Stress, Strain and Deformation
Bending Stresses – Visualisation

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07 November 2017

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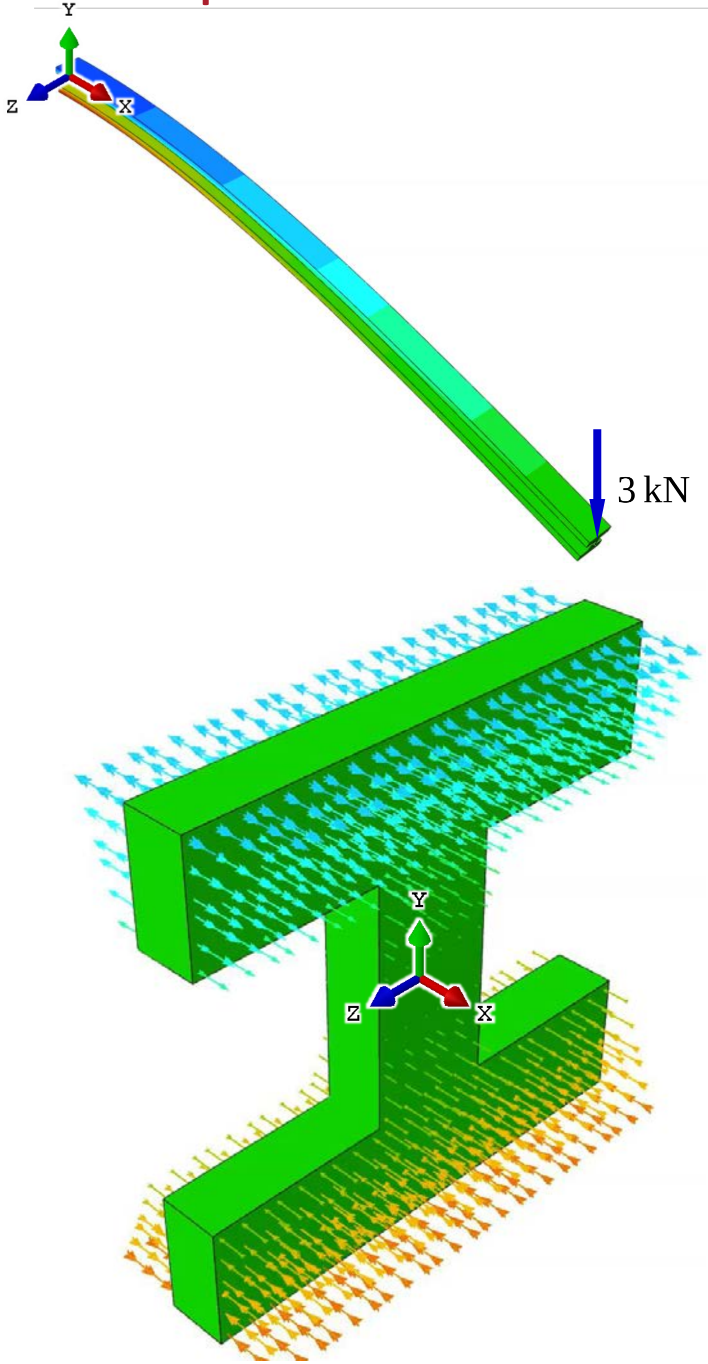
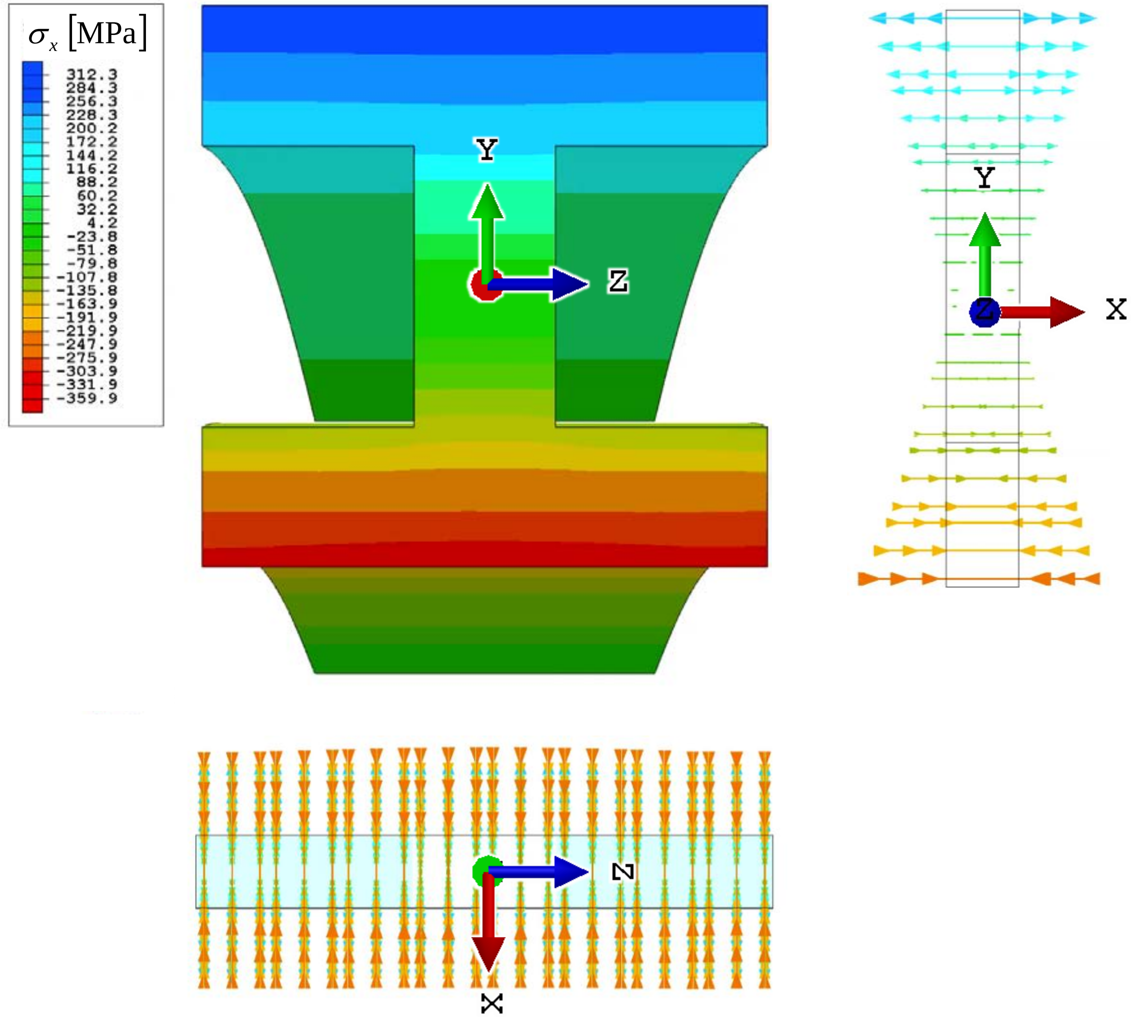
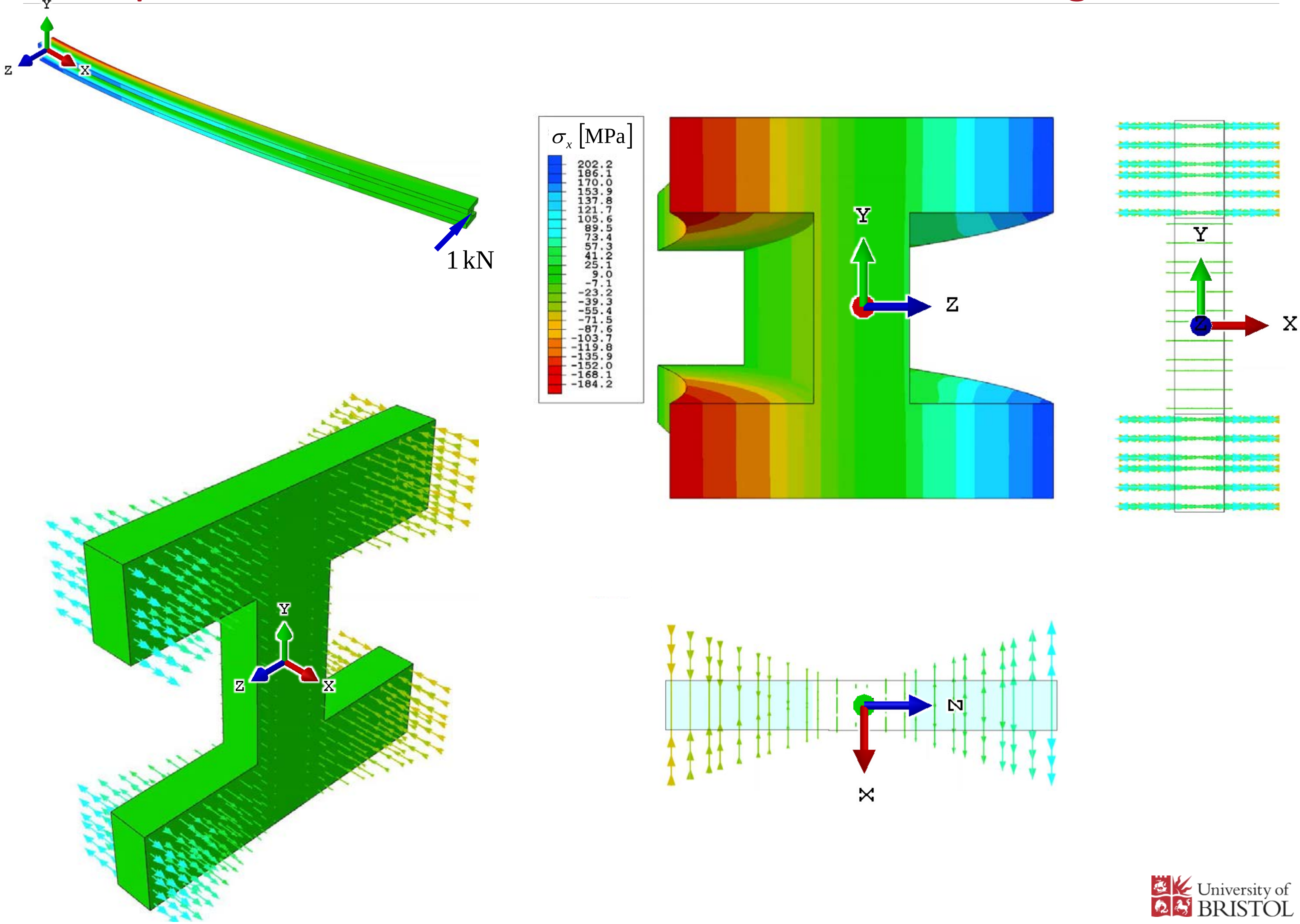
Example 2.2.8: I-section beam in combined bending





