Contents

Chapter 1

Type IIA generalities

The are six main types of strings theories: a bosonic string theory and five supersymmetric string theories, the latter include both bosons and fermions. In this thesis we will only work with the supersymmetric type IIA theory.

TIIA spectrum

Type IIA string theory requires ten space-time dimensions to be consistent. The flat 10 dimensional space-time bosinic spectrum of type IIA can be classified according to the boundary conditions of the strings, whether we consider Ramond (R) or Neveu–Schwarz (NS) conditions. In the NS-NS sector, we have the dilaton ϕ, a two-form B_2 and a graviton $G_{\mu\nu}$, while in the R-R sector we have the 1- and 3-forms c_1, c_3. The fermions, which belong to the NS-R and R-NS sectors, are two opposite-chirality gravitinos ψ and two opposite-chirality dilatinos λ.

D-brane introduction

A generalization of strings are Dp-branes, which are p-dimensional extended objects. Thus, a D1-brane would correspond to a string, a D2-brane would be a membrane and so on. The existence of Dp-branes can be motivated, in the weak coupling limit, as objects where open strings end, so they are a way to impose Dirichlet boundary conditions on open strings. In fact, D-branes should be though as new non-perturbative states in their own right.

More subtle than this. In perturbative regime branes are fully described in terms of strings. f-strings, T and S dualities
lead to a more democratic formulation.

We can study the dynamics of a Dp-branes in terms of the dynamics of open strings with endpoints attached to the Dp-brane. Let us consider the open string excitations of a Dp-brane, the latter spanning $p+1$ dimensions and transverse to $d-p-1$ dimensions. The presence of the Dp-brane breaks the ten-dimensional Poincaré invariance of the theory, because particles propagate on the $(p+1)$-dimensional volume of the Dp-brane only. This implies that massless particles must transform under irreducible representations of $SO(p-2)$, instead of $SO(d-2)$. The massless spectrum in $(p+1)$ dimensions of the open string theory is composed by a gauge boson A^μ ($\mu = 0, \dots, p$), $9-p$ real scalars ϕ^i and some fermions λ_a. This particle content can be arranged into a vector supermultiplet of $U(1)$ with 16 supersymmetries in $(p+1)$ dimensions. Thus, a Dp-brane reduces the degree of supersymmetry of the type IIA theory by half.

Branes supported on sLag cycles represent BPS states

In order to find out the action of a Dp-brane we must realize that it corresponds to the $(p+1)$-dimensional effective action of the massless open string excitations of the Dp-brane. Since a Dp-brane breaks the translational symmetry of the vacuum, Goldstone bosons vev of the scalar fields determine the position of the Dp-brane, and the fluctuations of the scalar fields determine the dynamics of the Dp-brane. The resulting action of the bosonic sector of the Dp-brane is the sum of a Dirac-Born-Infeld term S_{DBI} and a Chern-Simons term S_{CS}.

Complete

The DBI term carries the information of how a Dp-brane interacts with the NSNS fields. It takes the form

$$S_{DBI} = -\frac{\alpha'^{-(p+1)/2}}{(2\pi)^p} \int_{W_{p+1}} d^{p+1}x f(\phi^i, A^\mu, G_{\mu\nu}, B_2) \tag{1.1}$$

where the precise expression of f is unimportant to us.

What is the background here and what is dynamical? BG: closed string fields and Dynamical: open string fields?

If we expand the DBI action in powers of α', we obtain the Yang-Mills term

$$S_{YM} = \frac{\alpha'^{-(p-3)/2}}{4g_s(2\pi)^{p-2}} \int d^{p+1}x\sqrt{-g}\,\mathrm{Tr}\,F_{\mu\nu}F^{\mu\nu} \tag{1.2}$$

which allows us to identify the Yang-Mills coupling as

$$g_{YM}^2 = g_s\alpha'^{(p-3)/2}(2\pi)^{p-2} \tag{1.3}$$

The Chern-Simons term is topological in nature and describes how Dp-branes interact with RR-fields.

