MATH 10A 8/30/18
GSI Discussion Notes Thursday

Session 3 – Limits

Definition of Limit

<u>Definition of the Limit on Pg. 103 – (2)</u>

Let f be a function defined on some interval (a, infinity). Then $\lim_{x\to\infty} f(x) = L$ means that the values of f(x) can be made arbitrarily close to L by taking x sufficiently large.

Review through Section 2.2 Exercise: 1

1) Explain in your own words the meaning of each of the following:
   a. $\lim_{x\to\infty} f(x) = 5$:

   b. $\lim_{x\to-\infty} f(x) = 3$:

Finding Limits at Infinity

Practice finding limits at infinity through Section 2.2 Exercises: 5, 9, 15, 25

5) $\lim_{x\to\infty} \frac{1}{2x+3}$ =

9) $\lim_{x\to\infty} \frac{1-x-x^2}{2x^2-7}$ =

15) $\lim_{x\to\infty} \frac{(2x^2+1)^2}{(x-1)^2(x^2+x)}$ =

25) $\lim_{x\to\infty} e^{-1/t^2}$ =

Application in Practice: Section 2.2, Exercise 31)

The number of new infections produced by an individual infected with a pathogen such as influenza depends on the mortality rate that the pathogen causes. This pathogen-induced mortality rate is referred to as the pathogen's virulence. Extremely high levels of virulence result in very little transmission because the infected individual dies before infecting other individuals. Under certain assumptions, the number of new infections *N* is related to virulence *v* by the function:

$$N(v) = \frac{8v}{1 + 2v + v^2}$$

Where *v* is the mortality rate (that is, virulence) and *v* >= 0. Evaluate the $\lim_{v\to\infty} N(v)$ and interpret your result.

One Sided Limits

Definition on pg. 117 – (2)

$\lim_{x \to a^-} f(x) = L$, which means, "left-hand limit of f(x) as x approaches a" is equal to L if we can make the values of f(x) arbitrarily close to L by taking x to be sufficiently close to a and x less than a.

Formula for using One Sided Limits to Evaluate Finite Limits on pg. 118 – (3)

$\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^-} f(x) = L$ and $\lim_{x \to a^+} f(x) = L$

Review using Section 2.3 Exercise:

1) Explain in your own words what is meant by the equation $\lim_{x \to 2} f(x) = 5$

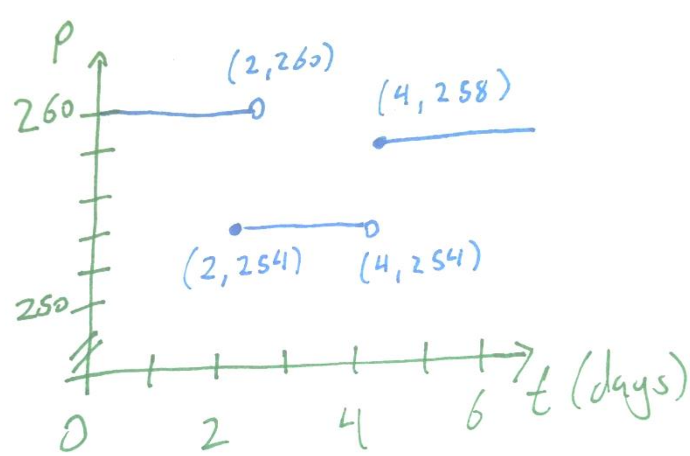
   a. Is it possible for this statement to be true and yet f(2) = 3? Explain.

7) The population of a village is P(t), t days after June 1. Use the graph of P to state the value of each limit, if it exists. If it doesn't exist, explain why.

a) $\lim_{t \to 2^-} P(t)$ = b) $\lim_{t \to 2^+} P(t)$= c) $\lim_{t \to 2} P(t)$=

d) $\lim_{t \to 4^-} P(t)$= e) $\lim_{t \to 4^+} P(t)$= f) $\lim_{t \to 4} P(t)$=

g) $\lim_{t \to 5} P(t)$ =



15) Sketch the graph of an example of a function f that satisfies all of the given conditions:

a) $\lim_{x \to 2} f(x) = -\infty$ b) $\lim_{x \to \infty} f(x) = \infty$ c) $\lim_{x \to -\infty} f(x) = 0$

d) $\lim_{x \to 0^+} f(x) = \infty$ e) $\lim_{x \to 0^-} f(x) = -\infty$

Infinite Limits

Definition (4) on pg 119 –

Let f be a function defined on both sides of a, except possibly at a itself. Then $\lim_{x \to a} f(x) = \infty$ means that the values of f(x) can be made arbitrarily large (as large as we please) by taking x sufficiently close to a but not equal to a. Vice versa for negative infinity and arbitrarily small values of f(x) (Def. 5 on 120).

Review Using Section 2.3 Exercise:

29) Determine the infinite limit: $\lim_{x \to -3^+} \frac{x+2}{x+3}$ =

31) $\lim_{x \to 1} \frac{2-x}{(x-1)^2}$ =

33) $\lim_{x \to 3^+} \ln(x^2 - 9)$ =

Limit Laws

Definitions of Limit Laws on pg. 125:

- Sum Law – Limit of a sum is the sum of the limits
- Difference Law – Limit of a difference is the difference of the limits
- Constant Multiple Law – Limit of a constant times a function is the constant times the limit of the function
- Product Law – Limit of a product is the product of the limits
- Quotient Law – Limit of a quotient is the quotient of the limits (provided that the limit of the denominator is not 0)

Practice applying Limit Laws on problems through Section 2.4 Exercises:

1) Given that $\lim_{x\to 2} f(x) = 4$, $\lim_{x\to 2} g(x) = -2$, $\lim_{x\to 2} h(x) = 0$, find the limits that exist. If the limit does not exist, explain why.

a. $\lim_{x\to 2} [f(x) + 5g(x)]$ =

b. $\lim_{x\to 2} [g(x)]^3$ =

c. $\lim_{x\to 2} \sqrt{f(x)}$ =

d. $\lim_{x\to 2} \frac{3f(x)}{g(x)}$ =

e. $\lim_{x\to 2} \frac{g(x)}{h(x)}$ =

f. $\lim_{x\to 2} \frac{g(x)h(x)}{f(x)}$ =

Evaluate the limit using limit laws:

3) $\lim_{x\to -2}(3x^4 + 2x^2 - x + 1)$ =

4) $\lim_{t\to -1} (t^2 + 1)^3(t + 3)^5$ =

11) $\lim_{x\to 5} \frac{x^2-5x+6}{x-5}$ =

15) $\lim_{h\to 0} \frac{(4+h)^2-16}{h}$ =

19) $\lim_{x\to -4} \frac{\frac{1}{4}+\frac{1}{x}}{4+x}$ =

Application in Practice

Section 2.4 Exercise 32) Gene Regulation

Genes produce molecules called mRNA that go on to produce proteins. High concentrations of protein inhibit the production of mRNA, leading to stable gene regulation. This process has been modeled to show that the concentration of mRNA over time is given by the equation:

$$m(t) = \frac{1}{2}e^{-t}(\sin(t) - \cos(t)) + \frac{1}{2}$$

a) Evaluate $\lim_{t\to 0} m(t)$ and interpret your result

b) Use the Squeeze Theorem to evaluate $\lim_{t\to\infty} m(t)$ and interpret your result.

<u>Squeeze Theorem Definition from pg. 132 (3) –</u>

If $f(x) \leq g(x) \leq h(x)$ when x is near a (except possibly at a) and $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$, then $\lim_{x\to a} g(x) = L$.