UNIVERSITY OF BRISTOL
FACULTY OF ENGINEERING

First Year Examination for the Degree of
Master of Engineering

MAY/JUNE 2015 2 Hours

FLUIDS 1

AENG11101

This paper contains *two* sections

SECTION 1
Answer *all* questions in this section
This section carries *40 marks*.

SECTION 2
This section has *three* questions.
Answer *two* questions.
All questions in this section carry *30 marks* each.
The maximum for this paper is *100 marks.*

Calculators may be used.

For air, assume R = 287 J/kgK. Take 0^{o}C as 273 K.
Use a gravitational acceleration of 9.81m/s^2
1 bar = 10^5 N/m^2

TURN OVER ONLY WHEN TOLD TO START WRITING

Useful Equations

The volume of a sphere: $\frac{4}{3}\pi r^3$ Area of a circle: πr^2

Roots of a quadratic: $ax^2 + bx + c = 0 \quad \rightarrow \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The equation of state for a perfect gas is:

$$p = \rho RT$$

Drag equation: $\text{Drag} = \text{Area} \times C_D \times \frac{1}{2}\rho V^2$

Equation of a streamline in 2D flow

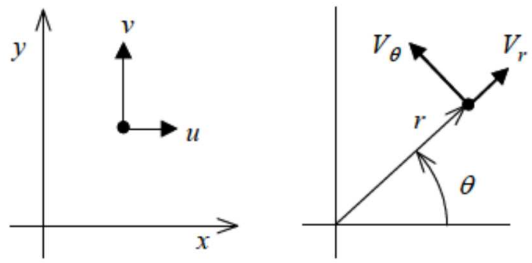
$$\frac{dy}{dx} = \frac{v}{u}$$

Change between Polar and Cartesian coordinate systems

$$r = \sqrt{x^2 + y^2}, \quad \theta = \tan^{-1}\left(\frac{y}{x}\right), \quad x = r\cos\theta, \quad y = r\sin\theta,$$

$$u = V_r \cos\theta - V_\theta \sin\theta, \qquad v = V_r \sin\theta + V_\theta \cos\theta$$

$$V_r = u\cos\theta + v\sin\theta, \qquad V_\theta = -u\sin\theta + v\cos\theta$$



2D Potential Flow

Velocity components in two-dimensional irrotational flow, in terms of the stream function and potential are given by

$$\underbrace{V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \qquad V_\theta = -\frac{\partial \psi}{\partial r} \qquad V_r = \frac{\partial \phi}{\partial r} \qquad V_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta}}_{\text{polar coordinates}} \qquad \underbrace{u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x} \qquad u = \frac{\partial \phi}{\partial x} \qquad v = \frac{\partial \phi}{\partial y}}_{\text{Cartesian coordinates}}$$

The stream function & velocity potential in Polar coordinates and the velocity distribution for

i) A uniform flow U_∞ parallel to the x axis:

$$\psi = U_\infty r\sin\theta, \qquad \phi = U_\infty r\cos\theta, \qquad V_r = U_\infty \cos\theta, \quad V_\theta = -U_\infty \sin\theta, \qquad u = U_\infty, \quad v = 0$$

ii) A source, of strength Λ at the origin:

$$\psi = \frac{+\Lambda\theta}{2\pi}, \quad \phi = \frac{+\Lambda}{2\pi}\ln r, \qquad V_r = \frac{+\Lambda}{2\pi r}, \quad V_\theta = 0, \qquad u = \frac{+\Lambda}{2\pi}\frac{x}{\left(x^2 + y^2\right)}, \quad v = \frac{+\Lambda}{2\pi}\frac{y}{\left(x^2 + y^2\right)}$$

iii) A doublet, of strength κ at the origin:

$$\psi = \frac{-\kappa}{2\pi}\frac{\sin\theta}{r}, \qquad \phi = \frac{+\kappa}{2\pi}\frac{\cos\theta}{r}, \qquad V_r = \frac{-\kappa}{2\pi r^2}\cos\theta, \qquad V_\theta = \frac{-\kappa}{2\pi r^2}\sin\theta,$$

