Element Design

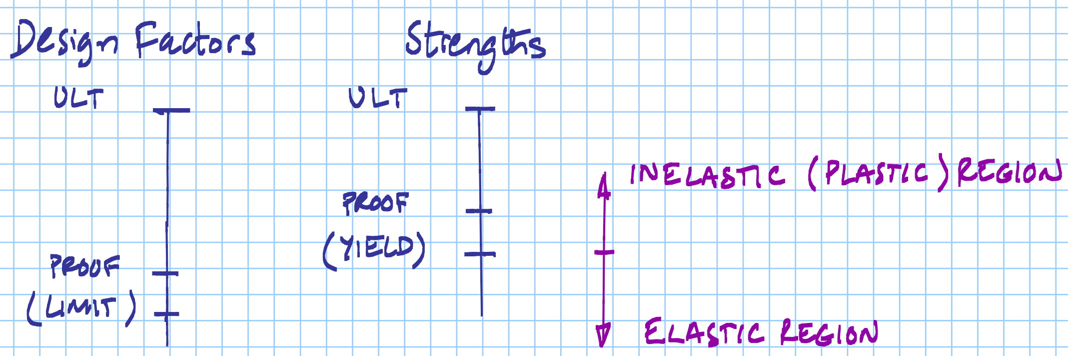
Using "Static Analysis" i.e. considering static "Load Cases" which represent "snap shots" of loading configurations, we can check stiffness, strength and stability against allowable values:

E.g.:
Stiffness: deflection at limit c/w allowable deflection
Strength: stress at proof or ultimate c/w yield, proof or ultimate* strength
Stability: stress at proof or ultimate c/w critical buckling strength

For most types of conventional design, e.g. land based machines or structures, the yield strength or proof stress is commonly used as the criterion for failure.

For airframe design we normally use the ultimate strength as the criterion for failure to allow greater efficiency. However, this means that before failure, the stress will exceed the elastic limit and enter the "inelastic" or "plastic" range.

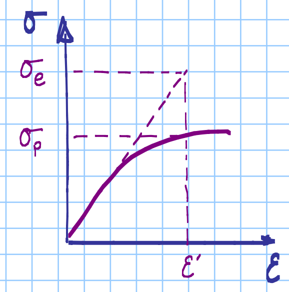
Note, if the ratio of ultimate factor: proof factor exceeds the ratio of ultimate strength: proof strength then the check at proof can be considered covered by the check at ultimate e.g.:



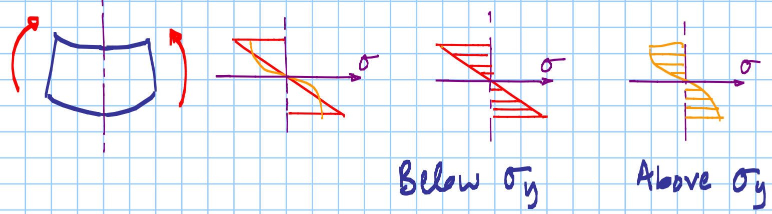
If this is not the case then the design at proof will need to be checked after initial design at ultimate.

Design in the Inelastic Region

For a given displacement or strain, linear elastic theory over-estimates stresses beyond the elastic limit, i.e. in the "inelastic" region.

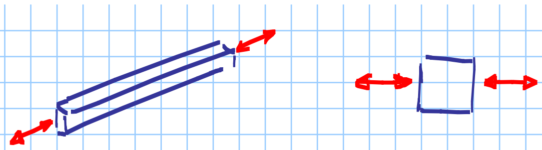


In particular, in bending or in torsion, as the material becomes plastic the stress distribution will become more uniform. I.e. the peak value will diminish.



Using elastic theory to design in the inelastic region will therefore result in some conservatism and a heavier design.

BARS: AXIAL LOADING



Stiffness: $F = kd$ ①, $\sigma = E\varepsilon$ ② : $k = \frac{AE}{L}$ ③ Assuming linear elasticity

Design for stiffness or deflection

Design statement: "Design not to exceed a deflection limit"

Design for $d < d^*$ @ limit

Stiffness is usually checked at limit or proof loading.

At limit load:

$$① : \frac{F}{k} < d^*$$

$$\hookrightarrow \quad k \geq \frac{F}{d^*}$$

$$③ \hookrightarrow \quad \frac{AE}{L} \geq \frac{F}{d^*}$$

I.e for a chosen material and associated modulus, E, we can now design the bar geometry to achieve the required stiffness.

The length may already be specified according to the allowable geometric envelope.

Strength: $\sigma = \frac{F}{A}$ ① Tension or compression axial loading.

Design Statement

"Design for stress not to exceed allowable strength

" Design for $\sigma \leq \sigma^*$

stress at limit, proof or ult* load

yield, proof or ultimate* strength $\sigma_y, \sigma_p, \sigma_u$

①: $\frac{F}{A} \leq \sigma^*$

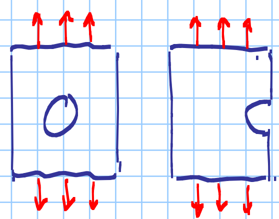
↳ $A \geq \frac{F}{\sigma^*}$

I.e.: for a chosen material we can now design the required cross-section dimensions.

But note, strength is usually driven by details!

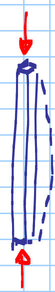
E.g. a stress concentration factor at a round hole or notch is typically x3 for an isotropic material. i.e. K_t = 3.

↳ $\sigma = K_T \cdot \frac{F}{A}$ where A is the net section area.



May need to Consder fatigue and fracture toughness" - Yr 3!

Stability Global: Euler $P_{CRIT} = k\frac{\pi^2 EI}{L^2}$



For a slender element we can assume that buckling occurs within the elastic range.

Usually more informative to work in terms of stress } $\sigma_{CRIT} = P_{CRIT}/A$

$= k\pi^2 E \frac{I}{A}\frac{1}{L^2}$

substituting: $\rho = \sqrt{\frac{I}{A}}$ "Radius of gyration"

$= k\pi^2 E\left(\frac{\rho}{L}\right)^2$

where L/ρ = "Slenderness ratio" and for a thin "Euler" strut: L/ρ >100

Design Statement

"Design for stress not to exceed the critical buckling strength"

Design for $\sigma \leq \sigma_{CRIT}$

stress at, lim, proof or ult

$\frac{F}{A} \leq k\pi^2 E\left(\frac{\rho}{L}\right)^2$

etc.

