Prog. Energy Combust. Sci. 1991, Vol. 17, pp. 67–82
Printed in Great Britain.

0360–1285/91 $0.00 + .50

MODELLING FIRE SPREAD THROUGH FUEL BEDS

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Received 9 July 1990

Abstract—This is a review of the essential ingredients needed to make a mathematical model of fire spread through a fuel bed. The physical problem is outlined in general terms. Previous models are classified as statistical, empirical, or physical in accordance with the methods used in their construction. Considerable attention is given to the many physical models proposed over the fifty years of model building. A unified mathematical treatment is presented, which permits a more objective comparison of the different physical models. The key theoretical and experimental expositions in the literature are interpreted as pointing to a missing ingredient for fire spread in still air: a short range heat transfer mechanism. A candidate for this is buoyant convection, which unfortunately is very difficult to model in terms of the fundamental fluid dynamic equations. However, the concept of turbulent transport by eddies can be of help here, resulting in flame spread rates which depend on values for the eddy diffusivity of the fuel bed.

CONTENTS

1. INTRODUCTION

Wildfires are an annually occurring threat to people and property in many nations of the world. The reasons for this include our ignorance of the basic chemical and physical processes for fire spread. An acute example of this is the (unknown) requirements for a change between modes of fire spread; such as when a fire in a vegetation crown will change from a passive effect of a surface fire into an independent propagating fire. The complexity of natural fuel configurations, their combustion, and the interaction with local meteorological conditions has prevented any comprehensive wild fire behaviour model from being developed; let alone incorporated into a practical management strategy. Rather, empirical

models based on test fires in the laboratory and in the field have been developed to estimate wildfire behaviour. While this has been a worthy and necessary first step in the understanding of wildfire behaviour, it is an unsatisfactory end point from both a scientific and a practical point of view.

For more than fifty years, beginning with Fons,[1] research has been undertaken to understand the behaviour of freely spreading fires in both regular and irregular arrays of fuel. Many people have contributed and tried to develop generally applicable models; for example: Albini,[2] de Mestre *et al.*,[3] Dorrer,[4] Emmons,[5] Fransden,[6] Grishin *et al.*,[7] Hottel *et al.*,[8] Pagni and Peterson,[9] Steward,[10] Telisin[11] and Thomas.[12] This review will endeavour to establish a unified account by concentrating on the conceptual advances made in the modelling of fire spread through fuel beds, both in the laboratory and in the field.

The motion of gases through the fuel bed matrix appears to be a key concept in understanding fire spread through a fuel bed. However, little work has been done to model this motion and its effect on fire spread. Future work in this direction would be valuable, both scientifically and in the practical prediction of the effect of wind on a spreading fire.

2. THE PHYSICAL PROBLEM

In general terms, the fundamental problem is to be able to predict the rate of spread of a fire through a fuel bed. The physical processes which are integral to fire spread can be described as follows:

(1)—an ignition source which causes the localized release of reactive gases in a fuel bed, followed by their gas phase combustion, which results in flames within and above the fuel bed,

(2)—the heat produced in this reaction is transferred by any available mechanism and a portion of which reaches unburnt fuel,

(3)—the absorption of energy by unburnt fuel which raises the enthalpy to a point where decomposition occurs, releasing fresh reactive gases (their subsequent combustion is observed as fire spread).

This conception of fire spread highlights the feedback nature of fire spread (indeed, of any sustained combustion) as sufficient heat must be liberated by the reaction and then transferred to cause unburnt fuel to ignite. The overall process can be modelled as a continuous process in the limit where the size of the elements of fuel involved is reduced to an infinitesimal amount yielding a differential equation model. However, before we attempt this, let us describe the fuel, the flames and the heat transfer in more detail, so that our problem is better defined.

2.1. *Fuel*

The fuel bed is an agglomeration of combustible, organic matter in a *metastable* state (metastable because it usually decomposes very slowly, unless a large perturbation, such as a lit match, is applied to it). The fuel bed could be a naturally occurring one or a man made one. Natural fuel beds can be homogeneous; for example, drying grassland and forest litter, or they can be composed of discrete elements such as salt bushes with large gaps between. A botanical classification of fuel types is usually available which can assist in modelling the amount of fuel available and its compactness.

Man made fuel beds can occur in the office or home, where socio-economic factors affect the quality of available fuel, and the fuel beds are inhomogeneous on any but the coarsest of scales.

Researches, in their quest for reproducible experimental results have built a variety of regular arrays from all manner of fuels and in some cases, such as the U.S. Forest Service, have tried to simulate natural fuel beds in the laboratory with pine needles and other foliage.

For the present review, only homogeneous fuel beds will be considered. To physically characterize an oven dry, homogeneous fuel bed composed of only one type of fuel, four measurements are sufficient. These are (Fons[1] or Thomas[12]):

(1) surface area per unit volume of the individual fuel particles
(2) density of the individual fuel particles
(3) ratio of the density of individual fuel particles to that of the whole bed (equivalent to estimating the voidage of the fuel bed)
(4) the depth of the fuel bed.

A fifth measurement is needed for fuel which is not oven dry; namely, the moisture content, usually expressed as a fraction of oven dry weight. Additional measurements are needed if the bed is composed of more than one fuel type.

Once a heat source is introduced, the thermal characteristics of the fuel become important. The minimum set of thermophysical variables would comprise the conductivity and heat capacity for the fuel (and possibly water). With these, one can calculate the enthalpy of the fuel bed, or any portion of the fuel bed, as a function of temperature. For the heat transfer models considered to date this has been sufficient information. However, if any chemical processes are to be modelled, kinetic data are required to determine at least the heat of reaction, the reaction rate constant and the Activation Energy; assuming that the Arrhenius law is a reasonable approximation to the reaction rate.

A further consideration which may be important is turbulent transport processes within the fuel bed. In this case it would be necessary to determine additional variables describing this mode of heat and mass transfer. As an example, the mean wind profile in a forest canopy cannot be described simply in terms of canopy structure (with our present theoretical understanding), but measurements of eddy diffusivity can yield a reasonable semi-empirical model; see Raupach and Thom.[13]

2.2. *Flame*

Once the enthalpy of any part of the fuel approaches critical values, which occur at temperatures of around 320°C for cellulosic fuels, thermal decomposition begins to occur at appreciable rates (pyrolysis). At this stage a whole branch of complex chemistry and physics comes into play if one wishes to understand the chemical and structural changes in the solid fuel and the chemistry of the reactive gases which are released. Considerable research into the gasification of cellulosic fuels, such as wood, has been undertaken (for example, see Ohlemiller *et al.*[14] and the references therein). Rates of gasification and the chemical species produced have been identified for particular fuels; however, it has proven difficult to devise a general model applicable to a variety of fuels. Because of this difficulty, an 'ignition temperature' is commonly assumed, and one can then calculate the energy required to raise the enthalpy of fuel from ambient to ignition values. It is also assumed that at the ignition temperature, flaming begins to occur and that any additional energy released in flaming combustion is accounted for in an energy balance for the flame, incorporating the amounts of energy transported by convection and radiation (and perhaps even any preferential directions for their transport). Thus, radiatively, a common assumption is that of a flame of a fixed size at a fixed temperature and with a given emmisivity. If a fire occurs when there is no imposed air flow (i.e. no wind) then it is usually assumed that convection energy transport only occurs vertically; however, this is not necessarily so, as will be discussed later.

In terms of the actual temperature history of the fuel, the ignition temperature has previously been defined as being the temperature at which there is an inflexion point in the temperature vs time graph. This can be used as a criterion for obtaining a prediction of the fire spread rate (Weber[15]). Nevertheless, it is important to appreciate that the concept of an ignition temperature is fictitious (see for example Williams[16]) albeit a useful approximate method. A model kinetic scheme, as alluded to earlier, would be a significant improvement in our understanding of fuel bed combustion.

2.3. *Heat Transfer*

There are three basic modes of heat transfer; namely conduction, convection and radiation. A review of experimental and theoretical methods of analysing each of these modes is beyond the scope of the present work; Gebhart[17] and Ozisik[18] are two of the many monographs which are of use.

