Structural Loads in Trusses
Method of Joints

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1.2.1 Method of Joints

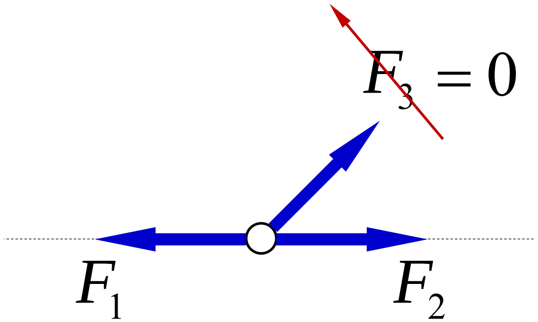
1.2.2 Method of Sections

1.2.3 Method of Tension Coefficients

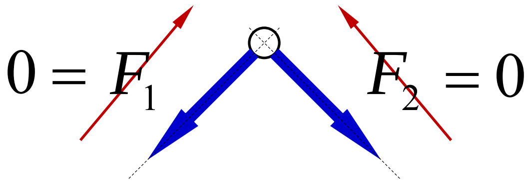
- In two dimensions (2D)
- In three dimensions (3D)

Some **internal forces** or some **reaction forces** in a pin-jointed truss might be **zero.** Often these 'zero loads' can be spotted early if we check for the **two collinearity rules**:

Rule 1:
If there are **exactly three forces** acting on a pin joint, an **two of these are collinear**, then the **non-collinear force must be zero**



Rule 2:
If there are **exactly two forces** acting on a pin joint and these are **not collinear**, then **both forces must be zero**



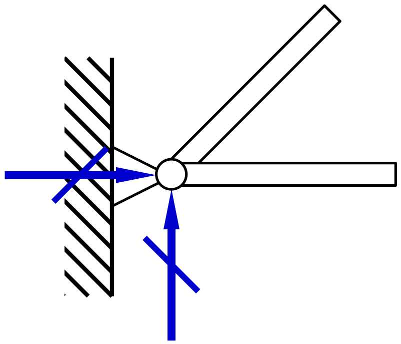
Default Load Directions or 'Sense'

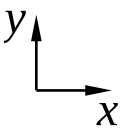
If the sense or direction of a force is unknown, **assume positive values**

- This applies to internal & external sign conventions

Reaction forces (external sign convention)**:**

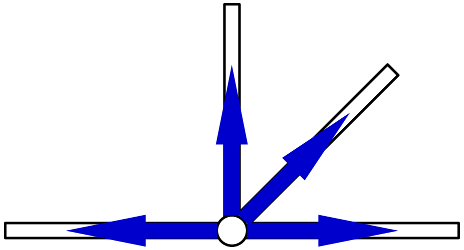
- **Horizontal**: positive 'to the right'
- **Vertical**: positive 'upwards'





Member forces (internal sign convention):

- Assume **tension**
  - As if forces were 'flowing out' of each pin joint



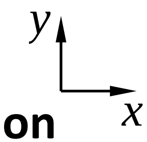
1.2.1 Method of Joints

KEY POINTS

1. Calculate the degree of redundancy **before** finding unloaded members!

2. Apply our **collinearity rules** to identify unloaded members

3. Create a **global FBD** for the entire structure



   – Draw positive reaction forces following the **external sign convention**

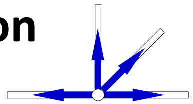
   – Write the **three** equilibrium equations: $\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M = 0$

     • Find the reaction forces at supports

Choose an appropriate joint as the reference point!

