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## ABSTRACT

This paper considers VAR models incorporating many time series that interact through a few dynamic factors. Several econometric issues are addressed including estimation of the number of dynamic factors and tests for the factor restrictions imposed on the VAR. Structural VAR identification based on timing restrictions, long run restrictions, and restrictions on factor loadings are discussed and practical computational methods suggested. Empirical analysis using U.S. data suggest several (7) dynamic factors, rejection of the exact dynamic factor model but support for an approximate factor model, and sensible results for a SVAR that identifies money policy shocks using timing restrictions.

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# 1. Introduction

A fundamental contribution of Sims (1980) was his constructive argument that many of the "incredible" identifying restrictions underlying the structural macroeconometric models of the 1960s and 1970s are unnecessary either for forecasting or for certain types of policy analysis. Instead of imposing large numbers of identifying restrictions that permitted system estimation by two- or three-stage least squares, Sims proposed that the system dynamics be left completely free. His key insight was that the effect of policy interventions – an autonomous increase in the money supply or an autonomous decrease in government spending – could be analyzed by examining the moving average representation relating macroeconomic reality (outcome variables of interest) directly to the structural economic shocks. To identify these policy effects, one only needed to identify the structural economic shocks; then the dynamic policy effects could be computed as the impulse response function obtained by inverting the vector autoregressive (VAR) representation of the data, linearly transformed to yield the moving average representation with respect to the structural shock. Restrictions on the dynamic structure were neither required nor desired – all that was needed was some scheme to sort through the VAR forecast errors, or innovations, in just the right way so that one can deduce the structural economic shock or shocks desired for undertaking the policy analysis.

This final requirement – moving from the VAR innovations to the structural shocks – is the hardest part of so-called structural VAR (SVAR) analysis, for it requires first that the structural shocks can in theory be obtained from the innovations, and second that there be some economic rationale justifying how, precisely, to distill the structural shocks from the innovations. The first of these requirements can be thought of as requiring that there is no omitted variable bias: if a variable is known to individuals, firms, and policy-makers and that variable contains information about a structural economic shocks distinct from what is already included in the VAR, then omitting that variable means that the VAR innovations will not in general span the space of the structural shocks, so the structural shocks cannot in general be deduced from the VAR innovations. This difficulty has long been recognized and indeed has been pointed to as the source of both practical problems in early VARs, including the "price puzzle" of Sims (1992) (see Christiano, Eichenbaum, and Evans (1999) for a discussion), and theoretical problems, such as the specter of noninvertibility (e.g. Lippi and Reichlin (1994)). The key to addressing these problems is to increase the amount of information in the VAR so that the innovations span the space of structural disturbances. For example, as recounted by Sims (1993), disappointing forecasts of inflation from the earliest real-time VAR forecasting exercises at the Federal Reserve Bank of Minnesota led Robert Litterman to add the trade-weighted exchange rate, the S&P 500, and a commodity price index to the original six-variable Minnesota VAR. This line of reasoning has led Sims and coauthors to consider yet larger VARs, such as the 13- and 18-variable VAR in Leeper, Sims, and Zha (1996). But increasing the number of variables in a VAR creates technical and conceptual complications, for the number of unrestricted VAR coefficients increases as the square of the number of variables in the system.

One approach to handling the resulting proliferation of parameters, spearheaded by Sims and his students, is to impose Bayesian restrictions and to estimate or calibrate the hyperparameters, so that the VAR is estimated by (possibly informal) empirical Bayes methods (see Doan, Litterman, and Sims (1984), Litterman (1986), Sims (1993), Leeper, Sims, and Zha (1996)). This is not a line of work for the computationally challenged. More importantly, because of the quadratic increase in complexity it is unclear that it can be pushed much beyond systems with a score or two of variables without, in effect, imposing the incredible (now statistical) identifying restrictions that SVAR analysis was designed to eschew. What if 18 variables are not enough to span the space of structural shocks? After all, in reality Fed economists track hundreds if not thousands of variables as they prepare for upcoming meetings of the Open Market Committee. Unless the staff economists are wasting their time, one must assume that these hundreds of variables help them isolate the structural shocks currently impacting the economy.

In this paper, we examine VAR methods that can be used to identify the space of structural shocks when there are hundreds of economic time series variables that potentially contain information about these underlying shocks. This alternative approach is based on dynamic factor analysis, introduced by John Geweke in his Ph.D. thesis (published as Geweke (1977)) under the supervision of Sims. The premise of the dynamic factor model (DFM) is that there are a small number of unobserved common dynamic factors that produce the observed comovements of economic time series. These common dynamic factors are driven by the common structural economic shocks, which are the relevant shocks that one must identify for the purposes of conducting policy analysis. Even if the number of common shocks is small, because the dynamic factors are unobserved this model implies that the innovations from conventional VAR analysis with a small or moderate number of variables will fail to span the space of the structural shocks to the dynamic factors. Instead, these shocks are only revealed when one looks at a very large number of variables and distills from them the small number of common sources of comovement.

There is a body of empirical evidence that the dynamic factor model, with a small number of factors, captures the main comovements of postwar U.S. macroeconomic time series data. Sims and Sargent (1977) examine a small system and conclude that two dynamic factors can explain 80% or more of the variance of major economic variables, including the unemployment rate, industrial production growth, employment growth, and wholesale price inflation; moreover, one of these dynamic factors is primarily associated with the real variables, while the other is primarily associated with prices. Empirical work using methods developed for many-variable systems has supported the view that only a few – perhaps two – dynamic factors explain much of the predictable variation in major macroeconomic aggregates (e.g. Stock and Watson (1999, 2002a), Giannone, Reichlin, and Sala (2004)). These new methods for estimating and analyzing dynamic factor models, combined with the empirical evidence that perhaps only a few dynamic factors are needed to explain the comovement of macroeconomic variables, has motivated recent research on how best to integrate factor methods into VAR and SVAR analysis (Bernanke and Boivin (2003), Bernanke, Boivin, and Eliasz (2005; BBE hereafter), Favero and Marcellino (2001), Favero, Marcellino, and Neglia (2004), Giannone, Reichlin, and Sala (2002, 2004), and Forni, Giannone, Lippi, and Reichlin (2004)); we return to this recent literature in Sections 2 and 5.

This paper has three objectives. The first is to provide a unifying framework that explicates the implications of DFMs for VAR analysis, both reduced-form (including forecasting applications) and structural. In particular we list a number of testable overidentifying restrictions that are central to the simplifications provided by introducing factors into VARs.

Our second objective is to examine empirically these implications of the DFM for VAR analysis. Is there support for the exact factor model restrictions or, if not, for an approximate factor model such as that of Chamberlain and Rothschild (1983)? If so, how many factors are needed: two, as suggested by Sargent and Sims (1977) and more recent literature, or more? Another implication of the DFM is that, once factors are included in the VAR, impulse responses with respect to structural shocks should not change upon the inclusion of additional observable variables; but is this borne out empirically?

Our third objective is to provide a unified framework and some new econometric methods for structural VAR analysis using dynamic factors. These methods build on the important initial work by Giannoni, Reichlin, and Sala (2002) and BBE (2005) on the formulation and estimation of structural VARs using factors obtained from large data sets, and we adopt BBE's term and refer to these system as FAVARs (Factor-Augmented VARs). We consider a variety of identifying schemes, including schemes based on the timing of shocks (as considered by BBE), on long run restrictions (as considered by Giannoni, Reichlin, and Sala (2002)), and on restrictions on the factor loading matrices (as considered by Kose, Otrok and Whiteman (2003), among others). We present feasible estimation strategies for imposing the potentially numerous overidentifying restrictions.

We have three main empirical findings, which are based on an updated version of the Stock-Watson (2002a) data set (the version used here has 132 monthly U.S. variables, 1959 – 2003). First, it appears that the number of dynamic factors present in our data set exceeds two; we estimate the number to be seven. This estimate is robust to details of the model specification and estimation method, and it substantially exceeds the estimates appearing in the earlier literature; we suggest that this estimate is not spurious but rather reflects the narrow scope of the data sets, combined with methodological limitations, in the early studies that suggested only one or two factors.

Second, we find that many of the implications of the DFM for the full 132-variable VAR are rejected, however these rejections are almost entirely associated with coefficients that are statistically significantly different from zero but are very small in an economic or practical sense.

Third, we illustrate the structural FAVAR methods by an empirical reexamination of the BBE identification scheme, using different estimation procedures. We find generally similar results to BBE, which in many cases accord with standard macroeconomic theory; but we also find many rejections of the overidentifying restrictions. These rejections suggest specific ways in which the BBE identifying assumptions fail, something not possible in exactly identified SVAR analysis.

The remainder of the paper is organized as follows. Section 2 lays out the DFM and its implications for reduced-form VAR analysis. Section 3 provides a treatment of identification and estimation in structural factor VARs. Sections 4 and 5 examine these implications empirically using the 132-variable data set, and Section 6 illustrates the structural FAVAR methods using the BBE identification scheme. Section 7 concludes.

## 2. The Dynamic Factor Model in VAR Form

This section summarizes the restrictions imposed by the dynamic factor model on the VAR representation of the variables. We do this by first summarizing the so-called static representation of the DFM, a representation of interest in its own right because it leads to estimation of the space spanned by the dynamic factors using principal components when $n$ is large. The static representation of the DFM is then used to derive two VAR forms of the DFM, expressed in terms of the (readily estimated) static factors.

### 2.1 The DFM and Reduced-Form VARs

Let $X_t$ be a $n\times 1$ vector of stationary time series variables observed for $t = 1,\ldots,T$.

***The exact dynamic factor model***. The exact DFM expresses $X_t$ as a distributed lag of a small number of unobserved common factors, plus an idiosyncratic disturbance that itself might be serially correlated:

$$X_{it} = \tilde{\lambda}_i(\mathrm{L})f_t + u_{it},\ i = 1,\ldots,n, \tag{1}$$

$$u_{it} = \delta_i(\mathrm{L})u_{it-1} + \nu_{it}, \tag{2}$$

where $f_t$ is the $q\times 1$ vector of unobserved dynamic factors, $\tilde{\lambda}_i(\mathrm{L})$ is a $1 \times q$ vector lag polynomial, called the "dynamic factor loadings," and $u_{it}$ is the idiosyncratic disturbance which we model as following an autoregression. The factors and idiosyncratic disturbances are assumed to be uncorrelated at all leads and lags, that is, $E(f_t u_{is}) = 0$ for all $i$, $t$, $s$. In addition, the idiosyncratic terms are taken to be mutually uncorrelated at all leads and lags, that is,

$$E(u_{it}u_{js}) = 0 \text{ for all } i, j, t, s, i \neq j \tag{3}$$

We briefly digress for a word on terminology. Chamberlain and Rothschild (1983) introduced a useful distinction between exact and approximate DFMs. The exact DFM – the version originally developed by Geweke (1977) and Sargent and Sims (1978) – adopts the strong uncorrelatedness assumption (3). In contrast, the approximate DFM relaxes this assumption to allow for a limited amount of correlation across the idiosyncratic terms for different $i$ and $j$ (see the survey by Stock and Watson (2004) for technical conditions). The focus of this paper is on the implications of the Geweke-Sargent-Sims exact DFM, and when we refer simply to "the DFM" this should be understood to mean the exact DFM; when we discuss instead the approximate DFM, we will make this explicit.

For our purposes it is convenient to work with a DFM in which the idiosyncratic errors are serially uncorrelated. This is achieved by multiplying both sides of (1) by $1 - \delta_i(\mathrm{L})\mathrm{L}$, which yields

$$X_{it} = \lambda_i(\mathrm{L})f_t + \delta_i(\mathrm{L})X_{it-1} + \nu_{it}, \tag{4}$$

where $\lambda_i(\mathrm{L}) = (1-\delta_i(\mathrm{L})\mathrm{L})\tilde{\lambda}_i(\mathrm{L})$.

The dynamic factor model consists of equation (4) and an equation describing the evolution of the factors, which we model as following a VAR. Accordingly, the DFM is, *The dynamic factor model*:

$$X_t = \lambda(\mathrm{L})f_t + D(\mathrm{L})X_{t-1} + v_t \tag{5}$$

$$f_t = \Gamma(\mathrm{L})f_{t-1} + \eta_t, \tag{6}$$

where,

$$\lambda(\mathrm{L}) = \begin{bmatrix} \lambda_1(\mathrm{L}) \\ \vdots \\ \lambda_n(\mathrm{L}) \end{bmatrix},\ D(\mathrm{L}) = \begin{bmatrix} \delta_1(\mathrm{L}) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \delta_n(\mathrm{L}) \end{bmatrix},\ v_t = \begin{bmatrix} v_{1t} \\ \vdots \\ v_{nt} \end{bmatrix}, \tag{7}$$

$\Gamma(\mathrm{L})$ is a matrix lag polynomial, and $\eta_t$ is a $q\times1$ disturbance vector, where $\mathrm{E}\eta_t v_{is} = 0$ for all $i, t, s$.

The DFM assumptions imply that the spectral density of $X$ has a factor structure:

$$S_X(\omega) = \tilde{\lambda}(e^{i\omega})S_f(\omega)\tilde{\lambda}(e^{-i\omega}) + S_u(\omega), \tag{8}$$

where $S_X(\omega)$, $S_f(\omega)$, and $S_u(\omega)$ are the spectral density matrices of $X$, $f$, and $u$ at frequency $\omega$, $S_u$ is diagonal, and $\tilde{\lambda}(z) = [\tilde{\lambda}_1(z) \ldots \tilde{\lambda}_n(z)]'$.

As written in (5) and (6), $\lambda(\mathrm{L})$ and $f_t$ are not separately identified; an observationally equivalent model is obtained by inserting a nonsingular $q\times q$ matrix and its inverse $H$ so that $\lambda(\mathrm{L})$ is replaced by $\lambda(\mathrm{L})H^{-1}$ and $f_t$ is replaced by $Hf_t$. In the treatment in this section, this ambiguity is handled by adopting an arbitrary statistical normalization which (implicitly) imposes an arbitrary $H$. In Section 3, we turn to structural economic DFMs, in which economic logic is used to identify $H$, so that $H$ can be thought of as embodying an economic model.

The unknown coefficients of the DFM (5) and (6) (with additional lag length and normalization restrictions) can be estimated by Gaussian maximum likelihood using the Kalman Filter (Engle and Watson (1981), Stock and Watson (1989, 1991), Sargent (1989), and Quah and Sargent (1993)). When $n$ is very large, however, this method is computationally burdensome. For this reason, alternative methods for estimation of the factors and DFM coefficients have been developed for large $n$. One approach is to use Brillinger's (1964, 1981) dynamic principal components; the theory of applying this method when $n$ is large is developed by Forni, Hallin, Lippi, and Reichlin (2000). However dynamic principal components analysis produces two-sided estimates of the factors and thus these estimates are not suitable for forecasting or for structural VAR analysis in which information set timing assumptions are used to identify shocks. This problem of two-sided estimates of the dynamic factors can be avoided by recasting the DFM in so-called static form.

***The DFM in static form***. In the static form of the DFM (Stock and Watson (2002)), there are $r$ static factors, $F_t$, that consist of current and (possibly) lagged values of the $q$ dynamic factors.

Suppose that $\lambda(\mathrm{L})$ has finite degree $p-1$, and let $F_t = [f_t' \; f_{t-1}' \; \ldots \; f_{t-p+1}']'$ or a subset of these lags of $f_t$ if not all dynamic factors appear with $p$ lags. Let the dimension of $F_t$ be $r$, where $q \le r \le qp$. Then the DFM (5) and (6) can be written,

*Static form of the DFM*:

$$X_t = \Lambda F_t + D(\mathrm{L})X_{t-1} + v_t \tag{9}$$

$$F_t = \Phi(\mathrm{L})F_{t-1} + G\eta_t, \tag{10}$$

where $\Lambda$ is a $n\times r$ matrix, the $i^{\text{th}}$ row of which consists of the coefficients of $\lambda_i(\mathrm{L})$, $\Phi(\mathrm{L})$ consists of the coefficients of $\Gamma(\mathrm{L})$ and zeros, and $G$ is $r\times q$. If the order of $\Gamma(\mathrm{L})$ is at most $p$, then the VAR for $F_t$ has degree one and $\Phi(\mathrm{L}) = \Phi$. In the terminology of state space models and Kalman filtering, equation (9) is the measurement equation and equation (10) is the state equation.

The representation (9) and (10) is called the "static" form of the DFM because $F_t$ appears in the $X$ equation without any lags, as it does in classical factor analysis in cross-sectional data. Note that if there are the same number of static and dynamic factors, that is, $r = q$, then it must be the case that $\lambda(\mathrm{L})$ in (5) has no lag terms, so $F_t = f_t$, $G = I$, and there is no difference between the static and dynamic forms.

The static form of the DFM implies that the variance of prefiltered $X_t$ has a conventional factor structure. Let $\tilde{X}_{it} = (1 - \delta_i(\mathrm{L})\mathrm{L})X_{it}$, $\Lambda = [\Lambda_1' \ldots \Lambda_n']'$ be the matrix of (static) factor loadings, $\tilde{X}_t = [\tilde{X}_{1t} \ \ldots \ \tilde{X}_{nt}]'$, $\nu_t = [\nu_{1t} \ldots \nu_{nt}]'$, and let $\Sigma_{\tilde{X}}$, $\Sigma_F$, and $\Sigma_\nu$ be the covariance matrices of $\tilde{X}_t$, $F_t$ and $\nu_t$. Then

$$\Sigma_{\tilde{X}} = \Lambda\Sigma_F\Lambda' + \Sigma_\nu. \tag{11}$$

This is the usual variance decomposition of classical factor analysis.

***The DFM in VAR form***. The VAR form of the DFM obtains by substituting (10) into (9) and collecting terms. The equation for $X_{it}$ in the VAR is,

$$X_{it} = \Lambda_i\Phi(\mathrm{L})F_{t-1} + \delta_i(\mathrm{L})X_{it-1} + \varepsilon_{X_it} \tag{12}$$

where $\varepsilon_{X_it} = \Lambda_i G\eta_t + \nu_{it}$ and $\varepsilon_{Ft} = G\eta_t$. Combining (12) with the factor evolution equation yields the complete DFM in VAR form:

*VAR form of the DFM (FAVAR):*

$$\begin{bmatrix} F_t \\ X_t \end{bmatrix} = \begin{bmatrix} \Phi(\mathrm{L}) & 0 \\ \Lambda\Phi(\mathrm{L}) & D(\mathrm{L}) \end{bmatrix} \begin{bmatrix} F_{t-1} \\ X_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{Ft} \\ \varepsilon_{Xt} \end{bmatrix} \tag{13}$$

where

$$\begin{bmatrix} \varepsilon_{Ft} \\ \varepsilon_{Xt} \end{bmatrix} = \begin{bmatrix} I \\ \Lambda \end{bmatrix} G\eta_t + \begin{bmatrix} 0 \\ \nu_t \end{bmatrix} \tag{14}$$

where $\varepsilon_{Xt} = [\varepsilon_{X_1t} \ \ldots \ \varepsilon_{X_nt}]'$. The covariance matrix of $\varepsilon_t \equiv [\varepsilon_{Ft}' \ \varepsilon_{Xt}']'$ is,

$$\mathrm{E}\varepsilon_t\varepsilon_t' \equiv \Sigma_\varepsilon = \begin{bmatrix} G\Sigma_\eta G' & G\Sigma_\eta G'\Lambda' \\ \Lambda G\Sigma_\eta G' & \Lambda G\Sigma_\eta G'\Lambda' + \Sigma_\nu \end{bmatrix} \tag{15}$$

where $\Sigma_\eta = \mathrm{E}\eta_t\eta_t'$.

We have adopted Bernanke, Boivin, and Eliasz's (2005) terminology in referring to (13) as a FAVAR; in contrast to their FAVAR, however, (13) incorporates the exclusion restrictions implied by the DFM.

A disciple of Sims (1980) might quibble with our use of the term "VAR form" for (13) – (14), for two reasons. First, this form imposes many restrictions on the lag dynamics and on the structure of the covariance matrix of the one-step ahead forecast errors; in contrast Sims (1980) introduced VARs as a way to avoid making any such restrictions and the term "VAR" typically refers to unrestricted structures. This said, the restrictions studied here are akin to the Bayesian restrictions, developed by Sims and his students, in which prior parametric restrictions are used to control the proliferation of parameters in high-dimensional VARs. Second, the $F_t$ variables in the VAR are unobserved, however because $n$ is large the factors are consistently estimable so we proceed as if they are observable.

***Impulse response functions and variance decompositions.*** Inverting the VAR representation (13) – (14) yields the moving average representation for $X_t$ in terms of current and lagged innovations $\eta_t$ to the dynamic factors and the idiosyncratic disturbances $\nu_t$:

*MA form of the DFM*:

$$X_t = B(\mathrm{L})\eta_t + u_t, \tag{16}$$

where $B(\mathrm{L}) = [\mathrm{I} - D(\mathrm{L})\mathrm{L}]^{-1}\Lambda[\mathrm{I} - \Phi(\mathrm{L})\mathrm{L}]^{-1}G$ and $u_t = [\mathrm{I} - D(\mathrm{L})\mathrm{L}]^{-1}\nu_t$. This moving average representation delivers impulse response functions and forecast error variance decompositions for $X_{t+h}$ as a function of the horizon $h$.

The impulse responses and variance decompositions based on (16) can be thought of as the factor version of impulse responses and variance decompositions with respect to Cholesky factorizations of conventional VAR innovations, in the sense that $\eta_t$ is identified using an arbitrary statistical normalization (like that produced by principal components analysis), not an economic model of structural shocks. Section 3 considers the further step from the moving average representation (16), which is in terms of $\eta_t$, to the structural impulse response function in terms of dynamic factor structural shocks.

### 2.3 Summary of VAR Restrictions Implied by the DFM

The static form and VAR form of the DFM incorporates several overidentifying restrictions.

