenomenon.
- Special attention was given to the phenomena of multiple-site damage, which can lead to widespread fatigue damage.
- This accident resulted in large-scale R&D by industry, the FAA and NASA on the subject of aging aircraft. This research confirmed that there is an escalating risk of failure of fail-safe structures as the aircraft ages.
- New FAA regulations were written, requiring a damage-tolerance assessment of aging fleets, which had not been originally designed to the damage-tolerance regulations.

Lockheed C130A Firefighting Tanker (1994,2002)

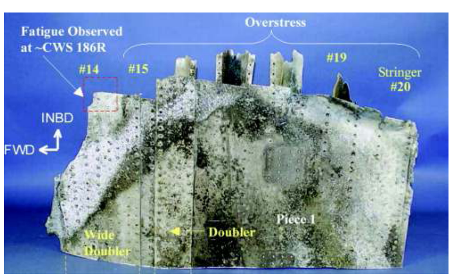


Lockheed C130A Firefighting Tanker (1994,2002)

- In 1994, a firefighting tanker aircraft had its right wing detached in flight. The accident was originally attributed to an explosion caused by leaking fuel.
- In 2002, a 45 year old C130A firefighting tanker aircraft had its wings fail in flight. Metallurgical investigation of the wreckage showed extensive fatigue damage to the wings.

Lockheed C130A Firefighting Tanker (1994,2002)

- It is believed that the low-level flight, resulting in severe turbulence, together with violent manoeuvres performed during firefighting, were probably responsible for the failures.
- The C130 was not originally designed for this loading spectrum.



Fatigue damage observed on C130A wing

Fatigue damage to center wing of C130A aircraft

Lessons Learned

- The lesson learned from the C130A failures is that new missions can result in new loading spectra that are significantly more severe than the original design spectrum!

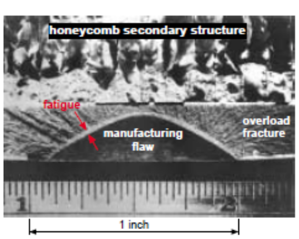
What can history tell us?

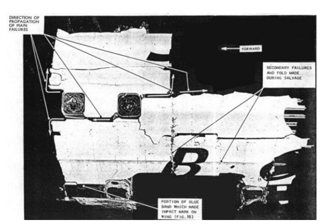
Aircraft Structural Integrity - History

- 1930 – 1940
  - Commercial development of metal aircraft for public transport. Design and analysis emphasized static strength, with little or no consideration of airframe fatigue
- 1940 – 1955
  - Increasing awareness of importance of fatigue for airframe safety. Materials with higher static strengths were developed without corresponding increases in fatigue strength. Design became based on both static and fatigue strengths.
- 1955 – present
  - Development of fail-safe and damage tolerance design methods, which recognise that airframe structures must withstand service loads even when damaged and cracked. Safety to be ensured by testing and analysis of damaged structures, pre-service and in-service inspections, and eventual repairs and replacements.

What can history tell us?

- Issues to address:
  - How do cracks that lead to failure form?
  - How is fracture resistance quantified?
  - How does the fracture resistances of different material classes compare?
  - How do we estimate the stress to fracture?
  - How do loading rate, loading history, and temperature affect the failure behaviour of materials?





Structures and Materials 3 (StM3)
Failure of Materials

Introduction to Failure

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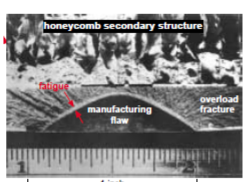
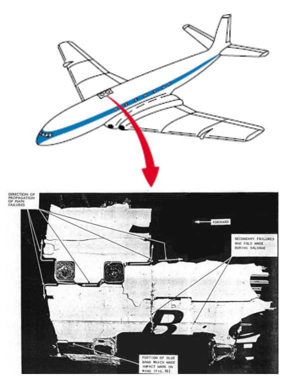
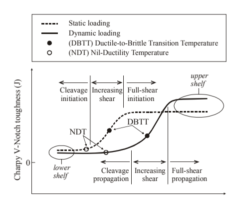
Last Time



Manufacturing Defects



Fatigue

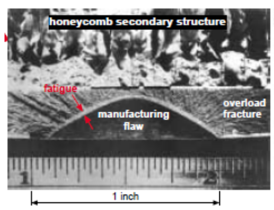


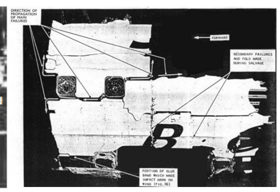
Environmental Conditions

Last Time

What can history tell us?

- **ISSUES TO ADDRESS...**
  - How do cracks that lead to failure?
  - How is fracture resistance quantified?
  - How do the fracture resistance of the different material classes compare?
  - How do we estimate the stress to fracture?
  - How do *loading history* and *temperature* affect the failure behaviour of materials?





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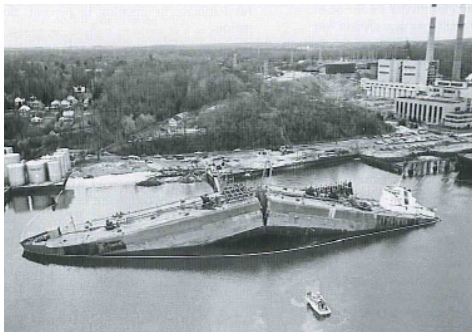
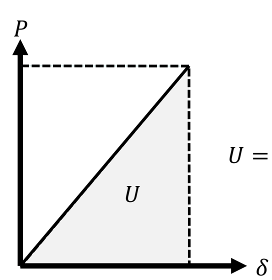


- It is important to understand the mechanisms for failure, especially to prevent in-service failures via design.
- This can be accomplished via
  - Materials selection,
  - Processing,
  - Design.

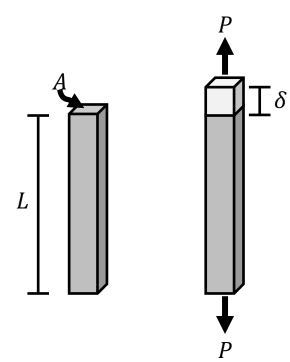
- Objective: Understand how flaws in a material initiate failure.
  - Describe crack propagation for ductile and brittle materials.
  - Explain why brittle materials are much weaker than theoretically possible.

Elastic Strain Energy

- When the bar is loaded you are inputting energy into the system; this is the elastic strain energy (U)
- This can be done through doing work (F)
- Energy conservation $F - U = 0$



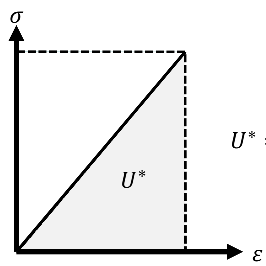
$$U = \frac{1}{2}P\delta$$



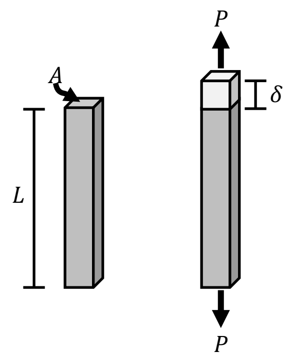
Elastic Strain Energy Density

- Elastic strain energy density (U^*) is the elastic strain energy per unit volume of material.

- On a stress (σ), strain (ε) plot, strain energy density is the area under the curve.

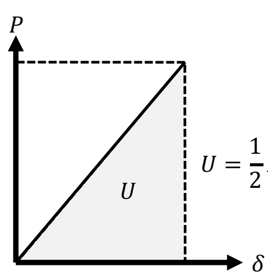


$$U^* = \frac{1}{2}\sigma\varepsilon$$

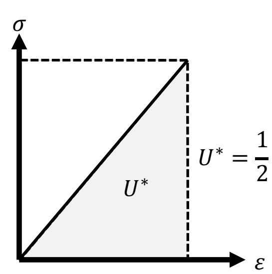


Elastic strain Energy Density

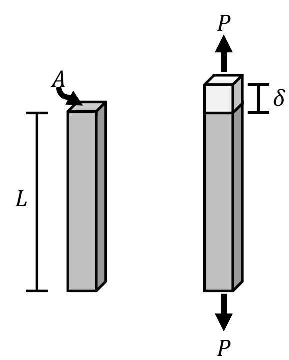
- The relationship between the input energy U and the strain energy density U^* is:



$$U = \frac{1}{2}P\delta$$



$$U^* = \frac{1}{2}\sigma\varepsilon$$



$$\sigma = \frac{P}{A} \qquad \varepsilon = \frac{\delta}{L} \qquad E = \frac{\sigma}{\varepsilon} \qquad U = \frac{1}{2}P\delta = \frac{\sigma^2}{2E}V = U^*V$$

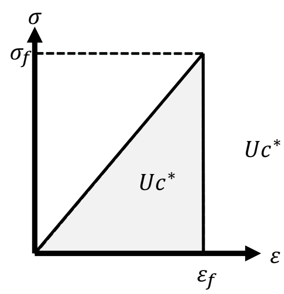
Key slide!

Toughness

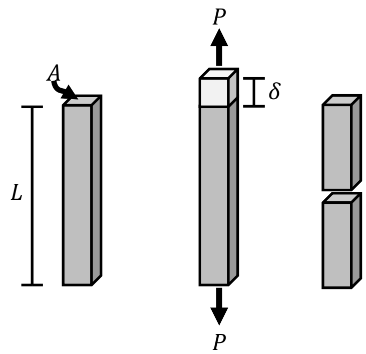
- We then continue loading until failure, where W is the work expended in fracturing material over da

$$\frac{d}{da}(F - U - W) = 0$$

- The energy dissipated per unit volume of the material **up to** failure is known as **Toughness**



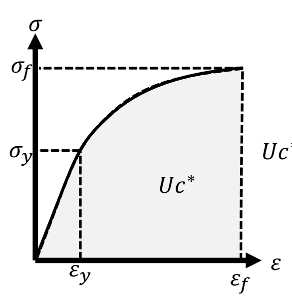
$$Uc^* = \frac{1}{2}\sigma\varepsilon$$



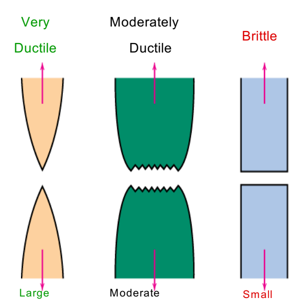
Ductile vs Brittle

- Ductile materials produce a non-linear stress, strain response

- Toughness (strain energy density before failure) definition is still the same but strain energy is no longer the same past the elastic limit (σ_y, ε_y)



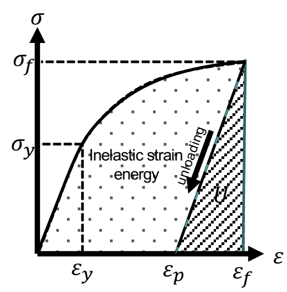
$$Uc^* = \int_0^{\varepsilon_f} \sigma \, d\varepsilon$$



Key slide!