The quantity and rate of heat transfer away from the combustion region and to unburnt fuel is the most extensively modelled feature of fire spread. The extent to which analytical methods for physically characterizing heat transfer have been used by different workers allows one to formally classify models as belonging to one of three types. I shall call these statistical, empirical, or physical, according to whether they involve no physics at all, no distinction between different modes of heat transfer, or account for each mechanism of heat transfer individually. This classification serves as the basic tool with which to understand the construction of models and with which to make comparisons between models. A summary of the models to be considered follows in the next section; they are listed according to the above classification in Table 1. The review of Catchpole and de Mestre[19] is in a similar vein, although it excludes any discussion of the mathematics used in the models. See also the review by Williams.[52]

TABLE 1. Examples of classification of fire spread models according to whether they are statistical (involving no physical mechanisms for heat transfer), empirical (lumping all physical mechanisms for heat transfer together), or physical (distinguishing between physical mechanisms for heat transfer)

Statistical	Empirical	Physical
Anderson *et al.*[23]	Fransden[6]	Albini[2,38,39]
Green[22]	Rothermel[28]	Cekirge[34]
McArthur[20]		de Mestre *et al.*[3]
Noble *et al.*[21]		Dorrer
Stauffer[24]		Emmons[5]
		Fons[1]
		Fujii *et al.*[33]
		Grishin[30]
		Grishin *et al.*[7]
		Hottel *et al.*[8]
		Pagni and Peterson[9]
		Steward[10]
		Telisin[11]
		Thomas[12]
		Weber[15,25]

3. CLASSIFICATION AND SUMMARY OF MODELS

3.1. *Statistical Models*

These models make no attempt to involve physical mechanisms, being merely a statistical description of test fires. The results can be very successful in predicting the outcome of similar fires to the test fires. However, the lack of a physical basis means that statistical models must be used cautiously outside of the test conditions.

The McArthur grassland and forest fire meters (McArthur[20] and Nobel *et al.*[21]) are of this kind. They were developed and tested using dry grassland and forest litter respectively, in conditions found in the New South Wales tablelands (South Eastern Australia) in winter.

As an example, the mathematical relationships derived by Nobel *et al.*[21] for the simplest McArthur

meter, the Mark 4 grassland fire danger meter, are:

$$F = 2\exp[-23.6 + 5.01\ln C_d + 0.0281T_a - 0.226H_r^{1/2} + 0.663U_{10}^{1/2}] \quad (1)$$

$$V = 0.036F \quad (2)$$

where C_d = degree of curing (%), T_a = air temperature (°C), H_r = relative humidity (%), U_{10} = wind velocity msec^{-1} at 10 m height, F = fire danger index and V = fire spread rate (msec^{-1}).

The use of the McArthur meters in conditions similar to those under which they were calibrated, has been very successful; for example low intensity fuel reduction burning. However, their use in other fuel types, such as heaths, has been markedly less successful.

A different sort of statistical modelling is obtained with simulation. In these, the objective is to simulate spreading fires qualitatively, as the actual fire spread rate is not predicted. Cellular Automata (Green[22]), Huygen's Principle (Anderson *et al.*[23]) and Percolation (Stauffer[24]) have all been used to simulate the two dimensional spreading of a fire through a bed of fuel; usually with a biasing such as a wind or slope included to add a touch of reality. (A novel variation of the simulation idea, which actually includes some physical mechanisms, has been given by Weber[51] and is based in part on Vogel and Williams.[26] It describes the convective plume from individual burning elements and the interaction with unburnt elements. A fire spread rate is predicted and simulation tests are encouraging.)

3.2. *Empirical Models*

This category is reserved for models which are based upon the principle of conservation of energy, but which do not differentiate between modes of heat transfer.

Williams[27] has given a generic form for the conservation of energy;

$$Q = \rho h_i V \quad (3)$$

where Q = net energy per unit area transported across the surface of fire inception (Wm^{-2}), ρ = fuel density (kgm^{-3}), h_i = enthalpy per unit mass required for ignition of fuel (Jkg^{-1}) and V = fire spread rate (msec^{-1}).

The empirical models, of which Fransden[6] and the practical implementation by Rothermel[28] are the quintessential examples, do not describe the processes which contribute to Q (left hand side of Eq. (3)). Rather, laboratory measurements over a range of environmental and fuel conditions have been used to empirically determine Q (which is then called the 'propagating flux') as a function of the reaction intensity I_R:

$$\frac{Q}{I_R} = (192 + 7.894s)^{-1}\exp[(0.792 + 3.760s^{0.5})(\beta + 0.1)] \quad (4)$$

where s = surface area to volume ratio (cm^{-1}) and β = fuel bed density/solid fuel density.

The reaction intensity I_R is related to the fuel mass loss rate per unit area, $\frac{dw}{dt}$, by

$$I_R = -h_i\frac{dw}{dt}. \quad (5)$$

Substituting Eqs (4 and 5) into Eq. (3) yields an equation for the fire spread rate:

$$V = f(s,\beta)\frac{dw}{dt}\rho^{-1} \quad (6)$$

where the left hand side of Eq. (4) has been denoted by $f(s, \beta)$. Equation (6) demonstrates that the important fuel bed parameters are s, β and ρ, as anticipated in Section 2.1. The empirical nature of the model is reflected in the functional forms derived by experimentation. The wide range of fuel parameters and external conditions over which Rothermel[28] conducted experiments with which to derive his model, has allowed it to be successfully applied in many different fuels under diverse meteorological conditions; for example, see Catchpole.[29] It has been built into an operational fire-management tool and is an integral part of the BEHAVE fire-behaviour prediction package available on a variety of personal computers and programmable calculators.

Note that wind and slope effects upon fire spread rate have also been included by substituting $Q(1 + \phi_w + \phi_s)$ for Q in Eq. (3) where ϕ_w and ϕ_s are wind and slope correction factors. These are determined experimentally to be (Rothermel[28])

$$\phi_w = C_w U_{10}^{B_w}(\beta/\beta_0)^{-E_w} \quad (7)$$

$$\phi_s = 5.275\beta^{-0.3}\tan^2\theta \quad (8)$$

where B_w, C_w and E_w are parameters involving s, β_0 is the 'optimum packing ratio', and θ is the angle of the slope measured from horizontal. The equation for the fire spread rate, Eq. (6), is then replaced by

$$V = f(s,\beta)\frac{dw}{dt}\rho^{-1}(1 + \phi_w + \phi_s). \quad (9)$$

For most conditions this turns out to be a reasonable model, albeit a little difficult to understand physically. However, for many fuels the value of B_w, the exponent to which wind speed is raised, is greater than 1.0 (see Table 2). The possibility then arises that, for large enough windspeed, the fire can move faster than the wind! In the model of Rothermel[27] this is corrected for by placing an upper bound upon the value of ϕ_w. While this is a reasonable pragmatic solution, it is hardly satisfactory from a scientific point of view; a good model should not suffer from such problems.

3.3. *Physical Models*

This category contains those models which differentiate among the modes of heat transfer and

TABLE 2. Representative values for C_w, B_w, E_w in Eq. (7), from Beer[50]

s (cm^{-1})	B_w	C_w	E_w
285	3.38	1.46	0.03
154	2.43	2.52	0.13
69	1.57	3.97	0.34
40	1.17	4.77	0.46
4	n/a	n/a	n/a

attempt to predict the fire spread rate using more fundamental physical and mathematical means. There have been many models of this type constructed, and they will be reviewed in detail in the next section. For now, some general comments and an instructive example will suffice.

Usually, the result of the combustion process is assumed to be known. That is, flame and ember properties, such as emissivity, temperature and physical dimensions, are assigned. (Exceptions to this are models by Grishin *et al.*[7] and Grishin[30] which will be discussed separately in Section 4.3) The problem of predicting the fire spread rate is then solely a heat transfer calculation. Radiative, convective and conductive/diffusive transfer of heat from the combustion region to unburnt fuel can then be modelled with familiar mathematical expressions. An energy balance for an element of fuel ahead of the fire relates the enthalpy of the fuel as a function of time and position to the energy input from the flame, and the energy losses to the environment. The result is a differential equation model for the enthalpy of the fuel, which needs to be supplemented with an enthalpy–temperature relationship. The fire spread rate is then found by solving the differential equation, subject to appropriate boundary and/or initial conditions.

As an example, consider the equation

$$\rho c \frac{\partial \tau}{\partial t} = D \frac{\partial^2 \tau}{\partial x^2} + Af(x, R(t)) - \mathrm{H}\tau \quad (10)$$

where the co-ordinate system is shown schematically in Fig. 1.

