Weekly Thesis Presentation

A project on Non-linear loudspeaker estimation

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3. First weekly meeting

Overview

3.1.1. Literature

Review of non-linear loudspeaker litterature

- Hugos Thesis
- Klippels papers
- Jens Brehm Nielsen's repport on Thermal models
- Alexander Weider King's paper on fractional derivatives

Relevant models

3.2.1. Models of a loudspeaker in increasing complexity (decreasing number of assumptions)

- **Lumped model**
  - ***Linearized*** at x=0 and i=0
  - ***Nonlinear***:
    - Accounting for displacement
    - Accounting for current and displacement
    - Accounting for current, displacement and structural modes (diaphragm modes)
  - **Universal differential equation** (ODE with universal approximator)
- **BEM/FEM** (Finite/boundary element method)

3.2.2. Universal differential equations (graybox modelling)

3.2.3. Model discovery

Sparse Identification of Nonlinear Dynamics (SINDy)

Relevant models (ii)

A data-driven method for discovering governing equations of dynamical systems directly from measurements.

Given time-series data $x(t)$, assume dynamics: $\dot{x} = \Theta(x)\xi$

where:

- $\Theta(x)$: library of candidate basis functions (e.g. polynomials, trigonometric, rational, etc.)
- ξ: sparse coefficient vector $\longrightarrow$ selects which terms are active in the dynamics

The algorithm:

1. Build $\Theta(x)$ from data
2. Solve sparse regression for ξ
3. Recover the governing ODE/PDE structure

Relevant models (iii)

Symbolic Regression (SR)

A machine-learning approach for identifying analytic expressions that fit observed data by searching over both model structure and parameters.

Given data $\{(x_i, y_i)\}_{i=1}^{N}$, find an expression: $y \approx F(x; \theta)$ where:

- $F(x; \theta)$ is built from a predefined function set $\mathcal{F} = \{+, -, \times, \div, \sin, \cos, \exp, \log, \ldots\}$
- θ are tunable parameters in the candidate expression

Algorithm (general form):

1. Generate candidate expressions by composing elements of $\mathcal{F}$
2. Estimate parameters θ for each candidate (e.g. regression, least squares)
3. Score candidates using loss (e.g. MSE, AIC, BIC, complexity penalties)
4. Search expression space via genetic programming, evolutionary search, Monte-Carlo tree search, or differentiable approaches

Relevant models (iv)

Dynamic Mode Decomposition (DMD)

A data-driven method for approximating the dynamics of complex systems by decomposing time-series data into spatial-temporal modes.

Given snapshot data $X = [x_1, x_2, ..., x_m]$, assume linear evolution: $x_{k+1} \approx A x_k$ where:

- A: best-fit linear operator mapping between successive states
- Eigenvalues of A: temporal dynamics
- Eigenvectors of A: spatial modes

The algorithm:

1. Collect snapshot matrices X and X'
2. Compute reduced operator A via SVD
3. Extract DMD modes and frequencies

Relevant models (v)

3.2.4. Model approximation

Universal Differential Equations (UDEs)

A differential equation with a universal approximator embedded in the right-hand side. Form:

$$\dot{u} = \mathrm{NN}(u, p, t)$$

where: NN is ODE partially or fully explained using a neural network

Training:

1. Define a loss between simulated and observed data
2. Differentiate through the solver (adjoint sensitivities)
3. Optimize p and the weights of NN

Notes:

- Works for ODEs, SDEs, DAEs, and DDEs; extend to PDEs via method of lines

Relevant models (vi)

- Structure (bounds, symmetries, conservation) can be enforced in f or via regularization

3.2.5. Results

DISCLAIMER - Results are all in comparison to dynax model

Relevant research questions

3.3.1. Possible Research questions

1. Combine models from manuel and Alex
2. Adressing thermal effect on parameters using either gray or blackbox modelling!
3. Adressing Magnetic hysterisis
4. Adressing polymer hysteresis
   - phenomena: speaker params after having been hot are different than cold after a long time

4. MLP without activation

Quick Summary:

4.1.1. Wrote and verified

- Testsignals for simulation and estimation
  - complex sine
  - Bandpassed pinknoise
- Simulation of MassSpringDamperModel in package LoudspeakerModels.jl
  - Currently Only MassSpringDamperModel
- Estimation of Dynamic systems in package AbstractEstimation.jl
  - Currently Only estimate_with_ann with parametrized activation (currently identity) and optimizer currently adam with bfgs after and cubic interpolation

Results:

4.2.1. Complex sine

4.2.2. Pinknoise Bandpassed at 5 to 2000Hz

Suggestions to future models and simulations

4.3.1. Future Simulation models to look at

- Linear Loudspeaker model (assuming x=0)
  - would enable fitting to real data and comparison to old work
  - would make a good baseline for further models

4.3.2. Future Estimation models to look at

Models of interest

- MLP with activation
- Surrogates
- Resovoir Computing

Methods of incoporation of interest

- end to end gradient estimation
- UDE/NODE formulation
  - see as end to end

5. Sceeening papers, Making plan and Fixing code

Screening papers

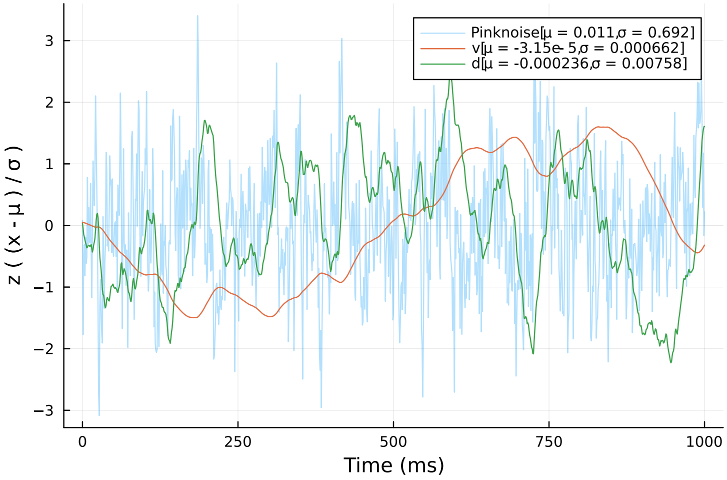
5.1.1. quaried questions (690)

(loudspeaker AND nonlinear AND model* AND ODE) OR (loudspeaker AND nonlinear AND model*) NOT (piezoelectric OR imaging OR Crystal OR Arrival OR array OR droplets OR “echo cancel*”)

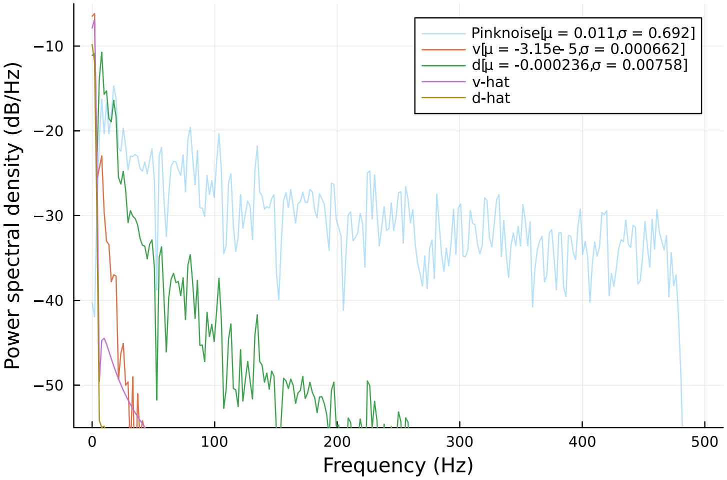
5.1.2. Main takeaways so far

- Joint state + parameter estimation
  - Application of Kalman and RLS adaptive algorithms to non-linear loudspeaker controler parameter estimation: A case study
- Nonlinear AR with exogeneous input (NARX)
- Parameter Estimation via PEM

