

# **GSERM - Oslo 2019**

## GLS-ARMA and Dynamics

January 7, 2019 (afternoon session)

For:

$$Y_{it} = \mathbf{X}_{it}\beta + u_{it}$$

i.i.d.  $u_{it}$ s require:

$$\mathbf{u}\mathbf{u}' \equiv \mathbf{\Omega} = \sigma^2 \mathbf{I}$$

$$= \begin{pmatrix} \sigma^2 & 0 & \dots & 0 \\ 0 & \sigma^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma^2 \end{pmatrix}$$

That is, within units:

- $\text{Var}(u_{it}) = \text{Var}(u_{is}) \forall t \neq s$  (temporal homoscedasticity)
- $\text{Cov}(u_{it}, u_{is}) = 0 \forall t \neq s$  (no within-unit autocorrelation)  
"temporal correlation"

and between units:

- $\text{Var}(u_{it}) = \text{Var}(u_{jt}) \forall i \neq j$  (cross-unit homoscedasticity)
- $\text{Cov}(u_{it}, u_{jt}) = 0 \forall i \neq j$  (no between-unit / spatial correlation)

Estimator:

$$\hat{\beta}_{GLS} = (\mathbf{X}'\Omega^{-1}\mathbf{X})^{-1}\mathbf{X}'\Omega^{-1}\mathbf{Y}$$

with:

$$\widehat{V(\beta_{GLS})} = (\mathbf{X}'\Omega^{-1}\mathbf{X})^{-1}$$

Two approaches:

- Use OLS  $\hat{u}_{it}$ s to get  $\hat{\Omega}$  ("feasible GLS")

estimate errors iteratively to  
get estimation of empirical error

- Use substantive knowledge about the data to structure  $\Omega$

-> assume substantive knowledge to infer  
distribution of errors.

# Parks' Approach

Assume:

- $E(u_{it}^2) = E(u_{is}^2) \forall t \neq s$
- $E(u_{it}, u_{jt}) = \sigma_{ij} \forall i \neq j,$
- $E(u_{it}, u_{js}) = 0 \forall i \neq j, t \neq s$
- $E(u_{it}, u_{is}) = \rho$  or  $\rho_i$

within unit temporal  
homoscedasticity in expectation

spatial correlation is purely  
contemporaneous

(B&K: “panel error assumptions”).

Then

1. Use OLS to generate  $\hat{u}s \rightarrow \hat{\rho} (\rightarrow \hat{\Omega}),$
2. Use  $\hat{\rho}$  for Prais-Winsten.

This method was widely used prior to B&K (1995)

$$\Omega = \begin{pmatrix} \Sigma & 0 & \cdots & 0 \\ 0 & \Sigma & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Sigma \end{pmatrix} = \Sigma \otimes \mathbf{I}_N$$

where

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{12} & \sigma_2^2 & \cdots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{1N} & \sigma_{2N} & \cdots & \sigma_N^2 \end{pmatrix}$$

Means:

- $\frac{N(N-1)}{2}$  distinct contemporaneous correlations,
- $NT$  observations,
- $\rightarrow 2T/(N+1)$  observations per  $\hat{\sigma}$

N needs to be smaller than  $2^*T$   
to have enough observations for  
estimation

# Panel-Corrected Standard Errors

Key to PCSEs:

$$\hat{\sigma}_{ij} = \frac{\sum_{t=1}^T \hat{u}_{it} \hat{u}_{jt}}{T}$$

Define:

$$\mathbf{U}_{T \times N} = \begin{pmatrix} \hat{u}_{11} & \hat{u}_{21} & \cdots & \hat{u}_{N1} \\ \hat{u}_{12} & \hat{u}_{22} & \cdots & \hat{u}_{N2} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{u}_{1T} & \hat{u}_{2T} & \cdots & \hat{u}_{NT} \end{pmatrix}$$

$$\hat{\Sigma} = \frac{(\mathbf{U}'\mathbf{U})}{T}$$

$$\hat{\Omega}_{PCSE} = \frac{(\mathbf{U}'\mathbf{U})}{T} \otimes \mathbf{I}_T$$

# Panel-Corrected Standard Errors

Correct formula:

$$\text{Cov}(\hat{\beta}_{PCSE}) = (\mathbf{X}'\mathbf{X})^{-1}[\mathbf{X}'\mathbf{\Omega}\mathbf{X}](\mathbf{X}'\mathbf{X})^{-1}$$