Ductile vs Brittle

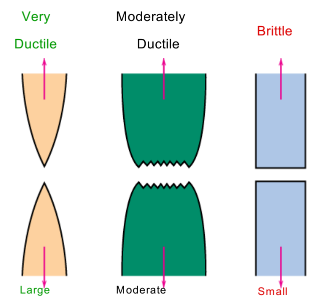
- Once a material has plastically deformed, the elastic strain energy is the energy that is **recovered** when the material is unloaded
- The inelastic strain energy ($U_{\text{inelastic}}$) is the energy that is **absorbed** by the material through **plastic deformation**
- The elastic strain energy (*U*) can drive the crack to grow



$$Uc^* = \int_0^{\varepsilon_f} \sigma d\varepsilon$$

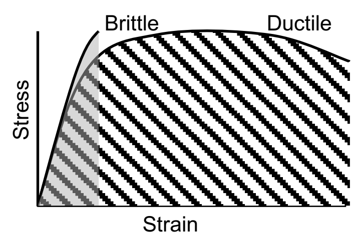
$$U = \frac{1}{2}\sigma(\varepsilon - \varepsilon_P)V$$

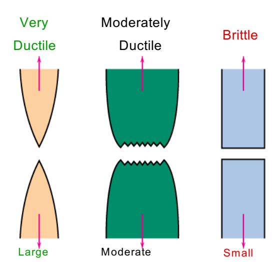
$$U_{\text{inelastic}} = Uc^*V - U$$



Modes of Failure

- Two general classes of fracture can be seen in engineering materials.
- These are **brittle** and **ductile.** The distinction is based on a material's ability to exhibit plastic deformation
- Once failed, the area under the stress-strain curves indicates the level of energy dissipation or absorption per unit volume
- Ductile materials exhibit extensive plastic deformation and high energy absorption; brittle materials exhibit little or no **plastic deformation** and low energy dissipation or absorption





Ductile: Warning before fracture	Brittle: No warning

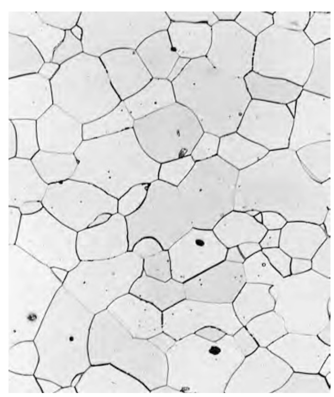
Imperfections

- Defects and flaws limit the potential elastic strain energy that can be stored by the material
- However, extra deformation (strain) prior to failure would mean the material is tougher
- For example, dislocations in crystalline structure results in slip planes
- Extra energy is absorbed when the crystalline structure slides, this is what we refer to as **plastic deformation**

Transmission electron micrograph of a titanium alloy in which the dark lines are dislocations. 51,450 ×
W. Callister, Materials Science and Engineering

Atomic bonds and Grain boundaries

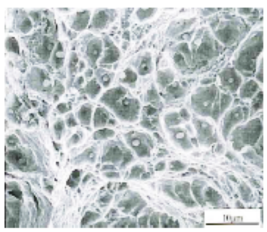
Photomicrograph of a polycrystalline brass specimen.
W. Callister, Materials Science and Engineering



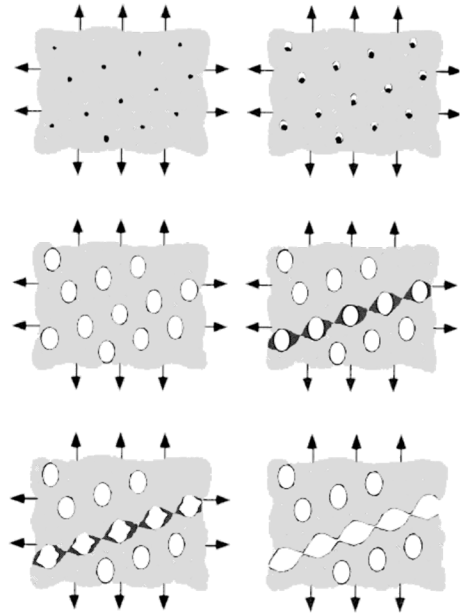
Photomicrograph of the surface of a polished and etched polycrystalline specimen of an ironchromium alloy in which the grain boundaries appear dark.
W. Callister, Materials Science and Engineering

Microvoid formation, growth and coalescence

- Microvoids are easily formed at inclusions, intermetallic or second-phase particles and grain boundaries
- These are simply pockets of air or foreign particles with little to no bond strength
- Growth and coalescence of microvoids progress as the local applied load increases



Ductile dimples centre on spherical particles



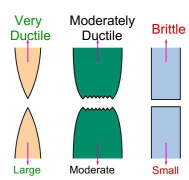
Fractography: Study of fracture surfaces

Ductile fracture

- Accompanied by significant plastic deformation

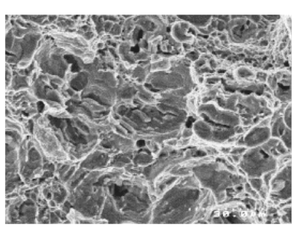
Brittle fracture

- Little or no plastic deformation
- Catastrophic, usually strain is < 5%

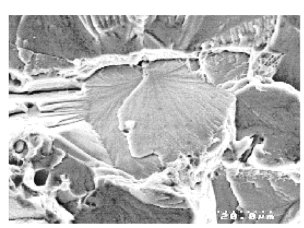


High energy absorbed by microvoid coalescence during ductile failure (high energy fracture mode)

Low energy absorbed during transgranular cleavage fracture (low energy fracture mode)



→ Less catastrophic

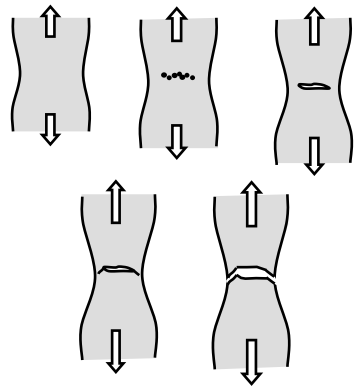


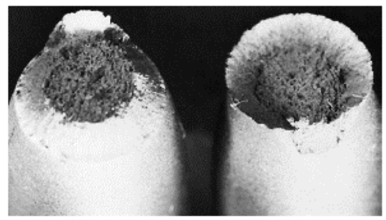
→ More catastrophic

Key slide!

Ductile Fracture

- Under uniaxial tensile force in ductile materials:
  - **Necking** caused by dislocation movements or polymer chain sliding
  - **Atomic debonding** and **microvoid init**
  - These **coalesce**(join) to form larger cracks
  - Eventually propagate in the direction normal to the tensile axis.
- **Ductile fracture** is much less critical in engineering
- Failure can be detected beforehand due to observable **plastic deformation**
- For round coupons: a crack eventually propagates through the periphery along the shear plane at 45º, leaving the typical **cup and cone** pattern





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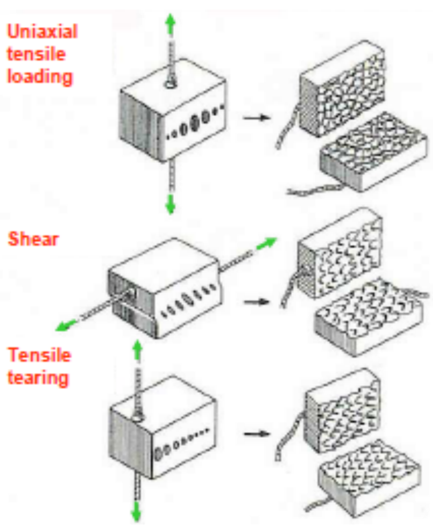
Micro-void shape

- Microvoid shape is strongly influenced by type of loading



- Uniaxial tensile loading
  - Equiaxed dimples
- Shear loading
  - Elongated and parabolic dimples pointing in the opposite directions on matching fracture surfaces
- Tensile tearing
  - Elongated dimples pointing in the same direction on the matching fracture surface

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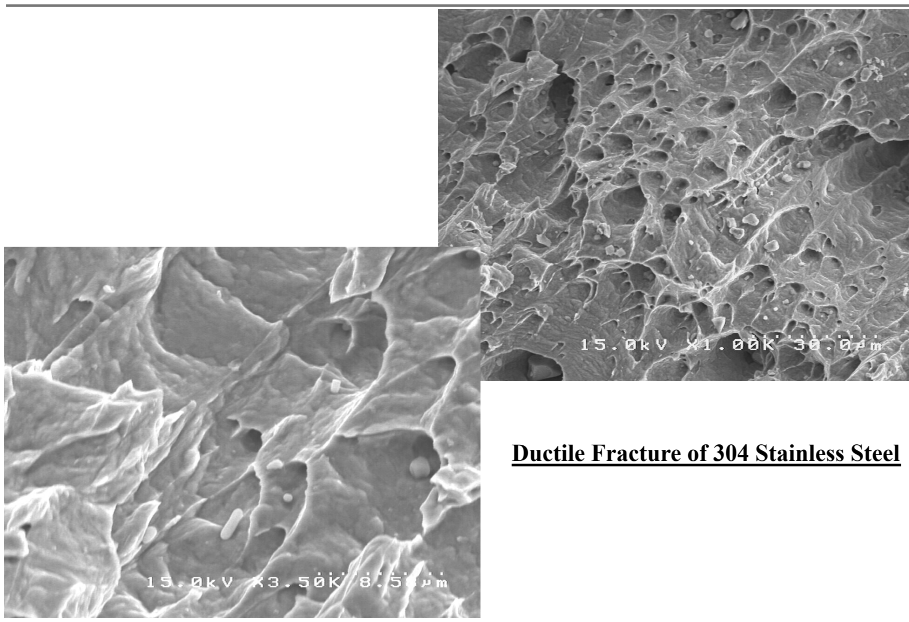
Fracture Surfaces – 304 Stainless Steel



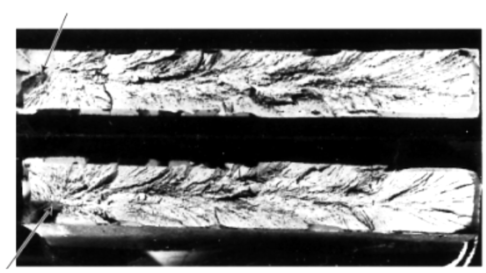
Ductile Fracture of 304 Stainless Steel

Key slide!

Brittle Fracture

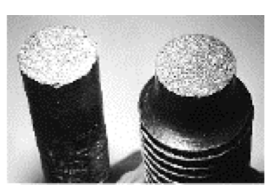
- The process of cleavage fracture consists of three steps:

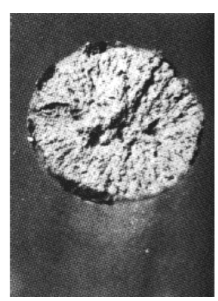
1. Plastic deformation to produce dislocation pile-ups.
2. Crack initiation
3. Crack propagation to failure



- Distinct characteristics of brittle fracture surfaces:

1. The absence of gross plastic deformation
2. Grainy or Faceted texture
3. River marking or stress lines

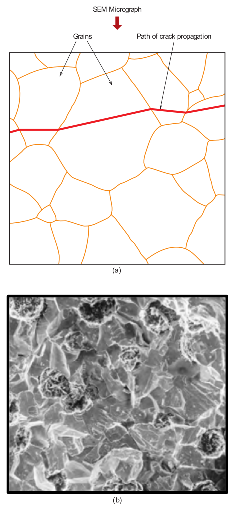




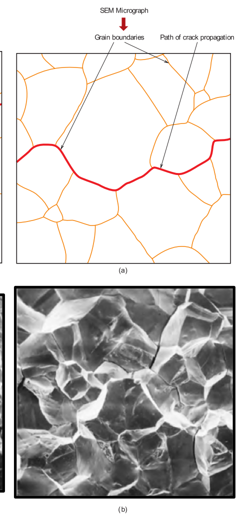
Fractography in metals

21

- **Cleavage** fracture is the breaking of atomic bonds along crystallographic planes (**Transgranular**)
  - Surface: Rough and textured, with river and feather patterns
  - Moderate to high strength brittle fracture mode
- In some metal alloys cracks form along grain boundaries (**Intergranular**)
  - Surface: Sharp and 3D faceted grains
  - Moderate to low energy brittle fracture mode



Transgranular fracture



Intergranular fracture

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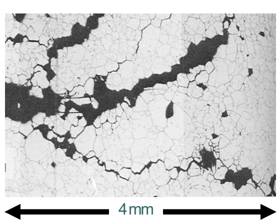
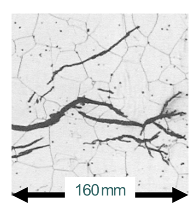
Brittle Fracture Surfaces

22

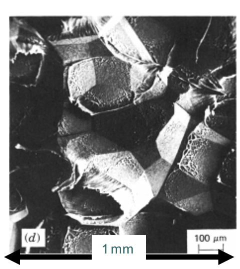
Intergranular (between grains) | Transgranular (through grains)



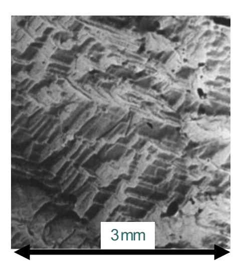
304 S. Steel (metal)



316 S. Steel (metal)



Polypropylene (polymer)



Al Oxide (ceramic)

