Before Maxwell, the equations that became Maxwell's equations didn't form a single consistent system. They were:

- Gauss's Law (1773/1813)
- The Law with No Name[1] ("There are no magnetic monopoles")
- Faraday's Law (1831)
- Ampere's Law (1827)

Maxwell made an addition to Ampere's Law in 1861. Ampere's original law (in modern notation) is:

$$\vec{\nabla} \times \vec{B} = \frac{\vec{j}}{\epsilon_0 c^2}$$

Maxwell noticed that if you take the divergence of each side, you get this (since the divergence of any curl is zero):

$$\vec{\nabla} \cdot \vec{j} = 0$$

Since this has to be true in general, it must mean that there can't be any current anywhere, which can't be right. Maxwell added another term to fix this:

$$c^2 \vec{\nabla} \times \vec{B} = \frac{\vec{j}}{\epsilon_0} + \frac{\partial \vec{E}}{\partial t}$$

This is Ampere's Law with Maxwell's Addition[2]. Now if we take the divergence of both sides, we can follow a few steps that lead to the equation for conservation of charge:

$$0 = \vec{\nabla} \cdot \frac{\vec{j}}{\epsilon_0} + \vec{\nabla} \cdot \frac{\partial \vec{E}}{\partial t}$$ Take the divergence of both sides

$$0 = \vec{\nabla} \cdot \frac{\vec{j}}{\epsilon_0} + \frac{\partial}{\partial t} \vec{\nabla} \cdot \vec{E}$$ Interchange the order of differentiation

$$0 = \vec{\nabla} \cdot \frac{\vec{j}}{\epsilon_0} + \frac{\partial}{\partial t} (\rho / \epsilon_0)$$ Apply Gauss's Law

$$0 = \vec{\nabla} \cdot \vec{j} + \frac{\partial \rho}{\partial t}$$ Cancel $1/\epsilon_0$

$$\vec{\nabla} \cdot \vec{j} = -\frac{\partial \rho}{\partial t}$$ Conservation of charge

In his lectures, Feynman says that the new term "doesn't do anything interesting on it's own" and this is what he means. All it really does is enforce conservation of charge.

Footnotes

[1]: I originally wanted to list all the equations that became Maxwell's equations and the years in which they were originally formulated. I was a little bothered when I couldn't find a name or year for the equation that says that the divergence of the magnetic field is zero. As I thought about it, I decided that I actually kind of like just calling it "The Law with No Name".

[2]: Maxwell's equations show up in a lot of different forms and I like to make them stand out, so I color-code them. Ampere's Law with Maxwell's Addition shows is red, for example.