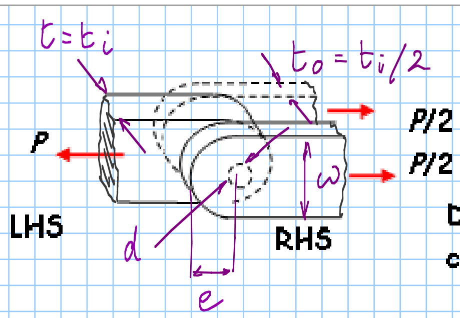
StM1 Structural Design - Joint Examples : Solutions.

IRF
13.3.2013

Note Title 13/03/2013

1

Q1 Pinned Joint



$P = 10\,kN$

Double shear lug configuration

<u>Lugs</u> – Checking inner lug (Outer lugs covered by this check since load $= P/2$ and $t_o = t_i/2$)

Tension : $\sigma_T = \dfrac{P}{(w-d)t} = \dfrac{10'000}{(30-6)\times 10} = 41.7\ N/mm^2$: $\boxed{RF = \dfrac{400}{41.7} = 9.6}$

Shear : $\tau = \dfrac{P}{2et} = \dfrac{10'000}{2\times 15\times 10} = 33.3\ N/mm^2$: $\boxed{RF = \dfrac{230}{33.3} = 6.9}$

Bearing : $\sigma_{br} = \dfrac{P}{dt} = \dfrac{10'000}{6\times 10} = 166.7\ N/mm^2$: $\boxed{RF = \dfrac{600}{166.7} = 3.6}$

2

<u>Pin</u>

Shear : $\tau = \dfrac{P}{2\pi \frac{d^2}{4}} = \dfrac{10'000}{2\pi \frac{6^2}{4}} = 176.8\ N/mm^2$: $\boxed{RF = \dfrac{580}{176.8} = 3.3}$

Bearing : Assumed 2ndary to lug here.

Bending : $\sigma_B = \dfrac{My}{I}$

where $M = \dfrac{P}{2}\left(\dfrac{t_o}{2} + \dfrac{t_i}{4} + g\right) = \dfrac{10'000}{2}\left(\dfrac{5}{2} + \dfrac{10}{4} + 0\right) = 25'000\ Nmm$

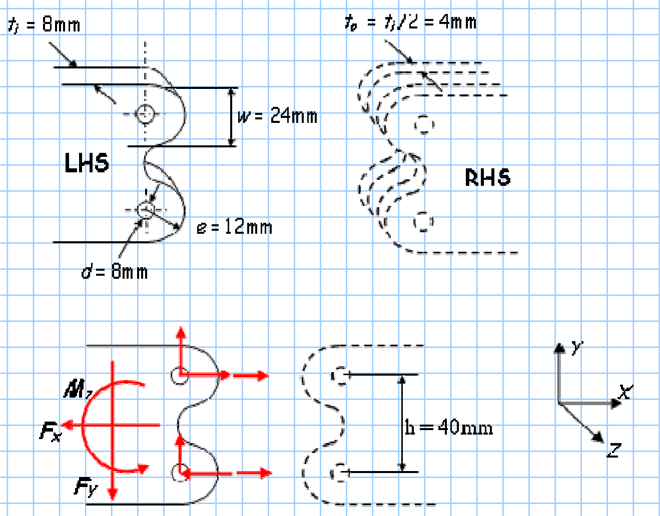
$y = \dfrac{d}{2} = \dfrac{6}{2} = 3\,mm$

$I = \dfrac{\pi d^4}{64} = \dfrac{\pi \times 6^4}{64} = 63.6\ mm^4$

$\hookrightarrow \sigma_b = \dfrac{25'000\times 3}{63.6} = \underline{1'179\ N/mm^2} \rightarrow \boxed{RF = \dfrac{1000}{1'179} = 0.85}$ *

RF<1 but conservative.

Q2 Fixed Joint – two fasteners



M_z = 0.5kNm, F_y = 1.5kN, F_x = 5kN at limit load

Calculate lug loading then proceed as for Q1

Here max lug load will be on upper lug since F_x and M_z/h couple loads will add as a tensile force

I.e. max lug load $P_x = \frac{F_x}{2} + \frac{M_z}{h} = \frac{5000}{2} + \frac{500'000}{40} = 15'000 \text{ N} = 15 \text{ kN}$

$$P_y = \frac{F_y}{2} = \frac{1.5}{2} = 0.75 \text{ kN}$$

Resultant $P = \sqrt{P_x^2 + P_y^2} = \sqrt{15^2 + 0.75^2} = 15.02 \text{ kN @ Limit}$

