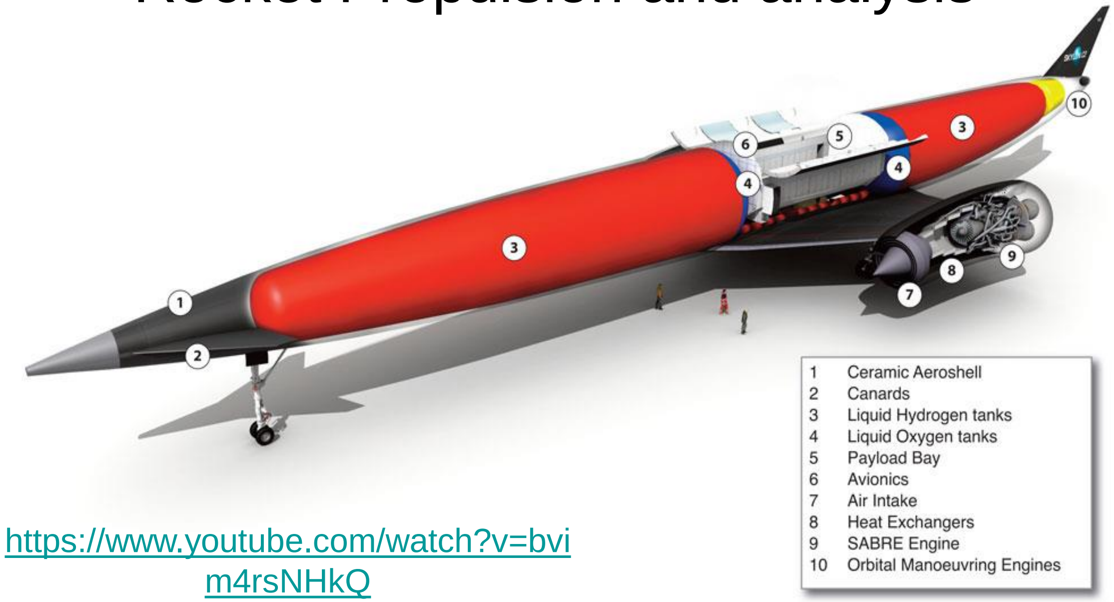
Space Systems

Rocket Propulsion and analysis



https://www.youtube.com/watch?v=bvim4rsNHkQ

Learning Objectives

1. Principles of a rocket
2. The rocket equation
3. Specific impulse
4. Performance parameters (thrust coefficient, C*,C)

Be able to describe what a rocket is, how it is different and why that is important for space
Be familiar with the derivation of the Rocket equation and know and be able to use it
Be able to define and use Specific impulse in calculations
Be familiar with and able to use critical performance parameters (thrust coefficient, C*,C)

Reading

Have a look at:

- "Fundamentals of Space Systems" by Vincent Pisacane, 2nd Edition, Publ. Oxford University Press, 2005
- http://www.braeunig.us/space/propuls.htm

What is a Rocket?

- Any motor that carries *its own reaction mass (ie: mass to expel) and which carries its own oxidant*
- One of the few propulsion methods that will work in the vacuum of space
- Rocket = launcher/subsystem
- Propellant = Fuel+Oxidiser

Rockets will also work underwater, early rocket went into water, fizzed around and then came back out heading towards launch crew!

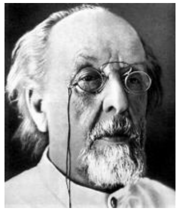
Rocket propulsion

- From Newton: $F = ma = \frac{d(mV)}{dt}$ (8-1)
- *Force = time derivative of linear momentum*
- So, if a small amount of mass is thrown out the back at a high velocity, the larger mass remaining will accelerate slightly in the opposite direction
- The increment in velocity achievable by a rocket is defined by the *Rocket Equation*

All acceleration requires an exchange of momentum. Momentum is related to mass and velocity. The velocity of a body is easily changeable, but in most cases the mass is not, which makes it important.

Rocket Equation

- Simple application of Newton's laws, also named *Tsiolkovsky's Equation* after Konstantin Tsiolkovsky.



Mass m and velocity V

Discharge with mass dm and *relative* velocity V_e

From Newton: $$\Sigma F = \frac{d(mV)}{dt} = m\frac{dV}{dt} + V\frac{dm}{dt} \qquad (8\text{-}2)$$

If we apply this to the rocket where no external forces are acting:

$$\Sigma F = m\frac{dV}{dt} + V\frac{dm}{dt} = 0$$

$$dV = -V\frac{dm}{m}$$

where *V* is assumed constant, and is the *effective exhaust velocity relative to the rocket* (positive in direction of motion). It is often written 'C'.

Imagine you are sitting on the rocket, so V=0
Note that Ve is relative to rocket, so Vobs= V+Ve

Integrate over burn time:

$$\int_{V}^{V+\Delta V} dV = -C \int_{m_0}^{m_0-\Delta m} \frac{dm}{m} \qquad (8\text{-}3)$$

$$\Delta V = -C.\ln\left(\frac{m_0-\Delta m}{m_0}\right) \qquad (8\text{-}4)$$

$$\Delta V = C.\ln\left(\frac{m_0}{m_0-\Delta m}\right) \qquad (8\text{-}5)$$

(mass ratio)

m0 is the start mass, whereas m0-Δm is the final mass (the Δm being the propellant mass) and we know that the integral (or antiderivative) of 1/x is ln(x)

Rocket Equation

$$\Delta V = C.\ln\left(\frac{m_0}{m_0 - \Delta m}\right) \qquad (8\text{-}6)$$

- The only things that affect ΔV are:
  - The effective exhaust velocity, 'C'
  - The mass ratio: initial to final mass
- Explains why mass is so critical!

