# STAT4106 Homework 1: Due September $5^{th}$

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All submitted work is expected to be your own. Violations to this will be at the mercy of the Virginia Tech Honor Court.

Submitted assignments should be very neat and easy to read. For this reason, LaTeX is preferred. Microsoft Word documents must be rendered as pdf or html files, or resubmission will be required with a loss of 10 points. Hand written assignments are accepted so long as they are neat and legible. Homeworks must be stapled or 10 points will be lost.

## Problem 1

Suppose that $X_1, X_2, ..., X_N$ are i.i.d. random variables with probability density function $f(x) = 3x^2, 0 \leq x \leq 1$.

### Part A

Find the distribution of $X_{(1)}$.

### Part B

Find the distribution of $X_{(N)}$.

### Part C

Find the distribution of $X_{(k)}$.

### Part D

Compute $\mathbb{E}[X]$.

## Problem 2

Let $Y_1, Y_2, ..., Y_N \sim$ i.i.d. $Unif(0, 1)$.

### Part A

Find the distribution of $Y_{(k)}$. What type of distribution is this? Specify a name and give values for the parameters.

### Part B

What are $\mathbb{E}[Y_{(k)}]$ and $\mathbb{V}[Y_{(k)}]$?

## Problem 3

Let $X_1, ..., X_N$ by independent normal random variables, with $X_i \sim N(\mu_i, \sigma_i^2)$. Show that any **linear combination** of normal random variables is also normally distributed, i.e. for any $a_i \in \mathbb{R}$, show that $U = \sum_{i=1}^{N} a_i X_i$ is also normal, and give its mean and variance. You may use the fact that the Normal MGF is given by $M_X(t) = \exp\left(\mu t + \frac{\sigma^2 t^2}{2}\right)$.

## Problem 4

Let $X$ be an exponential random variable with p.d.f. $f(x) = \lambda \exp(-\lambda x)$.

### Part A

Compute $M_X(t)$.

### Part B

Using properties of the MGF, compute $\mathbb{E}[X]$ and $\mathbb{V}[X]$.

### Part C

Let $X_1, X_2, ..., X_N \sim$ i.i.d $exponential(\lambda)$. Let $U = \sum_{i=1}^{N} X_i$. Find the distribution of $U$.

### Part D

Now suppose that independence still holds, but $X_i \sim exponential(\lambda_i)$. Let $U = \sum_{i=1}^{N} X_i$. Find the distribution of $U$.