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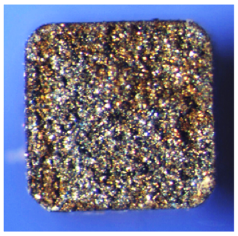
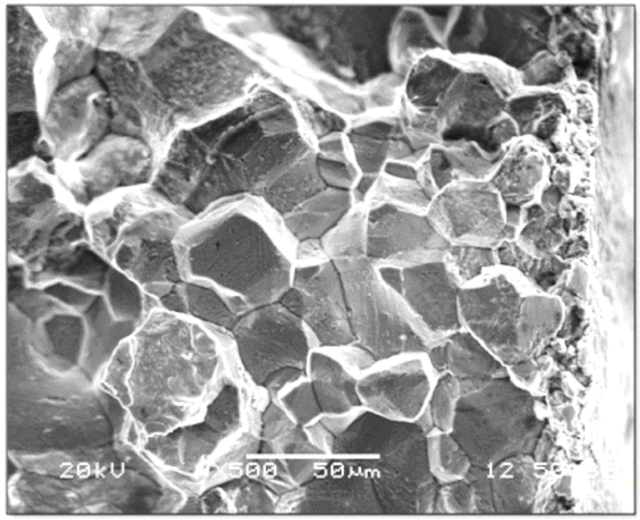
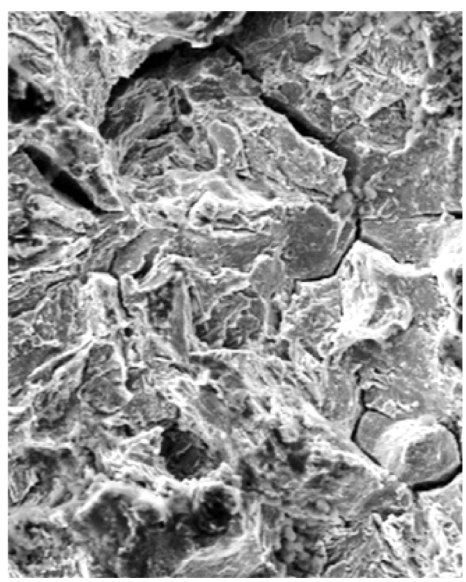
Brittle Fracture Surface - Faceted



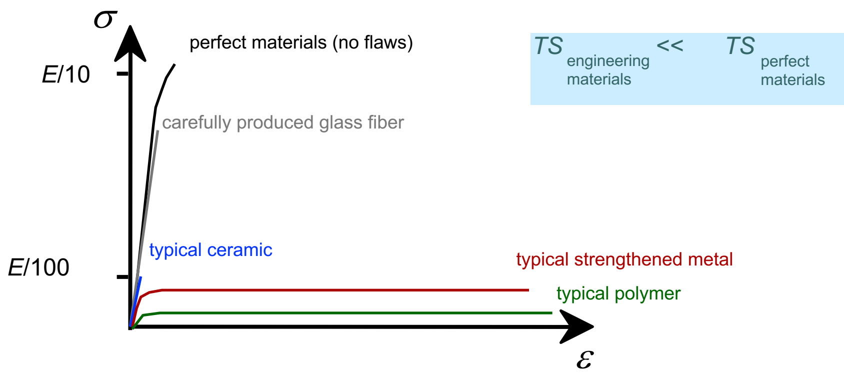


Brittle Fracture Surface - Cleavage



Ideal vs Real Materials

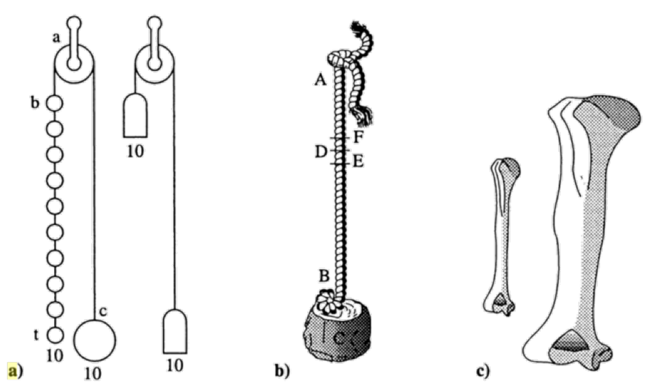
- Stress-strain behavior (room temperature):



- Flaws or defects leading to **size effects**

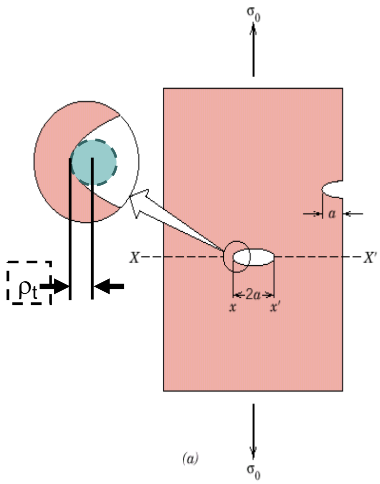
What are size effects?

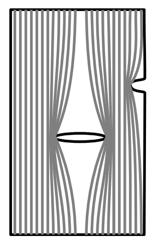
- A size effect is the change in strength with specimen dimensions.
- Leonardo da Vinci in the 1500s stated that "Among cords of equal thickness the longest is the least strong". (a, b)
- Galileo in 1638 pointed out that a size effect is indicated in the fact that large animals have relatively bulkier bones than small ones, which he called the "weakness of giants" . (c)



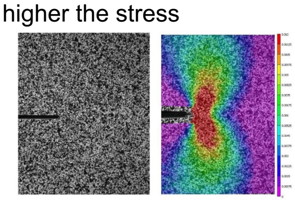
Flaws introduce Stress Concentrations

- Griffith Crack (this will be discussed in detail next time)
- Some important parameters
  - a = crack size
  - ρ_t = radius of curvature
  - σ_o = applied stress
  - σ_m = stress at crack tip

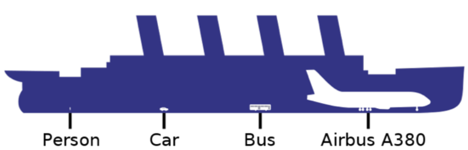


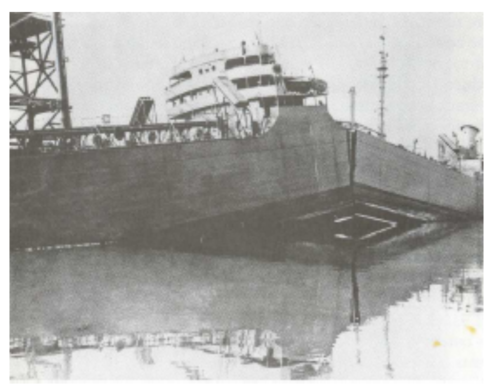


Closer the line spacings, higher the stress



Effect of Temperature

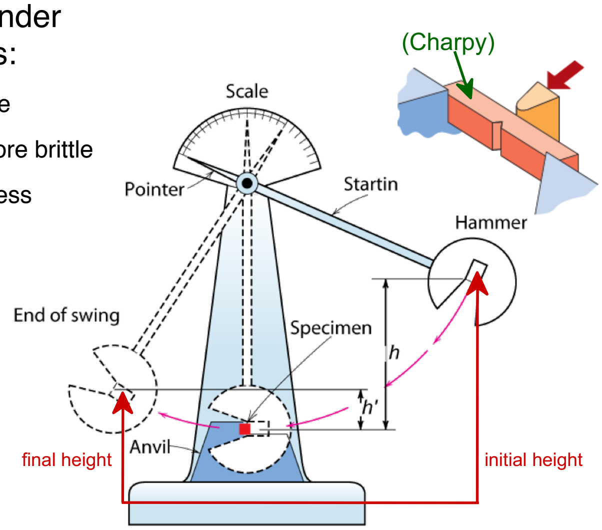




Characterisation of transition temperature

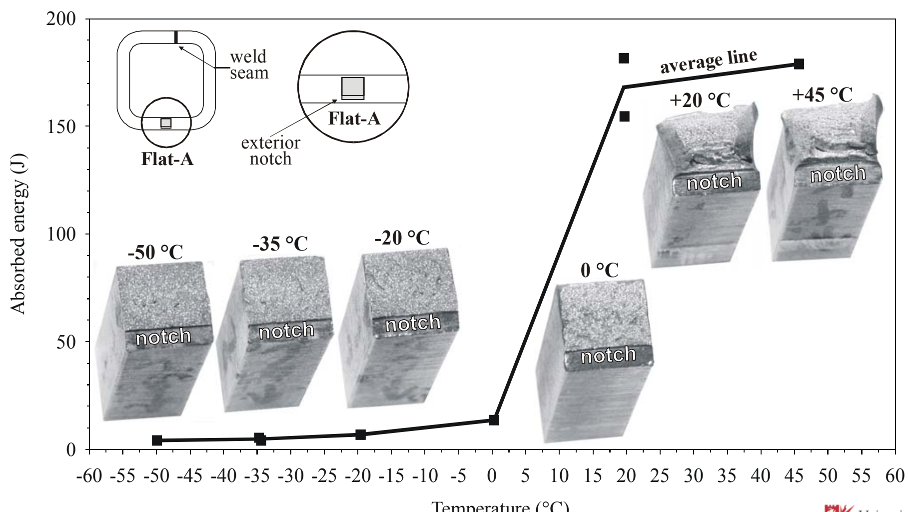
Charpy Impact Testing

- Impact loading under low temperatures:
  - severe testing case
  - makes material more brittle
  - decreases toughness



Key slide! Ductile to brittle transition behaviour

- Absorbed energy versus temperature behaviour of steel
- Increasing temperature allows more slip systems to operate, yielding general plastic deformation to occur prior to failure



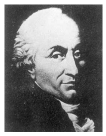
Other types of failure

- Fatigue failure
  - Fracture by **slow crack growth**
  - Happens when the part is subjected to **many repetitions** of a stress below that for static crack growth
- Corrosion fatigue failure
  - Combined actions of a cyclic stress and a corrosive environment
  - Fatigue resistance will decrease in the presence of an aggressive chemical environment
    - NB. 'aggressive' is a relative term; for some materials plain water has a very significant effect!
- Stress corrosion cracking
  - Similar to corrosion fatigue – combines mechanical and chemical failure processes
  - However in this case the stress is **not cyclic** (but still below the yield stress for a metal)
- Creep and stress rupture failure
  - A result of a static load applied over **long periods of time**

Concluding Remarks

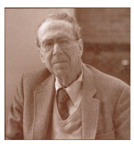
- We've learned the concept of failure
- We've learned that the presence of defects, cracks or flaw is associated with a loss of potential strain energy
- We've seen how defects and flaws may result in ductile failure
- Fractography is an important method to analyse fracture surfaces
- The effects of temperature, stress concentrations and other factors on the failure

Fracture Mechanics



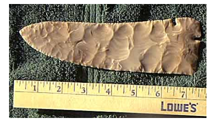
Charles Augustin de Coulomb (1736 – 1806)

Alan Arnold Griffith (1893 – 1963)



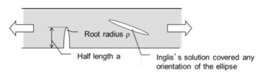
Irwin

Stone Age → Middle Age → Coulomb 1776 → Inglis 1913 → Griffith Model 1920 → Irwin's Model 1957

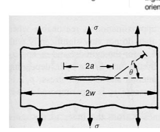


Flint Knapping

Crack propagation in stones under compression



$$\sigma_f = \sqrt{\frac{E\gamma}{4a}}$$



$$\sigma_f = \sqrt{\frac{2E\gamma}{\pi a}}$$

$$\sigma_f = \sqrt{\frac{2E\gamma}{\pi a(1-\nu^2)}}$$

Irwin line-crack

$$K = \sigma\sqrt{\pi a}$$

Fracture Energy

$$G = \frac{P^2}{2B}\frac{\partial C}{\partial a}$$

Stress Intensity Factor

Linear Elastic Fracture Mechanics

Part I: Stress based approach

Dr Giuliano Allegri
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Last Time

- It is important to understand the mechanisms of fracture and failure, especially to prevent in-service failure via design bad designs
- This can be accomplished via
  - Materials selection,
  - Processing,
  - Design.

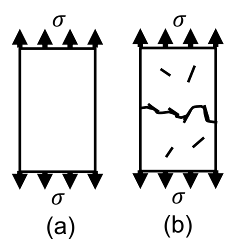
- Objective: Understand how flaws in a material initiate failure.
  - Describe crack propagation for ductile and brittle materials.
  - Explain why brittle materials are much weaker than theoretically possible.

