

The relative system states for  $S_r$  have been changed from eigenfunctions of  $A^r (\{\phi_e^r\})$  to those of  $B (\{n_m^r\})$ . We notice further, that, with respect to our measure on the superposition, the memory sequences still have the character of random sequences, but for which the individual terms are no longer independent. The memory states  $\beta_m^r$  now depend upon the states  $\alpha_e^r$ . The joint square amplitude distribution for this pair of quantities, conditioned by fixed values for the remaining quantities is  $(M_{\alpha_e^r, \beta_m^r}^{n_m^r, \phi_e^r}(\alpha_e^r, \beta_m^r))$

$$\rightarrow = \frac{M(\text{whole sequence})}{M(\text{diluted sequence})} = |(\phi_e^r, \psi^{S_r})|^2 / |(n_m^r, \phi_e^r)|^2$$

This is, first of all independent of the memory characterization for the remaining systems. Second, the dependence of  $\beta_m^r$  on  $\alpha_e^r$  is equivalent to that given by the stochastic process <sup>(4)</sup> which converts the states  $\phi_e^r$  into states  $n_m^r$ , with transition probabilities  $T_{e,m} = P(\phi_e^r \rightarrow n_m^r) = |(n_m^r, \phi_e^r)|^2$ .

<sup>(3)</sup> see previous discussion  
 since they are highly correlated

for which  
the number  
of observations  
of each quantity  
in each system  
is very large!

with

We can therefore summarize the situation for a completely arbitrary sequence of measurements, upon the same or different systems, in any order, with the following result: Almost all memory sequences contained in the superposition (as the number of observations <sup>of each type on</sup> on each system goes to infinity) satisfy the criterion for a random sequence, with independent probabilities upon the elements corresponding to different systems, and with <sup>conditional</sup> probabilities generated by the stochastic process ( ) upon the successive observations of a single system.

That is, all averages of functions over a memory sequence can be correctly computed by the use of probabilities for the individual elements which are the usual  $\langle \rangle$  for the first element referring to a single element, and are computed according to the stochastic process  $(\cdot)$  for subsequent elements referring to the same system, for almost all memory sequences.

We have thus shown that the assertions of Process I appear to hold to almost all observers.

incomplete

We note further that this result is not restricted to sequences of observations upon identical systems.

For an Arbitrary series of Measurements of the quantities  ${}^1 A \ {}^2 A \ {}^3 A \dots$  eigenfunctions  $\{\psi_1, \psi_2, \dots\}$  applied to the systems  $S_1, S_2, \dots S_n$

$$\sum_{\vec{i}}$$

$$\Psi \phi^0 \rightarrow \sum_i (\phi_i \psi_i) \phi_i \psi^0$$

measure second,  $\eta_j$

$$\sum_j -\eta_j \sum_i (\phi_i \psi_i) \phi_i \left( \eta_j \psi^0_{[x_i]} \right)$$

$$= \sum_{ij} (\phi_i \psi_i) (\eta_j \phi_i) \eta_j \psi^0_{[x_i; \beta_{ij}]} \quad [x_i; \beta_{ij}]$$

Thus, if we allow at some stage a new observation of a quantity  $B$ , (eigenfunctions  $\eta_m$ ), memory characterization  $\beta_i$ , upon a system  $S_r$  for which  $A$  has already been determined, then the state

$$\sum_{i_1 i_2 \dots i_k} (\phi_{i_1}^1 \psi_{i_1}^s) \dots (\phi_{i_r}^r \psi_{i_r}^{s_r}) \dots (\phi_{i_k}^m \psi_{i_k}^{s_m}) \phi_{i_1}^{s_1} \phi_{i_2}^{s_2} \dots \phi_{i_k}^{s_k} \psi^0_{[x_{i_1}^{s_1}, x_{i_2}^{s_2}, \dots, x_{i_k}^{s_k}]} \quad [x_{i_1}^{s_1}, x_{i_2}^{s_2}, \dots, x_{i_k}^{s_k}]$$

