

6520 Project

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```
rm(list=ls())
set.seed(6520)
library(ggplot2)

library(expm)
```

```
## Loading required package: Matrix
```

```
##
```

```
## Attaching package: 'expm'
```

```
## The following object is masked from 'package:Matrix':
```

```
##
```

```
##      expm
```

```
setwd("/Users/jwz34/Documents/Github/6520project/onlinegrad")
devtools::install()
```

```
## -- R CMD build -----
```

```
##   checking for file '/Users/jwz34/Documents/Github/6520project/onlinegrad/DESCRIPTION' ... v  checking
```

```
##   - preparing 'onlinegrad':
```

```
##   checking DESCRIPTION meta-information ... v  checking DESCRIPTION meta-information
```

```
##   - checking for LF line-endings in source and make files and shell scripts
```

```
##   - checking for empty or unneeded directories
```

```
##   - building 'onlinegrad_0.0.0.9000.tar.gz'
```

```
##
```

```
## Running /Library/Frameworks/R.framework/Resources/bin/R CMD INSTALL \
```

```
##   /var/folders/3l/795v58rx6110172qtnz6rwmc0000gp/T/Rtmp6MTKrY/onlinegrad_0.0.0.9000.tar.gz \
```

```
##   --install-tests
```

```
## * installing to library '/Users/jwz34/Library/R/arm64/4.3/library'
```

```
## * installing *source* package 'onlinegrad' ...
```

```
## ** using staged installation
```

```
## ** R
```

```
## ** byte-compile and prepare package for lazy loading
```

```
## ** help
```

```
## *** installing help indices
```

```
## ** building package indices
```

```
## ** testing if installed package can be loaded from temporary location
```

```
## ** testing if installed package can be loaded from final location
```

```
## ** testing if installed package keeps a record of temporary installation path
```

```
## * DONE (onlinegrad)
```

```
library(onlinegrad)
```

Simulate data for regression and classification

```
# simulate data: regression
n = 100 # sample size
p = 200 # number of predictors

# beta
k = round(0.05*p, 0) # number of nonzero coefficients
sd_beta = 0.01
nonzero_indexes = sample.int(n=p, size=k)
beta = rep(0, p)
beta[nonzero_indexes] = rnorm(n=k, mean=100, sd=sd_beta)
sum(which(beta !=0) != sort(nonzero_indexes)) # test that we made the right indexes nonzero
```

```
## [1] 0
```

```
beta = as.matrix(beta)

# x
X = matrix(rnorm(n=n*p, mean=0, sd=5), nrow=n)

# epsilon
E = matrix(rnorm(n=n, mean=0, sd=1), nrow=n)

# y
Y = X%%beta + E

# note that in the online setting, each t-th row of X and Y is for time t

# simulate data: classification
# X, beta same as above
probs = 1/(1+exp(-X%%beta))
Y = rbinom(n=n, size=1, prob = probs) # Bernoulli
```

Analysis of $\hat{\beta}$'s

Plots + Prediction error vs iterations + Estimation error vs iterations + Betahats for each dimension, nonzero vs zero indexes - Comparison of different learning rates - Run time of full vs diagonal Adagrad + Run time of OGD, Adagrad, adam - Variance of betahats across iterations?

```
# plot prediction or estimation error
# X: rows are observations, columns are predictors
# Y: response variable
# betahats: n x p matrix where each ith row is the coefficients for the ith iteration and the columns a
# beta: true beta coefficient px1 vector
```

```

# title: string for the title of the plot
# type: "prediction" or "estimation" for prediction error or estimation error
plot_prediction_error = function(betahats, beta, X, Y, title, type) {
  n = nrow(X)
  p = ncol(X)
  if ((type!="prediction") && (type!="estimation")) {
    stop("type parameter must be 'prediction' or 'estimation'")
  }
  if (type=="prediction") {
    # prediction error
    err = colSums((X%*%t(betahats) - matrix(rep(Y, n), nrow=n, ncol=n, byrow=F))^2) # row of the inside
    ylab = "Prediction error"
  } else {
    # estimation error
    beta = t(beta)
    beta = matrix(rep(beta, n), nrow=n, byrow=T) # row combine n number of t(beta)'s
    err = sqrt(rowSums((betahats - beta)^2))
    ylab = "Estimation error"
  }
  err = as.matrix(err)
  plot(err, xlab="Iteration", ylab=ylab, main=title)
  return(err)
}

```

```

# plot last iteration of betahat for nonzero vs zero indexes, true beta overlaid
# nonzero_indexes: kx1 vector of indexes where the k-sparse vector true beta has nonzero values
# nonzero: boolean, if true plot only nonzero indexes (k indexes of the true k-sparse beta), otherwise
plot_betas = function(betahats, beta, nonzero_indexes, nonzero=T) {
  n = nrow(betahats)
  p = ncol(betahats)
  dat = data.frame("p"=1:p, "bethat_n"=betahats[n, ], "beta"=beta)
  if (nonzero) {
    dat = dat[nonzero_indexes, ]
    title = "Last iteration of bethat (orange) and true beta (black) at nonzero indexes"
  } else {
    dat = dat[-nonzero_indexes, ]
    title = "Last iteration of bethat (orange) and true beta (black) at zero indexes"
  }
  ggplot(dat) + geom_point(aes(x=p, y=bethat_n), color="orange") +
    geom_point(aes(x=p, y=beta), color="black", shape=4) +
    xlab("p") +
    ylab("") +
    ggtitle(title)
}

```