### PCSEs:

- Do not fix unit-level heterogeneity (a la “fixed” / “random” effects)
- Do not deal with dynamics
- Depend critically on the “panel data assumptions” of Park / B&K

# Panel Assumptions and Numbers of Parameters to be Estimated

number of parameters

| Panel Assumptions   | No AR(1)               | Common $\hat{\rho}$            | Separate $\hat{\rho}_i$ s   |
|---|------------------------|--------------------------------|-----------------------------|
| $\sigma_i^2 = \sigma^2, \text{Cov}(\sigma_{it}, \sigma_{jt}) = 0$       | $k + 1$                | $k + 2$                        | $k + N + 1$                 |
| $\sigma_i^2 \neq \sigma^2, \text{Cov}(\sigma_{it}, \sigma_{jt}) = 0$    | $k + N$                | $k + N + 1$                    | $k + 2N$                    |
| $\sigma_i^2 \neq \sigma^2, \text{Cov}(\sigma_{it}, \sigma_{jt}) \neq 0$ | $\frac{N(N-1)}{2} + k$ | $\frac{N(N-1)}{2} + k + N + 1$ | $\frac{N(N-1)}{2} + k + 2N$ |

R slides following up refer to different cells of this table.

## Example: Central Banks, Unions, Unemployment

- Hall and Franzese (1998 *IO*)
- 18 OECD countries, 1955-1990 ( $N = 18$ ,  $T = 36$ ,  $NT = 648$ )
- $Y$  = unemployment
- Covariates: GDP, openness, union density, left cabinets, central bank independence, coordinated wage bargaining, interaction

# Example: Data

```
> summary(HF)
```

| country      | year         | ue           | inf          | cbi          |
|--------------|--------------|--------------|--------------|--------------|
| Min. : 1.0   | Min. :1955   | Min. : 0.0   | Min. : -1.7  | Min. :0.12   |
| 1st Qu.: 5.0 | 1st Qu.:1964 | 1st Qu.: 1.6 | 1st Qu.: 3.2 | 1st Qu.:0.41 |
| Median : 9.5 | Median :1972 | Median : 3.0 | Median : 4.9 | Median :0.47 |
| Mean :10.3   | Mean :1972   | Mean : 4.0   | Mean : 6.0   | Mean :0.50   |
| 3rd Qu.:15.0 | 3rd Qu.:1981 | 3rd Qu.: 5.7 | 3rd Qu.: 7.7 | 3rd Qu.:0.61 |
| Max. :21.0   | Max. :1990   | Max. :17.5   | Max. :27.2   | Max. :0.93   |

