Contents

Chapter 1

Generalities of type IIA string theory

In this thesis we will only work with the supersymmetric type IIA theory. The study of string theory in Minkowski spacetime has lead to the identification of five consistent string theories, which all turn out to be supersymmetric and give rise to massless bosonic and fermionic excitations in their spectrum. The five string theories were given their name according to their own specificities: Type heterotic HE and HO, Type I, Type IIB and Type IIA string theory. In this thesis we will only concentrate on the last one in the list.

Type IIA spectrum

Type IIA string theory requires ten space-time dimensions to be consistent. Furthermore, it has a 10-dimensional supersymmetry with 32 supercharges, which corresponds to $\mathcal{N} = (1, 1)$. The flat 10-dimensional space-time bosonic spectrum of type IIA can be classified according to the boundary conditions of the strings, whether we consider Ramond (R) or Neveu–Schwarz (NS) conditions. We list only the massless (closed) string states. In the NS-NS sector, we find the dilaton ϕ, a two-form B_2 and a graviton $G_{\mu\nu}$, while in the R-R sector we identify the 1- and 3-forms c_1, c_3. The fermions, which belong to the NS-R and R-NS sectors, are two opposite-chirality gravitinos ψ and two opposite-chirality dilatinos λ.

Type IIA SUGRA

The D-brane

The two-dimensional strings can be generalized to $(p+1)$-dimensional extended object, which are called Dp-branes. Thus, a D1-brane would correspond to a D-string, a D2-brane would be a three-dimensional membrane and so on. The existence of Dp-branes can be motivated, in the weak coupling limit, as objects where open strings end, so they are a way to impose Dirichlet boundary conditions on open strings. In type IIA string theory, only even-dimensional Dp-branes are physical, which are: the D0, D2, D4, D6 and D8-branes.

We can study the dynamics of a Dp-branes in terms of the open string excitations with endpoints attached to the Dp-brane. Let us consider the open string excitations of a Dp-brane, the latter spanning $p+1$ dimensions and transverse to $d-p-1$ dimensions. The presence of the Dp-brane breaks the ten-dimensional Poincaré invariance of the theory, because open string excitations propagate on the $(p+1)$-dimensional volume of the Dp-brane only. This implies that massless particles must transform under irreducible representations of $SO(p-2)$, instead of $SO(d-2)$. The massless spectrum in $(p+1)$ dimensions of the open string theory is composed by a gauge boson A^μ $(\mu = 0, \ldots, p)$ (corresponding to longitudinal oscillations to the brane), $9-p$ real scalars ϕ^i (corresponding to transverse oscillations to the brane) and some fermions λ_a. This particle content can be arranged into a vector supermultiplet of $U(1)$ with 16 supersymmetries in $(p+1)$ dimensions. Thus, a Dp-brane reduces the degree of supersymmetry of the type IIA theory by half.

In order to find out the action of a Dp-brane we must realize that it corresponds to the $(p+1)$-dimensional effective action of the massless open string excitations of the Dp-brane. As an illustration of this, a Dp-brane breaks the translational symmetry of the vacuum, which allows us to conclude that the ϕ^i scalar fields are the Goldstone bosons associated to the broken symmetry. The vev of these scalar fields determine the position of the Dp-brane in the transverse space, and the fluctuations of the scalar fields determine the evolution of the Dp-brane worldvolume W_{p+1} (the generalization of the particle worldline to the case of higher-dimensional branes). The resulting action of the bosonic sector of the Dp-brane is the sum of a Dirac-Born-Infeld term S_{DBI} and a Chern-Simons term S_{CS}.

The DBI term carries the information of how a Dp-brane interacts with the NSNS fields. It takes the form

$$S_{DBI} = -\frac{\alpha'^{-(p+1)/2}}{(2\pi)^p} \int_{W_{p+1}} d^{p+1}x f(\phi^i, A^\mu, G_{\mu\nu}, B_2, \phi) \tag{1.1}$$

where the precise expression of f is unimportant for the purposes of this work.

What is the background here and what is dynamical? BG: closed string fields and Dynamical: open string fields?

