MATH 10B – Spring 2019
Quiz 12 – Prepared by John Yirong Zhen
Date:04/23/2018

You are to finish this quiz in 10 minutes. You are allowed one single-sided letter-size cheat sheet. No calculators or other notes/books/devices are allowed.
Your cheatsheet must be handwritten by you, no photocopying or preprinted (unless you have written permission from the instructor). Try your best! Stay calm and good luck!

I. True/False (2 pts)

Circle T or F in the space provided in front of the statement to indicate whether it is true or false respectively. You get +1 for a correct answer, −1 for incorrect, and 0 for leaving it blank. (You should not guess if you don't know the answer.)
You do not need to justify your answers for T/F statements.

- T Ⓕ With the same null and alternative hypothesis and the same observation, if we reject the the null at the 10% significant level, we also reject the null at the 5% significant level.
  If we reject the the null at the 10% significant level, that means the p-values is less than 10%, which is not necessarily less that $\alpha = 5\%$. Thus, we might not have enough evidence to reject the null at the 5% significant level.

- T Ⓕ For hypothesis testing, p-value is the probability that a more extreme event happens assuming the alternative hypothesis is true.
  By definition, for hypothesis testing, p-value is the probability that a more extreme event happens assuming the null hypothesis is true.

II. Written problems (10pts)

- You MUST **justify your answer** to undoubtably convince me that you solved and not guessed it. Partial credit will be given to good work and progress even if there is no final answer or the answer is incorrect. On the other hand, bogus justification for a correct answer will receive a 0.

- Keep your scratch work separate. Cross out writing you don't want to be graded and clearly label the parts you want to be graded.

- Points will be deducted for incorrect writings that you "forget to cross out."

See problem on back.

(a) Suppose X is an exponential random variable with unknown parameter λ and PDF $f(x) = \lambda e^{-\lambda x}$. You randomly sample X three times and get the values $1, 2, 3$. What is the MLE estimate for λ given this data?

$L(\hat{\lambda}|x_1, \ldots, x_3) = \hat{\lambda}^3 e^{-\hat{\lambda}(x_1+x_2+x_3)}$. Thus $\log L(\hat{\lambda}|x_1, x_2, x_5) = 3\log(\hat{\lambda}) - \hat{\lambda}(x_1 + x_2 + x_3)$. Differentiating gives us $\frac{3}{\hat{\lambda}} - (x_1 + x_2 + x_3)$ which has a single 0 at $\hat{\lambda} = \frac{3}{x_1+x_2+x_3} = \frac{1}{2}$, which is the MLE since the second derivative test give negative values at $\frac{1}{2}$.