# Predicting Economic Time-Series using Dynamic Factor Models under Structural Breaks

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# 1 Introduction

Factor models are used in a wide range of situations spanning from economic indices over prediction of high-dimensional datasets. The basic idea is that there are unobservable factors driving the change of the variables of interest. Put differently, it is assumed that the change of variables is composed of a common component affecting all variables in the set and an idiosyncratic component which is specific to the variable of interest.

If the common factors can be derrived consistently they can prove to be a valuable and possibly less noisy set of predictors for the variables of interest than the whole set of available variables. Nowadays, typically the factors are estimated using principal components analysis. Resultingly factor models harbor similarity to principal component regression including the feature of dimensionality reduction which allows both models to be estimated even if the number of columns exceeds the number of rows. This is precisely what makes factor models appealing in a macroeconomic forecasting context. There exist many international macroeconomic variables some of which are not covered over a very long time horizon. With factor models all this information can be used directly.

In a time-series framework an interesting question to ask is whether these common components are stable over time or if the relationship of the variables of interest with the underlying factors are prone to changes. While there are recent additions to the field, the behaviour of factor models under structural breaks is still being researched. There are at least two reasons why structural breaks are interesting in an econometric model. If we assume that strcutural breaks imply a change in the coefficients of the model at a certain period, this can be interesting in itself especially if the coefficients and the factors are to be interpreted directly. Additionally, from a forecasting perspective it can be worthwhile to consider only the data after the structural break(s) has happened[1]. The hope is that the parameters of the model which are estimated relying only on the data after the last break might be closer to the true values which have generated the data to be forecast. This assumes that there will not be another structural break at the period which is be forecast. It should be notet that Pesaran and Timmermann (2007) have shown that it can be slightly better to include same observations before the last break.

This thesis deals with testing for structural breaks in factor models and using factor models for forecasting in a situation where structural breaks might be present.[2]
The contribution is twofold: Firstly, the influence of structural breaks on factor models is examined. To this end feasible testing procedures developed by Breitung and Eickmeier (2011) are presented. Bootstrap versions of these chow type tests are developed and their performance is evaluated. Secondly, the information about the structural breaks is used for forecating. The performance of different model specifications is tested in the form of horse races. The competing models are:

1. A static factor model with no structural breaks (but with more factors)
2. A static factor model with structural break(s)
3. A dynamic factor model with no structural breaks
4. A dynamic factor model with structural break(s)

The paper is structured into X parts. Firstly factor models are introduced. Both static and dynamic models are presented along with some theory and estimation methods. Secondly structural

[1] Alternatively, instead of splitting the data into multiple periods for each break, a single forecasting equation can be constructed which includes the structural break by introducing a dummy variable.

[2] Structural breaks can refer to changes in the means and variances of variables or a change in the relationship of variables. I focus on the latter. More specificity will be added in section 3.

breaks are introduced, their effects on the factors and loadings are looked at and testing methods are considered. Thirdly aforementioned models are estimated using both syntetic and real data. Finally the fourth part concludes.

Throughout this thesis I will assume that $X$ is a matrix of potentially very many predictors. To stay consistent with existing literature I will index $X$ with $i$ or $t$ interchangeably where $i$ stands for the column index and $t$ for the row index. Also as is customary, throughout this paper capital letters denote matrices while lower case letters denote vectors except for innovation terms which are always in lower case letters. All variables containing data are assumed to be demeaned and normalized to have unit variance[3].

## 2 Factor Models

As mentioned, the idea behind factor models or diffusion index models is that there are underlying latent factors which explain the evolution of the observable variables $X$.

They are ususally written in the form of a factor equation (1) and a forecasting equation (2).

$$X_t = \lambda(L) f_t + e_t \tag{1}$$

$$y^h_{t+h} = \alpha(L) W_t + \beta(L) \hat{f}_t + \varepsilon_{t+h} \tag{2}$$

$X_t$ is a $N \times 1$ vector of potentially very many predictors. $\lambda(L)$ is a $N \times q$ matrix of lag polynomials, $f_t$ is a $q \times 1$ vector of latent factors. $e_t$ is a $N \times 1$ vector of idiosyncratic errors which may or may not be serially auto-correlated. $W_t$ consists of variables which the researcher knows to be sufficiently relevant to enter the forecasting equation directly. Usually $W_t$ is taken to consist of s lags of $y_t$ and a constant term. $\hat{f}_t$ is an estimate of the factors in the factor equation (1). Estimation of the factors is treated below.

Most authors assume the idiosyncratic component to be a white noise process (i.e. independent and of mean 0), however "weak" dependence is also commonly assumed. In the vocabulary of factor models these are models with uncorrelated idiosyncratic components $e_t$ (more specifically $E(e_t e_t') = \Sigma = \text{diag}(\sigma^2_{e_1}, ..., \sigma^2_{e_N})$ and $E(e_t) = 0$).

Geweke (1977) and Sargent and Sims (1977) distinguish between exact or strict Factor Models and approximate Factor Models. In essence the difference is composed of different assumptions for the error terms. Approximated factor models allow for limited correlatedness of the idiosyncratic innovations $e_{it}$ between periods while strict factor models do not. More precise deifinitions follow now and are taken from Breitung and Eickmeier (2006)

### Strict factor models

Breitung and Eickmeier (2006) list the assumptions for strict factor models as follows: For the innovations it is assumed $E(e_t) = 0$ and $E(e_t e_t') = \sigma = \text{diag}(\sigma^2_1, ..., \sigma^2_N)$. Also $E(f_t) = 0$ and $E(f_t f_t') = \Omega$. If $E(f_t) = 0$ it follows that $E(X_t) = 0$ which is enforced by demeaning the variables. Additionally $E(f_t e_t') = 0$ i.e. the innovations and the error terms are uncorrelated. It follows directly that $E(X_t X_t') = \Lambda \Omega \Lambda' + \Sigma$[4]

[3] I.e. every variable $y_t$ is derrived as $y_t = \frac{y^*_t - \bar{y}_t}{\sigma_y}$ where $y^*_t$ is the vector containing the original data and $\sigma_y$ is the (sample) standard deviation of $y$

[4] $E(X_t X_t') = E[(\Lambda f_t + e_t)(\Lambda f_t + e_t)'] = E(\Lambda f_t f_t' \Lambda' + \Lambda f_t e_t' + e_t f_t' \Lambda' + e_t e_t') = \Lambda \Omega \Lambda' + \Sigma$

**Approximate factor models**

econo Approximate factor models losen the assumptions of strict factor models at the cost of assuming $N \to \infty$. $E(e_{it}e_{js})$ is allowed to be different from 0 but only weakly. Similarly the factors $f_t$ and and the idiosyncratic error terms are allowed to be correlated but again only weakly, so that $E(f_t e_t')$ does not have to be 0. But $N^{-1}\Lambda'\Lambda$ must converge to a positive definite limiting matrix $\Sigma_\Lambda$ which ensures that the factors influence each variable with a similar order of magnitude. Otherwise it could be that the loadings for some variables are 0 (Breitung and Eickmeier, 2006). Similiary it is assumed about the factors that $T^{-1}\sum_{t=1} TF_tF_t' \xrightarrow{p} \Sigma_F$. This allows for stationary processes e.g. an ARMA process.
Some more assumptions are necessary for the inferential theory[5] of Bai (2003) to hold. Namely these assumptions are Assumptions A through D in Bai (2003):

**Assumption A:** $E||F_t||^4 \leq M < \infty$ and $T^{-1}\sum_{t=1}^{T} F_tF_t' \xrightarrow{p} \Sigma_F$ for some $r \times r$ positive matrix $\Sigma_F$

**Assumption B:** $||\lambda_1|| \leq \bar{\lambda} < \infty,$ and $||\Lambda'\Lambda/N - \Sigma_\Lambda|| \to 0$ for some $r \times r$ positive definite matrix $\Sigma_\Lambda$

**Assumption C:** $\exists M : 0 < M < \infty$ such that $\forall N, T,$

1.) $E(e_{it}) = 0, E|e_{it}|^8 \leq M$

2.) $E(e_s'e_t/N) = E(N^{-1}\sum_{i=1}^{N} e_{is}e_{it}) = \gamma_N(s,t), |\gamma_N(s,s)| \leq M \forall s$ and

$T^{-1}\sum_{s=1}^{T}\sum_{t=1}^{T}|\gamma_N(s,t) \leq M$

3.) $E(e_{it}'e_{jt}) = \tau_{ij,t}$ with $|\tau_{ij,t}| \leq |\tau_{ij}|$ for some $\tau_{ij}$ and for all $t$ and

$N^{-1}\sum_{i=1}^{N}\sum_{j=1}^{N}|\tau_{ij} \leq M$

4.) $E(e_{it}'e_{js}) = \tau_{ij,ts}$ and $(NT)^{-1}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T}|\tau_{ij,ts}| \leq M$

$\forall(t,s)E\left|N^{-1/2}\sum_{i=1}^{N}[e_{is}e_{it} - E(e_{is}e_{it})]\right|^4 \leq M$

[5]Most notably the results of Bai (2003) are consistency and normality of the factor estimations. The rate of convergence is shown to be $\min(\sqrt{N}, \sqrt{T})$.

**Assumption D:** $E(\frac{1}{N}\sum_{i=1}^{N}||\frac{1}{\sqrt{N}}\sum_{t=1}^{T}F_t e_{it}||^2) \leq M$

Assumption C allows for some degree of dependence and heteroskedasticity among the innovation terms which will be used below to simplify the calculation of the dynamic factor model.

As mentioned, an important advantage of factor models compared to simpler econometric models is that they do not necessarily suffer if the fraction $\frac{T}{N}$ becomes small whereas most models can not cope with more variables than observations. VAR models for example have the problem that the number of parameters to estimate becomes too high quickly if $N$ is high relative to $T$. The reason why factor models do not have this problem is that firstly the factor equation is estimated. This gives estimates $\hat{f}_t$ of the factors. Then only the first $r$ eigenvectors are used in the forecasting equation where $r < N$[6]. This feature allows researchers in principle to append $X$ by further variables derived from the initial set. Bai and Ng (2008) for example use what they call squared principal components and quadratic principal components. The first of which appends squares of the $X_i$ to the initial matrix $X$ such that $X_t^{spc} = \{X_{it}, X_{it}^2\}$ and the second of which adds the cross products of $X_i$ such that $X_t^{qpc} = \{X_{it}, X_{it}X_{jt}\}\, \forall i \neq j$. The idea is to change the link function between $X_t$ and $f_t$ from being linear to allow for a nonlinear relationship. Bai and Ng (2008) also consider adding a squared term of the factors $f_t$ to the forecasting equation (2) to allow the volatility of the factors to influence the forecast. Naturally, the value of these approaches depends heavily on what is being forecasted.

Apart from the distinction into strict or approximate factor models, a further distinction can be made between static and dynamic factor models. This distinction concerns the relationship between lags of $f_t$ and $X$. Static models set $\alpha(L)$ to be of lag order 0 while dynamic models alow for dependence with other lag orders.

## 2.1 Static Factor Models

For static factor models $X_t$ depends only on $f_t$ and not on lags of $f_t$ i.e. factors enter only contemporaneously. Resultingly a static factor model is the case in which the lag order in equation (1) is 0.

$$X_t = \Lambda F_t + \bar{e}_t \tag{3}$$

Where $X$ is a $T \times N$ Matrix of predictors. $F$ is a $T \times r$ factor matrix. $\Lambda$ is a $N \times r$ loadings matrix. $\bar{e}$ is a $T \times N$ matrix of idiosyncratic errors. $C$ is called the common component. Note that depending on whether authors index by row, column or both, several alternative ways of writing down the factor model equation are used in the literature.

$$X_{it} = \Lambda_i' F_t + \bar{e}_{it} \tag{4}$$

$$X = F\Lambda' + e = C + \bar{e} \tag{5}$$

$$X_i = F_i\lambda + \bar{e}_i \tag{6}$$

[6]Typically $r$ is much smaller than $N$. E.g. the famous one factor model of intelligence by Spearman (1904) (in which factor models were developed the first time) has $r = 1$.