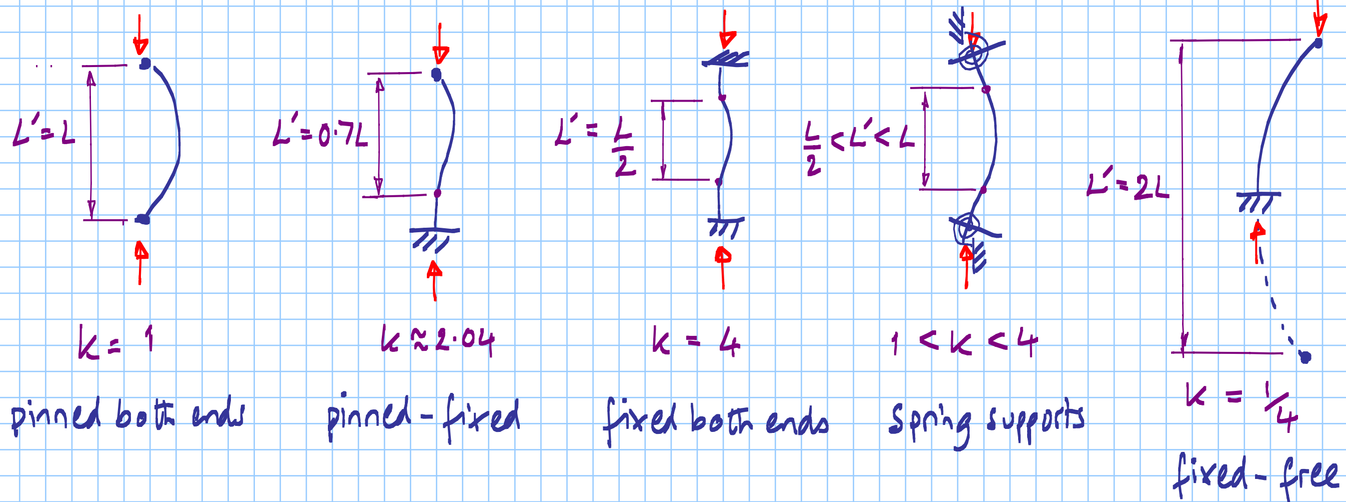
I.e. for a chosen material and end fixity conditions we can now design our strut geometry to avoid global buckling.

Note, we can use: $\sigma_{CRIT} = k\pi^2 E\left(\frac{\rho}{L}\right)^2$ where k is the end fixity constant

or: $\sigma_{CRIT} = \pi^2 E\left(\frac{\rho}{L'}\right)^2$ where L' is the "effective Euler length"

I.e.: $k = \left(\frac{L}{L'}\right)^2$

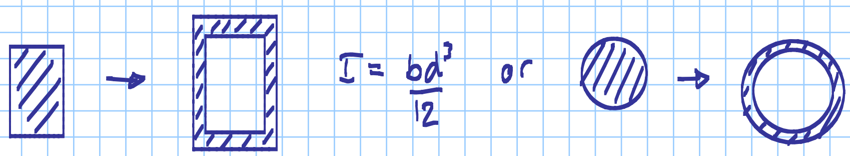
k or L' refer to the end fixity conditions, i.e.:



Note, if the applied stress, σ, or predicted critical elastic buckling strength, σ_{CRIT}, aproaches the yield or proof strength of to material, σ_y or σ_p then, based on the assumption of linear elasticity, the predicted elastic value may not be valid and must be corrected for plasticity.
Failure may even occur by yielding before buckling if the bar is stocky, i.e. if it has a low slenderness, ratio, L/ρ (More in yr2)

For efficiency it would be sensible to increase the cross section dimensions:

E.g.

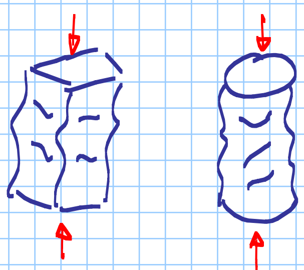


$I = \frac{bd^3}{12}$ or $I = \frac{\pi R^4}{4}$

But if we "thin out" the section too much then failure may occur due to a local buckling mode. E.g. "panel buckling" or "skin wrinkling"!

More on this in yr 2

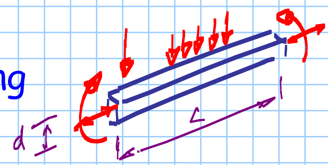
E.g.

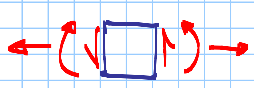


E.g. try crushing a beer can!

BEAMS

Transverse and axial loading





Sign convention!

Stiffness $\delta = \dfrac{KPL^3}{EI}$

Assuming linear elasticity,
slender beam, plane sections, etc.

$\dfrac{L}{d} > 20$

Design statement

Design for deflection not to exceed deflection limit

Design for $\delta \leq \delta^*$

Usually at limit

$①: \; K\dfrac{PL^3}{EI} \leq \delta^*$

etc.

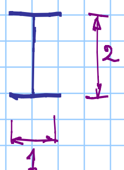
I.e for a chosen material and associated modulus, E, we can now design the beam geometry achieve the required stiffness.

For efficiency we are driven to increase the 2nd moment of area, I.
To achieve this efficiently we displace the material from to beam neutral axis, making the section from relatively slender walls, i.e. "thin wall "section design, typical of light aircraft.

But note, the section then becomes susceptible to local instability in the form of panel buckling.

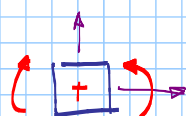
"I-beams" are classically used in section design where bending is predominantly in one plane.

For optimisation the ratio of I-beam web depth to flange width should be ~ 2:1



Strength $\dfrac{-\sigma}{y} = \dfrac{M}{I} = \dfrac{E}{R}$

Assumptions: elasticity etc.
-ve sign due to sign convention:



Design statement

Design for stress not to exceed allowable strength

Design for $\sigma \leq \sigma^*$

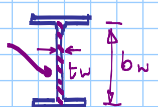
$\left.\begin{array}{l}\dfrac{My}{I} \leq \sigma^*\end{array}\right\}$ etc.

Also, as a first estimate we can check the shear stress, assuming the transverse shear load is carried by the aligned web elements only:

Design for $\tau \leq \tau^*$

where: τ^* = allowable shear strength.

Eg. $A_s \approx b_w . t_w$



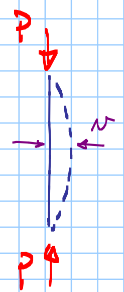
$\dfrac{V}{A_s} \leq \tau^*$

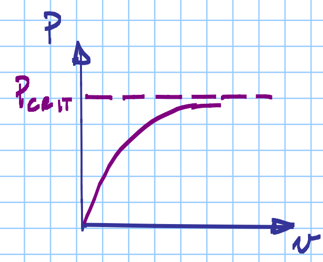
Typically: $\tau^* = \dfrac{\sigma^*}{\sqrt{3}}$ for ductile material

Stability   Global Euler Buckling

If the beam is subjected to a significant axial compressive load as well as transverse loading then a global 'Euler' mode of buckling may occur.

-A true "bifurcation" will not be reached because the beam will already be deflected under the transverse loading.





The lateral deflection will result in a further secondary moment due to the axial offset i.e. P.v

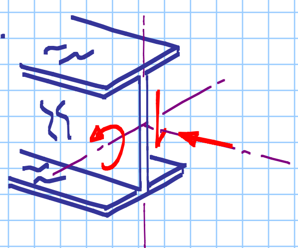
Such beams are often referred to as "Beam Columns"

For simple initial design we can make an estimate of this secondary bending' moment as: $M' = \dfrac{M}{1 - P/P_{CRIT}}$

Where M is the primary bending moment due to transverse loading
P is the applied axial compressive load
Pcrit is the critical Euler buckling load of the beam as a strut

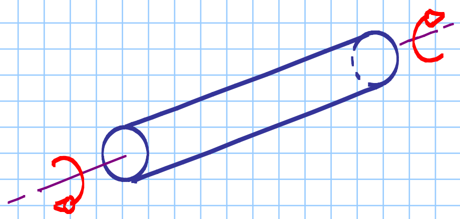
Local buckling of beam column elements, i.e. flanges and webs can also occur as panel buckling under the action of bending and shear leads as well as axial.

