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# ROmodel: Modeling robust optimization problems in Pyomo

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# Robust optimization

Consider a generic robust optimization problem:

$$\min_{\boldsymbol{x}\in\mathcal{X},\boldsymbol{y}(\boldsymbol{\xi})} \max_{\boldsymbol{\xi}\in\mathcal{U}(\boldsymbol{x})} \quad f(\boldsymbol{x},\boldsymbol{y}(\boldsymbol{\xi}),\boldsymbol{\xi}) \tag{1a}$$

$$\text{s.t} \quad g(\boldsymbol{x},\boldsymbol{y}(\boldsymbol{\xi}),\boldsymbol{\xi}) \leq 0 \qquad \forall \boldsymbol{\xi} \in \mathcal{U}(\boldsymbol{x}) \tag{1b}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ are "here and now" variables, $\boldsymbol{y}(\boldsymbol{\xi})$ are adjustable variables, and the uncertain parameter vector $\boldsymbol{\xi}$ is bounded by the uncertainty set $\mathcal{U}(\boldsymbol{x})$.

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**Solution approaches**

1. Robust reformulation: based on duality
2. Cutting planes: iterative approach

# Existing tools

There are a number of existing tools for solving robust optimization problems:

[1] Goh and Sim (2011), [2] Chen et al. (2020), [3] Isenberg et al. (2020), [4] Vayanos et al. (2020), [5] Dunning (2016)

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Designed to *solve* RO problems

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- PyROS [3]: Python/Pyomo
- ROC++ [4]: C++

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## Modeling languages

Designed for *modeling* RO problems

- JumPeR [5]: Julia/JumP
- AIMMS

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## ROmodel

Focuses on modeling. ROmodel is open source, tightly integrated with Pyomo, and allows modeling uncertainty sets with Pyomo constraints.

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These components can be used in Pyomo models like any Pyomo modeling component.

# Modeling uncertain parameters

Uncertain parameters are the central component of robust optimization problems:

```
import pyomo.environ as pe
import romodel as ro
# Construct model
m = pe.ConcreteModel()
# Add uncertain parameters
m.c = ro.UncParam(range(3), nominal=[0.1, 0.2, 0.3], uncset=m.U)
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```

Arguments:

- `index`: (optional), `UncParam` can be indexed or not
- `nominal`: specifies nominal values for the uncertain parameters
- `uncset`: specifies an uncertainty set for the uncertain parameters

# Modeling uncertainty sets

Uncertainty sets can be specified in two ways:

1. Generic sets with Pyomo constraints

```
# Define uncertainty set & uncertain parameters
m.U = ro.UncSet()
m.c = UncParam(range(2), uncset=m.U, nominal=[0.5, 0.5])
# Add constraints to uncertainty set
m.U.cons1 = Constraint(expr=m.c[0] + m.c[1] <= 1)
m.U.cons2 = Constraint(expr=m.c[0] - m.c[1] <= 1)
```

2. Library sets

```
# Define polyhedral set
m.U = ro.uncset.PolyhedralSet(rhs=[1, 1, 1, 1], mat=[[ 1,  1],
                                                     [ 1, -1],
                                                     [-1,  1],
                                                     [-1,
-1]])
```

# Modeling uncertain constraints

Users can model uncertain constraints implicitly by using `UncParam` objects in Pyomo constraints.

Consider a deterministic Pyomo constraint:

```
# deterministic
m.x = Var(range(3))
c = [0.1, 0.2, 0.3]
m.cons = Constraint(expr=sum(c[i]*m.x[i] for i in m.x) <= 0)
```

If `c` is uncertain, the robust formulation is:

```
# robust
m.x = Var(range(3))
m.c = UncParam(range(3), nominal=[0.1, 0.2, 0.3], uncset=m.U)
m.cons = Constraint(expr=sum(m.c[i]*m.x[i] for i in m.x) <= 0)
```

# Modeling adjustable variables

Adjustable variable: Decision variable who's value is determined after the uncertainty has been revealed.

Defining adjustable variables in ROmodel with the `AdjustableVar` class:

```
# Define uncertain parameters and adjustable variables
m.w = UncParam(range(3), nominal=[1, 2, 3], uncset=m.U)
m.y = AdjustableVar(range(3), uncparams=[m.w], bounds=(0, 1))
```

Arguments `uncparams` specifies which uncertain parameters are revealed before the decision is made. This can be set individually for each adjustable variable:

