Factors Influencing the Cooking time of Eggs

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1 Introduction

Although the consumption of eggs is common worldwide and considered a staple in many households due to its richness in protein, the cooking time of an egg is widely disputed. The question of how to maximise the cooking time of an egg often crosses the mind but is often neglected due to the lack of consensus worldwide. It is this exact question that we set out to investigate and shed light onto with our experimental study. In particular, we aim to answer the question of whether factors such as water temperature and water salinity influence the internal temperature of an egg given a controlled 10 minutes of boiling time. The results that can be obtained from this study are essential as it will reveal what factors impact the cooking time of an egg and help individuals boil their eggs to perfection.

2 Methodology

2.1 Variables

In the experiment, three variables were considered: two explanatory (independent variables) and one response (dependent variable). The explanatory variables were the salinity and starting temperature of the water in which the egg was boiled. The salinity of the water was binary, with two possible levels: 0 grams/litre or 50 grams/litre, making it a categorical variable. Similarly, the starting temperature of the water was categorical, with two possible levels: 10 degrees Celsius or 20 degrees Celsius. The response variable which was measured was the internal temperature of an egg in degrees Celsius after being boiled, which is a quantitative variable. To measure the salinity of the water, 37 grams of salt was measured with a cooking scale and then added to 750ml of water, otherwise the water was left unsalted.

2.2 Sample Size

To compute the sample size, testing parameters such as power, significance level and effect size must first be chosen. A significance level of 0.05 and an effect size of 0.5 were chosen so that relatively large effects are detected in the test. A power level of 0.7 was chosen due to budgetary limitations. With the use of R, a sample size of 10 observations per group proved to be sufficient for the testing parameters that were chosen.

2.3 Observation and Treatments

The total number of observations in this experiment was 40; there were 40 experimental units. The experimental unit of this study was a large sized egg. Each explanatory variable has two levels. Since this experiment is crossed, all possible combinations of factors and their levels were possible. Hence there were four possible treatments all together. They are listed in the table below.

	Unsalted	Salted
10C	(10C, Unsalted)	(10C, Salted)
20C	(20C, Unsalted)	(20C, Salted)

Table 1: Treatment effects and factors

The 40 experimental units were numbered one to forty. Then a random number generator in R was applied without replacement to pick a random integer within this range, and the egg with that number was selected. The selected eggs were assigned to treatments one, two, three and four in this order. This cycle was repeated until all eggs were assigned a treatment.

2.4 The Experiment

Forty large sized eggs were randomly selected and boiled one at a time in a pot containing 750ml of water with four possible treatments. After placing the egg into the pot of water, the pot was placed onto a preheated hot plate set at medium high heat, and the 10 minute timer was then started. After 10 minutes had elapsed, the egg was removed from the pot and allowed to rest for thirty seconds in a bowl at room temperature. The temperature was then measured using a digital instant read thermometer by inserting the probe approximately 11 millimeters into the bottom of the egg. Since the temperature fluctuated constantly, the temperature was recorded for two minutes and the maximum reading was taken as the result for that specific treatment.

To prevent nuisance factors, randomization, replication, and control were employed in the experiment. As all experimental units were homogeneous, blocking was not applicable for this study. Randomization was employed as all forty eggs were numbered one to forty and distributed to a treatment with the help of the random number generator in R as mentioned earlier. This is also demonstrative of treatment level replication as each treatment group contained 10 randomly selected experimental units or eggs, which in turn reduces the total variability within treatment groups. Finally, a multitude of control variables were considered and kept constant throughout the experiment. These consisted of using the same pot and initial pot temperature, hotplate top and hotplate temperature, constant volume of water at 750 ml per egg, egg size and brand of egg, constant egg starting temperature, boiling one egg at a time, checking the egg was not expired, cooking time at ten minutes, placing the egg in the same location in the pot, type of salt, and finally the thermometer used was the same throughout.

