STAT4106 Homework 3: Due September 20^{th}

John W Smith Jr

All submitted work is expected to be your own. Violations to this will be at the mercy of the Virginia Tech Honor Court.

Submitted assignments should be very neat and easy to read. For this reason, LaTeX is preferred. Microsoft Word documents must be rendered as pdf or html files, or resubmission will be required with a loss of 10 points. Hand written assignments are accepted so long as they are neat and legible. Homeworks must be stapled or 10 points will be lost.

Problem 1: A Central Limit Theorem Simulation

Here we will perform a Central Limit Theorem simulation similar to the ones done in class. That is:

- Pick a distribution (that was not presented in class)
- Justify that the distribution will abide by the central limit theorem
- Find a parameter set and value for N where we can see that the central limit theorem clearly applies
- Find a parameter set and value for N where we can see that the central limit theorem does not apply
- Code must be submitted (preferably in R, but MATLAB, Python, ForTran, and C++ will also be accepted).

Problem 2: Comparing Binomial Estimators

Suppose that $X_1, ..., X_N \sim Bern(p)$ are independent. Then, by definition, $U = \sum_{i=1}^{N} X_i \sim Binom(N, p)$. Let us consider two estimators for p,

$$\hat{p}_1 = \bar{X}$$

$$\hat{p}_2 = \frac{\sum_{i=1}^{N} X_i + .5}{N+1}.$$

The first estimator is the sample mean, and is a common estimator for p in a binomial distribution. The second estimator is the result of performing a Bayesian analysis with a reference prior. We will compare them here.

Part 1

Compute the MSE of each estimator.

Part 2

Which estimator performs better? If they perform better in different parts of the space, compute the respective intervals where each estimator performs better (as a function of N).

Problem 3: Uniform estimators

Suppose $X_1, ..., X_N \sim$ i.i.d. $Unif(0, \theta)$.

Part 1

Compute the MSE of $\hat{\theta} = X_{(N)}$. You may use results from Homework 1 to simplify calculations. How does this compare to the estimator $\hat{\theta} = 2 \cdot \bar{X}$ that we worked with in class?

Part 2

Find $c \in \mathbb{R}$ that minimizes that minimizes MSE for estimators of the form $\hat{\theta} = cX_{(N)}$

Part 3 (Extra Credit)

In class we briefly discussed that a property we wanted for $\hat{\theta}$ was that $\hat{\theta} \in (0, \theta)$. If we restrict $\hat{\theta}$ to being a random variable, we can show that there is no unbiased estimator of θ with this property. Prove this fact.