

93. Reversibility and Irreversibility

Let us return, for the moment, to the probabilistic interpretation of quantum mechanics based on Process 1 as well as Process 2. Suppose that we have a large number of identical systems (ensemble), and that the fraction of systems in the state ψ_j is p_j and that the j^{th} system is in the state ψ_j . Then for purposes of calculating expectation values for operators over the ensemble, the ensemble is represented by the mixture of states ψ_j weighted with $\frac{1}{N}$, where N is the number of systems, for which the density operator is

$$(3.1) \quad \rho = \frac{1}{N} \sum_i [\psi_i]$$

where $[\psi_i]$ denotes the projection operator on ψ_i . This density operator, in turn, is equivalent to a density operator which is a sum of projections on orthogonal states (the eigenstates of ρ):²

$$(3.2) \quad \rho = \sum_i p_i [\eta_i] \quad , \quad (\eta_i, \eta_j) = \delta_{ij} \quad , \quad \sum_i p_i = 1$$

So that any ensemble is always equivalent to a mixture of orthogonal states, which representation we shall henceforth assume.

Suppose that a quantity A_j with (non-degenerate) eigenstates $\{\phi_j\}$ is measured in each system of the ensemble. This measurement has the effect of transforming each state η_i into the state ϕ_j with probability $|(\phi_j, \eta_i)|^2$; i.e. it will transform ^{large} ensemble of systems in the state η_i into an ensemble represented by the mixture whose density operator is $\sum_i |(\phi_j, \eta_i)|^2 [\phi_j]$. Extending this result

to the case where the original ensemble is a mixture of the N_i weighted by P_i ((3.2)), we find that the density operator ρ is transformed by the measurement of A into the new density operator ρ' :

$$(3.3) \quad \begin{aligned} \rho' &= \sum_i P_i \sum_j |(N_i, \phi_j)|^2 [\phi_j] = \sum_j \left(\sum_i P_i (\phi_j, (N_i, \phi_j) N_i) \right) [\phi_j] \\ &= \sum_j (\phi_j, \sum_i P_i [N_i] \phi_j) [\phi_j] = \sum_j (\phi_j, \rho \phi_j) [\phi_j] \end{aligned}$$

This is the general law by which mixtures change through Process 1.

However, even when no measurements are taking place the states of an ensemble are changing according to Process 2, so that after a time interval t each state ψ will be transformed into the state $\psi' = U_t \psi$, where U_t is a unitary operator. This natural motion has the consequence that each mixture $\rho = \sum_i P_i [N_i]$ is carried into the mixture $\rho' = \sum_i P_i [U_t N_i]$ after a time t . But for every state ξ ,

$$(3.4) \quad \begin{aligned} \rho' \xi &= \sum_i P_i [U_t N_i] \xi = \sum_i P_i (U_t N_i \xi) U_t N_i \\ &= U_t \sum_i P_i (N_i U_t^{-1} \xi) N_i = U_t \sum_i P_i [N_i] (U_t^{-1} \xi) \\ &= (U_t \rho U_t^{-1}) \xi \end{aligned}$$

Therefore

$$(3.5) \quad \rho' = U_t \rho U_t^{-1}$$

which is the general law for the change of a mixture according to process 2.

We are now interested in whether or not we get from any mixture to another by means of these two processes, i.e., if for any pairs P, P' , there exist quantities A which can be measured and unitary (time dependence) operators V such that P can be transformed into P' by suitable applications of Processes 1 and 2. We shall see that this is not always possible, and that Process 1 can cause irreversible changes in mixtures.

For each mixture P we define a quantity I_P :

$$(3.6) \quad I_P = \text{Trace}(P \ln P)$$

This number, I_P , has the character of information. If $P = \sum_i P_i [N_i]$, a mixture of orthogonal states N_i weighted with P_i , then I_P is simply the information of the distribution P_i over the eigenstates of P (relative to the uniform measure). ($\text{Trace}(P \ln P)$ is a unitary invariant and is proportional to the negative of the entropy of the mixture, as discussed in II-§2.)

