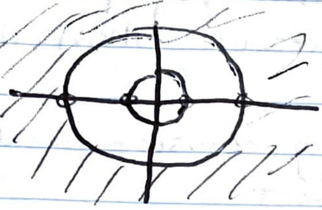
3.21) a) The ROC of H(z) is $|z| > .5$ which is the outer ring of the two poles due to the causal property of the system



b) The system is stable since $z=1 \in$ ROC determined in part a)

d) on next page ⇒

c) $$H(z) = \frac{Y(z)}{X(z)} = \frac{4 + 0.25z^{-1} - 0.5z^{-2}}{(1-0.25z^{-1})(1+0.5z^{-1})}$$

$$Y(z)\left((1-0.25z^{-1})(1+0.5z^{-1})\right) = X(z)\left(4 + 0.25z^{-1} - 0.5z^{-1}\right)$$

$$\Downarrow$$

$$Y(z)\left(1 + 0.25z^{-1} - 0.125z^{-2}\right) = X(z)\left(4 + 0.25z^{-1} - 0.5z^{-1}\right)$$

$$y[n] + 0.25y[n-1] - 0.125y[n-2] = 4x[n] + 0.25x[n-1] - 0.5x[n-2]$$

$$y[n] = 4x[n] + 0.25x[n-1] - 0.5x[n-2] - 0.25y[n-1] + 0.125y[n-2]$$

d) $$H(z) = \frac{4 + 0.25z^{-1} - 0.5z^{-2}}{(1 + 0.25z^{-1} + 0.125z^{-2})} = -4 + \frac{8 - 0.75z^{-1}}{(1-0.25z^{-1})(1+0.5z^{-1})}$$

$$-4 + \frac{A}{(1-0.25z^{-1})} + \frac{B}{(1+0.5z^{-1})}$$

$$A = \frac{8 - 0.75(4)}{1 + 0.5(4)} = 5/2$$

$$B = \frac{8 - 0.75(-2)}{1 - 0.25(-2)} = \frac{6.8}{1.8} = 13/3$$

$$H(z) = -4 + \frac{5/2}{(1-0.25z^{-1})} + \frac{13/3}{(1+0.5z^{-1})}$$

$$h[n] = -4\delta[n] + 5/2\,(1/4)^n u[n] + 13/3\,(-1/2)^n u[n]$$

d) $H(z) = \dfrac{4 + 0.25z^{-1} + 0.5z^{-2}}{1 + 0.25z^{-1} - 0.125z^{-2}} = 4 + \dfrac{(-0.75z^{-1})}{(1-0.25z^{-1})(1+0.5z^{-1})}$

$$\frac{-0.75z^{-1}}{(1-0.25z^{-1})(1+0.5z^{-1})} = \frac{A}{(1-0.25z^{-1})} + \frac{B}{(1+0.5z^{-1})}$$

$$B = \left.\frac{-0.75z^{-1}}{1-0.25z^{-1}}\right|_{z^{-1}=-2} = \frac{1.5}{1.5} = 1$$

$$A = \left.\frac{-0.75z^{-1}}{1+0.5z^{-1}}\right|_{z^{-1}=4} = \frac{-3}{3} = -1$$

$$H(z) = 4 + \frac{-1}{(1-0.25z^{-1})} + \frac{1}{(1+0.5z^{-1})}$$

$$\Rightarrow h[n] = 4\delta[n] - (1/4)^n u[n] + (1/2)^n u[n]$$

$16 + z^{-1} - 2z^{-2}$

e) $x[n] = u[-n-1] \Rightarrow X(z) = \frac{1}{1-z^{-1}}$ ROC: $|z| < 1$

$$Y(z) = H(z) \cdot X(z)$$

$$= \frac{4 + 0.25z^{-1} - 0.5z^{-2}}{(1 - 0.25z^{-1})(1 + 0.5z^{-1})} \cdot \frac{1}{(1-z^{-1})}$$

