Q2 A uniform horizontal beam $ABCD$ is simply supported at A and C and has vertical loads applied at B and D, as shown in Figure Q2(a). The cross-section of the beam is shown in Figure Q2(b).

(a) Sketch carefully the distribution of bending moments along the length of the beam due to the application of the loads, giving principal values and ensuring that the sketch is reasonably to scale.

[8 marks]

(b) Determine the maximum tensile and compressive stresses in the beam due to the bending moment, giving their values and positions where they occur, both along the beam and across the cross-section.

[12 marks]

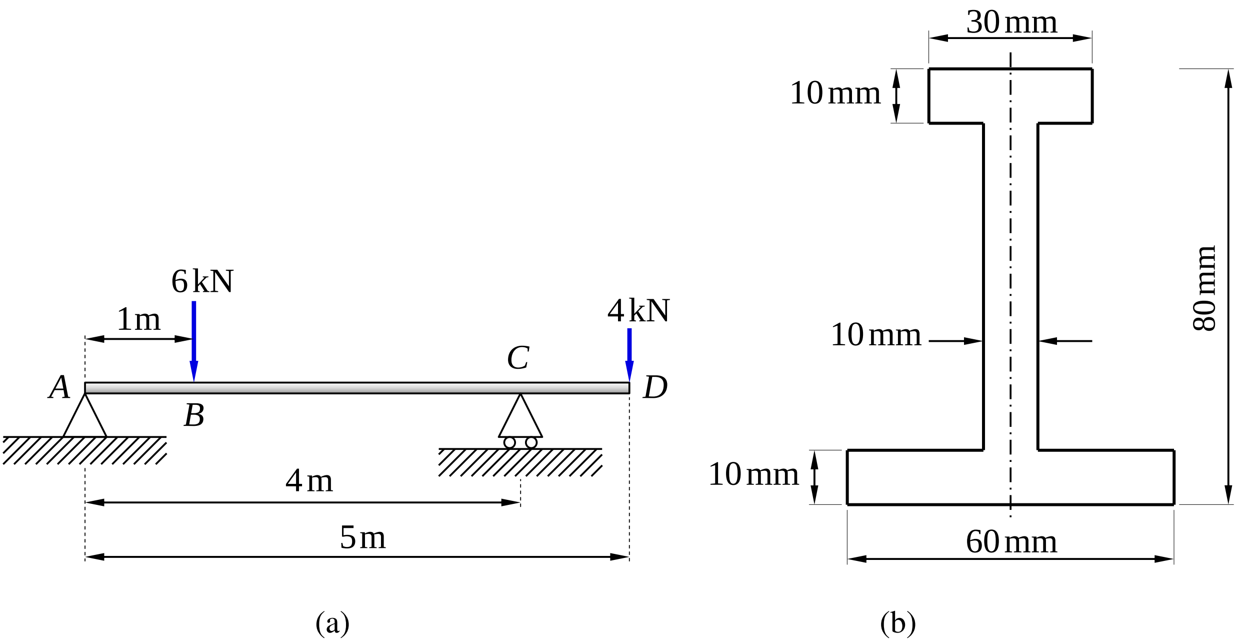
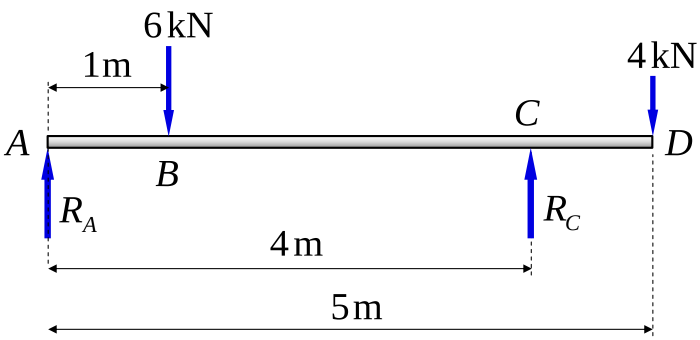


Figure Q2: A simply-supported beam (a) and detail of its cross-section (b).