3 Analysis and Discussion

3.1 Model Assumptions

3.1.1 Normality

As there are only ten observations per group, we check model normality for each group separately, using the Shapiro-Wilk test of normality at the 0.05 significance level with the following hypothesis:

H_0: Data is normally distributed
H_a: Data is not normally distributed

Groups 1, 2 and 3 passed the Shapiro - Wilk test of normality. (we fail to reject the null hypothesis for each test at the 0.05 significance level). Group 4 fails the test at the 0.05 significance level with a p-value 0f 0.00154148, thus the assumption of normality is violated. This paper will proceed to use data analysis techniques that rely on the assumption of normality even though the assumption has been violated as non parametric alternatives have not been discussed in class

3.1.2 Constant Variance

To test for constant variance between groups, we proceed with the Bartlett test for homogeneity of variance to test the following hypothesis.

$$H_0 : V(\epsilon_{ij}) = V(\epsilon_{ij'}),\ i \in \{1, 2, 3, ..., 10\},\ \ j, j' \in \{1, 2, 3, 4\}.\, j \neq j'$$

$$H_a : V(\epsilon_{ij}) \neq V(\epsilon_{ij'}),\ i \in \{1, 2, 3, ..., 10\},\ \ j, j' \in 1, 2, 3, 4.\, j \neq j'$$

We note that the results of the Bartlett test may be suspect due to the violation of normality indicated by the Shapiro - Wilk test. With a p-value of 0.905 reported from the Bartlett test, the assumption of homogeneous variance between groups is satisfied.

