MATH 10B – Spring 2019
Quiz 6 – Prepared by John Yirong Zhen
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You are to finish this quiz in 10 minutes. You are allowed one single-sided letter-size cheat sheet. No calculators or other notes/books/devices are allowed.
Your cheatsheet must be handwritten by you, no photocopying or preprinted (unless you have written permission from the instructor). Try your best! Stay calm and good luck!

I. True/False (2 pts)

Circle T or F in the space provided in front of the statement to indicate whether it is true or false respectively. You get +1 for a correct answer, −1 for incorrect, and 0 for leaving it blank. (You should not guess if you don't know the answer.)
You do not need to justify your answers for T/F statements.

- T Ⓕ Let $P(A) > 0$ and $P(B) > 0$ and $A \cap B = \emptyset$. Then, A and B are independent. Disjoint event are usually not independent. When $P(A) > 0$ and $P(B) > 0$, if A happens, B won't happen, and if B happens, A won't happen.

- Ⓣ F Let A be an event such that $P(A) = 1$. Then A is independent of any event B. $P(A)P(B) = 1 \times P(B) = P(B)$. $P(A \cap B) = P(B)$ since $B \subset A$. Thus, $P(A \cap B) = P(B)P(A)$, and A and B are independent.

II. Written problems (10pts)

- You MUST **justify your answer** to undoubtably convince me that you solved and not guessed it. Partial credit will be given to good work and progress even if there is no final answer or the answer is incorrect. On the other hand, bogus justification for a correct answer will receive a 0.

- Keep your scratch work separate. Cross out writing you don't want to be graded and clearly label the parts you want to be graded.

- Points will be deducted for incorrect writings that you "forget to cross out."

See problem on back.

Suppose we toss two fair 6-sided dice. Let E_1 be the event that the first roll comes up 5. Let E_2 be the event that the sum of the two rolls is 5. Let E_3 be the event that the sum of the two dice is 7. *(a)* Are events E_1 and E_2 independent? *(b)* Are events E_1 and E_3 independent? Show your reasoning.

(a) We compute that

$$P(E_1) = \tfrac{1\cdot 6}{6\cdot 6} = \tfrac{1}{6}, \qquad P(E_2) = \tfrac{\{(1,4),(2,3),(3,2),(4,1)\}}{\Omega} = \tfrac{4}{36} = \tfrac{1}{9}, \qquad P(E_1 \cap E_2) = 0,$$

where $P(E_1 \cap E_2) = 0$ since E_1 and E_2 cannot both happen at the same time. Therefore, $P(E_1 \cap E_2) = 0 \neq \frac{1}{72} = P(E_1)P(E_2)$, so $\boxed{E_1 \text{ and } E_2 \text{ are not independent}}$.

(b) We have:

$$P(E_1) = \tfrac{1}{6}, \quad P(E_3) = \tfrac{\{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}}{\Omega} = \tfrac{6}{6^2} = \tfrac{1}{6}, \quad P(E_1 \cap E_3) = \tfrac{\{(5,2)\}}{\Omega} = \tfrac{1}{36}.$$

Therefore, $P(E_1 \cap E_3) = \frac{1}{36} = \frac{1}{6} \cdot \frac{1}{6} = P(E_1)P(E_3)$, so $\boxed{E_1 \text{ and } E_3 \text{ are independent}}$. □