4. Analyse **mini FBDs** of each individual joint

   – Draw positive internal forces following the **internal sign convention**



   – Start by joints with known forces or reactions

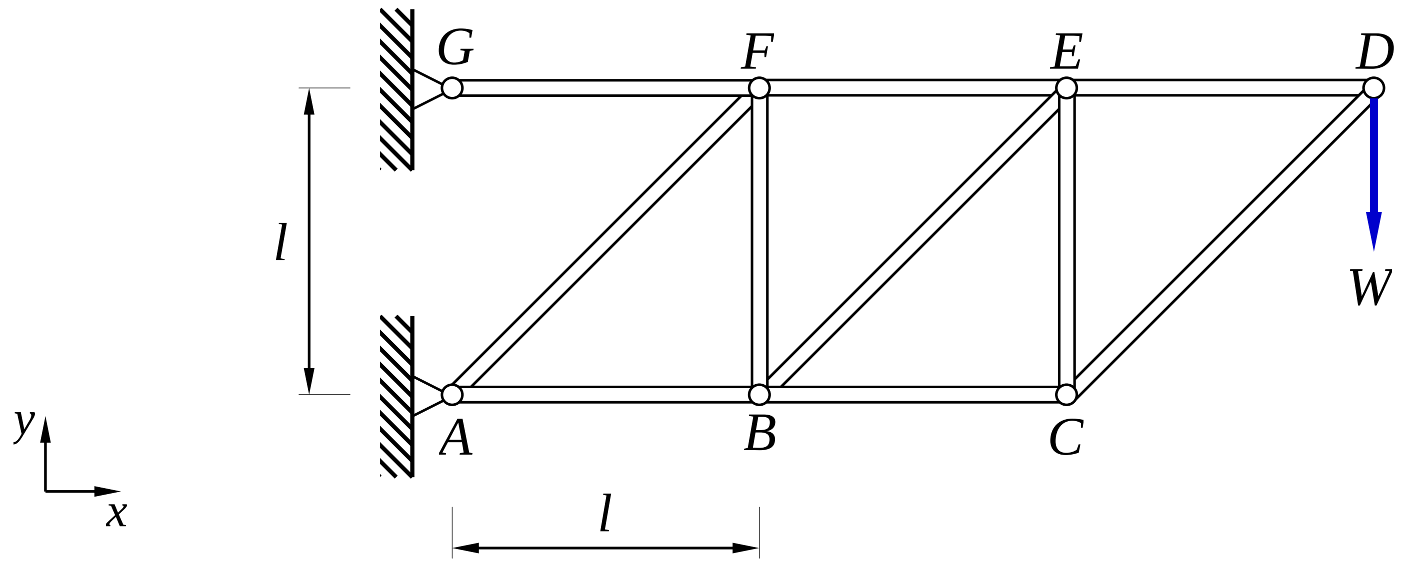
   – Write the **two** equilibrium equations: $\sum F_x = 0 \quad \sum F_y = 0$

     • Find the unknown internal forces

   – Move on to the next joint (again, choose an ‘easy’ joint!)

Example

For the pin-jointed truss below:

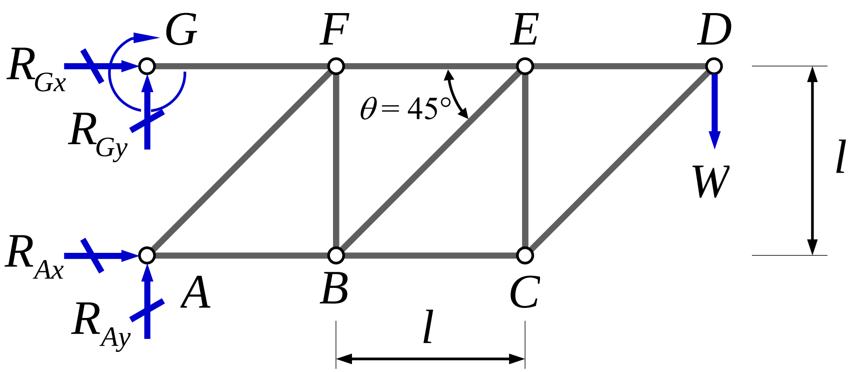


a) Find the magnitude and sense (up/down/left/right) of the **reaction forces** at **all supports**

b) Find the magnitude and sense (tension/compression) of the **internal forces** in **every member**

1.2.1 Method of Joints – Example

Step 1.1: Global FDB $\longrightarrow$ find reaction forces



The entire truss can then be seen as a single 'rigid body' which should not rotate about our reference joint

- Assume **positive reactions** following the **external sign convention**
- Considering the **balance of moments** about a joint, e.g. G:

$$\sum M_G = 0$$

CW (arbitrary)

1.2.1 Method of Joints – Example

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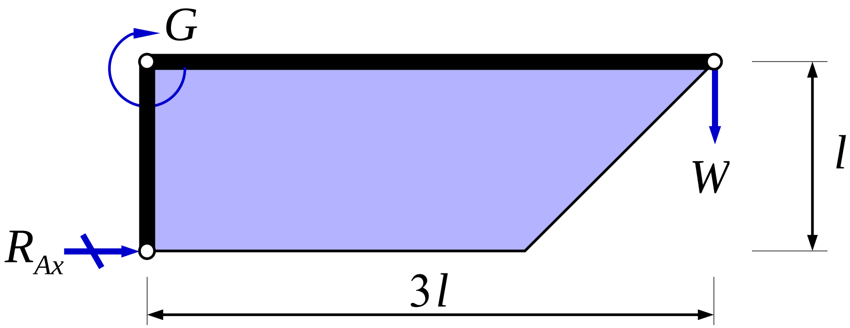

