Universität Konstanz

Predicting Economic Time-Series using Dynamic Factor Models under Structural Breaks

Author:
Johannes Degn

Supervisor:
Ralf Brüggemann

A thesis submitted in fulfilment of the requirements for the Masters degree in the

Department of Economics at the University of Konstanz

July 22, 2014

Contents

List of Figures

List of Tables

1 Introduction

Factor models are used in a wide range of situations spanning from economic indices over prediction of high-dimensional datasets. The basic idea is that there are unobservable factors driving the change of the variables of interest. Put differently, it is assumed that the change of variables is composed of a common component affecting all variables in the set and an idiosyncratic component which is specific to the variable of interest.

If the common factors can be derrived consistently they can prove to be a valuable and possibly less noisy set of predictors for the variables of interest than the whole set of available variables. Nowadays, typically the factors are estimated using principal components analysis. Resultingly factor models harbor similarity to principal component regression including the feature of dimensionality reduction which allows both models to be estimated even if the number of columns exceeds the number of rows. This is precisely what makes factor models appealing in a macroeconomic forecasting context. There exist many international macroeconomic variables some of which are not covered over a very long time horizon. With factor models all this information can be used directly.

In a time-series framework an interesting question to ask is whether these common components are stable over time or if the relationship of the variables of interest with the underlying factors are prone to changes. While there are recent additions to the field, the behaviour of factor models under structural breaks is still being researched. There are at least two reasons why structural breaks are interesting in an econometric model. If we assume that strcutural breaks imply a change in the coefficients of the model at a certain period, this can be interesting in itself especially if the coefficients and the factors are to be interpreted directly. Additionally, from a forecasting perspective it can be worthwhile to consider only the data after the structural break(s) has happened[1]. The hope is that the parameters of the model which are estimated relying only on the data after the last break might be closer to the true values which have generated the data to be forecast. This assumes that there will not be another structural break at the period which is be forecast. It should be notet that Pesaran and Timmermann (2007)

[1] Alternatively, instead of splitting the data into multiple periods for each break, a single forecasting equation can be constructed which includes the structural break by introducing a dummy variable.

have shown that it can be slightly better to include same observations before the last break.

This thesis deals with testing for structural breaks in factor models and using factor models for forecasting in a situation where structural breaks might be present.[2]
The contribution is twofold: Firstly, the influence of structural breaks on factor models is examined. To this end feasible testing procedures developed by Breitung and Eickmeier (2011) are presented. Bootstrap versions of these chow type tests are developed and their performance is evaluated. Secondly, the information about the structural breaks is used for forecating. The performance of different model specifications is tested in the form of horse races. The competing models are:

1. A static factor model with no structural breaks (but with more factors)

2. A static factor model with structural break(s)

3. A dynamic factor model with no structural breaks

4. A dynamic factor model with structural break(s)

The paper is structured into X parts. Firstly factor models are introduced. Both static and dynamic models are presented along with some theory and estimation methods. Secondly structural breaks are introduced, their effects on the factors and loadings are looked at and testing methods are considered. Thirdly aforementioned models are estimated using both syntetic and real data. Finally the fourth part concludes.

Throughout this thesis I will assume that X is a matrix of potentially very many predictors. To stay consistent with existing literature I will index X with i or t interchangeably where i stands for the column index and t for the row index. Also as is customary, throughout this paper capital letters denote matrices while lower case letters denote vectors except for innovation terms which are always in lower case letters. All variables containing data are assumed to be demeaned and normalized to have unit variance[3].

[2]Structural breaks can refer to changes in the means and variances of variables or a change in the relationship of variables. I focus on the latter. More specificity will be added in section 3.

[3]I.e. every variable y_t is derrived as $y_t = \frac{y_t^* - \bar{y}_t}{\sigma_y}$ where y_t^* is the vector containing the original data and σ_y is the (sample) standard deviation of y

2 Factor Models

As mentioned, the idea behind factor models or diffusion index models is that there are underlying latent factors which explain the evolution of the observable variables X.

They are ususally written in the form of a factor equation (1) and a forecasting equation (2).

$$X_t = \lambda(L) f_t + e_t \tag{1}$$

$$y_{t+h}^h = \alpha(L) W_t + \beta(L) \hat{f}_t + \varepsilon_{t+h} \tag{2}$$

X_t is a $N \times 1$ vector of potentially very many predictors. $\lambda(L)$ is a $N \times q$ matrix of lag polynomials, f_t is a $q \times 1$ vector of latent factors. e_t is a $N \times 1$ vector of idiosyncratic errors which may or may not be serially auto-correlated. W_t consists of variables which the researcher knows to be sufficiently relevant to enter the forecasting equation directly. Usually W_t is taken to consist of s lags of y_t and a constant term. $\hat{f}_t$ is an estimate of the factors in the factor equation (1). Estimation of the factors is treated below.

Most authors assume the idiosyncratic component to be a white noise process (i.e. independent and of mean 0). In the vocabulary of factor models these are models with uncorrelated idiosyncratic components e_t (more specifically $E(e_t e_t') = \Sigma = \text{diag}(\sigma_{e_1}^2, ..., \sigma_{e_N}^2)$ and $E(e_t) = 0$). However, "weak" dependence is also commonly assumed. Concretely, Geweke (1977) and Sargent and Sims (1977) distinguish between exact or strict Factor Models and approximate Factor Models. In essence the difference is composed of different assumptions for the error terms. Approximated factor models allow for limited correlatedness of the idiosyncratic innovations e_{it} between periods while strict factor models do not. More precise deifinitions follow now and are taken from Breitung and Eickmeier (2006)

Strict factor models

Breitung and Eickmeier (2006) list the assumptions for strict factor models as follows: For the innovations it is assumed $E(e_t) = 0$ and $E(e_t e_t') = \sigma = \text{diag}(\sigma_1^2, ..., \sigma_N^2)$. Also $E(f_t) = 0$ and $E(f_t f_t') = \Omega$. If $E(f_t) = 0$ it follows that $E(X_t) = 0$ which is enforced by demeaning the variables. Additionally $E(f_t e_t') = 0$ i.e. the innovations and the error terms are uncorrelated. It follows directly that $E(X_t X_t') = E[(\Lambda f_t + e_t)(\Lambda f_t + e_t)'] =$

$E(\Lambda f_t f_t' \Lambda' + \Lambda f_t e_t' + e_t f_t' \Lambda' + e_t e_t') = \Lambda\Omega\Lambda' + \Sigma$

Approximate factor models

Approximate factor models losen the assumptions of strict factor models at the cost of assuming $N \to \infty$. $E(e_{it}e_{js})$ is allowed to be different from 0 but only weakly. Similarly the factors f_t and and the idiosyncratic error terms are allowed to be correlated but again only weakly, so that $E(f_t e_t')$ does not have to be 0. But $N^{-1}\Lambda'\Lambda$ must converge to a positive definite limiting matrix Σ_Λ which ensures that the factors influence each variable with a similar order of magnitude. Otherwise it could be that the loadings for some variables are 0 (Breitung and Eickmeier, 2006). Similiary it is assumed about the factors that $T^{-1}\sum_{t=1} T F_t F_t' \xrightarrow{p} \Sigma_F$. This allows for stationary processes e.g. an ARMA process.

