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ON THE HYPOTHESIS THAT THE SCHROEDINGER
EQUATION IS EXACT

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1. INTRODUCTION

In traditional quantum mechanics the linear Schroedinger equation is not supposed to be exactly true. Indeed, it is not supposed to hold at all during "observation", when it is replaced by the rather vague but definitely non-linear "reduction of the wave packet". This is the way in which the theory recognizes facts - the observing of one possibility rather than another. Since it is unlikely that "observation", any more than other physical processes, can be sharply bounded in time, the linear Schroedinger equation is presumably never better than a very good approximation. A natural line of development, therefore, is towards a non-linear equation in which the wave-packet reduction would have precise mathematical form. There have been interesting speculations in this direction ¹⁾. However, there are also theories in which the linear Schroedinger equation is supposed to be exact, and in which no wave-packet reduction occurs. Of course, the wave function is then no longer held to be a complete description of physical reality. It is still necessary to recognize facts, and since these are not now incorporated into the wave function by reduction, some additional variables have to be invoked to represent them. This paper is an analysis of two theories of this kind, mainly due, respectively, to de Broglie ²⁾ and to Everett ^{3),4)}. It seems to me that the close relationship of the Everett theory to that of de Broglie has not been appreciated, and that as a result the really novel element in the Everett theory has not been identified. This really novel element, in my opinion, is the acceptance of the consequences of a skeptical analysis of the concept of the past, which could be considered to be in the same liberating tradition as Einstein's analysis of the concept of simultaneity.

It must be said that the versions to be presented here might not be accepted by the authors cited. This is to be feared particularly in the case of Everett. His theory was for long completely obscure to me. The obscurity was lightened by the expositions of De Witt ⁵⁾⁻⁸⁾. But I am not sure that my present understanding coincides with that of De Witt, or with that of Everett, or that a simultaneous coincidence with both would be possible ⁹⁾. Part of the difficulty lies in the very ambitious character of their expositions. Everything is supposed to follow from a very few assumptions, and the assertion is even made that the theory "yields its own interpretation". In an area as notoriously difficult as this it seems better to risk a redundant axiom than lack of communication. So the presentation here will be the reverse of axiomatic.

The paper starts with a review of some relevant aspects of ordinary quantum mechanics, in terms of a simple particular application. The problem which the unconventional formulations try to solve is then stated in more detail, and finally the de Broglie and Everett theories are formulated and compared.

In connection with this material I have profited from discussion with Professor B. d'Espagnat, and with the other participants in a seminar on this subject which he organized at Orsay in June, 1971. For useful comments on the manuscript I thank Professors M. Chanowitz, P. Csonka and K. Gottfried.

2. COMMON GROUND

To illustrate some points which are not in question, before coming to some which are, let us look at a particular example of quantum mechanics in actual use. A nice example for our purpose is the theory of formation of an α particle track in a set of photographic plates. The essential ideas of the analysis have been around at least since 1929 when Mott ¹⁰⁾ and Heisenberg ¹¹⁾ discussed the theory of Wilson cloud chamber tracks ¹²⁾. Yet somehow many students are left to rediscover for themselves ideas of this kind. When they do so it is often with a sense of revelation; this seems to be the origin of several published papers.

Let the α particle be incident normally on the stack of plates and excite various atoms or molecules in a way permitting development of blackened spots. In a first approach ¹¹⁾ to the problem only the α particle is considered as a quantum mechanical system, and the plates are thought of as external measuring equipment permitting a sequence of measurements of transverse position of the α particle. Associated with each such measurement there is a "reduction of the wave packet" in which all of the incident de Broglie wave except that near the point of excitation is eliminated. If the "position measurement" were of perfect precision the reduced wave would emerge in fact from a point source and, by ordinary diffraction theory, then spread over a large angle. However, the precision is presumably limited by something like the atomic diameter $a \approx 10^{-8}$ cm. Then the angular spread can be as little as

$$\Delta\theta \approx (ka)^{-1}$$

With an α particle of about one MeV, for example, $k \approx 10^{13}$ cm⁻¹, and with $a \approx 10^{-8}$ cm

$$\Delta\theta \approx 10^{-5} \text{ radians.}$$

In this way, one can understand that the sequence of excitations in the different plates approximate very well a straight line pointing to the source.

This first approach may seem very crude. Yet in an important sense it is an accurate model of all applications of quantum mechanics.

In a second approach we can regard the photographic plates also as part of the quantum mechanical system. As Heisenberg remarks "this procedure is more complicated than the preceding method, but has the advantage that the discontinuous change in the probability function recedes one step and seems less in conflict with intuitive ideas". To minimize the increased complication we will consider only highly simplified "photographic plates". They will be envisaged as zero temperature mono-atomic layers of atoms each with only one possible excited state, the latter supposed to be rather long-lived. Moreover, we will continue to neglect the possibility of scattering without excitation, (i.e., elastic scattering), which is not very realistic.