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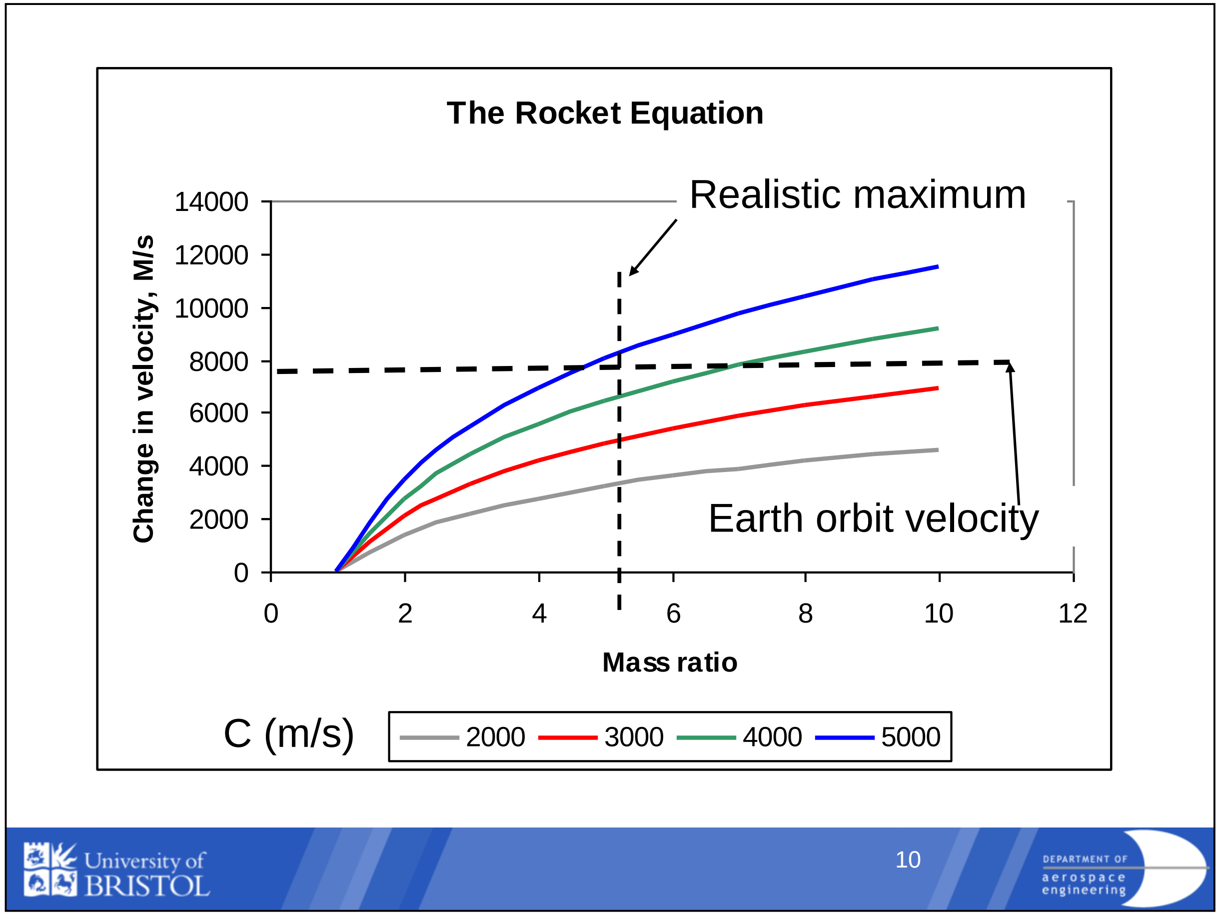
m_0 is the initial mass (rocket plus contents plus propellant)
m_0 -Δm is the final mass (rocket plus contents)



We can rewrite the equation: mass ratio=$e^{dV/C}$

We know that Earth orbit velocity is around 7.7km/s (depending on r). Typical maximum mass ratio for a single stage rocket is 5.5.

Even with Hydrogen which has a C of 4500m/s you would need a mass ratio of 6. The mass ratio is a useful quantity for back-of-the-envelope rocketry calculations. You can see it is virtually impossible to launch a rocket to orbit like this, which is why we have staging.