The entire truss can then be seen as a single 'rigid body' which should not rotate about our reference joint

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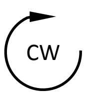


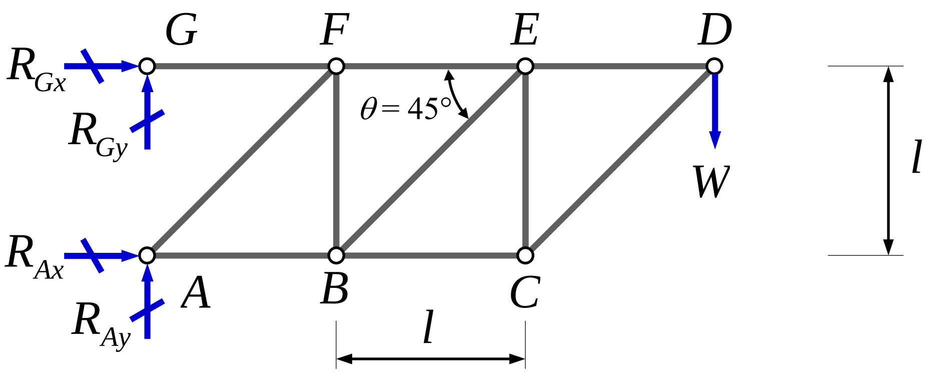
(arbitrary)

W × (horizontal moment arm) – R_A^x × (vertical moment arm) = 0

$$W(3\,l) - R_{Ax}(l) = 0 \qquad \therefore \qquad \boxed{R_{Ax} = 3\,W}$$

1.2.1 Method of Joints – Example

Step 1.2: Global FDB ⟶ find reaction forces



- Now the equilibrium of **horizontal forces**:

$$\sum F_x = 0$$

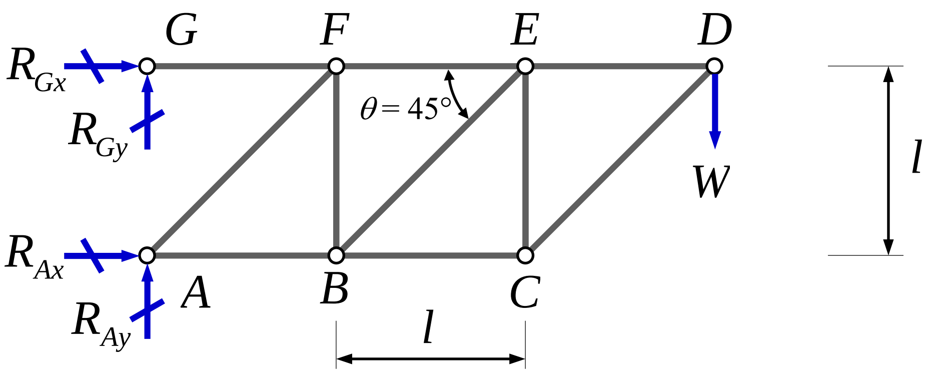
- There are only two terms:

$$R_{Ax} + R_{Gx} = 0 \qquad \therefore \qquad \boxed{R_{Gx} = -3W}$$

⟶ A negative sign means that the actual force is in the opposite direction!

1.2.1 Method of Joints – Example

Step 1.3: Global FDB $\longrightarrow$ find reaction forces



- Finally, the equilibrium of **vertical forces**:

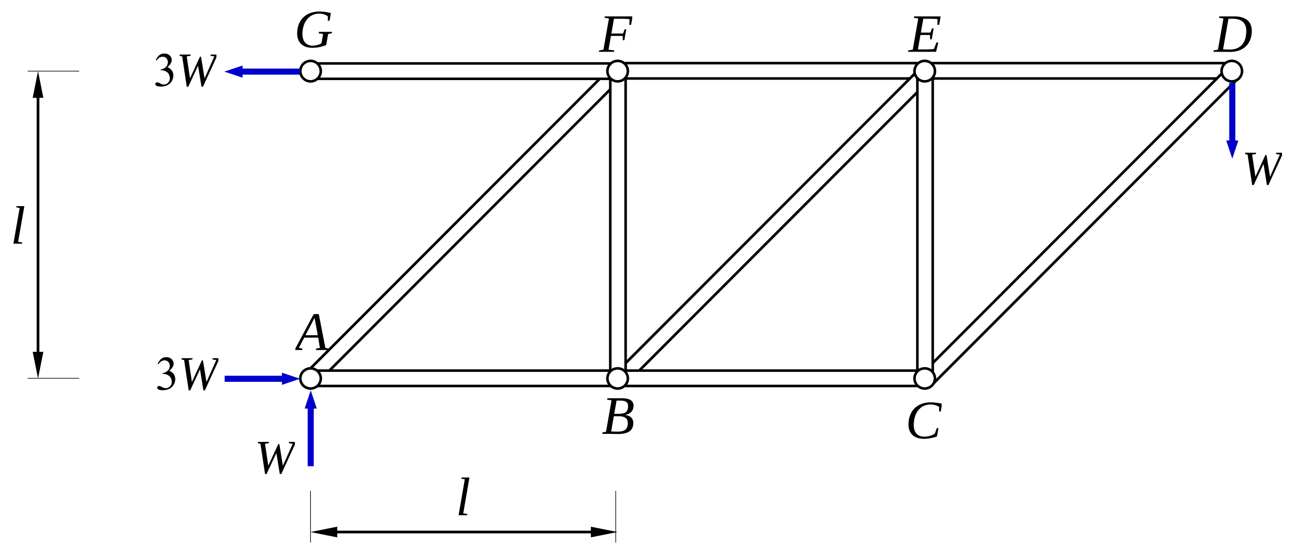
$$\sum F_y = 0$$

- Joint G **has no vertical or diagonal members**, hence $\boxed{R_{Gy} = 0}$

- And the two remaining terms give

$$R_{Ay} - W = 0 \qquad \therefore \qquad \boxed{R_{Ay} = W}$$

All reactions are now known:

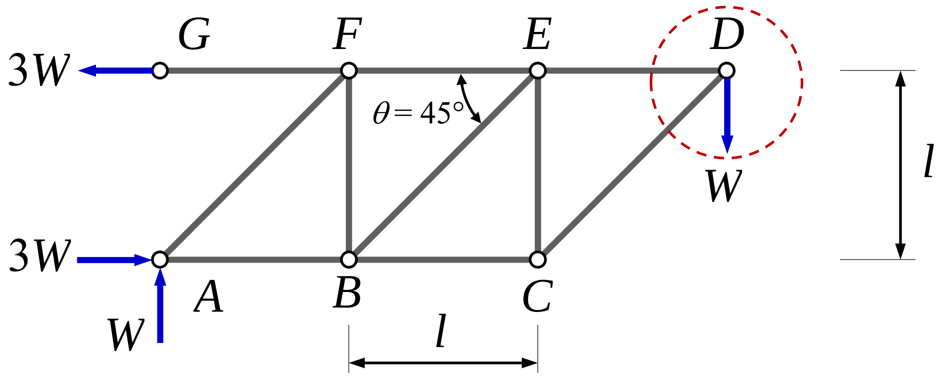


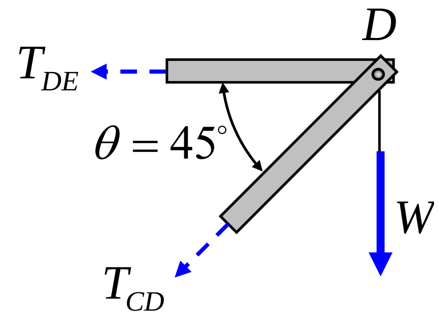
a) Find the magnitude and sense (up/down/left/right) of the **reaction forces** at **all supports** → **Done**

b) Find the magnitude and sense (tension/compression) of the **internal forces** in **every member**