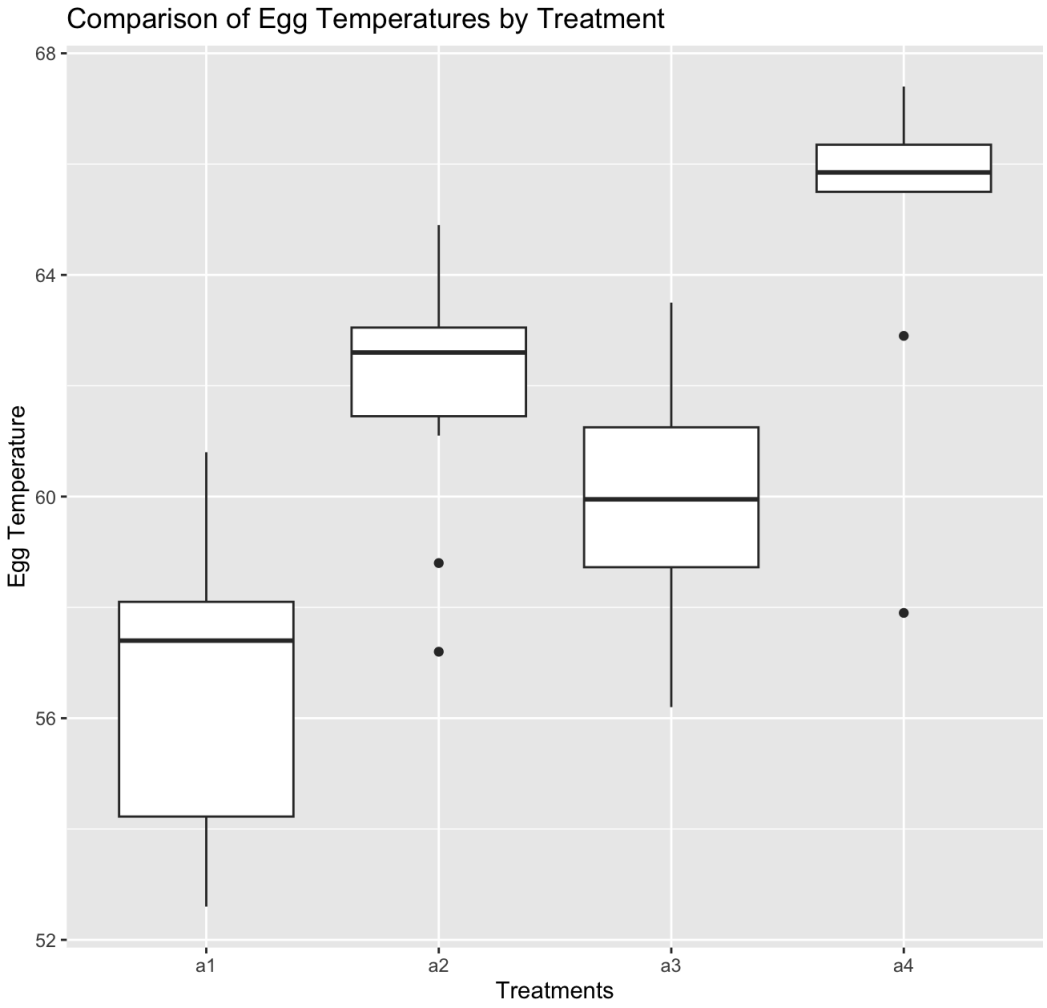


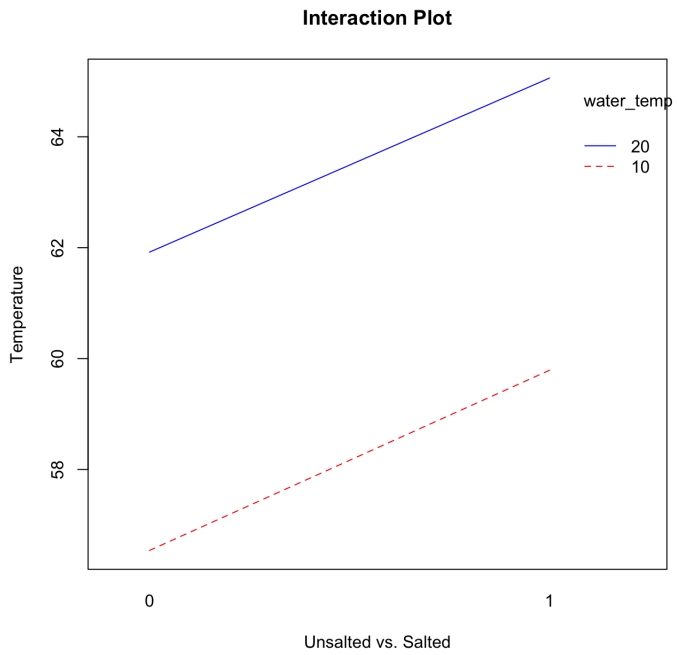
Figure 1: Boxplot of Treatments (Treatments in same order as Code book

However, it is worthy to note that the box plot of the treatments suggests that the four treatment's variances do vary.

3.2 Analysis/Discussion

There were three main methods that were used to analyse this data. First, two-way ANOVA was used on a model that included the interaction between the two factors. This was done to formally test if there was a significant interaction effect between water salinity and water temperature. Next, two-way ANOV+A was again used, now on an additive model. Finally, some contrasts of interest were then tested using the Bonferroni correction to control type 1 error inflation.

We first informally test the interaction between water salinity and water temperature using an interaction plot. The interaction plot below indicates that there is no significant interaction between the factors.



The interaction between the two factors is tested more formally with two-way ANOVA on the interaction model below. In the following, factor A refers to water temperature, the levels of water temperature being: {10C, 20C}. Factor B refers to water salinity, the levels of water salinity being : {0 grams/ liter, 50 grams/liter}.

$$Y_{ijk} = \mu + \alpha_j + \beta_k + (\alpha\beta)_{jk} + \epsilon_{ijdk}$$

Where

$$\begin{aligned} Y_{ijk} &: i^{th} \text{ obs from } j^{th} \text{ level of Factor A and } k^{th} \text{ level of Factor B} \\ \mu &: \text{Grand mean} \\ \alpha_j &: \text{Treatment effect of Factor A} \\ \beta_k &: \text{Treatment effect of Factor B} \\ (\alpha\beta)_{jk} &: \text{Effect of interaction between Factor A and Factor B} \\ \epsilon_{ijk} &: \text{error/noise/uncertainty} \end{aligned}$$

The null hypothesis of interest is that there is no interaction effect between water salinity and water temperature on the response.

$$H_0 : (\alpha\beta)_{jk} = 0$$

Assuming a significance of 0.05, the p-value from the ANOVA table is 0.9452302, which is larger than 0.05. Therefore, we fail to reject the null hypothesis at the 0.05 significance level, and we conclude that the interaction effect between factor A and factor B is not significant.

