

§4. Appropriate Measurement

A phenomenon which is difficult to understand within the framework of the probabilistic interpretation of quantum mechanics is the result of an appropriate measurement. In the abstract formulation of the usual theory there are two fundamental processes, the discontinuous, probabilistic Process 1 corresponding to precise measurement, and the continuous, deterministic Process 2 corresponding to absence of any measurement. What mixture of probability and causality are we to apply to the case where only an appropriate measurement is effected (i.e., where the apparatus or observer interacts only weakly and for a finite time with the object-system)?

In the case of appropriate measurement we need to be supplied with rules which will tell us, for any initial object-system state, first what probability one can expect the various possible ¹⁶ apparatus readings, and second what new state to ascribe to the system after the value has been observed. We shall see that it is generally impossible to give these rules within a framework which considers the apparatus or observer as performing an (abstract) observation subject to process 1, and that it is necessary, in order to give a full account of appropriate measurements, to treat the entire system, including apparatus or observer, wave mechanically.

The position that an approximate measurement results in the situation that the object-system state is changed into an eigenstate of the exact measurement, but for which ^{particular} the observer has only imprecise information, is manifestly false. This fact that we can make successive approximate position measurements of particles (in cloud chambers, for example) and use the results for ~~the approximate~~ ^{somewhat reliable} predictions of future positions. However if either of these measurements left the particle in an "eigenstate" of position (8 function), even though the particular one remained unknown, the momentum would have such a variance that no such prediction would be possible. (The possibility of such predictions lies in the correlations between position and momentum at one time ~~which at a later time~~ ^{to} ~~for wave packets~~ position and momentum at a later time ~~for~~ ^{for wave packets} correlations which are totally destroyed by precise measurements of either quantity.)

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we can
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of either quantity.

P. Instead of continuing the discussion of the inadequacy of the probabilistic formulation, let us first

~~To~~ investigate what actually happens in approximate measurements, from the viewpoint of pure wave mechanics. An approximate measurement consists of an interaction, for a finite time, which only imperfectly correlates the apparatus (or observer) with the object-system. We can deduce the desired rules in any particular case by the following ~~method~~ method: For fixed interaction and initial apparatus state and for any initial object-system state we solve the wave equation for the time of interaction in question. The result will be a superposition of apparatus (observer) states and relative ~~object~~ object-system states. Then (according to the method of Chap II for assigning a measure to a superposition) we assign a probability to each observed result equal to the square-amplitude of the coefficient of the element which contains the apparatus (observer) state representing the registering of that result. Finally the object-system is assigned the new state which is its relative state in that element.

For example, let us consider the measuring process described in III-§3, which is an excellent model for an approximate measurement. After the interaction the total state was found to be (II-(3.12)):

$$(4.1) \quad \psi_i^{\text{STA}} = \int \frac{1}{N_r} \cdot \xi'(q) \delta(r-r') dr'$$

Then, according to our prescription, we assign the probability, $P(r')$ to the observation of the apparatus coordinate r' density

$$(4.2) \quad P(r') = \left| \frac{1}{N_r} \right|^2 = \int \phi^* \phi(q) \eta^* \eta(r'-q) dq$$

which is the square amplitude of the coefficient $(\frac{1}{N_r})$

of the element $\xi'(q)\delta(r-r')$ of the superposition (9.1) in which the apparatus coordinate has the value $r=r'$. Then, depending upon the observed apparatus coordinate r' , we assign the object system the new state

$$(4.3) \quad \xi^{r'}(q) = N_r \phi(q) N(r'-gt)$$

(where $\phi(q)$ is the old state, and $N(r)$ is the initial apparatus state) which is the relative object-system state in (4.1) for apparatus coordinate r' .

This example supplies the counter-example to another conceivable method of dealing with approximate measurement within the framework of process 1. This is the position that when an approximate measurement of a quantity Q is performed in actuality another quantity Q' is precisely measured, where the eigenstates of Q' correspond to fairly well-defined (i.e., sharply peaked distributions for) Q values.² However, any such scheme based on process 1 always loses the prescription that after the measurement the (unnormalized) new state function results from the old by a projection (on an eigenstate or eigenspace), which depends upon the observed value. If this is true, then in the above example the new state $\xi'(q)$ must result from the old, $\phi(q)$, by a projection E :

$$(4.4) \quad \xi'(q) = N E \phi(q) = N_r \phi(q) N(r'-gt)$$

(where N, N_r are normalization constants) ~~But~~

²
 (F. Von
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 (approximate
 Measurement
 action))