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Informal Mid-Summer Letter

Having the pleasure to be at this quiet spot overlooking the ocean until September 7, I have had leisure to think of many questions in which we share an interest. I consider myself most fortunate to have had so many interesting discussions with my colleagues this past year. I cannot remember a period in which more fascinating issues arose.

The center of interest possessed a remarkably sharp focus, in the issue: Is curved space only an arena for physics, or is it in an appropriately extended sense the building material for all of physics? Thanks especially to Charles Misner, it became clear during the year that all of classical physics can be described in terms of curved empty space and nothing more: gravitation without gravitation (in Einstein's standard way, through the metric quantities  $g_{\mu\nu}$ ); electromagnetism without electromagnetism (again no invention, thanks to G. Y. Rainich; the Maxwell stress tensor determines the Ricci curvature  $R_{\mu\nu}$ ); conversely, from purely geometrical quantities associated with the curvature of space -- the  $R_{\mu\nu}$ 's and their first derivative -- one can find all that needs to be known about the electromagnetic field; charge without charge (lines of force in a multiply connected space); mass without mass (geons). Our long paper summarizing these considerations got done too late to make the July 1957 special relativity issue of Reviews of Modern Physics and is to appear in the December Annals of Physics. In that number is also to appear a shorter paper "On the Nature of Quantum Geometrodynamics". This paper assumes that one can look forward to the quantization of general relativity along the lines of Charles Misner's paper on the Feynman method in the July R. M. P. It asks whether

there is any possibility to account for elementary particles in terms of quantized curved empty space. It is concluded that this hope is not obviously misguided. However, it is emphasized that this hope can be maintained only if spin presents itself naturally and inevitably in the process of quantizing general relativity.

In discussions with Charles Misner and Robert Ewema and Dieter Brill three alternative suggestions have come up how spin, or Pauli's "non-classical two-valuedness", might conceivably make its appearance in the quantization of geometrodynamics (source free electrodynamics plus general relativity): (1) Two signs of the square root of such a factor as  $(-g)^{1/2}$  in the expression for the "field" or metric configuration on a space-like surface. Otherwise stated, two orientations for the elementary space-like volume element at a point. In classical theory there is a principle of continuity which demands the same sign everywhere. In quantum theory the variation of sign from point to point might be the essential origin of spin. (2) The two valuedness might not be point property, but a property of wormholes only: there are two directions in which the electric flux can run through a wormhole. Specifically consider (Charles Misner) two configurations  $C_1$  and  $C_2$  of the geometric field which look alike except that through a single one of the wormholes the flux is reversed. Can one make an argument that the probability amplitudes for these two configurations should be equal up to a + or - sign, the second sign corresponding to a spin excitation? (3) A property of the basic geometrical field itself already at the classical level, so that at every point the field is endowed with a spin-like property -- a characteristic of the field that has to be introduced to get the law of combination of two fields to make a third field. Analogy: To combine two rotations to get a third rotation, one multiplies the quaternions or spinors for the two rotations individually to get the quaternion or spinor that belongs to the third rotation. The spinor in this example is forced upon one by studying the combinatorics of the problem. What are the combinatorics of two metric configurations  $C_1$  and  $C_2$  to give a configuration  $C_3$ ? This question takes one back again to that great unsolved issue of classical relativity theory: What are the freely disposable initial value data on two space-like surfaces? (Even this wording of the question undoubtedly needs revision!) These two questions (1) the nature of spin and (2) freely disposable initial value data may well be centers of active discussion during the coming year.

Dieter Brill and Robert Ewema are discussing issue (1) and endeavoring to assess which view of spin (1,2 or 3) makes more sense. I enjoyed a good two day visit from them and Rick Werner around July 4. Charles Misner is discussing the quantization of general relativity with Oskar Klein, Stanley Deser and Bryce De Witt at the Institute in Copenhagen part of this summer. Let us hope that 2 pm Friday September 27 will not be too early to hear a report of his findings.

