# Data-driven optimization of processes with degrading equipment


Johannes Wiebe[1]

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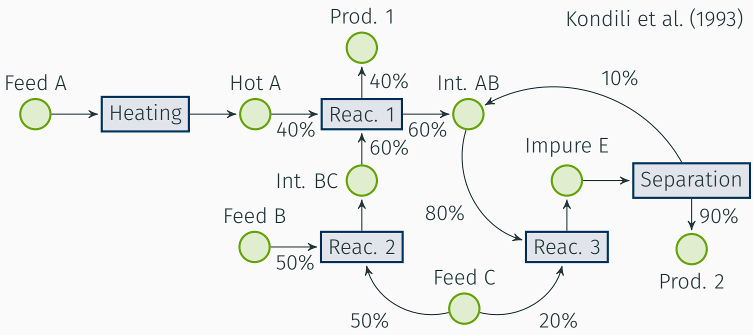
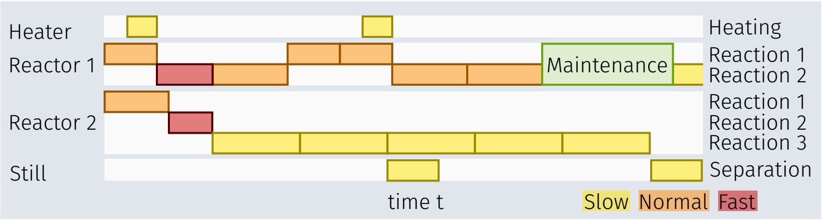
Supervisors: Ruth Misener[1], Ines Cecilio[2]

[1]Department of Computing, Imperial College London, London, UK

[2]Schlumberger Research Cambridge, Cambridge, UK London

# Motivation: Why degradation matters



# Starting point: Process level MI(N)LP model

$$\begin{aligned}
\min_{\boldsymbol{x},\boldsymbol{m}} \quad & \mathrm{cost}(\boldsymbol{x}, \boldsymbol{m} \quad ) \\
\text{s.t.} \quad & \text{process model}(\boldsymbol{x}, \boldsymbol{m} \quad ) && \text{(eg. balance equations)} \\
& \text{maintenance model}(\boldsymbol{x}, \boldsymbol{m} \quad ) && \text{(eg. types of maint.)}
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where $\boldsymbol{x}$ are process variables, $\boldsymbol{m}$ are maintenance variables

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& \text{health model}(\boldsymbol{x}, \boldsymbol{m}, \boldsymbol{h}), & \text{(eq. prognosis model)}
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where $\boldsymbol{x}$ are process variables, $\boldsymbol{m}$ are maintenance variables, and $\boldsymbol{h}$ are health related variables.

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### Related Work

Vassiliadis and Pistikopoulos (2001); Liu et al. (2014); Xenos et al. (2016); Aguirre and Papageorgiou (2018); Biondi et al. (2017); Yildirim et al. (2017); Başçiftci et al. (2018)

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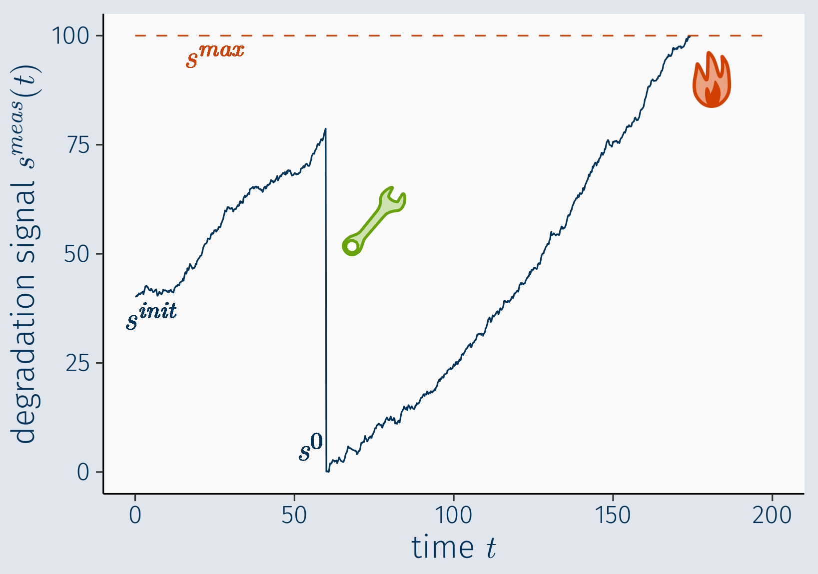
where $\boldsymbol{x}$ are process variables, $\boldsymbol{m}$ are maintenance variables, and $\boldsymbol{h}$ are health related variables.