The left hand side represents the rate of rise of temperature per unit volume of the fuel ahead of the fire front, $\tau(x, t)$ being the temperature of the fuel, at position x and time t, above ambient. It has been assumed that fuel can be described with a constant value of the product ρc, albeit an average.

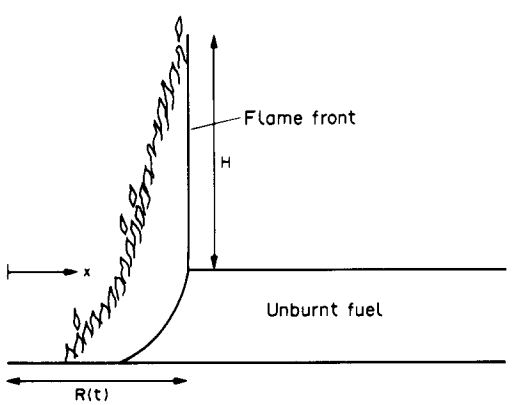

FIG. 1. Schematic representation of co-ordinate system for a fire front at position $R(t)$.

The first term on the right hand side of Eq. (10) represents conductive or diffusive transfer of heat through the fuel bed, with diffusivity D. This could be used to model the effect of eddies on heat transfer with D being a so-called 'eddy diffusivity'.

The second term on the right hand side of Eq. (10) represents the radiative heat input to a fuel element at position x, the fire front is at a position $R(t)$, t being the time after ignition. We shall assume that the only time dependence is through the position of the fire front—this excludes variable environmental conditions and means we are looking at *intrinsic* properties of fire propagation. The constant A is composed of the absorption coefficient for radiation of the fuel (α) and the intensity of radiation from the source (either the flame or the combustion zone). Examples for the 'view factor' $V(x, r(t))$ will be given in subsequent sections; for the moment we need merely appreciate that *it is dimensionless*, it depends on the geometry of the flame and fuel bed and that it represents a fraction of the energy output from combustion being transferred to the fuel.

The third term on the right hand side of Eq. (10) is a Newtonian cooling term. It includes free convective cooling due to the fuel being hotter than the ambient air, with the constant H being the surface area to volume ratio of the fuel bed (s) times the convective cooling coefficient (h). It can also be used to approximately account for radiative cooling of the fuel if we make the expansion

$$(T^4 - T_a^4) = (T^2 + T_a^2)(T + T_a)\tau \quad (11)$$

and include an additional contribution $\varepsilon\sigma(T^2 + T_a^2)(T + T_a)$, taking some average value for the temperature T over the range of interest (e.g. $\frac{1}{2}(T_i + T_a)$).

Equation (10) is a linear, second order, partial differential equation in two variables x, t and with two unknown functions, $\tau(x, t)$ and $R(t)$. Our objective is to solve this equation in order to determine the fire spread rate, that is, $\dot{R}(t)$, a dot denoting differentiation with respect to time. We also have boundary conditions on how our temperature profile should behave with position, and an initial condition for the position of the fire:

$$\tau(x = R(t), t) = \tau_i \quad (12)$$

$$\tau(x \to \infty, t) = 0 \quad (13)$$

$$R(0) = 0. \quad (14)$$

The first of these is a statement of the common assumption that when the temperature of the element has reached ignition, the flame has advanced to that element. A more sophisticated model might replace

Eq. (12) with the two conditions

$$\tau(x = R^+(t), t) = \tau_i,$$

$$\tau(x = R^-(t), t) = \tau_f;$$

considering the ignition interface to be a temperature discontinuity separating burning and non-burning materials. This requires a model for the energy production from combustion which is beyond the scope of the present example. We shall be content to assume that sufficient energy is liberated to change fuel at ignition temperature into burning material with a flame.

To obtain a solution one could formally integrate Eq. (10) over time; however, since $R(t)$ is unknown, this appears to be of little value. Rather, we shall use the notion that a flame is a travelling combustion wave, which seems to be well motivated physically (Berlad *et al.*[31]) and accords with condition (12). That is, suppose $\tau(x, t) = \tau(x - R(t))$. Then, one seeks solutions which obey

$$\frac{\partial \tau}{\partial t} + \dot{R}\frac{\partial x}{\partial x} = 0 \tag{15}$$

in addition to Eq. (10). Mathematically, we are interpreting the problem as wave propagation whence the value of τ_i is attained on a characteristic curve whose equation is $x = R(t)$. Substituting Eq. (15) into Eq. (10) leads one to reformulate the problem as follows:

$$\text{Solve } D\frac{\partial^2 \tau}{\partial x^2} + \rho c\dot{R}\frac{\partial \tau}{\partial x} - H\tau + Af(x, R(t)) = 0 \tag{16}$$

subject to conditions (12–14), for the temperature rise above ambient, $\tau(x - R(t))$.

Notice that this version of the problem has no explicit time dependence; time only enters through $R(t)$. Therefore, as far as the dependence on x is concerned, one can treat Eq. (16) as an ordinary differential equation of second order. Moreover, the homogeneous part has 'constant' coefficients (constant in x). The associated characteristic equation, found by substituting $e^{\lambda x}$ in the homogeneous part of Eq. (16) is

$$D\lambda^2 + \rho c\dot{R}\lambda - H = 0, \tag{17}$$

which has the roots (λ_1 positive, λ_2 negative)

$$\lambda_1 = -\frac{\rho c\dot{R}}{2D} + \frac{\sqrt{(\rho c\dot{R})^2 + 4DH}}{2D},$$

$$\lambda_2 = -\frac{\rho c\dot{R}}{2D} - \frac{\sqrt{(\rho c\dot{R})^2 + 4DH}}{2D}. \tag{18}$$

The solution to the homogeneous part of Eq. (16) is

$$c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x} \tag{19}$$

where c_1 and c_2 are arbitrary constants of integration. This can be verified by calculating the Wronskian:

$$W(e^{\lambda_1 x}, e^{\lambda_2 x}) = \begin{vmatrix} e^{\lambda_1 x} & e^{\lambda_2 x} \\ \lambda_1 e^{\lambda_1 x} & \lambda_2 e^{\lambda_2 x} \end{vmatrix} = (\lambda_2 - \lambda_1)e^{(\lambda_2 + \lambda_1)x}, \tag{20}$$

which is non-vanishing for all x. The general solution to Eq. (16) can then be found using the method of variation of parameters. Choosing the lower limit of integration to be $R(t)$ for definiteness, one finds

$$\tau(x - R(t)) = e^{\lambda_1 x}\left[c_1 + \frac{A}{(\lambda_2 - \lambda_1)} \times \int_{R(t)}^{x} e^{-\lambda_1 \varepsilon} f(\varepsilon, R(t))\, d\varepsilon\right] + e^{\lambda_2 x}\left[c_2 - \frac{A}{(\lambda_2 - \lambda_1)} \times \int_{R(t)}^{x} e^{-\lambda_2 \varepsilon} f(\varepsilon, R(t))\, d\varepsilon\right]. \tag{21}$$

The boundary conditions, Eqs (12 and 13), can be satisfied by requiring

$$c_1 + \frac{A}{(\lambda_2 - \lambda_1)} \int_{R(t)}^{\infty} e^{-\lambda_1 \varepsilon} f(\varepsilon, R(t))\, d\varepsilon = 0 \tag{22}$$

to satisfy $\tau(x - R(t))_{x \to \infty} = 0$; and

$$\tau_i = -e^{\lambda_1 R(t)}\left[\frac{A}{(\lambda_2 - \lambda_1)} \int_{R(t)}^{\infty} e^{-\lambda_1 \varepsilon} f(\varepsilon, R(t))\, d\varepsilon\right] + c_2 e^{\lambda_2 R(t)} \tag{23}$$

to satisfy $\tau(0) = \tau_i$.

