

# Data-driven optimization of processes with degrading equipment

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Johannes Wiebe<sup>1</sup>

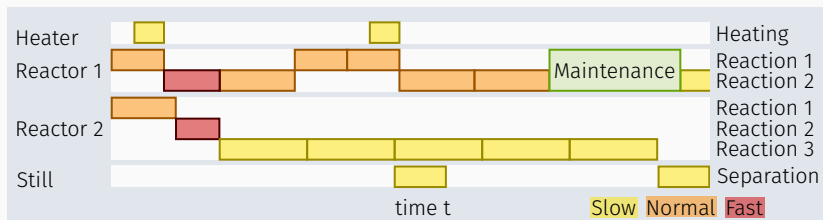
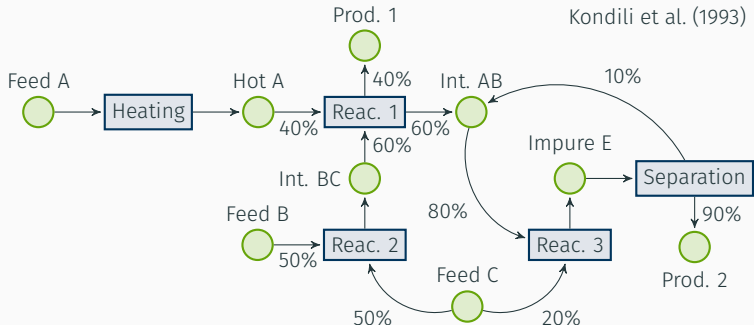
Wednesday 1<sup>st</sup> August, 2018

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# Motivation: Why degradation matters



## Starting point: Process level MI(N)LP model

$$\begin{array}{ll}\min_{\boldsymbol{x}, \boldsymbol{m}} & \text{cost}(\boldsymbol{x}, \boldsymbol{m}) \\ \text{s.t.} & \text{process model}(\boldsymbol{x}, \boldsymbol{m}) \quad (\text{eg. balance equations}) \\ & \text{maintenance model}(\boldsymbol{x}, \boldsymbol{m}) \quad (\text{eg. types of maint.})\end{array}$$

where  $\boldsymbol{x}$  are process variables,  $\boldsymbol{m}$  are maintenance variables

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where  $\mathbf{x}$  are process variables,  $\mathbf{m}$  are maintenance variables, and  $\mathbf{h}$  are health related variables.

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### Related Work

Vassiliadis and Pistikopoulos (2001); Liu et al. (2014); Xenos et al. (2016); Aguirre and Papageorgiou (2018); Biondi et al. (2017); Yildirim et al. (2017); Başıftci et al. (2018)

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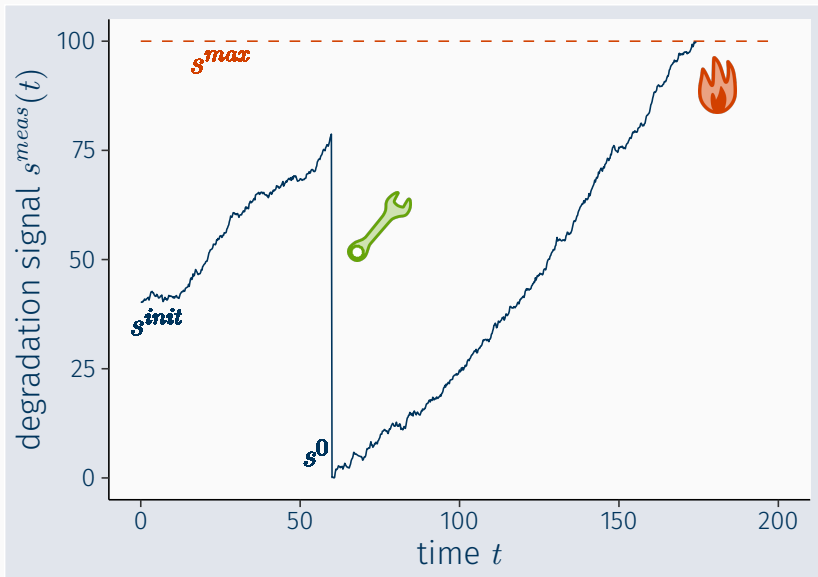
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### Idea

Combine process level MI(N)LP scheduling & planning with more sophisticated (stochastic) degradation modelling and robust optimization.