Specific impulse

- Impulse = Thrust 'F' x time
- So specific impulse is:

$$\boxed{I_{sp} = \frac{F\Delta t}{\Delta m} = \frac{F}{\dot{m}}} \qquad (8\text{-}7)$$

- Units- there are three alternatives:

$$\frac{Ns}{kg}$$

Or from F = ma: $\dfrac{Ns}{kg} = \dfrac{kgms}{s^2kg} = \dfrac{m}{s}$

I_{sp} is sometimes divided by '*g'* to give units of 's'

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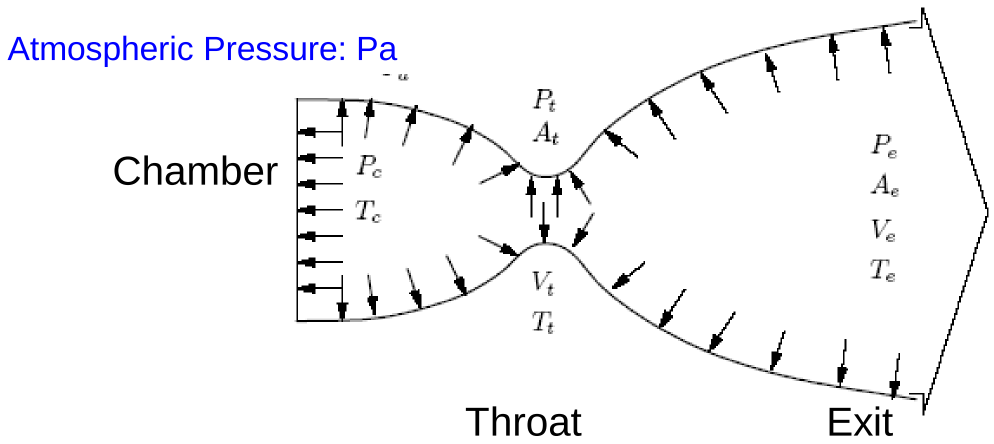
Remember that N= kgm/s2
'Specific' things are the thing divided by mass (e.g. specific enthalpy, etc.).
Remember that thrust is force.
g is the acceleration due to gravity ie: 9.8m/s2
Specific impulse is the impulse per unit propellant (a bit like fuel consumption of a car), thrust is very different.

Motor efficiency measures



Integrating forces acting on thrust chamber gives:

$$F = \dot{m}V_e + A_e(P_e - P_a) = \dot{m}C \qquad (8\text{-}8)$$

Actual 1D exhaust Velocity

Effective 1D exhaust Velocity

We will see that *Ve* and *Pe* are inversely proportional, that is, as one increases the other decreases. If you got a radar gun and measure the exhaust velocity you would get a reading of Ve. C is exhaust velocity but it is a generalised “effective” velocity and wraps up several losses of efficiency in to the equation. We will see that C and Ve are only the same for a perfectly expanded nozzle where Pe-Pa=0 ie: Pe=Pa. This is when we get maximum thrust.

However,

- Total Impulse:

$$I_t = I_{sp}\Delta m = \frac{F\Delta t}{\dot{m}} = F\Delta t = \dot{m}C\Delta t \qquad (8\text{-}9)$$

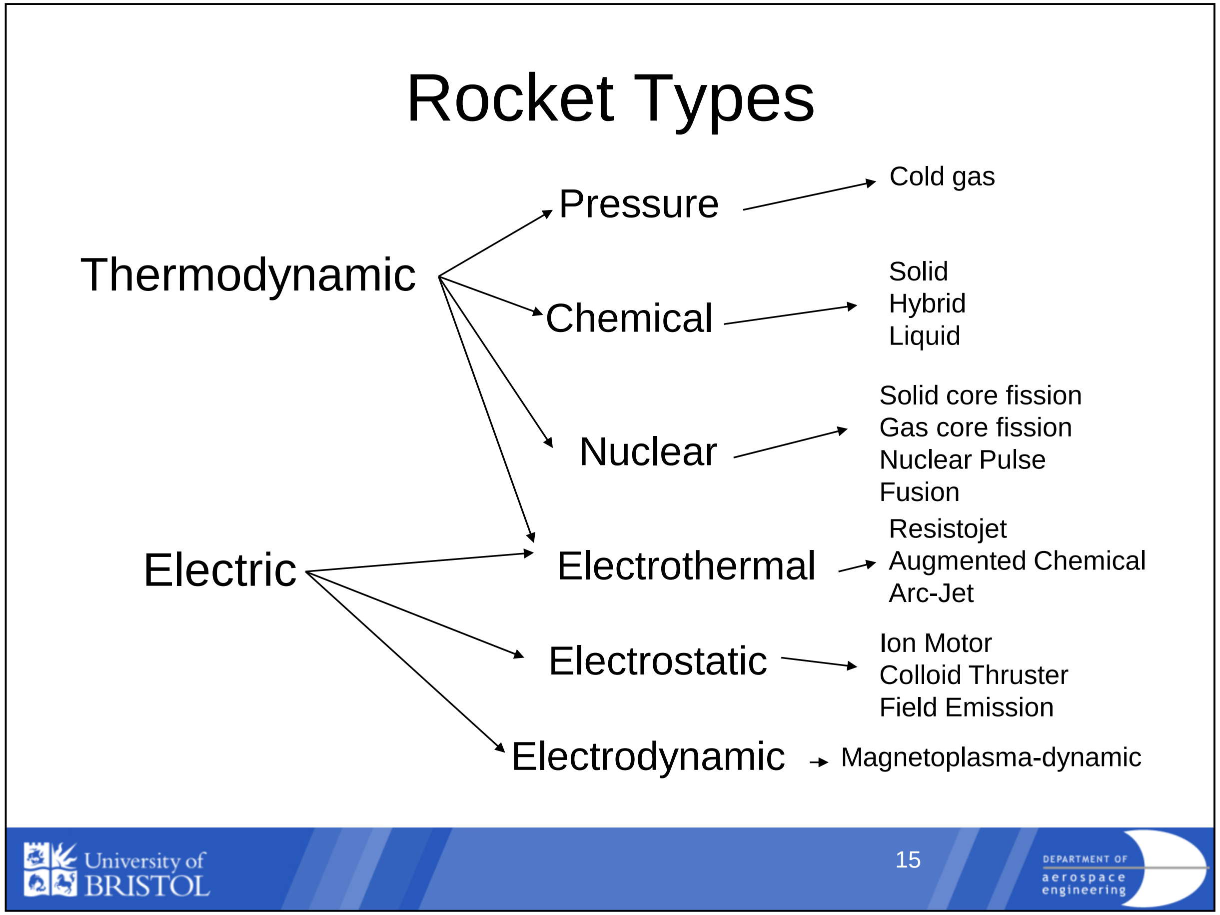
- So $C = \frac{F}{\dot{m}}$ =specific impulse = I_{sp} (8-10)