It is convenient to generalize the single Dp-brane configuration to N parallel Dp-branes. In order to determine the spectrum of a stack of Dp-branes, we consider open strings with endpoints attached to either a single brane or two of them.

In the case of N coincident Dp-branes, all configurations lead to massless states, so the gauge symmetry is increased from $U(1)$ to $U(N)$. The massless spectrum is composed of $(p-1)$-dimensional $U(n)$ gauge bosons, $(9-p)$ real scalars in the adjoint representation and several fermions in the adjoint representation.

If all branes are separated from each other, strings that stretch from a brane to itself correspond to massless gauge bosons that belong to $U(1)^N$. In contrast, strings that stretch from one brane to another lead to massive particles whose mass increases with the distance between branes. The lightest of these particles have opposite charges with respect to the $U(1)$ of each brane.

Maybe mention orientation

Intersecting branes

Reorganize

In our work, D6-branes are interesting because they are used as a basis to obtain 4d chiral fermions. A basic configuration of two stacks of D6-branes that intersect over a 4d subspace of their volumes.

Strings that stretch from a coincident stack of N D6-branes lead to 7d $U(N)$ gauge bosons, three real adjoint scalars and their fermion superpartners.

String that stretch from a stack of N_1 D6-branes to another stack N_2 are localized at the interesection, in order to minimize their energy. They lead to a 4d fermion charged in the $(\mathbf{N_1}, \mathbf{N_2})$ of $U(N_1) \times U(N_2)$.

SUSY condition

SUGRA

The low-energy theory of the ten-dimensional type IIA string theory is type IIA SUGRA. The spectrum of Type IIA SUGRA has as elementary fermions, which belong to the massless spectrum (NS-R and R-NS) of type IIA theory, two Majora-Weyl gravitinos of the same chirality ψ_M and two dilatinos λ.

Complete bosons

32 supersymmetries $\mathcal{N} = (1, 1)$ in ten dimensions

Chapter 2

Type IIA compactifications

In the following section we motivate the requirement that additional dimensions are compactified over a Calabi-Yau manifold.

Since Type IIA requires nine spatial dimensions but we only observe three, we need to compactify six of them over a small region. We assume that the manifold M is factorizable into a four-dimensional maximally symmetric space-time T and a six-dimensional compact space K, $M = T \times K$.

Type IIA string theory on 10 dimensional flat space has a large degree of supersymmetry, but the compactification choice can either preserve some degree of supersymmetry in four dimensions or remove it completely. We will consider compactifications over an internal manifold that leave some supersymmetry in four dimensions intact. A historical motivation for this choice is that they provide a nice way to obtain realistic particle physics models. In particular, we will see that a $\mathcal{N} = 1$ supersymmetric theory allows for chiral fermions in four dimensions, while field theories with a higher mumber of supersymmetry in four dimensions do not. In addition, supersymmetric configurations are easier to study before tackling more general compactifications. Indeed, supersymmetric compactifications of string theory allow for stable dimensional reductions, whose higher-dimensional corrections can be systematically studied.

The algebra of a N=1 supersymmetric theory in four-dimensional Minkoswki spacetime is an extension of the Poincare-algebra by adding supersymmetry generators which satisfy specific anti-commutation relations, instead of commutation relations.

Reformulate. We want to obtain a condition for a susy vacuum in 4d. We can obtain a classical field theory constraint.

A conserved charge Q associated to an unbroken supersymmetry annihilates the vacuum $|\Omega\rangle$, so $Q|\Omega\rangle = 0$. This in turn means that for any operator U, $\langle\Omega|\{Q,U\}|\Omega\rangle = 0$. If U is a fermionic operator, we derive that the variation of the operator under the supersymmetry transformation is $\delta U = \{Q,U\}$. Taking this as the classical limit, $\delta U = \langle\Omega|\delta U|\Omega\rangle$. Thus, we conclude that at the classical level $\delta U = \langle\Omega|\{Q,U\}|\Omega\rangle = 0$ for any fermionic field U.

