

Measuring the value of groundwater and other forms of natural capital

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Valuing natural capital is fundamental to measuring sustainability. The United Nations Environment Programme, World Bank, and other agencies have called for inclusion of the value of natural capital in sustainability metrics, such as inclusive wealth. Much has been written about the importance of natural capital, but consistent, rigorous valuation approaches compatible with the pricing of traditional forms of capital have remained elusive. We present a guiding quantitative framework enabling natural capital valuation that is fully consistent with capital theory, accounts for biophysical and economic feedbacks, and can guide interdisciplinary efforts to measure sustainability. We illustrate this framework with an application to groundwater in the Kansas High Plains Aquifer, a rapidly depleting asset supporting significant food production. We develop a 10-y time series (1996–2005) of natural capital asset prices that accounts for technological, institutional, and physical changes. Kansas lost approximately \$110 million per year (2005 US dollars) of capital value through groundwater withdrawal and changes in aquifer management during the decade spanning 1996–2005. This annual loss in wealth is approximately equal to the state's 2005 budget surplus, and is substantially more than investments in schools over this period. Furthermore, real investment in agricultural capital also declined over this period. Although Kansas' depletion of water wealth is substantial, it may be tractably managed through careful groundwater management and compensating investments in other natural and traditional assets. Measurement of natural capital value is required to inform management and ongoing investments in natural assets.



I. Derivation of Dynamic Equation

$$p(s) = \frac{MD(s, x(s)) + \dot{p}(s)}{\delta - [MG(s) - MHI(s, x(s))]}$$

Intuition,

$$\frac{\text{Marginal ecosystem service flow} + \text{anticipated price (scarcity)}}{\text{discount rate} - \text{adjustment to natural capital (Growth minus Impact)}}$$

Step 1: Define resource changes over time,

$$\frac{ds}{dt} = \dot{s} = \underbrace{G(s(t))}_{\text{Growth Fn}} - \underbrace{HI(s(t), x(s(t)))}_{\text{Human Impact Fn}}$$

Step 2: Define index of net benefits,

$$D(s, x(s(t)))$$

Step 3. Define net present value of benefits generated by natural capital at time t

$$V(s(t)) = \int_t^\infty e^{\delta(\tau-t)} D(s(\tau), x(s(\tau))) d\tau$$

Step 4. Express V as a function of $s(t)$, substitute economic program, $x(s)$, in D . (Envelope Theorem Derivation)

$$D_S(s(t), x(s(t))) = \frac{\partial D}{\partial s} + \underbrace{\frac{\partial D}{\partial x} \frac{dx}{ds}}_{=0} = \frac{\partial D}{\partial s} = D_S^* = MD$$

Step 5: Define accounting price for capital (benefit from marginal increase in stock of capital),

$$p(s(t)) = \frac{\partial V(s(t))}{\partial s(t)}$$

Step 6. Differentiate $V(s(t))$ w.r.t. t ,

$$\frac{\partial V}{\partial t} = \delta V - D(s, x(s)) = \frac{\partial V}{\partial s} \frac{ds}{dt} = p(s(t)) \frac{ds}{dt}$$

rewrite,

$$\delta V = D(s, x(s)) + p(s(t)) \dot{s} = H^*(s, p)$$

where $H^*(s, p)$ is the current value Hamiltonian with economic program $x(s)$, with flow of benefits $D(s, x(s))$, and the value of increments to stock, $p\dot{s}$

rearrange,

$$V(s) = \frac{D(s, x(s)) + p(s(t)) \dot{s}}{\delta}$$

Now, differential $V(s)$ w.r.t. s and rearrange,

$$p(s) = \frac{D(s, x(s)) + \dot{p}(s)}{\delta - (G_s(s) - HI_s(s, x(s)))}$$

Remove subscripts, s , and replace with marginal impacts of the stock with M .

$$p(s) = \frac{MD(s, x(s)) + \dot{p}(s)}{\delta - [MG(s) - MHI(s, x(s))]}$$

Done. ■

II. Approximate Dynamic Equation

Need to approximate unknown parameters, p and \dot{p}

$$p \approx \mu(s(t)) = \sum_{n=0}^{k-1} \beta_n \phi_n(s(t))$$

where $\mu(s(t))$ is a Chebyshev polynomial in s and k basis functions (approximations). $\phi_n(s(t))$ are basis functions, and β_n are a set of parameters determining the weighting of these basis functions.

$$\dot{p} \approx \mu_s(s(t)) \dot{s}(t)$$

where $\mu_s(s(t)) = \sum_{n=0}^{k-1} \beta_n \Delta \phi_n(s(t))$ and $\Delta \phi_n(s(t))$ is the derivative of the basis function w.r.t. $s(t)$. \dot{p} requires higher-order derivatives.

