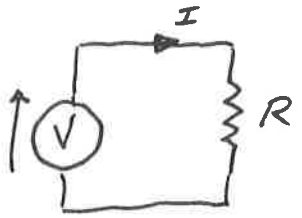
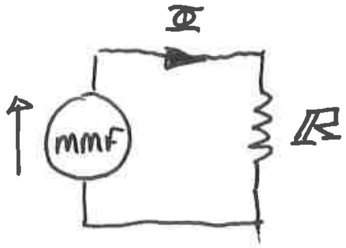
Magnetic Circuits - an analogy

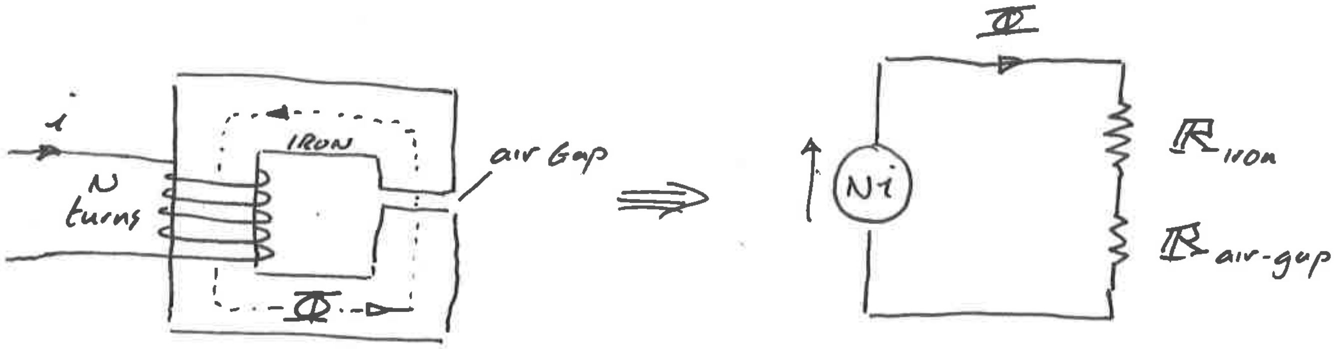


In an electric circuit a voltage, 'V', drives a current, 'I', through a resistance 'R'



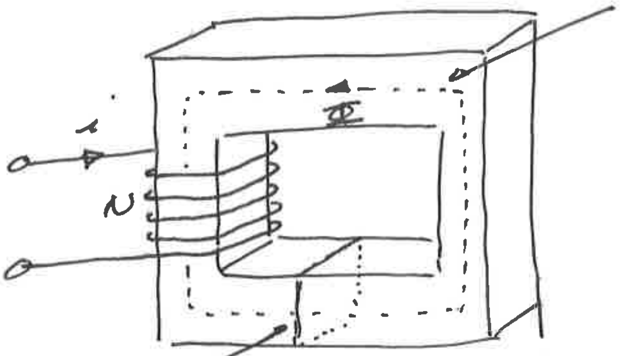
We can draw an equivalent magnetic circuit where an 'MMF' (Magneto, Magnetic Force) drives a flux, Φ, through a reluctance $\mathcal{R}$

The MMF could be created by a pemanent magnet or by an electromagnet. The MMF produced by an electromagnet is the product of the coil current and number of turns.



Unlike electrical circuits where all the current is confined to the conductors, a magnetic circuit is 'leaky' and flux extends into the surrounding space. Because of this simple 2D approximations of magnetic circuits have limitations and complex arrangements need numerical (finite element) techniques to solve.

Magnetic problems are often described by field quantities. We turn the mmf into a gradient around the magnetic circuit, denoted 'H', and we turn flux into a flux density over the surface through which it passes.



flux path of length 'l'

$$H = \frac{mmf}{l} = \frac{Ni}{l}$$

or more generally:

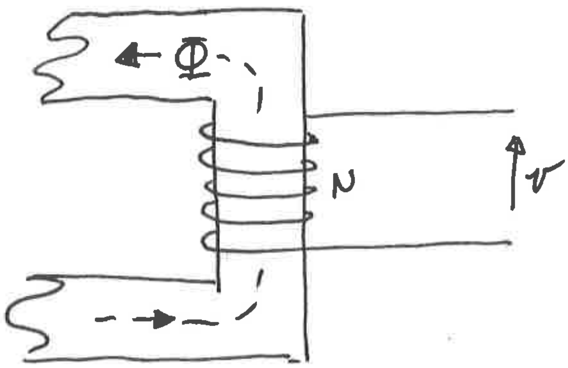
$$mmf = \int H\,dl$$

At this section, the magnetic circuit has a cross sectional area of 'A'. The flux density is then $B = \dfrac{\Phi}{A}$

B & H are related by the permeability of a material – A magnetic field H creates a flux density everywhere it exists according to the relation:

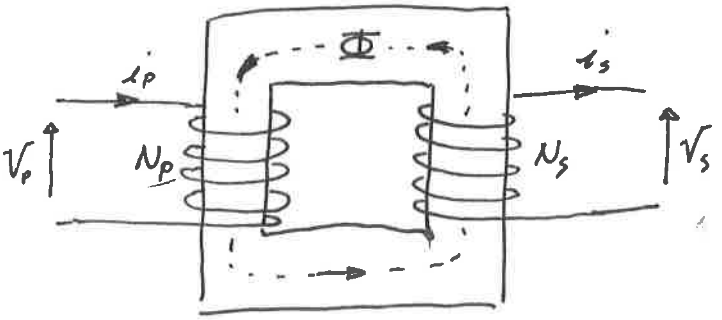
$B = \mu H$ where μ is the permeability and is often written $\mu_0 \mu_r$, where μ_0 is the permeability of free space and μ_r is the relative permeability.

Both examples of magnetic sensor we have seen – the LVDT and gear-tooth sensor – have an output coil linking flux. This situation is described by Faraday's Law of Induction;



$$v = N\frac{d\Phi}{dt}$$

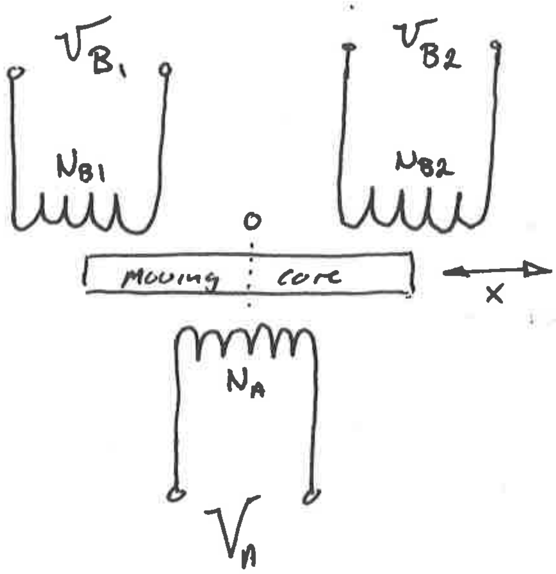
The LVDT is based around a transformer – two coils coupled by a magnetic circuit.



$$v_s = \frac{N_s}{N_p} v_p$$

$$i_s = \frac{N_p}{N_s} i_p$$

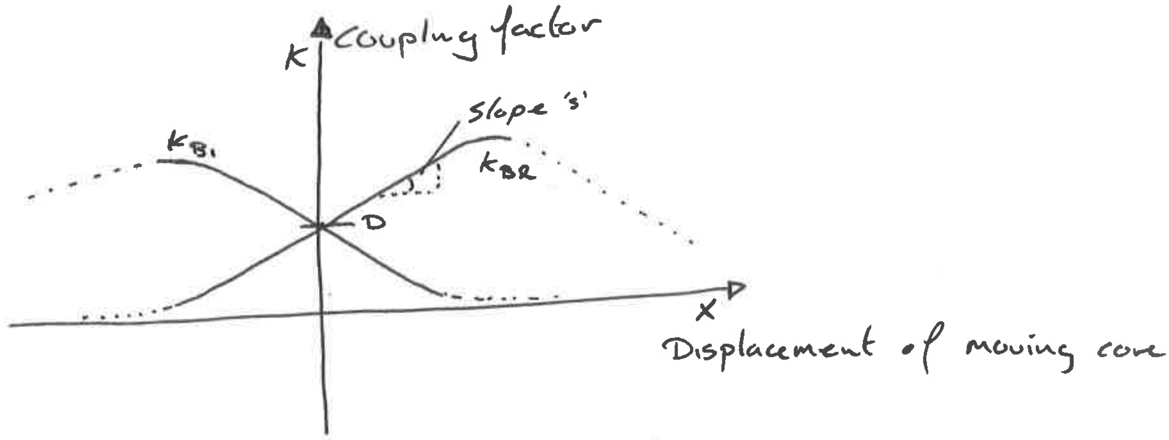
For a basic transformer we consider that all of the magnetic flux generated by the input (primary) winding links with the output (secondary) winding. In the LVDT we deliberately design a magnetic circuit where moving the core changes the amount of flux coupled between the primary and secondary windings.