```

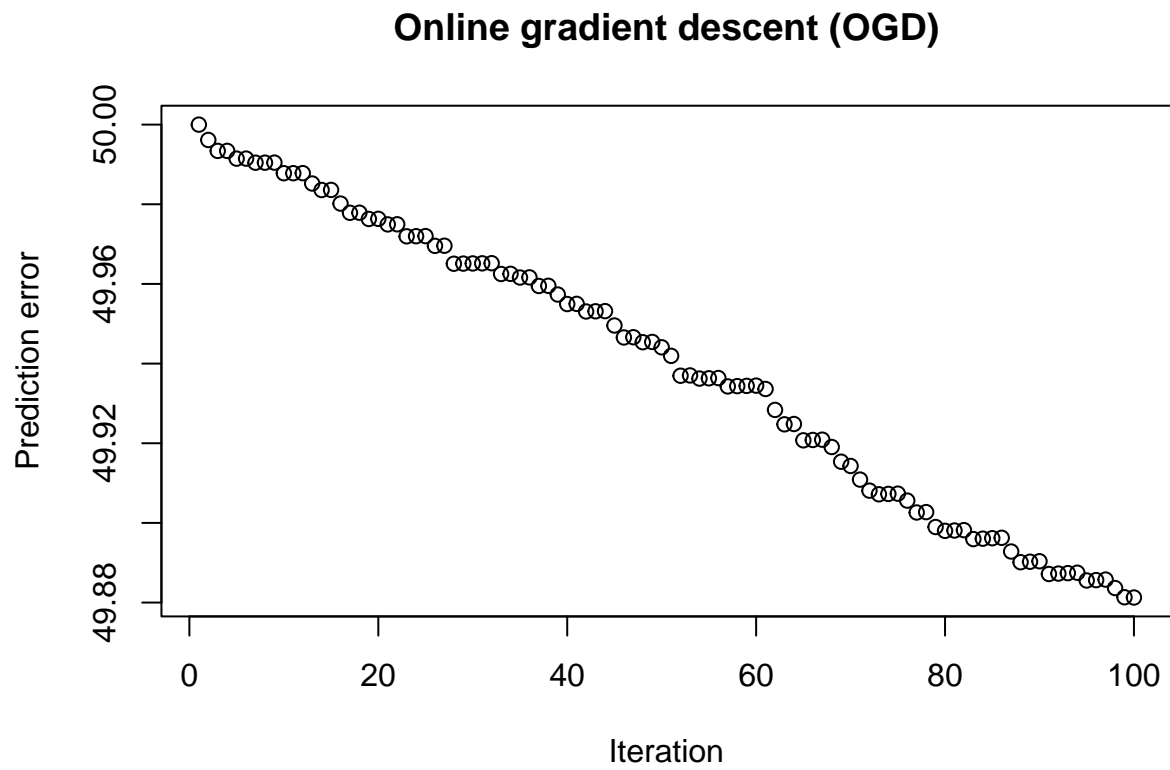
runtime_df = data.frame("n"=1:n)
pred_df = data.frame("n"=1:n) # prediction error
est_df = data.frame("n"=1:n) # estimation error

# OGD
temp = my_OGD(X=X, Y=Y, lr=0.0000001, beta_0=rep(0, p))
betahats = temp[[1]]
runtimes = temp[[2]]

```

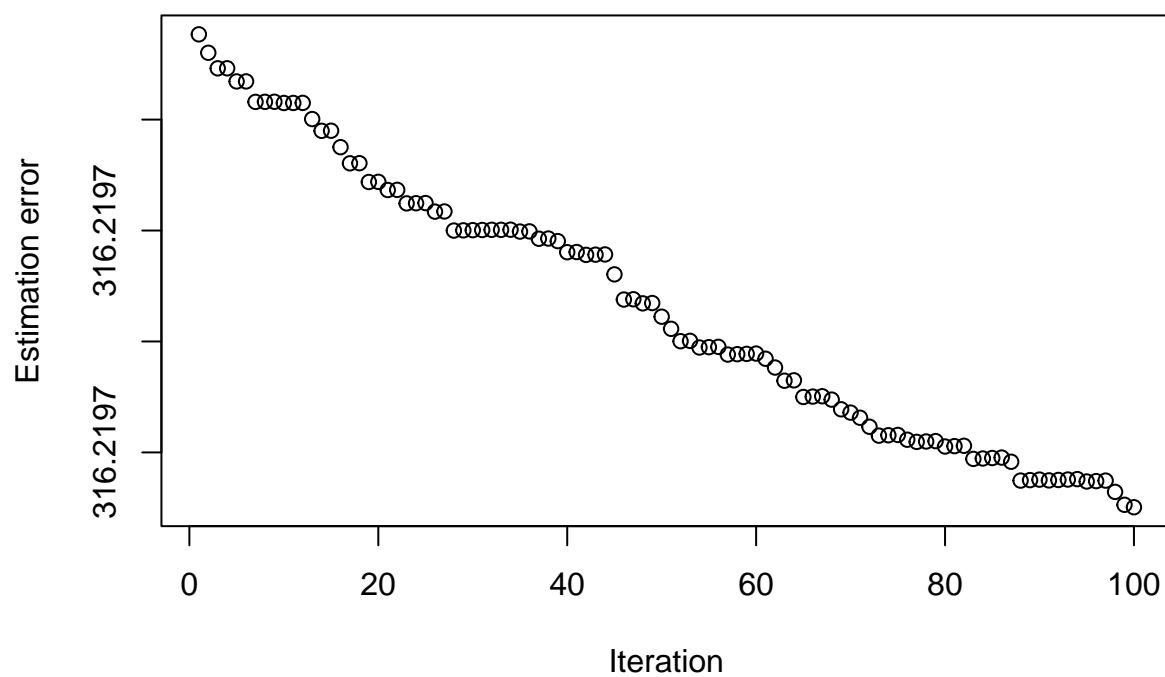
```
runtime_df$ogd = runtimes
```