$$u = \frac{-\kappa}{2\pi}\frac{\left(x^2 - y^2\right)}{\left(x^2 + y^2\right)^2}, \qquad v = \frac{-\kappa}{2\pi}\frac{2xy}{\left(x^2 + y^2\right)^2}$$

iv) A vortex, of strength Γ, at the origin:

$$\psi = \frac{+\Gamma}{2\pi}\ln r, \quad \phi = \frac{-\Gamma}{2\pi}\theta,$$

$$V_r = 0, \quad V_\theta = \frac{-\Gamma}{2\pi r}, \qquad u = \frac{+\Gamma}{2\pi}\frac{y}{\left(x^2 + y^2\right)}, \quad v = \frac{-\Gamma}{2\pi}\frac{x}{\left(x^2 + y^2\right)}$$

Useful integrals

$$\int \sin^2(ax)dx = \frac{x}{2} - \frac{\sin 2ax}{4a} + C$$

$$\int \cos^2(ax)dx = \frac{x}{2} + \frac{\sin 2ax}{4a} + C$$

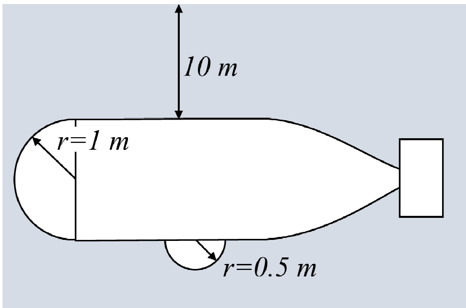
turn over …

SECTION 1
Answer all questions in this section

Q1 Oil, in a storage container, has become contaminated with water. The gauge pressure at the bottom of the container is 0.123 bar. The distance from the bottom of the container to the surface of the oil is 1.5m. What depth of water is in the container assuming the oil has a specific gravity of 0.8 and water has a density of 1000 $kg\ m^{-3}$?

(6 marks)

Q2 A small submarine has two hemispherical observation domes. One, with radius *1m*, is positioned vertically at the front of the submarine while the other, with radius *0.5m*, is placed horizontally at the bottom. When the submarine is at a depth of 10m, find the horizontal and vertical force on the front and bottom domes respectively. You may assume that: the relative positions of the domes are as shown in the diagram; that the pressure is atmospheric inside the sub and that the water has a density of 1000 $kg\ m^{-3}$.

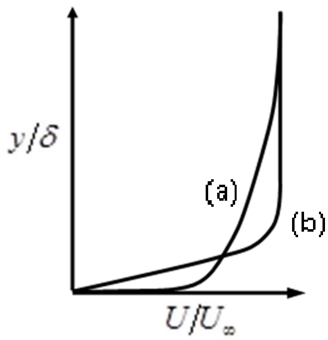


(6 marks)

Q3 State the assumptions that must be made for Bernoulli's equation to be valid.

(4 marks)

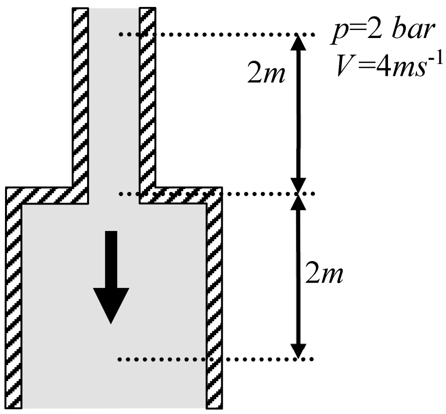
Q4 The diagram shows two boundary layer velocity profiles, (a) & (b), non-dimensionalised by free stream velocity and boundary layer thickness. Using (a) and (b) to refer to each profile, state which represents a laminar boundary layer and which represents a turbulent boundary layer. Also state which profile is the most resistant to separation and explain why?



