

$$= \langle [\phi_i] \rangle \psi$$

The Quantum Mechanics

we shall now develop our tools in Q.M.

In this chapter we assume that states of a physical system are represented by points in a Hilbert space, and that for isolated systems the time dependence of the state is given by a linear wave equation. The first section deals with the representation of composite system states, density matrices, and the concept of relative states. In the second section, the concept of the information of an operator for a given state is introduced, and its role as a measure of the "nearness" of a state to an eigenfunction is

a - a a o o

It is convenient at this point to introduce some notational conventions. We shall be concerned with points ψ in a Hilbert space \mathcal{D} , with scalar product (ψ_1, ψ_2) . For any linear operator A we define a ~~functional~~ functional, $\langle A \rangle \psi$, called the expectation of A for ψ , to be:

$$\langle A \rangle \psi = (\psi, A\psi)$$

A particular class of operators of interest is the class of projection operators. $[P_\phi]$ called the projection on $|\phi\rangle$ is defined through:

$$[P_\phi]\psi = (\phi, \psi)\phi$$

For a complete orthonormal set $\{\phi_i\}$ and a state ψ we define a square-amplitude distribution P_i called the distribution of ψ over $\{\phi_i\}$ through

$$P_i = |(\phi_i, \psi)|^2$$

2

add to end - This means for any possible measurement, if until
stop not later than either final -- only superposition
with apparatus bound to system -- no definiteness --
how can we interpret this, since we know definite things to happen --
toss up question of observation in next chapter.

of the formalism of composite systems, and the connection of composite system states with their derived joint distributions with the various ^{and marginal} possible ~~conditional~~ subsystem conditional distributions. We shall see that there exist relative state functions which correctly give the conditional distributions for all operators, while marginal distributions cannot generally be represented by state functions, but only by density matrices.

In section 2 the concepts of information and correlation, developed in the preceding chapter, are applied to quantum mechanics. It is shown that for any system there ^{always} exists a fundamental quantity which can be thought of as the correlation between subsystems, and a closely connected canonical representation of the composite system state. A stronger form of the uncertainty principle, phrased in information language, is indicated.

In section 3 we take up the question of measurement in quantum mechanics, viewed as a correlation producing interaction between physical systems. A single example of such a ^{measurement} ~~interaction~~ is given and discussed. Finally some consequences of the superposition principle are considered.

It is convenient, at this --

Information and Correlation are developed for operators on systems with prescribed states.

In the probabilistic interpretation this distribution represents the probability distribution over the results of a measurement with eigenstates Φ_i , performed upon a system in state ψ . (Note, hereafter when referring to prob. interps. we shall say briefly "the prob of that the system will be found in Φ_i " rather than the more cumbersome "The probability that the measurement of a quantity B , with eigenfunctions Φ_i , shall yield the eigenvalue corresponding to Φ_i ")
"which
is meant."

For two Hilbert spaces \mathcal{H}_1 and \mathcal{H}_2 , we ~~form~~ construct the direct product Hilbert space $\mathcal{H}_1 \otimes \mathcal{H}_2 = \mathcal{H}_3$ (Tensor product) which is taken to be the space of all possible ¹sums of formal products of points of \mathcal{H}_1 and \mathcal{H}_2 , i.e., the elements of \mathcal{H}_3 are those of the form $\sum_i a_i \xi_i \eta_i$ where $\xi_i \in \mathcal{H}_1$ and $\eta_i \in \mathcal{H}_2$. The scalar product in \mathcal{H}_3 is taken to be $(\sum_i a_i \xi_i \eta_i, \sum_j b_j \xi_j \eta_j) = \sum_{ij} a_i^* b_j (\xi_i, \xi_j)(\eta_i, \eta_j)$. It is then easily seen that if $\{\xi_i\}$ and $\{\eta_j\}$ form complete orthonormal sets in \mathcal{H}_1 and \mathcal{H}_2 respectively, that the set of all formal products $\{\xi_i \eta_j\}$ is a complete orthonormal set in \mathcal{H}_3 . For any pair of operators A, B in \mathcal{H}_1 and \mathcal{H}_2 there corresponds an operator $C = A \otimes B$, the direct product of A and B , in \mathcal{H}_3 , which can be defined by its effect on the elements $\xi_i \eta_j$ of \mathcal{H}_3 :

$$C \{ \xi_i \eta_j \} = A \otimes B \{ \xi_i \eta_j \} = (A \xi_i)(B \eta_j)$$

1) More rigorously, one forms the space of all finite sums, then completes it to obtain $\mathcal{H}_1 \otimes \mathcal{H}_2$.