UNIVERSITY OF BRISTOL
FACULTY OF ENGINEERING

First Year Examination for the Degree of Master of Engineering

MAY/JUNE 2011 3 Hours

AENG11101

FLUIDS I

This paper contains *two* sections

SECTION 1
Answer *all* questions in this section

This section carries *40 marks*.

SECTION 2
This section has *five* questions.

Answer *three* questions.

All questions in this section carry *20 marks* each.

The maximum for this paper is *100 marks.*

Calculators may be used.

For air, assume R = 287 J/kgK. Take 0^oC as 273^oK.
Use a gravitational acceleration of $9.81 m/s^2$

.

Useful Equations

The volume of a sphere: $\frac{4}{3}\pi r^3$ Area of a circle: πr^2

Roots of a quadratic: $ax^2 + bx + c = 0 \quad \rightarrow \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The equation of state for a perfect gas is:

$$p = \rho R T$$

Drag equation: $\text{Drag} = \text{Area} \times C_D \times \frac{1}{2}\rho V^2$

Equation of a streamline in 2D flow

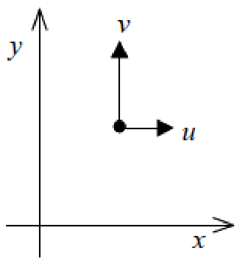
$$\frac{dy}{dx} = \frac{v}{u}$$

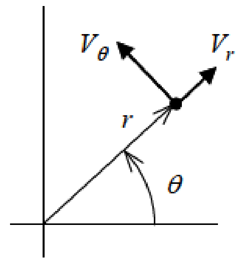
Change between Polar and Cartesian coordinate systems

$$r = \sqrt{x^2 + y^2}, \quad \theta = \tan^{-1}\left(\frac{y}{x}\right), \quad x = r\cos\theta, \quad y = r\sin\theta,$$

$$u = V_r \cos\theta - V_\theta \sin\theta, \qquad v = V_r \sin\theta + V_\theta \cos\theta$$

$$V_r = u\cos\theta + v\sin\theta, \qquad V_\theta = -u\sin\theta + v\cos\theta$$





2D Potential Flow

Velocity components in two-dimensional irrotational flow, in terms of the stream function and potential are given by

$$\underbrace{V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \qquad V_\theta = -\frac{\partial \psi}{\partial r} \qquad V_r = \frac{\partial \phi}{\partial r} \qquad V_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta}}_{\text{polar coordinates}}$$

$$\underbrace{u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x} \qquad u = \frac{\partial \phi}{\partial x} \qquad v = \frac{\partial \phi}{\partial y}}_{\text{Cartesian coordinates}}$$

The stream function & velocity potential in Polar coordinates and the velocity distribution for

i) A uniform flow U_∞ parallel to the x axis:

$$\psi = U_\infty r\sin\theta, \qquad \phi = U_\infty r\cos\theta, \qquad V_r = U_\infty \cos\theta, \quad V_\theta = -U_\infty \sin\theta, \qquad u = U_\infty, \quad v = 0$$

ii) A source, of strength Λ, at the origin:

$$\psi = \frac{+\Lambda\theta}{2\pi}, \quad \phi = \frac{+\Lambda}{2\pi}\ln r, \qquad V_r = \frac{+\Lambda}{2\pi r}, \quad V_\theta = 0, \qquad u = \frac{+\Lambda}{2\pi}\frac{x}{\left(x^2 + y^2\right)}, \quad v = \frac{+\Lambda}{2\pi}\frac{y}{\left(x^2 + y^2\right)}$$

iii) A doublet, of strength κ, at the origin:

$$\psi = \frac{-\kappa}{2\pi}\frac{\sin\theta}{r}, \qquad \phi = \frac{+\kappa}{2\pi}\frac{\cos\theta}{r}, \qquad V_r = \frac{-\kappa}{2\pi r^2}\cos\theta, \qquad V_\theta = \frac{-\kappa}{2\pi r^2}\sin\theta,$$

$$u = \frac{-\kappa}{2\pi}\frac{\left(x^2 - y^2\right)}{\left(x^2 + y^2\right)^2}, \qquad v = \frac{-\kappa}{2\pi}\frac{2xy}{\left(x^2 + y^2\right)^2}$$

iv) A vortex, of strength Γ, at the origin:

