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Trajectory Optimization for Planetary Multi-Point Powered Landing

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Abstract: Next-generation powered landing guidance for planetary soft landing needs multi-point-guidance capabilities. A novel trajectory optimization framework for planetary multi-point powered landing is presented in this study. First, the powered landing trajectory optimization problem with three-dimensional dynamics model, boundary conditions, and path constraints is formulated. The finite-element collocation approach with collocation based on Radau points is chosen to transcribe the formulated trajectory optimization problem into a nonlinear programming problem solved by Interior Point OPTimizer (IPOPT). The proposed trajectory optimization framework utilizes optimal sensitivity based on IPOPT to select the fuel-optimal landing aim point from candidate landing aim points by one optimization computation. Then, the optimal trajectory to the best landing aim point can be efficiently obtained based on the good initial value guess provided by the sensitivity analysis. Numerical results show that the proposed algorithm effectively and efficiently addresses the planetary multi-point powered landing problem and has potential for onboard implementation.

Keywords: trajectory optimization, multi-point powered landing, finite-element collocation approach, IPOPT, optimal sensitivity

1. INTRODUCTION

As a result of increased focus on the exploration of planets in the solar system, planetary soft landing is gaining renewed interest. The main focus of planetary soft landing is to make landers precisely land at scientifically interesting locations (Açıkmeşe et al., 2013). Soft landing on other planetary bodies generally requires powered landing guidance to plan a path onboard a lander for the final kilometres of descent. The path must be planned onboard because the state of the lander at the start of powered landing cannot be adequately predicted beforehand due to atmospheric uncertainties and/or the limits of deep space navigation (Scharf et al., 2015). A list of possibly tens to hundreds of safe landing sites is stored onboard. The list of safe sites could be reduced by considering only sites near the lander's position, but this may still leave tens of sites to evaluate (Scharf et al., 2015; Trawny, 2015). For autonomous landers, the candidate landing aim points need to be transformed into the guidance system's operating frame and used directly to guide the lander to the best landing aim point (Cohanim and Collins, 2009). Thus, the efficient multi-point trajectory optimization algorithm is of great significance for multi-point powered landing missions.

Existing studies mainly deal with single-point pinpoint landing. Meditch (1964) presented a closed-form solution to the lunar soft landing problem for the one-dimensional case. Klumpp (1974) presented a trajectory based on a quartic polynomial in time for the Apollo mission. More recently, Topcu et al. (2005) developed first-order necessary conditions for the fuel efficient Mars landing, and it is shown that the optimal thrust profile has a maximum-minimum-maximum profile. Direct collocation and direct multiple shooting methods were used to obtain numerical solutions of a constrained nonconvex parameter optimization problem converted from the original guidance problem. Sostaric and Rea (2005) developed a numerical solution to the pinpoint landing guidance problem by Legendre pseudospectral methods. Najson and Mease (2005) developed an approximate analytical solution to soft landing problem by solving a related optimal control problem that does not use a minimum fuel cost functional. Açıkmeşe et al. (2007, 2011, 2013) and Blackmore et al. (2010) developed powered descent guidance using convex optimization. Lossless convexification of nonconvex control bound and pointing constraints on the soft landing optimal control problem was proposed. The resulting convex optimal problem was solved by efficient second-order cone programming solvers with deterministic convergence properties. Ma et al. (2016) proposed a direct trajectory optimization framework with a Hamiltonian-based adaptive mesh refinement strategy to address problems of lunar soft landing.

In this study, a trajectory optimization framework for planetary multi-point powered landing is proposed. Finite-element collocation approaches (Kameswaran and Biegler, 2006, 2007; Biegler, 2007) are chosen to transcribe the trajectory optimization problem of powered landing process into a nonlinear programming (NLP) problem solved by a highly-efficient NLP solver. The time interval is partitioned into subintervals and polynomials are used to approximate the state and control profiles over each subinterval in the finite-element collocation approach. For planetary multi-point powered landing, the first mission is to select the best landing aim point from candidate landing aim points. Fuel consumption is regarded as the criterion to select the best landing aim point. It is quite time-consuming to calculate fuel

