

51 Probability distributions

with
respect
 $P_{X,Y}$

Let $P(x, y, \dots, z)$ represent a joint probability density for the random variables X, Y, \dots, Z , whose values are real numbers, so that $P(x, y, \dots, z) dx dy \dots dz$ is interpreted to mean the probability that X will take a value in the interval $[x, x+dx]$ and Y will take a value in $[y, y+dy]$ and ... and Z will take a value ~~between~~ in $[z, z+dz]$.

We now define a marginal probability for any subset of the original random variables, say X, Y to be the integral of the total joint density with respect to the remaining variables, i.e.

$$P(x, y) = \int_{z \dots w} P(x, y, z \dots w) dz \dots dw$$

which represents the density for the joint event $X \in [x, x+dx]$ and $Y \in [y, y+dy]$ with no restrictions on the remaining variables. It is the relative density over $X \dots Y$ when we have no information about the other variables.

Finally, we define a conditional density for any subset $X \dots Y$, conditioned on any remaining variables having prescribed values, say $Z = z \dots W = w$, denoted by $P_{x \dots y}^{z \dots w}$ to be:

$$P_{x \dots y}^{z \dots w} = \frac{P_{x \dots y \dots z \dots w}}{P_{z \dots w}}$$

which represents the density for $x \dots y$ relative to the knowledge that $Z \dots W$ have the definite values $z \dots w$.

We shall say that the Variables X and Y are independent if and only if $P(X, Y) = P(X)P(Y)$ except for a set of measure zero, and more generally we shall say that the Groups S, T, U, V, \dots, W, X are mutually of random variables are group-wise mutually independent if $P(s, t, u, v, \dots, w, x) = P(s)P(t)P(u, v) \dots P(w, x)$ except for a set of measure zero. Independence then implies that the conditional distribution for a group, conditioned on values in another independent group, is the same as the marginal for the group, i.e. X, Y indep $\Rightarrow P_{X|Y}^y = P_X$, or S, T, U, V, \dots, W, X mutually independent $\Rightarrow P_{W|S, T, U, V, \dots}^w = P_W$ almost everywhere.

that is, we learn nothing about W, \dots, X by being told values of variables in groups independent of W, \dots, X .

\rightarrow include Exp and def = (and and exp exp)

§2 Information

We now define a functional for probability distributions, $I_{X, Y} = I\{P_{x, y}\}$ to be:

$$I_{X, Y} = \iint P_{x, y} \ln P_{x, y} dx dy \\ = \text{Exp}\{\ln P_{x, y}\}$$

i.e. we define the information of a probability distribution to be the Expected logarithm of the probability density.

We can now define Marginal Prob. measures
over the product spaces of subclasses of X, Y, Z, \dots

$$\text{ie } P_m(X_i, Y_j) = M_p(X_i, Y_j, Z, \dots) \quad \text{whole sets}$$

and similarly from this Marginal Measures we derive

Marginal densities $P(x, y) = \int P(x, y, z, \dots) dz \dots dx$

We further define Conditional measures $M_p(X_i, Y_j, \dots | Z_k, \dots, W_l)$

$$\text{to be } = \frac{M_p\{X_i, Y_j, \dots | Z_k, \dots, W_l\}}{M_p\{Z_k, \dots, W_l\}}$$

which leads to Conditional densities $P^{Z \dots W_l}_{X, Y, \dots} = \frac{P(X, Y, \dots, Z \dots, W)}{P(Z \dots, W)}$

finally we define $I^{Z \dots W}_{X, Y} = \exp\{\ln P^{Z \dots W}_{X, Y}\}$

with respect to the conditional measure $M_p\{X, Y | Z \dots, W\}$

9 Probability distributions:

We shall assume that we have a collection of sets X, Y, Z, \dots , each with a measure assigned $M(X), M(Y), M(Z) \dots$

(ie a non-negative, countably additive set function over some of the subsets X_i, Y_j, \dots)

and furthermore we assign the product measure to the direct product of these sets.

We further assume that we have a probability measure over this direct product, which we shall denote by $P(X, Y, \dots)$, which we ~~will~~

is now a ~~set function~~ function of n -tuples of sets, one in X , one in Y , etc., and which we think of as being the Probability that a selection of one element from each set will result in the event that $x_i \in X_i, y_j \in Y_j, \dots$

Now it is known that this always results in the existence of a Probability density function $f(x, y, z, \dots)$, which is a point (or element) function rather than a set function, such that the integral of $f(x, y, z, \dots)$ over any ~~set~~ of the product space, with respect to our original product measure, is the Probability measure of that set.