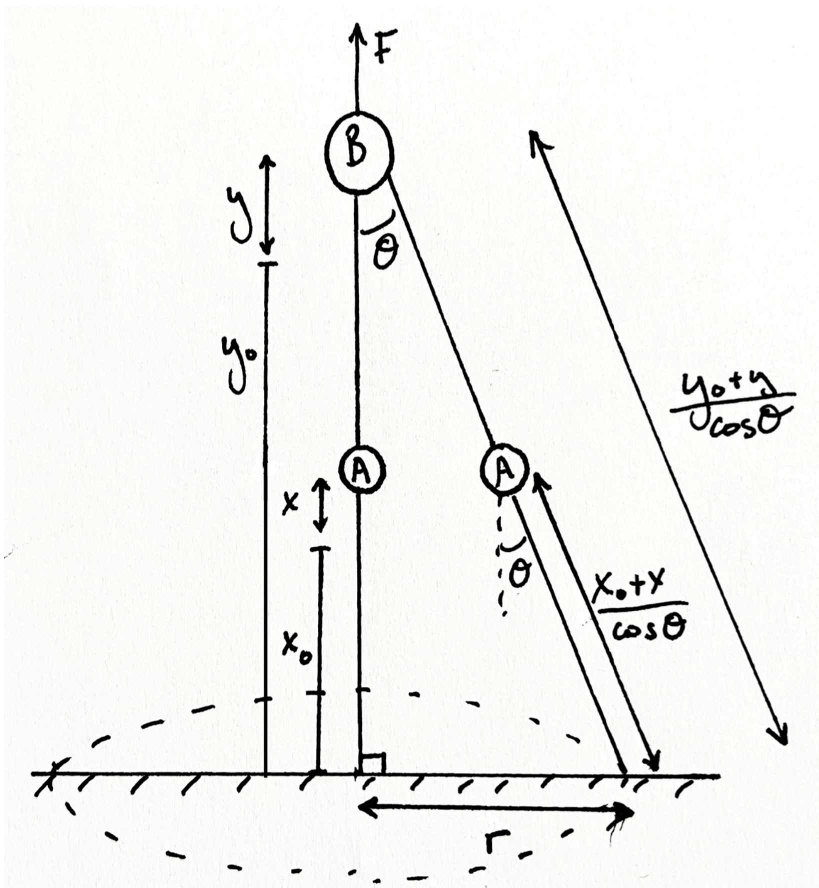
1 Antigen Tug of War Mobility



1.1 Potentials

$$U(x_a, x_b, \theta(r)) = U_a(x_a, \theta(r)) + U_b(x_b(x_a), \theta(r)) - Fcos(\theta)y \tag{1}$$

N.B. Not 100% confident on the pulling force tacked on to the end here

1.2 Bond Extension Coordinates

$$x_a = \frac{x_0 + x}{cos(\theta)} - x_0 \tag{2}$$

Where, x_0 is the equilibrium bond length and x the bond extension due to thermal fluctuations of the APC-antigen bond.

$$x_b = \frac{y_0 + y}{cos(\theta)} - y_0 - x_a \tag{3}$$

Similarly, y_0 is the total equilibrium bond length and y the bond extensions due to thermal fluctuations of the APC-antigen-BCR complex.

1.3 Angle Coordinates

$$\theta = tan^{-1}\left(\frac{r}{y + y_0}\right) \tag{4}$$

$$\frac{\partial \theta}{\partial r} = \frac{y + y_0}{(y + y_0)^2 + r^2} \tag{5}$$

θ is defined as the deflection angle due to horizontal displacement, r, at the tether. We assume the body is rigid, so that the two angles at A and B are equal.

1.4 Langevin Equations

$$\gamma_a \dot{x}_a = -U_a'(x_a, \theta(r)) + U_b'(x_b, \theta(r)) + \xi_a \tag{6}$$

$$\gamma_b(\dot{x}_a + \dot{x}_b) = -U_b'(x_b, \theta(r)) + Fcos(\theta) + \xi_b \tag{7}$$

$$\gamma_r \dot{r} = -\frac{\partial U}{\partial r} sin(\theta) + \xi_r = -\frac{\partial U}{\partial \theta}\frac{\partial \theta}{\partial r} sin(\theta) + \xi_r \tag{8}$$

The equations of motions summarise the conservative and non-conservative (velocity-dependent) forces acting on the particles. γ_a, γ_b, γ_r are the drag coefficients on A, B and the tether respectively.
N.B. not sure if in (8) force due to angular deformation needs to be resolved horizontally here, or do the equations take care of this? In which case: $\gamma_r \dot{r} = -\frac{\partial U}{\partial r} + \xi_r = -\frac{\partial U}{\partial \theta}\frac{\partial \theta}{\partial r} + \xi_r$

1.5 Noise Terms

$$\langle \xi_a(t) \xi_a(t') \rangle = 2k_B T \gamma_a \delta(t - t') \tag{9}$$

$$\langle \xi_b(t) \xi_b(t') \rangle = 2k_B T \gamma_b \delta(t - t') \tag{10}$$

$$\langle \xi_r(t) \xi_r(t') \rangle = 4D \gamma_r^2 \delta(t - t') \tag{11}$$

T is the temperature, k_B the Boltzmann constant, and D the diffusion coefficient of the tether undergoing 2d brownian motion on the APC surface.