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consumption of the lander arriving at each candidate landing aim point. Thus, sensitivity analysis is utilized to estimate the fuel consumption of landing at each candidate point. Sensitivity analysis provides information on regularity and curvature conditions at Karush–Kuhn–Tucker (KKT) points, assesses which variables play dominant roles in the optimization, and evaluates first-order sensitivities (i.e., first derivatives or directional derivatives) of the solution vector with respect to perturbation parameters (Pirnay et al., 2012). Here, the sensitivity analysis is coupled to Interior Point OPTimizer (IPOPT) (Wächter and Biegler, 2006) in which exact second derivatives are used. Therefore, sensitivity can be implemented very efficiently within IPOPT and provide valuable information with very little added computation. After the best landing aim point is determined, the final trajectory can be calculated efficiently based on the good initial value guess given by the sensitivity analysis.

The rest of this paper is organized as follows. The trajectory optimization problem of point-mass powered landing is described with the main features in Sec. 2. The proposed trajectory optimization framework for planetary multi-point powered landing is given in Sec. 3. Results and discussions are given in Sec. 4. Finally, the conclusions are presented in Sec. 5.

2. PROBLEM FORMULATION

The planetary powered landing problem searches for the thrust profile and an accompanying translational state trajectory which guides a lander from an initial position and velocity to a prescribed target location on the planet while minimizing the fuel consumption (Açıkmeşe et al., 2013). Since powered landing phase of a planetary landing mission starts at a low altitude relative to planet's radius, the uniform gravity assumption is appropriate. Other forces such as aerodynamic forces due to winds are neglected in optimal trajectory design and they will be treated as disturbances (Açıkmeşe and Ploen, 2007).

The translation dynamics of the lander are given as follows:

$$\begin{aligned} &dr(t)/dt = V(t) \\ &dV(t)/dt = g + T(t)/m(t) \\ &dm(t)/dt = -\alpha \|T(t)\| \end{aligned} \tag{1}$$

where $r = (x, y, z)$ is the position vector, and $V = (Vx, Vy, Vz)$ is the velocity vector. $g \in R^3$ is the constant gravitational acceleration vector of the planet. $T = (Tx, Ty, Tz)$ is the thrust vector. m is the lander mass, and α is a positive constant describing the fuel consumption rate.

The magnitude of T is bounded as

$$T_{\min} \le \|T\| \le T_{\max} \tag{2}$$

where $T_{\min}$ and $T_{\max}$ denote the minimum and maximum thrust value. Onboard sensors for terrain-relative navigation generally require specific viewing orientations, which imposes a constraint on the lander attitude (Açıkmeşe et al., 2013). This may imply that the thrust direction must not deviate more than γ degrees from the positive x direction. This type of constraint can easily be expressed as follows:

$$\begin{aligned} &\hat{n}^T T(t) \ge \|T(t)\| \cos\gamma \\ &\hat{n}^T = (1,0,0) \end{aligned} \tag{3}$$

where $\cos\gamma$ and $\hat{n}$ describe the cone that the thrust vector must point into. Also, it is required that the trajectory does not go below the surface during the maneuver, i.e.,

$$x(t) \ge 0 \tag{4}$$

Fuel consumption cannot be more than the mass of fuel, thus the constraint is given by the following:

$$m_0 - m_{fuel} \le m(t) \le m_0 \tag{5}$$

where m_{fuel} is total mass of fuel and m_0 is the initial mass of the lander. The initial and final position and velocity, and initial mass are specified as follows:

$$\begin{aligned} &m(0) = m_0, r(0) = r_0, V(0) = V_0 \\ &r(t_f) = r_f, V(t_f) = 0 \end{aligned} \tag{6}$$

where r_0, r_f, and V_0 are constant vectors. Therefore, the problem of single-point powered landing is obtained as follows:

$$\begin{aligned} &\max\ m(t_f) \\ &\text{s.t.}\ \ dr(t)/dt = V(t) \\ &\qquad dV(t)/dt = g + T(t)/m(t) \\ &\qquad dm(t)/dt = -\alpha\|T(t)\| \\ &\qquad x(t) \ge 0, \hat{n}^T T(t) \ge \|T(t)\|\cos\gamma \\ &\qquad T_{\min} \le \|T\| \le T_{\max}, m_0 - m_{fuel} \le m(t) \le m_0 \\ &\qquad m(0) = m_0, r(0) = r_0, V(0) = V_0 \\ &\qquad r(t_f) = r_f, V(t_f) = 0 \end{aligned} \tag{7}$$

For multi-point powered landing missions, the problem of single-point powered landing is generally solved for each candidate landing aim point, then the fuel-optimal candidate point will be chosen as the landing point, which will be quite time-consuming when the number of candidate points is large. Thus, an efficient trajectory optimization framework for planetary multi-point powered landing is proposed in Sec. 3.