## 2.2 Dynamic Factor Models

In applications there will usually be some kind of time dependence structure. Thus, dynamic factor models introduce a dynamic process into the factors.

$$f_t = \Psi(L)f_{t-1} + \eta_t \qquad \text{(VAR representation)} \tag{7}$$

In practice it is usually easier to ignore the dynamic process in the factors $f_t$ and follow e.g. Stock and Watson (2005) among others in assuming a dynamic proccess in the innovation term of the factor equation (1) instead. This captures the dynamic process while avoiding the technical difficulty in estimating the dynamic factors themselves. The underlying thought is that there will be some residual autocorrelation if the dynamic process of the factors is not modelled explicitely which can be caught by an AR(p) process for the innovation term (Breitung and Tenhofen, 2011). Additionally the lag polynomial $\lambda(L)$ in the factor equation (1) captures the noncontemporaneous interactions of the factors $f_t$ and $X_t$.

For the purpose of this paper I assume the following AR processes for the innovation term of the dynamic factor models (Breitung and Eickmeier, 2011).

$$e_{it} = \varrho_{i,1}e_{i,t-1} + ... + \varrho_{i,p_i}e_{i,t-p_i} + \xi_{it} \text{ where } \xi_{it} \text{ is white noise} \tag{8}$$

### 2.2.1 Static interpretation of dynamic factor models

Dynamic factor models can be written as static factor models in a linear state space environment. This gives the static factor equation (3) and $\Phi(L)F_t = G\eta_t$ where $\Phi(L)$ is defined such that equivalence to (7) holds and $F_t$ is defined below as the stacked lagged dynamic factors $f_t$ (Stock and Watson (2011)).

Solving the lag operator, we can write equation (1) in the form

$$X_{it} = \lambda'_{i1}f_t + ... + \lambda'_{im}f_{t-m} + e_{it} \tag{9}$$

Following Bai and Ng (2002) this can be rewritten as

$$X_{it} = \Lambda_i^{*'}F_t + e_{it}$$

$$\text{where } \Lambda_i^* = \begin{bmatrix} \lambda_{i1} \\ \lambda_{i2} \\ \vdots \\ \lambda_{im} \end{bmatrix} \text{ and } F_t = \begin{bmatrix} f_t \\ f_{t-1} \\ \vdots \\ f_{t-m} \end{bmatrix}$$

To keep consistency between variable naming schemes denote the dimension of $F_t$ as $r \times 1$ and the dimension of $\Lambda_i^*$ as $N \times r$. Thus we can always rewrite a dynamic factor model in static form and resultingly we can use the procedure introduced below to estimate the $r$ "static factors"[7] $F_t$ as specified below. It can be seen that the dimension of the resulting stacked vector of "static" factors is $q(m+1)$ thus we can set $r = q(m+1)$. This shows that there is a relationship between $r$ and $q$ if the dynamic factor model is written as a static factor model. To identify $q$ it remains to estimate $m$.

[7] Breitung et al. (2004) call the $f_t$ "structural factors" and the $F_t$ "reduced form" factors.

There exist several ways to identify the dynamic factors from there[8]. Firstly Giannone et al. (2002) suggest estimating the VAR $F_t = CF_{t-1} + \kappa_t$ and then to apply principal component analysis on an estimate of the residual covariance matrix (see Breitung et al. (2004) or Giannone et al. (2002) for details). Breitung et al. (2004) exploit the relationship between $q$ and $r$ in order to find information criteria which can determine $q$ and the lag order of the $\lambda$. This paper does not estimate the dynamic factors directly as in the application the factors are primarily used for the second stage regression.

Imagine the true data would be generated by a dynamic factor model. If instead of using the dynamic factor model specification and finding the dynamic factors from the static factors, a researcher simply estimates a static factor model it can be seen above that the number of factors $r$ will be larger than the number of dynamic factors $q$.

## 2.3 Estimation of the factors

### 2.3.1 Static factors

There exist now a number of ways to estimate the static factors $F$ in (5). Ultimately what we are looking for is a solution to the minimization of the squared error.

$$\min_{F_1,\ldots,F_T,\Lambda} \frac{1}{NT} \sum_{t=1}^{T} (X_t - \Lambda F_t)'(X_t - \Lambda F_t) \tag{10}$$

Since both $\Lambda$ and $F_t$ are unspecified, there are infinitely many solutions to (10) and the problem is not identified. To achieve identification assumptions can most conveniently be put on either $\Lambda$ or $F_t$[9]. Usually $\Lambda$ is normalized such that $N^{-1}\Lambda'\Lambda = I_r$. Alternatively $T^{-1}F'F = I_r$ is also frequently used. The choice of normalization can have a noticeable impact on the computational demand of calculating a solution for the minimization problem in (10). The columns spaces spanned by the estimates of $F$ are equivalent with both normalizations (Stock and Watson, 2011).

Stock and Watson (2011) list four ways which are being applied in the literature to get a solution $\hat{F}$ and $\hat{\Lambda}$ to the minimization problem (10). Firstly a Gaussian process can be assumed which allows for maximum likelihood estimation using the Kalman filter. To do this a dynamic factor can be rewritten as a static factor model in a linear state space writing as mentioned above.
Secondly principal components (and other nonparametric averaging methods) can be used to estimate both $\hat{F}$ and $\hat{\Lambda}$ at the same time. This requires relatively weak assumptions and is particularly easy to and computationally efficient which explains both the popularity of this approach and why this approach is applied in this thesis. Additionally the principal component estimation can be generalized by weighting the minimization problem with the variance matrix of the error term to take account of error variances which are not prooportional to the identity matrix (see Stock and Watson (2011) for details). Evidence of performance improvements by using the generalized principal components approach over the standard principal components estimator are mixed. Resultingly the simpler principal components estimator will be used. It will be described in more detail below.

[8]Note that identifying the dynamic factors might not be of interest for forecasting but rather that it is a necessary condition in order to interpret the factors in a dynamic factor model. For forecasting purposes, in order to choose the number of factors and the number of the factor lags used in the forecasting equation (2) one can also follow Bai and Ng (2008) in using the BIC criterion to choose both simultaneously.

[9]Bai and Ng (2013) consider several alternative normalizations. They also show that direct interpretation of the factors and loading matrices are possible if one is willing to additionally assume diagonality of $\Lambda'\Lambda$ or that $\Lambda$ is a block matrix of 2 submatrices with one submatrix being triangular.

An unattractive feature of principal component estimation is thaint it can not identify the "true" factors $f_t$ but rather a transformation $Qf_t$ for some undefined rotation matrix $Q$ which results in some difficulty in interpreting the factors directly. Bai and Ng (2013) show a minimum set of assumptions which are required in order to make valid interpretations of the estimated factor and loading matrices.

Principal Components estimation of the factors is asymptotically equivalent to maximum likelihood estimation if normality is assumed (Bai, 2003).
Thirdly mixture approaches between the two methods from above can be constructed.
Fourthly and finally Bayesian methods can be used to get estimations of the factors and loadings which can have computational advantages over the maximum likelihood approach and can be useful if the assumption of Gaussian error terms is unappealing.

The principal component method estimates $\hat{F}$ and $\hat{\Lambda}$ as follows. Normalizing $F$ such that $T^{-1}F'F = I_r$ gives $\hat{F} = \sqrt{T} * eigenvectors_r(XX')$ where $eigenvectors_i(A)$ returns the i eigenvectors corresponding to the i largest eigenvalues of a square matrix $A$. Since $\hat{X} = \hat{F}'\hat{\Lambda}$, it follows directly that we can estimate $\Lambda$ by $\hat{\Lambda}' = (\hat{F}'\hat{F})^{-1}\hat{F}'X$. due to the normalization $\hat{F}'\hat{F} = T$ and we can write $\hat{\Lambda} = X'\hat{F}/T$.
If, alternatively, we apply the normalization $N^{-1}\Lambda'\Lambda = I_r$ the solution to (10) becomes $\hat{\Lambda} = \sqrt{N} * eigenvectors_r(X'X)$. This solution to the optimization problem is shown in appendix B). Then we can follow from $\hat{X} = \hat{F}\hat{\Lambda}'$ that $\hat{F} = X\hat{\Lambda}(\hat{\Lambda}'\hat{\Lambda})^{-1}$. Again due to the normalization $\Lambda'\Lambda = N$ and we can write $\hat{F} = X\hat{\Lambda}/N$.

Note that the normalization $T^{-1}F'F = I_r$ requires us to calculate the eigenvectors of $XX'$ which has dimension $T \times T$ while the alternative $N^{-1}\Lambda'\Lambda = I_r$ requires us to calculate the eigenvectors of $X'X$ which is $N \times N$. Depending on the relative sizes of $T$ and $N$ it can make a considerable difference in terms of computational burden which method is applied.

**The number of factors** One important issue that remains to be addressed is the question of how many factors should be used in the factor equation. There exist many different ways that this choice can be made. Firstly there are simple rules of thumb. E.g. some researchers only consider factors with eigenvalues greater than 1. Then there are methods which rely on a similar idea but employ graphical methods. The idea is to inspect the resulting scree plots[10]. The intuition is to only take factors into account if the eigenvalue corresponding to the next factor is smaller by a sufficient amount. This idea is also behind the cut-off value of 1 for eigenvalues. Basing on this, formal tests have been derrived which rely on the eigenvalues calculated above (Stock and Watson, 2011).

The information criteria developed by Bai and Ng (2002) are perhaps more appealing as they formalize the costs and benefits of an additional factor explicitly (Stock and Watson, 2011). Bai and Ng (2002) consider 6 criteria, the first three of which they call $PCp$ criteria (Panel $C_p$ criteria). These criteria result from a generalization of Mallows' $C_p$ (Mallows, 1973). The second set of 3 criteria refine the usual criteria used in time-series analysis to depend on both $N$ and $T$. Bai and Ng (2002) highlight that the criteria $IC_{p1}$, $IC_{p2}$, $PC_{p1}$ and $PC_{p2}$ are specifically suited for principal component estimation. Bai and Ng (2002) present a montecarlo study to highlight the differences between the criteria. Notably the $PC_p$ criteria require the specification of a maximum number of factors, the choice of which is quite arbitrary[11]. Resultingly this paper uses the $IC_p$

[10] Scree plots plot the ordered eigenvalues against the number of factors (i.e. eigenvalues and corresponding eigenvectors) which results in a falling curve.

[11] In their montecarlo study Bai and Ng (2002) set the maximum number of factors to be 8 noting that they used the

criteria. Specifically the $IC_{p2}$ criterion because it seems to be more conservative (i.e. it usually underestimates $r$ whereas $IC_{p2}$ overestimates at times).

In order to understand the criteria better, tables 1 and 2 of Bai and Ng (2002) have been replicated using both different values for $r$ and $kmax$. If $r$ is chosen as in Bai and Ng (2002), the replicated results are almost identical to the original calculations and thus are not reported[12]. The results can be found in two tables in appendix C. Tables I through VIII in Bai and Ng (2002) in essence show 3 points. Firstly, traditional criteria perform poorly in estimating the number of factors if compared to the 6 presented criteria. Of the newly developed information criteria $PC_{p1}$, $PC_{p2}$, $IC_{p1}$ and $IC_{p2}$ perform well even under heteroskedasticity and autocorrelation. Secondly, if $\min\{N,T\}$ becomes too small, the performance of the criteria suffers which is highlighted by the bottom 5 results in the tables. Especially $PC_{p3}$ suffers in this regard. In the given specifications the cut-off value for $\min\{N,T\}$ appears to be 40 or 60 if the variance of the innovations is high. Thirdly, $PC_{p3}$ and $IC_{p3}$ are less robust to a small $\min\{N,T\}$.
The replication results in appendix Bai and Ng (2002) harbor three additional insights. Firstly as $r$ increases, the requirements for $\min\{N,T\}$ are also higher. For $r = 7$ and $r = 9$ it seems $\min\{N,T\}$ should be at least 100. Secondly the choice of $kmax$ can matter a lot. Especially for small samples the specification of $kmax = 8$ used in Bai and Ng (2002) choose the maximum number of factors frequently when the $N$ and $T$ are too small. Almost all results in the last 5 rows set $\hat{k} = 8$ whereas the replicated results often report a lower number of factors if $kmax = \text{ceil}(\min\{T,N\})$. Noticeably, this also happens in the replicated results for $r \in \{1,3,5\}$ and $kmax = \text{ceil}(\min\{T,N\})$. If $kmax = \text{ceil}(\min\{T,N\})$ $IC_{p3}$ predicts $\hat{k} = 50$ in the $T = 100, N = 100$ case for $r \in \{1,3,5,7,9\}$. Thus, the choice of $kmax$ can have a huge impact on the prediction of the number of factors. Luckily it is typically easy to see when the information criteria choose all the factors. In those cases the plausibily of the results should be checked by hand.

