Contents

Chapter 1

D6-branes on Calabi-Yau manifolds

Types of string theories

Compactification

In the following section we motivate the requirement that additional dimensions are compactified over a Calabi-Yau manifold (a compact complex manifold of $SU(3)$ holonomy).

We assume that the manifold is factorizable into a four-dimensional spacetime N and a compact space K, $\mathcal{M} = N \times K$.

Do we need N to be maximally symmetric.

Out of all the possible ways to compactify a theory, we pick out the ones that preserve some degree of supersymmetry. There are several reasons for this choice:

- Gauge hierarchy problem.

  Is it relevant today?

- As a way to solve the equations of motion.

  What does this mean? A SUSY configurations satisfies the eom -> SUGRA. Classically?

- It gives a nice phenomenological description.

A four dimensional theory with $\mathcal{N} = 1$ supersymmetry allows for massless fermions that transform in a complex representation of the gauge group associated to the supersymmetry. Since $\mathcal{N} \geq 2$ in four dimensions all fermions must transform in a real representation of the gauge group, we shall only consider the case $\mathcal{N} = 1$.

Understand the reason for different possible representations.

Does this mean that at higher energies there are no chiral fermions?

Understand the meaning of SUSY parameter. How does it exponentiate.

Every supersymmetry transformation is parametrized by an infinitesimal parameter $\eta_\alpha(X)$ which is anti-commuting, two-component Weyl fermion that has an associated conserved supercharge Q at every space-time point.

Fill in the proof. We translate field equations into operator equations.

A conserved charge Q associated to an unbroken supersymmetry annihilates the vacuum $|\Omega\rangle$, so $Q\,|\Omega\rangle = 0$, since This in turn means that for any operator U, $\langle\Omega|\{Q, U\}|\Omega\rangle = 0$.

$$U' = U + \delta U = e^{-iQ\eta} U e^{iQ\eta} = (1 + iQ\eta)U(1 - iQ\eta) = \cdots \tag{1.1}$$

If U is a fermionic operator, we derive that the variation of the operator under the supersymmetry transformation is $\delta U = \{Q, U\}$. Taking this as the classical limit, $\delta U = \langle\Omega|\delta U|\Omega\rangle$. Thus, we conclude that at tree level $\delta U = 0$ for any fermionic field U.

What is really U?. Read about tree level.

Read some details on how to obtain this. Fill in the gaps. Check the implication direction.

The low energy spectrum of a ten-dimensional theory has as elementary fermions the gravitino ψ_M, the dilatino λ and the gluino ξ.

This is 10d N=1 SUGRA

Their variation is

$$\begin{aligned}
\delta\psi_M &= \frac{1}{\kappa} D_M \eta + \frac{\kappa}{32 g^2 \phi}(\Gamma_M^{NPQ} - 9\delta_M^N \Gamma^{PQ})\eta H_{NPQ} + (\text{Fermi})^2 \\
\delta\xi^a &= -\frac{1}{4g\sqrt{\phi}}\Gamma^{MN} F^a_{MN}\eta + (\text{Fermi})^2 \\
\delta\lambda &= -\frac{1}{\sqrt{2}\phi}(\Gamma\partial\phi)\eta + \frac{\kappa}{8\sqrt{2}g^2\phi}\Gamma^{MNP}\eta H + (\text{Fermi})^2
\end{aligned} \tag{1.2}$$

Where the Dirac matrices for the ten-dimensional space time are

$$\Gamma^M = e_A^M \Gamma^A. \tag{1.3}$$

Here e_A^M denotes the vielbein that describes the graviton and Γ^A are elements of a Clifford algebra, so $\{\Gamma^A, \Gamma^B = 2\eta^{AB}$.

Wtf is Γ?

Supersymmetry preservation means that all variations must be zero. For convenience, we set $H = 0$ and $\phi = \text{const.}$. This leads to the constraints

$$\begin{aligned} \delta\psi_M &= \frac{1}{\kappa} D_M \eta \\ \delta\xi^a &= -\frac{1}{4g\sqrt{\phi}} \Gamma^{MN} F^a_{MN} \eta \end{aligned} \tag{1.4}$$

The first equation implies that there exists $[D_M, D_N]\eta = 0 \; R_{MNPQ}\Gamma^{PQ}\eta$. If we particularize to T, which is a maximally symmetric space, the second equation imposes that T is Minkowski space, which is not surprising. Cosmological constant blah, blah, blah. We can now use the first equation to conclude that η does not depend on the uncompactified coordinates, $\partial_T \eta = 0$.

`https://groups.google.com/forum/#!topic/sci.physics.research/rrBoIXk9Rw0`

We proceed to examine what the existence of a covariantly constant spinor field imposes on the compact space.

Let us consider a Riemannian manifold K of dimension n with a spin connection ω, which is in general a $SO(n)$ gauge field. If we parallel transport a field ψ around a contractible closed curve γ, the field becomes $U\psi$ where $U = \mathcal{P} e^{\int_\gamma dx\omega}$, where $\mathcal{P}$ is the path-ordered product.

The set of the transformation matrices associated to all possible loops form the holonomy group of the manifold, which must be a subgroup of $SO(n)$.

Fill in Group Theory discussion

We now consider how the $SU(3)$ holonomy translates into the manifold.

Fill in Group Theory discussion

The only $U(3)$ invariants in the **6** representation of $SO(6)$ are the identity and $\bar{I}$.

U(3) holonomy implies complex manifold

Check the different spaces we consider.