# What is degradation modelling?



## What is degradation modelling?

The degradation signal  $s^{meas}(t)$  can be modelled by a stochastic process :

$$S(t) = \{S_t : t \in T\},$$

where  $S_t$  is a random variable (Alaswad and Xiang, 2017).



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### Often used: Lévy type processes (Applebaum, 2004)

- Independent increments:  $S_{t_2} - S_{t_1}, \dots, S_{t_n} - S_{t_{n-1}}$  are independent for any  $0 < t_1 < t_2 < \dots < t_n < \infty$
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- Stationary increments:  $S_t - S_s$  and  $S_{t-s}$  have the same distribution for any  $s < t$

Therefore  $S_t - S_{t-\Delta t} = D \sim \mathcal{D}(\Theta, \Delta t)$ , where  $\Theta$  are parameters of distribution  $\mathcal{D}$ .

## A health model based on Lévy processes

### Assumption

The health of each unit  $j$  can be described by a Lévy process  $S_j(t)$  with increments  $S_{j,t} - S_{j,t-\Delta t} = D_j \sim \mathcal{D}_j(\Theta, \Delta t)$ .

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$$\text{s.t.} \quad \text{process model}(\mathbf{x}, \mathbf{m}, \mathbf{h})$$

$$\text{maintenance model}(\mathbf{x}, \mathbf{m}, \mathbf{h})$$

$$S_{j,t} \leq s_j^{\max} \quad \forall t, j \in J$$

$$S_{j,t} = \begin{cases} S_{j,t-1} + D_j, & \text{if } m_{j,t} = 0 \\ s_j^0, & \text{otherwise} \end{cases} \quad \forall t, j \in J$$

where  $m_{j,t} = 1$  if maintenance is performed on unit  $j$  at time  $t$ .

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## Accounting for effects of process variables

### Assumption (Liao and Tian, 2013)

All relevant operating variables are piecewise constant – i.e. the process has a set of discrete operating modes  $k \in K$ .

$$\begin{aligned} \min_{\mathbf{x}, \mathbf{m}, \mathbf{h}} \quad & \text{cost}(\mathbf{x}, \mathbf{m}, \mathbf{h}) \\ \text{s.t.} \quad & \text{process model}(\mathbf{x}, \mathbf{m}, \mathbf{h}) \\ & \text{maintenance model}(\mathbf{x}, \mathbf{m}, \mathbf{h}) \\ & S_{j,t} \leq s_j^{max} \quad \forall t, j \in J \\ & S_{j,t} = \begin{cases} S_{j,t-1} + \sum_{k \in \mathcal{K}} x_{j,k,t} \cdot D_{j,k}, & \text{if } m_{j,t} = 0 \\ s_j^0, & \text{otherwise} \end{cases} \quad \forall t, j \in J \end{aligned}$$

where  $x_{j,k,t} = 1$  if unit  $j$  operates in mode  $k$  at time  $t$ .

## Deriving a robust counterpart (Lappas and Gounaris, 2016)

### Idea

Replace random variables  $D_{j,k}$  and  $S_{j,t}$  by uncertain parameter  $\tilde{d}_{j,k} \in \mathcal{U}$  and second stage variable  $s_{j,t}(\tilde{d}_{j,k})$ .

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$\forall \tilde{d}_{j,k} \in \mathcal{U}$ . Approximate  $s_{j,t}(\tilde{d}_{j,k})$  by linear decision rule. Utilize Robust Optimization reformulation techniques.



How do we choose  $\mathcal{U}$ ?

