

## § 4 Several observers

We shall now consider the consequences of our scheme when several observers are allowed to interact with the same system, as well as with one another (communication). In the following discussion observers shall be denoted by  $O_1, O_2, \dots$ , other systems by  $S_1, S_2, \dots$ , and observables by operators  $A, B, C, \dots$ , with eigenfunctions  $\{\phi_i\}, \{n_j\}, \{\xi_k\}$  respectively.

The symbols  $\alpha_i, \beta_j, \gamma_k, \dots$  occurring in memory sequences shall refer to characteristics of the states  $\phi_i, n_j, \xi_k, \dots$  respectively. ( $\psi_{[... \alpha_i]}^{O_j}$  is interpreted as describing an observer,  $O_j$ , who has just observed the eigenvalue corresponding to  $\phi_i$  (i.e., who is "aware" that the system is in state  $\phi_i$ ).

We shall also wish to allow communication among the observers, which we view as an interaction by which the memory sequences of different observers become correlated. (For example the transfer of impulses from the magnetic tape of a mechanical observer to that of another constitutes such a transfer of information.) We shall regard these processes as observations made by one observer on another and shall use the notation that

$$\psi_{[... \alpha_i]}^{O_j}$$

represents a state function describing an observer  $O_j$  who has obtained the information  $\alpha_i$  from another observer,  $O_k$ . Thus the obtaining of information about  $A$  from  $O_1$  by  $O_2$  will transform the state

$$\psi_{[... \alpha_i]}^{O_1} \psi_{[...]}^{O_2}$$

$$\psi_{[... \alpha_i]}^{O_1} \psi_{[... \alpha_i]}^{O_2}$$

(Put this below  
into the state)

into the state

(4.1)

(No number)

Rules 1 and 2 are, of course, equally applicable to these interactions.

We shall now illustrate the possibilities for several observers, by considering several cases.

Case 1: We allow two observers to separately observe  
the same quantity in a system, and then  
compare results. [We suppose that first observer  $O_1$   
observes the quantity  $A$  for the system  $S$ . Then  
by rule 1 the original state  
 $\psi^{S+O_1 O_2} = \psi^S \psi^{O_2} \psi^{O_2}$

regular  
(state  
precept)

is transformed into the state

$$(4.2) \quad \psi' = \sum_i d_i \phi_i \psi^{O_1} \psi^{O_2}$$

We now suppose that  $O_2$  observes  $A$ , and  
by rule 2 the state becomes;

$$(4.3) \quad \psi'' = \sum_i d_i \phi_i \psi^{O_1} \psi^{O_2}$$

We now allow  $O_2$  to "consult"  $O_1$ , which  
leads in the same fashion, from (4.3) and rule 2, to the final state

$$(4.4) \quad \psi''' = \sum_i d_i \phi_i \psi^{O_1} \psi^{O_2}$$

Thus for every element of the superposition  
the information obtained from  $O_1$  agrees with that  
obtained directly from the system. This means  
that observers who have separately observed  
the same quantity will always agree with  
each other.

It is furthermore obvious at this point that the same result, (4.1), is obtained if  $O_2$  first consults  $O_1$ , then performs the direct observation, except that the memory sequence <sup>for  $O_2$</sup>  is reversed ( $[\alpha_i, \alpha_i]$  instead of  $[\alpha_i, \alpha_i^{O_1}]$ ). There is still perfect agreement in every element of the superposition. Therefore information obtained from another observer is always reliable, since subsequent direct observation will always verify it. We thus see the central role of correlations <sup>in wave function</sup> for preserving consistency in such cases. This is the transitivity of correlation in these cases (that if  $S_1$  is correlated to  $S_2$ , and  $S_2$  to  $S_3$ , then so is  $S_1$  to  $S_3$ ) which is responsible for this consistency.

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Indirect case 2 | We allow two observers to measure

Regular paragraph | separately two different, non-commuting quantities in the same system. Assume that first  $O_1$  observes A for the system, so that, as before, the state  $\psi^{\text{initial}} \psi^{O_1} \psi^{O_2}$  is transformed to:

$$(4.5) \quad \psi' = \left( \sum_i \alpha_i \phi_i \psi^{O_1}_{[\alpha_i]} \right) \psi^{O_2}_{[E_j]}$$

Now let  $O_2$  determine B for the system, where  $N_j$  are the eigenfunctions of B,  ~~$\sum_i b_i N_j$~~ . Then by application of rule 2 the result is

$$(4.6) \quad \psi'' = \sum_{i,j} \alpha_i (\eta_j, \phi_i) N_j \psi^{O_1}_{[\alpha_i]} \psi^{O_2}_{[\beta_j]}$$

$O_2$  is now perfectly correlated with the system, since a redetermination by him will lead to agreeing results. This is no longer the case for  $O_1$ , however, since a redetermination of  $A$  by him will result in (by rule 2)

$$(4.2) \Psi''' = \sum_{i,j,k} d_i b_j^* \cancel{b_k^*} \phi_k \Psi^{O_2} \Psi^{O_1} \begin{bmatrix} \psi_j \\ \psi_i \end{bmatrix} \begin{bmatrix} x_k \\ x_i \end{bmatrix}$$

Hence the second measurement of  $O_1$  does not in all cases agree with the first, and has been upset by the intervention of  $O_2$ . The statistical relation between the first and second determinations is easily seen to be that of the usual form of quantum theory (Principle 1) where  $O_2$ 's observation is regarded as converting the state  $\phi_j$  into the mixture of states  $\psi_j$ , weighted with  $b_j^* b_j$  upon which  $O_1$  makes his second observation.