Some more assumptions are necessary for the inferential theory[4] of Bai (2003) to hold. Namely these assumptions are Assumptions A through D in Bai (2003):

Assumption A: $E||F_t||^4 \leq M < \infty$ and $T^{-1}\sum_{t=1}^{T} F_t F_t' \xrightarrow{p} \Sigma_F$ for some $r \times r$ positive matrix Σ_F

Assumption B: $||\lambda_1|| \leq \bar{\lambda} < \infty$, and $||\Lambda'\Lambda/N - \Sigma_\Lambda|| \to 0$ for some $r \times r$ positive definite matrix Σ_Λ

[4]Most notably the results of Bai (2003) are consistency and normality of the factor estimations. The rate of convergence is shown to be $\min(\sqrt{N}, \sqrt{T})$.

Assumption C: $\exists M : 0 < M < \infty$ such that $\forall N, T$,

1.) $E(e_{it}) = 0, E|e_{it}|^8 \leq M$

2.) $E(e_s' e_t / N) = E(N^{-1} \sum_{i=1}^{N} e_{is} e_{it}) = \gamma_N(s,t), |\gamma_N(s,s)| \leq M \forall s$ and

$$T^{-1} \sum_{s=1}^{T} \sum_{t=1}^{T} |\gamma_N(s,t) \leq M$$

3.) $E(e_{it}' e_{jt}) = \tau_{ij,t}$ with $|\tau_{ij,t}| \leq |\tau_{ij}|$ for some τ_{ij} and for all t and

$$N^{-1} \sum_{i=1}^{N} \sum_{j=1}^{N} |\tau_{ij} \leq M$$

4.) $E(e_{it}' e_{js}) = \tau_{ij,ts}$ and $(NT)^{-1} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{t=1}^{T} \sum_{s=1}^{T} |\tau_{ij,ts}| \leq M$

$$\forall (t,s) E \left| N^{-1/2} \sum_{i=1}^{N} [e_{is} e_{it} - E(e_{is} e_{it})] \right|^4 \leq M$$

Assumption D: $E(\frac{1}{N} \sum_{i=1}^{N} || \frac{1}{\sqrt{N}} \sum_{t=1}^{T} F_t e_{it} ||^2) \leq M$

Assumption C allows for some degree of dependence and heteroskedasticity among the innovation terms which will be used below to simplify the calculation of the dynamic factor model.

As mentioned, an important advantage of factor models compared to simpler econometric models is that they do not necessarily suffer if the fraction $\frac{T}{N}$ becomes small whereas most models can not cope with more variables than observations. VAR models for example have the problem that the number of parameters to estimate becomes too high quickly if N is high relative to T. The reason why factor models do not have this problem is that firstly the factor equation is estimated. This gives estimates $\hat{f}_t$ of the factors. Then only the first r eigenvectors are used in the forecasting equation where $r < N$[5]. This feature allows researchers in principle to append X by further variables derived from the initial set. Bai and Ng (2008) for example use what they call squared principal components and quadratic principal components. The first of which appends squares of the X_i to the initial matrix X such that $X_t^{spc} = \{X_{it}, X_{it}^2\}$ and the second of which adds the cross products of X_i such that $X_t^{qpc} = \{X_{it}, X_{it}X_{jt}\}\ \forall i \neq j$. The idea is to change the link function between X_t and f_t from being linear to allow for a nonlinear relationship. Bai and Ng (2008) also consider adding a squared term of the factors f_t to the forecasting equation (2) to allow the volatility of the factors to influence the forecast. Naturally, the value of these approaches depends heavily on what is being forecasted.

Apart from the distinction into strict or approximate factor models, a further distinction can be made between static and dynamic factor models. This distinction concerns the relationship between lags of f_t and X. Static models set $\alpha(L)$ to be of lag order 0 while dynamic models alow for dependence with other lag orders.

2.1 Static Factor Models

For static factor models X_t depends only on f_t and not on lags of f_t i.e. factors enter only contemporaneously. Resultingly a static factor model is the case in which the lag order in equation (1) is 0.

$$X_t = \Lambda F_t + \bar{e}_t \tag{3}$$

Where X is a $T \times N$ Matrix of predictors. F is a $T \times r$ factor matrix. Λ is a $N \times r$ loadings matrix. $\bar{e}$ is a $T \times N$ matrix of idiosyncratic errors. C is called the common component. Note that depending on whether authors index by row, column or both, several alternative ways of writing down the factor model equation are used in the literature.

[5]Typically r is much smaller than N. E.g. the famous one factor model of intelligence by Spearman (1904) (in which factor models were developed the first time) has $r = 1$.

$$X_{it} = \Lambda_i' F_t + \bar{e}_{it} \tag{4}$$

$$X = F\Lambda' + e = C + \bar{e} \tag{5}$$

$$X_i = F_i \lambda + \bar{e}_i \tag{6}$$

2.2 Dynamic Factor Models

In applications there will usually be some kind of time dependence structure. Thus, dynamic factor models introduce a dynamic process into the factors.

$$f_t = \Psi(L) f_{t-1} + \eta_t \qquad \text{(VAR representation)} \tag{7}$$

In practice it is usually easier to ignore the dynamic process in the factors f_t and follow e.g. Stock and Watson (2005) among others in assuming a dynamic proccess in the innovation term of the factor equation (1) instead. This captures the dynamic process while avoiding the technical difficulty in estimating the dynamic factors themselves. The underlying thought is that there will be some residual autocorrelation if the dynamic process of the factors is not modelled explicitely which can be caught by an AR(p) process for the innovation term (Breitung and Tenhofen, 2011). Additionally the lag polynomial $\lambda(L)$ in the factor equation (1) captures the noncontemporaneous interactions of the factors f_t and X_t.