Example 2.2.8: I-section beam in combined bending



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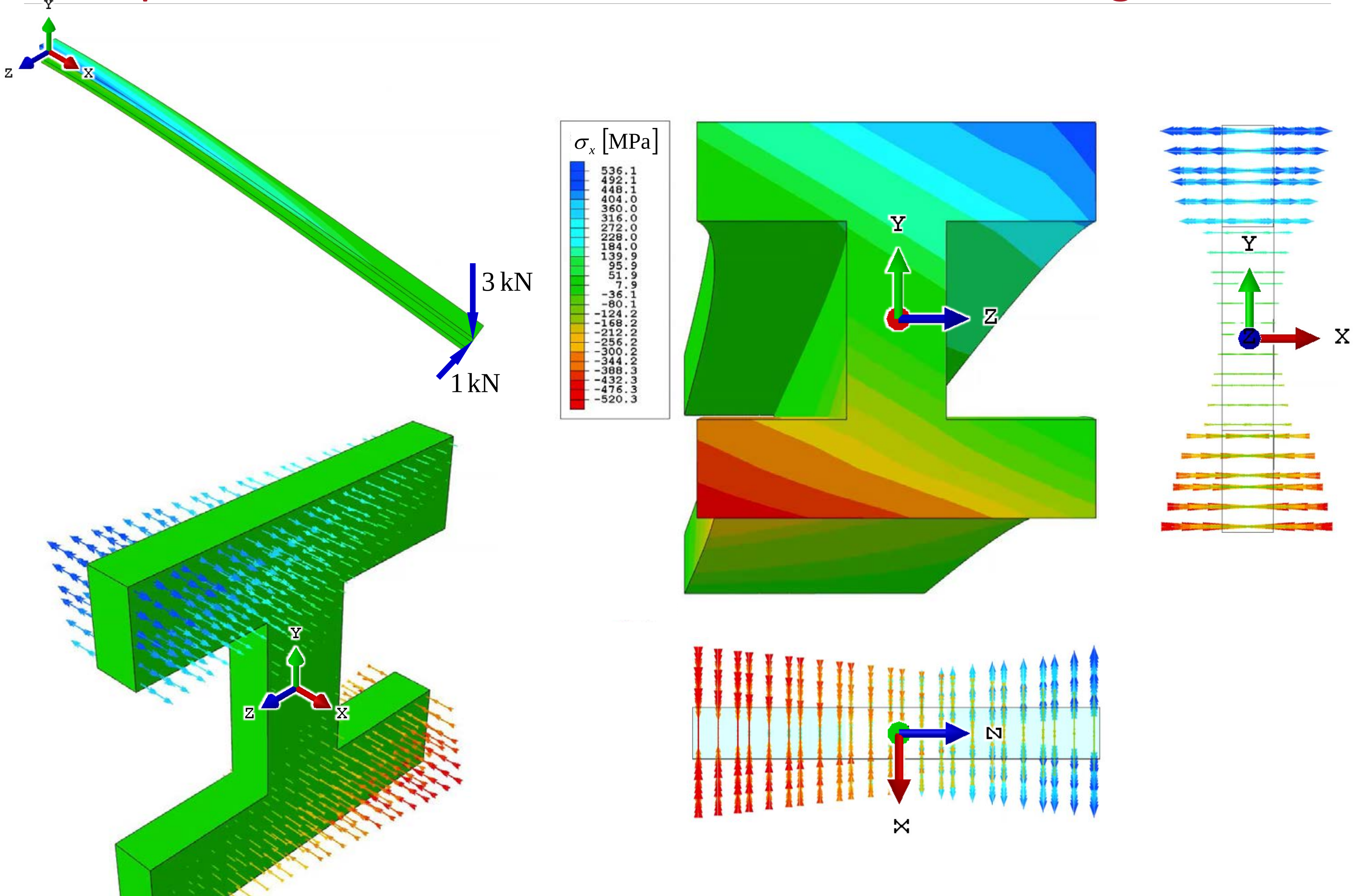
Example 2.2.8: I-section beam in combined bending



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Stress, Strain and Deformation
Bending Deflections

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Bending Deflection – Definitions

KEY POINTS

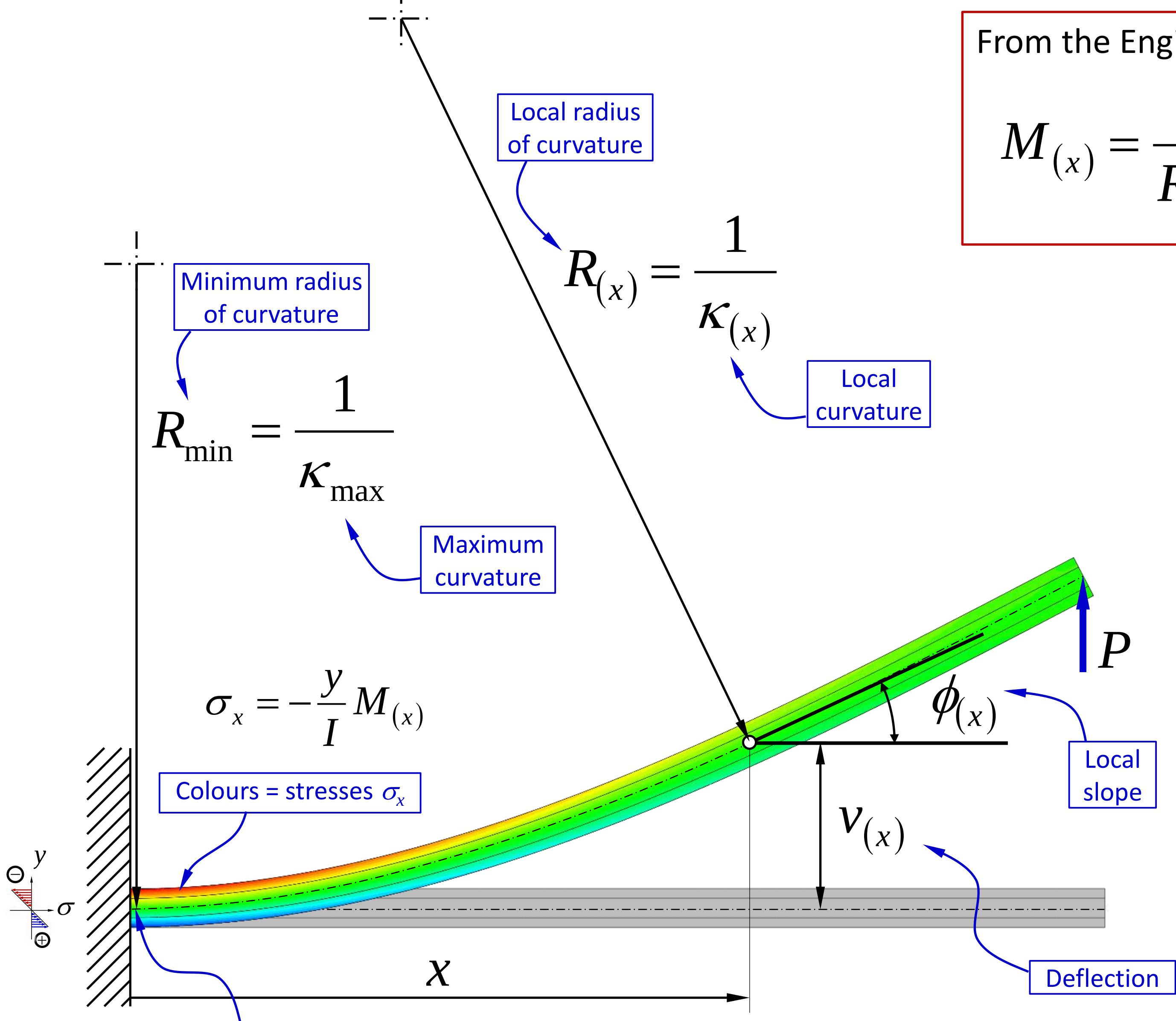
From the Engineer's Theory of Bending:

