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Forecasting Economic Time Series Using Dynamic Factor Models under
Structural Breaks

Author:

Johannes Degn

Assessors:

Prof. Dr. Ralf Brüggemann

Dr. Roxana Halbleib

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Abstract

This thesis applies recently developed methods for finding structural breaks in factor models. Bootstrap versions of these tests are considered. Using a large set of German and US macroeconomic predictors, it is found that removing variables which undergo breaks in the corresponding factor loadings has a considerable positive effect on the root mean squared forecasting error. This improvement is less pronounced if noisy predictors are taken care off beforehand by the method of targeted predictors introduced by Bai and Ng (2008). Both methods focus on a similar goal. Structural break tests can thus be considered a promising preselection mechanism which can complement other variable selection methods such as targeted predictors.

Introduction

Factor models are used in a wide range of situations spanning from the calculation of economic indicators over the prediction of high-dimensional data sets. The basic idea is that there are unobservable factors driving the change of all variables in the data set. Put differently, it is assumed that the variables are composed of a common component affecting all variables in the set and an idiosyncratic component which is specific to each variable in the set.

If the common factors can be derived consistently they prove to be a valuable and more practical set of predictors for the variables of interest than the whole set of available variables. Nowadays, typically the factors are estimated using principal components analysis. Resultantly factor models harbor similarity to principal component regression¹ including the feature of dimensionality reduction which allows factor models to be estimated even if the number of columns exceeds the number of rows. This is precisely what makes factor models appealing in a macroeconomic forecasting context as there exist many international macroeconomic variables some of which are not covered over a very long time horizon. With factor models all this information can be used while traditional models have to rely on variable selection in order to reduce the

¹In essence factor models where the factors are calculated using principal components are equivalent to principal component regressions with additional predictors. Dynamic factor models introduce dynamic process in the factors.

number of parameters in many situations.

In a time-series framework an interesting question to ask is whether the common components are stable over time or if the relationship of the variables of interest with the underlying factors is prone to changes. Specificity about what is meant when structural breaks are referenced is added in section 3.

There are recent additions to the field and the behavior of factor models under structural breaks is actively being researched. There are at least two reasons why structural breaks are interesting in an econometric model. If we assume that structural breaks imply a change in the coefficients of the model at a certain period, this can be interesting in and of itself especially if the coefficients and the factors are to be interpreted directly or if factor models are estimated in the context of some economic theory. Additionally, from a forecasting perspective it is likely that a model estimated over a certain time interval does not generalize well onto another time frame if parameters are unstable. This makes it attractive to limit the observations to those which do not include periods with parameter instability. For forecasting into the future it can e.g. be worthwhile to consider only the data after the structural break(s) has/have happened². The hope is that the parameters of the model which are estimated relying only on the data after the last break might be closer to the true values which have generated the data to be forecast. This assumes that there will not be another structural break at the period which is to be predicted. E.g. Bai and Ng (2008) employ a similar method in their empirical application by considering estimates based on several sub samples. This allows them to reason about parameter stability. It should be noted that Pesaran and Timmermann (2007) have shown that it can be slightly better to include some observations before the last break and not to fully cut off all unstable periods.

Instead of cutting the length of the data set by excluding observations before the structural break, the width of the data set can be decreased, as well. This treatment is perhaps more tailored towards factor models because they are usually employed in a high-dimensional environment where many predictors are replaced by relatively few common factors. It is thus possible that the overall information content does not suffer too much if some of the variables are excluded and the factors are instead calculated on a

²Alternatively, instead of splitting the data into multiple periods for each break, a single forecasting equation can be constructed which includes the structural break by introducing a dummy variable.

limited subset. This approach can be especially attractive if the number of observations is limited relative to the number of variables.

Or argued more directly and as is made explicit below, factor models assume the variable of interest to depend on the factors which in turn depend on the complete data set. If there is a structural break in the relationship of one of the variables with the factors, it is conceivable that this break propagates to the relationship between the factors and the variable of interest. Resultingly removing the variable with the break can help to keep the relationship stable. In the empirical application presented in this paper I find that this approach has a more favorable effect on forecasting accuracy than limiting the number of observations.

Structural breaks can be thought of as adding noise to the predictors. There exist already several methods on how to approach reducing the number of uninformative predictors prior to the estimation of factor models. Bai and Ng (2008) have developed the method of targeted predictors which is in essence a preselection tool for the predictors based on relevance for forecasting a variable of interest. It relies on regressing the variable of interest on the predictors to ascertain and test the strength of the dependence. Resultantly the approach to remove variables with associated structural breaks can be thought of as an addition to these methods.

This thesis treats testing for structural breaks in factor models and using knowledge about these breaks for forecasting.

The contribution is twofold: Firstly, the influence of structural breaks on factor models is examined. To this end feasible testing procedures developed by Breitung and Eickmeier (2011) are presented. Bootstrap versions of these Chow type tests are developed and their performance is evaluated. Secondly, the information about the structural breaks is used for forecasting. The performance of different model specifications is tested in the form of horse races³. The competing models are:

1. A static factor model which ignores structural breaks
2. A dynamic factor model which ignores structural breaks
3. A dynamic factor model which takes structural breaks into account

³I.e. the competing models are pitted against each other and performance is evaluated according to some criterion with the goal of establishing a winner.

4. Targeted dynamic factor models which ignore structural breaks
 - (a) Under hard thresholding
 - (b) Under soft thresholding
5. Targeted dynamic factor model which take structural breaks into account
 - (a) Under hard thresholding
 - (b) Under soft thresholding

The empirical section shows that removing variables with associated breaks in the relationship usually improves the forecasting accuracy considerably. The targeted models which take structural breaks into account are interesting as a tool to analyze the interaction between the two different variable preselection schemes. The improvement in terms of forecasting precision is reduced if the data set is targeted prior to estimation which shows that targeted predictors and the removable of variables under the influence of structural breaks target a similar goal.

The remainder of this paper is structured into 4 parts. Firstly factor models are introduced. Both static and dynamic models are presented along with some theory and estimation methods. Secondly structural breaks are introduced into the framework, their effects on the factors and loadings are looked at and testing methods are considered. Thirdly an empirical application is presented. Aforementioned models are estimated using a set of 169 German and US American variables and prediction accuracy is compared. Finally the fourth part concludes.

Some notes on notation are in order. Throughout this thesis I refer to X as a $T \times N$ matrix of potentially very many predictors. To stay consistent with existing literature I index X with i or t interchangeably where i stands for the column index and t for the row index. In other words X_t is the $N \times 1$ data vector for observation t i.e. t indexes along the longitudinal dimension, X_i is the $T \times 1$ data vector for variable i i.e. i indexes along the cross-sectional dimension. Also as is customary, throughout this paper unindexed capital letters denote matrices while unindexed lower case letters denote vectors except for innovation terms which are always in lower case letters. All

variables containing data are assumed to be demeaned and normalized to have unit variance⁴.

1 Factor models

As mentioned in the introduction, the idea behind factor models or diffusion index models is that there are underlying latent factors which explain the evolution of the observable variables collected in X . In other words all variables X_i are thought to consist of a common component driven by the factors and an idiosyncratic component which is particular to each variable.

If factor models are to be used for forecasting they are usually written in the form of a factor equation (1) and a forecasting equation (2).

$$X_t = \lambda(L)f_t + e_t \quad (1)$$

$$y_{t+h}^h = \alpha(L)W_t + \beta(L)\hat{f}_t + \varepsilon_{t+h} \quad (2)$$

X_t is a $N \times 1$ vector of potentially very many predictors. $\lambda(L)$ is a $N \times q$ matrix of lag polynomials, f_t is a $q \times 1$ vector of latent factors. e_t is a $N \times 1$ vector of idiosyncratic errors which may or may not be serially auto-correlated. W_t consists of variables which the researcher knows to be sufficiently relevant to enter the forecasting equation directly. Usually W_t is taken to consist of s lags of y_t and a constant term. \hat{f}_t is an estimate of the factors in the factor equation (1). Estimation of the factors is treated below.

It is sometimes assumed that the error e_t term follows a white noise process (i.e. independent and of mean 0). In the vocabulary of factor models these are models with uncorrelated idiosyncratic components e_t (more specifically $E(e_t e_t') = \Sigma = \text{diag}(\sigma_{e_1}^2, \dots, \sigma_{e_N}^2)$ and $E(e_t) = 0$). However, "weak" dependence is more commonly assumed. Concretely, Geweke (1977) and Sargent and Sims (1977) distinguish between exact or strict Factor Models and approximate Factor Models. In essence the difference is composed of different assumptions for the error terms. Approximate factor models allow for limited correlatedness of the idiosyncratic innovations e_t between periods

⁴I.e. every variable y_t is derived as $y_t = \frac{y_t^* - \bar{y}_t}{\sigma_y}$ where y_t^* is the vector containing the original data, \bar{y}_t is the sample average and σ_y is the corrected sample standard deviation of y

while strict factor models do not. More precise definitions follow now and are taken from Breitung and Eickmeier (2006)

Strict factor models

Breitung and Eickmeier (2006) list the assumptions for strict factor models as follows: For the innovations it is assumed that $E(e_t) = 0$ and $E(e_t e_t') = \sigma = \text{diag}(\sigma_1^2, \dots, \sigma_N^2)$. Also $E(f_t) = 0$ and $E(f_t f_t') = \Omega$. If $E(f_t) = 0$ it follows that $E(X_t) = 0$ which is enforced by demeaning the variables. Additionally $E(f_t e_t') = 0$ i.e. the innovations and the error terms are uncorrelated. It follows directly that $E(X_t X_t') = E[(\Lambda f_t + e_t)(\Lambda f_t + e_t)'] = E(\Lambda f_t f_t' \Lambda' + \Lambda f_t e_t' + e_t f_t' \Lambda' + e_t e_t') = \Lambda \Omega \Lambda' + \Sigma$

Approximate factor models

Approximate factor models loosen the assumptions of strict factor models at the cost of assuming $N \rightarrow \infty$. $E(e_{it} e_{js})$ are allowed to be different from 0 but only weakly so. Similarly the factors f_t and the idiosyncratic error terms are allowed to be correlated but again only weakly so, such that $E(f_t e_t')$ does not have to be 0. But $N^{-1} \Lambda' \Lambda$ must converge to a positive definite limiting matrix Σ_Λ which ensures that the factors influence each variable with a similar order of magnitude. Otherwise it could be that the loadings for some variables are 0 (Breitung and Eickmeier, 2006). Similarly it is assumed about the factors that $T^{-1} \sum_{t=1}^T T F_t F_t' \xrightarrow{p} \Sigma_F$. This allows for stationary processes e.g. an ARMA process.

Some more assumptions are necessary for the inferential theory⁵ of Bai (2003) to hold. Namely the required assumptions are Assumptions A through D from Bai (2003) which follow:

Assumption A: $E||F_t||^4 \leq M < \infty$ and $T^{-1} \sum_{t=1}^T F_t F_t' \xrightarrow{p} \Sigma_F$ for some $r \times r$
positive matrix Σ_F

⁵Most notably the results of Bai (2003) are consistency and normality of the factor estimations. The rate of convergence is shown to be $\min(\sqrt{N}, \sqrt{T})$.

Assumption B: $\|\lambda_i\| \leq \bar{\lambda} < \infty$, and $\|\Lambda' \Lambda / N - \Sigma_\Lambda\| \rightarrow 0$ for some $r \times r$ positive definite matrix Σ_Λ

Assumption C: $\exists M : 0 < M < \infty$ such that $\forall N, T$,

$$1.) E(e_{it}) = 0, E|e_{it}|^8 \leq M$$

$$2.) E(e'_s e_t / N) = E(N^{-1} \sum_{i=1}^N e_{is} e_{it}) = \gamma_N(s, t), |\gamma_N(s, s)| \leq M \forall s \text{ and}$$

$$T^{-1} \sum_{s=1}^T \sum_{t=1}^T |\gamma_N(s, t)| \leq M$$

$$3.) E(e'_{it} e_{jt}) = \tau_{ij,t} \text{ with } |\tau_{ij,t}| \leq |\tau_{ij}| \text{ for some } \tau_{ij} \text{ and for all } t \text{ and}$$

$$N^{-1} \sum_{i=1}^N \sum_{j=1}^N |\tau_{ij}| \leq M$$

$$4.) E(e'_{it} e_{js}) = \tau_{ij,ts} \text{ and } (NT)^{-1} \sum_{i=1}^N \sum_{j=1}^N \sum_{t=1}^T \sum_{s=1}^T |\tau_{ij,ts}| \leq M$$

$$\forall(t, s) E \left| N^{-1/2} \sum_{i=1}^N [e_{is} e_{it} - E(e_{is} e_{it})] \right|^4 \leq M$$

$$\textbf{Assumption D: } E\left(\frac{1}{N} \sum_{i=1}^N \left\| \frac{1}{\sqrt{T}} \sum_{t=1}^T F_t e_{it} \right\|^2\right) \leq M$$

Note that the notation has been changed a little from Bai (2003). Specifically the true values are not indicated by a superscript ⁰.

Assumption A allows the factors to follow a stationary AR(p) process, Assumption B forces each factor to contribute to the variance of X_i in a similar order of magnitude

via the respective loading, Assumption C allows for some degree of dependence and heteroskedasticity among the innovation terms. This will be used below to simplify the calculation of the dynamic factor model. Finally Assumption D allows some weak dependence between the factors and the innovation terms and replaces the uncorrelatedness assumption of the strict factor model. More details can be found in Bai (2003).

Using these assumptions Bai (2003) shows that the factors and loadings can be estimated consistently. By placing some additional constraints on serial dependence (Assumption C), the eigenvalues, and by invoking a central limit theorem Bai (2003) also proves normality of the principal component estimator using a simultaneous limit theorem. More precisely: as $T, N \rightarrow \infty$, Bai (2003) get the following limiting distributions for the principal components factor estimates:

(i) If $\sqrt{N}/T \rightarrow 0$ then for each t

$$\sqrt{N}(\tilde{F}_t - H'F_t) \xrightarrow{d} N(0, V^{-1}Q\Gamma_tQ'V^{-1})$$

Where Q, Δ_t and V are defined in Bai (2003), H is some rotation matrix

and

(ii) If $\liminf \sqrt{N}/T \geq \tau > 0$, then

$$T(\tilde{F}_t - H'F_t) = O_p(1)$$

The first part establishes normality for the principal component estimator. Note that the true value of the factors can only be estimated up to a rotation H which is due to the common component being separated into factors and loadings. This facts is further discussed below when principal component estimation is introduced. The normality result establishes that nonetheless, the factor estimates \tilde{F}_t are asymptotically normally distributed around a rotation of the true factors F_t . It is important because it allows researchers to establish significance and construct confidence intervals which is especially vital if factor models are used for the construction of indices (Bai, 2003). Bai and Ng (2006) uses similar assumptions to show \sqrt{T} consistency of the parameter

estimates from factor augmented regressions⁶ and to construct confidence intervals in those models.

As mentioned, an important advantage of factor models compared to simpler econometric models is that they do not necessarily suffer if the fraction $\frac{T}{N}$ becomes small whereas most models can not cope with more variables than observations. VAR models for example have the problem that the number of parameters to estimate becomes too high quickly if N is big relative to T . The reason why factor models do not have this problem is that they follow a two stage procedure. At first the factor equation is estimated. This gives estimates \hat{f}_t of the factors. Then only the first r eigenvectors are used in the forecasting equation where $r < N$ ⁷. In principle this feature allows researchers to append X by further variables derived from the initial set. Bai and Ng (2008) for example use what they call squared principal components and quadratic principal components. The first of which appends squares of the X_i to the initial matrix X such that $X_t^{spc} = \{X_{it}, X_{it}^2\}$ and the second of which adds the cross products of X_i such that $X_t^{qpc} = \{X_{it}, X_{it}X_{jt}\} \forall i \neq j$. The idea is to change the link function between X_t and f_t from being linear to allow for a nonlinear relationship. Bai and Ng (2008) also consider adding a squared term of the factors f_t to the forecasting equation (2) to allow the volatility of the factors to influence the forecast. Naturally, the value of these approaches depends heavily on the environment under which forecasts are undertaken. I.e. what is being forecast by which data.

Apart from the distinction into strict and approximate factor models, a further distinction can be made between static and dynamic factor models. This distinction concerns the relationship between lags of f_t to the X_t . Static models set $\alpha(L)$ to be of lag order 0 while dynamic models allow for dependence with other lag orders.

⁶Factor augmented regressions are essentially what we do in the forecasting equation (2) where a normal regression model is augmented by estimates of the factors to account for the common component of the whole set of predictors.

⁷Typically r is much smaller than N . E.g. the famous one factor model of intelligence by Spearman (1904) (in which factor models were developed the first time) has $r = 1$.

1.1 Static factor models

For static factor models X_t depends only on f_t and not on lags of f_t i.e. factors enter only contemporaneously. Thus a static factor model is the case in which the lag order in equation (1) is set to 0.

$$X_t = \Lambda F_t + \bar{e}_t = C_t + e_t \quad (3)$$

Where again X is a $T \times N$ Matrix of predictors. F is a $T \times r$ factor matrix. Λ is a $N \times r$ loadings matrix. \bar{e} is a $T \times N$ matrix of idiosyncratic errors. Resultingly X_t is a $N \times 1$ vector and F_t is of dimension $r \times 1$.

C_t is the common component. Note that depending on whether authors index by row, column or both, several alternative ways of writing down the static factor equation are used in the literature.

$$X_{it} = \Lambda'_i F_t + e_{it} = C_{it} + e_{it} \quad (4)$$

$$X = F\Lambda' + e = C + e \quad (5)$$

$$X_i = F\Lambda_i + e_i = C_i + e_i \quad (6)$$

For the sake of completeness X_i and e_i are $T \times 1$ vectors while Λ_i is $r \times 1$. In the literature Λ_i is also sometimes written as λ .