An interesting question if one is ultimately interested in forecasting is whether the "true" number of factors is of any interest at all or if the number of factors used in the forecasting equation (2) should perhaps be different from the predicted $r$ in the static model or $q$ in the dynamic model. The answer can not be easily given. Similar to Bai and Ng (2008) we can dodge this question somewhat by using the BIC criterion to choose a model among a list of models with differing number of factors (or number of lags of the variable of interest).

Breitung and Eickmeier (2011) among others, however, note that if one is primarily interested in forecasting, the number of dynamic factors can be ignored and a static factor model with a higher number of factors can be estimated with equivalent success.

## 2.4 Forecasting

The forecasting equation (2) can be estimated using a simple linear regression of $y_{t+h}$ on $\hat{F}_t$, $y_t$, lags of $y_t$ and depending on the model specification lags of $\hat{F}_t$.

The performance of factor models for forecasting purposes are most pronounced if the number of variables used in the factor equation is high (**?**). This makes sense because more variables improve the fit of the factor equation. However, there is a trade-off between quantity and quality of variables[13] that should not be ignored. This case will be adressed in the next subsection.

rule in Schwert (2002) and that one could consider $kmax = \text{int}[(\min\{N,T\}/100)^{1/4}]$. Tests which are not presented here show that choosing a different $kmax$ can result in the $PC_p$ criteria to become quite unstable. Especially high values of $kmax$ often result in the maximum number of factors chosen such that $r = kmax$

[12] I.e. differences are so small that they can be explained by sampling variation.

[13] I.e. a smaller set of variables with a high predictive power can outperform a larger set of variables which includes

### 2.4.1 Targeted predictors

Bai and Ng (2008) follow the idea that if factor models are ultimately going to be used for forecasting it might make sense to "target" the set of predictors $X_t$ to the task of forecasting the variable of interest $y_t$. The idea stems from Boivin and Ng (2006) who find that selecting only the "informative" variables can yield better results than taking a bigger set of variable which include the same "informative" variables but also additional ones. Bai and Ng (2008) thus provides a formal rule for which variables should be used for forecasting. To that end they use hard and soft thresholding rules to preselect among the predictors $X_t$ those that have a meassurable influence on $y_t$. Resultingly, a subset $\tilde{X}_t \subset X_t$ is selected for estimating the factor equation, the hope being that then the factors are better suited for predicting $y_t$.

For hard thresholding they regress $y_t^h$ on $W_{t-h}$ and $X_{it-h}$ for each $i = 1, ..., N$[14]. Then they compare the resulting t-statistics to a threshold and discard those predictors whose t-statistics do not exceed that threshold. For soft thresholding Bai and Ng (2008) perform penalized regressions, LASSO, elastic net and Least Angle Regression (see Tibshirani (1996), Zou and Hastie (2005), Efron et al. (2004) for details) using the fact that these regression specifications set the parameters of variables which are not important to 0.

It should be noted that this approach, while attractive in principle, requires additional thought in the presence of structural breaks.

## 2.5 Interpreting the factors

TODO: should I include such a chapter? It would involve finding out which variables the factors load onto.

## 2.6 Applications

Factor models have been applied in a wide range of different domains apart from economics. Most notably psychology and biology. Staying in the realms of economics, prominently, there are applications in the calculation of indices. The is based on the idea that the factors consist of underlying tendencies applying to all variables which makes them appealing as tools to generate indices.

Schumacher (2010) employs a similar idea in that he uses international predictors to forecast German GDP. Firstly it is intuitive that international data has an influence on an open economy directly. But secondly it is conceivable that there are underlying factors which influence both German GDP growth and international macroeconomic variables at the same time. However, to make sure that the information conveyed in the additional international predictors is of relevance for forecasting, Schumacher (2010) "targets" the predictors prior to esimating the factor equation, as described above.

TODO: see Bai (2003) page 1 for more examples!

# 3 Structural breaks

The treatment of structural breaks follows Breitung and Eickmeier (2011) closely. The question of interest is how a researcher should react to structural breaks in the factor equation.

the variables from the smaller set

[14] Note that the $N$ regressions include only the lags and single predictors $X_i$ as independent variables.

The factor model under a structural breaks in period $T^*$ can be written as follows:

$$y_{it} = f_t' \lambda_i^{(1)} + \varepsilon_{it} \text{ for } t = 1, ..., T^* \tag{11}$$

$$y_{it} = f_t' \lambda_i^{(2)} + \varepsilon_{it} \text{ for } t = T^* + 1, ..., T \tag{12}$$

The intuition is the same as with simpler models: a structural break implies a different relationship between the variable of interest and the factors and hence different factor loadings for the periods before and after the breaks. Multiple structural breaks can be specified accordingly. In theory, structural breaks could be also thought of as affecting the forecasting equation (2). This is ignored here because structural breaks in equations of this type have been thoroughly researched.

Interestingly, estimating a static factor model without structural breaks if the true data generating process includes structural breaks, yields an analogy to ignoring dynamic factors if they were present in the data generating process: the space estimated by the factors will be of higher dimension. This means that the number of factors $r$ predicted e.g. by the Bai and Ng (2002) information criteria will be higher than if no structural breaks were present. Breitung and Eickmeier (2011) note that similar to dynamic factor models it is enough to increase the number of factors if one is primarily interested in decomposing the common component from the idiosyncratic component (that is to say if one is interested foremost in forecasting).

If structural breaks are detected in one or more of the factors the question remains what should be done next. Since structural breaks consist of an instability in the factor loadings at a certain period it can be optimal in a forecasting sense to estimate the respective parameter after the structural break has occured in the hope that this new parameter value contains more accurate information about the future. There is of course a trade-off here because the data series is shorter afterwards which has a negative effect on prediction accuracy. An alternative approach consists of removing the variable containing the structural break from the data set. This approach allows all other series to enter the forecasting equation at the same length as before while the added noise through the structural breaks is removed. The idea is similar to targeted predictors which remove uninformative predictors from the data set in order to improve the ratio of information to noise. Because of the typically high dimension of the data matrix $X$, factor models appear to be well suited for the second approach. Which method should be chosen depends on the size of the structural break, the relative position of the break and the relative information content of the other variables. In practice it is necessary to test in order to weigh the benefits and costs. This is done in the empirical section.

### 3.1 Testing for structural breaks

Breitung and Eickmeier (2011) develop several Chow type tests for the static factor model under the assumptions of the strict factor model. They also develop Quandt-Andrews type supremum tests. The test statistics are as follows.

$$\text{lr}_i = T[\log(S_{0i}) - \log(S_{1i+S_{2i}})] \tag{13}$$

Where

$$S_{0i} = \sum_{t=1}^{T} (y_{it} - \hat{f}_t' \hat{\lambda}_i)^2,\ S_{1i} = \sum_{t=1}^{T_1^*} (y_{it} - \hat{f}_t' \hat{\lambda}_i^{(1)})^2 \text{ and } S_{2i} = \sum_{t=T_i^*+1}^{T_1^*} (y_{it} - \hat{f}_t' \hat{\lambda}_i^{(2)})^2$$

denote the residual sum of squares for the whole date range, the first subperiod up until the structural break and the period from the break to the end of the sample respectively. $\hat{\lambda}^{(1)}$ and $\hat{\lambda}^{(2)}$

denote the esimated factor loadings calculated on the data before the break and after the break respectively.

$$\text{lm}_i = TR_i^2 \text{ where } R_i^2 \text{ is the r-squared from a regression } \hat{\varepsilon}_{it} = \theta_i' \hat{f}_t + \phi \hat{f}_t^* + \tilde{\varepsilon}_{it} \tag{14}$$

$$\text{wald}_i = \text{ Wald statistic for } H_0 : \Psi_i = 0 \text{ in the regression } y_{it} = \lambda_i' \hat{f}_t + \psi \hat{f}_t^* + \nu_{it} \tag{15}$$

Where

$$\hat{f}_t^* = \begin{cases} 0 \text{ for } t = 1, ..., T_i^* \\ \hat{f}_t \text{ for } t = T_i^* + 1, ..., T \end{cases}$$

$$\text{LM}^* = \frac{\left(\sum_{i=1}^N s_i\right) - rN}{\sqrt{2rN}} \tag{16}$$

The LM* statistic pertains to the joint test of no structural break in all the factor loadings. Note, however, that the assumptions required for the LM* statistic are those of a strict factor model, namely no interdependence in the error terms $e_t$ in (1).

The LR, LM and Wald statistics defined above can be used to define a test for a unknown break date as in Andrews (1993). Under mild assumptions Breitung and Eickmeier (2011) show that the supremum statistic defined over the LM statistics is distributed using the nonstandard distribution given in Andrews (1993) and tabulated again in Andrews (2003).

$$\mathscr{S}_{i,T}(\tau_0) = \sup_{\tau \in [\tau_0, 1-\tau_0]} (s_i^\tau) \text{ where } s_i^\tau \text{ refers to either the LR, LM or Wald statistic} \tag{17}$$

As mentioned in the introduction of the dynamic factor model, I follow Breitung and Eickmeier (2011) in assuming an AR(p) process for the innovation term as in equation (18):

$$e_{it} = \varrho_{i,1} e_{i,t-1} + ... + \varrho_{i,p_i} e_{i,t-p_i} + \xi_{it} \tag{18}$$

This is in line with the assumptions of the approximate factor model introduced above.

For dynamic factor models[15] Breitung and Eickmeier (2011) propose to use a GLS transformed model, i.e. to perform the regression:

$$\varrho_i(L) y_{it} = \lambda_i' [\varrho(L) \hat{f}_t] + \psi' [\varrho_i(L) \hat{f}_t^*] + \varepsilon_{it}^*$$

The lag polynomials are obtained by applying some information criterion to the $N$ regressions

$$\hat{\varepsilon}_{it} = \varrho_{i,t-1} \hat{u}_{i,t-1} + ... + \varrho_{i,p_i} \hat{u}_{i,t-p_i} + \tilde{\varepsilon}_{i,t}$$

Similar test statistics as above can then be applied to the results of the GLS regressions. Breitung and Eickmeier (2011) only present the LM-statistic.

$$\hat{\varrho}_i(L) y_{it} = \lambda_i' \left[\varrho_i(L) \hat{f}_t\right] + \psi_i' \left[\hat{\varrho}_i(L) \hat{f}_t^*\right] + \tilde{\varepsilon}_{it}^* \text{ for } t = p_i + 1, ..., T_i$$

[15] Note that Breitung and Eickmeier (2011) call a model where the innovations are generated by an AR(p) model a dynamic model whereas the literature usually refers to a dynamic process in the factors as in equation (7). This is similar in effect but not quite the same.