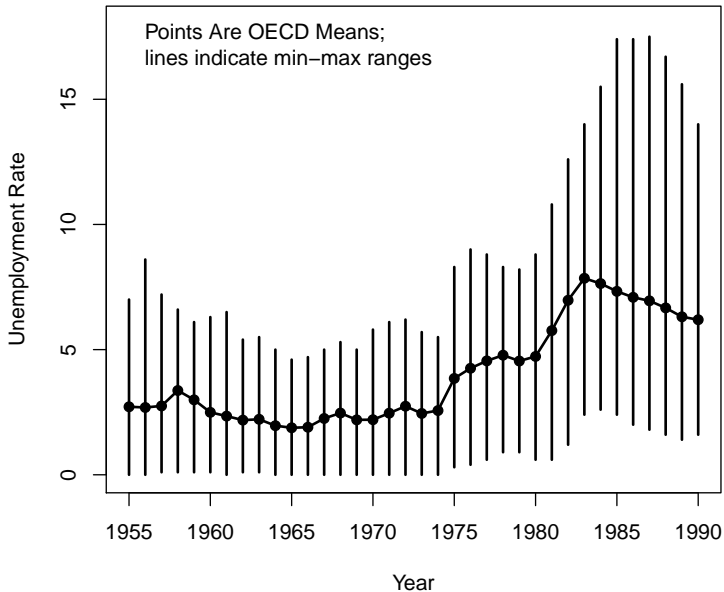
  

| cwagebrg     | GDP_PC      | open         | uden         | lcab         |
|--------------|-------------|--------------|--------------|--------------|
| Min. :0.00   | Min. :7.6   | Min. :0.07   | Min. :0.10   | Min. :0.00   |
| 1st Qu.:0.25 | 1st Qu.:8.9 | 1st Qu.:0.31 | 1st Qu.:0.32 | 1st Qu.:0.00 |
| Median :0.50 | Median :9.2 | Median :0.43 | Median :0.41 | Median :0.07 |
| Mean :0.49   | Mean :9.1   | Mean :0.46   | Mean :0.44   | Mean :0.31   |
| 3rd Qu.:0.75 | 3rd Qu.:9.4 | 3rd Qu.:0.54 | 3rd Qu.:0.56 | 3rd Qu.:0.58 |
| Max. :1.00   | Max. :9.8   | Max. :1.40   | Max. :0.85   | Max. :1.00   |

| wagexcbi     | HasLCAB   |
|--------------|-----------|
| Min. :0.00   | Min. :1   |
| 1st Qu.:0.04 | 1st Qu.:1 |
| Median :0.21 | Median :1 |
| Mean :0.25   | Mean :1   |
| 3rd Qu.:0.37 | 3rd Qu.:1 |
| Max. :0.70   | Max. :1   |

# Unemployment in 18 Nations, 1955-1990



# Example: OLS

```
> summary(HF.OLS)
Oneway (individual) effect Pooling Model

Balanced Panel: n=18, T=36, N=648

Residuals :
    Min. 1st Qu.  Median 3rd Qu.    Max.
-5.120  -1.500  -0.241   1.230   9.290

Coefficients :
              Estimate Std. Error t-value    Pr(>|t|)
(Intercept)  -13.579      2.328   -5.83 0.0000000086 ***
GDP_PC        1.603      0.263    6.09 0.0000000020 ***
open         5.119      0.418   12.24 < 2e-16 ***
uden         0.709      0.808    0.88    0.38
lcab         0.236      0.293    0.81    0.42
cbi          5.169      1.097    4.71 0.0000030150 ***
cwagebrg     -1.292      0.792   -1.63    0.10
wagexcbi     -7.030      1.505   -4.67 0.0000036327 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1 1

Total Sum of Squares:    6500
Residual Sum of Squares: 3730
R-Squared:               0.426
Adj. R-Squared: 0.421
F-statistic: 67.9634 on 7 and 640 DF, p-value: <2e-16
```

## Example: Prais-Winsten

```
> HF.prais <- prais.winsten(ue~GDP_PC+open+uden+lcab+cbi+cwagebrg+wagexcbi,  
                             data=HF,iter=100)
```

```
> HF.prais
```

Residuals:

|  | Min    | 1Q     | Median | 3Q    | Max   |
|--|--------|--------|--------|-------|-------|
|  | -8.456 | -0.431 | -0.144 | 0.314 | 4.615 |

Coefficients:

|           | Estimate | Std. Error | t value | Pr(> t ) |     |
|-----------|----------|------------|---------|----------|-----|
| Intercept | -15.2783 | 2.2128     | -6.90   | 1.2e-11  | *** |
| GDP_PC    | 2.3415   | 0.2515     | 9.31    | < 2e-16  | *** |
| open      | -0.3491  | 0.7950     | -0.44   | 0.6608   |     |
| uden      | 5.5466   | 1.1492     | 4.83    | 1.7e-06  | *** |
| lcab      | -0.0593  | 0.1861     | -0.32   | 0.7501   |     |
| cbi       | -3.4801  | 2.4753     | -1.41   | 0.1602   |     |
| cwagebrg  | -10.5954 | 2.0019     | -5.29   | 1.7e-07  | *** |
| wagexcbi  | 10.6805  | 3.4942     | 3.06    | 0.0023   | **  |

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Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.95 on 640 degrees of freedom

