

(Representations)

II Quantum Formalism Mechanics

Q.M. of Composite Systems

intro

Composite systems |

§1 State of composite system (representation with subsystem states
density matrix
relative states) Op. Research

§2 Canonical correlation

Info in P.M.

Mod Uncertainty Principle

§3 Canonical Correlation

Idea of macroscopic states

combine

nearest to eigenstate effect of correlation

§4 Example - Von Neumann (of how measurement can take place!
Example - Bohm include definition of measurement)

§5 Atomic constitution - amplification processes

§6 general Principle of superposition
Def of Measurement - (good "meas.")

Idea of macro state

IV Observation

introduction paragraph

assume states points in Hilbert space

In this section we study some consequences of the quantum mechanical formalism of composite systems -

buoyant 1) measurement as corr. interaction

2) any interaction produces some correlation

3) Cor., if we choose, at any instant put comp system in canon rep. and say that \tilde{A} is measured by \tilde{B} (or vice-versa), since there is exact correspondence.

4) However, this is not most convenient. We think of a measurement as a fixed interaction. Then what is con. corr. depends upon time of interaction. We should, rather, prefer to think of this measurement as an approximation for finite times of interaction, which approached exact meas. as time $\rightarrow \infty$. (This leads to ideal that what is con. corr. in limit is what is measured.)

Repeatability

5) we thus have a requirement that turning on interaction again must bring yet closer to ideal exact meas. (seems to imply a repeatability)
(Should all θ implied by $\lim_{t \rightarrow \infty} \tilde{A}, \tilde{B}$ exist)
 $\text{indep. of initial state}$

6) Let us consider an example ^{due to} of Von Neumann of a simplified requirement.

7) discussion of effect of $\{ \hat{I} \} \hat{I} \rightarrow \hat{I}^{\frac{1}{2}}$

and near to eigen \hat{I} suggests taking $\hat{I}^{\frac{1}{2}}$ as meas. of Nearest neighbor state,

$$\sum_i \phi_i n_i$$

U

$$\Rightarrow \sum_i U(\phi_i n_i)$$

$$\text{must} \rightarrow \sum_i a_i \phi_i$$

U

must be unitary since

unit \Rightarrow orthog preserved

\Rightarrow here also (?)

$$\text{ie } U \phi_i n_i = e^{ia_i} \phi_i U n_i$$

$$\Rightarrow U_{mn,ij} = (\phi_m n_n, \overline{U} \phi_i n_j)$$

$$= (\phi_m, e^{ia_i} \phi_i) (n_n, \overline{U} n_j)$$

$$= \delta_{im} \overline{U}_{nj}$$

$$\frac{\partial}{\partial}$$

$$AB$$

it interacts crosses

$$Y = \sum_i c_i n_i$$

$$\Rightarrow \sum_i c_i \phi_i n_i$$

$$\text{Limit } \bar{U}_t^{-1} \bar{A} \bar{U}_t \text{ at } t \rightarrow \infty \therefore = \bar{A}$$

Lim exists

$$\Rightarrow \bar{U}_t^{-1} \bar{A} \bar{U}_t = \bar{A} \quad \text{for all } \underline{\bar{U}_t}$$

$$\Rightarrow \bar{A} \bar{U}_t = \bar{U}_t \bar{A}$$

$$\Rightarrow [\bar{A}, \bar{U}_t] = 0$$

Requirement is Stationarity

$$\text{or back to } \psi_0 = \sum_i \alpha_i \phi_i n_i \quad \tilde{A}_i, \tilde{B}_i \text{ canonical proj}$$

$$\Rightarrow \langle \tilde{A}_i \tilde{B}_j \rangle \psi_0 = \cancel{\alpha_i^* \alpha_j} \delta_{ij}$$

now,

what for $U\psi_0$?

$$(\psi_0, \tilde{A}_i \tilde{B}_j \psi_0) = \alpha_i^* \alpha_j \delta_{ij}$$

$$= (\psi_0, U \tilde{U}^{-1} \tilde{A}_i \tilde{B}_j \tilde{U} U^{-1} \psi_0) = (U\psi_0, U \tilde{A}_i \tilde{B}_j \tilde{U}^{-1} U^{-1} \psi_0)$$

$$= \langle U \tilde{A}_i \tilde{B}_j \tilde{U}^{-1} \rangle U\psi_0$$

$$\text{therefore, therefore } \langle U \tilde{A}_i \tilde{B}_j \tilde{U}^{-1} \rangle = C_{ij} \text{ Proj for } \alpha_i^* \alpha_j \delta_{ij}$$

$$\text{if } C_{ij} = \bar{A}_i \bar{B}_j \text{ then } \bar{A} \bar{B} \text{ canonical}$$