If we expand the DBI action in powers of α', we obtain the Yang-Mills term

$$S_{YM} = \frac{\alpha'^{-(p-3)/2}}{4g_s(2\pi)^{p-2}} \int d^{p+1}x\sqrt{-g}\,\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} \tag{1.2}$$

which allows us to identify the Yang-Mills coupling as

$$g_{YM}^2 = g_s \alpha'^{(p-3)/2}(2\pi)^{p-2} \tag{1.3}$$

The Chern-Simons term is topological in nature and describes how Dp-branes interact with RR-fields.

Anything more to add?

Multiple D-branes

It is convenient to generalize the single Dp-brane configuration to N parallel Dp-branes. In order to determine the spectrum of a stack of Dp-branes, we consider open strings with endpoints attached to either a single brane or two of them.

In the case of N coincident Dp-branes, all configurations lead to massless states, so the gauge symmetry is increased from $U(1)$ to $U(N)$. The massless spectrum is composed of $(p-1)$-dimensional $U(n)$ gauge bosons, $(9-p)$ real scalars in the adjoint representation and several fermions in the adjoint representation.

If all branes are separated from each other, strings that stretch from a brane to itself correspond to massless gauge bosons that belong to $U(1)^N$. In contrast, strings that stretch from one brane A to another brane B lead to massive particles whose masses increases with the distance between branes. The lightest of these particles have opposite have charge $(1,-1)$ under $U(1)_A \times U(1)_B$. Since Type IIA strings carry an orientation, string that stretch from B to A would have opposite charges.

Let us now suppose Dp-branes which are not parallel, so they can intersect each other. This situation is relevant because in the case of D6-branes, it leads to 4-dimensional chiral fermions. We are interested in describing the open string spectrum of two stacks of D6-branes that intersect over a 4-dimensional subspace of their volumes.

Strings that stretch from a coincident stack of N D6-branes to itself lead to 7-dimensional $U(N)$ gauge bosons, three real adjoint scalars and their fermion superpartners.

String that stretch from a stack of N_1 D6-branes to another stack of N_2 D6-branes are localized at the intersection, in order to minimize their energy. They lead to a 4-dimensional fermion charged in the $(\mathbf{N_1}, \mathbf{N_2})$ of $U(N_1) \times U(N_2)$ or its conjugate, depending on the orientation of the intersection.

Not all geometric configurations preserve supersymmetry. Let us decompose space-time as $M_4 \times \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2$. The D6-branes span all M_4 and a line in each $\mathbb{R}^2$ plane, such that the angle between two stacks is given by θ_i for each plane. It can be shown that the condition $\theta_1 \pm \theta_2 \pm \theta_3 = 0(\mathrm{mod} 2\pi)$ implies $\mathcal{N} = 1$ supersymmetry in 4 dimensions, provided that no angle vanishes. If some of the angles vanish, the supersymmetry would we enhanced.

The reason we have used D6-branes and no other dimension of Dp-branes is that they would not lead to chiral fermions in 4 dimensions. Intuitively, two D6-branes allow to define an orientation in the transverse 6-dimensional space, which would not be possible with two other type of Dp-branes.

So have we seen that no Calabi-Yau is needed to obtain 4d chiral fermions if we consider a theory with intersecting D6-branes? Of course, we still would have to cancel RR charges in some way.

Add tadpole cancellation here?

SUGRA

The low-energy theory of the ten-dimensional type IIA string theory is type IIA SUGRA. The spectrum of Type IIA SUGRA has as elementary fermions, which belong to the massless spectrum (NS-R and R-NS) of type IIA theory, two Majorana-Weyl gravitinos of the same chirality ψ_M^a and two dilatinos λ^a.

Isn't the spectrum the same as the high-energy TIIA? Might not need to mention SUGRA here then.

Chapter 2

Type IIA compactifications

In the following section we motivate the requirement that additional dimensions are compactified over a Calabi-Yau manifold.

Since Type IIA requires nine spatial dimensions but we only observe three, we need to compactify six of them over a small region. We assume that the manifold M is factorizable into a four-dimensional maximally symmetric space-time T and a six-dimensional compact space K, $M = T \times K$.

Type IIA string theory on 10 dimensional flat space has a large degree of supersymmetry, but the compactification choice can either preserve some degree of supersymmetry in four dimensions or remove it completely. We will consider compactifications over an internal manifold that leave some supersymmetry in four dimensions intact. A historical motivation for this choice is that they provide a nice way to obtain realistic particle physics models. In particular, we will see that a $\mathcal{N} = 1$ supersymmetric theory allows for chiral fermions in four dimensions, while field theories with a higher mumber of supersymmetry in four dimensions do not. In addition, supersymmetric configurations are easier to study before tackling more general compactifications. Indeed, supersymmetric compactifications of string theory allow for stable dimensional reductions, whose higher-dimensional corrections can be systematically studied.