Suppose that the α particle originates in a long-lived radioactive source at position \vec{r}_0 and can be represented initially by the steady state wave function

$$\psi(\vec{r}) = \frac{e^{ik_0 |\vec{r} - \vec{r}_0|}}{|\vec{r} - \vec{r}_0|}$$

Let ϕ_0 denote the ground state of the stack of plates. Let $n (= 1, 2, 3, \dots)$ enumerate the atoms of the stack and let

$$\phi(n_1, n_2, n_3, \dots)$$

denote a state of the stack in which atoms n_1, n_2, n_3, \dots are excited. In the absence of α particle stack interaction the combined state would be simply

$$\phi_0 \frac{e^{ik_0 |\vec{r} - \vec{r}_0|}}{|\vec{r} - \vec{r}_0|}$$

To this must be added, because of the interaction, scattered waves determined by solution of the many-body Schroedinger equation. In a conventional multiple scattering approximation the scattered waves are

$$\sum_N \sum_{n_1, n_2, \dots, n_N} \phi(n_1, n_2, \dots, n_N) \frac{e^{ik_N |\vec{r} - \vec{r}_N|}}{|\vec{r} - \vec{r}_N|} f_N(\theta_N) \times \\ \frac{e^{ik_{N-1} |\vec{r}_N - \vec{r}_{N-1}|}}{|\vec{r}_N - \vec{r}_{N-1}|} f_{N-1}(\theta_{N-1}) \times \dots \dots \dots \frac{e^{ik_0 |\vec{r}_1 - \vec{r}_0|}}{|\vec{r}_1 - \vec{r}_0|} \quad (1)$$

The general term here is a sum over all possible sequences of N atoms, with \vec{r}_1 denoting the position of atom n_1 , \vec{r}_2 of atom n_2 , and so on; $k_n = (k_{n-1}^2 - \epsilon)^{\frac{1}{2}}$ where ϵ is a measure of atomic excitation energy; θ_n is the angle between $\vec{r}_n - \vec{r}_{n-1}$, and $\vec{r}_{n+1} - \vec{r}_n$ (or $\vec{r} - \vec{r}_N$ for $n = N$). Finally $f_n(\theta)$ is the inelastic scattering amplitude for an α particle of momentum k_{n-1} incident on a single atom; in the Born approximation for example we could give an explicit formula for $f(\theta)$ in terms of atomic wave functions, and would indeed find for it an angular spread

$$\Delta\theta \approx (ka)^{-1}$$

The relative probabilities for observing that various sequences of atoms n_1, n_2, \dots have been excited are given by the squares of the moduli of the coefficients of

$$\phi(n_1, n_2, \dots)$$

It is again clear that because of the forward peaking of $f(\theta)$ excited sequences will form essentially straight lines pointing towards the source.

We considered here only the location, and not the timing, of excitations. If timing also had been observed then in the first kind of treatment the reduced wave after each excitation would have been an appropriate solution of the time-dependent Schroedinger equation, limited in extent in time as well as space. In the second kind of treatment some physical device for registering and recording times would have been included in the system. We will not go further into this here. The comparison between the first and second kinds of treatment would still be essentially along the following lines. But before coming to this comparison it will be useful later to have pointed out two of the several general features of quantum mechanics which are illustrated in the example just discussed.

The first concerns the mutual consistency of different records of the same phenomenon. In the stack of plates of the above example we have a sequence of "photographs" of the α particle, and because the particle is not too greatly disturbed by the photographing, the sequence of records is fairly continuous. In this way, there is no difficulty for quantum mechanics in the continuity between successive frames of a movie film nor in the consistency between two movie films of the same phenomenon. Moreover, if instead of recording such information on a film, it is fed into the memory of a computer (which can incidentally be thought of as a model for the brain) there is no difficulty for quantum mechanics in the internal coherence of such a record - e.g., in the "memory" that the α particle (or instrument pointer, or whatever) has passed through a sequence of adjacent positions. These are all just "classical" aspects of the world which emerge from quantum mechanics at the appropriate level. They are called to attention here because later on we come to a theory which is fundamentally precisely about the contents of "memories".

The second point is the following. When the whole stack of plates is treated as a single quantum mechanical system, each α particle track is a single experimental result. To test the quantum mechanical probabilities requires then many such tracks. At the same time a single track, if sufficiently long, can be regarded as a collection of many independent single scattering events, which can be used to test the quantum mechanics of the single scattering process. That this is so is seen to emerge from the more complete treatment whenever interactions between plates are negligible (and when the energy loss ϵ is negligible). Of course, there could be statistical freaks, tracks with all scatterings up, or all down, etc., but the typical track, if long enough, will serve to test predictions for $|f(\theta)|^2$. The relevance of this remark is that later we will be concerned with theories of the universe as a whole. Then there is no opportunity to repeat the experiment; history is given to us once only. We are in the position of having a single track, and it is important that the theory has still something to say - provided that this single track is not a freak, but a typical member of the hypothetical ensemble of universes that would exhibit the complete quantum distribution of tracks¹³⁾.

We return now to the comparison of the two kinds of treatment. The second treatment is clearly more serious than the first. But it is by no means final. Just as at first we supposed without analysis that the photographic plates could effect position measurements on the α particle, so we have now supposed without analysis the existence of equipment allowing the observation of atomic excitation. We can therefore contemplate a third treatment, and a fourth, and so on. Any natural end to this sequence is excluded by the very language of contemporary quantum theory, which never speaks of events in the system but only of the outcome of observations upon the system, implying always the existence of external equipment adapted to the observable in question. Thus the logical situation does not change in going from the first treatment to the second. Nor would it change on going further, although many people have been intimidated simply by increasing complexity into imagining that this might be so. In spite of its manifest crudity, therefore, we have to take quite seriously the first treatment above, as a faithful model of what we have to do in the end anyway.