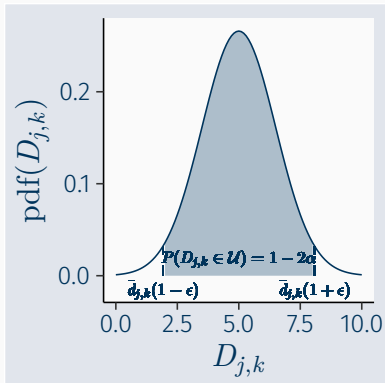
Assumption:  $\mathcal{U}$  is a box uncertainty set

$$\mathcal{U} = \{\tilde{d}_{j,k} | \bar{d}_{j,k}(1 - \epsilon_{j,k}) \leq \tilde{d}_{j,k} \leq \bar{d}_{j,k}(1 + \epsilon_{j,k})\}$$

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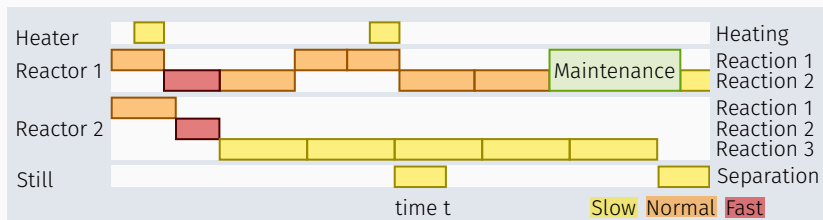
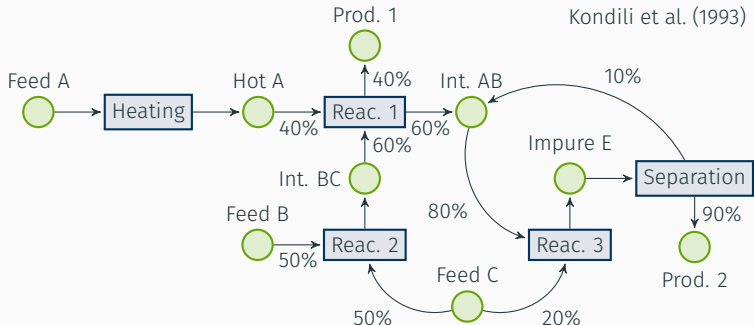


Choose  $\epsilon_{j,k}$  from distribution  $\mathcal{D}_{j,k}$ :

$$\epsilon_{j,k} = 1 - F^{-1}(\alpha) / \bar{d}_{j,k}$$

Size of  $\mathcal{U}$  depends on a single parameter  $\alpha$ !

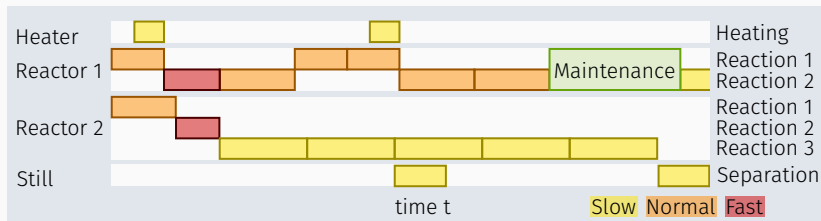
# Case study: State-Task-Network (Kondili et al., 1993)



### Case study: State-Task-Network (Kondili et al., 1993)

Biondi et al. (2017) extend the STN to include...

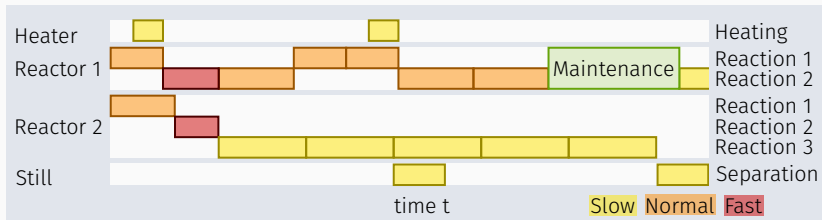
- ...unit health and maintenance scheduling
- ...integrated scheduling and planning
- ...multiple operating modes per task