Multiple R-squared: 0.279, Adjusted R-squared: 0.27

F-statistic: 31 on 8 and 640 DF, p-value: <2e-16

Rho Rho.t.statistic Iterations

|      |    |    |
|------|----|----|
| 0.94 | 73 | 10 |
|------|----|----|

# Example: GLS with Homoscedastic AR(1) Errors

```
> HF.GLS <- gls(ue~GDPPC+open+uden+lcab+cbi+cwagebrg+wagexcbi,
               HF,correlation=corAR1(form=~1|country))
> summary(HF.GLS)
Generalized least squares fit by REML
Model: ue ~ GDPPC + open + uden + lcab + cbi + cwagebrg + wagexcbi
Data: HF
    AIC   BIC logLik
1484 1529   -732
```

each unit can be autocorrelated  
with different values per unit.

```
Correlation Structure: AR(1)
Formula: ~1 | country
Parameter estimate(s):
Phi
0.99
```

Coefficients:

|             | Value | Std.Error | t-value | p-value |
|-------------|-------|-----------|---------|---------|
| (Intercept) | 43    | 7.3       | 5.8     | 0.000   |
| GDPPC       | -4    | 0.7       | -5.5    | 0.000   |
| open        | -1    | 0.8       | -1.8    | 0.072   |
| uden        | 1     | 2.2       | 0.3     | 0.792   |
| lcab        | 0     | 0.1       | -0.7    | 0.473   |
| cbi         | -1    | 7.5       | -0.2    | 0.848   |
| cwagebrg    | -5    | 6.4       | -0.8    | 0.402   |
| wagexcbi    | 3     | 11.7      | 0.3     | 0.770   |

Correlation:

|          | (Intr) | GDPPC  | open   | uden   | lcab   | cbi    | cwgbrg |
|----------|--------|--------|--------|--------|--------|--------|--------|
| GDPPC    | -0.827 |        |        |        |        |        |        |
| open     | 0.124  | -0.207 |        |        |        |        |        |
| uden     | -0.145 | 0.017  | -0.048 |        |        |        |        |
| lcab     | 0.041  | -0.054 | 0.005  | -0.003 |        |        |        |
| cbi      | -0.421 | -0.100 | 0.033  | 0.069  | 0.009  |        |        |
| cwagebrg | -0.371 | -0.065 | 0.018  | -0.084 | -0.014 | 0.721  |        |
| wagexcbi | 0.334  | 0.082  | -0.028 | 0.017  | 0.004  | -0.813 | -0.905 |

Standardized residuals:

|  | Min   | Q1    | Med   | Q3   | Max  |
|--|-------|-------|-------|------|------|
|  | -1.49 | -0.64 | -0.20 | 0.46 | 2.55 |



# More GLS: Unit-Wise Heteroscedasticity

```
> HF.GLS2 <- gls(ue~GDPPC+open+uden+lcab+cbi+cwagebrg+wagexcbi,  
+ HF,correlation=corAR1(form=~1|country),  
+ weights = varIdent(form = ~1|country))  
> summary(HF.GLS2)  
Generalized least squares fit by REML  
   AIC   BIC logLik  
1326 1446   -636
```

```
Correlation Structure: AR(1)  
Formula: ~1 | country  
Parameter estimate(s):  
Phi  
0.98
```

the model fits better for some  
units than for others

```
Variance function:  
Structure: Different standard deviations per stratum  
Formula: ~1 | country  
Parameter estimates:  
  1    2    3    4    5    6    7    8    9   10   11   13   14   15   18   19   20   21  
1.00 0.19 0.75 0.54 0.60 1.00 0.95 0.32 0.88 0.89 0.78 1.09 0.92 0.57 0.33 0.38 0.86 0.60
```

Coefficients:

|             | Value | Std.Error | t-value | p-value |
|-------------|-------|-----------|---------|---------|
| (Intercept) | 21.1  | 4.7       | 4.5     | 0.0000  |
| GDPPC       | -1.6  | 0.4       | -4.3    | 0.0000  |
| open        | -2.2  | 0.6       | -3.4    | 0.0008  |
| uden        | 0.9   | 1.5       | 0.6     | 0.5415  |
| lcab        | -0.1  | 0.1       | -1.2    | 0.2206  |
| cbi         | -1.9  | 7.3       | -0.3    | 0.7984  |
| cwagebrg    | -6.3  | 4.7       | -1.3    | 0.1794  |
| wagexcbi    | 5.0   | 9.5       | 0.5     | 0.5996  |
| .           |       |           |         |         |
| .           |       |           |         |         |
| .           |       |           |         |         |