For the purpose of this paper I assume the following AR processes for the innovation term of the dynamic factor models (Breitung and Eickmeier, 2011).

$$e_{it} = \varrho_{i,1} e_{i,t-1} + ... + \varrho_{i,p_i} e_{i,t-p_i} + \xi_{it} \text{ where } \xi_{it} \text{ is white noise} \tag{8}$$

2.2.1 Static interpretation of dynamic factor models

Dynamic factor models can be written as static factor models in a linear state space environment. This gives the static factor equation (3) and $\Phi(L)F_t = G\eta_t$ where $\Phi(L)$ is defined such that equivalence to (7) holds and F_t is defined below as the stacked lagged dynamic factors f_t (Stock and Watson (2011)).

Solving the lag operator, we can write equation (1) in the form

$$X_{it} = \lambda'_{i1} f_t + ... + \lambda'_{im} f_{t-m} + e_{it} \tag{9}$$

Following Bai and Ng (2002) this can be rewritten as

$$X_{it} = \Lambda_i^{*'} F_t + e_{it}$$

$$\text{where } \Lambda_i^* = \begin{bmatrix} \lambda_{i1} \\ \lambda_{i2} \\ \vdots \\ \lambda_{im} \end{bmatrix} \text{ and } F_t = \begin{bmatrix} f_t \\ f_{t-1} \\ \vdots \\ f_{t-m} \end{bmatrix}$$

To keep consistency between variable naming schemes denote the dimension of F_t as $r \times 1$ and the dimension of Λ_i^* as $N \times r$. Thus we can always rewrite a dynamic factor model in static form and resultingly we can use the procedure introduced below to estimate the r "static factors"[6] F_t as specified below. It can be seen that the dimension of the resulting stacked vector of "static" factors is $q(m+1)$ thus we can set $r = q(m+1)$. This shows that there is a relationship between r and q if the dynamic factor model is written as a static factor model. To identify q it remains to estimate m.

There exist several ways to identify the dynamic factors from there[7]. Firstly Giannone et al. (2002) suggest estimating the VAR $F_t = CF_{t-1} + \kappa_t$ and then to apply principal component analysis on an estimate of the residual covariance matrix (see Breitung et al. (2004) or Giannone et al. (2002) for details). Breitung et al. (2004) exploit the relationship between q and r in order to find information criteria which can determine q and the lag order of the λ. This paper does not estimate the dynamic factors directly as in the application the factors are primarily used for the second stage regression.

Imagine the true data would be generated by a dynamic factor model. If instead of

[6] Breitung et al. (2004) call the f_t "structural factors" and the F_t "reduced form" factors.

[7] Note that identifying the dynamic factors might not be of interest for forecasting but rather that it is a necessary condition in order to interpret the factors in a dynamic factor model. For forecasting purposes, in order to choose the number of factors and the number of the factor lags used in the forecasting equation (2) one can also follow Bai and Ng (2008) in using the BIC criterion to choose both simultaneously.

using the dynamic factor model specification and finding the dynamic factors from the static factors, a researcher simply estimates a static factor model it can be seen above that the number of factors r will be larger than the number of dynamic factors q.

2.3 Estimation of the factors

2.3.1 Static factors

There exist now a number of ways to estimate the static factors F in (5). Ultimately what we are looking for is a solution to the minimization of the squared error.

$$\min_{F_1,\ldots,F_T,\Lambda} \frac{1}{NT}\sum_{t=1}^{T}(X_t - \Lambda F_t)'(X_t - \Lambda F_t) \tag{10}$$

Since both Λ and F_t are unspecified, there are infinitely many solutions to (10) and the problem is not identified. To achieve identification assumptions can most conveniently be put on either Λ or F_t[8]. Usually Λ is normalized such that $N^{-1}\Lambda'\Lambda = I_r$. Alternatively $T^{-1}F'F = I_r$ is also frequently used. The choice of normalization can have a noticeable impact on the computational demand of calculating a solution for the minimization problem in (10). The columns spaces spanned by the estimates of F are equivalent with both normalizations (Stock and Watson, 2011).

Stock and Watson (2011) list four ways which are being applied in the literature to get a solution $\hat{F}$ and $\hat{\Lambda}$ to the minimization problem (10). Firstly a Gaussian process can be assumed which allows for maximum likelihood estimation using the Kalman filter. To do this a dynamic factor can be rewritten as a static factor model in a linear state space writing as mentioned above.

Secondly principal components (and other nonparametric averaging methods) can be used to estimate both $\hat{F}$ and $\hat{\Lambda}$ at the same time. This requires relatively weak assumptions and is particularly easy to and computationally efficient which explains both the popularity of this approach and why this approach is applied in this thesis. Additionally the principal component estimation can be generalized by weighting the minimization problem with the variance matrix of the error term to take account of error

[8] Bai and Ng (2013) consider several alternative normalizations. They also show that direct interpretation of the factors and loading matrices are possible if one is willing to additionally assume diagonality of $\Lambda'\Lambda$ or that Λ is a block matrix of 2 submatrices with one submatrix being triangular.

variances which are not prooportional to the identity matrix (see Stock and Watson (2011) for details). Evidence of performance improvements by using the generalized principal components approach over the standard principal components estimator are mixed. Resultingly the simpler principal components estimator will be used. It will be described in more detail below. An unattractive feature of principal component estimation is thaint it can not identify the "true" factors f_t but rather a transformation Qf_t for some undefined rotation matrix Q which results in some difficulty in interpreting the factors directly. Bai and Ng (2013) show a minimum set of assumptions which are required in order to make valid interpretations of the estimated factor and loading matrices.

Principal Components estimation of the factors is asymptotically equivalent to maximum likelihood estimation if normality is assumed (Bai, 2003).
Thirdly mixture approaches between the two methods from above can be constructed. Fourthly and finally Bayesian methods can be used to get estimations of the factors and loadings which can have computational advantages over the maximum likelihood approach and can be useful if the assumption of Gaussian error terms is unappealing.