- So, effective exhaust velocity and Specific Impulse are the same thing!

Types of Rocket

- Everything so far applies to all rockets
- In principle, all a rocket motor does is transfer energy from a source to a fluid, which is then ejected
- This means rockets may be thermodynamic (heat a gas adiabatically), or electric (use electro-magnetic forces to accelerate propellant)

Adiabatic = net heat transfer to or from fluid is zero



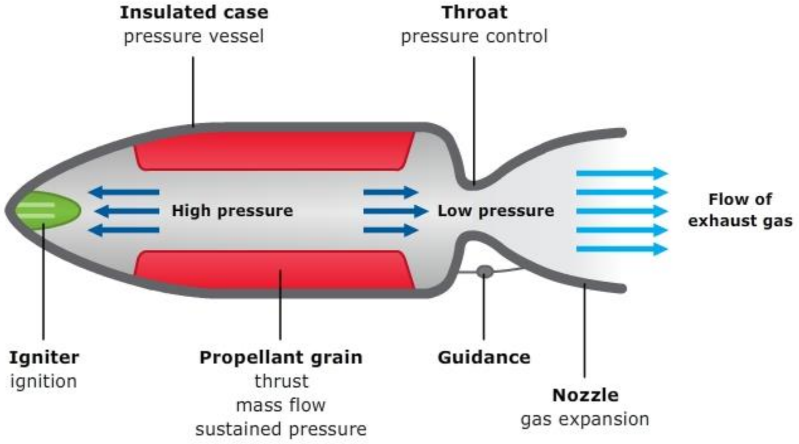
We will concentrate here on thermodynamic rockets as they have dominated the field so far.
In particular, we shall focus on Chemical rockets. However, in advanced space we will look electrical rockets, as they may become more common in the future.