$$M_{(x)} = \frac{1}{R_{(x)}} E\,I = \kappa_{(x)}\,E\,I$$

And from Calculus:

$$\phi_{(x)} = \frac{\mathrm{d}\,v}{\mathrm{d}\,x}$$

$$\kappa_{(x)} = \frac{\mathrm{d}\,\phi}{\mathrm{d}\,x} = \frac{\mathrm{d}^2\,v}{\mathrm{d}\,x^2}$$



$$M_{max} = \frac{1}{R_{min}} E\,I = \kappa_{max}\,E\,I$$

Bending Deflection – Differential Relations

differentiating

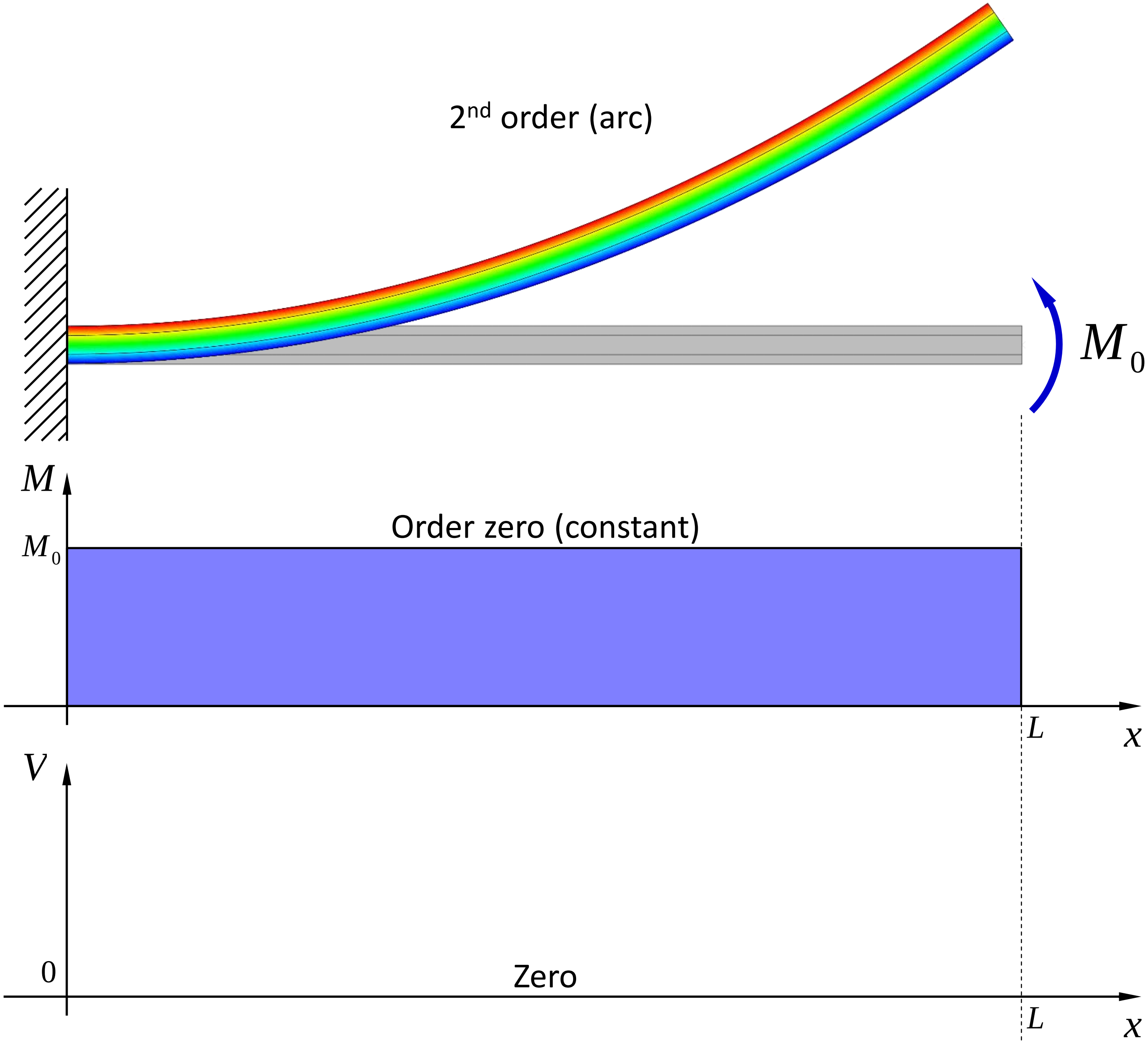
Rate of change of the rate of change of curvature (!)	$\dfrac{d^2\kappa_{(x)}}{dx^2} = \dfrac{d^4 v}{dx^4}$	$= +\dfrac{1}{EI} w_{(x)}$	(Sign depends on assumed sense of w) External load distribution
'Rate of change' of curvature	$\dfrac{d\kappa_{(x)}}{dx} = \dfrac{d^3 v}{dx^3}$	$= -\dfrac{1}{EI} V_{(x)}$	(Sign depends on assumed x-direction) Shear force
Curvature	$\kappa_{(x)} = \dfrac{d^2 v}{dx^2}$	$= \dfrac{1}{EI} M_{(x)}$	Moment
Slope	$\phi_{(x)} = \dfrac{dv}{dx}$		
Deflection	$v_{(x)} = v$		

