

① Example of Correlation for measurement:

In order to illustrate the manner in which correlations are built up in a system, we consider a simplified measuring process introduced by Von-Neumann. Let g represent the system variable of interest and r the apparatus variable. Assume that they are initially independent so that the combined wave function is $\Psi = \phi(g) \eta(r)$ where $\phi(g)$ is the initial system wave function and $\eta(r)$ is the initial apparatus function. Furthermore assume that the masses are sufficiently large, or the time of measurement (interaction) so small that the kinetic portion of the energy may be neglected, so that during the time of measurement the hamiltonian shall consist only of an interaction, which we shall take to be:

$$\text{.1} \quad H_I = -i\hbar g \frac{\partial}{\partial r}$$

Then according to the Schrödinger equation:

$$\text{.2} \quad -i\hbar \frac{\partial}{\partial t} \Psi_e(g, r) = i\hbar g \frac{\partial}{\partial r} \Psi_e(g, r)$$

which has the general solution:

$$\text{.3} \quad \Psi_e(g, r) = f(g, r - gt)$$

Since our initial wave function was $\Psi = \phi(g) \eta(r)$
we arrive at the final solution:

$$\text{.4} \quad \Psi_e(g, r) = \phi(g) \eta(r - gt)$$

or, translating into Square Amplitudes ("Probabilities")

$$.5 \quad P_t(z, r) = P_1(z) P_2(r - tz)$$

$$\text{where } P_1(z) = \phi^*(z) \phi(z), \quad P_2(r) = N^*(r) N(r)$$

and we note that for a fixed time, t , the distribution of r is translated by an amount depending upon the value of z , while the marginal distribution of z is unchanged. We see that a correlation has been introduced between z and r by this interaction. It is instructive to see quantitatively how fast the correlation takes place. We note that:

$$\begin{aligned} .6 \quad I_{QR}(t) &= \int P_t(z, r) \ln P_t(z, r) dz dr = \int P_1(z) P_2(r - tz) \ln P_1(z) P_2(r - tz) dz dr \\ &\quad \text{changing variables so that } \omega = r - tz \text{ for } r \\ &= \int \left[\int P_1(z) P_2(\omega) \ln P_1(z) P_2(\omega) d\omega \right] dz \\ &= I_{QR}(0) \end{aligned}$$

So that the information in the joint distribution does not change. Furthermore, since the marginal distribution for z is unchanged, $I_Q(t) = I_Q(0)$, and the only quantity which can change is the marginal information $I_R(t)$ which is:

$$.7 \quad I_R(t) = I \left\{ P_t(r) = \int_{-\infty}^{\infty} P_t(z, r) dz = \int_{-\infty}^{\infty} P_1(z) P_2(r - tz) dz \right\}$$

using theorem (T-6-Appendix) this implies that

$$.8 \quad I_R(t) \leq I_R(0) - \ln t$$

so that, except for an additive constant for initial information I_R tends to diminish at least as fast as $\ln t$ with time. This implies that for the correlation:

$$.9 \quad \{Q, R\}_t = I_{RQ}^{(t)} - I_R^{(t)} I_Q^{(t)} = I_{RQ}(0) - I_Q(0) - I_R(t)$$

but at $t=0$ the distributions for R and Q were independent so that $I_{RQ}(0) = I_R(0) + I_Q(0)$, and substitution of this relation together with .8 into .9 results in:

$$.10 \quad \{Q, R\}_t \geq \frac{\ln t}{I_R^{(0)} - I_Q^{(0)} + \ln t \text{ (at present)}} \quad (\text{check this})$$

See section

We notice that this measurement has the property that it does not change the marginal system distribution nor does the apparatus indicate any definite system value. However, one can look at the combined wave function as a superposition of states, consisting definite of values, with correspondingly displaced apparatus states — i.e. (C.F. discussion in III, but § discussion of relative WF's.)

$$.11 \quad \Psi = \int \phi(z') \delta(z-z') N(r-z't) dz'$$

which is a superposition of states $\Psi_{z'} = \delta(z-z') N(r-z't)$ in which the system has the definite eigenvalue z' ; and in which the apparatus is displaced from its original position by an amount $z't$; superposed with amplitude $\phi(z')$.

Conversely, if we transform to the representation where the apparatus is in the definite state we write:

$$.12 \quad \psi = \int \frac{1}{N_{r'}} \xi_{r'}(q) \delta(r-r') dr'$$

$$\text{where } \xi_{r'}(q) = N_{r'} \phi(q) \mathcal{N}(r'-qt), \quad \frac{1}{N_{r'}} = \sqrt{\int \phi^*(q) \mathcal{N}^*(r') dr'}$$

where the functions $\xi_{r'}(q)$ are nearly eigenstates for $q = \frac{r'}{t}$, i.e. as $t \rightarrow \infty$, or if $\mathcal{N}(r')$ is sufficiently peaked (near $\delta(r')$) then $\xi_{r'}(q)$ approaches $\delta(q - \frac{r'}{t})$. Hence the state of the combined system can be regarded as a superposition of states each of which corresponds to a definite apparatus value, r' , and for which the system state is nearly the eigenfunction corresponding to $q = \frac{r'}{t}$. In other words we are confronted with a superposition of apparatus, each of which has recorded the definite value, and relative to which the system is left in an approximate eigenstate of the measurement. The discontinuous "jump" into one eigenstate is then only a relative proposition, dependent upon our method of decomposition of the total wave function into the superposition, and relative to a particularly chosen apparatus variable. So far as the complete theory is concerned all elements of the superposition exist simultaneously, and the entire process was quite continuous.

Extend immediately to General Observer, formalized as a servomechanism with memory.

remarks upon the generality of this result:

namely for any interaction

which sends eigenstates ϕ_i system state N

$$\phi \cdot N \rightarrow u_i \bar{N}^i$$

then sends $(\sum_i p_i \phi_i) N$

into $\rightarrow \sum_i \alpha_i u_i \bar{N}^i$

That is if any particular results are being
to be expected when the system is in one
or another of a set of eigenstates ϕ_i .

then the result for system state $\sum_i \alpha_i \phi_i$

will simply be the superposition of the special results
superposed with amplitude α_i .

This is completely general, and depends
only upon the linearity of the wave equation,
i.e. the superposition principle for state functions
satisfying the wave equation. Hence it holds
for interactions in any quantum theory, field theories
or what have you.

This in turn implies that any situation for which an observer will observe a definite system value when the system is in an eigenstate is such that if the system is not in an eigenstate the result of observation will be a superposition of observers who observe each particular system value, superposed with the system superposed amplitude. This is an inescapable conclusion of Quantum mechanical treatment of observers.