MIT 18.01 Problem Set 6 Unofficial Solutions

Q1) Do 7.4/12 and 13.

Skipped. We do not have the textbook.

Q2) The voltage V of the house current is given by

$$V(t) = Csin(120\pi t)$$

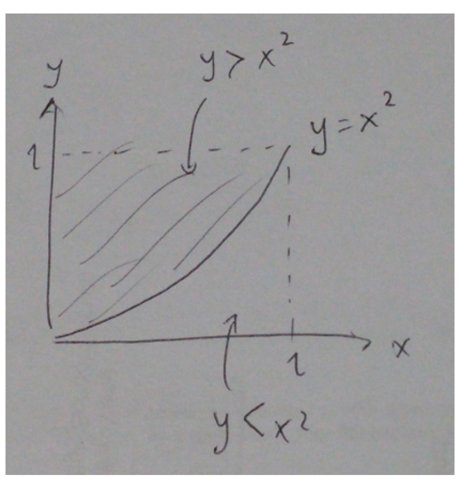
where t is time, in seconds and C is a constant amplitude. The square root of the average value of V^2 over one period of $V(t)$ (or cycle) is called the *root-mean-square* voltage, abbreviated RMS. This is what the voltage meter on a house records. For house current, find the RMS in terms of the constant C. (The peak voltage delivered to the house is $\pm C$. The units of V^2 are square volts; when we take the square root again after averaging, the units become volts again.)

Every cycle of the sin function corresponds to 2π. This happens every $t = \frac{2\pi}{120\pi} = \frac{1}{60}$ seconds.
Since $V(t) = Csin(120\pi t)$, $V^2(t) = C^2 sin^2(120\pi t)$. The average value of V^2 over 1 cycle of $V(t)$ is:

$$\begin{aligned}
\frac{1}{\frac{1}{60} - 0}\int_0^{\frac{1}{60}} C^2 sin^2(120\pi t)dt &= 60C^2 \int_0^{\frac{1}{60}} sin^2(120\pi t)dt \\
&= 60C^2\left(\frac{1}{2}t - \frac{1}{240\pi}sin(120\pi t)cos(120\pi t))\right]_0^{1/60} \\
&= 60C^2(\frac{1}{2}\cdot\frac{1}{60} - \frac{1}{240\pi}sin(2\pi)cos(2\pi) - (\frac{1}{2}\cdot 0 - \frac{1}{240\pi}sin(0)cos(0))) \\
&= 60C^2 \cdot \frac{1}{120} \\
&= \frac{C^2}{2}
\end{aligned}$$

Then RMS $= \sqrt{\frac{C^2}{2}} = \frac{C}{\sqrt{2}} = \frac{\sqrt{2}C}{2}$

Q3a) What is the probability that $x^2 < y$ if (x, y) is chosen from the unit square $0 \leq x \leq 1$, $0 \leq y \leq 1$ with probability equal to the area.



$$
\begin{aligned}
P(x^2 < y) &= \frac{\int_0^1 1dx - \int_0^1 x^2dx}{\int_0^1 1dx} \\
&= \frac{x\Big]_0^1 - \frac{x^3}{3}\Big]_0^1}{x\Big]_0^1} \\
&= \frac{1 - \frac{1}{3}}{1} \\
&= \frac{2}{3}
\end{aligned}
$$