Starting point (usually)

'Double Integration Method'

Curvature	$\kappa_{(x)}$	$= \dfrac{d^2 v}{d x^2}$	$= \dfrac{1}{EI} M_{(x)}$	← Moment
Slope	$\phi_{(x)}$	$= \dfrac{d v}{d x}$		
Deflection	$v_{(x)}$	$= v$		

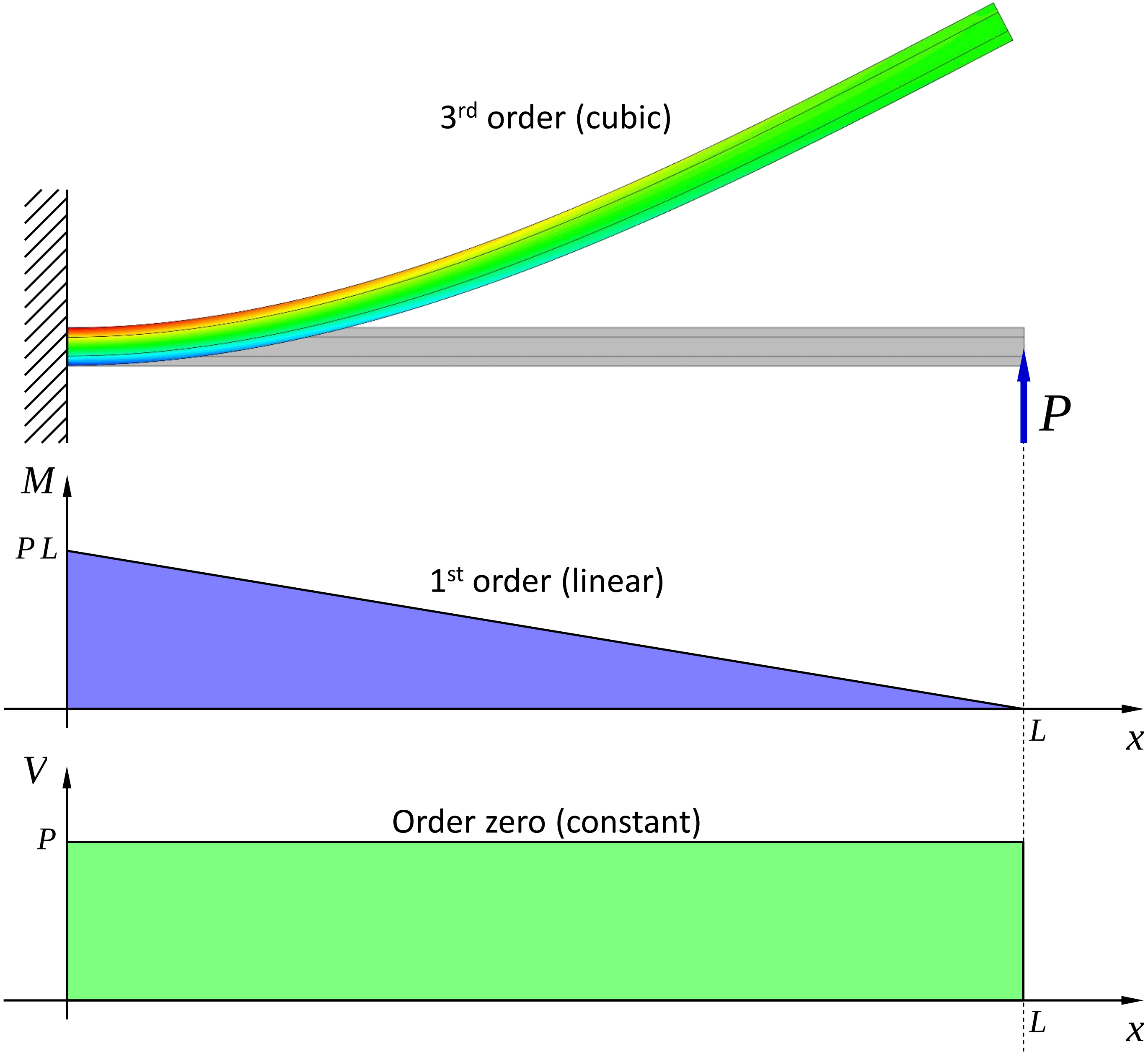
Bending Deflection – Pure Bending



$$\boxed{M_{(x)} = \kappa_{(x)} E I}$$