```
pred_df$ogd = plot_prediction_error(betahats, beta, X, Y, title="Online gradient descent (OGD)", type="e")
```



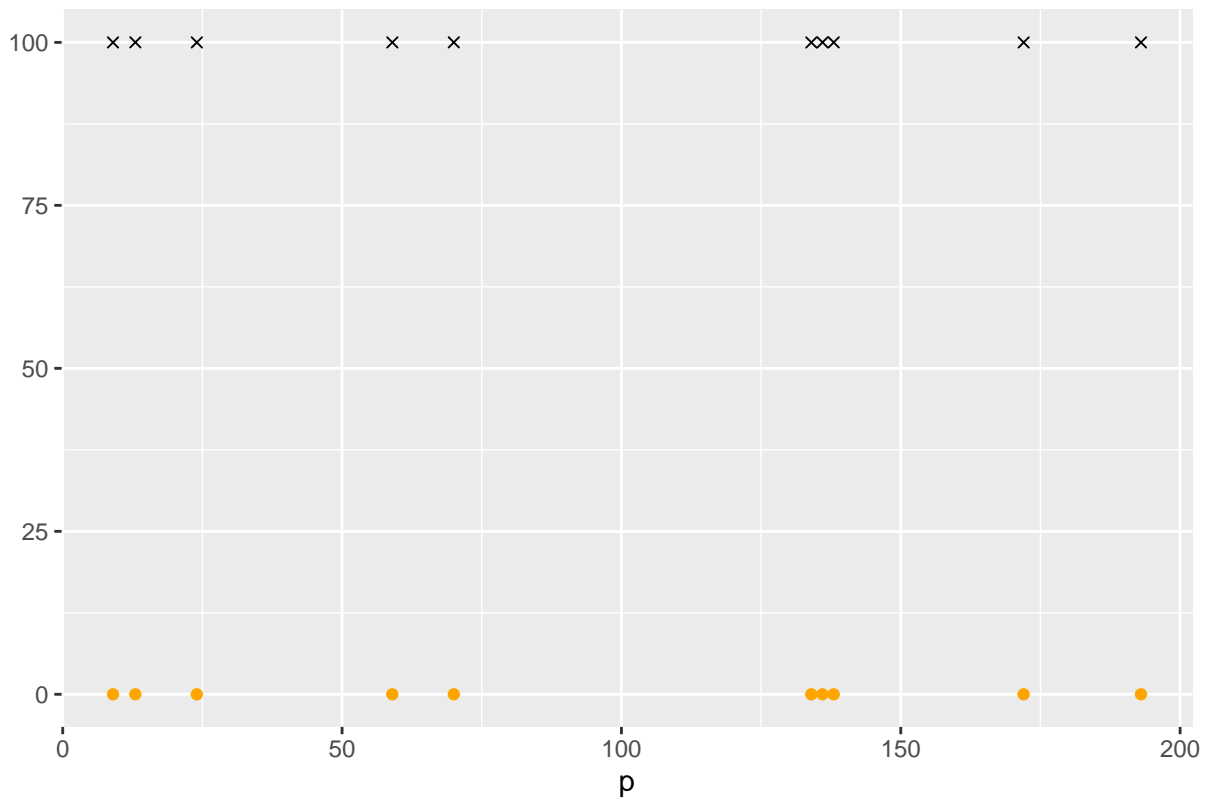
```
est_df$ogd = plot_prediction_error(betahats, beta, X, Y, title="Online gradient descent (OGD)", type="e")
```

Online gradient descent (OGD)



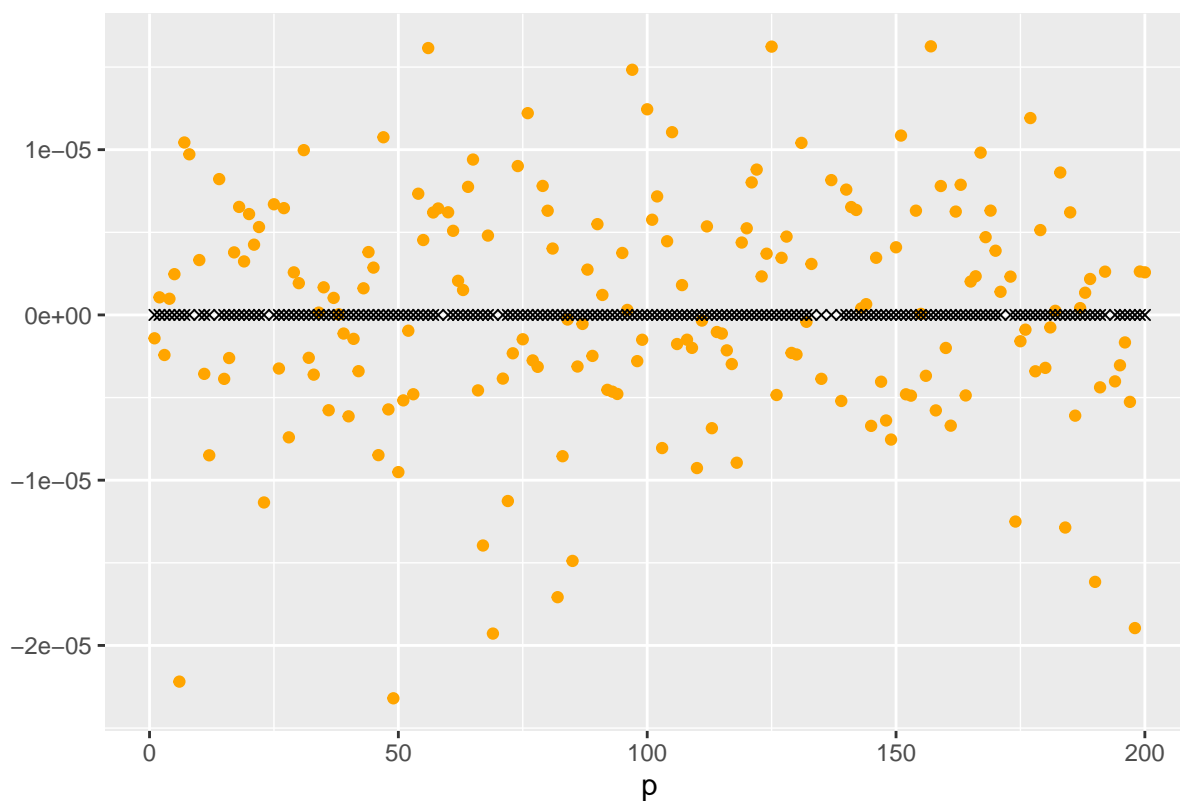
```
plot_betas(betahats, beta, nonzero_indexes, nonzero=T) # nonzero indexes
```