Not two, but four papers were too late for the July R. M. P. The other two amongst these 4 included one of Komar on Mach's principle. It may be that he will decide to have a look at the implications of the initial value problem for Mach's principle and have a new go at this fascinating question. The other paper -- by Tullio Regge on the stability of a Schwarzschild singularity -- has gone to the Physical Review. It should help greatly in analyzing gravitational waves in a spherically symmetrical metric. It may well be that this paper is an adequate starting point for a treatment of geons that derive all their mass

from gravitational radiation. Tullio Regge is working now with Luigi Favella on the stability of a Schwarzschild singularity endowed with charge -- i.e., the stability of the Reissner-Nordström solution. A paper with Joseph Weber on cylindrical gravitational waves should appear in July R. M. F.. A longer set of notes with him and Peter Putnam on gravitational radiation in general still needs much to make it complete.

This question of pure gravitational radiation seems to me not a side issue, but an especially good and simple situation where one can try to get hold of the very important initial value problem (2 above). The simplest case that has yet shown itself concerns "a time symmetric pure gravitational wave" which has a certain analogy with "a time symmetric pure electromagnetic wave" in the sense of the following table:

Time-symmetric disturbances		
	Electromagnetic	Gravitational
Quantity which has reached its extremal value at $T = 0$	$H$	(3) $\tilde{g}_{ik}$ on space-like surface
Quantity which is then zero	$E$	$\partial \tilde{g}_{ik} / \partial T$
Initial value data for first quantity arbitrary except for condition at right	$\text{div } H = 0$	(3) $R = 0$ (curvature invariant built from the metric on the 3 dimensional surface)
That quantity or "potential" which is freely disposable and out of which one can generate the most general solution of the foregoing equation	$A$	Not known
The generating equation	$H = \text{curl } A$	Not known

The condition to be satisfied in both cases is a single scalar equation; and the analysis deals exclusively with three dimensions and a positive definite metric. To find the requisite freely disposable potential for the gravitational problem is a most interesting issue which I discussed a little with Richard Lindquist in June and more with John Fletcher and Kent Harrison and John Klauder when I met them by rearrangement at the July 15-16 Brandeis University symposium on modern developments in quantum theory. If view (3) of the origin of spin is correct, it may well either show up or be disproved in this simplest version of the initial value problem. I am adding a few notes at the end about this problem.

Closely related both to the initial value problem and to the formulation of the Feynman method of quantization is work that John Fletcher and John Klauder have been doing. Their method, applied to the simpler problem of a particle with one degree of freedom, envisages a more detailed account of the history than that seen in the familiar Feynman bookkeeping. Feynman divides the time interval up into  $n$  steps, at each of which the coordinate is specified in one history. The sum over histories is accomplished by integrating over all these coordinates. Klauder specifies the history by giving also at the half integral times the value of the momentum. The element of action is written no longer as  $L(q, \dot{q})dt$  but as  $p\dot{q} - H(p, q)dt$ . Fletcher and Klauder's generalization of this idea to general relativity has real interest and they are working on a formulation on paper in some definitive form.

It has always been puzzling what happens when a sufficiently large amount of cold matter is piled together. The gravitational forces will ultimately overwhelm all other forces. Kent Harrison has calculated the equation of state,  $p = p(\rho)$ , of cold matter -- catalyzed to the end point of thermonuclear evolution -- all the way from normal uncompressed iron, through iron at very high pressure, past the point where electrons are squeezed onto the iron, through the point where the density of nuclear matter is attained, up to a relativistic Fermi gas composed  $8/9$  of neutrons and  $1/9$  of protons and electrons. Masao Wakano has solved on the Institute Maniac the general relativity equation of hydrostatic equilibrium with this equation of state. No solution exists when the number of nucleons or the mass exceed a certain critical amount, comparable to the mass of the sun. However, nothing in the world can keep one from adding more than this critical number of nucleons. Therefore a new state of matter must exist that is not included in our present equation of state at high densities. The two principal possibilities are these: (1) collapse to an elementary particle, perhaps with violation of the law of conservation of nucleons (2) attainment of a limiting density, as would be given by a hard core repulsion, in violation of the laws of a Fermi gas. In this latter case Masao Wakano shows that the mass will never exceed a certain critical amount but the number of nucleons can be augmented as much as one pleases. Neither case seems to present any difficulty of principle.

