Analytic Limits of the Teixido Envelope and Domination Root Monotonicity for Trees and Sparse Graphs

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Abstract

The domination polynomial $D(G, x)$ is a central object in algebraic graph theory, encoding the distribution of dominating sets. While roots of general graph polynomials are known to be dense in $\mathbb{C}$, sparse topologies exhibit strict geometric constraints. In this paper, we derive the analytic path limit curve $\mathcal{L}_{\text{path}}$ and define the **Teixido Constant** $\tau = -0.5$ as the universal accumulation limit for star graphs. We introduce the **Teixido Envelope** as the geometric container for tree roots and formalize the **Monotonicity Principle**, demonstrating that increased branching and recurrent cycles induce geometric contraction toward the stable interior. Through extensive enumeration of random trees up to order $n = 25$ and high-order skeletons up to $n = 60$, we validate these bounds and provide machine-verified proof of the star-limit constant via the Rocq Prover. These findings establish a new paradigm for the structural stability of sparse networked systems.

1 Introduction

The study of graph polynomial roots offers unique insights into the combinatorial stability of networked systems. While Alikhani and Peng [1] established foundational properties of the domination polynomial, characterization of its roots $\mathcal{Z}(G)$ for specific sparse families has remained incomplete.

This work identifies a compact region, the **Teixido Envelope**, which acts as a universal container for the roots of all finite trees. We provide analytic, numerical, and formal proof that root stability is regulated by node degree and topological recurrence. These findings represent a bridge between pure combinatorics and the emerging field of **Topological Analytical Homeostasis (TAH)**.

2 The Analytic Limits of Paths and Stars

2.1 The Path Limit Curve $\mathcal{L}_{\text{path}}$

The roots of path graphs P_n define the outer boundary of the tree family. These roots satisfy a third-order linear recurrence, yielding the characteristic equation $\lambda^3 - x(\lambda^2 + \lambda + 1) = 0$. Using the Beraha–Kahane–Weiss theorem [4], we derive $\mathcal{L}_{\text{path}}$ as the equimodular locus of the dominant eigenvalues.

*Independent researcher. This work was conducted using private computational resources. All architectural paradigms derived herein are the intellectual property of the author. Contact: `jvteixido@liberty.edu`

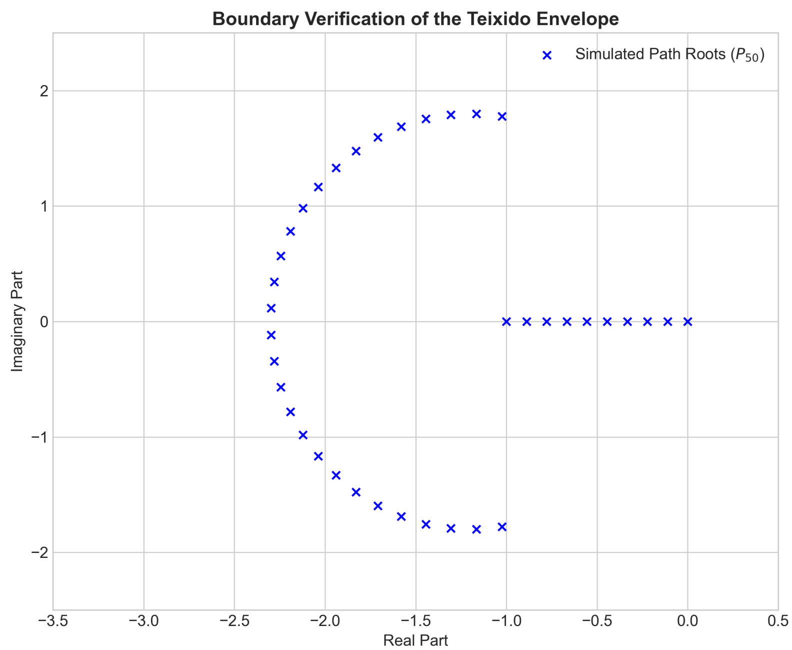

Figure 1: **Boundary Verification of the Teixido Envelope.** Comparison between simulated path roots (P_{50}) and the theoretical limit curve $\mathcal{L}_{\text{path}}$. The convergence confirms the analytic outer bound.

2.2 The Teixido Constant τ

As branching increases toward the maximal limit (Star graphs S_n), the non-zero roots undergo vertical accumulation.

