MATH 10B – Spring 2019
Quiz 6 – Prepared by John Yirong Zhen
Date:03/05/2018

You are to finish this quiz in 10 minutes. You are allowed one single-sided letter-size cheat sheet. No calculators or other notes/books/devices are allowed.
Your cheatsheet must be handwritten by you, no photocopying or preprinted (unless you have written permission from the instructor). Try your best! Stay calm and good luck!

I. True/False (2 pts)

Circle T or F in the space provided in front of the statement to indicate whether it is true or false respectively. You get +1 for a correct answer, −1 for incorrect, and 0 for leaving it blank. (You should not guess if you don't know the answer.)
You do not need to justify your answers for T/F statements.

- T F Let A be an event such that $P(A) = 0$. Then A is independent of any event B.
  $P(A)P(B) = P(B) \times 0 = 0$. $P(A \cap B) = 0$ since $B \cap A = \emptyset$. Thus, $P(A \cap B) = P(B)P(A)$, and A and B are independent.

- T F Let X be the random variable of the sum of two rolled die. Then, $X^{-1}(11) = \{(5,6)\}$.
  The inverse function should include all the premiages. In other words, $X^{-1}(11) = \{(5,6),(6,5)\}$.

II. Written problems (10pts)

- You MUST **justify your answer** to undoubtably convince me that you solved and not guessed it. Partial credit will be given to good work and progress even if there is no final answer or the answer is incorrect. On the other hand, bogus justification for a correct answer will receive a 0.
- Keep your scratch work separate. Cross out writing you don't want to be graded and clearly label the parts you want to be graded.
- Points will be deducted for incorrect writings that you "forget to cross out."

See problem on back.

Let E and F be the events that a family of n children has children of both sexes and has at most one boy, respectively. Are E and F independent if $n = 3$?
$P(E^c)$ represents the probability of the family having children of only one sex. In those three following cases, we will have $P(E^c) = 2/2^n$. So $P(E) = 1 - P(E^c) = 1 - \frac{1}{2^{n-1}}$
$P(F)$ represents the probability of the family having 0 or 1 boy. Those are two mutually exclusive cases. The probability of having 0 boy is $P(0 \text{ boy}) = \frac{1}{2^n}$. The probability of having exactly 1 boy is $P(1 \text{ boy}) = \frac{\binom{n}{1}}{2^n} = \frac{n}{2^n}$. In those three following cases, we will have $P(F) = \frac{1}{2^n} + \frac{n}{2^n}$
$P(E \cap F)$ represents the probability of having both sexes and at most one boy. That means having exactly one boy. Thus, $P(E \cap F) = \frac{\binom{n}{1}}{2^n} = \frac{n}{2^n}$
n=3 :
In this case, $P(E) = 1 - \frac{1}{2^{3-1}} = \frac{3}{4}$ and $P(F) = \frac{1}{2^3} + \frac{3}{2^3} = 1/8 + 3/8 = 1/2$
$P(E) \times P(F) = 3/8$
$P(E \cap F) = \frac{3}{2^3} = 3/8$
Thus E and F are independent.

$\square$