Solution

dividing

Check. 1. $\lim_{x\to\infty}\frac{\sqrt{9x+6}}{\sqrt{x+1}+2} = \lim_{x\to\infty}\frac{\sqrt{9+6/x}}{\sqrt{1+1/x}+\sqrt{4/x}} = \sqrt{9}/\sqrt{1} = 3$

2. $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = \lim_{n\to\infty} e^{\ln\left(1+\frac{1}{n}\right)^n} = e^{\lim_{n\to\infty}\frac{\ln\left(1+\frac{1}{n}\right)}{1/n}}$ $\quad \frac{0}{0}$

1^∞

$= e^{\lim_{n\to\infty}\frac{\frac{1}{1+1/n}\cdot\left(-\frac{1}{n^2}\right)}{-(1/n^2)}} = e^{\lim_{n\to\infty}\frac{1}{1+1/n}} = e^1 = e$

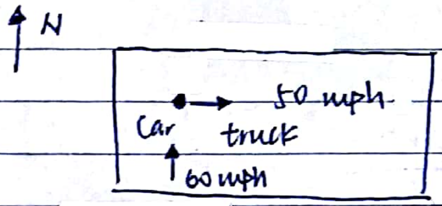
3. $y = x^{e^x}$.

$\ln y = e^x \ln x$

$\frac{1}{y}\cdot y' = e^x \ln x + \frac{e^x}{x}$

$y' = \left(e^x \ln x + \frac{e^x}{x}\right) x^{e^x}$

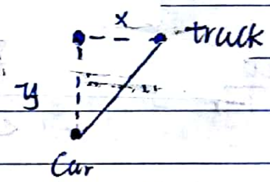
4.



x – dis traveled by truck.
y – dis traveled by car
z – dis between truck & car

$x^2 + y^2 = z^2$

$x(t)^2 + y(t)^2 = z(t)^2$



$2x\cdot\frac{dx}{dt} + 2y\cdot\frac{dy}{dt} = 2z\cdot\frac{dz}{dt}$

$4\times 50 + 3\times(-60) = 5\cdot\frac{dz}{dt}$.

$\Rightarrow dz/dt = 4$ mph.

Part I. 1.(a). $f(x) = \sqrt[3]{x^2}\,2x - x^2 = 2x^{\frac{5}{3}} - x^{\frac{8}{3}}$ when $x=1$, $f(x)=0$

$f'(x) = \frac{10}{3}x^{\frac{2}{3}} - \frac{8}{3}x^{\frac{5}{3}}$ when $x=1$. $f'(x) = \frac{2}{3}$

$y - 0 = \frac{2}{3}(x-1) \Rightarrow y = \frac{2}{3}x - \frac{2}{3}$

1. (b). $f(x) = \frac{2t^5+t^2-\frac{5}{t^2}}{t^2} = 2t^3+1-5t^{-2}$ when $x=1$. $f(x) = -2$

$f'(x) = 6t^2 + 10t^{-3}$ when $x=1$. $f'(x) = 16$

$y+2 = 16(x-1) \Rightarrow y = 16x-18$

2. $(0.9998)^7$ $\quad f(x) = x^7$. linearization approximation

$f'(x) = 7x^6$. when $x=1$ $\begin{cases} f(x) = 1 \\ f'(x) = 7 \end{cases}$

$\Rightarrow y-1 = 7(x-1) \Rightarrow y = 7x-6$

$(0.9998)^7 \approx 7\times 0.9998 - 6 = 0.9986$

3. $f(x) = \frac{1}{1-x}$ $n=3$, $a=0$ $f(0)=1$

$f'(x) = -1 \cdot \frac{1}{(1-x)^{2}} \cdot (-1) = (1-x)^{-2}$ $f'(0)=1$

$f''(x) = -2 \cdot \frac{1}{(1-x)^3} \cdot (-1) = 2(1-x)^{-3}$ $f''(0) = 2!$

$f'''(x) = 2 \cdot (-3) \cdot \frac{1}{(1-x)^4} \cdot (-1) = 2 \cdot 3 \cdot (1-x)^{-4}$ $f'''(0) = 3!$

$\vdots$

$f^{(n)}(x) = n!\,(1-x)^{-(n+1)}$ $f^{(n)}(0) = n!$

$$T_3(x) = f(0) + f'(0)(x-0) + \frac{f''(0)}{2}(x-0)^2 + \frac{f'''(0)}{6}(x-0)^3$$
$$= 1 + x + x^2 + x^3$$
$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^n$$

Part 2: 1. $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^n$

let $x = r$

$$\frac{1}{1-r} = 1 + r + r^2 + r^3 + \cdots + r^n$$

$\times\ a$

$$\frac{a}{1-r} = a + ar + ar^2 + ar^3 + \cdots + ar^n$$

$$a + ar + ar^2 + \cdots + ar^n = \frac{a(1-r^{n+1})}{1-r}$$

$$\lim_{n\to\infty} \frac{a(1-r^{n+1})}{1-r} = \lim_{n\to\infty}\left[\frac{a}{1-r} - \frac{ar^{n+1}}{1-r}\right]$$

$-1 < r < 1$

$$= \frac{a}{1-r} - \frac{a}{1-r}\lim_{n\to\infty} r^{n+1} \to 0.$$

$$= \frac{a}{1-r}.$$

2. $\lim_{n\to\infty}\left[\left(\frac{1}{2}\right)^{-n} + 6^{-n}\right] = \lim_{n\to\infty}\left[2^n + \left(\frac{1}{6}\right)^n\right] = \infty + 0 = \infty.$

3. $4.1\overline{6} = 4.1 + 0.06 + 0.006 + 0.0006 + \cdots$

$$= 4.1 + 0.06 + 0.06(0.1) + 0.06(0.1)^2 + 0.06(0.1)^3 + \cdots$$
$$= 4.1 + \frac{0.06}{1-0.1}$$
$$= 4.1 + \frac{6}{90}$$
$$= \frac{41}{10} + \frac{1}{15}$$
$$= \frac{125}{30} = 25/6$$

$\frac{2x - x^3}{x^2} = \frac{2}{x} - x$ $\qquad$ $\frac{2x^4 - \sqrt{x}}{x} = 2x^3 - x^{-\frac{1}{2}}$

Part 3.

1. $F(x) = \frac{1}{3}x^3 + \frac{5}{2}x^2 + 7x + C.$
2. $F(x) = 2\ln x - \frac{1}{2}x^2 + C$
3. $F(x) = e^{\sin x} + C$
4. $F(x) = 3\sin x + \frac{1}{2}x^4 - 2\sqrt{x}$