3. TRAJECTORY OPTIMIZATION FRAMEWORK FOR MULTI-POINT POWERED LANDING

3.1 Finite-Element Collocation Approach

The finite-element collocation approach fully discretizes the state and control variables, leading to large-scale NLP problems. Without loss of generality, the following general dynamic optimization problem is considered (Kameswaran and Biegler, 2006, 2007; Biegler, 2007):

$$\begin{aligned} &\min\ \Phi(z(t_f)) \\ &s.t.\quad \frac{dz}{dt} = f(z(t), y(t), u(t)),\ z(t_0) = z_0 \\ &\qquad g(z(t), y(t), u(t)) = 0 \\ &\qquad u_L \le u(t) \le u_U \\ &\qquad \psi(z(t_f)) \le 0 \end{aligned} \tag{8}$$

where $z(t)$ and $y(t)$ are the differential and algebraic state profiles, respectively, and $u(t)$ denotes the control profiles. t_f is the final time. $\Phi(z(t_f))$ is an objective function. g are algebraic equation constraints. ψ are terminal constraints. The differential-algebraic equations model is assumed to be index-1 and given in semi-explicit form. The whole time horizon is divided to NE finite elements, which satisfies

$$t_0 < t_1 < t_2 < ... < t_{NE-1} < t_{NE} = t_f \quad (9)$$

First, $K + 1$ Gauss or Radau interpolation points are selected in finite element i. The differential, control, and algebraic profiles in a specified finite element i are approximated by Lagrange polynomial as follows:

$$\begin{aligned} & z^K(t) = \sum_{j=0}^{K} L_j(\tau) z_{ij} \\ & u^K(t) = \sum_{j=1}^{K} \bar{L}_j(\tau) u_{ij}, y^K(t) = \sum_{j=1}^{K} \bar{L}_j(\tau) y_{ij} \\ & L_j(\tau) = \prod_{k=0,\neq j}^{K} \frac{(\tau - \tau_k)}{(\tau_j - \tau_k)}, \bar{L}_j(\tau) = \prod_{k=1,\neq j}^{K} \frac{(\tau - \tau_k)}{(\tau_j - \tau_k)} \\ & t \in [t_{i-1}, t_i], t = t_{i-1} + h_i \tau, \tau \in [0,1] \end{aligned} \quad (10)$$

where h_i refers to the length of the finite element i, and $0 < \tau_j \le 1, j = 1,...,K$ are the shifted Gauss or Radau points. This polynomial representation has the following property:

$$z^K(t_{ij}) = z_{ij}, t_{ij} = t_{i-1} + \tau_j h_i \quad (11)$$

The continuity of the differential state profile at the finite element boundaries is enforced by the following expression:

$$\begin{aligned} & z_{i+1,0} = \sum_{j=0}^{K} L_j(1) z_{ij}, i = 1,..., NE-1 \\ & z_f = \sum_{j=0}^{K} L_j(1) z_{Nj}, z_{1,0} = z_0 \end{aligned} \quad (12)$$

Substituting Eqs. (10), (11), and (12) into Problem (8), collocation equations can be derived as follows:

$$\begin{aligned} & \sum_{j=0}^{K} \frac{dL_j(\tau_k)}{d\tau} z_{ij} - h_i f(z_{ik}, y_{ik}, u_{ik}) = 0 \\ & z_{i+1,0} - \sum_{j=0}^{K} L_j(1) z_{ij} = 0 \\ & g(z_{ik}, y_{ik}, u_{ik}) = 0, i = 1,..., NE, k = 1,..., K \end{aligned} \quad (13)$$

