

Everett's marginalia in: DeWitt, Bryce S. and Graham, Neill, ed. The Many-Worlds Interpretation of Quantum Mechanics: a fundamental exposition by HUGH EVERETT, III, with papers by J. A. WHEELER, B. S. DEWITT, L. N. COOPER AND D. VAN VECHTEN, and N. GRAHAM. Princeton University Press. Princeton, New Jersey. 1973.

P.55: Typo corrected

P.161: Everett emphasizes a paragraph with a vertical line beside it

P. 185: ! Only to you! [you underlined]

Responding to B. S. De Witt's comment: "Everett's original derivation of this result [I] invokes the formal equivalence of measure theory and probability theory, and is rather too brief to be entirely satisfying."

P. 21^q~~z~~: Underlined sentence in a footnote, words insert [? "nor less than 17"], exclamation point

P. 21²~~q~~: Typo corrected

P. 236: bullshit

Reacting to Neill Graham: "In short, we criticize Everett's interpretation on the grounds of insufficient motivation. Everett gives no connection between his measure and the actual operations involved in determining a relative frequency, no way in which the value of his measure can actually influence the reading of, say, a particle counter. Furthermore, it is extremely difficult to see what significance such a measure can have when its implications are completely contradicted by a simple count of the worlds involved, worlds that Everett's own work assures us must all be on the same footing."

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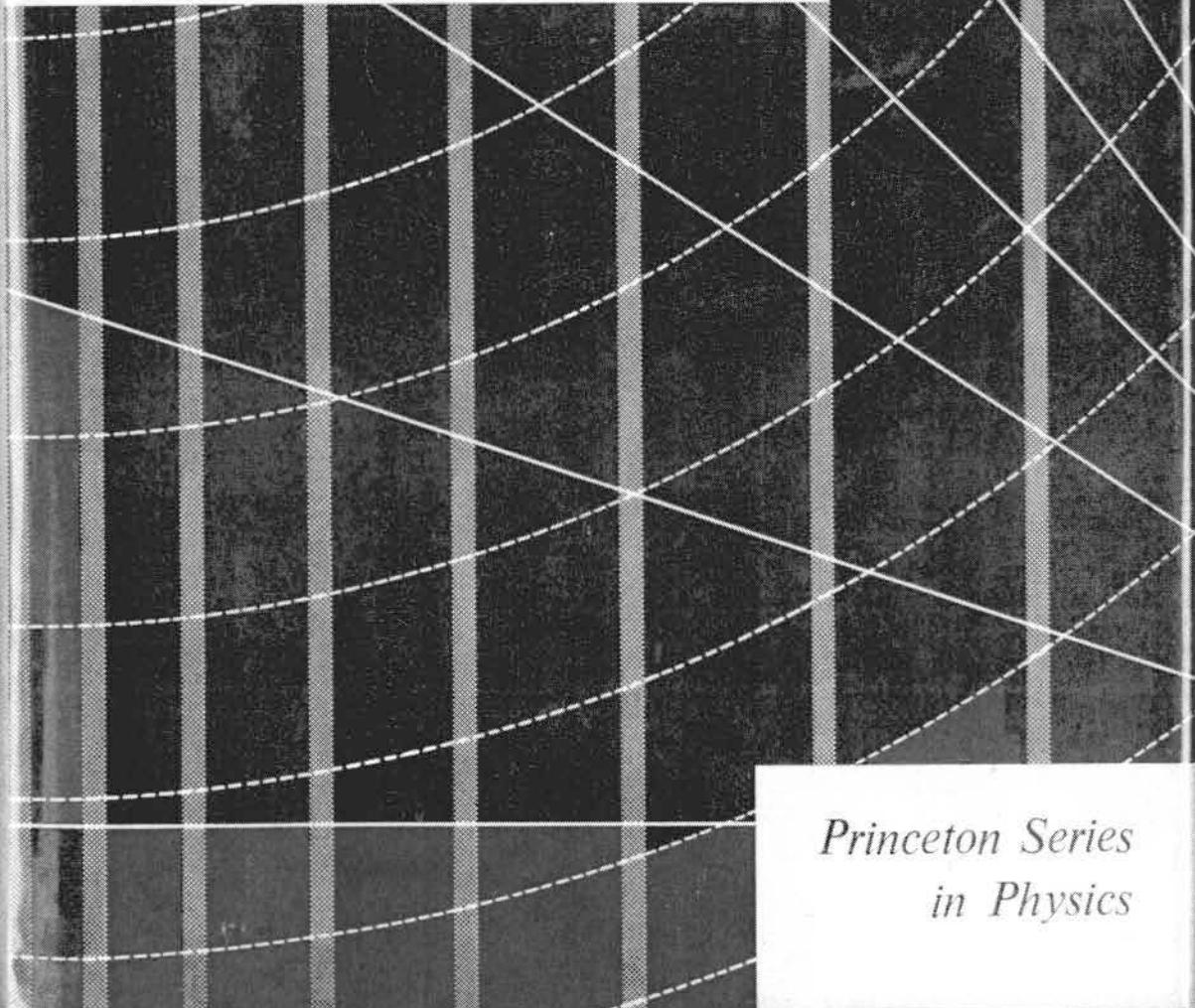
The Many-Worlds
Interpretation
of Quantum Mechanics