Loosely and graphically speaking each Λ_i maps the variables factors F to the corresponding variable X_i which is the i -th column of X .

1.2 Dynamic factor models

In applications there will usually be some kind of time dependence structure in the data. Thus, dynamic factor models introduce a dynamic process into the factors.

$$f_t = \Psi(L)f_{t-1} + \eta_t \quad (\text{VAR representation}) \quad (7)$$

In practice it is usually easier to ignore the dynamic process in the factors f_t and follow e.g. Stock and Watson (2005) among others in assuming a dynamic process in the

innovation term of the factor equation (1) instead. This captures the dynamic process while avoiding the technical difficulty in estimating the dynamic factors themselves. The underlying thought is that there will be some residual autocorrelation if the dynamic process of the factors is not modelled explicitly which can be caught by an AR(p) process for the innovation term (Breitung and Tenhofen, 2011). Additionally the lag polynomial $\lambda(L)$ in the factor equation (1) captures the non-contemporaneous interactions of the factors f_t and X_t .

For the purpose of this paper I assume the following AR processes for the innovation term of the dynamic factor models borrowed from Breitung and Eickmeier (2011).

$$e_{it} = \varrho_{i,1}e_{i,t-1} + \dots + \varrho_{i,p_i}e_{i,t-p_i} + \xi_{it} \text{ where } \xi_{it} \text{ is white noise} \quad (8)$$

1.2.1 Static interpretation of dynamic factor models

Dynamic factor models can be written as static factor models in a linear state space environment. This gives the static factor equation (3) and $\Phi(L)F_t = G\eta_t$ where $\Phi(L)$ is defined such that equivalence to (7) holds and F_t is defined below as the stacked lagged dynamic factors f_t (Stock and Watson (2011)). The result are as follows:

Solving the lag operator, we can write equation (1) in the form

$$X_{it} = \lambda'_{i1}f_t + \dots + \lambda'_{im}f_{t-m} + e_{it} \quad (9)$$

Following Bai and Ng (2002) this can be rewritten as

$$X_{it} = \Lambda_i^{*'} F_t + e_{it}$$

$$\text{where } \Lambda_i^* = \begin{bmatrix} \lambda_{i1} \\ \lambda_{i2} \\ \vdots \\ \lambda_{im} \end{bmatrix} \text{ and } F_t = \begin{bmatrix} f_t \\ f_{t-1} \\ \vdots \\ f_{t-m} \end{bmatrix}$$

To keep consistency between variable naming schemes denote the dimension of F_t as $r \times 1$ and the dimension of Λ_i^* as $r \times 1$. Thus a dynamic factor model can always

be rewritten in static form and resultantly principal components as is introduced below can be used to estimate the r "static factors"⁸ F_t and the loadings Λ_i^* . Keeping in mind that the dimension of $\lambda(L)$ is $N \times q$, it can be seen that the dimension of the resulting stacked matrix of lagged "static" factors is $q(m + 1)$ thus we can set $r = q(m + 1)$. This shows that there is a relationship between r and q if the dynamic factor model is written as a static factor model. To identify q it remains to estimate the lag order m and the number of "static" factors r .

There exist several ways to identify the dynamic factors from there⁹. Firstly Giannone et al. (2002) suggest estimating the VAR $F_t = CF_{t-1} + \kappa_t$ and then to apply principal component analysis on an estimate of the residual covariance matrix (see Breitung et al. (2004) or Giannone et al. (2002) for details). Breitung et al. (2004) exploit the relationship between q and r in order to find information criteria which can determine q and the lag order of the λ .

This paper does not estimate the dynamic factors directly as in the empirical application the factors are primarily used for the second stage regression to estimate the forecasting equation.

Imagine that the true data has been generated by a dynamic factor model. If instead of using the dynamic factor model specification and finding the dynamic factors from the static factors, a researcher simply estimates a static factor model it can be seen above that the number of factors r will be larger than the number of dynamic factors q . This is intuitive because a higher r implies a higher dimensional factor space which is necessary to catch the additional variation introduced by the dynamic process.

Most authors use the approach briefly hinted at above to mimic the dynamic process in the factors by lagging the static factors. This does not reproduce the exact model if the true data generating process follows (7) but it accounts for some time dependence and is much simpler because it only entails estimation of the static factors from the data and lagging them in the first stage regression.

⁸Breitung et al. (2004) call the f_t "structural factors" and the F_t "reduced form" factors.

⁹Note that identifying the dynamic factors might not be of interest for forecasting but rather that it is a necessary condition in order to interpret the factors in a dynamic factor model. For forecasting purposes, in order to choose the number of factors and the number of the factor lags used in the forecasting equation (2) one can also follow Bai and Ng (2008) in using the BIC criterion to choose both simultaneously.

1.3 Estimation of the factors

1.3.1 Static factors

There exist now a number of ways to estimate the static factors F in (5). Ultimately what is looked for is a solution to the minimization of the squared error.

$$\min_{F_1, \dots, F_T, \Lambda} \frac{1}{NT} \sum_{t=1}^T (X_t - \Lambda F_t)' (X_t - \Lambda F_t) \quad (10)$$

Since both Λ and F_t are unspecified, there are infinitely many solutions to (10) and the problem is not uniquely identified. To achieve identification, assumptions can most conveniently be put on either Λ or F_t ¹⁰. Usually Λ is normalized such that $N^{-1}\Lambda'\Lambda = I_r$. Alternatively $T^{-1}F'F = I_r$ is also frequently used. The choice of normalization can have a noticeable impact on the computational demand of calculating a solution for the minimization problem in (10) especially if the factors are to be calculated many times in some simulation study. The column spaces spanned by the estimates of F are equivalent with both normalizations (Stock and Watson, 2011). Since a regression will give the same fit regardless if all predictors are rotated in the same way both normalizations give the same final result.

Stock and Watson (2011) list four ways which are being applied in the literature to get a solution \hat{F} and $\hat{\Lambda}$ to the minimization problem (10). Firstly a Gaussian process can be assumed which allows for maximum likelihood estimation using the Kalman filter. To do this a dynamic factor model can be rewritten as a static factor model in a linear state space form as introduced above.

Secondly principal components (and other nonparametric averaging methods) can be used to estimate both \hat{F} and $\hat{\Lambda}$ at the same time. This requires relatively weak assumptions and is particularly easy and computationally efficient to implement which explains the popularity of this approach. This is also the approach taken in this thesis. Additionally the principal component estimation can be generalized by weighting the minimization problem with the variance matrix of the error term to take account of error variances which are not proportional to the identity matrix (see Stock and Watson

¹⁰Bai and Ng (2013) consider several alternative normalizations. They also show that direct interpretation of the factors and loading matrices are possible if one is willing to additionally assume diagonality of $\Lambda'\Lambda$ or that Λ is a block matrix of 2 submatrices with one submatrix being triangular.

(2011) for details). Evidence of performance improvements by using the generalized principal components approach over the standard principal components estimator are mixed. Resultingly the simpler principal components estimator is used here. It is described in more detail below and in appendices A and B. As mentioned above, an unattractive feature of principal component estimation is that it can not identify the "true" factors f_t but rather a transformation Qf_t for some undefined rotation matrix Q which results in some difficulty in interpreting the factors directly. This is due to the solution of the minimization problem not being unique. If the factors are used for forecasting this feature is not of much importance, however.

It should be noted that principal components estimation of the factors is asymptotically equivalent to maximum likelihood estimation if normality of the error terms is assumed (Bai, 2003).

Thirdly mixture approaches between the two methods from above can be constructed. Fourthly and finally Bayesian methods can be used to get estimations of the factors and loadings. These can have computational advantages over the maximum likelihood approach and can be useful if the assumption of Gaussian error terms is unappealing (Stock and Watson, 2011).

1.3.2 Principal component estimation of the static factors

The principal component method estimates \hat{F} and $\hat{\Lambda}$ as follows. Normalizing F such that $T^{-1}F'F = I_r$ gives $\hat{F} = \sqrt{T} * \text{eigenvectors}_r(XX')$ as a solution where $\text{eigenvectors}_i(A)$ returns the i eigenvectors corresponding to the i largest eigenvalues of a square matrix A^{11} . Since $\hat{X} = \hat{F}'\hat{\Lambda}$, it follows directly that we can estimate Λ by $\hat{\Lambda}' = (\hat{F}'\hat{F})^{-1}\hat{F}'X$. due to the normalization $\hat{F}'\hat{F} = T$ and we can write $\hat{\Lambda} = X'\hat{F}/T$.

If, alternatively, we apply the normalization $N^{-1}\Lambda'\Lambda = I_r$ the solution to (10) becomes $\hat{\Lambda} = \sqrt{N} * \text{eigenvectors}_r(X'X)$. This solution to the optimization problem is shown in appendix B). Then we can follow from $\hat{X} = \hat{F}\hat{\Lambda}'$ that $\hat{F} = X\hat{\Lambda}(\hat{\Lambda}'\hat{\Lambda})^{-1}$. Again due to the normalization $\Lambda'\Lambda = N$ and we can write $\hat{F} = X\hat{\Lambda}/N$.

Note that the normalization $T^{-1}F'F = I_r$ requires us to calculate the eigenvectors

¹¹Eigenvalues and eigenvectors can be calculated with most standard statistical packages. The analytical solution can be seen in appendix A.

of XX' which has dimension $T \times T$ while the alternative $N^{-1}\Lambda'\Lambda = I_r$ requires us to calculate the eigenvectors of $X'X$ which is of dimension $N \times N$. Depending on the relative sizes of T and N it can make a considerable difference in terms of computational burden which method is applied.

Also note that what is being estimated here are in fact rotations FH of the true factors F and rotations ΛH^{-1} of the true loadings matrix. However, $\hat{F}\hat{\Lambda}' = (FH)(H^{-1}\Lambda' = F\Lambda')$ estimates the true common component consistently for any adequate rotation matrix H (Bai and Ng, 2013).

Stock and Watson (1998) and Stock and Watson (2002) show that the principal component estimator remains consistent if Λ varies a bit over time as long as $\frac{T}{N} \rightarrow 0$. The case where Λ varies strongly and for which we speak of structural breaks is discussed below.

1.3.3 The number of factors

One important issue that remains to be addressed is the question of how many factors should be used in the factor equation. There exist many different ways in which this choice can be made. Firstly there are simple rules of thumb. E.g. some researchers only consider factors with eigenvalues greater than 1 (see Larsen and Warne (2010)). The intuition behind this rule is that the variance of the variables has been normalized to 1. Since the variance of the factors is equal to the corresponding eigenvalues (see appendix A) this implies that only factors with more information about variation than any single predictor are included. Other methods also focus on the percentage of variation explained but simply require the cumulative variance to be higher than a certain threshold value e.g. 90% of variation. Then there are methods which rely on a similar idea but employ graphical methods. One approach is to inspect the resulting scree plots¹². The intuition is to only take factors into account until the addition of another factor adds less additional variance than a certain threshold or in other words, additional factors are not informative enough to be added. Basing on this, formal tests

¹²Scree plots plot the ordered eigenvalues against their rank (i.e. eigenvalues and corresponding eigenvectors) which results in a falling curve.

have been derived which rely on the relative size of the eigenvalues (Stock and Watson, 2011).

Additionally Larsen and Warne (2010) stress that since the eigenvalues are summary statistics they are prone to sampling errors which allows for the construction of confidence intervals.

This list is not comprehensive, there are many more such methods and they are not all listed here. However, an additional method needs to be mentioned. The information criteria developed by Bai and Ng (2002) are perhaps more appealing than the approaches considered so far as they formalize the costs and benefits of an additional factor explicitly (Stock and Watson, 2011). Cragg and Donald (1997) show in a monte-carlo analysis that $T \rightarrow \infty$ and $N \rightarrow \infty$ makes that classical theory for predicting the number of factors performs badly. As a result information criteria as in Bai and Ng (2002) should be used which take into account that both T and N can grow. Bai and Ng (2002) show that their results hold under heteroskedasticity and weak serial correlation and are thus applicable to approximate factor models.

Bai and Ng (2002) consider 6 criteria in two triplets, the first triplet of which they call PC_p criteria (Panel C_p criteria). These criteria result from a generalization of Mallows' C_p (Mallows, 1973). The second set of 3 criteria refine the usual criteria used in time-series analysis to depend on both N and T . Bai and Ng (2002) highlight that the criteria IC_{p1} , IC_{p2} , PC_{p1} and PC_{p2} are specifically suited for principal component estimation. Bai and Ng (2002) present a monte-carlo study to highlight the differences between the criteria. Notably the PC_p criteria require the specification of a maximum number of factors k_{max} , the choice of which seems quite arbitrary¹³. This is why this paper uses the IC_p criteria. Specifically the IC_{p2} criterion because it appears to be more conservative (i.e. it usually underestimates r whereas IC_{p2} overestimates at times) which will usually lead to a more parsimonious model.

In order to understand the criteria better, tables 1 and 2 of Bai and Ng (2002) have been replicated using both different values for r and k_{max} . If r is chosen as in Bai and

¹³In their monte-carlo study Bai and Ng (2002) set the maximum number of factors to be 8 noting that they used the rule in Schwert (2002) and that one could consider $k_{max} = \text{int}[(\min\{N, T\}/100)^{1/4}]$. Tests which are not presented here show that choosing a different k_{max} can result in the PC_p criteria to become quite unstable. Especially high values of k_{max} often result in the maximum number of factors chosen such that the estimated number of factors is $\hat{k} = k_{max}$

Ng (2002), the replicated results are almost identical to the original calculations and thus are not reported. The differences are so small that they are likely to be due to sampling variation. The results can be found in the two tables in appendix C. Tables I through VIII in Bai and Ng (2002) in essence show 3 points. Firstly, traditional criteria perform poorly in estimating the number of factors if compared to the 6 presented criteria. Of the newly developed information criteria PC_{p1} , PC_{p2} , IC_{p1} and IC_{p2} perform well even under heteroskedasticity and autocorrelation. Secondly, if $\min\{N, T\}$ becomes too small, the performance of the criteria suffers which is highlighted by the bottom 5 results in the tables. Especially PC_{p3} suffers in this regard. In the given specifications the cut-off value for $\min\{N, T\}$ appears to be 40 or 60 if the variance of the innovations is high. Thirdly, PC_{p3} and IC_{p3} are less robust to a small $\min\{N, T\}$. The replication results in appendix C for $r = 7$ and $r = 9$ harbor three additional insights. Firstly as r increases, the requirements for $\min\{N, T\}$ are also higher. For $r = 7$ and $r = 9$ it seems $\min\{N, T\}$ should be at least 100. Secondly the choice of k_{max} can matter a lot. Especially for small samples the specification of $k_{max} = 8$ used in Bai and Ng (2002) choose the maximum number of factors frequently when the N and T are too small. Almost all results in the last 5 rows estimate $\hat{k} = 8$ whereas the replicated results often report a lower number of factors if $k_{max} = \text{ceil}(\min\{T, N\})$. Noticeably, this also happens in the replicated results for $r \in \{1, 3, 5\}$ and $k_{max} = \text{ceil}(\min\{T, N\})/2$. If $k_{max} = \text{ceil}(\min\{T, N\})/2$, IC_{p3} predicts $\hat{k} = 50$ in the $T = 100, N = 100$ all considered choices of r , that is for $r \in \{1, 3, 5, 7, 9\}$. Thus, the choice of k_{max} can have a sizeable impact on the prediction of the number of factors. Luckily it is typically easy to see when the information criteria choose all the factors. In those cases the plausibility of the results should be checked by hand.

An interesting question if one is ultimately interested in forecasting is whether the "true" number of factors is of any interest at all or if the number of factors used in the forecasting equation (2) should perhaps be different from the predicted r in the static model or q in the dynamic model. The answer can not be easily given. Similar to Bai and Ng (2008) dodge this question somewhat by using the BIC criterion and the out-of-sample error to choose a model among a list of models with differing number of

factors (or number of lags of the variable of interest).

Breitung and Eickmeier (2011) among others, however, note that if one is primarily interested in forecasting, the number of dynamic factors can in principle be ignored and a static factor model with a higher number of factors can be estimated instead. The results presented in the empirical section indicate that the forecasting accuracy of models which estimate the static factors and then take dynamic effects into account using the lagged static factors to account for the dynamic process is considerably higher than static models that ignore the dynamic effects. It should be noted that models which identify the true dynamic factors have not been compared.

1.3.4 Interpreting the factors

If the factors resulting from factor models are to be interpreted, it is necessary to identify the true factors rather than a transformation.

As has been noted, a difference has to be made for dynamic factor models and static factor models. Breitung et al. (2004) argue and as has been made explicit in section 1.2.1 in a setting where the factors follow a dynamic process the biggest challenge is identifying the dynamic factors from the static factor estimates if the principal components approach is taken. If one of the several approaches described above is taken, interesting economic insights can be gotten from the factor estimates. The relative importance of the factors for the variable of interest can be seen in the significance and size of the respective parameters in the second stage regression (2). Additionally the composition of the factors in terms of the initial variables contained in X can be seen in the factor loadings Λ . Most authors who employ factor models try to examine the composition of the factors through the loadings matrix and the correlation of the factors with the X_i ¹⁴.

As an example for an interpretation, Breitung et al. (2004) identify two static factors, one of which has a dynamic component and enters with a lag. They find that the first factor is given by short term interest rates while the second factor is composed mostly

¹⁴It is instructive to read the original paper by Spearman (1904) which finds that many different sources of intelligence depend on a single common factor. While this claim has been refuted since and the result seems to depend strongly on the chosen data set, Spearman (1904) provides an interesting example of how the factor loadings can be interpreted and the correlation of the factors to the variable of interest can be interpreted.

of PPI inflation of different countries.