$$\boxed{V_{(x)} = -\frac{\mathrm{d}\kappa_{(x)}}{\mathrm{d}x} E I}$$

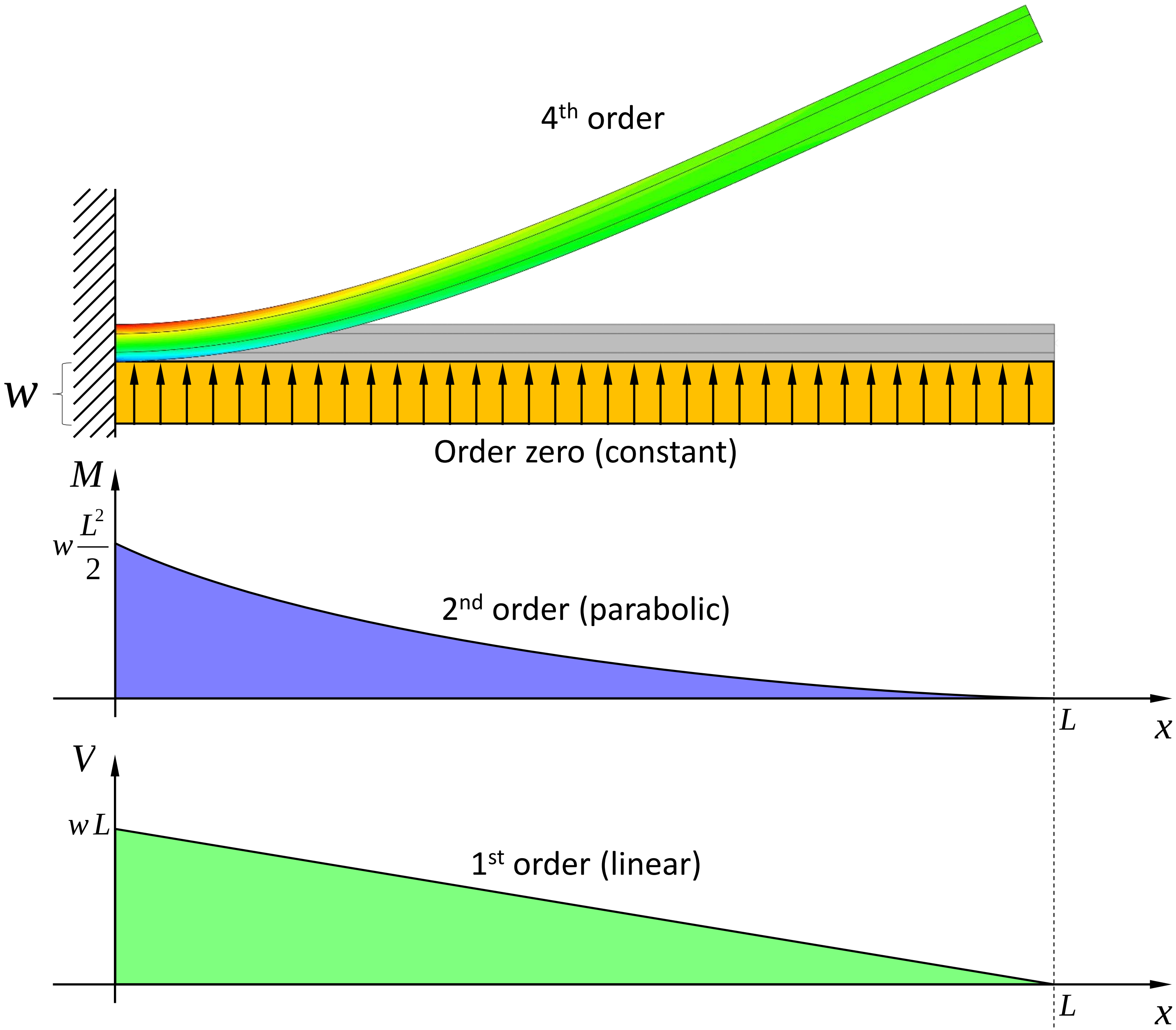
Bending Deflection – Point Load



$$M_{(x)} = \kappa_{(x)} \, E \, I$$

$$V_{(x)} = -\frac{\mathrm{d}\,\kappa_{(x)}}{\mathrm{d}\,x} \, E \, I$$

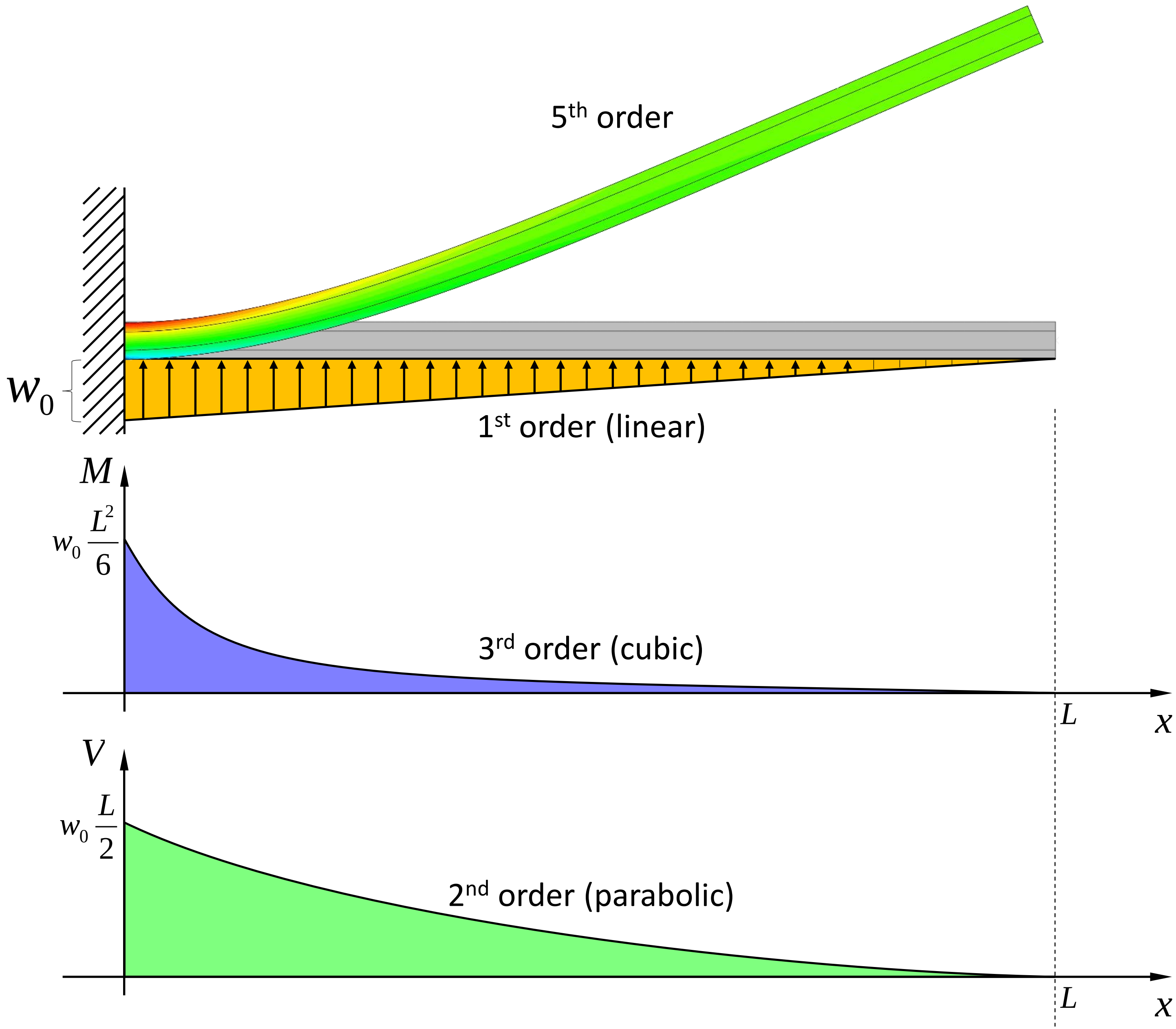
Bending Deflection – Uniform Load Distribution