We can add a coupling factor 'k' into our transformer expression and describe how the primary winding of the LVDT couples into each secondary.

$$V_{B1} = \frac{N_{B1}}{N_A} \cdot K_{B1} \cdot V_A \qquad V_{B2} = \frac{N_{B2}}{N_A} \cdot K_{B2} \cdot V_A$$

Lets assume $N_{B1} = N_A = N_{B2}$ then we end up with $V_{B1} = k_{B1} V_A$ or $V_{B2} = k_{B2} \cdot V_A$ where $0 < (k_{B1}, k_{B2}) < 1$.



The sketch above shows how the coupling factors will alter with core position. If we restrict travel to the linear range then we get;

$$k_{B1} = D - SX$$

$$k_{B2} = SX + D$$

where s is the slope and D is the coupling at $x=0$.

The output coils B_1 & B_2 are connected so the output is:

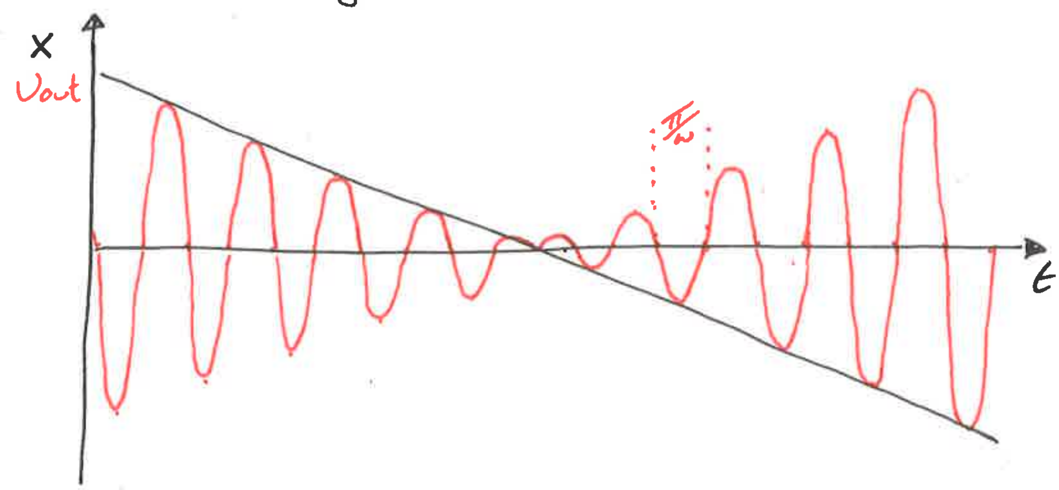
$$v_{out} = v_{B1} - v_{B2}$$

if the input $v_A = A\sin(\omega t)$ then

$$v_{out} = (D - SX) A \sin(\omega t) - (SX + D) A \sin(\omega t)$$
$$= -2SXA \sin(\omega t)$$

S is set by the geometry of the LVDT and $A\sin(\omega t)$ is set by the input. We can work out the magnitude of x from the magnitude of v_{out} and the direction from the phase relative to the input.

$\text{mag.}(v_{out}) \propto$ magnitude of x
$\text{sign}(v_{out}) =$ direction of x



The above sketch shows how the output voltage changes as x changes, both over time. The amplitude of the input voltage is modulated by the displacement x.