A walk through moduli space with SLags

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Madrid, 2017

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Chapter 1

Generalities of type IIA string theory

In this thesis we will only work with the supersymmetric type IIA theory. The study of string theory in Minkowski spacetime has lead to the identification of five consistent string theories, which all turn out to be supersymmetric and give rise to massless bosonic and fermionic excitations in their spectrum. The five string theories were given their name according to their own specificities: Type heterotic HE and HO, Type I, Type IIB and Type IIA string theory. In this thesis we will only concentrate on the last one in the list.

Type IIA spectrum

Type IIA string theory requires ten space-time dimensions to be consistent. Furthermore, it has a 10-dimensional supersymmetry with 32 supercharges, which corresponds to $\mathcal{N} = (1, 1)$. The flat 10-dimensional space-time bosonic spectrum of type IIA can be classified according to the boundary conditions of the strings, whether we consider Ramond (R) or Neveu–Schwarz (NS) conditions. There would be an infinite tower of massive string states, but restrict the discussion to the massless states only. In the NS-NS sector, we find the dilaton ϕ, a two-form B_2 and a graviton $G_{\mu\nu}$, while in the R-R sector we identify the 1- and 3-forms c_1, c_3. The fermions, which belong to the NS-R and R-NS sectors, are two opposite-chirality gravitinos ψ and two opposite-chirality dilatinos λ.

Type IIA SUGRA

The low-energy theory of the ten-dimensional type IIA string theory is type IIA SUGRA. The spectrum of Type IIA SUGRA has as elementary fermions, which belong to the massless spectrum (NS-R and R-NS) of type IIA theory, two Majorana-Weyl gravitinos of the same chirality ψ^a_M and two dilatinos λ^a.

Action

The D-brane

The two-dimensional strings can be generalized to $(p+1)$-dimensional extended object, which are called Dp-branes. Thus, a D1-brane would correspond to a D-string, a D2-brane would be a three-dimensional membrane and so on. The existence of Dp-branes can be motivated, in the weak coupling limit, as objects where open strings end, so they are a way to impose Dirichlet boundary conditions on open strings. In type IIA string theory, only even-dimensional Dp-branes are physical, which are: the D0, D2, D4, D6 and D8-branes.

We can study the dynamics of a Dp-branes in terms of the open string excitations with endpoints attached to the Dp-brane. Let us consider the open string excitations of a Dp-brane, the latter spanning $p+1$ dimensions and transverse to $d-p-1$ dimensions. The presence of the Dp-brane breaks the ten-dimensional Poincaré invariance of the theory, because open string excitations propagate on the $(p+1)$-dimensional volume of the Dp-brane only. This implies that massless particles must transform under irreducible representations of $SO(p-2)$, instead of $SO(d-2)$. The massless spectrum in $(p+1)$ dimensions of the open string theory is composed by a gauge boson A^μ $(\mu = 0, \ldots, p)$ (corresponding to longitudinal oscillations to the brane), $9-p$ real scalars ϕ^i (corresponding to transverse oscillations to the brane) and some fermions λ_a. This particle content can be arranged into a vector supermultiplet of $U(1)$ with 16 supersymmetries in $(p+1)$ dimensions. Thus, a Dp-brane reduces the degree of supersymmetry of the type IIA theory by half.

In order to find out the action of a Dp-brane we must realize that it corresponds to the $(p+1)$-dimensional effective action of the massless open string excitations of the Dp-brane. As an illustration of this, a Dp-brane breaks the translational symmetry of the vacuum, which allows us to conclude that the

ϕ^i scalar fields are the Goldstone bosons associated to the broken symmetry. The vev of these scalar fields determine the position of the Dp-brane in the transverse space, and the fluctuations of the scalar fields determine the evolution of the Dp-brane worldvolume W_{p+1} (the generalization of the particle worldline to the case of higher-dimensional branes). The resulting action of the bosonic sector of the Dp-brane is the sum of a Dirac-Born-Infeld term S_{DBI} and a Chern-Simons term S_{CS}.