Last iteration of bethat (orange) and true beta (black) at nonzero indexes



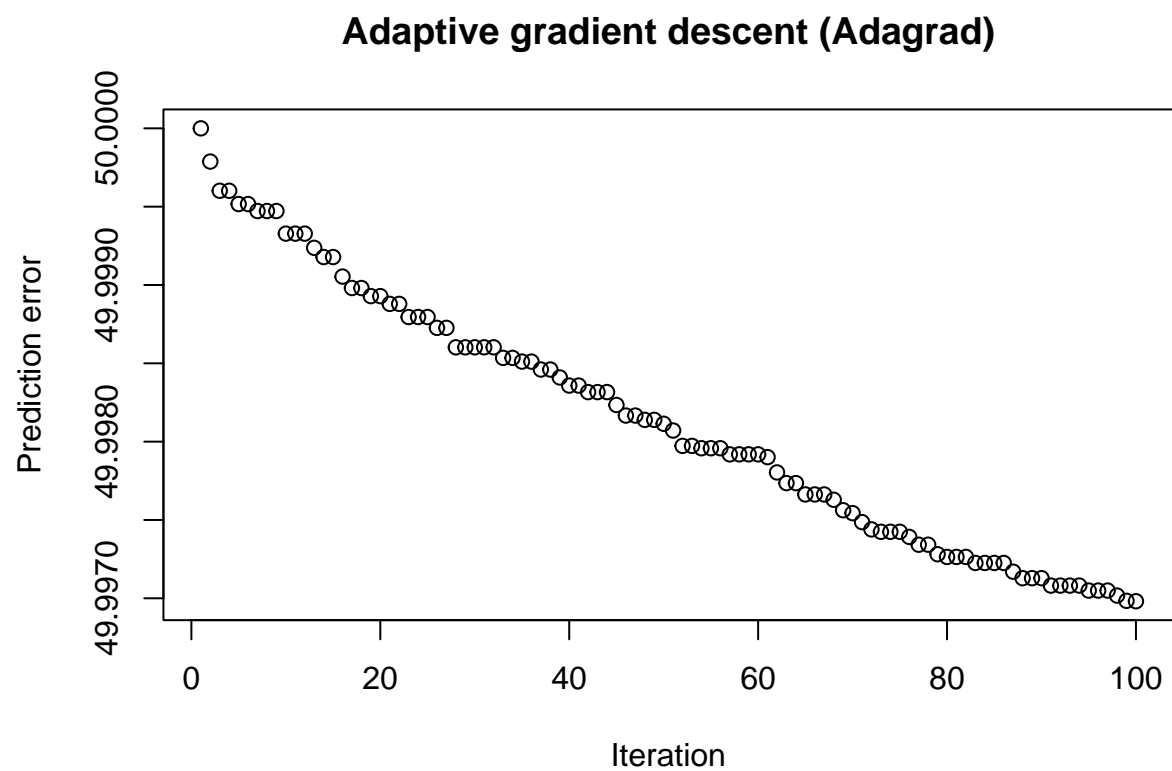
```
plot_betas(betahats, beta, nonzero_indexes, nonzero=F) # zero indexes
```

Last iteration of bethat (orange) and true beta (black) at zero indexes



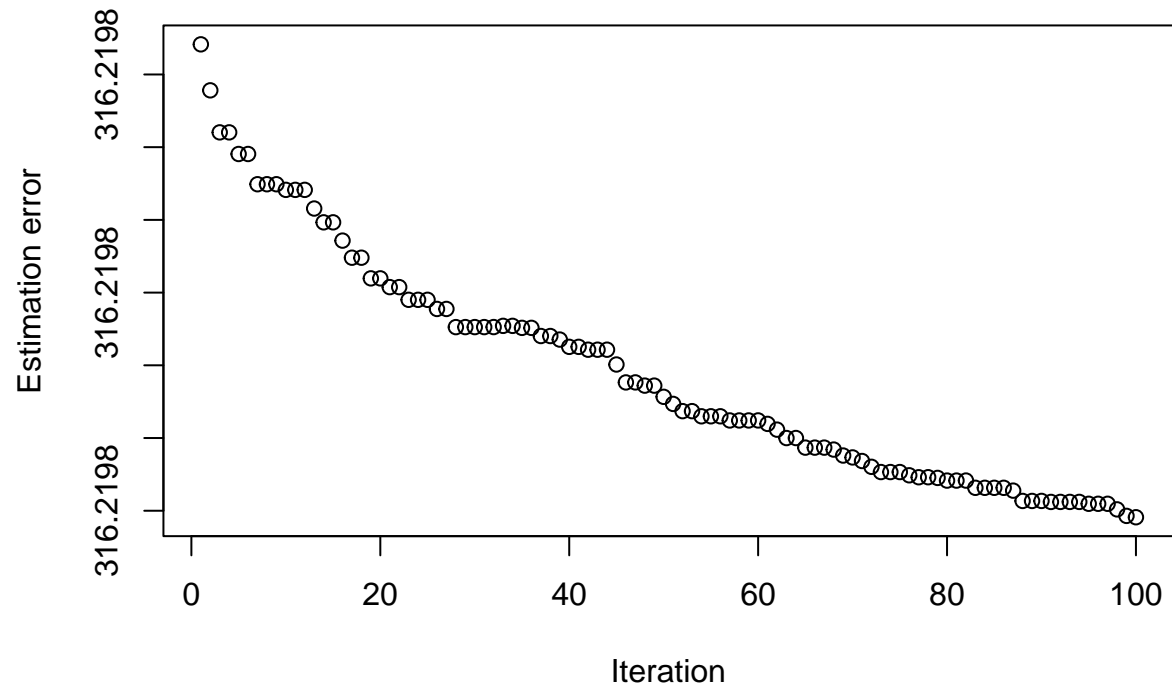
```
# Adagrad
temp = my_adagrad(X=X, Y=Y, lr=0.0000001, beta_0=rep(0, p), full=F)
betahats = temp[[1]]
runtimes = temp[[2]]
runtime_df$adagrad = runtimes