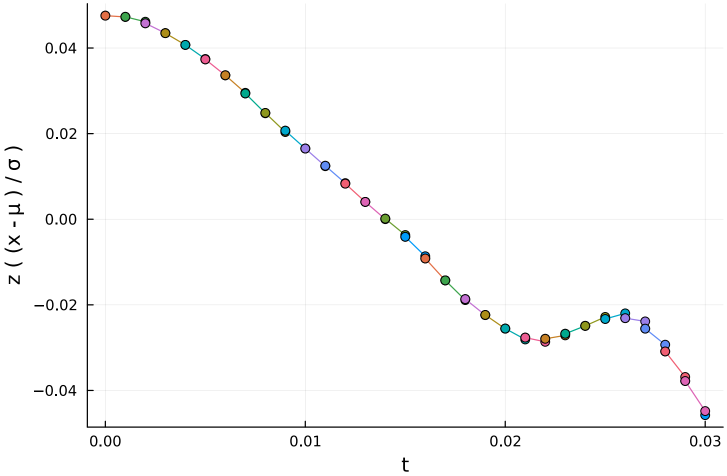
MLP results without Nonlinearity



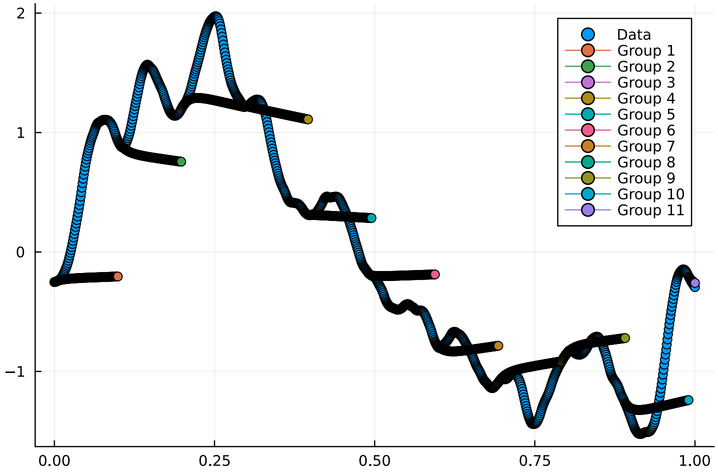
MLP results without Nonlinearity (ii)



MLP results without Nonlinearity multishot

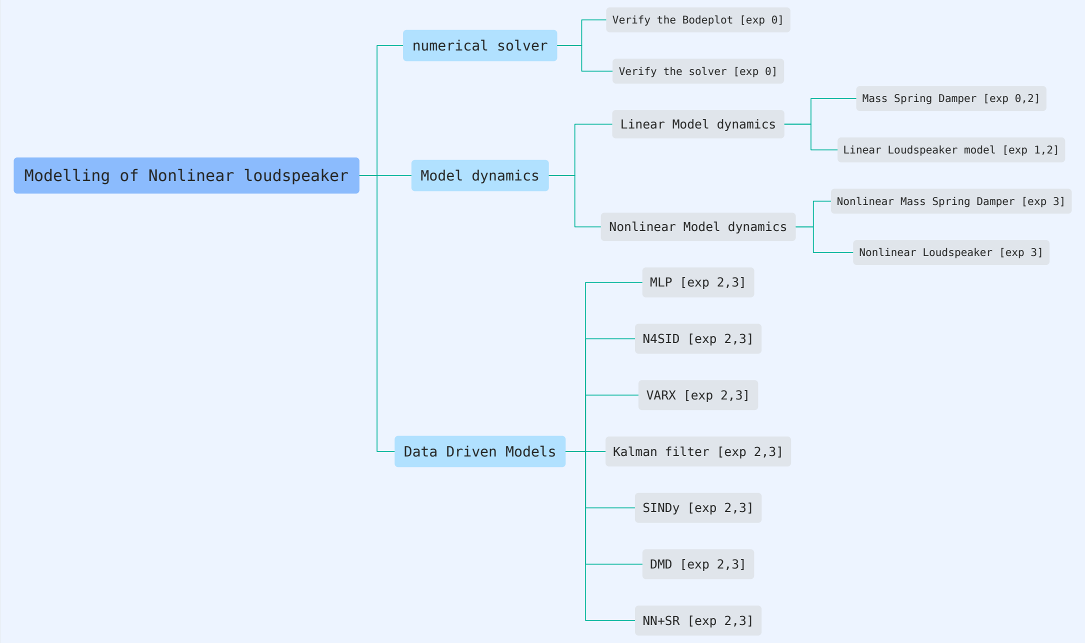


MLP results without Nonlinearity multishot (ii)



6. First principle Experiments & Studyplan

Experiment Overview



Experiment Overview (ii)

6.1.1. Notes on experimental principles and strategies

To uncover the complexities and minimize increase the chances of success experiments are created based on the scientific method and principles of decomposing complexity.

Experiment 0: Mass Spring Damper system + varifying setup

6.2.1. Theory

Definition

A mass spring damper is a physical system which has a mass, a spring and viscous damping! Assuming an ideal mass, spring and damper its reaction to an external force can be described using newtons 1st and 2nd law, Hooke's law and using the formula of viscous damping.