Under heteroskedasticity the regression has to be weighted by the covariance matrix. If we also take account of the possibility of a break in the covariance matrix we get two distinct regressions

$$\frac{1}{\hat{\sigma}^{(1)}}\varrho_i(L)y_{it} = \lambda_i' \left[\frac{1}{\hat{\sigma}_i^{(1)}}\varrho_i(L)\hat{f}_t\right] + \left[\frac{1}{\hat{\sigma}_i^{(1)}}\varrho_i(L)\hat{f}_t^*\right] + \tilde{\varepsilon}_{it}^* \text{ for } t = p_i + 1, ..., T_i^*$$

$$\frac{1}{\hat{\sigma}^{(2)}}\varrho_i(L)y_{it} = \lambda_i' \left[\frac{1}{\hat{\sigma}_i^{(2)}}\varrho_i(L)\hat{f}_t\right] + \left[\frac{1}{\hat{\sigma}_i^{(2)}}\varrho_i(L)\hat{f}_t^*\right] + \tilde{\varepsilon}_{it}^* \text{ for } t = T_i * +1, ..., T$$

### 3.2 Bootstrapping Breitung and Eickmeier (2011)

In order to test whether the properties of the above test statistics can be improved upon I have repeated the montecarlo study presented in Breitung and Eickmeier (2011) tables 2 and 3 with the difference that the test statistics have been bootstrapped.

The idea behind bootstrapping test statistics comes from the definition of the p-value. Denoting with $\tau$ the test statistic of interest, by $\hat{\tau}$ the realization of that test statistic given the observed data and by $F$ the cumulative distribution function $\tau$ under the null hypothesis we can borrow the notation from Davidson and MacKinnon (2004) and write the p-value as

$$p(\hat{\tau}) = 1 - F(\hat{\tau})$$

Resultingly the p-value can be estimated by estimating $F(\hat{\tau})$. To do this we will simulate new data which follows the DGP under $H_0$. The p-value is then the (estimated) probability of the test statistic exceeding the calculated value $\hat{\tau}$ if the hypothesis $H_0$ were true.

Again following Davidson and MacKinnon (2004) we denote with stars, values that are calculated from simulated data, $B$ denotes the number of simulated samples. We can then estimate the cumulative distribution function as

$$\hat{F}^*(x) = \frac{1}{B}\sum_{j=1}^{B} I(\tau_j^* \leq x) \text{ and } \hat{F}^*(\hat{\tau}) = \frac{1}{B}\sum_{j=1}^{B} I(\tau_j^* \leq \hat{\tau})$$

Resultingly

$$\hat{p}^*(\hat{\tau}) = 1 - \hat{F}^*(\hat{\tau}) = 1 - \frac{1}{B}\sum_{j=1}^{B} I(\tau_j^* \leq \hat{\tau}) = \frac{1}{B}\sum_{j=1}^{B} I(\tau_j^* > \hat{\tau})$$

In the following two specifications of the bootstrap are considered. Firstly, the often used resampling of the residuals is applied. Secondly the wild bootstrap is used.

Resampling the residuals uses the definition of the factor equation (1) to generate new data. The innovation term $e_t$ is replaced with resampled residuals[16] gotten from the estimation of the model using the initial data.

$$X_t^* = \hat{f}_t\hat{\Lambda}' + \hat{e}_t^* \tag{19}$$

The wild bootstrap serves to deal with heteroskedasticity. The idea is to rescale the residuals using a white noise random variable $\nu_t$.

[16] Resampling is done with replacement to model the empirical distribution. So the resampled residuals $e_t^*$ are given by setting $\hat{e}_t^* = \hat{e}_s$ where $s$ is drawn from $\{1, ..., T\}$ with replacement.

$$X_t^* = \hat{f}_t \hat{\Lambda}' + \hat{e}_t^+$$

Where $\hat{e}_t^+$ is set such that to $\hat{e}_{it}^+ = \hat{e}_{it}\nu_{it}$ and $\nu_{it}$ is distributed with mean 0 and variance 1. For the application I choose $\nu$ to be standard normal.

# 4 Notes on the Literature

Cragg and Donald (1997) shows in a Monte Carlo analysis that $T \to \infty$ and $N \to \infty$ makes that classical theory for predicting the number of factors performs badly. Resultingly information criteria as in Bai and Ng (2002) should be used. These results hold under heteroskedasticity and weak serial correlation. But: structural breaks! TODO!!

# 5 Empirical application

In this section I will approach the issue of forecasting economic time series in the presence of structural breaks. Specifically, I will focus on forecasting German GDP using German und US American macroeconomic time-series. The reason for including American time-series to predict German GDP is twofold. Firstly, Germany is active on international markets with the US being the second most important export market after France. Coupled with the big importance of the US American market in the world economy it is plausible that US data plays a role in predicting German economic development. Secondly, we are interested in forecasting time-series under structural breaks. There are at least two periods commonly associated with breaks in the world economy: the dot-com bubble of 2000 and the financial crisis of 2008, both of which originated in the US. The hope is that big ruptures in the world economy make a break of relationships between variables (and ultimately of the factor loadings) likely.

There are several ways to address the issue of structural breaks from a forecasting viewpoint. Firstly structural breaks can be ignored. As has been stated above, in this case factor models will tend to choose the a higher number of factors than if no structural breaks are present which is means that the space the factors span is of a higher dimension as the factors are orthogonal.[17]

Secondly, structural breaks can be included in the model in the form of a dummy parameter interacting with the factor estimations $\hat{f}_t$. The forecasting equation of the model then results as

$$y_{t+h}^h = \alpha(L)W_t + \beta(L)\hat{f}_t + \gamma(L)\hat{f}_t^* + \varepsilon_{t+h} \tag{20}$$

where $\hat{f}_t^*$ is defined as above.

Thirdly, if predictors are to be targeted prior to estimating the factor equation it is conceivable to find that a subset of the predictors $\tilde{X}_t^{(1)} \subset X_t$ peforms well before and a different subset $\tilde{X}_t^{(2)} \subset X_t$ performs better after the structural break. This finding would imply that it could be beneficial to estimate the factor equation again using only the data after $T^*$ and discarding the rest.

## 5.1 Data choice and treatment

The data used for the horse race has been collected from the German Bundesbank and the the American Federal Reserve Bank of St. Louis.[18] It consists of $T = 82$ observations from $N = 169$

[17] How this affects the information criteria of Bai and Ng (2002) can be seen in table 1 of Breitung and Eickmeier (2011).

[18] More specifically the FRED2 public API has been used to extract selected series. The data from the Bundesbank have been scraped from the Bundesbank website.

quarterly variables. The series are listed below in appendix D. The series contain macroeconomic variables such as wages and salaries, employment levels and survey data from Germany and the US as well as financial variables.

Additionally an emphasis has been put on selecting several time-series which are well known to be subjected to big changes in the abovementioned periods such as "Housing starts".

They have been downloaded in seasonally adjusted form where appropriate. The data series have then been differenced (zero times, once or twice) until stationarity. To this end the number of differencing operations necessary to make the series stationary have been tested using the KPSS method developed in Kwiatkowski et al. (1992).[19]

The series have been demeaned and normalized such that $\frac{1}{T}\sum_{t=1}^{T} X_{it} = 0\ \forall i$ and $\frac{1}{T}\sum_{t=1}^{T} X_{it}^2 = 1 \Leftrightarrow \sum_{t=1}^{T} X_{it}^2 = T\ \forall i$.

## 5.2 Model selection and evaluation

This section explores the different methods to selected and evaluate a model. The ultimate goal is always forecasting the true data as closely as possible. To this end I use the root mean squared forecasting error (RMSE) as an evaluation mechanism. $RMSE = \sqrt{\frac{1}{w}\sum_{s=t}^{t+h}[(y_s - \hat{y}_s)^2]}$ where $w$ denotes the length of the window to be forecasted and $t$ the period preceding the window being forecasted.

If the number of static factors and the number of factor lags in the forecasting is predetermined using criteria defined above the number of lags of the variable of interest $y_t$ is still open. In the following I refer to the number of lags of $y_t$ as $s$. I follow Bai and Ng (2008) in selecting among model candidated using the BIC criterion which is of course an in-sample criterion. In addition we also consider out-of-sample forecasting performance. Since the point of the exercise is forecasting, the models' out-of-sample performance is monitored using the root mean squared error of one step ahead pseudo out-of-sample forecasts is computed.[20] This evaluation method follows Forni et al. (2005) and Bai and Ng (2008) among others. The foreacasting window $w$ over which the pseudo out-of-sample-forecasts are performed is set to $w = 10$ here.

TODO: put BIC and FPE definitions here

All models are compared to a benchmark model which consists of a simple moving average over the forecasting window $w$. The length of this window is set to $w = 15$. While this is a very short forecasting horizon, it makes sure that we will have more observations than parameters for all possible models. Since the length of the data series is $T = 82$ this leaves only 67 observations for the first forecast in the pseudo out-of-sample procedure highlighting that factor models are adequate in a situation with $N >> T$.

The first panel in table **??** shows the results of the in sample selection procedure for a static

[19] The method consists of constructing an LM-test for the hypothesis that the variance $\sigma_\varepsilon^2$ is 0 (or equivalently that $\mu_t$ is constant) in

$$y_t = \beta' D_t + \mu_t + u_t$$
$$\mu_t = \mu_{t-1} + \varepsilon_t$$

where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ and $D_t$ captures deterministic components and time trends. This implies that the difference of $y_t$ denoted $\Delta y_t$ is stationary.

[20] Pseudo out-of-sample forecasts are a method to simulated a forecasting situation in that the forecaster does as if he would not know the true realizations of the data after $t$. The forecaster tries to predict the value $y_{t+h}$ given all the information up to $t$. $t$ is then successively moved one period ahead. Here we set $h = 1$.

factor model. Three variants for choosing the number of factors are considered. Firstly the $\mathrm{IC}_{p2}$ criterion from Bai and Ng (2002) is used to estimate the number of static factors for both the factor equation and the forecasting equation. The information criteria (BIC and FPE) are then used to select $s$, the number of lags of $y_t$. As a second choice, the decision over the number of factors *and* the number of lags of $y_t$ in the forecasting equation are left to the information criteria. I.e. the information criteria are minimized over different choices of $r$ and $s$. The same number of factors is used in the factor equation. A third possibility consists of leaving the choice of the number of factors for the factor equation to a Bai and Ng (2002) criterion and using the BIC or FPE criterion to choose the number of static factors in the forecasting equation along with the number of lags of $y_t$. I.e. the BIC or FPE are minimized over $r$ and $s$.

Both the BIC and FPE as well as the $IC_{p2}$ criteria select a large number of factors. The fact that conventional information criteria should not be used for selecting the number of static factors has been extensively documented in the factor model literature and is no surprise. Since the variance of the data series has been normalized to 1 and the mean to 0 the RMSE of the simple average $y_t$ in the given window can be expected to be close to 1. The RMSE of the resulting out-of-sample forecasts is very large with 1.30 and 1.22 respectively. The benchmark model of a moving average reports and RMSE of 0.86 and an AR(4) model achieves an RMSE of 0.60 for the same time horizon.

The best performing static factor model in terms of forecasting can be got if the BIC criterion for model selecting is replaced by the RMSE as can be seen in the second panel. I.e. the RMSE is minimized over choices of $r$ and $s$ rather than the BIC. This implies using the out-of-sample performance as an evaluation criterion. In that case both $r$ and $s$ are shrunken to 1 which greatly reduces the RMSE. While this outperforms the simple benchmark model, the fact that the most parsimonious static factor model performs best in out-of-sample prediction hints at possible performance increases if dynamic effects are allowed.

Table 1: Static factor model, model selection

| | $r_{factor}$ | $r_{forecast}$ | $s$ | RMSE |
|---|---|---|---|---|
| | | *In-Sample* | | |
| BIC | 13 | 13 | 5 | 1.30 |
| FPE | 15 | 15 | 5 | 1.23 |
| $IC_{p2}$ and BIC | 4 | 13 | 5 | 1.30 |
| $IC_{p2}$ and FPE | 4 | 15 | 5 | 1.23 |
| $IC_{p2}$ | 4 | 4 | 10 | 1.21 |
| | | *Out-of-Sample* | | |
| RMSE | 1 | 1 | 1 | 0.71 |
| RMSE and $IC_{p2}$ | 4 | 1 | 1 | 0.71 |

***Notes:*** The results are derrived by minimizing the information criteria in the first column. If both an Bai and Ng (2002) criterion and a standard information criterion is given the former is applied to the factor equation and the latter to the forecasting equation. The maximum number of factors $r$ is set to 15. The maximum number number of the lags of the variable of interest $s$ is set to 10.