pred_df$adagrad = plot_prediction_error(betahats, beta, X, Y, title="Adaptive gradient descent (Adagrad)")
```



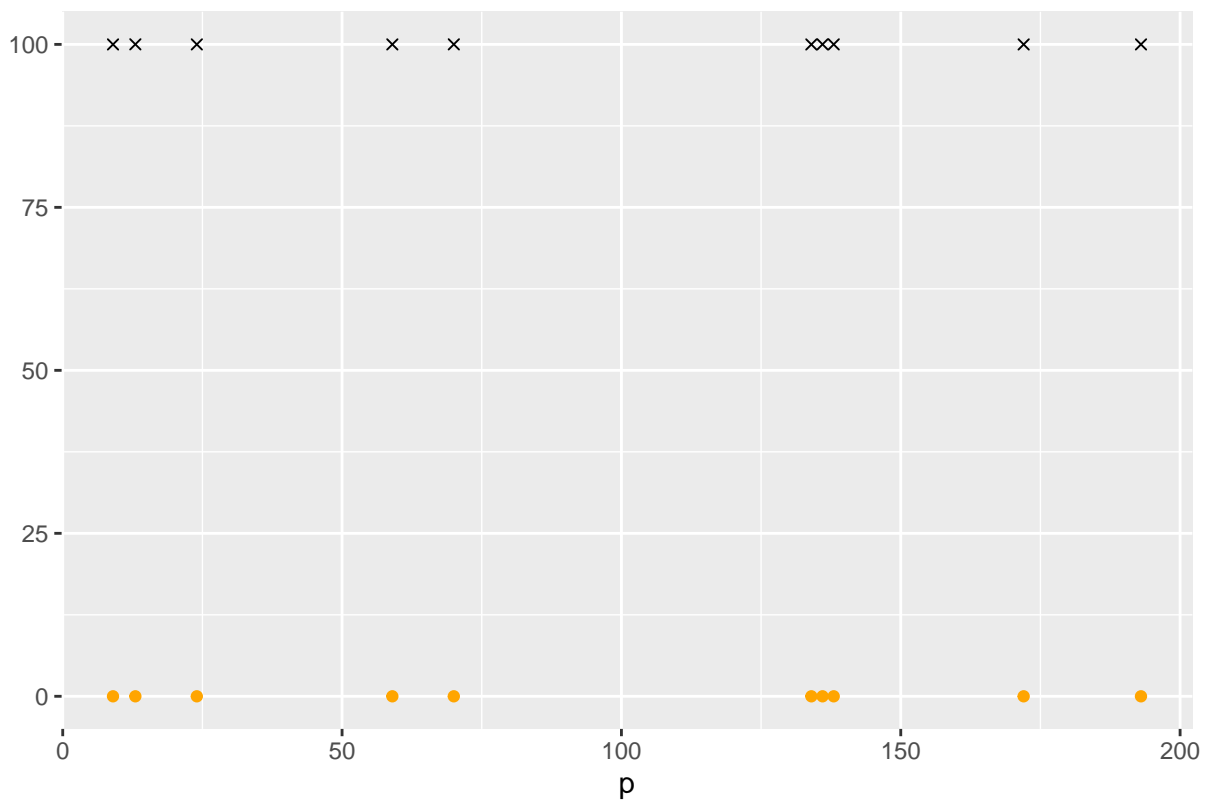
```
est_df$adagrad = plot_prediction_error(betahats, beta, X, Y, title="Adaptive gradient descent (Adagrad)
```


Adaptive gradient descent (Adagrad)



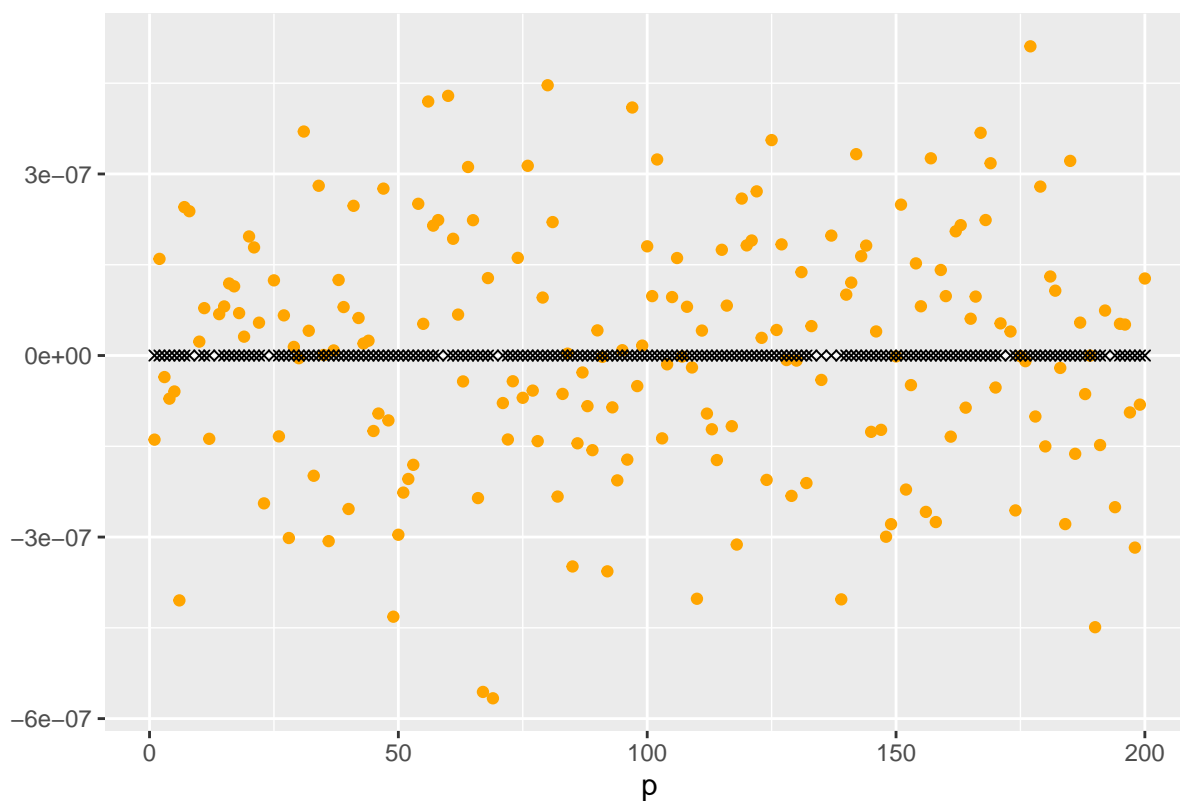
```
plot_betas(betahats, beta, nonzero_indexes, nonzero=T)
```

Last iteration of bethat (orange) and true beta (black) at nonzero indexes