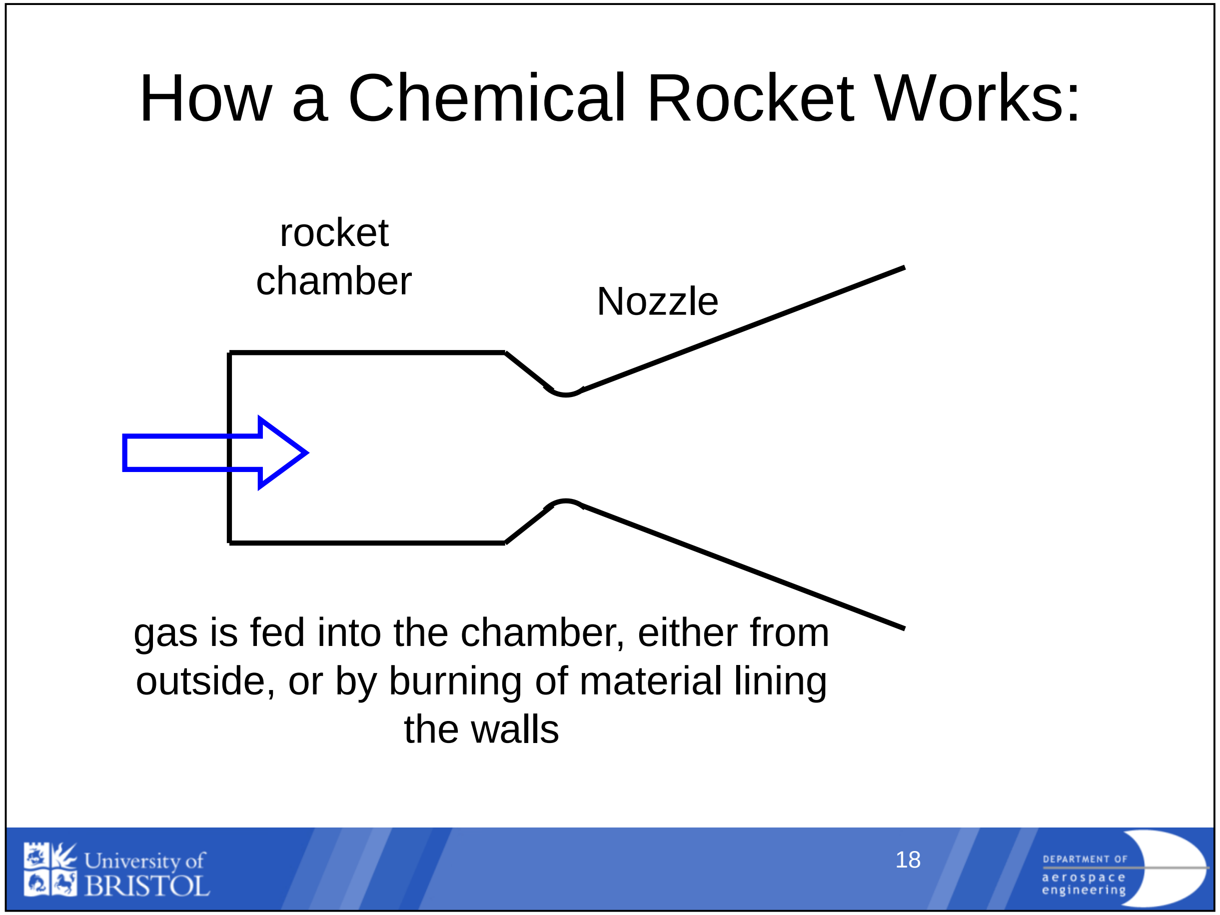
Chemical Rockets

- In simple terms, potential energy stored in chemical bonds is released through some form of reaction, heating up a gas. This hot gas expands to high velocity, creating thrust.
- Various types:
  - Liquid mono-, bi- (even tri-) propellants
  - Solids
  - Hybrids

Monopropellant eg: hydrazine (needs catalyst for combustion)
Bi propellants eg: UDMH and N_2O_4 have to keep the fuel and the oxidizer apart
Solid propellants eg: a composite of rubber +ammonium perchlorate baked together, are not re-startable eg: ABMs
Hybrids are where you use one solid, one gas/liquid.

