Model Predictive Control

The Model

In this project, I use the same model taught in the course. The state $[x, y, \psi, v, cte, e\psi]$ contains the following value:

- x, the x coordinate of the vehicle
- y, the y coordinate of the vehicle
- ψ, the heading of the vehicle
- v, the speed of the vehicle
- cte, the cross track error
- $e\psi$, the heading error

The actuators $[\delta, a]$ contains the following two values:

- δ, the steering angle of the vehicle, in this project it is set to be between -25 degrees and 25 degrees.
- a, the acceleration of the vehicle, in this project it is set to be between -1.0 and 1.0.

And the update equations are as following:

- $x_{t+1} = x_t + v_t * \cos(\psi_t) * dt$
- $y_{t+1} = y_t + v_t * \sin(\psi_t) * dt$
- $\psi_{t+1} = \psi_t + \frac{v_t}{L_f} * \delta_t * dt$
- $v_{t+1} = v_t + a_t * dt$
- $cte_{t+1} = f(x_t) - y_t + v_t * \sin(e\psi_t) * dt$, here $f(x_t)$ is applying x_t to the polynomial curve fit using given way points.
- $e\psi_{t+1} = \psi_t - \psi dest_t + \frac{v_t}{L_f} * \delta_t * dt$, here $\psi dest_t$ is the degree of the tangent at time t on the polynomial curve fit using given way points.

Time step length and elapsed duration

In my final submission, I set the N to be 10 and dt to be 0.1 seconds, therefore the total time length T is 1 seconds.

In this project, there is a 100 milliseconds delay between when the actuation is sent and when it is applied to the car in the simulator. That means the time duration between any two subsequent actuations should be >= 0.1 seconds. Therefore, I set dt to be 0.1 seconds and this seems to be the minimum dt meaningful in prediction.

Considering T, it should be as large as possible. However, as the car goes faster as faster, the T value can not be too large. In my final submission, I set the reference speed to be 80 mph. And after trying different T values, I set T to be 1 second. For big T value, the car can run out of the track on the sharp turnings. And N is set to be 10.

Polynomial Fitting and MPC Preprocessing

In this project, to make the processing easier, I convert all way points from global coordinate to car coordinate at the beginning. Given the car position $[x_{car}, y_{car}]$, and heading ψ, for a point $[x, y]$ in global coordinate, its position in the car coordinate $[x', y']$ can be calculated as following:

$$diff_x = x - x_{car}$$
$$diff_y = y - y_{car}$$
$$x' = diff_x * \cos(\psi) + diff_y * \sin(\psi)$$
$$y' = -diff_x * \sin(\psi) + diff_y * \cos(\psi)$$

After converting way points to the car coordinate, the vehicle state has to change in vehicle coordinate too. For position and heading, they should all be 0 $[x_{car}, y_{car}, \psi] = [0,0,0]$. For v, it should not change, and for cte and epsi, it is calculated as the following formula by using the new $[x_{car}, y_{car}, \psi]$:

$$cte_{t+1} = f(x_t) - y_t$$
$$e\psi_{t+1} = \psi_t - \psi dest_t$$

After that, I fit the way points to a polynomial of degree 2.

Model Predictive Control with Latency

In this project, my solution can handle the 100 millisecond latency. The way how I handle the latency is as taught in the course. I first predict a new state $[x_{new}, y_{new}, \psi_{new}, v_{new}, cte_{new}, e\psi_{new}]$ from the input state $[x, y, \psi, v, cte, e\psi]$ after 100 milliseconds using the following equations:

$$x_{new} = x + v * \cos(\psi) * time$$
$$y_{new} = y + v * \sin(\psi) * time$$
$$\psi_{new} = \psi + \frac{v}{L_f} * \delta * time$$
$$v_{new} = v + a * time$$
$$cte_{new} = cte + v * \sin(e\psi) * time$$
$$e\psi_{new} = e\psi + \frac{v}{L_f} \delta * time$$
$$time = 0.1\, seconds$$

Then I use this predicted state as the input to the Model Predictive Control.