**Idea**

Combine process level MI(N)LP scheduling & planning with more sophisticated (stochastic) degradation modelling and robust optimization.

# What is degradation modelling?



# What is degradation modelling?

The degradation signal $s^{meas}(t)$ can be modelled by a stochastic process :

$$S(t) = \{S_t : t \in T\},$$

where $S_t$ is a random variable (Alaswad and Xiang, 2017).

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### Often used: Lévy type processes (Applebaum, 2004)

- Independent increments: $S_{t_2} - S_{t_1}, ..., S_{t_n} - S_{t_{n-1}}$ are independent for any $0 < t_1 < t_2 < ... < t_n < \infty$
- Stationary increments: $S_t - S_s$ and $S_{t-s}$ have the same distribution for any $s < t$

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- Stationary increments: $S_t - S_s$ and $S_{t-s}$ have the same distribution for any $s < t$

Therefore $S_t - S_{t-\Delta t} = D \sim \mathcal{D}(\boldsymbol{\Theta}, \Delta t)$, where $\boldsymbol{\Theta}$ are parameters of distribution $\mathcal{D}$.

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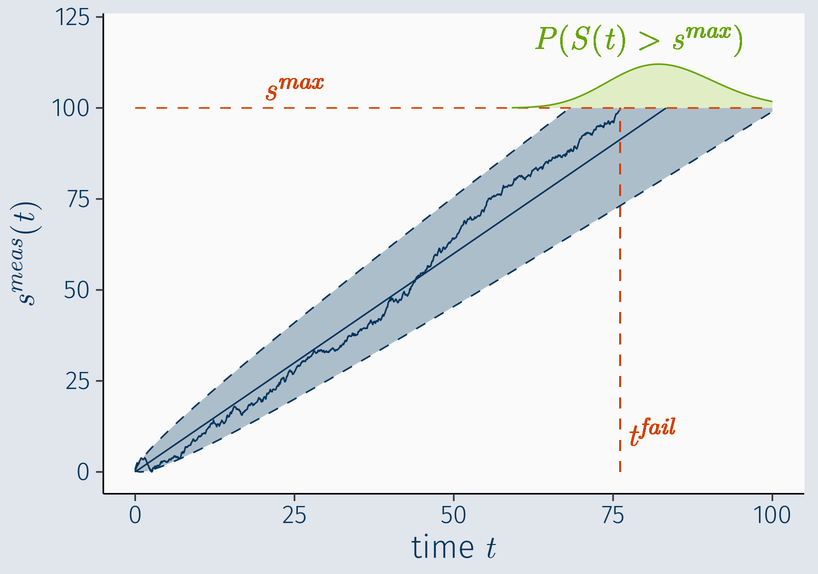

# A health model based on Lévy processes

**Assumption**

The health of each unit $j$ can be described by a Lévy process $S_j(t)$ with increments $S_{j,t} - S_{j,t-\Delta t} = D_j \sim \mathcal{D}_j(\mathbf{\Theta}, \Delta t)$.

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\text{s.t.} \quad & \text{process model}(\boldsymbol{x}, \boldsymbol{m}, \boldsymbol{h}) \\
& \text{maintenance model}(\boldsymbol{x}, \boldsymbol{m}, \boldsymbol{h}) \\
& S_{j,t} \leq s_j^{max} && \forall t, j \in J \\
& S_{j,t} = \begin{cases} S_{j,t-1} + D_j, & \text{if } m_{j,t} = 0 \\ s_j^0, & \text{otherwise} \end{cases} && \forall t, j \in J
\end{aligned}
$$

where $m_{j,t} = 1$ if maintenance is performed on unit $j$ at time $t$.

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# Accounting for effects of process variables

**Assumption (Liao and Tian, 2013)**

All relevant operating variables are piecewise constant – i.e. the process has a set of discrete operating modes $k \in K$.

$$
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$$

where $x_{j,k,t} = 1$ if unit $j$ operates in mode $k$ at time $t$.

# Deriving a robust counterpart (Lappas and Gounaris, 2016)

**Idea**

Replace random variables $D_{j,k}$ and $S_{j,t}$ by uncertain parameter $\tilde{d}_{j,k} \in \mathcal{U}$ and second stage variable $s_{j,t}(\tilde{d}_{j,k})$.

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& s_{j,t}(\tilde{d}_{j,k}) \leq s_j^{max} && \forall t, j \in J \\
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$\forall \tilde{d}_{j,k} \in \mathcal{U}.$

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\end{aligned}
$$

$\forall \tilde{d}_{j,k} \in \mathcal{U}$. Approximate $s_{j,t}(\tilde{d}_{j,k})$ by linear decision rule. Utilize Robust Optimization reformulation techniques.

