MATH 10A with Prof. Stankova
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OH: Wed 1-3pm, Evans 753

Derivatives and Differentiation

This document is a collective effort of multiple GSIs for Math10A Fall 2018. Make sure you also go through the homework thoroughly and read the assigned readings and do the "Word Problems" from the textbooks.

I. Concepts/Facts

1. Derivatives (left-over from last time)
   a) What is the slope of a curve? That of the tangent line at that curve, IF it exsists.
   b) Name three other phrases that mean the derivative of the function at a point.Inst. Rate of Change; slope of the tangent line; limit of the secant slope. the latter two are more mathy than the first.
   c) Give a famous function that is NOT differentiable. Favorite in the class: $|x|$
2. Differentiation
   a) Review your differentiation laws, including Addition, Constant-Jumping, Power, Product and Quotient laws (these laws are all theorems!) and BE CAREFUL with their conditions!
   b) **If two functions f, g are both not differentiable at $x = a$, does that imply $f + g$ is also not diff'ble at $x = a$? NO. $|x| - |x|$ is identically 0 and diff'able.

II. Challenge

Any problem with at least one * throughout the paper.

1. Think: Are derivatives a local property or a global property? That is, do derivatives tell you something about the function as a whole or just at some point?
   Local. Note the difference between the derivative function and a global property of the original.
2. Give an example of a cts function that is NOT differentiable.
   Favorite in the class: $|x|$
3. Give an example of a function that is differentiable at $x = 3$ but not continuous at $x = 3$. Impossible. Note the idea of contrapositive.
4. *Is composition of differentiable functions differentiable? Yes. Make connections to composition of cts functions
5. *If a function is NOT differentiable at $x = 3$, then its reflection, translation and dilation are also NOT differentiable at the transformed point. True. Use the idea of contrapositive again and the fact that these transformations preserve indifferentiability.
6. SuperChallenge***: If a function is differentiable everywhere on $\mathbb{R}$, will the derivative function be differentiable? If a function is differentiable AND the derivative function is continuous everywhere on $\mathbb{R}$, does that guarantee the derivative function to be differentiable?
   The answer is both NO. favorite cts but non-diff'able function: $|x|$. Cook up a diff'able function s.t. its derivative is exactly $|x|$, which is $y = x^2/2$, but reflect the left half across the $x - axis$. You could also write the piecewise f'la, something like $-x^2/2$ when $x < 0, x^2/2$ otherwise.

III. Practice Problems

1. Find the continuous intervals of the following functions Individual work, $1/2 \times 1/2$
   a) * $f(x) = \begin{cases} \frac{x^2-x-2}{x-2} & \text{if } x \neq 2 \\ 3 & \text{if } x = 2 \end{cases}$
   Check for continuity at $x = 2$, confirm that f is cts at 2. interval: $\mathbb{R}$
   b) $f(x) = \dfrac{lnx + e^x}{x^2 - 1}$
   f is a quotient of 2 cts functions and therefore cts on its domain. interval: $\mathbb{R}\backslash\{\pm 1\}$

2. Using the definition of the derivative, find the equation of tangent line of the function at the given point G4Pick
   a) $y = 4x - 3x^2$; $(2, -4)$.
   Remind students of difference quotient and taking limit of such. Answer: $y = -8(x-2) + 4 = -8x + 20$.
   b) $y = \dfrac{4}{x+1}$; $(3, 1)$
   Don't use the derivatives yet. Shortcut Answer: $y = (\frac{4}{x+1})'|_{x=3}(x-3) - 1 = -1/4(x-3) - 1 = -1/4x - 1/4$

3. Find derivatives of the function using definition of derivatives
   a) $f(x) = \sqrt{x}$
   $1/2\sqrt{x}$. Students should memorize this, per Zvezda's instructions.
   b) $f(x) = x^2 + 2/x$
   $2x - 2/x^2$

4. Find the derivative of the following functions: Pairs
   a) $e^x x$. $e^x(x+1)$
   b) $(sinx)(cosx) + x^2$. $2x - sin^2x + cos^2x$
   c) $(tanx)e^x$. $e^x(tanx + sec^2x)$
   d) $e^x x^{-2}$ $\frac{e^x(x-2)}{x^3}$
   e) $x^3 - x^{\sqrt{3}} + 2x^{\ln 2}$. What is the domain of the derivative function?
   Do you notice a pattern/fun fact about the power rule?
   $3x^2 - \sqrt{3}x^{\sqrt{3}-1} + 2\ln 2 x^{\ln 2 - 1}$.
   Domain: $\mathbb{R}\backslash\{0\}$. Note: Powers < 1 blow the derivative function up at 0.

5. Find the derivative of:2ways, 5 groups
   a) $\dfrac{x+3}{x+4}$. $\dfrac{1}{(x+4)^2}$
   b) $\dfrac{sinx}{x}$. $\dfrac{xcosx - sinx}{x^2}$
   c) $\dfrac{cosx}{sinx}$. $-csc^2x$
   d) $\dfrac{e^x}{x^2}$. $\frac{e^x(x-2)}{x^3}$
   e) $\dfrac{x^2}{x+1}$. $\dfrac{x^2+2x}{(x+1)^2}$