$$M_{(x)} = \kappa_{(x)}\, E\, I$$

$$V_{(x)} = -\frac{\mathrm{d}\,\kappa_{(x)}}{\mathrm{d}\,x}\, E\, I$$

Bending Deflection – Linear Load Distribution



$$\boxed{M_{(x)} = \kappa_{(x)}\, E\, I}$$

$$\boxed{V_{(x)} = -\frac{\mathrm{d}\,\kappa_{(x)}}{\mathrm{d}\,x}\, E\, I}$$

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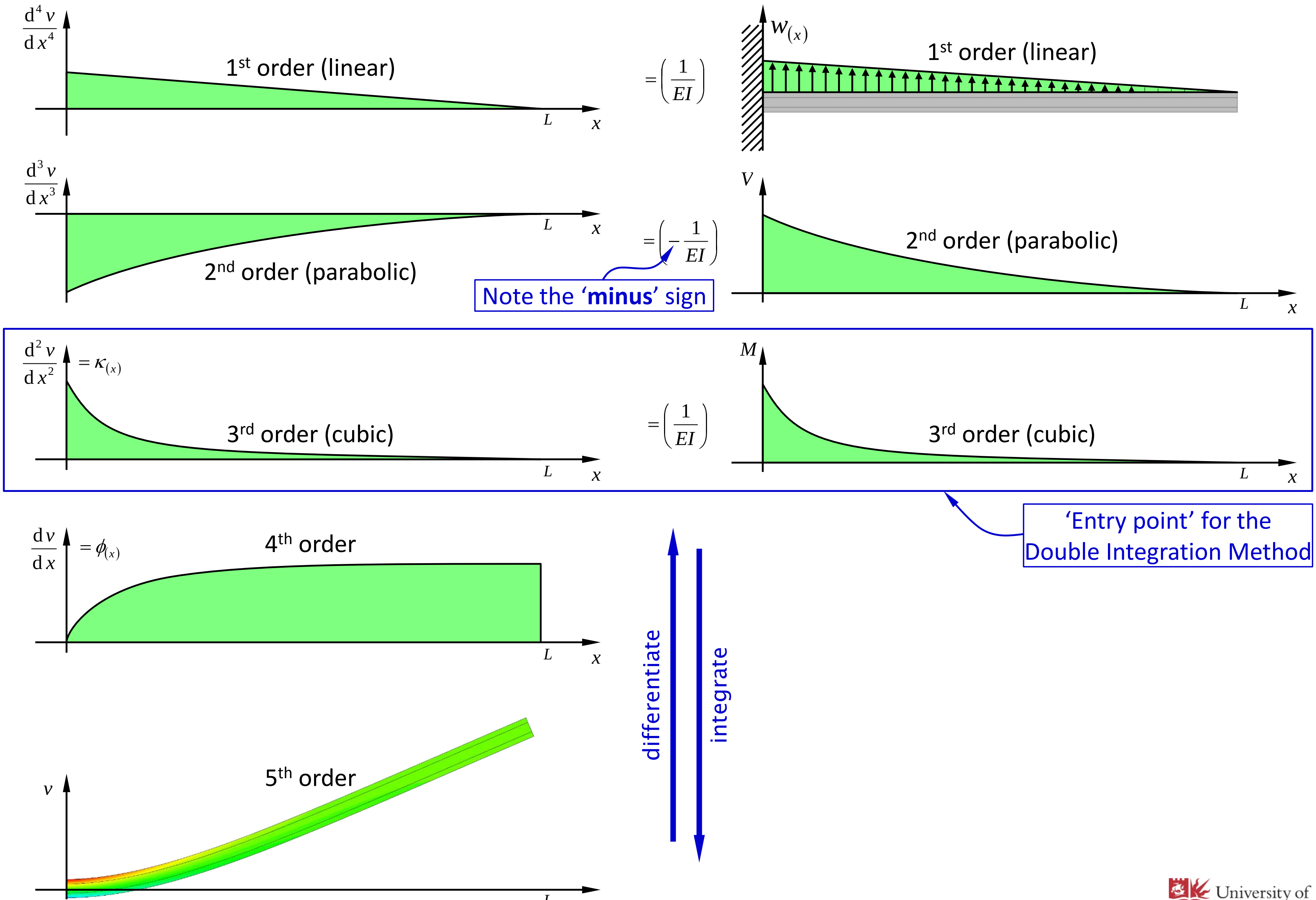
Derivatives and their physical meaning