# How do we choose $\mathcal{U}$?

**Assumption: $\mathcal{U}$ is a box uncertainty set**

$$\mathcal{U} = \{\tilde{d}_{j,k} | \bar{d}_{j,k}(1 - \epsilon_{j,k}) \leq \tilde{d}_{j,k} \leq \bar{d}_{j,k}(1 + \epsilon_{j,k})\}$$

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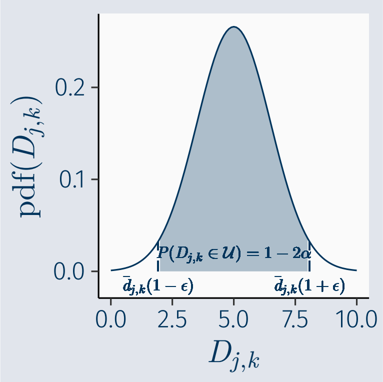
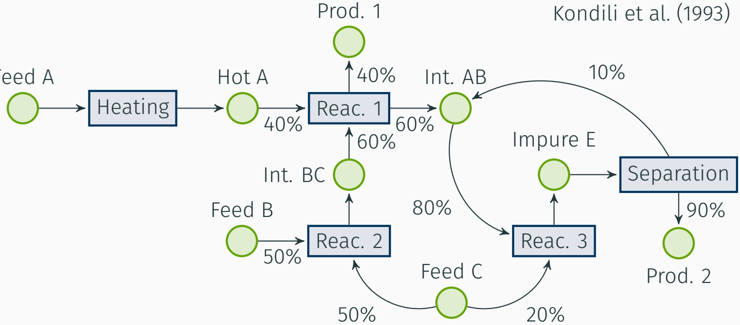


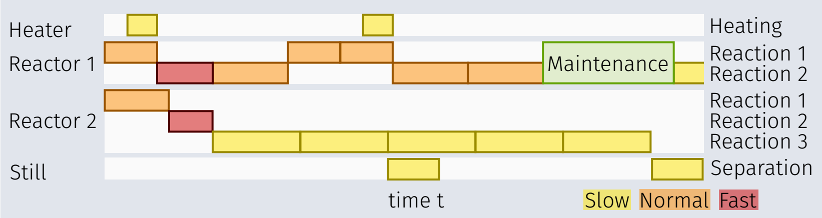

Choose $\epsilon_{j,k}$ from distribution $\mathcal{D}_{j,k}$:

$$\epsilon_{j,k} = 1 - F^{-1}(\alpha)/\bar{d}_{j,k}$$

Size of $\mathcal{U}$ depends on a single parameter $\alpha$!

# Case study: State-Task-Network (Kondili et al., 1993)

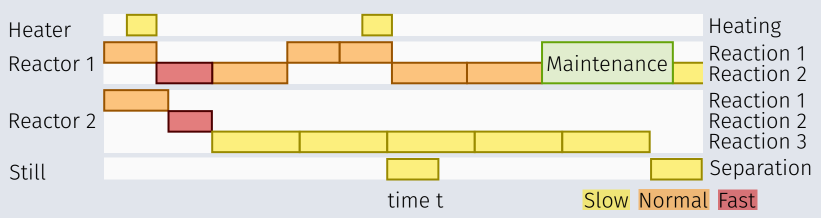


# Case study: State-Task-Network (Kondili et al., 1993)

Biondi et al. (2017) extend the STN to include...

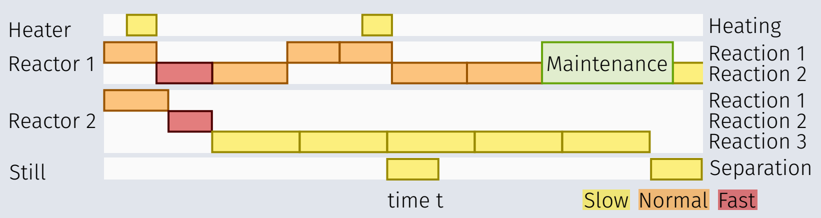
- ...unit health and maintenance scheduling
- ...integrated scheduling and planning
- ...multiple operating modes per task