Process 2 therefore has the property that it leaves I_P unchanged, because

$$(3.7) \quad \begin{aligned} I_{P'} &= \text{Trace}(P' \ln P') = \text{Trace}(V_t P V_t^{-1} \ln V_t P V_t^{-1}) \\ &= \text{Trace}(V_t P \ln P V_t^{-1}) = \text{Trace}(P \ln P) = I_P \end{aligned}$$

Process 1, on the other hand, can decrease I_P but never increase it. According to (3.3):

$$(3.8) \quad \rho' = \sum_j (\phi_{ij}, \rho \phi_j) [\phi_j] = \sum_{i,j} p_i |(n_i, \phi_j)|^2 [\phi_j]$$

$$= \sum_j p'_j [\phi_j]$$

where $p'_j = \sum_i p_i T_{ij}$ and $T_{ij} = |(n_i, \phi_j)|^2$ is a doubly-stochastic matrix³. But $I_{\rho'} = \sum_j p'_j \ln p'_j$ and $I_\rho = \sum_i p_i \ln p_i$, with the p_i, p'_j connected by T_{ij} , implies by the theorem of information decrease for stochastic processes (II-86) that:

$$(3.9) \quad I_{\rho'} \leq I_\rho$$

Moreover, it can easily be shown by a slight strengthening of the theorem of II-6 that strict inequality must hold unless (for each i such that $p_i > 0$) $T_{ij} = 1$ for one j and 0 for the rest ($T_{ij} = \delta_{ik_j}$). This means that $|(n_i, \phi_j)|^2 = \delta_{ik_j}$, which implies that $n_i = \phi_{k_j}$ -- which says that the original mixture was already a mixture of eigenstates of the measurement.

~~Therefore~~ We have answered our question, and it is not possible to get from any mixture to another by means of processes 1 and 2. There is an essential irreversibility to process 1, since it corresponds to a stochastic process, which cannot be compensated by process 2, which is reversible like classical mechanics?

Our theory of pure wave mechanics, to which we now return, must give equivalent results on the subjective level, since it leads to Process 1 there. Therefore measuring processes will appear to be irreversible to any observers (even though the composite system including the observer changes its state reversibly).

There is another way of looking at this apparent irreversibility within our theory which recognizes only process 2. When an observer performs an observation the result is a superposition, each element of which describes an observer who has perceived a particular value. From this time forward there is no interaction between the separate elements of the superposition (which describe the observer as having perceived different results), since each element separately obeys the wave equation ^{continues to}. Each observer described by a particular element of the superposition behaves in the future completely independently of any events in the remaining elements, and he can no longer obtain any information whatsoever concerning these other elements (they are completely unobservable to him).

The irreversibility of the measuring process is therefore, within our framework, simply a subjective manifestation reflecting the fact that in observation processes the state of the observer is transformed into a superposition of observer states, each element of which describes an observer who is irrevocably cut off from the remaining elements. While it is conceivable that some outside agency could reverse the total wave function, such a change cannot be brought about by any observer which is represented by a single element of a superposition, since he is entirely powerless to have any influence on any other elements.

restrictions

There are therefore fundamental restrictions to the knowledge that an observer can obtain about the state of the universe. It is impossible for any observer to discover the total state function of any physical system, since the process of observation itself leaves no ^{independent} states for the system or the observer, but only a composite system state in which the ~~system~~ object-system states are inevitably bound ~~with~~ up with the observer state. As soon as the observation is performed the composite state is split into a superposition for which each element describes a different object-system state and an observer with (different) knowledge of it. Only the totality of these observer states, with their diverse knowledge, contains complete information about ^{the} original object-system state, - but there is no possible communication between the observers described by these separate states. Any single observer can therefore possess knowledge only of the relative state function (relative to his state) ^{of any system}, of which is in any case all that is of any importance to him.

We conclude this section by commenting on another question which might be raised concerning irreversible processes: Is it necessary for the existence of measuring apparatus, ^{which can be correlated to other systems}, to have frictional processes which involve systems of a large number of degrees of freedom? Are such thermodynamically irreversible processes possible in the framework of pure wave mechanics with a reversible wave equation, and if so does this circumstance pose any difficulties for our treatment of measuring processes?

In the first place it is certainly not necessary for dissipative processes involving additional degrees of freedom to be present before an interaction which correlates an apparatus to an object-system can take place.

The counter-example is supplied by the simplified measuring process of III-§3, which involves only a system of one coordinate and an apparatus of one coordinate and no further degrees of freedom, supplies the counter-example.

To the question of whether such processes are possible within reversible wave mechanics we answer yes, in the same sense that they are present in classical mechanics, where the microscopic equations of motion are also reversible. This type of irreversibility, which might be called Macroscopic irreversibility, arises from a failure to separate "macroscopically indistinguishable" states into "true" microscopic states.⁵ It has a fundamentally different character than the irreversibility of Process 1, which applies to micro-states as well and is peculiar to quantum mechanics. Macroscopic irreversible phenomena are common to both classical and quantum mechanics, since they arise from our incomplete information concerning a system, not from any intrinsic behavior of the system.^{6,7}

Finally, even when such fictional processes are involved they present no new difficulties for the treatment of measuring and observation processes given here. We imposed no restrictions on the complexity or number of degrees of freedom of measuring apparatus or observers, and if any of these processes are present (such as heat reservoirs, etc.) then these systems

are to be simply included as part of the apparatus or observer.