Student Name:

MATH 10A with Prof. Stankova – Fall 2018
Discussion 22

Geometry of $\mathbb{R}^3$

1. What shapes do the equations $x^2+y^2+z^2=9$ and $x+y+z=0$ describe? How would you describe the set of points satisfying both equations?
2. If U and V are planes in $\mathbb{R}^3$, what forms can $U \cap V$ take? Construct explicit examples - for each possible form of $U \cap V$, find two planes whose intersection is if that form. What does this tell you about solutions to systems of 2 linear equations in 3 variables?
3. Describe the part of the plane $x+2y+z=4$ contained within the first octant.
4. Write down a system of inequalities describing a cone with base of radius 2 in the xy-plane centered at the origin and height 2 with top point $(0,0,2)$.

Vectors and the Dot Product

1. What are the lengths of the vectors $v = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $u = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$. Compute $v \cdot v$ and $u \cdot u$ - what do you notice?
2. Find a vector that has the same direction as $\begin{pmatrix} 8 \\ -1 \\ 4 \end{pmatrix}$ of length 2.
3. Consider the triangle in $\mathbb{R}^3$ with vertices $0, v = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, u = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$. What are the side lengths of the edges of this triangle? (Challenge) What is the area of this triangle?
4. Determine whenther angles between the following pairs of vectors are acute, right, or obtuse: $v_1 = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}, v_3 = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}, v_4 = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$.
5. Find the equation of the plane that passes through the origin and is perpendicular to the vector $\begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$.
6. (Challenge) Use the CSB inequality and the formula for the dot product to prove the triangle inequality for vectors: $|u+v| \leq |u|+|v|$ for vectors u, v.