4.4.2024

$\mathbb{R}^{1+3}$ | QM: $C(\mathbb{R}^{1+3}, \mathbb{C})$

N — Galilei-invariant

Conservation of momentum

$$\sum_{i=1}^{n} \dot p_i = \sum_{i=1}^{n} F_i$$

$$H + L = \dot x \cdot p$$

Conservation of information

$$\nabla \cdot \begin{bmatrix} \dot x \\ \dot p \end{bmatrix} = 0$$

$\dot p = F$ $\frac{d}{dt}\frac{\partial L}{\partial \dot x} = \frac{\partial L}{\partial x}$	$\Leftarrow$ $A[x] = \int L(x,\dot x)\,dt \to \min!$	$\frac{\partial H}{\partial x} = -\dot p \quad \frac{\partial H}{\partial p} = \dot x$ $\{\cdot, H\} = \frac{d}{dt}$ ← Conservation of energy	$\hat x = x$ $\hat p = -i\frac{\partial}{\partial x}$ $\hat E = -i\frac{\partial}{\partial t}$	Heisenberg: $\Delta \hat A \cdot \Delta \hat B \geq \frac{1}{2}\langle[\hat A, \hat B]\rangle$

	Newton		Lagrange		Hamilton		QM	
	$\dot p = F$		$L(x,\dot x)$		$E = H(x,p)$		$\hat E = -i\hat H$	← Schrödinger
	$m\ddot x = -\partial V$		$m/2\,\dot x^2 - V(x)$		$p^2/2m + V$		$\frac{d}{dt} = -\frac{1}{2m}\nabla^2 + V$	
	$x(t) = -a t^2$	$\Leftarrow$	$m/2\,\dot x^2 - amx$	$\Rightarrow$	$p^2/2m + amx$	$\Rightarrow$	$d/dt = -\frac{1}{2m}\nabla^2 + x$	← Double slit
Hooke →	$x(t) = -\cos\omega t$		$m/2\,\dot x^2 - \frac{k}{2}x^2$		$p^2/2m + \frac{k}{2}x^2$			
	$\dots$		$\dots$		$\dots$			
magnetic field →	$m\ddot x = \dot x \times B$, $B = \nabla \times A$		$m/2\,\dot x + A\cdot\dot x$		$\frac{1}{2m}(p-A)^2 = \frac{1}{2}mv^2$		$d/dt = \frac{i}{2m}(\nabla - A)^2$	

SR — Lorentz-invariant | QFT

$\nabla \cdot p = F$ $\nabla \cdot \frac{\partial L}{\partial(\nabla\phi)} = \frac{\partial L}{\partial \phi}$	$\Leftarrow$	$A[\phi] = \int L(\phi, \nabla\phi)\,dx \to \min$	$\Rightarrow$	$\frac{\partial H}{\partial p} = \nabla\phi \quad \frac{\partial H}{\partial \phi} = \nabla\cdot p$

			$-m\sqrt{1-\dot x^2} - V$		$E^2 - p^2 = m^2$		$\partial^2 = m^2$	← Gordon-Klein
wave equation →	$\partial^2\phi + \partial V = 0$	$\Leftarrow$	$\frac{1}{2}(\partial\phi)^2 - V(\phi)$	$\Rightarrow$		$\Rightarrow$		
Maxwell →	$\partial F = j$		$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + jA = \frac{1}{2}(E^2 - B^2)$				QED	