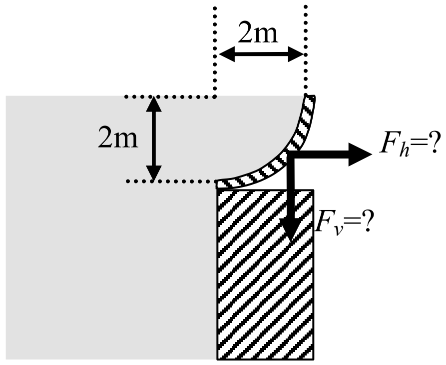
$$\psi = \frac{+\Gamma}{2\pi}\ln r, \quad \phi = \frac{-\Gamma}{2\pi}\theta, \qquad V_r = 0, \quad V_\theta = \frac{-\Gamma}{2\pi r}, \qquad u = \frac{+\Gamma}{2\pi}\frac{y}{\left(x^2 + y^2\right)}, \quad v = \frac{-\Gamma}{2\pi}\frac{x}{\left(x^2 + y^2\right)}$$

SECTION 1 Answer <u>all</u> questions in this section

Q1.1 A pressure transducer at the bottom of an oil storage container records a pressure of 1.9 bar. How far below the surface is the pressure transducer, if the atmospheric pressure at the surface of the oil is 1.023bar, the oil has a specific gravity of 0.9 and water density is 1000 kg m^{-3}?

(4 *marks*)

Q1.2 A dam has a sluice gate whose cross section is a quarter circle of radius 2m. The gate is 6m wide. If the water just reaches the top of the gate, find the vertical and horizontal components of thrust on the gate.



(4 *marks*)

Q1.3 Under which assumptions is Bernoulli's equation valid for the flow along a duct?

(4 marks)

Q1.4 A horizontal straight sectioned duct carries air with an average speed of 5m/s at a pressure of $0.98\text{x}10^{5}\text{Nm}^{-2}$. The duct then enters a smooth contraction before reaching a second straight sectioned duct with a similar but scaled cross section to the original duct. If the average velocity in the second duct is 80m/s, what is the ratio of original to final duct dimensions? What is the pressure downstream of the contraction? Take the density of the air as 1.2 kg m^{-3}

(*4 marks*)

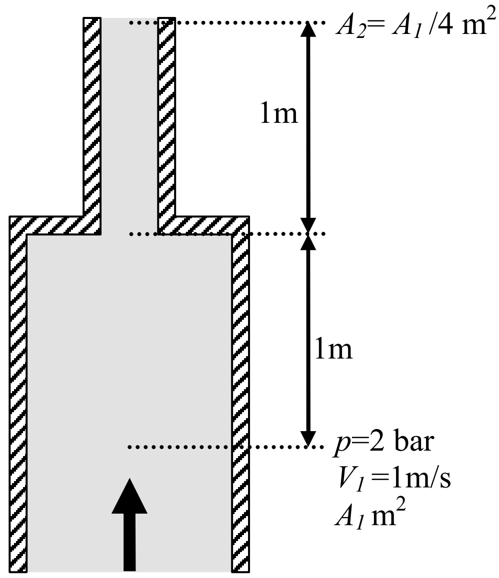
Q1.5 (a) Name the transformation that allows the steady flow of air over a fixed body to be compared with the steady movement of that body through still air.
(b) A 1/6th scale model of a formulae one car is placed in a wind tunnel, what would the wind tunnel air speed need to be to reproduce the Reynolds number of the flow around the full scale car moving at 50m/s (assume the viscosity and density are unchanged).
(c) If the speed of sound is approximately 340 m/s^2 for both cases, what is the difficulty involved with the use of the wind tunnel model.

(*4 marks*)

Q1.6 The Drag force on a static elliptical airship body in a 4m/s wind is 70N. Assuming the orientation of the balloon into the wind and the drag coefficient do not change, estimate the drag on the body when the wind speed increases to 7m/s. At 4m/s the drag coefficient is 0.39, when the wind speed increases to 7m/s the drag coefficient actually drops to 0.21, explain the probable cause of this result.