```
# Set uncertain parameters for individual indicies
m.y[0].set_uncparams([m.w[0]])
m.y[1].set_uncparams([m.w[0], m.w[1]])
```

ROmodel currently only implements linear decision rules.

## Solvers

ROmodel includes three solvers:

1. Robust reformulation:

```
solver = SolverFactory('romodel.reformulation')
```

2. Cutting planes:

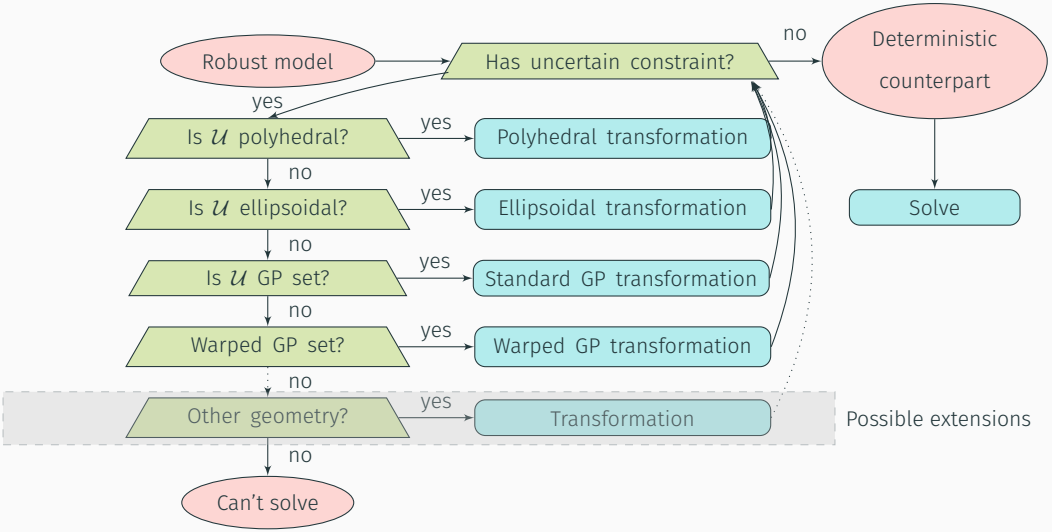
```
solver = SolverFactory('romodel.cuts')
```

3. Nominal:

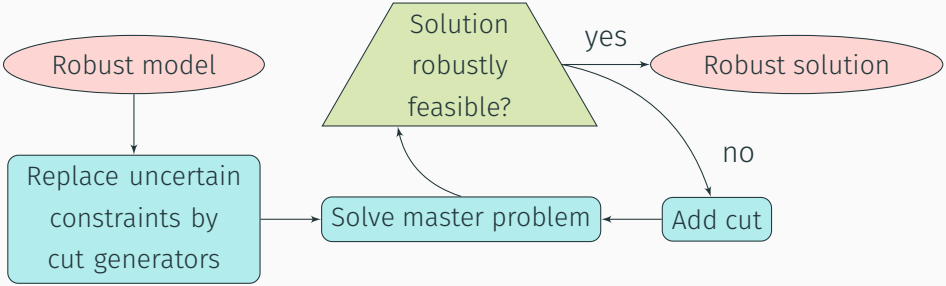
```
solver = SolverFactory('romodel.nominal')
```

```
# Set master & sub solver
solver.options['solver'] = 'gurobi'
solver.options['subsolver'] = 'ipopt'
# Solve
solver.solve(m)
```

# Solvers: reformulation



# Solvers: cutting planes



# Extending ROmodel

ROmodel can be extended in a number of ways:

1. Implementing new library uncertainty set
2. Adding new reformulations

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**Gaussian process-based uncertainty sets**

Gaussian processes are often used as surrogates for uncertain black-box constraints. We have developed reformulation approaches for chance constraints based on (warped) Gaussian processes [1].

[1] Wiebe et al. (2020)

# Extending ROmodel: adding library sets

Class which collects relevant data:

```
    class EllipsoidalSet(UncSet):
        '''
        Defines an ellipsoidal uncertainty set of shape:
            (param - mu)^T * A * (param - mu) <= 1
        '''
        def __init__(self, mean, cov, *args, **kwargs):
            self.mean = mean
            self.cov = cov
            super().__init__(*args, **kwargs)
```

Make compatible with cutting planes:

```
    def generate_cons_from_lib(self, param):
        raise NotImplementedError
```

# Extending ROmodel: adding reformulations

```
    def _reformulate(self, c, param, uncset, counterpart):
        """
        Reformulate an uncertain constraint or objective
            c: Constraint or Objective
            param: UncParam
            uncset: UncSet
            counterpart: Block
        """
        return counterpart
```

Make compatible with generic uncertainty sets (`UncSet`):