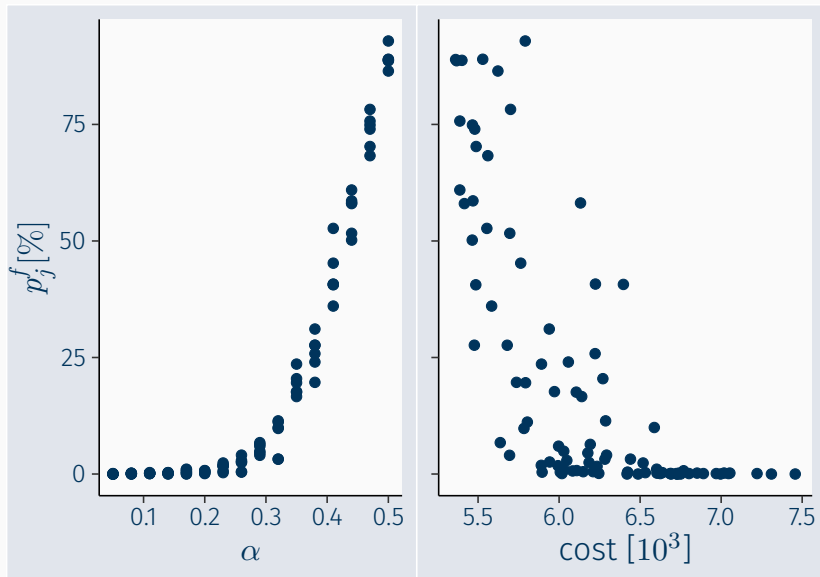
## Case study: State-Task-Network (Kondili et al., 1993)

This work...

- ...replaces their deterministic health model by the proposed approach based on degradation modelling.
- ...utilizes robust optimization to obtain a solution that is likely to remain feasible.



## The price of robustness



## Choosing $\alpha$ is its own optimization problem

We optimize  $\alpha$  by solving

$$\min_{\alpha} c^*(\alpha) + \sum_j p_j^f(\alpha) \cdot c_j^f$$

- $c^*(\alpha)$  is the objective value of a MILP solution given  $\alpha$ .
- $p_j^f(\alpha)$  is the corresponding probability of failure (of unit  $j$ ).
- $c_j^f$  is the cost of an unexpected failure.

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### Idea: Use Bayesian Optimization (BO)

Both  $c^*$  and  $p_j^f$  can be viewed as expensive black box functions. BO is very suitable for this setting.



## Saving time: a deterministic approximation

### Assumption

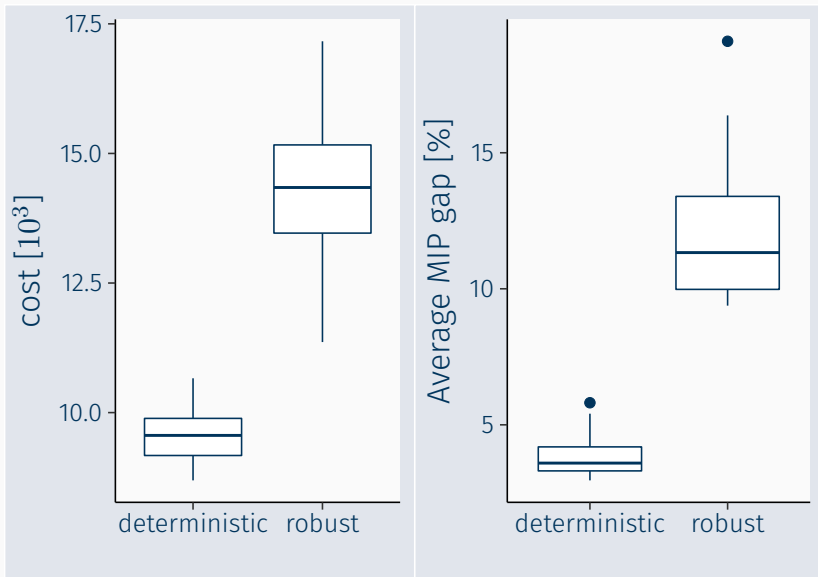
Only the health model depends on  $\tilde{d}_{j,k}$  and  $\tilde{d}_{j,k} \geq 0$ .