III. Replication Code

Estimate $MD(s, x(s))$ and $MHI(s, x(s))$ where economic program $x(s)$ reflects decisions of crop choice and irrigation pumping as a function of the groundwater stock under an acre

$MD(s, x(s))$ links the change in the water in the aquifer with the change in the field-level net revenues.

$MHI(s, x(s))$ is influenced indirectly through the economic program by the irrigation requirements of various crop choices.

$MHI(s)$ computed as the change in water in the aquifer resulting from water withdrawal.

Recharge rate = 1.25 per year.

Discount rate = 3%

$MG(s) = 0$ because hydrologic theory does not support stock-dependent recharge rates.

Estimate,

$$p(s) = \frac{MD(s, x(s)) + \dot{p}}{0.03 + MHI(s, x(s))}$$

$MD(s, x(s))$ and $MHI(s, x(s))$ can be estimated using econometric methods or simple derivation of functional form.

\dot{p} is estimated using a Chebychev Polynomial to forecast capital gains from observed price data.

1. Environment Setup

```
In [65]: ### Load libraries
library(tidyverse)
library(capn)
library(repmis)

### Set git branch
gitbranch <- "master"

### Load external functions (system_fns.R)
source(paste0("https://raw.githubusercontent.com/efenichel/capn_stuff/",gitbranch,"/system_fns.R")) #will need to fix in the end

### Load script to process raw data (data_setup.R)
source(paste0("https://github.com/efenichel/capn_stuff/raw/",gitbranch,"/data_setup.R"))

### Get data
source_data(paste0("https://github.com/efenichel/capn_stuff/raw/",gitbranch,"/KSwater_data.RData")) #Rdata file upload
ksdata <- KSwater_data #save KSwater_data as ksdata

## STRUCTURE of ksdata
#The object ksdata is a list of 7 lists. Each of the 7 lists corresponds to a groundwater management district (1-5),
# the outgroup(6), or the entire state of Kansas(7).
# Each list will have 11 named elements, the number reference the elements in the list:
# [1] $gmdnum: int    1:7
# [2] $mlogitcoeff: df with dim = (#cropcodes x 23) containing the coefficients and intercept terms from the multinomial logit model
# [3] $mlogitmeans: df containing the mean for each of the variables run in the mlogit model
# [4] $cropamts: df containing the mean acres planted to each of the 5 crop cover types for each cropcode
# [5] $watercoeff: df containing the water withdrawal regression coefficients and intercept
# [6] $wwdmeans: df containing the means for each of the variables in the water withdrawal regression
# [7] $costcropacre: df dim=(1x5) containing the cost per acre of planting each of the 5 crops
# [8] $cropprices: df dim=(5x1) containing the per unit prices of each of the crops
# [9] $meanwater: num mean AF water in the region of interest
# [10] $recharge: num recharge rate for the region of interest
# [11] $watermax: num upper bound of domain for node space, max water observed in region.
```

Downloading data from: https://github.com/efenichel/capn_stuff/raw/master/KSwater_data.RData

SHA-1 hash of the downloaded data file is:
b739fc2e4800300edb8c2d90da79d86d557a58df

'KSwater_data'

2. Data Step

```
In [66]: ### Set region
my.region <- 7

### Create data structure
if (!exists("region")){ region <- my.region }
region_data <- ksdata[[region]]

### Process data
gw.data <- datasetup(region)

# the gw.data data repackages parameters and means, see the datasetup
code.
# the return is of the form
# list(crop.coeff, crop.amts, alpha, beta, gamma, gamma1, gamma2, cro
p.prices, cost.crop.acre)
```