We now consider the SUGRA theory of type IIA string theory and the condition that some fourd-supersymmetry remains. In the same way that a translation generated by the momentum operator is parametrized by a vector and a rotation is a parametrized by an antisymmetric tensor, a supersymmetry transformation generated by Q_α is parametrized by a spinor η_α. The variation of the gravitino field under a supersymmetry transformation is

$$\delta\psi_M = D_M\eta + (\text{fluxes}) \tag{2.1}$$

Where D_M is the covariant derivative on M. Supersymmetry preservation means that all variations must be zero. We assume that all fluxes vanish. This leads to the constraint that η is a covariantly constant spinor

$$\delta\psi_M = D_M\eta = 0 \tag{2.2}$$

If we particularize this equation to the four-dimensional space-time T, which is a maximally symmetric space, it imposes that T is Minkowski space and thus, η only depends on the compact coordinates.

To study the implication of this equation to the four-dimensional space-time T, we employ the fact that T is maximally symmetric, so we can decompose the metric as

$$ds^2 = e^{2A(y)}\tilde{g}_{\mu\nu}dx^\mu dx^\nu + g_{mn}dy^m dy^n, \qquad \mu = 0,1,2,3 \quad m = 1,\dots,6 \tag{2.3}$$

where x^μ are the compact coordinates, y^m the internal coordinates and $\tilde{g}_{\mu\nu}$ can be either the de Sitter, anti-de Sitter or the Minkowski metric in four dimensions.

Particularizing to the space-time components, equation (2.2) can be written as

$$\widetilde{\nabla}_\mu \eta + \frac{1}{2}(\tilde{\gamma}_\mu \gamma_5 \otimes \nabla\!\!\!/ A)\eta = 0 \tag{2.4}$$

where $\widetilde{\nabla}$ is the derivative with respect $\tilde{g}_{\mu\nu}$.

What is the gamma

Should I explain where this comes from? integrability

We then obtain

$$\left[\widetilde{\nabla}_\mu, \widetilde{\nabla}_\nu\right]\eta = \frac{1}{2}(\nabla_m A)(\nabla^m A)\gamma_{\mu\nu}\eta \tag{2.5}$$

$$\left[\widetilde{\nabla}_\mu, \widetilde{\nabla}_\nu\right]\eta = \frac{1}{4}\tilde{R}_{\mu\nu\lambda\rho}\gamma^{\lambda\lambda}\eta \tag{2.6}$$

since for a maximally symmetric space, the Riemman tensor is $R_{\mu\nu\lambda\rho} = k(g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda})$. Combining the previous two equations and inverting $\gamma^{\mu\nu}$, we obtain the condition

$$k + \nabla_m A \nabla^m A = 0 \tag{2.7}$$

constant value etc

The existence of a single covariantly constant spinor on the compact manifold can be reformulated as a topological condition, namely that the holonomy group (whose precise definition is given in the next chapter) of the compact manifold is $SU(3)$. A compact manifold of $SU(3)$ (local) holonomy is the definition of a Calabi-Yau manifold. The holonomy group being a proper subgroup of $SU(3)$ is equivalent to having more than one covariantly constant spinor, which would lead to a larger degree of supersymmetry preserved.

The existence of a covariantly constant spinor implies for a TIIA theory that there are two four-dimensional supersymmetry parameters and therefore, $\mathcal{N} = 2$.

2.1 Type IIA on Calabi-Yau manifolds

We examine more closely what the existence of a covariantly constant spinor field implies on the compact space.

Let us consider a Riemannian manifold K of dimension six with a spin connection ω, which is in general a $SO(6)$ gauge field. If we parallel transport a field ψ around a contractible closed curve γ, the field becomes $\psi' = U\psi$ where $U = \mathcal{P}e^{\int_\gamma dx\omega}$ and $\mathcal{P}$ denotes the path ordering of the exponential. The set of transformation matrices associated to all possible loops form the holonomy group of the manifold, which must be a subgroup of $SO(6)$.

