

An interesting point is the conservation of information application yields
in classical mechanics of a group of interacting particles.

The system is represented by a point, $(x_1^1, \dot{x}_1^1, p_x^1, p_y^1, p_z^1, \dots, x_n^m, \dot{x}_n^m, p_x^n, p_y^n, p_z^n)$ at any instant in the phase space of all position + momentum coordinates. The natural motion of the system then carries each point into another, defining a continuous transformation of the phase space into itself. According to Liouville's theorem the measure of a set of points of the phase space is invariant with respect to this under this transformation. This invariance of measure implies that if we begin with a probability distribution over the phase space, rather than a single point, the total information (the information of the joint distribution for all positions and momenta) remains constant in time.

$$I_{\text{total}} = I_{X^1 Y^1 Z^1 P_x^1 P_y^1 P_z^1} \dots I_{X^n Y^n Z^n P_x^n P_y^n P_z^n}$$

$$\{ \text{total} \} \quad I_{\text{Marginal}} = I_{X^1} + \dots + I_{P_x^1} + \dots + I_{X^n} + \dots + I_{P_x^n}$$

remains constant in time. In order to see this, consider any partition P of the phase space at one time, t_0 , with its information relative to the phase space measure, $I^{P(t_0)}$. At a later time t_1 , a partition P' into the image sets of P under the mapping of the space into itself is induced, for which the probabilities for the sets of P' are the same as the corresponding sets of P , and furthermore for which the measures are the same, by Liouville's theorem. Thus corresponding to each partition P at time t_0 with information $I^{P(t_0)}$ there is a corresponding partition P' at time t_1 with information $I^{P'(t_1)}$ which is the same:

$$(2.1) \quad I^{P'(t_1)} = I^{P(t_0)}$$

thus the supremums of each overall partitions must be the same, and by (2.1) we have deduced that:

$$(2.2) \quad I_{\substack{xx'x'' \\ \text{Total}}} \dots I_{\substack{pp'p'' \\ \text{Total}}} = I_{\substack{xx'x'' \\ \text{Total}}} \dots I_{\substack{pp'p'' \\ \text{Total}}} \quad (t_0)$$

Suppose each term is constant

which says that the total information is conserved.

(P.D.) It is known that the individual (marginal) position, and momentum, distributions decay (except for rare fluctuations) into the uniform and Maxwellian distributions respectively, for which the classical entropy is a maximum. The classical entropy is, however, simply the negative of the marginal information, which then tends toward a minimum. But this decay of marginal information,

$I_{x_1} + I_{x_2} + I_{x_3} + \dots$ is, by (2.2) exactly compensated by an increase of $\frac{\text{Total}}{\text{Total}}$ correlation information, $\{I_{x_1 x_2}, I_{x_2 x_3}, \dots\}$,

$$\{\text{Total}\} = I_{\text{Total}} - I_{\text{Marginal}}$$

B give reference to
Von Neumann's extropes
for density matrix.

since according to (2.2) the total information remains constant. Therefore, if one were to define the total entropy to be the negative of the total information, one replaces ^{could} the usual second law of thermodynamics by a law of conservation of total entropy, ~~where~~ where the increase in the standard (marginal) entropy is exactly compensated by a (negative) correlation entropy. The usual second law then results simply from our renunciation of all correlation knowledge, ^(Stosszustand) and not from any intrinsic behavior.



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The situation is slightly more complicated when interactions with fields are allowed, since in this case the conservation of total particle information need not hold. However, for proper definition of field information one again expects a conservation law of total (particle + field) information. Thus when particles interact electromagnetically, correlations of the particle coordinates can be built up more rapidly than the decay of marginal information, but this should be compensated by a loss of information for the electromagnetic field corresponding to the existence of ensuing radiation of uncertain whereabouts.

We have already seen that in composite systems of quantum mechanics one form of such a principle holds, namely that the entropy of mixtures is always compensated by system correlation (chap III). Furthermore, as shown by von Neumann⁽³⁾, for a system which is a mixture characterized by density matrix ρ , the entropy - trace $S_{\text{ent}} = -\text{Tr}[\rho \ln \rho]$ remains constant so long as the system is isolated (no measurements performed upon it). Thus the conservation of entropy law (or mixture entropy) holds true here also.

§7 Example: Conservation of Information in Classical mechanics

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As a second illustrative example we consider briefly the classical mechanics of a group of particles.

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The situation for classical mechanics is thus in sharp contrast to that of stochastic processes, which are intrinsically irreversible.