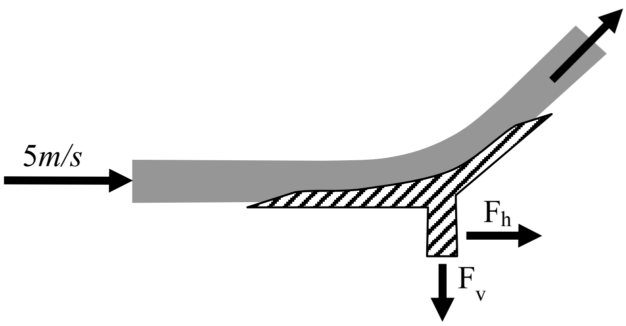
(4 marks)

Q1.7 Water flows up a straight smooth vertical pipe with a circular cross-section, the pressure and average velocity in this pipe are 2 bar and 1m/s. The water enters another straight smooth vertical pipe, with an area 4 times smaller than the first section, via a sudden contraction. If the loss factor through the contraction is 0.5, find the pressure 1m upstream of the contraction. What causes the pressure loss associated with the loss factor?



(4 marks)

Q1.8 A horizontal circular water jet of diameter 30cm and speed 5m/s hits a stationary turning vane that smoothly turns the water through 30^o. By using a suitable control volume, find the horizontal and vertical forces on the turning vane. Assume the water has a density of 1000 kg m^{-3}.



(4 marks)

Q1.9 Sketch the equipotentials and streamlines for an isolated source and an isolated vortex, marks will not be awarded for untitled sketches.

(4 marks)

Q1.10 The non-lifting flow over an oval can be modelled as a combination of which three flows? The lifting flow over a cylinder can be modelled as a combination of which three flows?

(4 marks)

SECTION 2 Answer *three* questions in this section

Q2.1 A simple diving bell, shown in figure 2.1 below, can be split into three sections. The ballast tank has an interior volume V_b m^3 and is entirely open to the water at the bottom of the vessel. Two air tanks, each with an interior volume of V_a m^3, are connected to the ballast tank by pumps that can create a pressure r times greater than that in the ballast tank at the maximum depth. The final section is sealed. At the point when the diving bell is just lowered into the water (figure 2.1a) the air in all sections is at the surface pressure and temperature, p_s N/m^2 and T_s ^{o}K respectively. Also the volume and mass of the vessel and all contained air is V m^3 and M kg. The density of the sea water, ρ_w, is considered constant throughout.

(a) Assuming that the ballast tanks will not be completely filled, show that at the maximum depth the vessel can reach, the volume of air in the ballast tanks is given by

$$V_h = \frac{T}{T_s}\frac{p_s}{p_s + \rho_w g h_{\max}}\left(V_b + 2V_a\right) - 2rV_a$$

and that the depth measured to the water surface from the ballast tanks is given by

$$h_{\max} = \frac{p_s}{\rho_w g}\left(\frac{T}{T_s}\cdot\frac{\left(V_b + 2V_a\right)\rho_w}{\left[M - \left(V - V_b - 2rV_a\right)\rho_w\right]} - 1\right)$$

where T (in ^{o}K) is the temperature of the surrounding sea water and equals the temperature in the ballast and air tanks. *Note that the total mass of air in the ballast and air tanks remains constant.*

(13 marks)

(b) The total mass of the vessel described above, including trapped air, is 4000kg. The interior volume of the ballast tank is 2.31 m^3 while the interior volumes of the air tanks is 0.01m^3 each and the total volume of the vessel is 6.2 m^3.The temperature of the sea water varies linearly from 20^{o}C at the surface to 4^{o}C at a depth of 2000m. The atmospheric pressure is 1.02x10^5 N/m^2. The density of the water is constant at 1028 kg/m^3. Find the maximum depth the vessel may reach if a pressure ratio of 2:1 is maintained between the air tanks and the ballast chamber. You can assume that the maximum depth reached is less than 2000m.