It is therefore important to consider to what extent the first treatment is actually consistent with the second, and not simply superseded by the latter. The consistency is in fact quite high, especially if we incorporate into the rather vaguely "reduced" wave function of the first treatment the correct angular factor $f(\theta)$ from the second. Then the first method will give exactly the same distribution of excitations, and the same correlations between those in different plates. However, it must be stressed that this perfect agreement is only a result of idealizations that we have made, for example, the neglect of interactions between atoms (especially in different plates). To take accurate account of these we are simply obliged to adopt the second procedure, of regarding α particle and stack together as a single quantum mechanical system. The first kind of treatment would be manifestly absurd if we were concerned with an α particle incident on two atoms forming a single molecule. It is perhaps not absurd, but it is not exact, when we have 10^{23} atoms with somewhat larger spaces between. Therefore, the placing of the inevitable split, between quantum system and observing world, is not a matter of indifference.

So we go on displacing this Heisenberg split to include more and more of the world in the quantum system. Eventually we come to a level where the required observations are simply of macroscopic aspects of macroscopic bodies. For example, we have to observe instrument readings, or a camera may do the observing, then we may observe the photographs of the instrument readings, and so on. At this stage, we know very well from everyday experience that it does not matter whether we think of the camera as being in the system or in the observer - the transformation between the two points of view being trivial, because the relevant aspects of the camera are "classical" and its reaction on the relevant aspects of the instrument negligible. Then at this level it becomes of no practical importance just where we put the Heisenberg split - provided of course that these "classical" features of the macroscopic world emerge also from the quantum mechanical treatment. There is no reason to doubt that this is the case.

This is already illustrated in the example that we analyzed above. Thus the α particle is already largely "classical" in its behaviour - preserving its identity, in a sense, as it is seen to move along a practically continuous and smooth path. Moreover, the different parts of the complete wave function (1) associated with different tracks can be to a considerable extent regarded as incoherent, as indicated by the success of the first kind of treatment. These "classical" features can be expected to be still more pronounced for macroscopic bodies. The possibilities of seeing quantum interference phenomena are reduced not only by the shortness of de Broglie wave length, which would make any such pattern extremely fine grained, but also by the tendency of such bodies to record their passage in the environment. With macroscopic bodies it is not necessary to ionize atoms; we have the steady radiation of heat for example, which would leave a "track" even in the vacuum, and we have the excitation of the close packed low lying collective levels of both the body in question and neighbouring ones¹⁴⁾.

So there is no reason to doubt that the quantum mechanics of macroscopic objects yields an image of the familiar everyday world. Then the following rule for placing the Heisenberg split, although ambiguous in principle, is sufficiently unambiguous for practical purposes:

put sufficiently much into the quantum system that the inclusion of more would not significantly alter practical predictions.

To ask whether such a recipe, however adequate in practice, is also a satisfactory formulation of fundamental physical theory, is to leave the common ground.

3. THE PROBLEM

The problem is this: quantum mechanics is fundamentally about "observations". It necessarily divides the world into two parts, a part which is observed and a part which does the observing. The results depend in detail on just how this division is made, but no definite prescription for it is given. All that we have is a recipe which, because of practical human limitations, is sufficiently unambiguous for practical purposes. So we may ask with Stapp¹⁵⁾: "How can a theory which is fundamentally a procedure by which gross macroscopic creatures, such as human beings, calculate predicted probabilities of what they will observe under macroscopically specified circumstances ever be claimed to be a complete description of physical reality ?". Rosenfeld¹⁶⁾ makes the point with equal eloquence: "... the human observer, whom we have been at pains to keep out of the picture, seems irresistibly to intrude into it, since after all the macroscopic character of the measuring apparatus is imposed by the macroscopic structure of the sense organs and the brain. It thus looks as if the mode of description of quantum theory would indeed fall short of ideal perfection to the extent that it is cut to the measure of man".

Actually these authors feel that the situation is acceptable. As indicated by the quotations, they are among the more thoughtful of those who do so. Stapp finds reconciliation in the pragmatic philosophy of William James. On this view, the situation in quantum mechanics is not peculiar. But rather the concepts of "real" or "complete" truth are quite generally mirages. The only legitimate notion of truth is "what works". And quantum mechanics certainly "works". Rosenfeld seems to take much the same position, preferring however to keep academic philosophy out of it: "we are not facing here any deep philosophical issue, but the plain common sense fact that it takes a complicated brain to do theoretical physics". That is to say, that theoretical physics is quite necessarily cut to the measure of theoretical physicists.

In my opinion, these views are too complacent. The pragmatic approach which they exemplify has undoubtedly played an indispensable role in the evolution of contemporary physical theory. However, the notion of the "real" truth, as distinct from a truth that is presently good enough for us, has also played a positive role in the history of science. Thus Copernicus found a more intelligible pattern by placing the sun rather than the earth at the centre of the solar system. I can well imagine a future phase in which this happens again, in which the world becomes more intelligible to human beings, even to theoretical physicists, when they do not imagine themselves to be at the centre of it.

