2. Kinematics of mechanisms

Design 2

AENG21350

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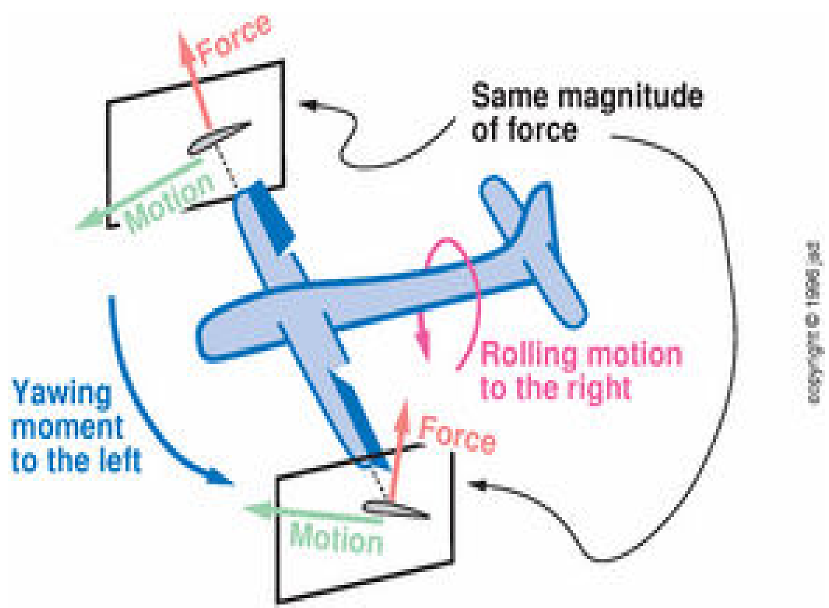
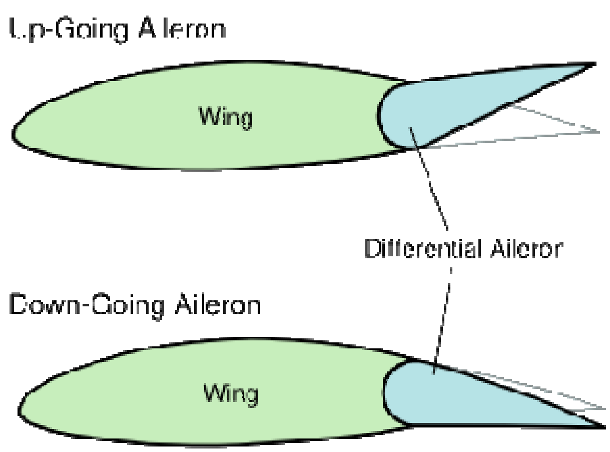
2.1 Adverse yaw moment



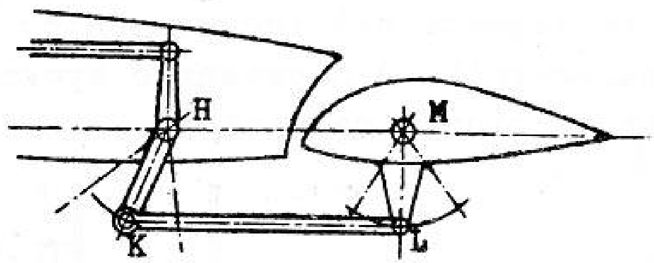


http://en.wikipedia.org/wiki/Image:DifferentialAileron.svg

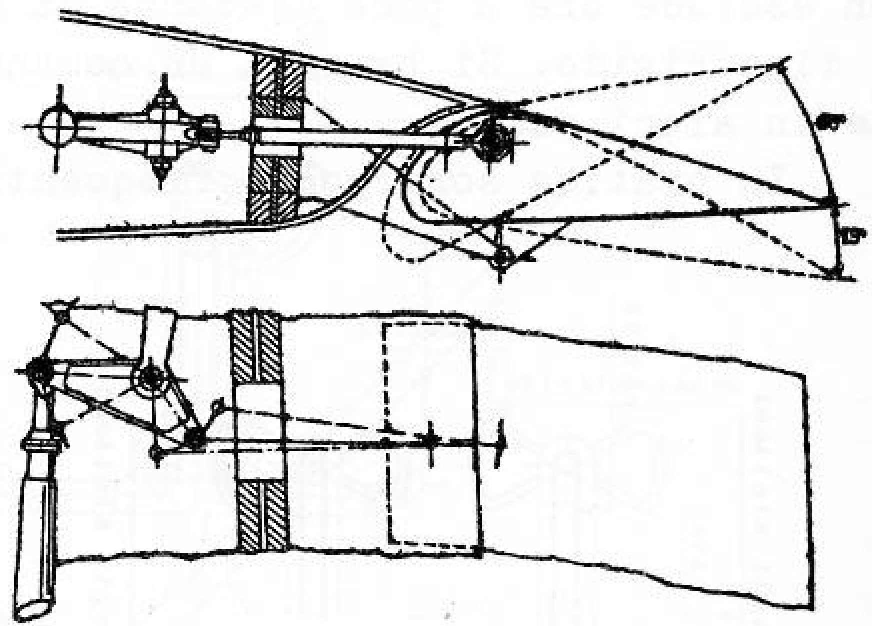
•Consequence of aerodynamic spanload over finite wings
•The outer (upper) wing of an airplane which is performing a banked turn generates
•more lift and therefore more drag than the inner wing. This results in a yaw moment which is counter
•to the desired turn direction.