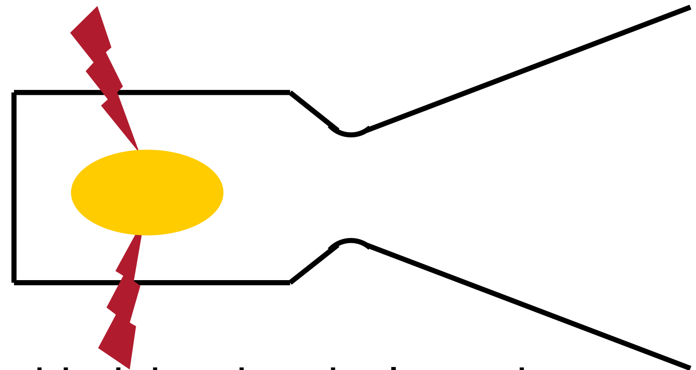
Typical (solid) rocket engine



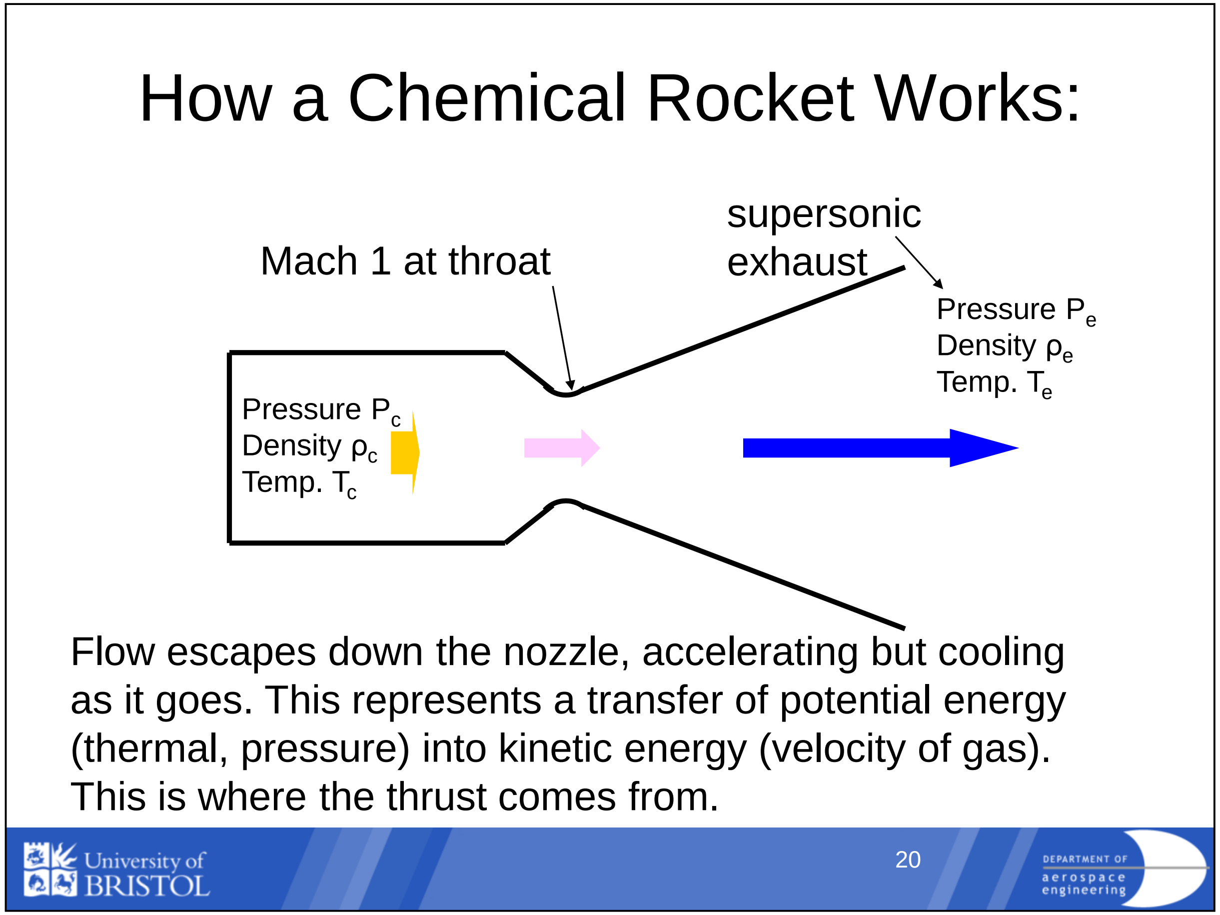


This is rocket chamber, this is nozzle.
Expansion of hot gas against walls of chamber does the work
If it is liquid, the propellant and oxidant are pumped in,
If solid, the propellant and oxidant are premixed and combustion takes place on inner surface of solid propellant.

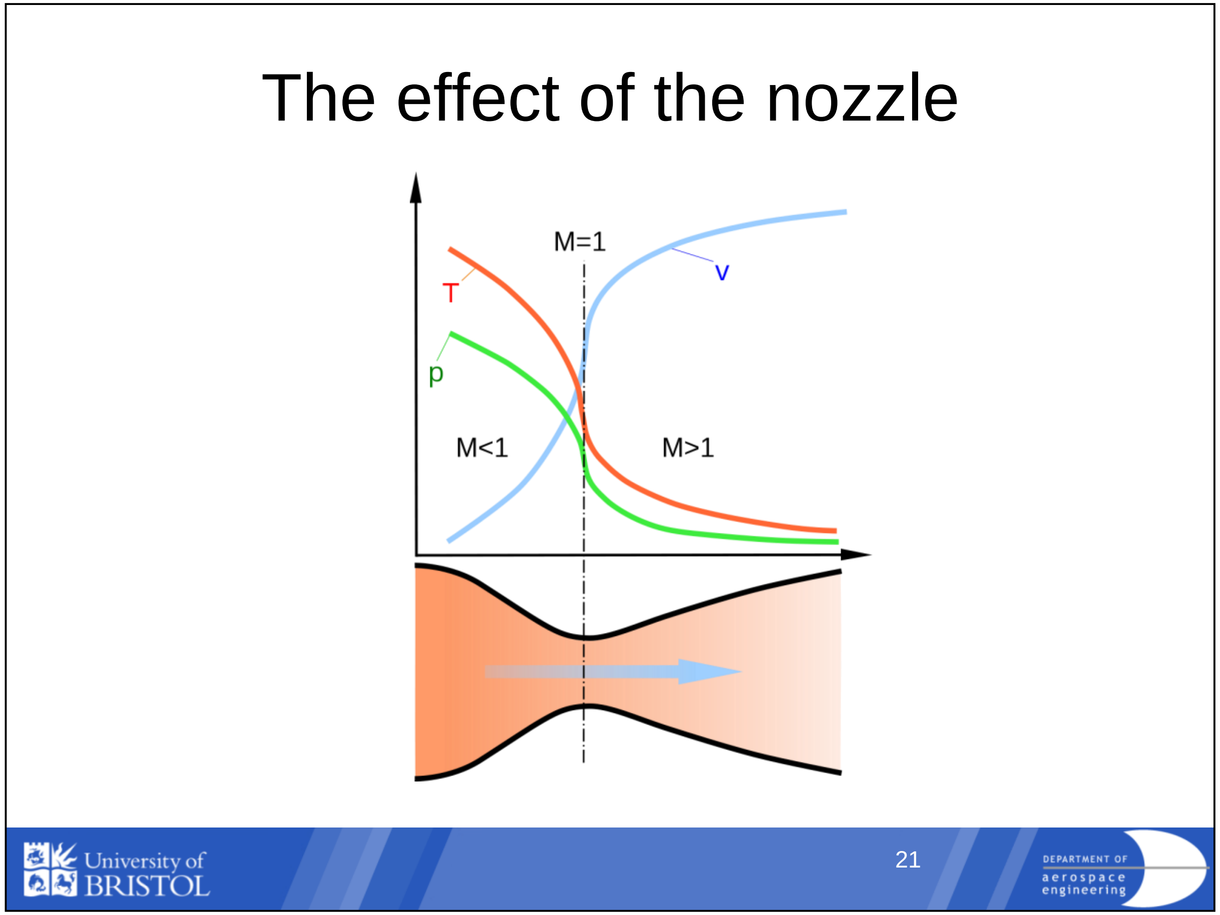
How a Chemical Rocket Works:



heat added, by chemical reactions or other means. Random kinetic motion of molecules increases (this is what temperature is)



The function of the nozzle is to convert the chemical-thermal energy generated in the combustion chamber into kinetic energy. The nozzle converts the slow moving, high pressure, high temperature gas in the combustion chamber into high velocity gas of lower pressure and temperature. Gas velocities from 2-4.5 km/s can be obtained in rocket nozzles. These nozzles are called DeLaval nozzles and consist of a convergent and divergent section. The minimum flow area between the convergent and divergent section is called the nozzle throat. The flow area at the end of the divergent section is called the nozzle exit area.



