Worksheet 0 Solution

Basics of counting

1. How many different three-letter initials are there that begin with an A? contain an A? $1 \times 26 \times 26 = 676$, $3 \times 26 \times 26 = 2028$
2. How many 6-element RNA sequences
   (a) end with GU? $4^4 = 256$
   (b) contain only A or U? $2^6 = 64$
3. (a) How many different functions are there from a set with n elements to a set with m elements? n^m (Ex. 7 p. 407)
   (b) How many different injective functions are there from a set with n elements to a set with m elements? You may assume $n \geq m$. $n(n-1)\cdots(n-m-1)$ (Ex. 7 p. 407)
4. How many positive integers between 100 and 999 inclusive
   (a) are divisible by 7? $\lfloor 900/7 \rfloor = 128$
   (b) are divisible by 3 or 4? $\lfloor 900/3 \rfloor + \lfloor 900/4 \rfloor - \lfloor 900/(3 \times 4) \rfloor = 300 + 225 - 75 = 450$
   (c) are divisible by 3 but not by 4? $\lfloor 900/3 \rfloor - \lfloor 900/(3 \times 4) \rfloor = 300 - 75 = 225$
5. How many strings of 5 decimal digits
   (a) contain at least one 4? $10^5 - 9^5 = 40951$
   (b) do not have the same digit? $10 \times 9 \times 8 \times 7 \times 6 = 30240$
   (c) do not have two consecutive digits that are the same? $10 \times 9 \times 9 \times 9 \times 9 = 65610$
   (d) either end in 4, or start with 6? $10^4 + 10^4 - 10^3 = 19000$
6. How many diagonals does a convex polygon with n sides have? $n(n-1)/2 - n = n(n-3)/2$
7. In how many ways can a photographer at a wedding arrange six people in a row, including the bride and groom, if
   (a) the bride must be next to the groom? $2 \times 5 \times 4! = 240$
   (b) the bride is not next to the groom? $6! - 240 = 480$
   (c) the bride is positioned somewhere to the left of the groom? $6!/2 = 360$
8. ** Todd has ten apples and he hopes to finish them within ten days. If Todd will eat at least one apple a day, how many ways can he choose to achieve his goal? (This question uses a trick that we haven't seen yet. Don't worry if you can't solve it, but it is an interesting problem and worths thinking about!) $10/1! + (10 \times 9)/2! + (10 \times 9 \times 8)/3! + \cdots + 10!/10!$

Source: Rosen's *Discrete Mathematics and its Applications*.