1. **Factor structure of $X_t$.** The covariance matrix $\Sigma_{\tilde{X}}$ has the factor structure (11), where the rank of $\Lambda\Sigma_F\Lambda'$ is $r$, the number of static factors. This restriction (under the weaker conditions of an approximate dynamic factor model) is used by the Bai-Ng (2002) information criteria methods for estimation of $r$.
2. **Reduced rank of $\varepsilon_{Ft}$.** The rank of $E\varepsilon_{Ft}\varepsilon_{Ft}'$ (the (1,1) block of (15)) is $q$, the number of dynamic factors. Giannone, Reichlin, and Sala (2004) use informal methods based on this restriction to make inferences about $q$. Bai and Ng (2005b) develop formal procedures for estimating $q$ using this restriction, and we discuss the Bai-Ng (2005b) approach further below.
3. **Factor structure of $\varepsilon_{Xt}$.** The innovations in $X_t$, $\varepsilon_{Xt}'$, obey a classical factor model, that is, they are serially uncorrelated and the (2,2) block of (15) has a factor structure, where the number of factors is the number of dynamic factor innovations, $q$. We discuss below how this restriction can be used to estimate $q$.
4. ***X* does not predict *F* given lagged *F*.** That is, the upper right block in (13) is zero, a restriction tested in Section 5.
5. **$X_j$ does not predict $X_i$ given lagged $F$.** That is, D(L) in (7) is diagonal, so $E(X_{it}|F_{t-1}, F_{t-2},\ldots X_{it-1}, X_{it-2},\ldots, X_{jt-1}, X_{jt-2},\ldots) = E(X_{it}|F_{t-1}, F_{t-2},\ldots, X_{it-1}, X_{it-2},\ldots)$. This restriction is tested Section 5.
6. **$X_j$ does not explain $X_i$ given current $F$.** That is, $X_j$ does not appear in(9), so $E(X_{it}|F_t, X_{it-1}, X_{it-2},\ldots, X_{jt}, X_{jt-1},\ldots) = E(X_{it}|F_t, X_{it-1}, X_{it-2},\ldots)$. This is a key implication for SVAR analysis using factors because it says that, given the factors, the VAR need not include any other $X$'s except the $X$ of

interest, that is, excluding other observable variables from the VAR does not produce omitted variable bias. This restriction is tested Section 5.

7. **Cross-equation restrictions in the *X* equations.** If Φ(L) has degree one or more, there are overidentifying cross-equation restrictions across the rows in the lower left block of (13). If however $F_t$ follows a VAR(1) so Φ(L) = Φ then there are no overidentifying restrictions. Because of the sensitivity of this restriction to subsidiary lag restrictions we do not examine this restriction empirically.

## 2.4 Estimation of Static Factors, Restricted VAR Coefficients, and Dynamic Factor Innovations

In principle the coefficients of the VAR representation (with additional lag length restrictions and normalizations) could all be estimated by restricted quasi-maximum likelihood estimation. However that would be computationally cumbersome and we instead adopt a stepwise approach that first entails estimation of the static factors, then estimation of the VAR coefficients, and finally estimation of the dynamic factor innovations.

***Estimation of static factors and the number of static factors.*** The static factors $F_t$ can be estimated as the principal components of the filtered observables $\tilde{X}_t$ (where $X_t$ is standardized to have sample mean zero and unit standard deviation). Specifically the estimators of $\{F_t\}$ and Λ solve the minimization problem,

$$\min_{F_1,\ldots,F_T,\Lambda,D(L)} T^{-1}\sum_{t=1}^{T}[(I-D(L)L)X_t-\Lambda F_t]'[I-D(L)L)X_t-\Lambda F_t] \tag{17}$$

where $D$(L) is given in (7). The minimization in (17) is conveniently done iteratively. Given a preliminary estimator of D(L), $\{F_t\}$ can be computed as the first $r$ principal components of $(I - D(\mathrm{L})\mathrm{L})X_t$; given the estimate of $\{F_t\}$, $\delta_i$(L) and Λ are estimated by $n$ individual regressions of $X_{it}$ on ($F_t$, $X_{it-1}$,…, $X_{it-m_i+1}$), where $m_i$ is the order of $\delta_i$(L). Each step of this procedure reduces (does not increase) the sum of squares in (17) and the procedure can be iterated to convergence.[1] This produces estimators $\hat{F}_t$, $\hat{\Lambda}$, and $\hat{D}(\mathrm{L})$.

The number of static factors can be estimated using the Bai-Ng (2002) information criteria. These can be applied either to the sample covariance matrix of $X_t$ (the method proposed by Bai and Ng (2002)) or alternatively to the covariance matrix of $(I - \hat{D}(\mathrm{L})\,\mathrm{L})X_t$.

***Estimation of restricted VAR coefficients***. Given the estimates $\hat{F}_t$, the restricted VAR coefficients are estimated by first regressing $\hat{F}_t$ onto the desired number of lags to obtain the estimator of $\Phi(\mathrm{L})$, $\hat{\Phi}(\mathrm{L})$, then using $\hat{\Phi}(\mathrm{L})$, $\hat{\Lambda}$, and $\hat{D}(t)$ to construct the restricted VAR coefficient matrix in (13).

***Estimation of the number of dynamic factors***. The number of dynamic factors, $q$, can be estimated in two ways.

The first method, developed here, exploits restriction #3 in Section 2.3, that $\mathrm{E}\varepsilon_{Xt}\varepsilon_{Xt}' = \Lambda G\Sigma_\eta G'\Lambda' + \Sigma_v$ so that the innovations of $X_t$ have a factor structure. First, the innovations in $X_{it}$, $\varepsilon_{X_it}$, are estimated by $\hat{\varepsilon}_{X_it}$, constructed using (13) as the residuals from the regression of $X_{it}$ onto lags of $X_{it}$ and lags of $\hat{F}_t$. Second, the number of dynamic factors $q$ is estimated by applying the Bai-Ng (2002) procedure to the sample covariance matrix of $\hat{\varepsilon}_{Xt}$, yielding an estimator $\hat{q}$.

The second method, developed by Bai and Ng (2005b), exploits restriction #2 in Section 2.3, that $\mathrm{E}\varepsilon_{Ft}\varepsilon_{Ft}' = G\Sigma_\eta G$, so that the $r\times r$ matrix of innovations of the static factors has rank $q$. Their method entails a spectral decomposition on the sample analogue of $\mathrm{E}\varepsilon_{Ft}\varepsilon_{Ft}'$ and estimating its rank using an information criterion approach.

***Estimation of space spanned by dynamic shocks***. Given $q$, there are several ways to estimate the dynamic factor innovations. The algorithm used here chooses $G$ such that the innovations are uncorrelated and that they maximize the trace $R^2$ of $X$, ordered so that the first dynamic factor makes the largest variance reduction, the second the second-largest, and so forth. Specifically, write $B(\mathrm{L})$ in equation (16) as $B(\mathrm{L}) =$

[1]This estimator modifies the static principal components estimator of Stock and Watson (2002), in which $\delta_i(\mathrm{L}) = 0$ in the notation here.

$A$(L)$G$, where $A(\mathrm{L}) = [\mathrm{I} - D(\mathrm{L})\mathrm{L}]^{-1}\Lambda[\mathrm{I} - \Phi(\mathrm{L})\mathrm{L}]^{-1}$, and normalize $\Sigma_\eta = I$; then $\mathrm{trE}[(B(\mathrm{L})\eta_t)(B(\mathrm{L})\eta_t)'] = \mathrm{trE}[(A(\mathrm{L})G\eta_t)(A(\mathrm{L})G\eta_t)'] = \mathrm{tr}(\sum_{j=0}^{\infty} A_j G\Sigma_\eta G' A_j') =$ $\mathrm{tr}(\sum_{j=0}^{\infty} A_j GG' A_j') = \mathrm{tr}[G'\left(\sum_{j=1}^{\infty} A_j A_j'\right)G]$. Then

$$\mathrm{tr}(\Sigma_X) = \mathrm{tr}[G'\left(\sum_{j=1}^{\infty} A_j A_j'\right)G] + \mathrm{tr}(\Sigma_u) \tag{18}$$

so that choosing $G$ to maximize the trace $R^2$ explained by the factors is equivalent to choosing $G$ to be the eigenvectors of $\sum_{j=1}^{\infty} A_j A_j'$ that correspond to the largest $q$ eigenvalues. The estimator of $G$, $\hat{G}$, is the sample analog of this matrix of eigenvectors, computed using $\hat{A}(\mathrm{L}) = [I - \hat{D}(\mathrm{L})\mathrm{L}]^{-1}\hat{\Lambda}[I - \hat{\Phi}(\mathrm{L})\mathrm{L}]^{-1}$.

Another way to estimate the dynamic factor innovations, which we do not use, is to estimate them as the first $\hat{q}$ principal components of $\hat{\varepsilon}_{Xt}$. In this case, the dynamic factor innovations sequentially maximize the trace $R^2$ of $\hat{\varepsilon}_{Xt}$.

### 2.5 Distribution Theory and Inference

The foregoing procedures are justified by a body of distribution theory concerning the performance of principle components in large panels. Stock and Watson (2002b) proved consistency of the principle components estimator for the space spanned by the factors for the approximate DFM for $N, T \to \infty$. Bai (2003) shows the asymptotic normality of $\hat{\Lambda}_i$ under approximate DFM assumptions and the rate condition $N, T \to \infty$, $T^{1/2}/N \to 0$. Bai and Ng (2005a) consider the distribution of the coefficients of factor-augmented regressions, in which $X_{it}$ is regressed against $\hat{F}_t$ and a fixed number of additional observed stationary regressors, and show that standard $T^{1/2}$ inference applies if $N, T \to \infty$, $T^{1/2}/N \to 0$. These final results justify the application of standard testing methods to regression tests of the FAVAR exclusion restrictions, in which the estimated factors are treated as if they were known and observed.

## 3. Structural DFMs and Structural FAVARs

Structural VAR analysis requires deducing one or more structural shocks from the VAR innovations. The same requirement arises in structural DFMs, except that the innovations and structural shocks are those of the dynamic factors. Let $\zeta_t$ denote the $q$ structural shocks to the dynamic factors. Analogously to structural VAR analysis, the dynamic factor structural shocks are assumed to be linearly related to the reduced form dynamic factor innovations $\eta_t$ by

$$\zeta_t = H\eta_t \tag{19}$$

where $H$ is an invertible $q\times q$ matrix. In this notation, the task of structural FAVAR analysis is to identify $H$ or, if one is interested in just one economic shock, a row of $H$. Throughout we assume that $E\zeta_t\zeta_t' = I$, so that $H\Sigma_\eta H' = I$.

There are several approaches available for the identification of $H$ in structural VAR analysis, and these plus more are available for structural FAVAR analysis. This section begins by laying out the structural DFM and discusses in general identification and estimation of structural FAVARs. The remaining subsections develop special cases, several of which provide significant computational gains over the general approach. These subsections separately consider identification using exclusion restrictions on the contemporaneous incidence of structural shocks; identification using long-run restrictions; and identification directly from the factor loadings.

### 3.1 Identification, Estimation, and Testing in Structural FAVARs

We adopt the notational convention that "*" denotes a parameter in the structural system, for example $\lambda$(L) is the ("reduced-form") DFM lag polynomial in (5) and $\lambda^*$(L) is its counterpart in the structural form of the DFM. Accordingly, the structural form of the DFM is,

*The DFM – structural form*:

$$X_t = \lambda^*(\mathrm{L})\, f_t^* + D(\mathrm{L})X_{t-1} + v_t \tag{20}$$

$$f_t^* = \Gamma^*(\mathrm{L})\, f_t^* + \zeta_t, \tag{21}$$

where $f_t^* = Hf_t$, $\lambda^*(\mathrm{L}) = \lambda(\mathrm{L})H^{-1}$, and $\Gamma^*(\mathrm{L}) = H\Gamma(\mathrm{L})H^{-1}$.

The moving average representation of the structural form is,

*The DFM – structural MA form*:

$$X_t = B^*(\mathrm{L})\zeta_t + u_t, \tag{22}$$

where $B^*(\mathrm{L}) = B(\mathrm{L})H^{-1} = [\mathrm{I} - D(\mathrm{L})\mathrm{L}]^{-1}\Lambda[\mathrm{I} - \Phi(\mathrm{L})\mathrm{L}]^{-1}GH^{-1}$. The structural moving average lag polynomial $B^*(\mathrm{L})$ is the impulse response function with respect to the structural shocks $\zeta_t$; this is the primary object of ultimate interest in structural DFM analysis.

Identification of $H$ can be achieved by imposing restrictions on $B^*(\mathrm{L})$ and/or on $\lambda^*(\mathrm{L})$. Both these lag polynomial matrices have dimension $n \times q$, so in principle identification schemes can range from exact identification to identification using very many overidentifying restrictions, which can be tested.

Without additional structure, there is little that can be said about identification and estimation. One approach that is always available is to recognize that the system (20) and (21) comprise a linear state space system with unobserved components $f_t^*$. The parameters can be estimated subject to the identifying (or overidentifying) restrictions by using the Kalman filter to construct the Gaussian likelihood. In many cases of interest, however, the computational is greatly reduced using a two-step procedure in which principal components methods are used in the first step, to estimate the reduced-form factor model, $G$, and $\{\eta_t\}$ as described in the previous section; in the second step the estimated factors and factor innovations are treated as data and the identifying restrictions are used to estimate $H$, $\{\zeta_t\}$, and thus the structural impulse response function $B^*(\mathrm{L})$. These cases are now described for different families of identifying restrictions.

### 3.2 Contemporaneous Timing Restrictions

Timing restrictions typically are exclusion restrictions stating that certain structural shocks do not affect certain $X$ variables contemporaneously, for example, the monetary policy shock does not affect output within the month. This approach, which is standard in the structural VAR literature, here implies that the innovations in some of the $X$'s depend on only some of the $\zeta$'s. Specifically, from (22) we have that

$$\varepsilon_{Xt} = B_0^* \zeta_t + v_t \tag{23}$$

where $B_0^*$ is the coefficient matrix that is the leading (zero-lag) term in $B^*(L)$. We illustrate identification and estimation in three examples of increasing complexity, beginning with the factor version of the Sims (1980) Wold causal ordering.

***Example #1: Exact identification/Cholesky factorization.*** Suppose the identification scheme is such that $B_0^*$ is lower triangular:

$$B_0^* = \begin{bmatrix} x & 0 & \cdots & 0 \\ x & x & \ddots & 0 \\ x & x & \ddots & 0 \\ x & x & \cdots & x \\ \vdots & \vdots & \vdots & \vdots \\ x & x & \cdots & x \end{bmatrix} \tag{24}$$

where $x$ denotes an unrestricted nonzero element. There are $q(q-1)/2$ exclusion restrictions and $H$ is exactly identified.

This example is analogous to achieving exact identification by ordering the variables in a standard VAR in a particular Wold causal chain, however it is not the same thing because there is the additional idiosyncratic innovation $v_t$. Identification and estimation proceeds using a Cholesky factorization of the factor innovation variance matrix. Specifically, let $B_{0;q}^*$ denote the $q\times q$ matrix of the first $q$ rows of $B_0^*$, which contain all the identifying restrictions, and let $B_{0;q}$ denote the $q\times q$ matrix of the first $q$

rows of $B_0$. Because $B_0^* \zeta_t = B_0 \eta_t$, $B_{0;q}^* B_{0;q}^{*}{}' = B_{0;q} \Sigma_\eta B_{0;q}{}'$. But $B_{0;q}^*$ is lower triangular, so it must be the case that

$$B_{0;q}^* = \text{Chol}(B_{0;q} \Sigma_\eta B_{0;q}{}'). \tag{25}$$

Now $B^*(\text{L}) = [\text{I} - D(\text{L})\text{L}]^{-1}\Lambda[\text{I} - \Phi(\text{L})\text{L}]^{-1}GH^{-1}$, so $B_{0;q}^* = \Lambda_q GH^{-1}$, where $\Lambda_q$ denotes the first $q$ rows of $\Lambda$; thus

$$H = [\text{Chol}(B_{0;q} \Sigma_\eta B_{0;q}{}')]^{-1}\Lambda_q G. \tag{26}$$

The matrices on the right hand side of (26) can all be deduced from the population DFM parameters, as discussed in Section 2, showing that the lower triangular assumption (24) exactly identifies $H$ and thus (by (19)) $\{\zeta_t\}$.

Estimation of $H$ and of $\{\zeta_t\}$ proceeds by replacing the population matrices in (26) with their sample estimates, computed as described in Section 2.4.

***Example #2: Partial identification via block lower-triangular exclusion restrictions.*** BBE introduce a scheme for identifying a single shock in a structural FAVAR by adopting a block lower triangular structure for $B_0^*$. They partition the structural shocks and variables into three groups, slow variables, an interest rate, and fast variables. The economic content of the BBE restrictions is discussed in Section 6.

Denote the three groups of variables "$S$", "$R$," and "$F$." The structural shocks are $\zeta_t = (\zeta_t^{S}{}', \zeta_t^R, \zeta_t^{F}{}')'$, where $\zeta_t^S$ is $q_S\times1$, $\zeta_t^R$ is a scalar, and $\zeta_t^F$ is $q_F\times1$. The $\zeta_t^S$ shocks potentially can affect all the variables within a period, $\zeta_t^R$ affects all but $n_S$ "slow" variables within a period, and $\zeta_t^F$ affects only the remaining $n_F = n - n_S - 1$ "fast" variables within the period. The variables can be organized in the ordered groups $S$, $R$, and $F$, so that $B_0^*$ has the block lower triangular form,

$$B_0^* = \begin{bmatrix} B_{0,SS}^* & 0 & 0 \\ B_{0,RS}^* & B_{0,RR}^* & 0 \\ B_{0,FS}^* & B_{0,FR}^* & B_{0,FF}^* \end{bmatrix} \tag{27}$$

where $B_{0,SS}^*$ is $n_S \times q_S$, $B_{0,RS}^*$ is $1 \times q_S$, $B_{0,RR}^*$ is a scalar, and $B_{0,FS}^*$, $B_{0,FR}^*$, and $B_{0,FF}^*$ are respectively $n_F \times q_S$, $n_F \times 1$, and $n_F \times q_S$.

The block triangular restrictions in (27) identify $\zeta_t^R$ (the shock of interest), the space spanned by $\zeta_t^S$, and the space spanned by $\zeta_t^F$. Because $\zeta_t^R$ is identified, the column of $B^*(L)$ associated with $\zeta_t^R$ is identified and thus the structural impulse response of $X_t$ with respect to $\zeta_t^R$ is identified. To show this identification algebraically, partition $H$ conformably with $\zeta_t$ so that (19) becomes

$$\begin{bmatrix} \zeta_t^S \\ \zeta_t^R \\ \zeta_t^F \end{bmatrix} = \begin{bmatrix} H_S' \\ H_R' \\ H_F' \end{bmatrix} \eta_t. \tag{28}$$

With the notation in (28) and the restrictions in (27), the innovations expression (23) becomes,

$$\begin{aligned} \varepsilon_{Xt}^S &= B_{0,SS}^* H_S' \eta_t + v_t^S \\ \varepsilon_{Xt}^R &= B_{0,RS}^* H_S' \eta_t + B_{0,RR}^* H_R' \eta_t + v_t^R \\ \varepsilon_{Xt}^F &= B_{0,FS}^* H_S' \eta_t + B_{0,FR}^* H_R' \eta_t + B_{0,FF}^* H_F' \eta_t + v_t^F \end{aligned} \tag{29}$$

where $\varepsilon_{Xt}^S$ is the $n_S \times 1$ vector of innovations in the $S$ variables, etc. Because $B_{0,SS}^*$ is $n_S \times q_S$ and $H_S'$ is $q_S \times q$, the rank of $B_{0,SS}^* H_S'$ is $q_S$ (assuming $n_S \geq q_S$). Thus the population projection of $\varepsilon_{Xt}^S$ onto $\eta_t$ spans a $q_S$-dimensional space, which is the space spanned by $\zeta_t^S$. Similarly, because $B_{0,RR}^* H_R'$ is $1 \times q$, and because the space spanned by $\zeta_t^S$ is now identified, $\zeta_t^R$ is identified (up to scale) as $\zeta_t^R$ = Proj( $\varepsilon_{Xt}^R | \eta_t$) – Proj( $\varepsilon_{Xt}^R | \zeta_t^S$ ). The space spanned by $\zeta_t^F$ is the span of $\eta_t$ that is orthogonal to $\zeta_t^S$ and $\zeta_t^R$.

One way to estimate $H$, which imposes the overidentifying restrictions in (27), parallels the preceding discussion of identification:

1. Estimate the innovations to $X_t$ and the dynamic factor innovations $\eta_t$ as described in Section 2.4; denote these $\hat{\varepsilon}_{Xt}^S$ (etc.) and $\hat{\eta}_t$.
2. Estimate (or impose based on *a-priori* grounds) the number of $S$ shocks, $q_S$. The dimension $q_S$ can be estimated using the method described in Section 2.4 for the estimation of $q$, except applied only to the $S$ variables.
3. Estimate $B_{0,SS}^*$ and $H_S$ by reduced rank regression of $\hat{\varepsilon}_{Xt}^S$ onto $\hat{\eta}_t$, imposing the rank $q_S$. This produces $\hat{\zeta}_t^S$.
4. Estimate $\zeta_t^R$ by $\hat{\zeta}_t^R$ = Proj( $\hat{\varepsilon}_{Xt}^R | \hat{\eta}_t$ ) – Proj( $\hat{\varepsilon}_{Xt}^R | \hat{\zeta}_t^S$ ), where the projections are implemented by OLS regression. The OLS regression of $\hat{\zeta}_t^R$ onto $\hat{\eta}_t$ yields the estimated coefficients $\hat{H}_R$.

This procedure produces the Gaussian maximum likelihood estimator of $H$ based on the innovations under the assumption that $v_t$ is i.i.d. homoskedastic Gaussian with unknown and unrestricted covariance matrix and $\eta_t$ are observed regressors. Note that these are different assumptions than those that underly exact ML estimation of the DFM using the Kalman filter, in which $\eta_t$ is treated as unobserved. However, because $\eta_t$ is well estimated when $n$ is large, the approximate ML interpretation of the algorithm 1 – 4 suggests that it will produce estimates with good sampling properties.