The algebra of a N=1 supersymmetric theory in four-dimensional Minkoswki spacetime is an extension of the Poincare-algebra by adding supersymmetry generators which satisfy specific anti-commutation relations, instead of commutation relations.

Reformulate. We want to obtain a condition for a susy vacuum in 4d. We can obtain a classical field theory constraint.

A conserved charge Q associated to an unbroken supersymmetry annihilates the vacuum $|\Omega\rangle$, so $Q|\Omega\rangle = 0$. This in turn means that for any operator U, $\langle\Omega|\{Q, U\}|\Omega\rangle = 0$. If U is a fermionic operator, we derive that the variation of the operator under the supersymmetry transformation is $\delta U = \{Q, U\}$. Taking this as the classical limit, $\delta U = \langle\Omega|\delta U|\Omega\rangle$. Thus, we conclude that at the classical level $\delta U = \langle\Omega|\{Q, U\}|\Omega\rangle = 0$ for any fermionic field U.

We now consider the SUGRA theory of type IIA string theory and the condition that some fourd-supersymmetry remains. In the same way that a translation generated by the momentum operator is parametrized by a vector and a rotation is a parametrized by an antisymmetric tensor, a supersymmetry transformation generated by Q_α is parametrized by a spinor η_α. The variation of the gravitino field under a supersymmetry transformation is

$$\delta\psi_M = D_M\eta + (\text{fluxes}) \tag{2.1}$$

Where D_M is the covariant derivative on M. Supersymmetry preservation means that all variations must be zero. We assume that all fluxes vanish. This leads to the constraint that η is a covariantly constant spinor

$$\delta\psi_M = D_M\eta = 0 \tag{2.2}$$

If we particularize this equation to the four-dimensional space-time T, which is a maximally symmetric space, it imposes that T is Minkowski space and thus, η only depends on the compact coordinates.

To study the implication of this equation to the four-dimensional space-time T, we employ the fact that T is maximally symmetric, so we can decompose the metric as

$$ds^2 = e^{2A(y)}\tilde{g}_{\mu\nu}dx^\mu dx^\nu + g_{mn}dy^m dy^n, \qquad \mu = 0, 1, 2, 3 \quad m = 1, \ldots, 6 \tag{2.3}$$

where x^μ are the compact coordinates, y^m the internal coordinates and $\tilde{g}_{\mu\nu}$ can be either the de Sitter, anti-de Sitter or the Minkowski metric in four dimensions.

Particularizing to the space-time components, equation (2.2) can be written as

$$\widetilde{\nabla}_\mu \eta + \frac{1}{2}(\tilde{\gamma}_\mu \gamma_5 \otimes \nabla\!\!\!\!/ A)\eta = 0 \tag{2.4}$$

where $\widetilde{\nabla}$ is the derivative with respect $\tilde{g}_{\mu\nu}$.

What is the gamma

Should I explain where this comes from? integrability

We then obtain

$$\left[\widetilde{\nabla}_\mu, \widetilde{\nabla}_\nu\right]\eta = \frac{1}{2}(\nabla_m A)(\nabla^m A)\gamma_{\mu\nu}\eta \tag{2.5}$$

$$\left[\widetilde{\nabla}_\mu, \widetilde{\nabla}_\nu\right]\eta = \frac{1}{4}\tilde{R}_{\mu\nu\lambda\rho}\gamma^{\lambda\lambda}\eta \tag{2.6}$$

since for a maximally symmetric space, the Riemman tensor is $R_{\mu\nu\lambda\rho} = k(g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda})$. Combining the previous two equations and inverting $\gamma^{\mu\nu}$, we obtain the condition

$$k + \nabla_m A \nabla^m A = 0 \tag{2.7}$$

constant value etc

The existence of a single covariantly constant spinor on the compact manifold can be reformulated as a topological condition, namely that the holonomy group (whose precise definition is given in the next chapter) of the compact manifold is $SU(3)$. A compact manifold of $SU(3)$ (local) holonomy is the definition of a Calabi-Yau manifold. The holonomy group being a proper subgroup of $SU(3)$ is equivalent to having more than one covariantly constant spinor, which would lead to a larger degree of supersymmetry preserved.

