Worksheet 27

Partial Derivative

Simple Partial Derivative

1. If $f(x, y) = 7 - x^2y^3$, what are its two partial derivatives at $(1, 2)$?
2. $\mu = \ln(x^2 + y^2)$. Find the two derivatives with respective to x and y, that is μ_x and μ_y.
3. Same setup at the last part, find $(\mu_x)_y$, $(\mu_x)_x$, $(\mu_y)_x$, $(\mu_y)_y$.

Chain Rule and PDE

1. If $z = x^2y + 5y^2, x = \sin t, y = \ln t$, find $z(t)$ two ways:
   (a) rewriting $z = z(t)$ as a single-variable function of t and taking its ordinary derivative $z'(t)$
   (b) applying the Chain Rule $\mathbf{CR}_1$: $\dfrac{dz}{dt} = \dfrac{\partial z}{\partial x} \cdot \dfrac{dx}{dt} + \dfrac{\partial z}{\partial y} \cdot \dfrac{dy}{dt}$.
2. Verify that the given functions are solutions to the corresponding PDEs.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

   (a) $u(x, y) = e^x \cos y$
   (b) $u(x, y) = x^2 - y^2$
   (c) $u(x, y) = x^4 - 6x^2y^2 + y^4$

T/F

1. Let X and Y be random variables with finite means and variances. Then $Cov[10X, 10Y] = Cov[X, Y]$.

2. The least squares line of best fit is the line which minimizes the sum of the distances from the observed data points (x_i, y_i) to the line.

Source: Homework 37.