ROC: $0.5 < |z| < 1$

f)

$$Y(z) = \frac{4 + 0.25z^{-1} - 0.5z^{-2}}{(1 - 0.25z^{-1})(1 + 0.5z^{-1})(1-z^{-1})} =$$

$$\frac{A}{(1-0.25z^{-1})} + \frac{B}{(1+0.5z^{-1})} + \frac{C}{1-z^{-1}}$$

$$A = \left.\frac{4 + 0.25z^{-1} - 0.5z^{-2}}{(1+0.5z^{-1})(1-z^{-1})}\right|_{z^{-1}=4} = \frac{4+1-\cancel{8}^{-3}}{(3)(-3)} = \cancel{}\ 1/3$$

$$B = \left.\frac{4 + 0.25z^{-1} - 0.5z^{-2}}{(1-0.25z^{-1})(1-z^{-1})}\right|_{z^{-1}=-2} = \frac{4 - 1/2 \cancel{+2}}{(1+\frac{1}{2})(1+2)} = \cancel{}\ \frac{1}{3}$$

$$C = \left.\frac{4 + 0.25z^{-1} - 0.5z^{-2}}{\underset{1.5}{(1-0.25z^{-1})}\ \underset{1+2}{(1+0.5z^{-1})}}\right|_{z^{-1}=1} = \frac{3.75}{(.75)(1.5)} = \frac{10}{3}$$

$$y[n] = 1/3\,(1/4)^n u[n] + 1/3\,(-1/2)^n u[n] + \frac{10}{3}(-u[-n-1])$$

$\frac{\pi}{2}(n-2)$

3.22) $H(z) = \dfrac{1-4z^{-2}}{1+0.5z^{-1}}$

$\rightarrow$ Find steady state ~~gain~~

$Y(z)(1+0.5z^{-1}) = X(z)(1-4z^{-2})$

$y[n] + \frac{1}{2}y[n-1] = x[n] - 4x[n-2]$

$\lim_{n\to\infty} x[n] - 4x[n-2] = (1-2\cos(\frac{\pi}{2}n)) - 4(1-2\cos(\frac{\pi}{2}(n-2)))$

$= 1 - 2\cos(\frac{\pi n}{2}) - 4 - 8\cos(\frac{\pi n}{2} - \pi)$

$= 1 - 2\cos(\frac{\pi n}{2}) - 4 + 8\cos(\frac{\pi n}{2})$

$= -3 + 6\cos(\frac{\pi n}{2}) = y[n] + \frac{1}{2}y[n-1]$ at large n

$$H(e^{j\pi/2}) = \frac{1-4e^{-j\pi}}{1+\frac{1}{2}e^{-j\pi/2}} = \frac{5}{1-j\frac{1}{2}} \cdot \frac{1+j^1/_2}{1+j^1/_2} = \frac{5+j\,5/2}{1+\frac{1}{4}}$$

$$= \frac{5+j\,5/2}{5/4}$$

$$= 4+2j$$

$|H(e^{j\pi/2})| = \sqrt{18}$

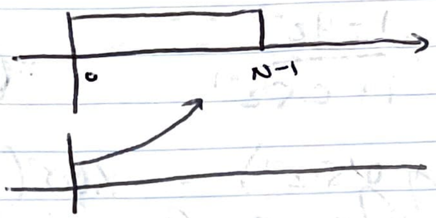
$\angle H(e^{j\pi/2}) = .464$ rad. } GAIN for cosine

$H(e^{j0}) = H(1) = \dfrac{1-4}{1+0.5} = -2$ } gain for $u[n]$

$\lim_{n\to\infty} y[n] = -2 + 2\sqrt{18}\cos(\pi/2\, n + .464)$ } STEADY STATE

3.23) a) $x[n]$:

$h[n]$:



$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

for $n < 0$:

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

because $x[n] = 0$ for all negative values

$$y[n] = 0$$

for $0 \le n \le N-1$

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k] = \sum_{k=0}^{n} a^k = \frac{(1-a^{n+1})}{1-a}$$

for $n \geq N$:

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k] = \sum_{k=n-(N-1)}^{n} a^k$$

$$= \sum_{k=0}^{n} a^k - \sum_{k=0}^{n-N} a^k = \frac{(1-a^{n+1})}{1-a} - \frac{(1-a^{n-N+1})}{1-a}$$

$$= \frac{-(a^{n+1} - a^{n-N+1})}{1-a} = -a^{n+1}(1-a^{-N}) / 1-a$$

b) $h[n] = a^n u[n]$

$x[n] = u[n] - u[n-N]$

$$H(z) = \frac{1}{1-az^{-1}} \qquad X(z) = \frac{1}{1-z^{-1}} - \frac{z^{-N}}{1-z^{-1}} = \frac{1-z^{-N}}{1-z^{-1}}$$

$$Y(z) = H(z)X(z) = \frac{1-z^{-N}}{(1-z^{-1})(1-az^{-1})}$$

$$Y(z) = H(z)X(z) = \frac{1 - z^{-N}}{(1-az^{-1})(1-z^{-1})}$$

$$= \underbrace{\frac{1}{(1-az^{-1})(1-z^{-1})}}_{\text{Non shifted}} - \underbrace{\frac{z^{-N}}{(1-az^{-1})(1-z^{-1})}}_{\text{shifted}}$$

$$\frac{1}{(1-az^{-1})(1-z^{-1})} = \frac{A}{(1-az^{-1})} + \frac{B}{1-z^{-1}}$$

$$\left.\begin{aligned} A &= \frac{1}{1-a^{-1}} \\ B &= \frac{1}{1-a} \end{aligned}\right\}$$

$$= \frac{1}{1-a}\left(\frac{-a}{1-az^{-1}} + \frac{1}{1-z^{-1}}\right)$$

$$\hookrightarrow \frac{1}{1-a}\left(-a\left(a^n u[n]\right) + u[n]\right)$$

$$= \frac{1}{1-a} \cdot u[n]\left(1 - a^{n+1}\right) = \frac{1-a^{n+1}}{1-a} u[n]$$

$$\frac{1-a^{n+1}}{1-a} u[n] - \frac{1-a^{n-N+1}}{1-a} u[n-N] = y[n]$$

3.25) a) $\sum_{k=-\infty}^{\infty} \delta(n-4k) = \cdots + \delta(n-4) + \delta(n) + \delta(n+4) + \cdots$

$\Rightarrow \cdots + z^{-4} + 1 + z^{4} \cdots$

$X(z) = \sum_{k=-\infty}^{\infty} z^{4k} \Big\}$ ROC(X) = entire z plane but not $z=0, z=\infty$

b) $\frac{1}{2}\left[e^{j\pi n} + \cos(\pi/2\, n) + \sin\left(\frac{\pi}{2} + 2\pi n\right)\right] u[n]$

$= \frac{1}{2}\left[(-1)^n + \cos(\pi/2\, n) + 1\right] u[n] \longrightarrow$ periodic with 4

$n=0 \Rightarrow (1+1+1)\frac{1}{2} = 3/2$

$n=1 \Rightarrow (-1+0+1) = 0$

$n=2 \Rightarrow (1-1+1)\frac{1}{2} = \frac{1}{2}$

$n=3 \Rightarrow (-1+0+1) = 0$

$= \frac{3}{2}\sum_{k=0}^{\infty} \delta(n-4k) + \frac{1}{2}\sum_{k=0}^{\infty} \delta(n-(4k+2))$

$\Downarrow z$

$\frac{3}{2}\sum_{k=0}^{\infty} z^{-4k} + \frac{1}{2}\sum_{k=0}^{\infty} z^{-4k-2}$

$= \frac{3}{2}\sum_{k=0}^{\infty} z^{-4k} + \frac{1}{2}z^{-2}\sum_{k=0}^{\infty} z^{-4k} = \dfrac{\frac{3}{2} + \frac{1}{2}z^{-2}}{1 - z^{-4}}$

ROC → converges only if $|z| > 1$

$\cancel{3.26 \quad X(z) = \frac{z^2}{(z-a)(z-b)} = \frac{A}{z-a} + \frac{B}{z-b}}$

$\cancel{A = \frac{2a\,a - ab}{a-b} = \frac{2a^2 b - ab}{a-b}} \qquad \cancel{B = \frac{2ab\,b - ab}{b-a} = \frac{2ab^2 - ab}{b-a}}$