Repeating the exercise but allowing for lags of the factors to enter the forecasting equation we

get the results in table 2.[21] Since the data used for the exercise consists of time-series it makes sense that a dynamic factor model is better suited for prediction. In the case of the dynamic model the FPE criterion seems to select too many lags of $y_t$ which results in a higher RMSE. The $IC_{p2}$ criterion chooses $r = 4$ for the factor equation. For the forecasting equation, however, it seems to be better to reduce the number of factors to 2 if the factors are also lagged once in the forecasting equation. This can be seen in the row showing the results for the BIC and the $IC_{p2}$. The resulting RMSE is RMSE= 0.58 which is slightly lower thatn 0.59, the RMSE for $r = 4$ factors in the forecasting equation. While the difference is small, this shows that it can be optimal to choose a different number of factors in the forecasting equation than in the factor equation.

In both the static and the dynamic factor model, choosing the number of static factors using the $\mathrm{IC}_{p2}$ criterion has at least slightly outperformed the other methods in terms of forecasting accuracy.

Table 2: Dynamic factor model, model selection

| | $r_{factor}$ | $r_{forecast}$ | $s$ | $q$ | RMSE |
|---|---|---|---|---|---|
| BIC | 2 | 2 | 3 | 1 | 0.58 |
| FPE | 10 | 10 | 10 | 2 | 0.82 |
| $IC_{p2}$ and BIC | 4 | 2 | 3 | 1 | 0.58 |
| $IC_{p2}$ and FPE | 4 | 10 | 10 | 2 | 0.82 |
| $IC_{p2}$ | 4 | 4 | 3 | 1 | 0.59 |
| | *Out-of-Sample* | | | | |
| RMSE | 7 | 7 | 1 | 2 | 0.55 |
| RMSE and $IC_{p2}$ | 4 | 2 | 1 | 1 | 0.54 |

***Notes:*** The results are derrived by minimizing the information criteria in the first column. If both an Bai and Ng (2002) criterion and a standard information criterion is given the former is applied to the factor equation and the latter to the forecasting equation. The maximum number of factors $r$ is set to 10. The maximum number number of the lags of the variable of interest $s$ is set to 10. $q$ here refers to the number of lags of the factors in the forecasting equation, not to the number of dynamic factors as defined above.

### 5.3 Structural breaks

Structural breaks are estimated using Quandt-Andrews type supremum tests as defined above. The critival values for the test statistics are given in table I of Andrews (2003). The results can be seen in table 3.

It is apparent that there are two periods with major changes in the loadings. Firstly, there are structural breaks reported in the loadings of 8 variables in the first quarter of 1999 which coincides approximately with the Russian default of 1998 and maybe the aftermath of the Asian financial crisis in 1997. Secondly, a total of 22 changes in the loadings are reported for the period of the second quatal of 2008 until the second quarter of the 2009. TODO: talk about the series that are most affected (for that I need to fix the variable indices)

[21] Essentially, table 1 uses the same models as are used in table 2 while restricting $q = 0$.

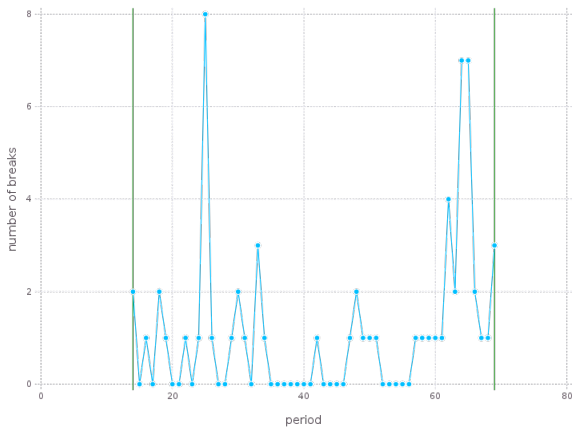
Table 3: Number of variable breaks per period

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 96:1 | 96:3 | 97:2 | 97:3 | 98:2 | 98:4 | 99:1 | 99:2 | 00:1 | 00:2 | 00:3 | 01:1 | 01:2 | 03:2 | 04:3 | 04:3 |
| 2 | 1 | 2 | 1 | 1 | 1 | 8 | 1 | 1 | 2 | 1 | 3 | 1 | 1 | 1 | 2 |
| 05:1 | 05:2 | 05:3 | 07:1 | 07:2 | 07:3 | 07:4 | 08:1 | 08:2 | 08:3 | 08:4 | 09:1 | 09:2 | 09:3 | 09:4 | 10:1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 | 2 | 7 | 7 | 2 | 1 | 1 | 3 |

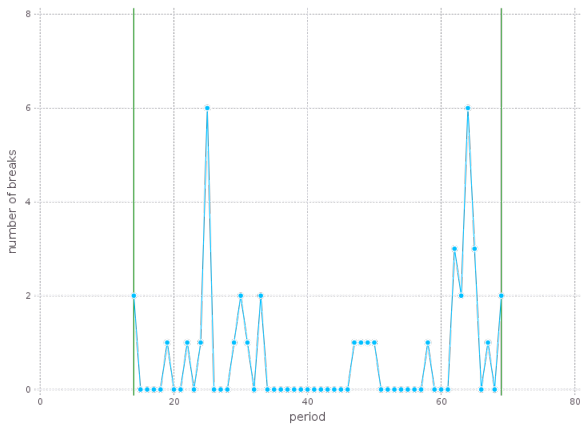
***Notes:*** The entries report how many of the 169 variables are predicted to undergo a structural break by period. Reported are the numbers of significant values of $\mathscr{S}_{i,T}$ at the 5% significance level. The total number of structural breaks found in the 169 variables is 64.

Figure 1: Structural breaks per quarter

(a) 5% significance level




(b) 1% significance level




Given the fact there are two periods where a majority of structural breaks occur, each of which affects a different set of factor loadings, the values before the break can not simply be cut-off and the model reestimated using data on the subperiod after the break as was proposed above as one way to reacting to structural breaks. While both periods could be treated as in the sense that only data after both structural breaks is used, this, however, would leave us with 17 observations of which we have to remove another 15 for the frist step in the pseudo out-of-sample forecasts.

In principle there are at least three ways to overcome this issue. A first way consists of as proceeding as above. Then it can be remembered that some of the data series are monthly series which have been merged into the quarterly data set, dropping two thirds of the observations. The data could be cast back into monthly form interpolating the missing data. While this method solves the problem in a technical sense[22], it has several drawbacks: firstly no additional information is added although it appears to the model as if there are more observations. At the same time the imputation adds additional uncertainty. Traditional standard error estimates do not account for this and resultingly they may be downward biased (cf. Gelman and Hill (2006, chapter 25). It is thus likely that prediction based on this short data set performs badly in a forecasting sense. On the other hand, if predictions resulting from this shaved data set should perform better, it

[22]For the first step of the pseudo out-of-sample period there are only 8 data points for estimation. For every quarterly step of the pseudo out-of-sample procedure there are an additional 4 observations. Thus, the ratio of observations to predictors improves quickly with each step of the pseudo out-of-sample prediction.

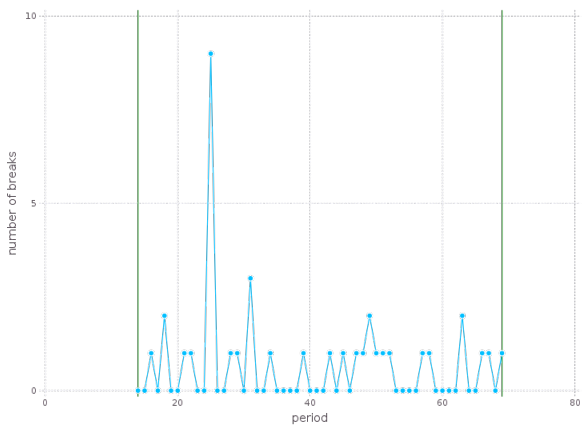
would constitute a strong indication that the structural breaks uncovered by the tests should not be ignored. In this case doing so clearly leads to a significant degradation of forecasting results.

For the purpose of this paper we are interested in forecasting performance of factor models which ignore structural breaks versus factor models which take the structural breaks into account. Since the data set is very wide and there are only relatively few series undergoing a structural break around 2008 a second possibility consists of dropping the series which are affected and redoing the predictions exercise from above based on this limited data set. Focussing on variables without parameter instability for prediction could improve the forecasting accuracy.

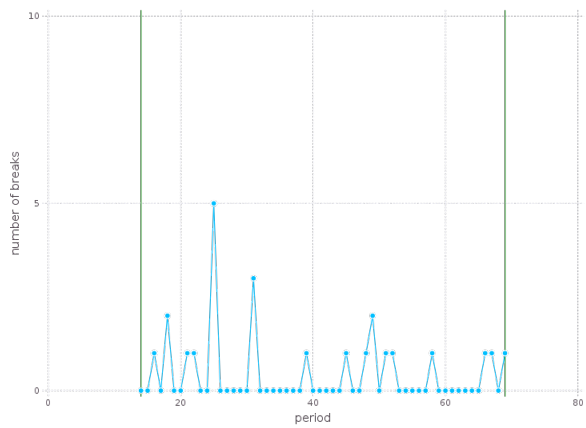
Figure 2 shows that the sup LM statistic $\mathscr{S}_{i,T}$ reports one clear period where most structural breaks are centered for the reduced data set. In the following both models are compared in terms of forecasting precision based on the reduced data set.

Figure 2: Structural breaks per quarter, reduced data set

(a) 5% significance level




(b) 1% significance level



# Apendix

## A Derrivation of Principal Components

The principal components of a data matrix $X$ are defined as the transformation of $X$ into the space spanned by the loadings [23] $u_i$ where each $u_i$ is a unit vector. The transformation is defined such that the transformed columns are uncorrelated (as are the loadings $u_i$) and such that the first transformed column (i.e. the first principal component) contains the largest amount of the variance of the original data, the second column the second largest amount (i.e. the second principal component), etc.

Let $X$ have mean 0 (i.e. subtract the mean from each observation if it does not). For the first principal component, we want to find a loading vector vector $u_1$ such that the variance along the projection of $x$ onto the first principal component direction $u_1$ is maximized where $u_1$ is a unit norm vector.

$$\max_{u_1:||u_1||=1} \frac{1}{T}\sum_{i=1}^{T}(x_i'u_1)^2 = \max_{u_1:u_1'u_1=1} u_1'(\frac{1}{T}\sum_{i=1}^{T} x_i'x_i)u_1 = \max_{u_1:u_1'u_1=1} u_1'(\Sigma)u_1$$

Setting up the Lagrangian yields

$$L(u_1, \lambda) = u_1'\Sigma u_1 - \lambda(u_1'u_1 - 1)$$

After taking the derivative with respect to $u_1$ and dividing by 2 we are left with

$$\frac{\partial L}{\partial u_1} = \Sigma u_1 - \lambda u_1 \stackrel{!}{=} 0$$

in other words $u_1$ is an eigenvector of $\Sigma$, the covariance matrix of x. The corresponding eigenvalue is $\lambda$ and we must have $\Sigma = \lambda$ for the variance of the principal component to be maximal under the constraint.
The maximal variance of $x'u_1$ is thus $V(x'u_1) = u_1'\Sigma u_1 = u_1'\lambda u_1$ and it is achieved for the highest eigenvalue $\lambda$ of the covariance matrix of x. The derrivation of the following principal components is accordingly subject to the additional constraint that the principal components are uncorrelated. In the case of the second principal component this implies $\text{cov}(u_1'x, u_2'x) = u_1'\Sigma u_2 = 0$. The solution for the third and all following principal component is more complex but goes accordingly. The derrivations can be seen e.g. in Jolliffe (2005) or any other textbook on the subject. The solution for the $i$-th principal component is to set the loadings $u_i$ to be eigenvector which belongs to the $i$-th highest eigenvalue. Also, for each principal component $i$ we get that $\text{var}(u_i'x) = \lambda_i$.

[23] The $u_i$ are also called scores or principal component directions.