1.2.1 Method of Joints – Example

Step 2: Draw mini-FBDs for each joint





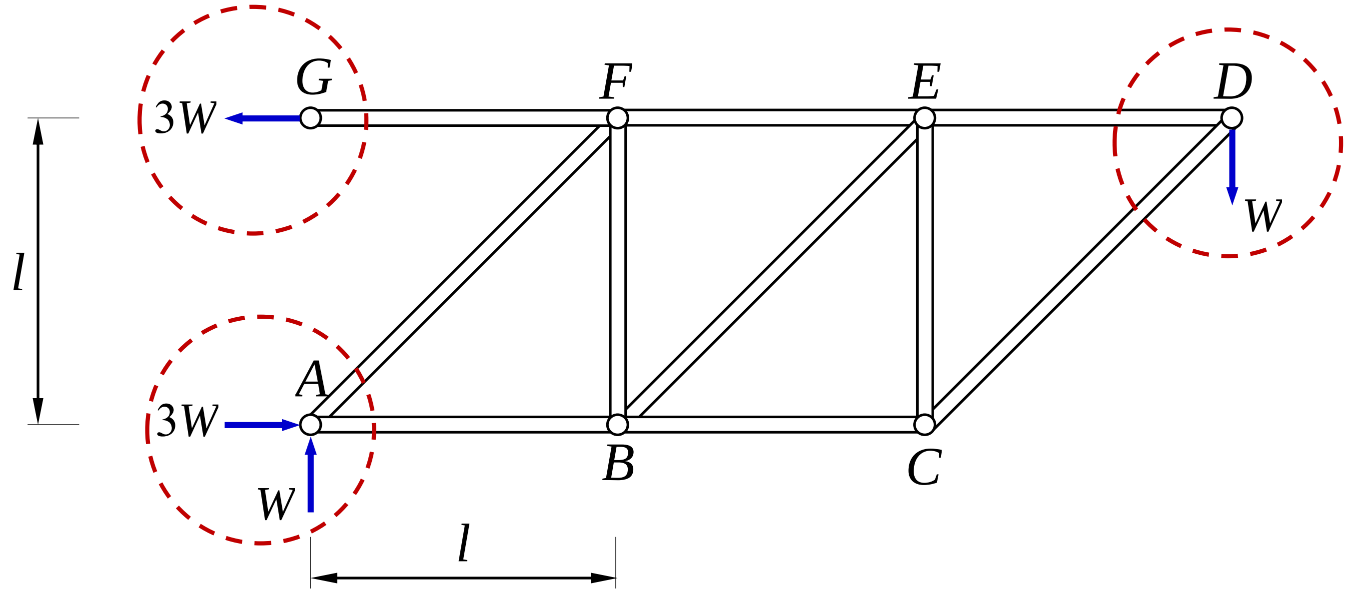
- Create FBDs at joints by **cutting across** joining members

- **Always** assume unknowns to act in positive sense (tension or ‘pulling’)
  - The correct sense will be determined by the solution of the equilibrium equations

- Next step: write the vertical and horizontal equilibrium for each joint
  - **Important: always start from joints with only 2 unknowns!**

1.2.1 Method of Joints – Example

Which joints have initially two (or less) unknown forces?

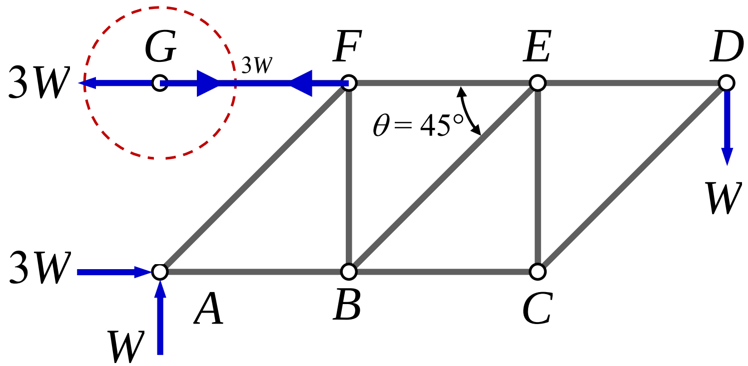


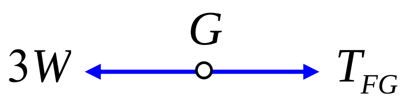
Answer:

- Usually the joints with known **applied forces** or **reaction forces**

1.2.1 Method of Joints – Example

Step 3.1: Equilibrium @ joint G

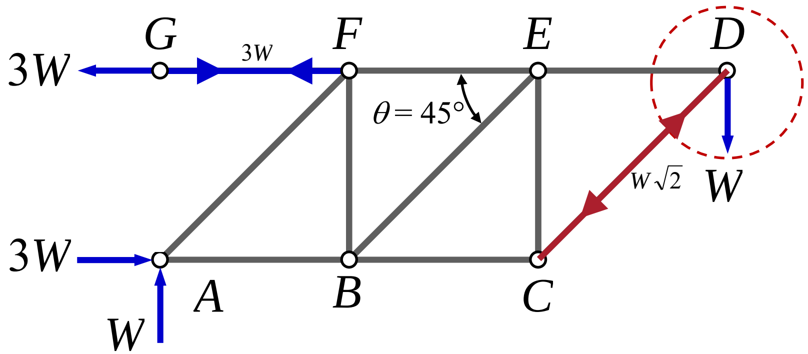


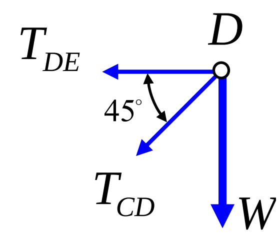


$$\sum F_x = 0 \qquad \therefore \qquad T_{FG} - 3W = 0 \qquad \therefore \qquad \boxed{T_{FG} = 3W}$$