## Example: PCSEs

```
> library(lmtest)
> coeftest(HF.OLS,vcov=vcovBK)
```

t test of coefficients:

|             | Estimate | Std. Error | t value | Pr(> t ) |     |
|-------------|----------|------------|---------|----------|-----|
| (Intercept) | -13.579  | 7.320      | -1.86   | 0.064    | .   |
| GDP_PC      | 1.603    | 0.821      | 1.95    | 0.051    | .   |
| open        | 5.119    | 1.304      | 3.93    | 0.000096 | *** |
| uden        | 0.709    | 2.518      | 0.28    | 0.778    |     |
| lcab        | 0.236    | 0.668      | 0.35    | 0.724    |     |
| cbi         | 5.169    | 3.439      | 1.50    | 0.133    |     |
| cwagebrg    | -1.292   | 2.478      | -0.52   | 0.602    |     |
| wagexcbi    | -7.030   | 4.726      | -1.49   | 0.137    |     |

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

estimates stay the same,  
standard errors change  
with corrected errors.

# Alternative Approach: pcse

```
> HF.lm<-lm(ue~GDP_PC+open+uden+lcab+cbi+cwagebrg+wagexcbi,data=HF)
> HF.pcse<-pcse(HF.lm,groupN = HF$country, groupT = HF$year)
> summary(HF.pcse)
```

Results:

|             | Estimate | PCSE | t value | Pr(> t ) |
|-------------|----------|------|---------|----------|
| (Intercept) | -13.58   | 4.74 | -2.87   | 4.3e-03  |
| GDP_PC      | 1.60     | 0.53 | 3.01    | 2.7e-03  |
| open        | 5.12     | 0.53 | 9.71    | 7.0e-21  |
| uden        | 0.71     | 0.53 | 1.35    | 1.8e-01  |
| lcab        | 0.24     | 0.27 | 0.88    | 3.8e-01  |
| cbi         | 5.17     | 0.85 | 6.10    | 1.9e-09  |
| cwagebrg    | -1.29    | 0.77 | -1.67   | 9.6e-02  |
| wagexcbi    | -7.03    | 1.05 | -6.68   | 5.3e-11  |

results are different.  
whv?? --> look into code.

-----

```
# Valid Obs = 648; # Missing Obs = 0; Degrees of Freedom = 640.
```

# General advice...

look into error structure and your assumptions.  
what do you know substantively about your data and the data generating process?

# Lagged: Y?

function of Y yesterday and other variables affecting it.

$$Y_{it} = \phi Y_{it-1} + \mathbf{X}_{it}\beta_{LDV} + \epsilon_{it}$$

If  $\epsilon_{it}$  is perfect...

- $\hat{\beta}_{LDV}$  is biased (but consistent),
- $O(\text{bias}) = \frac{-1+3\beta_{LDV}}{T}$

importance of bias gets smaller with large T  
coz it takes part of the autocorrelation from  
the error.

If  $\epsilon_{it}$  is autocorrelated...

- $\hat{\beta}_{LDV}$  is biased and inconsistent
- IV is one (bad) option...

large T does not change bias.

# Lagged $Y$ s and GLS-ARMA

Can rewrite:

$$\begin{aligned}Y_{it} &= \mathbf{X}_{it}\boldsymbol{\beta}_{AR} + u_{it} \\u_{it} &= \phi u_{it-1} + \eta_{it}\end{aligned}$$

rewrite

as

autocorrelated errors  
is autoregressive in  $Y$

$$\begin{aligned}Y_{it} &= \mathbf{X}_{it}\boldsymbol{\beta}_{AR} + \phi u_{it-1} + \eta_{it} \\&= \mathbf{X}_{it}\boldsymbol{\beta}_{AR} + \phi(Y_{it-1} - \mathbf{X}_{it-1}\boldsymbol{\beta}_{AR}) + \eta_{it} \\&= \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta}_{AR} + \mathbf{X}_{it-1}\psi + \eta_{it}\end{aligned}$$

where  $\psi = \phi\boldsymbol{\beta}_{AR}$  and  $\psi = 0$  (by assumption).