Solutions:
•*Using rudder*
•**Differential aileron.** It is the downwards deflection of an aileron that causes aileron drag
→ a simple way of eliminating adverse yaw would be to rely solely on the upward deflection of the opposite wing to cause the aircraft to roll
→ slow roll rate → better solution is to make a compromise between adverse yaw and roll rate. This is what occurs in Differential ailerons

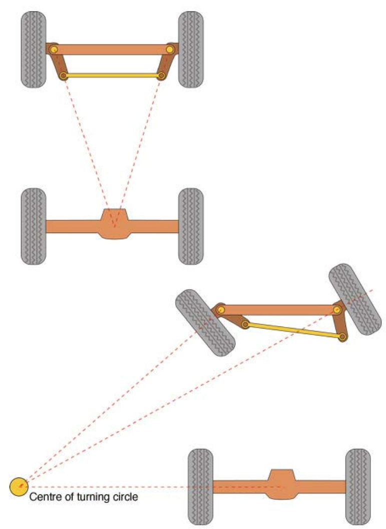
2.2 Differential aileron mechanism - 1



Differential ailerons produced by 4-points linkage **HKLM.**
This solution is used in training gliders – light aircraft.
Disadvantage: external mechanism – possible ice formation

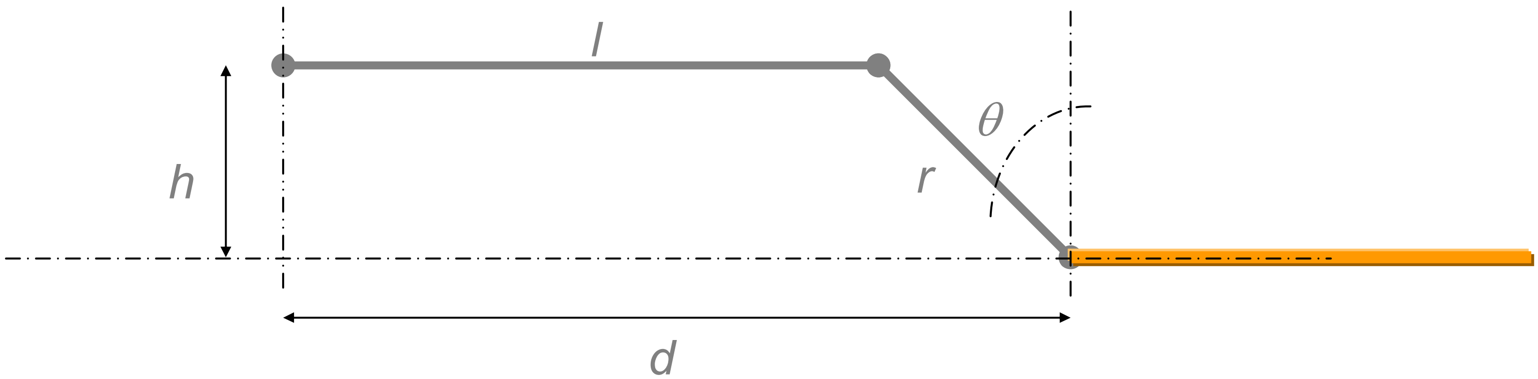


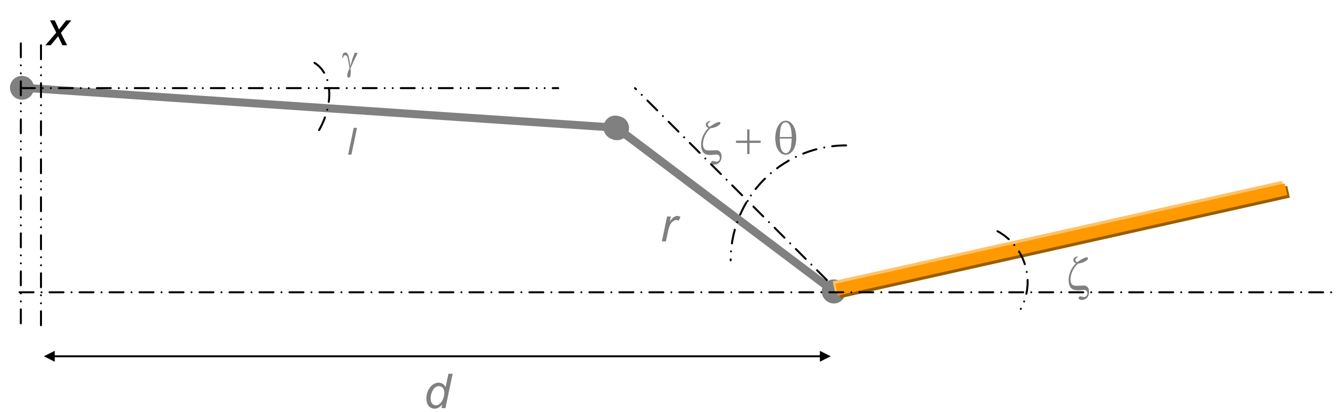
Hidden mechanism solution – used in light aircraft and most WWII bombers-CAS aircrafts



Used also for general turns in suspension systems

2.2 Differential aileron mechanism - 2





Example of **Ackermann steering mechanism**

2.2 Differential aileron mechanism - 3

Initial position:

$$d = l + r\sin\theta \qquad (1)$$

$$h = r\cos\theta \qquad (2)$$

After control movement x:

$$d + x = l\cos\gamma + r\sin(\theta + \xi) \qquad (3)$$

$$h = l\sin\gamma + r\cos(\theta + \xi) \qquad (4)$$

Back substituting (3), (4) in (1), (2):