This yields the complete solution for the temperature profile as

$$\tau(x - R(t)) = -\frac{Ae^{\lambda_1 x}}{(\lambda_2 - \lambda_1)} \int_{x}^{\infty} e^{-\lambda_1 \varepsilon} f(\varepsilon, R(t))\, d\varepsilon + e^{\lambda_2 x}\left[\tau_i e^{-\lambda_2 R(t)} + \frac{Ae^{-(\lambda_2 - \lambda_1)R(t)}}{(\lambda_2 - \lambda_1)} \times \int_{R(t)}^{\infty} e^{-\lambda_1 \varepsilon} f(\varepsilon, R(t))\, d\varepsilon - \frac{A}{(\lambda_2 - \lambda_1)} \times \int_{R(t)}^{x} e^{-\lambda_2 \varepsilon} f(\varepsilon, R(t))\, d\varepsilon\right]. \tag{24}$$

This is a somewhat complicated expression, and though it is of interest, we still have not found our primary objective, the fire spread rate. Indeed, we cannot find the fire spread rate until we place a further constraint upon the temperature profile; Eq. (24). There have been various suggestions in the literature as to the physically correct condition to impose.

Itoh and Kurosaki[32] have argued that for a flame spreading above a solid fuel surface, one should use

$$\left.\frac{d\tau}{dx}\right|_{x = R(t)} = 0 \tag{25}$$

However, measurements of fuel temperature do not indicate a flat temperature profile when ignition occurs, so this is not very well motivated on physical grounds.

Weber[25] has noted that, physically, the point in the temperature profile at which ignition occurs is an inflexion point and suggested using

$$\left.\frac{d^2\tau}{dx^2}\right|_{x=R(t)} = 0 \qquad (26)$$

This was found to be a useful model for dust flames and it should be appropriate for a spreading fire in a fuel bed, as the fuel bed is usually composed of thermally thin fuel elements. Equation (26) applied to the differential equation, Eq. (16), yields

$$\rho c\dot{R}\left.\frac{\partial\tau}{\partial x}\right|_{x=R(t)} = H\tau(0) - Af(R(t), R(t)) \quad (27)$$

This is somewhat similar in form to a Stefan condition (see, for example, Fujii *et al.*[33] and Cekirge[34]) or to the condition derived by Pagni[35] for a thin solid fuel. The principal advantages of Eq. (26) are that it reflects what is thought to occur at ignition, and that it is easy to implement. (A further noteworthy feature is that it is allied to asymptotic methods used in laminar flame theory (e.g. Williams[16]) as it represents the matching condition one might use if one didn't know the reaction chemistry!)

Using Eq. (24) for $\tau(x - R(t))$ in Eq. (26) yields the equation

$$\lambda_2^2\tau_i - Af(R(t), R(t)) + A(\lambda_2 + \lambda_1)e^{\lambda_1 R(t)}\int_{R(t)}^{\infty} e^{-\lambda_1\varepsilon}f(\varepsilon, R(t))\,d\varepsilon = 0. \quad (28)$$

Recalling that λ_1 and λ_2 depend on $\dot{R}(t)$ (Eq. (18)), one notices that Eq. (28) is a first order integro-differential equation for $R(t)$ which needs to be solved subject to the condition in Eq. (14); namely $R(0) = 0$. Clearly this is a well posed problem as the integral in Eq. (28) is convergent ($\lambda_1 > 0$) and one expects a unique solution for $R(t)$ and therefore a unique solution for the fire spread rate $\dot{R}(t)$. To proceed one requires further information about the radiation function, $f(x, R(t))$.

A particularly interesting and relevant special case is when the radiation depends only upon the separation of the source at $R(t)$ and the element at x:

$$f(x, R(t)) = f(x - R(t)). \qquad (29)$$

An example of this is a spreading line fire, which happens to be the most frequently modelled case due to the simplified geometry. For this case, the transformation

$$\varepsilon \to \varepsilon - R(t)$$

in (the integral in) Eq. (28) results in the simplified *algebraic* equation for $\dot{R}(t)$:

$$\lambda_2^2\tau_i - Af(0) + A(\lambda_2 + \lambda_1)\int_0^{\infty} e^{-\lambda_1\varepsilon}f(\varepsilon)\,d\varepsilon = 0. \qquad (30)$$

That is, Eq. (30) is solved for $\dot{R}$, which turns out to be a constant. In that case $R(t) = \dot{R}.t$; the fire moves at a constant rate. In all the work reviewed this will be the case, with one exception. The exception is an accelerating circular fire model given by Weber.[15] There the radiation function cannot be written as in Eq. (29) and a time varying fire spread rate must be found from the integro-differential equation, Eq. (28). It needs to be emphasized that this is what one would usually expect, and that constant spread rates are a useful laboratory idealization which most researchers have employed.

The derivation of Eq. (30) assumed the presence of a diffusive heat transfer process; that is, $D \neq 0$ in Eq. (10). However, many researchers have ignored such a process (see the next section) arguing that convection is much more important than conduction for a fuel bed and that radiation can be adequately described with the function $f(x, R(t))$. It is of interest to compare the prediction of this assumption with the results in Eqs (28 and 30). Note that we cannot simply let $D \to 0$ in the final result as we are dealing with a singular perturbation problem. Rather, one should set $D = 0$ in Eq. (10) and repeat the (simpler) calculation. The analogue of Eq. (28), for $R(t)$ is found to be

$$\dot{R}\rho c\,\tau_i = A\int_{R(t)}^{\infty} e^{-H(\varepsilon - R(t))/\dot{R}\rho c}f(\varepsilon, R(t))\,d\varepsilon \quad (31)$$

and the result when f is as in Eq. (29), giving an algebraic equation for $\dot{R}$, is

$$\dot{R}\rho c\,\tau_i = A\int_0^{\infty} e^{-H\varepsilon/\dot{R}\rho c}f(\varepsilon)\,d\varepsilon. \qquad (32)$$

One should note that Eqs (31 and 32) are more typical representatives of the models found in the next section, and that they are derived without requiring the point of ignition in the temperature profile to be an inflection point. This is because the two conditions given in Eqs (12 and 13), namely $\tau(x = R(t), t) = \tau_i$ and $\tau(x \to \infty, t) = 0$, are sufficient when one is only integrating a first order differential equation. The presence of a Laplace Transform in these equations is also noteworthy. This occurs because of the presence of cooling ($H \neq 0$) and is necessary for convergence.

4. PHYSICAL MODELS

In this section, the main models are reviewed in chronological order. Key concepts and equations are discussed and a critical evaluation is given.

4.1. *Basic Physical Models*

4.1.1. *Fons*[1]

Fons[1] considered an idealized bed, composed of equal distances, s, from each other. The time for the n^{th} particle to ignite after the $(n - 1)^{th}$ particle has

ignited is denoted t_i. The fire spread rate is then

$$R = \frac{s}{t_i}$$

and the main objective of the paper is to calculate the time to ignition t_i, allowing for conduction, convection and (approximately) radiation transfer of energy. The total energy absorbed per unit volume in unit time is written as

$$q = q_c + q_r = \sigma(f_c + f_r)(T_f - T).$$

The parameters f_c and f_r are estimated film conductance for forced convection and radiation, respectively. The time to ignition is then computed by solving the first order differential equation

$$\rho c_p \frac{dT}{dt} = q$$

for $T(t)$ and find the time at which $T = t_i$. The final equation is

$$R = \frac{(f_c + f_r)\sigma s}{\rho c_p \ln\left(\dfrac{T_f - T_\infty}{T_f - T_i}\right)} \tag{33}$$

and one should note that average values for ρ and c_p applicable to moist wood are used. T_∞ is the initial temperature of the fuel. T_1, which Fons takes to be the arithmetic mean of T_i and ambient temperature T_∞, is used in the calculation of f_r (and f_c) and Fons acknowledges that this is only an approximation used in lieu of the correct quartic dependence. An ignition temperature of 550°F/288°C is recommended for all light forest fuels and used throughout by Fons.[1]

s, the spacing between discrete elements, is related to measurable fuel, and fuel-bed variables using geometrical arguments. Fons[1] derives a relationship between s and λ (the ratio of voids to fuel-surface area):

$$\sigma s = E(1 + \sigma\lambda)^{1/2}$$

where E is independent of the size of the fuel particle and depends only upon the shape of the cross section. $E = 2\pi^{1/2}$ for round sticks, $E = 4.0$ for square sticks, and for sticks with more irregular cross sections one calculates

$$E = (\sigma p)^{1/2} > 4.0$$

where p is the perimeter.