(6 marks)

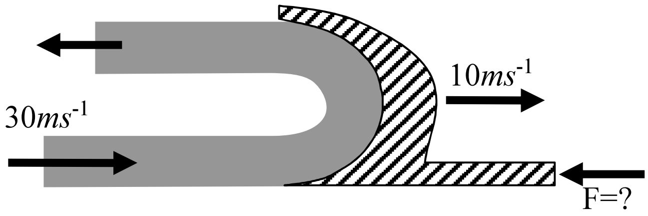
Q5 Water flows down a straight smooth vertical pipe with a circular cross-section. At a particular point, 2*m* upstream of a sudden enlargement, the pressure and velocity are 2 *bar* and 4ms^{-1} respectively.

Given that the area ratio across the enlargement is 4, find the loss coefficient using: $\left(1 - A_1/A_2\right)^2$. Hence, find the static pressure 2m downstream of the expansion, where the flow may be assumed to have returned to uniform. Assume the water has a density of 1000 $kg\ m^{-3}$.



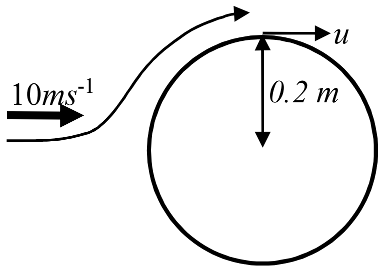
(6 marks)

Q6 A horizontal circular water jet of diameter 10 *cm* and speed 30 ms^{-1} hits a plate moving with a constant velocity of 10 ms^{-1} away from the jet. By using a suitable control volume, find the horizontal force that must be applied to the plate to keep it moving at this constant velocity if all the water leaving the plate is turned through an angle of 180^o to the plate with no losses. Assume the water has a density of 1000 $kg\ m^{-3}$.



(6 marks)

Q7 An air flow over a long circular cylinder, with a radius of 0.2*m*, can be modelled by a horizontal onset flow of 10 ms^{-1} and a doublet of strength κ m^2s^{-1}. Find the strength of the doublet and the velocity, *u*, at the top of the cylinder assuming potential flow. What would be the value of *u* at the surface for the real viscous flow?



(6 marks)

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SECTION 2
Answer *two* questions in this section

Q8 **(a)** An idealised water system (shown in figures Q8a & b below) has been punctured so that water is escaping through a circular sharp edged hole. The resulting jet of water is vertical and the sharp edged hole is such that a vena contracta forms at a fixed height H from the top of a large water storage tank. Initially the water tank is open so that the surface is at atmospheric pressure. Ignoring the variation of atmospheric pressure with height and assuming the water surface in the tank can be treated as stationary, show that the mass flow rate when the tank is completely full is given by:

$$c_d A\rho\sqrt{2gH}$$

where c_d is the discharge coefficient, A is the area of the hole and ρ is the density of water .

(8 marks)

(b) Water escapes the system until the surface in the tank is a height h_0 above the vena contracta. At this point the water tank is sealed (see figure Q8b). Show that, if the vena contracta position remains fixed, the mass flow rate after the tank is sealed becomes

$$c_d \rho A\sqrt{2gh - 2\frac{p_a}{\rho}\frac{(h_0 - h)}{(H-h)}}$$

where h is the instantaneous height of the tank surface above the vena contracta. Also show that the height of the jet of water, h_{jet}, after the tank is sealed is given by

$$h_{jet} = h - \frac{p_a}{\rho g}\frac{(h_0 - h)}{(H-h)}$$

(12 marks)

(c) A water system with a hole, as described in parts (a) and (b) above is such that the height of the surface of the water of the full tank, H, is 5m above the vena contracta. What will be the initial height of the jet of water and the height at which the jet has twice the cross sectional area of the hole? Use a value of $c_d = 0.6$.
The tank is sealed when the water level has dropped $10\,cm$. How much further will the water level drop if the vena contracta is level with the outer surface of the pipe? Use values of atmospheric pressure and water density of $1\,bar$ and $1000\,kg\,m^{-3}$ respectively.