Temperature T and Pressure P decrease.
Velocity v increases from <Mach 1 before the throat, to Mach 1 at the throat, to >Mach 1 after the throat.

Let's try and find the exhaust velocity 'v_e' from conditions at chamber and exit

- We can use:
  - Laws of thermodynamics
  - Ideal gas laws
  - Isentropic flow
  - The fact that initial velocity is zero (in chamber)

Terms

p = pressure

V = volume of container

v = velocity

T = absolute temperature (K)

R = universal gas constant: 8314.472 J/kmol.K

h = sp.enthalpy

M = molecular mass of propellant (kg/kmol)

γ = ratio of specific heat (const P) to specific heat (const vol)

Revision: 1st law of thermodynamics and ideal gas laws:

$$h_i - h = \tfrac{1}{2}v^2 - \tfrac{1}{2}v_i^2 = \frac{\gamma.R(T_i - T)}{(\gamma - 1)M} \qquad (8\text{-}11)$$

We can rewrite this:

$$v = \left[\frac{2\gamma.RT_i}{(\gamma-1)M}\left(1 - \frac{T}{T_i}\right) + v_i^2\right]^{\frac{1}{2}} \qquad (8\text{-}12)$$

The *first law of thermodynamics* is the application of the conservation of energy principle to heat and *thermodynamic* processes: because energy is conserved, the internal energy of a system changes as heat flows in or out of it. Delta U=Q-W. If you want a derivation of 8-11, you could look at:
http://www.grc.nasa.gov/WWW/k12/airplane/thermo1f.html
V here is exhaust velocity
Look at two points in the flow initial ‘i’ and some point after…

From isentropic flow (adiabatic and reversible) and the ideal gas law:

$$\frac{P_e}{P_c} = \left(\frac{T_e}{T_c}\right)^{\frac{\gamma}{\gamma-1}}, \quad \text{or} \quad \frac{T_e}{T_c} = \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}} \qquad (8\text{-}13)$$

in the case of initial condition in the Chamber= c, and final condition at the exhaust=e.

We will also need from continuity:

$$\dot{m} = \rho A v = constant \qquad (8\text{-}14)$$

If the flow is gradually compressed (area decreases) and then gradually expanded (area increases), the flow conditions return to their original values. We say that such a process is reversible. From a consideration of the second law of thermodynamics, a reversible flow maintains a constant value of entropy. We call this type of flow an isentropic flow. Continuity equation gives us a constant value for mass flow rate in terms of density, area and velocity. Equation for an ideal gas: PV=nRT. Adiabatic – no heat transfer. Notice V=volume in ideal gas equation, but otherwise v=velocity

- Substituting (8-13) into (8-12), $v_i \rightarrow 0$, gives:

$$v_e^2 = \frac{2\gamma . RT_c}{(\gamma - 1)M}\left(1 - \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right) \qquad (8\text{-}15)$$

We want a big v_e so:

- We need $P_e/P_c \rightarrow 0$, Molecular mass M↓ and T_c↑
- These can be grouped into two numbers characterising the engine: C* and C_F

This will always be quoted for you, you do not need to learn it by heart. C* and CF can be measured
You can see why Hydrogen is so desirable as rocket propellant
1.7 to 2.9 km/s for liquid monopropellants
2.9 to 4.5 km/s for liquid bipropellants
2.1 to 3.2 km/s for solid propellants

Thrust Coefficient, C_F

Measure of efficiency (“figure of merit”) for nozzle design:

- Defined by:

$$C_F = \frac{F}{P_c A_t} \qquad (8\text{-}16)$$

- Or:

$$C_F = \frac{\dot{m} V_e + A_e (P_e - P_a)}{P_c A_t} \qquad (8\text{-}17)$$

P in pascals, A in m^2. We can substitute our equation (8-8) in for thrust F to get (8-17). Thrust coefficient depends primarily on (P_c/P_a) so it is a good indicator of *nozzle* performance.