(7 marks)

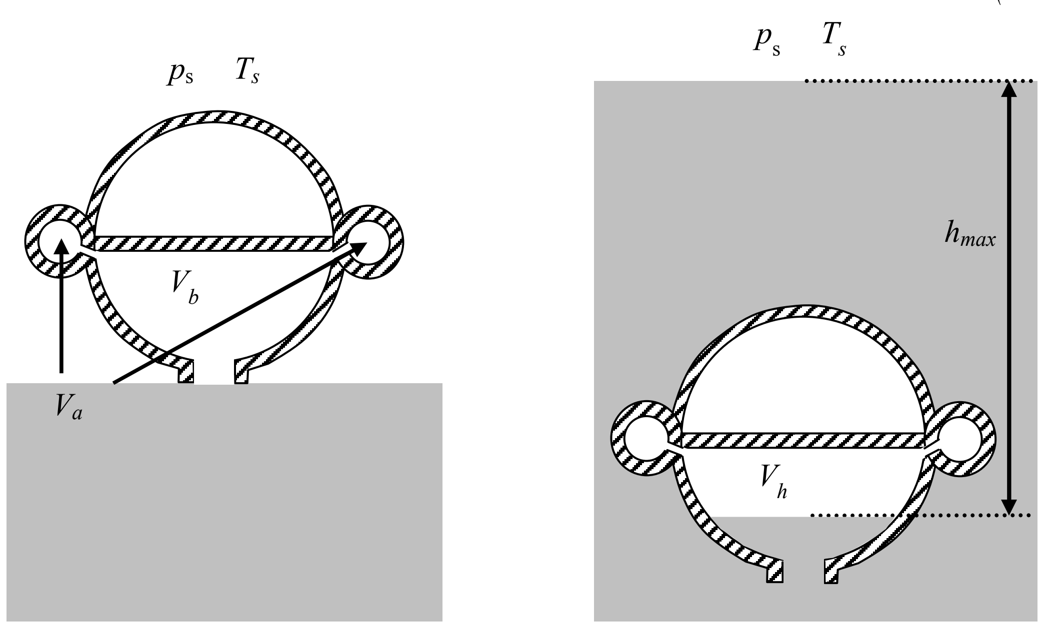


Figure 2.1a) diving bell at the surface, 2.1b) diving bell at its maximum depth h_{max}

Q2.2 Water is syphoned from a large tank that has a smaller reservoir at the bottom as shown in figure 2.2 below. The highest point of the pipe is h m above the top of the small reservoir, while the exit of the pipe is at a height H m below the top of the small reservoir. The cross sectional area of the pipe is A_e at the exit while the difference in cross-sectional area between the interior of the small reservoir and the outside of the pipe is given by A_c. There is a constriction in the pipe at its highest point so that cross sectional area becomes A_h.

(a) Show that the mass flow rate, $\dot{m}$, when the height of the water surface is $H+\delta$ above the exit of the pipe is

$$\dot{m} = \rho A_e \sqrt{2g(H+\delta)}$$

Where ρ is the density of the water and g is the gravitational constant,

(5 marks)

(b) Show that the mass flow rate when the water surface is δ **below** the top of the small reservoir (i.e. $H-\delta$ above the exit of the pipe) is

$$\dot{m} = \rho \frac{A_c A_e}{\sqrt{A_c^2 - A_e^2}} \sqrt{2g(H-\delta)}$$

(7 marks)

(c) Show that, as the surface of the fluid is falling and just reaches the top of the small reservoir, the pressure at the constriction suddenly changes by an amount given by

$$\Delta p_h = \rho g H \left(\frac{A_e^2 - A_h^2}{A_h^2} \right) \left(\frac{A_e^2}{A_c^2 - A_e^2} \right)$$

(8 marks)