```
    def _check_applicability(self, uncset):
        """
        Returns ‘True‘ if the reformulation is applicable to ‘uncset‘
        """
        return uncset.__class__ == GPSet
```

# Extending ROmodel: Gaussian process-based sets

Train (warped) Gaussian process with GPy:

```
import GPy
# Set up kernel
kernel = GPy.kern.RBF(input_dim=1)
# Set up GP and train
gp = GPy.models.WarpedGP(x, y, kernel=kernel, warping_terms=3)
gp.optimize()

```

Use GPy model to construct uncertainty set:

```
from romodel.uncset import WarpedGPSet
m.z = pe.Var(range(3), within=pe.NonNegativeReals)
# Set up GP-based uncertainty set
m.uncset = WarpedGPSet(gp, m.z, 0.95)

```

# Case studies

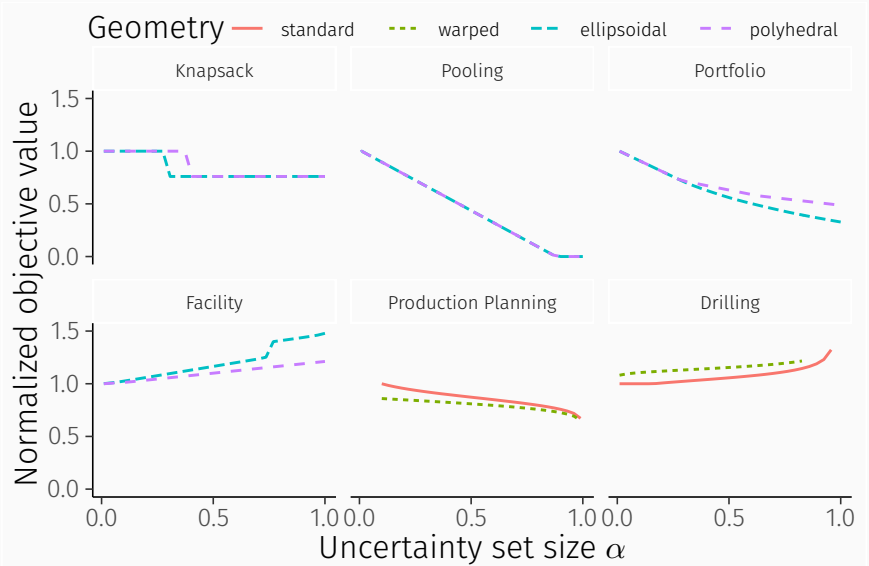
1. Portolio optimization with uncertain returns [1],
2. Knapsack problem with uncertain item weights,
3. Pooling problem instance with uncertain product demands [2],
4. Capacitated facility location problem with uncertain demand,
5. Production planning problem with prices dependent on uncertain black-box function [3],
6. Drill scheduling problem with uncertain degradation rate dependent on black-box function [3]

[1] Bertsimas and Sim (2004), [2] Adhya et al. (1999), [3] Wiebe et al. (2020)

# Results: median times

| | | Reformulation | Cuts | Overall |
|---|---|---|---|---|
| Knapsack | Polyhedral | 54 | 272 | 85 |
| | Ellipsoidal | 50 | 183 | 91 |
| Pooling | Polyhedral | 74 | 329 | 173 |
| | Ellipsoidal | 638 | 331 | 349 |
| Portfolio | Polyhedral | 50 | 276 | 126 |
| | Ellipsoidal | 49 | 1659 | 129 |
| Facility | Polyhedral | 261 | 13353 | 5588 |
| | Ellipsoidal | – | 31275 | 31275 |
| Planning | Standard | 2776 | NA | 2776 |
| | Warped | 8536 | NA | 8536 |
| Drilling | Standard | 13646 | NA | 13646 |
| | Warped | 75325 | NA | 75325 |
| Overall | | 74 | 330 | 271 |

# Results: price of robustness



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ROmodel...

- ...makes modeling & solving robust problems with Pyomo intuitive.
- ...makes it easy to compare different uncertainty sets & solution approaches.
- ...is open source and can be extended to other uncertainty sets & reformulations.

# Thank you!

Try ROmodel: https://github.com/cog-imperial/romodel

Paper: https://arxiv.org/abs/2105.08598

Twitter: @CogImperial

Funding: EP/L016796/1, EP/R511961/1 no. 17000145, and EP/P016871/1

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