The existence of a covariantly constant spinor implies for a TIIA theory that there are two four-dimensional supersymmetry parameters and therefore, $\mathcal{N} = 2$.

2.1 Type IIA on Calabi-Yau manifolds

We examine more closely what the existence of a covariantly constant spinor field implies on the compact space.

Let us consider a Riemannian manifold K of dimension six with a spin connection ω, which is in general a $SO(6)$ gauge field. If we parallel transport a field ψ around a contractible closed curve γ, the field becomes $\psi' = U\psi$ where $U = \mathcal{P}e^{\int_\gamma dx\omega}$ and $\mathcal{P}$ denotes the path ordering of the exponential. The set of transformation matrices associated to all possible loops form the holonomy group of the manifold, which must be a subgroup of $SO(6)$.

A covariantly constant spinor is left unchanged when parallel transported along a contractible closed curve, so the holonomy matrices of a manifold that admits a covariantly constant spinor must satisfy $U\eta = \eta$. Taking into account the Lie algebra isomorphism $\mathfrak{so}(6) \simeq \mathfrak{su}(4)$ we identify the positive (negative)-chirality spinors of $SO(6)$ with the fundamental $\mathbf{4}$ ($\bar{\mathbf{4}}$) of $SU(4)$. Let us consider that η is a positive chirality spinor, so it transforms according with the $\mathbf{4}$ of $SU(4)$. In order to have a covariantly constant spinor, the holonomy group must be such that the $\mathbf{4}$ representation decomposes into a singlet. This decomposition is achieved if the holonomy group is $SU(3)$ so that

$$SO(6) \to SU(3) \tag{2.8}$$

$$\mathbf{4} \to \mathbf{3} \oplus \mathbf{1} \tag{2.9}$$

We can also check that the 2-form $\mathbf{15}$ and the 3-form $\mathbf{20}$ decompositions contain a singlet, $\mathbf{15} \to \mathbf{8} \oplus \mathbf{3} \oplus \bar{\mathbf{3}} \oplus \mathbf{1}$ and $\mathbf{20} \to \mathbf{6} \oplus \bar{\mathbf{6}} \oplus \mathbf{3} \oplus \bar{\mathbf{3}} \oplus \mathbf{1} \oplus \mathbf{1}$, so they are globally well defined. We refer to the 2-form as J and the 3-form as the holomorphic three-form Ω. Raising an index of J we obtain an almost-complex structure, which satisfies $(J^2)^i_j = -\delta^i_j$. For a particular point of the manifold, we can form a basis of complex coordinates z^i from the real coordinates x^i, as $z^1 = x^1 + ix^2$, $z^2 = x^3 + ix^4$ and $z^3 = x^5 + ix^6$, in which $J = idz^i \otimes dz^i - id\bar{z}^{\bar{i}} \otimes d\bar{z}^{\bar{i}}$. If we can extend this particular form of J to the neighborhood of any point, J is said to be integrable and the manifold is complex. An integrable almost-complex structure is referred to as a complex structure. The integrability condition is equivalent to the Nijenhuis tensor

$$N^k_{ij} = J^l_i(\partial_l J^k_j - \partial_j J^k_l) - J^l_j(\partial_l J^k_i - \partial_i J^k_l) \tag{2.10}$$

vanishing everywhere. It is possible to formulate an alternative definition of a complex manifold, as

It is useful to define with the aid of the metric the form $k = g_{i\bar{j}} dz^i \wedge d\bar{z}^{\bar{j}}$. A manifold is Kähler if $dk = 0$ and k is then called the Kähler form. It can be shown that the holonomy group being contained in $U(N)$ implies that the manifold is Kähler.

Cohomology

It is useful to introduce some algebraic topology tools.

A differential p-form ω_p is $(0,p)$-rank tensor which has completely anti-symmetric components. A p-form is expanded as a linear combination of the basis $\{dx^\nu\}_{\nu=1\ldots p}$

$$\omega_p = \frac{1}{p!}\omega_{\nu_1\ldots\nu_p} dx^{\nu_1} \wedge \cdots \wedge dx^{\nu_p} = \frac{1}{p!}\omega_{\nu_1\ldots\nu_p} dx^{[\nu_1} \otimes \cdots \otimes dx^{\nu_p]} \tag{2.11}$$

where the square brackets denote antisymmetrization.