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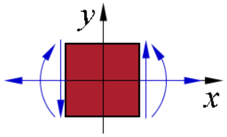
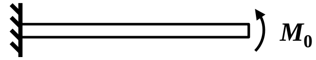
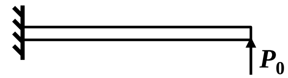
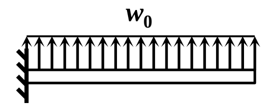
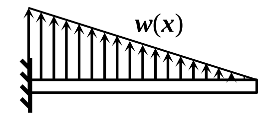
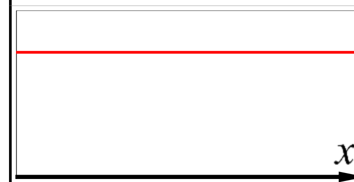
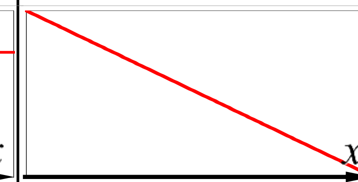
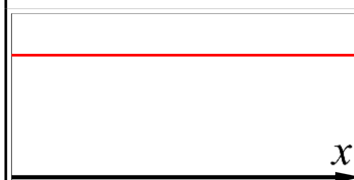
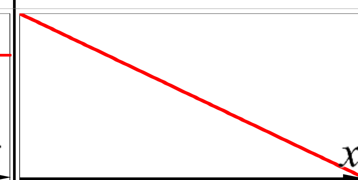
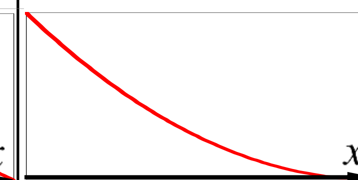
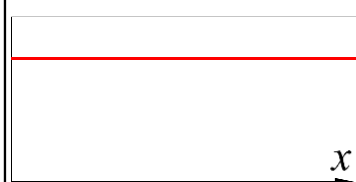
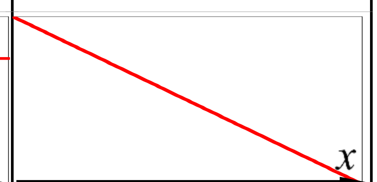
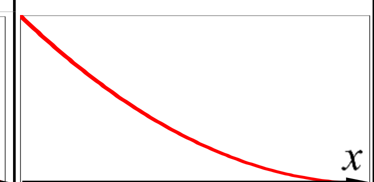
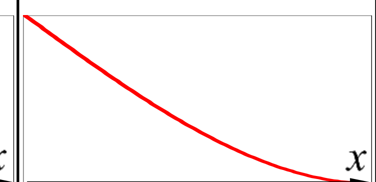
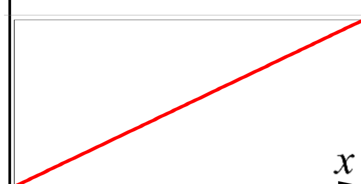
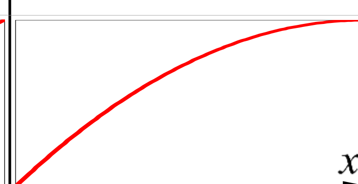
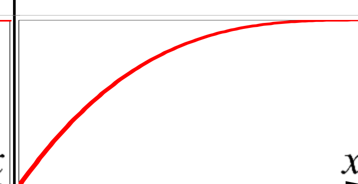
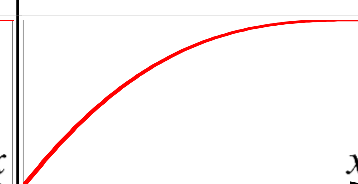
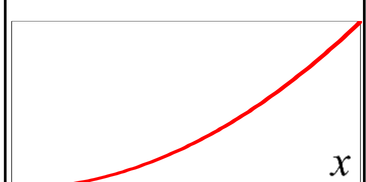
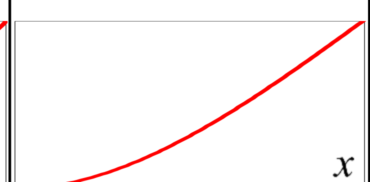
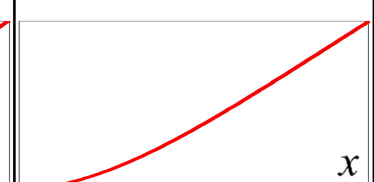
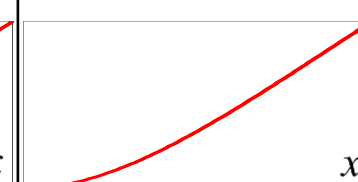
					
$EI\dfrac{d^4 v}{dx^4} = w_{(x)}$ **4th order ODE**	**External load distribution**	**0**	**0**	 **constant**	 **1st order polynomial**
$-EI\dfrac{d^3 v}{dx^3} = V_{(x)}$ **3rd order ODE**	**Shear force**	**0**	 **constant**	 **1st order polynomial**	 **2nd order polynomial**
$EI\dfrac{d^2 v}{dx^2} = M_{(x)}$ **2nd order ODE**	**Moment**	 **constant**	 **1st order polynomial**	 **2nd order polynomial**	 **3rd order polynomial**
$\dfrac{dv}{dx} = \phi_{(x)}$ **1st order ODE**	**Slope**	 **1st order polynomial**	 **2nd order polynomial**	 **3rd order polynomial**	 **4th order polynomial**
$v_{(x)}$ **polynomial**	**Deflection**	 **2nd order polynomial (arc)**	 **3rd order polynomial**	 **4th order polynomial**	 **5th order polynomial**