***Example #3: Partial identification via block lower-triangular exclusion restrictions.*** The logic of the Wold causal ordering in example #1 could be applied to groups of similar variables, for example all employment variables could be in the same group. Let $X_t$ be partitioned into $q$ groups, each with $n_i$ elements, where $n_1 + \ldots + n_q = n$. Under this scheme, $B_0^*$ has the block lower triangular form,

$$B_0^* = \begin{bmatrix} B_{0,11}^* & 0 & \cdots & 0 \\ B_{0,12}^* & B_{0,22}^* & \ddots & 0 \\ \vdots & \vdots & \ddots & 0 \\ B_{0,1q}^* & B_{0,2q}^* & \cdots & B_{0,qq}^* \end{bmatrix}, \tag{30}$$

where $B_{0,ij}^*$ in (30) is a $n_i$×1 vector. Let $H_i'$ denote the $i^{\text{th}}$ row of $H$. Partition $\varepsilon_{Xt}$ conformably with $B_0^*$. Then the innovations equations corresponding to (30) are,

$$\begin{aligned} \varepsilon_{Xt}^1 &= B_{0,11}^* H_1 ' \eta_t + v_t^1 \\ \varepsilon_{Xt}^2 &= B_{0,21}^* H_1 ' \eta_t + B_{0,22}^* H_2 ' \eta_t + v_t^2 \\ &\vdots \\ \varepsilon_{Xt}^q &= B_{0,q1}^* H_1 ' \eta_t + B_{0,q2}^* H_2 ' \eta_t + \cdots + B_{0,qq}^* H_q ' \eta_t + v_t^q \end{aligned} \tag{31}$$

Identification of $\zeta_t^1 = H_1' \eta_t$ is achieved (up to scale) by noting that $B_{0,11}^* H_1'$ has rank 1 so that $\zeta_t^1$ = Proj($\varepsilon_{Xt}^1 \mid \eta_t$). Similarly, $\zeta_t^2$ is identified (up to scale) as $\zeta_t^2$ = Proj($\varepsilon_{Xt}^2 \mid \eta_t$) – Proj($\varepsilon_{Xt}^2 \mid \zeta_t^1$). The remaining elements of $\zeta_t$ are identified by extending this projection argument. This entirely identifies $H$.

The structural shocks and $H$ can be estimated by an extension of the reduced rank regression algorithm presented for example #2. Specifically:

1. Estimate $\hat{\varepsilon}_{Xt}$ and $\hat{\eta}_t$.
2. Estimate $B_{0,11}^*$ and $H_1$ by reduced rank regression of $\hat{\varepsilon}_{Xt}^1$ onto $\hat{\eta}_t$, imposing rank 1. This produces $\hat{H}_1$ and $\hat{\zeta}_t^1$.
3. Construct the $q$×($q$–1) matrix $\hat{H}_1^\perp$ such that $\hat{H}_1^{\perp}{}' \hat{H}_1 = 0$ and $\hat{H}_1^{\perp}{}' \hat{H}_1^\perp = I$, then estimate the reduced rank regression of $\hat{\varepsilon}_{Xt}^2$ on $\hat{\zeta}_t^1$ and $\hat{H}_1^{\perp}{}' \hat{\eta}_t$. The ($q$–1)×1 vector of reduced rank weights on $\hat{H}_1^{\perp}{}' \hat{\eta}_t$ from this regression, times $\hat{H}_1^{\perp}{}'$, produces the estimators $\hat{H}_2'$ and $\hat{\zeta}_t^2 = \hat{H}_2' \hat{\eta}_t$.
4. Repeat this process for each of the remaining blocks, yielding $\hat{H}$.

Unlike example #2, this algorithm does not have the interpretation of being Gaussian maximum likelihood, given $\eta_t$ (for example, varying $H_1$ affects the fit of the second block of equations but only the first block of equations is used to estimate $H_1$).

***General restrictions***. Little can be said about general identification restrictions, beyond standard rank and order condition requirements from the theory of simultaneous equations. It is worth noting, however, that working with the innovation equations and treating $\eta_t$ as observed still can result in significant computational advantages relative to full Gaussian ML via the Kalman filter. Specifically, the innovations equations permit a computationally efficient iterative algorithm for estimation of $H$ subject to general exclusion restrictions:

1. Estimate $\hat{\varepsilon}_{Xt}$ and $\hat{\eta}_t$ using all $n$ variables.
2. Use a subset of exactly identifying restrictions to obtain initial estimates of $H$ and $\zeta_t$, $\hat{H}^{(1)}$ and $\hat{\zeta}_t^{(1)}$.
3. Regress $\hat{\varepsilon}_{Xt}$ onto $\hat{\zeta}_t^{(1)}$, imposing all the identifying restrictions on the matrix of coefficients $B_0^*$. If there are no cross-equation restrictions, this can be done either by restricted least squares, equation by equation, which imposes the exact DFM restriction that $\Sigma_v$ is diagonal, or by SURE, which allows $\Sigma_v$ to be unrestricted. This produces the value of the Gaussian likelihood given $\hat{H}^{(1)}$, $L(\hat{H}^{(1)})$.
4. Search numerically over $H$ to minimize $L(H)$, subject to $H\Sigma_\eta H' = I$.

***Testing overidentifying restrictions.*** In all but example #1, the structural FAVAR is overidentified, and these overidentifying restrictions are testable.

One approach to testing overidentifying restrictions is to do so equation by equation. As discussed in Section 2.5, under suitable conditions the estimated factors, and thus $\hat{\eta}_t$, can be treated as observed regressors for the purposes of hypothesis tests on the coefficients in a single equation. In this case the overidentifying restrictions can be tested by standard regression methods, treating $\hat{\zeta}_t^R$ as observed. For example, in the BBE identifying scheme, each innovation equation in the $S$ block has $q_F$+1 exclusion restrictions, that $\zeta_t^R$ and $\zeta_t^F$ not enter that equation. For a given equation, this exclusion restriction can be testing by regressing the $i^{\text{th}}$ estimated innovation, $\hat{\varepsilon}_{Xt,i}^S$, onto $\hat{\zeta}_t^S$, $\hat{\zeta}_t^R$, and $\hat{\zeta}_t^F$, then testing using a standard Wald test to the hypothesis that the coefficients on $\hat{\zeta}_t^R$ and $\hat{\zeta}_t^F$ are zero. Strictly this test will have an asymptotic $\chi^2_{q_F+1}$ distribution under the assumptions referred to in Section 2.5 if this $i^{\text{th}}$ equation was not used in the estimation of $\hat{\zeta}_t^S$, $\hat{\zeta}_t^R$, and $\hat{\zeta}_t^F$; however, if the number of overidentifying restrictions is very large it might reasonably be assumed that the $\chi^2_{q_F+1}$ is a useful (possibly conservative) approximation to the distribution of the Wald statistic.

Another approach to testing the overidentifying restrictions would be to test all the restrictions simultaneously. In the BBE example, there are $(q_F+1)(n_S - q_F/2)$. Because $\eta_t$ is not observed and because $n_S$ is large, we would not expect the distribution of the joint Wald test of these restrictions to be well approximated by a $\chi^2_{(q_F+1)(n_S-q_F/2)}$ distribution. More work on the statistical properties of joint tests of large number of such restrictions is warranted. Based on existing distribution theory, single-equation tests or tests of a small number of restrictions seem likely to be more reliable than joint tests of very many restrictions.

***Imposing the overidentifying restrictions on the levels regression.*** Because $B_0^* = \Lambda G H^{-1}$, rank restrictions on $B_0^*$, such as those in examples 2 and 3, imply rank restrictions on $\Lambda$. These restrictions can be imposed and tested directly on the levels regression (9), and can be used to estimate $H$. To implement these restrictions, it is useful to perform an additional transformation. Let $W$ be the $r\times r$ matrix

$$W = \begin{bmatrix} (G'G)^{-1}G' \\ G^{\perp}{}' \end{bmatrix} \tag{32}$$

where $G^{\perp}{}'$ is $(r-q)\times q$. Let $C_t = WF_t$, and let $C_t = [C_{1t}' \;\; C_{2t}']'$, where $C_t$ is partitioned conformably with $W$. Then

$$C_t = \Pi(\mathrm{L})C_{t-1} + \begin{bmatrix} \eta_t \\ 0 \end{bmatrix}. \tag{33}$$

where $\Pi(L) = W\Phi(L)W^{-1}$. The $r–q$ rotated factors $C_{2t}$ are dynamically redundant, in the sense that they are predetermined given $C_{t–1}$; equivalently, their innovation is zero. Thus $\Lambda F_t = \Lambda W^{-1}C_t = \Lambda_1^C C_{1t} + \Lambda_2^C C_{2t}$, so the measurement equation of the static DFM (9) can be written,

$$\tilde{X}_t = \Lambda_1^C C_{1t} + \Lambda_2^C C_{2t} + v_t. \tag{34}$$

and $B_0^* = \Lambda_1^C H^{-1}$. Constraints on $B_0^*$ are then imposed via reduced rank restrictions on $\Lambda_1^C$ in the regression (34).

For example, the restrictions in example 2 can be imposed using (34) and the following algorithm:

1. Estimate $\eta_t$, $F_t$, and $G$ as described in Section 2.4, and construct $\hat{C}_t$.
2. Estimate (or impose based on *a-priori* grounds) the number of $S$ shocks, $q_S$.
3. Estimate $\Lambda_1^{C,S}$ and $H_S$ by reduced rank regression (with rank $q_S$) of the slow moving elements in $\tilde{X}_t$ onto $\hat{C}_{1t}$, including without restrictions the additional regressor $\hat{C}_{2t}$. This produces $\hat{H}_S$ and $\hat{\zeta}_t^S$.
4. Estimate $\zeta_t^R$ by $\hat{\zeta}_t^R$ = Proj($\hat{\varepsilon}_{Xt}^R \mid \hat{\eta}_t$) – Proj($\hat{\varepsilon}_{Xt}^R \mid \hat{\zeta}_t^S$), where the projections are implemented by OLS regression. The OLS regression of $\hat{\zeta}_t^R$ onto $\hat{\eta}_t$ yields the estimated coefficients $\hat{H}_R$.
5. The overidentifying restrictions imposed in (4) can tested as exclusion restrictions on the regressors $\hat{H}_S^{\perp}\hat{C}_{1t}$ (equivalently $\hat{H}_R\hat{C}_{1t}$ and $\hat{H}_F\hat{C}_{1t}$) in each of the regressions involving slow-moving variables.

### 3.3 Identification Using Long-Run Restrictions

A second family of identification methods uses long run restrictions to identify $H$ (or rows of $H$). These methods build on the VAR identification schemes of Blanchard and Quah (1989) and King, Plosser, Stock, and Watson (1991). These restrictions were first used in FAVAR models by Giannone, Reichlin, and Sala (2002), who considered exactly identified systems and systems with a small number of overidentifying restrictions by imposing restrictions on only seven of the variables in $X_t$ and leave the remaining equations unrestricted. Here, we discuss structural FAVARs, both in the case that the number of identifying conditions is small, and in the case that that the system is heavily overidentified.

Long run restrictions are restrictions, typically exclusion restrictions, on the long-run effect of individual shocks on groups of $X_t$ variables. These correspond to restrictions on the cumulative (or long-run) structural moving average coefficients, that is, on $B^*(1)$. The long-run restrictions can be expressed in an innovations representation. Let $\omega_{Xt}$ be the innovation in the forecast of the cumulative value of $X_t$ into the infinite future, that is, let $\omega_{Xt} = \sum_{k=0}^{\infty}(X_{t+k|t} - X_{t+k|t-1})$. Then it follows from (22) that

$$\omega_{Xt} = B^*(1)\zeta_t + [I - D(1)]^{-1}v_t. \tag{35}$$

The forecast innovations $\omega_{Xt}$ can be constructed from the reduced form MA representation, specifically, $\omega_{Xt} = B(1)\eta_t + [I - D(1)]^{-1}v_t$. Thus in population $\omega_{Xt}$ can be treated as observable. Because $v_t$ in (35) is serially uncorrelated, the discussion of identification and estimation in examples 1 – 3 of Section 3.2, as well as the concluding discussion about general exclusion restrictions and testing the overidentifying restrictions, applies directly, with $\omega_{Xt}$ replacing $\varepsilon_{Xt}$ and with $\hat{\omega}_{Xt}$ = $\hat{B}(1)\hat{\eta}_t + [I - \hat{D}(1)]^{-1}\hat{v}_t$ replacing $\hat{\varepsilon}_{Xt}$. Note that the moments involved in the reduced rank regression computations can alternatively be computed directly from the estimated reduced-form DFM moments so that it is not actually necessary to construct $\hat{\omega}_{Xt}$ to compute the desired projections and reduced rank regression eigenvectors.

### 3.4 Identification from Factor Loading Restrictions

A different approach is to identify $H$ directly from restrictions on the factor loadings $\lambda^*(L)$ in the structural DFM (20). This is the approach taken in small systems by Sargent (1989) and Stock and Watson (1991), and it has been extended to large systems by Kose, Otrok and Whiteman (2003) and by Boivin and Giannoni (2005).

We are not aware of any general method for fast (one-pass) computation of $H$ under general conditions on $\lambda(\mathrm{L})$, even if $\eta_t$ is treated as known based on large-$n$ asymptotics. In some leading special cases, however, fast computational methods are available, and we now give two examples.

***Example #4. Contemporaneous loadings, block lower triangular structure***. When $\lambda^*(\mathrm{L})$ has no lags, then the static and dynamic models coincide; thus $f_t = F_t$, $f_t^* = F_t^*$, and the measurement equation of the structural DFM (20) and (21) becomes,

$$\tilde{X}_t = \Lambda^* f_t^* + v_t \tag{36}$$

where $\Lambda^* = \lambda^*$. Because $f_t^* = Hf_t = HF_t$ (the second equality follows because $\lambda^*(\mathrm{L})$ has no lags), because $F_t$ can be estimated consistently, and because $v_t$ is serially uncorrelated, the system (36) has the same mathematical structure as the innovations representation (23) studied in Section 3.2, with $\tilde{X}_t$ replacing $\varepsilon_{Xt}$, $f_t^*$ replacing $\zeta_t$, and $f_t$ (or $F_t$) replacing $\eta_t$ in Section 3.2. Thus the discussion of identification and estimation in Section 3.2 (examples 1, 2, and 3, also the concluding remarks) extends directly to block lower triangular exclusion restrictions on the dynamic factor loadings when $\lambda^*(\mathrm{L})$ has no lags.

***Example #5: Contemporaneous loadings on distinct blocks of variables***. As in the previous example, suppose that $\lambda^*(\mathrm{L})$ has no lags; in addition, suppose that $\lambda^*$ is block diagonal:

$$\lambda^*(\mathrm{L}) = \mathrm{diag}(\lambda_{11}^*, \ldots, \lambda_{qq}^*) \tag{37}$$

where $\lambda_{11}^*$ is a $n_1 \times 1$ vector (etc.), where $n_1 + \ldots + n_q = n$. Thus each factor loads only on subset of variables, which can be thought of as indicators (with measurement error) of an unobserved economic variable. Sargent (1989) implemented a low-dimensional version of this identification scheme, and Boivin and Giannoni (2005) recently implemented a high-dimensional version.

Because $\lambda^*(L)$ has no lags, under (37), the DFM and the static form of the DFM coincide and the measurement equation is given by (36). Substituting (37) into (36), we have,

$$\begin{aligned} \tilde{X}_t^1 &= \lambda_{11}^* H_1' F_t + v_t^1 \\ &\vdots \\ \tilde{X}_t^q &= \lambda_{qq}^* H_q' F_t + v_t^q \end{aligned} \tag{38}$$

The weights $H$ can be estimated using the following algorithm:

1. Estimate $\hat{F}_t$ using all $n$ variables.
2. Regress $\tilde{X}_t^1$ on $\hat{F}_t$ by reduced rank regression with rank 1, yielding $\hat{H}_1$.
3. Repeat step 2 for each block of equations, yielding $\hat{H}$.
4. $\hat{H}$ does not satisfy $\hat{H}\,\hat{\Sigma}_{\hat{\eta}}\,\hat{H}' = I$, however it can serve as the target in method of moments estimation of $H$ that imposes this restriction.

Instead of step 4, $\hat{H}$ from step 3 could be used as a starting value in maximum likelihood estimation of the restricted DFM using the Kalman filter.

### 3.5 Other Identification Schemes

***Uhlig's (2005) sign identification scheme.*** Uhlig (2005) uses an entirely different approach to identification, in which identification conditions are imposed on the time path of the impulse response functions, not just the impact or long-run cumulative effect. For example, one identifying restriction might be that prices cannot rise in response to a contractionary monetary policy shock. This corresponds to placing sign restrictions on the coefficients of $B^*(L)$. Uhlig (2005) implements this approach by placing a diffuse prior over (in our notation) $B(L)$ and $\Sigma_\eta$, drawing realizations of these from their posterior distribution, and retaining only those realizations that accord with the sign restrictions. This approach in general yields set identification, and as implemented yields realizations and a probability distribution over the set of impulse responses that do not violate the sign conditions. This can be implemented here, however as a practical matter it appears to be inconvenient to compute the posterior distribution of $B^*(L)$; a modification of Uhlig's (2005) approach would be to compute the set of impulse responses by drawing only from the posterior of $\Sigma_\eta$, and computing the non-violating set for each of those draws. Computationally this is no more difficult than the computations in Uhlig (2005) because the dimension of $\Sigma_\eta$ (and of $H$) is comparable to the dimension of the conventional VARs he considers.

***The Favero and Marcellino (2005)/Favero, Marcellino, Neglia (2004) identification scheme.*** These authors first estimate the static factors using a large panel of data, then construct a low-dimensional VAR that includes these static factors and a small number of observable variables; in Favero, Marcellino, and Neglia (2004), these additional variables are the output gap, inflation, commodity price inflation, an exchange rate, and the monetary policy instrument (the short rate). The monetary policy shock is identified by ordering the interest rate last in a Cholesky decomposition. In terms of Section 2, this scheme relaxes the DFM implications and is equivalent to allowing the variables in the VAR to be observable (static) factors. The Favero, Marcellino, and Neglia (2004) scheme orders both slow- and fast shocks ahead of the monetary policy shock, and assumes that there is no idiosyncratic or measurement error component $v_t^R$. This general approach – augmenting a low-dimensional VAR by estimated factors, then performing standard VAR identification – is inconsistent with the primitives of the DFM used to compute the factors, and it is unclear how this approach could provide a basis for compelling structural identification.

## 4. Empirical Results I: Number of Factors and Reduced-Form Variance Decompositions

We begin the empirical analysis by estimating the number of factors, the number of dynamic factors, and the dynamic factor innovations, and by computing forecast error variance decompositions with respect to the dynamic factor innovations, using the methods of Section 2.4.

### 4.1 The Data and Transformations

The data set consists of monthly observations on 132 U.S. macroeconomic time series from 1959:1 through 2003:12. The predictors include series in 14 categories: real output and income; employment and hours; real retail, manufacturing and trade sales; consumption; housing starts and sales; real inventories; orders; stock prices; exchange rates; interest rates and spreads; money and credit quantity aggregates; price indexes; average hourly earnings; and miscellaneous. The series are transformed by taking logarithms and/or differencing so that the transformed series are approximately stationary. In general, first differences of logarithms (growth rates) are used for real quantity variables, first differences are used for nominal interest rates, and second differences of logarithms (changes in growth rates) for price series. Specific transformations and the list of series is given in Appendix A.

Both outlier-adjusted and outlier-unadjusted versions of the series were used. The outlier adjustment entailed replacing observations of the transformed series with absolute median deviations larger than 6 times the inter quartile range by with the median value of the preceding 5 observations. The outlier-adjusted series were used for the estimation of the number of static and dynamic factors, the estimation of the static factors, and the estimation of the matrix $G$ relating the dynamic and static factor innovations. All other analysis (VAR estimation, estimating structural impulse responses, exclusion tests, etc.) was conducted using the outlier-unadjusted series.

### 4.2 Number of Static and Dynamic Factors

***Number of static factors***. The Bai-Ng information criteria $IC_{p1}$ and $IC_{p2}$ were computed both for the sample covariance matrix of $X_t$ and for the sample covariance matrix of the filtered $X_t$, $[I - \hat{D}(\mathrm{L})\,\mathrm{L}]X_t$, where the filter was computed using 6 lags for $\delta_i(\mathrm{L})$ ($m_i = 6$ for all $i$). When applied to $X_t$, the Bai-Ng criteria estimated there to be 7 static factors, although the criteria are nearly flat for $6 \le q \le 10$. When applied to the filtered $X_t$, the criteria estimate 9 static factors, and again the criterion is nearly flat for $6 \le q \le 10$. These results are robust to using 4 lags in $D(\mathrm{L})$ instead of 6. Our interest is in the space of dynamic factors, not the number of static factors, so to be conservative we choose the larger of these two estimates and adopt $\hat{q}$ = 9 static factors.

***Number of dynamic factors.*** Following the procedure of Section 2.4, the Bai-Ng information criteria $IC_{p2}$ was used to estimated the number of dynamic factors from the innovation matrix of $\hat{\varepsilon}_{Xt}$. The results are summarized in Table 1 for the baseline case in which the $X_t$ are prefiltered, $D$(L) having degree 5 ($m$ = 6), and in which the factors $F_t$ follow a VAR(2) (Φ(L) has degree one). For 7 or fewer static factors, the number of static and dynamic factors are estimated to be the same, however for more than 7 static factors, the number of dynamic factors is estimated to be 7. These results are robust to using instead 4 lags for $D$(L) or a VAR(1) for $F_t$.

We also estimated the number of static factors using unfiltered data ($X_t$, not $\tilde{X}_t$). If the number of static factors is taken to be less than or equal to 7, then the number of dynamic factors is estimated to equal the number of static factors; if the number of static factors is taken to be 8, 9, or 10, then 7 dynamic factors are estimated. Because the number of static factors is estimated to be 7 using the unfiltered data, the use of unfiltered data again result in an estimate of 7 dynamic factors.