# Lagged Ys and World Domination

In:

$$Y_{it} = \phi Y_{it-1} + \mathbf{X}_{it}\beta_{LDV} + \epsilon_{it}$$

Achen: Bias “deflates”  $\hat{\beta}$  relative to  $\hat{\phi}$ , “suppress” the effects of  $\mathbf{X}$ ...

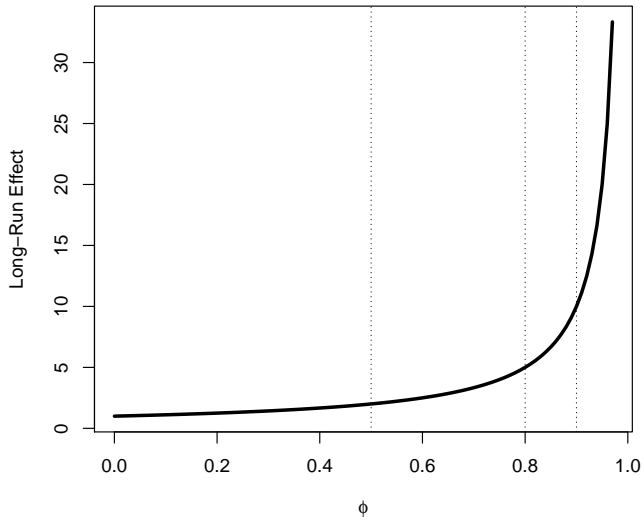
lagged Y takes up too much of the variation as it tends to highly correlate the Y.

Keele & Kelly (2006):

- Contingent on  $\epsilon$ s having autocorrelation
- Key: In LDV, *long-run impact of a unit change in X is:*

$$\hat{\beta}_{LR} = \frac{\hat{\beta}_{LDV}}{1 - \hat{\phi}}$$

# Long-Run Impact for $\hat{\beta} = 1$





# Lagged $Y$ s and Unit Effects

Consider:

$$Y_{it} = \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta} + \alpha_i + u_{it}.$$

e.g. walmart (400 ppl)  
and facebook (10 ppl)  
legal departments

If we omit the unit effects, we have:

$$Y_{it} = \phi Y_{it-1} + \mathbf{X}_{it}\boldsymbol{\beta} + u_{it}^*$$

with

$$u_{it}^* = \alpha_i + u_{it}$$

Lagging yields:

$$Y_{it-1} = \phi Y_{it-2} + \mathbf{X}_{it-1}\boldsymbol{\beta} + \alpha_i + u_{it-1}$$

which means

$$\text{Cov}(Y_{it-1}, u_{it}^*) \neq 0.$$

specification bias

Bias in  $\hat{\phi}$  is

- toward zero when  $\phi > 0$ ,
- increasing in  $\phi$ .

Including unit effects still yields bias in  $\hat{\phi}$  of  $O(\frac{1}{T})$ , and bias in  $\hat{\beta}$ .

Solutions:

- Difference/GMM estimation
- Bias correction approaches

# First Difference Estimation (gmm)

$$\begin{aligned}Y_{it} - Y_{it-1} &= \phi(Y_{it-1} - Y_{it-2}) + (\mathbf{X}_{it} - \mathbf{X}_{it-1})\beta + (\alpha_i - \alpha_i) + (u_{it} - u_{it-1}) \\ \Delta Y_{it} &= \phi \Delta Y_{it-1} + \Delta \mathbf{X}_{it}\beta + \Delta u_{it}\end{aligned}$$

Anderson/Hsiao: If  $\nexists$  autocorrelation, then use  $\Delta Y_{it-2}$  or  $Y_{it-2}$  as instruments for  $\Delta Y_{it-1}$ ...

- Consistent in theory,
  - in practice, the former is preferred, and both have issues if  $\phi$  is high;
  - both are inefficient.
- works better with long T

Arellano & Bond (also Wawro): Use *all* lags of  $Y_{it}$  and  $\mathbf{X}_{it}$  from  $t - 2$  and before.

