

**MATH 10B – Spring 2019**  
**Quiz 3 – Prepared by John Yirong Zhen**  
**Date:02/12/2018**

You are to finish this quiz in 10 minutes. You are allowed one single-sided letter-size cheat sheet. No calculators or other notes/books/devices are allowed.

Your cheatsheet must be handwritten by you, no photocopying or preprinted (unless you have written permission from the instructor). Try your best! Stay calm and good luck!

**I. True/False (2 pts)**

Circle T or F in the space provided in front of the statement to indicate whether it is true or false respectively. You get +1 for a correct answer, -1 for incorrect, and 0 for leaving it blank. (You should not guess if you don't know the answer.)

You do not need to justify your answers for T/F statements.

- T    ☐ The equation  $x_1 + x_2 + x_3 = 8$  in nonnegative numbers where the order of the variables does not matter has as many solutions as the number of ways to split 8-tuplets (8 identical kids) into 3 identical playpens, where each playpen has at least one kid.  
Both problems are same as distributing 8 identical balls to 3 identical boxes, but the first one is arbitrary but the second one is surjective.
- T    ☐ There are  $\binom{18}{4}$  solution for the equation  $x_1 + x_2 + x_3 + x_4 = 18$ , where  $x_1, x_2, x_3, x_4$  are nonnegative integers.  
It should be  $\binom{21}{3}$  since it is the dist. bin, indist. ball and arbitrary case.

**II. Written problems (10pts)**

- You **MUST justify your answer** to undoubtably convince me that you solved and not guessed it. Partial credit will be given to good work and progress even if there is no final answer or the answer is incorrect. On the other hand, bogus justification for a correct answer will receive a 0.
- Keep your scratch work separate. Cross out writing you don't want to be graded and clearly label the parts you want to be graded.
- Points will be deducted for incorrect writings that you "forget to cross out."

See problem on back.

- (a) How many solutions are there to the inequality

$$x_1 + x_2 + x_3 + x_4 \leq 14$$

where  $x_1, x_2, x_3$  and  $x_4$  are nonnegative integers? [Hint: Introduce an auxiliary variable  $x_5$  such that  $x_1 + x_2 + x_3 + x_4 + x_5 = 14$ .]

We can introduce a nonnegative integer variable  $x_5$  such that  $x_1 + x_2 + x_3 + x_4 + x_5 = 14$ , which will have the same number of solutions as the original problem. It will have  $\binom{18}{4}$  different number of solution.

- (b) How many different strings can be made from the letters in AAACCCCDDB, using all the letters?

There are 3 A's, 1 B, 4 C's and 2 D's. Therefore,  $\frac{10!}{1!2!3!4!}$  different permutations.