STAT4106 Homework 7: Due Wednesday, Dec 11^{th}

John W Smith Jr

Problem 1

Suppose that $X_1, X_2, ..., X_n \sim$ i.i.d. $N(\mu, \sigma^2)$, with σ^2 known.

Part A

Compute the LRT statistic for the hypotheses $H_0 : \mu \leq \mu_0$ vs $H_a : \mu > \mu_0$.

Part B

Show that this test statistic is equivalent to the standard one-sided z-test.

Part C

Suppose now that σ^2 is unknown. Find the new LRT statistic and show that this test statistic and rejection rule are equivalent to a test based on the Student's $t-$statistic.

Problem 2

Suppose that a factory foreman is interested in testing whether or not one of his pieces of machinery needs to be calibrated. To do this, he looks at $n = 50$ items $(X_1, ..., X_n)$ and tests whether or not they are defective. He picks $H_0 : p = .02$ vs $H_a : p > .02$

Part A

Suppose that the foreman decides to use $RR = \{x | \sum_{i=1}^{n} x \geq 4\}$. Compute α, the Type I error rate.

Part B

Suppose that the true value for p is $p = .03$. Compute β, the Type II error rate, as well as the power of the test.

Part C

The factory foreman decides that instead of using an exact test with the binomial distribution, he will use a normal approximation to the binomial distribution. Give the form of the test statistic.

Part D

The foreman decides that we wants to keep an equivalent rejection rule based on his exact binomial test. Find the new rejection region, RR^* for the normal approximation. Is the Type I error rate the same? Justify your answer.

Problem 3

Suppose that we have a random variable with pdf given by

$$f(x) = \frac{\exp(-\sqrt{x})}{2\sqrt{x}}, x \in (0, \infty)$$

Part A

Find the CDF, $F(x)$.

Part B

Find the inverse cdf, $F^{-1}(x)$

Part C

Write psuedo-code to outline how to generate a random sample from the pdf $f(x)$.

Part D

Using the software of your choice, generate 10,000 random samples from $f(x)$. Overlay the density of $f(x)$ and confirm that we have generated random samples from the desired pdf. Are these exact samples?