A covariantly constant spinor is left unchanged when parallel transported along a contractible closed curve, so the holonomy matrices of a manifold that admits a covariantly constant spinor must satisfy $U\eta = \eta$. Taking into account the Lie algebra isomorphism $\mathfrak{so}(6) \simeq \mathfrak{su}(4)$ we identify the positive (negative)-chirality spinors of $SO(6)$ with the fundamental $\mathbf{4}$ ($\bar{\mathbf{4}}$) of $SU(4)$. Let us consider that η is a positive chirality spinor, so it transforms according with the $\mathbf{4}$ of $SU(4)$. In order to have a covariantly constant spinor, the holonomy group must be such that the $\mathbf{4}$ representation decomposes into a singlet. This decomposition is achieved if the holonomy group is $SU(3)$ so that

$$SO(6) \rightarrow SU(3) \tag{2.8}$$

$$\mathbf{4} \rightarrow \mathbf{3} \oplus \mathbf{1} \tag{2.9}$$

We can also check that the 2-form $\mathbf{15}$ and the 3-form $\mathbf{20}$ decompositions contain a singlet, $\mathbf{15} \rightarrow \mathbf{8} \oplus \mathbf{3} \oplus \bar{\mathbf{3}} \oplus \mathbf{1}$ and $\mathbf{20} \rightarrow \mathbf{6} \oplus \bar{\mathbf{6}} \oplus \mathbf{3} \oplus \bar{\mathbf{3}} \oplus \mathbf{1} \oplus \mathbf{1}$, so they are globally well defined. We refer to the 2-form as J and the 3-form as the holomorphic three-form Ω. Raising an index of J we obtain an almost-complex structure, which satisfies $(J^2)^i_j = -\delta^i_j$. For a particular point of the manifold, we can form a basis of complex coordinates z^i from the real coordinates x^i, as $z^1 = x^1 + ix^2$, $z^2 = x^3 + ix^4$ and $z^3 = x^5 + ix^6$, in which $J = idz^i \otimes dz^i - id\bar{z}^{\bar{i}} \otimes d\bar{z}^{\bar{i}}$. If we can extend this particular form of J to the neighborhood of any point, J is said to be integrable and the manifold is complex. An integrable almost-complex structure is referred to as a complex structure. The integrability condition is equivalent to the Nijenhuis tensor

$$N^k_{ij} = J^l_i(\partial_l J^k_j - \partial_j J^k_l) - J^l_j(\partial_l J^k_i - \partial_i J^k_l) \tag{2.10}$$

vanishing everywhere. It is possible to formulate an alternative definition of a complex manifold, as

It is useful to define with the aid of the metric the form $k = g_{i\bar{j}} dz^i \wedge d\bar{z}^{\bar{j}}$. A manifold is Kähler if $dk = 0$ and k is then called the Kähler form. It can be shown that the holonomy group being contained in $U(N)$ implies that the manifold is Kähler.

Algebraic geometry

It is useful to introduce some algebraic topology tools.

A differential p-form ω_p is $(0,p)$-rank tensor which has completely antisymmetric components. A p-form is expanded as a linear combination of the basis $\{dx^\nu\}_{\nu=1\ldots p}$

$$\omega_p = \frac{1}{p!}\omega_{\nu_1\ldots\nu_p} dx^{\nu_1} \wedge \cdots \wedge dx^{\nu_p} = \frac{1}{p!}\omega_{\nu_1\ldots\nu_p} dx^{[\nu_1} \otimes \cdots \otimes dx^{\nu_p]} \tag{2.11}$$

where the square brackets denote antisymmetrization.

The wedge product of a p-form ω_p and a q-form α_q is a $(p+q)$-form

$$\omega_p \wedge \alpha_q = \frac{1}{p!q!}\omega_{\nu_1\ldots\nu_p}\alpha_{\mu_1\ldots\mu_p} dx^{\nu_1} \wedge \cdots \wedge dx^{\nu_p} \wedge dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_p} \tag{2.12}$$

The exterior derivative of a p-form yields a $(p+1)$-form

$$d\omega_p = \frac{1}{p!}\partial_\mu \omega_{\nu_1\ldots\nu_p} dx^\mu \wedge dx^{\nu_1} \wedge \cdots \wedge dx^{\nu_p} \tag{2.13}$$

A p-form whose exterior derivative vanishes is called closed and a p-form that is the exterior derivative of a $(p-1)$-form is exact.

