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Chapter 1

D6-branes on Calabi-Yau manifolds

Types of string theories

Compactification

In the following section we motivate the requirement that additional dimensions are compactified over a Calabi-Yau manifold (a compact complex manifold of $SU(3)$ holonomy).

We assume that the manifold \mathcal{M} is factorizable into a four-dimensional maximally symmetric space-time N and a six-dimensional compact space K , $\mathcal{M} = N \times K$.

Out of all the possible ways to compactify a theory, we pick out the ones that preserve some degree of supersymmetry. Our main reason for this choice is that they provide a nice way to obtain realistic particle physics models. In particular, we will see that an $\mathcal{N} = 1$ supersymmetric theory allows for chiral fermions. In addition, supersymmetric configurations are easier to study before tackling more general compactifications.

The supersymmetry condition means there is a transformation that relates fermions to bosons, and vice versa.

Superspace Roughly, extend a Poincaré algebra to include supersymmetry generators which satisfy specific anti-commutation relations, instead of commutation

relations

$$\{Q, Q\} = \{\bar{Q}, \bar{Q} = 0, \{Q, \bar{Q} = \quad (1.1)$$

A finite SUSY transformation is

$$e^{i(\eta Q + \bar{\eta} \bar{Q} - x_\mu p^\mu)} \quad (1.2)$$

Every supersymmetry transformation is parametrized by an infinitesimal parameter $\eta_\alpha(X)$, so that

$$\delta |boson\rangle = |fermion\rangle \quad (1.3)$$

Insert definition of η . Is it a Grassmann variable, a spinor, an operator...

Fill in the proof. We translate field equations into operator equations.

What is the relation of the charge to the full SUSY transformation.

A conserved charge Q associated to an unbroken supersymmetry annihilates the vacuum $|\Omega\rangle$, so $Q|\Omega\rangle = 0$, since This in turn means that for any operator U , $\langle\Omega|\{Q, U\}|\Omega\rangle = 0$.

$$U' = U + \delta U = e^{-iQ\eta} U e^{iQ\eta} = (1 + iQ\eta)U(1 - iQ\eta) = \dots \quad (1.4)$$

If U is a fermionic operator, we derive that the variation of the operator under the supersymmetry transformation is $\delta U = \{Q, U\}$. Taking this as the classical limit, $\delta U = \langle\Omega|\delta U|\Omega\rangle$. Thus, we conclude that at tree level $\delta U = 0$ for any fermionic field U .

What is really U ?. Read about tree level.

The low-energy theory of the ten-dimensional type IIA string theory is type IIA SUGRA, which although it has $\mathcal{N} = 2$ instead of $\mathcal{N} = 1$, we will see a workaround. The type IIA string theory has as NS-NS fields the dilaton ϕ , the metric G and the two-form field B . In the RR sector we have the one-form and three-form, c_1 and c_3 . For convenience, we define employ the democratic formulation

Understand democratic formulation

We also define the field strength of the NS form $H = dB$ and the F^{10} as the formal sum

$$F^{10} = dC - H \wedge C + m e^B \quad (1.5)$$

where C is the formal sum of odd differential form fields.

What is m?

$$F_n^{(10)} = (-1)^{\lfloor n/2 \rfloor} \star F_{10-n}^{(10)} \quad (1.6)$$

Type IIA SUGRA has as elementary fermions two Majora-Weyl gravitinos of the same chirality ψ_M and two dilatinos λ . Their variation under a supersymmetry transformation is

$$\delta\psi_M = D_M\eta + \frac{1}{4}\not{H}_M\Gamma_{11}\eta + \frac{1}{16}e^\phi\sum_n\not{F}_n^{(10)}\Gamma_M\Gamma_{11}^{n/2}\sigma^1\eta \quad (1.7)$$

Where $H = dB$ is the field strength of the antisymmetric two-form B . $\not{F}_n^{(10)} = \frac{1}{n!}F_{P_1\dots P_N}\Gamma^{P_1\dots P_N}$ and $\not{H}_M = \frac{1}{2}H_{MNP}\Gamma^{NP}$. The $\Gamma\sigma^1$

Where the Dirac matrices for the ten-dimensional space time are $\Gamma^M = e_A^M\Gamma^A$. Here e_A^M denotes the vielbein that describes the graviton and Γ^A are elements of a Clifford algebra, so $\{\Gamma^A, \Gamma^B\} = 2\eta^{AB}$.