Less thoughtful physicists sometimes dismiss the problem by remarking that it was just the same in classical mechanics. Now if this were so it would diminish classical mechanics rather than justify quantum mechanics. But actually, it is not so. Of course, it is true that also in classical mechanics any isolation of a particular system from the world as a whole involves approximation. But at least one can envisage an accurate theory, of the universe, to which the restricted account is an approximation. This is not possible in quantum mechanics, which refers always to an outside observer, and for which therefore the universe as a whole is an embarrassing concept. It could also be said (by one unduly influenced by positivistic philosophy) that even in classical mechanics the human observer is implicit, for what is interesting if not experienced? But even a human observer is no trouble (in principle) in classical theory - he can be included in the system (in a schematic way) by postulating a "psycho-physical parallelism" - i.e., supposing his experience to be correlated with some functions of the co-ordinates. This is not possible in quantum mechanics, where some kind of observer is not only essential, but essentially outside. In classical mechanics we have a model of a theory which is not intrinsically inexact, for it neither needs nor is embarrassed by an observer.

Classical mechanics does, however, have the grave defect, as applied on the atomic scale, of not accounting for the data. For this good reason it has been abandoned on that scale. However, classical concepts have not thereby been expelled from physics. On the contrary, they remain essential on the "macroscopic" scale, for¹⁷⁾ "... it is decisive to recognize that, however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms". Thus contemporary theory employs both quantum wave

functions ψ and classical variables x , and a description of any sufficiently large part of the world involves both:

$$(\psi, x_1, x_2, \dots)$$

In our discussion of the α particle track, for example, implicit classical variables specified the position of the various plates, and the degrees of excitation of the atoms were also considered as classical variables for which probability distributions could be extracted from the calculations. In a more thorough treatment the degrees of excitation of atoms would be replaced as classical variables by the degrees of blackening of the developed plates. And so on. It seems natural to speculate that such a description might survive in a hypothetical accurate theory to which the contemporary recipe would be a working approximation. The ψ 's and x 's would then presumably interact according to some definite equations. These would replace the rather vague contemporary "reduction of the wave packet" - intervening at some ill-defined point in time, or at some ill-defined point in the analysis, with a lack of precision which, as has been said, is tolerable only because of human grossness.

Before coming to examples of such theories I would like to suggest two general principles which should, it seems to me, be respected in their construction. The first is that it should be possible to formulate them for small systems. If the concepts have no clear meaning for small systems it is likely that "laws of large numbers" are being invoked at a fundamental level, so that the theory is fundamentally approximate. The second, related, point is that the concepts of "measurement", or "observation", or "experiment", should not appear at a fundamental level. The theory should of course allow for particular physical set-ups, not very well defined as a class, having a special relationship to certain not very well-defined subsystems - experimenters. But these concepts appear to me to be too vague to appear at the base of a potentially exact theory. Thus the x 's then would not be "macroscopic" "observables" as in the traditional theory, but some more fundamental and less ambiguous quantities.

4. THE PILOT WAVE

The duality indicated by the symbol

$$(\psi, x)$$

is a generalization of the original wave-particle duality of wave mechanics. The mathematics had to be done with waves ψ extending in space, and then had to be interpreted in terms of probabilities for localized events. At an early stage de Broglie²⁾ proposed a scheme in which particle and wave aspects were more closely integrated. This was resurrected in 1952 by Bohm¹⁸⁾. Despite some curious features it remains, in my opinion, well worth attention as a model of what might be the logical structure of a quantum mechanics which is not intrinsically inexact.

To avoid arbitrary division of the world into system and apparatus, we must work straight away with some model of the world as a whole. Let this "world" be simply a large number N of particles, with Hamiltonian

$$H = \sum_n \frac{\vec{p}_n^2}{2M_n} + \sum_{m>n} V_{mn} (\vec{r}_m - \vec{r}_n) \quad (2)$$

The world wave function $\Psi(r, t)$, where r stands for all the \vec{r} 's, evolves according to

$$\frac{\partial}{\partial t} \Psi(r, t) = -iH\Psi \quad (3)$$

We will need the purely mathematical consequence of this that

$$\frac{\partial}{\partial t} \rho(r, t) + \sum_n \frac{\partial}{\partial \vec{r}_n} \cdot \vec{j}_n(r, t) = 0 \quad (4)$$

where

$$\rho(r, t) = |\Psi(r, t)|^2 \quad (5)$$

$$\vec{j}_n(r, t) = M_n^{-1} \operatorname{Im} \left\{ \Psi^*(r, t) \frac{\partial}{\partial \vec{r}_n} \Psi(r, t) \right\} \quad (6)$$

We have to add classical variables. A democratic way to do this is to add variables $\vec{x}_1, \vec{x}_2, \dots, \vec{x}_N$ in one-to-one correspondence with the \vec{P} 's. The \vec{x} 's are supposed to have definite values at any time and to change according to

$$\frac{d}{dt} \vec{x}_n = \vec{j}_n(x, t) / \rho(x, t) \quad (7)$$

We then have a deterministic system in which everything is fixed by the initial values of the wave Ψ and the particle configuration x . Note that in this compound dynamical system the wave is supposed to be just as "real" and "objective" as say the fields of classical Maxwell theory - although its action on the particles, (7), is rather original. No one can understand this theory until he is willing to think of Ψ as a real objective field rather than just a "probability amplitude".

From the "microscopic" variables \vec{x} can be constructed "macroscopic" variables x

$$x_n = F_n(\vec{x}_1, \dots, \vec{x}_N) \quad (8)$$

- including in particular instrument readings, image density on photographic plates, ink density on computer output, and so on. Of course, there is some ambiguity in defining such quantities - e.g., over precisely what volume should the discrete particle density be averaged to define the smooth macroscopic density? However, it is the merit of the theory that the ambiguity is not in the foundation, but only at the level of identifying objects of particular interest to macroscopic observers, and the ambiguity arises simply from the grossness of these creatures.