$$l\cos\gamma = l + r\sin\theta - r\sin(\theta + \xi) + x \qquad (5)$$

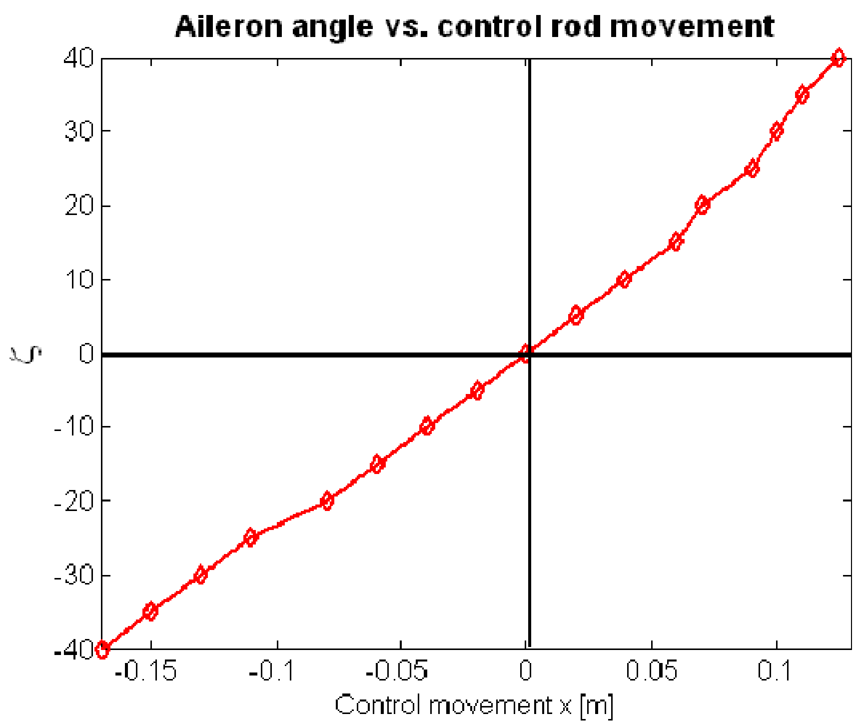
$$l\sin\gamma = r\cos\theta - r\cos(\theta + \xi) \qquad (6)$$

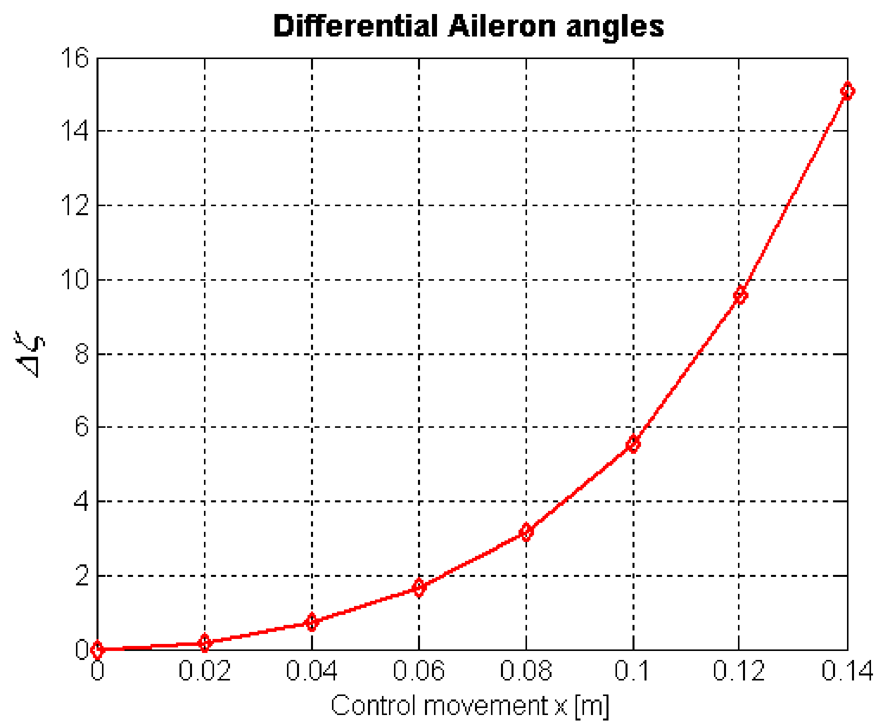
Squaring and adding (5) and 6:

$$l^2 = [l + r\sin\theta - r\sin(\theta + \xi) + x]^2 + [r\cos\theta - r\cos(\theta + \xi)]^2$$

$$\boxed{x = \sqrt{l^2 - [r\cos\theta - r\cos(\theta + \xi)]^2} - l - r\sin\theta + r\sin(\theta + \xi)}$$

2.2 Differential aileron mechanism - 4

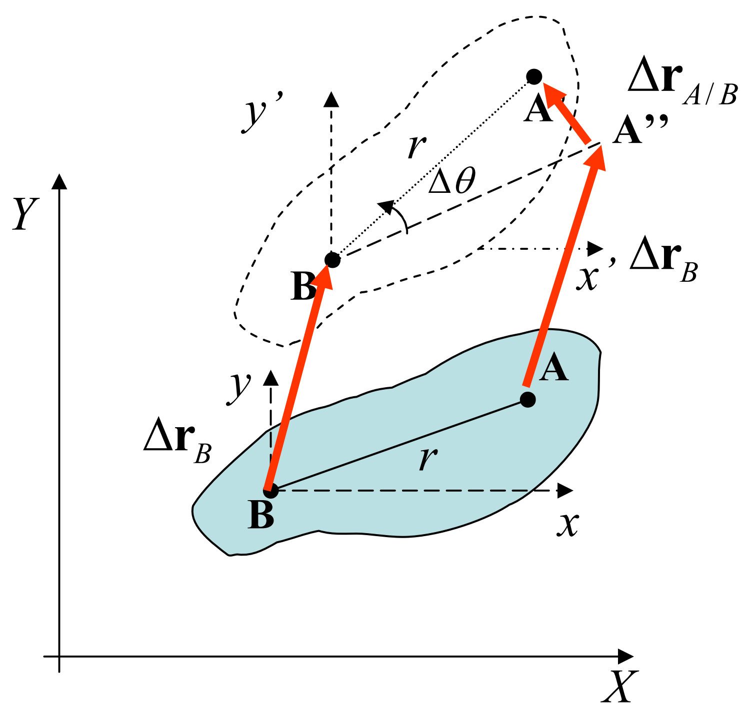




l = 1 m

r = 0.25 m

2.3 Relative velocity



$$\Delta \mathbf{r}_A = \Delta \mathbf{r}_B + \Delta \mathbf{r}_{A/B}$$

Lim Δt → 0:

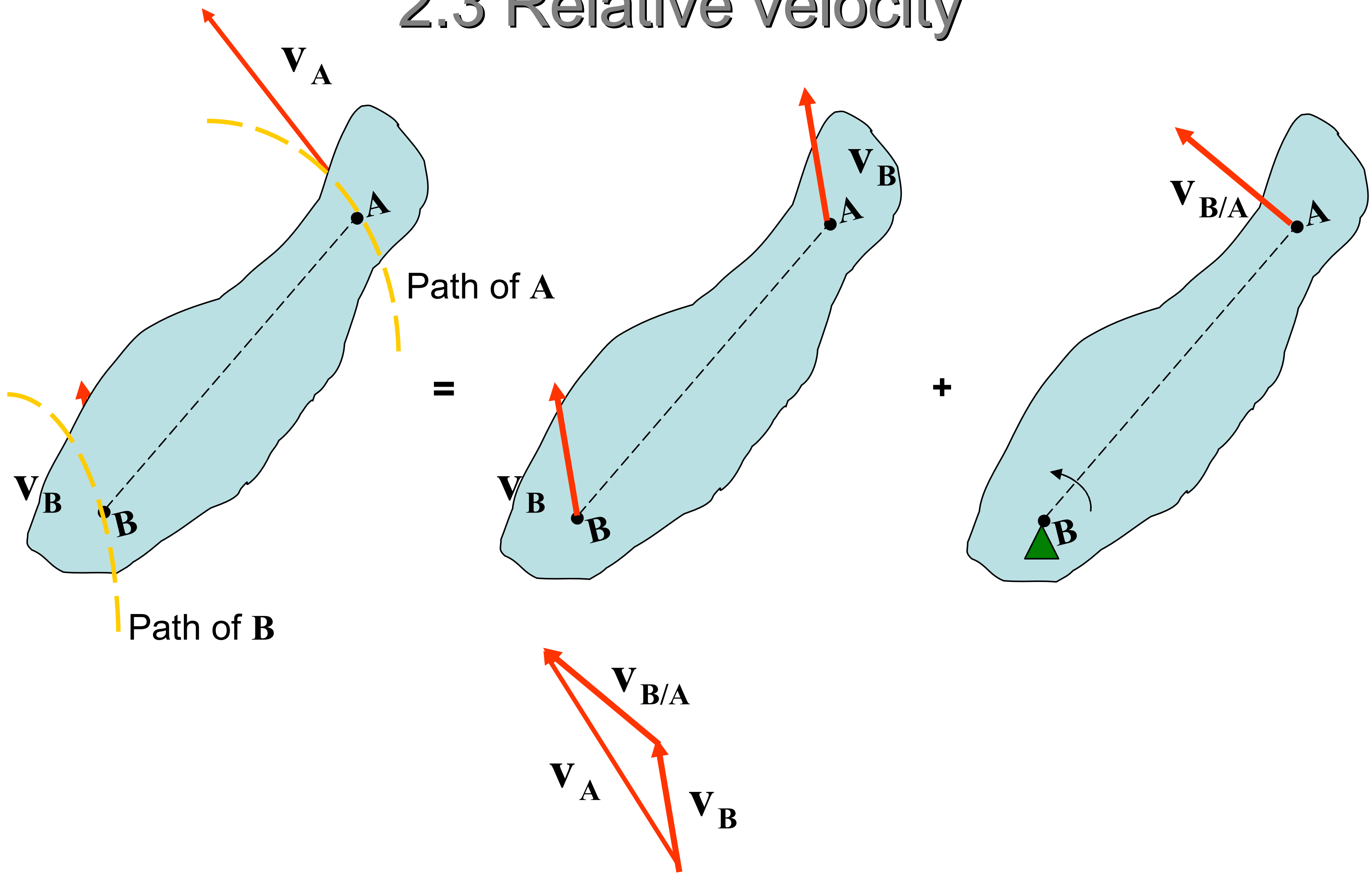
$$\underline{\mathbf{v}}_A = \underline{\mathbf{v}}_B + \underline{\mathbf{v}}_{A/B}$$

But:

$$v_{A/B} = \lim_{\Delta t \to 0}\left(\frac{\Delta \mathbf{r}_{A/B}}{\Delta t}\right) = \lim_{\Delta t \to 0}\left(\frac{r\Delta\theta}{\Delta t}\right) = r\omega$$

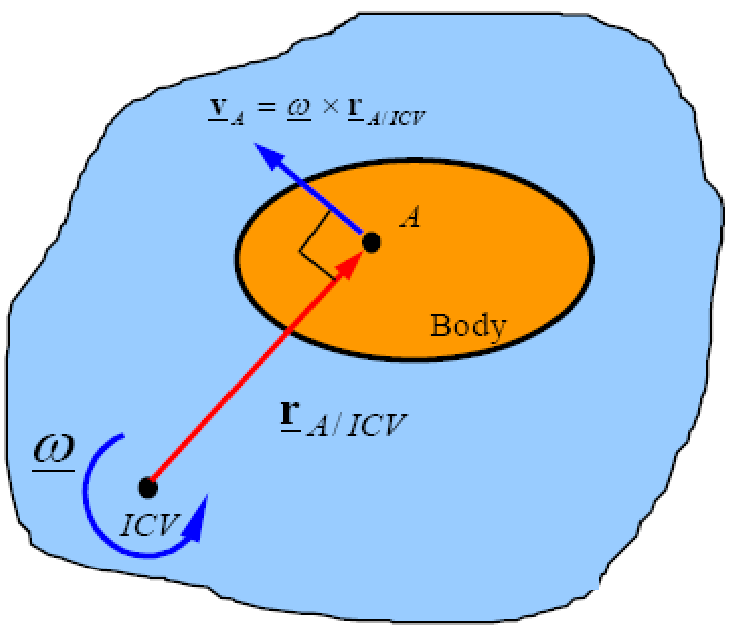
$$\boxed{\underline{\mathbf{v}}_{A/B} = \boldsymbol{\omega} \times \mathbf{r}}$$