In independent work reported contemporaneously with the first draft of this paper, Bai and Ng (2005b) developed a different estimator of $q$ (described in Section 2.4) and applied it to a similar, but not identical, large U.S. monthly macro data set. Strikingly, they also estimated seven dynamic factors. For this draft of this paper, we also implemented the Bai-Ng (2005b) estimator of $q$. Using our unfiltered data and the estimated value of 7 static factors, the Bai-Ng (2005b) method estimates 7 dynamic factors; using the filtered data and the estimated value of 9 static factors, the Bai-Ng (2005b) method estimates 9 dynamic factors.

Taken together, these results clearly point to a large number of dynamic factors in these data, with the modal estimate being 7; none of the estimates produced by any of our sensitivity checks or by using other methods is less than 7.

### 4.3. Why So Many Factors?

Our estimates of the number of static and dynamic factors exceeds those typically found in the literature (discussed in Section 1) and exceeds those we have found when we have focused on forecasting the main macroeconomic aggregates (Stock and Watson (1999, 2002a)). Is there actually only a few factors but the estimators spuriously indicate many? If not, are we simply detecting factors that are present in a statistical sense but are unimportant economically? If not, what are these extra factors doing?

To investigate the first question – the possibility that the estimators are biased upwards, and/or are quite imprecise –we undertook a Monte Carlo study to examine its performance. The study was calibrated to the data used in this analysis. In the Monte Carlo experiment, the estimator correctly estimated the number of dynamic factors with high probability, typically exceeding 94% and never less than 88%, and the estimate was within 1 of the true value of $q$ in more than 99% of Monte Carlo realizations. The design and results are presented in Appendix B. While these results are for just one design and more work is needed, they suggest that the performance of this estimator is promising and that the estimate of many factors is not spurious.

This preliminary Monte Carlo evidence suggests that the factors are statistically meaningful, but are they economically meaningful? To address this question, we examine the roles played by the various factors in explaining the movement of different macro variables. We begin by computing variance decompositions with respect to the different factor innovations. If only two of the factor innovations were important in an economic sense, the remaining five innovations ought to have a negligible role in explaining the variation of $X_t$.

The results are summarized in Table 2. Part A presents a summary of the cumulative forecast error variance decompositions for the $X$ variables at several horizons, and part B provides detailed results for the marginal contribution of each factor to each series at the 24 month horizon (similar results obtain at the 48 month horizon, and for a bandpass-filtered component over business cycle frequencies). Several features of these variance decompositions stand out. Although the first two factors explain on average 42% of the variation of these series at the 24 month horizon, the remaining factors together also explain a great deal of the variation, so that the average cumulative fraction explained by all seven factors rises to 56%. The percentages at other horizons are similar. Inspection of part B reveals that the first factor explains nearly all the variation in the major aggregates measuring production and hours; for example, the first factor explains 93% of the 24-month ahead forecast error variance of total industrial production, 91% of this quantity for capacity utilization, and 94% of this quantity for total employment. In contrast, this dynamic factor explains very little of the variation in inflation or stock returns at this horizon. The second factor explains movements in interest rates, consumption, and stock prices. The variation in inflation is mainly explained by the second and third factors, which together account for 65% of the forecast error variance of overall CPI inflation and 56% of the variance of the PCE deflator. The fourth factor is mainly associated with movements in interest rates. The fifth factor is associated with swings in long-term unemployment. The sixth and seventh factors mainly affect exchange rates, stock returns, and average hourly earnings. Although the factors explain much or most of the forecast error variance of most series, some series appear to be simply unrelated to these overall economic and financial factors. For example, employment in mining, medical price inflation, services price inflation, and growth of the monetary base are in the main unrelated to the overall economic conditions measured by the seven dynamic factors.

Figure 1 plots, for selected series, the business cycle component (computed using a bandpass filter with pass band of 24 – 96 months) and that part explained by various factors. These graphs confirm that the first factor explains most of the medium-run variation in industrial production, and the second and third factor explain most of the variation in price inflation. The fourth factor explains much of the variation of the 10-year T-bond rate. Taken together, the fifth, sixth, and seventh factors explain much of the variation in exchange rates and contribute to explaining the largest swings in long-term unemployment.

These results provide a more nuanced view of the general findings, surveyed in the introduction, that only two or three factors are needed to explain the covariation in U.S. economic time series. For the leading measures of real economic activity and prices, this appears to be true. Starting with Sargent and Sims (1977), many of the papers in this literature have focused on these series. In addition, for the purposes of forecasting either inflation or output growth, these forecast error variance decompositions suggest that perhaps only two or three factors are needed, a result consistent with the small number of factors in Stock and Watson (1999, 2002). The additional dynamic factors account for additional movements of the remaining series, which are mainly financial series such as interest rates, stock returns, and exchange rates. For the purposes of forecasting, it may suffice to use a small number of dynamic (and possibly static) factors, but for the purpose of structural VAR modeling the dimension of the space of dynamic factor innovations appears to be larger.

For the rest of the empirical analysis, we adopt a baseline specification of 9 static factors and 7 dynamic factors.

## 5. Empirical Results II: Testing VAR exclusion restrictions

This section examines empirically the restrictions on the reduced-form factor VAR summarized in section (2.3): that $X$ does not predict $F$ given lagged $F$, that $X_j$ does not predict $X_i$ given lagged $F$, and that $X_j$ does not explain $X_i$ given current $F$.

### 5.1 Restriction #4: *X* does not predict *F* given lagged *F*

We examine this restriction by sequentially including $X_j$ in (10), so that the factor prediction equation is,

$$F_t = \Phi(\mathrm{L})F_{t-1} + \Psi_j(\mathrm{L})X_{jt-1} + \varepsilon_{Ft,} \tag{39}$$

where $\Psi_j(\mathrm{L})$ is a $9\times1$ vector lag polynomial of degree five (so each row of $\Psi_j(\mathrm{L})$ has six unrestricted coefficients). Restriction #4 is that $\Psi_j(\mathrm{L}) = 0$, $j = 1,\ldots, 132$. For each of the nine equations in (39), we computed the six degree-of-freedom heteroskedasticity-robust chi-squared test of the hypothesis that the relevant row of $\Psi_j(\mathrm{L})$ is zero, along with the marginal $R^2$ (the increase in the $R^2$) from including $X_{jt-1}, \ldots, X_{jt-6}$. This yields $9\times132 = 1188$ separate test statistics and marginal $R^2$s. We do not report $p$-values for full test of $\Psi_j(\mathrm{L}) = 0$ because of doubts about accuracy of large-sample distribution theory in approximating the distribution of this test, which has 54 degrees of freedom.

The results of these 1188 exclusion tests are summarized in Table 3, which reports the percentiles of the marginal empirical distribution of these 1188 $p$-values and marginal $R^2$s. Under the hypothesis, one would expect 5% of the $p$-values to be less than

.05. Empirically, however, there are many more rejections than would be expected under the null: 10% of the $p$-values being less than .004 and 25% being less than .057. But these rejections are almost entirely associated with economically small improvements in the ability to predict $F$, with only one percent of the regressions being associated with improvements in the $R^2$ by .05 or more; 90% of the regressions have a marginal $R^2$ of.028 or less.

We continue the discussion of these results in Section 5.3, after examining the other VAR exclusion restrictions.

### 5.2 Restriction #5: $X_j$ does not predict $X_i$ given lagged $F$

We examine this restriction by augmenting (12) with lagged values of $X_{jt}$:

$$X_{it} = \Lambda_i \Phi(\mathrm{L}) F_{t-1} + \delta_i(\mathrm{L}) X_{it-1} + \delta_{ij}(\mathrm{L}) X_{jt-1} + \varepsilon_t \tag{40}$$

Restriction #5 is that $\delta_{ij}(\mathrm{L}) = 0$, $i, j = 1,\ldots, 132$, $i \neq j$.

We examine this restriction by estimating equation (40) for different dependent variables where, for each dependent variable, six lags of the remaining 131 $X$'s were included sequentially, yielding 131 separate heteroskedasticity-robust chi-squared statistics and marginal $R^2$s for each dependent variable. The estimation imposes no restrictions on $\Lambda_i\Phi(\mathrm{L})$. Taken across all 132 dependent variables, this produces 132×131 = 17,292 test statistics and marginal $R^2$s.

The results of these tests are reported in row (b) of Table 4(i) and 4(ii), which respectively presents the marginal distribution of these 17,292 $p$-values and marginal $R^2$s. For purposes of comparison, row (a) of the Table presents the corresponding marginal distributions of $p$-values and marginal $R^2$s for the regression specification omitting lagged $F$ (that is, omitting $\Lambda_i\Phi(\mathrm{L})F_{t-1}$ in (40)). The results indicate that there many more rejections of the exclusion restriction than would be expected under the null hypothesis: 10% of the $p$-values are less than .017. The marginal $R^2$s are generally small, with only 5% exceeding .026. Despite this evidence of statistically significant departures from the null, these departures are estimated to be quantitatively small. Moreover, there is evidence that including the factors substantially reduces the predictive content of $X_j$ for $X_i$

Inspection of the individual tests (not reported here to save space) revealed only one systematic pattern of rejections, which occurred when interest rates were used to predict other interest rates. One interpretation of this finding is that the estimated factors might not fully capture the dynamics of interest rate spreads. It might be that, consistent with results in the finance literature, a three-factor model is needed just to explain term structure dynamics, and our seven dynamic factors do not completely span these factors.

### 5.3 Restriction #6: $X_j$ does not explain $X_i$ given current $F$

If this restriction fails, then equation (9) becomes

$$X_{it} = \Lambda_i^j F_t + \delta_i^j(L) X_{it} + \alpha_{ij}(\mathrm{L}) X_{jt} + \nu_{it}^j, \tag{41}$$

where the superscript $j$ distinguishes these coefficients from those in (9) without $X_j$. Restriction #6 is that $\alpha_{ij}(\mathrm{L}) = 0$, which we examine by computing heteroskedasticity-robust chi-squared tests of the hypothesis that $X_{jt}$,.., $X_{jt-6}$ do not enter equation (41). The results are summarized in Table 4, row (d) of panels (i) and (ii). For comparison purposes, the corresponding results for the specification excluding $F_t$ are presented in row (c) of each panel. As is the case for the previous tests, there are an excess of rejections of $\alpha_{ij}(\mathrm{L}) = 0$ over what would be expected under the null. At the same time, there are substantially fewer rejections of the $X_j$ exclusion restrictions, once the factors are included in the regression. Including the factors produces a very large reduction in the marginal $R^2$s in these regressions.

The importance of restriction 6 is that if it holds, then the impulse responses with respect to dynamic factor structural shocks can be computed without including any other lags of $X$ in the VAR. Restriction 6, however, is sufficient but not necessary to justify the exclusion of $X_{jt}$ from (41). The necessary condition is simply that $\Lambda_i^j = \Lambda_i$, in which case the impulse responses and variance decompositions with respect to the dynamic factor structural shocks will not change upon inclusion of $X_{jt-1}$ in the VAR even if $\alpha_{ij}(\mathrm{L}) \neq 0$.

We therefore test directly the hypothesis that $\Lambda_i^j = \Lambda_i$ for all 17,292 case using a Hausman test testing for significant changes in the estimated values of $\Lambda_i$ when $X_{jt}$,.., $X_{jt-6}$

are included or excluded from the regression. The results are summarized in the final line of Table 4. There are many fewer excess rejections of this hypothesis than of the exclusion restriction hypotheses. Thus there is statistically significant evidence that the $X_j$ exclusion restrictions in the factor equations do not hold, but that these departures from the exact DFM result in few statistically significant changes in the coefficients on the factors in these equations; by implication, the impulse response functions with respect to the dynamic factor structural shocks would not change were $X_j$ to be included in (41).

### 5.4 Discussion

Taken at face value, the results of this section indicate widespread rejection of the exclusion restrictions of the DFM, yet at the same time the economic importance of these violations – as measured by marginal $R^2$s or statistically significant changes in the factor loadings $\Lambda$ upon including observable variables in the $X_j$ equations – generally is small.

There are at least three possible sources for these many violations: certain features of the data might make the exact DFM inapplicable, at least to some series; these many rejections might be statistical artifacts of the tests rejecting too often under the null; or the exact DFM might in fact not hold. We consider these possibilities in turn.

The first possibility is that these results reflect weaknesses in the data set. One specific weakness is that these data contain some series with overlapping coverage, for example the data set contains some series with several overlapping levels of aggregation (IP for consumer durables, IP for manufacturing, and Total IP), and mean unemployment duration is approximately a weighted average of the unemployment rate by length of spell. In the original units, if the DFM holds at the disaggregated level then the idiosyncratic disturbance in the aggregate will equal the sum of the idiosyncratic disturbances in the subaggregates, and the idiosyncratic terms will be correlated across series with overlapping scope. Thus the exact DFM might hold at a disaggregated level and we would still expect to see violations of the DFM within blocks of variables in this data set. For this reason, the approximate DFM might be a better description of these data than the exact DFM.

The second possibility is that these apparent violations might in fact be statistical artifacts. There are three reasons to believe that this might be an important issue. First, these regressions all involve estimated factors. Although the factor estimates are consistent, in finite samples the factors will contain estimation error. Standard errors-in-variables reasoning suggests that the estimation of the factors will reduce their predictive content and as a result the individual variables will retain some predictive content, even if in population they follow an exact DFM. This interpretation is consistent with the large fraction of rejections combined with the small marginal $R^2$s when individual $X$'s are included in either the $F$ or $X$ static DFM equations.

Second, most of these regressions contain quite a few regressors, which raises concerns about the applicability of conventional large-sample asymptotic theory.

Third, although some of the predictive relations uncovered by these tests – such as short rates having additional predictive content for long rates, given the factors – make economic sense, many do not. For example, residential building permits in the South has a relatively large marginal $R^2$ for predicting the first factor, but building permits in the Northeast or the Midwest, or housing starts in the south, do not. Although building permits in the South might in fact contain special information useful for forecasting this aggregate real output factor, its relatively high in-sample marginal $R^2$ could just be a statistical artifact.

The final possibility is that these tests have correctly detected violations of DFM restrictions. In this regard, we make three comments.

First, if $X_{jt}$ enters the $F_t$ equation only with a lag (restriction #4 fails), then this can still be consistent with estimating 9 static factors using the Bai-Ng (2002) criterion. Specifically, consider the modified model (9) and (39), where $\varepsilon_{Ft} = G\eta_t$. Then $\mathrm{E}F_t v_t = 0$ and the covariance matrix of $\tilde{X}_t$ still has the factor structure (11) and the Bai-Ng (2002) will estimate the dimension of the factor matrix to be $r$, the number of static factors. However, the spectral density matrix of $X_t$ does not have a factor structure (exact or approximate) at every frequency and in this sense the DFM fails. Moreover, the covariance matrix of $X_t$ (as opposed to $\tilde{X}_t$) does not have a factor structure (exact or approximate), so the estimated number of factors should differ, possibly substantially, depending on whether the series are filtered. But our estimates of the number of static factors are comparable whether the series are filtered or not, in fact the estimate is slightly less (not more) when the series are unfiltered.

Second, if current or lagged $X_{jt}$ enters the $X_{it}$ equation after conditioning on $F_t$ (restriction #5 fails), then neither the covariance matrices of $X_t$ nor that of $\tilde{X}_t$ will have a factor structure. In this case statistically significant evidence against restriction #5 is inconsistent with estimation of a handful of factors, at least in large samples. If there are only a few observable variables that predict $F_t$, then then those variables would be observable static factors; however the rejections are widespread, so this interpretation is not consistent with the empirical evidence.

Third, perhaps the series in fact obey a DFM but the Bai-Ng (2002) procedure has identified too few static factors. This would be consistent with the widespread rejections, and would indicate a difference between the Bai-Ng (2002) information criterion approach to the estimation of the factors and the significance testing approach of this section. But changing the number of static factors in this analysis does not substantially change the number of rejections of the DFM restrictions, so this explanation also is not fully consistent with the empirical results.

Taken together, these considerations and the results of Sections 4 and 5 lead us to conclude that the exact DFM model is an imperfect description of these data: many of its restrictions are violated. This said, there is strong evidence that there are a reduced number of linear combinations of the data – seven factors – that have considerable explanatory content for all the series. Given the factors, the violations of the exact DFM are small in an economic and quantitative sense. These findings are consistent with these series following an approximate DFM, in which there is some small correlation among the idiosyncratic components, given the factors.

As discussed in Section 2.5, the conceptual basis for the estimation of the factors and the factor innovations, and the associated distribution theory, has been developed for the approximate factor model. Moreover, the structural FAVAR innovation identification schemes and the associated two-step estimates (based on preliminary estimation of the factor innovations) hold under the approximate DFM, assuming $\Lambda_i = \Lambda_i^j$ in the notation of (41), a restriction that we found to be infrequently violated using the Hausman test. Although there remain some loose ends, such as the substantial rejections of the DFM restrictions among interest rate equations, we therefore interpret these results as supporting taking the next step of identifying and estimating structural FAVARs.

## 6. Empirical Results III: The BBE Structural FAVAR

This section illustrates the use of structural FAVARs by adopting the BBE identification scheme discussed in Section 4.2. We briefly review the economics of the BBE identification scheme, then turn to the empirical results.

### 6.1 The BBE Identification Scheme

The purpose of the BBE identification scheme is to identify a single structural shock, the monetary policy shock. Their scheme entails partitioning the series into three groups, slow, the interest rate, and fast. The slow moving variables, such as output and employment, are assumed to be unaffected within the month by the monetary policy shock or by shocks to financial markets. The $q_S$ shocks to the slow variables are the "slow shocks," $\zeta_t^S$. These slow shocks are assumed to be observed by the Fed, so that the monetary policy instrument (the Federal Funds rate) is a function of $\zeta_t^S$, the monetary policy shock $\zeta_t^R$, and an idiosyncratic disturbance. Finally, the remaining fast variables – stock returns, other interest rates, exchange rates, etc. – are assumed to be affected by the slow and monetary policy shocks and, in addition, to $q_F$ additional "fast" structural shocks to financial markets. These assumptions produce the identification scheme discussed in Section 4.2.[2]

Although most of this reasoning is conventional, one noteworthy point is that the Fed Funds specification allows for an idiosyncratic disturbance, a feature not present in a standard structural VAR implementation. This allows for institutional features that introduce slippage between monetary policy and monthly movements in the Fed Funds rate, for example the fact that the Fed Funds rate moves in 25 basis point increments and the tendency of the Fed to smooth a large interest rate movement over several quarters rather than to implement a large movement after a single meeting of the FOMC. Whether

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[2] In a precursor to this large-*n* approach, Leeper, Sims, and Zha (1996) identify the monetary policy shock as not affected by a large number of "sluggish" private sector variables in their 13- and 18-variable VARs.

the idiosyncratic disturbance is small or large quantitatively is an empirical matter that can be determined by applying the BBE identification scheme to the structural FAVAR.

The two differences between our implementation and BBE are differences in the data (which we believe to be minor) and differences in the estimation method. The estimation method used here is that described in Section 4.2; for the BBE estimation method, see their paper.

### 6.2 Baseline Empirical Results

Following BBE, our slow variables are output, employment, inventories, and broad-based price indexes (for a total of 67 slow variables) and the fast variables are interest rates, exchange rates, commodity prices, and stock returns (64 fast variables). The list is in Appendix A.

The first step is to estimate the number of dynamic factors among the slow variables, $q^S$. Like the estimation of the total number of dynamic factors (reported in Table 1), this was done by applying the Bai-Ng (2002) $IC_{p2}$ criterion to the sample covariance matrix of the estimated innovations $\hat{\varepsilon}_{Xt}^{S}$ in the slow variables. The results for the filtered data are summarized in Table 5. If fewer than four static factors are used, $q^S$ is estimated to be the number of static factors; if four or more static factors are used, $q^S$ is estimated to be 4. These estimates were computed for a VAR(2) for $F$ and 6 lags for $D$(L), and are robust to using either a VAR(1) for $F$ or 4 lags for $D$(L). The total number of possible static factors cannot exceed the total for the full panel, 9, and application of the Bai-Ng (2002) criterion to only the slow variables yields an estimate of 6 static factors; these statistics taken together therefore estimate $\hat{q}_S$ = 4 dynamic factors among the slow variables. This said, the Bai-Ng (2004) criterion is fairly flat in the region of $2 \leq q^S \leq 4$ so these results are consistent with the Sims-Sargent (1977) finding of only two quantitatively important dynamic factors among the slow variables.

As a robustness check, we also estimated the number of dynamic factors using the unfiltered data. For the unfiltered slow data, the number of static factors is estimated to be three, and the number of dynamic factors is estimated to be three. If we allow instead for up to 9 static factors to enter the unfiltered slow variables, the estimate of $q_S$ is either 4 or 5, depending on the number of static factors. These results suggest some ambiguity about $q_S$, which is estimated to be between 3 and 5; for the sequel, we adopt the modal estimate, based on the filtered data, of $\hat{q}_S$ = 4; the estimated number of fast shocks therefore is $\hat{q}_F = \hat{q} - \hat{q}_S - 1 = 2$.

Empirical results for the structural FAVAR with $q_S = 4$ and $q_F = 2$ are summarized in Table 6. The first block of columns reports impulse responses to a monetary policy shock, normalized so to correspond to a 1 percentage point increase in the Federal Funds rate. The second block of columns reports the fraction of the forecast error variance explained by the monetary policy shock at different horizons. The next block of columns examines the overidentifying restrictions, equation by equation. The final two columns report the fraction of the innovation variance explained by the slow and fast shocks, respectively. The results in Table 6 were computed using the "levels" algorithm for the BBE identification scheme of Section 4.3. The overidentifying restrictions were imposed for identification of the shocks, however the impulse response functions were not estimated subject to that restriction; that is, the impulse response function was estimated as $\hat{B}^*(\mathrm{L}) = [I - \hat{D}(\mathrm{L})\mathrm{L}]^{-1}\hat{\Lambda}[I - \hat{\Phi}(\mathrm{L})\mathrm{L}]^{-1}\hat{G}\hat{H}^{-1}$, where all matrices except $\hat{H}$ were estimated using the methods of Section 2. Thus the first column, the impact effect of the monetary policy shock, can be estimated to be nonzero even though the shock is identified by assuming this effect is nonzero. Repeating the estimation using the "innovations" algorithm of Section 4.3 yielded results similar to those from the "levels" algorithm, so to save space the discussion here focuses on the results in Table 6.