3. Model Setup

```
In [67]: ### Economic Parameters
dr <- 0.03      # discount rate

### System parameters
recharge <- region_data[['recharge']]      # units are inches per year
constant rate

### capN parameters
order <- 10      # approximaton order
NumNodes <- 100   # number of nodes
wmax <- region_data[['watermax']]           # This sets the the upper bo
und on water amount to consider

### Prepare capN
Aspace <- aproxdef(order, 0, wmax, dr)      # defines the approximat
on space
nodes <- chebnodegen(NumNodes, 0, wmax)     # define the nodes

### Prepare for simulation
simuData <- matrix(0, nrow = NumNodes, ncol = 5)
```

4. Estimation

```

In [68]: ### Simulate at nodes
for(j in 1:NumNodes){
  simuData[j, 1] <- nodes[j] # water depth
nodes
  simuData[j, 2] <- sdot(nodes[j], recharge, gw.data) # change in s
tock over change in time
  simuData[j, 3] <- 0 - WwdDs1(nodes[j], gw.data) # d(sdot)/ds,
of the rate of change in the change of stock
  simuData[j, 4] <- ProfDs1(nodes[j], gw.data) # Change in p
rofit with the change in stock
  simuData[j, 5] <- profit(nodes[j], gw.data) # profit
}

### Recover approximating coefficients
pC <- paprox(Aspace, simuData[, 1], simuData[, 2], simuData[, 3], simuData[, 4]) #the approximated coefficient vector for prices

### Project shadow prices, value function, and inclusive wealth
waterSim <- psim(pcoeff = pC,
                 stock = simuData[,1],
                 wval = simuData[,5],
                 sdot = simuData[,2])

### Convert to data.frame
waterSim <- as.data.frame(waterSim)

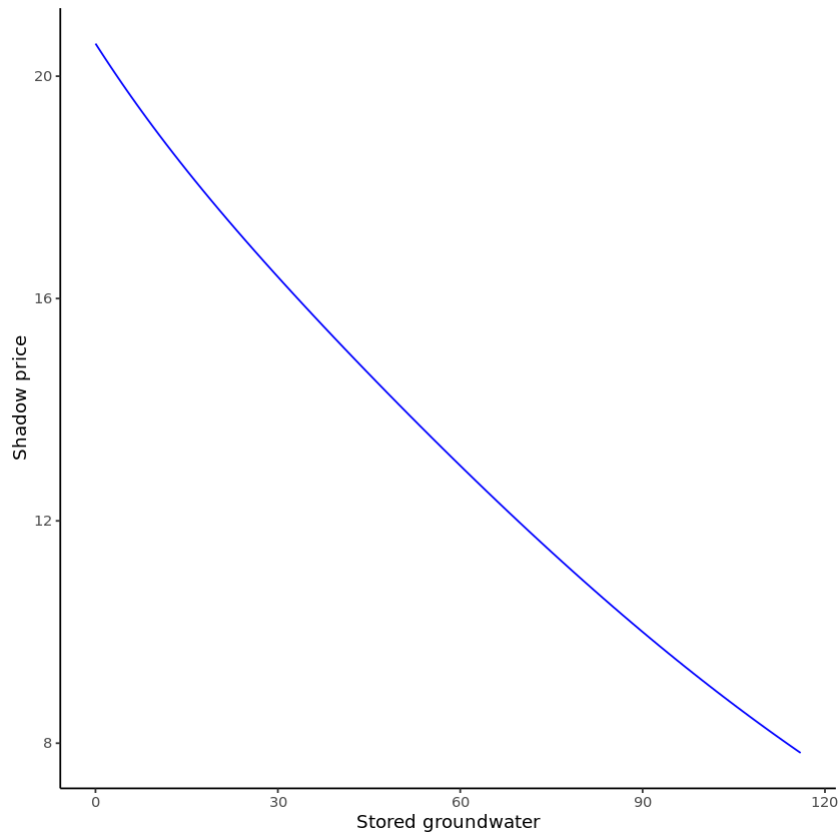
cat("if everything runs well the next line should say 17.44581", "\n"
)
cat("At 21.5 acre feet of water, the shadow price is" , psim(pC, 21.5)
)$shadowp, "\n")

if everything runs well the next line should say 17.44581
At 21.5 acre feet of water, the shadow price is 17.44581

```

5. Plot Results

```
In [69]: ### Plot water shadow price function  
ggplot() +  
  geom_line(data = waterSim, aes(x = stock, y = shadowp), color =  
    'blue') +  
  labs(x= "Stored groundwater", y = "Shadow price") +  
  theme(axis.line = element_line(color = "black"),  
        panel.background = element_rect(fill = "transparent", colour  
= NA),  
        plot.background = element_rect(fill = "transparent", colour  
= NA))
```