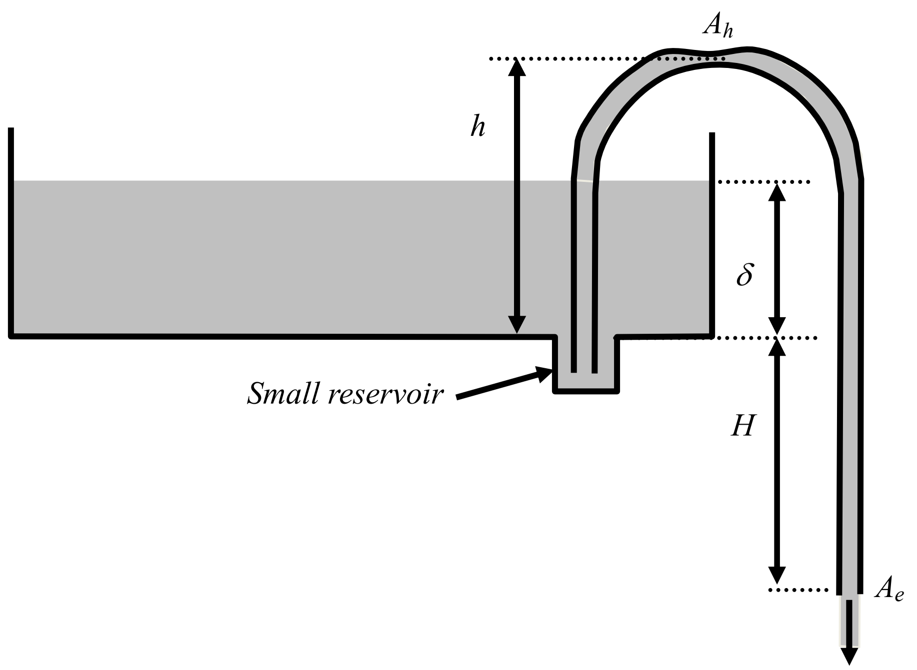


Figure 2.2: Diagram of syphon, tank and small reservoir used in question 2.2

Q2.3 Figure 2.3a below shows a low speed, open section wind tunnel. The air is drawn from static atmospheric conditions into a parallel section of area A_i, through a smooth contraction designed to eliminate total pressure losses, into a parallel working section of area A_w and uniform velocity of V_w. The air then passes over the fan before flowing through an expansion and straight section, with an area of A_e, and exiting to atmospheric conditions.

(a) Find the differential height of manometer fluid, Δh, in terms of the air velocity V_w, air density ρ_a, the manometer fluid density ρ_m, the areas A_i & A_w and the acceleration due to gravity g. State all the assumptions you have made.

(7 marks)

(b) Derive an expression for the change in static pressure across the fan, Δp_f, in terms of only: A_w, A_e, ρ_a & V_a. Assume the pressure at the exit is atmospheric

(7 marks)

(c) A honeycomb section is introduced in the inlet section, as shown in figure 2.3b, to ensure smooth flow in the working section of the wind tunnel but produces a pressure loss. Show that if the air speed in the working section is unchanged, the pressure across the fan becomes Δp_{fh} so that the change in pressure drop becomes

$$\Delta p_f - \Delta p_{fh} = \tfrac{1}{2}\rho_a\left[1-\left(\frac{A_w}{A_i}\right)^2\right]V_w^2 - \rho_m g\Delta h$$

(6 marks)