Although we use a somewhat different identification strategy and a different estimation method, the results in Table 6 generally accord with those of BBE and, as do theirs, with standard theory. A monetary policy shock that initially increases the Fed Funds rate by 100 basis points is estimated to be highly persistent, with the Fed Funds rate still elevated by 80 basis points after three years. Output and employment contract, with total employment falling by 0.5%, and IP falling by 1.0% after one year, relative to the no-shock benchmark. The contraction is felt more strongly in some sectors, for example construction and goods-producing sectors, than in others, for example finance and services. The stock market enters a pronounced decline in response to the contraction, with the S&P500 loosing 11% of its value within 6 months. As in Eichenbaum and Evans (1995), a contractionary monetary policy shock leads to a large and persistent appreciation of the dollar relative to other currencies.

On the other hand, there are some curious features of the responses. There remains some puzzling price behavior: while PCE inflation responds immediately by falling .2% (annual rate) and continues to fall at the rate of 0.2% per year thereafter, CPI inflation initially rises by 0.2% and does not appear to fall thereafter. Also, the contraction is associated with a temporary steepening of the yield curve.

The fraction of the variance explained by the monetary policy shock is estimated to be small for most of the real quantity variables and for prices. These estimates are somewhat smaller than results found in conventional (observable variable) SVAR analysis (see Christiano, Eichenbaum, and Evans (1999), for example). The monetary policy shocks are estimated to account for a substantial fraction of the variability of interest rates and, at horizons of one to three years, for substantial fractions of the variability of retail sales, residential building permits, and the growth of M2.

The tests of the identifying restrictions in the final columns, along with the estimated impact effect of the monetary policy shock on the slow variables, provides a way to assess how well the BBE overidentifying assumptions fit the data. For most of the slow-moving variables, the assumption that the shock has no immediate effect is not rejected at the 5% significance level, and the estimated impact effect of the monetary policy shock is small. Exceptions to this general statement include the NAPM production and employment indexes and the short-term unemployment rate (but, oddly, not unemployment insurance claims). Notably, the PCE deflator for durables increases sharply within the month in response to the monetary policy shock (*p*-value = .026). Also, there are widespread rejections of the restriction that the fast shocks not enter the slow variables. For the slow variables, such as total consumption, retail sales, and IP for consumer durables, the fast shocks explain nearly 10% or more of the innovation variance.

## 7. Summary

One of our three main empirical findings is that there seem to be a relatively large number of dynamic factors that account for the movements in these data: between two and four that account for the movement in output, employment, and price inflation, and between 3 and 5 more that account for additional movements in financial variables. These are many more factors than have been found by previous researchers, starting with Sims and Sargent (1977). A partial resolution of this conflict is that early researchers, including Sims and Sargent (1977), mainly focused on output, employment, and inflation, for which a small number of factors is plausible, but conflicts remain between our results and those of researchers (e.g. Giannone, Reichlin, and Sala (2004)) who have also used large data sets with a diverse range of variables.

A second empirical finding is evidence against the VAR restrictions implied by the exact DFM. Although many of these violations are estimated to be small from an economic perspective, a few of them are large enough to suggest possible misspecification in our base model. We interpret these results as suggesting that these data are well described by an approximate factor model, but not an exact factor model, however further work along the lines indicated at the end of Section 5 is needed.

Our third main finding is that the support for the BBE identification scheme is mixed. On the one hand, most of the impulse responses accord with standard macroeconomic theories. The full set of impulse responses in Table 6 demonstrate, as do BBE, that these methods can be used to map out the path for many variables after a single shock, thereby addressing the common criticism of structural VARs that they are silent about many of the variables of interest to policymakers. On the other hand, many of the overidentifying restrictions are violated, and some of the estimated impulse responses do not accord with monetary theory. This situation could be a statistical artifact, it could be a feature readily addressed by modifying the data set (perhaps changing the composition of the slow variables), or it might be a fundamental flaw in the recursive identification scheme. Understanding the source of these rejections is an obvious next step for structural FAVAR research. From a methodological perspective, finding mixed support for the BBE identification scheme represents an advance over exactly identified structural VAR analysis: the structural FAVAR framework permits examination of overidentifying restrictions and diagnosis of modeling problems.

## Appendix A: Data

Table A.1 lists the short name of each series, its mnemonic (the series label used in the source database), the transformation applied to the series, and a brief data description. All series are from the Global Insights Basic Economics Database, unless the source is listed (in parentheses) as TCB (The Conference Board's Indicators Database) or AC (author's calculation based on Global Insights or TCB data). In the transformation column, ln denotes logarithm, Δln and $\Delta^2$ln denote the first and second difference of the logarithm, lv denotes the level of the series, and Δlv denotes the first difference of the series.

### Table A.1 Data sources, transformations, and definitions

| Short name | Mnemonic | Fast or Slow? | Tran | Description |
|---|---|---|---|---|
| PI | a0m052 | S | Δln | Personal Income (AR, Bil. Chain 2000 $) (TCB) |
| PI less transfers | a0m051 | S | Δln | Personal Income Less Transfer Payments (AR, Bil. Chain 2000 $) (TCB) |
| Consumption | a0m224_r | S | Δln | Real Consumption (AC) a0m224/gmdc (a0m224 is from TCB) |
| M&T sales | a0m057 | S | Δln | Manufacturing And Trade Sales (Mil. Chain 1996 $) (TCB) |
| Retail sales | a0m059 | S | Δln | Sales Of Retail Stores (Mil. Chain 2000 $) (TCB) |
| IP: total | ips10 | S | Δln | Industrial Production Index - Total Index |
| IP: products | ips11 | S | Δln | Industrial Production Index - Products, Total |
| IP: final prod | ips299 | S | Δln | Industrial Production Index - Final Products |
| IP: cons gds | ips12 | S | Δln | Industrial Production Index - Consumer Goods |
| IP: cons dble | ips13 | S | Δln | Industrial Production Index - Durable Consumer Goods |
| IP: cons nondble | ips18 | S | Δln | Industrial Production Index - Nondurable Consumer Goods |
| IP: bus eqpt | ips25 | S | Δln | Industrial Production Index - Business Equipment |
| IP: matls | ips32 | S | Δln | Industrial Production Index - Materials |
| IP: dble matls | ips34 | S | Δln | Industrial Production Index - Durable Goods Materials |
| IP: nondble matls | ips38 | S | Δln | Industrial Production Index - Nondurable Goods Materials |
| IP: mfg | ips43 | S | Δln | Industrial Production Index - Manufacturing (Sic) |
| IP: res util | ips307 | S | Δln | Industrial Production Index - Residential Utilities |
| IP: fuels | ips306 | S | Δln | Industrial Production Index - Fuels |
| NAPM prodn | pmp | S | lv | Napm Production Index (Percent) |
| Cap util | a0m082 | S | Δlv | Capacity Utilization (Mfg) (TCB) |
| Help wanted indx | lhel | S | Δlv | Index Of Help-Wanted Advertising In Newspapers (1967=100;Sa) |
| Help wanted/emp | lhelx | S | Δlv | Employment: Ratio; Help-Wanted Ads:No. Unemployed Clf |
| Emp CPS total | lhem | S | Δln | Civilian Labor Force: Employed, Total (Thous.,Sa) |
| Emp CPS nonag | lhnag | S | Δln | Civilian Labor Force: Employed, Nonagric.Industries (Thous.,Sa) |
| U: all | lhur | S | Δlv | Unemployment Rate: All Workers, 16 Years & Over (%,Sa) |
| U: mean duration | lhu680 | S | Δlv | Unemploy.By Duration: Average(Mean)Duration In Weeks (Sa) |
| U < 5 wks | lhu5 | S | Δln | Unemploy.By Duration: Persons Unempl.Less Than 5 Wks (Thous.,Sa) |
| U 5-14 wks | lhu14 | S | Δln | Unemploy.By Duration: Persons Unempl.5 To 14 Wks (Thous.,Sa) |
| U 15+ wks | lhu15 | S | Δln | Unemploy.By Duration: Persons Unempl.15 Wks + (Thous.,Sa) |
| U 15-26 wks | lhu26 | S | Δln | Unemploy.By Duration: Persons Unempl.15 To 26 Wks (Thous.,Sa) |
| U 27+ wks | lhu27 | S | Δln | Unemploy.By Duration: Persons Unempl.27 Wks + (Thous,Sa) |
| UI claims | a0m005 | S | Δln | Average Weekly Initial Claims, Unemploy. Insurance (Thous.) (TCB) |
| Emp: total | ces002 | S | Δln | Employees On Nonfarm Payrolls: Total Private |
| Emp: gds prod | ces003 | S | Δln | Employees On Nonfarm Payrolls - Goods-Producing |
| Emp: mining | ces006 | S | Δln | Employees On Nonfarm Payrolls - Mining |
| Emp: const | ces011 | S | Δln | Employees On Nonfarm Payrolls - Construction |
| Emp: mfg | ces015 | S | Δln | Employees On Nonfarm Payrolls - Manufacturing |
| Emp: dble gds | ces017 | S | Δln | Employees On Nonfarm Payrolls - Durable Goods |

| | | | | |
|---|---|---|---|---|
| Emp: nondbles | ces033 | S | $\Delta$ln | Employees On Nonfarm Payrolls - Nondurable Goods |
| Emp: services | ces046 | S | $\Delta$ln | Employees On Nonfarm Payrolls - Service-Providing |
| Emp: TTU | ces048 | S | $\Delta$ln | Employees On Nonfarm Payrolls - Trade, Transportation, And Utilities |
| Emp: wholesale | ces049 | S | $\Delta$ln | Employees On Nonfarm Payrolls - Wholesale Trade |
| Emp: retail | ces053 | S | $\Delta$ln | Employees On Nonfarm Payrolls - Retail Trade |
| Emp: FIRE | ces088 | S | $\Delta$ln | Employees On Nonfarm Payrolls - Financial Activities |
| Emp: Govt | ces140 | S | $\Delta$ln | Employees On Nonfarm Payrolls - Government |
| Emp-hrs nonag | a0m048 | S | $\Delta$ln | Employee Hours In Nonag. Establishments (AR, Bil. Hours) (TCB) |
| Avg hrs | ces151 | S | lv | Avg Weekly Hrs of Prod or Nonsup Workers On Private Nonfarm Payrolls - Goods-Producing |
| Overtime: mfg | ces155 | S | $\Delta$lv | Avg Weekly Hrs of Prod or Nonsup Workers On Private Nonfarm Payrolls - Mfg Overtime Hours |
| Avg hrs: mfg | aom001 | S | lv | Average Weekly Hours, Mfg. (Hours) (TCB) |
| NAPM empl | pmemp | S | lv | Napm Employment Index (Percent) |
| Starts: nonfarm | hsfr | S | ln | Housing Starts:Nonfarm(1947-58);Total Farm&Nonfarm(1959-)(Thous.,Saar) |
| Starts: NE | hsne | F | ln | Housing Starts:Northeast (Thous.U.)S.A. |
| Starts: MW | hsmw | F | ln | Housing Starts:Midwest(Thous.U.)S.A. |
| Starts: South | hssou | F | ln | Housing Starts:South (Thous.U.)S.A. |
| Starts: West | hswst | F | ln | Housing Starts:West (Thous.U.)S.A. |
| BP: total | hsbr | F | ln | Housing Authorized: Total New Priv Housing Units (Thous.,Saar) |
| BP: NE | hsbne* | F | ln | Houses Authorized By Build. Permits:Northeast(Thou.U.)S.A |
| BP: MW | hsbmw* | F | ln | Houses Authorized By Build. Permits:Midwest(Thou.U.)S.A. |
| BP: South | hsbsou* | F | ln | Houses Authorized By Build. Permits:South(Thou.U.)S.A. |
| BP: West | hsbwst* | F | ln | Houses Authorized By Build. Permits:West(Thou.U.)S.A. |
| PMI | pmi | F | lv | Purchasing Managers' Index (Sa) |
| NAPM new ordrs | pmno | F | lv | Napm New Orders Index (Percent) |
| NAPM vendor del | pmdel | F | lv | Napm Vendor Deliveries Index (Percent) |
| NAPM Invent | pmnv | F | lv | Napm Inventories Index (Percent) |
| Orders: cons gds | a0m008 | F | $\Delta$ln | Mfrs' New Orders, Consumer Goods And Materials (Bil. Chain 1982 $) (TCB) |
| Orders: dble gds | a0m007 | F | $\Delta$ln | Mfrs' New Orders, Durable Goods Industries (Bil. Chain 2000 $) (TCB) |
| Orders: cap gds | a0m027 | F | $\Delta$ln | Mfrs' New Orders, Nondefense Capital Goods (Mil. Chain 1982 $) (TCB) |
| Unf orders: dble | a1m092 | F | $\Delta$ln | Mfrs' Unfilled Orders, Durable Goods Indus. (Bil. Chain 2000 $) (TCB) |
| M&T invent | a0m070 | F | $\Delta$ln | Manufacturing And Trade Inventories (Bil. Chain 2000 $) (TCB) |
| M&T invent/sales | a0m077 | F | $\Delta$lv | Ratio, Mfg. And Trade Inventories To Sales (Based On Chain 2000 $) (TCB) |
| M1 | fm1 | F | $\Delta^2$ln | Money Stock: M1(Curr,Trav.Cks,Dem Dep,Other Ck'able Dep)(Bil$,Sa) |
| M2 | fm2 | F | $\Delta^2$ln | Money Stock:M2(M1+O'nite Rps,Euro$,G/P&B/D Mmmfs&Sav&Sm Time Dep(Bil$,Sa) |
| M3 | fm3 | F | $\Delta^2$ln | Money Stock: M3(M2+Lg Time Dep,Term Rp's&Inst Only Mmmfs)(Bil$,Sa) |
| M2 (real) | fm2dq | F | $\Delta$ln | Money Supply - M2 In 1996 Dollars (Bci) |
| MB | fmfba | F | $\Delta^2$ln | Monetary Base, Adj For Reserve Requirement Changes(Mil$,Sa) |
| Reserves tot | fmrra | F | $\Delta^2$ln | Depository Inst Reserves:Total, Adj For Reserve Req Chgs(Mil$,Sa) |
| Reserves nonbor | fmrnba | F | $\Delta^2$ln | Depository Inst Reserves:Nonborrowed,Adj Res Req Chgs(Mil$,Sa) |
| C&I loans | fclnq | F | $\Delta^2$ln | Commercial & Industrial Loans Oustanding In 1996 Dollars (Bci) |
| $\Delta$C&I loans | fclbmc | F | lv | Wkly Rp Lg Com'l Banks:Net Change Com'l & Indus Loans(Bil$,Saar) |
| Cons credit | ccinrv | F | $\Delta^2$ln | Consumer Credit Outstanding - Nonrevolving(G19) |
| Inst cred/PI | a0m095 | F | $\Delta$lv | Ratio, Consumer Installment Credit To Personal Income (Pct.) (TCB) |
| S&P 500 | fspcom | F | $\Delta$ln | S&P's Common Stock Price Index: Composite (1941-43=10) |
| S&P: indust | fspin | F | $\Delta$ln | S&P's Common Stock Price Index: Industrials (1941-43=10) |
| S&P div yield | fsdxp | F | $\Delta$lv | S&P's Composite Common Stock: Dividend Yield (% Per Annum) |
| S&P PE ratio | fspxe | F | $\Delta$ln | S&P's Composite Common Stock: Price-Earnings Ratio (%,Nsa) |
| Fed Funds | fyff | F | $\Delta$lv | Interest Rate: Federal Funds (Effective) (% Per Annum,Nsa) |
| Comm paper | cp90 | F | $\Delta$lv | Cmmercial Paper Rate (AC) |
| 3 mo T-bill | fygm3 | F | $\Delta$lv | Interest Rate: U.S.Treasury Bills,Sec Mkt,3-Mo.(% Per Ann,Nsa) |
| 6 mo T-bill | fygm6 | F | $\Delta$lv | Interest Rate: U.S.Treasury Bills,Sec Mkt,6-Mo.(% Per Ann,Nsa) |
| 1 yr T-bond | fygt1 | F | $\Delta$lv | Interest Rate: U.S.Treasury Const Maturities,1-Yr.(% Per Ann,Nsa) |
| 5 yr T-bond | fygt5 | F | $\Delta$lv | Interest Rate: U.S.Treasury Const Maturities,5-Yr.(% Per Ann,Nsa) |
| 10 yr T-bond | fygt10 | F | $\Delta$lv | Interest Rate: U.S.Treasury Const Maturities,10-Yr.(% Per Ann,Nsa) |
| Aaa bond | fyaaac | F | $\Delta$lv | Bond Yield: Moody's Aaa Corporate (% Per Annum) |
| Baa bond | fybaac | F | $\Delta$lv | Bond Yield: Moody's Baa Corporate (% Per Annum) |
| CP-FF spread | scp90 | F | lv | cp90-fyff (AC) |
| 3 mo-FF spread | sfygm3 | F | lv | fygm3-fyff (AC) |
| 6 mo-FF spread | sfygm6 | F | lv | fygm6-fyff (AC) |
| 1 yr-FF spread | sfygt1 | F | lv | fygt1-fyff (AC) |
| 5 yr-FF spread | sfygt5 | F | lv | fygt5-fyff (AC) |
| 10 yr-FF spread | sfygt10 | F | lv | fygt10-fyff (AC) |
| Aaa-FF spread | sfyaaac | F | lv | fyaaac-fyff (AC) |

| | | | | |
|---|---|---|---|---|
| Baa-FF spread | sfybaac | F | lv | fybaac-fyff (AC) |
| Ex rate: avg | exrus | F | $\Delta$ln | United States;Effective Exchange Rate(Merm)(Index No.) |
| Ex rate: Switz | exrsw | F | $\Delta$ln | Foreign Exchange Rate: Switzerland (Swiss Franc Per U.S.$) |
| Ex rate: Japan | exrjan | F | $\Delta$ln | Foreign Exchange Rate: Japan (Yen Per U.S.$) |
| Ex rate: UK | exruk | F | $\Delta$ln | Foreign Exchange Rate: United Kingdom (Cents Per Pound) |
| EX rate: Canada | exrcan | F | $\Delta$ln | Foreign Exchange Rate: Canada (Canadian $ Per U.S.$) |
| PPI: fin gds | pwfsa | F | $\Delta^2$ln | Producer Price Index: Finished Goods (82=100,Sa) |
| PPI: cons gds | pwfcsa | F | $\Delta^2$ln | Producer Price Index: Finished Consumer Goods (82=100,Sa) |
| PPI: int mat'ls | pwimsa | F | $\Delta^2$ln | Producer Price Index:I ntermed Mat.Supplies & Components(82=100,Sa) |
| PPI: crude mat'ls | pwcmsa | F | $\Delta^2$ln | Producer Price Index: Crude Materials (82=100,Sa) |
| Spot market price | psccom | F | $\Delta^2$ln | Spot market price index: bls & crb: all commodities(1967=100) |
| Sens mat'ls price | psm99q | F | $\Delta^2$ln | Index Of Sensitive Materials Prices (1990=100)(Bci-99a) |
| NAPM com price | pmcp | F | lv | Napm Commodity Prices Index (Percent) |
| CPI-U: all | punew | S | $\Delta^2$ln | Cpi-U: All Items (82-84=100,Sa) |
| CPI-U: apparel | pu83 | S | $\Delta^2$ln | Cpi-U: Apparel & Upkeep (82-84=100,Sa) |
| CPI-U: transp | pu84 | S | $\Delta^2$ln | Cpi-U: Transportation (82-84=100,Sa) |
| CPI-U: medical | pu85 | S | $\Delta^2$ln | Cpi-U: Medical Care (82-84=100,Sa) |
| CPI-U: comm. | puc | S | $\Delta^2$ln | Cpi-U: Commodities (82-84=100,Sa) |
| CPI-U: dbles | pucd | S | $\Delta^2$ln | Cpi-U: Durables (82-84=100,Sa) |
| CPI-U: services | pus | S | $\Delta^2$ln | Cpi-U: Services (82-84=100,Sa) |
| CPI-U: ex food | puxf | S | $\Delta^2$ln | Cpi-U: All Items Less Food (82-84=100,Sa) |
| CPI-U: ex shelter | puxhs | S | $\Delta^2$ln | Cpi-U: All Items Less Shelter (82-84=100,Sa) |
| CPI-U: ex med | puxm | S | $\Delta^2$ln | Cpi-U: All Items Less Midical Care (82-84=100,Sa) |
| PCE defl | gmdc | S | $\Delta^2$ln | Pce, Impl Pr Defl:Pce (1987=100) |
| PCE defl: dlbes | gmdcd | S | $\Delta^2$ln | Pce, Impl Pr Defl:Pce; Durables (1987=100) |
| PCE defl: nondble | gmdcn | S | $\Delta^2$ln | Pce, Impl Pr Defl:Pce; Nondurables (1996=100) |
| PCE defl: service | gmdcs | S | $\Delta^2$ln | Pce, Impl Pr Defl:Pce; Services (1987=100) |
| AHE: goods | ces275 | S | $\Delta^2$ln | Avg Hourly Earnings of Prod or Nonsup Workers On Private Nonfarm Payrolls - Goods-Producing |
| AHE: const | ces277 | S | $\Delta^2$ln | Avg Hourly Earnings of Prod or Nonsup Workers On Private Nonfarm Payrolls - Construction |
| AHE: mfg | ces278 | S | $\Delta^2$ln | Avg Hourly Earnings of Prod or Nonsup Workers On Private Nonfarm Payrolls - Manufacturing |
| Consumer expect | hhsntn | F | $\Delta$lv | U. Of Mich. Index Of Consumer Expectations(Bcd-83) |

## Appendix B: Monte Carlo Results on the Estimation of *q*

This Monte Carlo study examines the statistical performance of the new estimator, described in Section 2.4, of the number of dynamic factors $q$. The values of $T$ and $N$ are the same as those in the data ($T = 528$ [1960:1-2003:12], $N = 132$). The factor loadings $\Lambda$ were set at the fitted values from the data, and $D(\mathrm{L})$ was set at the fitted value. The static factors $F_t$ were generated by AR(2) with parameter values set to fitted values from data. The idiosyncratic disturbance follows the process,

$$v_{it} = \sigma_{v_i} \times [0.2\times e_{i-1,t} + (0.92)^{1/2} e_{it} + 0.2 e_{i+1,t}]$$

where

$$e_{it} = \sigma_{it}^{1/2}\, a_{it}\ ;\ a_{it} \text{ iid } N(0,1);\ \sigma_{it} = 0.1 + 0.45\times\sigma_{it-1} + 0.45\times e_{it-1}^2$$

where $\sigma_{v_i}$ is the estimated standard deviation of $v_{it}$ in the data. (The parameters are such that $\mathrm{var}(e_{it}) = 1$ in the simulations.) Note that the unconditional correlation between adjacent uniquenesses is 0.4, slightly greater than the correlation of 0.3 in the data.