(except don't require same α_i, α_j)

$$(AB)^n = A^n B^n$$

$$e^x = \sum \frac{x^n}{n!}$$

$$U = e^{iABt} = \sum \frac{(iABt)^n}{n!} = \sum \frac{(i^n(A^n)(B^n)t^n)}{n!}$$

$$\text{if } A\xi = \lambda_i \xi_i$$

$$(AB)^n$$

$$(AB)^{n-1}(A\xi)(B^n)$$

$$= (AB)^{n-1}(A^2\xi)(B^2n)$$

$$\stackrel{\text{ind}}{=} (\underline{A^n\xi})(\underline{B^n\xi}) = A^n B^n$$

$$= \sum \frac{i^n \lambda_i^n \xi_i (B^n n) t^n}{n!}$$

$$= \sum_i f(n)$$

has
prop of
good meas

$$(AB + A_1 B) \xi$$

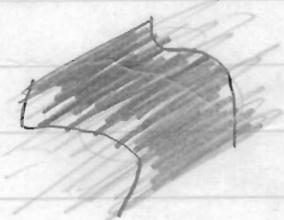
$$= (A_1 \xi) B n + (A_2 \xi) B n$$

$$= (A_1 + A_2) B$$

e.g. if $H = AB$

$$\Rightarrow H(\phi n) = A\phi = \lambda \phi$$

$$= AB(\phi n) = \lambda \phi (B n) = i \frac{\partial}{\partial t} \phi n$$

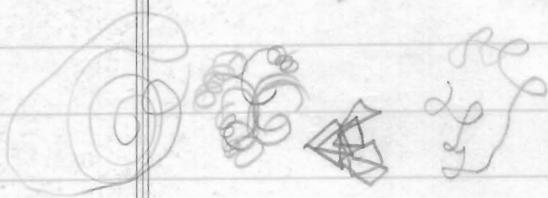


$$Y_0 + \frac{\partial Y}{\partial E} dE$$

$$= \phi n \cdot \lambda \phi (B n)$$

$$= (i \frac{\partial}{\partial t}) \phi (1-\theta) n$$

modifying ϕ



Substate ϕ eigenstate of Composite operator (V)

$$\text{if } U\phi n = \phi (\bar{\phi} n) \quad \text{at } \theta = 0$$

now

$$\psi_0 = \sum_i \phi_i \xi_i \eta_i$$

Eigen of \tilde{A}
 η_i eigen of \tilde{B}

$$\rightarrow (U_i^1)(U^2 \eta_i)$$

$$\rightarrow \sum_i \phi_i (U_i^1) (U^2 \eta_i)$$

$$= \sum_i \phi_i \phi_i \theta_i$$

and still can

with $U^{-1} \tilde{A} U$ $(U^{-1})^2 \tilde{B} U^2$
new commutator

Def: No interaction if $U = U^1 U^2$

$$\text{note } H = e^{-i \frac{Ht}{\hbar}}$$

$$U_1 = e^{-i \frac{H_1 t}{\hbar}}$$

$$U = e^{-i \frac{(H_1 + H_2)t}{\hbar}}$$

$$= e^{-i \frac{H_1 t}{\hbar}} e^{-i \frac{H_2 t}{\hbar}} = \underline{U^1 U^2}$$

(so O.K. for no
interaction)

More general

$$U = e^{-i \frac{AB^2 t}{\hbar}}$$

\downarrow
H arm
 ϵN_y

$$\psi = \sum c_i \xi_i \eta_i$$

$$\xi_t = \{c_{ij} \phi_j\}$$

i.e $(U^t U^2)$ is operator for time dependence

$$U U^2 \{ \eta_i \} = (U^2) (\eta_i)$$

then

$$\psi' = U \psi$$

$$\langle A \rangle \psi' = \langle A \rangle U \psi = (U \psi^0, A U \psi^0)$$

$$= (\psi^0, U^* A U \psi^0)$$

$$U^* = U^{-1}$$

$$= \underbrace{\langle U^* A U \rangle}_{\psi^0}$$

∴

$$\overline{A' A^2 B' B^2} \{ \eta \} = (A' B') \{ \} (B^2 A^2 \eta)$$

Now, what about $U A_i U$

$$\text{i.e } A_i \psi = \lambda_i \psi \quad \psi \text{ eigenstate}$$

$$\Rightarrow U A_i \psi = \lambda_i U \psi$$

$$\Rightarrow U A_i U^{-1} (U \psi) = \lambda_i (U \psi)$$

OK

$$U_t^S \Psi_0^S = \Psi_t^S \quad \text{what operators are in limit case?}$$

• V must be somehow separable.