- “Good” estimates, better as  $T \rightarrow \infty$ ,
- Easy to handle higher-order lags of  $Y$ ,
- Easy software (plm in R , xtabond in Stata ).
- Model *is* fixed effects...
- $\mathbf{Z}_i$  has  $T - p - 1$  rows,  $\sum_{i=p}^{T-2} i$  columns  $\rightarrow$  difficulty of estimation declines in  $p$ , grows in  $T$ .

Kiviet (1995, 1999; Bun and Kiviet 2003; Bruno 2005a,b): Derive the bias in  $\hat{\phi}$  and  $\hat{\beta}$ , then correct it...

- More accurate than the instrumental-variables/GMM estimators of A&H/A&B...
- ...especially when  $T$  is small; but not as  $T$  gets reasonably large ( $T \approx 20$ )

choose model based on how long  $T$  is.  
( $N$  does not matter)

# Stationarity: Quick Intro

"how to think of panel data as timeseries"

Mean stationarity:

$$E(Y_t) = \mu \forall t$$

Variance stationarity:

$$\text{Var}(Y_t) = E[(Y_t - \mu)^2] \equiv \sigma_Y^2 \forall t$$

Covariance stationarity:

$$\text{Cov}(Y_t, Y_{t-s}) = E[(Y_t - \mu)(Y_{t-s} - \mu)] = \gamma_s \forall s$$

intervals of same length  $s$  have same covariance  
(more precisely two values apart  $s$  steps)

# I(1) Series and Unit Roots

I(1) ("integrated") series:

$$Y_t = Y_{t-1} + u_t$$

autoregressiveness of 1.0

vs. AR(1) series:

$$Y_t = \rho Y_{t-1} + u_t$$

autoregression with coefficient !  
= 1.0

or *trending* series:

$$Y_t = \beta t + u_t$$

Differencing:

$$\begin{aligned}\Delta Y_t \equiv Y_t - Y_{t-1} &= Y_t + u_t - Y_{t-1} \\ &= u_t\end{aligned}$$

and

differencing changes  
with integrated vs  
trending series.

$$\begin{aligned}\Delta Y_t \equiv Y_t - Y_{t-1} &= \beta t + u_t - (\beta(t-1) + u_{t-1}) \\ &= \beta t + u_t - \beta t + \beta - u_{t-1} \\ &= u_t - u_{t-1} + \beta\end{aligned}$$

# I(1) series (continued)

More generally:

- $|\rho| > 1$ 
  - Series is nonstationary / *explosive*
  - Past shocks have a greater impact than current ones
  - Uncommon
- $|\rho| < 1$ 
  - *Stationary* series
  - Effects of shocks die out exponentially according to  $\rho$
  - Is mean-reverting
- $|\rho| = 1$ 
  - Nonstationary series
  - Shocks persist at full force
  - Not mean-reverting; variance increases with  $t$



# Unit Root Tests: Dickey-Fuller

Two steps:

- Estimate  $Y_t = \rho Y_{t-1} + u_t$ ,
- test the hypothesis that  $\hat{\rho} = 0$ , *but*
- this requires that the  $u$ s are uncorrelated.

But suppose:

$$\Delta Y_t = \sum_{i=1}^p d_i \Delta Y_{t-i} + u_t$$

which yields

$$Y_t = Y_{t-1} + \sum_{i=1}^p d_i \Delta Y_{t-i} + u_t.$$

D.F. tests will be incorrect.

## Augmented Dickey-Fuller Tests:

- Estimate

$$\Delta Y_t = Y_{t-1} + \sum_{i=1}^p d_i \Delta Y_{t-i} + u_t$$

- Test  $\hat{\rho} = 0$

## Phillips-Perron Tests:

- Estimate:

$$\Delta Y_t = \alpha + \rho Y_{t-1} + u_t$$

- Calculate modified test statistics ( $Z_\rho$  and  $Z_t$ )
- Test  $\hat{\rho} = 0$

all these approaches work for high T (asymptotics in T)  
but with panel data T may be small