Ten cases were considered, in which the true number of factors was set to $r = 1,\ldots,10$. The maximum number of dynamic factors considered in each case was 10. The procedure of Section 2.4 was then applied, in brief: the static factors, filters, and factor loadings were estimated; the $X_t$ innovations were estimated; and the Bai-Ng (2002) $IC_{p2}$ procedure was used to estimate $q$. The number of Monte Carlo replications was 500 (the slow step in this process is the estimation of the filter $D(\mathrm{L})$).

The results are summarized in Table B.1. In all cases, the true number of factors was correctly estimated with high probability.

**Table B.1 Monte Carlo distribution of the estimated number of dynamic factors**

| Estimated Number of Factors | True Number of Factors | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | 1.00 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 2 | 0.00 | 0.98 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 3 | 0.00 | 0.00 | 1.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 4 | 0.00 | 0.00 | 0.00 | 0.99 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 5 | 0.00 | 0.00 | 0.00 | 0.00 | 0.99 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 6 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.98 | 0.00 | 0.00 | 0.00 | 0.00 |
| 7 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.97 | 0.00 | 0.00 | 0.00 |
| 8 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.03 | 0.96 | 0.00 | 0.00 |
| 9 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.04 | 0.88 | 0.00 |
| 10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.12 | 1.00 |

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Table 1
Estimation of the Number of Dynamic Factors $q$

| # dynamic factors ($q$) | **$r$ = 1** | **$r$ = 2** | **$r$ = 3** | **$r$ = 4** | **$r$ = 5** | **$r$ = 6** | **$r$ = 7** | **$r$ = 8** | **$r$ = 9** | **$r$ = 10** |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | **-0.577** | -0.589 | -0.593 | -0.604 | -0.615 | -0.624 | -0.630 | -0.637 | -0.642 | -0.649 |
| 2 | . | **-0.637** | -0.641 | -0.649 | -0.659 | -0.664 | -0.668 | -0.677 | -0.680 | -0.686 |
| 3 | . | . | **-0.676** | -0.683 | -0.694 | -0.699 | -0.703 | -0.710 | -0.714 | -0.719 |
| 4 | . | . | . | **-0.693** | -0.704 | -0.708 | -0.713 | -0.720 | -0.724 | -0.730 |
| 5 | . | . | . | . | **-0.712** | -0.717 | -0.722 | -0.729 | -0.733 | -0.739 |
| 6 | . | . | . | . | . | **-0.719** | -0.723 | -0.731 | -0.735 | -0.741 |
| 7 | . | . | . | . | . | . | **-0.726** | **-0.734** | **-0.738** | **-0.744** |
| 8 | . | . | . | . | . | . | . | -0.732 | -0.738 | -0.743 |
| 9 | . | . | . | . | . | . | . | . | -0.734 | -0.740 |
| 10 | . | . | . | . | . | . | . | . | . | -0.736 |

Notes: Entries are the Bai-Ng (2002) $IC_{p2}$ criterion, evaluated using the sample covariance matrix of the estimated innovations in $X_t$ from the restricted VAR implied by the DFM. Each entry reports the $IC_{p2}$ for the number of static factors $r$ given in the column heading and the number of dynamic factors $q$ given in the row. Estimates of $q$ given $r$ (the column maximum of $IC_{p2}$) are presented in bold.

**Table 2**
**Forecast Error Variance Decomposition with respect to Factor Innovations**

**A. Forecast Error Variance Decompositions, Averaged over All Series**

| Horizon | Idiosyncratic | Cumulative fraction of the variance explained by dynamic factors 1, …, *q*: | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | *q* = 1 | *q* = 2 | *q* = 3 | *q* = 4 | *q* = 5 | *q* = 6 | *q* = 7 |
| 6 month | 0.49 | 0.28 | 0.36 | 0.40 | 0.44 | 0.47 | 0.49 | 0.51 |
| 12 month | 0.45 | 0.33 | 0.40 | 0.45 | 0.47 | 0.51 | 0.53 | 0.55 |
| 24 month | 0.44 | 0.35 | 0.42 | 0.46 | 0.49 | 0.52 | 0.54 | 0.56 |
| 48 month | 0.43 | 0.36 | 0.43 | 0.47 | 0.50 | 0.53 | 0.55 | 0.57 |
| | | | | | | | | |
| Bus cycle freqs | 0.42 | 0.37 | 0.44 | 0.49 | 0.52 | 0.55 | 0.56 | 0.58 |

Notes: For the first four rows of the table, the entry in the first numeric column is the fraction of the variance of the forecast error explained by the idiosyncratic disturbance $v_{it}$. The entries in the remaining columns are the cumulative fraction of the variance explained by the dynamic innovations, up to and including the dynamic innovation in the column heading. The final row presents analogous results for the business cycle band-passed series. The seven dynamic factors were computed as described in Section 2.4.

## Table 2 (Continued)

## B. 24 Month Ahead Forecast Error Decompositions for Individual Series

| $X_i$ Series | Idio-syncratic | Fraction of variance explained by dynamic factors 1, …, *q*: | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Total |
| PI | 0.53 | **0.40** | 0.01 | 0.01 | 0.00 | 0.03 | 0.02 | 0.00 | 0.47 |
| PI less transfers | 0.41 | **0.55** | 0.00 | 0.01 | 0.00 | 0.01 | 0.02 | 0.00 | 0.59 |
| Consumption | 0.32 | **0.48** | **0.14** | 0.01 | 0.00 | 0.04 | 0.01 | 0.01 | 0.68 |
| M&T sales | 0.12 | **0.81** | 0.05 | 0.01 | 0.00 | 0.01 | 0.00 | 0.00 | 0.88 |
| Retail sales | 0.29 | **0.47** | **0.15** | 0.01 | 0.00 | 0.05 | 0.00 | 0.02 | 0.71 |
| IP: total | 0.02 | **0.93** | 0.01 | 0.00 | 0.01 | 0.02 | 0.00 | 0.01 | 0.98 |
| IP: products | 0.03 | **0.92** | 0.01 | 0.00 | 0.02 | 0.01 | 0.01 | 0.01 | 0.97 |
| IP: final prod | 0.04 | **0.89** | 0.01 | 0.00 | 0.04 | 0.01 | 0.01 | 0.01 | 0.96 |
| IP: cons gds | 0.07 | **0.76** | 0.01 | 0.01 | 0.06 | 0.04 | 0.03 | 0.02 | 0.93 |
| IP: cons dble | 0.12 | **0.73** | 0.01 | 0.00 | 0.05 | 0.06 | 0.00 | 0.04 | 0.88 |
| IP:cons nondble | 0.38 | **0.44** | 0.01 | 0.01 | 0.07 | 0.02 | 0.09 | 0.00 | 0.62 |
| IP:bus eqpt | 0.15 | **0.82** | 0.01 | 0.00 | 0.01 | 0.00 | 0.00 | 0.01 | 0.85 |
| IP: matls | 0.10 | **0.85** | 0.01 | 0.00 | 0.01 | 0.03 | 0.00 | 0.01 | 0.90 |
| IP: dble mats | 0.11 | **0.82** | 0.00 | 0.00 | 0.01 | 0.03 | 0.00 | 0.02 | 0.89 |
| IP:nondble mats | 0.24 | **0.74** | 0.01 | 0.01 | 0.00 | 0.01 | 0.00 | 0.00 | 0.76 |
| IP: mfg | 0.02 | **0.94** | 0.01 | 0.00 | 0.01 | 0.01 | 0.00 | 0.01 | 0.98 |
| IP: res util | 0.78 | 0.00 | 0.01 | 0.01 | 0.04 | 0.00 | **0.12** | 0.04 | 0.22 |
| IP: fuels | 0.88 | 0.03 | 0.00 | 0.07 | 0.00 | 0.01 | 0.00 | 0.00 | 0.12 |
| NAPM prodn | 0.35 | **0.56** | 0.03 | 0.00 | 0.02 | 0.02 | 0.00 | 0.01 | 0.65 |
| Cap util | 0.03 | **0.91** | 0.00 | 0.00 | 0.01 | 0.03 | 0.00 | 0.01 | 0.97 |
| Hlp want. indx | 0.30 | **0.65** | 0.02 | 0.00 | 0.02 | 0.00 | 0.00 | 0.00 | 0.70 |
| Hlp want./emp | 0.18 | **0.77** | 0.01 | 0.00 | 0.01 | 0.01 | 0.00 | 0.01 | 0.82 |
| Emp CPS total | 0.12 | **0.76** | 0.01 | 0.00 | 0.01 | 0.06 | 0.01 | 0.03 | 0.88 |
| Emp CPSnonag | 0.12 | **0.76** | 0.01 | 0.00 | 0.01 | 0.07 | 0.00 | 0.03 | 0.88 |
| U: all | 0.08 | **0.85** | 0.01 | 0.00 | 0.00 | 0.06 | 0.00 | 0.00 | 0.92 |
| U: mean dur. | 0.31 | **0.31** | 0.01 | 0.01 | 0.00 | **0.30** | 0.01 | 0.06 | 0.69 |
| U < 5 wks | 0.49 | **0.46** | 0.01 | 0.00 | 0.01 | 0.00 | 0.00 | 0.02 | 0.51 |
| U 5-14 wks | 0.22 | **0.74** | 0.00 | 0.00 | 0.01 | 0.04 | 0.00 | 0.00 | 0.78 |
| U 15+ wks | 0.06 | **0.70** | 0.00 | 0.00 | 0.00 | **0.21** | 0.00 | 0.02 | 0.94 |
| U 15-26 wks | 0.15 | **0.67** | 0.01 | 0.00 | 0.00 | **0.16** | 0.00 | 0.02 | 0.85 |
| U 27+ wks | 0.15 | **0.58** | 0.00 | 0.01 | 0.00 | **0.23** | 0.01 | 0.02 | 0.85 |
| UI claims | 0.22 | **0.71** | 0.03 | 0.01 | 0.01 | 0.02 | 0.00 | 0.01 | 0.78 |
| Emp: total | 0.04 | **0.94** | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.01 | 0.96 |
| Emp: gds prod | 0.04 | **0.94** | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.96 |
| Emp: mining | 0.94 | 0.03 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.06 |
| Emp: const | 0.24 | **0.66** | 0.00 | 0.00 | 0.01 | 0.01 | 0.02 | 0.06 | 0.76 |
| Emp: mfg | 0.07 | **0.92** | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.93 |
| Emp: dble gds | 0.10 | **0.88** | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.90 |
| Emp: nondbles | 0.19 | **0.78** | 0.01 | 0.00 | 0.00 | 0.02 | 0.00 | 0.01 | 0.81 |
| Emp: services | 0.20 | **0.73** | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.06 | 0.80 |
| Emp: TTU | 0.15 | **0.79** | 0.00 | 0.00 | 0.01 | 0.01 | 0.01 | 0.03 | 0.85 |
| Emp: wholesale | 0.23 | **0.72** | 0.01 | 0.00 | 0.01 | 0.02 | 0.00 | 0.01 | 0.77 |
| Emp: retail | 0.26 | **0.65** | 0.00 | 0.00 | 0.01 | 0.00 | 0.01 | 0.07 | 0.74 |
| Emp: FIRE | 0.79 | **0.18** | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.03 | 0.21 |
| Emp: Govt | 0.90 | 0.00 | 0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.07 | 0.10 |
| Emp-hrs nonag | 0.11 | **0.84** | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.03 | 0.89 |
| Avg hrs | 0.33 | **0.62** | 0.01 | 0.01 | 0.01 | 0.01 | 0.00 | 0.01 | 0.67 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Overtime: mfg | 0.36 | **0.61** | 0.01 | 0.00 | 0.00 | 0.01 | 0.00 | 0.01 | 0.64 |
| Avg hrs: mfg | 0.30 | **0.66** | 0.01 | 0.01 | 0.01 | 0.01 | 0.00 | 0.00 | 0.70 |
| NAPM empl | 0.28 | **0.66** | 0.02 | 0.00 | 0.01 | 0.00 | 0.00 | 0.02 | 0.72 |
| HStarts: Total | 0.57 | **0.21** | **0.10** | 0.03 | 0.01 | 0.04 | 0.02 | 0.01 | 0.43 |
| HStarts: ne | 0.80 | 0.08 | 0.03 | 0.01 | 0.00 | 0.03 | 0.02 | 0.03 | 0.20 |
| HStarts: MW | 0.72 | **0.17** | 0.04 | 0.01 | 0.01 | 0.01 | 0.03 | 0.01 | 0.28 |
| HStarts: South | 0.64 | **0.18** | 0.07 | 0.02 | 0.02 | 0.05 | 0.01 | 0.00 | 0.36 |
| HStarts: West | 0.79 | **0.10** | 0.05 | 0.03 | 0.01 | 0.01 | 0.00 | 0.01 | 0.21 |
| BP: total | 0.63 | **0.12** | **0.13** | 0.05 | 0.01 | 0.06 | 0.00 | 0.00 | 0.37 |
| BP: ne | 0.71 | **0.19** | 0.04 | 0.02 | 0.00 | 0.03 | 0.01 | 0.01 | 0.29 |
| BP: MW | 0.64 | **0.12** | 0.10 | 0.07 | 0.01 | 0.03 | 0.02 | 0.01 | 0.36 |
| BP: South | 0.74 | 0.05 | 0.09 | 0.03 | 0.01 | 0.08 | 0.00 | 0.00 | 0.26 |
| BP: West | 0.80 | 0.06 | **0.10** | 0.01 | 0.00 | 0.02 | 0.00 | 0.00 | 0.20 |
| PMI | 0.29 | **0.58** | 0.03 | 0.00 | 0.03 | 0.03 | 0.00 | 0.03 | 0.71 |
| NAPM ordrs | 0.37 | **0.47** | 0.04 | 0.01 | 0.03 | 0.05 | 0.00 | 0.03 | 0.63 |
| NAPM vend. del | 0.65 | **0.31** | 0.01 | 0.00 | 0.01 | 0.01 | 0.00 | 0.02 | 0.35 |
| NAPM Invent | 0.61 | **0.36** | 0.01 | 0.00 | 0.01 | 0.01 | 0.00 | 0.00 | 0.39 |
| Orders: con. gds | 0.12 | **0.80** | 0.02 | 0.01 | 0.00 | 0.02 | 0.00 | 0.02 | 0.88 |
| Orders: dble gds | 0.16 | **0.78** | 0.02 | 0.01 | 0.00 | 0.00 | 0.01 | 0.03 | 0.84 |
| Orders: cap gds | 0.46 | **0.49** | 0.00 | 0.00 | 0.00 | 0.02 | 0.00 | 0.01 | 0.54 |
| Unf orders: dble | 0.52 | **0.41** | 0.02 | 0.00 | 0.00 | 0.01 | 0.01 | 0.04 | 0.48 |
| MT invent | 0.43 | **0.39** | 0.04 | 0.01 | 0.02 | **0.11** | 0.00 | 0.00 | 0.57 |
| MT invent/sales | 0.16 | **0.55** | **0.12** | 0.02 | 0.00 | **0.13** | 0.00 | 0.01 | 0.84 |
| M1 | 0.80 | 0.06 | 0.03 | 0.00 | 0.01 | 0.00 | 0.09 | 0.00 | 0.20 |
| M2 | 0.65 | **0.12** | 0.04 | 0.00 | 0.03 | 0.00 | **0.14** | 0.01 | 0.35 |
| M3 | 0.85 | 0.03 | 0.01 | 0.00 | 0.01 | 0.00 | 0.09 | 0.01 | 0.15 |
| M2 (real) | 0.52 | 0.07 | 0.16 | 0.09 | 0.06 | 0.00 | **0.10** | 0.01 | 0.48 |
| MB | 0.90 | 0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.07 | 0.00 | 0.10 |
| Reserves tot | 0.93 | 0.01 | 0.02 | 0.00 | 0.00 | 0.00 | 0.03 | 0.00 | 0.07 |
| Reser. nonbor | 0.83 | 0.04 | 0.00 | 0.01 | 0.03 | 0.01 | 0.05 | 0.02 | 0.17 |
| C&I loans | 0.87 | **0.10** | 0.00 | 0.01 | 0.00 | 0.01 | 0.00 | 0.00 | 0.13 |
| C&I loans | 0.92 | 0.04 | 0.01 | 0.01 | 0.00 | 0.01 | 0.00 | 0.00 | 0.08 |
| Cons credit | 0.78 | **0.16** | 0.01 | 0.00 | 0.00 | 0.01 | 0.00 | 0.03 | 0.22 |
| Inst cred/PI | 0.80 | 0.06 | 0.02 | 0.00 | 0.00 | 0.08 | 0.03 | 0.01 | 0.20 |
| S&P 500 | 0.22 | 0.06 | **0.42** | 0.04 | **0.14** | 0.00 | **0.12** | 0.01 | 0.78 |
| S&P: indust | 0.22 | 0.06 | **0.40** | 0.03 | **0.16** | 0.00 | 0.11 | 0.01 | 0.78 |
| S&P div yield | 0.26 | 0.02 | **0.46** | 0.05 | **0.11** | 0.00 | 0.09 | 0.01 | 0.74 |
| S&P PE ratio | 0.47 | 0.02 | **0.29** | 0.05 | 0.08 | 0.02 | 0.07 | 0.00 | 0.53 |
| FedFunds | 0.29 | **0.48** | **0.10** | 0.05 | 0.03 | 0.03 | 0.01 | 0.00 | 0.71 |
| Commpaper | 0.21 | **0.42** | **0.19** | 0.09 | 0.08 | 0.01 | 0.01 | 0.00 | 0.79 |
| 3 mo T-bill | 0.21 | **0.39** | **0.16** | 0.07 | **0.17** | 0.00 | 0.01 | 0.00 | 0.79 |
| 6 mo T-bill | 0.14 | **0.40** | **0.18** | 0.08 | **0.18** | 0.00 | 0.01 | 0.00 | 0.86 |
| 1 yr T-bond | 0.10 | **0.38** | **0.20** | **0.10** | **0.21** | 0.00 | 0.00 | 0.01 | 0.90 |
| 5 yr T-bond | 0.13 | **0.21** | **0.22** | 0.09 | **0.31** | 0.04 | 0.00 | 0.01 | 0.87 |
| 10 yr T-bond | 0.18 | **0.12** | **0.24** | 0.08 | **0.31** | 0.05 | 0.01 | 0.01 | 0.82 |
| Aaabond | 0.24 | 0.06 | **0.33** | 0.08 | **0.22** | 0.05 | 0.02 | 0.00 | 0.76 |
| Baa bond | 0.29 | 0.02 | **0.35** | **0.10** | **0.16** | 0.05 | 0.03 | 0.00 | 0.71 |
| CP-FF spread | 0.71 | **0.11** | 0.03 | 0.01 | 0.02 | **0.10** | 0.00 | 0.00 | 0.29 |
| 3 mo-FF spread | 0.57 | **0.24** | 0.02 | 0.01 | 0.05 | **0.11** | 0.00 | 0.01 | 0.43 |
| 6 mo-FF spread | 0.58 | **0.24** | 0.01 | 0.00 | 0.05 | **0.11** | 0.00 | 0.01 | 0.42 |
| 1 yr-FF spread | 0.63 | **0.18** | 0.01 | 0.00 | 0.06 | **0.11** | 0.01 | 0.00 | 0.37 |
| 5 yr-FFspread | 0.51 | **0.34** | 0.02 | 0.01 | 0.01 | 0.09 | 0.01 | 0.00 | 0.49 |
| 10yr-FF spread | 0.45 | **0.40** | 0.03 | 0.02 | 0.00 | 0.07 | 0.02 | 0.00 | 0.55 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Aaa-FF spread | 0.40 | **0.46** | 0.03 | 0.03 | 0.01 | 0.06 | 0.02 | 0.00 | 0.60 |
| Baa-FF spread | 0.35 | **0.51** | 0.03 | 0.03 | 0.01 | 0.05 | 0.02 | 0.00 | 0.65 |
| Ex rate: avg | 0.27 | 0.00 | 0.04 | 0.11 | 0.00 | **0.11** | **0.18** | **0.28** | 0.73 |
| Ex rate: Switz | 0.40 | 0.00 | 0.01 | 0.05 | 0.02 | **0.11** | **0.20** | **0.19** | 0.60 |
| Ex rate: Japan | 0.55 | 0.02 | 0.02 | 0.09 | 0.00 | 0.05 | **0.12** | **0.15** | 0.45 |
| Ex rate: UK | 0.53 | 0.00 | 0.01 | 0.06 | 0.00 | 0.07 | **0.12** | **0.22** | 0.47 |
| EX rate: Canada | 0.77 | 0.00 | 0.07 | 0.03 | 0.01 | 0.07 | 0.00 | 0.06 | 0.23 |
| PPI: fin gds | 0.56 | 0.05 | 0.12 | **0.23** | 0.02 | 0.00 | 0.01 | 0.00 | 0.44 |
| PPI: cons gds | 0.56 | 0.03 | 0.10 | **0.27** | 0.02 | 0.00 | 0.01 | 0.01 | 0.44 |
| PPI: int mat'ls | 0.54 | **0.18** | 0.07 | **0.18** | 0.02 | 0.00 | 0.00 | 0.00 | 0.46 |
| PPI: crd mat'ls | 0.77 | 0.02 | 0.04 | **0.16** | 0.01 | 0.00 | 0.00 | 0.01 | 0.23 |
| Com. spot price | 0.78 | **0.10** | 0.01 | 0.02 | 0.04 | 0.00 | 0.00 | 0.03 | 0.22 |
| Sen mat'ls price | 0.75 | **0.11** | 0.03 | 0.02 | 0.06 | 0.02 | 0.01 | 0.01 | 0.25 |
| NAPM com prce | 0.56 | **0.32** | 0.05 | 0.04 | 0.01 | 0.00 | 0.01 | 0.00 | 0.44 |
| CPI-U: all | 0.23 | 0.09 | **0.28** | **0.37** | 0.02 | 0.00 | 0.00 | 0.01 | 0.77 |
| CPI-U: apparel | 0.88 | 0.04 | 0.03 | 0.04 | 0.00 | 0.00 | 0.01 | 0.00 | 0.12 |
| CPI-U: transp | 0.44 | 0.02 | **0.16** | **0.36** | 0.01 | 0.00 | 0.00 | 0.00 | 0.56 |
| CPI-U: medical | 0.96 | 0.02 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.04 |
| CPI-U: comm. | 0.21 | 0.05 | 0.25 | **0.45** | 0.02 | 0.00 | 0.00 | 0.01 | 0.79 |
| CPI-U: dbles | 0.90 | 0.01 | 0.07 | 0.01 | 0.01 | 0.01 | 0.00 | 0.00 | 0.10 |
| CPI-U: services | 0.81 | **0.12** | 0.04 | 0.01 | 0.00 | 0.02 | 0.00 | 0.00 | 0.19 |
| CPI-U: ex food | 0.40 | 0.09 | **0.24** | **0.25** | 0.01 | 0.00 | 0.00 | 0.01 | 0.60 |
| CPI-U: ex shltr | 0.23 | 0.05 | **0.25** | **0.43** | 0.02 | 0.00 | 0.00 | 0.01 | 0.77 |
| CPI-U: ex med | 0.24 | 0.09 | **0.26** | **0.38** | 0.02 | 0.00 | 0.00 | 0.01 | 0.76 |
| PCE Deflator | 0.34 | 0.05 | **0.19** | **0.37** | 0.04 | 0.00 | 0.01 | 0.00 | 0.66 |
| PCE D: dlbes | 0.91 | 0.03 | 0.04 | 0.01 | 0.02 | 0.00 | 0.00 | 0.00 | 0.09 |
| PCE D: nondble | 0.24 | 0.05 | **0.22** | **0.46** | 0.02 | 0.00 | 0.00 | 0.01 | 0.76 |
| PCE D: services | 0.96 | 0.00 | 0.01 | 0.02 | 0.01 | 0.00 | 0.00 | 0.00 | 0.04 |
| AHE: goods | 0.82 | 0.04 | 0.01 | 0.00 | 0.00 | 0.01 | 0.00 | 0.11 | 0.18 |
| AHE: const | 0.94 | 0.01 | 0.01 | 0.01 | 0.00 | 0.01 | 0.01 | 0.02 | 0.06 |
| AHE: mfg | 0.73 | 0.07 | 0.02 | 0.00 | 0.01 | 0.02 | 0.00 | **0.16** | 0.27 |
| Cons. expect | 0.68 | 0.03 | **0.11** | 0.01 | 0.06 | 0.04 | 0.05 | 0.02 | 0.32 |