is transformed, by rule 2, into

$$\sum_{i_1 i_2 i_3 \dots i_k} (\phi_{i_1}^1 \psi_{i_1}^s) \dots (\phi_{i_r}^r \psi_{i_r}^{s_r}) \dots (\phi_{i_k}^m \psi_{i_k}^{s_m}) \underbrace{(\eta_{m_r} \phi_{i_1}^{s_1})}_{\eta_m} \phi_{i_2}^{s_2} \dots \phi_{i_{k-1}}^{s_{k-1}} \eta_{m_r} \phi_{i_k}^{s_k} \phi_{i_k}^{s_k} \psi^0_{[x_{i_1}^{s_1}, x_{i_2}^{s_2}, \dots, x_{i_k}^{s_k}, \beta_{m_r}^r]} \quad [x_{i_1}^{s_1}, x_{i_2}^{s_2}, \dots, x_{i_k}^{s_k}, \beta_{m_r}^r]$$

## IV Measurement and observation

In accordance with our plan to develop quantum mechanics within the framework of pure wave mechanics, we must investigate the result of regarding the process of measurement itself as a natural process, and treating it entirely wave-mechanically. Thus from our point of view a measurement, or observation, is to be regarded simply as an interaction between a system and a measuring apparatus, or observer, which which correlates the two systems. As an example of such a process we shall first discuss a simplified situation given by Von-Neumann:

### § 1

#### Simplified Measurement

Suppose that we have a system of only one coordinate,  $q$ , (such as position of a particle), and an apparatus of one coordinate  $r$  (for example the position of a meter needle). Further suppose that they are initially independent, so that the combined wave function is  $\Psi_{S+A} = \Phi(q) N(r)$ , where  $\Phi(q)$  is the initial system wave function, and  $N(r)$  the initial apparatus function, and that the masses are sufficiently large or the time of interaction sufficiently small that the kinetic portion of the energy may be neglected, so that during the time of measurement the Hamiltonian shall consist only of an interaction, which we shall take to be:

$$(1.1) \quad H_I = -i\hbar q \frac{\partial}{\partial r}$$

Then according to the Schrödinger equation, it is easily verified that the solution at time  $t$ , for the specified initial conditions is:

$$(1.2) \quad \Psi_{S+A}^t(z, r) = \phi(z) N(r - gt)$$

or, translating into square amplitudes:

$$(1.3) \quad P_t(z, r) = P_1(z) P_2(r - gt)$$

where  $P_1(z) = \phi^*(z)\phi(z)$ ,  $P_2(r) = N^*(r)N(r)$

and we note that for a fixed time,  $t$ , the conditional square amplitude distribution for  $r$  has been translated by an amount depending upon the value of  $z$ , while the marginal distribution for  $z$  has been unaltered. We see thus that a correlation has been introduced between  $z$  and  $r$  by this interaction, which allows us to interpret it as a measurement. It is instructive to see quantitatively how fast this correlation takes place. We note that:

$$(1.4) \quad I_{QR}^*(t) = \iint P_t(z, r) \ln P_t(z, r) dz dr = \iint P_1(z) P_2(r - gt) \ln P_1(z) P_2(r - gt) dz dr$$

$$= \iint P_1(z) P_2(w) \ln P_1(z) P_2(w) dw dz = I_{QR}(0)$$

So that the information of the joint distribution does not change. Furthermore, the marginal distribution for  $z$  is unchanged:

$$(1.6) \quad I_Q(t) = I_Q(0)$$

and the only quantity which can change is the marginal information,  $I_R$ , of  $r$ , whose marginal distribution is:

$$(1.7) \quad P_t(r) = \int P_t(r, q) dq = \int P_1(q) P_2(r - qt) dq$$

*new sentence*  
 from  
 2 for which, using (1.5, appendix), we have that:

$$(1.8) \quad I_R(t) \leq I_Q(0) - \ln t$$

so that, except for the additive constant  $I_Q(0)$ , the marginal information  $I_R$  tends to decrease at least as fast as  $\ln t$  with time during the interaction. This implies the relation for the correlation:

$$(1.9) \quad \{Q, R\}_t = I_{RQ}(t) - I_Q(t) - I_R(t) \geq I_{RQ}(t) - I_Q(0) - I_Q(0) + \ln t$$