Course Content

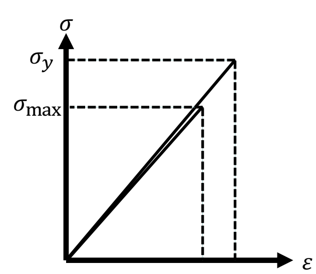
- Lecture 1: Case Studies
  - Why study failure?
  - Historical Examples
- Lecture 2: Modes of failure
  - Concept of strain energy and toughness
  - Ductile, brittle failure
  - Fractography
  - Factors affecting ductile to brittle transition
- Lecture 3: Introduction to fracture mechanics – Part 1
  - Introduction to fracture mechanics
  - Theoretical stress approach to fracture
  - Stress intensity factor
- Lecture 4: Introduction to fracture mechanics – Part 2
  - Griffith's energy balance approach
  - Irwin's energy balance approach
- Lecture 5: Measuring fracture toughness
  - Fracture process zone and geometrical considerations
  - Measuring toughness
  - Anisotropical materials

Introduction to Fracture Mechanics

- The presence of defects modify the local stress field in the material
- Therefore, elastic stress analyses assuming perfectly homogeneous and flawless materials are not suitable for designs using high-strength materials

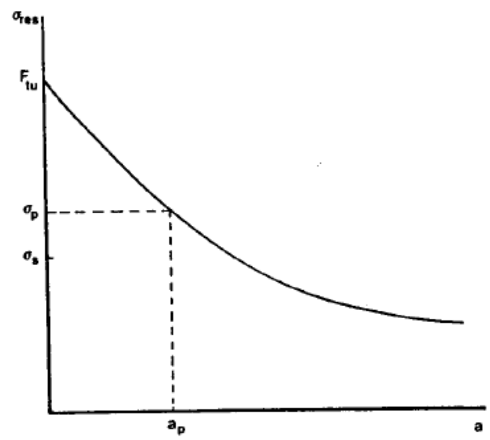


- When a crack reaches a certain critical length, it can propagate catastrophically through the structure
- This can happen at gross stresses which are much less than the yield stress of the material

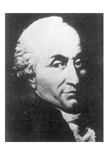


Introduction to Fracture Mechanics

- The term “fracture mechanics” refers to a vital specialisation within solid mechanics in which the presence of a flaw is assumed
- Relationships are established between the material’s inherent resistance to crack growth, crack length, and the ‘far field’ stress at which the crack propagates to cause structural failure



Milestones in Fracture Mechanics



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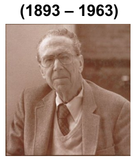

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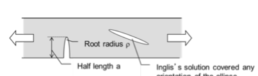

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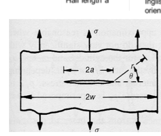

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Irwin line-crack

$$K = \sigma\sqrt{\pi a}$$

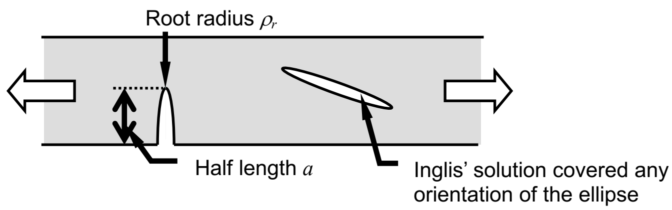
Fracture Energy

$$G = \frac{P^2}{2B}\frac{dC}{da}$$

Stress Intensity Factor

Elliptical Notch – Inglis' Solution

- In 1913 Charles Inglis (OBE, FRS) derived the stress field around an elliptical opening at any orientation in a plate subjected to a tensile stress:



- Inglis showed that the maximum stress at the tip of the notch has the form:

$$\sigma_{max} = R\left(1 + 2\sqrt{\frac{a}{\rho_r}}\right)$$

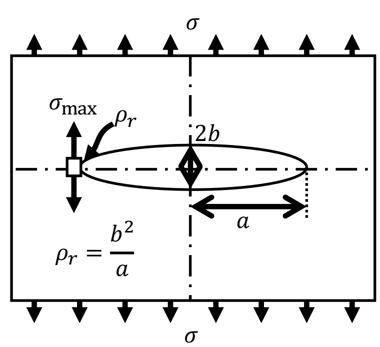
- where R depends on the orientation of the notch and the details of the stress field etc.

Key slide!

Elliptical Notch - Normal Loading

- For an elliptical notch normal to the direction of a uniform ('far field') tensile stress (σ) in an infinite plate:

$$\sigma_{max} = \sigma\left(1 + 2\sqrt{\frac{a}{\rho_r}}\right)$$



- e.g. for a circular hole where $b = a$

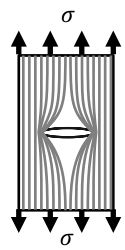
$$\sigma_{\max} = \sigma(1 + 2\sqrt{1}) = 3\sigma$$

- The ratio of maximum stress to the nominal applied stress is known as the stress concentration factor, $\boldsymbol{k_t}$

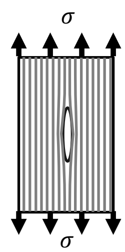
$$k_t = \frac{\sigma_{\max}}{\sigma}$$

Elliptical Notch - Example

- Consider these two cases below where the ellipse is 30 mm long and 10 mm wide. What are their respective concentration factors k_t?



$$k_t = 1 + 2\sqrt{\frac{15}{25/15}} = 7$$

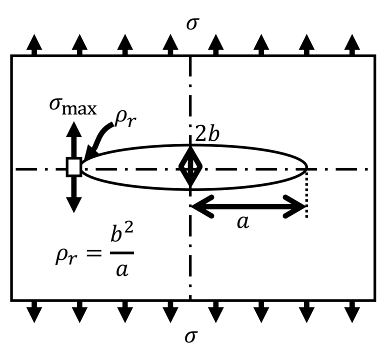


$$k_t = 1 + 2\sqrt{\frac{5}{45}} = 1.67$$

Sharp Notches

- Recalling the elliptical notch normal to the direction of a uniform tensile stress:

$$\sigma_{max} = \sigma\left(1 + 2\sqrt{\frac{a}{\rho_r}}\right)$$



- As b becomes smaller, ρ_r vanishes (i.e. tends towards zero)
- i.e. crack becomes infinitely sharp, so $a/\rho_r \gg 1$
- Therefore the equation above can be simplified to:

$$\sigma_{tip} \approx 2\sigma\sqrt{\frac{a}{\rho_r}}$$

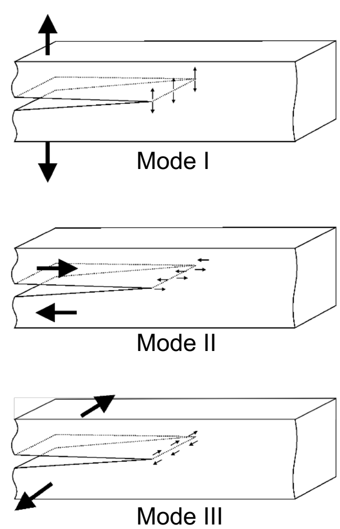
Crack Tip Singularity

- At the limit of a perfectly sharp crack, the stresses tend to be infinit at the crack tip – the stress field becomes singular
- Using such a result would predict that materials would have near-zero strength!
- This is obviously non-physical
- In reality the material generally undergoes **local yielding** which **blunts** the crack tip

Key slide!

Stress Intensity Factor

- The stress at the tip of the crack can undergo three modes of loading:
  - I. Tensile opening,
  - II. Shear sliding
  - III. Tearing



- Many real fractures in components are a mixture of these three modes, but mode I and mode II tend to dominate

Stress Intensity Factor

- Measure that captures the singular nature of the stress field

Singularity

$$\frac{1}{\sqrt{2\pi r}}\,\sigma\sqrt{\pi a}\,f(\theta)$$

The angular dependence

Geometrical relationship and the applied stress which **George R. Irwin** defined (in 1957) as the **stress intensity factor**, K

$$K = \sigma\sqrt{\pi a}$$

- For a specific crack geometry and applied stress, K will be a constant

Stress Intensity Factor

- Near the crack tip, material will yield, thus the maximum stress will have a limit.

- During fracture large part of the energy is associated to this plastic flow near known as 'plastic zone'

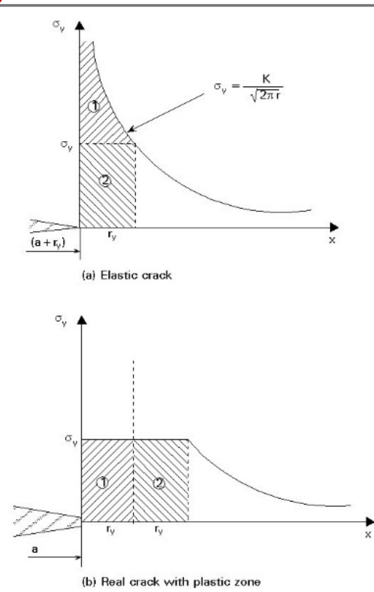


Figure 6 Plastic zone correction

Key slide!

Stress Intensity Factor

- For the initial case of an infinite plate with internal crack of length $2a$ subjected to uniform stresses, stress intensity factor is:

$$K = \sigma\sqrt{\pi a}$$

- However in general K is highly dependent on the geometry of the cracked body, so it usual to express it as:

$$K = \beta\sigma\sqrt{\pi a}$$

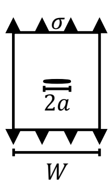
- Where β is a function determined depending on the geometrical configuration of the cracked body.

- Critical K when fracture occurs, is referred to as Fracture Toughness, Kc

$$Kc = \beta\sigma_s\sqrt{\pi a}$$

Shape factor β for common geometries

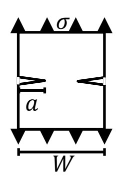
Centre Cracked Plate



$$K_I = \beta\sigma\sqrt{\pi a}$$

$$\beta = \sqrt{sec\frac{\pi a}{W}}$$

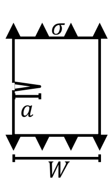
Double Edge Notched Plate



$$K_I = \beta\sigma\sqrt{\pi a}$$

$\beta = 1.12$ for small cracks

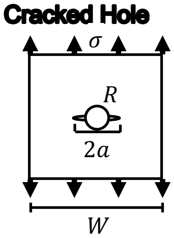
Single Edge Notched Plate



$$K_I = \beta\sigma\sqrt{\pi a}$$

$\beta = 1.12$ for small cracks

Cracked Hole



$$K_I = \beta\sigma\sqrt{\pi a}$$

$$\beta = f(R, a, W)$$

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An Example: Notched Composites

www.bris.ac.uk/composites

ADVANCED COMPOSITES CENTRE FOR INNOVATION & SCIENCE

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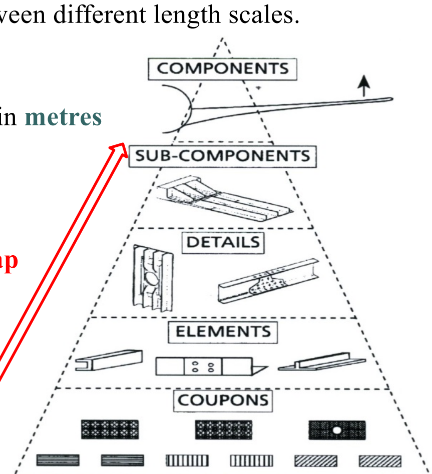
Why size effects matter?

- Size effect is the link between different length scales.

Components sized in **metres**

Dimension gap

Coupons in **centimetres**



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Why notched strengths matter?

- Notched strength is a design driver.

  - bolted joints

  - riveted connections

  - windows and doors

Boeing 787 fuselage
https://en.wikipedia.org/wiki/Boeing_787_Dreamliner

- Notches cause stress concentrations.

- The sizes of the holes vary - from about 1 mm to over 50 mm.

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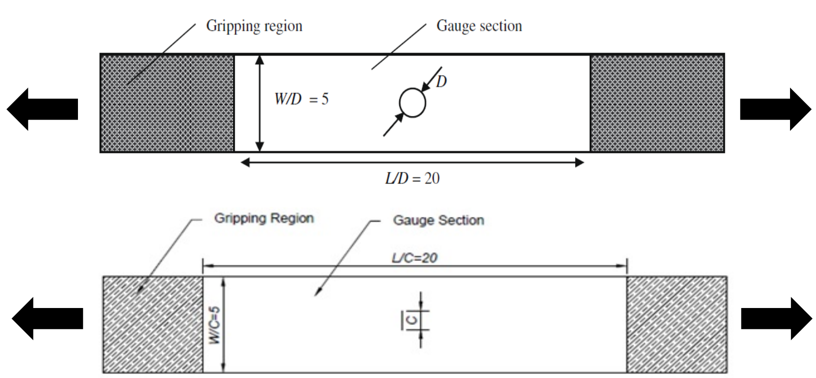
In-plane scaled notched tests

- In-plane scaled IM7/8552 $[45/90/-45/0]_{4s}$ notched tensile specimens



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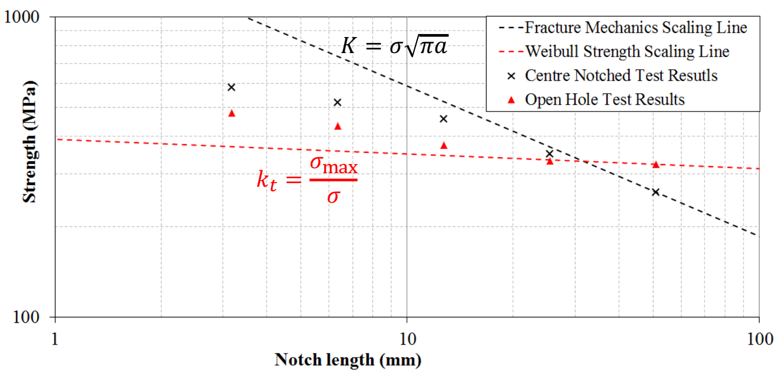
Scaling of notched strengths



- Notched strengths decrease with increasing notched length
- Large open-hole strengths –Weibull strength scaling, “constant” $\sigma_{\max}$
- Large centre-notched strengths – Fracture mechanics scaling, constant K

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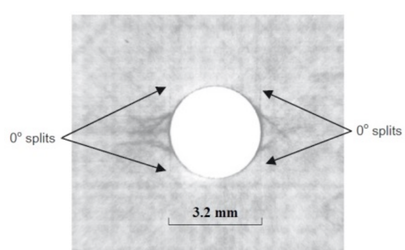
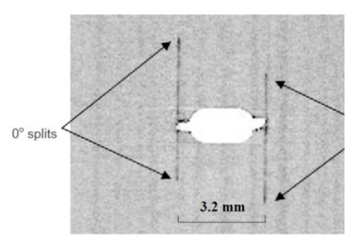
Why are sharp notches stronger?