The principal component method estimates $\hat{F}$ and $\hat{\Lambda}$ as follows. Normalizing F such that $T^{-1}F'F = I_r$ gives $\hat{F} = \sqrt{T} * eigenvectors_r(XX')$ where $eigenvectors_i(A)$ returns the i eigenvectors corresponding to the i largest eigenvalues of a square matrix A. Since $\hat{X} = \hat{F}'\hat{\Lambda}$, it follows directly that we can estimate Λ by $\hat{\Lambda}' = (\hat{F}'\hat{F})^{-1}\hat{F}'X$. due to the normalization $\hat{F}'\hat{F} = T$ and we can write $\hat{\Lambda} = X'\hat{F}/T$.
If, alternatively, we apply the normalization $N^{-1}\Lambda'\Lambda = I_r$ the solution to (10) becomes $\hat{\Lambda} = \sqrt{N} * eigenvectors_r(X'X)$. This solution to the optimization problem is shown in appendix B). Then we can follow from $\hat{X} = \hat{F}\hat{\Lambda}'$ that $\hat{F} = X\hat{\Lambda}(\hat{\Lambda}'\hat{\Lambda})^{-1}$. Again due to the normalization $\Lambda'\Lambda = N$ and we can write $\hat{F} = X\hat{\Lambda}/N$.

Note that the normalization $T^{-1}F'F = I_r$ requires us to calculate the eigenvectors of XX' which has dimension $T \times T$ while the alternative $N^{-1}\Lambda'\Lambda = I_r$ requires us to calculate the eigenvectors of $X'X$ which is $N \times N$. Depending on the relative sizes of T and N it can make a considerable difference in terms of computational burden which method is applied.

The number of factors One important issue that remains to be addressed is the question of how many factors should be used in the factor equation. There exist many different ways that this choice can be made. Firstly there are simple rules of thumb. E.g. some researchers only consider factors with eigenvalues greater than 1. Then there are methods which rely on a similar idea but employ graphical methods. The idea is to inspect the resulting scree plots[9]. The intuition is to only take factors into account if the eigenvalue corresponding to the next factor is smaller by a sufficient amount. This idea is also behind the cut-off value of 1 for eigenvalues. Basing on this, formal tests have been derrived which rely on the eigenvalues calculated above (Stock and Watson, 2011).

The information criteria developed by Bai and Ng (2002) are perhaps more appealing as they formalize the costs and benefits of an additional factor explicitly (Stock and Watson, 2011). Cragg and Donald (1997) show in a Monte Carlo analysis that $T \to \infty$ and $N \to \infty$ makes that classical theory for predicting the number of factors performs badly. Resultingly information criteria as in Bai and Ng (2002) should be used which take into account that both T and N can grow. Bai and Ng (2002) show that their results hold under heteroskedasticity and weak serial correlation and are thus applicable to approximate factor models.

Bai and Ng (2002) consider 6 criteria, the first three of which they call PCp criteria (Panel C_p criteria). These criteria result from a generalization of Mallows' C_p (Mallows, 1973). The second set of 3 criteria refine the usual criteria used in time-series analysis to depend on both N and T. Bai and Ng (2002) highlight that the criteria IC_{p1}, IC_{p2}, PC_{p1} and PC_{p2} are specifically suited for principal component estimation. Bai and Ng (2002) present a montecarlo study to highlight the differences between the criteria. Notably the PC_p criteria require the specification of a maximum number of factors, the choice of which is quite arbitrary[10]. Resultingly this paper uses the IC_p criteria.

[9] Scree plots plot the ordered eigenvalues against the number of factors (i.e. eigenvalues and corresponding eigenvectors) which results in a falling curve.

[10] In their montecarlo study Bai and Ng (2002) set the maximum number of factors to be 8 noting that they used the rule in Schwert (2002) and that one could consider $kmax = \text{int}[(\min\{N,T\}/100)^{1/4}]$. Tests which are not presented here show that choosing a different $kmax$ can result in the PC_p criteria to become quite unstable. Especially high values of $kmax$ often result in the maximum number of factors chosen such that $r = kmax$

Specifically the IC_{p2} criterion because it seems to be more conservative (i.e. it usually underestimates r whereas IC_{p2} overestimates at times).

In order to understand the criteria better, tables 1 and 2 of Bai and Ng (2002) have been replicated using both different values for r and $kmax$. If r is chosen as in Bai and Ng (2002), the replicated results are almost identical to the original calculations and thus are not reported[11]. The results can be found in two tables in appendix C. Tables I through VIII in Bai and Ng (2002) in essence show 3 points. Firstly, traditional criteria perform poorly in estimating the number of factors if compared to the 6 presented criteria. Of the newly developed information criteria PC_{p1}, PC_{p2}, IC_{p1} and IC_{p2} perform well even under heteroskedasticity and autocorrelation. Secondly, if $\min\{N, T\}$ becomes too small, the performance of the criteria suffers which is highlighted by the bottom 5 results in the tables. Especially PC_{p3} suffers in this regard. In the given specifications the cut-off value for $\min\{N, T\}$ appears to be 40 or 60 if the variance of the innovations is high. Thirdly, PC_{p3} and IC_{p3} are less robust to a small $\min\{N, T\}$. The replication results in appendix Bai and Ng (2002) harbor three additional insights. Firstly as r increases, the requirements for $\min\{N, T\}$ are also higher. For $r = 7$ and $r = 9$ it seems $\min\{N, T\}$ should be at least 100. Secondly the choice of $kmax$ can matter a lot. Especially for small samples the specification of $kmax = 8$ used in Bai and Ng (2002) choose the maximum number of factors frequently when the N and T are too small. Almost all results in the last 5 rows set $\hat{k} = 8$ whereas the replicated results often report a lower number of factors if $kmax = \text{ceil}(\min\{T, N\})$. Noticeably, this also happens in the replicated results for $r \in \{1, 3, 5\}$ and $kmax = \text{ceil}(\min\{T, N\})$. If $kmax = \text{ceil}(\min\{T, N\})$ IC_{p3} predicts $\hat{k} = 50$ in the $T = 100, N = 100$ case for $r \in \{1, 3, 5, 7, 9\}$. Thus, the choice of $kmax$ can have a huge impact on the prediction of the number of factors. Luckily it is typically easy to see when the information criteria choose all the factors. In those cases the plausibily of the results should be checked by hand.

An interesting question if one is ultimately interested in forecasting is whether the "true" number of factors is of any interest at all or if the number of factors used in the

[11] I.e. differences are so small that they can be explained by sampling variation.

forecasting equation (2) should perhaps be different from the predicted r in the static model or q in the dynamic model. The answer can not be easily given. Similar to Bai and Ng (2008) we try to dodge this question somewhat by using the BIC criterion to choose a model among a list of models with differing number of factors (or number of lags of the variable of interest).

