

ST5209/X Assignment 4

Due 1 Apr, 11.59pm

Set up

1. Make sure you have the following installed on your system: \LaTeX , R4.2.2+, RStudio 2023.12+, and Quarto 1.3.450+.
2. Pull changes from the course [repo](#).
3. Create a separate folder in the root directory of the repo, label it with your name, e.g. `yanshuo-assignments`
4. Copy the `assignment1.qmd` file over to this directory.
5. Modify the duplicated document with your solutions, writing all R code as code chunks.
6. When running code, make sure your working directory is set to be the folder with your assignment `.qmd` file, e.g. `yanshuo-assignments`. This is to ensure that all file paths are valid.¹

Submission

1. Render the document to get a `.pdf` printout.
2. Submit both the `.qmd` and `.pdf` files to Canvas.

1. AR polynomial

Consider the AR(2) model

$$X_t = 4 + 0.5X_{t-1} - 0.25X_{t-2} + W_t. \quad (1)$$

- a. What is the autoregressive polynomial?
- b. What are its roots?
- c. Is this model causal? Why?
- d. What is the period of the sinusoidal portion of its ACF?

¹You may view and set the working directory using `getwd()` and `setwd()`.

- e. Given the representation

$$X_t = \sum_{j=0}^{\infty} \psi_j W_{t-j},$$

solve for $\psi_0, \psi_1, \psi_2, \psi_3$.

2. Likelihood

Consider the AR(1) model

$$X_t = 1 - 0.6X_{t-1} + W_t,$$

where $W_t \sim WN(0, 0.25)$ is Gaussian white noise. We are given observations $x_1 = 0.2, x_2 = -0.3, x_3 = 0.4$.

- What is the mean of X_t ?
- Write the full likelihood of this model.
- Write the conditional likelihood of this model, when conditioning on the value of x_1 .

3. Reversibility

- Create a sample trajectory of length $n = 200$ from the AR(2) model from Problem 1 using `arma.sim()`.
- Reverse the time index of the vector you obtain using `rev()`.
- Fit an AR(2) model to the reversed time series using `fable`. Hint: You may use the code snippet `model(AR(X ~ order(2)))`.
- Inspect the model parameters using `tidy()`. Why are they similar to the those in Equation 1?
- Make a forecast with $h = 10$. What does this correspond to in terms of the original time series?

4. Yule-Walker

- Write the Yule-Walker equations for the AR(2) model from Problem 1.
- Arrange the equations in the following matrix form (i.e. fill in the missing entries):

$$\begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \begin{bmatrix} \gamma(0) \\ \gamma(1) \\ \gamma(2) \end{bmatrix} = \begin{bmatrix} \sigma^2 \\ 0 \\ 0 \end{bmatrix}.$$

- c. Solve the system from part b) for $\gamma(0), \gamma(1), \gamma(2)$ numerically using `solve()`.
- d. Given

$$\Gamma_2 = \begin{bmatrix} \gamma(0) & \gamma(1) \\ \gamma(1) & \gamma(0) \end{bmatrix},$$

what is the top left entry of Γ_2^{-1} in terms of $\gamma(0)$ and $\gamma(1)$?

- e. Write a 95% confidence interval for ϕ_1 using your answers for d) and 3d).

5. Real data analysis

- a. Load the dataset `globtemp.rds` in `_data/cleaned` and convert it into a tsibble.
- b. Filter the time series to include measurements only up to 1960.
- c. Fit an AR model using `model(AR(X))`.
- d. What is order of the model selected? What are the fitted parameters?
- e. Inspect the model residuals using `gg_tsresiduals()`. What can you say about the model fit?