(a) Open-hole specimen



(b) Centre-notched specimen

- Sharp notches have longer 0º splits
- 0º splitting blunts stress concentrations

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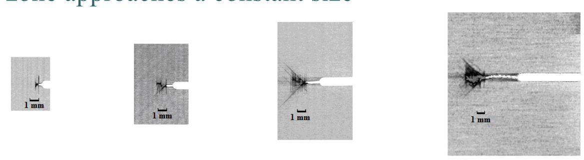
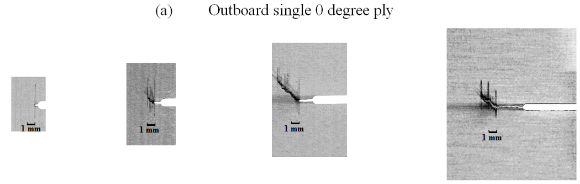
Scaling of damage process zones

- Damage zone approaches a constant size



C=3.2mm   *C*=6.4mm   *C*=12.7mm   *C*=25.4mm

(a) Outboard single 0 degree ply



C=3.2mm   *C*=6.4mm   *C*=12.7mm   *C*=25.4mm

(b) Central double 0 degree ply block

X-ray Computed Tomography (CT scan) images prior to failure

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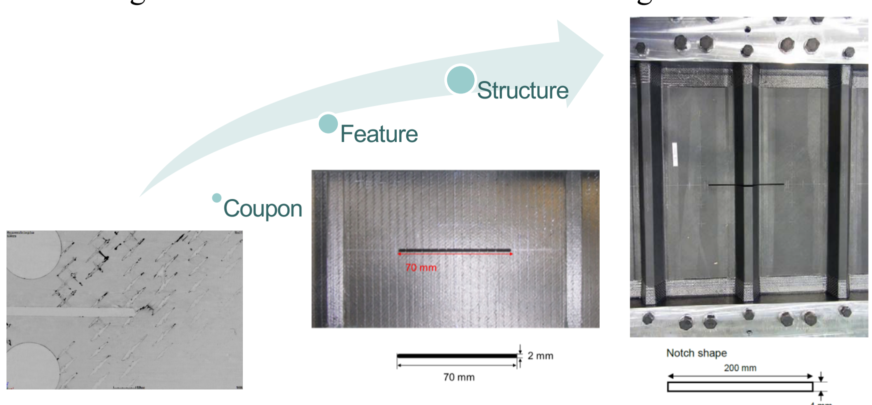
Engineering application

- Scaling effect is the link between different length scales



Coupon test at Bristol   Stiffened coupon test at ***JAXA***   Large panel test at ***JAXA***

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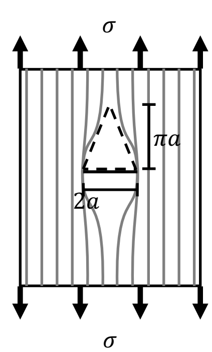
Summary

- The importance of fracture mechanics in understanding material failure has been introduced
- Inglis:
  - Stress singularity close to sharp crack $\sigma_{tip} \approx 2\sigma\sqrt{\frac{a}{\rho_r}}$
  - Stress concentration factor $k_t = \frac{\sigma_{\max}}{\sigma}$
- Stress intensity factor $K = \beta\sigma\sqrt{\pi a}$
- Fracture toughness $Kc = \beta\sigma_s\sqrt{\pi a}$
- Loading modes
  - I. opening
  - II. shear sliding
  - III. tearing

Next: Energy based approach

- The ideas from Inglis paper were further developed in 1920 by a young engineer, Alan Arnold Griffith, who analysed the phenomena from an energy point of view
- Consider an infinitely wide plate with a width of B, loaded in tension. We determined the elastic strain energy to be:

$$U_0 = \frac{1}{2}\frac{\sigma^2}{E}V$$



- Taking the same plate however with a crack. There will be a change in the strain energy for the same applied stress. Using the Inglis solution, Griffith was able to estimate this energy change by working out the volume of the material no longer being stressed.
- By visualising this as triangular region with a height of πa we can estimate the strain energy of the unstressed region to be:

$$U_{\text{unstressed}} = \frac{1}{2}\frac{\sigma^2}{E}2\pi a^2 B \quad = \frac{\sigma^2 \pi a^2 B}{E}$$

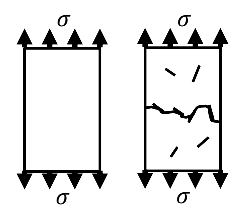
Linear Elastic Fracture Mechanics

Part II: Energy based approach

Dr Giuliano Allegri
giuliano.allegri@bristol.ac.uk

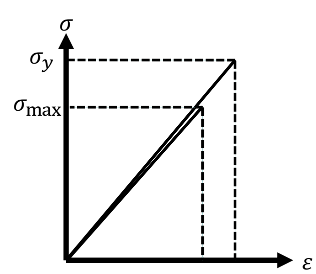
Last time

- The presence of defects modify the local stress field in the material
- Therefore, elastic stress analyses assuming perfectly homogeneous and flawless materials are not suitable for designs using high-strength materials



(a) (b)

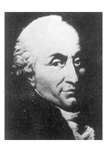
- When a crack reaches a certain critical length, it can propagate catastrophically through the structure
- This can happen at gross stresses which are much less than the yield stress of the material



Course Content

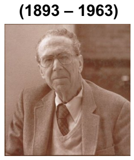
- Lecture 1: Case Studies
  - Why study failure?
  - Historical Examples
- Lecture 2: Modes of failure
  - Concept of strain energy and toughness
  - Ductile, brittle failure
  - Fractography
  - Factors affecting ductile to brittle transition
- Lecture 3: Introduction to fracture mechanics – Part 1
  - Introduction to fracture mechanics
  - Theoretical stress approach to fracture
  - Stress intensity factor
- Lecture 4: Introduction to fracture mechanics – Part 2
  - Griffith's energy balance approach
  - Irwin's energy balance approach
- Lecture 5: Measuring fracture toughness
  - Fracture process zone and geometrical considerations
  - Measuring toughness
  - Anisotropical materials

Milestones in Fracture Mechanics



Charles Augustin de Coulomb (1736 – 1806)

Alan Arnold Griffith (1893 – 1963)

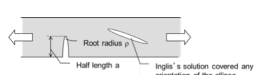


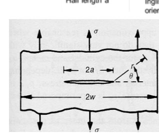
Irwin

Stone Age → Middle Age → Coulomb 1776 → Inglis 1913 → Griffith Model 1920 → Irwin's Model 1957

Flint Knapping

Crack propagation in stones under compression





$\sigma_f = \sqrt{\frac{E\gamma}{4a}}$

$\sigma_f = \sqrt{\frac{2E\gamma}{\pi a}}$

$\sigma_f = \sqrt{\frac{2E\gamma}{\pi a(1-\nu^2)}}$

Irwin line-crack

$K = \sigma\sqrt{\pi a}$

Fracture Energy

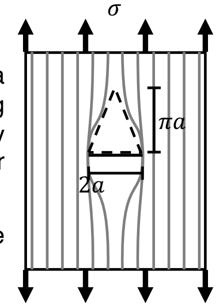
$G = \frac{P^2}{2B}\frac{dC}{da}$

Stress Intensity Factor

Griffith (1921) Theory

- The ideas from Inglis paper were further developed in 1920 by a young engineer, Alan Arnold Griffith, who analysed the phenomena from an energy point of view
- Consider an infinitely wide plate with a thickness of B, loaded in tension. We determined the elastic strain energy to be:

$$U_0 = \frac{1}{2}\frac{\sigma^2}{E}V$$



- Taking the same plate however with a crack. There will be a change in the strain energy for the same applied stress. Using the Inglis solution, Griffith was able to estimate this energy change by working out the volume of the material no longer being stressed.
- By visualising this as triangular region with a height of πa we can estimate the strain energy of the unstressed region to be:

$$U_{\text{unstressed}} = \frac{1}{2}\frac{\sigma^2}{E}2\pi a^2 B \quad = \frac{\sigma^2 \pi a^2 B}{E}$$

Griffith (1921) Theory

- Therefore the strain energy for the plate with a crack is:

$$U = U_0 - U_{unstressed}$$

$$U = U_0 - \frac{\sigma^2 \pi a^2 B}{E}$$

- Energy balance requires that the change is strain energy must be transferred somewhere.
- In fact this energy is equivalent to the energy needed in forming the fracture surface.
- This energy is in effect absorbed by the material. The surface energy S associated with a crack of length a is thus:

$$S = 2A\gamma = 4aB\gamma$$

where the energy needed to create a surface is γ.

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Griffith (1921) Theory

Let’s define the Griffith energy balance:

- The potential energy of the system is the sum of the strain energy U, plus the surface energy of the crack S occur when

$$Potential\ energy\ E = U + S$$

- The maximum of this total energy will occur when

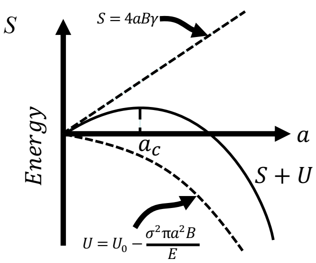
$$\frac{d(U+S)}{dA} = 0$$

- Where change of strain energy release per unit crack area is

$$\frac{dU}{dA} = -\frac{\sigma^2 \pi a}{E}$$

- and surface energy per area of crack area

$$\frac{dS}{dA} = 2\gamma$$



a_c is the critical crack length when two forms of energies are balanced.

Key slide! Griffith (1921) Theory

- Strain energy release rate G – change of strain energy per unit crack area

$$G = \left|\frac{dU}{dA}\right|$$

- When the material is stressed, unstable fracture will occur only if the strain energy release rate becomes more than the atomic bond surface energy

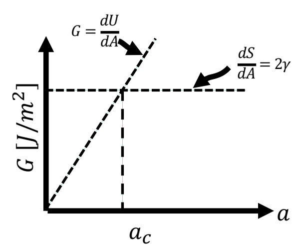
$$G > 2\gamma$$

- We can see that this corresponds to a critical crack length at which the material is expected to fracture unstably

$$\frac{d(U+S)}{dA} = 0 = -\frac{\sigma^2 \pi a}{E} + 2\gamma$$

- Re-writing the equation we get

$$\sigma = \sqrt{\frac{2E\gamma}{\pi a}} \qquad \sigma_s = \sqrt{\frac{2E\gamma}{\pi a_c}}$$



The energy balance is broken once a_c is reached.

Griffith (1921) theory

- This procedure helped define a structural strength to two inherent material properties, modulus and surface energy of the atomic bonds
- However, this procedure was found to agree well only to highly brittle materials, like **glass**
- This is because Griffith assumes an **infinitely sharp crack** where the stresses at the tip are very high
- When applied to **ductile materials** this procedure severely underestimated its fracture strength

Irwin (1957)

In 1957, a professor from the Leigh University, George Rankine Irwin, showed that Griffith's relation should include the work done in the plastic region, i.e. the crack will propagate if the strain energy is bigger than the total energy necessary (work done to create new crack surfaces and the work done in plastic region).

Key slide!