A slightly different way to evaluate and interpret the factors is to look at the percentage of variation explained by the factors for each variable individually and use this to draw conclusions about the factors. Stock and Watson (2005) forecast inflation in VAR and estimate impulse responses of a monetary policy shock applied by the central bank. To that end they use a factor model on 132 of macroeconomic variables. They find the first factor to explain a high percentage of variation in production and hours-worked related series such as industrial production and capacity utilization whereas the second factor explains variation in interest rates, consumption and stock prices. The second and the third factor in combination can explain variations in inflation (taken together they account for 65% of inflation in their data).

The fact that factors seem to "specialize" in a choice of predictors which they explain is possibly a convenient by-product of principal component estimation as a method to estimate the factors. Principal component estimation forces the factors to be orthogonal and to contain the highest variance along the first few principal components. Thus they make it likely for factors to undergo the observed "specialization" in the sense that they focus on explaining the variance of the most salient predictors in terms of variance.

While these approaches sometimes allow for insightful conclusions it is not always possible to relate a single or sufficiently few factors to the variation of the predictors X . Sometimes variables can only be sufficiently explained by a combination of several factors which does not allow for easy and clean economic interpretations. Additionally the factor estimates will change if for example a single variable is drop which brings the interpretation procedure somewhat into question. As a result interpretations of the factors should usually be taken with a grain of salt.

1.4 Forecasting

The forecasting equation (2) can be estimated using a simple linear regression of y_{t+h} on \hat{F}_t , y_t , lags of y_t and, depending on the model specification, lags of \hat{F}_t .

The performance of factor models for forecasting purposes is most pronounced if the number of variables used in the factor equation is high (Stock and Watson, 2011). This

makes sense because more variables improve the fit of the factor equation. However, there is a trade-off between quantity and quality of variables¹⁵ that should not be ignored. This case will be addressed in the next subsection.

The number of factors in the forecasting equation (2) and the factor equation (1) need not be the same. Similarly the number of factor lags which the forecasting equation (2) sets to be the same for all factors could be left flexible between factors. While this is likely to improve forecasting performance somewhat, this paper abstains from the additional complication.

Actual point forecasts for the variable of interest can be gotten by rescaling the demeaned and normalized forecast back to the original form i.e. by multiplying the original standard error of the series and then adding the population average. If required the variable has to be brought to an undifferenced form.

1.4.1 Targeted predictors

Bai and Ng (2008) follow the idea that if factor models are ultimately going to be used for forecasting it might make sense to "target" the set of predictors X_t to the task of forecasting the variable of interest y_t . The idea stems from Boivin and Ng (2006) who find that selecting only the "informative" variables can yield better results than taking a bigger set of variable which include the same "informative" variables but also additional, less informative variables. Bai and Ng (2008) thus provides a formal way to decide which variables should be used for forecasting. To that end they use hard and soft thresholding rules to preselect among the predictors X_t those that have a measurable influence on y_t . Resultingly, a subset $\tilde{X}_t \subset X_t$ is selected for estimating the factor equation, the hope being that then the factors are better suited for predicting y_t .

For hard thresholding they regress y_t^h on W_{t-h} and X_{it-h} for each $i = 1, \dots, N$ ¹⁶. Then they compare the resulting t-statistics to a threshold and discard those predictors whose t-statistics do not exceed that threshold.

For soft thresholding Bai and Ng (2008) perform penalized regressions in the form of LASSO, elastic net and Least angle regression (see Tibshirani (1996), Zou and Hastie

¹⁵I.e. a smaller set of variables with a high predictive power can outperform a larger set of variables which includes the variables from the smaller set

¹⁶Note that the N regressions include only the lags and single predictors X_i as independent variables. In other words N different regressions are performed to assess each variable i .

(2005), Efron et al. (2004) for details) using the fact that these regression specifications set the parameters of variables which are not "important" to 0. The idea is to perform a regularized regression i.e. a normal least squares regression with the addendum of a restriction on the size of the parameters β . LASSO sets the restriction to be that $||\beta||$ is not bigger than some constant. The resulting optimization problem reads

$$\min \text{RSS} + \lambda \sum_{i=1}^N |\beta_j|$$

Ridge regression takes a similar approach but uses the L_2 norm $\sum_{i=1}^N \beta_i^2 \leq c$ for some penalty term c as the restriction. The elastic net weighs between the two regularization mechanisms keeping both the sparsity of the LASSO estimator and the property of the Ridge estimator to include groups of variables which are correlated together (Zou and Hastie, 2005). Least angle regression is a variable selection mechanism which updates coefficients stepwise in a way that keeps the angle between variables which are most correlated with the residuals of the currently chosen set equal until (in the next step) a new variable is added to the set of the most correlated variables.

The sparsity property of these penalized regressions allows the choice of relevant variables to be those which have a coefficient different from 0. Note that ridge regression, although similar in construction, does not have this property because coefficients are shrunk in absolute size but not set to 0. In applications the three approaches named above typically give similar results. The practical problem with soft thresholding lies in the fact that the algorithms¹⁷ provide solutions to restricted regression problem in the form of a solution path consisting of several steps where in each step the "most relevant" variable is added in the sense that the respective parameter is estimated and set to be different from 0. After the final step all variables are included in the model. Typically one chooses the number of steps to be taken by the algorithm and thus the value of the restriction parameter using a cross-validation approach or alternatively sometimes Mallows C_p is used. Bai and Ng (2008) propose to use the BIC or the AIC criteria to choose the size of the active set in a general application of the LARS-EN algorithm.

¹⁷In practice the LARS-EN (Least angle regression and Elastic net) algorithm can be used which computes Elastic net solutions for all values of the penalization parameter using Least angle regressions. It has been shown by Efron et al. (2004) that the LARS-EN implementation includes the LASSO as special case (namely if the parameter λ_2 is set to 0. λ_2 weighs the penalty of the Elastic net between Ridge regression and the LASSO penalty).

However, in the framework of factor models it is of interest how the factors calculated from the chosen active set performs in the forecasting equation. This open question is answered here by considering the optimal size of the restriction parameter in a forward looking way. The RMSE of pseudo out-of-sample forecasts are calculated to choose the size of the targeted set of variables. This approach is presented in the empirical section.

The approach of targeting predictors, while attractive, requires additional thought in the presence of structural breaks. As has been stated, targeting the data prior to estimation can be interpreted as removing predictors which are not relevant or too "noisy" to be used in the prediction of the variable of interest. One of the approaches considered here in dealing with structural breaks once detected, is to remove the offending variables. The success of this method to deal with structural breaks is measured using the RMSE. It is thus unclear whether improved forecasting accuracy are the results of removing structural breaks or of removing a generally noisy predictor which happens to also have a structural break but would be removed also if it would not feature the structural break. Luckily this relationship can be somewhat untangled by targeting the data prior to testing the approach to deal with structural breaks. If the residual effect is positive it can be assumed that the method is successful especially since targeting the data in advance is likely to already account for some of the structural breaks. Put differently, if there is a residual positive effect of removing structural breaks it can be assumed that because some variable with structural breaks have already been removed by targeting the predictors, the approach of removing variables associated with breaks is successful.

1.5 Applications

Factor models have been applied in a wide range of problems both in economics and in other domains apart from economics, most notably perhaps psychology and biology. Specifically biology has an abundance of high-dimensional data sets on which factor models can be applied to great success. Within the realms of economics, prominently, there are applications in the calculation of indices. Analogously to the factors in factor models, indices try to capture underlying tendencies applying to a set of variables. The

underlying factors calculated from a data set can be seen as an index. Examples for this type of application include Langetieg (1978) who calculates a performance index of stockholder gains from mergers. Kim and Nelson (1998) use Bayesian methods to estimate a regime switching factor model which indicates boom and bust business cycles. Altissimo et al. (2001) calculate a monthly inflation index using time-series data of differing frequencies.

Factor augmented regressions can be used in which the factors are used as a way to account for a component common to all variables. There are many examples for applications. Forni and Reichlin (1998) use the factors to estimate structural VAR equations to identify economy wide shocks. Similarly and perhaps more prominently Bernanke et al. (2004) estimate a factor augmented regression model to assess the effects of monetary policy. Perhaps the most attractive feature of factor augmented regressions in these settings is that they are able to incorporate information contained in big data sets into traditional VAR models and thus in a sense combine the best of both worlds.

Schumacher (2010) employs international predictors to forecast German GDP. His reasoning is that it is intuitive that international data has an influence on an open economy directly and that there are underlying factors which influence both German GDP growth and international macroeconomic variables at the same time. However, to make sure that the information conveyed in the additional international predictors is of relevance for forecasting, Schumacher (2010) "targets" the predictors prior to estimating the factor equation. Schumacher (2010) does not deal with structural breaks. In a similar international setting, Eickmeier and Breitung (2006) identify the sources of fluctuations in the euro area.

2 Structural breaks

In a macroeconomic framework it is easy to picture reasons for why relationships between variables might change over time. Commonly cited reasons include the change of technology, different trade patterns, institutional and governmental changes. Given the relatively long time frame in macroeconomic data it is likely that most data sets contain breaks in at least a few of the variables. Several approaches to dealing with

structural breaks have been developed in the economic literature. Firstly structural breaks could be included into the model making the model more sophisticated in a sense but also more complicated. To this end e.g. the approach of Bai and Perron (2003) can be followed. Alternatively simpler model can be maintained while focussing on the part of the data which remains stable. Banerjee et al. (2007) among others consider the presence of structural breaks in a simulation study and find that using only the observations after the break is preferential. Pesaran and Timmermann (2007) find that including some observations before the break can improve upon removing all samples from unstable periods.

The treatment of structural breaks in this paper follows Breitung and Eickmeier (2011) closely and is introduced below. The question of interest is how a researcher should react to structural breaks in the factor equation.¹⁸

The factor model under a structural breaks in period T^* can be written as follows:

$$y_{it} = f_t' \lambda_i^{(1)} + \varepsilon_{it} \text{ for } t = 1, \dots, T^* \quad (11)$$

$$y_{it} = f_t' \lambda_i^{(2)} + \varepsilon_{it} \text{ for } t = T^* + 1, \dots, T \quad (12)$$

The intuition is the same as with simpler models: a structural break implies a different relationship between the variable of interest and the factors and hence different factor loadings for the periods before and after the breaks. Multiple structural breaks can be specified accordingly. In theory, structural breaks could be also thought of as affecting the forecasting equation (2). This is ignored here because structural breaks in equations of this type have been thoroughly researched.

Interestingly, estimating a static factor model without structural breaks if the true data generating process includes structural breaks, yields an analogy to ignoring dynamic factors if they were present in the data generating process: Breitung and Eickmeier (2011) note that the space estimated by the factors will be of higher dimension. This means that the number of factors r predicted e.g. by the Bai and Ng (2002) infor-

¹⁸Structural breaks can refer to changes in the means and variances of variables or a change in the relationship of variables. I focus on the latter. In this paper when it is referred to structural breaks it is always assumed to mean breaks in the parameters of the factor equation (1) unless stated otherwise. Additionally breaks in the parameters of the forecasting equation could be considered. This is not the focus of this paper. The literature dealing with this kind of breaks is already large and there exists a number of instructions for treatment.

mation criteria will be higher than if no structural breaks were present. Breitung and Eickmeier (2011) note that similar to dynamic factor models it is enough to increase the number of factors if one is primarily interested in decomposing the common component from the idiosyncratic component (that is to say if one is interested foremost in forecasting). While this is true for the factor equation, it might not hold for the forecasting equation. If the same number of factors is used in the forecasting equation as in the factor equation, there are now more parameters in the forecasting equation. Assuming a *ceteris paribus* situation where a break is introduced into a factor model setting, the same information content is spread over more variables in the forecasting equation which should decrease forecasting accuracy through additional variance. If the number of factors is allowed to differ the situation is more complex but a similar intuition should hold.

If structural breaks are detected in one or more of the factors the question remains what should be done next. Since structural breaks consist of an instability in the factor loadings at a certain period it can be optimal in a forecasting sense to estimate the respective parameter after the structural break has occurred in the hope that this new parameter value contains more accurate information about the future. There is of course a trade-off here because the data series is shorter afterwards which has a negative effect on prediction accuracy. An alternative approach consists of removing the variable containing the structural break from the data set. This approach allows all other series to enter the forecasting equation at the same length as before while the added noise through the structural breaks is removed. The idea to remove complete variables from the data set is similar to targeted predictors which remove uninformative predictors in order to improve the ratio of information to noise. Because of the typically high dimension of the data matrix X , factor models appear to be well suited for the second approach. Which method should be chosen depends on the size of the structural break, the relative position of the break and the relative information content of the other variables. In practice it is necessary to test in order to weigh the benefits and costs. This question is considered in the empirical section.

2.1 Testing for structural breaks

Breitung and Eickmeier (2011) develop several Chow type tests for the static factor model under the assumptions of the strict factor model. They also present Quandt-Andrews type supremum tests to identify breaks at unknown dates. The test statistics are as follows.

$$\text{lr}_i = T[\log(S_{0i}) - \log(S_{1i+S_{2i}})] \quad (13)$$

Where

$$S_{0i} = \sum_{t=1}^T (y_{it} - \hat{f}_t' \hat{\lambda}_i)^2, S_{1i} = \sum_{t=1}^{T_1^*} (y_{it} - \hat{f}_t' \hat{\lambda}_i^{(1)})^2 \text{ and } S_{2i} = \sum_{t=T_1^*+1}^{T_1^*} (y_{it} - \hat{f}_t' \hat{\lambda}_i^{(2)})^2$$

denote the residual sum of squares for the whole date range, the first subperiod up until the structural break and the period from the break to the end of the sample respectively. $\hat{\lambda}^{(1)}$ and $\hat{\lambda}^{(2)}$ denote the estimated factor loadings calculated on the data before the break and after the break respectively.

$$\text{lm}_i = TR_i^2 \text{ where } R_i^2 \text{ is the r-squared from a regression } \hat{\varepsilon}_{it} = \theta_i' \hat{f}_t + \phi \hat{f}_t^* + \tilde{\varepsilon}_{it} \quad (14)$$

$$\text{wald}_i = \text{Wald statistic for } H_0 : \Psi_i = 0 \text{ in the regression } y_{it} = \lambda_i' \hat{f}_t + \psi \hat{f}_t^* + \nu_{it} \quad (15)$$

Where

$$\hat{f}_t^* = \begin{cases} 0 & \text{for } t = 1, \dots, T_i^* \\ \hat{f}_t & \text{for } t = T_i^* + 1, \dots, T \end{cases}$$

$$\text{LM}^* = \frac{\left(\sum_{i=1}^N s_i \right) - rN}{\sqrt{2rN}} \quad (16)$$

The LM* statistic pertains to the joint test of no structural break in all the factor loadings. Note, however, that the assumptions required for the LM* statistic are those

of a strict factor model, namely no interdependence in the error terms e_t in (1).

The LR, LM and Wald statistics defined above can be used to define a test for a unknown break date as in Andrews (1993). Under mild assumptions Breitung and Eickmeier (2011) show that the supremum statistic defined over the LM statistics is distributed using the nonstandard distribution given in Andrews (1993) and retabulated due to calculation errors in Andrews (2003).

$$\mathcal{S}_{i,T}(\tau_0) = \sup_{\tau \in [\tau_0, 1-\tau_0]} (s_i^\tau) \text{ where } s_i^\tau \text{ refers to either the LR, LM or Wald statistic} \quad (17)$$

As mentioned in the introduction of the dynamic factor model, I follow Breitung and Eickmeier (2011) in assuming an AR(p) process for the innovation term as in equation (18):

$$e_{it} = \varrho_{i,1}e_{i,t-1} + \dots + \varrho_{i,p_i}e_{i,t-p_i} + \xi_{it} \quad (18)$$

It is assumed that the process is stationary. These assumptions are then in line with the assumptions of the approximate factor model introduced above.

For dynamic factor models¹⁹ Breitung and Eickmeier (2011) propose to use a GLS transformed model, i.e. to perform the regression:

$$\varrho_i(L)y_{it} = \lambda'_i[\varrho(L)\hat{f}_t] + \psi'_i[\varrho_i(L)\hat{f}_t^*] + \varepsilon_{it}^*$$

The lag polynomials are obtained by applying some information criterion to the N regressions

$$\hat{\varepsilon}_{it} = \varrho_{i,t-1}\hat{u}_{i,t-1} + \dots + \varrho_{i,p_i}\hat{u}_{i,t-p_i} + \tilde{\varepsilon}_{i,t}$$

Similar test statistics as above can then be applied to the results of the GLS regressions. Breitung and Eickmeier (2011) only present the LM-statistic arguing that it has the best size properties compared to the LR and Wald statistics.

$$\hat{\varrho}_i(L)y_{it} = \lambda'_i \left[\varrho_i(L)\hat{f}_t \right] + \psi'_i \left[\hat{\varrho}_i(L)\hat{f}_t^* \right] + \tilde{\varepsilon}_{it}^* \text{ for } t = p_i + 1, \dots, T_i$$

¹⁹Note that Breitung and Eickmeier (2011) call a model where the innovations are generated by an AR(p) model a dynamic model whereas the literature usually refers to a dynamic process in the factors as in equation (7). This is similar in effect but not quite the same.

