Worksheet 2

Basics of counting - Solutions

1. How many ways are there to choose a committee of 9 people, as well as the chairperson and vice chairperson (who are also members of the committee) from a group of 10 people?

$$10 \cdot 9 \cdot 8$$

2. How many six character strings of English letters

   i) contain an A? $26^6 - 25^6$

   ii) contain an E or an I? $26^6 - 24^6$

   iii) start with a C or start with a D? $2 \cdot 26^5$

   iv) start with a C or end with a D? $26^5 + 26^5 - 26^4$

   v) contain exactly one X? $6 \cdot 25^5$

3. Consider the set $\{1, 2, 3, ..., 99\}$.

   i) How many subsets are there that contain the numbers 1 or 99? $2^{98} + 2^{98} - 2^{97}$

   ii) How many subsets contain only even numbers? 2^{49}

   iii) How many subsets only contain numbers divisible by 3 but not divisible by 4? 2^{33-8}

   iv) How many subsets only contain numbers divisible by 4 or divisible by 6? 2^8

   v) ($\star\star$) How many subsets are there that have an even number of elements? 2^{98}

4. ($\star$) How many different ways could a Best of Seven series go?

$$2 \cdot (1 + 4 + 10 + 20) = 70$$

5. How many binary strings are there of length 7 with

   i) two consecutive zeroes in the beginning or two consecutive ones at the end? $2^5 + 2^5 - 2^3$

   ii) a one in the first, fourth, or last digit? $2^6 + 2^6 + 2^6 - 2^5 - 2^5 - 2^5 + 2^4$

   iii) exactly two zeroes? $6 + 5 + 4 + 3 + 2 + 1 = 21$

   iv) an even number of ones? 2^6

   v) ($\star\star$) exactly one pair of adjacent zeroes? 38

6. ($\star$) Suppose students at a school are able to take French, Latin, Spanish (or any combination of the three). There are 500 students; 200 students take French, 150 students take Latin, and 170 students take Spanish. Furthermore, you also know that 50 students take French and Latin, 85 students take French and Spanish, and 70 students take Spanish and Latin. If there are 150 students who take no language course, how many students take all three languages? 35

7. How many ways are there to split 10 people into a Red Team and Blue Team? The teams do not have to be even, but each side needs at least one person. $2^{10} - 2$

   ($\star\star$) What if we want to have a Green Team as well? $3^{10} - (2^{10} + 2^{10} + 2^{10}) + 3$