We know that if $V_t = V_0^1 V^2$ (^{operator}
direct product)
then no interaction (definition)

$$\sum_i \alpha_i \phi_i \Theta_i \rightarrow \sum_i \alpha_i \left(\sum_{k\ell} C_{ijk\ell} \phi_k \Theta_\ell \right)$$

$$\text{since } \phi_i \Theta_i \rightarrow \sum_{k\ell} C_{ijk\ell} \phi_k \Theta_\ell$$

want ~~$\phi_i \Theta_i \rightarrow \{n_i\}$~~ all this only for non-inter.

$$\sum_i \alpha_i \phi_i \Theta_i \rightarrow \sum_{k\ell} \left(\sum_i \alpha_i C_{ijk\ell} \right) \phi_k \Theta_\ell$$

~~ϕ_i~~

given that $S^{-1} A S = D$ diagonalizes A

what diagonalizes $U^\dagger A U$?

$$(S^{-1} U) U^\dagger A U (U^{-1} S) = D$$

∴ answer is if S diagonalizes A $(S^{-1} A S) = D$

then $U^{-1} S$ diag $U^{-1} A U$

$$\rho = \sum_n p_n [\psi_n] = \sum_n p_n (\psi_n, \phi_i) \psi_n$$

$$\begin{aligned} \Rightarrow \rho_{ij} &= (\phi_i, \rho \phi_j) = (\phi_i, \sum_n p_n (\psi_n, \phi_j) \psi_n) \\ &= \sum_n p_n (\psi_n, \phi_i) (\phi_i, \psi_n) \\ &= \sum_n p_n (\phi_j, \psi_n)^* (\phi_i, \psi_n) \end{aligned}$$

alternately :

$$\begin{aligned} \text{Trace } \rho A &= \sum_i (\phi_i, \rho A \phi_i) = \sum_i (\phi_i, A \rho \phi_i) \\ &= \sum_i (\phi_i, A \sum_n p_n (\psi_n, \phi_i) \psi_n) \\ &= \sum_{i,n} p_n (\psi_n, \phi_i) (\phi_i, A \psi_n) \\ &\quad = \sum_n p_n (\psi_n, A \psi_n) \end{aligned}$$

But $\boxed{\text{Theorem } \sum_i (\psi, \phi_i) (\phi_i, n) = (\psi, n)}$

Proof:

$$M = \sum_i (\phi_i, n) \phi_i$$

$$\Rightarrow (\psi, n) = (\psi, \sum_i (\phi_i, n) \phi_i)$$

$$= \sum_i (\phi_i, n) (\psi, \phi_i)$$

$$\text{Trace } \rho = 1$$

$$\text{Trace } \rho \ln \rho$$

is $\rho = \sum w_n [\psi_n]$ valid for $\{\psi_n\}$ not orthonormal?

$$\text{Trace } A = \sum_n (\phi_n, A \phi_n)$$

$$\text{Exp } A = \sum w_n (\psi_n, A \psi_n) = \cancel{\sum_m} \cancel{\phi_m}$$

so

$$\begin{aligned} \text{Trace } PA &= \sum_n (\phi_n, PA \phi_n) \\ &= AP \end{aligned}$$

$$= \sum_n (\phi_n, A \sum_m w_m [\psi_m] \phi_n)$$

$$= \sum_{n,m} w_m (\phi_n, A [\psi_m] \phi_n)$$

$$= \sum_{n,m} w_m ([\psi_m] \phi_n, A [\psi_m] \phi_n)$$

$$= \sum_n w_n \sum_n ([\psi_n] \phi_n, A [\psi_n] \phi_n)$$

??

$$([\psi_n, A \psi_n])?$$

$$[\psi_n] = (\psi_n,) \psi_n$$

$$A [\psi_n] \phi_n \xrightarrow{?} [A \psi_n] \phi_n ?$$

$$= \sum_n w_n \sum_n ((\psi_n, \phi_n) \psi_n, A (\psi_n, \phi_n) \psi_n)$$

no right ϕ
or wrong

$$= \sum_n w_n \sum_n (\psi_n, \phi_n) (\phi_n, \psi_n) (\psi_n, A \psi_n)$$

yes!