Eventually, the dynamic optimization problem is discretized into an NLP formulation with fixed finite element length h_i, as follows:

$$\begin{aligned} & \min \Phi(z_f) \\ s.t. \ & \sum_{j=0}^{K} \frac{dL_j(\tau_k)}{d\tau} z_{ij} - h_i f(z_{ik}, y_{ik}, u_{ik}) = 0 \\ & g(z_{ik}, y_{ik}, u_{ik}) = 0 \\ & u_L \le u_{ik} \le u_U, k = 1,..., K, i = 1,..., NE \\ & z_{i+1,0} = \sum_{j=0}^{K} L_j(1) z_{ij}, i = 1,..., NE-1 \\ & z_f = \sum_{j=0}^{K} L_j(1) z_{Nj}, z_{1,0} = z_0, \psi(z_f) \le 0 \end{aligned} \quad (14)$$

The resulting NLP problem can be solved by the highly-efficient solver IPOPT.

3.2 Multi-Point Trajectory Optimization Algorithm

If there exists N candidate landing aim points (x_{fi}, y_{fi}, z_{fi}), $i = 1 \dots N$, we define the midpoint M of these N candidate landing aim points as follows:

$$\begin{aligned} & M = (x_M, y_M, z_M) \\ & \min_{x_M, y_M, z_M} \sum_{i=1}^{N} \left\| (x_M - x_{fi}, y_M - y_{fi}, z_M - y_{fi}) \right\| \end{aligned} \quad (15)$$

where the sum of the distances from the midpoint M to each candidate landing aim point is minimized.

Here, the parameter vector p_i and p_0 represent the position of the candidate landing aim point and the position of the midpoint M, respectively. Thus, we define

$$\begin{aligned} & p_i = \left(x_{fi}, y_{fi}, z_{fi}\right), i = 1,..., N \\ & p_0 = M = (x_M, y_M, z_M) \end{aligned} \quad (16)$$

Next, optimal sensitivity will be utilized to estimate trajectories from the initial position of lander to each candidate landing aim point. For simplicity, consider formulation (17) as the parametric NLP problem as follows:

$$\begin{aligned} & \min_x f(x; p) \\ & \text{s.t. } c(x; p) = 0 \\ & \quad x \ge 0 \end{aligned} \quad (17)$$

with the variable vector $x \in R^{n_x}$ which consists of z_{ij}, y_{ij}, u_{ij}, the position of candidate landing aim point $p \in R^{n_p}$, and equality constraints $c(x; p): R^{n_x + n_p} \to R^m$. The interior point method is used to solve formulation (17). The inequality constraints are transformed into a barrier term and added to the objective function. The following sequence of barrier subproblems is solved with $\mu \to 0$ (Wächter and Biegler, 2006):

$$\begin{aligned} & \min_x f(x; p) - \mu \sum_{i=1}^{n_x} ln(x_i) \\ & \text{s.t. } c(x; p) = 0 \end{aligned} \quad (18)$$

The KKT conditions for formulation (18) are defined as

$$\begin{aligned} & \nabla_x L(x, \lambda, v; p) = \nabla_x f(x; p) + \nabla_x c(x; p)\lambda - v = 0 \\ & c(x; p) = 0 \\ & x^{(i)} v^{(i)} = \mu \end{aligned} \quad (19)$$

with the multiplier vector $\lambda \in R^{n_\lambda}$ and dual variable vector v; $x^{(i)}$ and $v^{(i)}$ denote the ith components of the corresponding vector.

The implicit function theorem can be applied to formulation (19) with respect to p_0. We define the quantities (Pirnay et al., 2012):

$$S_s(s(\mu; p_0)) = \begin{bmatrix} \nabla_{xx} L(s(\mu; p_0)) & \nabla_x c(x(\mu; p_0)) & -I_{nx} \\ \nabla_x c(x(\mu; p_0))^T & 0 & 0 \\ V(\mu; p_0) & 0 & X(\mu; p_0) \end{bmatrix}$$

$$s(\mu;p_0)^T=[x(\mu;p_0)^T\ \lambda(\mu;p_0)^T\ v(\mu;p_0)^T]$$
$$S_p(s(\mu;p_0))=\begin{bmatrix}\nabla_{xp}L(s(\mu;p_0))\\ \nabla_p c(x(\mu;p_0))^T\\ 0\end{bmatrix}\ S_\mu(s(\mu;p_0))=\begin{bmatrix}0\\0\\-I_{nx}\end{bmatrix} \quad (20)$$

where $X=diag\{x\}$, $V=diag\{v\}$, and I is identity matrix.