Breitung and Eickmeier (2011) among others, however, note that if one is primarily interested in forecasting, the number of dynamic factors can in principle be ignored and a static factor model with a higher number of factors can be estimated instead. The results presented in the empirical seciont, however, indicate that the forecasting accuracy of models which take dynamic effects into account is higher than static models.

2.4 Forecasting

The forecasting equation (2) can be estimated using a simple linear regression of y_{t+h} on $\hat{F}_t$, y_t, lags of y_t and depending on the model specification lags of $\hat{F}_t$.

The performance of factor models for forecasting purposes is most pronounced if the number of variables used in the factor equation is high (Stock and Watson, 2011). This makes sense because more variables improve the fit of the factor equation. However, there is a trade-off between quantity and quality of variables[12] that should not be ignored. This case will be adressed in the next subsection.

2.4.1 Targeted predictors

Bai and Ng (2008) follow the idea that if factor models are ultimately going to be used for forecasting it might make sense to "target" the set of predictors X_t to the task of forecasting the variable of interest y_t. The idea stems from Boivin and Ng (2006) who find that selecting only the "informative" variables can yield better results than taking a bigger set of variable which include the same "informative" variables but also additional ones. Bai and Ng (2008) thus provides a formal rule for which variables should be used for forecasting. To that end they use hard and soft thresholding rules to preselect among the predictors X_t those that have a meassurable influence on y_t.

[12] I.e. a smaller set of variables with a high predictive power can outperform a larger set of variables which includes the variables from the smaller set

Resultingly, a subset $\tilde{X}_t \subset X_t$ is selected for estimating the factor equation, the hope being that then the factors are better suited for predicting y_t.

For hard thresholding they regress y_t^h on W_{t-h} and X_{it-h} for each $i = 1, ..., N$[13]. Then they compare the resulting t-statistics to a threshold and discard those predictors whose t-statistics do not exceed that threshold.

For soft thresholding Bai and Ng (2008) perform penalized regressions in the form of LASSO, elastic net and Least angle regression (see Tibshirani (1996), Zou and Hastie (2005), Efron et al. (2004) for details) using the fact that these regression specifications set the parameters of variables which are not "important" to 0. The idea is to perform a regularized regression i.e. a normal least squares regression with the addendum of a restriction on the size of the parameters β. LASSO sets the restriction to be that $||\beta||$ is not bigger than some constant. The resulting optimization problem reads

$$\min \text{RSS} + \lambda \sum_{i=1}^{N} |\beta_j|$$

Ridge regression takes a similar approach but uses the L_2 norm $\sum_{i=1}^{N} \beta_i^2 \leq c$ for some penalty term c. The elastic net weighs between the two regularization mechanisms keeping both the sparsity of the LASSO estimator and the property of the Ridge estimator to include groups of variables which are correlated together Zou and Hastie (2005). Least angle regression is a variable selection mechanism which updates coefficients stepwise in a way that keeps the angle between variables which are most correlated with the residuals of the currently chosen set equal until (in the next step) a new variable has to be added to the set of most correlated variables.

The sparsity property of these penalized regressions allows the choice of relevant variables to be those which have a coefficient different from 0. Note that ridge regression, although similar in construction, does not have this property because coefficients are shrunk in absolute size but not set to 0. In applications the three approaches named above typically give similar results. The practical problem with soft thresholding lies in the fact that the algorithms[14] provide solutions to restricted regression problem in

[13] Note that the N regressions include only the lags and single predictors X_i as independent variables.

[14] In practice the LARS-EN (Least angle regression and Elastic net) algorithm can be used which computes Elastic Net solutions for all values of the penalization parameter using Least angle regressions. It has been shown by Efron et al. (2004) that LARS-EN includes the LASSO as special case.

the form of a solution path consisting of several steps where in each step the "most relevant" variable is added in the sense that the respective parameter is estimated and set to be different from 0. After the final step all variables are included in the model. Typically one choses the number of steps to be taken by the algorithm and thus the value of the restriction parameter using a cross-validation approach. Bai and Ng (2008) propose to use the BIC or the AIC criteria to choose the size of the active set in a general application of the LARS-EN algorithm. However, in the framework of factor models it is of interest how the factors calculated from the chosen active set performs in the forecasting equation. This open question is answered here by considering the optimal size of the restriction parameter in a forward looking way. The RMSE of pseudo out-of-sample forecasts are calculated to choose the size of the targeted set of variables. This approach is presented in the empirical section.

It should be noted that the approach of targeting predictors, while attractive, requires additional thought in the presence of structural breaks. Targeting the data prior to estimation can be interpreted as removing predictors which are not relevant or too "noisy" to be used in the prediction of the variable of interest. One of the approaches considered here in dealing with structural breaks once detected, is to remove the offending variables. The success of method to deal with structural breaks is meassured using the RMSE. Resultingly it is unclear whether improved forecasting accuracy are the results of removing structural breaks or of removing a generally noisy predictor which happens to also have a structural break. Luckily this issue can somewhat be resolved by targeting the data prior to testing the approach to deal with structural breaks. If the residual effect is positive it can be assumed that the method is succesfull especially since targeting the data in advance is likely to already account for some of the structural breaks.

2.5 Interpreting the factors

TODO: should I include such a chapter? It would involve finding out which variables the factors load onto.

2.6 Applications

Factor models have been applied in a wide range of different domains apart from economics. Most notably psychology and biology. Staying in the realms of economics, prominently, there are applications in the calculation of indices. The is based on the idea that the factors consist of underlying tendencies applying to all variables which makes them appealing as tools to generate indices.

Schumacher (2010) employs a similar idea in that he uses international predictors to forecast German GDP. Firstly it is intuitive that international data has an influence on an open economy directly. But secondly it is conceivable that there are underlying factors which influence both German GDP growth and international macroeconomic variables at the same time. However, to make sure that the information conveyed in the additional international predictors is of relevance for forecasting, Schumacher (2010) "targets" the predictors prior to esimating the factor equation, as described above.

TODO: see Bai (2003) page 1 for more examples!

3 Structural breaks

The treatment of structural breaks follows Breitung and Eickmeier (2011) closely. The question of interest is how a researcher should react to structural breaks in the factor equation.