Supersymmetry preservation means that all variations must be zero. For convenience, we set $H = 0$ and $\phi = \text{const.}$. This leads to the constraint that η is a covariantly constant spinor

$$\delta\psi_M = D_M\eta = 0 \quad (1.8)$$

Why this cov der.

The equation implies that there exists $[D_M, D_N]\eta = R_{MNPQ}\Gamma^{PQ}\eta = 0$. If we particularize to T , which is a maximally symmetric space, the last equation imposes that T is Minkowski space, which is not surprising. Cosmological constant blah, blah, blah. We can now use the first equation to conclude that η does not depend on the uncompactified coordinates, $\partial_T\eta = 0$.

<https://groups.google.com/forum/#!topic/sci.physics.research/rrBoIXk9Rw0>

We proceed to examine what the existence of a covariantly constant spinor field imposes on the compact space.

Let us consider a Riemannian manifold K of dimension n with a spin connection ω , which is in general a $SO(n)$ gauge field. If we parallel transport a field ψ around a contractible closed curve γ , the field becomes $U\psi$ where $U = \mathcal{P}e^{\int_\gamma dx\omega}$, where \mathcal{P} is the path-ordered product (complete).

The set of the transformation matrices associated to all possible loops form the holonomy group of the manifold, which must be a subgroup of $SO(n)$.

Fill in Group Theory discussion

We now consider how the $SU(3)$ holonomy translates into the manifold.

Fill in Group Theory discussion

The only $U(3)$ invariants in the **6** representation of $SO(6)$ are the identity and \bar{I} .

U(3) holonomy implies complex manifold

Check the different spaces we consider.

We can also form a tensor field on K of the type $J_j^i(y) = g^{ik}(y)\bar{\eta}\Lambda_{kj}\eta(y)$. For each point y , we can consider J_j^i as a matrix that acts on the tangent space, so $v^i \rightarrow J_j^i v^j$. In this sense, J_j^i is a real, traceless and $SU(3)$ invariant matrix, which means that it must be proportional to \bar{I} . We had already seen that $\bar{I} = -I$, this an example of an almost-complex structure, which is a tensor field J that satisfies $J^2 = -I$.

If we employ complex coordinates, we can diagonalize J so that the non-zero components are $J_b^a = i\delta_b^a$ and $J_{\bar{b}}^{\bar{a}} = -i\delta_{\bar{b}}^{\bar{a}}$. This particular choice is known as the canonical form.

It is always possible to choose particular coordinates to bring J to the canonical form at a particular point. But in general, the canonical form will not hold at an open neighborhood of a point. If a manifold admits a set of coordinates (called local holomorphic coordinates) such that at every point, the canonical form holds for an open neighborhood, then the almost complex structure is integrable.

The necessary and sufficient condition for integrability is that the Nijenhuis tensor

$$N_{ij}^k = J_i^l(\partial_l J_j^k - \partial_j J_l^k) - J_j^l(\partial_l J_i^k - \partial_i J_l^k) \quad (1.9)$$

vanishes. An integrable almost-complex structure is a complex structure and a manifold with a complex structure is a complex manifold.

Coordinate definition of complex manifold

SU(3) implies vanishing first Chern class

Probably not necessary

If we want chirality we need SU(3) exactly

Moduli space

Calabi-Yau definition: SU(n) global holonomy \leftrightarrow non-vanishing n-form \rightarrow vanishing first Chern class \leftrightarrow vanishing Ricci curvature \leftrightarrow local SU(n) holonomy. If simply connected, both definitions are equivalent.

We start from a complex manifold with a metric st. it is a Calabi-Yau manifold. We then deform it's metric while it remains CY

Given a Calabi-Yau manifold with a particular complex structure and metric, we could ask ourselves when the deformations

ORIENTIFOLD PLANES AND D-BRANES

Orientifold planes and D-branes

D-brane motivation through O-planes.