It is thus from the \vec{x} 's, rather than from ψ , that in this theory we suppose "observables" to be constructed. It is in terms of the \vec{x} 's that we would define a "psycho-physical parallelism" - if we were pressed to go so far. Thus it would be appropriate to refer to the \vec{x} 's as "exposed variables" and to ψ as a "hidden variable". It is ironic that the traditional terminology is the reverse of this.

It remains to compare the pilot-wave theory with orthodox quantum mechanics at the practical level, which is that of the x 's. A convenient device for this purpose is to imagine, in the context of the orthodox approach, a sort of ultimate observer, outside the world and from time to time observing its macroscopic aspects. He will see in particular other, internal, observers at work, will see what their instruments read, what their computers print out, and so on. In so far as ordinary quantum mechanics yields at the appropriate level a classical world, in which the boundary between system and observer can be rather freely moved, it will be sufficient to account for what such an ultimate observer would see. If he were to observe at time t a whole ensemble of worlds corresponding to an initial state

$$\psi(\vec{r}_1, \dots, \vec{r}_N, 0)$$

he would see, according to the usual theory, a distribution of x 's given closely by

$$\rho(x_1, x_2, \dots) = \int d\vec{r}_1 d\vec{r}_2 \dots d\vec{r}_N \delta(x_1 - F_1(r)) \delta(x_2 - F_2(r)) \dots | \psi(r, t) |^2 \quad (9)$$

with $\psi(t)$ obtained by solving the world Schrödinger equation. It would not be exactly this, for his own activities cause wave-packet reduction and spoil the Schrödinger equation. But macroscopic observation is supposed to have not much

effect on subsequent macroscopic statistics. Thus (4) is closely the distribution implied by the usual theory. Moreover, it is easy to construct in the pilot-wave theory an ensemble of worlds which gives the distribution (9) exactly. It is sufficient that the configuration \mathbf{x} should be distributed according to

$$|\psi(\mathbf{x}, t)|^2 d\vec{x}_1 d\vec{x}_2 \dots d\vec{x}_N \quad (10)$$

It is a consequence of Eqs. (4), (5) and (6) that (10) will hold at all times if it holds at some initial time. Thus it suffices to specify in the pilot-wave theory that the initial configuration \mathbf{x} is chosen at random from an ensemble of configurations in which the distribution is $|\psi(\mathbf{x}, 0)|^2$. It is only at this point, in defining a comparison class of possible initial worlds, that anything like the orthodox probability interpretation is invoked.

Then for instantaneous macroscopic configurations the pilot-wave theory gives the same distribution as the orthodox theory, insofar as the latter is unambiguous. However, this question arises: what is the good of either theory, giving distributions over a hypothetical ensemble (of worlds !) when we have only one world. The answer has been anticipated in the introductory discussion of the α particle track. A long track is on the one hand a single event, but is at the same time an ensemble of single scatterings. In the same way a single configuration of the world will show statistical distributions over its different parts. Suppose, for example, this world contains an actual ensemble of similar experimental set-ups. In the same way as for the α particle track it follows from the theory that the "typical" world will approximately realize quantum mechanical distributions over such approximately independent components¹³⁾. The role of the hypothetical ensemble is precisely to permit definition of the word "typical".

So much for instantaneous configurations. Both theories give also trajectories, by which instantaneous configurations at different times are linked up. In the traditional theory these trajectories, like the configurations, emerge only at the macroscopic level, and are constructed by successive wave-packet reduction. In the pilot-wave theory macroscopic trajectories are a consequence of the microscopic trajectories determined by the guiding formula (7).

To exhibit some features of these trajectories, consider a standard example from quantum measurement theory - the measurement of a spin component of a spin $\frac{1}{2}$ particle. A highly simplified model for this can be based on the interaction

$$H = g(t) \sigma \frac{1}{i} \frac{\partial}{\partial r} \quad (11)$$

where σ is the Pauli matrix for the chosen component and r is the "instrument reading" co-ordinate. For simplicity take the masses associated with both particle and instrument reading to be infinite. Then other terms in the Hamiltonian can be

neglected, and the time-dependent coupling $g(t)$ can be supposed to arise from the passage of the particle along a definite classical orbit through the instrument. Let the initial state be

$$\Psi_m(0) = \phi(r) a_m \quad (12)$$

where $\phi(r)$ is a narrow wave packet centred on $r = 0$ and $m (= 1, 2)$ is a spin index; we choose the representation in which σ is diagonal. The solution of the Schrödinger equation

$$\frac{\partial \Psi}{\partial t} = -iH\Psi$$

is

$$\Psi_m(t) = \phi(r - (-i)^m h) a_m \quad (13)$$

where

$$h(t) = \int_{-\infty}^t dt' g(t') \quad (14)$$

After a short time the two components of (13) will separate in r space. Observation of the instrument reading will then, in the traditional view, yield the values $+h$ or $-h$ with relative probabilities $|a_1|^2$ and $|a_2|^2$, and with small uncertainties given by the width of the initial wave packet. Because of wave-packet reduction, subsequent observation will reveal the instrument continuing along whichever of the two trajectories, $\pm h(t)$, was in fact selected.