The factor model under a structural breaks in period T^* can be written as follows:

$$y_{it} = f_t' \lambda_i^{(1)} + \varepsilon_{it} \text{ for } t = 1, ..., T^* \tag{11}$$

$$y_{it} = f_t' \lambda_i^{(2)} + \varepsilon_{it} \text{ for } t = T^* + 1, ..., T \tag{12}$$

The intuition is the same as with simpler models: a structural break implies a different relationship between the variable of interest and the factors and hence different factor loadings for the periods before and after the breaks. Multiple structural breaks can be specified accordingly. In theory, structural breaks could be also thought of as affecting the forecasting equation (2). This is ignored here because structural breaks in equations of this type have been thoroughly researched.

Interestingly, estimating a static factor model without structural breaks if the true data generating process includes structural breaks, yields an analogy to ignoring dynamic factors if they were present in the data generating process: the space estimated by the factors will be of higher dimension. This means that the number of factors r predicted e.g. by the Bai and Ng (2002) information criteria will be higher than if no structural breaks were present. Breitung and Eickmeier (2011) note that similar to dynamic factor models it is enough to increase the number of factors if one is primarily interested in decomposing the common component from the idiosyncratic component (that is to say if one is interested foremost in forecasting).

If structural breaks are detected in one or more of the factors the question remains what should be done next. Since structural breaks consist of an instability in the factor loadings at a certain period it can be optimal in a forecasting sense to estimate the respective parameter after the structural break has occured in the hope that this new parameter value contains more accurate information about the future. There is of course a trade-off here because the data series is shorter afterwards which has a negative effect on prediction accuracy. An alternative approach consists of removing the variable containing the structural break from the data set. This approach allows all other series to enter the forecasting equation at the same length as before while the added noise through the structural breaks is removed. The idea is similar to targeted predictors which remove uninformative predictors from the data set in order to improve the ratio of information to noise. Because of the typically high dimension of the data matrix X, factor models appear to be well suited for the second approach. Which method should be chosen depends on the size of the structural break, the relative position of the break and the relative information content of the other variables. In practice it is necessary to test in order to weigh the benefits and costs. This is done in the empirical section.

3.1 Testing for structural breaks

Breitung and Eickmeier (2011) develop several Chow type tests for the static factor model under the assumptions of the strict factor model. They also develop Quandt-Andrews type supremum tests. The test statistics are as follows.

$$\mathrm{lr}_i = T[\log(S_{0i}) - \log(S_{1i+S_{2i}})] \tag{13}$$

Where

$$S_{0i} = \sum_{t=1}^{T}(y_{it} - \hat{f}_t'\hat{\lambda}_i)^2,\ S_{1i} = \sum_{t=1}^{T_1^*}(y_{it} - \hat{f}_t'\hat{\lambda}_i^{(1)})^2 \text{ and } S_{2i} = \sum_{t=T_i^*+1}^{T_1^*}(y_{it} - \hat{f}_t'\hat{\lambda}_i^{(2)})^2$$

denote the residual sum of squares for the whole date range, the first subperiod up until the structural break and the period from the break to the end of the sample respectively. $\hat{\lambda}^{(1)}$ and $\hat{\lambda}^{(2)}$ denote the esimated factor loadings calculated on the data before the break and after the break respectively.

$$\mathrm{lm}_i = TR_i^2 \text{ where } R_i^2 \text{ is the r-squared from a regression } \hat{\varepsilon}_{it} = \theta_i'\hat{f}_t + \phi\hat{f}_t^* + \tilde{\varepsilon}_{it} \tag{14}$$

$$\mathrm{wald}_i = \text{ Wald statistic for } H_0 : \Psi_i = 0 \text{ in the regression } y_{it} = \lambda_i'\hat{f}_t + \psi\hat{f}_t^* + \nu_{it} \tag{15}$$

Where

$$\hat{f}_t^* = \begin{cases} 0 \text{ for } t = 1, ..., T_i^* \\ \hat{f}_t \text{ for } t = T_i^* + 1, ..., T \end{cases}$$

$$\mathrm{LM}^* = \frac{\left(\sum_{i=1}^{N} s_i\right) - rN}{\sqrt{2rN}} \tag{16}$$

The LM^* statistic pertains to the joint test of no structural break in all the factor loadings. Note, however, that the assumptions required for the LM^* statistic are those of a strict factor model, namely no interdependence in the error terms e_t in (1).

The LR, LM and Wald statistics defined above can be used to define a test for a unknown break date as in Andrews (1993). Under mild assumptions Breitung and Eickmeier (2011) show that the supremum statistic defined over the LM statistics is distributed using the nonstandard distribution given in Andrews (1993) and tabulated again in Andrews (2003).

$$\mathscr{S}_{i,T}(\tau_0) = \sup_{\tau \in [\tau_0, 1-\tau_0]} (s_i^\tau) \text{ where } s_i^\tau \text{ refers to either the LR, LM or Wald statistic} \quad (17)$$

As mentioned in the introduction of the dynamic factor model, I follow Breitung and Eickmeier (2011) in assuming an AR(p) process for the innovation term as in equation (18):

$$e_{it} = \varrho_{i,1} e_{i,t-1} + ... + \varrho_{i,p_i} e_{i,t-p_i} + \xi_{it} \quad (18)$$

This is in line with the assumptions of the approximate factor model introduced above.

For dynamic factor models[15] Breitung and Eickmeier (2011) propose to use a GLS transformed model, i.e. to perform the regression:

$$\varrho_i(L) y_{it} = \lambda_i' [\varrho(L) \hat{f}_t] + \psi' [\varrho_i(L) \hat{f}_t^*] + \varepsilon_{it}^*$$

The lag polynomials are obtained by applying some information criterion to the N regressions

$$\hat{\varepsilon}_{it} = \varrho_{i,t-1} \hat{u}_{i,t-1} + ... + \varrho_{i,p_i} \hat{u}_{i,t-p_i} + \tilde{\varepsilon}_{i,t}$$

Similar test statistics as above can then be applied to the results of the GLS regressions. Breitung and Eickmeier (2011) only present the LM-statistic arguing that it has the best size properties compared to the LR and Wald statistics.

$$\hat{\varrho}_i(L) y_{it} = \lambda_i' \left[\varrho_i(L) \hat{f}_t \right] + \psi_i' \left[\hat{\varrho}_i(L) \hat{f}_t^* \right] + \tilde{\varepsilon}_{it}^* \text{ for } t = p_i + 1, ..., T_i$$