The wedge product of a p-form ω_p and a q-form α_q is a $(p+q)$-form

$$\omega_p \wedge \alpha_q = \frac{1}{p!q!}\omega_{\nu_1\ldots\nu_p}\alpha_{\mu_1\ldots\mu_p} dx^{\nu_1} \wedge \cdots \wedge dx^{\nu_p} \wedge dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_p} \tag{2.12}$$

The exterior derivative of a p-form yields a $(p+1)$-form

$$d\omega_p = \frac{1}{p!}\partial_\mu \omega_{\nu_1\ldots\nu_p} dx^\mu \wedge dx^{\nu_1} \wedge \cdots \wedge dx^{\nu_p} \tag{2.13}$$

A p-form whose exterior derivative vanishes is called closed and a p-form that is the exterior derivative of a $(p-1)$-form is exact.

A fundamental property of the exterior derivative is Poincare's lemma, which states that for any diffential form α, $d(d\alpha) = 0$ holds. This can be rewritten as $d^2 = 0$. In other words, every exact form is closed. We could ask ourselves if the inverse statement is true, is every closed form exact? The answer for an arbitrary manifold is no.This information is encoded in the q-th deRham cohomology group, which is formed by considering the set of all closed q-forms defined on a manifold. Since given a closed form ω, we can always find another closed form by adding an exact form $\omega' = \omega + d\alpha$, we take the equivalence relation that two forms are equivalent if they differ by a closed form. The q-th deRham cohomology group is defined as the quotient

$$H^q_d(X, \mathbb{R}) = \{\omega | d\omega = 0\}/\{\alpha | \alpha = d\beta\} \tag{2.14}$$

dimension Betti number

DOLBEAULT COHOMOLOGY

We can easily make a generalization of the previous concepts to complex manifolds. $\bar{\partial}$

$$H^{r,s}_{\bar{\partial}}(X,\mathbb{C}) = \{\omega|\bar{\partial}\omega = 0\}/\{\alpha|\alpha = \bar{\partial}\beta\} \tag{2.15}$$

HODGE DECOMPOSITION / HODGE NUMBERS

$$\omega = d\beta + d^{\dagger}\gamma + \omega' \tag{2.16}$$

$$\omega = d\beta + \omega' \tag{2.17}$$

The dimension of $H^{p,q}$ is know as the Hodge number, $h_{p,q}$ and it is a topological invariant. For Calabi-Yau manifolds, $h_{d,0} = 1$ $h_{p,q} = h_{d-p,d-q}$ mirror symmetry

Hodge symmetry $h_{p,q} = h_{q,p}$

Homology

A very related construction to cohomology is homology. The basic element of homology is the p-chain

algebraic dual, in the sense that

Poincaré duality

Moduli space

TIIA spectrum on CY

2.2 Type IIA on orientifolds

TIIA orientifolds

If we compactify a type II string theory on a Calabi-Yau manifold, we obtain a four-dimensional $\mathcal{N} = 2$ supersymmetric theory. This degree of supersymmetry does not allow for chiral fermions, so Calabi-Yau compactifications of type II theories have little interest in the context of model building. An option to reduce the supersymmetry to $\mathcal{N} = 1$ is to apply the orientifold projection, which consists in modding out the action of $\Omega\mathcal{R}(-1)^{F_L}$, where Ω is the worldsheet parity (so strings become unoriented) and $\mathcal{R}$ is a particular $\mathbb{Z}_2$ symmetry of the compact six-dimensional space.

Explain worldsheet vs manifold

(-1) not necessary type IIB. Where does it act?

$\mathcal{R}$ condition anti-holomorphic involution squares to the identity $J \to -J$ and $\Omega \to \bar{\Omega}$.

The fixed points under $\mathcal{R}$ define the orientifold planes in the model and are denoted as Op-planes, where p is the spatial dimension. They span the entire four-dimensional Minkowski space and wrap a compact $(p-3)$-cycle on the internal space.

