

$$\{x_1, \dots, x_n\} = \lim \{x_1, \dots, x_n\}^P = \lim \left[I_{x_1, \dots, x_n}^P - I_{x_1}^P - \dots - I_{x_n}^P \right]$$

but for directed sets $\lim(f+g) = \lim f + \lim g$
if the latter exist.

Proof: $\lim f = a \Rightarrow$ for every $\epsilon \exists \alpha_1$
 $\exists \beta \leq \alpha_1$
 $|f(\beta) - a| < \epsilon$

$\lim g = b \Rightarrow \forall \epsilon \exists \alpha_2$
 $\exists \beta \leq \alpha_2$

$|g(\beta) - b| < \epsilon$ choose
 $\alpha_3 \leftarrow \text{both } (\alpha_1, \alpha_2) \text{ in}$ (always can
 $\alpha_3 \leftarrow \text{both } (\alpha_1, \alpha_2) \text{ in}$ directed)

$\Rightarrow \forall \epsilon \exists \alpha_3$ such that
 $\forall \beta \leq \alpha_3$

$|f(\beta) - a| < \epsilon$ and $|g(\beta) - b| < \epsilon$

$\Rightarrow |f(\beta) + g(\beta) - (a+b)| < 2\epsilon$

$\Rightarrow \lim(f+g) = a+b$

Ideal Observer

Assume that we have ^{some sort of} an automaton, or servomechanism, by which we mean a device which couples sensory apparatus with machinery so that the actions of the device will depend upon ~~the~~ stimuli through the signals which it receives from its sensory apparatus. Furthermore, let us endow the device with a "memory," by which we mean here some recording apparatus which will record the past sensory data and machine actions. A state of the apparatus will then consist of its present configuration plus the contents of its memory, and we assume that its action at any time depends upon its present configuration, the contents of its memory, and its current sensory data, in a deterministic manner. If we label all possible configurations by $\{C_i\}$, here assumed discrete for simplicity, and all possible memory states by $\{M_j\}$, further all possible sensory data (signals from the sensory apparatus) by $\{S_k\}$, then the (dual) functionality of the machine is described by a mapping of the product space $\{C_i\} \times \{M_j\} \times \{S_k\} \xrightarrow{F} \{C_i\} \times \{M_k\}$ where it is required that $M_k \supset M_j$ in the appropriate sense.

We can now couple this automaton with a measuring apparatus, so that it will perform measurements upon some micro system, record its results (memory), and so that its future measurements will be decided upon the basis of past results. For convenience we now assume that the measuring apparatus are included in the sensory levels of the automaton. For further convenience we shall label the (dual) machine states by G_{ij} which we intend to consist of Configuration C_i and memory M_j .

We shall assume that all of its measurements upon micro systems are brought about by interactions between the ~~accessory~~ apparatus and the system which terminates after a certain time so that they are uncoupled thereafter.

Suppose now that the machine "decides" to measure ψ_{ik} whose wave function is ψ_{ik} the attribute whose operator is A in a system S_m . Let the eigenfunctions of A be ϕ_i with eigenvalues a_i , and let the present machine state be ψ_{ijk} which is a function of the G_{jk} , so that the total WF is ψ_{ijk} .

We further suppose that the machine turns on, for a fixed time τ , an interaction between itself and the system, H_s , which has the property that it brings about a change in machine configuration, for each eigenstate ϕ_i which is definite, and different for each state ϕ_i , that is, we assume that the interaction is such that each combined wave function $\phi_i S_{jk}$ is carried over into another for which the ϕ_i is the same, and the S_{jk} is changed into S_{jm} which depends upon j, k and τ , and for fixed j, k is different for each i :

$$(1) \quad \phi_i S_{jk} \xrightarrow{\tau} \phi_i S_{jm} \rightarrow S_{(lm)} = S_{rs}^{em} G_{rs}$$

So that if the Machine is initially in configuration G_{jk} (its wave function is S_{jk} or nearly this, i.e. nearly all amplitude on G_{jk}) and the system is in eigenstate ϕ_i , then the ~~apparatus~~ ^{Machine} configuration after the interaction will be G_{em} , so that we can say that the machine has measured A and discovered and remembers the value a_i .

We now investigate what will happen in the event that the apparatus system is not an eigenstate of A , but in a state $\Psi_s = \sum_i b_i \phi_i$. Because of what we have just said, and the superposition principle, we can superpose our previous solutions (1) to obtain the total wave function after measurement:

$$(2) \quad \psi_{\text{SA}} = \sum_i b_i \phi_i S_i^{(em)}$$

We can now no longer speak of a definite apparatus state, but are dealing with a superposition of them. However, this is a correlated superposition, so that still for the eigenstates ϕ_i the relative machine state is definite and equal to $S_i^{(em)}$. So that for each eigenstate ϕ_i there is now a machine which records the result a_i . In a manner of speaking, when we began with a single machine, we are now left with a number of them, one for each system eigenstate, and which has perceived that particular eigenstate.

Furthermore, after the interaction has ceased each part, that is each $\phi_i S_i^{(em)}$ separately obeys the wave equation, and proceeds just as it would in the absence of the remainder, i.e. if we consider the i^{th} level, we find the machine $G_i^{(em)}$, which has perceived the value a_i , and which will behave in the future just as though the remainder did not exist, i.e. as though the system had spontaneously jumped into the state ϕ_i while being measured.

Define approximate eigenfunction for A ,

or "the value of A is nearly definite"

when T_A is very high, i.e.

approaching ∞ for discrete spectra

then backs up argument in ideal
measurement using ^{this} notion, and convolution

relative to how hot relative system info, conditioned
on def apparatus $\rightarrow 0$ (to discrete, ∞ for cont.)

Point of distinguishability
some value of $\alpha > 1/2$ need

$$T \gg \ln \frac{1}{\alpha}$$

$$\ln(x^{(1-\alpha)^{-1}}) = 0$$

$\Rightarrow 0$ may 7 will we need, α
all others have $< \infty$

Compare the above Information metric with usual Hilbert space metric

$$P = \sum_i p_i \phi_i \Rightarrow (\psi, \phi)^2 = \phi_i \phi_i^* \text{ near 1}$$

$$\text{while info is } - \sum_i p_i \phi_i^* h \phi_i \phi_i^* \text{ near 0}$$

ie if P_i near 1

remainder
near zero,

~~Defn~~
of info

16. in

P_i largest

$$\text{then } P_i \ln P_i + \left(\sum_i P_i' \ln P_i' \right) \leq f(P_i) ?$$

certainly
if $P_i \neq 1$
since $P=1$

con't
go to
 \rightarrow so

2

Hilbert space
relation

to Hilbert space
metric (good)
only info metric

Maximal
info for mat = P_i

if remaining distribution

$(1-P_i)$ on a single state

and expectation
could be 0 of P_i
given mat of P_i

inequality $f'(I)$ for P_i

$$\text{Therefore for any value } P_i \text{ inequality } I \leq P_i \ln P_i + (1-P_i) \ln(1-P_i)$$

Analogously for any set maximal when remainder is lumped on single other state

$$\frac{x}{m} \left(\frac{x}{m} - \frac{1}{m} \right)^{m-1}$$

||

$$x(-x) + m(-x)$$

$$35 \\ 8.5 \\ 7.25$$

for $P_i = 1/2$ for 10
for 100

If we allow a long series of measurements
then we see a branching with each measurement,
so that we are confronted with a "tree" for the machine.