Under heteroskedasticity the regression has to be weighted by the covariance matrix. If we also take account of the possibility of a break in the covariance matrix we get two distinct regressions

$$\frac{1}{\hat{\sigma}^{(1)}} \varrho_i(L)y_{it} = \lambda'_i \left[\frac{1}{\hat{\sigma}_i^{(1)}} \varrho_i(L)\hat{f}_t \right] + \left[\frac{1}{\hat{\sigma}_i^{(1)}} \varrho_i(L)\hat{f}_t^* \right] + \tilde{\varepsilon}_{it}^* \text{ for } t = p_i + 1, \dots, T_i^*$$

$$\frac{1}{\hat{\sigma}^{(2)}} \varrho_i(L)y_{it} = \lambda'_i \left[\frac{1}{\hat{\sigma}_i^{(2)}} \varrho_i(L)\hat{f}_t \right] + \left[\frac{1}{\hat{\sigma}_i^{(2)}} \varrho_i(L)\hat{f}_t^* \right] + \tilde{\varepsilon}_{it}^* \text{ for } t = T_i^* + 1, \dots, T$$

2.2 Bootstrapping Breitung and Eickmeier (2011)

In order to test whether the properties of the above test statistics can be improved upon I have repeated the montecarlo study presented in Breitung and Eickmeier (2011) tables 2 and 3 with the difference that the test statistics have been bootstrapped.

The idea behind bootstrapping test statistics comes from the definition of the p-value. Denoting with τ the test statistic of interest, by $\hat{\tau}$ the realization of that test statistic given the observed data and by F the cumulative distribution function τ under the null hypothesis we can borrow the notation from Davidson and MacKinnon (2004) and write the p-value as

$$p(\hat{\tau}) = 1 - F(\hat{\tau})$$

Resultantly the p-value can be estimated by estimating $F(\hat{\tau})$, the distribution of the test statistic. To do this we simulate new data from the given set which follows the DGP under H_0 . The estimated p-value is then the estimated probability of the test statistic exceeding the calculated value $\hat{\tau}$ if the hypothesis H_0 were true.

Again following Davidson and MacKinnon (2004) we denote with stars, values that are calculated from simulated data, B denotes the number of simulated samples. We can estimate the cumulative distribution function as

$$\hat{F}^*(x) = \frac{1}{B} \sum_{j=1}^B I(\tau_j^* \leq x) \text{ and } \hat{F}^*(\hat{\tau}) = \frac{1}{B} \sum_{j=1}^B I(\tau_j^* \leq \hat{\tau})$$

Resultingly

$$\hat{p}^*(\hat{\tau}) = 1 - \hat{F}^*(\hat{\tau}) = 1 - \frac{1}{B} \sum_{j=1}^B I(\tau_j^* \leq \hat{\tau}) = \frac{1}{B} \sum_{j=1}^B I(\tau_j^* > \hat{\tau})$$

In the following two specifications of the bootstrap are considered. Firstly, the often used resampling of the residuals is applied. Secondly the wild bootstrap is used.

Resampling the residuals uses the definition of the factor equation (1) to generate new data. The innovation term e_t is replaced with resampled residuals²⁰ gotten from the estimation of the model using the initial data.

$$X_t^* = \hat{f}_t \hat{\Lambda}' + \hat{e}_t^* \quad (19)$$

The wild bootstrap serves to deal with heteroskedasticity. The idea is to rescale the residuals using a white noise random variable ν_t .

$$X_t^* = \hat{f}_t \hat{\Lambda}' + \hat{e}_t^+$$

Where \hat{e}_t^+ is set such that to $\hat{e}_{it}^+ = \hat{e}_{it} \nu_{it}$ and ν_{it} is distributed with mean 0 and variance 1. For the application I choose ν to be standard normal.

Tables with size and power properties can be found in the appendix as tables 11, 12, 13, 14. The tables show modest size improvements especially for small samples and the Wald and LR statistics which are slightly oversized in their regular versions. The power properties of the bootstrapped tests are quite similar to the regular versions of the tests although it could be argued that in small samples the bootstrapped versions outperformed the normal versions. Especially the small sample size results of the wild bootstrap seem to be improved. This could be an artifact of the relatively small number of repetitions, however. The results are not encouraging enough as to warrant an increase of the number of repetitions especially in light of the high computational burden although it would be interesting to see if the size properties of the wild bootstrap remain as favourable.

In the following the regular versions of the test statistics are used.

²⁰Resampling is done with replacement to model the empirical distribution. So the resampled residuals \hat{e}_t^* are given by setting $\hat{e}_t^* = \hat{e}_s$ where s is drawn from $\{1, \dots, T\}$ with replacement.

3 Empirical application

In this section the issue of forecasting economic time-series in the presence of structural breaks is approached. Specifically, focus is put on forecasting German GDP using German and US American macroeconomic time-series. The reason for including American time-series to predict German GDP is twofold. Firstly, Germany is active on international markets with the US being the second most important export market after France. Coupled with the big importance of the US American market in the world economy it is plausible that US data plays a role in predicting the underlying factors to German economic development. Secondly, we are interested in forecasting time-series under structural breaks. There are several periods commonly associated with breaks in the world economy. Two of which are the dot-com bubble of 2000 and the financial crisis of 2008. Arguably, both originated or at least had a noticeable impact in the US. The hope is that big ruptures in the world economy make a break of relationships between variables (and ultimately of the factor loadings) providing us with meaningful data for the following analysis.

There are several ways to address the issue of structural breaks from a forecasting viewpoint. Firstly structural breaks can be ignored. As has been stated above, in this case factor models will tend to choose a higher number of factors than if no structural breaks are present which means that the space the factors span is of a higher dimension as the factors are orthogonal to each other and thus linearly independent.²¹

Secondly, structural breaks can be included in the model in the form of a dummy parameter interacting with the factor estimations for both subperiods. The forecasting equation of the model then results as

$$y_{t+h}^h = \alpha(L)W_t + \beta(L)\hat{f}_t^{(1)} + \gamma(L)\hat{f}_t^{(2)} + \varepsilon_{t+h} \quad (20)$$

where $\hat{f}_t^{(1)}$ and $\hat{f}_t^{(2)}$ are estimated on the respective subperiods and are 0 for time indices other than belonging to these subperiods. This is equivalent to trimming off the first subperiod in terms of forecasting. W_t has to be trimmed accordingly.

Thirdly, if predictors are to be targeted prior to estimating the factor equation it is

²¹How this affects the information criteria of Bai and Ng (2002) can be seen in table 1 of Breitung and Eickmeier (2011).

conceivable to find that a subset of the predictors $\tilde{X}_t^{(1)} \subset X_t$ performs well before and a different subset $\tilde{X}_t^{(2)} \subset X_t$ performs better after the structural break. This finding would imply that it could be beneficial to estimate the factor equation again using only the subset $\tilde{X}_t^{(2)}$ of the data.

3.1 Data choice and treatment

The data used for the horse race has been collected from the German Bundesbank (German central bank) and from the American Federal Reserve Bank of St Louis.²² Together the data set consists of $T = 82$ observations from $N = 169$ quarterly (or casted to quarterly) variables. The data spans from 1993-01 until 2013-04. The series are listed below in appendix F. The data set contains macroeconomic variables such as wages and salaries, employment levels, survey data as well as financial variables from Germany and the US.

Additionally an emphasis has been put on selecting several time-series which are well known to be subjected to big changes in the abovementioned periods such as US "Housing starts".

The data series have been downloaded in seasonally and calendar adjusted form where possible. The data series have then been differenced (zero times, once or twice) until stationarity was achieved. To this end the number of differencing operations necessary to make the series stationary has been tested using the KPSS method developed in Kwiatkowski et al. (1992).²³

Finally, the series have been demeaned and normalized such that $\frac{1}{T} \sum_{t=1}^T X_{it} = 0 \forall i$ and $\frac{1}{T} \sum_{t=1}^T X_{it}^2 = 1 \Leftrightarrow \sum_{t=1}^T X_{it}^2 = T \forall i$. This is achieved by subtracting from each series its sample average and dividing by its sample standard deviation.

²²More specifically the FRED2 public API has been used to extract selected series. The data from the Bundesbank have been scraped from the Bundesbank website. Variable descriptions have been left as returned from the system and have not been translated.

²³The method consists of constructing an LM-test for the hypothesis that the variance σ_ε^2 is 0 (or equivalently that μ_t is constant) in

$$\begin{aligned} y_t &= \beta' D_t + \mu_t + u_t \\ \mu_t &= \mu_{t-1} + \varepsilon_t \end{aligned}$$

where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ and D_t captures deterministic components and time trends. This implies that the difference of y_t denoted Δy_t is stationary.

3.2 Model selection and evaluation

This section explores the different methods to select and evaluate the models. The ultimate goal is always to forecast the true data as closely as possible. To this end the root mean squared forecasting error (RMSE) is used as an evaluation mechanism. $RMSE = \sqrt{\frac{1}{w} \sum_{s=t+1}^{t+w} [(y_s - \hat{y}_s)^2]}$ where w denotes the length of the window to be forecast and t the period preceding the window being forecast. The estimates \hat{y}_t are gotten from estimating the forecasting equation and predicting $h = 1$ steps into the future.

The IC_{p2} criterion finds 4 static factors for the complete series. The scree plot depicted in figure 1 shows that the size of the eigenvalues drops of at approximately 4 factors which makes that choice seem appropriate in the sense that the factors which explain the biggest portion of the variance are likely to be contained in the first 4 factors.

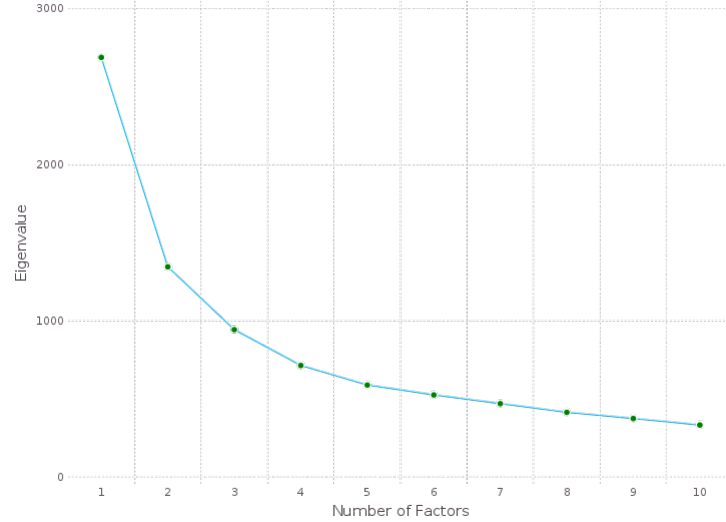


Figure 1: Scree plot of the eigenvalues calculated on the whole data set

The first factor is most strongly correlated with variables related to total US output and manufacturing. Namely the X_i with a large absolute correlation with the first factor are "Industrial Production Index", "Industrial Production: Durable Manufacturing (NAICS)", "Industrial Production: Manufacturing (NAICS)", "ISM Manufacturing: PMI Composite Index", "ISM Manufacturing Production Index" and "Capacity Utilization: Total Industry". The second factor mostly affects variables dealing with US

construction and private housing: "Housing Starts in Midwest Census Region", "Housing Starts: Total: New Privately Owned Housing Units Started", "New Privately-Owned Housing Units Authorized by Building Permits: Total" and "All Employees: Construction". The second factor is also correlated with some output related variables in Germany such as "Output in the production sector", "Orders received" but also "Orders received" of the German construction industry. For the third and the fourth factors the correlations are less strong in absolute variables but one can perhaps say that they are mostly correlated to hours worked and total employment variables in the US.

These results reflect the relative importance of US business cycles for the chosen set of macroeconomic variables which can on the one hand be explained by the fact that 120 of the 169 variables are composed of US macroeconomic variables but on the other hand due to the relative sizes of the economies it is more likely that US variables affect German variables and thus that they have a higher influence on the underlying factors driving the evolution of the set of X_i .

If the number of static factors in the forecasting equation is predetermined using the IC_{p2} criterion, the number of lags of the variable of interest y_t and the number of lags of the static factors are still open. In the following I refer to the number of lags of y_t as s .

I follow Bai and Ng (2008) in selecting among model candidates using the BIC and the FPE criterion which are of course in-sample criteria.

Bai and Ng (2008) write the BIC and FPE criteria as follows

$$\begin{aligned} \text{BIC} &= \log(\hat{\sigma}_n^2) + n \frac{\log T}{T} \\ \text{FPE} &= \log(\hat{\sigma}_n^2) + n \frac{2}{T} \end{aligned} \tag{21}$$

Where $\hat{\sigma}_n^2$ is the residual variance $\hat{e}'e$ from an estimation of the factor equation using n predictors.

Since the point of the exercise is forecasting, the optimal model in terms of out-of-sample forecasting performance is found. This is done by calculating the root mean

squared error of one step ahead pseudo out-of-sample forecasts.²⁴ This evaluation method follows Forni et al. (2005) and Bai and Ng (2008) among others. The forecasting window w over which the pseudo out-of-sample-forecasts are performed is set to $w = 15$ here.

This forecasting window makes sure that there are more observations than parameters for all possible models. Since the length of the data series is $T = 82$ this leaves 67 observations to learn the model with for the first forecast in the pseudo out-of-sample procedure. The maximum number of factors is set to 15 for static models and 10 for dynamic models. The maximum number of lags of y_t is set to 10. This brings the maximal number of parameters to $n_{max} = 25$ for static models and $n_{max} = 50$ for dynamic models. Note that the number of observations available has to be reduced by the maximal number of lagging performed in each model. This means in the worst cases (i.e. the highest number of lags of the variable of interest enters the equation) another 10 observations available for learning have to be deducted from both models. The original number of predictors is $N = 169$ which highlights that factor models can be used as a dimensionality reduction method and that they are an appropriate way to deal with the situation of $N \gg T$ in this data set. However, it also becomes clear that there is not a lot of room for trimming the length of the data set unless the maximum number of parameters is to be reconsidered.

All models are compared to a benchmark model which consists of a simple moving average of the w periods before period $t + h$ which is being forecast. The window size for the moving average benchmark is the same as the forecasting window w .

The first panel in table 1 shows the results of the in sample selection procedure for a static factor model. Three variants for choosing the number of factors are considered. Firstly the IC_{p2} criterion from Bai and Ng (2002) is used to estimate the number of static factors for both the factor equation and the forecasting equation. The information criteria (BIC and FPE) are then used to select s , the number of lags of y_t . As a second choice, the decision over the number of factors *and* the number of lags of y_t

²⁴Pseudo out-of-sample forecasts are a method to simulated a forecasting situation in that the forecaster does as if he would not know the true realizations of the data after t . The forecaster tries to predict the value y_{t+h} given all the information up to t . t is then successively moved one period ahead. Here we set $h = 1$.

in the forecasting equation are left to the information criteria. I.e. the information criteria are minimized over different choices of r and s . The same number of factors is used in the factor equation. A third possibility consists of leaving the choice of the number of factors for the factor equation to a Bai and Ng (2002) criterion and using the BIC or FPE criterion to choose the number of static factors in the forecasting equation along with the number of lags of y_t . I.e. the BIC or FPE are minimized over r and s .

Both the BIC and FPE as well as the IC_{p2} criteria select a large number of factors and are generally very erratic in their choice of a model. It can be seen that conventional information criteria mostly perform badly in selecting the number of static factors. Another approach to model selection for forecasting consists of testing the model in an out-of-sample forecasting situation and selecting the best performing model in terms of the forecasting error. In a forecasting application this implies that a certain fraction of the data has to be reserved for model selection. The number of parameters are then chosen by evaluating pseudo out-of-sample forecasts of this window.

Since the variance of the data series has been normalized to 1 and the mean to 0 the RMSE of the simple average y_t in the given window can be expected to be close to 1. The RMSE of the out-of-sample forecasts resulting from choosing the model using the BIC is very large with 1.30 and 1.29 respectively. The benchmark model of a moving average reports an RMSE of 0.86 and an AR(6) model achieves an RMSE of 0.50 for the same time horizon.²⁵

The best performing static factor model in terms of forecasting can be got if the BIC criterion for model selecting is replaced by the RMSE as can be seen in the second panel. I.e. the RMSE is minimized over choices of r and s rather than the BIC. This implies using the out-of-sample performance as an evaluation criterion. In that case both r and s are shrunk to 1. While this outperforms the simple benchmark model in form of the moving average, the fact that the most parsimonious static factor model performs best in out-of-sample prediction hints at possible performance increases if dynamic effects are allowed. It is interesting to consider why the AR(6) model outperforms the static factor model with an RMSE of about 0.50 compared to 0.71. The static factor

²⁵ $p = 6$ is the optimal lag number in terms of RMSE.

model does not allow for a dynamic interaction between the factors and the dependent variable. In other words the factors can only influence the X_i contemporaneously. As will be seen below, the dynamic factor model can improve on this result by allowing non-contemporaneous interaction.

Table 1: Static factor model, model selection

	r_{factor}	$r_{forecast}$	s	RMSE
<i>In-Sample</i>				
BIC	13	13	5	1.29
FPE	15	15	5	1.23
IC_{p2} & BIC	4	13	5	1.30
IC_{p2} & FPE	4	15	5	1.23
IC_{p2}	4	4	10	1.21
<i>Out-of-Sample</i>				
RMSE	1	1	1	0.71
RMSE & IC_{p2}	4	1	1	0.71

Notes: The results are derived by minimizing the criteria in the first column. If both an Bai and Ng (2002) criterion and a standard information criterion is given the former is applied to the factor equation and the latter to the forecasting equation. The maximum number of factors r is set to 15. The maximum number of lags of the variable of interest s is set to 10.