If $S_s(s(\mu;p_0))$ is nonsingular, we can have (Pirnay et al., 2012):

$$\begin{aligned} ds(\mu;p_0)^T/dp &= -S_s(s(\mu;p_0))^{-1}S_p(s(\mu;p_0)) \\ ds(\mu;p_0)^T/d\mu &= -S_s(s(\mu;p_0))^{-1}S_\mu(s(\mu;p_0)) \end{aligned} \quad (21)$$

For small values of μ and $\|p-p_0\|$, expand $s(\mu;p)$ at $(0;p_0)$:

$$\begin{aligned} s(0;p)=s(\mu;p_0)-S_s(s(\mu;p_0))^{-1}[S_p(s(\mu;p_0))(p-p_0) \\ +S_\mu(s(\mu;p_0))\mu]+o(\|p-p_0\|)+o(\mu) \end{aligned} \quad (22)$$

Thus, the trajectories from the initial position of lander to each candidate landing aim point can be estimated by the optimal trajectory from the initial position of lander to the defined midpoint p_0.

Eventually, the proposed trajectory optimization framework for planetary multi-point powered landing is concluded as follows:

Step 1: Obtain the position of the midpoint based on Formulation (15). Then obtain the fuel-optimal trajectory $s(0;p_0)$ from the initial position of the lander (x_0, y_0, z_0) to the midpoint M of the N candidate landing aim points.

Step 2: Estimate trajectories $s(0;p_i), i=1,...,N$ from the initial position of the lander (x_0, y_0, z_0) to each candidate landing aim point $\left(x_{fi},y_{fi},z_{fi}\right), i=1,...,N$ based on sensitivity analysis as follows:

$$\begin{aligned} s(0;p_i)=s(0;p_0)-S_s(s(0;p_0))^{-1}[S_{p_i}(s(0;p_0))(p_i-p_0)] \\ +o(\|p_i-p_0\|) \\ p_i=(x_{fi},y_{fi},z_{fi}), i=1,...,N \end{aligned} \quad (23)$$

Step 3: Select the optimal landing aim point $(x_{f^*},y_{f^*},z_{f^*})$ that consumes the least fuel based on the sensitivity result in Step 2.

Step 4: Calculate the fuel-optimal trajectory from the initial position of the lander (x_0, y_0, z_0) to the optimal landing aim point $(x_{f^*},y_{f^*},z_{f^*})$ based on good initial value guess $s(0;p^*)$, $p^*=(x_{f^*},y_{f^*},z_{f^*})$.

Step 5: Output the optimal results.

4. RESULTS AND DISCUSSIONS

This section presents the results of simulations performed in AMPL environment (Fourer et al., 1990), on a Lenovo Y430p running Windows 7 with an Intel® Core^{TM} i7-4710MQ 2.50 GHz processor and 4 GB RAM. The NLP solver is sIPOPT (Pirnay et al., 2012) which is a barrier NLP solver coupled with sensitivity strategy. The tolerance of sIPOPT is set as 1.0e−8. The collocation points are three-order Radau points (i.e., K = 3), and the number of finite elements is 15. The minimum thrust value $T_{\min}$ and maximum thrust value $T_{\max}$ are 4800.0 N and 19200.0 N, respectively. The constant gravitational acceleration vector of the planet g is (−3.71 m/s^2, 0, 0). The initial mass of the lander m is 2000.0 kg, and total mass of fuel m_{fuel} is 500 kg. The fuel consumption rate α is 1e−3. In the designed scenario, there are sixteen candidate landing aim points as shown in Table 1. The thrust direction must not deviate more than 45 degrees from the positive x direction.