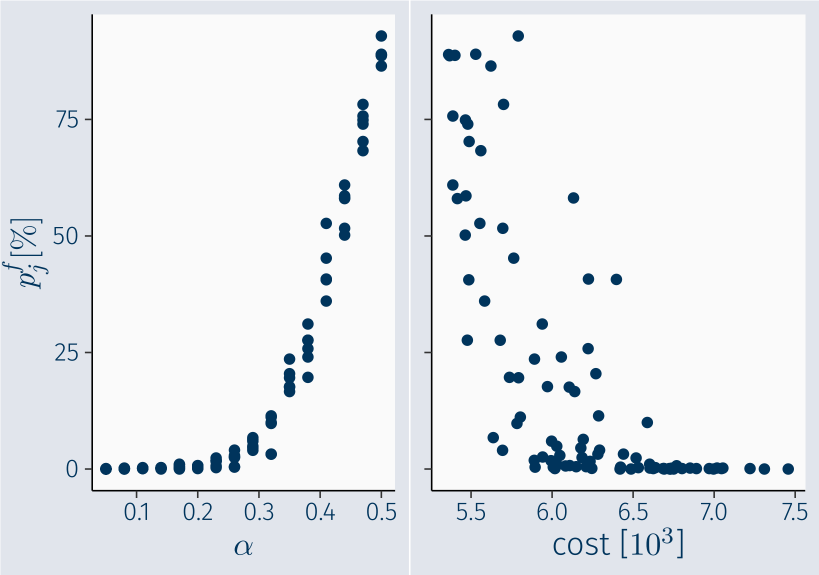
# Case study: State-Task-Network (Kondili et al., 1993)

This work...

- ...replaces their deterministic health model by the proposed approach based on degradation modelling.
- ...utilizes robust optimization to obtain a solution that is likely to remain feasible.



# The price of robustness



# Choosing $\alpha$ is its own optimization problem

We optimize $\alpha$ by solving

$$\min_{\alpha} c^*(\alpha) + \sum_j p_j^f(\alpha) \cdot c_j^f$$

- $c^*(\alpha)$ is the objective value of a MILP solution given $\alpha$.
- $p_j^f(\alpha)$ is the corresponding probability of failure (of unit $j$).
- $c_j^f$ is the cost of an unexpected failure.

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**Idea: Use Bayesian Optimization (BO)**

Both $c^*$ and $p_j^f$ can be viewed as expensive black box functions. BO is very suitable for this setting.

# Saving time: a deterministic approximation

**Assumption**

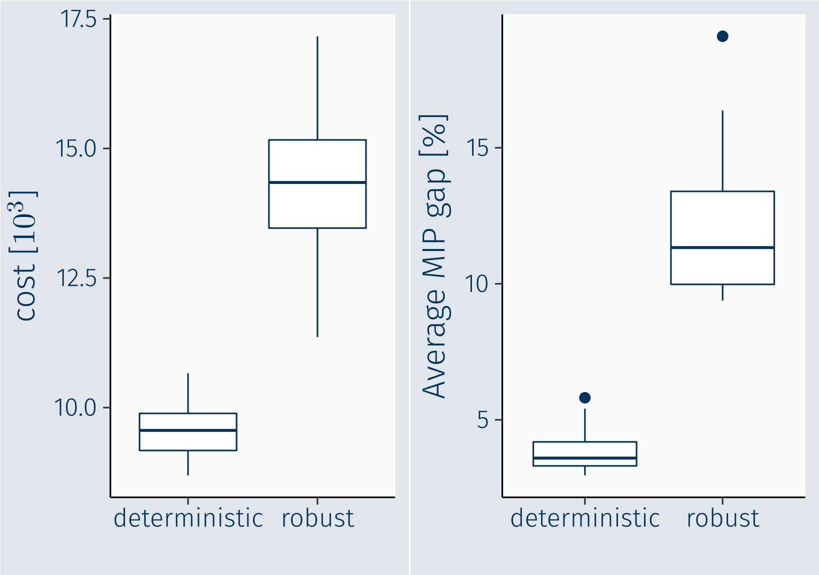
Only the health model depends on $\tilde{d}_{j,k}$ and $\tilde{d}_{j,k} \geq 0$.

Then we can prove that a solution to

$$
\begin{aligned}
\min_{\boldsymbol{x},\boldsymbol{m},\boldsymbol{h}} \quad & \operatorname{cost}(\boldsymbol{x}, \boldsymbol{m}, \cancel{\boldsymbol{h}}) \\
\text{s.t.} \quad & \text{process model, maint. model}(\boldsymbol{x}, \boldsymbol{m}, \cancel{\boldsymbol{h}}) \\
& s_{j,t} \leq s_j^{max} && \forall t, j \in J \\
& s_{j,t} = \begin{cases} s_{j,t-1} + \sum_{k \in \mathcal{K}} x_{j,k,t} \cdot d_{j,k}^{max}, & \text{if } m_{j,t} = 0 \\ s_j^0, & \text{otherwise} \end{cases} && \forall t, j \in J
\end{aligned}
$$