Irwin (1957) Theory

- Firstly, the Griffith equation satisfies only ideally brittle materials like glass.
- However, metals are not ideally brittle and normally fail with certain amounts of plastic deformation, the fracture stress is increased due to blunting of the crack tip.
- Irwin suggested that a crack will propagate if the strain energy release rate G is bigger than the critical work necessary to create new crack surfaces.

$$G = \frac{dU}{dA} > G_C$$

- G_C is known as the fracture energy of a material.

Key slide!

Irwin (1957) Theory

- Irwin simply argued that definition of $\frac{dS}{dA} = 2\gamma$ (the energy required to break bonds) requires an extra plastic energy, γ_P

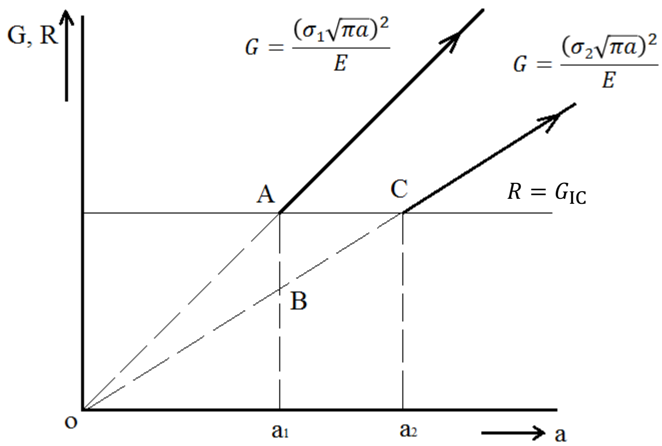
$$G_C = 2[\gamma_S + \gamma_P]$$

- The modified Griffith equation can then be rewritten in the form:

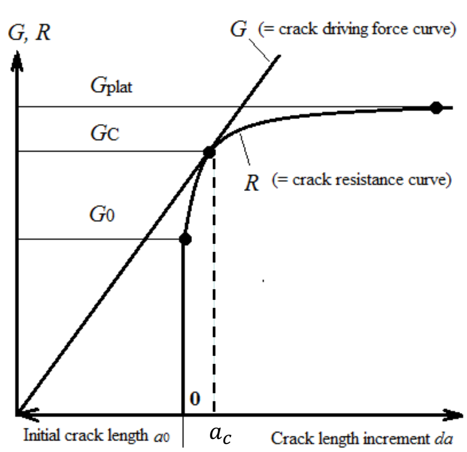
$$\sigma_s = \sqrt{\frac{E G_C}{\pi a}} \qquad G_C = \frac{(\sigma_s \sqrt{\pi a})^2}{E}$$

- This expression describes the relationship between three important parameters involved in the fracture process:
  - the material, as evidenced in the critical strain energy release rate G_C;
  - the stress level σ_s;
  - and the size, a, of the flaw.

- Materials with a constant G_C: Energy balanced at a_1 and a_2 for two different crack driving force curves



- Materials with a R-curve: Energy balanced at a_C for one crack driving force curve



Irwin (1957) Theory

- In a design situation, one might choose a value of a based on the smallest crack that could be easily detected.

- Then for a given fracture energy G_C of a material the safe level of stress σ_S can be determined. The structure would then be sized so as to keep the working stress comfortably below this critical value.

$$\sigma_S = \sqrt{\frac{E G_C}{\pi a}}$$

$$G_c = \frac{(\sigma_s \sqrt{\pi a})^2}{E}$$

Key slide!

Fracture toughness vs. Fracture energy

- As you can see stress intensity factor K can be related to the strain energy release rate G

$$K = \sigma\sqrt{\pi a} \qquad G = \frac{(\sigma\sqrt{\pi a})^2}{E} \qquad G = \frac{K^2}{E}$$

- For critical strain energy release rate G_C (fracture energy) [J/m^2], we have the relationship to the critical stress intensity factor $\boldsymbol{K_C}$ (fracture toughness) [Pa m$^{1/2}$]

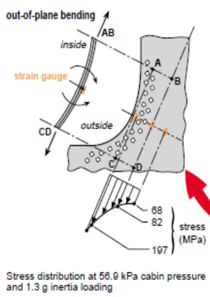
In Plane Stress

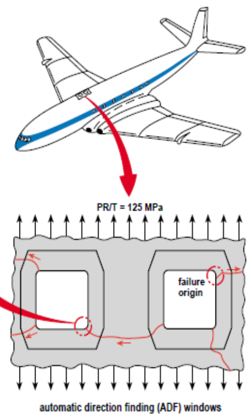
$$G_C = \frac{K_C^2}{E}$$

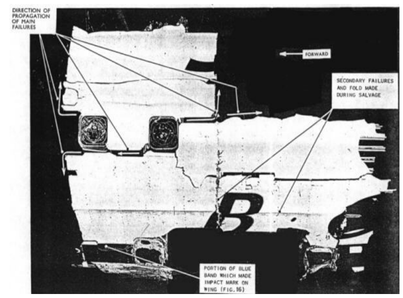
In Plane Strain

$$G_C = \frac{(1-\nu^2)K_C^2}{E}$$

Example: DeHavilland Comet Failure







Key slide! Example of FM: Comet Failure

- **Remember:** The story of the DeHavilland Comet aircraft of the early 1950's, in which at least two aircraft disintegrated in flight, provides a tragic but fascinating insight into the importance of fracture theory.

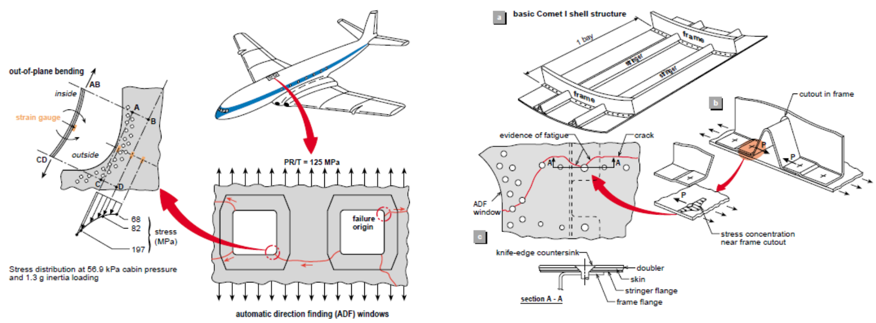
- The Comet aircraft had a aluminium fuselage, with $G_C \approx 300$ in-psi. The hoop stress due to relative cabin pressurization was 20,000 psi, and at that stress the length of crack that will propagate catastrophically is:

$$G_c = \frac{(\sigma_s\sqrt{\pi a})^2}{E} \quad \Rightarrow \quad a = \frac{G_C\,E}{\pi\,\sigma_s^{\,2}} = \frac{(300)(11\times10^6)}{\pi\left(20\times10^3\right)^2} = 2.62 \text{ inches} \cong 66.5 \text{ mm}$$

- A crack would presumably be detected in routine inspection long before it could grow to this length!

Example of FM: Comet Failure

- But in the case of the Comet, the cracks were propagating from rivet holes near the cabin windows. When the crack reached the window, the size of the window opening was effectively added to the crack length, plus the absence of any crack-arrest design features ultimately lead to catastrophic disaster.



Countersunk rivet heads + stress concentrations @ windows + absence of crack arrest design =
DISASTER!

Summary

- The importance of fracture mechanics in understanding material failure has been introduced
- Inglis:
  - Stress singularity close to sharp crack

$$\sigma_{tip} \approx 2\sigma\sqrt{\frac{a}{\rho_r}}$$

- Griffith
  - concept of an energy balance, the relationship between stress, crack length atomic bonds strength

$$\sigma = \sqrt{\frac{E\gamma}{2\pi a}}$$

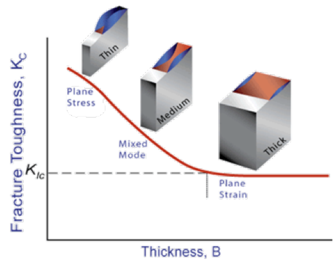
- Irwin
  - The plastic deformation ahead of crack contributes to toughness
  - Stress intensity factor and loading modes
    - I. opening
    - II. shear sliding
    - III. tearing

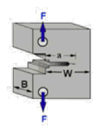
$$G = \frac{(\sigma\sqrt{\pi a})^2}{E}$$

$$G_c = \frac{(\sigma_s\sqrt{\pi a})^2}{E}$$

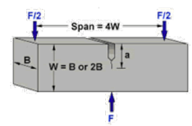
$$G_C = \frac{K_C^2}{E}$$

Next: Measurement of Fracture Toughness





CT Specimen



SENB Specimen

- It is important to understand the role of material thickness and application of loading direction in establishing fracture toughness
- This can be accomplished via understanding of
  - Plane Strain and Plane Stress
  - Crack tip state of tension
  - Isotropic vs Anisotropic Toughness
- Objective: Understand the influence of geometry and loading method on fracture toughness
  - Describe the importance of Plane Strain
  - Explain the role of material anisotropy in fracture toughness

Structures and Materials 3 (StM3)
Failure of Materials

Measuring fracture toughness

Dr Giuliano Allegri

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Last time

- The importance of fracture mechanics in understanding material failure has been introduced
- Inglis:
  - Stress singularity close to sharp crack
- Griffith
  - concept of an energy balance, the relationship between stress, crack length atomic bonds strength
- Irwin
  - The plastic deformation ahead of crack contributes to toughness
  - Stress intensity factor and loading modes
    - I. opening
    - II. shear sliding
    - III. tearing

$$\sigma_{tip} \approx 2\sigma\sqrt{\frac{a}{\rho_r}}$$

$$\sigma = \sqrt{\frac{E\gamma}{2\pi a}}$$

$$G = \frac{(\sigma\sqrt{\pi a})^2}{E}$$

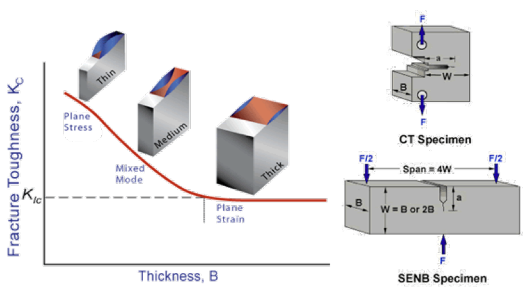
$$G_c = \frac{(\sigma_s\sqrt{\pi a})^2}{E}$$

$$G_C = \frac{K_C^2}{E}$$

Course Content

- Lecture 1: Case Studies
  - Why study failure?
  - Historical Examples
- Lecture 2: Modes of failure
  - Concept of strain energy and toughness
  - Ductile, brittle failure
  - Fractography
  - Factors affecting ductile to brittle transition
- Lecture 3: Introduction to fracture mechanics – Part 1
  - Introduction to fracture mechanics
  - Theoretical stress approach to fracture
  - Stress intensity factor
- Lecture 4: Introduction to fracture mechanics – Part 2
  - Griffith's energy balance approach
  - Irwin's energy balance approach
- Lecture 5: Measuring fracture toughness
  - Fracture process zone and geometrical considerations
  - Measuring toughness
  - Anisotropical materials

Measurement of Fracture Toughness



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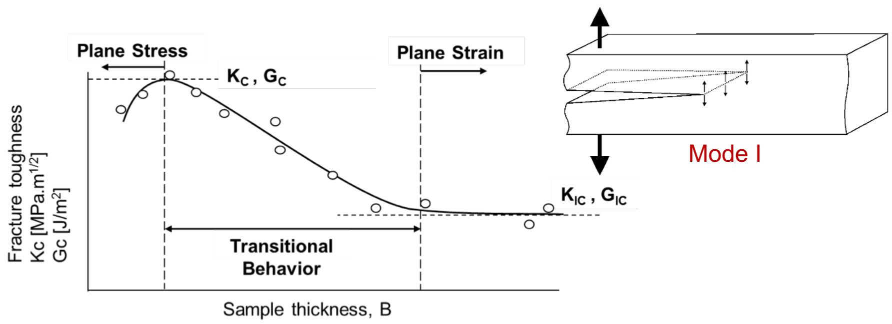
Measuring the mode I fracture toughness 5

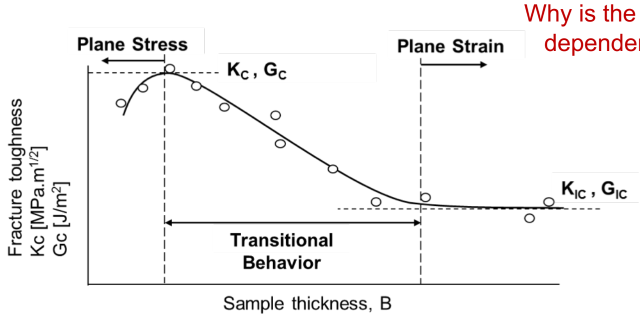
- The critical stress intensity factor required to extend the crack under a opening model is called the **mode I fracture toughness** K_C.
- But measured fracture toughness can depend on sample sizes, so we need to improve control of the test dimensions.



