MATH 10A with Prof. Stankova
Prepared by Yirong Zhen (GSI)
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Discussion on Sets and Functions

1 Practice Problems

1. Describe the following sets in words:
   - $A = \{2x | x \in \mathbb{Z}\}$
     A is all even numbers.
   - $B = \{x \in \mathbb{R} | x^2 - 5x + 6 = 0\}$
     $B = \{2, 3\}$
   - $C = \{x \in \mathbb{R} | x^2 + 1 = 0\}$
     $C = \emptyset$
   - $D = \mathbb{R} \backslash \mathbb{Q}$
     D contains all irrational numbers.
2. Are the functions $f(x) = \frac{x^2-1}{x+1}$ and $g(x) = x - 1$ equal?
   No, they have a different domain. Specifically, $D(f)$ doesn't contain -1.
3. What are the domains of the following functions?
   - $f(x) = \frac{x+1}{1+\frac{1}{x+1}}$
     $(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$
   - $g(x) = \sqrt{2 - \sqrt{p}}$
     $[0, 4]$
   - $h(x) = \arcsin(x)$
     $[-1, 1]$
4. What are the ranges of the following functions?
   - $f(x) = (x + 1)^2$
     $[0, \infty]$
   - $g(x) = +\sqrt{9 - (x - 3)^2}$
     $[0, 3]$
   - $h(x) = \frac{x+1}{1+\frac{1}{x+1}}$
     $(-\infty, -4) \cup (0, \infty)$
5. Are the following functions even or odd (or neither)?
   - $f(x) = x|x|$
     odd
   - $g(x) = (x + 2)^2 + (x - 2)^2$
     even
   - $h(x) = \cos(x) + \sin(x)$
     neither

2 Additional Practice Problems

1. Find the domain of the following functions
   (a) $f(x) = \frac{2x+1}{x^2+x-2}$
   $x \neq -2$ or 1
   (b) $g(x) = \frac{x^{\frac{1}{3}}}{x^2+1}$
   all real numbers
   (c) $k(x) = \sqrt{4-x} + \sqrt{x^2-1}$
   $(-\infty, -1] \cup [1, 4]$
2. Find the range of the following functions
   (a) $f(x) = x^2 + x - 2$
   (b) $g(x) = \frac{x^2-1}{x^2+1}$
   (c) $k(x) = \sqrt{x^2-1}$
3. Find the result of the following set operations:
   (a) $\mathbb{Z} \setminus \mathbb{Q}$
   empty set
   (b) $\mathbb{R} \setminus \mathbb{Q}$
   irrational numbers
   (c) $\mathbb{Z} \setminus (\mathbb{N} \cup 0)$
   all negative integers
   (d) $\mathbb{Z} \setminus (\mathbb{N} \cap 0)$
   all integers except 0
4. True or False:
   (a) $f(x) = \pm x$ is a function.
   false
   (b) $f(x) = |x|$ is a function.
   true
   (c) $\mathbb{Z} \subset (\mathbb{C} \setminus \mathbb{N})$
   false
   (d) For function $f(x) = \sqrt{x}$, the range is equal to the domain.
   true