Example 3.4

Calculate the shear stress distribution in the walls of the three-cell wing section shown in Figure 4, when it is subjected to an anticlockwise torque of 11.3 kN m.

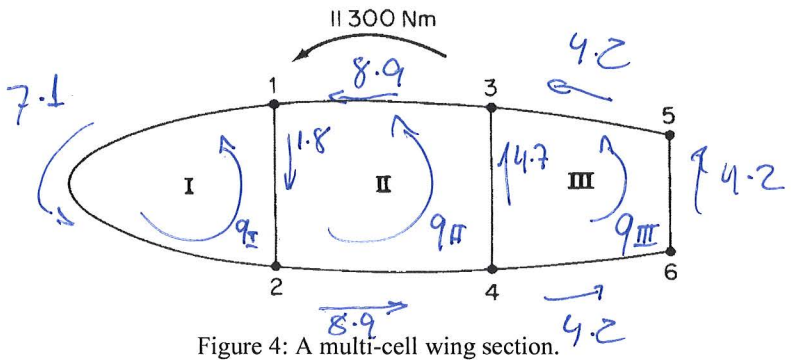

Figure 4: A multi-cell wing section.

Negson: $\delta_i = \frac{b_i}{(t_i)_{eff}}$

Table 4: Wall properties for the wing section.

Wall	Length (mm)	t_i Thickness (mm)	$(t_i)_{eff}$	G (N/mm^2)	Cell area (mm^2)	δ_i
Outer segment (leading edge) 12^o	1650	1.22	1.07	24 200	$A_I = 258\,000$	1542
Inner segment (vertical web) 12^i	508	2.03	2.03	27 600 = G_{ref}	$A_{II} = 355\,000$	250
13, 24	775	1.22	1.07	24 200	$A_{III} = 161\,000$	724
34	380	1.63	1.63	27 600 = G_{ref}		233
35, 46	508	0.92	0.69	20 700		736
56	254	0.92	0.69	20 700		368

"Scaled area Method"

Choose $G_{ref} = 27.6$ GPa (chosen arbitrarily)

Find $(t_i)_{eff}$ (effective thickness will depend on chosen Gref)

$$\frac{d\theta_j}{dz} = \frac{1}{2A_j G_{ref}} \oint q \frac{ds}{t_{eff}} \simeq \frac{1}{2A_j G_{ref}} \sum q_i \frac{b_i}{(t_i)_{eff}}$$

Cell (I)

$$\frac{d\theta_I}{dz} = \frac{q_I\left(\frac{1650}{1.07} + \frac{508}{2.03}\right) - q_{II}\left(\frac{508}{2.03}\right)}{2\left(258\,000\ mm^2\right)\left(27\,600\ \frac{N}{mm^2}\right)}$$

$$\left(1.424 \times 10^{10}\right)\frac{d\theta_I}{dz} = 1792\, q_I - 250\, q_{II}$$

Cell (II)

$$\frac{d\theta}{dz} = \frac{q_{II}\left[\frac{508}{2\cdot03} + \left(\frac{775}{1\cdot07}\right) \times 2 + \frac{380}{1\cdot63}\right]}{2(335\,000)(27600)}$$

$$(1.960 \times 10^{10})\frac{d\theta}{dz} = -250\,q_{I} + 1931\,q_{II} - 233\,q_{III}$$

Cell (III)

Using tabulated values of $\delta_i = \frac{b_i}{(t_i)_{eff}}$

$$\frac{d\theta}{dz} = \frac{1}{2(161\,000)(27\,600)}\left[q_{III}(736 + 233 + 736 + 368) - q_{II}(233)\right]$$

$$(0.889 \times 10^{10})\frac{d\theta}{dz} = -233\,q_{II} + 2073\,q_{III}$$

$$T = \int q\,r\,ds = \sum_j 2A_j q_j$$

$$11\cdot3 \times 10^{6}\ N\,mm = 2(258\,000\,q_{I} + 355\,000\,q_{II} + 161\,000\,q_{III})$$

$$5.65 \times 10^{3} = 258\,q_{I} + 335\,q_{II} + 161\,q_{III}$$

4 eqs., 4 unknowns :

q_I = 7089 N/m

q_{II} = 8880 N/m

q_{III} = 4154 N/m

$\frac{d\theta}{dz}$ = 7.36×10^{-4} rad/m