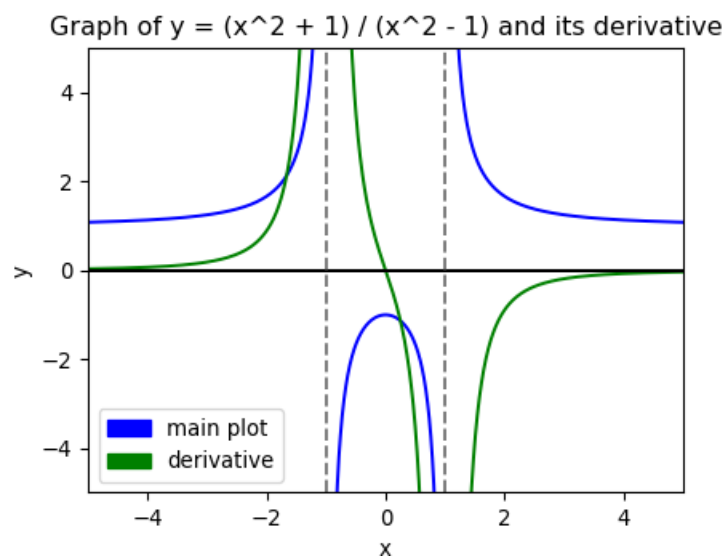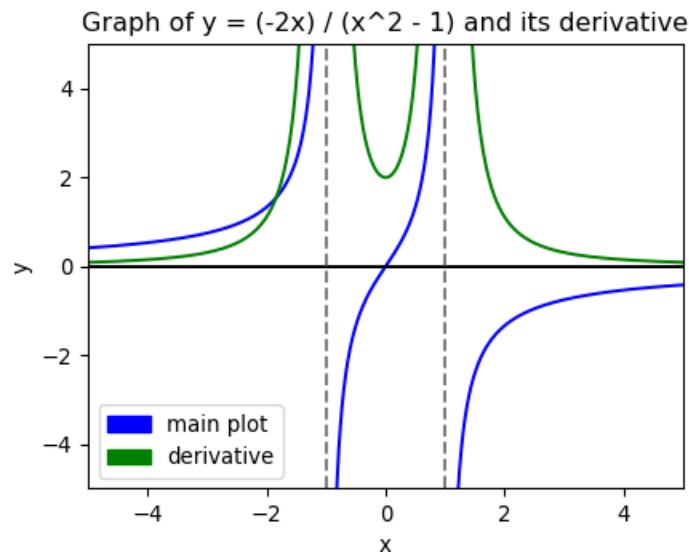# MIT 18.01 Problem Set 2A Unofficial Solutions

Q1) Graph the even and odd functions you found in Problem 1, Part II of PS1. Directly below, graph their derivatives. Do this qualitatively using your estimation of the slope. Do not use the formulas for the derivatives (except to check your work if you want). You can use a graphing calculator to check your answer, provided that you mention it in Problem 0. (Note, however, that you may not use books, notes or calculators during tests, so it is unwise to rely on a graphing calculator here.)

$$\frac{x-1}{x+1} = \frac{x-1}{x+1} \cdot \frac{x-1}{x-1}$$
$$= \frac{x^2 - 2x + 1}{x^2 - 1}$$
$$= \frac{x^2 + 1}{x^2 - 1} - \frac{2x}{x^2 - 1}$$

so the even function is $\frac{x^2+1}{x^2-1}$ and the odd function is $-\frac{2x}{x^2-1}$

Graph of y = (x^2 + 1) / (x^2 - 1) and its derivative

Graph of y = (-2x) / (x^2 - 1) and its derivative

Q2a) Compute $(d/dx)\tan^3(x^4)$

$$\frac{d}{dx}\tan^3(x^4) = \frac{d}{dx}(\tan(x^4))^3$$
$$= 3\cdot(\tan(x^4))^2\cdot\frac{d}{dx}\tan(x^4)$$
$$= 3(\tan(x^4))^2\cdot 4x^3\sec^2(x^4)$$
$$= 12x^3(\sec(x^4)\tan(x^4))^2$$

Q2b) Compute $(d/dy)(\sin^2 y\cos^2 y)$

$$\frac{d}{dy}(\sin^2 y\cos^2 y) = 2\sin(y)\cos(y)\cdot\cos^2 y + \sin^2 y\cdot 2\cos(y)\cdot -\sin(y)$$
$$= 2\sin(y)\cos^3(y) - 2\sin^3(y)\cos(y)$$
$$= 2\sin(y)\cos(y)(\cos^2(y) - \sin^2(y))$$

Q3a) If $y = uv$, show that $y'' = u''v + 2u'v' + uv''$

$$y' = u'v + uv'$$
$$y'' = u''v + u'v' + u'v' + uv''$$
$$= u''v + 2u'v' + uv''$$

Q3b) Find y'''.

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$$y''' = u'''v + u''v' + 2u''v' + 2u'v'' + u'v'' + uv'''$$
$$= u'''v + 3u''v' + 3u'v'' + uv'''$$

Let $y = \cos^{-1} x$. Then $\cos(y) = x$. Differentiating implicitly with respect to x:

$$-\sin(y) \cdot \frac{dy}{dx} = 1$$
$$\frac{dy}{dx} = -\frac{1}{\sin(y)}$$

$\cos(y) = x$ represents a triangle whose angle is y, adjacent side has length x and hypotenuse has length 1. Hence its opposite side has length $\sqrt{1 - x^2}$ and $\sin(y) = \frac{\sqrt{1-x^2}}{1} = \sqrt{1 - x^2}$. We take the positive square root because for $0 \leq x \leq \pi$, $\sin(x) \geq 0$.

$$\frac{dy}{dx} = -\frac{1}{\sin(y)}$$
$$= -\frac{1}{\sqrt{1 - x^2}}$$

I do not have the textbook. Skipped.

$$D(u_1 u_2 \cdots u_n) = D(e^{ln(u_1 u_2 \cdots u_n)})$$
$$= D(ln(u_1 u_2 \cdots u_n)) \cdot e^{ln(u_1 u_2 \cdots u_n)}$$
$$= D(ln(u_1) + ln(u_2) + \cdots ln(u_n)) \cdot (u_1 u_2 \cdots u_n)$$
$$= (D(ln(u_1)) + D(ln(u_2)) + \cdots + D(ln(u_n))) \cdot (u_1 u_2 \cdots u_n)$$
$$= (\frac{u_1'}{u_1} + \frac{u_2'}{u_2} + \cdots + \frac{u_n'}{u_n}) \cdot (u_1 u_2 \cdots u_n)$$
$$= u_1' u_2 \cdots u_n + u_1 u_2' \cdots u_n + \cdots + u_1 u_2 \cdots u_n'$$