In principle Eq. (33) could now be applied to a fire in a regular fuel bed. However, Fons'[1] interest is in natural homogeneous beds which are not as regular as the idealized bed. Invoking the theory of similitude one can show that there are only two independent proportionality constants which generalize Eq. (33); C_1 and C_2 such that

$$R = \frac{C_1(f_c + f_r)\sigma s}{\rho c_p \ln\left[C_2\left(\dfrac{T_f - T_\infty}{T_f - T_i}\right)\right]}. \tag{34}$$

C_1 and C_2 are then evaluated experimentally, using beds of ponderosa pine needles, and also beds of ponderosa pine twigs. It was found that C_2 varied with wind speed from 0.9 in still air to 0.7 at a wind speed of 12.5 miles hr^{-1}/20 km hr^{-1}/5.6 msec^{-1}, and that C_1 varied from 1.12 to 2.04. The theory was able to account reasonably well for the spread rates measured. However, in view of the freedom introduced with the constants C_1 and C_2, this is not surprising. Unfortunately, the most novel aspect of Fons[1] model, the logarithmic dependence on temperature, is not amenable to conclusive testing as flame and ignition temperatures are neither sensitive nor robust experimental quantities.

Equations for heterogeneous fuel beds were also derived; in much the same way as resistors in parallel are treated.

4.1.2. *Emmons*[5]

Emmons[5] attempted a fire spread model in which the fire produces an energy flux $q(x)$ horizontally into unburned fuel, and in which radiation heat transfer through the fuel bed is responsible for raising the temperature of unburned fuel to ignition. Emmons[5] appears to be the first researcher to use an exponential to characterize attenuation of radiation by a fuel bed; i.e.

$$q(x) = q_0 e^{-\alpha x}. \tag{35}$$

By integrating over the burning zone (from 0 to d) in a reference frame moving with the steadily propagating fire, a non-linear equation is obtained for the fire spread rate, R:

$$R = \frac{q_0}{\alpha\rho c_p(T_i - T_\infty)}(1 - e^{-\alpha\rho R/W}), \tag{36}$$

where W is the average burning rate per unit volume of fuel. In Eq. (36) the relationship $\rho R = Wd$ has been used. This is obtained by noticing that the characteristic time required to burn an element of fuel is

$$t = \frac{d}{R} = \frac{\rho}{W}.$$

Interestingly, the non-linear equation, Eq. (36), only has a solution when $q_0 > c_p(T_i - T_\infty)W$, and the solution for R increases as q_0 increases. For high values of $\alpha\rho R/W$, the solution to Eq. (36) is

$$R = \frac{q_0}{\alpha\rho c_p(T_i - T_\infty)} \tag{37}$$

and the fire moves at a speed which is independent of the average burning rate W.

Emmons[5] has attempted to apply the solution of his formulation to practical fire problems, such as the size of a fire break:

$$\frac{\text{width}}{\text{height}} \geqslant \alpha\rho R/W, \tag{38}$$

and fire build-up (in one dimension) where a criterion for the front to advance, based upon the size/width of the ignition region x_i, is developed:

$$\alpha x_i \geqslant -\ln\left(1 - \frac{\alpha\rho R}{W}\right). \tag{39}$$

Equations (38 and 39) are examples of the utility of mathematical models of fire spread. Criteria can be found which may provide practical advice to fire fighting agencies. Unfortunately, neither Emmons[5] nor any other researcher has followed up these criteria experimentally in order to test their applicability to real fire situations.

4.1.3. *Hottel, Williams and Steward*[8]

Four mathematical models were proposed by Hottel *et al.*[8] and tested by laboratory experiments with torn newsprint and computer cards. All the models have the same heating terms; namely, radiation from the flame of height H, from an external planar source of height H_w, and from the embers, and local turbulent convection from the flame. These were modelled with

$$q_f = \tfrac{1}{2}\alpha\varepsilon_f\sigma T_f^4\{1 - [1 + (H/x)^2]^{-1/2}\} \tag{40a}$$

$$q_s = (\alpha Q/2\pi H_w)[1 + (x/H_w)^2]^{-1} \tag{40b}$$

$$q_e = \alpha\varepsilon_e\sigma T_e^4 \exp(-\alpha|x|) \tag{40c}$$

$$q_c = \frac{k_c c_{pg}\rho_g D}{l}(T_f - T)\exp(-\tfrac{1}{4}\pi(x/l)^2) \tag{40d}$$

respectively, where the many constants are as defined in Hottel *et al.*[8] The view factors in Eqs (40a and 40b) were derived by the traditional method used for radiative exchange between solid, opaque surfaces. As discussed by de Mestre *et al.*,[3] this may require modification for semi-transparent fuel beds, composed of pine needles or other natural fuels (see also the later section on de Mestre *et al.*[3]). The ember radiation term, Eq. (40c) is essentially the same as Emmons[5] used. Inclusion of convection from the flame in Eq. (40d) is likely to be needed because of small scale turbulence (over length scales l) of the flame gas. Modelling it with an eddy diffusivity D is a handy first approximation; however, it highlights the danger of having parameters in the model which are difficult to estimate accurately. Hottel *et al.*[8] find a first integral of q_c by making the assumption

$$T_f - T \cong T_f - T_i,$$

that is, replacing the variable $T_f - T$ by an upper bound $T_f - T_i$! The first integral is then easily computed

$$\int_0^\infty q_c dx \cong K_c c_{pg}\rho_g D(T_f - T_i),$$

but one is left to wonder upon the accuracy of the result.

The four models studied by Hottel *et al.*[8] differ in the mechanisms included for cooling (linear or non-linear) and the idealization of the fuel bed (thin-slab or simi-infinite). As one might expect, the linear cooling models can be integrated analytically and an equation for the fire spread rate is obtained, albeit rather cumbersome. The interesting feature is that cooling appears as a dividing factor, rather than as a subtraction. The non-linear cooling models require numerical integration in order to determine the fire spread rate.

The primary reason for this elaborate modelling by Hottel *et al.*[8] is the description of their experimental results. In particular, the goal they set, but are unable to achieve, within the framework of any of the models, is to explain the decreasing effect of additional radiation from the external source on the fire spread rate. That is, as more and more external radiation was applied, Hottel *et al.*[8] observed that the effect of the additional increments on fire spread rate is not as dramatic. This suggests that there is a maximum possible fire spread rate; at least for these experiments in still air.

In this paper (Hottel *et al.*[8]) and a sequel (Hottel *et al.*[36]) various explanations were considered. These included the possibility that the net resultant heat gained by convection is negative due to induced cool air flowing inwards to replace the rising hot gases. However, this would seem to depend upon the length scale under consideration as very near to the fire the expansion of hot combustion products must produce an outflow. An alternative suggestion is that the reasonably complex kinetics which occurs, cannot be modelled with a single ignition temperature or flame convection parameter (or for that matter flame temperature). In view of the mystery in which the ignition temperature concept remains shrouded, this suggestion should be taken seriously. Hence, models for fuel bed combustion should be developed in which some model kinetic scheme is used (similar to the work of Joulin[37] and others for particle-laden gas flames).

In the sequel (Hottel *et al.*[36]) it is also noted that a brush fuel bed has a relatively high void volume. This means that heat transfer mechanisms operating solely within the fuel bed, such as ember radiation and eddy convection, become more important.

4.1.4. *Albini*[2]

Albini[2] presents an order of magnitude analysis of fire spread through brush, with radiative heating from the flame as the dominant heat transfer mechanism. The radiant flux incident upon the upper surface of the brush is taken to be Eq. (40a); i.e. the same as Hottel *et al.*[8] This is related to the nett rate of volatile production, per unit of upper surface area, by the equation

$$\dot{m}(x) = q_f/\alpha L \equiv \dot{m}_0\tfrac{1}{2}\{1 - [1 + (H/x)^2]^{-1/2}\}, \tag{41}$$

where $\dot{m}_0 \equiv \varepsilon_f \sigma T_f^4 / L$ is a characteristic volatile flux, and L is the mean latent heat of vaporisation.

The effect of wind is included via a 'flame tilt angle', calculated from the ratio of bouyant force to wind force:

$$\text{flame tilt angle} \cong \left(\frac{4W^2}{gd}\right)^{1/4}. \tag{42}$$

Equations (41 and 42), together with other equations for fuel moisture and relating variables, are solved numerically to obtain the dimensionless spread rate, flame length and burning zone.