Table 1. Positions of sixteen candidate landing aim points

No.	x, y, z (m)	No.	x, y, z (m)
1	0.6, −450, 450	9	0.2, −450, −150
2	0.6, −150, 450	10	0.2, −150, −150
3	0.8, 150, 450	11	0.4, 150, −150
4	0.8, 450, 450	12	0.4, 450, −150
5	0.6, −450, 150	13	0.2, −450, −450
6	0.6, −150, 150	14	0.2, −150, −450
7	0.8, 150, 150	15	0.4, 150, −450
8	0.8, 450, 150	16	0.4, 450, −450

In the scenario, the initial position of the lander (x_0, y_0, z_0) in the figures is (2500 m, 150 m, 150 m), and the initial velocity of the lander (V_{x0}, V_{y0}, V_{z0}) is (−10 m/s, −50 m/s, 10 m/s). The coordinate of the defined midpoint of these sixteen candidate landing aim points is (0.5 m, 0 m, 0 m). In the following figures, A denotes the initial position of the lander; C_1 to C_{16} denote the candidate landing aim points from No. 1 to No. 16; M refers to the defined midpoint of these candidate landing aim points.

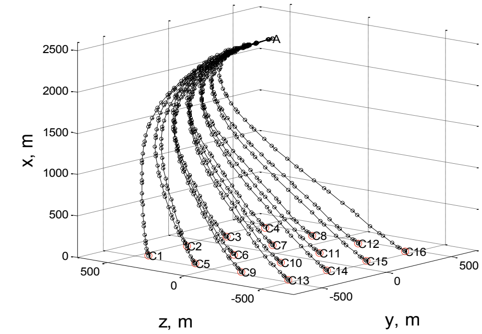


Fig. 1. Optimal trajectories obtained point by point.

Figure 1 shows the optimal trajectories from the initial position A to each candidate landing aim point. We optimize the trajectories from C_1 to C_{16} point by point. Then, we select the best trajectory that consumes the least fuel from these sixteen trajectories. When optimizing the trajectory from A to C_1, we set the initial value guess of state variables at each collocation as the initial values of state variables. When we optimize trajectories from A to C_2 – C_{16}, the initial value guess of state and control variables is set as the results of the adjacent point.

Figure 2 shows results obtained by the proposed trajectory optimization framework for planetary multi-point powered landing. First, the fuel-optimal trajectory from A to the midpoint *M* is obtained shown in Fig. 2a. Then, the estimated trajectories expressed by the red lines shown in Fig. 2b from A to each candidate landing aim point are obtained by the sensitivity analysis. We select the fuel-optimal trajectory from these estimated trajectories. The estimated trajectory from A to C_1 is fuel-optimal, thus C_1 is the best landing aim point. Finally, the trajectory from A to C_1 is optimized based on solving the NLP problem Eq. (15) from an initial value guess given by the estimated sensitivity-based trajectory from A to C_1 shown in Fig. 2c.

Figure 3 shows the estimated and optimized final mass of the lander at each candidate landing aim point. It is obvious that No. 1 point is the best landing aim point which consumes the least fuel. We can accurately select the best landing aim point in terms of the estimated final mass of the lander. The relative errors between the estimated final mass of the lander and the optimized final mass of the lander are less than 0.5%, thus the estimated final mass of the lander at each candidate landing aim point based on sensitivity analysis are acceptable, which verifies the rationality of the proposed multi-point powered landing algorithm.

