

$$\text{Define } \omega^2 = \frac{1}{2\tau^2}$$

Normal distrib = Likelihood Function

$$P(x)dx = \frac{\omega}{\sqrt{\pi}} e^{-\omega^2(x-\mu)^2} dx$$

$$= \frac{1}{\sqrt{\pi}} e^{-\omega^2(x^2 - 2\mu x + \mu^2) + \ln \omega} dx$$

Canonical Form:

$$N e^{-[a^2\omega^2 - 2bc\mu\omega^2 + c^2\mu^2] - d\ln\omega} d\omega$$

N = Normalizer

Bayesian Update to Canonical Form for observation X :

$$a'^2 = a^2 + X^2$$

$$bc = bc + X$$

$$c^2 = c^2 + 1$$

$$d = d + 1$$

Marginal Distribution of ω :

$$P(\omega)d\omega = N \frac{\sqrt{\pi}}{2c} \omega^{d-1} e^{-\omega^2(a^2-b^2)} d\omega$$

Unconditional Distribution of X :

$$P(x)dx = \frac{N}{\sqrt{c^2+1}} \int_0^\infty \omega^d e^{-\omega^2[x^2+a^2 - \frac{(bc+x)^2}{c^2+1}]} d\omega$$

(see later)

Expected Value of u :

$$\langle u \rangle = b/c \quad (= \sum X_i / \text{NoBS})$$

Standard dev (again w)

$$V\#R(u) = \frac{1}{2w^2c^2}$$

NoBS = NUMBER OBS.

$$S.D.(u) = \frac{1}{w c \sqrt{2}} = \frac{\sigma}{c} = \frac{\sigma}{\sqrt{NoBS}}$$

$\Rightarrow u$ is Normally distributed for any w !

Mode of Variance (most probable)

$$w^* = \frac{d-1}{2(a^2 - b^2)}$$

$$\sigma^* = \frac{a^2 - b^2}{d-1}$$

PHI FUNCTION

$n = [d/2]$ (truncate)

$$\Psi(d) = \begin{cases} \frac{1 \cdot 3 \cdot 5 \dots (2n-1)}{2^{n+1} \sqrt{\pi}} & d \text{ even} \\ \frac{n!}{2}, d \text{ odd} \\ \sqrt{\pi}/2 & d=0 \\ 1/2 \cancel{\text{Theta}} & d=1 \end{cases}$$

Summary

C

Canonical Normalizer :

$$\frac{1}{N} = \gamma(d-1) \frac{\sqrt{\pi}}{2c} \left(a^2 - b^2 \right)^{-\frac{d-3}{2}}$$

Standard Integrals

$$\int_0^\infty x^d e^{-ax^2} dx = \gamma(d)a^{-\frac{d+1}{2}}$$

$$\int_{-\infty}^\infty e^{-(ax-b)^2} dx = \frac{\sqrt{\pi}}{a} \quad \checkmark$$

Unconditioned π

$$\lambda P(x)dx = \frac{2c}{\sqrt{\pi}} \frac{\gamma(d)}{\gamma(d-1)} \left[\frac{(cx-b)^2}{(a^2-b^2)(c^2+1)} + 1 \right] \cdot \left[\frac{1}{(a^2-b^2)(c^2+1)} \right]^{-\frac{d+1}{2}}$$

Mean of w :

$$\langle w \rangle = \bar{w} = \frac{\gamma(d)}{\gamma(d-1)} \frac{1}{\sqrt{a^2 - b^2}}$$

Variance of w :

$$\text{VAR}(w) = \frac{\gamma(d+1)\gamma(d-1) - \gamma(d)^2}{(a^2 - b^2)\gamma(d-1)^2}$$

$$\text{Ratio } \frac{\text{StDev}(w)}{\langle w \rangle} = \left[\frac{\gamma(d+1)\gamma(d-1)}{\gamma(d)^2} - 1 \right]^{\frac{1}{2}}$$