The DBI term carries the information of how a Dp-brane interacts with the NSNS fields. It takes the form

$$S_{DBI} = -\frac{\alpha'^{-(p+1)/2}}{(2\pi)^p} \int d^{p+1}x e^{-\phi} \sqrt{-\det(P[G+B] - 2\pi\alpha' F)} \tag{1.1}$$

gauge field strength $F = d$

$$\mu_p = \frac{(\alpha')^{-(p+1)/2}}{(2\pi)^p} \tag{1.2}$$

pullback

If we expand the DBI action in powers of α', we obtain the Yang-Mills term

$$S_{YM} = \frac{\alpha'^{-(p-3)/2}}{4g_s(2\pi)^{p-2}} \int d^{p+1}x\sqrt{-g}\,\mathrm{Tr}\,F_{\mu\nu}F^{\mu\nu} \tag{1.3}$$

which allows us to identify the Yang-Mills coupling as

$$g_{YM}^2 = g_s \alpha'^{(p-3)/2}(2\pi)^{p-2} \tag{1.4}$$

The Chern-Simons term is topological in nature and describes how Dp-branes interact with RR-fields.

$$S_{CS} = \mu_p \int P\left[\sum_p c_q\right] \wedge e^{2\pi\alpha F_2 - B_2} \wedge \hat{A}(R) \tag{1.5}$$

Detail

A-roof polynomial

Multiple D-branes

It is convenient to generalize the single Dp-brane configuration to N parallel Dp-branes. In order to determine the spectrum of a stack of Dp-branes, we consider open strings with endpoints attached to either a single brane or two distinct ones.

If all branes are separated from each other, strings that stretch from a brane to itself correspond to massless gauge bosons that belong to $U(1)^N$. In contrast, strings that stretch from one brane A to another brane B lead to massive particles whose masses increases with the distance between branes. The lightest of these particles have opposite charge $(1, -1)$ under $U(1)_A \times U(1)_B$. Since Type IIA strings carry an orientation, a string stretching B to A would have opposite charges.

In the case of N coincident Dp-branes, all configurations lead to massless states, such that the gauge symmetry enhances from $U(1)^N$ to $U(N)$. The massless spectrum is composed of $(p-1)$-dimensional $U(N)$ gauge bosons, $(9-p)$ real scalars in the adjoint representation of $U(N$ and several fermions in the adjoint representation.

Let us now suppose Dp-branes which are not parallel, so they can intersect each other. This situation is relevant as it can lead to four-dimensional chiral fermions in the case of intersecting D6-branes. We are interested in describing the open string spectrum of two stacks of D6-branes that intersect over a 4-dimensional subspace of their volumes.

Strings that stretch from a coincident stack of N D6-branes to itself lead to 7-dimensional $U(N)$ gauge bosons, three real adjoint scalars and their fermion superpartners.

String that stretch from a stack of N_1 D6-branes to another stack of N_2 D6-branes are localized at the intersection, in order to minimize their energy. They lead to a 4-dimensional fermion charged in the $(\mathbf{N_1}, \mathbf{N_2})$ of $U(N_1) \times U(N_2)$ or its conjugate, depending on the orientation of the intersection.

Not all geometric configurations preserve supersymmetry. Let us decompose space-time as $M_4 \times \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2$. The D6-branes span all M_4 and a line in each $\mathbb{R}^2$ plane, such that the angle between two stacks is given by θ_i for each plane. It can be shown that the condition $\theta_1 \pm \theta_2 \pm \theta_3 = 0(\mathrm{mod} 2\pi)$ implies $\mathcal{N} = 1$ supersymmetry in 4 dimensions, provided that no angle vanishes. If some of the angles vanish, the supersymmetry would we enhanced.

The reason we have used D6-branes and no other dimension of Dp-branes is that they would not lead to chiral fermions in 4 dimensions. Intuitively, two D6-branes allow to define an orientation in the transverse 6-dimensional space, which would not be possible with two other type of Dp-branes in type IIA string theory.