E.g.



But more on this in yr 2!

SHAFTS Torsional loading



Stiffness $\frac{T}{J} = \frac{G\theta}{L}$ ①

Assuming linear elasticity, free torsion (neglecting warping), plane sections, etc.

Design Statement

Design for twist not to exceed an allowable value:

Design for $\theta \leq \theta^*$

①: $\frac{TL}{GJ} \leq \theta^*$ $\left\{ \begin{array}{l} \theta \text{ rad.} \\ \text{etc} \end{array} \right.$

Usually at limit load

Strength $\frac{\tau}{R} = \frac{T}{J}$ ①

Design Statement

Design for shear stress not to exceed allowable value:

Design for $\tau \leq \tau^*$

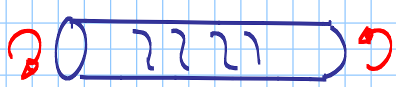
①: $\frac{TR}{J} \leq \tau^*$

where: τ^* = allowable shear strength.

Typically: $\tau^* = \frac{\sigma^*}{\sqrt{3}}$ for ductile material

Similar to beams, we are driven to increase the polar second moment of area, J, by using thin walled sections with large radius. But the section then becomes susceptible to local torsional buckling mode wrinkling along the shaft.

Stability

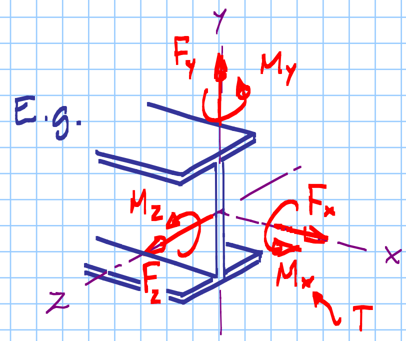


Stability becomes important as a local wrinkling mode for thin wall shafts.

COMBINED LOADING & SUPERPOSITION

For elements which carry combined loading, if we assume that the material and structural responses are linear then we can carry out a separate analysis for each loading and simply add the results by superposition.

E.g.



$$\sigma_x = \sigma_{F_x} + \sigma_{M_z} + \sigma_{M_y}$$

$$= \frac{F_x}{A} + \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\tau = \tau_{F_y} + \tau_{F_z} + \tau_T$$

For combined stresses of different types, e.g. σ, τ, that can not be added directly we need to refer to a "failure criterion" that accounts for the combination of direct and shear stresses that cause failure.

THE STRUCTURAL DESIGN PROCESS

Because there is usually more than one unknown quantity when designing a structure it is often convenient to start with a trial scheme and then check and modify it according to the resulting RF valves.

Evaluating the resulting deflections and stresses for each scheme will quickly build experience and help to develop your engineering judgement.

All scheme developments and checks should be carried out in an engineering "log book".

When the final design is arrived at, the structure should be fully defined by CAD and drawings should be presented in a "Stress Report" along with illustrations of the critical calculations of deflections and stresses and resulting RF values for your final design, based on your most refined methods.

JOINT DESIGN

The following notes illustrate basic joint stressing. Items include:

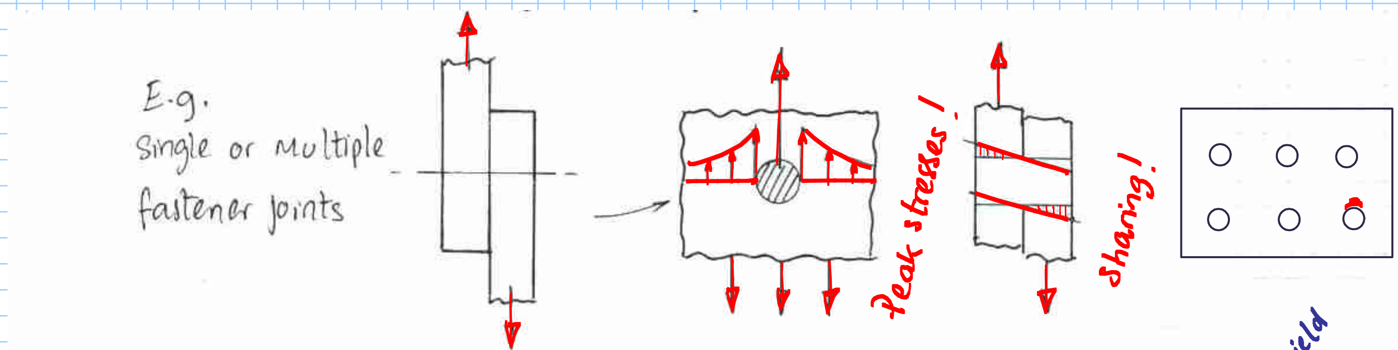
1. Design at Ultimate

2. Single fastener pinned joints

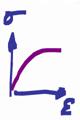
3. Multiple Fastener fixed joints
   a. ... with concentric loading
   b. ... with concentric and eccentric loading
   c. Fixed joint connection configurations

Basic assumptions:
Pins are perfectly rigid (i.e. they do not deform)
Pins have perfect fit (i.e. no tilt)

1. Design at Ultimate "Net-section average-stress"



Below elastic limit : Unequal load sharing between fasteners
Non uniform stress distribution
– In plane and thru-thickness



Above elastic limit : Plastic yielding
Equal load sharing
Uniform stress distributions

No alleviation below yield

Ductile alleviation of stress conc'n above yield

→ Design using averaged net section stress ok for ductile materials

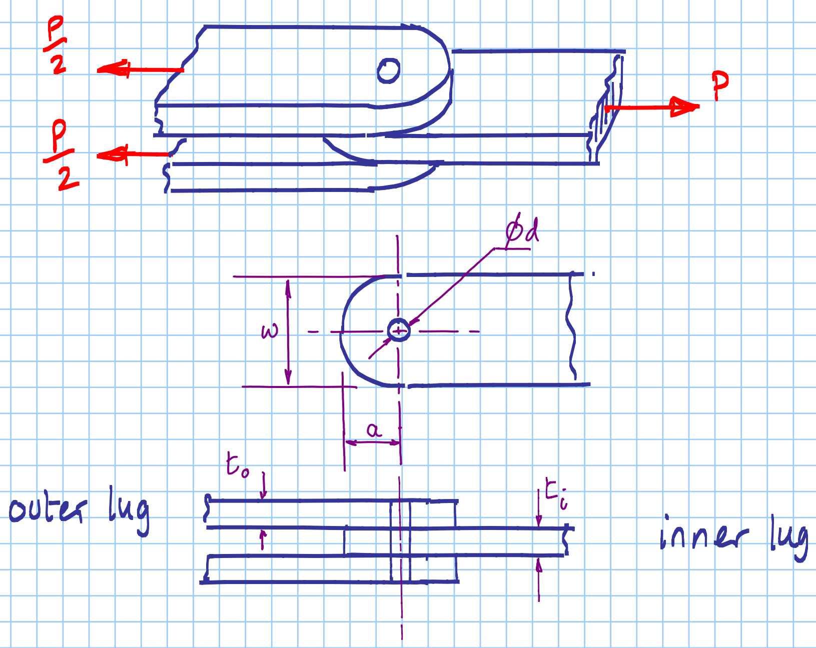
I.e. for an engineering alloy at ultimate loading we assume that yeilding dissipates stress concentrations and promotes even load sharing (but note, below yield this is not the case - with implications for fatigue!)