```
plot_betas(betahats, beta, nonzero_indexes, nonzero=F)
```

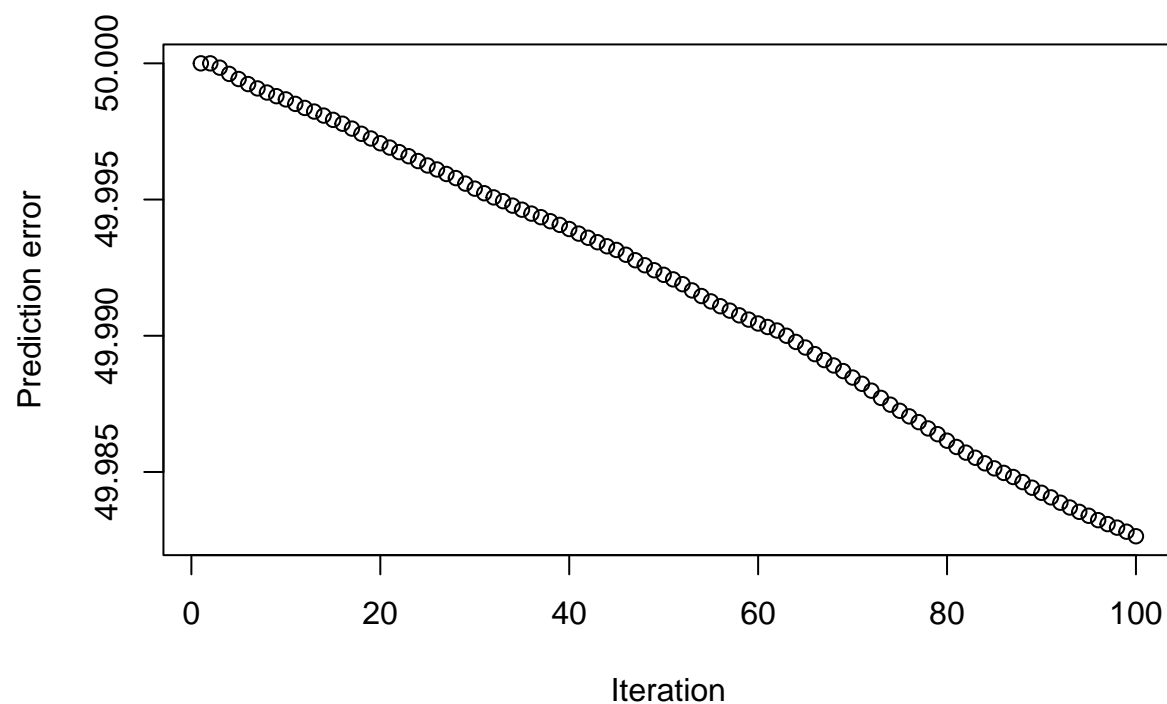
Last iteration of bethat (orange) and true beta (black) at zero indexes