Unfortunately, there is no comparison with experimental data. However, the model is of interest in that it approximately accounts for a range of variables that must all be included in a comprehensive model. It is also an interesting prelude to Albini[38,39] where the full integral equations for radiative transport are coupled to the enthalpy of the fuel—see the later discussion in Section 4.2.

4.1.5. *Thomas*[12]

Thomas[12] is in reality a review paper of experimental and theoretical work at the Fire Research Station, Borehamwood, U.K. The main aim of this work was to correlate the size of flames and the rates of burning; including the effect of wind. In this sense it is similar to Albini,[2] although more comprehensive, particularly in its comparison with experimental data in the laboratory and the field.

Fire spread in a fuel bed is then considered. The description of the fuel bed is identical to Fons,[1] leading to the observation that spread rate varies inversely with bulk density of the fuel bed. This was also observed by Bruce *et al.*[40] Further calculations showed that the heat transferred to the fuel bed by radiation from flames above the fuel bed was much less than heat transferred through the fuel bed. This latter phenomena was attributed to radiation through the fuel bed (although convection should not be dismissed *even* for fire spread in still air). McCarter and Broido[41] experimentally corroborated the view that fire spread through cribs in still air is largely independent of radiation transfer from the flames above the crib. Clearly this must depend upon the porosity of the crib, as in the limit of a solid slab this is no longer correct. However, for forest fuel beds the results of Thomas[12] are of significance to modellers.

Spread in a wind was also considered by Thomas.[12,42] Comparison with data in heather and gorse suggested a model where the fire spread rate is calculated from

$$\rho_b R = 0.07(1 + W) \tag{43a}$$

for wildland fuels, and

$$\rho_b R = 0.05(1 + W) \tag{43b}$$

for cribs. In a manner similar to Emmons,[5] the heat balance for fire spread in a wind is constructed. Thomas[12,42] ends up needing to solve the non-linear equation

$$Ax^{-1} + Bx^{-1/3}(1 - e^{-x}) = 1 \tag{44}$$

where x is related directly to the fire spread rate R, and where the constants A, B are related to the heat transfer and heat sink properties of the fire and fuel bed. However, Eq. (44) can have one or two solutions, depending upon the values for A and B. These are dubbed 'slow' and 'fast'. The 'slow' spreading is found to occur only if the flame radiation is less than $\frac{3}{5}$ of the total heat transfer. The 'fast' spreading is found to overpredict any experimental observations by an order of magnitude or more. There is speculation about the significance of this 'fast' mode to extreme fire events, but it seems that one should not make too much of this prediction.

Thomas[42] also notes that the drag of a fuel bed causes the wind speed in the fuel bed, V, to be less than W and suggests

$$\frac{V}{W} \cong \frac{1}{3} \tag{45}$$

which is of interest for modelling convective heat transfer.

4.1.6. *Van Wagner*[43]

A surface heating model, similar in spirit to Albini,[2] was proposed by van Wagner.[43] The only heat transfer mechanism was assumed to be flame radiation to the top surface of a thermally thin fuel bed. An extension to include combustion-zone radiation was suggested, but not implemented. The same view factor used by Hottel *et al.*[8] and Albini[2] was employed to write an energy balance for moist fuel. Water was modelled as 'frozen moisture'. That is, it was assumed that water is driven off in a two stage process. Firstly, all the water is heated to boiling point, and secondly, all the water is vapourized at its boiling point. This gives a good estimate of the overall heat required to dry fuel, although the discontinuity in the temperature that this implies is not borne out by experiments (de Mestre *et al.*[3] and Fig. 2).

Van Wagner[43] carried out a laboratory experiment to test the effect of flame angle on fire spread rate. However, it is not clear if the correlation observed is due to a causal relationship between flame angle and spread rate, or is merely a reflection of the magnitude of the (perpendicular) forces due to buoyancy and wind.

4.1.7. *Anderson*[44]

Anderson[44] attempted to construct a radiative heating model based upon the view factor

$$f(x) = e^{-ax}, \tag{46}$$

and including moisture in a more comprehensive way

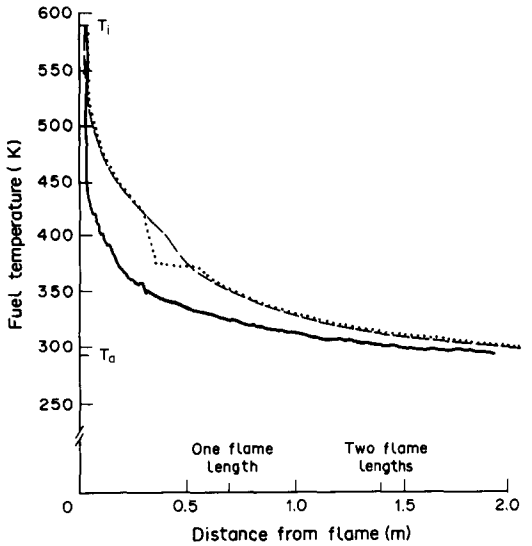

FIG. 2. Fuel surface temperature ahead of the fire front. Experimental (solid line), 'frozen' moisture model (dotted line), and averaged moisture model (dashed line) are shown. From de Mestre *et al.*[3]

than did previous workers. That is, heat of desorption and the variation in boiling temperature with moisture content were included in calculating the energy required to ignite fuel per unit mass.

Other novel features are a distinction between the temperatures of the flame and the combustion zone; use of a Reynolds number to calculate convective heat transfer; and use of the concept of porosity of a fuel bed, similar to that of Fons[1] and Thomas.[12]

Laboratory experiments were conducted to determine the fraction of total heat transfer due to radiation. However, since these were interpreted in terms of the view factor in Eq. (46), which is really only appropriate for radiation transport through the fuel bed, one cannot be completely confident of Anderson's[44] conclusion that radiant heat transfer accounted for only 40% of the total heat transfer. Nonetheless, Anderson's[44] study does demonstrate that radiative transport is not necessarily the sole heat transfer mechanism, even for fire spread in still air. It also demonstrates how devilishly clever one has to be in order to accurately measure the heat flux due to different mechanisms, but which act simultaneously.

4.1.8. *Pagni and Peterson*[9]

A very comprehensive analysis of possible heating mechanisms was performed by Pagni and Peterson[9] for a thermally thin, porous layer of fuel. Heat transfer mechanisms included were radiation from the flame, Eq. (40a), and embers, Eq. (40c), turbulent diffusion of flame eddies near the flame, Eq. (40d), wind driven convection through the fuel and onto the top surface of the fuel, based upon the local Reynolds numbers Re_c in the fuel bed and Re above the fuel bed,

$$q_{ci} = 2.1\lambda_g l_c N_c Re_c^{0.3}(T_f - T)(1 - 5vl_c N_c x/W)^{\beta} \quad (47a)$$

$$q_{cs} = 0.037 c_{pg}\rho_g W Re^{-0.2} Pr^{-0.67} l_f^{-1}(T_f - T) \times \exp(-0.185 Pr^{-0.67} x/L) \quad (47b)$$

where

$$\beta = 0.42 Re_c^{0.33} Pr^{-1},$$

and conduction in the gas-phase in the immediate vicinity of the flame;

$$q_{cg} = \delta(0)\lambda_g(T_f - T_i)/\Delta \quad (48)$$

where $\delta(0)$ is a delta function at the origin and the 'stand-off' distance of the flame from the fuel, Δ, is obtained by approximating the flow of volatiles from the fuel to the flame by pure diffusion;

$$\Delta = D_v\rho_v/\dot{m}_e. \quad (49)$$

It is further assumed that the mass flux of volatiles from the end of the fuel bed, $\dot{m}_e$, can be described by a single Arrhenius decomposition reaction as

$$\dot{m}_e \cong \rho\Delta A \exp(-E/RT_i). \quad (50)$$

Fuel moisture and the enthalpy of pyrolysis (assumed endothermic) were included in the total energy balance. A single integration over the space variable x, together with suitable boundary conditions yielded a predictive equation for the flame spread rate—the mathematics is identical to that already given in the lead up to Eq. (32). However, no cooling mechanism of any kind was included by Pagni and Peterson.[9]

The analysis was tested by comparison with the laboratory data of Rothermel and Anderson[45] as all the parameters necessary had been measured (there is some debate on this, see Catchpole and de Mestre[19]). The agreement between predicted and observed spread rates is strikingly good. However, no fuel temperature vs position from the flame is produced; rather it is assumed that fuel temperature decreases linearly with position from T_i at the flame front to T_∞ one flame length away!