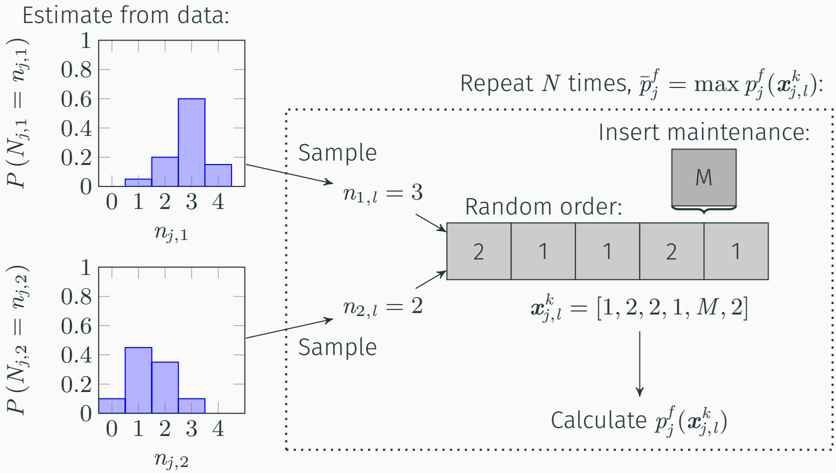
with $d_{j,k}^{max} = \max_{\mathcal{U}} \tilde{d}_{j,k}$ is also feasible in the robust problem.

# Saving time: a deterministic approximation

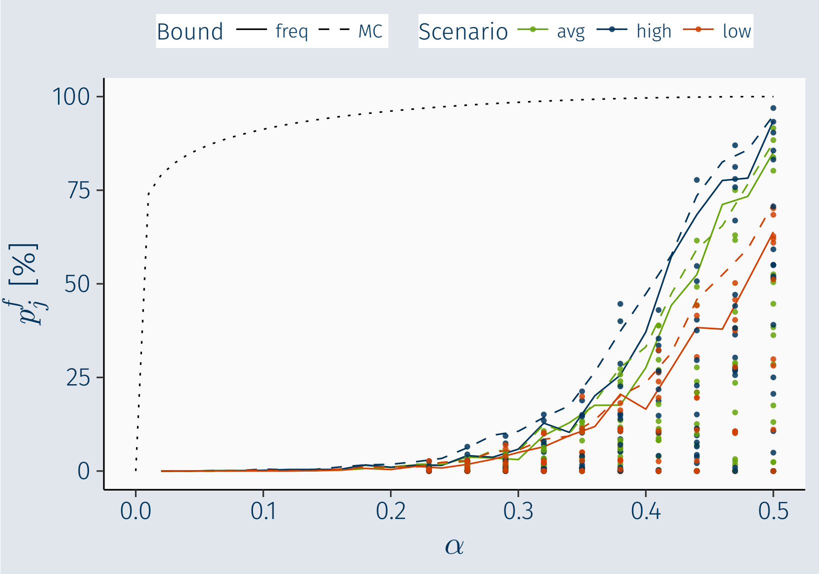


# Saving time: data-driven approximations

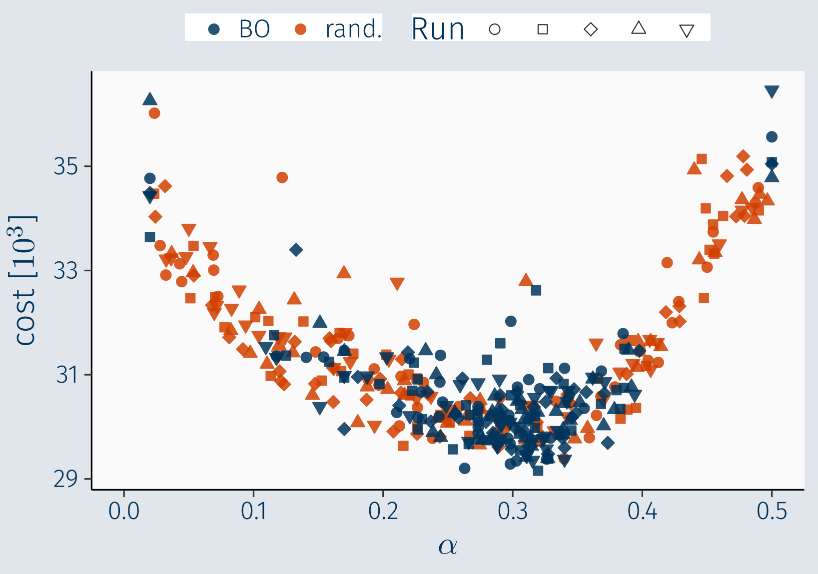
An upper bound on the probability of failure $p_j^f$ can be estimated from data (using logistic regression).



