

We now Assume the following Model:

We have a pari-mutual machine which operates in the following manner: It selects (uniformly) one of the N states (X_i) and Posts it.

It then selects a number Q uniformly from $[E]$ which gives the "odds" that X_i really obtains

A player then has the choice of either Betting $\frac{1}{Q}$ on X_i or refraining from betting. If he elects to bet then he wins amount $\left(\frac{1}{Q} - 1\right)$ if X_i actually obtains, and forfeits his dollar otherwise. If he pays $\frac{1}{Q}$ for ticket, gets back $\frac{1}{Q}$ if he wins, 0 otherwise.

We now assume that the individual possesses a subjective prob distrib P_i which indicates to him the prob. that X_i actually obtains.

Then in this case he will clearly ~~only~~
bet if and only if $P_i > Q$ (in case the
 i^{th} alternative is announced, with "odds" Q)

since ~~if~~ in this case his expectation is

$$\frac{P_i}{Q} - 1$$

~~the~~ Now his A priori expectation on the
 i^{th} event (before the odds are posted), is

clearly $\text{Exp}_i = \int_{\varepsilon}^{P_i} \left[\frac{P_i}{Q} - 1 \right] dQ$

$$= P_i \ln \left[\frac{P_i}{\varepsilon} \right] - (P_i - \varepsilon)$$

$$= P_i \ln P_i - P_i \ln \varepsilon - P_i + \varepsilon$$

since Q is chosen uniform over $[\varepsilon, 1]$

finally, since the event itself is chosen uniformly after the set $\{X_i\}$

the overall A priori Expectation of the Player is:

$$Ex = \frac{1}{N} \sum_i Ex P_i = \frac{1}{N} \left[\sum_i P_i \ln P_i - \sum_i P_i \ln E - \sum_i P_i + \sum_i E \right]$$

$$= \boxed{\frac{1}{N} \left[\sum_i P_i \ln P_i - \ln E - 1 + NE \right]}$$

(if model changed slightly so ~~he is free~~ Odds are announced

on each alternative, then he is free to bet or not on any or all alternatives, and his

Expectation is: $\underline{\sum_i P_i \ln P_i} - \ln E - 1 + NE$

so that in this model his Expectation is directly his information and he would be willing to pay a price dollars for bins of info.

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Further note, $N\varepsilon - 1 - \ln \varepsilon$

~~is always > 0 for $\varepsilon \leq 1$~~

(since his expectation ~~$\exp_i > 0 \Rightarrow$~~

not necessarily

but $O \geq I$ is always $\geq -\ln N$

$$\Rightarrow \sum_i p_i \ln p_i = \ln \varepsilon - 1 + N\varepsilon$$

$$\geq -\ln N - \ln \varepsilon - 1 + N\varepsilon$$

$$= \ln \frac{1}{N\varepsilon} + N\varepsilon - 1 \geq 0$$

but, min of $x - \ln x$ goes from $+\infty$ at 0

$$\frac{dx}{x} 1 - \frac{1}{x} = 0$$

$$\frac{1}{x} = 1 \quad x = 1 \text{ is min}$$

at 1 at 1

to ∞ at ∞

Since we have $-\ln x + x \geq 1$ all $x \geq 0$
 $= 1$ only $x = 1$

ie, in case the cutoff ε has the value $\frac{1}{N}$ - then Expect 0 if & only if the Information minimum (uniform distri^b) otherwise $\text{Exp} \stackrel{\text{Def}}{\geq} 0.$

To We consider only the information difference from that of the Uniform over the n objects, we will then always have positive inform, from 0 to $\log N$

ie $I' = \cancel{I + \ln N} \quad (\text{since inform uniform} = \ln N)$

so that here

$$\text{Exp} = \sum p_i \ln p_i - \ln \varepsilon - 1 + N\varepsilon$$

$$= I' - \ln N - \ln \varepsilon + N\varepsilon - 1$$

$$= I' + \left(N\varepsilon + \ln \frac{1}{N\varepsilon} - 1 \right)$$

(in case then
 $N\varepsilon = 1$)

$\text{Exp} = I' \cancel{\text{direct}}$

We can now make it an n -person game by requiring the n -players to support the bank mutually, in which case the different information of the players is a direct measure of their advantage in the game. Information can even be sold or traded in this case, with I being worth precisely 1 dollar. (provided no of players sufficiently large.)

in our model for differing Subjective
distributions, i.e. α Variable in Question,
chosen from Joint distrib $P_{\alpha\beta\dots\gamma}$, with Various
Players to be told other Variables. ~~other~~

i.e. Assume P_1 is to be told nothing, then his
subjective distribution is P_α with info $I(P_\alpha) = \sum_{\alpha} P_\alpha \ln P_\alpha$