A positive T_{FG} means that member FG is under **tension**

Step 3.2: Equilibrium @ joint D





$$\sum F_y = 0 \quad \therefore \quad \overbrace{-W - T_{CD}\sin\theta}^{\text{LHS}} = 0 \quad \therefore \quad \boxed{T_{CD} = -W\sqrt{2}}$$

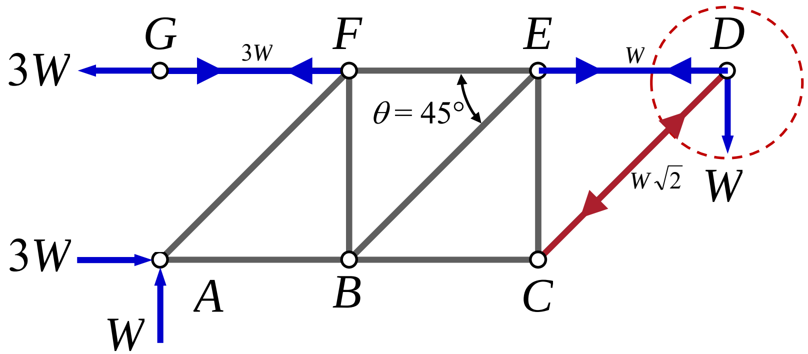
$$\boxed{\sin\theta = \frac{1}{\sqrt{2}}}$$

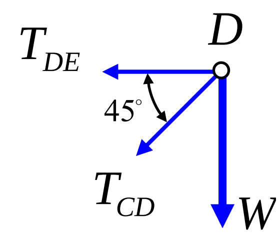
A negative T_{CD} means that member CD is under **compression**

- For the equilibrium equations write all terms on the left-hand side (LHS) to allow a consistent interpretation of signs
- Define the sign of each term on the LHS based on a certain reference (e.g. follow the external sign convention)

1.2.1 Method of Joints – Example

Step 3.2: Equilibrium @ joint D





And horizontal equilibrium gives:

$$\boxed{T_{CD} = -W\sqrt{2}}$$

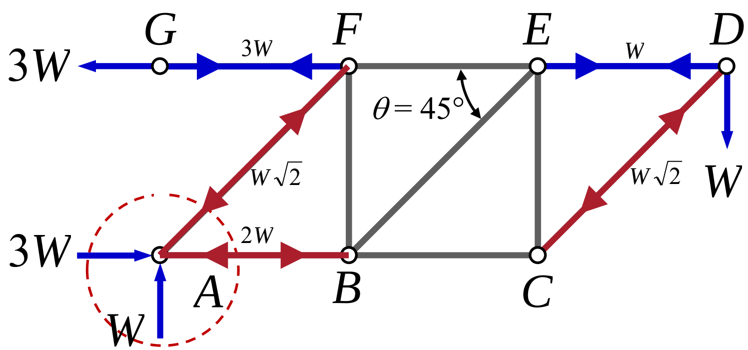
$$\sum F_x = 0 \quad \therefore \quad -T_{DE} - T_{CD}\cos\theta = 0 \quad \therefore \quad \boxed{T_{DE} = W}$$

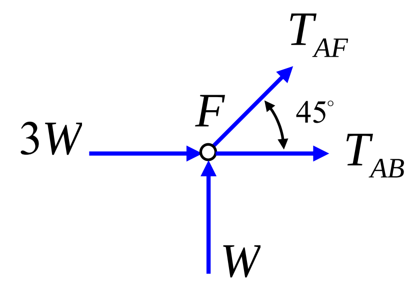
$$\boxed{\cos\theta = \frac{1}{\sqrt{2}}}$$

A positive T_{DE} means that member DE is under **tension**

1.2.1 Method of Joints – Example

Step 3.3: Equilibrium @ joint A





Vertical equilibrium:

$$\sin\theta = \frac{1}{\sqrt{2}}$$

$$\sum F_y = 0 \quad \therefore \quad W + T_{AF}\sin\theta = 0 \quad \therefore \quad \boxed{T_{AF} = -W\sqrt{2}}$$