2. Single-fastener pinned joints

A single pin fastener joint should always be double shear configuration to avoid excessive eccentricity and bending across the joint.

Configuration + Geometry:

E.g.:



Design Statement

Design to ensure that generated stresses do not exceed the allowable strengths of any of the failure modes at ultimate loading.

Design for: pin: $\tau, \sigma_{br}, \sigma_b \leq \tau^*, \sigma_{br}^*, \sigma_b^*$

lug: $\sigma_{br}, \sigma_{tens}, \tau_{so} \leq \sigma_{br}^*, \sigma_{tens}^*, \tau_{to}^*$ etc!

Stresses at ultimate — Allowable values

For initial joint design, to cope with the numerous possible modes of failure we often refer to guidelines to help us approach an optimum solution quickly, e.g.:

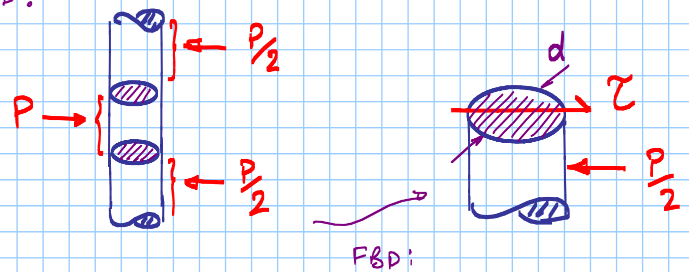
$$\frac{d}{t} \leq 3.5 \quad , \quad \frac{a}{d} \geq 1.5 \text{ to } 2 \quad , \quad \frac{w}{d} \geq 4$$

Consider each mode in turn:

- Pin Shear

Consider the bolt cross-section being sheared at the interface between the joint plates (lugs)

FBD:



Shear stress: $\tau = \dfrac{P}{2\pi\left(\frac{d}{2}\right)^2}$

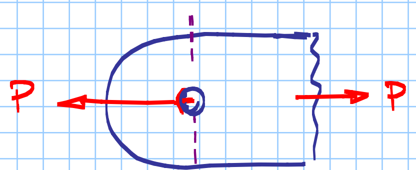
- design for $\underset{\text{actual}}{\tau} < \underset{\text{allowable}}{\tau^*}$

↳ "Reserve factor" $\boxed{RF = \dfrac{\text{Allowable}}{\text{Actual}}}$

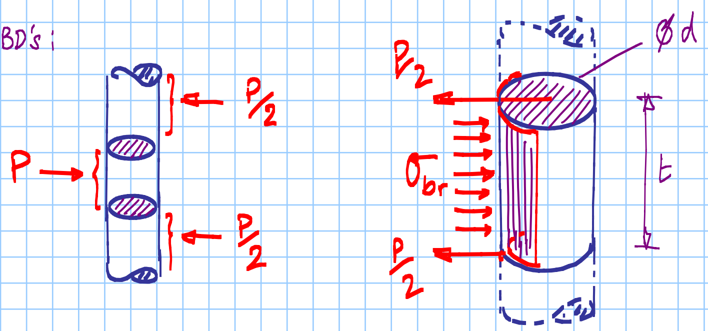
RF > 1 required.

- Pin Bearing (usually less critical than lug bearing)

Consider the lug bearing against the side of the bolt:



FBD's:

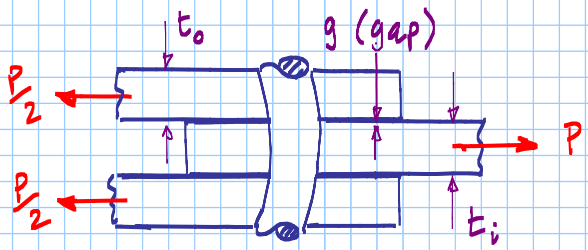


$\sigma_{br} = \dfrac{P}{dt}$ ← projected flat area

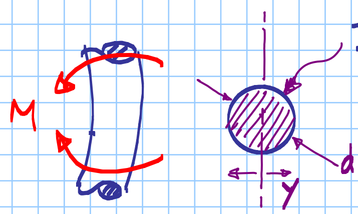
Design for $\sigma_{br} < \sigma_{br}^*$ ↳ $\boxed{RF = \dfrac{\sigma_{br}^*}{\sigma_{br}}}$

- Pin Bending

Consider the relative dispacement of the lug plates and the effective bending moment on the pin



Can be a significant design driver for thick, single shear or offset joints, e.g. with a filler.



$$I = \frac{\pi R^4}{4} = \frac{\pi d^4}{64}$$

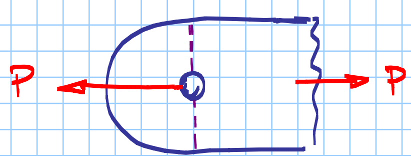
$$\sigma = \frac{M\left(\frac{d}{2}\right)}{I} \qquad M \approx \frac{P}{2}\left(\frac{t_o}{2} + \frac{t_i}{4} + g\right)$$ as a <u>conservative</u> estimate.

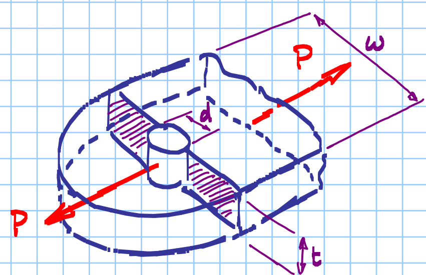
Design for $\sigma < \sigma^*$ $\hookrightarrow$ $\boxed{RF = \frac{\sigma^*}{\sigma}}$

- Lug Tension

Consider the direct tension carried by the net lug section at the bolt.

E.g.



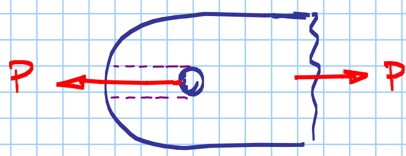


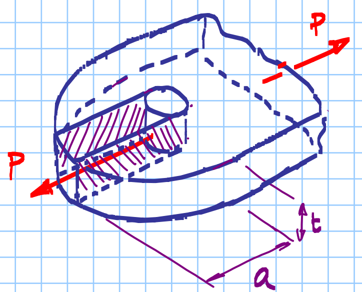
$$\sigma_t = \frac{P}{(w-d)t}$$

Design for $\sigma_t < \sigma^*$ $\hookrightarrow$ $\boxed{RF = \frac{\sigma^*}{\sigma_t}}$

- Lug Shear out

Consider the bolt shearing out through the end of the lug.