Under heteroskedasticity the regression has to be weighted by the covariance matrix. If we also take account of the possibility of a break in the covariance matrix we get two distinct regressions

$$\frac{1}{\hat{\sigma}^{(1)}} \varrho_i(L) y_{it} = \lambda_i' \left[\frac{1}{\hat{\sigma}_i^{(1)}} \varrho_i(L) \hat{f}_t \right] + \left[\frac{1}{\hat{\sigma}_i^{(1)}} \varrho_i(L) \hat{f}_t^* \right] + \tilde{\varepsilon}_{it}^* \text{ for } t = p_i + 1, ..., T_i^*$$

[15] Note that Breitung and Eickmeier (2011) call a model where the innovations are generated by an AR(p) model a dynamic model whereas the literature usually refers to a dynamic process in the factors as in equation (7). This is similar in effect but not quite the same.

$$\frac{1}{\hat{\sigma}^{(2)}}\varrho_i(L)y_{it} = \lambda_i' \left[\frac{1}{\hat{\sigma}_i^{(2)}}\varrho_i(L)\hat{f}_t\right] + \left[\frac{1}{\hat{\sigma}_i^{(2)}}\varrho_i(L)\hat{f}_t^*\right] + \tilde{\varepsilon}_{it}^* \text{ for } t = T_i * +1, ..., T$$

3.2 Bootstrapping Breitung and Eickmeier (2011)

In order to test whether the properties of the above test statistics can be improved upon I have repeated the montecarlo study presented in Breitung and Eickmeier (2011) tables 2 and 3 with the difference that the test statistics have been bootstrapped.

The idea behind bootstrapping test statistics comes from the definition of the p-value. Denoting with τ the test statistic of interest, by $\hat{\tau}$ the realization of that test statistic given the observed data and by F the cumulative distribution function τ under the null hypothesis we can borrow the notation from Davidson and MacKinnon (2004) and write the p-value as

$$p(\hat{\tau}) = 1 - F(\hat{\tau})$$

Resultingly the p-value can be estimated by estimating $F(\hat{\tau})$. To do this we will simulate new data which follows the DGP under H_0. The p-value is then the (estimated) probability of the test statistic exceeding the calculated value $\hat{\tau}$ if the hypothesis H_0 were true.

Again following Davidson and MacKinnon (2004) we denote with stars, values that are calculated from simulated data, B denotes the number of simulated samples. We can then estimate the cumulative distribution function as

$$\hat{F}^*(x) = \frac{1}{B}\sum_{j=1}^{B} I(\tau_j^* \leq x) \text{ and } \hat{F}^*(\hat{\tau}) = \frac{1}{B}\sum_{j=1}^{B} I(\tau_j^* \leq \hat{\tau})$$

Resultingly

$$\hat{p}^*(\hat{\tau}) = 1 - \hat{F}^*(\hat{\tau}) = 1 - \frac{1}{B}\sum_{j=1}^{B} I(\tau_j^* \leq \hat{\tau}) = \frac{1}{B}\sum_{j=1}^{B} I(\tau_j^* > \hat{\tau})$$

In the following two specifications of the bootstrap are considered. Firstly, the often used resampling of the residuals is applied. Secondly the wild bootstrap is used.

Resampling the residuals uses the definition of the factor equation (1) to generate new data. The innovation term e_t is replaced with resampled residuals[16] gotten from the estimation of the model using the initial data.

$$X_t^* = \hat{f}_t\hat{\Lambda}' + \hat{e}_t^* \tag{19}$$

The wild bootstrap serves to deal with heteroskedasticity. The idea is to rescale the residuals using a white noise random variable ν_t.

$$X_t^* = \hat{f}_t\hat{\Lambda}' + \hat{e}_t^+$$

Where $\hat{e}_t^+$ is set such that to $\hat{e}_{it}^+ = \hat{e}_{it}\nu_{it}$ and ν_{it} is distributed with mean 0 and variance 1. For the application I choose ν to be standard normal.

4 Empirical application

In this section I will approach the issue of forecasting economic time series in the presence of structural breaks. Specifically, I will focus on forecasting German GDP using German und US American macroeconomic time-series. The reason for including American time-series to predict German GDP is twofold. Firstly, Germany is active on international markets with the US being the second most important export market after France. Coupled with the big importance of the US American market in the world economy it is plausible that US data plays a role in predicting German economic development. Secondly, we are interested in forecasting time-series under structural breaks. There are at least two periods commonly associated with breaks in the world economy: the dot-com bubble of 2000 and the financial crisis of 2008, both of which originated in the US. The hope is that big ruptures in the world economy make a break of relationships between variables (and ultimately of the factor loadings) providing us with meaningful data for the following analysis.

There are several ways to address the issue of structural breaks from a forecasting viewpoint. Firstly structural breaks can be ignored. As has been stated above, in this

[16] Resampling is done with replacement to model the empirical distribution. So the resampled residuals e_t^* are given by setting $\hat{e}_t^* = \hat{e}_s$ where s is drawn from $\{1, ..., T\}$ with replacement.

case factor models will tend to choose a higher number of factors than if no structural breaks are present which means that the space the factors span is of a higher dimension as the factors are orthogonal to each other and thus linearly independent.[17]

Secondly, structural breaks can be included in the model in the form of a dummy parameter interacting with the factor estimations $\hat{f}_t$. The forecasting equation of the model then results as

$$y_{t+h}^h = \alpha(L)W_t + \beta(L)\hat{f}_t + \gamma(L)\hat{f}_t^* + \varepsilon_{t+h} \tag{20}$$

where $\hat{f}_t^*$ is defined as above.

Thirdly, if predictors are to be targeted prior to estimating the factor equation it is conceivable to find that a subset of the predictors $\tilde{X}_t^{(1)} \subset X_t$ peforms well before and a different subset $\tilde{X}_t^{(2)} \subset X_t$ performs better after the structural break. This finding would imply that it could be beneficial to estimate the factor equation again using only the data after T^* and discarding the rest.

4.1 Data choice and treatment

The data used for the horse race has been collected from the German Bundesbank and from the American Federal Reserve Bank of St. Louis.[18] Together the data set consists of $T = 82$ observations from $N = 169$ quarterly variables. The series are listed below in appendix F. The data set contains macroeconomic variables such as wages and salaries, employment levels and survey data from Germany and the US as well as financial variables.