	Df	Sum Sq	Mean Sq	F value	P value
salted	1	102.08	102.08	16.1496	0.0002855
water temp	1	283.56	283.556	44.8600	0.00000008207
salted:water temp	1	0.03	0.0048	0.9452302	
residuals	36	227.55	6.321		

Table 2: Two way ANOVA table for testing main effects and interaction effect

As the interaction effect was not significant, we proceeded to test the main effects with an additive model (without interaction) using two-way ANOVA. Note that the additive model is

$$Y_{ijk} = \mu + \alpha_j + \beta_k + \epsilon_{ijk}$$

where

Y_{ijk} : i^{th} obs from j^{th} level of Factor A and k^{th} level of Factor B
μ : Grand mean
α_j : deviation of Factor A's level mean from overall means
β_k : deviation of Factor B's level mean from overall means
ϵ_{ijk} : error/noise/uncertainty

Now testing for main effects, we have the following hypotheses for factors A and B respectively:

$$H_0\text{: } \alpha_j = 0 \text{ for all } j$$
$$H_a\text{: At least one } \alpha_j \neq 0$$

$$H_0\text{: } \beta = 0 \text{ for all } k$$
$$H_a\text{: At least one } \beta_k \neq 0$$

The p-value reported for the first hypothesis test for factor A (Water Temperature) is 0.00000005413, which is smaller than the significance level of 0.05. Hence, we reject H_0 at the 0.05 significance level and conclude that there is a water temperature effect.

The p-value reported for the second hypothesis test for factor B (Salinity) is 0.0002345, which is also smaller than the significance level of 0.05. Therefore, we reject H_0 at the 0.05 significance level and conclude that water salinity has an effect on the internal temperature of an egg.

From this, it is evident that water salinity and water temperature are both significant factors.

3.2.1 Post-Hoc Analysis

The goal of this post-hoc analsysis is to draw some directional conclusions about the different treatments that were applied in this experiment (please refer to Table 4 given below

	Df	Sum Sq	Mean Sq	F value	P value
salted	1	102.08	102.08	16.596	0.0002345
water temp	1	283.56	283.556	46.100	0.00000005413
residuals	37	227.58	6.151		

Table 3: Two way ANOVA table for testing main effects

for treatment specifications).

Treatment 1	**Treatment 2**	**Treatment 3**	**Treatment 4**
(10C, unsalted)	(20C, unsalted)	(10C, salted)	(20C, salted)

Table 4: Treatment specifications

There were two main contrasts of interest given as follows:

$$\psi_1 : \{-1, 0, 0, 1\}$$
$$\psi_2 : \{-1, -1, -1, 3\}$$

The following hypotheses were tested using a Bonferroni correction to control type 1 error inflation.

$$H_0 : \psi_1 \leq 0$$
$$H_a : \psi_2 > 0$$

$$H_0 : \psi_2 \leq 0$$
$$H_a : \psi_2 > 0$$

The p-value from the hypothesis test regarding ψ_1 is 0.0000000058 which implies that we reject H_0 at the 0.05 significance level. This indicates that the mean internal temperatures of the experimental units of treatment 4 is significantly larger than the mean internal temperatures of the experimental units of treatment 1.

The p-value from the hypothesis test regarding ψ_2 is 0.000000442 which implies that we reject H_0 at the 0.05 significance level. This indicates that the mean internal temperature of the eggs in treatment 4 is significantly larger than the mean internal temperatures of the eggs not in treatment 4.

4 Limitations

There were several limitations to the study as expected, however there was one limitation which likely violates one of the necessary assumptions made when applying the methods of data analysis used in this paper.

The source of this limitation came from equipment failure. The tool that was used to measure the response (an instant read thermometer), failed with 8 observations to go. The thermometer was replaced with another instant read thermometer of a different brand due to local providers no longer carrying the original thermometer that was used. The new instant read thermometer was then used to measure the last 8 observations (ie the last 2 observations per treatment). Since there is little reason to believe that the accuracy of the new thermometer is the same as the old one, it is unlikely that the measurement error of the last 8 observations are the same as the previous observations. In modelling terms, this indicates that the assumption of constant variance in the error terms within groups is almost certainly violated:

$$V(\epsilon_{i,j}) \neq V(\epsilon_{i',j}), \quad i \in 1, 2, ..., 8. \quad i' \in 9, 10.$$

5 Conclusion

The primary objective of this experimental study was to examine whether factors such as water temperature and water salinity had an influence on the cooking time of an egg. Our findings suggest that both the factors water temperature and water salinity had an impact on the cooking time of an egg. However, there were several limitations that impact the validity of this experimental study as previously discussed. The violation of normality in this study may be remedied by using non-parametric methods of data analysis. Furthermore, limitation introduced by equipment failure mentioned above will require data analysis methods that are robust to the violation of homogeneous variance within groups. The validity of this study may also be improved by increasing the number of eggs used for each treatment which would have yielded more accurate results. With the assistance of R code, we came to the conclusion that there was no interaction between the two factors, and that both factors on their own were statistically significant.