But at  $t=0$  the distributions for  $R$  and  $Q$  were independent, so that  $I_{RQ}(0) = I_R(0) + I_Q(0)$ . Substitution of this relation, (1.5), and (1.6), into (1.9) then yields the final result:

$$(1.10) \quad \{Q, R\}_t \geq I_R^0 - I_Q^0 + \ln t$$

Therefore we have shown that

~~that~~ the correlation is built up at least as fast as  $\ln t$ , except for an additive constant representing the difference of the information of the initial distributions  $P_2(r)$  and  $P_1(q)$ .

note that  
upon initial state of system  
for the relative functions

remain invariant later

ie thus even though  
they go to infinity or diverge  
contribution for field interaction  
there is some dependence upon  
initial's state,

We notice further, however, that this measurement has the property that it does not change the marginal system distribution, nor does the apparatus indicate any definite system value. One can, though, look upon the total wave function,  $\Psi_0(Z)$  as a superposition of ~~pairs of subsystem~~ system states, each of which has a definite  $q$  value and a correspondingly displaced apparatus state (see discussion of relative states in III-§2) : Thus we can write (1.2) as :

$$(1.11) \quad \Psi_{S+A}^{r,t} = \int \phi(q') \delta(q-q') N(r-q't) dq'$$

~~(1.11) at p. 1~~

which is a superposition of states  $\Psi_{q'} = \delta(q-q') N(r-q't)$ , in which the system has the definite value  $q=q'$ , and the apparatus is displaced from its original position by an amount  $q't$ , superposed with amplitude  $\phi(q')$ .

Conversely, if we transform to the representation where the apparatus is definite, we write (1.2) as :

$$(1.12) \quad \Psi_{S+A}^{r,t} = \frac{1}{N_r} \sum_{r'} \xi_{r'}^{r,t}(q) S(r-r') dr'$$

$$\text{where } \xi_{r'}^{r,t}(q) = N_r \phi(q) N(r-q't), \quad \frac{1}{N_r} = \sqrt{\int \phi^*(q) \phi(q) N^*(r'qt) N(r'qt) dq}$$

where the  $\xi_{r'}^{r,t}(q)$  are the relative state functions for the apparatus states  $S(r-r')$  of definite value  $r=r'$ . We can see also that if the degree of correlation is sufficiently high, the relative functions  $\xi_{r'}^{r,t}(q)$  are nearly eigenfunctions for the values  $q=\frac{r'}{t}$ , since as  $t \rightarrow \infty$ , or if  $N(r)$  is sufficiently peaked (near  $\delta(r)$ ) then  $\xi_{r'}^{r,t}(q)$  is nearly  $S(q-\frac{r'}{t})$ .

Insert 4

A better way to see that the relative system exists in  $\xi^r(Q)$  approach eigenfunctions of the measurement is to adopt the reasonable definition of "nearness" to an eigenfunction as the information of the distribution over eigenvalues for the state in question, since the greater the information, the sharper peaked the distribution, and hence the nearer to a  $\delta$  function. Thus as a measure of the nearness to eigenfunctions of the relative states  $\xi^r(Q)$  we adopt  $I_Q^r$ , the conditional information. Making use of the relation  $\text{Exp}[I_Q^r] - I_Q = \{R, Q\}$ , and (1.9) and (1.5) we get the relationships

$$(1.12) \quad \text{Exp}[I_Q^r] \cong I_R^\circ + \ln t$$

so that as  $t$  goes to infinity, or as the apparatus initial distribution becomes sufficiently sharp (so  $I_R^\circ \rightarrow \infty$ ) the expectation tends to infinity, and, at least, almost all relative functions approach eigenfunctions. (One can assert this result for all relative states so long as the initial q distribution  $\phi(q)\phi^*(q)$  is not too pathological, i.e. for continuity.)

Thus, by (1.12), we are confronted with a superposition of apparatuses, each of which has recorded a definite value, and relative to which the system is left in an approximate eigenstate of the measurement. The discontinuous "jump" into an eigenstate is thus only a relative proposition, dependent upon our decomposition of the total wave function into the superposition, and relative to a particularly chosen apparatus values.

5

O.K.