A walk through moduli space with SLags

John Xihua Liu Anta

Universidad Autónoma de Madrid
Instituto de Física Teórica

Adviser: Wieland Staessens

Madrid, 2017

Contents

Chapter 1

Type IIA on the quintic

Intersection numbers

We now study the intersection numbers between different SLags, which are used to determine the number of chiral fermions associated to the intersecting D6-branes.

First of all, we choose a coordinate patch of $\mathbb{CP}^4$ where $x_0 = 1$ and take one of the SLags to be $|0,0,0,0,0\rangle$ and the other SLag $|0,k_1,k_2,k_3,k_4\rangle$. The latter SLag can be obtained by applying successive rotations ω_k associated to the $\mathbb{Z}_5^4$ symmetry as $|0,k_1,k_2,k_3,k_4\rangle = \omega_1^{k_1}\omega_2^{k_2}\omega_3^{k_3}\omega_4^{k_4}\,|0,0,0,0,0\rangle$. The intersection numbers in this patch are:

- If $\omega_i^{k_i} \neq 1$ for all i, there are no intersections in this patch. The justification is that the coordinates of one of the SLags $[1 : x_1 : x_2 : x_3 : x_4]$ where $x_i \in \mathbb{R}$ take only real values, while some of the coordinates of the SLag $[1 : \omega^{k_1}x_1 : \omega^{k_2}x_2 : \omega^{k_3}x_3 : \omega^{k_4}x_4]$ take always complex numbers.
- If $\omega_i^{k_i} = 1$ for only one i, which we set to be $i = 4$, the intersection is at a single point. In particular, the intersection of $[1 : x_1 : x_2 : x_3 : x_4]$ with $[1 : \omega^{k_1}x_1 : \omega^{k_2}x_2 : \omega^{k_3}x_3 : \omega^{k_4}x_4]$ is located at $(1,0,0,0,x_4)$. When we restrict this line in $\mathbb{CP}^4$ to the quintic hypersurface, the intersection reduces to the point $(1,0,0,0,-1)$. The signature of the intersection is $\mathrm{sgn}(\mathrm{Im}\,\omega^{k_1}\,\mathrm{Im}\,\omega^{k_2}\,\mathrm{Im}\,\omega^{k_3})$.
- If $\omega_i^{k_i} = 1$ for at least two different values of i, we have to deform one of the SLags. The deformation must be normal to both SLags and through

McLean's theorem we identify the normal bundle of the intersection with its tangent space. Then, the intersection number is the number of zeros of the non-trivial vector fields that can be defined on the intersection locus.

- If $\omega_i^{k_i} = 1$ for two i's, the intersection is $[1 : 0 : 0 : x_3 : x_4]$, which is homeomorphic to $\mathbb{RP}^1$ and the circle S^1. We can form a nowhere vanishing vector field on the circle, embedded in $\mathbb{R}^2$, associating to every point $(\sin\phi, \cos\phi)$ the vector $X(\phi) = -\sin\phi\partial_x + \cos\phi\partial_y$. Thus, the intersection number is zero $I_{\mathbb{RP}^1} = 0$.
- If $\omega_i^{k_i} = 1$ for three i's, the intersection is $[1 : 0 : 0 : x_3 : x_4]$, which is homeomorphic to the $\mathbb{RP}^2$. The double cover of $\mathbb{RP}^2$ is the two-sphere S^2, which we consider embedded in $\mathbb{R}^3$. We define the vector field

$$X_\theta = \cos\phi\cos\theta\partial_x + \cos\theta\sin\phi\partial_y - \sin\theta\partial_z \tag{1.1}$$

$$X_\phi = \sin\theta(-\sin\phi\partial_x + \cos\phi\partial_y) \tag{1.2}$$

  where X_θ vanishes at the poles, while X_ϕ is not (uniquely) defined. These two zeros reduce to a single zero when we relate antipodal points of S^2 to a single point of $\mathbb{RP}^2$, so the intersection number is one $I_{\mathbb{RP}^2} = 1$. The orientation of the intersection is $\mathrm{sgn}(\mathrm{Im}\,\omega^{k_j})$ $j \neq i$.
- If $\omega^{k_i} = 1$ for all i's, the intersection is $\mathbb{RP}^3$, whose double cover is S^3. Since the vector fields on S^3 have no zeros, there are no intersections $I_{\mathbb{RP}^3} = 0$.

The total intersection number is obtained by repeating this procedure in all patches and then adding all the obtained intersection numbers. It can be shown that the total intersection number of intersections of $|0,0,0,0,0\rangle$ with $|0,k_1,k_2,k_3,k_4\rangle$ that preserve supersymmetry ($\sum_i k_i = 1$) is zero.

Volumes

We now try to compute the volume of SLag three-cycles, as they carry the information of the four-dimensional gauge theory supported by the wrapping D6-branes, such as the gauge coupling. It suffices to determine the volume of $|0,0,0,0,0\rangle$, since a generic SLag can be obtained through $\mathbb{Z}_4$ rotations. The holomorphic three-form on a patch $x_0 \neq 0$ in terms of the local coordinates y_i is

$$\Omega_3 = \frac{dy_1 \wedge dy_2 \wedge dy_3}{5y_4^4} \tag{1.3}$$

The volume of the SLag is obtained integrating Ω_3 over the three-cycle

$$\mathrm{Vol} = \int_{\mathbb{RP}^3} \frac{dy_1 \wedge dy_2 \wedge dy_3}{5y_4} = \int \frac{dy_1 \wedge dy_2 \wedge dy_3}{5(1 + y_1^4 + y_2^4 + y_3^4)^{4/5}} \tag{1.4}$$

It is not evident how to compute this integral, due to the non-trivial integration domain.

We can relate the volume of $|0,0,0,0,0\rangle$ to the volume of an associated three-sphere as follows. The SLag three-cycle is homeomorphic to $\mathbb{RP}^3$, which in turn is diffeomorphic to the the rotation group $SO(3)$. Considering that we there is a map from S^3 with antipodal points identified onto $SO(3)$, we conclude that the volume of a SLag is half the volume of a corresponding three-sphere on the quintic. In practice, we can make no further progress through this approach, since we do not know the Calabi-Yau metric of the quintic.

To complete

Bibliography