edited by

Bryce S. DeWitt and Neill Graham

with papers by

*Hugh Everett, III, J. A. Wheeler,
B. S. DeWitt, L. N. Cooper
and D. van Vechten, and N. Graham*



*Princeton Series
in Physics*

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Hugh Everett III

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never decrease the information in the marginal distribution of A. If H is to produce a measurement of A by correlating it with B, we expect that a knowledge of B shall give us more information about A than we had before the measurement took place, since otherwise the measurement would be useless. Now, H might produce a correlation between A and B by simply destroying the marginal information of A, without improving the expected conditional information of A given B, so that a knowledge of B would give us no more information about A than we possessed originally. Therefore in order to be sure that we will gain information about A by knowing B, when B has become correlated with A, it is necessary that the marginal information about A has not decreased. The expected information gain in this case is assured to be not less than the correlation $\{A, B\}$.

The restriction that H shall not decrease the marginal information of A has the interesting consequence that the eigenstates of A will not be disturbed, i.e., initial states of the form $\psi_0^S = \phi \eta_0$, where ϕ is an eigenfunction of A, must be transformed after any time interval into states of the form $\psi_t^S = \phi \eta_t$, since otherwise the marginal information of A, which was initially perfect, would be decreased. This condition, in turn, is connected with the *repeatability* of measurements, as we shall subsequently see, and could alternately have been chosen as the condition for measurement.

We shall therefore accept the following definition. An interaction H is a measurement of A in S_1 by B in S_2 if H does not destroy the marginal information of A (equivalently: if H does not disturb the eigenstates of A in the above sense) and if furthermore the correlation $\{A, B\}$ increases toward its maximum¹³ with time.

13 The maximum of $\{A, B\}$ is $-I_A$ if A has only a discrete spectrum, and ∞ if it has a continuous spectrum.

Wheeler¹¹ and has been elaborated by R. Neill constitutes the third way of the crisis posed by of infinite regression.

Wheeler and Graham state that the real world, part of it one may wish to regard as *the* world, presented solely by the mathematical objects: a vector space; a set of dyons (derived from a principle) for a set of operators in Hilbert space, and a relation relations for the derived from the Poisson classical theory by the rule, where classical analysis one additional postulate to give physical mathematics. This is complexity: The world is so complicated that it splits into systems and

is based on any external mathematics other than rules of logic, EWG are able to prove some postulates, to prove metatheorem: *The formalism of the quantum theory of yielding its own interpretation*. To prove this metatheorem must answer two questions:

conventional probability of quantum mechanics he formalism itself? Is there correspondence with reality if the state vector?

ce

ing the answers to these questions we note that the conventional interpretation of quantum mechanics is two concepts that really are quite distinct—probability as quantum mechanics and

probability as it is understood in statistical mechanics. Quantum mechanics is a theory that attempts to describe in mathematical language a world in which chance is not a measure of our ignorance but is absolute. It must inevitably lead to states, like that of equation 5, that undergo multiple fission, corresponding to the many possible outcomes of a given measurement. Such behavior is built into the formalism. However, precisely because quantum-mechanical chance is *not* a measure of our ignorance, we ought not to tamper with the state vector merely because we acquire new information as a result of a measurement.