Time Domain

Using the system description the following second order ODE in the time domain can be formulated as

$$\sum F = F_{\text{system}} + F_{\text{ext}} = F_{\text{mass}} + F_{\text{viscous}} + F_{\text{spring}} + F_{\text{ext}}$$

$$\ddot{x} = \frac{-(c\dot{x} + kx) + F_{\text{ext}}}{m}$$

Experiment 0: Mass Spring Damper system + varifying setup (ii)

State Space

Because the derivative is of 2nd order $x, \dot{x}, \ddot{x}$ can be derived using 2 states! Using the time domain formulations the state space formulation is formulated

$$\ddot{x} = \frac{F_{\text{ext}} - (c\dot{x} + kx)}{m}$$

$$\Leftrightarrow \dot{X} = AX + Bu \quad \text{where}$$

$$X = [x, \dot{x}]^T, \quad u = F_{\text{ext}}, \quad A = \begin{pmatrix} -\frac{k}{m} & 0 \\ 0 & -\frac{c}{m} \end{pmatrix}, B = \frac{1}{m}$$

Experiment 0: Mass Spring Damper system + varifying setup (iii)

Harmonic analysis

Using the Laplace transformation we obtain the transfer functions for a, v and d

$$\mathcal{L}\{\ddot{x}\} = Xs^2 - X(0) - X(0)s = \frac{-(cXs - X(0) + kX) + F_{\text{ext}}}{m} \Leftrightarrow \frac{F_{\text{ext}}}{m} = Xs^2 + \frac{X(cs+k)}{m}$$

$$\Leftrightarrow F_{\text{ext}} = mX\left(s^2 + \frac{cs+k}{m}\right)$$

$$\Leftrightarrow \frac{X}{F_{\text{ext}}} = \frac{\frac{k}{m}\frac{1}{k}}{s^2 + \frac{c}{m}s + \frac{k}{m}} \Leftrightarrow \text{2nd order Lowpass with: } \omega_n = \sqrt{\frac{k}{m}}, \quad \zeta = \frac{c}{2\sqrt{km}} \quad \text{gain} = \frac{1}{k}$$

$$\Leftrightarrow \frac{Xs}{F_{\text{ext}}} = \frac{2\frac{c}{2m\sqrt{\frac{k}{m}}}\sqrt{\frac{k}{m}}\frac{1}{c}s}{s^2 + \frac{c}{m}s + \frac{k}{m}} \Leftrightarrow \text{2nd order Bandpass with: } \omega_n = \sqrt{\frac{k}{m}}, \quad \zeta = \frac{c}{2\sqrt{km}} = \quad \text{gain} = \frac{1}{c}$$

$$\Leftrightarrow \frac{Xs^2}{F_{\text{ext}}} = \frac{\frac{1}{m}s^2}{s^2 + \frac{c}{m}s + \frac{k}{m}} \Leftrightarrow \text{2nd order Highpass with: } \omega_n = \sqrt{\frac{k}{m}}, \quad \zeta = \frac{c}{2\sqrt{km}} \quad \text{gain} = \frac{1}{m}$$

Experiment 0: Mass Spring Damper system + varifying setup (iv)

6.2.2. Hypothesis

Thus making a 50g mass spring damper with natural frequency at 250Hz and damping ratio 0.1 has the parameters

$$m = 0.05\text{Kg}, k = 0.05 \cdot (250 \cdot 2\pi)^2 = 123370.055N, c = 2\zeta m\omega_n = 2 \cdot 0.01 \cdot 0.05\text{Kg} \cdot 250 \cdot 2\pi\text{rad} = 1.571\frac{\text{Ns}}{m}$$

This theoretical mass spring damper would be very underdamp and thus we expect a ressonance at 250Hz aswell as a −12db rolloff after around 250Hz (not exactly 250 as the damping isn't $\omega = \omega_n$).

To verify this theory aswell as the solver and Bodeplot we will apply a pinknoise testsignal bandpassed to 10Hz-1000Hz aswell as a complex sine tone!

6.2.3. Results

- TODO make julia control statespace model

view results of normal model

6.2.4. Discussion

Experiment 2: Linear models

6.3.1. Theory

In ***experiment 0*** a linear mass spring damper was derived and tuned with a natural frequency of 250Hz!

Now a linear loudspeaker will be analyzed, and the findings will be compared to the mass spring damper:

Definition

A Linear Loudspeaker is a physical system comprised of 3 subsystems:

- an electrical system,
- a mechanical system
- an acoustical system

The electrical system

The electrical system is comprised of a voice coil which is assumed to have a parasitic Resistance, an inductance and to be coupled with the mechanical system by the Lorentz force (converting current to force) and the electromotive force (converting velocity to voltage). Thus this constitutes an “RL” circuit with a current dependent force and velocity dependent voltage source.