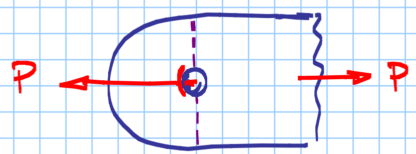


$$\tau = \frac{P}{2at}$$

Design for $\tau < \tau^*$ ↳ $\boxed{RF = \frac{\tau^*}{\tau}}$

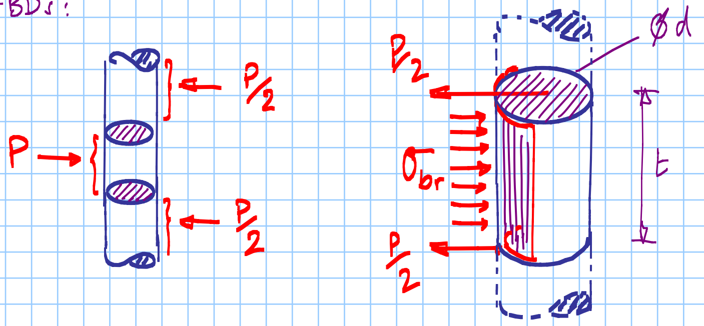
- Lug Bearing*

Consider the bolt bearing on the lug hole surface.



Preferable earliest failure mode since "benign" with warning of loose joint due to local deformation at hole.

FBD's:

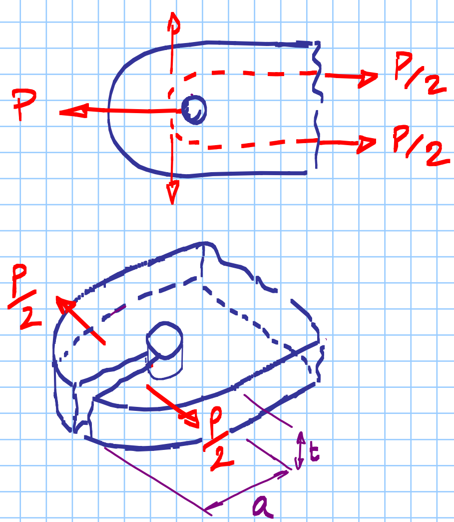


$$\sigma_{br} = \frac{P}{\underbrace{dt}_{\text{Projected flat area}}}$$

Design for $\sigma_{br} < \sigma_{br}^*$ ↳ $\boxed{RF = \frac{\sigma_{br}^*}{\sigma_{br}}}$

- Lug Bursting (Cleavage)

Consider the effective transverse tension across the lug



$$\sigma_{CL} = \frac{P}{2} \Big/ at$$

Design for $\sigma_{CL} \leq \sigma^*$ ↳ $\boxed{RF = \frac{\sigma^*}{\sigma_{CL}}}$

3. Multiple fastener fixed Joints

I.e. no rotation.

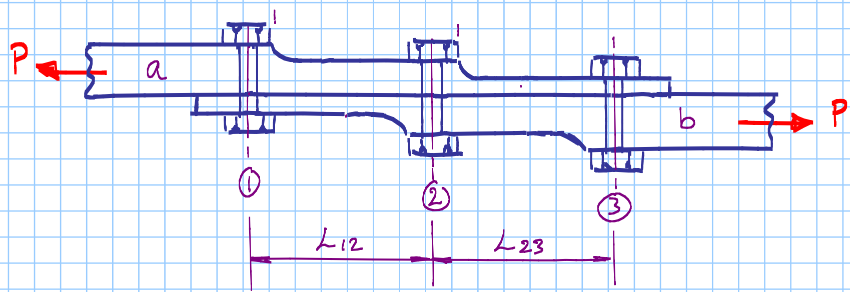
The basic failure modes defined for a simple pin joint also apply to fixed joints with multiple fasteners.

To relate to these modes we need to calculate the load carried by each fastener in a multiple fastener fixed joint.

Methods for estimating individual fastener loads in multiple fastener joints are outlined below.

3a. Multiple fastener fixed joint with Concentric loading

E.g. consider a three pin lap joint:



Note, for highly loaded multi-fastener joints the joint elements must be tailored to promote even load sharing.

Note stiffness of each element: $k_{a_{12}} = \left.\frac{AE}{L}\right|_{a_{12}}$ etc

For plates of the same material $E_a = E_b$

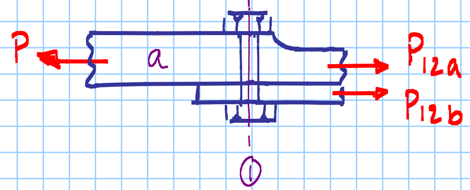
For equal length steps $L_{12} = L_{23}$

For equal width plates $A_{12} = w t_{12}$, $A_{23} = w t_{23}$

where: L = step length
w = step width
t = step thickness

Consider FBD's of sections revealing step loads and pin loads.

FBD1



Note this is a redundant structure, i.e. more than one load path.
So to solve we must consider: equilibrium, constitutive relationships

Equilibrium:

$$\Sigma \overset{+}{\rightarrow} = 0 : \quad -P + P_{12a} + P_{12b} = 0 \qquad (1)_{12}$$

Constitutive Relations: $P_{12a} = k_{12a} d_{12a}$ $(2)_{12a}$ where $k_{12a} = \left.\frac{AE}{L}\right|_{12a}$

$P_{12b} = k_{12b} d_{12b}$ $(2)_{12b}$ And $k_{12b} = \left.\frac{AE}{L}\right|_{12b}$

Compatibility: $d_{12a} = d_{12b}$ $(3)_{12}$ I.e. each side of step extends by the same amount.

$$(3),(2): \quad \frac{P_{12a}}{k_{12a}} = \frac{P_{12b}}{k_{12b}} \quad \text{ie} \quad \frac{P_{12a}}{P_{12b}} = \frac{k_{12a}}{k_{12b}} \qquad (4)_{12}$$

Note if plates are of same material, E, same step width, w, and step length, L

then: $(4)_{12}$: $\frac{P_{12a}}{P_{12b}} = \frac{k_{12a}}{k_{12b}} \rightarrow \frac{t_{12a}}{t_{12b}}$ i.e. load in each step proportional to

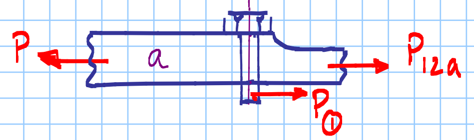
1,4 : Eliminating P_{12b}

$$\hookrightarrow P_{12a} + P_{12a}\frac{k_{12b}}{k_{12a}} = P \quad \rightarrow \quad P_{12a} = P \Big/ \left(1 + \frac{k_{12b}}{k_{12a}}\right) \qquad 5_{12a}$$

