# MIT 18.01 Problem Set 7 Unofficial Solutions

**Q1)** (from PS6) The voltage $V$ of house current is given by

$$V(t) = Csin(120\pi t)$$

where $t$ is time, in seconds and $C$ is a constant amplitude. The square root of the average value of $V^2$ over one period of $V(t)$ (or cycle) is called the *root-mean-square* voltage, abbreviated RMS. This is what the voltage meter on a house records. For house current, find the RMS in terms of the constant $C$. (The peak voltage delivered to the house is $\pm C$. The units of $V^2$ are square volts; when we take the square root again after averaging, the units become volts again.)

Average value of $V^2$ over 1 period of $V(t)$ is

$$\begin{aligned}
\frac{1}{60}\int_0^{\frac{1}{60}} C^2 sin^2(120\pi t)dt &= \frac{C^2}{60}\int_0^{\frac{1}{60}} sin^2(120\pi t)dt \\
&= \frac{C^2}{60}\int_0^{\frac{1}{60}} \frac{1-cos(240\pi t)}{2}dt \\
&= \frac{C^2}{120}\int_0^{\frac{1}{60}} 1-cos(240\pi t)dt \\
&= \frac{C^2}{120}(t-\frac{sin(240\pi t)}{240\pi})\Big]_0^{\frac{1}{60}} \\
&= \frac{C^2}{120}(\frac{1}{60}-\frac{sin(240\pi\cdot\frac{1}{60})}{240\pi}) \\
&= \frac{C^2}{120}(\frac{1}{60}-\frac{sin(4\pi)}{240\pi}) \\
&= \frac{C^2}{120}(\frac{1}{60}) \\
&= \frac{C^2}{7200}
\end{aligned}$$

Square root of average value of $V^2$ over 1 period of $V(t) = \sqrt{\frac{C^2}{7200}} = \frac{C}{\sqrt{3600*2}} = \frac{C}{60\sqrt{2}}$