Chapter 2

Type IIA compactifications

Moduli space

Starting from a particular choice of metric g on a Calabi-Yau manifold X, we could try to determine which deformations of the metric still preserve the Calabi-Yau condition. These deformations of the metric are known as moduli and play an important role in the physics of compactifications. We will restrict our discussion to Calabi-Yau manifolds of complex dimension three. An arbitrary deformation of the metric will consist of those with pure indices $g_{ij}dz^i dz^j$ and those with mixed indices $g_{i\bar{j}}dz^i dz^{\bar{j}}$. In order to preserve the Calabi-Yau condition they must lead to a vanishing Ricci tensor, $R_{i\bar{j}} = 0$. This constraint implies that:

A deformation of the type $g_{ij}dz^i \wedge dz^j$ must be harmonic, so it can be identified with an unique element of a cohomology class in $H^{1,1}$, the Kähler form. If we write the Kähler form in terms of the basis elements $\{t_a\}_{a=1,\dots,h_{1,1}}$

$$k = \sum_{a=1}^{h_{1,1}} t_a \omega_a \tag{2.1}$$

the $h_{1,1}$ real parameters t_a are the Kähler moduli of the manifold. The Kähler form is employed to calculate the volume of a Calabi-Yau manifold of complex dimension three as $\int k \wedge k \wedge k$, since $k \wedge k \wedge \wedge k$ has the same rank as the volume form, which is unique up to a proportionality constant.

Deformations of the type $\Omega_{ijk} g^{k\bar{k}} \delta_{\bar{k}\bar{l}} dz^i \wedge dz^j \wedge d\bar{z}^{\bar{l}}$ must be a harmonic form belonging to a cohomology class in $H^{2,1}$. These deformations correspond to deformations of the complex structure, since the choice of a complex structure is related to a $(2,1)$-form through the holomorphic 3-form $J_{ij\bar{k}} = \Omega_{ijl} J^{l}_{\bar{k}}$. We can expand the $(2,1)$-form in a basis $\{s_a\}_{a=1,\dots,h_{2,1}}$

$$J = \sum_{a=1}^{h_{2,1}} s_a \sigma_a \tag{2.2}$$

The $h_{2,1}$ complex parameters s_a are the complex structure moduli of the manifold and determine the volume of 3-cycles Π in the compact space through Ω_3

$$\mathrm{Vol}(\Pi) = \int_{\Pi} \Omega_3 \tag{2.3}$$

In conclusion, a Calabi-Yau metric is determined uniquely by the Kähler form and the holomorphic three-form. The former leads to $h_{1,1}$ real parameters while the latter requires $h_{2,1}$ complex parameters.

Why some some moduli real and others complex?

Type IIA spectrum on CY

In order to compute the 4-dimensional massless spectrum of type IIA theory on a Calabi-Yau, we consider the limit where the size of compact space tends to zero (Kaluza-Klein dimensional reduction).

The simplest example of KK reduction is based on a free scalar field $\phi(x^M)$ in ten dimensions. We first apply its Fourier expansion in terms of the eigenvectors $\phi_k(x^m)$ of the Laplace operator in the internal space with eigenvalues λ_k

$$\phi(x^M) = \sum_k \phi_k(x^\mu)\phi_k(x^m) \tag{2.4}$$

where the dimension of the mode is determined by the argument, x^μ for the 4-dimensional Minkowski space and x^m for the compact space. The masslessness condition of $\phi(x^M)$ implies that

$$\Box\phi(x^\mu) - \lambda_k \phi(x^\mu) = 0 \tag{2.5}$$

This equation permits us to identify λ_k as the squared mass of the 4-dimensional $\phi(x^\mu)$ field. Thus, the number of massless scalar fields is given by the number of solutions of $\Box\phi(x^\mu) = 0$ which in the case of compact manifolds is one. We conclude that a 10-dimensional scalar field leads to a massless scalar field in 4-dimensions.

Our next example is KK reduction of a p-form C_p with the expansion

$$C_p = \sum_{k,q} c_q^k(x^m) \wedge C_{p-q}^k(x^\mu) \tag{2.6}$$

Massless 4-dimensional $(p-q)$-form fields correspond to internal modes that satisfy $dc_q = d^\dagger c_q = 0$, so c_q is a harmonic form. Since there is a single harmonic q-form in each q-cohomology class, the number of 4-dimensional massless $(p-q)$-forms arising from a p-form is the dimension of the H_q cohomology group, the Betti number b_q.