Speaking of large masses, I should add that Rick Werner has practically finished his work on the stability of superheavy nuclei and this should shortly be going off for publication. The nucleus  $Z = 147$ ,  $A \sim 500$  is estimated to have decay rates of only  $10^4 \text{ sec}^{-1}$  with respect to both beta decay and fission. However, Robert Ewema has looked some at the stability of ring shaped nuclei. This geometry may permit even larger masses and charges, though this point remains to be firmly established insofar as anything can be firmly established on the basis of the semi-empirical mass formula!

I have studied with sympathy the memorandum of several graduate student friends about problems of graduate study. I look forward to a departmental meeting in the fall to discuss this question in considerable detail and to take appropriate positive steps.

I am finishing up papers with David Hill on nucleon energy levels and with Kenneth Ford on scattering theory and then have to complete my handbook article on fission, so I am trying to resist the lure of geometrodynamics this

summer. The hurricane of three years ago took down about one tree out of five here. If I spend an hour a day with chain saw and axe on cleaning up the woods, I can make this job last roughly fifty years. I send best wishes for a good summer.

John Wheeler

Notes on the Problem of  
Time-Symmetric Gravitational Waves

(The parts marked with stars need to be firmed up.)

From the paper with Joseph Weber I quote the initial value conditions of Lichnerowicz and Y. Fourès-Bruhat (*J. Rational Mech. Anal.* 5, 951, 1956) for pure gravitational waves -- i.e., gravitational waves not accompanied by electromagnetic disturbances (for which latter see Misner):

$$\partial g_{mn}/\partial T \equiv P_{mn} \quad (m,n = 1,2,3);$$

$$P_m^m \equiv P; \quad$$

$$(P_m^n - \delta_m^n P)_{;n} = 0;$$

$$P^2 - P_{mn} P^{mn} + R = 0.$$

Here  $R$  is the curvature invariant for the 3-space with metric  $g_{mn}$ . For time-symmetric waves all the  $P_{mn}$  vanish at  $T = 0$  so  $R$  must vanish.

Eisenhart's *Riemannian Geometry* (1926) deals in section 15, p. 43 with  $N$ -tuply orthogonal systems of hypersurfaces in a Riemannian manifold  $V$ . We assume from his discussion that it is always possible\* to construct 3 families of mutually orthogonal hypersurfaces throughout a finite region of a 3-space of arbitrary metric. Whether the surfaces can be extended to cover the whole space without singularity is not clear. Indeed, if the space is multiply connected it is clear from topological arguments that no coordinate system, orthogonal or not, can cover the whole space. However, we are interested for the present in wormhole-free gravitational waves -- in other words, in spaces topologically identical with Euclidean 3-space. We shall assume that the surfaces  $x = \text{const}$ ,  $y = \text{const}$  and  $z = \text{const}$  of Euclidean space can be deformed in our space so as to remain everywhere orthogonal.\* This is a strong assumption to make and badly needs justification. The arguments of H. Hopf (*Proc. Int. Cong. Math.*, Cambridge, Mass., Vol. I, Providence, 1952, p. 193) on existence of 3 orthogonal families of vector fields in 3 space deal with the special spaces  $S^3$  and  $P^3$  of uniform curvature and do not help us. Conceivably in Ricci's work of long ago can be found a proof of what we want. (See also Bianchi references, Eisenhart, p. 121.)