2.3 Relative velocity



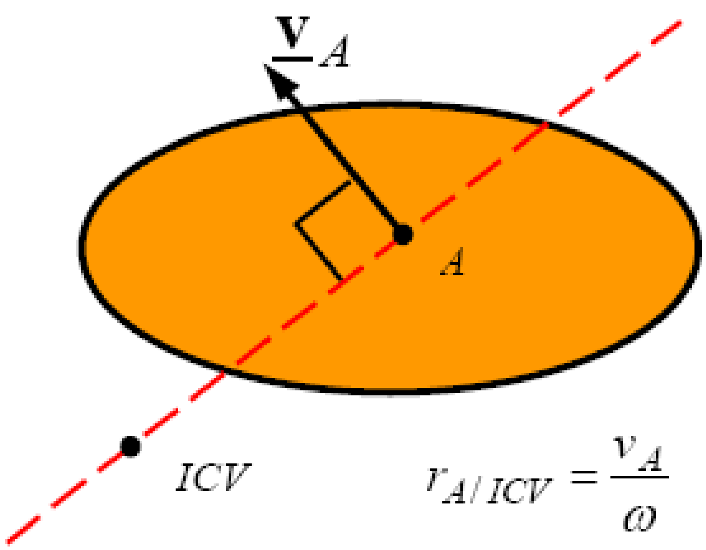
2.4 Instantaneous Centre of velocity (ICV)

We can choose a unique reference point which momentarily has zero velocity. **The body may be considered to be in pure rotation about an axis normal to the plane of motion passing through this point.** Assuming one knows the ICV of a body, one can calculate the velocity of any point A on the body using the equation:



$$\underline{\mathbf{v}}_A = \underline{\omega} \times \underline{\mathbf{r}}_{A/ICV}$$

2.4 Methods to identify ICV: -1

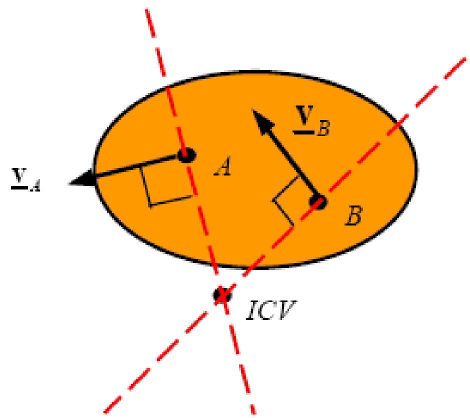


$$r_{A/ICV} = \frac{v_A}{\omega}$$

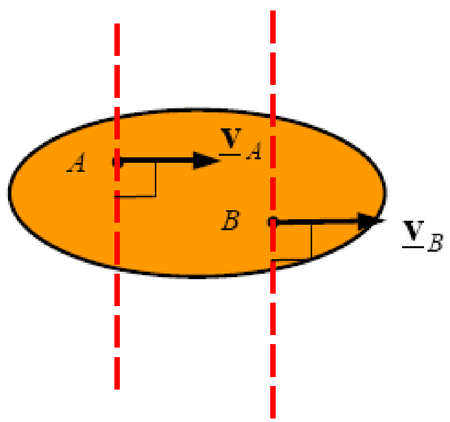
1. Given the velocity of point A on a rigid body and the angular velocity of the rigid body one can use the equation to find the distance $r_{A/ICV}$.
2. Draw a line perpendicular to the velocity and passing through A, and move along this line a distance $r_{A/ICV}$ to get to the ICV. The side on which the ICV is can be determined by the direction of the angular velocity

2.4 Methods to identify ICV: -2

Given the velocity of points A and B on a rigid body one can find the ICV by drawing a line perpendicular to $\underline{\mathbf{v}}_A$ and passing through A, and by drawing a line perpendicular to $\underline{\mathbf{v}}_B$ and passing through B. Several cases can occur:

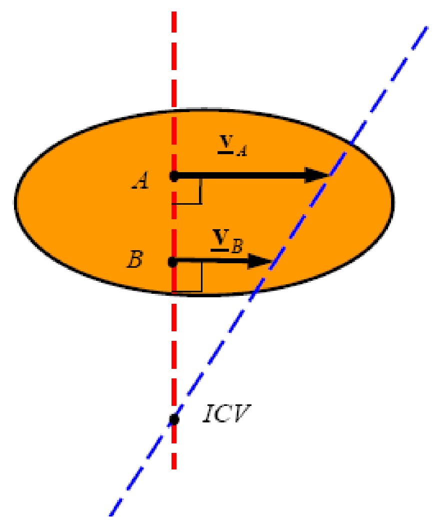


1. Lines intersect at one point.
Angular velocity calculated when ICV is determined



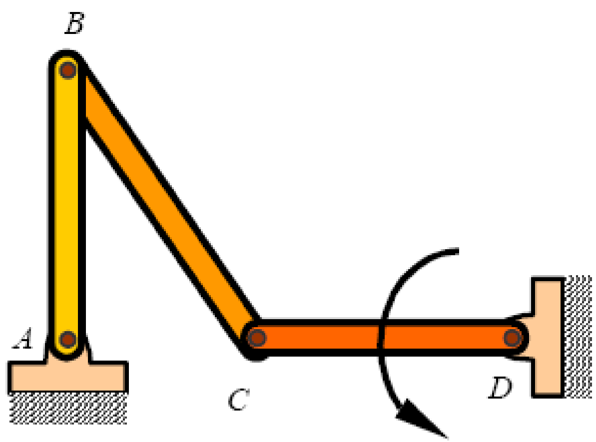
$$ICV\ at\ \infty \updownarrow \Rightarrow \omega = 0 \Rightarrow \underline{\mathbf{v}}_A = \underline{\mathbf{v}}_B$$