Suppose P_2 will be told the Value of β ,
then his distrib will be $P_\alpha^\beta = \frac{P_{\alpha\beta}}{P_\beta}$ which depends
upon β , and his information will be $\sum_{\alpha} P_\alpha^\beta \ln P_\alpha^\beta = I_\alpha^\beta$
and his Expected Information on α is $\sum_{\beta} P_\beta I_\alpha^\beta$
 $= I_\alpha + C_{\alpha;\beta}$

So that the advantage over P_1 given that he
will be told the Value of β is precisely $C_{\alpha;\beta}$
(the correlation of α with β)

(Similarly with further players - the

dvantage of a player who is informed of $\beta\delta\xi$
vers one who is simply informed of $\beta\gamma$

$$\text{is } C_{\alpha;\beta\delta\xi} - C_{\alpha;\beta\gamma}$$

(define $C_{\alpha;\phi} = C_{\phi;\alpha} = 0$)

$$= I_{\alpha\beta\delta\xi} - \cancel{I_\alpha} - I_{\beta\delta\xi} - \left[I_{\cancel{\beta\delta\gamma}} - \cancel{I_\alpha} - I_{\beta\gamma} \right]$$

$$= I_{\alpha\beta\delta\xi} - I_{\beta\delta\xi} + I_{\beta\gamma} \neq I_{\alpha\beta\gamma}$$

||

~~$I_{\alpha\beta\gamma} + I_{\beta\delta\xi} + C_{\alpha\beta\gamma;\delta\xi}$~~

~~$= I_{\alpha\beta\delta\xi} - I_\alpha - I_{\beta\xi} - C_{\beta\gamma;\delta\xi} + I_{\beta\gamma} \neq I_{\alpha\beta\gamma}$~~

||

~~$I_{\alpha\beta\gamma} + I_{\beta\xi} + C_{\alpha\beta\gamma;\delta\xi}$~~

P

~~ΔI_α~~

$$= C_{\alpha\beta\gamma;\delta\xi} - C_{\alpha\beta\gamma}$$

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$$\alpha \cancel{I_{\alpha\beta\gamma}} = C_{\alpha\beta\gamma} - C_{\alpha\beta\delta}$$

$$= I_{\alpha\beta\gamma} - I_\alpha - I_{\beta\gamma} - C_{\alpha\beta\gamma}$$

||

$$I_{\alpha\beta\gamma} + I_{\beta\gamma} + C_{\alpha\beta\gamma} - I_\alpha - I_{\beta\gamma} - C_{\alpha\beta\gamma}$$

||

$$\cancel{I_\alpha} + \cancel{I_{\beta\gamma}} + C_{\alpha\beta\gamma} + \cancel{I_{\beta\gamma}} + C_{\alpha\beta\gamma} - I_\alpha - \cancel{I_{\beta\gamma}} - \cancel{I_{\beta\gamma}} - C_{\alpha\beta\gamma}$$

$$= C_{\alpha\beta\gamma} - C_{\beta\gamma\alpha}$$

ans

Question is $C_{\alpha\beta} + C_{\alpha\gamma} - C_{\beta\gamma} = C_{\alpha\beta\gamma}$? (NO!)

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$$= I_{\alpha\beta} \cancel{(-I_\alpha - I_\beta)} + I_{\alpha\gamma} \cancel{(-I_\alpha - I_\gamma)} - I_{\beta\gamma} \cancel{(+I_\beta + I_\gamma)}$$

$$= I_{\alpha\beta} + I_{\alpha\gamma} + I_{\beta\gamma} - I_\alpha - I_\beta - I_\gamma$$

$$= I_{\alpha\beta} + I_{\alpha\gamma} - I_{\beta\gamma}$$

$$= \sum_{\alpha\beta} P_{\alpha\beta} \ln P_{\alpha\beta} + \sum_{\alpha\gamma} P_{\alpha\gamma} \ln P_{\alpha\gamma} - \sum_{\beta\gamma} P_{\beta\gamma} \ln P_{\beta\gamma}$$

$$\sum_{\beta\gamma} P_{\beta\gamma} \ln \left[\frac{P_{\alpha\beta} P_{\alpha\gamma}}{P_{\alpha\beta\gamma}} \right]$$

$$\frac{P_{\alpha\beta}}{P_\beta} = P_\alpha^\beta$$

~~not = 1~~

$$= \ln \left[\frac{\frac{P_\alpha^\beta}{P_\beta} \frac{P_\alpha^\gamma}{P_\gamma}}{\frac{P_\alpha}{P_{\beta\gamma}}} \right] = \left[\frac{\frac{P_\alpha^\beta P_\alpha^\gamma}{P_\alpha}}{\frac{P_\beta P_\gamma}{P_{\beta\gamma}}} \right] \left[\frac{\frac{P_\beta P_\gamma}{P_{\beta\gamma}}}{\frac{P_\alpha}{P_{\beta\gamma}}} \right]$$