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# Chapter 1

# Type IIA compactifications

sTRING THEORY GENERALITIES

In the following section we motivate the requirement that additional dimensions are compactified over a Calabi-Yau manifold (a compact complex manifold of $SU(3)$ holonomy).

Since Type IIA requires nine spatial dimensions but we only observe three, we need to compactify six of them over a small region. We assume that the manifold $M$ is factorizable into a four-dimensional maximally symmetric space-time $T$ and a six-dimensional compact space $K$, $M = T \times K$.

Type IIA string theory on 10 dimensional flat space has a large degree of supersymmetry, but the compactification choice can either preserve some degree of supersymmetry or remove it completely. We will consider compactifications which leave some supersymmetry. Our main reason for this choice is that they provide a nice way to obtain realistic particle physics models. In particular, we will see that a $\mathcal{N} = 1$ supersymmetric theory allows for chiral fermions. In addition, supersymmetric configurations are easier to study before tackling more general compactifications.

An approach to construct a supersymmetric theory is roughly to extend the Poincaré algebra into a super-Poincaré algebra by adding supersymmetry generators which satisfy specific anti-commutation relations, instead of commutation relations.

A conserved charge $Q$ associated to an unbroken supersymmetry annihilates the vacuum $|\Omega\rangle$, so $Q\,|\Omega\rangle = 0$. This in turn means that for any operator $U$,

$\langle\Omega|\{Q,U\}|\Omega\rangle = 0$. If $U$ is a fermionic operator, we derive that the variation of the operator under the supersymmetry transformation is $\delta U = \{Q,U\}$. Taking this as the classical limit, $\delta U = \langle\Omega|\delta U|\Omega\rangle$. Thus, we conclude that at the classical level $\delta U = \langle\Omega|\{Q,U\}|\Omega\rangle = 0$ for any fermionic field $U$.

The low-energy theory of the ten-dimensional type IIA string theory is type IIA SUGRA, which although it has $\mathcal{N}=2$ instead of $\mathcal{N}=1$, it is possible to obtain $\mathcal{N}=1$ through orientifold projections. N=2 or N=(1,1). Something about the worldsheet.

The spectrum of Type IIA SUGRA has as elementary fermions two Majora-Weyl gravitinos of the same chirality $\psi_M$ and two dilatinos $\lambda$. In the same way that a translation generated by the momentum operator is parametrized by a vector and a rotation is a parametrized by an antisymmetric tensor, a supersymmetry transformation generated by $Q_\alpha$ is parametrized by a spinor $\eta_\alpha$. The variation of the gravitino field under a supersymmetry transformation is

$$\delta\psi_M = D_M\eta + (\text{fluxes}) \tag{1.1}$$

Where $D_M$ is the covariant derivative on $M$. Supersymmetry preservation means that all variations must be zero. We assume that all fluxes vanish. This leads to the constraint that $\eta$ is a covariantly constant spinor

$$\delta\psi_M = D_M\eta = 0 \tag{1.2}$$

If we particularize this equation to the four-dimensional space-time $T$, which is a maximally symmetric space, it imposes that $T$ is Minkowski space and thus, $\eta$ only depends on the compact coordinates.

The existence of a single covariantly constant spinor on the compact manifold can be reformulated as a topological condition, namely that the holonomy group (whose precise definition is given in the next chapter) of the compact manifold is $SU(3)$. A compact manifold of $SU(3)$ (local) holonomy is the definition of a Calabi-Yau manifold. The holonomy group being a proper subgroup of $SU(3)$ is equivalent to having more than one covariantly constant spinor, which would lead to a larger degree of supersymmetry preserved.

The existence of a covariantly constant spinor implies for a TIIA theory that there are two four-dimensional supersymmetry parameters and therefore, $\mathcal{N}=2$.

## 1.1 Type IIA on Calabi-Yau manifolds

We examine more closely what the existence of a covariantly constant spinor field implies on the compact space.

Let us consider a Riemannian manifold $K$ of dimension six with a spin connection $\omega$, which is in general a $SO(6)$ gauge field. If we parallel transport a field $\psi$ around a contractible closed curve $\gamma$, the field becomes $\psi' = U\psi$ where $U = \mathcal{P}e^{\int_\gamma dx\omega}$ and $\mathcal{P}$ denotes the path ordering of the exponential. The set of transformation matrices associated to all possible loops form the holonomy group of the manifold, which must be a subgroup of $SO(6)$.

A covariantly constant spinor is left unchanged when parallel transported along a contractible closed curve, so the holonomy matrices of a manifold that admits a covariantly constant spinor must satisfy $U\eta = \eta$. Taking into account the Lie algebra isomorphism $\mathfrak{so}(6) \simeq \mathfrak{su}(4)$ we identify the positive (negative)-chirality spinors of $SO(6)$ with the fundamental $\mathbf{4}$ ($\bar{\mathbf{4}}$) of $SU(4)$. Let us take that $\eta$ is a positive chirality spinor, so it transforms according with the $\mathbf{4}$ of $SU(4)$. In order to have a covariantly constant spinor, the holonomy group must be such that the $\mathbf{4}$ representation decomposes into a singlet. This decomposition is achieved if the holonomy group is $SU(3)$ so that

$$SO(6) \to SU(3) \tag{1.3}$$

$$\mathbf{4} \to \mathbf{3} \oplus \mathbf{1} \tag{1.4}$$

We can also check that the 2-form $\mathbf{15}$ and the 3-form $\mathbf{20}$ decompositions contain a singlet, $\mathbf{15} \to \mathbf{8} \oplus \mathbf{3} \oplus \bar{\mathbf{3}} \oplus \mathbf{1}$ and $\mathbf{20} \to \mathbf{6} \oplus \bar{\mathbf{6}} \oplus \mathbf{3} \oplus \bar{\mathbf{3}} \oplus \mathbf{1} \oplus \mathbf{1}$, so they are globally well defined. We refer to the 2-form as $J$ and the 3-form as the holomorphic three-form $\Omega$. Raising an index of $J$ we obtain an almost-complex structure, which satisfies $(J^2)^i_j = -\delta^i_j$. For a particular point of the manifold, we can form a basis of complex coordinates $z^i$ from the real coordinates $x^i$, as $z^1 = x^1+ix^2$, $z^2 = x^3+ix^4$ and $z^3 = x^5+ix^6$, in which $J = idz^i \otimes dz^i - id\bar{z}^{\bar{i}} \otimes d\bar{z}^{\bar{i}}$. If we can extend this particular form of $J$ to the neighborhood of any point, $J$ is said to be integrable and the manifold is complex. An integrable almost-complex structure is referred to as a complex structure. The integrability condition is equivalent to the Nijenhuis tensor

$$N^k_{ij} = J^l_i(\partial_l J^k_j - \partial_j J^k_l) - J^l_j(\partial_l J^k_i - \partial_i J^k_l) \tag{1.5}$$

vanishing everywhere. Intuitively, a complex manifold can though as a manifold that can be covered by complex coordinate charts which are related at their intersections by holomorphic transition functions.

It is useful to define with the aid of the metric the form $k = g_{i\bar{j}} dz^i \wedge d\bar{z}^{\bar{j}}$. A manifold is Kähler if $dk = 0$ and $k$ is then called the Kähler form. It can be shown that the holonomy group being contained in $U(N)$ implies that the manifold is Kähler.

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## 1.2 Orientifold projections and D-branes

### Orientifold planes and D-branes

### D6-branes on a Calabi-Yau. Special Lagrangians.

### Model building

# Bibliography