2. Intersection at infinity. The body is in pure translation

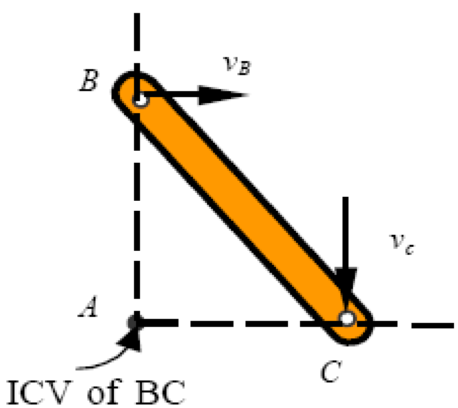


3. Lines fall on top of each other. ICV located using proportionality of the velocity vectors

2.5 ICV – Examples -1



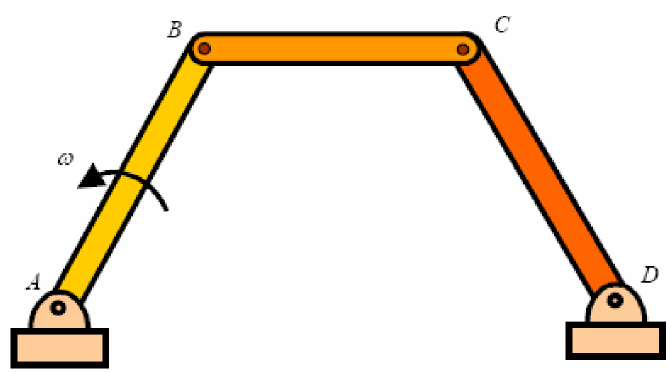
Q. ICV of BC



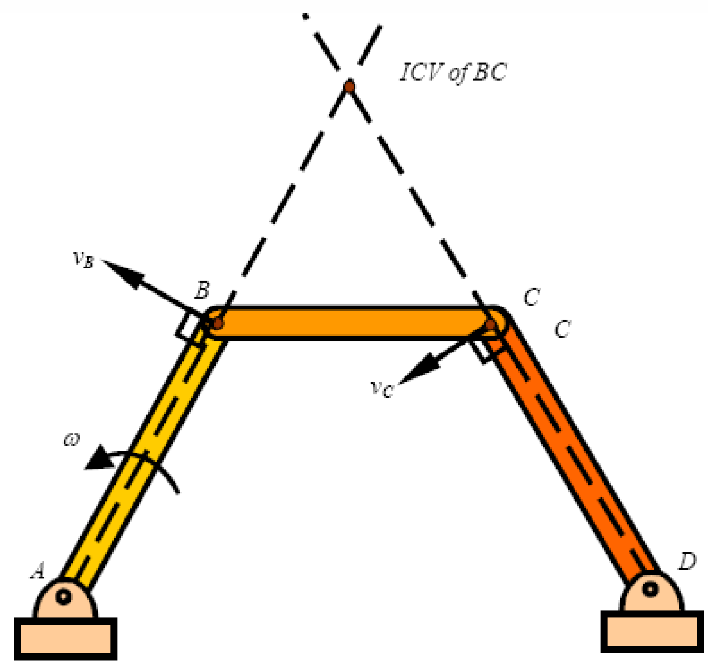
Ans. B is rotating around A → horizontal velocity

C is rotating in a circle around D → a vertical velocity. The ICV is at the intersection of the two perpendicular lines to the velocities.

2.5 ICV – Examples -2

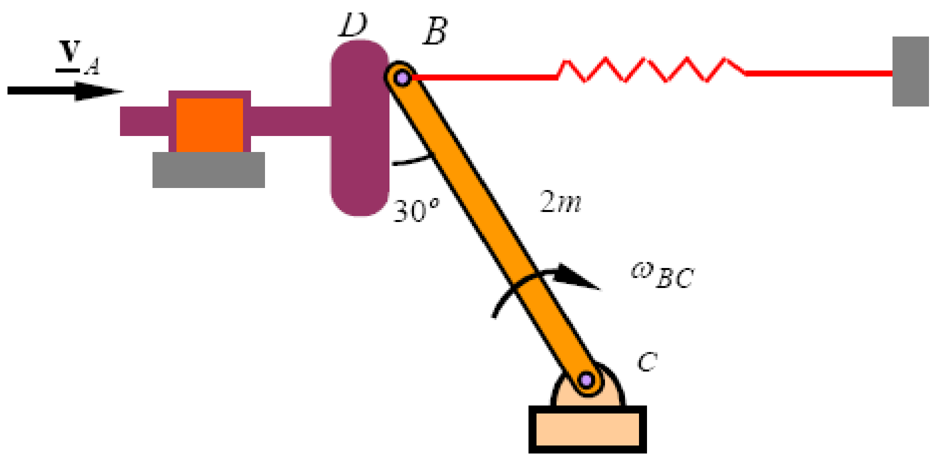


Q. ICV of BC

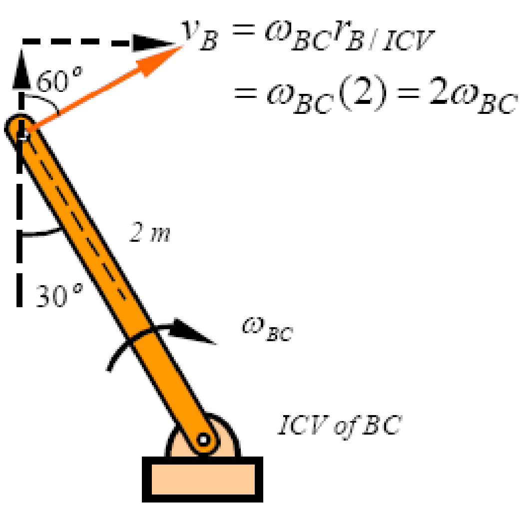


Ans.

