MIT 18.01 Problem Set 5 Unofficial Solutions

Q1a) Suppose that at the beginning of day 0, some time last summer, the temperature in Boston was $y(0) = 65°$ Fahrenheit and that over a 50-day period, the temperature increased according to the rule $y'(t) = y(t)/100$, with time t measured in days. Find the formula for y, and draw a graph of temperature on days 3 and 4, $3 <= t <= 5$, and label with the correct day and shade in the regions whose areas represent the average temperature each of the two days.[a]

[a] The continuous average of a function is $\frac{1}{b-1}\int_a^b f(x)dx$. In this case $b - a = 1$, so the average is the same as the integral. For more, see Notes, AV and Lecture 23.

Based on the rule given, we calculate the temperatures from $t = 0$ to $t = 5$:

t	$y(t)$
0	65
1	65 + 65 / 100 = 65.65
2	65.65 + 65.65 / 100 = 66.3065
3	66.3065 + 66.3065 / 100 = 66.969565
4	66.969565 + 66.969565 / 100 = 67.63926065
5	67.63926065 + 67.63926065 / 100 = 68.3156532565

From the formula $y'(t) = y(t)/100$, we get $\frac{dy}{dt} = \frac{y}{100}$. Then

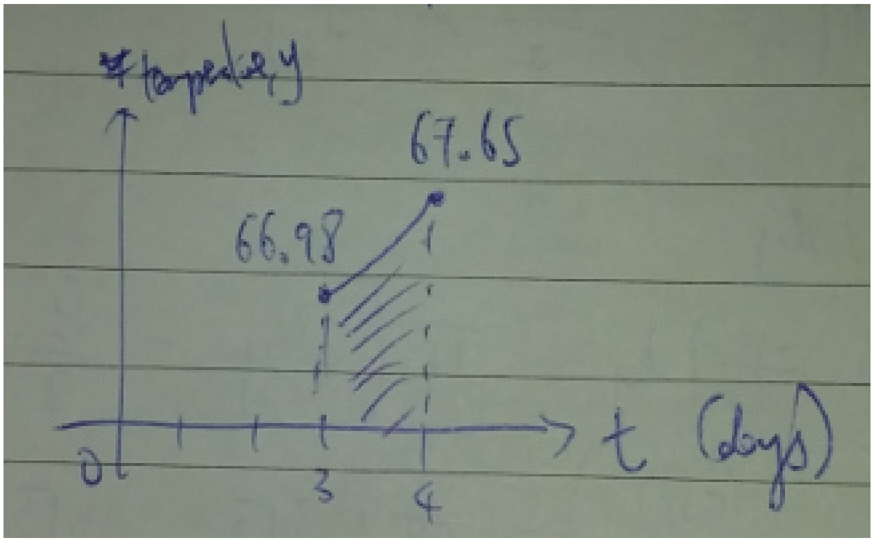
$$\begin{aligned}\frac{dy}{dt} &= \frac{y}{100}\\ \frac{1}{y}dy &= \frac{1}{100}dt\\ ln(|y|) &= \frac{1}{100}t + C\end{aligned}$$

Since $y > 0$ and is increasing,

$$\begin{aligned}ln(y) &= \frac{1}{100}t + C\\ y &= e^{\frac{1}{100}t+C}\\ &= e^C \cdot e^{\frac{1}{100}t}\\ &= Ae^{\frac{1}{100}t}\end{aligned}$$

At $t = 0, y = 65$. Hence $65 = Ae^{\frac{1}{100}\cdot 0} = A$. Then $ln(y) = 65e^{\frac{1}{100}t}$

Graph:



66.98
67.65
0
3
4
t (days)