Similarly eliminating P_{12a} :

$$P_{12b} = P \Big/ \left(1 + \frac{k_{12a}}{k_{12b}}\right) \qquad 5_{12b}$$

FBD1a

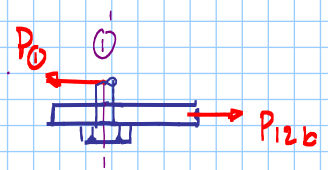


$$\Sigma \xrightarrow{+} = 0 : \quad -P + P_{12a} + P_{1} = 0$$

$$\hookrightarrow P_{1} = P - P_{12a} \quad \text{and } 1 : \; P_{12b} = P - P_{12a} \quad \text{So } P_{1} = P_{12b} \qquad 6_{12}$$

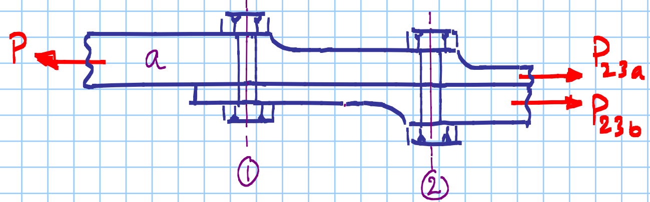
Pin transfer load (P_{1}) By-pass load (P_{12a})

FBD1b



$$\Sigma \xrightarrow{+} = 0 : \quad -P_{1} + P_{12b} = 0 \quad : \quad P_{1} = P_{12b}$$ agreeing with previous FBD result.

FBD2



Equilibrium:

$$\Sigma \xrightarrow{+} = 0 : \quad -P + P_{23a} + P_{23b} = 0 \qquad 1_{23}$$

Constitutive Relations: $P_{23a} = k_{23a}\, d_{23a}$ $\quad 2_{23a}$ where $k_{23a} = \left.\frac{AE}{L}\right|_{23a}$

$P_{23b} = k_{23b}\, d_{23b}$ $\quad 2_{23b}$ And $k_{23b} = \left.\frac{AE}{L}\right|_{23b}$

Compatibility: $d_{23a} = d_{23b}$ $\quad 3_{23}$ ie. each side of step extends by the same amount.

$$3,2 : \quad \frac{P_{23a}}{k_{23a}} = \frac{P_{23b}}{k_{23b}} \qquad 4_{23} \qquad \text{ie} \quad \frac{P_{23a}}{P_{23b}} = \frac{k_{23a}}{k_{23b}}$$

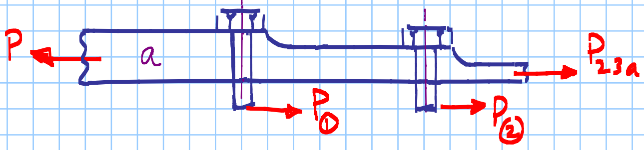
1,4 : Eliminating P_{23b}

$$\hookrightarrow P_{23a} + P_{23a}\frac{k_{23b}}{k_{23a}} = P \quad \rightarrow \quad P_{23a} = P \Big/ \left(1 + \frac{k_{23b}}{k_{23a}}\right) \qquad 5_{23a}$$

Similarly eliminating P_{23a} :

$$P_{23b} = P \Big/ \left(1 + \frac{k_{23a}}{k_{23b}}\right) \qquad 5_{23b}$$

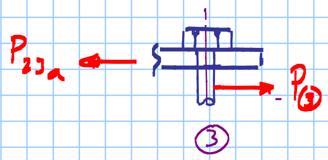
FBD2a



$$\Sigma \xrightarrow{+} = 0: \quad -P + P_{①} + P_{②} + P_{23a} = 0 \qquad ⑦$$

Where $⑥_{12}$, $⑤_{12b}$: $\quad P_{①} = P_{12b} = P \Big/ \left(1 + \dfrac{k_{12a}}{k_{12b}}\right)$

And $⑤_{23a}$: $\quad P_{23a} = P \Big/ \left(1 + \dfrac{k_{23b}}{k_{23a}}\right) = P_{③}$ ie:

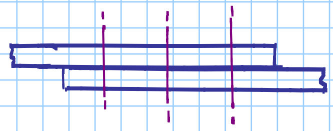


So ⑦: $\quad -P + P \Big/ \left(1 + \dfrac{k_{12a}}{k_{12b}}\right) + P_{②} + P \Big/ \left(1 + \dfrac{k_{23b}}{k_{23a}}\right) = 0$

$$\hookrightarrow P_{②} = P - P \Big/ \left(1 + \frac{k_{12a}}{k_{12b}}\right) - P \Big/ \left(1 + \frac{k_{23b}}{k_{23a}}\right)$$

ie. $P_{②} = P - P_{①} - P_{③}$

- For untailored joint, ie. no steps:



$k_{12a} = k_{12b} = k_{23b} = k_{23a}$

$$\hookrightarrow P_{②} = P - P/(1+1) - P/(1+1) = P - \frac{P}{2} - \frac{P}{2} = 0 \; !$$

I.e. middle bolt carries no load!

- For tailored joint we want equal load sharing:

E.g. for 3-pin example we want:

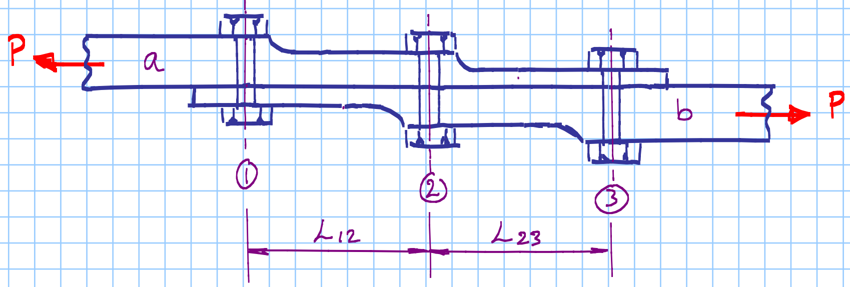
$$P \Big/ \left(1 + \frac{k_{12a}}{k_{12b}}\right) = P \Big/ \left(1 + \frac{k_{23b}}{k_{23a}}\right) = P/3$$

$$\hookrightarrow \left(1 + \frac{k_{12a}}{k_{12b}}\right) = \left(1 + \frac{k_{23b}}{k_{23a}}\right) = 3$$

$$\hookrightarrow \frac{k_{12a}}{k_{12b}} = \frac{k_{23b}}{k_{23a}} = 2$$

Ie: $\quad k_{12a} = 2k_{12b}$ and $k_{23b} = 2k_{23a}$

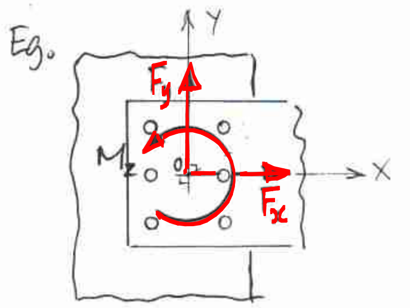
etc!



3b. Multiple fastener fixed joints with concentric and eccentric loading

E.g. rivet group loading

Eg.



