Student Name:

MATH 10A – Fall 2018
Discussion 23 Solutions

The Dot Product

1. The standard basis vectors are $\mathbf{i} = \begin{bmatrix}1\\0\\0\end{bmatrix}$, $\mathbf{j} = \begin{bmatrix}0\\1\\0\end{bmatrix}$, $\mathbf{k} = \begin{bmatrix}0\\0\\1\end{bmatrix}$

   (a) Show that $\mathbf{i}\cdot\mathbf{j} = \mathbf{j}\cdot\mathbf{k} = \mathbf{k}\cdot\mathbf{i} = \mathbf{0}$

   $\mathbf{i}\cdot\mathbf{j} = (\mathbf{1})(\mathbf{0}) + (\mathbf{0})(\mathbf{1}) + (\mathbf{0})(\mathbf{0}) = \mathbf{0}$
   similar for $\mathbf{j}\cdot\mathbf{k}, \mathbf{k}\cdot\mathbf{i}$

   (b) Show that $\mathbf{i}\cdot\mathbf{i} = \mathbf{j}\cdot\mathbf{j} = \mathbf{k}\cdot\mathbf{k} = \mathbf{1}$

   $\mathbf{i}\cdot\mathbf{i} = (\mathbf{1})(\mathbf{1}) + (\mathbf{0})(\mathbf{0}) + (\mathbf{0})(\mathbf{0}) = \mathbf{0}$
   similar for $\mathbf{j}\cdot\mathbf{j}, \mathbf{k}\cdot\mathbf{k}$

2. $|a| = \sqrt{(-8)^2 + 6^2} = 10, \quad |b| = \sqrt{(\sqrt{7})^2 + 3^2} = 4$
   $a\cdot b = (-8)(\sqrt{7}) + (1)(5) = 18 - 8\sqrt{7}$

   $cos(\theta) = \frac{a\cdot b}{|a||b|} = \frac{9-4\sqrt{7}}{20}$ so $\theta \approx 95^o$

3. (a) $\mathbf{a} = [\mathbf{1}, \mathbf{2}]$, $\mathbf{b} = [\mathbf{-4}, \mathbf{1}]$

   $|a| = \sqrt{5}$
   $comp_a b = \frac{a\cdot b}{|a|} = \frac{-2}{\sqrt{5}}$
   $proj_a b = \frac{a\cdot b}{|a|}\frac{a}{|a|} = \frac{-2}{\sqrt{5}}\frac{1}{\sqrt{5}}a = [\frac{-2}{5}, \frac{-4}{5}]$

   (b) $\mathbf{a} = [\mathbf{1}, \mathbf{1}, \mathbf{1}]$, $\mathbf{b} = [\mathbf{1}, \mathbf{-1}, \mathbf{1}]$

   $|a| = \sqrt{3}$
   $comp_a b = \frac{a\cdot b}{|a|} = \frac{1}{\sqrt{3}}$
   $proj_a b = \frac{a\cdot b}{|a|}\frac{a}{|a|} = \frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}a = [\frac{1}{3}, \frac{1}{3}, \frac{1}{3}]$

4. If $\mathbf{a} = [\mathbf{3}, \mathbf{0}, \mathbf{-1}]$, find a vector $\mathbf{b}$ stuch that comp$_a\mathbf{b} = \mathbf{2}$

   $\mathbf{a}\ \cdot b_{\overline{|a|=2}}$ **so** $a \cdot b = 2|a| = 2\sqrt{10}$
   We need: $(3)(b_1) + (0)(b_2) + (-1)(b_3) = 2\sqrt{10}$
   $b = [s, t, 3s - 2\sqrt{10}]$**,** $s, t \in R$
   Ex. $b = [0, 0, -2]$

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Eigenvectors and Eigenvalues

1. Let $u = [0, 2]$ and $v = [1, 0]$

   (a) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} u = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} v = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
   y-component changes sign, x-component stays the same. reflection about x-axis

   $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} u = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} v = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$
   x-component changes sign, y-component stays the same. reflection about y-axis

   (b) $\begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{bmatrix} u = \begin{bmatrix} 0 \\ \frac{2}{3} \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{bmatrix} v = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
   the x components of each vector remain the same and the y components decrease by $\frac{1}{3}$. The matrix causes vertical contraction by a factor of 1/3. This compresses the "L" in the y direction

   $\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} u = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} v = \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix}$
   the x and y components decrease by $\frac{1}{2}$. The matrix causes horizontal and vertical contraction by a factor of 1/2. This compresses the "L" in the x and y directions

2. Determine whether or not $\mathbf{x}$ is an eigenvector of A. If it is, determine the associated eigenvalue.

   (a) $A\mathbf{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$
   eigenvector with $= 1$

   (b) $A\mathbf{x} = \begin{bmatrix} -2 \\ -1 \\ -1 \end{bmatrix}$
   eigenvector with $= -1$

   (c) $A\mathbf{x} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$
   this vector cannot be expressed as a scalar of x. x is not an eigenvector

   (d) $A\mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$
   eigenvector with $\lambda = 0$