The obstacle to taking such a lofty view of things, of course, is that it forces us to believe in the reality of all the simultaneous worlds represented in the superposition described by equation 5, in each of which the measurement has yielded a different outcome. Nevertheless, this is precisely what EWG would have us believe. According to them the real universe is faithfully represented by a state vector similar to that in equation 5 but of vastly greater complexity. This universe is constantly splitting into a stupendous number of branches, all resulting from the measurementlike interactions between its myriads of components. Moreover, every quantum transition taking place on every star, in every galaxy, in every remote corner of the universe is splitting our local world on earth into myriads of copies of itself.

A splitting universe

I still recall vividly the shock I experienced on first encountering this multiworld concept. The idea of $10^{100} +$ slightly imperfect copies of oneself all constantly splitting into further copies, which ultimately become unrecognizable, is not easy to reconcile with common sense. Here is schizophrenia with a vengeance. How pale in comparison is the mental state of the imaginary

friend, described by Wigner,⁵ who is hanging in suspended animation between only two possible outcomes of a quantum measurement. Here we must surely protest. None of us feels like Wigner's friend. We do not split in two, let alone into $10^{100} +$! To this EWG reply: To the extent that we can be regarded simply as automata and hence on a par with ordinary measuring apparatuses, the laws of quantum mechanics do not allow us to feel the splits.

A good way to prove this assertion is to begin by asking what would happen, in the case of the measurement described earlier by equations 4 and 5, if one introduced a second apparatus that not only looks at the memory bank of the first apparatus but also carries out an independent direct check on the value of the system observable. If the splitting of the universe is to be unobservable the results had better agree.

The couplings necessary to accomplish the desired measurements are readily set up. The final result is as follows (see reference 13): The state vector at the end of the coupling interval again takes the form of a linear superposition of vectors, each of which represents the system observable as having assumed one of its possible values. Although the value varies from one element of the superposition to another, not only do both apparatuses within a given element observe the value appropriate to that element, but also, by straightforward communication, they agree that the results of their observations are identical. The splitting into branches is thus unobserved.

Probability interpretation

We must still discuss the questions of the coefficients c_s in equations 5 and 6. EWG give no *a priori* interpretation to these coefficients. In order to find an interpretation they introduce an apparatus that makes repeated measure-

Then, making use of (4.13), we find

$$(4.16) \quad \langle \chi_N^\varepsilon | \chi_N^\varepsilon \rangle = \langle \Psi | \Psi \rangle \sum_{\substack{s_1, \dots \\ \delta(s_1, \dots, s_N) \geq \varepsilon}} w_{s_1} w_{s_2} \dots = \langle \Psi | \Psi \rangle \sum_{\substack{s_1, \dots, s_N \\ \delta(s_1, \dots, s_N) \geq \varepsilon}} w_{s_1} \dots w_{s_N}$$

$$\leq \frac{1}{\varepsilon} \langle \Psi | \Psi \rangle \sum_{s_1, \dots, s_N} \delta(s_1, \dots, s_N) w_{s_1} \dots w_{s_N} = \frac{1}{N\varepsilon} \langle \Psi | \Psi \rangle \sum_s w_s (1 - w_s) \leq \frac{1}{N\varepsilon} \langle \Psi | \Psi \rangle.$$

$[_1 \dots \bar{A}_N, A_{N+1} \dots]$,

exactly the same state vectors and values for the superposition (4.8). The values for the

ideal analysis. The on of the *relative*

this function (see

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ne degree to which h weights w_s . Let he sequence $s_1 \dots s_N$.

l those elements for . Denote the result

$|s_1 \dots s_N\rangle$.