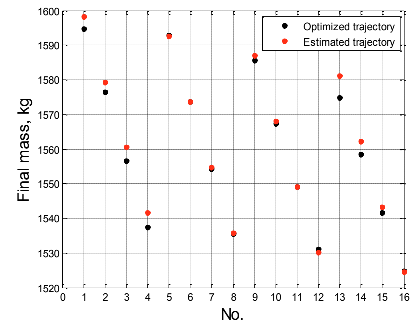
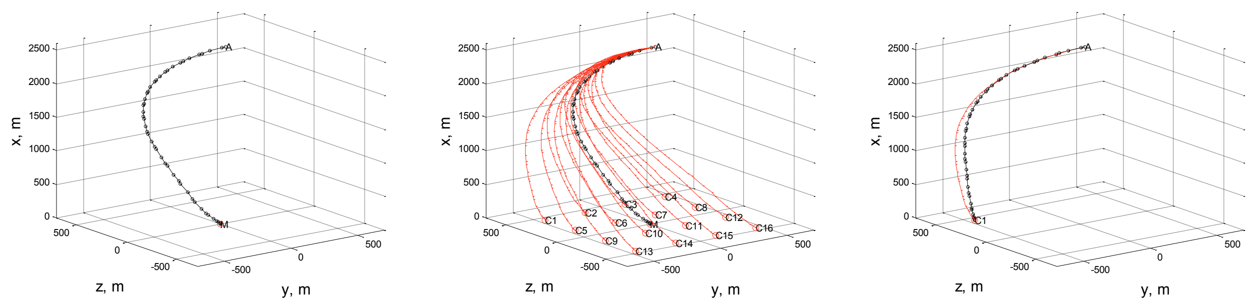
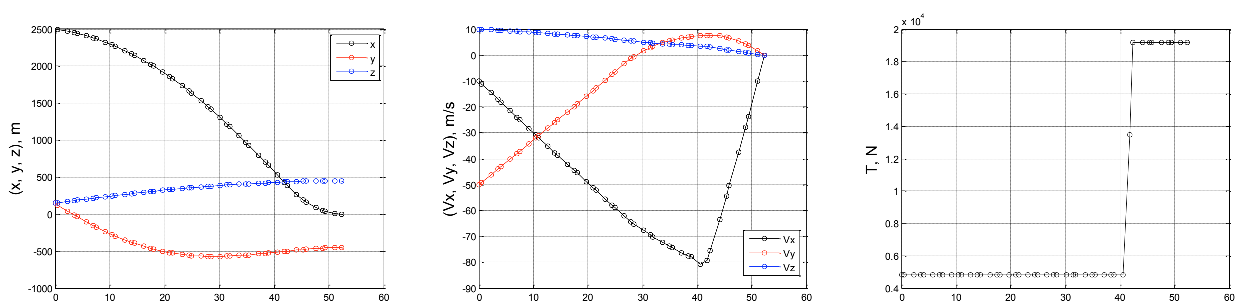


Fig. 3. Estimated and optimized final mass of the lander at each candidate landing aim point.



(a) (b) (c)

Fig. 2. Results obtained by the proposed multi-point powered landing algorithm.



(a) (b) (c)

Fig. 4. State and control profiles.

Table 2. Performance comparison

Approach	#toc	#iter	Total CPU, s
Proposed approach	2	61	1.008
General approach	16	315	2.512

The performance of the proposed approach for multi-point powered landing compared with that of the general approach (point by point) is presented in Table 2. This table includes the number of times of optimization computation (#toc), number of total iterations (#iter), and total CPU time cost for solving the problem (Total CPU). The general approach needs sixteen optimization computations with 315 iterations, while the proposed approach only needs two optimization computations with 61 iterations. The CPU time cost of the proposed approach is better than that of the general approach.

The subfigures in Fig. 4 show that optimal state and control profiles from the initial position of the lander to the best landing aim point C_1. Figure 4a shows the three-dimensional position of the lander vs. time. The lander finally lands at the specified landing point (0.6 m, −450 m, 450 m). Figure 4b shows that the velocity vector eventually equals zero when the lander lands on the specified position of the planetary surface. Figure 4c shows the thrust magnitude profile vs. time. The thrust magnitude of the lander is throttleable. It can be seen from Fig. 4c that the optimal thrust control consists of a period of minimum thrust after the mission starts, then a period of maximum thrust until touchdown

5. CONCLUSIONS

This study presents a novel trajectory optimization framework for planetary multi-point powered landing. The best landing aim point from the candidate landing aim points can be efficiently and accurately obtained by optimal sensitivity analysis based on IPOPT, which only needs one optimization computation. Then another optimization computation is implemented to obtain the optimal trajectory from the initial position of the lander to the selected best landing aim point, whose computation cost is quite low because of the good initial value guess provided by the sensitivity analysis. Therefore, only two optimization computations are required to obtain the optimal trajectory to the best landing aim point, which is quite efficient. Thus, the proposed approach has the potential for onboard implementation for multi-point powered landing missions. For the proposed algorithm, the estimation error will increase as the distance from the candidate point to the defined midpoint increases. Therefore, designing the multi-point powered landing algorithm when the candidate landing aim points are located in large landing region will be the future work.

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