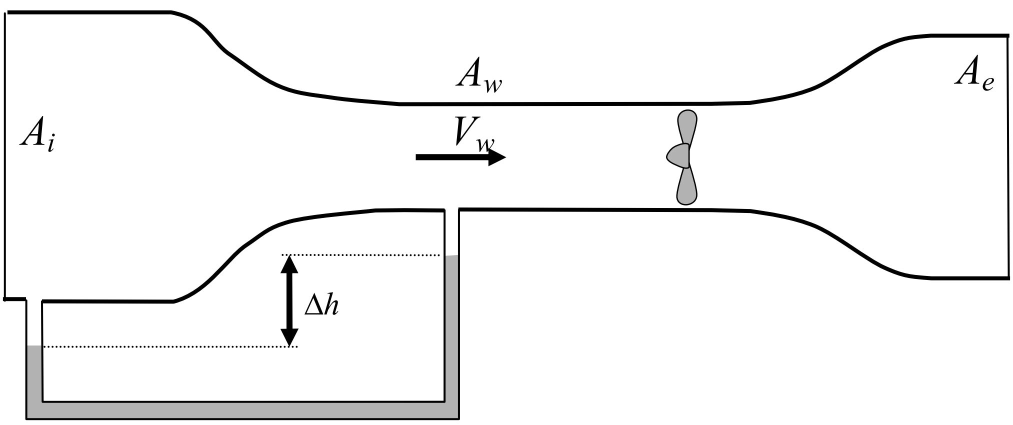


Figure 2.3a: Schematic diagram of empty wind tunnel and manometer

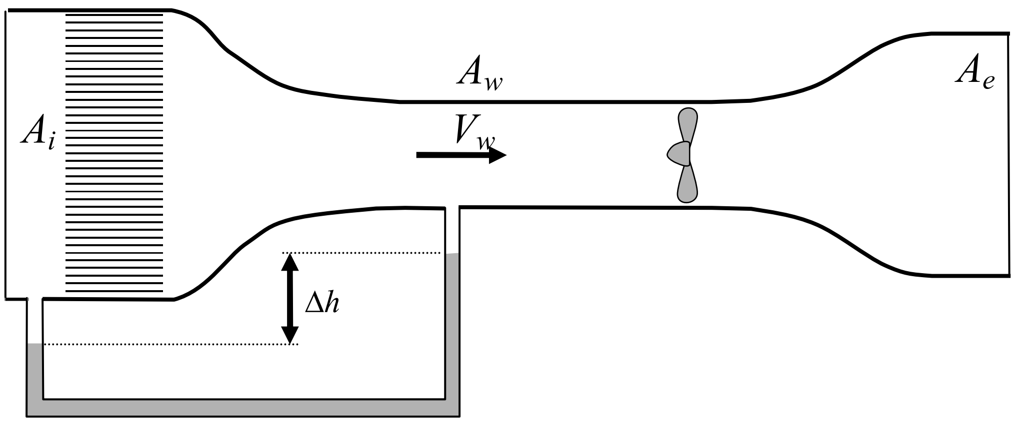


Figure 2.3b: Schematic diagram of the same wind tunnel with model and strut

Q2.4 Use actuator disc theory to analyse a helicopter, with a rotor disc radius r m, climbing vertically through still air. When the helicopter climbs at a speed of V m/s, the air at a point downstream of the rotor where the pressure has returned to atmospheric, has a velocity relative to the ground of V_o m/s. The velocity V_o can be rewritten as $V_o = aV$, where a is a constant that represents the ratio of the downstream wind speed to the vertical velocity.

(a) Show that if the density of air is ρ, the force supplied by the rotor can be written as.

$$F = \rho \frac{\pi}{2} r^2 V^2 a(a+2)$$

Clearly state all assumptions made during your derivation

(8 marks)

(b) Continuing the analysis of the rotor defined above, show that the efficiency when in vertical motion is given by

$$\eta = \frac{2}{(2+a)}$$

(7 marks)

(c) A helicopter of mass 7000kg has an ideal rotor of radius 3m. If the air density is 1.25 kg/m^3, find the airspeed through the disc when the helicopter is in hover.

(5 marks)

Q2.5 (a) Using Bernoulli's equation for potential flow, derive an expression for the pressure coefficient in terms of the velocity.

(2 marks)

(b) The non-lifting flow over a cylinder can be modelled using a free stream and a doublet. Determine the strength of the doublet in terms of the cylinder radius and free stream velocity, U_∞. What is the pressure coefficient distribution on the cylinder? Hence show that the pressure distribution on the cylinder is given by

$$p(\theta) = p_\infty + \frac{1}{2}\rho U_\infty^2 (1 - 4\sin^2\theta)$$

(9 marks)

(c) The cylinder, modelled with the doublet strength found from the free stream analysis in (b), is placed so that its centre is a distance h above a solid surface. See figure 2.5. Assuming inviscid potential flow, show that the pressure coefficient on a vertical line between the surface and the nearest point on the cylinder, can be written as

$$C_p = -4R^2 \frac{(y^2 + h^2)}{(y^2 - h^2)^2}\left(1 + R^2 \frac{(y^2 + h^2)}{(y^2 - h^2)^2}\right)$$

Note that, due to symmetry there is no vertical velocity along this line.

(9 marks)

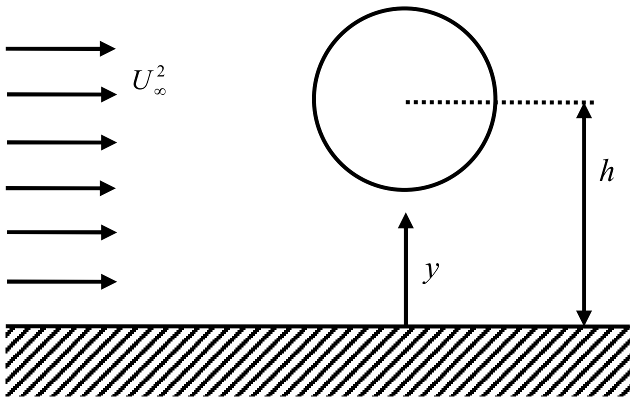


Figure 2.5: Cylinder in a freestream with the centre a distance h from a solid surface.