From this it follows that no matter how small we choose ε we can always find an N big enough so that the norm of $|\chi_N^\varepsilon\rangle$ becomes smaller than any positive number. This means that

$$(4.17) \quad \lim_{N \rightarrow \infty} |\Psi_N^\varepsilon\rangle = |\Psi\rangle.$$

It will be noted that, because of the orthogonality of the basis vectors $|s_1\rangle |s_2\rangle \dots$, this result holds regardless of the quality of the measurements, *i.e.* independently of whether or not the condition

$$(4.18) \quad \langle \Phi[s_1 \dots s_N] | \Phi[s'_1 \dots s'_N] \rangle = \langle \Phi | \Phi \rangle \prod_{n=1}^N \delta_{s_n s'_n}$$

for good measurements is satisfied.

A similar result is obtained if $|\Psi_N^\varepsilon\rangle$ is redefined by excluding, in addition, elements of the superposition (4.2) whose memory sequences fail to meet any finite combination of the infinity of other requirements for a random sequence. Moreover, no other choice for the w 's but (2.4) will work. *The conventional statistical interpretation of quantum mechanics thus emerges from the formalism itself.* Nonrandom memory sequences in the superposition (4.8) are of measure zero in the Hilbert space, in the limit $N \rightarrow \infty$ (*). Each automaton (that is, apparatus *cum* memory sequence) in the superposition sees the world obey the familiar statistical quantum laws. This conclusion obviously admits of immediate extension to the world of cosmology. Its state vector is like a tree with an enormous number of branches. Each branch corresponds to a possible universe-as-we-actually-see-it.

The alert student may now object that the above argument contains an element of circularity. In order to derive the *physical* probability interpreta-

(*) Everett's original derivation of this result [1] invokes the formal equivalence of measure theory and probability theory, and is rather too brief to be entirely satisfying. The present derivation is essentially due to GRAHAM [7] (see also ref. [8]). A more rigorous treatment of the statistical interpretation question, which deals carefully with the problem of defining the Hilbert space in the limit $N \rightarrow \infty$, has been given by HARTLE [9].

I think to go on

For simplicity let us consider the special case in which the commutator $[r, s]$ (or, equivalently, the Poisson bracket (r, s)) is a multiple of the identity operator, and \mathcal{X} and \mathcal{Y} have the forms (10.9), with (10.10) and (10.11) holding. The coupling (10.1) then transforms the undisturbed apparatus observables A and B into the disturbed observables:

$$(11.4) \quad \bar{A} = \exp[-ig(\mathcal{X} + \mathcal{Y})]A\exp[ig(\mathcal{X} + \mathcal{Y})] = A + gr - \frac{1}{2}g^2(r, s)Y,$$

$$(11.5) \quad \bar{B} = \exp[-ig(\mathcal{X} + \mathcal{Y})]B\exp[ig(\mathcal{X} + \mathcal{Y})] = B + gs + \frac{1}{2}g^2(r, s)X.$$

It will be convenient to introduce also the following operators:

$$(11.6) \quad \hat{r} = \exp[-ig(\mathcal{X} + \mathcal{Y})]r\exp[ig(\mathcal{X} + \mathcal{Y})] = r - g(r, s)Y,$$

$$(11.7) \quad \hat{s} = \exp[-ig(\mathcal{X} + \mathcal{Y})]s\exp[ig(\mathcal{X} + \mathcal{Y})] = s + g(r, s)X.$$

(These are *not* equal to the disturbed operators \bar{r} and \bar{s} respectively.)