Experiment 2: Linear models (ii)

The mechanical system

The mechanical system is comprised of the suspension and the motor system, and the diaphragm and the acoustical load on the diaphragm. Assuming a discretization of physical properties a lumped model of each component can be described which can be described as an equivalent circuit. Its also comprised of the electrical coupling by the Lorentz force and the electromotive force and an acoustical coupling through the diaphragm velocity and through the ! Thus the mechanical system then can be formulated as an effective mass spring and damper with a voltage dependent force, and a voltage dependent !

The acoustical system

The acoustical system can also be described as having an acoustical resistance,acoustic mass and acoustic compliance thus an equivalent circuit. In academia this system is often accounted for in the mechanical system parameters!

NOTE:

This has to be tested! A hypothesis as to why is that the acoustic resistance, mass and compliance is almost zero speaker in free air! This is a gross assumption which would have to be studied!

Experiment 2: Linear models (iii)

Time Domain

Using the system description the following second order ODE in the time domain can be formulated as an second order ode

$$\begin{aligned} u &= Bl\dot{x} + L_e \frac{\mathrm{d}}{\mathrm{d}t} i + R_e i \\ Bli &= K_m x + M_m \ddot{x} + R_m \dot{x} \end{aligned}$$

or as an first order ode by introducing $v = \dot{x}$

$$\begin{aligned} u &= Blv + L_e \frac{\mathrm{d}}{\mathrm{d}t} i + R_e i \\ Bli &= K_m x + M_m \dot{v} + R_m v \end{aligned}$$

Experiment 2: Linear models (iv)

State Space

We formulate the statespace model of Second Order ODE Loudspeaker model using the time domain formulation using 3 states as we observe that the highest derivative is 2 for x and 1 for i

$$u = Bl\dot{x} + L_e \frac{\mathrm{d}}{\mathrm{d}t} i + R_e i \Leftrightarrow \qquad \frac{\mathrm{d}}{\mathrm{d}t} i = \frac{u - (Bl\dot{x} + R_e i)}{L_e}$$

$$Bli = K_m x + M_m \ddot{x} + R_m \dot{x} \Leftrightarrow \qquad \ddot{x} = \frac{Bli - (K_m x + R_m \dot{x})}{M_m}$$

$$\ddot{x} = \frac{\mathrm{d}}{\mathrm{d}t} \dot{x}$$

$$\Leftrightarrow \dot{X} = AX + Bu, \text{where} \quad X = [i, x, \dot{x}]^T, \qquad A = \begin{pmatrix} -\frac{R_e}{L_e} & 0 & -\frac{Bl}{L_e} \\ 0 & 0 & 1 \\ \frac{Bl}{M_m} & -\frac{K_m}{M_m} & -\frac{R_m}{M_m} \end{pmatrix}, B = \left[\frac{1}{L_e}, 0, 0\right]^T$$

Experiment 2: Linear models (v)

Harmonic analysis 1/2

Using the Laplace transformation we obtain the transfer functions for i, x and $\dot{x}$

$$\mathcal{L}\{Bli\} = BlI = K_m X + M_m X s^2 - X(0)s - X(0) + R_m X s - X(0) \Leftrightarrow I = \frac{K_m + M_m s^2 + R_m s}{Bl} X$$

$$\mathcal{L}\{u\} = U = BlXs - X(0) + L_e I s - I(0) + R_e I \Leftrightarrow U = BlXs + (L_e s + R_e) I$$

$$\Leftrightarrow U = XBls + X(L_e s + R_e)\frac{K_m + M_m s^2 + R_m s}{Bl} = X\left(Bls + (L_e s + R_e)\frac{K_m + M_m s^2 + R_m s}{Bl}\right)$$

$$\frac{X}{U} = \frac{Bl}{Bl^2 s + (K_m + M_m s^2 + R_m s)(L_e s + R_e)} \Leftrightarrow 3 \text{ poles} \Rightarrow \text{Lowpass}$$

$$\frac{Xs}{U} = \frac{sBl}{Bl^2 s + (K_m + M_m s^2 + R_m s)(L_e s + R_e)} \Leftrightarrow 3 \text{ poles, } 1 \text{ zeros} \Rightarrow \text{Bandpass}$$

$$\frac{I}{U} = \frac{K_m + M_m s^2 + R_m s}{Bl^2 s + (K_m + M_m s^2 + R_m s)(L_e s + R_e)} \Rightarrow \text{p poles, } 2 \text{ zeros} \Rightarrow \text{Notch}$$

Experiment 2: Linear models (vi)

6.3.2. Experiment 2.a

6.3.2.1. Hypothesis

6.3.2.2. Results

6.3.2.3. Discussion

Nonlinear Loudspeaker system

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