Role of specimen thickness 6

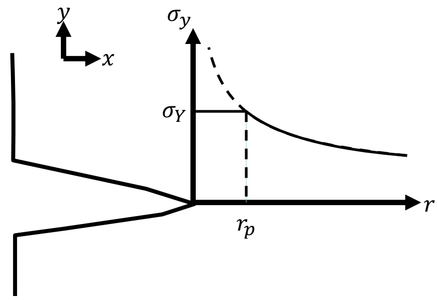
- Specimens having standard proportions but different in absolute size produce different values for K_C.
- The **stress states** depend on the specimen thickness (B) until the thickness exceeds some critical dimension.
- Once the thickness exceeds the critical dimension, the value of K_C becomes relatively constant, which is regarded as the true material property called the **plane-strain fracture toughness** K_{IC}.

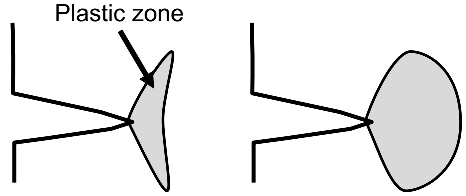
Why is the result so thickness dependent? – Constraint !



Plastic zone

- When a crack is loaded the stress increases exponentially, recall $1/\sqrt{r}$
- However a material will yield before the stress reaches this singularity
- This yield region is known as the **Plastic zone**, or **Fracture process zone**
- It is clear that this region depends on the nature of the stress ahead of the crack
- Plane Stress $r_p \approx$ 3x Plane Strain r_p





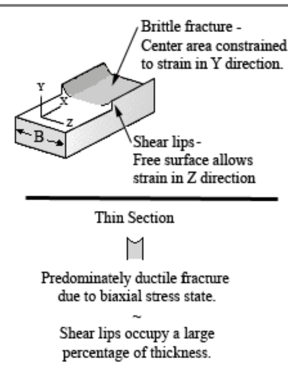
Plane strain Plane stress

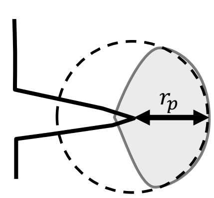
Key slide!

Plastic zone – Plane Stress

- For thin sheets there is no stress in thickness direction, i.e. $\sigma_{zz} = 0$
- This stress state is known as Plane Stress
- However there's still a ε_{zz} which results in a bi-axial stress state such that the material fractures in a characteristic ductile manner, with a 45º shear lip being formed at each free surface.
- The Plastic zone radius can be approximated:

$$r_p = \frac{1}{2\pi}\left(\frac{K_C}{\sigma_y}\right)^2$$



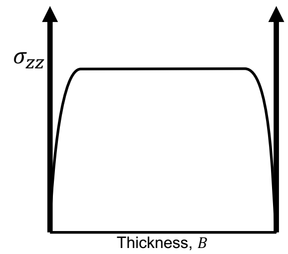


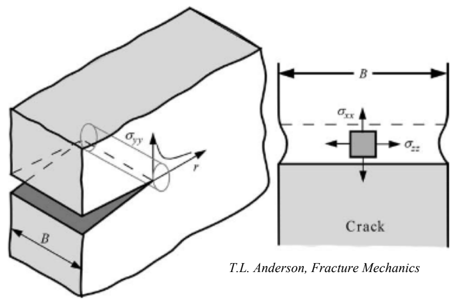
Plane stress

Key slide! Plastic zone – Plane Strain

- For a thick material the stress in the thickness direction becomes significant, $\sigma_{zz} \gg 0$ i.e. there's a tri-axial stress state
- With the thickness contraction effectively being $\varepsilon_{zz} = 0$
- This stress state is known as Plane Strain
- The Plastic zone radius can be approximated:

$$r_p = \frac{1}{6\pi}\left(\frac{K_{IC}}{\sigma_y}\right)^2$$

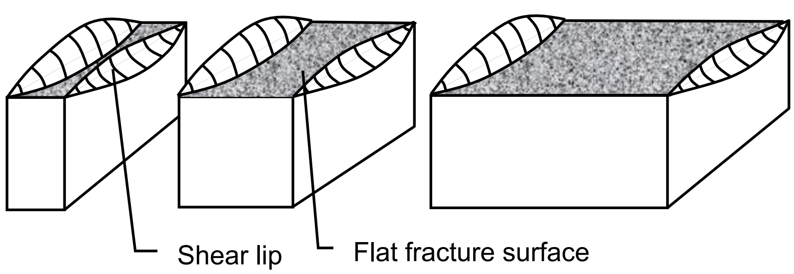




T.L. Anderson, Fracture Mechanics

Shear lips

- As the crack approaches the surface it tends to run out into a 'shear lip'.
- For small samples, the shear lips are the dominant failure mechanism.
- At the thickest sample size the contribution of the shear lips is trivial and we have a valid measurement of the plane strain fracture toughness.



Example

- High Strength Steel

$$\sigma_{YS} = 1640MPa$$

- Plane strain r_p

$$K_{IC} = 50MPa\sqrt{m}$$

- This is about twice the grain diameter, i.e. very localised plasticity

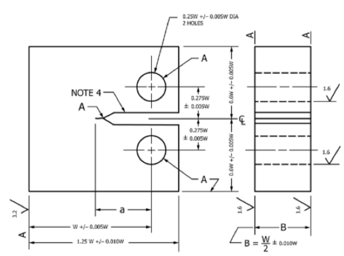
Material	Yield Strength MPa	Yield Strength ksi	K_{Ic} $MPa/\sqrt{m}$	K_{Ic} $ksi/\sqrt{in}$
Metals				
Aluminum Alloy[a] (7075-T651)	495	72	24	22
Aluminum Alloy[a] (2024-T3)	345	50	44	40
Titanium Alloy[a] (Ti-6Al-4V)	910	132	55	50
Alloy Steel[a] (4340 tempered @260°C)	1640	238	50.0	45.8
Alloy Steel[a] (4340 tempered @425°C)	1420	206	87.4	80.0
Ceramics				
Concrete	—	—	0.2–1.4	0.18–1.27
Soda-Lime Glass	—	—	0.7–0.8	0.64–0.73
Aluminum Oxide	—	—	2.7–5.0	2.5–4.6
Polymers				
Polystyrene (PS)	—	—	0.7–1.1	0.64–1.0
Poly(methyl methacrylate) (PMMA)	53.8–73.1	7.8–10.6	0.7–1.6	0.64–1.5
Polycarbonate (PC)	62.1	9.0	2.2	2.0

[a] **Source:** Reprinted with permission, *Advanced Materials and Processes,* ASM International, © 1990.

$$r_p = \frac{1}{6\pi}\left(\frac{50}{1640}\right)^2 = 0.049mm$$

Mode I fracture toughness ASTM standard

- Compact Tension Specimen
- ASTM Standard
  - ASTM-E399-12 *"Standard Test Method for Linear-Elastic Plane-Strain Fracture Toughness K_{IC} of Metallic Materials"*



- Must ensure plane strain conditions - thick specimens

$$K_{IC} = \frac{P_C}{B\sqrt{W}} f\left(\frac{a}{W}\right)$$

$$f\left(\frac{a}{W}\right) = \qquad \text{(A4.2)}$$

$$\frac{\left(2+\frac{a}{W}\right)\left[0.886+4.64\frac{a}{W} - 13.32\left(\frac{a}{W}\right)^2 + 14.72\left(\frac{a}{W}\right)^3 - 5.6\left(\frac{a}{W}\right)^4\right]}{\left(1-\frac{a}{W}\right)^{3/2}}$$

Mode I fracture toughness ASTM standard

- When you load the material typical load curves look as shown
- What value do you take for the critical load P_c ?
- You must make correct engineering decision on this quantity as this will directly affect the fracture toughness

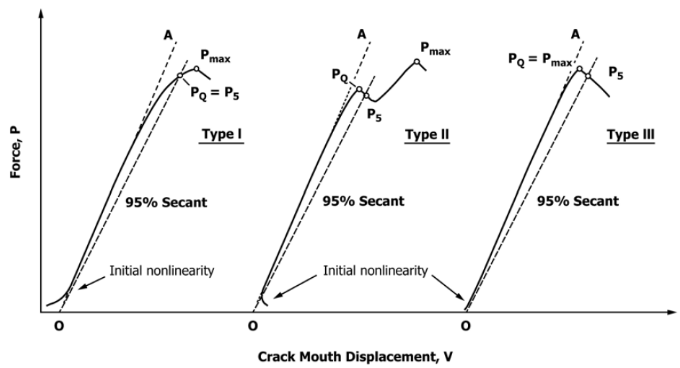
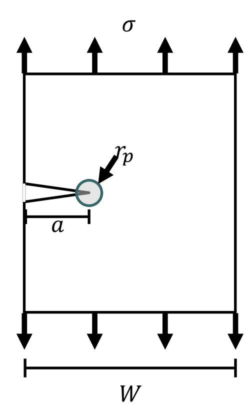


FIG. 7 Principal Types of Force-Displacement (CMOD) Records

LEFM validity

- For LEFM equations to be valid the approximate plastic zone radius ahead of the crack must be $\boldsymbol{r_p < a/50}$
- Similarly $\boldsymbol{r_p < (W - a)/50}$
- An additional requirement for Plane strain crack is the thickness, *B* relative to the plastic zone radius must satisfy $\boldsymbol{r_p < B/50}$



$$\sigma_s = \frac{K_{IC}}{\beta\sqrt{\pi a}}$$

Example

Material	K_{IC} [MPa m$^{1/2}$]	σ_y [MPa]	Plane strain r_p	B_{min}	LEFM Valid
High strength Aluminium alloy	25	500	0.013mm	6mm	Yes
High strength Steel	60	1500	0.1mm	5mm	Yes
PMMA	1.5	50	0.05mm	2.5mm	Yes
Medium strength structural steel	80	450	1.7mm	85mm	Yes*

*LEFM valid for specimen sizes with minimum 85mm thickness, minimum crack of 85mm and minimum width of 170mm

To characterise K_{IC} large and thick specimen for a laboratory coupon standard is needed

To recap

- Cracks and defects exist in all engineering structures
- Cracks can be represented as flat free surfaces in regions where tri-axial stresses dominate – which makes Linear Elastic Fracture Mechanics assumption correct
- This means the stress field can be characterised and the K_{IC} value at the onset of crack extension can be correctly accepted as a material property (plane-strain fracture toughness)
- Factors have been calculated for a variety of crack types, the most commonly used being the interior crack in a plate of infinite width and an edge crack in a plate of semi-infinite width.

Mode I fracture toughness energy approach

- Energy conservation $\frac{d}{dA}(U + W - F) = 0$

$$G = \frac{dW}{Bda} = \frac{d}{Bda}(F - U) \qquad U = \frac{1}{2}P\delta \qquad F = Pd\delta$$

- Under **fixed displacement**: $F = 0$
  - Energy dissipation $G = -\frac{dU}{Bda} = -\frac{\delta}{2B}\frac{dP}{da}$
- Under **fixed load**: $G = \frac{dF}{Bda} - \frac{dU}{Bda}$
  - Energy absorption $G = \frac{P}{B}\frac{d\delta}{da} - \frac{P}{2B}\frac{d\delta}{da} = \frac{P}{2B}\frac{d\delta}{da}$

Mode I fracture toughness energy approach

- Compliance is the inverse of stiffness
  - If something is compliant - It will deform more

$$C = \frac{\delta}{P} \Rightarrow CP = \delta$$

- Under **fixed displacement**: $\frac{d(CP)}{da} = \frac{d\delta}{da} \quad P\frac{dC}{da} + C\frac{dP}{da} = 0 \quad \frac{dP}{da} = -\frac{P}{C}\frac{dC}{da}$
  - Energy dissipation

$$G = -\frac{\delta}{2B}\frac{dP}{da} = -\frac{CP}{2B}\frac{dP}{da} = \frac{P^2}{2B}\frac{dC}{da}$$

- Under **fixed load:** $\frac{d(CP)}{da} = \frac{d\delta}{da} \quad P\frac{dC}{da} = \frac{d\delta}{da}$
  - Energy absorption

$$G = \frac{P}{2B}\frac{d\delta}{da} = \frac{P^2}{2B}\frac{dC}{da}$$

- Strain energy released rate *G* under both **fixed displacement** and **fixed load**, can now be written in the same form.