2.5 ICV – Examples -3



Given v_A = 10 m/s , calculate the angular velocity of BC. Assume that B is in contact with D at all times.



Ans. First find velocity of B Using the bar BC.

The non-penetration condition requires that the horizontal component of the velocity of B be equal to the horizontal component of velocity of D:

$$v_B \sin 60 = v_A \Rightarrow \quad 2\omega_{BC} \sin 60 = 10$$

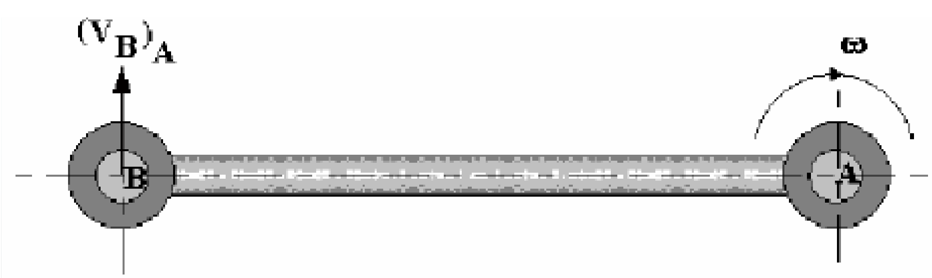
$$\omega_{BC} = \frac{10}{2\frac{\sqrt{3}}{2}} = \frac{10\sqrt{3}}{3} rad / s$$

2.6 Velocity diagrams - 1

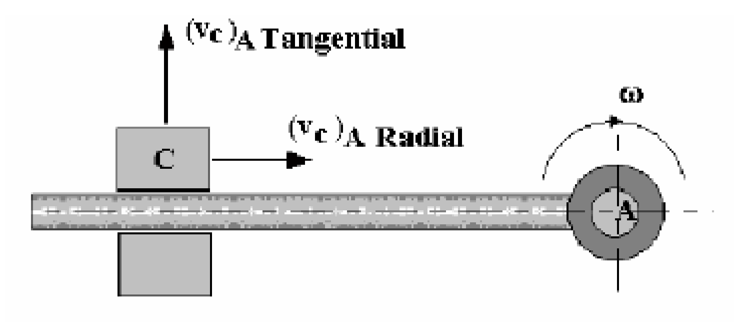
Velocity diagrams represent the state of the velocity vectors of the linkage **at a specific instant of time**

Definitions

•**Absolute and relative velocity**: see 2.1



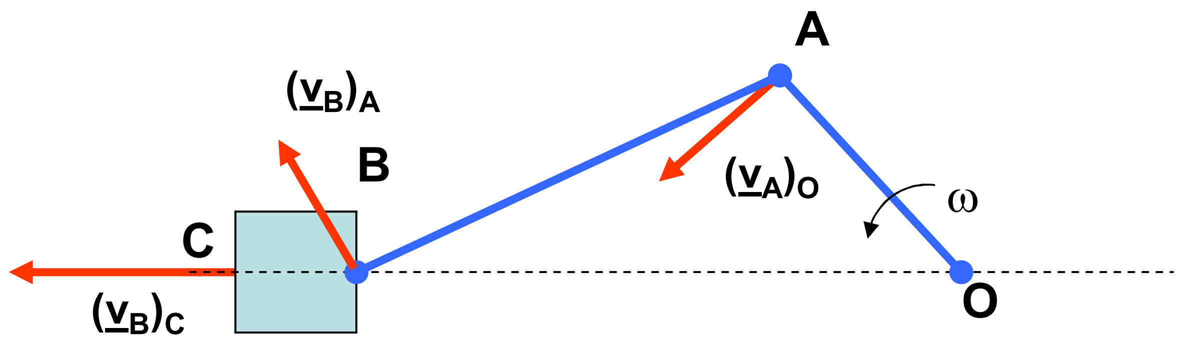
•**Tangential velocity**: consider the following link pinned in A and revolving at angular velocity w. Point B is always moving at 90° versus the link → tangential

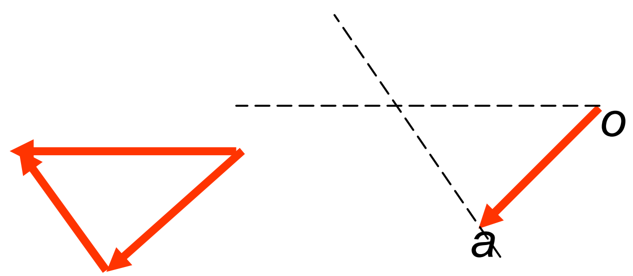


•**Radial velocity**: consider the sliding link C that can slide on link AB. The direction can only be radial. If there is also rotation around A there will be composition of velocities.