Op-planes carry RR charge, so in order to eliminate RR tadpoles we must also introduce Dp-branes, which carry opposite charge. It is important to note that Dp-branes do not need to wrap the same 3-cycles as the Op-planes.

D-branes on CY

Can a submanifold of a CY not be a cycle?

In order to obtain stable D6-brane configurations on a type IIA theory compactified on a Calabi-Yau manifold, we impose that they wrap around volume minimizing 3-cycles on the compact space. The volume minimizing condition means that the branes must wrap special Lagrangian 3-cycles Π, which satisfy

$$k|_\Pi = 0, \qquad \mathrm{Im}(e^{-i\phi}\Omega_3)|_\Pi = 0 \tag{2.18}$$

for some real ϕ. k is the Kähler two-form and Ω_3 the holomorphic three-form. The $e^{-i\phi}\Omega_3$ is referred as a calibration and the special Lagrangian is calibrated with respect to it, for every choice of ϕ. The volume of the special Lagrangian 3-cycle is

$$\mathrm{Vol}(\Pi) = \int_\Pi \mathrm{Re}(e^{-i\phi}\Omega_3) \tag{2.19}$$

D6-branes wrapped around an special Lagrangian cycle are guaranteed to preserve 4-dimensional $\mathcal{N}=1$. This preserved supersymmetry concides with that preserved by Op-planes only if $\phi = 0$.

Should relocate. What if otherwise?

The open string spectrum of D6_a-brane stacks with multiplicities N_a wrapping special Lagrangian 3-cycles Π_a can be classified into two sectors: strings that stretch from one stack to itself and those that stretch between to stacks, 6_a and 6_b. $b_1(\Pi_a)$ chiral fermion intersection number $I_{ab} = [\Pi_a][\Pi_b]$ which transform as $(\mathbf{N_a}, \bar{\mathbf{N}}_\mathbf{b})$ relative angle between.

Orientifold compactifications with intersecting D-branes

$z_i \to \bar{z}_i$

$$\theta_1 + \theta_2 + \theta_3 = 0 \tag{2.20}$$

$$\sum_a N_a[\Pi_a] + \sum_a N_a[\Pi_{a'}] - 4[\Pi_{O6}] = 0 \tag{2.21}$$

Spectrum, effective action and model building?

$$C \tag{2.22}$$

Chapter 3

Type IIA on the quintic

3.1 Quintic threefold motivation

Why the quintic?

Why study sLags?

3.2 sLags on Fermat's quintic

SM on the quintic

3.3 sLags on the deformed quintic

Quintic deformations

The consider the possible deformations of Fermat's quintic. There should be 101 independent deformations, since they correspond to different complex structures and are given be the Hodge number $h_{21} = 101$.

We can add terms of the following type to the quintic

$$x_i^5, x_i^4 x_j^1, x_i^3 x_j^2, x_i^3 x_j x_k, x_i^2 x_j x_k x_l, x_1 x_2 x_3 x_4 x_5 \tag{3.1}$$

Not all of these terms are independent, since a coordinate redefiniton $GL(5, \mathbb{C})$.

Deformation classification

Coordinate redefinition freedom

Singularities

As an example, we take as deformation $-5\phi z_1 z_2 z_3 z_4 z_5$

Change of variables to study geometry of the singularity

In order to determine the geometry near the singularity, we make the following change of variables

$$
\begin{aligned}
x_1 &= 1 + y_1/\sqrt{10} + y_2/5 + y_4/\sqrt{50} \\
x_2 &= 1 + y_1/\sqrt{10} - y_2/5 + y_4/\sqrt{50} \\
x_3 &= 1 + y_1/\sqrt{10} + y_3/5 - y_4/\sqrt{50} \\
x_4 &= 1 + y_1/\sqrt{10} - y_3/5 - y_4/\sqrt{50}
\end{aligned}
\tag{3.2}
$$

In these coordinates, the quintic becomes

$$5(\psi - 1) = y_1^2 + y_2^2 + y_3^2 + y_4^2 + O(\psi - 1) \tag{3.3}$$

Something about the branch

Volume of cycles wrapping singularities

Three-form integration

$$\Omega = \frac{x_5 dx_1 \wedge dx_2 \wedge dx_3}{\frac{\partial p}{\partial x_4}} \tag{3.4}$$

$$\int_{A^2} \Omega = \int \dots \tag{3.5}$$

Bibliography

How should I cite?