The conclusions of Pagni and Peterson[9] were

(1) energy absorbed in the pyrolysis is negligible,
(2) heating by turbulent diffusion of flame eddies is negligible,
(3) for low wind speeds, the dominant heating mechanism is flame radiation; with contributions from ember radiation and gas phase conduction,
(4) for higher wind speeds, convection is the dominant heating mechanism; with contributions from flame radiation.

Further work to obtain flame characteristics analytically was recommended so that the model would require less inputs.

4.1.9. *Telisin*[11]

Like the van Wagner[43] model, Telisin[11] has constructed a radiation heating only model. However, Telisin[11] has included ember radiation in addition to flame radiation. Furthermore, values for the emissivity of the flame and separately for the embers were calculated based upon the depth of the flame in the fuel bed, d:

$$\varepsilon_f = 1 - e^{-\alpha d} \tag{51a}$$

$$\varepsilon_e = 1 - e^{-\alpha(d+d_c)}. \tag{51b}$$

d_c is a correction factor to account for the effect of the glowing part of the solid fuel (which is in addition to the flaming part of the solid fuel bed).

A large data set (including published and unpublished data) was used to test the spread rate predictions of Telisin's[11] radiation only model. In particular, the effect of wind was included by allowing the radiation transfer between the flame and the fuel to depend upon the angle of the flame, which in turn is governed by the windspeed. The length of the burning zone, d, was predicted by multiplying the residence time and the spread rate. Both the spread rate and the burning zone length predictions were in satisfactory agreement with experimental observations; but one should note that some of Telisin's[11] parameters were estimated from the experimental data.

4.1.10. *Konev and Sukhinin*[46]

Konev and Sukhinin[46] have taken a discrete modelling approach, considering local flames which are formed around individual particles at the leading edge of the fire spread process. The local flames merge into the total flame above the fuel bed. Both radiation and convection transport of heat to unburnt fuel were included. It was found, in experiments with *Pinus silvestris* fallen needles, that the contribution of radiation ranged from around 8% near the flame spread limit, to 37% for heavily packed fuel beds. These numbers are estimates calculated solely within the theoretical framework.

4.1.11. *Cekirge*[34]

Cekirge[34] proposed a model similar to Pagni and Peterson[9] in that convection through the fuel bed was allowed, in addition to radiation through the fuel bed and from the flame above. The local energy balance used leads to a first-order partial differential equation for $T(x, t)$ which in principle could be solved to obtain the non-steady spread rate of a fire as it develops toward a steady state. Two different view factors are considered—one for the usual planar fire front, and another for a circular fire front. The latter is of interest in the early stages of a fire which has been ignited at a point, and the view factor is

$$f(x, R) = \frac{1}{\pi}\{\tan^{-1}((x + R)/(x - R)) - [(H^2 + x^2 - R^2)/(H^2 + x^2 + R^2)] \times \tan^{-1}((x - R)/(x + R))\} \tag{52}$$

A finite difference technique is used to numerically solve the differential equation—this is necessary because Cekirge[34] includes radiative cooling according to its T^4 law. The model, including convection, appears to provide better agreement than Telisin's[11] with the same data set. However, since it is based upon a wind parameter which is related to W by an arbitrary coefficient (less than 1.0) the agreement must be viewed as instructive rather than conclusive.

4.1.12. *De Mestre, Catchpole, Anderson and Rothermel*[3]

de Mestre *et al.*[3] propose a physical model for fire spread in the absence of wind, which they proceed to carefully compare with one detailed fire spread experiment in *Pinus Ponderosa* fallen needles. Heat transfer from the combustion region to unburnt fuel is assumed to occur only by radiation from the flames and embers. However, by integrating the energy transfer over space (and hence time, since quasi-steady spread is assumed), de Mestre *et al.*[3] find that there appears to be far too much energy transfer than expected based upon the observed spread rate. It is concluded that significant convective and radiative cooling of the fuel must occur; so this is included in the model. While this assists in obtaining the correct spread rate, the comparison of observed and calculated temperature ahead of the fire (see Fig. 2) indicates that heating occurs due to a short range mechanism, rather than a long range mechanism such as radiation.

An interesting secondary issue raised by de Mestre *et al.*[3] concerns the view factors used to calculate the contribution of flame radiation. As is correctly pointed out, the view factors previously employed, e.g. Eqs (40a, 40b and 52), are for radiation transfer between two opaque, solid surfaces. On the other hand a pine needle fuel bed is semi-transparent which necessitates the definition of a view factor to be changed to

$$f(x, R) = \int\int_S \frac{\cos\varphi \, dA}{\pi(x^2 - R^2)} \tag{53}$$

where the integral over S is over the flame surface. The difference between the usual definition and Eq. (53) is the absence of a second cosine factor due to penetration of the radiation into the semi-transparent fuel bed. The view factors obtained with this new definition have a different functional form (see de Mestre *et al.*[3] and Weber[25]), but once used in a conservation of energy equation and integrated out, there

are only minor variations. This is as it should be, since both definitions yield normalized transfer functions and the areas underneath the graphs of these functions are similar.

4.1.13. *Weber*[15,25]

Given the approximate nature of the physical modelling which has been performed, Weber[15,25] argues that analytical solutions are more valuable results than numerical solutions, as only analytical results provide relationships that can easily be tested in a wide range of circumstances. With this point of view, Weber[15,25] considers a simple, linear equation from a local energy balance and substitutes several different view factors to model particular situations governed by radiative heating.

The two main situations modelled are the spreading of a planar fire front, and the spreading of a circular fire front. The first of these has been considered by many authors, except that Weber[15,25] uses the view factor recommended by de Mestre *et al.*[15,25] The second is a model for fire build-up, and Weber[15,25] manages to provide an estimate of the lag-time for a circular fire to reach a steady spread rate.

The view factor considered is

$$f(x, R(t)) = \exp\left[-\alpha(x^2 - R^2)/x\right] \qquad (54)$$

which is simpler than that of Cekirge.[34] Substituting this into Eq. (32), one must take a Laplace transform in order to calculate the spread rate, $\dot{R}(t)$. It was not possible to do this analytically for Eq. (54), but it was possible to do for upper and lower bounds

$$\exp\left[-\alpha(x^2 - R^2)/R\right] \leqslant \exp\left[-\alpha(x^2 - R^2)/x\right] \times \leqslant \exp\left[-2\alpha(x - R)\right]. \qquad (55)$$

The upper bound yields a constant spread rate, which turns out to be the steady spread rate attained by the circular fire as it grows in time.

The lower bound yields a non-linear differential equation for $\dot{R}(t)$ as a function of $R(t)$ which must be solved subject to the condition in Eq. (14); $R(0) = 0$. The result describes a circle with large initial acceleration but zero initial velocity, and which approaches the steady rate of spread obtained in the upper bound, see Fig. 3. The acceleration has reduced to 5% of its initial value after a lag time

$$t \simeq \frac{6\rho c_p(T_i - T_\infty)}{\alpha I_r}. \qquad (56)$$

Weber[15] conjectured that the curvature of the circular fire front is crucial to this behaviour. Initially, the curvature is very large, but as the fire grows, the curvature of the fire front diminishes and the spread rate approaches a steady-state value from below.

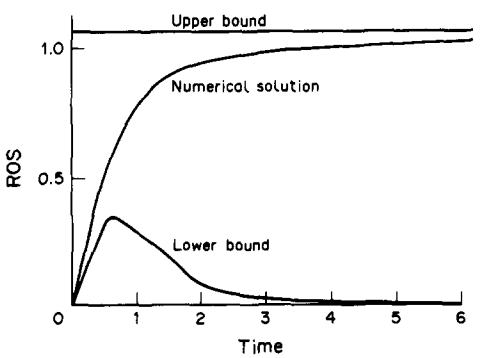

FIG. 3. Numerical solution and upper and lower bounds for the fire spread rate $\dot{R}(t)$ for fire build-up from a point ignition. Normalized units. From Weber.[15]

4.2. *Refinements*

4.2.1. *Fujii, Hasegawa, Pallop and Sakawa*[33]

Fujii *et al.*[33] consider a thermally thin, porous fuel bed and formulate the fire spreading process as a moving boundary problem (also known as Stefan problem). The moving boundary is, of course, the fire front, and a condition is imposed upon it:

$$\rho L \frac{dR}{dt} = \left(4\sigma\alpha T^3 \frac{\partial T}{\partial x} + \sigma(T_f^4 - T^4) + \gamma(T_f - T)\right)_{x=R(t)} \qquad (57)$$

where γ is the convective heat transfer coefficient, and one can (probably) make the identification $\gamma \equiv k_c c_{pg} \rho_g D$ by comparison with Hottel *et al.*[8] Formulation as a moving boundary problem necessitates including the heat of pyrolysis, which is one of the main differences in this formulation. The other main difference is that radiation through the fuel bed is treated as a non-linear conduction process, corresponding to the opaque limit of the full transfer equations (see e.g. Ozisik[18]). This is in contrast to flame radiation which Fujii *et al.*[33] describe with a view factor!