Repeating the exercise but allowing for lags of the factors to enter the forecasting equation, the results in table 2 are obtained.²⁶ Since the data used for the exercise consists of time-series it makes sense that a dynamic factor model is better suited for prediction here. In the case of the dynamic model the FPE criterion readily selects too many parameters which results in a higher RMSE. The IC_{p2} criterion chooses $r = 4$ for the factor equation. If the choice of the number of factors in both the factor and the forecasting equation is left to the IC_{p2} which sets $r = 4$, the RMSE increases to about 0.70. The BIC then selects 10 lags of the dependent variable and 2 factor lags. The RMSE drops to 0.55 if only 2 factors, 3 lags of the dependent variable and 1 factor lag is chosen which happens when the choice over the number of factors is left to the BIC criterion.

²⁶Essentially, table 1 uses the same models as are used in table 2 with the added restriction $q = 0$ and the fact the maximal number of factors has been reduced to 10 because the factors enter the forecasting equation also in lagged form now which increases the number of parameters of the model quickly.

Noteworthy, in this situation it does not make a difference in terms of RMSE if the number of factors differs in the factor equation and in the forecasting equation. This can be seen as in the optimal out-of-sample models. The number of factors in the factor equation usually differ but the RMSE stays exactly the same. This no surprise because the factor estimates do not change regardless of how many factors the IC_{p2} criterion chooses for the factor equation.

In the dynamic factor model, choosing the number of static factors r_{factor} in the factor equation using the IC_{p2} criterion outperforms the other in-sample methods in terms of forecasting accuracy. Looking at table 2 it becomes clear why this is the case. Comparing panels a) and b) it is evident that the maximum number of factors, factor lags and lags of the dependent variable has to be chosen somewhat conservatively or else the model with the maximal number of factors and factor lags will be chosen. This is because the residuals are very small in these cases. If the number of factors is set by the IC_{p2} criterion, the residuals are set "exogenously" to be big enough for the information criteria to perform well.

	r_{factor}	$r_{forecast}$	s	q	RMSE
<i>In-Sample</i>					
BIC	2	2	3	1	0.55
FPE	10	10	10	3	2.91
IC_{p2} & BIC	4	2	3	1	0.55
IC_{p2} & FPE	4	10	10	3	2.91
IC_{p2}	4	4	10	2	0.70
<i>Out-of-Sample</i>					
RMSE	7	7	1	3	0.47
RMSE & IC_{p2}	4	7	1	3	0.47

(a) Maximum number of factor lags q set to 3

	r_{factor}	$r_{forecast}$	s	q	RMSE
<i>In-Sample</i>					
BIC	10	10	10	4	19.16
FPE	10	10	10	3	19.16
IC_{p2} & BIC	4	10	10	4	19.16
IC_{p2} & FPE	4	10	10	4	19.16
IC_{p2}	4	4	10	4	0.78
<i>Out-of-Sample</i>					
RMSE	7	7	1	3	0.47
RMSE & IC_{p2}	4	7	1	3	0.47

(b) Maximum number of factor lags q set to 4

Notes: The results are derived by minimizing the criteria in the first column. If both a Bai and Ng (2002) criterion and a standard information criterion is given the former is applied to the factor equation and the latter to the forecasting equation. The maximum number of factors r is set to 10. The maximum number number of the lags of the variable of interest s is set to 10. q here refers to the number of lags of the factors in the forecasting equation, not to the number of dynamic factors as defined above. The maximum number of the factor lags q is set to 3 and 4 respectively. The results using the traditional information criteria hinge on the maximum number of factor lags.

Table 2: Dynamic factor model, model selection

3.3 Introducing structural breaks

Structural breaks are estimated using Quandt-Andrews type supremum tests as defined above. The critical values for the test statistics are given in table I of Andrews (2003). The number of estimated breaks per period can be seen in table 3.

It is apparent that there are two periods with major changes in the loadings. Firstly, there are structural breaks reported in the loadings of 8 variables in the first quarter of 1999 which coincides approximately with the Russian default of 1998 and maybe the aftermath of the Asian financial crisis in 1997. In the following this period will be referred to as the first cluster of structural breaks. Secondly, a total of 22 changes in the loadings are reported for the period of the second quarter of 2008 until the second quarter of 2009 which coincides with the recent financial crisis starting after an US housing bubble burst. This period will be referred to as the second cluster of structural breaks. It caused and was affected by severe macroeconomic changes which are quite obviously reflected in the given data set.

Variables with breaks in the periods around 2008 include several aggregate variables which deal with orders received by German companies, variables describing productivity and total output, variables describing the monetary base and loans given by banks, the consumer price index, the market rates for treasury bills of differing maturities and the "University of Michigan Consumer Sentiment" survey. These are all variables which can be expected to undergo structural breaks during the 2008 financial crisis. Eminently, "PERMITNSA" which counts the number of new privately-owned housing units authorized, undergoes a structural break in the third quarter of 2005. This is also the time when the housing bubble in the US was arguably at its peak in terms of housing price increases: Bernanke (2010) stresses that "[...] the most rapid price gains were in 2004 and 2005, when the annual rate of house price appreciation was between 15 and 17 percent".

Variables with breaks in the loadings at the first quarter of 1999 include variables describing German government debt, German exports and other trade related variables as well as variables summarizing bank term sheets and additionally, employment in the

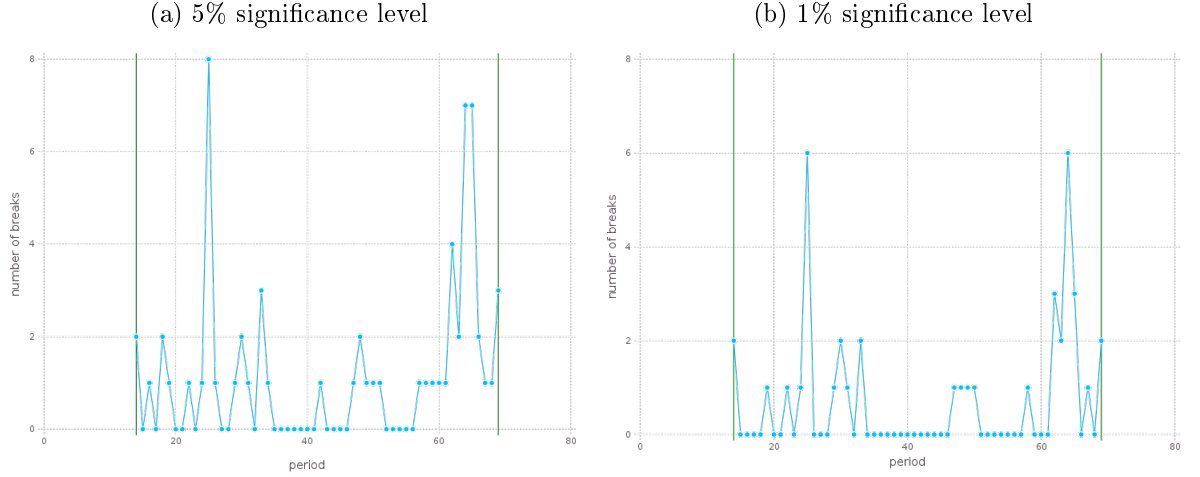
US durable goods sector.

Table 3: Number of structural breaks per period

96:1	96:3	97:2	97:3	98:2	98:4	99:1	99:2	00:1	00:2	00:3	01:1	01:2
2	1	2	1	1	1	8	1	1	2	1	3	1
03:2	04:3	04:3	05:1	05:2	05:3	07:1	07:2	07:3	07:4	08:1	08:2	08:3
1	1	2	1	1	1	1	1	1	1	1	4	2
08:4	09:1	09:2	09:3	09:4	10:1							
7	7	2	1	1	3							

Notes: The entries show how many of the $N = 169$ variables are predicted to undergo a structural break in each period. Reported are the numbers of significant values of $\mathcal{S}_{i,T}$ at the 5% significance level. The total number of structural breaks found in the 169 variables is 64.

Figure 2: Structural breaks per quarter



3.3.1 Removing variables which undergo structural breaks

In the following the question of how to react to the presence of structural breaks is approached as in section 2 above. Structural breaks in the factor loadings influence the performance of the forecasting equation through at least two channels. Firstly the presence of structural breaks decreases the signal to noise ratio in the data and thus the quality of the affected predictors. Secondly, Breitung and Eickmeier (2011) show that it is necessary to increase the number of factors r under the presence of structural breaks in order to estimate the space spanned by the factors. If the number of static factors in the forecasting equation is set equal to the number of static factors in the

factor equation it becomes necessary to estimate a bigger number of parameters which can be an issue of practical relevance if there are not enough observations relative to the number of parameters as has been seen above.

Either the length of the series is reduced by estimating the model only on data which is not subjected to structural breaks or the width of the data is reduced by removing the variables which are subjected to the structural break or a combination thereof. For the purpose of this paper we are interested in forecasting performance of factor models which ignore structural breaks versus factor models which take the structural breaks into account. Thus the results of models which restrict the amount of data to reduce the effect of structural breaks are compared with those that do not. It is worth noting that the comparison is not perfect because as has been argued in section 2 it can be better to reduce the number of predictors to a set which is "targeted" towards the prediction of the variable of interest. It is possible that removing the variables with structural breaks "accidentally" targets the set of variables towards predicting the variable of interest independent of the structural break. In other words an improved result is not necessarily only due to the structural breaks being removed but might be due to a noisy predictor being removed. To be precise a series with a structural break could be considered a noisy predictor as well. However, it might be noisy independent of the structural breaks. A robustness check consists thus of targeting the set of variables for both approaches prior to the estimation to remove that effect.

A practical difficulty has to be addressed before the results are presented. Given the fact that there are two periods where a majority of structural breaks occur, each of which affects a different set of factor loadings, cutting off the values before the significant break periods would leave very few observations for estimating the new model.

If both periods with major structural breaks are treated in a way that only data after the structural breaks is used, this would leave at most 17 observations of which another 15 have to be removed for the first step in the pseudo out-of-sample forecasts.

In principle there is a simple way to overcome this issue. It can be remembered that some of the data columns are monthly series which have been merged into the quarterly data set by dropping two thirds of the observations. The data could be cast back into monthly form. Most of the series will have missing data which can be interpolated

(e.g. GDP is only reported quarterly). While this method solves the problem in a technical sense²⁷, it has several drawbacks: firstly no additional information is added by the interpolation although there are more observations available to the model. At the same time the imputation adds additional uncertainty. Traditional standard error estimates do not account for this and they may be downward biased (cf. Gelman and Hill (2006, chapter 25)). It is thus likely that prediction based on this short data set performs badly in a forecasting sense. On the other hand, if predictions resulting from this shaved data set should perform better, it would constitute a strong indication that the structural breaks uncovered by the tests should not be ignored. In this case doing so clearly leads to a significant degradation of forecasting results. This has not been tested here.

Instead four approaches to removing the effects of structural breaks from the data are considered. First the variables in the second cluster of structural breaks between the second quarter of 2008 and the first quarter of 2009 are removed. Because this approach ignores the first cluster of structural breaks in the first quarter of 1999, a second model is estimated to explore what happens if these series are excluded as well. A third model removes all series with structural breaks in the loadings. Finally, the fourth model leaves the *variables* with breaks in the first cluster in 1999 in the model but removes *observations* prior to the second quarter of 1999.

For all four approaches the best performing dynamic factor model in terms of RMSE is reported where the number of static factors in the factor equation is calculated using the IC_{p2} criterion and the number of factors in the forecasting equation as well as the number of factor lags is chosen according to the minimum of the RMSE of out-of-sample forecasts. Structural breaks are detected at the 5% significance level.

The figures in appendix D show the number of significant structural breaks left if the above adjustments to the data have been made. The reason to check for structural breaks again after some variables have been removed or the observations prior to the second quarter of 1999 have been removed, is that the estimation of the factor equation changes with a reduced set of variables which has an impact on the statistics of the

²⁷For the first step of the pseudo out-of-sample period there are only 8 data points for estimation. For every quarterly step of the pseudo out-of-sample procedure there are an additional 3 observations. Thus, the ratio of observations to predictors improves quickly with each step of the pseudo out-of-sample prediction.

structural break tests.

Table 4 shows that the exclusion of variables improves the RMSE considerably compared to ignoring the structural breaks. For this data set, the more variables with structural breaks at the 5% significance level are removed, the lower the RMSE becomes. The two models with $N = 105$ are among the best performing models overall.

	r_{factor}	$r_{forecast}$	s	q	RMSE
	<i>Second break cluster removed</i>				
	$T = 82, N = 149$				
RMSE	7	7	1	3	0.45
RMSE & IC_{p2}	3	7	1	3	0.45
	<i>First and second break cluster removed</i>				
	$T = 82, N = 141$				
RMSE	6	6	4	2	0.44
RMSE & IC_{p2}	3	6	4	2	0.44
	<i>All variables with breaks removed</i>				
	$T = 82, N = 105$				
RMSE	6	6	3	2	0.40
RMSE & IC_{p2}	3	3	4	2	0.40
	<i>Second break cluster removed, length trimmed</i>				
	$T = 57, N = 149$				
RMSE	1	1	5	1	0.51
RMSE & IC_{p2}	4	1	5	1	0.51

Table 4: Dynamic factor model, data adjusted for structural breaks

Figures for the number of structural breaks per period after variables have been dropped or the data length has been trimmed can be found in appendix D. Notably figure 6 shows a modest number of structural breaks at the 5% significance level *after* all structural breaks had been removed from the data set. Iterating the procedure by removing the variables with structural breaks again increases the RMSE to 0.72 which makes clear that blindly removing all variables which contain structural breaks need not be a good idea even though the previous results in table 4 seemed to indicate that removing as many variables with structural breaks as possible has a positive effect on RMSE.

3.3.2 Removing variables with breaks after targeting the data

It was found in the previous section that removing all variables with structural breaks can improve the forecasting performance. To test if targeting the predictors and removing the structural breaks has a similar effect, the variables with structural breaks are removed after the data set has been targeted. To this end the following sub section considers the residual effect of removing the structural breaks after the effect of targeting the predictors has been accounted for. Both soft and hard thresholding rules are applied.

It should be noted as well that German GDP, -the variable of interest, is kept in the data set in all variable preselection schemes even if structural breaks are found to be present in the loadings of this variable. This case occurs sometimes when structural breaks are tested for on targeted data sets.

Starting with hard thresholding, targeting the predictors at the 10%, 5% and 1% significance level (see Bai and Ng (2008) for details) leaves 85, 78 and 43 variables in the data set respectively. Figure 3 shows that targeting the data removes a part of the two clusters of structural breaks found in the original data. It can be inspected in table 5 that with hard thresholding the RMSE is higher than it was for the best performing model of the original data set. For the complete data set the optimal model achieved an RMSE of 0.47. with hard thresholding the RMSE increases to about 0.50 for all three cut-off values. Regardless of the relatively bad performance of hard thresholding rules to target the predictors, removing the structural breaks from the targeted data set marginally improves the results.

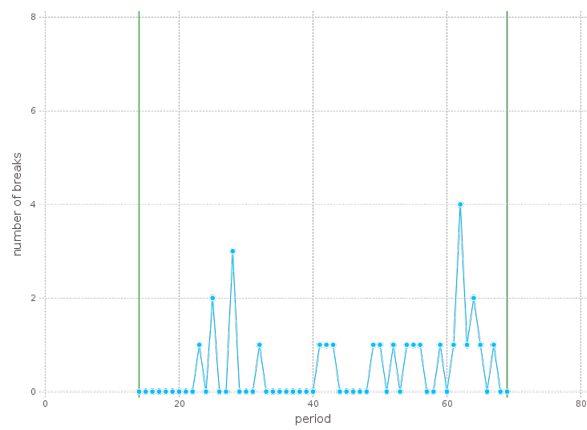
	Hard thresholding				
	r_{factor}	$r_{forecast}$	s	q	RMSE
	$\alpha = 0.1$				
	$T = 82, N = 85$				
RMSE	1	1	5	2	0.50
RMSE & IC_{p2}	6	1	5	2	0.50
	Hard Thresholding $\alpha = 0.05$				
	$T = 82, N = 78$				
RMSE	1	1	5	2	0.50
RMSE & IC_{p2}	6	1	5	2	0.50
	$\alpha = 0.01$				
	$T = 82, N = 43$				
RMSE	1	1	5	2	0.49
RMSE & IC_{p2}	7	1	5	2	0.49

Table 5: Dynamic factor model, targeted data, hard thresholding

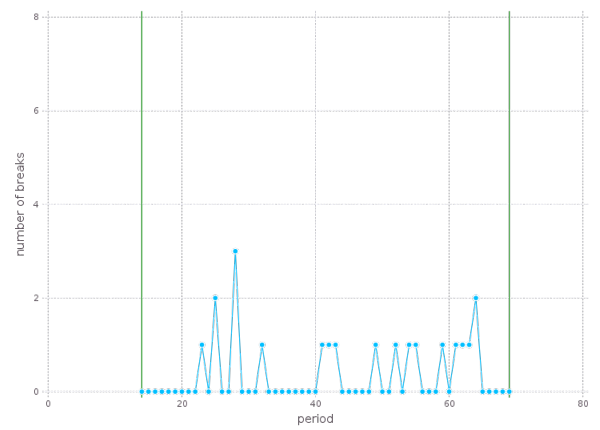
	Hard thresholding, breaks removed					
	r_{factor}	$r_{forecast}$	s	q	RMSE	Δ_{RMSE}
	$\alpha = 0.1$					
	$T = 82, N = 51$					
RMSE	11	11	2	1	0.44	0.06
RMSE & IC_{p2}	4	11	2	1	0.44	0.06
	$\alpha = 0.05$					
	$T = 82, N = 51$					
RMSE	1	1	5	2	0.50	0
RMSE & IC_{p2}	4	1	5	2	0.50	0
	$\alpha = 0.01$					
	$T = 82, N = 28$					
	Results omitted					

Notes: T and N denote the dimension of the targeted data sets where all variables with evidence of structural breaks found at the 5% level using the sup LM test have been removed. The column denoted with Δ_{RMSE} calculates the difference between the RMSE of the best factor model calculated on the targeted data with structural breaks removed and the results from table 5 where the structural breaks have not been removed. Note that the set of variables for hard thresholding with $\alpha = 0.05$ differs although the number of variables coincides. Also note that there are no results for $\alpha = 0.01$ because the estimated number of factors for this data set is 22 for which there are no tabulated values to calculate the structural break tests with in Andrews (2003). The critical values have not been simulated here.