Notes: Entries are the marginal contribution of each column variable to the 24-month ahead forecast error variance decomposition of the row variable. Marginal contributions of individual factors that exceed 0.10 appear in bold.

**Table 3**
**Percentiles of *p*-values and Marginal $R^2$ from $X \rightarrow F$ Granger Causality Tests**

| | **Percentile** | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Series** | **0.010** | **0.050** | **0.100** | **0.250** | **0.500** | **0.750** | **0.900** | **0.950** | **0.990** |
| *p*-value | 0.000 | 0.001 | 0.004 | 0.057 | 0.252 | 0.555 | 0.833 | 0.908 | 0.981 |
| Marginal $R^2$ | 0.002 | 0.003 | 0.004 | 0.007 | 0.012 | 0.018 | 0.028 | 0.036 | 0.050 |

Notes: The table summarizes results from 1188 Granger-causality tests for each of the 132 $X$ variables as a potential predictor for each of the 9 static factors. The first row of the table shows the percentiles of the 1188 p-values for the Granger-causality tests. The final row shows the percentiles for the marginal $R^2$ associated with including lags of $X_j$ in the forecasting equation for $F_k$.

**Table 4**
**Percentiles for *p*-values and Marginal $R^2$ from Excluding $X_j$ from $X_i$ Equation**

Specifications:

(a) $X_{it} = \Lambda_i\Phi(L)X_{it-1} + \beta_{ij}(L)X_{jt-1} + \varepsilon_t$

(b) $X_{it} = \Lambda_i\Phi(L)F_{t-1} + \delta_i(L)X_{it-1} + \beta_{ij}(L)X_{jt-1} + \varepsilon_t$

(c) $X_{it} = \delta_i^j(L)X_{it} + \beta_{ij}(L)X_{jt} + \nu_{it}^j$

(d) $X_{it} = \Lambda_i^j F_t + \delta_i^j(L)X_{it} + \beta_{ij}(L)X_{jt} + \nu_{it}^j$

(e) $X_{it} = \Lambda_i F_t + \delta_i(L)X_{it} + \nu_{it}$

| | **Percentile** | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | **0.010** | **0.050** | **0.100** | **0.250** | **0.500** | **0.750** | **0.900** | **0.950** | **0.990** |
| ***(i) P-values for testing $\delta_{ij}$(L) = 0 in specification:*** | | | | | | | | | |
| (a) | 0.000 | 0.000 | 0.000 | 0.005 | 0.084 | 0.364 | 0.682 | 0.820 | 0.964 |
| (b) | 0.000 | 0.004 | 0.017 | 0.093 | 0.306 | 0.603 | 0.825 | 0.910 | 0.980 |
| (c) | 0.000 | 0.000 | 0.000 | 0.000 | 0.028 | 0.247 | 0.597 | 0.763 | 0.946 |
| (d) | 0.000 | 0.000 | 0.001 | 0.028 | 0.195 | 0.497 | 0.756 | 0.862 | 0.967 |
| ***(ii) Marginal R-Squared associated with relaxing constraint that $\beta_{ijk}$ = 0 in specification:*** | | | | | | | | | |
| (a) | 0.001 | 0.002 | 0.002 | 0.006 | 0.013 | 0.026 | 0.049 | 0.069 | 0.120 |
| (b) | 0.000 | 0.001 | 0.001 | 0.003 | 0.007 | 0.013 | 0.020 | 0.026 | 0.041 |
| (c) | 0.001 | 0.002 | 0.004 | 0.008 | 0.018 | 0.043 | 0.098 | 0.163 | 0.420 |
| (d) | 0.000 | 0.001 | 0.001 | 0.003 | 0.005 | 0.010 | 0.018 | 0.027 | 0.088 |
| ***(ii) Hausman test for $\lambda$ in specifications (d) versus (e)*** | | | | | | | | | |
| | 0.000 | 0.003 | 0.063 | 0.413 | 0.780 | 0.937 | 0.983 | 0.992 | 0.999 |

Notes: The first two panels of the table summarize results from 17,292 heteroskedasticity-robust exclusion tests for each of the $X$ variables as a potential predictor of all of the other $X$ variables. The first panel shows the percentiles of the 17,292 *p*-values for the exclusion tests. The first row of this panel shows results for specification (a), the next row for specification (b), and so forth. The second panel shows the percentiles for the marginal $R^2$ associated with including $X_j$ in the equation for $X_i$ for each of the specifications. All lag polynomials have six lags. The final panel of the table shows the percentiles for the 17,292 *p*-values for the Hausman test of equality of $\Lambda_i$ and $\Lambda_i^j$ in specifications (d) and (e).

**Table 5**

**Estimation of the Number of Dynamic Factors $q^S$ among the Slow-Moving Variables**

| **# dynamic factors (*q*)** | ***r* = 1** | ***r* = 2** | ***r* = 3** | ***r* = 4** | ***r* = 5** | ***r* = 6** | ***r* = 7** | ***r* = 8** | ***r* = 9** | ***r* = 10** |
|---|---|---|---|---|---|---|---|---|---|---|
| 1.00 | **-0.555** | -0.567 | -0.572 | -0.580 | -0.592 | -0.598 | -0.602 | -0.608 | -0.612 | -0.620 |
| 2.00 | . | **-0.625** | -0.630 | -0.639 | -0.652 | -0.658 | -0.662 | -0.668 | -0.671 | -0.678 |
| 3.00 | . | . | **-0.641** | -0.649 | -0.666 | -0.673 | -0.677 | -0.683 | -0.686 | -0.692 |
| 4.00 | . | . | . | **-0.651** | **-0.669** | **-0.676** | **-0.681** | **-0.686** | **-0.691** | **-0.698** |
| 5.00 | . | . | . | . | -0.667 | -0.674 | -0.679 | -0.685 | -0.690 | -0.697 |
| 6.00 | . | . | . | . | . | -0.670 | -0.675 | -0.681 | -0.686 | -0.693 |
| 7.00 | . | . | . | . | . | . | -0.672 | -0.677 | -0.683 | -0.690 |
| 8.00 | . | . | . | . | . | . | . | -0.667 | -0.673 | -0.680 |
| 9.00 | . | . | . | . | . | . | . | . | -0.659 | -0.666 |
| 10.0 | . | . | . | . | . | . | . | . | . | -0.654 |

Notes: Entries are the Bai-Ng (2002) $IC_{p2}$ information criterion, computed using only the slow-moving variables. The estimates are based on the filtered data with 6 lags for $D$(L) and a VAR(1) for $F_t$. See the notes to Table 1.

Table 6
Summary of Results from BBE SFAVAR Model

| Variable | Impulse response to Fed Funds shock at horizon: | | | | | Percentage of variance explained by Fed Funds shock at horizon: | | | | | Test of overidentifying restriction: *p*-value | | | Unrestricted model fraction of innovation variance explained by | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **0** | **6** | **12** | **24** | **36** | **0** | **6** | **12** | **24** | **36** | $\zeta^R$&$\zeta^F$ | $\zeta^R$ | $\zeta^F$ | $\zeta^S$ | $\zeta^F$ |
| **Federal Funds Rate** | **1.0** | **0.8** | **0.8** | **0.7** | **0.7** | **14.9** | **16.1** | **10.4** | **7.2** | **6.1** | . | . | . | **0.090** | **0.000** |
| ***Slow variables*** | | | | | | | | | | | | | | | |
| PI | 0.2 | 0.0 | -0.1 | -0.1 | -0.1 | 0.7 | 0.2 | 0.1 | 0.1 | 0.1 | 0.000 | 0.024 | 0.000 | 0.233 | 0.018 |
| PI less transfers | 0.2 | 0.0 | -0.2 | -0.3 | -0.3 | 0.6 | 0.2 | 0.2 | 0.3 | 0.3 | 0.000 | 0.033 | 0.000 | 0.252 | 0.017 |
| Consumption | -0.1 | -0.8 | -0.8 | -0.8 | -0.8 | 0.2 | 6.3 | 8.1 | 9.1 | 9.4 | 0.000 | 0.156 | 0.000 | 0.260 | 0.117 |
| M&T sales | -0.4 | -1.7 | -1.8 | -1.8 | -1.8 | 0.6 | 4.7 | 6.5 | 7.4 | 7.7 | 0.000 | 0.001 | 0.000 | 0.518 | 0.063 |
| Retail sales | -0.3 | -1.6 | -1.6 | -1.6 | -1.6 | 0.3 | 7.7 | 9.9 | 11.0 | 11.5 | 0.000 | 0.059 | 0.000 | 0.349 | 0.124 |
| IP: total | 0.0 | -0.6 | -1.0 | -1.1 | -1.1 | 0.0 | 0.4 | 1.0 | 1.5 | 1.6 | 0.000 | 0.796 | 0.000 | 0.887 | 0.006 |
| IP: products | -0.1 | -0.6 | -0.9 | -0.9 | -0.9 | 0.0 | 0.4 | 1.1 | 1.6 | 1.7 | 0.000 | 0.243 | 0.000 | 0.842 | 0.017 |
| IP: final prod | -0.1 | -0.5 | -0.9 | -0.9 | -0.9 | 0.1 | 0.4 | 1.0 | 1.4 | 1.5 | 0.000 | 0.079 | 0.000 | 0.814 | 0.023 |
| IP: cons gds | -0.3 | -0.8 | -0.9 | -0.8 | -0.8 | 0.4 | 0.9 | 1.7 | 2.1 | 2.2 | 0.000 | 0.000 | 0.000 | 0.749 | 0.034 |
| IP: cons dble | -0.5 | -2.0 | -2.1 | -2.0 | -2.0 | 0.2 | 0.9 | 1.8 | 2.2 | 2.3 | 0.002 | 0.025 | 0.008 | 0.628 | 0.006 |
| IP: cons nondble | -0.1 | -0.3 | -0.3 | -0.3 | -0.3 | 0.1 | 0.3 | 0.6 | 0.8 | 0.9 | 0.000 | 0.194 | 0.000 | 0.298 | 0.095 |
| IP: bus eqpt | -0.1 | -0.5 | -1.3 | -1.7 | -1.7 | 0.0 | 0.2 | 0.5 | 1.0 | 1.2 | 0.010 | 0.532 | 0.004 | 0.439 | 0.010 |
| IP: matls | 0.1 | -0.7 | -1.2 | -1.2 | -1.2 | 0.0 | 0.3 | 0.9 | 1.3 | 1.4 | 0.090 | 0.500 | 0.047 | 0.596 | 0.003 |
| IP: dble mats | 0.0 | -1.2 | -2.0 | -2.0 | -2.0 | 0.0 | 0.3 | 1.0 | 1.4 | 1.5 | 0.000 | 0.795 | 0.000 | 0.578 | 0.023 |
| IP: nondble mats | 0.1 | -0.6 | -1.0 | -1.0 | -1.0 | 0.1 | 0.3 | 0.9 | 1.3 | 1.4 | 0.038 | 0.493 | 0.022 | 0.266 | 0.010 |
| IP: mfg | -0.1 | -0.8 | -1.3 | -1.3 | -1.3 | 0.0 | 0.5 | 1.3 | 1.8 | 1.9 | 0.000 | 0.189 | 0.000 | 0.879 | 0.005 |
| IP: res util | 0.9 | 0.6 | 0.7 | 0.8 | 0.8 | 0.4 | 0.9 | 0.9 | 1.0 | 1.0 | 0.000 | 0.065 | 0.000 | 0.202 | 0.032 |
| IP: fuels | -0.9 | -1.0 | -1.0 | -0.9 | -0.9 | 0.8 | 1.6 | 1.9 | 2.1 | 2.2 | 0.183 | 0.030 | 0.750 | 0.063 | 0.001 |
| NAPM prodn | 2.5 | -2.2 | -0.9 | -0.1 | 0.0 | 1.8 | 2.0 | 2.7 | 2.7 | 2.7 | 0.000 | 0.000 | 0.000 | 0.174 | 0.076 |
| Cap util | -0.1 | -0.7 | -1.1 | -1.1 | -1.1 | 0.1 | 0.6 | 1.4 | 2.0 | 2.1 | 0.001 | 0.092 | 0.002 | 0.848 | 0.003 |
| Help wanted indx | 1.0 | -2.4 | -3.7 | -4.0 | -4.0 | 0.8 | 1.3 | 2.4 | 3.2 | 3.4 | 0.000 | 0.013 | 0.000 | 0.150 | 0.080 |
| Help wanted/emp | 0.0 | -0.1 | -0.1 | -0.1 | -0.1 | 0.2 | 2.1 | 3.4 | 4.4 | 4.6 | 0.000 | 0.179 | 0.000 | 0.219 | 0.118 |
| Emp CPS total | 0.1 | -0.1 | -0.2 | -0.3 | -0.3 | 1.1 | 0.3 | 0.7 | 1.0 | 1.1 | 0.000 | 0.001 | 0.009 | 0.331 | 0.008 |
| Emp CPS nonag | 0.1 | -0.1 | -0.2 | -0.3 | -0.3 | 0.9 | 0.3 | 0.6 | 0.9 | 1.0 | 0.001 | 0.001 | 0.032 | 0.322 | 0.006 |
| U: all | -0.1 | 0.1 | 0.2 | 0.2 | 0.2 | 1.4 | 0.4 | 1.1 | 1.7 | 1.8 | 0.000 | 0.000 | 0.000 | 0.424 | 0.028 |
| U: mean duration | 0.1 | 0.1 | 0.4 | 0.5 | 0.5 | 0.3 | 0.3 | 0.6 | 1.3 | 1.6 | 0.000 | 0.042 | 0.000 | 0.281 | 0.021 |
| U < 5 wks | -3.0 | 0.9 | 1.2 | 1.1 | 1.1 | 1.6 | 0.8 | 0.8 | 0.7 | 0.7 | 0.000 | 0.000 | 0.000 | 0.109 | 0.033 |
| U 5-14 wks | 0.3 | 3.6 | 4.8 | 4.9 | 4.9 | 0.0 | 0.8 | 1.8 | 2.5 | 2.7 | 0.000 | 0.729 | 0.000 | 0.162 | 0.036 |
| U 15+ wks | 1.2 | 4.5 | 8.7 | 9.3 | 9.3 | 0.3 | 0.7 | 1.8 | 2.6 | 2.8 | 0.052 | 0.024 | 0.405 | 0.693 | 0.001 |
| U 15-26 wks | 1.7 | 4.5 | 7.6 | 7.9 | 7.9 | 0.2 | 0.7 | 1.7 | 2.5 | 2.7 | 0.109 | 0.144 | 0.196 | 0.337 | 0.003 |
| U 27+ wks | 0.5 | 4.3 | 9.4 | 10.7 | 10.7 | 0.0 | 0.4 | 1.2 | 2.1 | 2.3 | 0.430 | 0.616 | 0.267 | 0.418 | 0.003 |
| UI claims | 0.0 | 6.3 | 6.7 | 6.5 | 6.5 | 0.0 | 2.5 | 3.9 | 4.6 | 4.8 | 0.000 | 0.966 | 0.000 | 0.300 | 0.065 |
| Emp: total | 0.0 | -0.2 | -0.5 | -0.6 | -0.6 | 0.0 | 0.4 | 1.3 | 2.1 | 2.3 | 0.000 | 0.545 | 0.000 | 0.705 | 0.010 |
| Emp: gds prod | 0.0 | -0.3 | -0.8 | -0.9 | -0.9 | 0.0 | 0.2 | 0.9 | 1.5 | 1.7 | 0.000 | 0.928 | 0.000 | 0.645 | 0.025 |
| Emp: mining | 0.0 | 0.6 | 0.4 | 0.3 | 0.3 | 0.0 | 0.3 | 0.3 | 0.2 | 0.2 | 0.109 | 0.929 | 0.058 | 0.017 | 0.010 |
| Emp: const | -0.2 | -0.9 | -1.4 | -1.5 | -1.5 | 0.3 | 1.0 | 2.1 | 2.9 | 3.1 | 0.013 | 0.038 | 0.154 | 0.356 | 0.004 |
| Emp: mfg | 0.1 | -0.2 | -0.6 | -0.8 | -0.8 | 0.1 | 0.2 | 0.5 | 1.0 | 1.2 | 0.000 | 0.304 | 0.000 | 0.538 | 0.029 |
| Emp: dble gds | 0.1 | -0.2 | -0.7 | -1.0 | -1.0 | 0.1 | 0.2 | 0.4 | 0.8 | 0.9 | 0.000 | 0.188 | 0.000 | 0.494 | 0.034 |
| Emp: nondbles | 0.0 | -0.2 | -0.5 | -0.5 | -0.5 | 0.0 | 0.2 | 0.9 | 1.6 | 1.7 | 0.075 | 0.642 | 0.037 | 0.227 | 0.009 |
| Emp: services | 0.0 | -0.1 | -0.3 | -0.4 | -0.4 | 0.0 | 0.3 | 0.9 | 1.6 | 1.9 | 0.001 | 0.604 | 0.000 | 0.334 | 0.019 |
| Emp: TTU | 0.1 | -0.2 | -0.4 | -0.5 | -0.5 | 0.3 | 0.3 | 1.0 | 1.8 | 2.0 | 0.332 | 0.085 | 0.855 | 0.384 | 0.000 |
| Emp: wholesale | 0.1 | 0.0 | -0.2 | -0.4 | -0.4 | 0.6 | 0.2 | 0.2 | 0.6 | 0.8 | 0.020 | 0.025 | 0.158 | 0.175 | 0.005 |
| Emp: retail | 0.1 | -0.2 | -0.4 | -0.5 | -0.5 | 0.6 | 0.3 | 0.8 | 1.4 | 1.6 | 0.044 | 0.028 | 0.197 | 0.330 | 0.003 |
| Emp: FIRE | 0.0 | 0.0 | -0.1 | -0.2 | -0.3 | 0.0 | 0.0 | 0.1 | 0.3 | 0.4 | 0.036 | 0.791 | 0.014 | 0.029 | 0.015 |
| Emp: Govt | 0.0 | 0.3 | 0.4 | 0.4 | 0.4 | 0.0 | 1.2 | 1.5 | 1.5 | 1.5 | 0.001 | 0.731 | 0.000 | 0.015 | 0.060 |
| Emp-hrs nonag | -0.3 | -0.3 | -0.5 | -0.5 | -0.5 | 1.0 | 0.6 | 1.2 | 1.8 | 2.0 | 0.000 | 0.006 | 0.000 | 0.420 | 0.041 |
| Avg hrs | -0.1 | -0.1 | -0.1 | -0.1 | -0.1 | 1.1 | 1.2 | 1.7 | 2.1 | 2.2 | 0.014 | 0.041 | 0.005 | 0.289 | 0.016 |
| Overtime: mfg | 0.0 | 0.0 | -0.1 | -0.1 | -0.1 | 0.1 | 0.4 | 0.6 | 0.7 | 0.8 | 0.116 | 0.299 | 0.086 | 0.179 | 0.004 |
| Avg hrs: mfg | -0.1 | -0.1 | -0.1 | -0.1 | -0.1 | 0.5 | 0.8 | 1.3 | 1.7 | 1.8 | 0.139 | 0.183 | 0.067 | 0.290 | 0.009 |
| NAPM empl | 2.2 | -1.1 | -0.9 | -0.2 | 0.0 | 2.5 | 1.2 | 1.5 | 1.7 | 1.7 | 0.000 | 0.000 | 0.000 | 0.198 | 0.063 |
| CPI-U: all | 0.2 | 0.0 | -0.1 | 0.0 | 0.0 | 0.0 | 0.9 | 0.7 | 0.4 | 0.3 | 0.087 | 0.626 | 0.037 | 0.733 | 0.005 |
| CPI-U: apparel | 0.4 | 0.1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.2 | 0.2 | 0.1 | 0.1 | 0.294 | 0.632 | 0.214 | 0.074 | 0.005 |
| CPI-U: transp | -0.9 | -0.9 | -0.8 | -0.8 | -0.8 | 0.1 | 0.4 | 0.4 | 0.4 | 0.4 | 0.003 | 0.380 | 0.001 | 0.543 | 0.010 |