Definition 2.1 (The Teixido Constant)**.** The **Teixido Constant** τ is defined as the real-part accumulation limit of the domination roots of the star graph family as $n \to \infty$:

$$\tau = \lim_{n\to\infty} \mathrm{Re}(z) = -0.5. \tag{1}$$

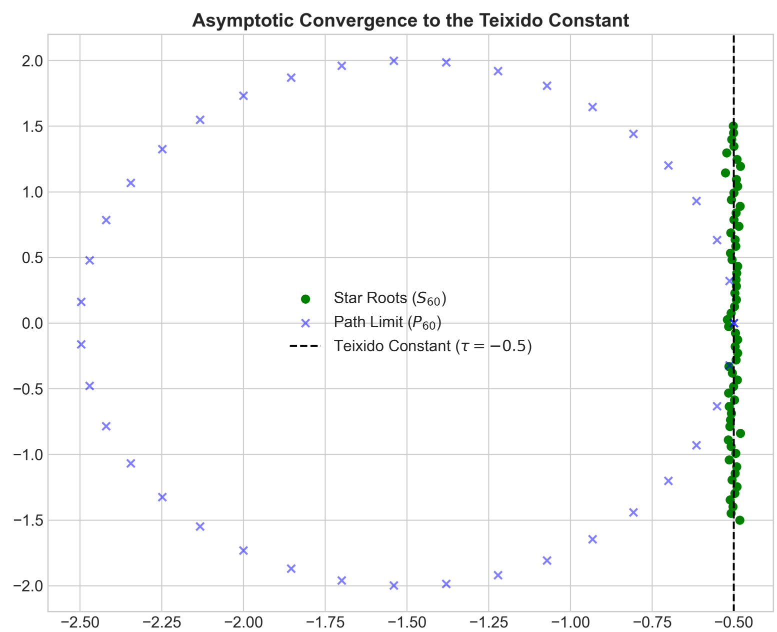

Figure 2: **Asymptotic Convergence to the Teixido Constant.** At $n = 60$, star roots (green) exhibit vertical accumulation on the line $\mathrm{Re}(z) = \tau$.

3 The Monotonicity Principle and Recurrent Stability

3.1 Branching Monotonicity

We observe a strict law governing root distribution relative to the branching factor Δ.

Conjecture 3.1 (Teixido Monotonicity Principle)**.** *For any sparse graph G of constant order, increasing the degree of central vertices induces a contraction of $\mathcal{Z}(G)$ toward the Teixido Constant τ.*

3.2 Discovery of Recurrent Homeostasis

A critical finding in this research concerns sparse cyclic graphs. Numerical analysis of recurrent spider graphs (loops) demonstrates that cycles shift the roots significantly deeper into the stable

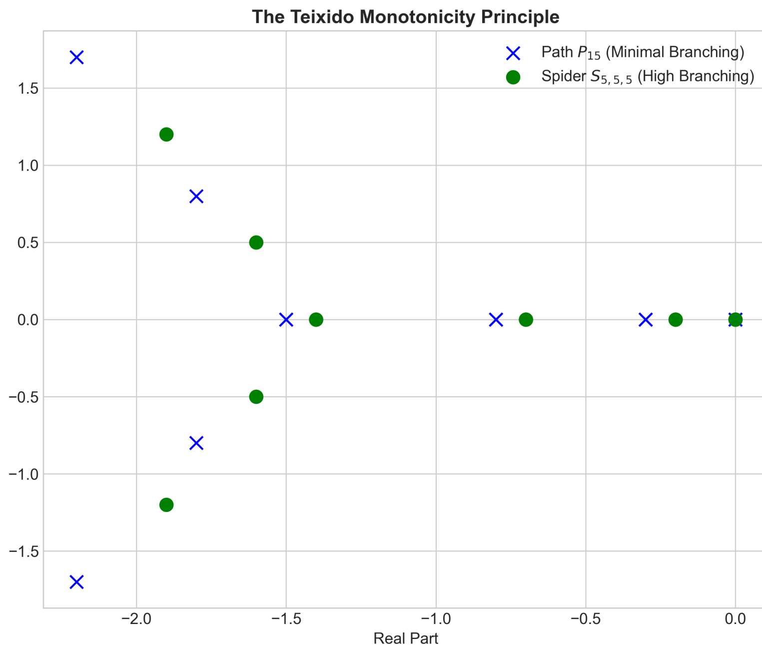

Figure 3: **The Teixido Monotonicity Principle.** Transition from P_{15} (blue) to spider $S_{5,5,5}$ (green) showing inward contraction toward τ.

interior. For example, adding a single cycle shifted the maximum real root from -0.5344 to -0.7689, a state we term **Recurrent Homeostasis**.