```
# Adam
temp = my_adam(X=X, Y=Y, lr=0.0000001, beta_0=rep(0, p), rho_1=0.9, rho_2=0.999, epsilon=1e-8)
betahats = temp[[1]]
runtimes = temp[[2]]
runtime_df$adam = runtimes

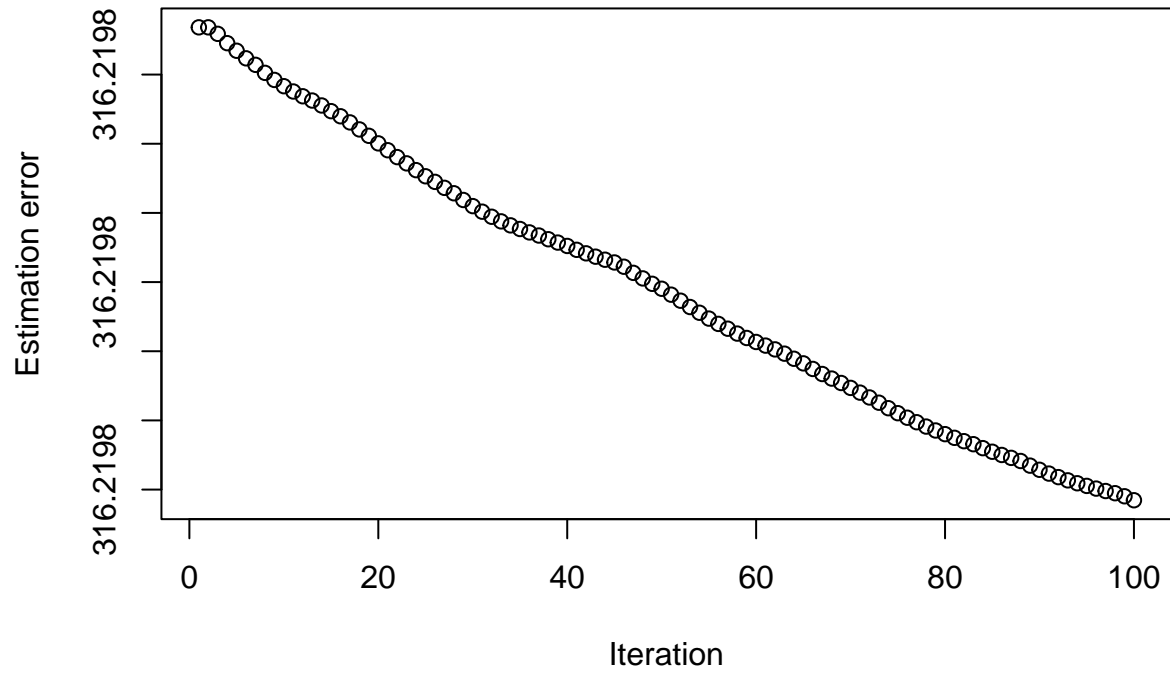
pred_df$adam = plot_prediction_error(betahats, beta, X, Y, title="Adaptive moment estimation (Adam)", t,
```

Adaptive moment estimation (Adam)



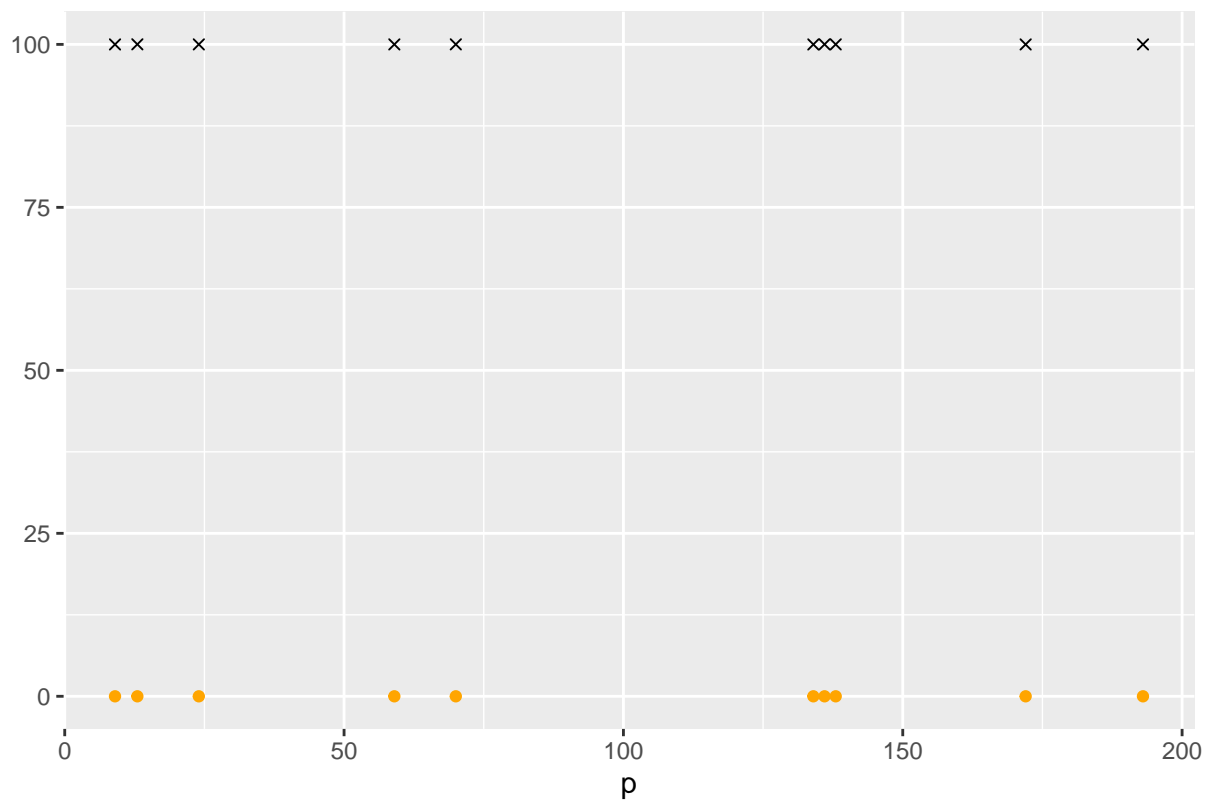
```
est_df$adam = plot_prediction_error(betahats, beta, X, Y, title="Adaptive moment estimation (Adam)", ty
```

Adaptive moment estimation (Adam)



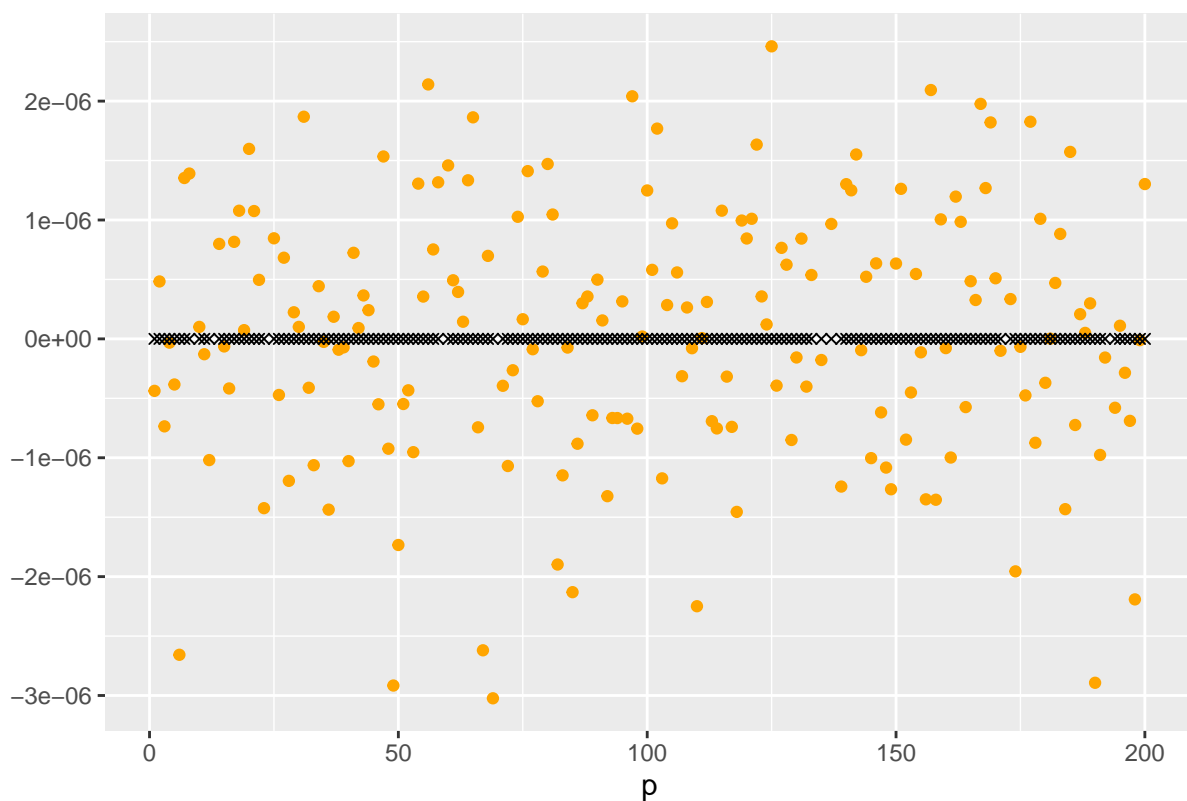
```
plot_betas(betahats, beta, nonzero_indexes, nonzero=T)
```

Last iteration of bethat (orange) and true beta (black) at nonzero indexes



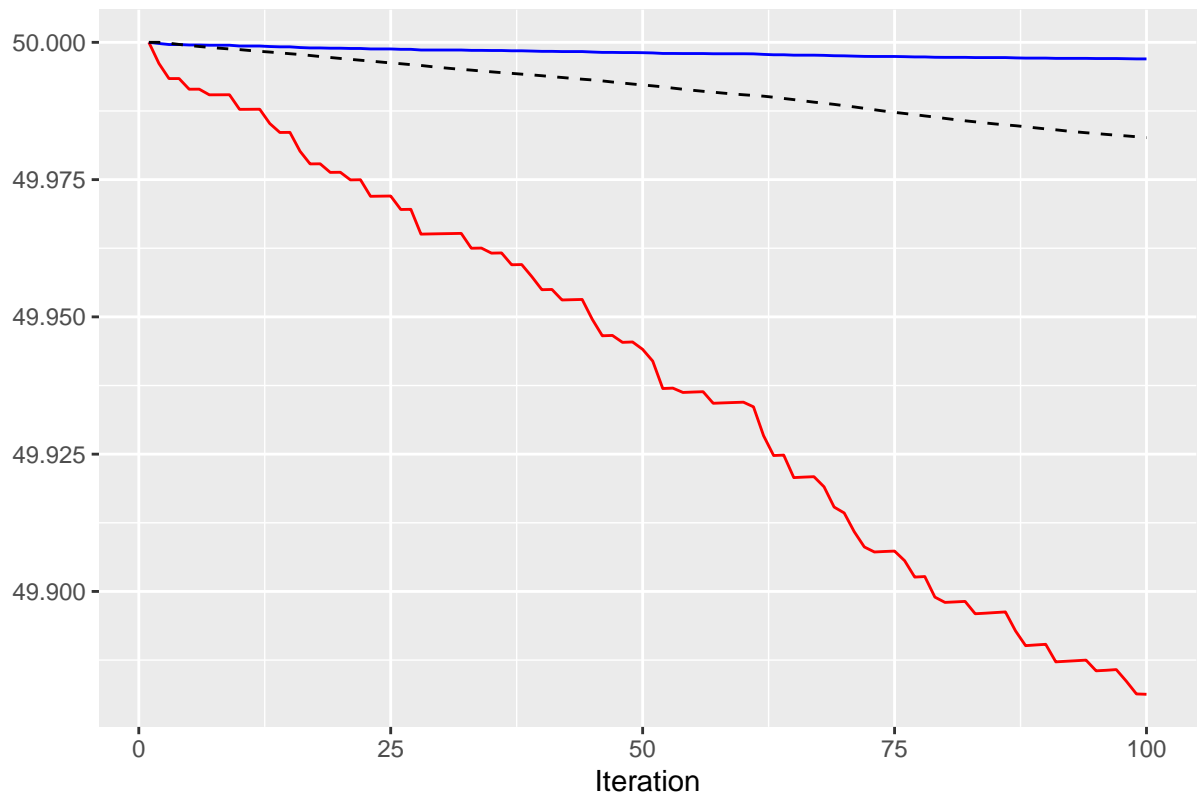
```
plot_betas(betahats, beta, nonzero_indexes, nonzero=F)
```

Last iteration of bethat (orange) and true beta (black) at zero indexes



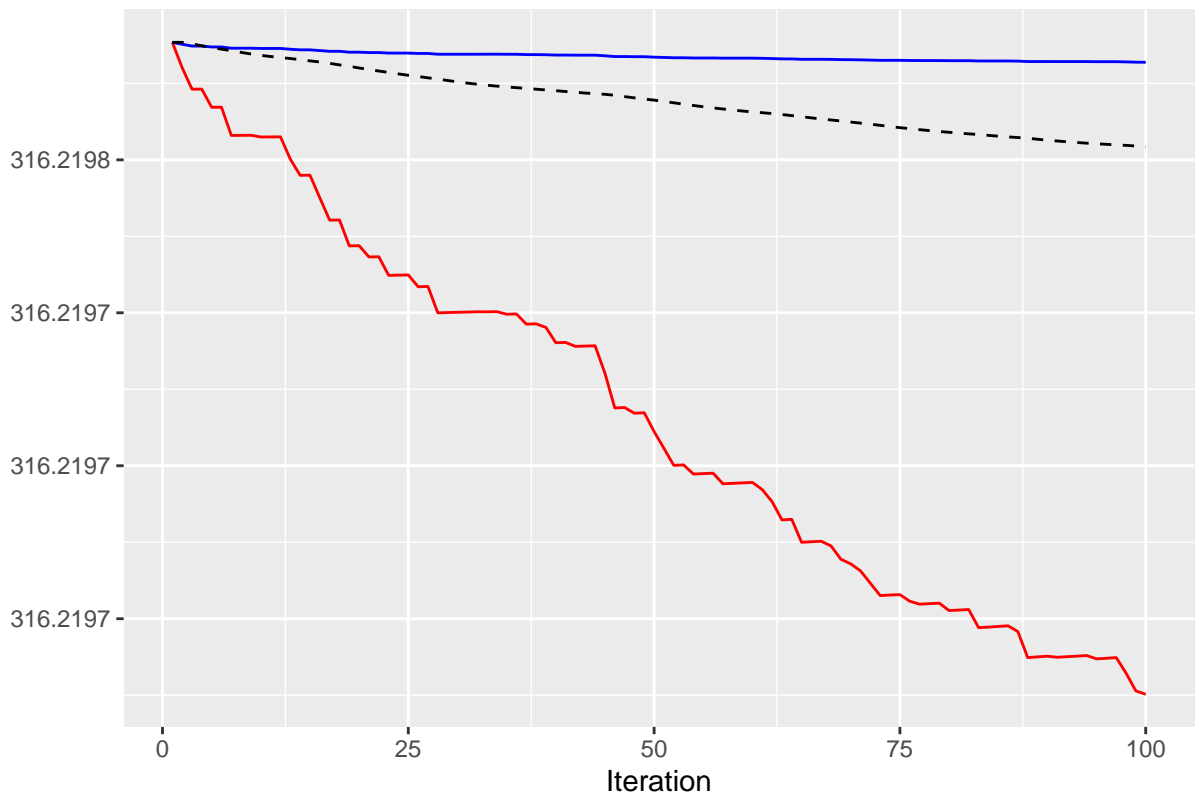
```
# plot prediction error for ogd, adagrad, adam
ggplot(pred_df, aes(x=n)) +