Consider now the pilot-wave version. Nothing new has to be said about the orbital motion of the particle, which was already taken to be classical and fixed. We do now have a classical variable χ for the instrument reading. We could consider introducing classical variables for the spin motion, but in the simplest version¹⁹⁾ this is not done; instead the spin indices of the wave function are just summed over in constructing densities and currents

$$\rho(r, t) = \Psi^*(r, t) \Psi(r, t) \quad (15)$$

$$j(r, t) = \Psi^*(r, t) g \sigma \Psi(r, t) \quad (16)$$

with the summation implied; the slightly surprising form of j follows from the gradient form of the coupling (11), and from the absence of the normal term (6) in the case of infinite mass. The motion of χ is then determined by

$$\frac{d\chi}{dt} = j(x, t) / \beta(x, t)$$

or explicitly

$$\frac{dx}{dt} = g \frac{\sum_m |a_{mn}|^2 |\phi(x - (-i)^m h)|^2 (-i)^m}{\sum_m |a_{mn}|^2 |\phi(x - (-i)^m h)|^2} \quad (17)$$

As soon as the wave packets have separated $\dot{x} = \pm g$, according to $\chi \approx \pm h$. Thus we have essentially the same two trajectories as the wave-packet reduction picture, and they will be realized with the same relative probabilities if χ is supposed to have an initial probability distribution $|\phi(x)|^2$ - this is the familiar general consequence, for instantaneous configurations, of the method of construction. In any individual case which trajectory is selected is actually determined by the given initial χ value in that case. However, if such fine details are ignored - say by introducing a macroscopic variable X by dropping all but a few decimal places in χ (in some suitable units) - the description becomes indeterministic and identical with that of the wave-packet reduction theory.

Consider now a slightly more complicated example, in which measurements of the above kind are made simultaneously on two spin $\frac{1}{2}$ particles. Denote by r_1 and r_2 the co-ordinates of the two instruments. If the initial state is

$$\Psi_{mn}(0) = \phi(r_1) \phi(r_2) a_{mn}$$

solution of the Schrödinger equation yields

$$\Psi_{mn}(t) = \phi(r_1 - (-i)^m h_1) \phi(r_2 - (-i)^n h_2) a_{mn} \quad (18)$$

with

$$h_1(t) = \int_{-\infty}^t dt' g_1(t'), \quad h_2(t) = \int_{-\infty}^t dt' g_2(t')$$

In the wave-packet reduction picture one of four possible trajectories, $(\pm h_1, \pm h_2)$, will be realized, the relative probabilities being given by $|a_{mn}|^2$. The pilot-wave picture will give again an identical account in terms of sufficiently coarse-grained variables X_1 and X_2 .

But when examined in detail the microscopic trajectories are quite peculiar during the brief initial period in which the different terms in (18) still overlap in (r_1, r_2) space. The detailed time development of the \dot{x} 's is given by

$$\begin{aligned}\dot{x}_1 &= g_1 \frac{\sum_{m,n} (-1)^m |a_{mn}|^2 |\phi(x_1 - (-1)^m h_1)|^2 |\phi(x_2 - (-1)^m h_2)|^2}{\sum_{m,n} |a_{mn}|^2 |\phi(x_1 - (-1)^m h_1)|^2 |\phi(x_2 - (-1)^m h_2)|^2} \\ \dot{x}_2 &= g_2 \frac{\sum_{m,n} (-1)^n |a_{mn}|^2 |\phi(x_1 - (-1)^n h_1)|^2 |\phi(x_2 - (-1)^n h_2)|^2}{\sum_{m,n} |a_{mn}|^2 |\phi(x_1 - (-1)^n h_1)|^2 |\phi(x_2 - (-1)^n h_2)|^2}\end{aligned}\quad (19)$$

These expressions simplify greatly when the two spin states are uncorrelated, i.e., when a_{mn} factorizes

$$a_{mn} = b_m c_n$$

The factors referring to the second particle then cancel out in the expression for \dot{x}_1 , and those referring to the first particle cancel in the expression for \dot{x}_2 , so that we have just two independent motions of the instrument pointers of the type already discussed. However, in general the spin state does not factorize. One can even envisage situations in which the two particles interact at short range and strong spin correlations are induced which persist when the particles subsequently move far apart. Then it follows from (19) that the detailed behaviour of x_1 and x_2 depends not only on the programmes h_1 and h_2 respectively of the local instruments, but also on those of the remote instruments h_2 and h_1 . The detailed dynamics is quite non-local in character.

In these examples the pilot-wave theory is seen to reproduce, on a sufficiently coarse-grained level, the macroscopic trajectories of the wave-packet reduction account. Perhaps this would be generally so, or could be secured by some modified prescription. However, it is known^{20), 21)} that the non-locality in the microscopic theory, exemplified above, cannot be removed by any prescription which leaves the theory both deterministic and in agreement with the statistical macroscopic predictions of the conventional theory. Since the microscopic trajectories are the distinctive element in the pilot-wave theory, their peculiar nature raises a question mark over it. Some people may find these peculiarities quite acceptable, at least so long as they are below the level of observability by gross macroscopic observers. Others may feel that the definition of trajectories should be restricted to some macroscopic level; the Schrödinger equation itself seems to give no hint of where such a fundamental macroscopic level might be found²²⁾. Still other people may feel that some element of indeterminism might be the thing. However, the boldest and simplest solution is that of Everett; he has no trouble with trajectories because he has no trajectories and finds no need for them.