Determine most highly loaded fastener

"O" = fastener group centroid

F_x, F_y = "Concentric" loading cpts. thru centroid

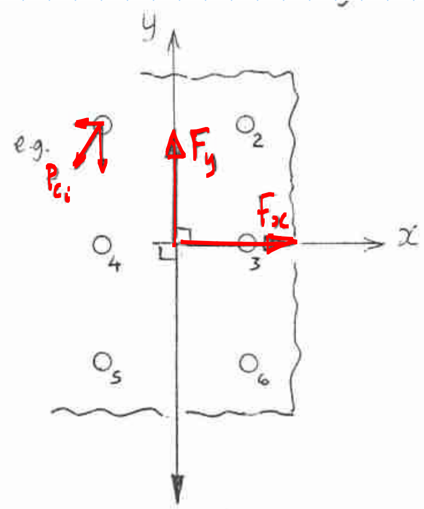
M_z = "Eccentric" loading cpt about centroid.

Obtain from FBD end reactions (or from FE model @ end of discts element)

– Consider separately

I.e. concentric + eccentric components summed by superposition.

Concentric loading



For Equilibrium w.r.t x, y directions:

$$\Sigma \xrightarrow{+} = 0 \quad : \quad \Sigma P_{c_{x_i}} + F_x = 0$$

$$\Sigma \uparrow^{+} = 0 \quad : \quad \Sigma P_{c_{y_i}} + F_y = 0$$

For n equal fasteners

Assuming uniform shear distribution *

I.e above elastic limit

ok for ultimate design using rivets to join plates with ductile characteristic.

$$P_{c_{x_i}} = -F_x/n$$

$$P_{c_{y_i}} = -F_y/n$$

Suffix c ⇒ concentric

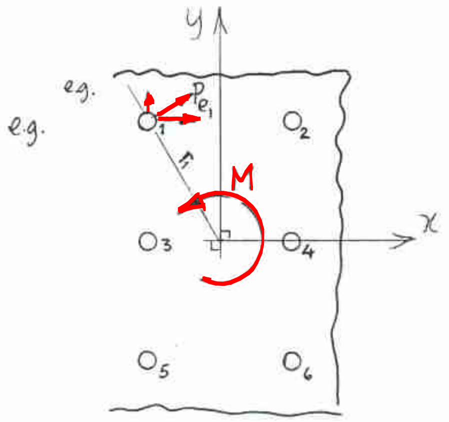
Sign convention:

Applied force cpts: F_y, F_x, +

Reaction force cpts: $P_{c_{y_i}}$, $P_{c_{x_i}}$, +, i

(* Applicable for rivets – but not necessarily for bolts)

Eccentric loading



For Equilibrium:

$$\sum \circlearrowright^{+} = 0 \quad : \quad \sum (P_{e_i} \cdot r_i) + M = 0 \qquad (1)$$

For n equal fasteners

Assuming fastener load proportional to fastener distance from group centroid

$$\rightarrow P_{e_i} = k \cdot r_i \qquad (2)$$

Eliminating k using equilibrium eqn.

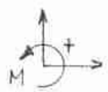
(1): $\sum (\overset{\text{CONST}}{k}\, r_i\, r_i) + M = 0$, (2) $\rightarrow P_{e_i} = -\dfrac{M . r_i}{\sum r_i^2}$

$\hookrightarrow k = -M / \sum r_i^2$

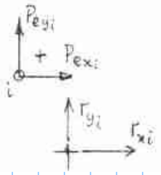
Suffix e ⇒ eccentric

Sign convention:

Applied moment:



Reaction force cpts:



Wrt x-y co-ords:

$$P_{e_{x_i}} = \frac{M r_{y_i}}{\sum (r_{x_i}^2 + r_{y_i}^2)}$$

$$P_{e_{y_i}} = -\frac{M r_{x_i}}{\sum (r_{x_i}^2 + r_{y_i}^2)}$$

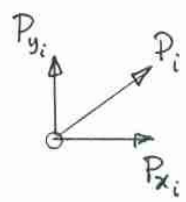
for M +ve a-clock

Summing concentric and eccentric x, y components:

at each fastener

$$P_{x_i} = P_{c_{x_i}} + P_{e_{x_i}}$$

$$P_{y_i} = P_{c_{y_i}} + P_{e_{y_i}}$$



Resultant

$$P_i = \sqrt{P_{x_i}^2 + P_{y_i}^2}$$

Spread sheet!

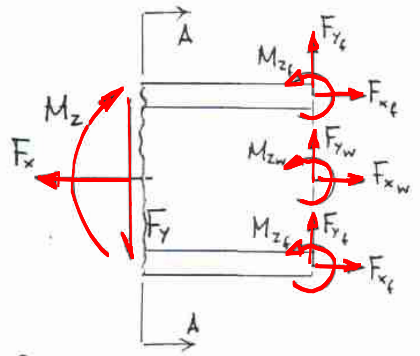
Rivet No.	r_x	r_y	P_{c_x}	P_{e_x}	P_x	P_{c_y}	P_{e_y}	P_y	P
–	–	–	–	–	–	–	–	–	–
–	–	–	–	–	–	–	–	–	–
–	–	–	–	–	–	–	–	–	–

Check joint failure modes @ most highly loaded fastener
(Fastener shear + plate bearing!)

other modes covered by spacing guidelines.

3c. Fixed Joint connection configurations

Transfer of beam loading



Flange and web loads + mmts to be transferred at end of joint.

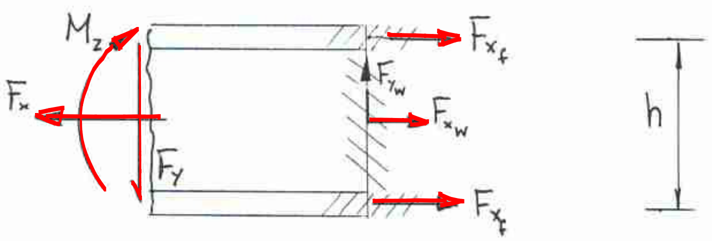
E.g. A-A = 

f = flange
w = web

Consider transfer @ flange and web connections

- based on sub-element @ end of joint element

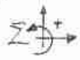
Using flange and web connections



Ie:

Flange joint loading : $F_{x_f} \simeq F_x \dfrac{A_f}{\Sigma(A_f + A_w)} \pm \dfrac{M_z}{h}$ — Bending mont reacted by couple between flange joints

$F_{y_f} \simeq 0$

From equilibrium sums  $= 0$

$M_{z_f} \simeq 0$

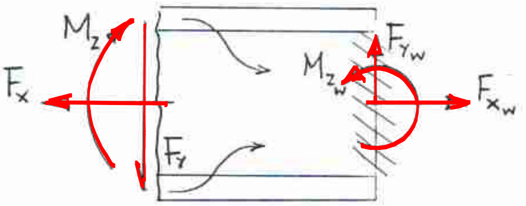
Axial load reacted by forces in flange + web joints in proportion to flange+web sections

Web joint loading : $F_{x_w} \simeq F_x \dfrac{A_w}{\Sigma(A_f + A_w)}$

$F_{y_w} \simeq F_y$ — Shear load reacted by force in web joint

$M_{z_w} \simeq 0$

Using web connection only



Here: flanges off-load into web @ joint
↳ must thicken or reinforce web!