Then we can prove that a solution to

$$\begin{aligned} \min_{\mathbf{x}, \mathbf{m}, \mathbf{h}} \quad & \text{cost}(\mathbf{x}, \mathbf{m}, \mathbf{h}) \\ \text{s.t.} \quad & \text{process model, maint. model}(\mathbf{x}, \mathbf{m}, \mathbf{h}) \\ & s_{j,t} \leq s_j^{\max} \quad \forall t, j \in J \\ & s_{j,t} = \begin{cases} s_{j,t-1} + \sum_{k \in \mathcal{K}} x_{j,k,t} \cdot d_{j,k}^{\max}, & \text{if } m_{j,t} = 0 \\ s_j^0, & \text{otherwise} \end{cases} \quad \forall t, j \in J \end{aligned}$$

with  $d_{j,k}^{\max} = \max_{\mathcal{U}} \tilde{d}_{j,k}$  is also feasible in the robust problem.

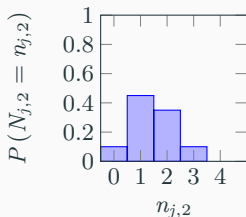
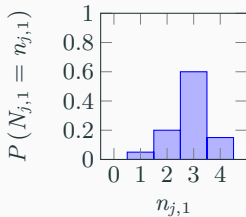
## Saving time: a deterministic approximation



## Saving time: data-driven approximations

An upper bound on the probability of failure  $p_j^f$  can be estimated from data (using logistic regression).

Estimate from data:



Sample

$n_{1,l} = 3$

Sample

$n_{2,l} = 2$

Repeat  $N$  times,  $\bar{p}_j^f = \max p_j^f(\mathbf{x}_{j,l}^k)$ :

Insert maintenance:

M

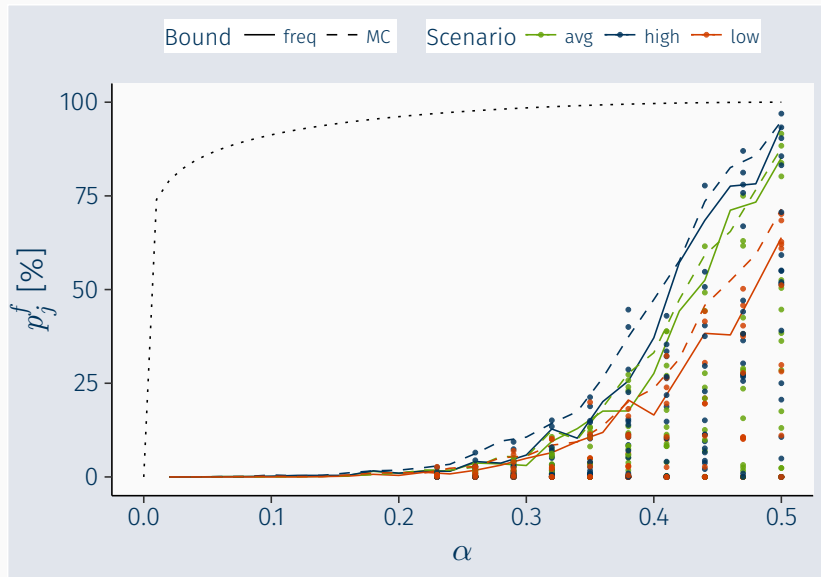
Random order:



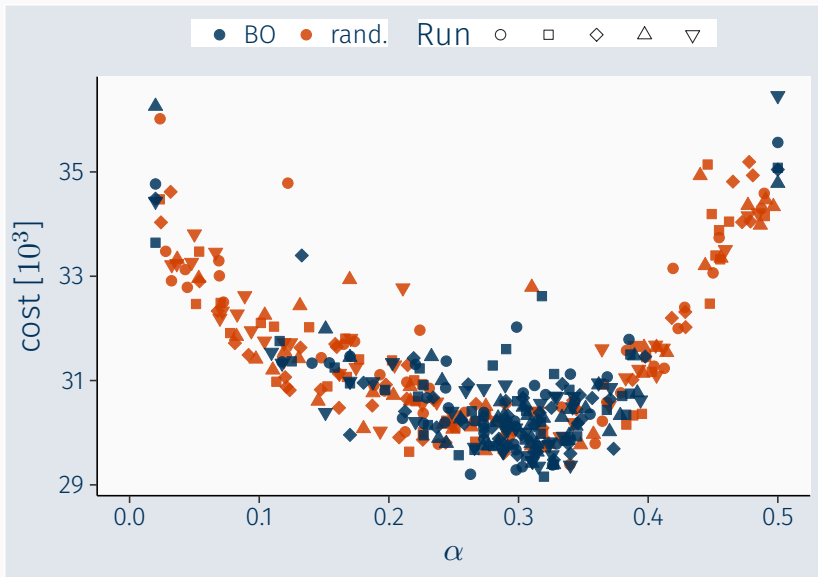
$\mathbf{x}_{j,l}^k = [1, 2, 2, 1, M, 2]$