The result is a complex non-linear model which must be solved numerically. Although Fujii *et al.*[33] anticipate a dynamical solution for spread rate, they only seem to find a quasi-steady solution. Considering the complexity of the model compared with previous investigators, and the lack of new insight, the model of Fujii *et al.*[33] seems to have little to recommend it. Temperature distributions are calculated and compared with the paper array experiments of Emmons and Shen.[47] However, the calculated results are dominated by longer range heating which is absent in the experimental results; just as de Mestre *et al.*[3] found for fuel beds (see Fig. 2).

4.2.2. *Albini*[38,39]

Albini[38,39] has presented a precise formulation of the propagation of a line fire by radiative heating.

The complete integral equations describing

radiation transport in a uniform layer of randomly distributed, thermally-thin, radiometrically black fuel particles are used to connect the combustion zone and unburnt fuel. Significantly Albini's[38,39] model is the first physical model which considers a two-dimensional ignition interface rather than simply a one-dimensional idealization (Fransden[6] considered a two-dimensional ignition interface, but not in connection with a physical model as it has been defined in the present paper). Furthermore, the shape of the ignition interface is an unknown which Albini[38,39] determines from the model.

The first paper, Albini,[38] considers radiation only. Since the full radiation transport equations are used, radiative cooling is naturally included in the model. In non-dimensional variables the equations are (see also Ozisik[18])

$$\left(\mu \frac{\partial}{\partial x} + \nu \frac{\partial}{\partial y} + \eta \frac{\partial}{\partial z}\right) i = C(\theta - i) \quad (58a)$$

$$-\Lambda \frac{\partial Q}{\partial x} = C\left(\frac{1}{\pi}\int\int_{4\pi} i\,dw' - 4\theta\right). \quad (58b)$$

The dimensionless parameter $C = \alpha\beta\delta$ describes the fuel bed in terms of its optical opacity and is related to the leaf area index for a homogeneous plant cover, such as a forest. The other parameter $\Lambda = Rq_i/\sigma T_f^4$ is an eigenvalue of the problem and has to be determined along with the ignition interface, which is a curve $x(z)$ on which

$$i = 1 - (T_\infty/T_f)^4, \quad (59a)$$

$$\theta = (T_i/T_f)^4 - (T_\infty/T_f)^4, \quad (59b)$$

$$Q = 1, \quad (59c)$$

$$x(0) = 0. \quad (59d)$$

Q and θ are related since Q is the normalized energy density of the fuel and θ the normalized temperature. In Albini[38] a stepwise model is used to represent moist fuel, just as used by Thomas[12] and others.

Albini's[38,39] models are then solved numerically by an iterative process. The ignition interface is given an initial form (e.g. an angled phase), θ is taken to be zero and the normalized radiation intensity is taken to be the exponential profile found in earlier models.

The first model, Albini,[38] is tested on laboratory fires. Physically reasonable values for the radiative intensities were chosen so that the model predicted the observed spread rates. The predicted shape of the ignition interface is compared with that observed and the agreement is poor. For this reason, in Albini,[39] natural convection cooling is also included which improves the agreement with observation.

Albini and Stocks[48] then use the Albini[39] version of the model to fit data from crown fires through Jack Pine forest. The experimental fires were observed under wind conditions, which are incorporated into the model by tilting the flame, but without the addition of convective heating. Modelling only the crown layer fire yields unsatisfactory ignition interfaces, although the spread rate predictions are reasonably good. To overcome this shortfall, a two-layer model is introduced, where a surface fire through small dead pines is modelled by the transparent limit (exponential decay of radiation) with a vertical ignition interface. This improves the agreement with observation. However, it was *not* possible to compare predicted and observed temperature profiles ahead of the fire.

4.3. *Incorporating Kinetics: The Future?*

4.3.1. *Grishin, Gruzin and Zverev*[7]

Grishin *et al.*[7] propose a mathematical model for a wind-blown forest fire which accounts for the basic physicochemical processes of heating, drying, pyrolysis and combustion. The model is in one space and one time dimension and uses first order Arrhenius kinetics to describe pyrolysis and combustion. It is assumed that turbulent transport processes in the vegetation can be modelled by effective turbulent exchange coefficients. For example, the heat transfer flux can be written as

$$q = \lambda_T \frac{\partial T}{\partial x} \quad (60)$$

where λ_T is an effective turbulent 'conductivity' or 'eddy diffusivity'. The energy equation then can be written as

$$\left(\sum_{i=1}^{4} \rho_i \varphi_i c_{pi} + \rho c_p\right)\frac{\partial T}{\partial t} + \rho c_p W \frac{\partial T}{\partial x} = \frac{\partial}{\partial x}\left(\lambda_T \frac{\partial T}{\partial x}\right) - H(T - T_\infty) + \text{Reaction Terms}. \quad (61)$$

The summation term includes dry organic matter, liquid water, condensed pyrolysis products and the mineral component of the forest fuel. Note that natural convective cooling is included with the term $H(T - T_\infty)$. This highlights the fact that the model of Grishin *et al.*[7] does not account for the hydrodynamical aspects of the flow, only the combustion.

Numerical analysis enabled Grishin *et al.*[7] to solve their time dependent model. The structure of the fire front is part of the solution and it was possible to model the development of the front from the moment of initiation until quasi-steady spread. The analysis indicated that the fire spread rate decreased strongly with an increase in moisture content, and that critical moisture contents exist above which a fire will not spread. This is analogous to the critical conditions observed in ignition/extinction theory and is a general feature of any model which incorporates a kinetic scheme, see e.g. Williams.[16]

In a later article, Grishin,[30] the steady state propagation was treated analytically. It was found that the fire spread rate depends upon the wind velocity in an almost linear fashion,

$$R \sim W \quad (62)$$

where the constant of proportionality depends upon physicochemical properties of the forest fuel. A linear relationship could have been anticipated on the grounds of dimensional analysis, and other investigators (Thomas[42] and Catchpole[29]) have found that a linear law is a good model for fire spread in a suspended layer of vegetation, e.g. heather, gorse and heath.

5. CONCLUSIONS

Until now, only the statistical and empirical models have been developed into operational fire management tools. However, there are many questions which cannot be addressed within the framework of such models, which happen to be specific to the experimental conditions in which such models were derived. Because of this, and natural curiosity, researchers have attempted to develop physical models of fire spread based upon well understood heat transfer mechanisms. Unfortunately, because there appears to be several heat transfer mechanisms operating at any one time this has led to confusion about which mechanisms are the principal ones. Many different models have been proposed and tested against experimental data. While it has been possible to adjust the models to predict the correct fire spread rate, other information, such as temperature distributions, has not been reproduced with good accuracy. In particular, the temperature of the fuel is found to rise more dramatically near the fire than any of the models predict. This could be due to the buoyant convective heat transfer mechanisms which operate for fire spread on a single fuel element (Weber and de Mestre[49]) being important for a conglomeration of fuel elements in a fuel bed. This will require further, careful research which has as its aim not only the prediction of spread rate, but also the temperature profile and the shape of the ignition interface.

The inclusion of a model chemical kinetic scheme by Grishin *et al.*[7] is a major advance. Any questions related to fire build-up, the effect of moisture on flame properties, extinction of fire spread, effect of combustion on hydrodynamics and hydrodynamics on combustion, can benefit from the inclusion of chemical kinetics. For this reason, research into the kinetics of combustion of natural forest fuels is to be encouraged.

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