Note, however, that this equivalence with the statistical results obtained by considering that  $O_2$ 's observation changes the <sup>system</sup> state into a mixture, holds

We conclude the statistical relation in this case by observing that the method of averaging to measure each element of the expectation value in (4.2) is the same as in (4.1).

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ture only so long as  $O_1$ 's second observation  
is restricted to the system. If he were to  
attempt to simultaneously determine property  
of the system as well as of  $O_2$ , interference  
effects might become important. The description  
of the states relative to  $O_1$ ,<sup>after  $O_2$ 's observation</sup> as non-interfering  
mixtures is therefore incomplete.

Case 3: We suppose that two systems  $S_1$  and  $S_2$  are correlated but no longer interacting, and that  $O_1$  measures property A in  $S_1$ , and  $O_2$  property B in  $S_2$ . We wish to see whether  $O_2$ 's intervention with

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$S_2$  can in any way affect  $O_1$ 's results in  $S_1$ , so that perhaps signals might be sent by these means.

← We shall assume that the initial state for the system pair is

$$(4.9) \quad \psi^{S_1 S_2} = \sum_i \alpha_i \phi_i^{S_1} \phi_i^{S_2}$$

We now allow  $O_1$  to observe A in  $S_1$ , so that after this observation the total state becomes:

$$(4.10) \quad \psi'_{S_1 S_2 O_1 O_2} = \sum_i \alpha_i \phi_i^{S_1} \phi_i^{S_2} \psi^{O_1} \psi^{O_2} \begin{bmatrix} \alpha_i \\ [..] \end{bmatrix} \begin{bmatrix} [..] \end{bmatrix}$$

$O_1$  can of course continue to repeat the determination, obtaining the same result each time.

We now suppose that  $O_2$  determines B in  $S_2$ , which results in

$$(4.11) \quad \psi'' = \sum_{i,j} \alpha_i \left( \frac{\phi_j}{\phi_i} \right) \phi_i^1 \eta_j^2 \begin{bmatrix} \alpha_i \\ [\phi_i] \end{bmatrix} \begin{bmatrix} \eta_j^2 \\ [\phi_j] \end{bmatrix} \psi^{O_1} \psi^{O_2}$$

However in this case, as distinct from case 2, we see that the intervention of  $O_2$  in no way affects  $O_1$ 's determinations, since  $O_1$  is still perfectly correlated to the states  $\phi_i$  of  $S_1$ , and any further observations by  $O_2$  will lead to the same results as the earlier (~~before  $O_2$  intervened~~) observations. Thus each

memory sequence for  $O_1$  continues without change due to  $O_2$ 's observation, and such a scheme could not be used to send any signals.

Furthermore, we see that the result (4.16) is arrived at even in the case that  $O_2$  should make his determination before that of  $O_1$ . Therefore any expectations for the outcome of  $O_1$ 's first observation are in no way affected by whether or not  $O_2$  performs his observation before that of  $O_1$ . ~~or not~~

This is true because the expectation of the outcome for  $O_1$  can be computed from (4.10), which is the same whether or not  $O_2$  performs his measurement before or after  $O_1$ .

It is therefore seen that one observer's observation <sup>upon</sup> ~~on~~ one system of a correlated, but non-interacting pair of systems, has no effect on the remote system, in the sense that the outcome or expected outcome of <sup>any</sup> experiments by another observer on the remote system are not effected. ~~This~~ Paradoxes like that of Einstein-Rosen-Podolski<sup>2</sup> which are concerned with such correlated, non-interacting, systems are thus easily understood in the present scheme.

Many further combinations of several observers and systems can be easily studied in the present framework, and all questions answered by first writing down the final state for the situation with the aid of the rules 1 and 2, and then noticing the relations between the elements of the memory sequences.

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1.

We thus see the central role played by correlations in wave functions for the preservation of consistency in situations where several observers are allowed to consult one another.

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We can deduce the statistical relation between  $O_1$ 's first and second results ( $\alpha_i$  and  $\alpha_k$ ) by our previous method of assigning the measure to the elements of the superposition (4.7). The measure assigned to the  $(i, j, k)^{\text{th}}$  element is then:

(4.8)

$$M_{ijk} = |f_i(N_j, \phi_i)(\phi_k, N_j)|^2$$

This measure is equivalent, in this case, to the probabilities by the orthodox theory (Process 1) where  $O_2$ 's observation is regarded as having converted each state  $\phi_i$  into a non-interfering mixture of states  $N_j$ , weighted with probabilities  $|f_i(N_j, \phi_i)|^2$ , upon which  $O_1$  makes his second observation.