# Saving time: data-driven approximations



# Bayesian Optimization



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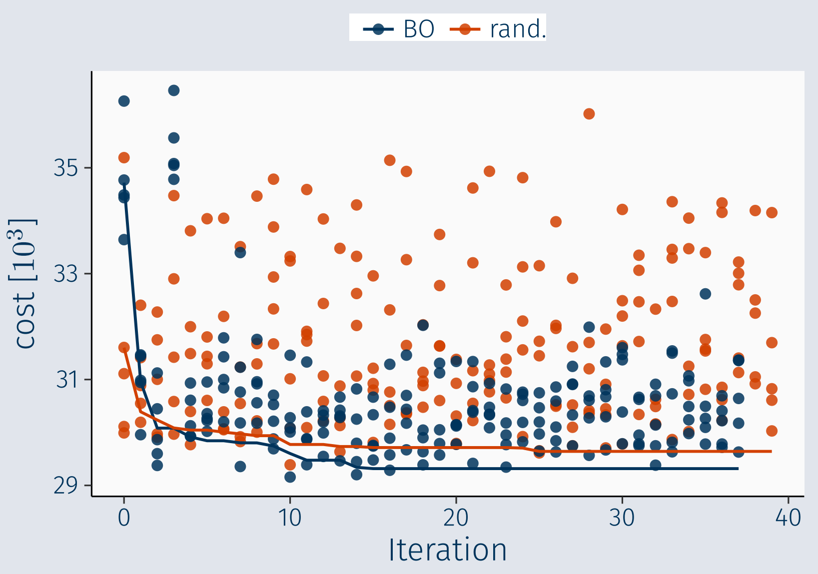
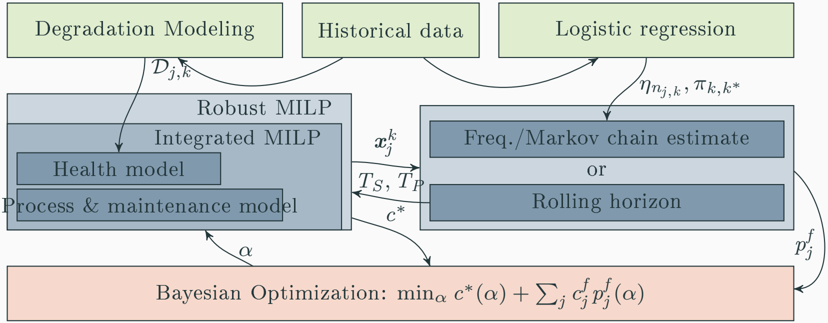


# Conclusion

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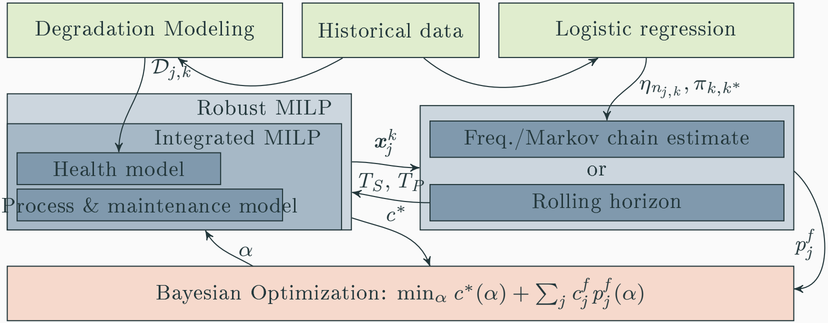
# Conclusion




Thank You!