Horizontal equilibrium:

$$T_{AF} = -W\sqrt{2}$$

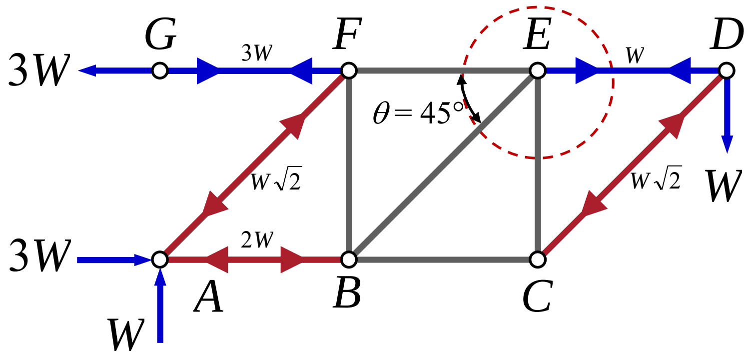
$$\sum F_x = 0 \quad \therefore \quad 3W + T_{AB} + T_{AF}\cos\theta = 0 \quad \therefore \quad \boxed{T_{AB} = -2W}$$

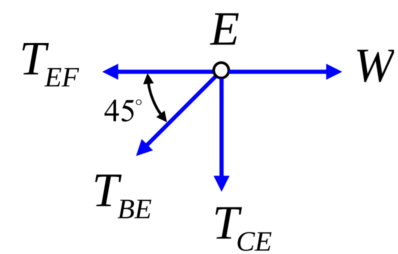
$$\cos\theta = \frac{1}{\sqrt{2}}$$

Negative T_{AF} and T_{AB} mean that members AF and AB are both under **compression**

1.2.1 Method of Joints – Example

~~**Step 3.4**: Equilibrium @ joint E~~ → Cannot be solved yet!





Vertical equilibrium:

$$\sum F_y = 0 \qquad \therefore \qquad -T_{CE} - T_{BE}\sin\theta = 0$$

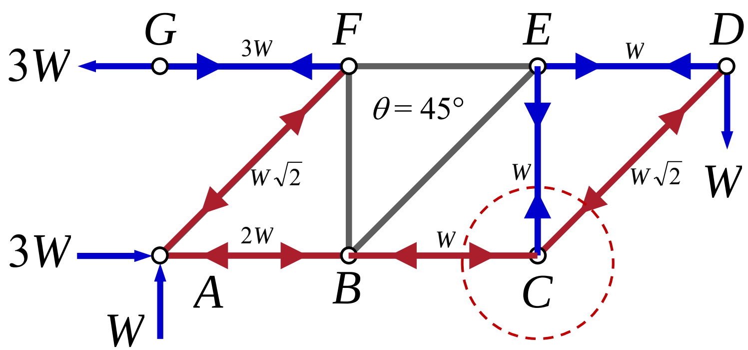
Horizontal equilibrium:

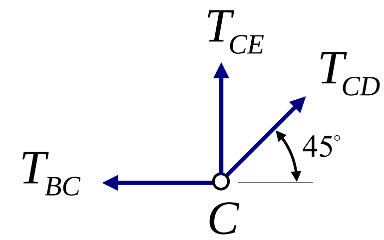
$$\sum F_x = 0 \qquad \therefore \qquad W - T_{EF} - T_{BE}\cos\theta = 0$$

Too many unknowns – no direct solution. Must choose another joint instead!

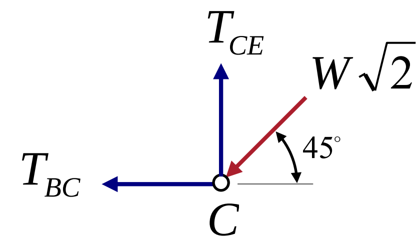
1.2.1 Method of Joints – Example

Step 3.4: Equilibrium @ joint C instead..





NB. We could also have drawn joint C like this:



Vertical equilibrium:

$$\sum F_y = 0 \quad \therefore \quad T_{CE} + \underbrace{\overbrace{T_{CD}}^{T_{CD} = -W\sqrt{2}} \sin\theta}_{\sin\theta = \frac{1}{\sqrt{2}}} = 0 \quad \therefore \quad \boxed{T_{CE} = W}$$

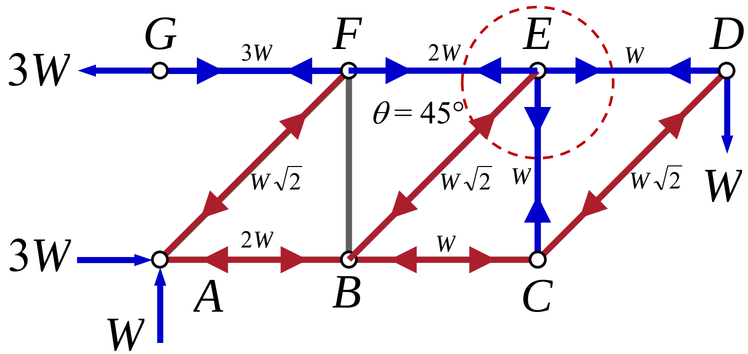
Horizontal equilibrium:

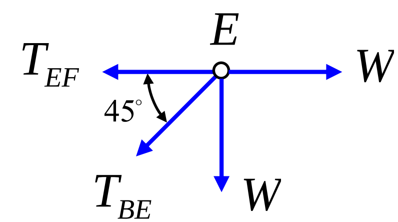
$$\sum F_x = 0 \quad \therefore \quad -T_{BC} + \underbrace{\overbrace{T_{CD}}^{T_{CD} = -W\sqrt{2}} \cos\theta}_{\cos\theta = \frac{1}{\sqrt{2}}} = 0 \quad \therefore \quad \boxed{T_{BC} = -W}$$

A negative T_{BC} means that member BC is under **compression**

1.2.1 Method of Joints – Example

Step 3.5: Equilibrium @ joint E (again)





Vertical equilibrium:

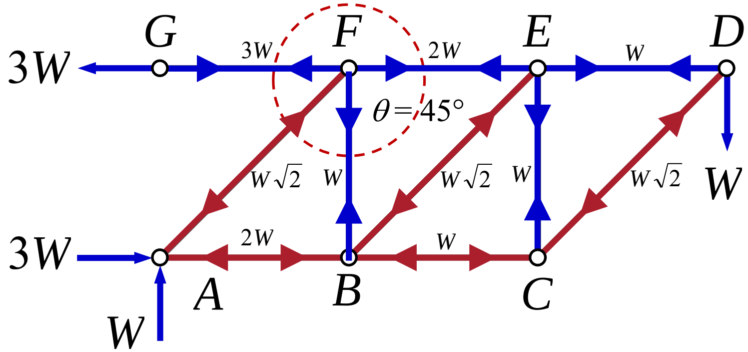
$$\sum F_y = 0 \quad \therefore \quad -W - T_{BE}\underbrace{\sin\theta}_{\boxed{\sin\theta = \frac{1}{\sqrt{2}}}} = 0 \quad \therefore \quad \boxed{T_{BE} = -W\sqrt{2}}$$

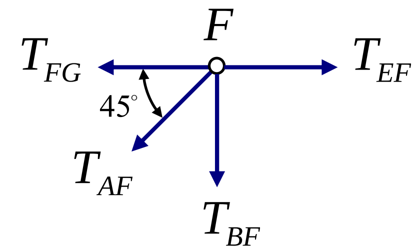
Horizontal equilibrium:

$$\sum F_x = 0 \quad \therefore \quad W - T_{EF} - \overbrace{T_{BE}}^{\boxed{T_{BE} = -W\sqrt{2}}}\underbrace{\cos\theta}_{\boxed{\cos\theta = \frac{1}{\sqrt{2}}}} = 0 \quad \therefore \quad \boxed{T_{EF} = 2W}$$

A negative T_{BE} means that member BE is under **compression**

Step 3.6: Equilibrium @ joint F





Vertical equilibrium:

$$\boxed{T_{AF} = -W\sqrt{2}}$$

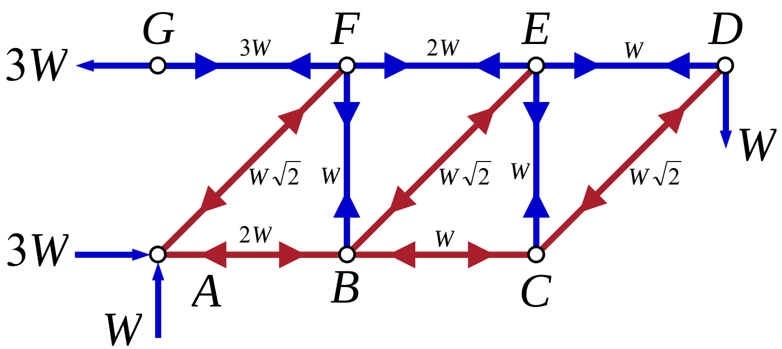
$$\sum F_y = 0 \quad \therefore \quad -T_{BF} - T_{AF}\sin\theta = 0 \quad \therefore \quad \boxed{T_{BF} = W}$$

$$\boxed{\sin\theta = \frac{1}{\sqrt{2}}}$$

And this completes our map of internal forces!

1.2.1 Method of Joints – Example

Result: Magnitude and sense of all forces:



Always conduct a final 'sanity check':

- Imagine the **deformed geometry** and check whether your forces have the correct sense
- Don't worry, you should get better at this with time and practice

Results of a computer (Finite Element) simulation:

- Member forces are in perfect agreement with our hand-calcs!