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CPI-U: medical | 0.3 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.238 | 0.511 | 0.148 | 0.010 | 0.005 |
| CPI-U: comm. | 0.3 | -0.2 | -0.3 | -0.2 | -0.2 | 0.0 | 0.5 | 0.4 | 0.3 | 0.3 | 0.000 | 0.500 | 0.000 | 0.771 | 0.014 |
| CPI-U: dbles | 2.1 | 0.6 | 0.8 | 0.9 | 0.9 | 1.6 | 2.8 | 2.8 | 2.8 | 2.8 | 0.040 | 0.008 | 0.776 | 0.054 | 0.001 |
| CPI-U: services | -0.2 | -0.1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.5 | 0.4 | 0.2 | 0.2 | 0.882 | 0.778 | 0.718 | 0.049 | 0.002 |
| CPI-U: ex food | 0.3 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.8 | 0.6 | 0.4 | 0.3 | 0.637 | 0.440 | 0.462 | 0.548 | 0.002 |
| CPI-U: ex shelter | 0.1 | -0.2 | -0.2 | -0.2 | -0.2 | 0.0 | 0.6 | 0.4 | 0.3 | 0.3 | 0.000 | 0.854 | 0.000 | 0.749 | 0.015 |
| CPI-U: ex med | 0.0 | -0.2 | -0.1 | -0.1 | -0.1 | 0.0 | 0.8 | 0.6 | 0.4 | 0.3 | 0.026 | 0.910 | 0.016 | 0.715 | 0.006 |
| PCE defl | -0.2 | -0.2 | -0.2 | -0.2 | -0.2 | 0.0 | 0.5 | 0.4 | 0.4 | 0.4 | 0.000 | 0.516 | 0.000 | 0.603 | 0.027 |
| PCE defl: dlbes | 1.2 | 0.5 | 0.4 | 0.4 | 0.4 | 0.6 | 0.8 | 0.8 | 0.7 | 0.6 | 0.008 | 0.026 | 0.038 | 0.029 | 0.010 |
| PCE defl: nondble | -0.4 | -0.6 | -0.6 | -0.5 | -0.5 | 0.0 | 0.5 | 0.6 | 0.6 | 0.6 | 0.000 | 0.325 | 0.000 | 0.747 | 0.014 |
| PCE defl: services | -0.1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.1 | 0.1 | 0.1 | 0.1 | 0.664 | 0.771 | 0.454 | 0.033 | 0.005 |
| AHE: goods | 0.9 | 0.1 | 0.1 | 0.1 | 0.1 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.000 | 0.298 | 0.000 | 0.084 | 0.052 |
| AHE: const | 2.5 | 0.5 | 0.4 | 0.4 | 0.4 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 | 0.098 | 0.076 | 0.197 | 0.045 | 0.006 |
| AHE: mfg | 1.3 | 0.1 | 0.1 | 0.2 | 0.2 | 0.4 | 0.4 | 0.3 | 0.3 | 0.2 | 0.000 | 0.147 | 0.000 | 0.153 | 0.056 |
| ***Fast variables*** | | | | | | | | | | | | | | | |
| HStarts: Total | -3.0 | -7.5 | -5.3 | -2.9 | -1.7 | 0.6 | 6.2 | 7.8 | 8.4 | 8.5 | . | . | . | 0.201 | 0.027 |
| HStarts: NE | -6.0 | -5.5 | -4.4 | -3.3 | -2.5 | 0.5 | 1.7 | 2.1 | 2.4 | 2.5 | . | . | . | 0.130 | 0.003 |
| HStarts: MW | -3.8 | -6.8 | -5.1 | -2.9 | -1.7 | 0.3 | 2.7 | 3.6 | 4.0 | 4.1 | . | . | . | 0.124 | 0.012 |
| HStarts: South | -1.1 | -6.3 | -4.6 | -2.8 | -1.8 | 0.1 | 3.5 | 4.6 | 4.9 | 5.0 | . | . | . | 0.145 | 0.029 |
| HStarts: West | -3.8 | -8.7 | -6.5 | -4.0 | -2.6 | 0.4 | 4.5 | 5.6 | 6.0 | 6.1 | . | . | . | 0.025 | 0.040 |
| BP: total | -2.8 | -8.8 | -6.3 | -3.8 | -2.5 | 0.8 | 8.5 | 10.0 | 10.3 | 10.4 | . | . | . | 0.131 | 0.035 |
| BP: NE | -2.8 | -6.1 | -5.2 | -3.9 | -3.0 | 0.2 | 2.7 | 3.6 | 4.1 | 4.2 | . | . | . | 0.111 | 0.004 |
| BP: MW | -7.6 | -10.9 | -8.2 | -5.1 | -3.4 | 2.0 | 9.2 | 10.8 | 11.4 | 11.6 | . | . | . | 0.158 | 0.022 |
| BP: South | -0.5 | -7.0 | -5.3 | -3.8 | -3.0 | 0.0 | 3.7 | 4.4 | 4.3 | 4.3 | . | . | . | 0.073 | 0.038 |
| BP: West | -2.6 | -9.0 | -6.7 | -4.4 | -3.1 | 0.3 | 5.7 | 6.6 | 6.7 | 6.7 | . | . | . | 0.028 | 0.032 |
| PMI | 1.9 | -2.0 | -1.2 | -0.3 | -0.1 | 2.6 | 1.8 | 2.6 | 2.7 | 2.7 | . | . | . | 0.194 | 0.116 |
| NAPM new ordrs | 2.5 | -2.5 | -1.0 | -0.2 | 0.0 | 1.7 | 2.6 | 3.1 | 3.2 | 3.2 | . | . | . | 0.169 | 0.109 |
| NAPM vendor del | 1.1 | -1.9 | -1.9 | -0.4 | -0.1 | 0.5 | 0.6 | 1.3 | 1.6 | 1.6 | . | . | . | 0.048 | 0.028 |
| NAPM Invent | 0.6 | -0.7 | -0.9 | -0.2 | 0.0 | 0.2 | 0.2 | 0.6 | 0.8 | 0.8 | . | . | . | 0.036 | 0.012 |
| Orders: cons gds | -0.4 | -2.7 | -2.7 | -2.6 | -2.6 | 0.1 | 3.3 | 4.7 | 5.5 | 5.7 | . | . | . | 0.478 | 0.069 |
| Orders: dble gds | -1.2 | -3.7 | -4.0 | -3.9 | -3.9 | 0.6 | 3.8 | 5.4 | 6.3 | 6.5 | . | . | . | 0.374 | 0.094 |
| Orders: cap gds | -2.1 | -3.3 | -3.7 | -3.7 | -3.7 | 0.4 | 1.5 | 2.5 | 3.3 | 3.6 | . | . | . | 0.084 | 0.027 |
| Unf orders: dble | -0.3 | -2.4 | -4.0 | -5.4 | -5.8 | 0.7 | 3.4 | 5.0 | 6.2 | 6.6 | . | . | . | 0.087 | 0.052 |
| M&T invent | -0.1 | 0.2 | -0.1 | -0.3 | -0.3 | 0.8 | 0.3 | 0.1 | 0.2 | 0.2 | . | . | . | 0.086 | 0.050 |
| M&T invent/sales | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.3 | 6.3 | 8.4 | 9.3 | 9.6 | . | . | . | 0.500 | 0.129 |
| M1 | 3.2 | -0.2 | -0.7 | -0.9 | -0.9 | 1.5 | 4.8 | 3.9 | 2.9 | 2.4 | . | . | . | 0.042 | 0.121 |
| M2 | 0.7 | -0.5 | -0.7 | -0.8 | -0.8 | 0.3 | 5.6 | 4.3 | 3.1 | 2.7 | . | . | . | 0.095 | 0.180 |
| M3 | 0.9 | -0.5 | -0.5 | -0.5 | -0.5 | 0.4 | 2.5 | 2.0 | 1.5 | 1.3 | . | . | . | 0.039 | 0.095 |
| M2 (real) | 0.1 | -0.7 | -0.8 | -0.9 | -1.0 | 0.2 | 3.0 | 2.3 | 1.6 | 1.3 | . | . | . | 0.324 | 0.113 |
| MB | 2.5 | 0.3 | 0.2 | 0.1 | 0.1 | 1.3 | 1.4 | 1.0 | 0.7 | 0.5 | . | . | . | 0.009 | 0.091 |
| Reserves tot | 14.7 | 2.2 | 2.0 | 2.1 | 2.1 | 1.1 | 1.3 | 1.2 | 1.0 | 0.9 | . | . | . | 0.023 | 0.028 |
| Reserves nonbor | -10.7 | -3.5 | -4.0 | -4.2 | -4.2 | 0.5 | 1.7 | 1.5 | 1.3 | 1.2 | . | . | . | 0.035 | 0.063 |
| C&I loans | 6.5 | 0.5 | -0.2 | -0.2 | -0.2 | 0.7 | 0.5 | 0.3 | 0.2 | 0.2 | . | . | . | 0.031 | 0.004 |
| C&I loans | 31.2 | 3.6 | -0.1 | -0.2 | 0.0 | 1.1 | 1.0 | 0.9 | 0.8 | 0.8 | . | . | . | 0.023 | 0.002 |
| Cons credit | 1.2 | -1.2 | -1.3 | -1.3 | -1.3 | 0.1 | 0.7 | 1.0 | 1.2 | 1.3 | . | . | . | 0.075 | 0.008 |
| Inst cred/PI | 0.0 | -0.1 | -0.2 | -0.3 | -0.3 | 0.4 | 2.4 | 3.5 | 4.5 | 4.9 | . | . | . | 0.108 | 0.050 |
| S&P 500 | -5.5 | -11.3 | -10.9 | -10.7 | -10.7 | 13.4 | 25.4 | 26.9 | 27.5 | 27.6 | . | . | . | 0.073 | 0.666 |
| S&P: indust | -5.1 | -11.2 | -10.8 | -10.6 | -10.6 | 10.9 | 23.2 | 24.8 | 25.4 | 25.6 | . | . | . | 0.072 | 0.684 |
| S&P div yield | 0.2 | 0.4 | 0.4 | 0.4 | 0.4 | 15.6 | 28.4 | 29.0 | 28.6 | 28.4 | . | . | . | 0.063 | 0.551 |
| S&P PE ratio | -5.6 | -13.2 | -12.5 | -12.0 | -12.0 | 6.5 | 16.7 | 16.6 | 15.4 | 14.9 | . | . | . | 0.018 | 0.340 |
| Commpaper | 1.5 | 1.3 | 1.2 | 1.2 | 1.2 | 40.9 | 34.7 | 25.4 | 19.9 | 18.1 | . | . | . | 0.066 | 0.001 |
| 3 mo T-bill | 1.4 | 1.2 | 1.0 | 1.0 | 1.0 | 45.1 | 32.5 | 24.3 | 19.5 | 17.8 | . | . | . | 0.049 | 0.039 |
| 6 mo T-bill | 1.5 | 1.3 | 1.1 | 1.1 | 1.1 | 59.6 | 38.7 | 29.2 | 23.8 | 21.9 | . | . | . | 0.046 | 0.038 |
| 1 yr T-bond | 1.6 | 1.5 | 1.3 | 1.3 | 1.3 | 69.4 | 43.1 | 33.3 | 27.7 | 25.8 | . | . | . | 0.034 | 0.045 |
| 5 yr T-bond | 1.3 | 1.4 | 1.3 | 1.3 | 1.3 | 71.8 | 48.0 | 41.7 | 38.1 | 36.9 | . | . | . | 0.006 | 0.059 |
| 10 yr T-bond | 1.1 | 1.2 | 1.2 | 1.2 | 1.2 | 67.4 | 48.5 | 44.0 | 41.7 | 41.0 | . | . | . | 0.010 | 0.060 |
| Aaabond | 0.8 | 1.1 | 1.1 | 1.1 | 1.1 | 62.9 | 52.5 | 48.9 | 46.9 | 46.2 | . | . | . | 0.017 | 0.039 |
| Baa bond | 0.7 | 1.3 | 1.3 | 1.3 | 1.3 | 58.7 | 55.5 | 53.5 | 52.2 | 51.8 | . | . | . | 0.021 | 0.023 |
| CP-FF spread | 0.5 | 0.2 | 0.2 | 0.1 | 0.1 | 10.0 | 10.3 | 10.4 | 10.7 | 10.8 | . | . | . | 0.026 | 0.002 |
| 3 mo-FF spread | 0.3 | 0.1 | 0.1 | 0.1 | 0.1 | 3.2 | 1.8 | 1.5 | 1.5 | 1.5 | . | . | . | 0.040 | 0.060 |
| 6 mo-FF spread | 0.4 | 0.1 | 0.1 | 0.1 | 0.1 | 4.6 | 2.4 | 2.1 | 2.0 | 2.0 | . | . | . | 0.044 | 0.045 |
| 1 yr-FF spread | 0.6 | 0.1 | 0.2 | 0.2 | 0.1 | 7.7 | 4.3 | 3.9 | 3.8 | 3.8 | . | . | . | 0.037 | 0.042 |
| 5 yr-FFspread | 0.2 | -0.1 | 0.1 | 0.1 | 0.1 | 0.7 | 1.1 | 0.7 | 0.6 | 0.5 | . | . | . | 0.077 | 0.021 |
| 10yr-FF spread | 0.0 | -0.2 | 0.0 | 0.0 | 0.0 | 0.0 | 2.1 | 1.2 | 0.7 | 0.6 | . | . | . | 0.090 | 0.014 |
| Aaa-FF spread | -0.2 | -0.2 | -0.1 | 0.0 | 0.0 | 0.8 | 3.6 | 2.1 | 1.3 | 1.1 | . | . | . | 0.101 | 0.005 |
| Baa-FF spread | -0.3 | -0.2 | -0.1 | 0.0 | 0.0 | 1.9 | 4.0 | 2.3 | 1.5 | 1.3 | . | . | . | 0.117 | 0.003 |

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex rate: avg | 3.8 | 6.8 | 6.8 | 6.8 | 6.8 | 28.2 | 33.7 | 34.5 | 35.1 | 35.2 | . | . | . | 0.007 | 0.553 |
| Ex rate: Switz | 5.6 | 8.7 | 9.0 | 9.0 | 9.0 | 20.9 | 21.5 | 21.5 | 21.8 | 21.9 | . | . | . | 0.008 | 0.463 |
| Ex rate: Japan | 4.6 | 7.8 | 7.7 | 7.6 | 7.6 | 17.2 | 19.3 | 20.4 | 21.1 | 21.2 | . | . | . | 0.007 | 0.332 |
| Ex rate: UK | -3.9 | -6.0 | -6.0 | -6.0 | -6.0 | 14.5 | 15.2 | 15.3 | 15.3 | 15.3 | . | . | . | 0.007 | 0.371 |
| EX rate: Canada | 1.4 | 2.7 | 2.8 | 2.8 | 2.8 | 7.7 | 13.8 | 14.2 | 14.3 | 14.4 | . | . | . | 0.029 | 0.092 |
| PPI: fin gds | 1.1 | -0.6 | -0.8 | -0.8 | -0.8 | 0.2 | 0.4 | 0.6 | 0.8 | 0.9 | . | . | . | 0.425 | 0.024 |
| PPI: cons gds | 0.8 | -1.1 | -1.2 | -1.2 | -1.2 | 0.1 | 0.5 | 0.8 | 1.1 | 1.3 | . | . | . | 0.445 | 0.023 |
| PPI: int mat'ls | 1.4 | -1.7 | -1.9 | -1.8 | -1.8 | 0.2 | 0.8 | 1.4 | 1.9 | 2.1 | . | . | . | 0.341 | 0.033 |
| PPI: crude mat'ls | -5.4 | -7.2 | -7.3 | -7.2 | -7.2 | 0.1 | 0.9 | 1.3 | 1.8 | 2.0 | . | . | . | 0.184 | 0.036 |
| Commod: spot price | 5.0 | -4.3 | -5.6 | -6.1 | -6.1 | 0.2 | 1.6 | 1.9 | 2.2 | 2.3 | . | . | . | 0.010 | 0.100 |
| Sens mat'ls price | 9.4 | -4.6 | -4.4 | -4.4 | -4.4 | 1.4 | 2.9 | 2.8 | 2.7 | 2.6 | . | . | . | 0.031 | 0.091 |
| NAPM com price | 1.2 | -1.1 | -1.7 | -0.7 | -0.3 | 0.3 | 0.4 | 0.6 | 0.8 | 0.8 | . | . | . | 0.113 | 0.026 |
| Consumer expect | 1.1 | -1.6 | -1.0 | -0.8 | -0.8 | 0.3 | 0.7 | 0.7 | 0.5 | 0.4 | . | . | . | 0.082 | 0.122 |

Notes: Estimated using the structural FAVAR with Bernanke-Boivin-Eliasz (2005) identification of the monetary policy shock. The model has 9 static factors, 7 dynamic factors, 4 slow-moving dynamic factors, a VAR(2) specification for $F_t$, and 6 lags in $D$(L). The first 6 numerical columns show the impulse responses and fraction of variance explained by the Federal Funds shock ($\zeta^R$) over different horizons. The *p*-value columns test the hypothesis that $\zeta^R$ and $\zeta^F$ have no contemporaneous effect on each slow series; that $\zeta^R$ has no contemporaneous effect; and that $\zeta^F$ has no contemporaneous effect, respectively. The final three columns show the fraction of 1-month ahead forecast error variance explained by $\zeta^S$ and $\zeta^F$ in a specification that allows all of the shocks to enter the equation.

Figure 1
Business Cycle Components of Selected Series
and the Part Explained by the Common Dynamic Factors

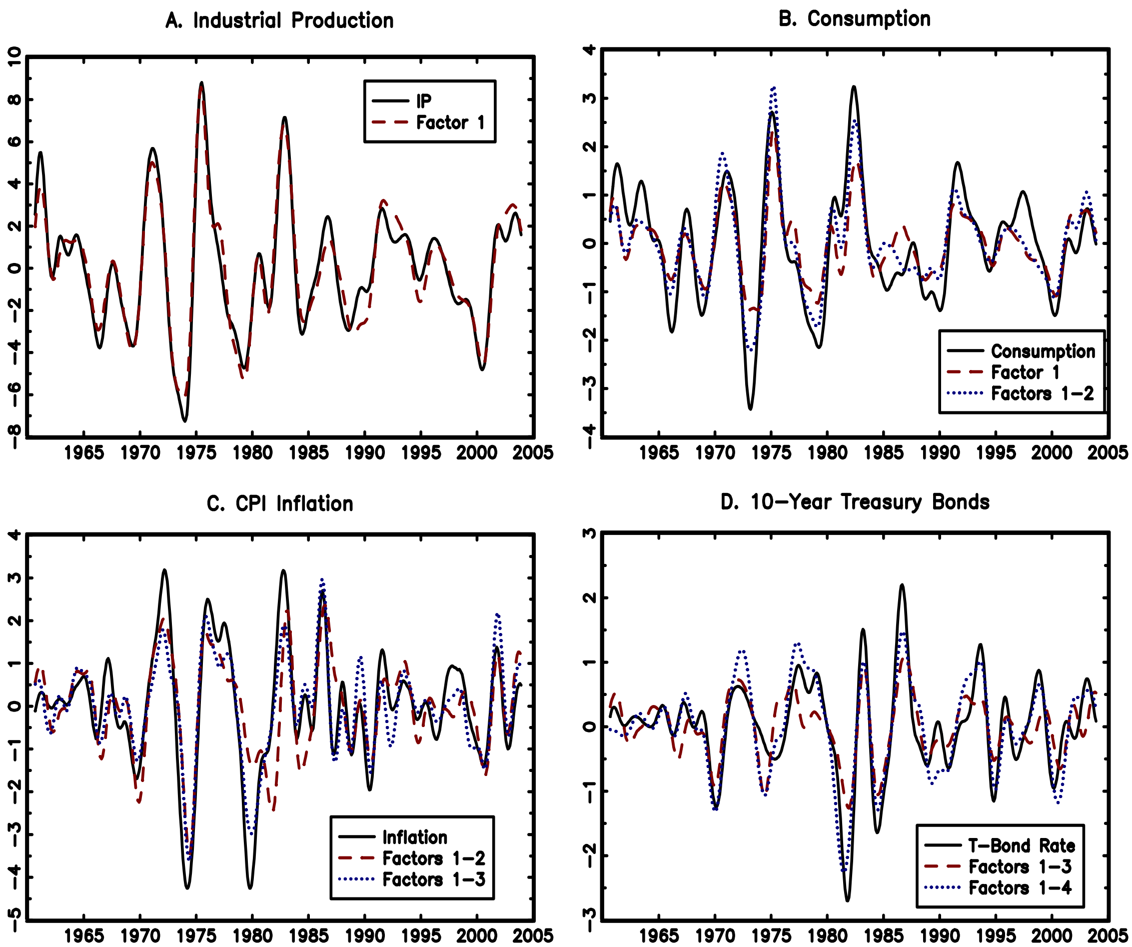

Figure 1 (continued)