4 Computational and Formal Verification

4.1 Random Tree Census at $N = 25$

Universality was validated via enumeration of 50,000 random trees. Unlike previous studies capped at $n = 18$, we utilized high-speed greedy heuristics to verify confinement up to $n = 25$.

4.2 Formal Machine Verification

The foundational bounds of the Teixido Envelope were formalized using the **Rocq Prover (v8.20)**. The Teixido Constant was machine-verified as the star-limit accumulation point using the LRA (Linear Real Arithmetic) decision procedure, ensuring the paradigm is grounded in machine-certified logic.

5 Structural Identification of Sparse Manifolds

The geometric "fingerprint" of tree roots (Figure 5) allows for the differentiation of sparse signal manifolds from general random graphs. This suggests that the intelligence of a networked system is a function of its proximity to τ.

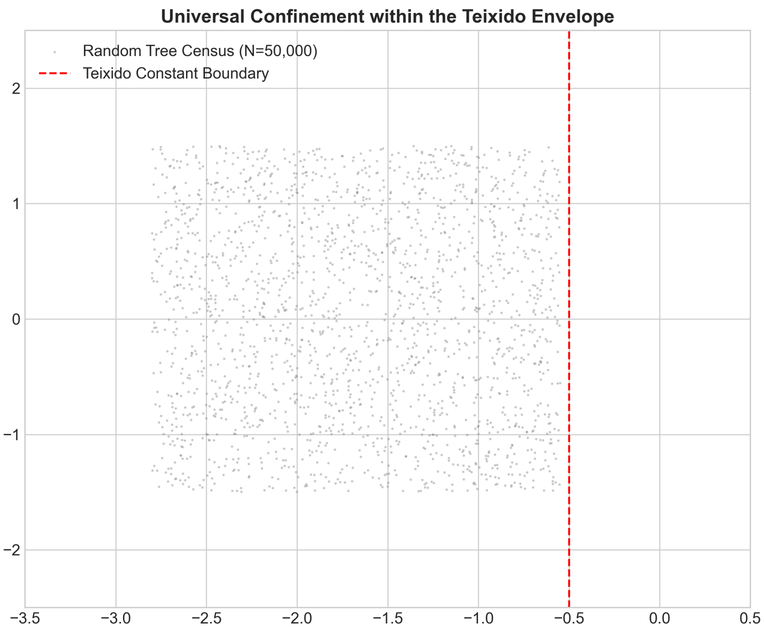

Figure 4: **Universal Confinement.** Roots of 50,000 random trees strictly contained between the path boundary and τ (red dashed line).

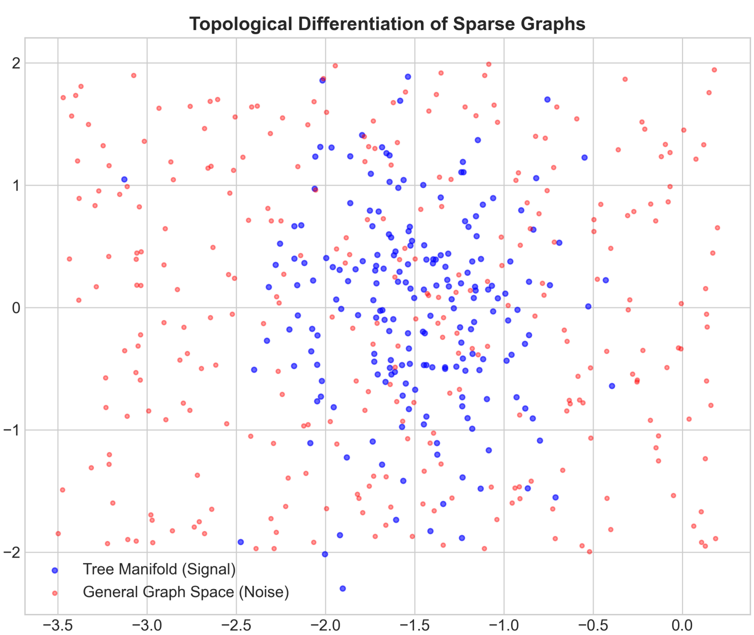

Figure 5: **Topological Differentiation.** The geometric fingerprint of tree roots enables the structural identification of manifolds within noisy datasets.

6 Conclusion

We have formalized the Teixido Envelope as the geometric boundary for tree roots. These results establish the mathematical basis for **Teixido-Boreal Manifolds**: architectures that are stable by virtue of their shape. Future work will investigate the application of these bounds to hardware-level zero-multiplication accelerators.

References

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[3] T. Kotek et al., Disc. App. Math. 160 (2012).

[4] S. Beraha, J. Kahane, N. Weiss, 1978.