Now note that one can obtain eight distinct commuting triplets of operators by choosing one from each of the pairs $\{r, s\}$, $\{A, X\}$, $\{B, Y\}$. Any one of the eight possible Hilbert-space bases determined by these triplets can be used in the description of the undisturbed state of the system-plus-apparatus. Because

$$(11.8) \quad \begin{cases} \exp[-ig(\mathcal{X} + \mathcal{Y})]X\exp[ig(\mathcal{X} + \mathcal{Y})] = X, \\ \exp[-ig(\mathcal{X} + \mathcal{Y})]Y\exp[ig(\mathcal{X} + \mathcal{Y})] = Y, \end{cases}$$

it follows that eight distinct commuting triplets of operators can also be obtained by choosing one from each of the pairs $\{\hat{r}, \hat{s}\}$, $\{\bar{A}, X\}$, $\{\bar{B}, Y\}$. Any one of the eight possible Hilbert-space bases determined by these triplets can be used in the description of the *disturbed* state of the system plus apparatus. Because the pairs $\{A, X\}$, $\{B, Y\}$, $\{\bar{A}, X\}$, $\{\bar{B}, Y\}$ are conjugate pairs, the transformation coefficients between the various bases may be taken in the forms

$$(11.9) \quad \langle X|A\rangle = \langle X|\bar{A}\rangle = (2\pi)^{-\frac{1}{2}}\exp[iXA] = (2\pi)^{-\frac{1}{2}}\exp[iX\bar{A}],$$

$$(11.10) \quad \langle Y|B\rangle = \langle Y|\bar{B}\rangle = (2\pi)^{-\frac{1}{2}}\exp[iYB] = (2\pi)^{-\frac{1}{2}}\exp[iY\bar{B}].$$

Let us assume an uncorrelated initial state for the system and apparatus, so that the total state vector takes the form

$$(11.11) \quad |\Psi\rangle = |\psi\rangle|\Phi\rangle,$$

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Then we have als

$$(11.13) \quad \langle A, Y|\Psi\rangle$$

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$$(11.18) \quad |\hat{r}, A\rangle$$

or, equivalently,

$$(11.19) \quad |\hat{s}, \bar{A}\rangle$$

In a similar man

$$(11.20) \quad |\hat{s}, X\rangle$$

$$(11.21) \quad |\hat{s}, \bar{X}\rangle$$

with $s = \hat{s}$, $B = \bar{B}$

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$$\begin{aligned} & {}^l G^{-im} G^{+kn}, \\ & \downarrow \\ & {}^m G^{-kn} G^{+il}. \end{aligned}$$

third term in (C.1)
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On the Interpretation of Measurement within the Quantum Theory*

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(Received 26 May 1969)

An interpretation of the process of measurement is proposed which can be placed wholly within the quantum theory. The entire system including the apparatus and even the mind of the observer can be considered to develop according to the Schrödinger equation. No separation, in principle, of the observer and the observed is necessary; nor is it necessary to introduce either the type I process of von Neumann or wave function reduction.

INTRODUCTION

Although the structure of the quantum theory in the opinion of almost all physicists is free from contradiction, questions about the consistency of its interpretation have been and continue to be posed. The view expressed in most texts and taught in many classes derives from the work of von Neumann¹ in the early thirties; it implies what is close to a Cartesian dualism dividing mind and body which, though consistent (and perhaps even respectable in the 17th century^{2,3}), seems somewhat of an anachronism at present.^{4,5}

* Supported in part by the Advanced Research Projects Agency, the National Science Foundation and the U. S. Atomic Energy Commission.

¹ J. von Neumann, *Mathematical Foundations of Quantum Mechanics*, transl. by Robert T. Beyer (Princeton University Press, Princeton, N. J., 1965).

² It was not accepted universally even then. Spinoza, for example, objected saying: "he [Descartes] accomplishes nothing beyond a display of the acuteness of his own great intellect."

³ B. Spinoza, *The Ethics*, D. D. Runes, Ed. (Wisdom Library, division of The Philosophical Library, New York, 1957), p. 24.