# Issues with Unit Roots in Panel Data

- Short series + Asymptotic tests → “borrow strength”
- Requires uniform unit roots across *is*
- Various alternatives:
  - Maddala and Wu (1999)
  - Hadri (2000)
  - Levin, Lin and Chu (2002)
- What to do?
  - Difference the data...
  - Error-correction models

borrow strength  
across units

# Example: HIV/AIDS in Africa, 1997-2001

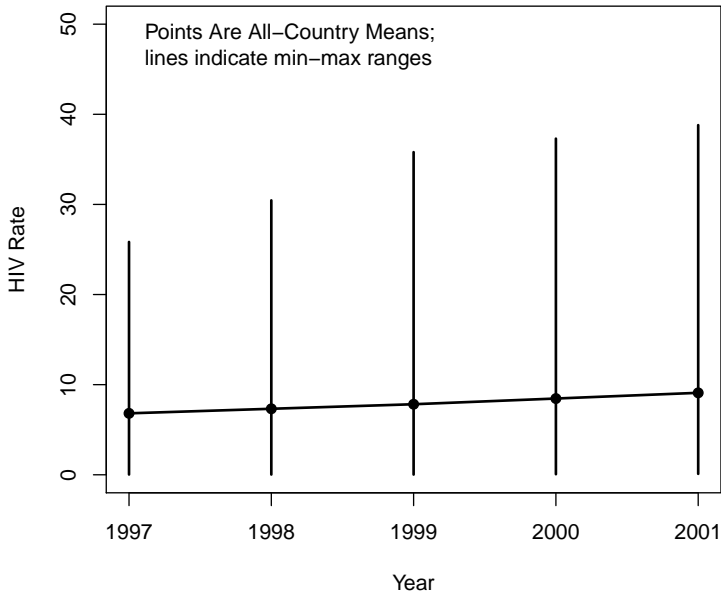
```
> summary(AIDS)
```

| cocode      | year         | lnAIDS       | lnAIDSlag  | warlag       | popden       |
|-------------|--------------|--------------|------------|--------------|--------------|
| Min. :404   | Min. :1997   | Min. : -3.8  | Min. : -4  | Min. :0.00   | Min. :0.00   |
| 1st Qu.:451 | 1st Qu.:1998 | 1st Qu.: 0.6 | 1st Qu.: 1 | 1st Qu.:0.00 | 1st Qu.:0.01 |
| Median :506 | Median :1999 | Median : 1.6 | Median : 2 | Median :0.00 | Median :0.03 |
| Mean :510   | Mean :1999   | Mean : 1.2   | Mean : 1   | Mean :0.14   | Mean :0.06   |
| 3rd Qu.:560 | 3rd Qu.:2000 | 3rd Qu.: 2.4 | 3rd Qu.: 2 | 3rd Qu.:0.00 | 3rd Qu.:0.07 |
| Max. :651   | Max. :2001   | Max. : 3.7   | Max. : 4   | Max. :1.00   | Max. :0.57   |

| refsin      |
|-------------|
| Min. : 0    |
| 1st Qu.: 1  |
| Median : 10 |
| Mean : 56   |
| 3rd Qu.: 46 |
| Max. :543   |

# HIV/AIDS in Africa, 1997-2001



# Panel Unit Root Tests: R

```
> lnAIDS<-cbind(AIDS$ccode,AIDS$year,AIDS$lnAIDS)
> purtest(lnAIDS,exo="trend",test=c("levinlin"))
```

Levin-Lin-Chu Unit-Root Test (ex. var.: Individual Intercepts and Trend)

```
data: lnAIDS
z.x1 = 3e+12, p-value <2e-16
alternative hypothesis: stationarity
```

```
> purtest(lnAIDS,exo="trend",test=c("hadri"))
```

Hadri Test (ex. var.: Individual Intercepts and Trend)

```
data: lnAIDS
z = 60, p-value <2e-16
alternative hypothesis: at least one series has a unit root
```

```
> purtest(lnAIDS,exo="trend",test=c("ips"))
```

Im-Pesaran-Shin Unit-Root Test (ex. var.: Individual Intercepts and Trend)

```
data: lnAIDS
z = 4, p-value = 0.0002
alternative hypothesis: stationarity
```

unit root = non-stationarity  
test results are conflicting.

# Final Thoughts: Dynamic Panel Models

accept limitations of data, e.g.  
small  $T$

- $N$  vs.  $T$ ...
- Are dynamics nuisance or substance? impacts if you specify dynamics as errors or covariates.
- What problem(s) do you *really* care about?