5. EVERETT (?)

The Everett (?) theory of this section will simply be the pilot-wave theory without trajectories. Thus instantaneous classical configurations \mathbf{x} are supposed to exist, and to be distributed in the comparison class of possible worlds with probability n_1^2 . But no pairing of configurations at different times, as would be effected by the existence of trajectories, is supposed. And it is pointed out that no such continuity between present and past configurations is required by experience.

I would really prefer to leave the formulation at that, and proceed to elucidate the last sentence. But some additional remarks must be made for readers of Everett and De Witt, who may not immediately recognize the formulation just made.

- A) First there is the "many-universe" concept given prominence by Everett and De Witt. In the usual theory it is supposed that only one of the possible results of a measurement is actually realized on a given occasion, and the wave function is "reduced" accordingly. But Everett introduced the idea that all possible outcomes are realized every time, each in a different edition of the universe, which is therefore continually multiplying to accommodate all possible outcomes of every measurement. The psycho-physical parallelism is supposed such that our representatives in a given "branch" universe are aware only of what is going on in that branch. Now it seems to me that this multiplication of universes is extravagant, and serves no real purpose in the theory, and can simply be dropped without repercussions. So I see no reason to insist on this particular difference between the Everett theory and the pilot-wave theory - where, although the wave is never reduced, only one set of values of the variables \mathbf{x} is realized at any instant. Except that the wave is in configuration space, rather than ordinary three-space, the situation is the same as in Maxwell-Lorentz electron theory²³⁾. Nobody ever felt any discomfort because the field was supposed to exist and propagate even at points where there was no particle. To have multiplied universes, to realize all possible configurations of particles, would have seemed grotesque.
- B) Then it could be said that the classical variables \mathbf{x} do not appear in Everett and De Witt. However, it is taken for granted there that meaningful reference can be made to experiments having yielded one result rather than another. So instrument readings, or the numbers on computer output, and things like that, are the classical variables of the theory. We have argued already against the appearance of such vague quantities at a fundamental level. There is always some ambiguity about an instrument reading; the pointer has some thickness and is subject to Brownian motion. The ink can smudge in computer output, and it is a matter of practical human judgement that one figure has been printed rather than another. These distinctions are unimportant in practice, but surely the theory should be more precise. It was for that reason that the hypothesis was made of fundamental variables \mathbf{x} , from which instrument readings and so on can

be constructed, so that only at the stage of this construction, of identifying what is of direct interest to gross creatures, does an inevitable and unimportant vagueness intrude. I suspect that Everett and De Witt wrote as if instrument readings were fundamental only in order to be intelligible to specialists in quantum measurement theory.

- What's new?*
- Why?*
- As above
classical
state model*
- And why?*
- c) Then there is the surprising contention of Everett and De Witt that the theory "yields its own interpretation". The hard core of this seems to be the assertion that the probability interpretation emerges without being assumed. In so far as this is true it is true also in the pilot-wave theory. In that theory our unique world is supposed to evolve in deterministic fashion from some definite initial state. However, to identify which features are details critically dependent on the initial conditions (like whether the first scattering is up or down in an α particle track) and which features are more general (like the distribution of scattering angles over the track as a whole) it seems necessary to envisage a comparison class. This class we took to be a hypothetical ensemble of initial configurations with distribution $|\psi|^2$. In the same way Everett has to attach weights to the different branches of his multiple universe, and in the same way does so in proportion to the norms of the relevant parts of the wave function. Everett and De Witt seem to regard this choice as inevitable. I am unable to see why, although of course it is a perfectly reasonable choice with several nice properties. *Re-read Prof. [unclear]*

- And why?*
- After w/*
- d) Finally there is the question of trajectories, or of the association of a particular present with a particular past. Both Everett and De Witt do indeed refer to the structure of the wave function as a "tree", and a given branch of a tree can be traced down in a unique way to the trunk. In such a picture the future of a given branch would be uncertain, or multiple, but the past would not. But, if I understand correctly, this tree-like structure is only meant to refer to a temporary and rough way of looking at things, during the period that the initially unfilled locations in a memory are progressively filled, labelling the different branches of the tree only by the macroscopic-type variables describing the contents of the locations. When a more fundamental description is adopted there is no reason to believe that the theory is more asymmetric in time than classical statistical mechanics. There also apparent irreversibility can arise (e.g., the increase of entropy) when coarse-grained variables are used. Moreover, De Witt says "... every quantum transition taking place on every star, in every galaxie, in every remote corner of the universe is splitting our local world in myriads of copies of itself". Thus De Witt seems to share our idea that the fundamental concepts of the theory should be meaningful on a microscopic level, and not only on some ill-defined macroscopic level. But at the microscopic level there is no such asymmetry in time as would be indicated by the existence of branching and non-existence of debranching. Thus the structure of the wave function is not fundamentally tree-like. It does not associate a particular branch at the present time with any particular branch in the past any more than with any particular branch in the future. Moreover, it even seems reasonable to regard the

Early *After* *Concise!*
only *more* *way*

coalescence of previously different branches, and the resulting interference phenomena, as the characteristic feature of quantum mechanics. In this respect an accurate picture, which does not have any tree-like character, is the "sum over all possible paths" of Feynman.

Thus in our interpretation of the Everett theory there is no association of the particular present with any particular past. And the essential claim is that this does not matter at all. For we have no access to the past. We have only our "memories" and "records". But these memories and records are in fact present phenomena. The instantaneous configuration of the χ 's can include clusters which are markings in notebooks, or in computer memories, or in human memories. These memories can be of the initial conditions in experiments, among other things, and of the results of those experiments. The theory should account for the present correlations between these present phenomena. And in this respect we have seen it to agree with ordinary quantum mechanics, in so far as the latter is unambiguous.