  geom_line(aes(y=ogd), color="red") +
  geom_line(aes(y=adagrad), color="blue") +
  geom_line(aes(y=adam), linetype="dashed") +
  xlab("Iteration") +
  ylab("") +
  ggtitle("Prediction error for OGD (red), Adagrad (blue), Adam (black)")
```

Prediction error for OGD (red), Adagrad (blue), Adam (black)



```
# plot estimation error for ogd, adagrad, adam
ggplot(est_df, aes(x=n)) +
  geom_line(aes(y=ogd), color="red") +
  geom_line(aes(y=adagrad), color="blue") +
  geom_line(aes(y=adam), linetype="dashed") +
  xlab("Iteration") +
  ylab("") +
  ggtitle("Estimation error for OGD (red), Adagrad (blue), Adam (black)")
```


Estimation error for OGD (red), Adagrad (blue), Adam (black)



```
# plot runtime for ogd, adagrad, adam
ggplot(runtime_df, aes(x=n)) +
  geom_line(aes(y=ogd), color="red") +
  geom_line(aes(y=adagrad), color="blue") +
  geom_line(aes(y=adam), linetype="dashed") +
  xlab("Iteration") +
  ylab("") +
  ggtitle("Runtime for OGD (red), Adagrad (blue), Adam (black)")
```

Runtime for OGD (red), Adagrad (blue), Adam (black)