If we compactify a Type II string theory on a Calabi-Yau manifold, we obtain a four-dimensional $\mathcal{N} = 2$ supersymmetric theory. In order to allow for chirality, we must obtain a $\mathcal{N} = 1$ theory. This can be done through the orientifold projection, which consists in modding out $\Omega\mathcal{R}$ acting on the 10d manifold, where Ω is the worldsheet parity and \mathcal{R} is a particular \mathbb{Z}_2 symmetry.

Understand RR charges.

$$S = -\frac{1}{4\pi\alpha'} \int d^2\sigma \epsilon^{\alpha\beta} B_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu \quad (1.10)$$

D6-branes in flat 10d space

We have seen that if we compactify an heterotic string theory on a Calabi-Yau manifold, we obtain $\mathcal{N} = 1$ which allows chirality. This is not the case of Type II theories, which lead to $\mathcal{N} = 2$. In order to reduce the degree of supersymmetry and thus obtain chiral 4d fermions, two D6-branes in flat 10d can intersect over

a 4d region. Before considering D6-branes over a Calabi-Yau compactification, we consider the flat space case.

The open string spectrum of an intersection of a stack of N_1 D6-branes and a stack of N_2 D6-branes can be classified as:

- Strings stretching from one stack to itself, which lead to 7d $U(N_i)$ gauge bosons, three real adjoint scalars and their fermion superpartners, .
- String that go from one stack to the other are localized at the intersection. Their associated fields are charged under the bi-fundamental representation (N_1, \bar{N}_2) of $U(N_1) \times U(N_2)$, which includes a 4d chiral fermion in (N_1, \bar{N}_2) .

D6-branes on a torus

Let us consider type IIA theory compactified on a 6-torus $T^6 = T^2 \times T^2 \times T^2$.

D6-branes on a Calabi-Yau

In order to obtain stable D6-brane configurations, we impose that they wrap around volume minimizing 3-cycles, which are special Lagrangian 3-cycles and satisfy

$$J|_{\Pi} = 0, \quad \text{Im}(e^{-i\phi}\Omega_3)|_{\Pi} = 0 \quad (1.11)$$

The volume of the special Lagrangian 3-cycle is

$$\text{Vol}(\Pi) = \int_{\Pi} \text{Re}(e^{-i\phi}\Omega_3) \quad (1.12)$$

Spectrum

INTRODUCE NON-ABELIAN GAUGE BOSONS

Model building

Quintic deformations

The consider the possible deformations of Fermat's quintic. There should be 101 independent deformations, since they correspond to different complex structures and are given by the Hodge number $h_{2,1} = 101$.

We can add terms of the following type to the quintic

$$x_i^5, x_i^4 x_j^1, x_i^3 x_j^2, x_i^3 x_j x_k, x_i^2 x_j x_k x_l, x_1 x_2 x_3 x_4 x_5 \quad (1.13)$$

Not all of these terms are independent, since a coordinate redefinition $GL(5, \mathbb{C})$.

Deformation classification

Coordinate redefinition freedom

Singularities

As an example, we take as deformation $-5\phi z_1 z_2 z_3 z_4 z_5$

Change of variables to study geometry of the singularity

In order to determine the geometry near the singularity, we make the following change of variables

$$\begin{aligned} x_1 &= 1 + y_1/\sqrt{10} + y_2/5 + y_4/\sqrt{50} \\ x_2 &= 1 + y_1/\sqrt{10} - y_2/5 + y_4/\sqrt{50} \\ x_3 &= 1 + y_1/\sqrt{10} + y_3/5 - y_4/\sqrt{50} \\ x_4 &= 1 + y_1/\sqrt{10} - y_3/5 - y_4/\sqrt{50} \end{aligned} \quad (1.14)$$

In these coordinates, the quintic becomes

$$5(\psi - 1) = y_1^2 + y_2^2 + y_3^2 + y_4^2 + O(\psi - 1) \quad (1.15)$$

Something about the branch

Volume of cycles wrapping singularities

Three-form integration

$$\Omega = \frac{x_5 dx_1 \wedge dx_2 \wedge dx_3}{\frac{\partial p}{\partial x_4}} \quad (1.16)$$

$$\int_{A^2} \Omega = \int \dots \quad (1.17)$$

Bibliography