Student Name:

MATH 10A with Prof. Stankova – Fall 2018
Discussion 11

1. Find the derivatives of the following functions (using FTC1):
   - $f_1(x) = \displaystyle\int_1^x \frac{1}{t^3+1}dt$ ($f_1 : (-1,\infty) \to \mathbb{R}$).

     Directly applying FTC1, we get $f_1'(x) = \frac{1}{x^3+1}$.
   - $f_2(x) = \displaystyle\int_0^{\sin(x)} \sqrt{1-t^2}dt$

     Using the FTC1 with the chain rule, we get $f_2'(x) = \cos(x)\sqrt{1-\sin^2(x)} = \cos(x)|\cos(x)|$.
   - $f_3(x) = \displaystyle\int_{x^2}^{x^3} \frac{1}{t}dt$ ($f_3 : (0,\infty) \to \mathbb{R}$).

     Split the integral up as $\displaystyle\int_1^{x^3} - \int_1^{x^2}$, then apply FTC1 with the chain rule to get $f_3'(x) = \frac{3x^2}{x^3} - \frac{2x}{x^2} = \frac{1}{x}$. You can also note that integrating directly gives you $f_3(x) = \ln(x)$.

2. Find an antiderivative of e^{-x^2} (Use FTC1!).

   An antiderivative of e^{-x^2} is $\displaystyle\int_0^x e^{-t^2}dt$ (check this with FTC1).

3. Given an integrable function $f : \mathbb{R} \to \mathbb{R}$, find an antiderivative of f.

   An antiderivative of f is $\int_0^x f(t)dt$ (check with FTC1).

4. Evaluate the following definite and indefinite integrals. If you use u substitution, state your variable u. Think about how you determined which substitution u to use.
   - $\displaystyle\int \frac{(\ln(x))^2}{x}dx$

     Let $u = \ln(x), du = \frac{1}{x}dx$. Then $\displaystyle\int \frac{\ln^2(x)}{x}dx = \int u^2du = \frac{1}{3}u^3 + C = \frac{1}{3}\ln^3(x) + C$.
   - $\displaystyle\int \frac{\sin(2x)}{1+\cos^2(x)}dx$

     Let $u = \cos^2(x)$, $du = 2\cos(x)\sin(x)dx = \sin(2x)dx$, so $\displaystyle\int \frac{\sin(2x)}{1+\cos^2(x)}dx = \int \frac{1}{1+u}du = \ln(1+u) + C = \ln(1+\cos^2(x)) + C$.
   - $\displaystyle\int_1^2 x\sqrt{x-1}dx$

     Let $u = x-1$, $du = dx$. Then $\displaystyle\int_1^2 x\sqrt{x-1}dx = \int_0^1 (u+1)\sqrt{u}du = \frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\Big|_0^1 = \frac{2}{5} + \frac{2}{3} = \frac{16}{15}$.
   - $\displaystyle\int_{-\pi/4}^{\pi/4} (x^3 + x^4\tan(x))dx$

     The integral is 0 as the integrand is an odd function.

- $\displaystyle\int_0^1 \frac{1}{(1+\sqrt{x})^4}dx$

  Let $u = 1+\sqrt{x}$, $du = \frac{dx}{2\sqrt{x}}$ (so $2(u-1)du = dx$). Then $\displaystyle\int_0^1 \frac{1}{(1+\sqrt{x})^4}dx = \int_1^2 \frac{2(u-1)}{u^4}du = \frac{-1}{u^2} + \frac{2}{3u^3}\Big|_1^2 = \frac{1}{6}$.

5. If $a, b > 0$, prove that $\displaystyle\int_0^1 x^a(1-x)^b dx = \int_0^1 x^b(1-x)^a dx$.

   Let $u = 1-x$, $du = -dx$. Then $\displaystyle\int_0^1 x^a(1-x)^b dx = \int_1^0 -(1-u)^a u^b du = \int_0^1 u^b(1-u)^a du$.