Characteristic Exhaust Velocity C*

- Represents the 1D flow velocity at the throat
- A measure of efficiency of propellant and combustion chamber design
- Defined as:

$$C^* = \frac{C}{C_F} = \frac{F/\dot{m}}{F/_{P_c A_t}\, P_c}$$

$$C^* = \frac{P_c A_t}{\dot{m}} \quad (8\text{-}18)$$

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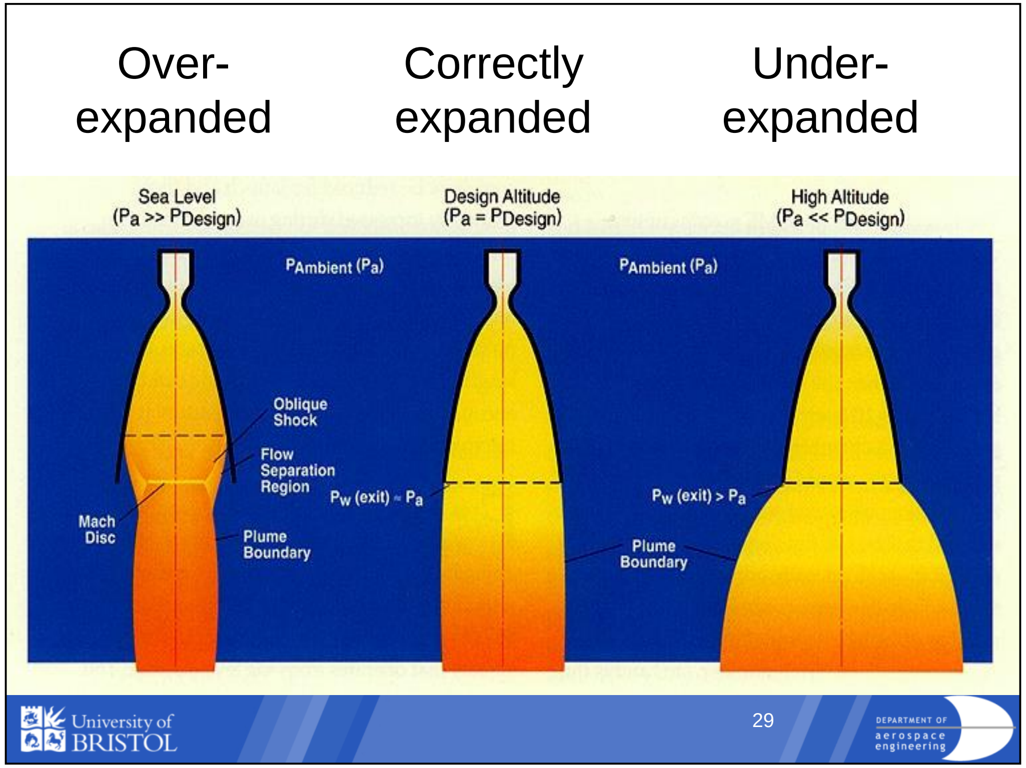
If we substituted in for mass flow=ro.A.v, we would see that C* also depends on R/M, Tc, and gamma so it is affected by propellants, mixture ratio and combustion efficiency. Need – high T_c, high R/M (low molecular weight). Not much can be done about gamma. C* is independent of nozzle performance. High T_c value desirable for high Isp - but gives problems with heat transfer into case walls and dissociation of combustion products – practical limit 2750 -3500 K, depending on propellant. So, if you burn hydrogen and oxygen the gases will be a given temperature, but actually if they are only 96% as hot as they should be, because they aren't mixing well, it would be accounted for in the C* value.



Nozzles are only optimum for a certain altitude. At sea level, the atmospheric pressure is greater than exit pressure and the flow is called ‘over expanded’. The pressure term becomes negative, you get suction and shocks forming with separation. This can be dangerous. In space, however, Pa=0, so flow is ‘underexpanded’. If the atmospheric pressure is lower than the exit pressure, it is called under-expanded. In this case, the flow continues to expand outward after it has exited the nozzle. This behaviour reduces efficiency because the external expansion does not exert any force on the nozzle wall. This energy can therefore not be converted into thrust and is lost. Ideally, the nozzle should be longer to capture this expansion and convert it into thrust. If the nozzle is under or overexpanded then loss of efficiency occurs. Altitude-compensating nozzles have been developed to overcome this.

Nozzle performance

- Expression for V_e (8-15) implies low P_e useful
- But if P_a high (low altitude), pressure term becomes negative and get suction (over-expansion)
- If this is too severe, get separation
  - unstable
  - dangerous
- Most nozzles are designed not to separate

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Summary

- A rocket is any motor that carries *its own reaction mass (ie: mass to expel) and its own oxidant*
- Tsiolkovsky's rocket equation: $\Delta V = C.\ln\left(\dfrac{m_0}{m_0 - \Delta m}\right)$
- Specific impulse: $I_{sp} = \dfrac{F\Delta t}{\Delta m} = \dfrac{F}{\dot{m}}$

$$F = \dot{m}V_e + A_e(P_e - P_a) = \dot{m}C$$

- Propellant and combustion chamber design: $C^* = \dfrac{P_c A_t}{\dot{m}}$
- Nozzle design, thrust coefficient: $C_F = \dfrac{F}{P_c A_t}$

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Test Yourself! (Feedback)

1. What conditions are required for maximum thrust?
2. A spacecraft's engine ejects mass at a rate of 30 kg/s with an exhaust velocity of 3.1 km/s. The pressure at the nozzle exit is 5 kPa and the exit diameter is 0.95 m. What is the thrust of the engine in a vacuum?
3. Find the Characteristic Exhaust Velocity (C*) for perfect expansion if:

Throat diameter= 0.16 m	Nozzle exit diameter= 0.23 m
Constant Thrust= 18450N	Chamber pressure = 8 x 10^5 Pa
Time of burn = 80 sec	Mass of solid propellant = 400 kg