Dis_ 19_ Solution

Quick Check:

① $xy' = y + x^2 \sin x$ 1^{st} order linear ODE

$y' - \frac{1}{x} y = x \sin x$ $y' + p(x) y = q(x)$.

$p(x) = -\frac{1}{x}$. $I(x) = e^{\int -\frac{1}{x} dx} = e^{-\ln x} = \frac{1}{x}$

$\Rightarrow \frac{1}{x} y' - \frac{1}{x^2} y = \sin x$

$(\frac{1}{x} y) = \int \sin x \, dx = -\cos x + C$

$y = -x \cos x + xC$

② $\frac{dy}{dx} = \frac{x^2 y - 4y}{x+2} = \frac{(x+2)(x-2)y}{x+2}$ $x \neq -2$

$= (x-2) y$

① if $y = 0$. check. $y' = 0$

$\frac{dy}{dx} = 0 = \frac{x^2 \cdot 0 - 4 \cdot 0}{x+2} = 0$. → OK!

② if $y \neq 0$, then

$\frac{1}{y} dy = (x-2) dx$

$\int \frac{1}{y} dy = \int (x-2) dx$

$\ln|y| = \frac{1}{2} x^2 - 2x + B$

$y = \pm e^{\frac{1}{2} x^2 - 2x + B}$

$= \pm e^B \cdot e^{\frac{1}{2} x^2 - 2x}$

$= C \cdot e^{\frac{1}{2} x^2 - 2x}$ $(C \neq 0)$

consider case ① & ②.

$y = C \cdot e^{\frac{1}{2} x^2 - 2x}$.

③ $y'' + 25y = 0$. $y'(0) = 6$. $y'(\pi) = -8$

$r^2 + 25 = 0$

$r = \pm 5i$

$\alpha = 0$ $\beta = 5$

$y = C_1 \cos(5x) + C_2 \sin(5x)$

$y' = -C_1 \sin(5x) + C_2 \cos(5x)$

$\begin{cases} 6 = y'(0) = 5C_2 & \Rightarrow C_2 = \frac{6}{5} \\ -8 = y'(\pi) = -5C_2 & \Rightarrow C_2 = \frac{8}{5} \end{cases}$ ⇒ No solution

(3) $$\begin{cases} \dfrac{dx}{dt} = 0.08x\left(1-\dfrac{x}{5000}\right) - 0.001xy \\ \dfrac{dy}{dt} = -0.02y - 0.00002xy \end{cases}$$

x : prey . constrained by carrying capacity .
y: predator

check logstic model

★ Find equilibrium.

$$\begin{cases} \dfrac{dx}{dt} = 0 \\ \dfrac{dy}{dt} = 0 \end{cases} \Rightarrow \begin{cases} 0.001x\left(80 - \dfrac{80}{5000}x - y\right) = 0 \\ 0.0002y(-1000+x) = 0 \end{cases}$$

$$\begin{cases} \text{Group 1}: & x=0. & y=0 \\ \text{Group 2}: & x=5000 & y=0 \\ \text{Group 3}: & x=1000 & y=64 \end{cases}$$

1. Matrix Addition & Scalar Multiplication

(a) (1) $\left[\begin{array}{ccc|c} 3 & 9 & 1 & 0 \\ 0 & 4 & 2 & 3 \\ 6 & 3 & 0 & 7 \end{array}\right]$

(2) $\begin{cases} x + 8y + 4z = 2 \\ 2x - 3y = 4 \end{cases}$

(b) $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 1 + 4 + 9 = 14$

$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$$

2. ① $A - 2BC = \begin{bmatrix} 7 & 0 \\ 1 & 9 \end{bmatrix} - 2 \begin{bmatrix} 0 & 2 \\ 13 & 6 \end{bmatrix} \begin{bmatrix} 15 & 0 \\ 0 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 7 & 0 \\ 1 & 9 \end{bmatrix} - 2 \begin{bmatrix} 0 & 4 \\ 195 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 0 \\ 1 & 9 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ 390 & 24 \end{bmatrix} = \begin{bmatrix} 7 & -8 \\ -389 & -15 \end{bmatrix}$$

② $EG = \begin{bmatrix} x & 5 & 3 \\ 0 & 9 & 1 \\ 6 & 4 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 5 & 4 & 6 \\ 9 & 7 & 8 \end{bmatrix}$

$$= \begin{bmatrix} x+25+27 & 2x+20+21 & 3x+30+24 \\ 0+45+9 & 0+36+7 & 0+54+8 \\ 6+20+0 & 12+16+0 & 18+24 \end{bmatrix}$$

$$= \begin{bmatrix} x+52 & 2x+41 & 3x+54 \\ 54 & 43 & 62 \\ 26 & 28 & 42 \end{bmatrix}$$

③ $FK - 2K = \begin{bmatrix} y & 0 & y \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 9 & 2 \\ 1 & 5 \\ 7 & 8 \end{bmatrix} - 2 \begin{bmatrix} 9 & 2 \\ 1 & 5 \\ 7 & 8 \end{bmatrix}$

$$= \begin{bmatrix} 16y & 10y \\ 1 & 5 \\ 8 & 13 \end{bmatrix} - \begin{bmatrix} 18 & 4 \\ 2 & 10 \\ 14 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} 16y-18 & 10y-4 \\ -1 & -5 \\ -6 & -3 \end{bmatrix}$$

④ $HEA = H(EA)$

$$\underset{3\times 3}{H}\,(\underset{2\times 2}{EA})$$

by property 1 of matrix multiplication
not defined

2. Transpose of matrix and identity matrix

① $\begin{bmatrix} 1 & 4 \\ 5 & 8 \end{bmatrix}^T = \begin{bmatrix} 1 & 5 \\ 4 & 8 \end{bmatrix}$

$\begin{bmatrix} 7 & 8 \\ 0 & 2 \end{bmatrix}^T = \begin{bmatrix} 7 & 0 \\ 8 & 2 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$

$\begin{bmatrix} 5 & 4 & 7 \\ 8 & 4 & 2 \\ 0 & 9 & 5 \end{bmatrix}^T = \begin{bmatrix} 5 & 8 & 0 \\ 4 & 4 & 9 \\ 7 & 2 & 5 \end{bmatrix}$

$\begin{bmatrix} 1 & 8 & 4 \\ 2 & -3 & 0 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ 8 & -3 \\ 4 & 0 \end{bmatrix}$

$\begin{bmatrix} 7 & 4 \\ 2 & 8 \\ 1 & 3 \end{bmatrix}^T = \begin{bmatrix} 7 & 2 & 1 \\ 4 & 8 & 3 \end{bmatrix}$

② $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow I^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 1+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow I^2 = I$

③ $(AB)^T = \left(\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \right)^T$

$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}^T$$

$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{21}b_{11} + a_{22}b_{21} \\ a_{11}b_{12} + a_{12}b_{22} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

$B^TA^T = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}^T \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^T$

$$= \begin{bmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \end{bmatrix} \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix}$$

$$= \begin{matrix} a_{11}b_{11} + a_{12}b_{21} & b_{11}a_{21} + b_{21}a_{22} \\ a_{11}b_{12} + a_{12}b_{22} & b_{12}a_{21} + b_{22}a_{22} \end{matrix}]$$

$\therefore (AB)^T = B^TA^T$