3.26) $X(z) = \dfrac{z^2}{(z-a)(z-b)} = 1 + \dfrac{(a+b)z - ab}{(z-a)(z-b)}$

$$\frac{(a+b)z - ab}{(z-a)(z-b)} = \frac{A}{z-a} + \frac{B}{z-b}$$

$$A = \frac{(a+b)a - ab}{a-b} = \frac{a^2}{a-b}$$

$$B = \frac{(a+b)b - ab}{b-a} = \frac{b^2}{b-a}$$

$$X(z) = 1 + \frac{1}{a-b}\left(\frac{a^2}{z-a} - \frac{b^2}{z-b}\right)$$

$$= 1 + \frac{1}{a-b}\left(\frac{a^2 z^{-1}}{1-az^{-1}} - \frac{b^2 z^{-1}}{1-bz^{-1}}\right)$$

$$= \delta[n] + \frac{a^2}{a-b} a^{n-1} u[n-1] - \frac{b^2}{a-b} b^{n-1} u[n-1]$$

$$= \delta[n] + \frac{1}{a-b}\left(a^{n+1} u[n+1] - b^{n+1} u[n-1]\right)$$

3.31) a) $1+\frac{1}{3}z^{-1} \overline{\big) 1-\frac{1}{3}z^{-1}}$ quotient: $1-\frac{2}{3}z^{-1}+\frac{2}{9}z^{-2}-\frac{2}{27}z^{-3}$

$$-1-\frac{1}{3}z^{-1}$$

$$-\frac{2}{3}z^{-1}$$

$$-\left(-\frac{2}{3}z^{-1}-\frac{2}{9}z^{-2}\right)$$

$$\frac{2}{9}z^{-2}$$

$$-\frac{2}{9}z^{-2}-\frac{2}{27}z^{-3}$$

$$H(z)=1-\frac{2}{3}z^{-1}+\frac{2}{9}z^{-2}-\frac{2}{27}z^{-3}\ldots$$

$$=2\sum_{k=0}^{\infty}\left(-\frac{1}{3}z^{-1}\right)^{k}-1$$

$$=\frac{2}{1+\frac{1}{3}z^{-1}}-1 \Rightarrow 2\left(-\frac{1}{3}\right)^{n}u[n]-\delta[n]$$

b) $$\frac{3}{\cancel{z}-\frac{1}{4}-\frac{1}{8}z^{-\cancel{1}}}=\frac{3z^{-1}}{\cancel{1}-\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}}=\frac{3z^{-1}}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{4}z^{-1})}$$

$$=\frac{A}{1-\frac{1}{2}z^{-1}}+\frac{B}{1+\frac{1}{4}z^{-1}}$$

$$B=\left.\frac{3z^{-1}}{1-\frac{1}{2}z^{-1}}\right|_{z^{-1}=-4}=\frac{-12}{3}=-4$$

$$A=\left.\frac{3z^{-1}}{1+\frac{1}{4}z^{-1}}\right|_{z^{-1}=2}=\frac{6}{1.5}=4$$

$$=\frac{4}{1-\frac{1}{2}z^{-1}}-\frac{4}{1+\frac{1}{4}z^{-1}}$$

$$=4\left(\frac{1}{2}\right)^{n}u[n]-4\left(\frac{-1}{4}\right)^{n}u[n]$$ causal due to stability

c) $X(z) = \ln(1-4z)$

$$= \sum_{k=1}^{\infty} \frac{-(4z)^k}{k} = -4z - \frac{4^2}{2}z^2 - \frac{4^3}{3}z^3 \ldots$$

$$x[n] = \frac{1}{n}(4)^{-n} \underbrace{u[-n-1]}_{\text{Anti causal}}$$

d) $X(z) = \dfrac{1}{1-\frac{1}{3}z^{-3}}$

$$\begin{array}{r} 1 + \frac{1}{3}z^{-3} + \frac{1}{9}z^{-6} \ldots \\ 1-\frac{1}{3}z^{-3} \overline{)\,1 \qquad\qquad\qquad} \\ -1 + \frac{1}{3}z^{-3} \\ \hline \frac{1}{3}z^{-3} \\ -\frac{1}{3}z^{-3} + \frac{1}{9}z^{-6} \end{array}$$

$$X(z) = 1 + \frac{1}{3}z^{-3} + \frac{1}{9}z^{-6} \ldots$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{3}z^{-3}\right)^k \Rightarrow x[n] = \delta[n] + \frac{1}{3}\delta[n-3] + \frac{1}{9}\delta[n-6] \ldots$$

$$= \begin{cases} \left(\frac{1}{3}\right)^{n/3} u[n] & \forall n \text{ multiple of } 3 \\ 0 & \text{otherwise} \end{cases}$$