## B Estimation of the Factor Equation

Estimators of the factor equation solve (10).

$$\min_{F_1,\ldots,F_T,\Lambda} V_r(\Lambda, F) = \min_{F_1,\ldots,F_T,\Lambda} \frac{1}{NT}\sum_{t=1}^{T}(X_t - \Lambda F_t)'(X_t - \Lambda F_t)$$

$$\text{s.t. } N^{-1}\Lambda'\Lambda = I_r$$

The solution can be gotten by first optimizing over $F_t$. The first order condition is:

$$\frac{\partial V_r}{\partial F_t} = -2X_t'\Lambda + 2\hat{F}_t'\Lambda'\Lambda \stackrel{!}{=} 0$$

Resultingly $\hat{F}_t = (\Lambda'\Lambda)^{-1}\Lambda' X_t$. Inserting that back into the optimization problem gives $\min_\Lambda \frac{1}{NT} X_t'X_t - 2X_t'\Lambda(\Lambda'\Lambda)^{-1}\Lambda' X_t + X_t'\Lambda(\Lambda'\Lambda)^{-1}\Lambda'\Lambda(\Lambda'\Lambda)^{-1}\Lambda' X_t = \min_\Lambda \frac{1}{NT} X_t'[I - \Lambda(\Lambda'\Lambda)^{-1}\Lambda']X_t$.
This is equivalent to the problem $\max_\Lambda \operatorname{tr}\left\{(\Lambda'\Lambda)'^{-\frac{1}{2}}\lambda'(T^{-1}\Sigma_{t=1}^{T} X_t X_t')\Lambda(\Lambda'\Lambda)^{-\frac{1}{2}}\right\}$ which is the same as $\max_\Lambda \Lambda'\hat{\Sigma}_{XX}\Lambda$ s.t. $N^{-1}\Lambda'\Lambda = I_r$ where $\hat{\Sigma}_{XX}$ is the usual sample equivalent of the variance of $X$. This can be solved by setting $\hat{\Lambda}$ to eigenvectors of the $r$ largest eigenvalues of $\hat{\Sigma}_{XX}$.

# C Replication of Bai and Ng (2002) for $r = 7$ and $r = 9$

Table 4: Replication of Table I of Bai and Ng (2002) for $r = 7$

DGP: $X_{it} = \sum_{j=1}^{r} \lambda_{ij} F_{tj} + \sqrt{\theta} e_{it}, r = 7$

| N | T | $PC_{p1}$ | $PC_{p2}$ | $PC_{p3}$ | $IC_{p1}$ | $IC_{p2}$ | $IC_{p3}$ |
|---|---|---|---|---|---|---|---|
| 100 | 40 | 6.4 | 5.9 | 6.97 | 4.93 | 3.46 | 6.73 |
| 100 | 60 | 6.72 | 6.27 | 7.02 | 6.34 | 4.74 | 6.99 |
| 200 | 60 | 6.93 | 6.87 | 7.0 | 6.78 | 6.46 | 6.96 |
| 500 | 60 | 6.99 | 6.96 | 7.0 | 6.94 | 6.92 | 6.99 |
| 1000 | 60 | 7.0 | 6.99 | 7.0 | 6.99 | 6.97 | 7.0 |
| 2000 | 60 | 7.0 | 6.99 | 7.0 | 6.98 | 6.99 | 7.0 |
| 100 | 100 | 7.0 | 6.77 | 7.35 | 6.89 | 6.32 | 50.0 |
| 200 | 100 | 7.0 | 7.0 | 7.0 | 7.0 | 6.99 | 7.0 |
| 500 | 100 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 |
| 1000 | 100 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 |
| 2000 | 100 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 |
| 40 | 100 | 6.39 | 6.12 | 7.02 | 4.86 | 3.49 | 6.78 |
| 60 | 100 | 6.79 | 6.37 | 7.01 | 6.01 | 5.0 | 7.0 |
| 60 | 200 | 6.93 | 6.85 | 7.0 | 6.82 | 6.58 | 6.98 |
| 60 | 500 | 6.96 | 6.97 | 7.0 | 6.97 | 6.9 | 6.99 |
| 60 | 1000 | 7.0 | 6.99 | 7.0 | 6.99 | 6.96 | 7.0 |
| 60 | 2000 | 7.0 | 7.0 | 7.0 | 6.97 | 6.98 | 6.99 |
| 4000 | 60 | 7.0 | 7.0 | 6.99 | 6.98 | 6.99 | 6.99 |
| 4000 | 100 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 |
| 8000 | 60 | 6.99 | 7.0 | 7.0 | 7.0 | 6.98 | 7.0 |
| 8000 | 100 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 |
| 60 | 4000 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 6.99 |
| 100 | 4000 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 |
| 60 | 8000 | 6.99 | 7.0 | 7.0 | 6.99 | 7.0 | 6.99 |
| 100 | 8000 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 | 7.0 |
| 10 | 50 | 5.0 | 5.0 | 5.0 | 4.76 | 3.95 | 5.0 |
| 10 | 100 | 5.0 | 5.0 | 5.0 | 4.58 | 4.18 | 4.96 |
| 20 | 100 | 7.04 | 6.5 | 7.98 | 3.14 | 2.07 | 6.94 |
| 100 | 10 | 5.0 | 5.0 | 5.0 | 5.0 | 4.96 | 5.0 |
| 100 | 20 | 7.1 | 6.64 | 8.05 | 3.3 | 2.08 | 8.2 |

***Notes:*** Estimated number of factors averaged over 1000 simulations. The true number of factors is $r$ and the maximum number of factors is ceil(min$\{T, N\}$)/2).

Table 5: Replication of Table I of Bai and Ng (2002) for $r = 9$

DGP: $X_{it} = \sum_{j=1}^{r} \lambda_{ij} F_{tj} + \sqrt{\theta} e_{it}, r = 9$

see Bai and Ng (2002) for specification of the DGP

| N | T | $PC_{p1}$ | $PC_{p2}$ | $PC_{p3}$ | $IC_{p1}$ | $IC_{p2}$ | $IC_{p3}$ |
|---|---|---|---|---|---|---|---|
| 100 | 40 | 6.51 | 5.93 | 8.06 | 3.55 | 1.49 | 7.89 |
| 100 | 60 | 7.02 | 6.25 | 8.77 | 5.4 | 2.74 | 8.9 |
| 200 | 60 | 7.74 | 7.31 | 8.65 | 7.33 | 6.26 | 8.74 |
| 500 | 60 | 8.09 | 8.02 | 8.52 | 8.18 | 7.97 | 8.58 |
| 1000 | 60 | 8.42 | 8.38 | 8.59 | 8.45 | 8.25 | 8.62 |
| 2000 | 60 | 8.47 | 8.32 | 8.59 | 8.67 | 8.56 | 8.64 |
| 100 | 100 | 8.03 | 6.93 | 9.01 | 7.68 | 4.85 | 50.0 |
| 200 | 100 | 8.85 | 8.48 | 8.99 | 8.92 | 8.58 | 9.0 |
| 500 | 100 | 9.0 | 8.99 | 9.0 | 9.0 | 8.99 | 9.0 |
| 1000 | 100 | 9.0 | 8.99 | 9.0 | 9.0 | 9.0 | 9.0 |
| 2000 | 100 | 9.0 | 9.0 | 9.0 | 9.0 | 9.0 | 9.0 |
| 40 | 100 | 6.63 | 6.06 | 8.08 | 3.38 | 1.66 | 7.79 |
| 60 | 100 | 7.03 | 6.33 | 8.85 | 5.51 | 2.68 | 8.94 |
| 60 | 200 | 7.79 | 7.43 | 8.6 | 7.1 | 6.05 | 8.73 |
| 60 | 500 | 8.16 | 7.98 | 8.61 | 8.28 | 7.96 | 8.66 |
| 60 | 1000 | 8.41 | 8.28 | 8.51 | 8.51 | 8.34 | 8.69 |
| 60 | 2000 | 8.4 | 8.4 | 8.58 | 8.57 | 8.56 | 8.74 |
| 4000 | 60 | 8.52 | 8.44 | 8.57 | 8.61 | 8.51 | 8.73 |
| 4000 | 100 | 9.0 | 9.0 | 9.0 | 9.0 | 9.0 | 9.0 |
| 8000 | 60 | 8.44 | 8.52 | 8.55 | 8.66 | 8.64 | 8.68 |
| 8000 | 100 | 9.0 | 9.0 | 9.0 | 9.0 | 9.0 | 9.0 |
| 60 | 4000 | 8.53 | 8.54 | 8.55 | 8.62 | 8.6 | 8.57 |
| 100 | 4000 | 9.0 | 9.0 | 9.0 | 9.0 | 9.0 | 9.0 |
| 60 | 8000 | 8.61 | 8.47 | 8.51 | 8.66 | 8.58 | 8.76 |
| 100 | 8000 | 9.0 | 9.0 | 9.0 | 9.0 | 9.0 | 9.0 |
| 10 | 50 | 5.0 | 5.0 | 5.0 | 4.66 | 3.29 | 5.0 |
| 10 | 100 | 5.0 | 5.0 | 5.0 | 4.32 | 3.59 | 4.77 |
| 20 | 100 | 7.25 | 6.84 | 8.1 | 1.76 | 1.28 | 7.21 |
| 100 | 10 | 5.0 | 5.0 | 5.0 | 4.96 | 4.74 | 5.0 |
| 100 | 20 | 7.37 | 7.02 | 8.28 | 1.92 | 1.25 | 8.89 |

***Notes:*** Estimated number of factors averaged over 1000 simulations. The true number of factors is $r$ and the maximum number of factors is ceil(min$\{T, N\}$)/2).

# D Data series

Table 6: Bundesbank data series

| | Series Id | Series Title |
|---|---|---|
| 1 | BBNA2.Q.DE.N.I. 021.CA.D.V.I05.A, | Actual earnings/ Overall economy/ Wages and salaries per employee (workplace concept)/ Index/ Germany |
| 2 | BBK01.EA4220, | Balance on current account |
| 3 | BBK01.DU7834, | Basic pay rates, overall economy, excluding ancillary benefits, excluding one-off payments, on a monthly basis, Germany |
| 4 | BBXS1.A.I7.N.EUBUCS. MANU.MAN013.TT.PCT.I00 | Business and consumer surveys (European Commission) / Capacity utilization (man) / Euro area 18 (fixed composition) |
| 5 | BBXS1.A.DE.N.EUBUCS. MANU.MAN013.TT.PCT.I00 | Business and consumer surveys (European Commission) / Capacity utilization (man) / Germany |
| 6 | BBK01.OU7744, | Capital / Total / All categories of banks |
| 7 | BBK01.BQ1351, | Central government debt - total debt |
| 8 | BBDE1.M.DE.Y.JAA9. P2XF50500.CB.V.ABA.A | Construction permits - estimated construction cost / At current prices, flows / Germany / Total (including construction work on existing buildings) - residential and non-residential buildings, total |
| 9 | BBXR1.M.HWWI.N.EURO. TOT-NXNGY.INDBEU.HE.M00 | Construction price index / Germany / Unadjusted figure / Total |
| 10 | BBK01.OXA8R3 | domestic Financial Vehicle Corporations - All categories of banks |
| 11 | BBDP1.M.DE.Y.VPI. C.A00000.I10.L | Consumer price index / Germany / Unadjusted figure / Total |
| 12 | BBK01.WX9141 | DAX price index / End 1987 = 1000 / End of month |
| 13 | BBK01.WT4065 | DE0001102358 / Prices / 1,500 Bund 14/24 |
| 14 | BBK01.SUD106S | Effective interest rates of German banks / New business (estimated) / Households' deposits, overnight |
| 15 | BBK01.SUD210 | Effective interest rates of German banks / New business / Non-financial corporations' deposits, overnight |
| 16 | BBDE1.M.DE.Y.GUA1. P2XF13020.BA.V.ABA.A | Employed persons (including proprietors and family workers) / Persons / Germany / Main construction industry |
| 17 | BBK01.US8CA0 | Employees subject to social security contributions / Total / Eastern Germany |
| 18 | BBK01.US80CA | Employees subject to social security contributions / Total / Western Germany |