2.6 Velocity diagrams - 2

Example: crank, connecting rod and piston





Methodology:

1. Calculate velocity $(\underline{v}_A)_O$ from ω X radius

2. Impose a point *o* in the space as the origin of your velocity diagram

3. Draw vector *oa* in the correct direction

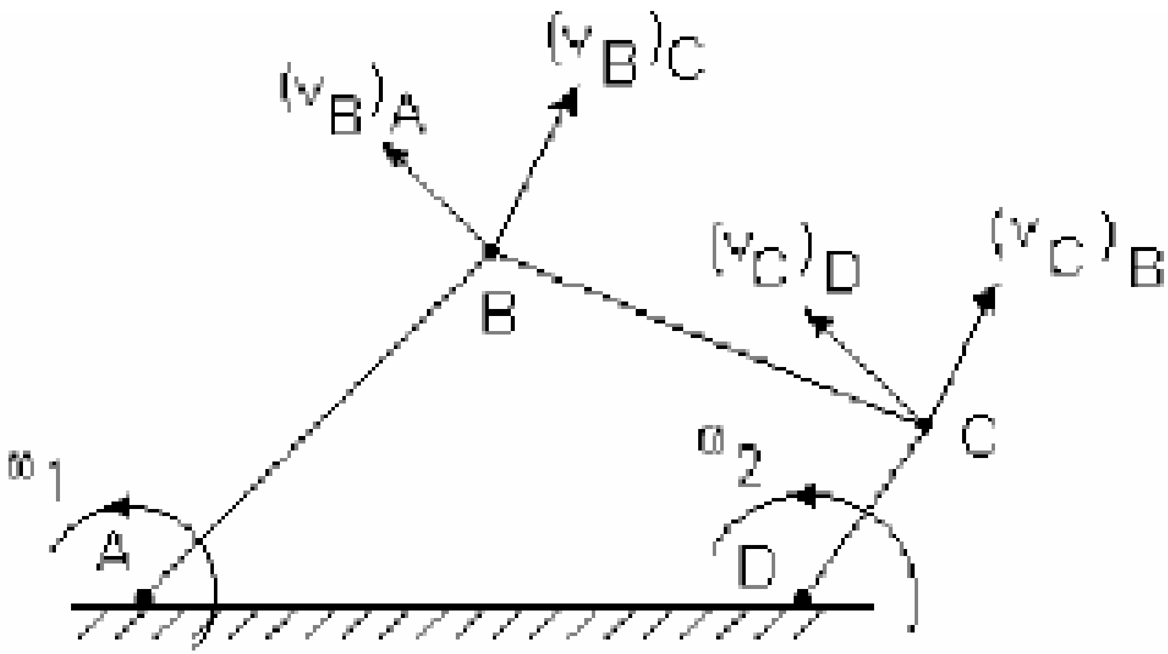
4. Velocity B relative to A has to be added → from a draw line perpendicular to AB with unknown length

5. Velocity B versus O is horizontal and absolute → start from *o* an horizontal line with unknown length

6. Compose triangle of velocities

2.6 Velocity diagrams - 3

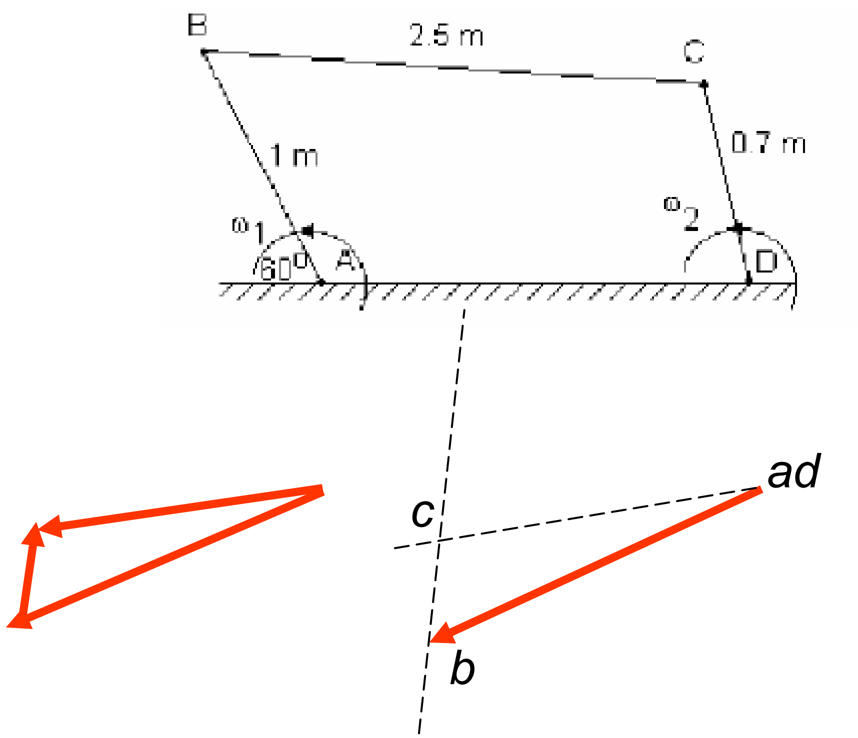
Example: 4 bar linkage



Each velocity vector is at right angle with the link. <u>Points A and D are fixed and appear as the same point in the velocity diagram</u>

2.6 Velocity diagrams - 4

Example: 4 bar linkage



ω_1 = 500 rpm ? ω_2

Ans.

1. ω_1 = 500 rpm = 2 X π X 500/60 = 52. 36 rad/s

$(\underline{v}_B)_A = \omega_1$ X AB = 52.36 m/s

2. Draw $(\underline{v}_B)_A$ **in a suitable scale**

3. Draw the velocity line of C relative to B ⊥ to BC passing through point *b*

4. Draw the velocity line of C relative to D ⊥ to DC passing through point *ad*

5. Identify point *c*; close velocity diagram

6. Measure $(\underline{v}_C)_D$ → 43.5 m/s
ω_2 = 43.5/0.7 = 62 rad/s

List of concepts to know

- Ackermann steering and differential aileron
- Relative velocity concept
- ICV
- Principle of velocity diagrams