Q2

a) The free body diagram is:



$$\sum M^{CW}_{@C} = 0$$

$$R_A(4\,\text{m}) - (6\,\text{kN})(3\,\text{m}) + (4\,\text{kN})(1\,\text{m}) = 0$$

$$R_A = 3.5\,\text{kN}$$

$$\sum F = 0$$

$$R_A + R_C = 6\,\text{kN} + 4\,\text{kN}$$

$$R_C = 6.5\,\text{kN}$$

$$\sum M^{CW}_{@B} = 0$$

$$R_A(1\,\text{m}) - M_B = 0$$

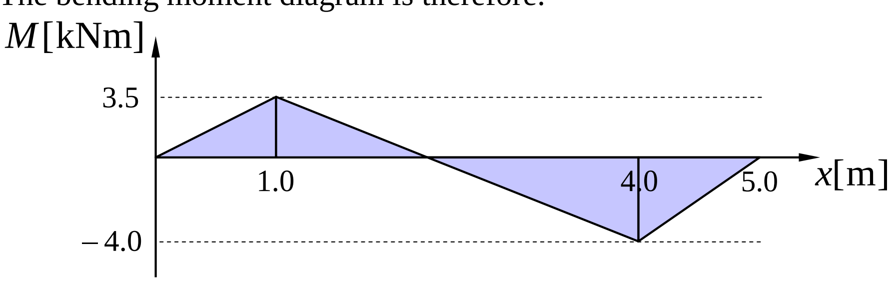
$$M_B = 3.5\,\text{kNm}$$

$$\sum M^{CW}_{@C} = 0$$

$$M_C + (4\,\text{kN})(1\,\text{m}) = 0$$

$$M_C = -4\,\text{kNm}$$

The bending moment diagram is therefore:



Q2 (cont.)

b) The second moment of area must be found. The neutral axis is given by:

$$\bar{y}[(60)(10)+(10)(60)+(30)(10)]=(60)(10)(5)+(10)(60)(40)+(30)(10)(75)$$
$$\bar{y}=33\text{ mm}$$

And using the parallel axis theorem:

$$I_{xx}=\left[\frac{1}{12}(60)(10)^3\right]+[5-33]^2[(60)(10)]+\left[\frac{1}{12}(10)(60)^3\right]+[40-33]^2[(10)(60)]+\left[\frac{1}{12}(30)(10)^3\right]+[75-33]^2[(30)(10)]$$
$$I_{xx}=1.2165\times10^6\text{ mm}^4=1.2165\times10^{-6}\text{ m}^4$$

The stresses due to bending are given by: $\sigma=\dfrac{-M\,y}{I}$

At $x=1\text{m}$, $M_{(x)}=3.5\text{kNm}$ and:

$$\sigma_{\text{top}}=\frac{-(3.5\text{kNm})(80-33)\times10^{-3}\text{ m}}{1.2165\times10^{-6}\text{ m}^4}$$
$$\sigma_{\text{top}}=-1.3522\times10^8\frac{\text{N}}{\text{m}^2}=-1.3522\times10^2\frac{\text{N}}{\text{mm}^2}=-135.22\text{MPa}$$

$$\sigma_{\text{bottom}}=\frac{-(3.5\text{kNm})(-33)\times10^{-3}\text{ m}}{1.2165\times10^{-6}\text{ m}^4}$$
$$\sigma_{\text{bottom}}=9.4945\times10^7\frac{\text{N}}{\text{m}^2}=9.4945\times10^1\frac{\text{N}}{\text{mm}^2}=94.94\text{MPa}$$

At $x=4\text{m}$, $M_{(x)}=-4\text{kNm}$ and:

$$\sigma_{\text{top}}=\frac{-(-4\text{kNm})(80-33)\times10^{-3}\text{ m}}{1.2165\times10^{-6}\text{ m}^4}$$
$$\sigma_{\text{top}}=1.5454\times10^8\frac{\text{N}}{\text{m}^2}=1.5454\times10^2\frac{\text{N}}{\text{mm}^2}=154.54\text{MPa}$$

$$\sigma_{\text{bottom}}=\frac{-(-4\text{kNm})(-33)\times10^{-3}\text{ m}}{1.2165\times10^{-6}\text{ m}^4}$$
$$\sigma_{\text{bottom}}=-1.0851\times10^8\frac{\text{N}}{\text{m}^2}=-1.0851\times10^2\frac{\text{N}}{\text{mm}^2}=-108.51\text{MPa}$$

Therefore the maximum tensile stress is 154.54MPa in the upper surface at point *C*, and the maximum compressive stress is -135.22MPa in the upper surface at point *B*.