| | Series Id | Series Title |
|---|---|---|
| 19 | BBK01.USCA80 | Employment in Germany / Employed persons in Germany / Germany |
| 20 | BBK01.JAB938 | Enterprises |
| 21 | BBXN1.A.I7.N.NAG. B1QG00.0000.Z0N.IND.I00 | Euro area 18 (fixed composition) / National accounts - Gross domestic product / Gross domestic product / Total / Indices |
| 22 | BBK01.XS5609, | Exports / European countries |
| 23 | BBK01.XX3382, | Exports of the Federal Republic of Germany / Special Trade / Total / 1 / 2 / |
| 24 | BBK01.EU4370, | Financial transactions / Other investment / Balance |
| 25 | BBK01.EU2518, | Foreign investment in Germany / Other investment / Total / Balance |
| 26 | BBK01.EU4510 | Foreign trade / Exports (fob) |
| 27 | BBMF1.M.D0.EUR. GOVT.WYWG.Y10.BID | Geldmenge M1 / Veränderung saisonbereinigt / Jahresrate / EWU |
| 28 | BBK01.BQ2190 | General government budgetary position (Germany) - total revenue |
| 29 | BBK01.BQM403 | General government debt - of which currency and deposits |
| 30 | BBK01.BQM208 | General government debt - of which loans |
| 31 | BBK01.BQ8068 | General government debt as defined in the Maastricht Treaty - Germany - overall |
| 32 | BBK01.BJ8072 | General government deficit / surplus as defined in the Maastricht Treaty - Germany - overall |
| 33 | BBK01.XS4204 | German exports / special trade / values / 1 / 2 / 3 / 4 |
| 34 | BBK01.XS4206 | German foreign trade balance / special trade / values / 1 / 2 / 3 / 4 |
| 35 | BBK01.XS4205 | German imports / special trade / values / 1 / 2 / 3 / 4 |
| 36 | BBK01.EC7750 | German investment abroad / EU member states (28) / Total / Balance |
| 37 | BBK01.ED9065 | German investment abroad / Emerging markets and developing countries / Total / Balance |
| 38 | BBK01.ED9004 | German investment abroad / Euro-area member states / Total / Balance |
| 39 | BBK01.ED9001 | German investment abroad / Europe / Total / Balance |
| 40 | BBK01.ED1096 | German investment abroad / Other EU member states / Total / Balance |
| 41 | BBK01.ED9031 | German investment abroad / Other European countries / Total / Balance |
| 42 | BBK01.JBA161 | Gross domestic product / at current prices / Seasonally and working-day adjusted |
| 43 | BBK01.JAA026 | Gross national income (GNP) / at current prices |
| 44 | BBK01.WU0042 | Gross sales of domestic debt securities at nominal value / Total |
| 45 | BBK01.JAA327 | Gross wages and salaries |
| 46 | BBXP1.M.U2.N.HICP. 000000.IND.I00 | Hamonized Index of Consumer Prices / Euro area (changing composition) / Overall index / Unadjusted figure / Monthly |
| 47 | BBK01.TQ7162 | Households - Household debt to GDP FSI-I033 |
| 48 | BBK01.XS8953 | Imports / Product Classification for Production / Statistics from 2009 Total excluding energy |

| | Series Id | Series Title |
|---|---|---|
| 49 | BBK01.XS8857 | Imports / Value / Total, excluding energy |
| 50 | BBK01.EU4169 | Income / Total / Receipts |
| 51 | BBDE1.Q.DE.Y.LCB1. A2N200000.A.L.I08.A1 | Index of gross wages and salaries / Germany / Mining and quarrying, manufacturing and service activities (B-S) |
| 52 | BBDE1.Q.DE.Y.LCA1. A2N200000.A.L.I08.A1 | Index of labour costs / Germany / Mining and quarrying, manufacturing and service activities (B-S) |
| 53 | BBDE1.Q.DE.Y.LCC1. A2N200000.A.L.I08.A1 | Index of non-wage costs / Germany / Mining and quarrying, manufacturing and service activities (B-S) |
| 54 | BBDP1.M.DE.Y. VPI.C.SVXR.I10.A | Index of producer prices of industrial products sold on the domestic market / Germany |
| 55 | BBEE1.M.DE.AAA. XY0U12.R.AACPB.M00 | Indicator of the German economy's price competitiveness against 24 selected industrial countries, based on the deflators of total sales |
| 56 | BBK01.XSC430 | Indices of foreign trade prices - exports, total / Germany |
| 57 | BBXE1.M.I7.W. PROD.NS0020.IND.I00 | Industrieproduktion / Gesamte Industrie (ohne Baugewerbe) / Index / nur kalenderbereinigt / Euro-Währungsgebiet-18 |
| 58 | BBK01.EB0057 | International Investment Position, all countries, net, total (from end 1997) |
| 59 | BBK01.OEAA56G | Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Baden-Württemberg |
| 60 | BBK01.OEAA21H | Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Bayern |
| 61 | BBK01.OEAA03I | Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Berlin |
| 62 | BBK01.OEAA52S | Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Brandenburg |
| 63 | BBK01.OEAA56J | Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Bremen |
| 64 | BBK01.OEAA01K | Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Hamburg |
| 65 | BBK01.OEAA56L | Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Hessen |
| 66 | BBK01.OEAA37R | Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Mecklenburg-Vorpommern |
| 67 | BBK01.OEAA56M | Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Niedersachsen |
| 68 | BBK01.OEAA56N | Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Nordrhein-Westfalen |
| 69 | BBK01.OEAA03O | Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Rheinland-Pfalz |
| 70 | BBK01.OEAA03P | Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Saarland |
| 71 | BBK01.OEAA56V | Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Sachsen |
| 72 | BBK01.OEAA02T | Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Sachsen-Anhalt |

| | Series Id | Series Title |
|---|---|---|
| 73 | BBK01.OEAA56Q | Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Schleswig-Holstein |
| 74 | BBK01.OEAA56U | Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Thüringen |
| 75 | BBK01.OEAB01G | Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Baden-Württemberg |
| 76 | BBK01.OEBB21H | Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Bayern |
| 77 | BBK01.OEAB01I | Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Berlin |
| 78 | BBK01.OEAB01S | Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Brandenburg |
| 79 | BBK01.OEAB01J | Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Bremen |
| 80 | BBK01.OEBB12K | Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Hamburg |
| 81 | BBK01.OELB02L | Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Hessen |
| 82 | BBK01.OEAB01R | Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Mecklenburg-Vorpommern |
| 83 | BBK01.OEAB01M | Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Niedersachsen |
| 84 | BBK01.OEAB01N | Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Nordrhein-Westfalen |
| 85 | BBK01.OELB03O | Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Rheinland-Pfalz |
| 86 | BBK01.OEAB01P | Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Saarland |
| 87 | BBK01.OEYB19V | Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Sachsen |
| 88 | BBK01.OEAB01T | Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Sachsen-Anhalt |
| 89 | BBK01.OEAB01Q | Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Schleswig-Holstein |
| 90 | BBK01.OEAB01U | Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Thüringen |
| 91 | BBK01.SU0009 | Lending rates of banks / Current account credit less than EUR 100,000 / Average interest rate |
| 92 | BBK01.SU0511 | Lending rates of banks / Long-term fixed-rate loans to enterprises and self-employed persons, EUR 100,000 and more but less than EUR 500,000, effective interest rate / Average interest rate |
| 93 | BBK01.SU0012 | Lending rates of banks / Mortgage loans secured by residential real estate with interest rates fixed for 2 years, effective interest rate / Average interest rate |
| 94 | BBK01.OU7677 | Lending to banks (MFIs) / Balances and loans / All categories of banks |

| | Series Id | Series Title |
|---|---|---|
| 95 | BBK01.OU7676 | Lending to banks (MFIs) / Total / All categories of banks |
| 96 | BBK01.PC3767 | Lending to computer and related activities, research and development / Total / All categories of banks |
| 97 | BBK01.PC3723 | Lending to construction / Total/ All categories of banks |
| 98 | BBK01.OU7712 | Lending to domestic banks (MFIs) / Credit balances and loans / All categories of banks |
| 99 | BBK01.TUD364 | MONEY STOCK M1 (FROM 2002, EXCLUDING CURRENCY IN CIRCULATION) / GERMAN CONTRIBUTION |
| 100 | BBK01.TQ7036 | Market liquidity - Average bid-ask spread in the securities market - government bills FSI-I035 |
| 101 | BBK01.OU8045 | Medium and long-term lending / to general government / Securities / All categories of banks |
| 102 | BBK01.OU8041 | Medium and long-term lending to general government / Loans / Long-term / All categories of banks |
| 103 | BBK01.OU9829 | Medium and long-term lending to general government / Loans / Medium-term / All categories of banks |
| 104 | BBK01.TS1303 | Monetary aggregate M3 (from January 2002, excluding currency in circulation; from June 2010, excluding repos with central counterparties) / German contribution / Outstanding amounts at the end of the month (stocks) |
| 105 | BBK01.OUP001 | Money market paper issued by banks (MFIs) / All categories of banks |
| 106 | BBK01.SU0295 | Money market rates / EONIA / Monthly average |
| 107 | BBK01.SU0334G | Money market rates / EURIBOR three-month funds / Moving monthly average |
| 108 | BBK01.TVE309 | Money stock M3 / EMU / Total / Changes |
| 109 | BBK01.JB5001 | National accounts - domestic demand (price adjusted) |
| 110 | BBK01.JB5007 | National accounts - employment / Germany / 1 / 2 / |
| 111 | BBK01.JB5005 | National accounts - exports (price adjusted) / |
| 112 | BBK01.JQ5003 | National accounts - government consumption (price adjusted) |
| 113 | BBK01.JB5004 | National accounts - gross fixed capital formation (price adjusted) |
| 114 | BBK01.JB5008 | National accounts - labour costs per employee / |
| 115 | BBK01.JQ5002 | National accounts - private consumption (price adjusted) |
| 116 | BBK01.BJ9195 | National accounts - total general government revenue (ESA 95) - Germany as a whole (western Germany up to and including 1990) |
| 117 | BBK01.BJ9049 | National accounts - total general government revenue (GFS as defined by the ECB) - Germany as a whole |
| 118 | BBNA2.Q.DE.N.I. 041.CA.D.V.I05.A | National accounts / Gross wages and salaries per hour worked by employees / Whole economy / Germany |
| 119 | BBK01.JJA327 | National accounts/Households' income/Germany/ Gross wages and salaries (residence concept) |
| 120 | BBK01.JQA024 | National accounts/Labour productivity per hour worked by persons in employment/ Whole economy |
| 121 | BBK01.JJB071 | National accounts/Origin of GDP/Chain-linked index/ Production sector (excluding construction) |