Table 6: Dynamic factor model, targeted data, structural breaks removed



(a) 5% significance level



(b) 1% significance level

Figure 3: Structural breaks per quarter, targeted predictors

As noted in the introduction on targeted predictors above, the difficulty with soft thresholding lies in the choice of the number of steps k in the LARS-EN algorithm. This coincides with the number of variables chosen as each step adds one variable to the active set. It is conceivable that if the thresholding is too strict, i.e. the regularization parameter too large, the LASSO will have kicked out most of the variables which contain structural breaks. A chosen set of results of the removal of structural breaks after applying soft thresholding using the LASSO can be seen in table 7.

The value of k which yields the best performing targeted predictor model is $k = 39$ which gives an RMSE of just below 0.4 as can be inspected in table 7. The fact that the maximum number of factors is chosen is suspicious. However, if the maximum number of factors is increased the results remain constant. The same holds for $k = 30$ and for the number of factor lags.

	Soft thresholding					
	r_{factor}	$r_{forecast}$	s	q	RMSE	Δ_{RMSE}
	$k = 39$					
	$T = 82, N = 39$					
RMSE	15	15	3	1	0.4000	-0.0811
RMSE & IC_{p2}	2	15	3	1	0.4000	-0.0811
	$k = 32$					
	$T = 82, N = 32$					
RMSE	13	13	1	1	0.4014	-0.0125
RMSE & IC_{p2}	2	13	1	1	0.4014	-0.0125
	$k = 40$					
	$T = 82, N = 40$					
RMSE	15	15	3	1	0.4141	-0.0716
RMSE & IC_{p2}	2	15	3	1	0.4141	-0.0716

Notes: The entries report detailed results of the three best performing optimal models for $k \in \{30, 32, 39\}$. Less detailed results for more values of k can be found in table 15 in the appendix. Δ_{RMSE} denotes the change in RMSE if the optimal model is calculated from the targeted data set with variables containing structural breaks removed. The maximum number of factors is set to 15, the maximum number of factor lags is set to 3 and the maximum number of lags of y_t is set to 5

Table 7: Dynamic factor model, targeted data, soft thresholding

The 39 variables contained in the best performing targeted predictor model include German output and employment variables, several variables describing German exports

and the trade balance. The US variables included contain output and employment variables, some production related financial variables such as "Commercial and Industrial Loans", the consumer and producer price indices, construction and housing variables as well as monetary variables and the "6-Month Treasury Bill: Secondary Market Rate". Interestingly the foreign exchange rate between Switzerland and the US is also added to the set of variables. It is likely that this variable captures the lack of other exchange rates in the data set.

The second best performing model removes some of the variables but because the second best model is nothing but an earlier point on the solution path, the other variables remain the same. The removed variables include one of the German export variables, the US labor force participation rate, one of the US housing starts variables and markedly the producer price index.

Relative improvements of removing structural breaks for more values of k and Diebold-Mariano test statistics²⁸ are shown in table 15. The table starts with $k = 26$ which is the first value in the LASSO solution path for which the resulting number of factors calculated by the IC_{p2} criterion returns sensible values.²⁹ It can be seen that removing the variables with structural breaks can only improve the RMSE for some of the different sets of targeted data. This can be seen in the positive entries in the Δ_{RMSE} column.

However, significant improvements in forecasting accuracy can only be confirmed for $k = 49$ at customary levels of significance and different forecasting performance can only be detected for 3 choices of k . It should also be noted that even this one rejection has to be taken with a grain of salt although significant at the 1% level. This is because if the 5% significance level is taken, at least one rejection can be expected due to chance in 25 tests and at the 1% level 1 false rejection is less likely but possible nonetheless.

²⁸The models are estimated on different subsets of the data set. Strictly speaking the models could be seen as nested and another comparison procedure such as Clark and McCracken (2001) should be used depending on the specificity of the restriction. This corner case is ignored here.

²⁹Sensible here means smaller than 20 which is the last value for which Andrews (2003) has tabulated critical values.

To further study the interaction between targeted predictors and structural breaks in a confined environment, in a final step, a montecarlo study is performed in which the simulated data contains a structural break in only one of the factor loadings. The size of the break and the information content for the prediction of y is varied in one predictor. The idea is to measure the difference in RMSE in this contained environment if the variable is removed versus keeping the variable in the model for varying sizes of the break and levels of information for predicting y . More precisely the Cholesky decomposition is used in order to generate data with varying degree of dependence δ between the dependent variable and the predictor which experiences a structural break of size b . All values of δ which are bigger than 0.1 imply that the last column is typically chosen first if the predictors are targeted. Table 8 shows that a higher δ decreases the likelihood of an improved forecasting accuracy which is intuitive because removing an informative predictor should decrease performance. It seems that an increase in the break size b has the same effect but this effect seems to be less pronounced.

$$\text{DGP: } X_{it} = \sum_{j=1}^r \lambda_{ij} F_{tj} + \text{chol}(v(\delta)) e_{it}, r = 1$$

$$\text{where } v(\delta) = \begin{bmatrix} 1 & 0.1 & \dots & \dots & 0.1 & \delta \\ 0.1 & 1 & 0.1 & \dots & 0.1 & 0.1 \\ \vdots & 0.1 & \ddots & 0.1 & \vdots & \vdots \\ \vdots & \vdots & 0.1 & \ddots & 0.1 & \vdots \\ 0.1 & \vdots & \vdots & 0.1 & \ddots & 0.1 \\ \delta & 0.1 & \dots & \dots & 0.1 & 1 \end{bmatrix}$$

		δ									
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
b	0.0	0.040	0.036	0.018	0.020	0.024	0.024	0.020	0.018	0.014	0.028
	0.1	0.032	0.024	0.026	0.026	0.026	0.008	0.026	0.010	0.020	0.004
	0.2	0.026	0.020	0.020	0.024	0.018	0.018	0.018	0.014	0.014	0.008
	0.3	0.042	0.040	0.020	0.008	0.014	0.020	0.010	0.024	0.022	0.002
	0.4	0.040	0.036	0.020	0.026	0.020	0.008	0.010	0.026	0.010	0.018
	0.5	0.030	0.028	0.022	0.016	0.016	0.018	0.020	0.018	0.014	0.008
	0.6	0.040	0.040	0.014	0.012	0.004	0.016	0.030	0.004	0.008	0.012
	0.7	0.042	0.036	0.020	0.028	0.014	0.020	0.018	0.016	0.010	0.012
	0.8	0.036	0.034	0.024	0.016	0.024	0.016	0.018	0.016	0.006	0.010
	0.9	0.030	0.034	0.018	0.014	0.026	0.014	0.014	0.014	0.010	0.012
	1.0	0.028	0.024	0.016	0.012	0.020	0.026	0.012	0.020	0.020	0.018

Notes: The entries report the average rejection rate of $R = 500$ Diebold-Mariano tests of improved forecasting performance at the 5% significance level. The two competing models are static factor models, one of which includes the column with the structural break, the other does not. Optimal models are chosen using the RMSE as before. The structural break occurs in the mid observation $T^* = 25$ in the last column. The size of the break is given by b and the correlation of the dependent variable in the first column with last column is given by δ . $T = 50, N = 20$

Table 8: Montecarlo of DM-tests for varying break size and information

4 Conclusion

While factor models perform quite well in forecasting economic time-series, structural breaks can, if not accounted for, undermine the forecasting performance significantly. Not only does one have to be careful with the actual prediction results but also interpreting the factors directly can become unfeasible as the number of static factors changes under structural breaks.

Estimating different model specifications under structural breaks on a large macroeconomic data set including German and US macroeconomic variables shows that removing variables with structural breaks tends to improve forecasting accuracy considerably. In this data set the improvement of removing the variables which undergo structural breaks improves the performance more so than removing the pre-break observations. This result must not necessarily hold for different data sets and could be an artifact of its relative shortness.

The best performing models with breaks removed handily beat the best performing benchmark AR(p) model and the benchmark moving average model as well as dynamic factor models which do not account for breaks.

The best overall results are achieved using targeted predictors with soft thresholding with $k = 39$ which amounts to keeping only roughly 23% of the original predictors.

Removing all structural breaks from the data yields an equivalently well performing model but leaves 105 variables or approximately 62% of the original predictors. It is not clear which approach is to be preferred. A case can maybe be made that soft-thresholding ultimately uses less predictors and thus is in a sense more simple. However, the model which removes all breaks is able to use two factor lags and less static factors. The total number of parameters in the models is 33 for the targeted predictors model as compared to 21 or 13 for the two versions of the model which removes the structural breaks. In this sense the latter is more parsimonious.

It is noteworthy that the number of factor lags employed is higher if structural breaks are removed than if the predictors are targeted. Perhaps, the removal of the sources of parameter instability enables dynamic factor models to exploit the time-series structure better.

It should perhaps be noted that the approach to remove all variables which have sig-

nificant structural breaks after the data has been targeted is crude and the comparison to targeted predictors is somewhat unfair. Targeting the predictors under soft thresholding consists of a whole solution path of which the best solution is picked. As an alternative and perhaps more fairly one could consider removing variables with breaks step by step starting e.g. with variables which have a high associated sup LM statistic. This is likely to yield additional gains in forecasting performance.

The removal of variables which feature structural breaks works similarly to the method of targeted predictors. While each method individually improves forecasting precision, there is little evidence that the two methods used in conjunction further improve the forecasting results. This is due to them performing a similar task. The method of removing structural breaks ignores the information contained in the predictor while soft-thresholding focusses on correlatedness over the whole time-frame without taking into account that the linear relationship might break down over time. Put differently: as long as two predictors are sufficiently correlated *overall* it does not matter if the most recent data follow a different data generating process if targeted predictors are used for variable preselection. Further research into variable preselection methods for factors models could also yield a method which generalizes targeted predictors and the approach to remove variables featuring structural breaks.

Additionally and independently from the treatment of structural breaks, the model selection framework can be further refined. It is likely for example that allowing factors to enter the forecasting equation with different lag levels can help to boost the forecasting performance.

Appendix

A Derivation of principal components

The principal components of a data matrix X are defined as the orthogonal transformation of X into the space spanned by the loadings Λ_i where each Λ_i is a unit vector³⁰. The transformation is defined such that the transformed columns are uncorrelated (as are the loadings) and such that the first transformed column (i.e. the first principal component or the first factor if principal components is used in factor models) contains the largest amount of the variance of the original data, the second column the second largest amount (i.e. the second principal component), etc.

Let X have mean 0 (i.e. subtract the mean from each column if it does not) and denote as U the rotation which transforms X into the space spanned by the Λ_i as described above. For the first principal component, we want to find a rotation vector U_1 such that the variance along the projection of X onto the first principal component direction U_1 is maximized where U_1 is a unit norm vector.

$$\max_{U_1: \|U_1\|=1} \frac{1}{T} \sum_{t=1}^T (X'_t U_1)^2 = \max_{U_1: U'_1 U_1 = 1} U'_1 \left(\frac{1}{T} \sum_{t=1}^T X'_t X_t \right) U_1 = \max_{U_1: U'_1 U_1 = 1} U'_1 (\Sigma) U_1$$

Setting up the Lagrangian yields

$$L(U_1, \lambda_1) = U'_1 \Sigma U_1 - \lambda_1 (U'_1 U_1 - 1)$$

After taking the derivative with respect to U_1 and dividing by 2 we are left with

$$\frac{\partial L}{\partial U_1} = \Sigma U_1 - \lambda_1 U_1 \stackrel{!}{=} 0$$

in other words U_1 is an eigenvector of Σ , the covariance matrix of x . The corresponding eigenvalue is λ_1 and we must have $\Sigma = \lambda_1$ for the variance of the principal component to be maximal under the constraint.

The maximal variance of $X'U_1$ is thus $V(X'U_1) = U'_1 \Sigma U_1 = U'_1 \lambda_1 U_1$ and it is achieved

³⁰The Λ_i are also called principal component directions.

for the highest eigenvalue λ_1 of the covariance matrix of X . The derivation of the following principal components is accordingly subject to the additional constraint that the principal components are uncorrelated. In the case of the second principal component this implies $\text{cov}(U_1'X, U_2'X) = U_1'\Sigma U_2 = 0$. The solution for the third and all following principal component is more complex but goes accordingly. The derivations can be seen e.g. in Jolliffe (2005) or any other textbook on the subject. The solution for the i -th principal component is to set the loadings U_i to be eigenvector which belongs to the i -th highest eigenvalue. Also, for each principal component i we get that $\text{var}(U_i'X) = \lambda_i$.

Since U is the matrix that rotates X into the space spanned by the loadings, it follows that Λ can be set to equal U'^{-1} in the notation of factor models used throughout the paper. An easy way to see this is to write the estimation of the factor equation as $X = F\Lambda'$ ignoring for a second the error term. Resultingly we must have that $X\Lambda'^{-1} = F$. That is if $\Lambda = U'^{-1}$, the factor matrix F is simply the transformed data matrix.

B Estimation of the factor equation

Estimators of the factor equation solve (10).

$$\min_{F_1, \dots, F_T, \Lambda} V_r(\Lambda, F) = \min_{F_1, \dots, F_T, \Lambda} \frac{1}{NT} \sum_{t=1}^T (X_t - \Lambda F_t)' (X_t - \Lambda F_t)$$

$$\text{s.t. } N^{-1} \Lambda' \Lambda = I_r$$

The solution can be gotten by first optimizing over F_t . The first order condition is:

$$\frac{\partial V_r}{\partial F_t} = -2X_t' \Lambda + 2\hat{F}_t' \Lambda' \Lambda \stackrel{!}{=} 0$$

Resultingly $\hat{F}_t = (\Lambda' \Lambda)^{-1} \Lambda' X_t$. Inserting that back into the optimization problem gives $\min_{\Lambda} \frac{1}{NT} X_t' X_t - 2X_t' \Lambda (\Lambda' \Lambda)^{-1} \Lambda' X_t + X_t' \Lambda (\Lambda' \Lambda)^{-1} \Lambda' \Lambda (\Lambda' \Lambda)^{-1} \Lambda' X_t = \min_{\Lambda} \frac{1}{NT} X_t' [I - \Lambda (\Lambda' \Lambda)^{-1} \Lambda'] X_t$.

This is equivalent to the problem $\max_{\Lambda} \text{tr} \left\{ (\Lambda' \Lambda)^{-\frac{1}{2}} \lambda' (T^{-1} \sum_{t=1}^T X_t X_t') \Lambda (\Lambda' \Lambda)^{-\frac{1}{2}} \right\}$ which is the same as $\max_{\Lambda} \Lambda' \hat{\Sigma}_{XX} \Lambda$ s.t. $N^{-1} \Lambda' \Lambda = I_r$ where $\hat{\Sigma}_{XX}$ is the usual sample equivalent of the variance of X . This can be solved by setting $\hat{\Lambda}$ to eigenvectors of the r largest eigenvalues of $\hat{\Sigma}_{XX}$.

C Replication of Bai and Ng (2002) for $r = 7$, $r = 9$

Table 9: Replication of Table I of Bai and Ng (2002) for $r = 7$

$$\text{DGP: } X_{it} = \sum_{j=1}^r \lambda_{ij} F_{tj} + \sqrt{\theta} e_{it}, r = 7$$

N	T	PC_{p1}	PC_{p2}	PC_{p3}	IC_{p1}	IC_{p2}	IC_{p3}
100	40	6.4	5.9	6.97	4.93	3.46	6.73
100	60	6.72	6.27	7.02	6.34	4.74	6.99
200	60	6.93	6.87	7.0	6.78	6.46	6.96
500	60	6.99	6.96	7.0	6.94	6.92	6.99
1000	60	7.0	6.99	7.0	6.99	6.97	7.0
2000	60	7.0	6.99	7.0	6.98	6.99	7.0
100	100	7.0	6.77	7.35	6.89	6.32	50.0
200	100	7.0	7.0	7.0	7.0	6.99	7.0
500	100	7.0	7.0	7.0	7.0	7.0	7.0
1000	100	7.0	7.0	7.0	7.0	7.0	7.0
2000	100	7.0	7.0	7.0	7.0	7.0	7.0
40	100	6.39	6.12	7.02	4.86	3.49	6.78
60	100	6.79	6.37	7.01	6.01	5.0	7.0
60	200	6.93	6.85	7.0	6.82	6.58	6.98
60	500	6.96	6.97	7.0	6.97	6.9	6.99
60	1000	7.0	6.99	7.0	6.99	6.96	7.0
60	2000	7.0	7.0	7.0	6.97	6.98	6.99
4000	60	7.0	7.0	6.99	6.98	6.99	6.99
4000	100	7.0	7.0	7.0	7.0	7.0	7.0
8000	60	6.99	7.0	7.0	7.0	6.98	7.0
8000	100	7.0	7.0	7.0	7.0	7.0	7.0
60	4000	7.0	7.0	7.0	7.0	7.0	6.99
100	4000	7.0	7.0	7.0	7.0	7.0	7.0
60	8000	6.99	7.0	7.0	6.99	7.0	6.99
100	8000	7.0	7.0	7.0	7.0	7.0	7.0
10	50	5.0	5.0	5.0	4.76	3.95	5.0
10	100	5.0	5.0	5.0	4.58	4.18	4.96
20	100	7.04	6.5	7.98	3.14	2.07	6.94
100	10	5.0	5.0	5.0	5.0	4.96	5.0
100	20	7.1	6.64	8.05	3.3	2.08	8.2

Notes: Estimated number of factors averaged over 1000 simulations. The true number of factors is r and the maximum number of factors is $k_{max} = \text{ceil}(\min\{T, N\}/2)$.