(10 marks)

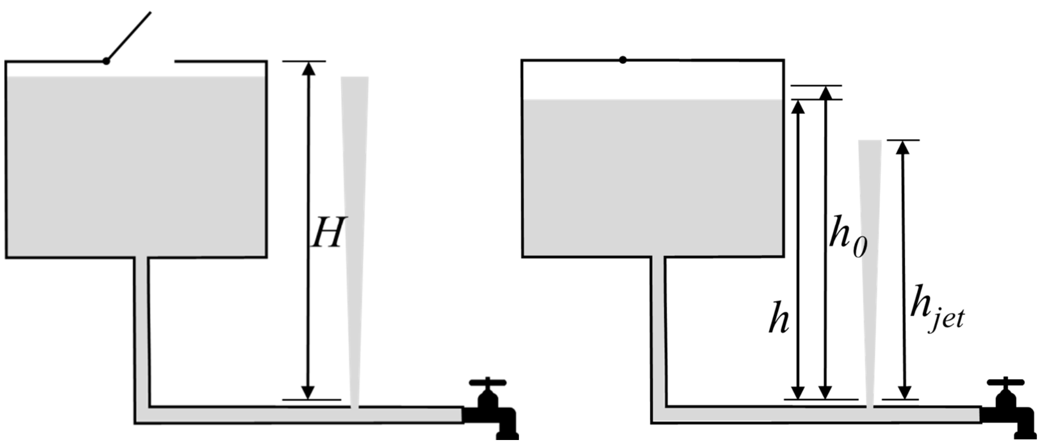

***Figure Q8**: Diagram of system used in question 8, before (a) and after (b) the tank is sealed*

Q9 A propeller sweeps out a circular disc of diameter d while flying horizontally through still air at a speed of $V\ ms^{-1}$. The ratio of the drop in streamtube area (between far upstream and the disc) to the area of the propeller disk is given by a - the inflow factor.

(a) Use the actuator disc theory for an ideal propeller to show from first principles that the force supplied by the disc can be written as.

$$F = \tfrac{1}{2}\rho A_d\left(V_4^2 - V^2\right)$$

where ρ is the density of the air and V_4 is the downstream velocity relative to the disc. Clearly state all assumptions made during your derivation.

(8 marks)

(b) Further, show that the force can be rewritten as

$$F = \tfrac{1}{2}\pi\rho d^2 V^2 a(a+1)$$

and that the efficiency is given by

$$\eta = \frac{1}{(1+a)}$$

(14 marks)

(c) A light aircraft is being designed and a force of 8,000N is required to allow a design flight speed of 85 ms^{-1}. Find the diameter for the ideal propeller required for this aircraft assuming the air density is given by 1.2 $kg\ m^{-3}$ and the inflow factor is 0.2. Further, find the power required to drive the propeller.

(8 marks)

turn over..

Q10 (a) Briefly describe the physical significance of the stream function ψ in incompressible flow. What are the limits on its application, compared with the potential function ϕ?

(4 marks)

(b) A wind of speed 10 m/s is blowing over a rounded cliff; the situation is modelled as a source/sink of strength $\Lambda = -130\pi\ m^2/s$ at the origin combined with a uniform stream $U_\infty = 10\ m/s$ in the x – direction.

i) Draw a sketch of the streamlines of the flow (internal & external to the cliff).

(3 marks)

ii) Find the location of the stagnation point.

(6 marks)

iii) Find the height of the cliff surface where it intersects the y axis.

(4 marks)

iv) The height of the cliff as $x \to -\infty$

(4 marks)

v) What is the percentage difference, relative to the free stream velocity, of the velocity measured by a sensor located 1m above the surface of the cliff directly above the origin?

(9 marks)

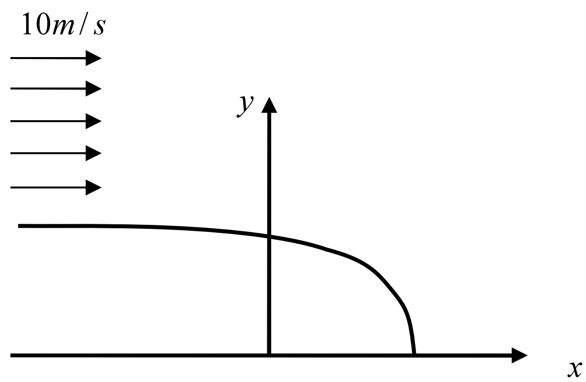

Figure Q10: Coordinate system for the cliff