Calculate  $p_j^f(\mathbf{x}_{j,l}^k)$

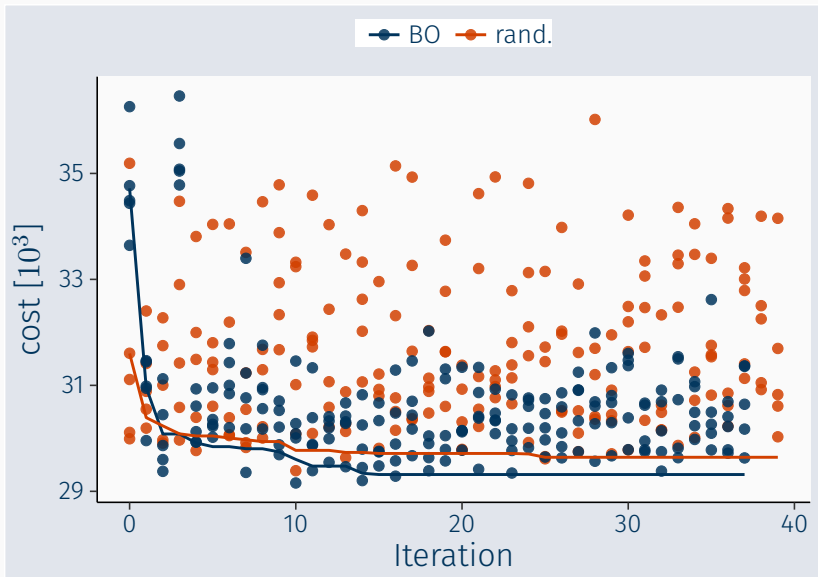
## Saving time: data-driven approximations



# Bayesian Optimization



# Bayesian Optimization



# Conclusion

- Something smart.
- Something really smart.
- Something really really smart.

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## Thank You!



