STAT4106 Homework 6, Due Monday November 4^{th}

John W Smith Jr

Problem 1

Suppose that $X_1, ..., X_n \sim$ iid $Gamma(\alpha, \beta)$ and α is fixed. Let the pdf be given by

$$f(x|\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} \exp(-x\beta)$$

Part A

Compute the maximum likelihood estimator for β.

Part B

The variance of a Gamma distribution is given by $\alpha\beta^2$. What is the maximum likelihood estimator for $\alpha\beta^2$?

Part C

Suppose that we perform a Bayesian analysis with $\pi(\beta) \sim Exp(\lambda_0)$, with pdf

$$f(\beta|\lambda_0) = \lambda_0 \exp(-\lambda_0\beta)$$

What is the distribution of the posterior? Give a name and parameter(s).

Part D

What is the Bayes estimator for β?

Part E

Suppose that α is no longer fixed. Find the method of moments estimator for (α, β). Is this the MVUE? Give full justification.

Problem 2

Suppose that $X_1, ..., X_n \sim$ iid $Bern(p)$.

Part A

Find the maximum likelihood estimator for p.

Part B

Find the method of moments estimator for p.

Part C

The variance of a Bernoulli distribution is given by $p(1-p) = p - p^2$. Compute the maximum likelihood estimator for $p(1-p)$.

Part D

Suppose that we want to perform a Bayesian analysis, with $\pi(p) \sim Beta(a, b)$. What distribution does the posterior follow? Give a name and parameter(s).

Part E

What is the resulting Bayes estimator from your posterior in Part D?