

Conservation of Information

We consider momentarily the classical mechanics of a group of particles interacting through central forces. We consider the system to be represented by a point in phase space, whose coordinates are the positions and momenta of all the particles. Then according to Liouville's theorem the measure of a set of points in the phase space is invariant with respect to the natural motion of the phase space into itself. If instead of a single system we consider a probability distribution over the phase space, then we have a total information for the system which consists of the integral of $P \ln P$ over the whole phase space, where P is the probability density. Then it is an easy consequence of Liouville's theorem that this information is constant in time, as is seen by consideration ^{the following} ~~of approximations~~ to the distribution by ~~discrete distributions~~. Divide the phase space into sets S_i , with probability P_i and measure $M(S_i)$, then the information is given by $\sum_i P_i \ln P_i M(S_i)$ in the limit as all of the $M(S_i) \rightarrow 0$.

But at a future time the set S_i is transformed into a set T_i with the same probability, and by Liouville's theorem the same measure, so

that at the future time the information is the limit of $\sum_i P_i \ln P_i M(T_i) = \sum_i P_i \ln P_i M(S_i)$

which is the same, so that ^{Total} information is invariant with time. (where the random variables are

Total
Information
and
Measure
of
Phase Space

This result is interesting of itself. It is known that the marginal ^{individual} position and momentum distributions decay ^{except for occasional fluctuations} into the uniform and Maxwell distributions, respectively, and that the classical entropy is simply the negative of this marginal information. However, at the same time this decrease of marginal information is exactly compensated by the total correlation of all the positions and momenta, since the joint information is constant. If one were to define the total entropy as the negative of the total information, then instead of the usual second law we have a conservation of entropy law, where the ~~increase~~ in the standard (marginal) entropy is exactly compensated by a (negative) correlation entropy. The standard second law is thus seen ~~to~~ to result simply from ~~the~~ ^{our} renunciation of all correlation knowledge, and not from any intrinsic behavior.

The situation is slightly more complicated when interactions with fields are allowed, since in this case the conservation of total particle information no longer holds, and may even increase. However, for proper definition of field information one again expects a conservation of total (particle+field) information law. It can sometimes happen that when particles interact electromagnetically that correlations ^{of the particle coordinates} are built up faster than the marginal information decays, but this should be compensated by a loss of information of the electromagnetic field corresponding to the existence of radiation of uncertain whereabouts.

For a single free particle, the loss of position information is compensated simply by the correlation between the position and momentum.

When we turn to Quantum mechanics, however, a new difficulty appears; namely, how are we to consider correlations between non-commuting variables? If one considers a ^{true} gaussian wave packet centered at the origin at time zero, then as time progresses the momentum ^{amplitude} density remains constant, while the position amplitude density spreads rapidly, causing a decrease in the marginal information. In the case of a classical particle, this loss of marginal information is exactly compensated, as we have seen, by a correlation between the position and momentum. The present formalism of Quantum mechanics, however, does not allow us to construct a joint amplitude density for noncommuting variables, so that we have no immediate way to define position-momentum correlation information in general. Still it is a fact that it makes sense to talk about such correlations. If, in the wave packet example above, one waits for a time, and then makes an approximate position measurement one knows something about subsequent momentum measurements, namely, that if the particle is found at a great distance its momentum will be found to be high. One can in cloud chambers estimate both position and momentum of particles, and make predictions about their future trajectories. We know that, at least as long as our measurements are coarse, Liouilles theorem holds approximately, and there are correlations between position and momentum.

This existence of this correlation, however, is seen to be dependent upon the somewhat coarse nature of the measurements. If either position or momentum is measured with complete accuracy, then the information about the conjugate variable is completely lost, and there is no correlation.

We are in a position then of knowing that such correlations are meaningful, but that they elude us in the usual framework of exact measurements.

To adequately treat this point one would need an adequate theory of approximate measurement, so that one might give a formal definition of this correlation, and hence of the ^{total} joint phase space information for a wave function, and be able to justify it as a limit perhaps as a limit attainable by suitable approximate measurements. This problem has not yet been adequately solved, but it seems does not seem insuperable. Once solved one would expect to be able then to extend the law of Conservation of information to all quantum mechanical systems.

This seems quite plausible, since we know ^{already} that it nearly holds, at least in the sense to the degree that particles obey classical mechanics. One would also expect to extend it to electrodynamics, obtaining a definition of quantum field information as a result. We note that ^(as elsewhere in Physics + Mathematics) it is not so much a matter of proving the theorem as it is a matter of defining the information involved so that the theorem is true. This point of view can ~~yield useful~~ yield some useful criteria for acceptable measures on the space of fields, since we should probably only consider

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those measures which are invariant with time
so that Thirring's theorem (and conservation of info) hold.
At any rate there seems to be a basic principle after
~~which~~ ^{which is a conservation law,} which is more general than the usual
second law, and from which the second law
results simply from neglect of correlations. It should
receive fuller investigation.