⁴ The number of papers on this subject is large and the individual contributions not always easy to understand. We have not made an exhaustive study of the literature and make no claim that every concept presented is written down for the first time; however we have never seen the entire matter discussed in this light. Our own primary references were Wigner's 1963 article, von Neumann's book, and what a somewhat reversible memory told us we had read and been taught over the years. We were directed to Prof. K. Gottfried's excellent discussion in his book on Quantum Mechanics (Ref. 5) somewhat later. There he has stated the relation between measurement and irreversibility in a very clear and elegant fashion. We would like to express our gratitude to Prof. H. P. Stapp for a very interesting correspondence and for bringing our attention to an article of Hugh Everett III, Rev. Mod. Phys. **29**, 454 (1957). Everett, whose views do not seem to be generally known, recognizes the necessity of retaining all branches of the wave-function; in this respect his ideas are quite similar to our own.

Just as Everett we retain all branches of the wave function. However, it is not the wave function itself which is put into

Von Neumann proposed that the interpretation of measurement—or the means by which we come to know that something has happened—requires a process which does not develop according to the Schrödinger equation. He says:⁶ "We therefore have two fundamentally different types of interventions which can occur in a system S or in an ensemble (S_1, \dots, S_N) . First, the arbitrary changes by measurements which are given by the formula

$$U \rightarrow U' = \sum_{n=1}^{\infty} (U\phi_n, \phi_n) P_{\{\phi_n\}}. \quad (I)$$

Second, the automatic changes which occur with passage of time. These are given by the formula

$$U \rightarrow U_t = \exp\{-2\pi i/h)tH\} U \exp\{(2\pi i/h)tH\}. \quad (II)$$

Further:⁷...we must always divide the world into two parts, the one being the observed system, the other the observer. In the former, we can follow up all physical processes (in principle at least) arbitrarily precisely. In the latter, this is meaningless."

Wigner recently has written:⁸ "...one must conclude that the only known theory of measurement which has a solid foundation is the orthodox one and that this implies the dualistic theory concerning the changes of the state vector. It implies, in particular, the so-called reduction of the state vector."

correspondence with experience. Rather this correspondence is made via the amplitude 35. Thus there are amplitudes which give the probability for any particular sequence of events that might constitute an evolving world. These is nothing, however, which necessitates that more than one of these come to pass. (Last paragraph of footnote added November 1972.)

⁶ K. Gottfried, *Quantum Mechanics*, (W. A. Benjamin, Inc., New York, 1966), vol. I, pp. 165-189.

⁷ See Ref. 1, p. 351.

⁸ See Ref. 1, p. 420.

⁹ E. P. Wigner, Amer. J. Phys. **31**, 6 (1963), p. 12.

If we again refer to Table 1.1, we see that the measure of those worlds with R successes does indeed have a peak at $R = NP = 36$. But this is of little comfort when we observe the degree to which these worlds are in a numerical minority.

In short, we criticize Everett's interpretation on the grounds of insufficient motivation. Everett gives no connection between his measure and the actual operations involved in determining a relative frequency, no way in which the value of his measure can actually influence the reading of, say, a particle counter. Furthermore, it is extremely difficult to see what significance such a measure can have when its implications are completely contradicted by a simple count of the worlds involved, worlds that Everett's own work assures us must all be on the same footing.

(To be sure Everett argues that the measure defined by (1.13) is unique. But remember that Gleason [8] has shown that the probabilities defined by the Born interpretation, considered as a measure on a Hilbert space, are themselves unique. Nevertheless, this (hopefully) does not deter anyone from inquiring into the connection between those probabilities and experiments that measure relative frequency.)

It thus appears that, in the "one step" measurement we have described, any attempt to show that the probability interpretation holds in the majority of the resulting Everett worlds is doomed to failure. As mentioned earlier, we shall attempt to improve matters by considering instead a "two step" measurement, in which a macroscopic apparatus mediates between a microscopic system and a macroscopic observer. To better motivate the technical work that follows, we now give a brief outline of this approach.

To begin, define the relative frequency of ℓ in the sequence m_1, \dots, m_N by

$$f_\ell(m_1, \dots, m_N) = \frac{1}{N} \sum_{i=1}^N \delta_{m_i \ell}. \quad (1.15)$$

Since relative frequency is an observable, on the same footing with any other observable in quantum mechanics, we may associate with it a Hermitean operator. We define this operator, F_ℓ , in the obvious way:

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