Table 10: Replication of Table I of Bai and Ng (2002) for $r = 9$

$$\text{DGP: } X_{it} = \sum_{j=1}^r \lambda_{ij} F_{tj} + \sqrt{\theta} e_{it}, r = 9$$

see Bai and Ng (2002) for specification of the DGP

N	T	PC_{p1}	PC_{p2}	PC_{p3}	IC_{p1}	IC_{p2}	IC_{p3}
100	40	6.51	5.93	8.06	3.55	1.49	7.89
100	60	7.02	6.25	8.77	5.4	2.74	8.9
200	60	7.74	7.31	8.65	7.33	6.26	8.74
500	60	8.09	8.02	8.52	8.18	7.97	8.58
1000	60	8.42	8.38	8.59	8.45	8.25	8.62
2000	60	8.47	8.32	8.59	8.67	8.56	8.64
100	100	8.03	6.93	9.01	7.68	4.85	50.0
200	100	8.85	8.48	8.99	8.92	8.58	9.0
500	100	9.0	8.99	9.0	9.0	8.99	9.0
1000	100	9.0	8.99	9.0	9.0	9.0	9.0
2000	100	9.0	9.0	9.0	9.0	9.0	9.0
40	100	6.63	6.06	8.08	3.38	1.66	7.79
60	100	7.03	6.33	8.85	5.51	2.68	8.94
60	200	7.79	7.43	8.6	7.1	6.05	8.73
60	500	8.16	7.98	8.61	8.28	7.96	8.66
60	1000	8.41	8.28	8.51	8.51	8.34	8.69
60	2000	8.4	8.4	8.58	8.57	8.56	8.74
4000	60	8.52	8.44	8.57	8.61	8.51	8.73
4000	100	9.0	9.0	9.0	9.0	9.0	9.0
8000	60	8.44	8.52	8.55	8.66	8.64	8.68
8000	100	9.0	9.0	9.0	9.0	9.0	9.0
60	4000	8.53	8.54	8.55	8.62	8.6	8.57
100	4000	9.0	9.0	9.0	9.0	9.0	9.0
60	8000	8.61	8.47	8.51	8.66	8.58	8.76
100	8000	9.0	9.0	9.0	9.0	9.0	9.0
10	50	5.0	5.0	5.0	4.66	3.29	5.0
10	100	5.0	5.0	5.0	4.32	3.59	4.77
20	100	7.25	6.84	8.1	1.76	1.28	7.21
100	10	5.0	5.0	5.0	4.96	4.74	5.0
100	20	7.37	7.02	8.28	1.92	1.25	8.89

Notes: Estimated number of factors averaged over 1000 simulations. The true number of factors is r and the maximum number of factors is $k_{max} = \text{ceil}(\min\{T, N\})/2$.

$$\text{DGP: } X_{it} = \sum_{j=1}^r \lambda_{ij} F_{tj} + \sigma e_{it}, r = 1, \sigma_i \sim U(0.5, 1.5)$$

N	LR	LM	W	LR	LM	W
	T=50			T=100		
20	0.0445	0.0445	0.0483	0.0405	0.0435	0.0443
50	0.0462	0.0456	0.0488	0.0524	0.0512	0.0508
100	0.0513	0.0516	0.0549	0.0458	0.0458	0.0475
150	0.0481	0.0487	0.0519	0.0483	0.0493	0.0487
200	0.0497	0.0498	0.0509	0.0481	0.0483	0.0472
	T=150			T=200		
20	0.0510	0.0540	0.0540	0.0515	0.0540	0.0525
50	0.0458	0.0474	0.0464	0.0504	0.0514	0.0460
100	0.0473	0.0463	0.0478	0.0472	0.0464	0.0478
150	0.0491	0.0493	0.0491	0.0499	0.0499	0.0511
200	0.0505	0.0514	0.0511	0.0489	0.0499	0.0497

Notes: Average rejection frequencies of $R = 100$ bootstrapped Wald, Lagrange multiplier and Likelihood ratio tests with $B = 100$ bootstrap repetitions. Results are averaged over the N tests for each variable. The DGP features no structural break ($b = 0$). The tests are calculated at the 5% significance level.

Table 11: Size of Chow-tests, Table II of Bai and Ng (2002) Residual resampling

$$\text{DGP: } X_{it} = \sum_{j=1}^r \lambda_{ij} F_{tj} + \sigma e_{it}, r = 1, \sigma_i \sim U(0.5, 1.5)$$

N	LR	LM	W	LR	LM	W
	T=50			T=100		
20	0.0251	0.0250	0.0552	0.0252	0.0351	0.0353
50	0.0522	0.04602	0.0501	0.0461	0.0483	0.0462
100	0.0421	0.0370	0.0502	0.0440	0.0481	0.0591
150	0.0420	0.0401	0.0553	0.0547	0.0547	0.0601
200	0.0435	0.0421	0.0495	0.0505	0.0502	0.0565
	T=150			T=200		
20	0.0551	0.0496	0.0581	0.0352	0.0403	0.0593
50	0.0498	0.0574	0.0553	0.0346	0.0436	0.0374
100	0.0569	0.0466	0.0559	0.0457	0.0455	0.0487
150	0.0472	0.04464	0.0518	0.0447	0.0428	0.0453
200	0.0485	0.0432	0.0515	0.0465	0.0492	0.0555

Notes: Average rejection frequencies of $R = 100$ bootstrapped Wald, Lagrange multiplier and Likelihood ratio tests with $B = 100$ bootstrap repetitions. Results are averaged over the N tests for each variable. The DGP features no structural break ($b = 0$). The tests are calculated at the 5% significance level.

Table 12: Size of Chow-tests, Table II of Bai and Ng (2002) Wild bootstrap

$$\text{DGP: } X_{it} = \sum_{j=1}^r \lambda_{ij} F_{tj} + \sigma e_{it}, r = 1, \sigma_i \sim U(0.5, 1.5)$$

b	LR	LM	W	LR	LM	W
	T=50			T=100		
0.1	0.0532	0.0542	0.0564	0.0642	0.0682	0.0693
0.2	0.0788	0.0784	0.0746	0.1134	0.1096	0.1112
0.3	0.1076	0.1064	0.1006	0.1598	0.1616	0.1541
0.5	0.1836	0.1806	0.1698	0.2934	0.2894	0.2828
	T=150			T=200		
0.1	0.0721	0.0743	0.0712	0.0814	0.0804	0.0838
0.2	0.1466	0.1474	0.1442	0.1744	0.1716	0.1742
0.3	0.2298	0.2284	0.2262	0.2644	0.2644	0.2564
0.5	0.3654	0.3618	0.3662	0.4232	0.4264	0.4194

Notes: Average rejection frequencies of $R = 100$ bootstrapped Wald, Lagrange multiplier and Likelihood ratio tests with $B = 100$ bootstrap repetitions. Results are averaged over the N tests for each variable. The DGP features structural breaks of size b . The tests are calculated at the 5% significance level.

Table 13: Power of Chow-tests, Table III of Bai and Ng (2002) Residual resampling

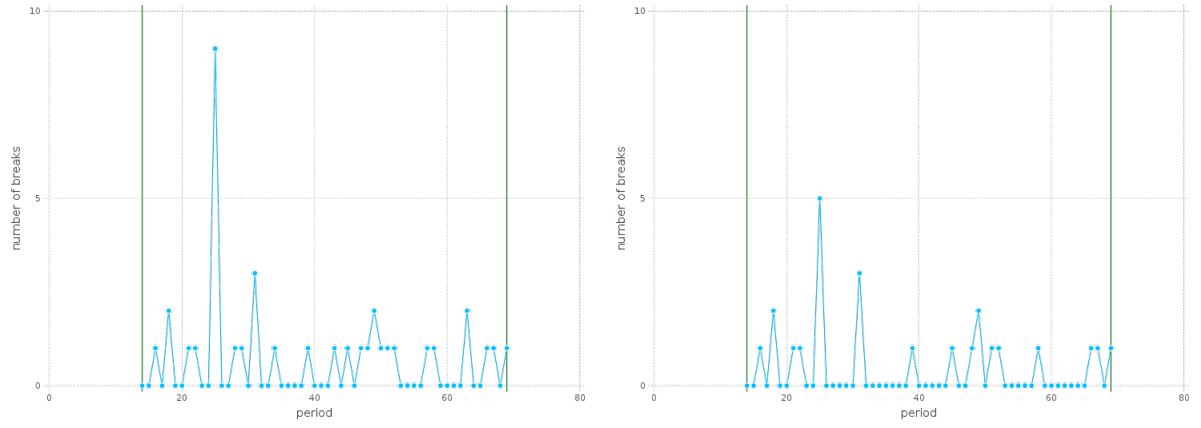
$$\text{DGP: } X_{it} = \sum_{j=1}^r \lambda_{ij} F_{tj} + \sigma e_{it}, r = 1, \sigma_i \sim U(0.5, 1.5)$$

b	LR	LM	W	LR	LM	W
	T=50			T=100		
0.1	0.0625	0.0684	0.0622	0.0921	0.0942	0.0961
0.2	0.0644	0.0562	0.0741	0.1042	0.1310	0.1142
0.3	0.0946	0.1041	0.1141	0.1583	0.1543	0.1665
0.5	0.1644	0.1622	0.1785	0.3122	0.3312	0.3264
	T=150			T=200		
0.1	0.0581	0.0664	0.0686	0.1167	0.1041	0.1048
0.2	0.1528	0.1581	0.1684	0.1824	0.1823	0.1921
0.3	0.1981	0.2089	0.2147	0.2448	0.2389	0.2389
0.4	0.4583	0.3965	0.4117	0.4246	0.4243	0.4347

Notes: Average rejection frequencies of $R = 100$ bootstrapped Wald, Lagrange multiplier and Likelihood ratio tests with $B = 100$ bootstrap repetitions. Results are averaged over the N tests for each variable. The DGP features structural breaks of size b . The tests are calculated at the 5% significance level.

Table 14: Power of Chow-tests, Table III of Bai and Ng (2002) Wild bootstrap

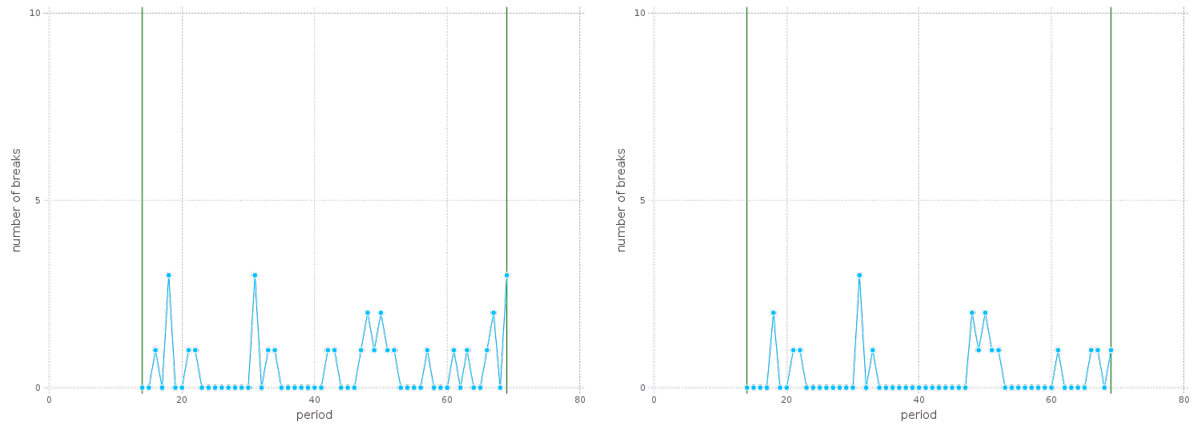
D Figures - Removal of breaks



(a) 5% significance level

(b) 1% significance level

Figure 4: Second cluster removed



(a) 5% significance level

(b) 1% significance level

Figure 5: First and second cluster removed

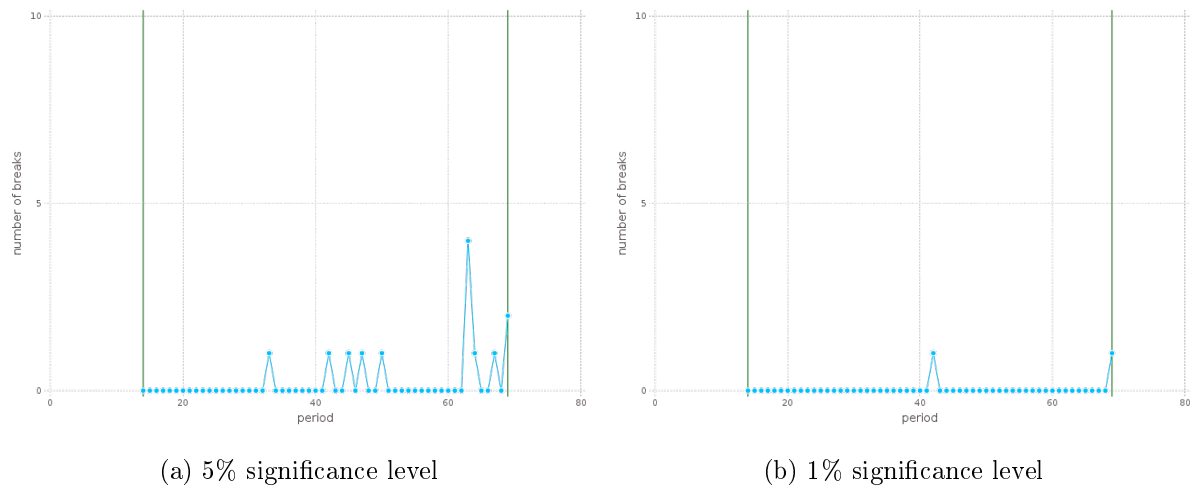


Figure 6: All variables with breaks removed

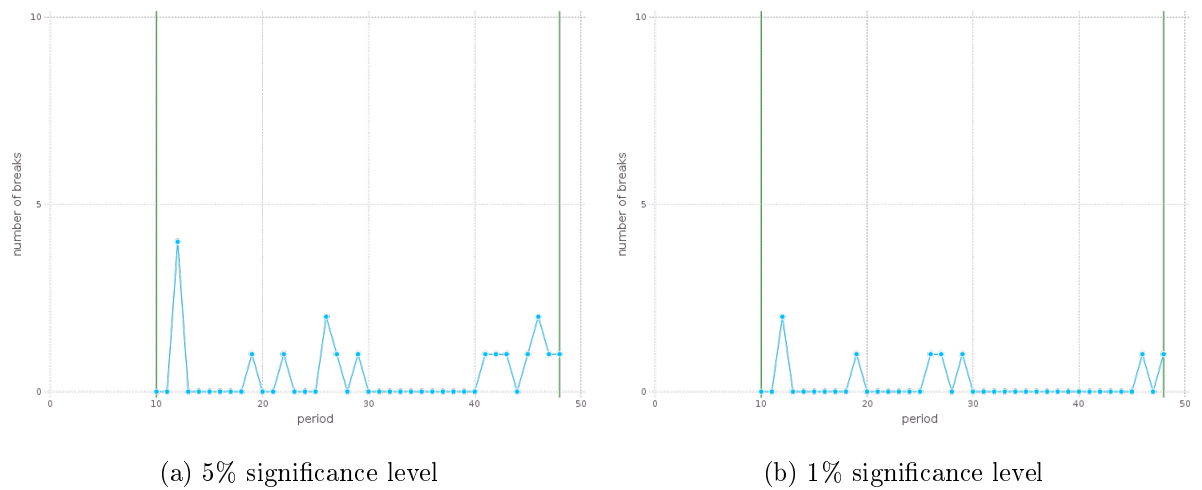


Figure 7: Second cluster removed, data set trimmed before 1991 quarter 1

E Interaction: Targeted predictors, removal of breaks

k	N _{breaks removed}	RMSE _{tar}	RMSE _{br}	Δ_{RMSE}	DM	$p_{>}$	p_{\neq}
26	18	0.4809	0.4564	0.0245	0.4965	0.3136	0.6272
27	19	0.4823	0.4602	0.0221	-0.4547	0.6718	0.6563
28	18	0.4957	0.4528	0.0429	-0.2979	0.6149	0.7701
29	18	0.4927	0.4806	0.0121	0.0883	0.4654	0.9309
30	17	0.4925	0.4959	-0.0034	0.5863	0.2835	0.5670
31	18	0.4943	0.4999	-0.0055	-0.1871	0.5729	0.8543
32	21	0.4014	0.4139	-0.0125	-0.5408	0.7014	0.5971
33	21	0.4747	0.4295	0.0452	-1.9228	0.9625	0.0751
34	22	0.4849	0.4501	0.0349	-1.9577	0.9647	0.0705
35	24	0.4217	0.5230	-0.1013	-0.7701	0.7730	0.4541
36	24	0.4826	0.4620	0.0206	-1.1949	0.8740	0.2520
37	25	0.4442	0.4530	-0.0088	-1.2802	0.8894	0.2213
38	26	0.4312	0.4785	-0.0473	-1.0385	0.8417	0.3167
39	27	0.4000	0.4811	-0.0811	-1.3551	0.9016	0.1968
40	28	0.4141	0.4856	-0.0716	-1.2543	0.8849	0.2303
41	27	0.4269	0.5076	-0.0807	-1.9239	0.9625	0.0749
42	28	0.4490	0.5048	-0.0558	-1.9349	0.9633	0.0735
43	28	0.4485	0.5084	-0.0600	-1.6533	0.9397	0.1205
44	29	0.4360	0.5192	-0.0832	-2.0236	0.9687	0.0625
45	30	0.4309	0.5237	-0.0928	-1.0432	0.8427	0.3145
46	31	0.4786	0.4948	-0.0163	-1.5163	0.9242	0.1517
47	32	0.4851	0.4943	-0.0092	-0.9166	0.8126	0.3748
48	33	0.4716	0.5000	-0.0284	-3.0493	0.9957	0.0087
49	33	0.5100	0.5000	0.0100	4.7581	0.0002	0.0003
50	35	0.5204	0.4579	0.0625	-2.3135	0.9818	0.0364

Critical values for the Diebold-Mariano test of improved and differing accuracy:

$DM_{1\%} = 2.62$, $DM_{5\%} = 1.76$, $DM_{10\%} = 1.35$

$DM_{1\%} = 2.98$, $DM_{5\%} = 2.14$, $DM_{10\%} = 1.76$ (the relevant Statistic is $\text{abs}(\text{DM})$)

Notes: Entries report the performance of the optimal model achieved through targeting the predictors only (RMSE_{tar}) and removing the structural breaks from the targeted data sets afterwards (RMSE_{br}). The data sets have been targeted using soft thresholding with k steps. N_{breaks removed} is the number of variables left in the model after structural breaks are removed. DM denotes the small sample corrected Diebold Mariano statistic from Harvey et al. (1997). $p_{>}$ and p_{\neq} denote the p-values of a test for improved and differing forecasting accuracy respectively.