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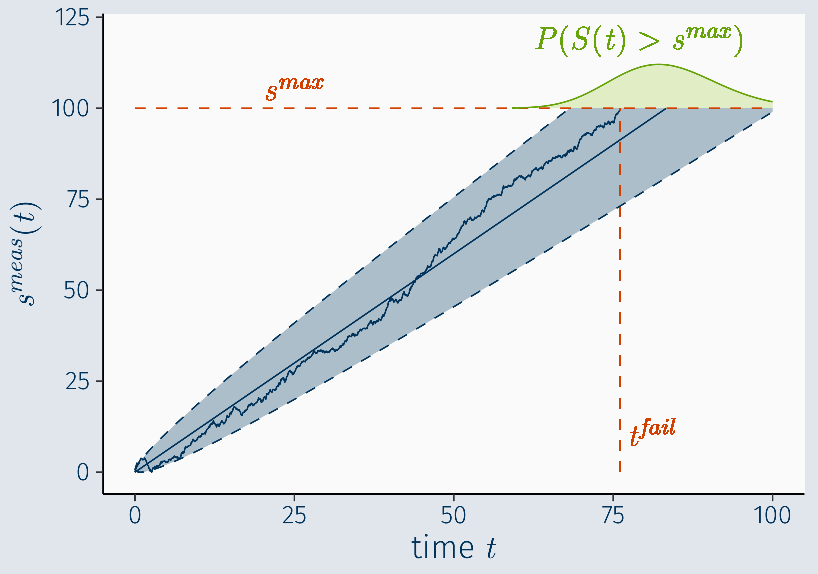
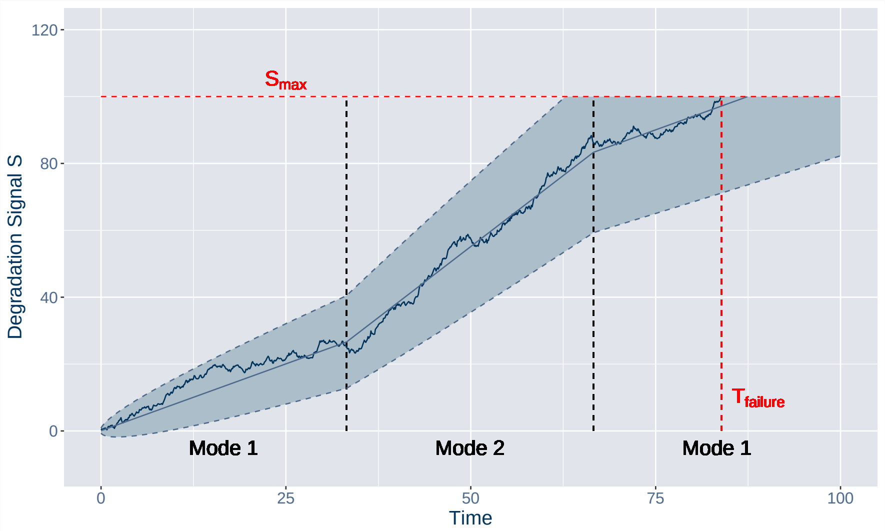
# Degradation modelling



# Degradation modelling with multiple operating modes



# How does robust optimization work?

## General idea

- Make constraints hold for all values in $\mathcal{U}$:
  $\sum_j \tilde{a}_{ij} x_j \leq b_i, \forall \tilde{a}_{ij} \in \mathcal{U}$
- Reformulate semi-infinite constraint:
  $\sum_j a_{ij} x_j + \text{protection}\,(\mathcal{U}) \leq b_i$
- How do we choose the right protection level?

## Example: Soyster's method (worst case) [1973]

$$\begin{aligned} \max_{x_1, x_2} \quad & x_1 + x_2 \\ \text{s.t.} \quad & \tilde{a}_{11} x_1 + \tilde{a}_{12} x_2 \leq b_1, \\ & \forall \tilde{a}_{ij} \in \mathcal{U} \end{aligned}$$

$$\max_{x_1, x_2} \quad x_1 + x_2$$

# Formulation

## Scheduling

$$M_{j,t}S_{j,0} \leq S_{j,t} \leq S_{j,max} + M_{j,t} \cdot (S_{j,0} - S_{j,max}) \quad \forall t, j \in J, D \in \mathcal{D}$$

$$S_{j,t} \geq S_{j,t-\Delta t} + \sum_k Z_{j,k,t}D_{j,k,t} + M_{j,t} \cdot (S_{j,0} - S_{j,max}) \quad \forall t, j \in J, D \in \mathcal{D}$$

$$S_{j,t} \leq S_{j,t-\Delta t} + \sum_k Z_{j,k,t}D_{j,k,t} \quad \forall t, j \in J, D \in \mathcal{D}$$

## Planning

$$S_{j,t} \leq S_{j,max} \quad \forall t, j \in J$$

$$S_{j,t} \geq S_{j,t-\Delta t} + \sum_k N_{j,k,t}D_{j,k,t} + M_{j,t} \cdot (S_{j,0} - S_{j,max}) \quad \forall t, j \in J$$

$$S_{j,t} \leq S_{j,t-\Delta t} + \sum_k N_{j,k,t}D_{j,k,t} \quad \forall t, j \in J$$

# Adjustable robust optimization

## Affine decision rule

$$S_{j,t} = [S_{j,t}]_0 + \sum_k \sum_{t'=0}^{t} [S_{j,t}]_{k,t'} D_{j,k,t'}. \tag{1}$$