Web joint loading: $F_{x_w} = F_x$

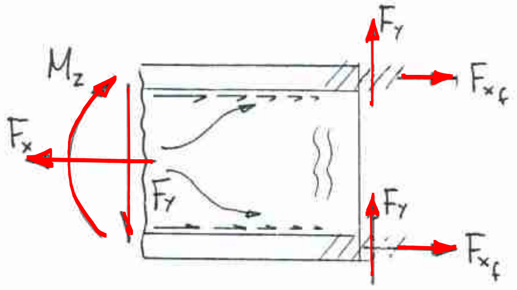
$$F_{y_w} = F_y$$

$$M_{z_w} = M_z$$

Web transfers bending moment as eccentric loading.

Note stability of web plate under axial + shear load + bending moment!

Using flange connections only



Here, web off-loads into flanges

Flange joint loading: $F_{x_f} = F_x \dfrac{A_f}{\Sigma A_f} \pm \dfrac{M_z}{h}$ Bending mom't reacted by couple between flanges

$$F_{y_f} = F_y \frac{A_f}{\Sigma A_f}$$

etc.

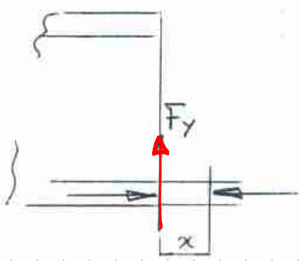
↳ transfer of shear load through flange joint needs careful consideration.

$$M_{z_f} = 0$$

Note, stability of flange joint plate under compression and bending due to offset shear load

Also, note stability of web plate @ free edge

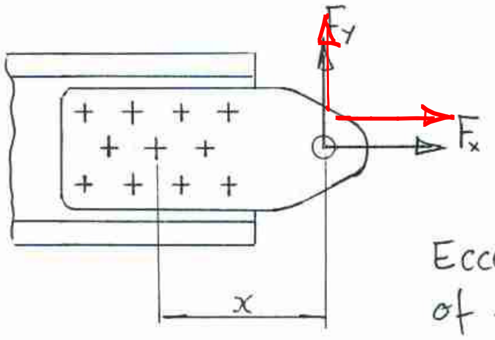
Eg:



Further considerations:

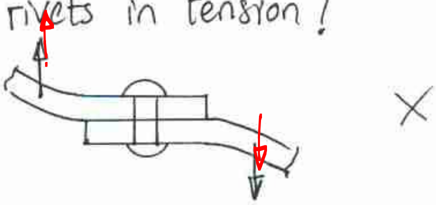
- Eccentric loading in pinned joint fittings

Eg.



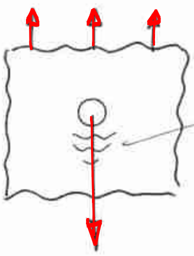
Eccentric loading @ centroid of fitting fastener group.
-due to offset of pin loading
ie. $F_y . x$

- Avoid putting rivets in tension!



ctd.

- Beware of local instability @ pin loading in thin plates

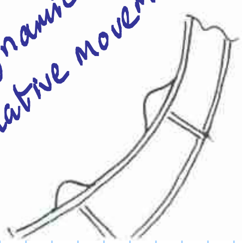


Check d/t ratio guidelines
eg. $d/t \leq 3.5$ etc.

- Must use multifastener joints / fittings in thin plates to disperse concentrated loads
- Consider bushing in pin joint holes to improve bearing
- Avoid using mixed fasteners in same joint eg. rivets + bolts
- For bolted joints check "fitting factors" – Apply to bolted joints especially when subjected to dynamic loading or relative movement.

  ALTERNATIVELY CONSIDER MINIMUM TARGET RF VALUES.
- Use integral fittings where possible
- Use local reinforcement

Eg:



Fitting details

IRF 22.4.09

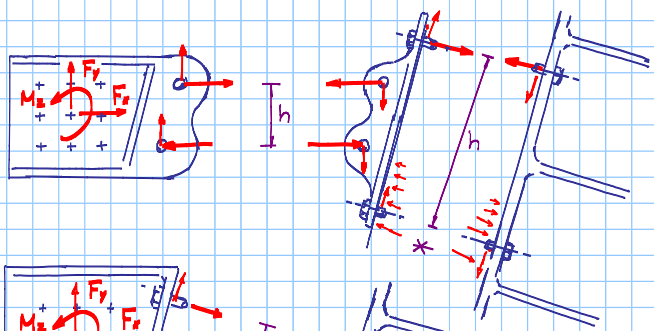
A fixed joint can be created by using multiple fastener connections (a minimum of two to create couple forces).

Fittings can be separate items or integral with the structure to be joined.

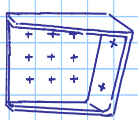
FBD's must be created to understand load transfer as direct and shear loads.

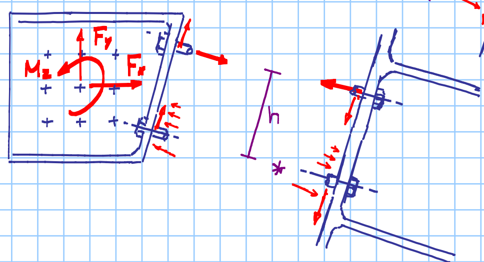
Direct loads will have contributions from F_x and M_z/h couple loads:

E.g. Shear lug fitting
Double shear connection needed



Eg. Tension end plate fitting.





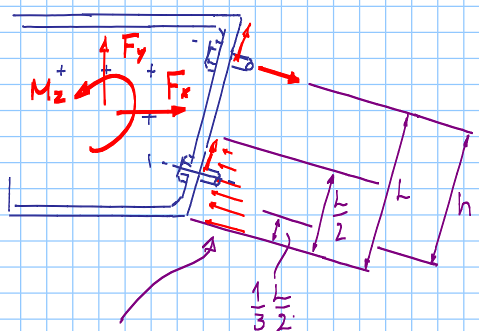
FBD's !

* Note the distribution of the compressive couple load between separate fittings

* Assume the effective centre of the compressive couple load acts at

- <u>either</u> the outer bolt line
- <u>or</u> the centre of an assumed triangular distribution of stress from the edge of the fitting face on the compression side
- whichever is most conservative.

Eg.:



Assumed triangular distribution of compressive stress

Further consideration would be needed for more rows of fasteners.

Joint fittings can

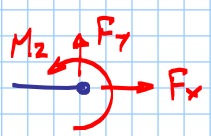
- either be separate items riveted or bolted to the beams to be joined as illustrated (but this requires further parts and fasteners)
- or as integral items within the beam or frame (but this requires significant machining).

Joint analysis includes concentric + eccentric rivet group analysis, lug and pin analysis and fitting analysis.

Depending on your chosen configuration you will need to give some thought to the diffusion of loads into these fittings and their potential failure modes.

Connecting to flanges only or web only or web + flanges presents significantly different schemes with conflicting pros and cons in terms of load transfer and ease of manufacture and assembly.

Connection load transfer



Combinations of Fx, Fy, Mz must be transferred at the ends of a beam, depending on the load case and configuration considered,

Values of Fx, Fy, Mz can be obtained from beam FBD and AF, SF, BM diagrams.

The end loads can be interpreted as joint loadings and further consideration of the fittings can be achieved by FBD's of suitable sections to illustrate loading as shown in the examples above.