The question of making a Lorentz invariant theory on these lines raises intriguing questions²⁴⁾. Leaving these aside it would seem that the Everett theory provides a resting place for those who do not like the pilot-wave trajectories but who would regard the Schrödinger equation as exact. But a heavy price has to be paid. We would live in a present which had no particular past, nor indeed any particular (even if unpredictable) future. If such a theory were taken seriously it would hardly be possible to take anything else seriously. So much for the social implications. *(what difference from probability? also no unique past?)*

In conclusion it is perhaps interesting to recall another occasion when the presumed accuracy of a theory required that the existence of present historical records should not be taken to imply that any past had indeed occurred. The theory was that of the creation of the world in 4004 B.C. During the 18th century growing knowledge of the structure of the earth seemed to indicate a more lengthy evolution. But it was pointed out that God in 4004 B.C. would quite naturally have created a going concern. The trees would be created with annular rings, although the corresponding number of years had not elapsed. Adam and Eve would be fully grown, with fully grown teeth and hair²⁵⁾. The rocks would be typical rocks, some occurring in strata and bearing fossils - of creatures that had never lived. Anything else would not have been reasonable:²⁶⁾

Si le monde n'eut été à la fois jeune et vieux, le grand, le sérieux, le moral, disparaissaient de la nature, car ces sentiments tiennent par essence aux choses antiques. L'homme-roi naquit lui-même à trente années, afin de s'accorder par sa majesté avec les antiques grandeurs de son nouvel empire, de même que sa compagne compta sans doute seize printemps, qu'elle n'avait pourtant point vécu, pour être en harmonie avec les fleurs, les oiseaux, l'innocence, les amours, et toute la jeune partie de l'univers.

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G. Ludwig, in "Werner Heisenberg und die Physik unserer Zeit" (Vieweg & Sohn, Braunschweig, 1961);
D. Bohm and J. Bub, Revs. Mod. Phys. 38, 453 (1966).
- 2) L. de Broglie, "Tentative d'Interprétation causale et non-linéaire de la Mécanique ondulatoire" (Gauthier-Villars, Paris, 1956).
- 3) H. Everett, Revs. Mod. Phys. 29, 454 (1957).
- 4) J.A. Wheeler, Revs. Mod. Phys. 29, 463 (1957).
- 5) B.S. De Witt, in "Battelle Rencontres", 1967 Lectures in Mathematics and Physics (C. De Witt, J.A. Wheeler, eds.), W.A. Benjamin Inc., New York (1968).
- 6) B.S. De Witt, in Proceedings of the International School of Physics "Enrico Fermi", Course III: "Foundations of Quantum Mechanics", Academic, New York, 1971.
- 7) B.S. De Witt, Physics Today 23, No. 9, 30 (1970).
- 8) B.S. De Witt and others, Physics Today 24, No. 4, 36 (1971).
- 9) In particular it is not clear to me that Everett and De Witt conceive in the same way the division of the wave function into "branches". For De Witt this division seems to be rather definite, involving a specific (although not very clearly specified) choice of variables (instrument readings) to have definite values in each branch. This choice is in no way dictated by the wave function itself (and it is only after it is made that the wave function becomes a complete description of De Witt's physical reality). Everett on the other hand (at least in some passages) seems to insist on the significance of assigning an arbitrarily chosen state to an arbitrarily chosen subsystem and evaluating the "relative state" of the remainder. It is when arbitrary mathematical possibilities are given equal status in this way that it becomes obscure to me that any physical interpretation has either emerged from, or been imposed on, the mathematics.
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- 10) N.F. Mott, Proc. Roy. Soc. A126, 79 (1929).
- 11) W. Heisenberg, "Physical Principles of the Quantum Theory", Chicago, (1930).
- 12) The particularly instructive nature of this example has been stressed by E.P. Wigner.
- 13) For elaboration of this point see Everett ³⁾, De Witt ^{6),7),8)} and J.B. Hartle, Am. J. Phys. 36, 704 (1968).
- 14) The high probability of exciting collective levels is emphasized by H.D. Zeh, Foundations of Physics 1, 69 (1970).
- 15) H. Stapp, preprint UCRL-20294, to be published.
- 16) L. Rosenfeld, Suppl. of Progr. of Theor. Phys., extra number 222 (1965).
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- 21) J.S. Bell, in Proceedings of the International School of Physics "Enrico Fermi", Course IL: "Foundation of Quantum Mechanics", Academic, New York, 1971.
- 22) See, however, H.D. Zeh, preprint "Dynamical Separation of Quantum Systems".
- 23) But the following difference of detail is notable. In the Maxwell-Lorentz electron theory particles and field interacted in a reciprocal way. In the pilot-wave theory the wave influences the particles but is not influenced by them. Finding this peculiar, de Broglie, Ref. 2), always regarded the pilot-wave theory as just a stepping-stone on the way towards a more serious theory which would be in appropriate circumstances experimentally distinct from ordinary quantum mechanics.
- 24) Would it be necessary to restrict attention to the here as well as the now ?
- 25) They would also have navels, although they had not been born. See, for example, P.H. Gosse, *Omphalos* (1857).
- 26) F. de Chateaubriand, "Génie du Christianisme" (1802).