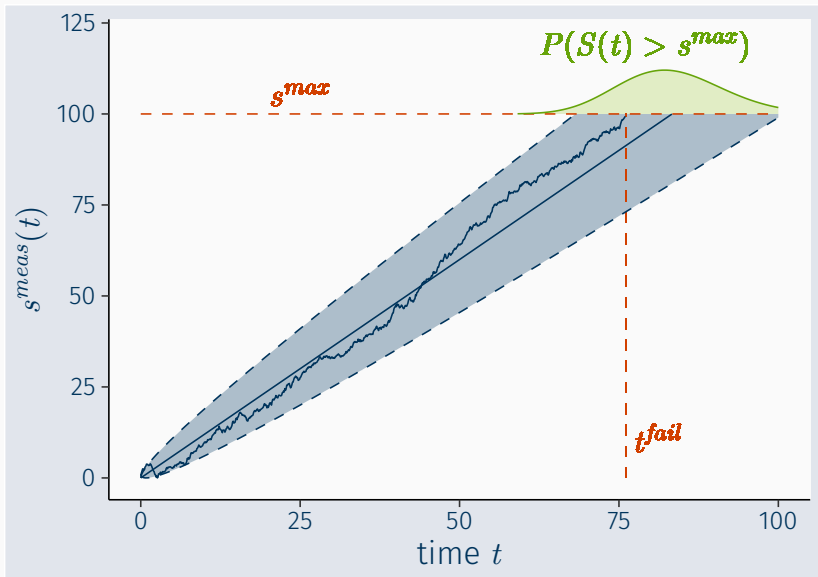
# References

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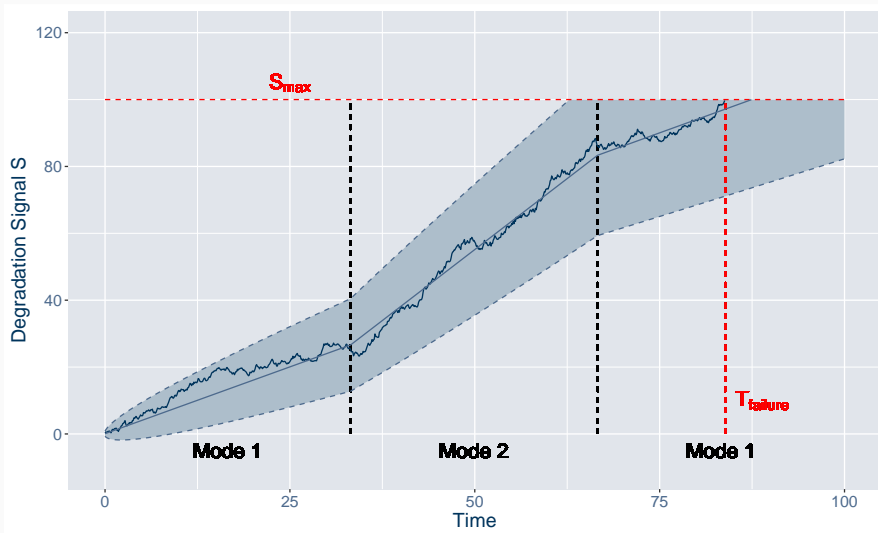
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## Degradation modelling



# Degradation modelling with multiple operating modes



# How does robust optimization work?

## General idea

- Make constraints hold for all values in  $\mathcal{U}$ :

$$\sum_j \tilde{a}_{ij} x_j \leq b_i, \forall \tilde{a}_{ij} \in \mathcal{U}$$

- Reformulate semi-infinite constraint:

$$\sum_j a_{ij} x_j + \text{protection}(\mathcal{U}) \leq b_i$$

- How do we choose the right protection level?

## Example: Soyster's method (worst case) [1973]

$$\max_{x_1, x_2} \quad x_1 + x_2$$

$$\text{s.t.} \quad \tilde{a}_{11} x_1 + \tilde{a}_{12} x_2 \leq b_1,$$

$$\forall \tilde{a}_{ij} \in \mathcal{U}$$

$$\max_{x_1, x_2} \quad x_1 + x_2$$

# Formulation

## Scheduling

$$M_{j,t} S_{j,0} \leq S_{j,t} \leq S_{j,max} + M_{j,t} \cdot (S_{j,0} - S_{j,max}) \quad \forall t, j \in J, D \in \mathcal{D}$$

$$S_{j,t} \geq S_{j,t-\Delta t} + \sum_k Z_{j,k,t} D_{j,k,t} + M_{j,t} \cdot (S_{j,0} - S_{j,max}) \quad \forall t, j \in J, D \in \mathcal{D}$$

$$S_{j,t} \leq S_{j,t-\Delta t} + \sum_k Z_{j,k,t} D_{j,k,t} \quad \forall t, j \in J, D \in \mathcal{D}$$

## Planning

$$S_{j,t} \leq S_{j,max} \quad \forall t, j \in J$$

$$S_{j,t} \geq S_{j,t-\Delta t} + \sum_k N_{j,k,t} D_{j,k,t} + M_{j,t} \cdot (S_{j,0} - S_{j,max}) \quad \forall t, j \in J$$

$$S_{j,t} \leq S_{j,t-\Delta t} + \sum_k N_{j,k,t} D_{j,k,t} \quad \forall t, j \in J$$

# Adjustable robust optimization

## Affine decision rule

$$S_{j,t} = [S_{j,t}]_0 + \sum_k \sum_{t'=0}^t [S_{j,t}]_{k,t'} D_{j,k,t'}. \quad (1)$$

