I: 1. Harvesting Renewable Resources

$$\frac{dP}{dt} = 2\left(1 - \frac{P}{1000}\right)P - 100 \qquad \text{when } H = 100$$

At eq$^m$, $\frac{dP}{dt} = 0$

$$2\hat{P} - \frac{\hat{P}^2}{500} - 100 = 0$$

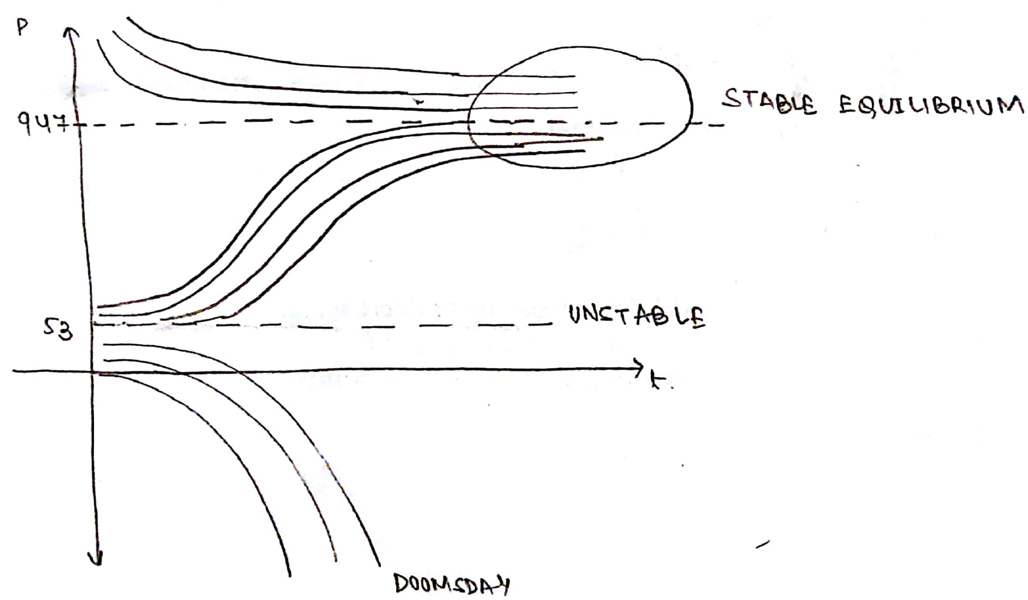
$$1000\hat{P} - \hat{P}^2 - 50000 = 0$$

$$\hat{P}^2 - 1000\hat{P} + 50000 = 0$$

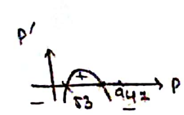
$$\hat{P} = \frac{1000 \pm \sqrt{1000^2 - 4(1)(50000)}}{2(1)} = 500 \pm 200\sqrt{5}$$

$$\hat{P}_1 = 500 + 200\sqrt{5} \approx 947$$

$$\hat{P}_2 = 500 - 200\sqrt{5} \approx 53$$




| P | ↘ | 53 | ↗ | 947 | ↘ |
|---|---|---|---|---|---|
| P' | − | 0 | + | 0 | − |



II. Predator-Prey DE systems.

1.(a) x - predators

y → prey

growth is restricted by predators, which feed only on prey.

(b) x → prey

y → predators

growth is restricted by carrying capacity and by predators which feed only on prey.

2. (a) $dL/dA = \dfrac{dL/dt}{dA/dt} = \dfrac{-0.5L + 0.0001AL}{2A - 0.01AL}$

(b) Equilibrium point (5000, 200) inside

$\dfrac{dL}{dA} = \dfrac{L(-0.5 + 0.0001A)}{A(2 - 0.01L)}$

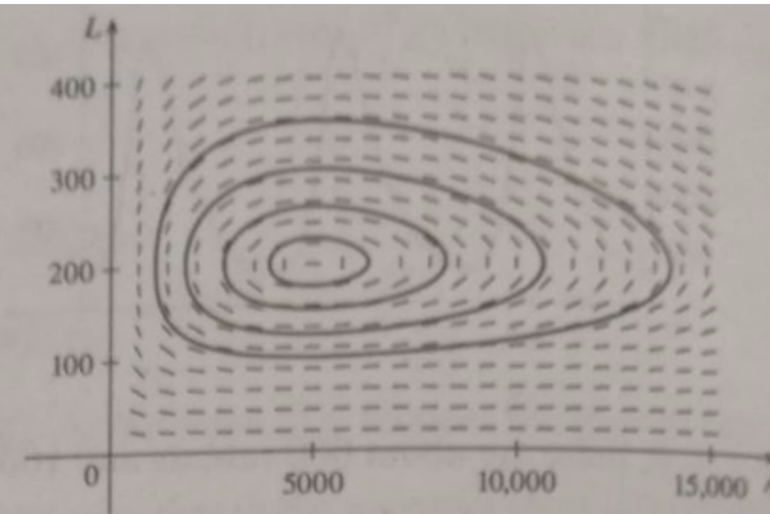
$\dfrac{dL}{dt} = 0$ $L = 200$

$A = 5000$

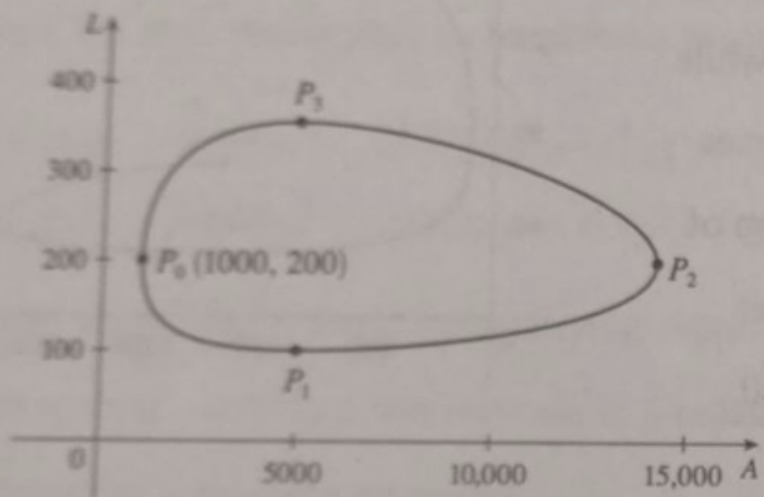
$\dfrac{dA}{dt} = 0$

(0,0) is also an equilibrium.

(b) The solution curves (phase trajectories) are all closed curves that have the equilibrium point $(5000, 200)$ inside them.




(c)




At $P_0(1000, 200)$, $dA/dt = 0$ and $dL/dt = -80 < 0$, so the number of ladybugs is decreasing and hence, we are proceeding in a counterclockwise direction. At $P_0$, there aren't enough aphids to support the ladybug population, so the number of ladybugs decreases and the number of aphids begins to increase. The ladybug population reaches a minimum at $P_1(5000, 100)$ while the aphid population increases in a dramatic way, reaching its maximum at $P_2(14, 250, 200)$.

Meanwhile, the ladybug population is increasing from $P_1$ to $P_3(5000, 355)$, and as we pass through $P_2$, the increasing number of ladybugs starts to deplete the aphid population. At $P_3$ the ladybugs reach a maximum population, and start to decrease due to the reduced aphid population. Both populations then decrease until $P_0$, where the cycle starts over again.

(d) Both graphs have the same period and the graph of $L$ peaks about a quarter of a cycle after the graph of $A$.