In the case of a Calabi-Yau manifold, from the relation of the Betti numbers with the Hodge numbers, we determine $b_0 = h_{0,0} = 1$, $b_1 = h_{1,0} + h_{0,1} = 0$, $b_2 = h_{1,1} + h_{2,0} + h_{0,2} = h_{1,1}$ and $b_3 = h_{3,0} + h_{0,3} + h_{2,1} + h_{1,2} = 2h_{2,1} + 1$. Thus, c_1 leads to a 4-dimensional 1-form A_μ, B_2 leads to a 2-form $B_{\mu\nu}$ and $h_{1,1}$ scalar fields B_0 and c_3 leads to a 3-form, $h_{1,1}$ 1-forms and $2h_{2,1} + 2$ scalar fields.

Why c_3 form is missing from the 4d spectrum?

The KK reduction of the 10-dimensional metric is applied considering its components separately:

- The $G_{\mu\nu}$ components correspond to scalar fields in the internal space satisfying the Laplace equation and whose solution is unique for compact spaces. Thus, a 10-dimensional graviton reduces to a 4-dimensional graviton.

- The $G_{\mu m}$ components would correspond to 4-dimensional vector bosons, associated to 6-dimensional vector fields in the compact space. The masslessness condition of the 4-dimensional field would imply that the 6-dimensional vectors are Killing vectors associated to continuous isometries of the compact space, which in the case of Calabi-Yau manifolds are non-existent. As a consequence, the $G_{\mu m}$ components do not lead to any massless fields in 4 dimensions.

- The G_{mn} components reduce to 4-dimensional scalar fields associated to the moduli of the internal space, whose vev determine the geometry of the internal space. In the case of Calabi-Yau manifolds, why have seen that there are h_{21} real scalar fields and h_{11}

The complex moduli scalars combine with the other scalars

Explain moduli fields

$\{\omega_P\}_{P=1,\ldots,h_{1,1}}$ $\{\alpha_K, \beta^K\}_{K=1,\ldots,h_{2,1}}$

$\int \alpha_K \wedge \beta^L = \delta_K^L$

Having seen how the bosonic fields of type IIA behave under KK reduction, we proceed to describe the spectrum of type IIA theory compactified on a Calabi-Yau manifold. The closed string spectrum can be arranged into supermultiplets of 4-four dimensional $\mathcal{N} = 2$ supersymmetry, which are:

A single supergravity multiplet, composed of a graviton $G_{\mu\nu}$, a gauge boson A_μ and two gravitinos ψ with opposite chiralities.

$h_{1,1}$, vector multiplets, composed of a gauge boson $C_{i\bar{j}\mu}$, a complex scalar and two Majorana fermions.

$h_{2,1}$ vector hypermultiplets composed of two complex scalar and two left-handed fermions.

A single vector hypermultiplet composed of two complex scalar and two left-handed fermions.

Add moduli notation

2.1 Type IIA on orientifold projections

Generalities of orientifolds

If we compactify a type II string theory on a Calabi-Yau manifold, we obtain a four-dimensional $\mathcal{N} = 2$ supersymmetric theory. This degree of supersymmetry does not allow for chiral fermions, so Calabi-Yau compactifications of type II theories have no straightforward application in the context of model building. An option to reduce the supersymmetry to $\mathcal{N} = 1$ is to apply the orientifold projection, which consists in modding out the action of ΩR, where Ω is the worldsheet parity, so strings become unoriented, and R is a particular $\mathbb{Z}_2$ symmetry of the compact six-dimensional space

In type IIA string theory we define $R = \mathcal{R}(-1)^{F_L}$. $\mathcal{R}$ satisfies the condition that it is an involution (squares to the identity) and acts antiholomorphically on the complex coordinates of the internal space ($\mathcal{R} : z_i \to \bar{z}_i$). This implies that the Kähler and the holomorphic three-form transform as $J \to -J$ and $\Omega_3 \to \bar{\Omega}_3$.

(-1)F anticommutes with the fermionic coordinates of the string. (-1)FL only anticommutes with left moving spectrum? Where do we apply it to?

What points belong to a plane or another?

The fixed points under $\mathcal{R}$ define the orientifold planes in the model and are denoted as Op-planes, where p is the spatial dimension. They span the entire four-dimensional Minkowski space and wrap a compact $(p-3)$-cycle on the internal space.

In order to have a stable compactification, we expect all RR and NSNS charges to vanish so the compactification is stable. Furtheremore, RR tadpole cancellation implies that the 4-dimensional theory is free of non-abelian gauge anomalies. Op-planes carry RR charge, so in order to eliminate RR tadpoles we must also introduce Dp-branes, which carry opposite charge. It is important to note that Dp-branes do not need to wrap the same 3-cycles as the Op-planes to remove RR tadpoles.