Table 15: RMSE after removing structural breaks from targeted data

F Data series

Table 16: Federal Reserve Saint Louis data series

	Series ID	Series Description
1	A229RX0	Real Disposable Personal Income: Per capita
2	AAA	Moody's Seasoned Aaa Corporate Bond Yield©
3	ACNGNO	Value of Manufacturers' New Orders for Consumer Goods: Consumer Nondurable Goods Industries
4	AHECONS	Average Hourly Earnings Of Production And Nonsupervisory Employees: Construction
5	AHEMAN	Average Hourly Earnings Of Production And Nonsupervisory Employees: Manufacturing
6	AHETPI	Average Hourly Earnings of Production and Nonsupervisory Employees: Total Private
7	AWHMAN	Average Weekly Hours of Production and Nonsupervisory Employees: Manufacturing
8	AWOTMAN	Average Weekly Overtime Hours of Production and Nonsupervisory Employees: Manufacturing
9	BAA	Moody's Seasoned Baa Corporate Bond Yield©
10	BOGMBASE	Monetary Base; Total
11	BOGNONBR	Non-Borrowed Reserves of Depository Institutions (DISCONTINUED SERIES)
12	BUSLOANS	Commercial and Industrial Loans, All Commercial Banks
13	BUSLOANSNSA	Commercial and Industrial Loans, All Commercial Banks
14	CCRETT01USM661N	Real Effective Exchange Rates Based on Manufacturing Consumer Price Index for the United States©
15	CES06000000007	Average Weekly Hours of Production and Nonsupervisory Employees: Goods-Producing

16	CES0600000008	Average Hourly Earnings of Production and Nonsupervisory Employees: Goods-Producing
17	CES2000000008	Average Hourly Earnings of Production and Nonsupervisory Employees: Construction
18	CES3000000008	Average Hourly Earnings of Production and Nonsupervisory Employees: Manufacturing
19	CEU0500000008	Average Hourly Earnings of Production and Nonsupervisory Employees: Total Private
20	CEU0600000007	Average Weekly Hours of Production and Nonsupervisory Employees: Goods-Producing
21	CEU0600000008	Average Hourly Earnings of Production and Nonsupervisory Employees: Goods-Producing
22	CEU1000000001	All Employees: Mining and Logging
23	CEU2000000001	All Employees: Construction
24	CEU3000000001	All Employees: Manufacturing
25	CEU3000000007	Average Weekly Hours of Production and Nonsupervisory Employees: Manufacturing
26	CEU3000000009	Average Weekly Overtime Hours of Production and Nonsupervisory Employees: Manufacturing
27	CEU3100000001	All Employees: Durable Goods
28	CEU3200000001	All Employees: Nondurable Goods
29	CEU9000000001	All Employees: Government
30	CIVPART	Civilian Labor Force Participation Rate
31	CLF16OV	Civilian Labor Force
32	CPALTT01USM661S	Consumer Price Index: Total All Items for the United States©
33	CPIAPPNS	Consumer Price Index for All Urban Consumers: Apparel
34	CPIAPPSL	Consumer Price Index for All Urban Consumers: Apparel
35	CPIAUCNS	Consumer Price Index for All Urban Consumers: All Items
36	CPIAUCSL	Consumer Price Index for All Urban Consumers: All Items
37	CPILFENS	Consumer Price Index for All Urban Consumers: All Items Less Food & Energy
38	CPILFESL	Consumer Price Index for All Urban Consumers: All Items Less Food & Energy
39	CPIMEDNS	Consumer Price Index for All Urban Consumers: Medical Care
40	CPIMEDSL	Consumer Price Index for All Urban Consumers: Medical Care

41	CPITRNNS	Consumer Price Index for All Urban Consumers: Transportation
42	CPITRNSL	Consumer Price Index for All Urban Consumers: Transportation
43	CURRNS	Currency Component of M1
44	CURRSL	Currency Component of M1
45	DGORDER	Manufacturers' New Orders: Durable Goods
46	DMANEMP	All Employees: Durable goods
47	EXCAUS	Canada / U.S. Foreign Exchange Rate
48	EXJPUS	Japan / U.S. Foreign Exchange Rate
49	EXSZUS	Switzerland / U.S. Foreign Exchange Rate
50	EXUSUK	U.S. / U.K. Foreign Exchange Rate
51	FEDFUNDS	Effective Federal Funds Rate
52	GS1	1-Year Treasury Constant Maturity Rate
53	GS5	5-Year Treasury Constant Maturity Rate
54	HOUST	Housing Starts: Total: New Privately Owned Housing Units Started
55	HOUSTMW	Housing Starts in Midwest Census Region
56	HOUSTMWNSA	Housing Starts in Midwest Census Region
57	HOUSTNE	Housing Starts in Northeast Census Region
58	HOUSTNSA	Housing Starts: Total: New Privately Owned Housing Units Started
59	HOUSTS	Housing Starts in South Census Region
60	HOUSTSNSA	Housing Starts in South Census Region
61	HOUSTW	Housing Starts in West Census Region
62	HOUSTWNSA	Housing Starts in West Census Region
63	INDPRO	Industrial Production Index
64	IPBUSEQ	Industrial Production: Business Equipment
65	IPCONGD	Industrial Production: Consumer Goods

66	IPDCONGD	Industrial Production: Durable Consumer Goods
67	IPDMAN	Industrial Production: Durable Manufacturing (NAICS)
68	IPFINAL	Industrial Production: Final Products (Market Group)
69	IPFUELN	Industrial Production: Fuels
70	IPFUELS	Industrial Production: Fuels
71	IPMAN	Industrial Production: Manufacturing (NAICS)
72	IPMAT	Industrial Production: Materials
73	IPMINE	Industrial Production: Mining
74	IPNCONGD	Industrial Production: Nondurable Consumer Goods
75	ISRATIO	Total Business: Inventories to Sales Ratio
76	LNS12032197	Employment Level - Part-Time for Economic Reasons, Non-agricultural Industries
77	LNU01000000	Civilian Labor Force
78	LNU01300000	Civilian Labor Force Participation Rate
79	M1NS	M1 Money Stock
80	M1SL	M1 Money Stock
81	M2NS	M2 Money Stock
82	M2SL	M2 Money Stock
83	MABMM301USM189S	M3 for the United States©
84	MANEMP	All Employees: Manufacturing
85	MCUMFN	Capacity Utilization: Manufacturing (NAICS)
86	MICH	University of Michigan Inflation Expectation©
87	NAPM	ISM Manufacturing: PMI Composite Index©
88	NAPMEI	ISM Manufacturing: Employment Index©
89	NAPMII	ISM Manufacturing: Inventories Index©
90	NAPMNOI	ISM Manufacturing: New Orders Index©

91	NAPMPI	ISM Manufacturing: Production Index©
92	NAPMSDI	ISM Manufacturing: Supplier Deliveries Index©
93	NDMANEMP	All Employees: Nondurable goods
94	NEWORDER	Manufacturers' New Orders: Nondefense Capital Goods Excluding Aircraft
95	PAYEMS	All Employees: Total nonfarm
96	PAYNSA	All Employees: Total nonfarm
97	PERMITNSA	New Privately-Owned Housing Units Authorized by Building Permits: Total
98	PPIACO	Producer Price Index: All Commodities
99	PPICRM	Producer Price Index: Crude Materials for Further Processing
100	PPIFCF	Producer Price Index: Finished Consumer Foods
101	PPIFGS	Producer Price Index: Finished Goods
102	PPIITM	Producer Price Index: Intermediate Materials: Supplies & Components
103	RRSFS	Real Retail and Food Services Sales
104	SRVPRD	All Employees: Service-Providing Industries
105	TB3MS	3-Month Treasury Bill: Secondary Market Rate
106	TB6MS	6-Month Treasury Bill: Secondary Market Rate
107	TCU	Capacity Utilization: Total Industry
108	TOTBUSIMNSA	Total Business Inventories
109	TOTBUSIRNSA	Total Business: Inventories to Sales Ratio
110	UCNGNO	Value of Manufacturers' New Orders for Consumer Goods: Consumer Nondurable Goods Industries
111	UEMPMEAN	Average (Mean) Duration of Unemployment
112	UMCSENT	University of Michigan: Consumer Sentiment©
113	UNEMPLOY	Unemployed
114	UNRATE	Civilian Unemployment Rate
115	UNRATENSA	Civilian Unemployment Rate
116	USCONS	All Employees: Construction
117	USGOOD	All Employees: Goods-Producing Industries
118	USGOVT	All Employees: Government
119	USMINE	All Employees: Mining and logging
120	W875RX1	Real personal income excluding current transfer receipts

Table 17: Bundesbank data series

	Series ID	Series Description
121	BBK01. JQA000	National accounts/Overall economic view/At current prices/ GDP
122	BBDE1. M.DE.N. AEA1. A2P300000. F.C.I10.L	Orders received / At constant prices / Germany / Industry / Unadjusted figure
123	BBDE1. M.DE.N. BAA1. P2XF00000. G.C.I10.L	Output in the production sector / At constant prices / Ger- many / Main construction industry / Unadjusted figure
124	BBDE1. M.DE.W. AEA1. A2P310000. F.V.I10.A	Orders received / At current prices / Germany / Industry / Calendar adjusted only
125	BBDE1. M.DE.W. AEA1. P2XF00000. B2.V.I10.A	Orders received / At current prices, flows / Germany / Main construction industry / Calendar adjusted only
126	BBDE1. M.DE.Y. AEA1. A2P300000. F.V.I10.A	Orders received / At current prices / Germany / Industry / Calendar and seasonally adjusted
127	BBDE1. M.DE.Y. AEA1. A2R200050. F.V.I10.A	Orders received / At current prices, flows / Germany / 20+21 Manufacture of chemicals, chemical products; basic pharma- ceutical products and pharmaceutical preparations / Calen- dar and seasonally adjusted
128	BBDE1. M.DE.Y. AEA1. P2XF00000. B2.C.I10.A	Orders received / At constant prices / Germany / Main con- struction industry / Calendar and seasonally adjusted
129	BBDE1. M.DE.Y. AEA1. P2XF02000. B2.V.I10.A	Orders received / At current prices, flows / Germany / Main construction industry / Calendar and seasonally adjusted
130	BBDE1. M.DE.Y. AEB1. A2P310000. F.V.I10.A	Orders received from the domestic market / At current prices / Germany / Industry / Calendar and seasonally adjusted

131	BBDE1. M.DE.Y. AEB1. N2C290000. F.C.I10.A	Orders received from the domestic market / At constant prices / Germany / 20+21 Manufacture of chemicals, chemical products; basic pharmaceutical products and pharmaceutical preparations / Calendar and seasonally adjusted
132	BBDE1. M.DE.Y. AEB5. A2P300000. F.V.I10.A	Orders received from abroad / At current prices / Germany / Industry / Calendar and seasonally adjusted
133	BBDE1. M.DE.Y. BA01. A2P200000. F.C.I10.A	Produktionsergebnis je Beschäftigten / Deutschland / Bergbau und Gewinnung von Steinen und Erden sowie Verarbeitendes Gewerbe (B + C) / Angaben für fachliche Betriebsteile / kalender- und saisonbereinigt
134	BBDE1. M.DE.Y. BAA1. A2P000000. G.C.I10.A	Output in the production sector / Germany / Production sector including construction (B -F) / Calendar and seasonally adjusted
135	BBDE1. M.DE.Y. BEA1. P2XF00000. BA.P.ABA.A	Employed persons (including proprietors and family workers) / Persons / Germany / Main construction industry / Calendar and seasonally adjusted
136	BBDP1. M.DE.N. APA1.G. GP09SA000000. I10.A	Construction price index / Germany / Unadjusted figure / Total
137	BBDP1. M.DE.N. VPI.C. A00000. I10.L	Consumer price index / Germany / Unadjusted figure / Total
138	BBDP1. M.DE.Y. APA1.G. GP09SA000000. I10.A	Indices of foreign trade prices - exports, total / Germany / Calendar and seasonally adjusted / Total
139	BBDP1. M.DE.Y. EPL.L. L1A00000000. I10.A	Index of producer prices of industrial products sold on the domestic market / Germany / Calendar and seasonally adjusted / Total
140	BBEE1. M.DE.AAA. XY0B02. R.AACPB. M00	Indicator of the German economy's price competitiveness against 24 selected industrial countries, based on the deflators of total sales
141	BBEX3. M.XAU. USD.EA. AC.C06	Price of gold in London / morning fixing * / 1 ounce of fine gold = USD ...
142	BBK01. BU1132	Central government debt - total debt
143	BBK01. DU7802	Basic pay rates, overall economy, excluding ancillary benefits, excluding one-off payments, on a monthly basis, Germany
144	BBK01. ES4710	Balance on current account / Seasonally adjusted
145	BBK01. EU2001	Foreign trade / Exports (fob)

146	BBK01. EU4170	Income / Total / Receipts
147	BBK01. EU4670	Financial transactions / Other investment / Balance
148	BBK01. EU4680	Foreign investment in Germany / Other investment / Total / Balance
149	BBK01. OU0092	Lending to domestic banks (MFIs) / Credit balances and loans / All categories of banks
150	BBK01. OU0144	Medium and long-term lending to general government / Loans / Medium-term / All categories of banks
151	BBK01. OU0149	Medium and long-term lending to general government / Loans / Long-term / All categories of banks
152	BBK01. OU0337	Medium and long-term lending / to general government / Securities / All categories of banks
153	BBK01. OU0778	Lending to banks (MFIs) / Total / All categories of banks
154	BBK01. OU1429	Lending to banks (MFIs) / Balances and loans / All categories of banks
155	BBK01. OU1825	Capital / Total / All categories of banks
156	BBK01. TUD317	MONEY STOCK M1 (FROM 2002, EXCLUDING CURRENCY IN CIRCULATION) / GERMAN CONTRIBUTION
157	BBK01. TVS303J	Geldmenge M1 / Veränderung saisonbereinigt / Jahresrate / EWU
158	BBK01. TXI355	TOTAL ASSETS OR LIABILITIES / GERMAN CONTRIBUTION
159	BBK01. US02CC	Employees subject to social security contributions / Total / Western Germany / Seasonally adjusted / 1 / 2
160	BBK01. US0CC1	Employees subject to social security contributions / Total / Eastern Germany / Seasonally adjusted / 1 / 2
161	BBK01. USBA14	Employment in Germany / Employed persons in Germany / Germany / Seasonally adjusted / 1 / 2
162	BBK01. UUCY01	Unemployment rate (unemployment as a percentage of the civilian labour force) / Germany / 1 / 2 / 3
163	BBK01. WU0028	Gross sales of domestic debt securities at nominal value / Total
164	BBK01. WU3141	DAX price index / End 1987 = 1000 / End of month
165	BBK01. XS7085	Exports / European countries / Seasonally adjusted
166	BBK01. XU4204	German exports / special trade / values / 1 / 2 / 3 / 4
167	BBK01. XU4205	German imports / special trade / values / 1 / 2 / 3 / 4
168	BBK01. XU4206	German foreign trade balance / special trade / values / 1 / 2 / 3 / 4
169	BBK01. XX3739	Exports of the Federal Republic of Germany / Special Trade / Total / 1 / 2 /

G Replication and data files

All replication files can be found at <https://github.com/joidegn/Masterarbeit> or can be gotten from the author by request.

A result from writing this thesis is the FactorModels Julia package which has been released in the public domain as free software licensed under the MIT License a copy of which can be found at <http://opensource.org/licenses/MIT>. Julia, is a powerful, open-source technical computing language which can be found at <http://julialang.org/>. The FactorModels package does the heavy lifting for the replication files and has to be installed in order to run them. Instructions on how to replicate the results are provided both with the package and the replication files.

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