We can also form a tensor field on K of the type $J^i_j(y) = g^{ik}(y)\bar{\eta}\Lambda_{kj}\eta(y)$. For each point y, we can consider J^i_j as a matrix that acts on the tangent space, so $v^i \to J^i_j v^j$. In this sense, J^i_j is a real, traceless and $SU(3)$ invariant matrix, which means that it must be proportional to $\bar{I}$. We had already seen that $\bar{I} = -I$, this an example of an almost-complex structure, which is a tensor field J that satisfies $J^2 = -I$.

If we employ complex coordinates, we can diagonalize J so that the non-zero components are $J^a_b = i\delta^a_b$ and $J^{\bar{a}}_{\bar{b}} = -i\delta^{\bar{a}}_{\bar{b}}$. This particular choice is know as the canonical form.

It is always possible to choose particular coordinates to bring J to the canonical form at a particular point. But in general, the canonical form will not hold at an open neighborhood of a point. If a manifold admits a set of coordinates (called local holomorphic coordinates) such that at every point, the canonical form holds for an open neighborhood, then the almost complex structure is integrable.

The necessary and sufficient condition for integrability is that the Nijenhuis tensor

$$N^k_{ij} = J^l_i(\partial_l J^k_j - \partial_j J^k_l) - J^l_j(\partial_l J^k_i - \partial_i J^k_l) \tag{1.5}$$

vanishes. An integrable almost-complex structure is a complex structure and a manifold with a complex structure is a complex manifold.

Coordinate definition of complex manifold

SU(3) implies vanishing first Chern class

Probably not necessary

If we want chirality we need SU(3) exactly

Moduli space

Read about the manifold they form

Orientifold planes and D-branes

D-brane motivation through O-planes.

If we compactify a Type II string theory on a Calabi-Yau manifold, we obtain a four-dimensional $\mathcal{N} = 2$ supersymmetric theory. In order to allow for chirality, we must obtain a $N = 1$ theory. This can be done through the orientifold projection, which consists in modding out $\Omega\mathcal{R}$.

Couldn't we start from a modded out manifold? It seems there is more than geometry here.

Understand RR charges.

$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma\epsilon^{\alpha\beta}B_{\mu\nu}\partial_\alpha X^\mu\partial_\beta X^\nu \tag{1.6}$$

D6-branes in flat 10d space

We have seen that if we compactify an heterotic string theory on a Calabi-Yau manifold, we obtain $\mathcal{N} = 1$ which allows chirality. This is not the case of Type II theories, which lead to $\mathcal{N} = 2$. In order to reduce the degree of supersymmetry and thus obtain chiral 4d fermions, two D6-branes in flat 10d can intersect over a 4d region. The open string spectrum of an intersection of a stack of N_1 D6-branes and a stack of N_2 D6-branes can be classified as:

- Strings stretching from one stack to itself, which lead to 7d $U(N_i)$ gauge boson, three real adjoint scalars and their fermion superpartners, .
- String that go from one stack to the other are localized at the intersection. Their associated fields are are charged under the bi-fundamental representation $(N_1, \bar{N}_2)$ of $U(N_1) \times U(N_2)$, which includes a 4d chiral fermion in $(N_1, \bar{N}_2)$.

D6-branes on a torus

Let us consider type IIA theory compactified on a 6-torus $T^6 = T^2 \times T^2 \times T^2$.

D6-branes on a Calabi-Yau

In order to obtain stable D6-brane configurations, we impose that they wrap around volume minimizing 3-cycles, which are special Lagrangian 3-cycles and satisfy

$$J|_{\Pi} = 0, \qquad \mathrm{Im}(e^{-i\phi}\Omega_3)|_{\Pi} = 0 \tag{1.7}$$

The volume of the special Lagrangian 3-cycle is

$$Vol(\Pi) = \int_{\Pi} \mathrm{Re}(e^{-i\phi}\Omega_3) \tag{1.8}$$

Spectrum

INTRODUCE NON-ABELIAN GAUGE BOSONS

Model building

Quintic deformations

The consider the possible deformations of Fermat's quintic. There should be 101 independent deformations, since they correspond to different complex structures and are given be the Hodge number $h_{21} = 101$.

We can add terms of the following type to the quintic

$$x_i^5, x_i^4 x_j^1, x_i^3 x_j^2, x_i^3 x_j x_k, x_i^2 x_j x_k x_l, x_1 x_2 x_3 x_4 x_5 \tag{1.9}$$

Not all of these terms are independent, since a coordinate redefiniton $GL(5, \mathbb{C})$.

Deformation classification

Coordinate redefinition freedom

Singularities

As an example, we take as deformation $-5\phi z_1 z_2 z_3 z_4 z_5$

Change of variables to study geometry of the singularity

In order to determine the geometry near the singularity, we make the following change of variables

$$\begin{aligned} x_1 &= 1 + y_1/\sqrt{10} + y_2/5 + y_4/\sqrt{50} \\ x_2 &= 1 + y_1/\sqrt{10} - y_2/5 + y_4/\sqrt{50} \\ x_3 &= 1 + y_1/\sqrt{10} + y_3/5 - y_4/\sqrt{50} \\ x_4 &= 1 + y_1/\sqrt{10} - y_3/5 - y_4/\sqrt{50} \end{aligned} \tag{1.10}$$

In these coordinates, the quintic becomes

$$5(\psi - 1) = y_1^2 + y_2^2 + y_3^2 + y_4^2 + O(\psi - 1) \tag{1.11}$$

Something about the branch

Volume of cycles wrapping singularities

Three-form integration

$$\Omega = \frac{x_5 dx_1 \wedge dx_2 \wedge dx_3}{\frac{\partial p}{\partial x_4}} \tag{1.12}$$

$$\int_{A^2} \Omega = \int \ldots \tag{1.13}$$

Bibliography