↳ 22.53 kN @ Ultimate

Note, influence of F_y is low so just consider as an axial lug load

Now, proceed as for Q1. ↳ Lug : $\sigma_t = 176 \text{ N/mm}^2$ $\boxed{RF = 2.5}$

$\sigma_{br} = 352$ " $\boxed{RF = 1.9}$

$\tau_{so} = 117$ " $\boxed{RF = 2.2}$

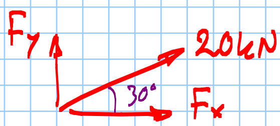
Pin : $\tau = 224$ " $\boxed{RF = 3.1}$

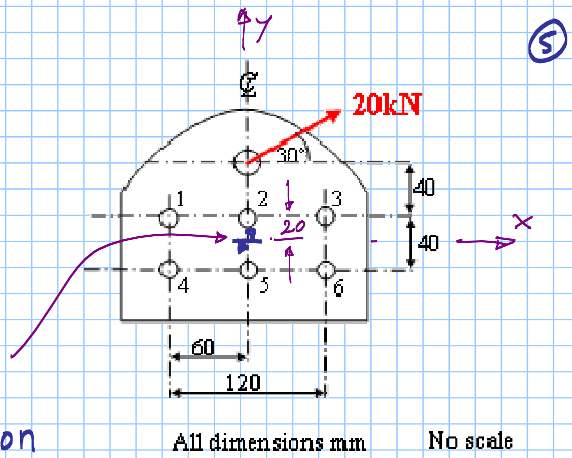
$\sigma_B = 896.3$ " $\boxed{RF = 1.3}$ ← CONSERVATIVE

⑤

Q3 Fixed Joint - multiple fasteners.

Applied load as concentric + eccentric components:





$F_x = 20 \cos 30° = 17.32 \text{ kN}$

$F_y = 20 \sin 30° = 10 \text{ kN}$

$M_z = 20 \cos 30° \times (-60) = -1039 \text{ kNmm}$

	Applied	×1.5		
	LIM	LIM	ULT	
Fx	17320.51	17320.51	25980.7621	N
Fy	10000	10000	15000	N
Mz	-1039230	-1039230	-1558845.7	Nmm

Note -ve since +ve Mmt is a.clock according to RH rule

Next, calculate the concentric and eccentric reactions at each fastener.

Tabulate! Use spreadsheet if pc available. (But check by hand!)

⑥

Note, signs based on applied forces + mmts and reacted fastener loads

Rivet	Co-ordinates (mm)			Loads (N) ULT				Totals		Resultants
	r_x	r_y	$r_x^2+r_y^2$	$-\frac{F_x}{n}$	$-\frac{F_y}{n}$	$\frac{M r_{y_i}}{\Sigma(r_x^2+r_y^2)}$	$\frac{-M r_{x_i}}{\Sigma(r_x^2+r_y^2)}$	$P_{cx_i}+P_{ex_i}$	$P_{cy_i}+P_{ey_i}$	$\sqrt{P_{x_i}^2+P_{y_i}^2}$
				Concentric		Eccentric				
i	rx	ry	rx^2+ry^2	Pcxi=-...	Pcyi=-...	Pexi=+...	Peyi=-...	Pxi	Pyi	Pi
	mm	mm		N	N	N	N	N	N	kN
1	-60	20	4000	-4330.1	-2500.0	-1855.8	-5567.3	-6185.9	-8067.3	10.17 *
2	0	20	400	-4330.1	-2500.0	-1855.8	0.0	-6185.9	-2500.0	6.67
3	60	20	4000	-4330.1	-2500.0	-1855.8	5567.3	-6185.9	3067.3	6.90
4	-60	-20	4000	-4330.1	-2500.0	1855.8	-5567.3	-2474.4	-8067.3	8.44
5	0	-20	400	-4330.1	-2500.0	1855.8	0.0	-2474.4	-2500.0	3.52
6	60	-20	4000	-4330.1	-2500.0	1855.8	5567.3	-2474.4	3067.3	3.94
Σ			Σ(rx^2+ry^2)							
6			16800							

$\Sigma(r_x^2+r_y^2)$

Loads calculated at ULT

Minimum RF's:

Fastener 1: Shear: $\tau = \frac{P_i}{\frac{\pi d^2}{4}} = \frac{10'170}{\frac{\pi 6^2}{4}} = 360 \text{ N/mm}^2$: $\boxed{RF = \frac{500}{360} = 1.39}$

Bearing $\sigma_{br} = \frac{P_i}{dt} = \frac{10'170}{6 \times 2} = 847 \text{ N/mm}^2$: $\boxed{RF = \frac{1000}{847} = 1.18}$