A fundamental property of the exterior derivative is Poincare's lemma, which states that for any diffential form α, $d(d\alpha) = 0$ holds. This can be rewritten as $d^2 = 0$. In other words, every exact form is closed. We could ask ourselves if the inverse statement is true, is every closed form exact? The answer for an arbitrary manifold is no.This information is encoded in the q-th deRham cohomology group, which is formed by considering the set of all closed q-forms defined on a manifold. Since given a closed form ω, we can always find another closed form by adding an exact form $\omega' = \omega + d\alpha$, we take the equivalence relation that two forms are equivalent if they differ by a closed form. The q-th deRham cohomology group is defined as the quotient

$$H^q_d(X, \mathbb{R}) = \{\omega | d\omega = 0\}/\{\alpha | \alpha = d\beta\} \tag{2.14}$$

DOLBEAULT COHOMOLOGY

We can easily make a generalization of the previous concepts to complex manifolds. $\bar{\partial}$

$$H^{r,s}_{\bar{\partial}}(X,\mathbb{C}) = \{\omega|\bar{\partial}\omega = 0\}/\{\alpha|\alpha = \bar{\partial}\beta\} \tag{2.15}$$

<++>

HODGE DECOMPOSITION / HODGE NUMBERS

$$\omega = d\beta + d^{\dagger}\gamma + \omega' \tag{2.16}$$

$$\omega = d\beta + \omega' \tag{2.17}$$

The dimension of $H^{p,q}$ is know as the Hodge number, $h_{p,q}$ and it is a topological invariant. For Calabi-Yau manifolds, $h_{d,0} = 1$ $h_{p,q} = h_{d-p,d-q}$ mirror symmetry

HOMOLOGY / INTERSECTION NUMBERS

A very related construction to cohomology is homology. The basic element of homology is the p-chain

algebraic dual, in the sense that

MODULI SPACE

2.2 Type IIA on orientifolds

Orientifold planes and D-branes

D6-branes on a Calabi-Yau. Special Lagrangians.

Model building

Chapter 3

Type IIA on the quintic

3.1 Quintic threefold motivation

Why the quintic?

Why study sLags?

3.2 sLags on Fermat's quintic

SM on the quintic

3.3 sLags on the deformed quintic

Quintic deformations

The consider the possible deformations of Fermat's quintic. There should be 101 independent deformations, since they correspond to different complex structures and are given be the Hodge number $h_{21} = 101$.

We can add terms of the following type to the quintic

$$x_i^5, x_i^4 x_j^1, x_i^3 x_j^2, x_i^3 x_j x_k, x_i^2 x_j x_k x_l, x_1 x_2 x_3 x_4 x_5 \tag{3.1}$$

Not all of these terms are independent, since a coordinate redefiniton $GL(5, \mathbb{C})$.

Deformation classification

Coordinate redefinition freedom

Singularities

As an example, we take as deformation $-5\phi z_1 z_2 z_3 z_4 z_5$

Change of variables to study geometry of the singularity

In order to determine the geometry near the singularity, we make the following change of variables

$$
\begin{aligned}
x_1 &= 1 + y_1/\sqrt{10} + y_2/5 + y_4/\sqrt{50} \\
x_2 &= 1 + y_1/\sqrt{10} - y_2/5 + y_4/\sqrt{50} \\
x_3 &= 1 + y_1/\sqrt{10} + y_3/5 - y_4/\sqrt{50} \\
x_4 &= 1 + y_1/\sqrt{10} - y_3/5 - y_4/\sqrt{50}
\end{aligned}
\tag{3.2}
$$

In these coordinates, the quintic becomes

$$5(\psi - 1) = y_1^2 + y_2^2 + y_3^2 + y_4^2 + O(\psi - 1) \tag{3.3}$$

Something about the branch

Volume of cycles wrapping singularities

Three-form integration

$$\Omega = \frac{x_5 dx_1 \wedge dx_2 \wedge dx_3}{\frac{\partial p}{\partial x_4}} \tag{3.4}$$

$$\int_{A^2} \Omega = \int \ldots \tag{3.5}$$

Bibliography