MATH 10A with Prof. Stankova
10/04/2018

Discussion on Partial Fraction and Improper Integral

1 Partial Fraction

1. Evaluate the following integrals.
   - $\int \frac{4x^2-7x-12}{x(x+2)(x-3)}dx = \int \frac{2}{x} + \frac{9}{5(x+2)} + \frac{1}{3(x-3)}dx = 2\ln|x| + \frac{9}{5}\ln|x+2| + \frac{1}{3}\ln|x-3| + C$
   - $\int \frac{e^{2x}}{e^{2x}+3e^x+2}dx$
     let $u = e^x$, then $du = e^x dx$
     $\int \frac{e^{2x}}{e^{2x}+3e^x+2}dx = \int \frac{u}{u^2+3u+2}du = \int \frac{2}{u+2} - \frac{1}{u+1}du = 2\ln|e^x+2| - \ln|e^x+1| + C$
   - $\int \frac{x^3-4x-10}{x^2-x-6} = \int x + 1 + \frac{2}{x+2} + \frac{1}{x-3}dx = \frac{1}{2}x^2 + x + 2\ln|x+2| + \ln|x-3| + C$

2 Improper integrals

1. Find if the following integrals converge. If so, evaluate it.
   (a) $\int_{2\pi}^{\infty} \sin\theta d\theta = \lim_{t\to\infty} \int_{2\pi}^{t} -\cos\theta d\theta$, which diverges.
   (b) $\int_{-\infty}^{\infty} xe^{-x^2}dx = \lim_{t\to\infty} -\frac{1}{2}e^{-x^2}|_0^t + \lim_{l\to\infty} -\frac{1}{2}e^{-x^2}|_l^0 = \frac{1}{2} + (-\frac{1}{2}) = 0$
   (c) $\int_{1}^{\infty} \frac{x+1}{x^2+2x}dx = \lim_{t\to\infty} \frac{1}{2}\ln(x^2+2x)|_1^t$, which diverges.
2. Conceptual Problem
   (a) Show that $\int_{-\infty}^{\infty} xdx$ is divergent.
   If the limit approach positive and negative infinity at different rate, for example, $\int_{-\infty}^{\infty} xdx = \lim_{t\to\infty} \frac{1}{2}x^2|_0^{2t} + \lim_{t\to-\infty} \frac{1}{2}x^2|_t^0$ diverges.
   (b) Show that $\lim_{t\to\infty} \int_{-t}^{t} xdx = 0$
   $\lim_{t\to\infty} \int_{-t}^{t} xdx = \lim_{t\to\infty} \frac{1}{2}x^2|_{-t}^{t} = \frac{1}{2}t^2 - \frac{1}{2}(-t)^2 = 0$
3. Word Problem(if time allows)
   The plasma drug concentration of a new drug was modeled by the function $C(t) = 23te^{-2t}$, where t is measured in hours and C in mg/mL.
   (a) What is the maximum drug concentration and when did it occur?
   $C'(t) = 23e^{-2t} - 46te^{-2t} = 0 \implies t = 0.5$ and $C(0.5) = 4.23$
   (b) Calculate $\int_0^{\infty} C(t)dt$ and explain its significance.
   Integration by part and we have $\int_0^{\infty} 23te^{-2t}dt = -\frac{23}{2}te^{-2t} - \frac{23}{4}e^{-2t}|_0^{\infty} = \frac{23}{4}$
   this is the long-term availability of a single drug dose.