6 Appendix

```
# ================================================================ #
# ============================ Preamble ============================ #
# ================================================================ #
# Please manually install any missing packages using install.package()
library(rstatix)
library(ggplot2)
library(ggpubr)
library(readxl)
library(pwr)

data = read_excel("STA305 Group Project Data.xlsx")

dat = data[,-1]
a = factor(rep(c('a1', 'a2', 'a3', 'a4'), each=10))
data_egg = data.frame(cbind(a, dat))
names(data_egg) = c('a', 'salted', 'water_temp', 'y')

# ========================================================== #
# ============== Calculating Sample Size ================== #
# ========================================================== #
pwr.anova.test(k = 4, f = 0.5, sig.level = 0.05, power = 0.70)
# from the output, a sample size of 10 per group is sufficient

# ========================================================== #
# ========== Randomization of Experimental Units =========== #
# ========================================================== #
set.seed(2000)
eggs = 1:40
sample(eggs, replace = FALSE, size = 40)

# ========================================================== #
# ================== Model Assumptions ===================== #
# ========================================================== #

##############################################
################ Test for Normality ##########
##############################################
# We split up the response for the groups, and test if each of the responses for
# the four groups are normally distributed or not.
g1 = data_egg$y[1:10]
g2 = data_egg$y[11:20]
g3 = data_egg$y[21:30]
```

```
g4 = data_egg$y[31:40]
#testing group 1
shapiro_test(g1)

#testing group 2
shapiro_test(g2)

#testing group 3
shapiro_test(g3)

#testing group 4
shapiro_test(g4)


#####################################
#### Test for constant variance #####
#####################################

## The normality assumption for the bartlett.test is not satisfied
bartlett.test(y ~ a, data=data_egg)

## Creating boxplot for the four treatments in order as codebook
ggplot(data_egg, aes(x = a, y = y)) +
  geom_boxplot() +
  labs(title = "Comparison of Egg Temperatures by Treatment",
       x = "Treatments",
       y = "Egg Temperature")

# ============================================================
# ========================== Section 1 ======================= #
# ============================================================

library(rstatix)
library(readr)
library(readxl)

data = read_excel("STA305 Group Project Data.xlsx")
dat = data[,-1]
a = factor(rep(c('a1', 'a2', 'a3', 'a4'), each=10))
data_egg = data.frame(cbind(a, dat))
names(data_egg) = c('a', 'salted', 'water_temp', 'y')
attach(data_egg)

###############################################
############### TWO WAY ANOVA ###############
```

```
##############################################
#Means
with(data_egg, tapply(y, salted, mean))
with(data_egg, tapply(y, water_temp, mean))
with(data_egg, tapply(y, list(salted, water_temp), mean))


#Interaction Plot
with(data_egg, interaction.plot(salted, water_temp, y, col=c("red", "blue"),
                                main="Interaction Plot", xlab="Salted mean", ylab="Yield
with(data_egg, interaction.plot(water_temp, salted, y, col=c("red", "blue"),
                                main="Interaction Plot", xlab="Salted mean", ylab="Yield


#Model with interaction
model1 <- lm(y ~ salted*water_temp, data = data_egg)
summary(model1)
anova(model1)
# Reject H0, interaction not significant

##############################################
############### OVERALL TEST ################
##############################################

#Additive Model
model2 <- lm(y~ salted + water_temp, data = data_egg)
summary(model2)
anova(model2)
# Fail to reject H0, both factors on their own are significant


##############################################
################# POST HOC ###################
##############################################

# using Bonferroni coerrection to test general contrast
fit <- lm(y ~ a- 1, data=data_egg)
L <- matrix(c(
  -1, 0, 0, 1,
  -1, -1, -1, 3), byrow=T, nrow=2)
summary(glht(fit, L), test=adjusted('bonferroni'))

# the p-values are divided by 2 and we take note of the sign of estimates reported as we
```