| | Series Id | Series Title |
|---|---|---|
| 122 | BBK01.JQA001 | National accounts/Overall economic view/ Compensation of employees - residents |
| 123 | BBK01.JQC000 | National accounts/Overall economic view/At current prices/ GDP |
| 124 | BBK01.JJB949 | National accounts/Unit labour costs per hour/ Whole economy (excluding agriculture, forestry and fishing, public services, education and health and other services) |
| 125 | BBK01.JQA111 | National accounts/Use of GDP/At current prices/ Private consumption |
| 126 | BBK01.JQC111 | National accounts/Use of GDP/Price index/ Private consumption |
| 127 | BBK01.OXA8T5 | off-balance true sale of domestic banks (MFIs) - All categories of banks |
| 128 | BBEE1.A.I7.AAA. XZE021.R.AACPE.M00 | Nominal effective exchange rate of the euro against the currencies of the EER-20 group |
| 129 | BBK01.TQ7032 | Non-financial corporations sector - Total debt to equity FSI-I028 |
| 130 | BBDE1.M.DE.Y.AEA1. A2P300000.F.C.I10.L | Orders received / At constant prices / Germany / Industry / Unadjusted figure |
| 131 | BBDE1.M.DE.Y.AEA1. P2XF04000.B2.C.I10.A | Orders received / At constant prices / Germany / Main construction industry |
| 132 | BBDE1.M.DE.W.AEA1. A2P340000.F.V.I10.A | Orders received / At current prices / Germany / Industry / Calendar adjusted only |
| 133 | BBDE1.M.DE.Y.AEA1. A2P300000.F.C.I10.A | Orders received / At current prices / Germany / Industry |
| 134 | BBDE1.M.DE.Y.AEA1. A2Q501000.F.V.I10.A | Orders received / At current prices, flows / Germany / 20+21 Manufacture of chemicals, chemical products; basic pharmaceutical products and pharmaceutical preparations |
| 135 | BBDE1.M.DE.W.AEA1. P2XF04000.B2.V.I10.A | Orders received / At current prices, flows / Germany / Main construction industry / Calendar adjusted only |
| 136 | BBDE1.M.DE.Y.AEA1. P2XF13010.B2.V.I10.A | Orders received / At current prices, flows / Germany / Main construction industry |
| 137 | BBDE1.M.DE.W.AEN1. A2P340000.F.V.I10.A | Orders received excluding large orders, domestic / At current prices, flows / Germany / Industry / Calendar adjusted only |
| 138 | BBDE1.M.DE.Y.AEN1. A2P350000.F.C.I10.A | Orders received excluding large orders, domestic / At current prices, flows / Germany / Industry |
| 139 | BBDE1.M.DE.Y.AEP1. A2P350000.F.C.I10.A | Orders received excluding large orders, euro-area countries / At current prices, flows / Germany / Industry |
| 140 | BBDE1.M.DE.W.AEN5. A2P340000.F.V.I10.A | Orders received excluding large orders, foreign / At current prices, flows / Germany / Industry / Calendar adjusted only |
| 141 | BBDE1.M.DE.Y.AEN5. A2P350000.F.C.I10.A | Orders received excluding large orders, foreign / At current prices, flows / Germany / Industry |
| 142 | BBDE1.M.DE.Y.AEP5. A2P350000.F.C.I10.A | Orders received excluding large orders, non-euro-area countries / At current prices, flows / Germany / Industry |
| 143 | BBDE1.M.DE.W.AEM1. A2P340000.F.V.I10.A | Orders received excluding large orders, total / At current prices, flows / Germany / Industry / Calendar adjusted only |
| 144 | BBDE1.M.DE.Y.AEM1. A2P350000.F.C.I10.A | Orders received excluding large orders, total / At current prices, flows / Germany / Industry |

| | Series Id | Series Title |
|---|---|---|
| 145 | BBDE1.M.DE.Y.AEB5. A2P350000.F.C.I10.A | Orders received from abroad / At current prices / Germany / Industry |
| 146 | BBDE1.M.DE.Y.AEB1. A2Q501000.F.C.I10.A | Orders received from the domestic market / At constant prices / Germany / 20+21 Manufacture of chemicals, chemical products; basic pharmaceutical products and pharmaceutical preparations |
| 147 | BBDE1.M.DE.Y.AEB1. A2P350000.F.C.I10.A | Orders received from the domestic market / At current prices / Germany / Industry |
| 148 | BBDE1.M.DE.Y.BAA1. P2XF00000.G.C.I10.L | Output in the production sector / At constant prices / Germany / Main construction industry / Unadjusted figure |
| 149 | BBDE1.M.DE.Y.BAA1. A2P300000.G.C.I10.A | Output in the production sector / Germany / Production sector including construction (B -F) |
| 150 | BBEX3.M.XAU. DEM.EA.AC.C02, | Price of gold in London / morning fixing * / 1 ounce of fine gold = USD ... |
| 151 | BBDE1.M.DE.Y.FG30. A2P200000.F.N.I10.A | Produktionsergebnis je Beschäftigten / Deutschland / Bergbau und Gewinnung von Steinen und Erden sowie Verarbeitendes Gewerbe (B + C) / Angaben für fachliche Betriebsteile / kalender- und saisonbereinigt |
| 152 | BBK01.TQ7040 | Real estate markets - Residential real estate loans to total loans FSI-I039 |
| 153 | BBK01.TXI362 | TOTAL ASSETS OR LIABILITIES / GERMAN CONTRIBUTION |
| 154 | BBK01.TUB618 | Total assets |
| 155 | BBK01.TUE365 | Total assets or liabilities / Euro area |
| 156 | BBK01.TUB641 | Total liabilities |
| 157 | BBK01.UJCY01 | Unemployment rate (unemployment as a percentage of the civilian labour force) / Germany |

Table 7: FRED data series

| | Series Id | Series Title |
|---|---|---|
| 158 | A229RX0 | Real Disposable Personal Income: Per capita |
| 159 | AAA | Moody's Seasoned Aaa Corporate Bond Yield© |
| 160 | AHECONS | Average Hourly Earnings Of Production And Nonsupervisory Employees: Construction |
| 161 | AHEMAN | Average Hourly Earnings Of Production And Nonsupervisory Employees: Manufacturing |
| 162 | AWHMAN | Average Weekly Hours of Production and Nonsupervisory Employees: Manufacturing |
| 163 | AWOTMAN | Average Weekly Overtime Hours of Production and Nonsupervisory Employees: Manufacturing |
| 164 | BAA | Moody's Seasoned Baa Corporate Bond Yield© |
| 165 | BOGMBASE | Monetary Base; Total |
| 166 | BUSLOANS | Commercial and Industrial Loans, All Commercial Banks |
| 167 | BUSLOANSNSA | Commercial and Industrial Loans, All Commercial Banks |
| 168 | CES0600000007 | Average Weekly Hours of Production and Nonsupervisory Employees: Goods-Producing |
| 169 | CES0600000008 | Average Hourly Earnings of Production and Nonsupervisory Employees: Goods-Producing |
| 170 | CES2000000008 | Average Hourly Earnings of Production and Nonsupervisory Employees: Construction |
| 171 | CES3000000008 | Average Hourly Earnings of Production and Nonsupervisory Employees: Manufacturing |
| 172 | CEU0600000007 | Average Weekly Hours of Production and Nonsupervisory Employees: Goods-Producing |
| 173 | CEU0600000008 | Average Hourly Earnings of Production and Nonsupervisory Employees: Goods-Producing |
| 174 | CEU1000000001 | All Employees: Mining and Logging |
| 175 | CEU2000000001 | All Employees: Construction |
| 176 | CEU3000000001 | All Employees: Manufacturing |
| 177 | CEU3000000007 | Average Weekly Hours of Production and Nonsupervisory Employees: Manufacturing |
| 178 | CEU3000000009 | Average Weekly Overtime Hours of Production and Nonsupervisory Employees: Manufacturing |
| 179 | CEU3100000001 | All Employees: Durable Goods |
| 180 | CEU3200000001 | All Employees: Nondurable Goods |
| 181 | CEU9000000001 | All Employees: Government |
| 182 | CIVPART | Civilian Labor Force Participation Rate |
| 183 | CLF16OV | Civilian Labor Force |
| 184 | CPIAPPNS | Consumer Price Index for All Urban Consumers: Apparel |
| 185 | CPIAPPSL | Consumer Price Index for All Urban Consumers: Apparel |
| 186 | CPIAUCNS | Consumer Price Index for All Urban Consumers: All Items |
| 187 | CPIAUCSL | Consumer Price Index for All Urban Consumers: All Items |

| | Series Id | Series Title |
|---|---|---|
| 188 | CPILFENS | Consumer Price Index for All Urban Consumers: All Items Less Food & Energy |
| 189 | CPILFESL | Consumer Price Index for All Urban Consumers: All Items Less Food & Energy |
| 190 | CPIMEDNS | Consumer Price Index for All Urban Consumers: Medical Care |
| 191 | CPIMEDSL | Consumer Price Index for All Urban Consumers: Medical Care |
| 192 | CPITRNNS | Consumer Price Index for All Urban Consumers: Transportation |
| 193 | CPITRNSL | Consumer Price Index for All Urban Consumers: Transportation |
| 194 | CURRNS | Currency Component of M1 |
| 195 | CURRSL | Currency Component of M1 |
| 196 | DMANEMP | All Employees: Durable goods |
| 197 | FEDFUNDS | Effective Federal Funds Rate |
| 198 | GS1 | 1-Year Treasury Constant Maturity Rate |
| 199 | GS5 | 5-Year Treasury Constant Maturity Rate |
| 200 | HOUST | Housing Starts: Total: New Privately Owned Housing Units Started |
| 201 | HOUSTMW | Housing Starts in Midwest Census Region |
| 202 | HOUSTMWNSA | Housing Starts in Midwest Census Region |
| 203 | HOUSTNE | Housing Starts in Northeast Census Region |
| 204 | HOUSTNSA | Housing Starts: Total: New Privately Owned Housing Units Started |
| 205 | HOUSTS | Housing Starts in South Census Region |
| 206 | HOUSTSNSA | Housing Starts in South Census Region |
| 207 | HOUSTW | Housing Starts in West Census Region |
| 208 | HOUSTWNSA | Housing Starts in West Census Region |
| 209 | INDPRO | Industrial Production Index |
| 210 | IPBUSEQ | Industrial Production: Business Equipment |
| 211 | IPCONGD | Industrial Production: Consumer Goods |
| 212 | IPDCONGD | Industrial Production: Durable Consumer Goods |
| 213 | IPFINAL | Industrial Production: Final Products (Market Group) |
| 214 | IPFUELN | Industrial Production: Fuels |
| 215 | IPFUELS | Industrial Production: Fuels |
| 216 | IPMAT | Industrial Production: Materials |
| 217 | IPNCONGD | Industrial Production: Nondurable Consumer Goods |
| 218 | LNS12032197 | Employment Level - Part-Time for Economic Reasons, Nonagricultural Industries |
| 219 | LNU01000000 | Civilian Labor Force |
| 220 | LNU01300000 | Civilian Labor Force Participation Rate |
| 221 | M1NS | M1 Money Stock |
| 222 | M1SL | M1 Money Stock |
| 223 | M2NS | M2 Money Stock |
| 224 | M2SL | M2 Money Stock |
| 225 | MANEMP | All Employees: Manufacturing |

| | Series Id | Series Title |
|---|---|---|
| 226 | NAPM | ISM Manufacturing: PMI Composite Index© |
| 227 | NAPMEI | ISM Manufacturing: Employment Index© |
| 228 | NAPMII | ISM Manufacturing: Inventories Index© |
| 229 | NAPMNOI | ISM Manufacturing: New Orders Index© |
| 230 | NAPMPI | ISM Manufacturing: Production Index© |
| 231 | NAPMSDI | ISM Manufacturing: Supplier Deliveries Index© |
| 232 | NDMANEMP | All Employees: Nondurable goods |
| 233 | PAYEMS | All Employees: Total nonfarm |
| 234 | PAYNSA | All Employees: Total nonfarm |
| 235 | PERMITNSA | New Privately-Owned Housing Units Authorized by Building Permits: Total |
| 236 | PPIACO | Producer Price Index: All Commodities |
| 237 | PPICRM | Producer Price Index: Crude Materials for Further Processing |
| 238 | PPIFCF | Producer Price Index: Finished Consumer Foods |
| 239 | PPIFGS | Producer Price Index: Finished Goods |
| 240 | PPIITM | Producer Price Index: Intermediate Materials: Supplies & Components |
| 241 | SRVPRD | All Employees: Service-Providing Industries |
| 242 | TB3MS | 3-Month Treasury Bill: Secondary Market Rate |
| 243 | TB6MS | 6-Month Treasury Bill: Secondary Market Rate |
| 244 | UEMPMEAN | Average (Mean) Duration of Unemployment |
| 245 | UNEMPLOY | Unemployed |
| 246 | UNRATE | Civilian Unemployment Rate |
| 247 | UNRATENSA | Civilian Unemployment Rate |
| 248 | USCONS | All Employees: Construction |
| 249 | USGOOD | All Employees: Goods-Producing Industries |
| 250 | USGOVT | All Employees: Government |
| 251 | USMINE | All Employees: Mining and logging |
| 252 | W875RX1 | Real personal income excluding current transfer receipts |

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