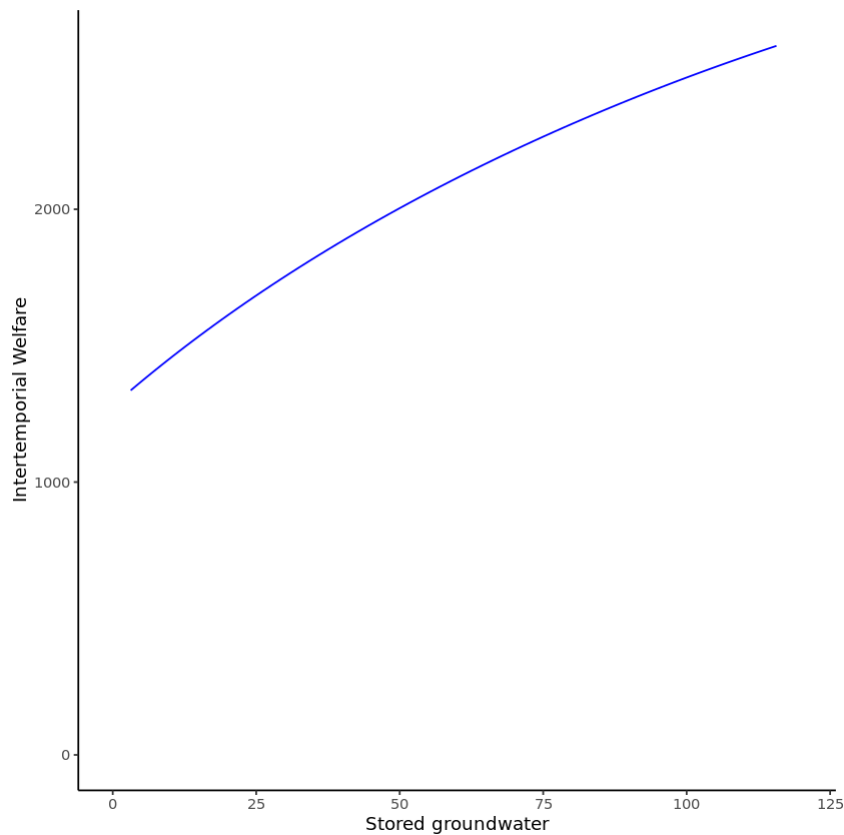



```
In [70]: ### Plot value function
lrange <- 6 # the closest nodes to zero have some issues.

ggplot() +
  geom_line(data = waterSim[lrange :100,], aes(x = stock[lrange :100]
], y = vfun[lrange :100]), color = 'blue') +
  xlim(0, 120) +
  ylim(0, 2600) +
  labs(x= "Stored groundwater", y = "Intertemporal Welfare") +
  theme(axis.line = element_line(color = "black"),
        panel.background = element_rect(fill = "transparent",colour =
NA),
        plot.background = element_rect(fill = "transparent",colour =
NA))
```

Warning message:

“Removed 8 row(s) containing missing values (geom_path).”



```
In [71]: ### Check on results
testme <- psim(pcoeff = pC,
              stock = c(18.5, 21.5),
              wval = c(profit(18.5, gw.data), profit(21.5, gw.data)),
              sdot = c(sdot(18.5, recharge, gw.data), sdot(21.5, recharge, gw.data)))
print(testme)
testme
```

\$shadowp

```
      [,1]  
[1,] 17.83600  
[2,] 17.44581
```

\$iw

```
      [,1]  
[1,] 329.9659  
[2,] 375.0848
```

\$vfun

```
      [,1]  
[1,] 1588.393  
[2,] 1633.214
```

\$stock

```
      [,1]  
[1,] 18.5  
[2,] 21.5
```

\$wval

```
[1] 91.71122 92.96710
```

\$shadowp

A matrix: 2
× 1 of type
dbl

17.83600

17.44581

\$iw

A matrix: 2
× 1 of type
dbl

329.9659

375.0848

\$vfun

A matrix: 2
× 1 of type
dbl

1588.393

1633.214

\$stock

A
matrix:
2 × 1
of type
dbl

18.5

21.5

\$wval

91.7112163590698 · 92.9670976298727