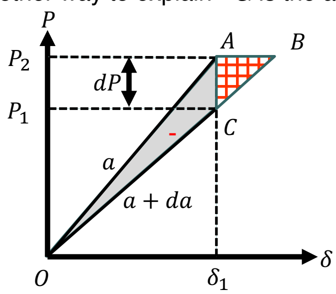
Key slide!

Strain energy, compliance and crack growth

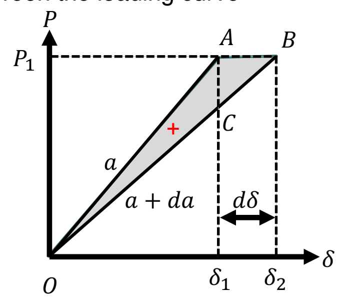
- This form of the equation is very useful as now we can relate the fracture energy to the compliance of the system

$$G = \frac{P^2}{2B}\frac{dC}{da}$$

- Another way to explain - *G* is the area between the loading curve

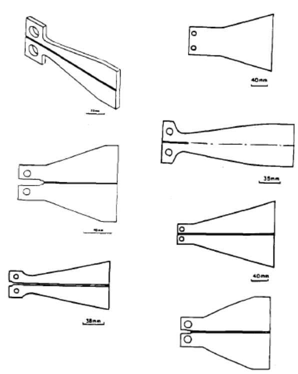


Under **fixed displacement**



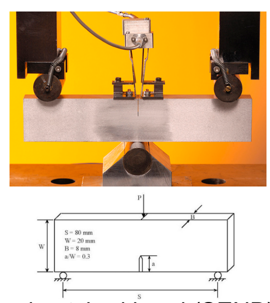
Under **fixed load**

Test configurations



Tapered Double Cantilever Bend (TDCB)

Compact Tension (CT)

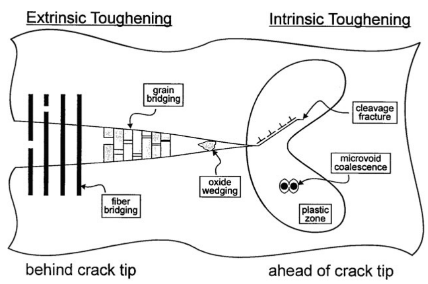


Single edged notched bend (SENB)

Double Cantilever Bend (DCB)

Factors that influence fracture toughness

- Extrinsic and Intrinsic toughness mechanisms
  - Schematic illustration of the mutual competition between intrinsic mechanisms of damage which act ahead of the crack tip to promote crack advance and extrinsic mechanisms of crack-tip shielding that act mainly behind the crack tip to impede crack advance.
- Crack tip radius
  - Notching techniques
  - Does it create small enough crack tip?
  - Is it uniform, no external damage to specimen?



Elastic Plastic Fracture Mechanics (EPFM)

- There will come a time where it is required to characterize a ductile or highly tough material.
- To do this, a procedure has been developed called the Crack Tip Opening Displacement (CTOD)
- Crack tip plasticity makes the crack behave as if it where longer, a+r_p
- This method calculates the displacement at the physical crack tip

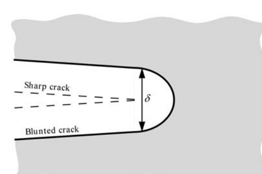


FIGURE 3.1 Crack-tip-opening displacement (CTOD). An initially sharp crack blunts with plastic deformation, resulting in a finite displacement (δ) at the crack tip.

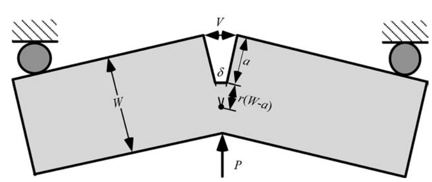
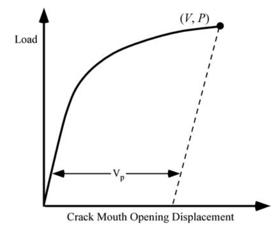


FIGURE 3.5 The hinge model for estimating CTOD from three-point bend specimens.

T.L. Anderson, Fracture Mechanics

Elastic Plastic Fracture Mechanics (EPFM)

- Crack Tip Opening Displacement (CTOD) Procedure
- Separating the elastic and plastic components of the CTOD we get:



$$\delta = \delta_{el} + \delta_p = \frac{K_I^2}{m\sigma_{YS}E'} + \frac{r_p(W-a)V_p}{r_p(W-a)+a}$$

Where m is a dimensionless constant ~1 for plane strain and ~2 for plane stress

r_p is a rotational factor 0.44 for typical materials and test specimen geometries

E' is effective modulus:

$E' = E$ for plane stress $\quad E' = \frac{E}{1-\nu^2}$ for plane strain

- Elastic strain energy dissipation or absorption determines the fracture energy J_C.

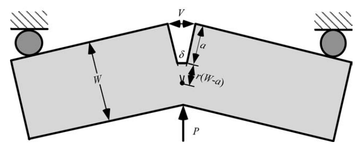


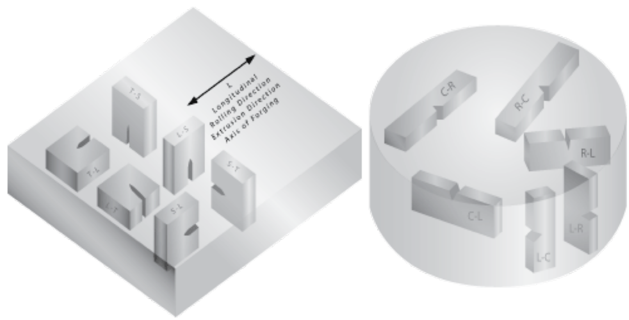
FIGURE 3.5 The hinge model for estimating CTOD from three-point bend specimens.

T.L. Anderson, Fracture Mechanics

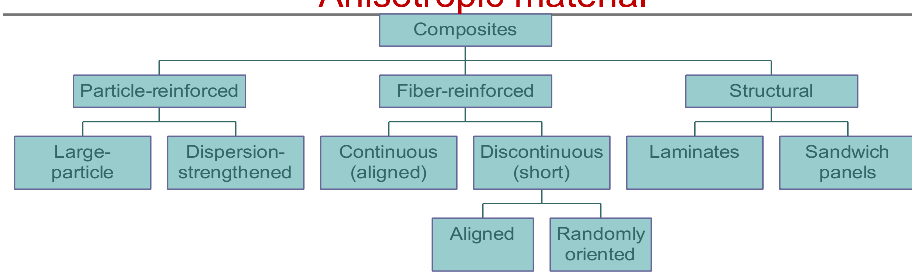
Isotropic Materials vs Anisotropic Materials

Isotropic material

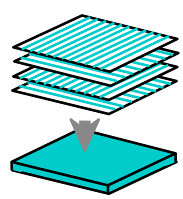
- Remember an Isotropic material (steel, aluminium) has a stress-strain relationship that is independent of orientation of the coordinate system at that point,
  - Same elastic properties (E, ν) in all directions



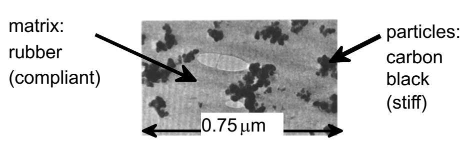
Anisotropic material



- Laminates -
  - stacked and bonded fiber-reinforced sheets
  - stacking sequence: e.g., 0º/90º
  - benefit: balanced in-plane stiffness



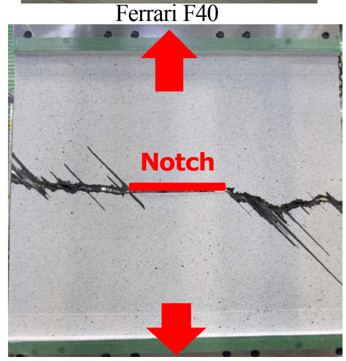
- Automobile tire rubber



Why composites?

- The main advantages of composites
  - High stiffness
  - High strength
  - Light weight
- Understanding failure of composite structures
  - Strength scaling
  - Notch sensitivity
  - Trans-laminar fracture

Ferrari F40



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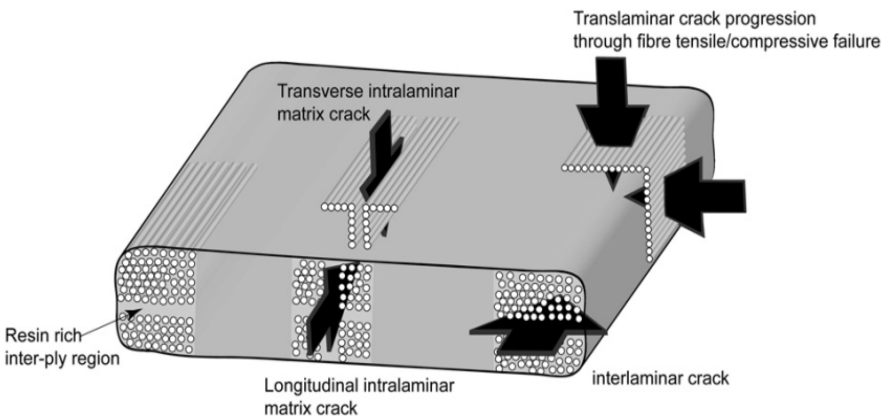
Failure of composite laminates

- A general lack of knowledge about how damage initiates and progresses



Overview of ply-level failure

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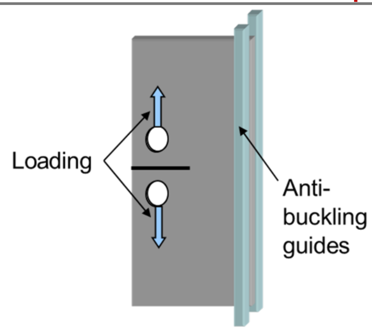
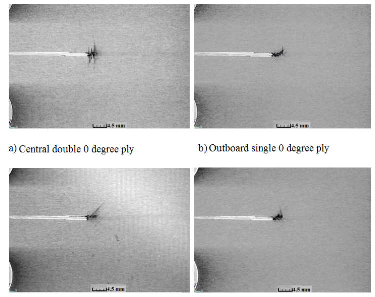
Test example - Composites



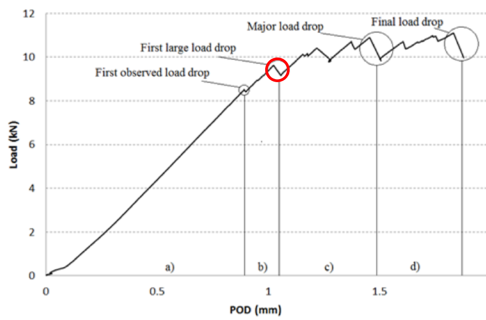


a) Central double 0 degree ply

b) Outboard single 0 degree ply

c) Typical 45 degree ply

d) Typical -45 degree ply



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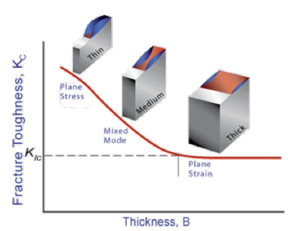
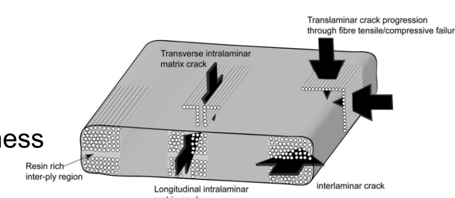
Concluding remarks

- It is important to understand the role of material thickness and application of loading direction in establishing fracture toughness



- This has been demonstrated via the introduction of the following concepts:
  - Plane Strain and Plane Stress
  - LEFM vs. EPFM
  - Isotropic vs. Anisotropic Toughness



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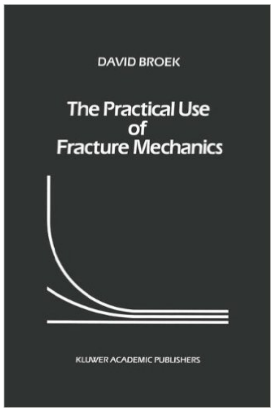
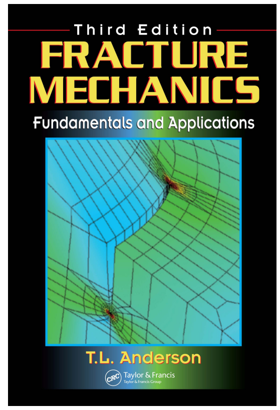
Recommended Books





- The Practical Use of Fracture Mechanics
  - D. Broek
- Fracture Mechanics Fundamentals and Applications – T.L. Anderson (3rd Edition)