# Size of toy problem

| | deterministic | robust $D \neq f(t)$ | robust $D = f(t)$ |
|---|---|---|---|
| # vars | 913 | 3011 | 27719 |
| # binaries | 338 | 338 | 338 |
| # constraints | 1198 | 2356 | 13300 |
| time to solve [s] | 2 | 0.3-10 | 0.3-10 |
| gap [%] | 0 | 0 | 0 |
| scheduling periods | 30 | 30 | 30 |
| planning periods | 8 | 8 | 8 |
| task-unit-op. mode combinations | 6 | 6 | 6 |

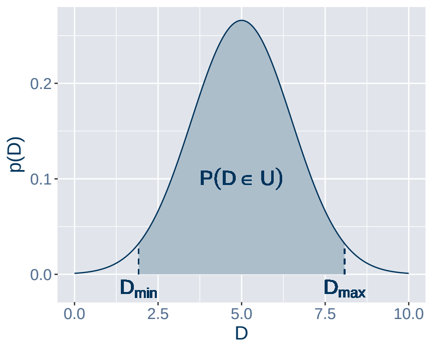
# Size of realistic problem

| | deterministic | robust $D \neq f(t)$ | robust $D = f(t)$ |
|---|---|---|---|
| # vars | 5389 | | 397361 |
| # binaries | 2492 | | 2492 |
| # constraints | 6798 | | 180858 |
| time to solve [s] | 7883 | | 16756 |
| gap [%] | 3.62 | | 31.02 |
| scheduling periods | 56 | 56 | 56 |
| planning periods | 24 | 24 | 24 |
| task-unit-op. mode combinations | 24 | 24 | 24 |

# How do we choose $\mathcal{U}$?

## Choose $\mathcal{U}$ from distribution

- Choose parameter $\alpha$
- Choose $D_{min}$ such that $P(D \leq D_{min}) = \alpha$
- Choose $D_{max}$ such that $P(D \geq D_{max}) = \alpha$
- $\mathcal{U} = \{D | D_{min} \leq D \leq D_{max}\}$



# Deriving a robust counterpart

Replace $D_{j,k}$ by an uncertain parameter $\tilde{d}_{j,k}$ bounded by a set $\mathcal{U}$:

$$s_{j,t} \leq s_j^{max} \qquad \forall t, j \in J$$

$$s_{j,t} = \begin{cases} s_{j,t-1} + \sum_{k \in \mathcal{K}} x_{j,k,t} \cdot \tilde{d}_{j,k}, & \text{if } m_{j,t} = 0 \\ s_j^0, & \text{otherwise} \end{cases} \qquad \forall \tilde{d}_{j,k} \in \mathcal{U}, t, j \in J$$

Reformulate:

$$m_{j,t} s_j^0 \leq s_{j,t} \leq s_j^{max} + m_{j,t} \cdot (s_j^0 - s_{j,max}) \qquad \forall t, j \in J, \tilde{d}_{j,k} \in \mathcal{U}$$

$$s_{j,t} \geq s_{j,t-\Delta t} + \sum_k x_{j,k,t} \tilde{d}_{j,k} + m_{j,t} \cdot (s_j^0 - s_j^{max}) \qquad \forall t, j \in J, \tilde{d}_{j,k} \in \mathcal{U}$$

$$s_{j,t} \leq s_{j,t-\Delta t} + \sum_k x_{j,k,t} \tilde{d}_{j,k} \qquad \forall t, j \in J, \tilde{d}_{j,k} \in \mathcal{U},$$

Replace $s_{j,t}$ by linear decision rule $s_{j,t} = [s_{j,t}]_0 + \sum_k [s_{j,t}]_k \tilde{d}_{j,k}$.

# Results: metrics data-driven approximation

| instance | bound | rms_all | rms_max | p_out |
|---|---|---|---|---|
| toy | freq | 8.00 | 1.53 | 29.40 |
| toy | mc | 10.41 | 3.08 | 21.27 |
| P1 | freq | 12.61 | 3.52 | 17.54 |
| P1 | mc | 17.25 | 4.39 | 9.62 |
| P2 | freq | 7.40 | 2.31 | 18.08 |
| P2 | mc | 13.68 | 4.98 | 10.13 |
| P4 | freq | 9.17 | 3.27 | 47.78 |
| P4 | mc | 11.43 | 2.84 | 32.50 |
| P6 | freq | 18.75 | 8.94 | 12.17 |
| P6 | mc | 20.84 | 10.09 | 10.98 |
| all | freq | 11.19 | 3.91 | 24.99 |
| all | mc | 14.72 | 5.08 | 16.90 |