MATH 10B – Spring 2019

Quiz 4 – Prepared by John Yirong Zhen
Date:02/19/2018

You are to finish this quiz in 10 minutes. You are allowed one single-sided letter-size cheat sheet. No calculators or other notes/books/devices are allowed.
Your cheatsheet must be handwritten by you, no photocopying or preprinted (unless you have written permission from the instructor). Try your best! Stay calm and good luck!

I. True/False (2 pts)

Circle T or F in the space provided in front of the statement to indicate whether it is true or false respectively. You get +1 for a correct answer, −1 for incorrect, and 0 for leaving it blank. (You should not guess if you don't know the answer.)
You do not need to justify your answers for T/F statements.

- T F Let Ω be the probability space of getting a card of spade, diamond, heart and club from a deck of 52 cards. Then $|\Omega| = 13^4$.
  Each suit contains 13 different outcomes, and in total there are 13^4 different outcomes by the product rule.
- T F For any induction proofs, the base case only contains the case where $n = 1$.
  One counterexample is the written problem in this quiz where the base case contains the case where $n = 1$ and $n = 2$.

II. Written problems (10pts)

- You MUST **justify your answer** to undoubtably convince me that you solved and not guessed it. Partial credit will be given to good work and progress even if there is no final answer or the answer is incorrect. On the other hand, bogus justification for a correct answer will receive a 0.
- Keep your scratch work separate. Cross out writing you don't want to be graded and clearly label the parts you want to be graded.
- Points will be deducted for incorrect writings that you "forget to cross out."

See problem on back.

Given $a_1 = 2$, $a_2 = 3$ and $a_{n+2} = 3a_{n+1} + 4a_n$, show that $a_n = 4^{n-1} + (-1)^{n-1}$.
Base case: $a_1 = 4^0 + (-1)^0 = 2$ and $a_2 = 4^1 + (-1)^1 = 3$. Both are true.
Assume $a_n = 4^{n-1} + (-1)^{n-1}$ and $a_{n+1} = 4^n + (-1)^n$ for some n.
Then, $a_{n+2} = 3a_{n+1} + 4a_n = 3(4^n + (-1)^n) + 4(4^{n-1} + (-1)^{n-1}) = 3 \times 4^n + 4^n + 3 \times (-1)^n - 4 \times (-1)^n = (3+1)4^n + (3-4)(-1)^n = 4^{n+1} + (-1)^{n+1}$.
Therefore, by induction, $a_n = 4^{n-1} + (-1)^{n-1}$ for $n \geq 1$.