MATH 10A 10/09/18

Student Section Sheet Tuesday

Session 14 – Area and Volume between Curves

Definition of Area between Curves (Pg. 389, (2))

The area A of the region bounded by the curves $y = f(x), y = g(x)$ and the lines $x = a, x = b$, where f and g are continuous and $f(x) \geq g(x)$ for all x in $[a, b]$, is:

$$A = \int_a^b [f(x) - g(x)]dx$$

Review – Section 6.1, Exercise 13

13) Sketch the region that lies between the curves $y = \cos(x)$ and $y = \sin(2x)$ and between $x = 0$ and $= \frac{\pi}{2}$. Notice that the region consists of two separate parts. Find the area of this region.

Application in Practice – Section 6.1, Exercise 25

25) Birth and Death Rates:

If the birth rate of a population is $b(t) = 2200e^{0.024t}$ people per year and the death rate is $d(t) = 1460e^{0.018t}$ people per year, find the area between these curves for $0 \leq t \leq 10$. What does this area represent?

Definition of Volume of Solid Objects (Pg. 407)

Let S be a solid that lies between $x = a$ and $x = b$. If the cross-sectional area of S in the plane P_x through x and perpendicular to the x-axis is $A(x)$, where A is a continuous function, the volume of S:

$$V = \lim_{n \to \infty} \sum_{i=1}^{n} A(x_i^*) x\Delta = \int_a^b A(x)dx$$

Review – Section 6.4, Example 4

The region enclosed by the curves $y = x$ and $y = x^2$ is rotated about the x-axis. Find the volume of the resulting solid:

a) Sketch a graph of $y = x$ and $y = x^2$ and find their point of intersection
b) Find the area between the two curves (just a review, not necessary for volume)
c) Determine the area function of the two circles created by the functions
d) Use these area functions within the integral to determine the volume of the rotated solid

Practice – Section 6.4, Exercises 5 & 6

Find the volume of the solid obtained by rotating the region bounded by the given curves about the x-axis. Sketch the region, the resulting solid, and evaluate the volume.

5) $y = x,\ y = x^3, x \geq 0$

6) $y = \frac{1}{4}x^2, y = 5 - x^2$

Challenge Problem – Section 6.4, Exercise 17

17) Find the volume common to two spheres, each with radius r, if the center of each sphere lies on the surface of the other sphere. (Remember: V = $\frac{4}{3}\pi r^3$)