D-branes on Calabi-Yau manifolds

Can a submanifold of a CY not be a cycle?

In order to obtain stable D6-brane configurations on a type IIA theory compactified on a Calabi-Yau manifold, we impose that they wrap around volume minimizing 3-cycles on the compact space, so energy is minimized too. The volume minimizing condition means that the branes must wrap special Lagrangian 3-cycles in the internal space. Special Lagrangian 3-cycles Π, are defined by

$$k|_\Pi = 0, \qquad \mathrm{Im}(e^{-i\phi}\Omega_3)|_\Pi = 0 \tag{2.7}$$

for some real ϕ, where k is the Kähler two-form and Ω_3 the holomorphic three-form. The $e^{-i\phi}\Omega_3$ is referred as a calibration and the special Lagrangian is calibrated with respect to it.The volume of the special Lagrangian 3-cycle is

$$\mathrm{Vol}(\Pi) = \int_\Pi \mathrm{Re}(e^{-i\phi}\Omega_3) \tag{2.8}$$

D6-branes wrapped around a special Lagrangian cycle are guaranteed to preserve 4-dimensional $\mathcal{N} = 1$ supersymmetry. This preserved supersymmetry coincides with that preserved by Op-planes only if $\phi = 0$.

Should relocate? What if no common susy with branes or Oplanes?

The open string spectrum of stacks of N_a D6_a-branes wrapping special Lagrangian 3-cycles Π_a can be classified into two sectors: strings that stretch from one stack to itself and those that stretch between to different stacks, 6_a and 6_b.

Strings that stretch over 6_a lead to $U(N_a)$ vector multiplets of 4-dimensional $\mathcal{N} = 1$ supersymmetry. There are also $b_1(\Pi_a)$ chiral multiplets in the adjoint representation, which are composed of the internal components of the gauge field along Π_a combined with the geometric moduli of the 3-cycle, and their fermion superpartners.

Clarify. Susy can be N=4?

Strings that stretch between 6_a and 6_b lead to $I_{ab} = [\Pi_a][\Pi_b]$ chiral fermions, where I_{ab} is the intersection number between 3-cycles. These fermions transform in the $(\mathbf{N_a}, \bar{\mathbf{N}}_\mathbf{b})$ representation.

Is everything massless? Additional fields?

Orientifold compactifications with intersecting D-branes

$z_i \to \bar{z}_i$

Introduce

The condition that D6-branes preserve the same $\mathcal{N} = 1$ supersymmetry as the O6-planes is that the local relative angles between them obey

$$\theta_1 + \theta_2 + \theta_3 = 0 \tag{2.9}$$

The condition for RR tadpole cancellation is

$$\sum_a N_a[\Pi_a] + \sum_a N_a[\Pi_{a'}] - 4[\Pi_{O6}] = 0 \tag{2.10}$$

Briefly include spectrum

Effective action of intersecting D-branes on Calabi-Yau orientifolds

$$\begin{aligned} C_1 &= C_1(x) \\ B_2 &= b_2(x) + b^p\omega_p \\ C_3 &= A_1^P(x)\wedge\omega_P + C^K(x)\alpha_K - \tilde{C}_K(x)\beta^K \end{aligned} \tag{2.11}$$

Dimensional reduction

$$S_{DBI} = -\mu_p \int_{D_p} e^{-\phi}\sqrt{\det(G+B-2\pi\alpha' F)} \tag{2.12}$$

$$S = \mu_p \int_{D_p} \sum_q \mathrm{Tr}\, e^{2\pi\alpha' F - B} + \cdots \tag{2.13}$$

$$\frac{1}{g^2} = e^{-\phi}\frac{(\alpha')^{(3-p)/2}}{(2\pi)^{p-2}}\mathrm{Vol}(\Pi_{p-3}) \tag{2.14}$$

$$f_a = \frac{(\alpha')^{3/2}}{(2\pi)^4}\left[e^{-\phi}\int_{\Pi_a}\mathrm{Re}(e^{-i\phi_a}\Omega_3) + i\int_{\Pi_a} c_3\right] \tag{2.15}$$

Bibliography