## Size of toy problem

	deterministic	robust $D \neq f(t)$	robust $D = f(t)$
# vars	913	3011	27719
# binaries	338	338	338
# constraints	1198	2356	13300
time to solve [s]	2	0.3-10	0.3-10
gap [%]	0	0	0
scheduling periods	30	30	30
planning periods	8	8	8
task-unit-op. mode combinations	6	6	6



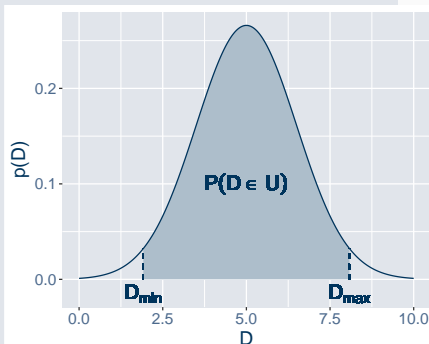
## Size of realistic problem

	deterministic	robust $D \neq f(t)$	robust $D = f(t)$
# vars	5389		397361
# binaries	2492		2492
# constraints	6798		180858
time to solve [s]	7883		16756
gap [%]	3.62		31.02
scheduling periods	56	56	56
planning periods	24	24	24
task-unit-op. mode combinations	24	24	24

## How do we choose $\mathcal{U}$ ?

### Choose $\mathcal{U}$ from distribution

- Choose parameter  $\alpha$
- Choose  $D_{min}$  such that  $P(D \leq D_{min}) = \alpha$
- Choose  $D_{max}$  such that  $P(D \geq D_{max}) = \alpha$
- $\mathcal{U} = \{D | D_{min} \leq D \leq D_{max}\}$



# Deriving a robust counterpart

Replace  $D_{j,k}$  by an uncertain parameter  $\tilde{d}_{j,k}$  bounded by a set  $\mathcal{U}$ :

$$\begin{aligned} s_{j,t} &\leq s_j^{max} && \forall t, j \in J \\ s_{j,t} &= \begin{cases} s_{j,t-1} + \sum_{k \in \mathcal{K}} x_{j,k,t} \cdot \tilde{d}_{j,k}, & \text{if } m_{j,t} = 0 \\ s_j^0, & \text{otherwise} \end{cases} && \forall \tilde{d}_{j,k} \in \mathcal{U}, t, j \in J \end{aligned}$$

Reformulate:

$$\begin{aligned} m_{j,t} s_j^0 &\leq s_{j,t} \leq s_j^{max} + m_{j,t} \cdot (s_j^0 - s_{j,max}) && \forall t, j \in J, \tilde{d}_{j,k} \in \mathcal{U} \\ s_{j,t} &\geq s_{j,t-\Delta t} + \sum_k x_{j,k,t} \tilde{d}_{j,k} + m_{j,t} \cdot (s_j^0 - s_j^{max}) && \forall t, j \in J, \tilde{d}_{j,k} \in \mathcal{U} \\ s_{j,t} &\leq s_{j,t-\Delta t} + \sum_k x_{j,k,t} \tilde{d}_{j,k} && \forall t, j \in J, \tilde{d}_{j,k} \in \mathcal{U}, \end{aligned}$$

Replace  $s_{j,t}$  by linear decision rule  $s_{j,t} = [s_{j,t}]_0 + \sum_k [s_{j,t}]_k \tilde{d}_{j,k}$ .

## Results: metrics data-driven approximation

instance	bound	rms_all	rms_max	p_out
toy	freq	8.00	1.53	29.40
toy	mc	10.41	3.08	21.27
P1	freq	12.61	3.52	17.54
P1	mc	17.25	4.39	9.62
P2	freq	7.40	2.31	18.08
P2	mc	13.68	4.98	10.13
P4	freq	9.17	3.27	47.78
P4	mc	11.43	2.84	32.50
P6	freq	18.75	8.94	12.17
P6	mc	20.84	10.09	10.98
all	freq	11.19	3.91	24.99
all	mc	14.72	5.08	16.90