In the assumed orthogonal coordinates the metric has the form

$$ds^2 = (H_i)^2 (dx^i)^2$$

and the curvature invariant and Ricci tensor have the values\* (Eisenhart, p. 119, Eq. 37.4)

$$R_{hk} = \frac{\partial^2 \ln g^{1/2}}{\partial x^h \partial x^k} - \frac{\partial \ln g^{1/2}}{\partial x^h} \frac{\partial \ln H_h}{\partial x^k} - \frac{\partial \ln g^{1/2}}{\partial x^k} \frac{\partial \ln H_k}{\partial x^h} + \frac{\partial \ln H_i}{\partial x^h} \frac{\partial \ln H_i}{\partial x^k},$$

$$R = \frac{1}{(H_h)^2 (\partial x^h)^2} \frac{\partial^2 \ln g^{1/2}}{} - \frac{2}{(H_h)^2} \frac{\partial \ln g^{1/2}}{\partial x^h} \frac{\partial \ln H_h}{\partial x^h} + \frac{1}{(H_h)^2} \frac{\partial \ln H_i}{\partial x^h} \frac{\partial \ln H_i}{\partial x^h}.$$

The solution we want has the following character: lots of ripples in the curvature out to a distance of the order of  $m^{\#}$ , to represent gravitational wave energy; from there out, a metric approaching the Schwarzschild value, with factors  $(1 - 2m^{\#}/r)^{1/2}$ , representing the static gravitational field due to the mass-energy of the gravitational waves. At great distances the Ricci tensor in spherical polar coordinates has the qualitative form

$$R_m^h = (2m^{\#}/r^3) \begin{vmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

From the circumstance that this curvature tensor at large distances takes a diagonal form only in polar coordinates one might be tempted to believe that the orthogonal coordinate system itself must also be essentially polar in character at large distances. But the angular part of a polar coordinate system in 3-space always has singularities at the poles (cf. topology) --- singularities that we would be very unhappy to have in our coordinate system. The following two arguments suggest that we are not pushed into any such polar coordinate system. (1) The Ricci tensor  $R_{hk}$  calculated above in the orthogonal coordinate system is not in general diagonal in that coordinate system. In other words, the axes that make  $R_{hk}$  diagonal (polar at large distances) are not in general the same as the local axes of the orthogonal coordinate system. Therefore polar axes for diagonal  $R_{hk}$  do not imply polar axes for orthogonality of our coordinate system. (2) There is a perfectly good non-polar but orthogonal coordinate system in terms of which one knows how to write the Schwarzschild metric:

$$ds^2 = \frac{dx^2 + dy^2 + dz^2}{f(x, y, z)}$$

(Eisenhart, theorem at end of section 28; Eq. 28.14; and both  $R_{ik}$  and  $R$  vanish for the Schwarzschild solution.

So far only the framework for the discussion has been mapped out. It would not seem to make sense to try to solve directly the one equation  $R = 0$  for the three unknown functions  $H_i(x, y, z)$ . Instead one needs some construction algorithm of a geometrical character that will manufacture such solutions easily and automatically. To spy out this geometrical construction is the heart of the problem. It is the analog of writing  $\mathbf{H} = \text{curl } \mathbf{A}$  in the electromagnetic problem. Stäckel's work on the separation of coordinates in classical mechanics (Encyk. der Math. Wiss.) and Eisenhart's work on the same problem in quantum mechanics (Phys. Rev. 74, 87, 1948) remind one of two more cases where one has found construction procedures. An even closer analog is provided by the single non linear scalar equation

$$(1 + \varphi_y^2)\varphi_{xx} - 2\varphi_x\varphi_y\varphi_{xy} + (1 + \varphi_x^2)\varphi_{yy} = 0$$

for extremal surfaces. Again one doesn't bang his head against the direct problem of solving this equation. Instead (Lipman Bers, Int. Cong. of Math. cited above, Vol. II, p. 157) takes "any real plane algebraic curve  $C$  in the  $s, t$  plane" (the analog of a potential) and constructs the Abelian integrals

$$x = \operatorname{Re} \int \frac{2s}{1-s^2} it \, ds ;$$

$$y = \operatorname{Re} \int \frac{1-s^2}{1+s^2} it \, ds$$

$$\varphi = \operatorname{Re} \int t \, ds$$

and has automatically the desired solution. This is the kind of procedure we want to solve the equation  $R = 0$ .

Incidentally one asks, will the orthogonal coordinate system be essentially unique -- or will it be open to something like a global rotation?

What an interesting problem!