Additionally an emphasis has been put on selecting several time-series which are well known to be subjected to big changes in the abovementioned periods such as "Housing starts".

They have been downloaded in seasonally adjusted form where appropriate. The data series have then been differenced (zero times, once or twice) until stationarity. To this end the number of differencing operations necessary to make the series stationary

[17] How this affects the information criteria of Bai and Ng (2002) can be seen in table 1 of Breitung and Eickmeier (2011).

[18] More specifically the FRED2 public API has been used to extract selected series. The data from the Bundesbank have been scraped from the Bundesbank website.

have been tested using the KPSS method developed in Kwiatkowski et al. (1992).[19]

Finally, the series have been demeaned and normalized such that $\frac{1}{T}\sum_{t=1}^{T} X_{it} = 0 \ \forall i$ and $\frac{1}{T}\sum_{t=1}^{T} X_{it}^2 = 1 \Leftrightarrow \sum_{t=1}^{T} X_{it}^2 = T \ \forall i$.

4.2 Model selection and evaluation

This section explores the different methods to selected and evaluate a model. The ultimate goal is always forecasting the true data as closely as possible. To this end I use the root mean squared forecasting error (RMSE) as an evaluation mechanism. $RMSE = \sqrt{\frac{1}{w}\sum_{s=t}^{t+h}[(y_s - \hat{y}_s)^2]}$ where w denotes the length of the window to be forecasted and t the period preceding the window being forecasted. The estimates $\hat{y}_t$ are gotten from estimating the forecasting equation and predicting h steps into the future.

If the number of static factors in the forecasting equation is predetermined using criteria defined above, the number of lags of the variable of interest y_t and the number of lags of the static factors are still open. In the following I refer to the number of lags of y_t as s.

I follow Bai and Ng (2008) in selecting among model candidates using the BIC and the FPE criterion which are of course in-sample criteria.

Bai and Ng (2008) write the BIC and FPE criteria as follows

$$\begin{aligned} \text{BIC} &= \log(\hat{\sigma}_n^2) + n\frac{\log T}{T} \\ \text{FPE} &= \log(\hat{\sigma}_n^2) + n\frac{2}{T} \end{aligned} \tag{21}$$

Where $\hat{\sigma}_n^2$ is the residual variance $\hat{e}'e$ from an estimation of the factor equation using n predictors.

Since the point of the exercise is forecasting, additionally, the optimal model in

[19] The method consists of constructing an LM-test for the hypothesis that the variance σ_ε^2 is 0 (or equivalently that μ_t is constant) in

$$\begin{aligned} y_t &= \beta' D_t + \mu_t + u_t \\ \mu_t &= \mu_{t-1} + \varepsilon_t \end{aligned}$$

where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ and D_t captures deterministic components and time trends. This implies that the difference of y_t denoted Δy_t is stationary.

terms of out-of-sample forecasting performance is found. This is done by calculating the root mean squared error of one step ahead pseudo out-of-sample forecasts.[20] This evaluation method follows Forni et al. (2005) and Bai and Ng (2008) among others. The foreacasting window w over which the pseudo out-of-sample-forecasts are performed is set to $w = 15$ here.

All models are compared to a benchmark model which consists of a simple moving average over the forecasting window w. This forecasting window makes sure that there are more observations than parameters for all possible models. Since the length of the data series is $T = 82$ this leaves 67 observations for the first forecast in the pseudo out-of-sample procedure. The maximum number of factors is set to 15 for static models and 10 for dynamic models. The maximum number of lags of y_t is set to 10. This brings the maximal number of parameters to $n_{max} = 25$ for static models and $n_{max} = 50$ for dynamic models. The original number of predictors is $N = 169$ which highlights that factor models can be used as a dimensionality reduction method and that they are are an appropriate way to deal with the situation of $N >> T$ in this data set.

The first panel in table 1 shows the results of the in sample selection procedure for a static factor model. Three variants for choosing the number of factors are considered. Firstly the IC_{p2} criterion from Bai and Ng (2002) is used to estimate the number of static factors for both the factor equation and the forecasting equation. The information criteria (BIC and FPE) are then used to select s, the number of lags of y_t. As a second choice, the decision over the number of factors *and* the number of lags of y_t in the forecasting equation are left to the information criteria. I.e. the information criteria are minimized over different choices of r and s. The same number of factors is used in the factor equation. A third possibility consists of leaving the choice of the number of factors for the factor equation to a Bai and Ng (2002) criterion and using the BIC or FPE criterion to choose the number of static factors in the forecasting equation along with the number of lags of y_t. I.e. the BIC or FPE are minimized over r and s.

Both the BIC and FPE as well as the IC_{p2} criteria select a large number of factors

[20] Pseudo out-of-sample forecasts are a method to simulated a forecasting situation in that the forecaster does as if he would not know the true realizations of the data after t. The forecaster tries to predict the value y_{t+h} given all the information up to t. t is then successively moved one period ahead. Here we set $h = 1$.

and are generally very erratic in their choice of a model. The fact that conventional information criteria should not be used for selecting the number of static factors has been extensively documented in the factor model literature and is no surprise. Another approach to model selection for forecasting consists of testing the model in an out-of-sample forecasting situation and selecting the best performing model in terms of the forecasting error. In a forecasting application this would imply that a certain fraction of the data has to be used for

The results of optimal forecasting model are reported in the second panel of table 1.

Since the variance of the data series has been normalized to 1 and the mean to 0 the RMSE of the simple average y_t in the given window can be expected to be close to 1. The RMSE of the resulting out-of-sample forecasts is very large with 1.30 and 1.22 respectively. The benchmark model of a moving average reports an RMSE of 0.86 and an AR(6) model achieves an RMSE of 0.50 for the same time horizon.[21]

The best performing static factor model in terms of forecasting can be got if the BIC criterion for model selecting is replaced by the RMSE as can be seen in the second panel. I.e. the RMSE is minimized over choices of r and s rather than the BIC. This implies using the out-of-sample performance as an evaluation criterion. In that case both r and s are shrunken to 1. While this outperforms the simple benchmark model, the fact that the most parsimonious static factor model performs best in out-of-sample prediction hints at possible performance increases if dynamic effects are allowed.

Repeating the exercise but allowing for lags of the factors to enter the forecasting equation, the results in table 2 are obtained.[22] Since the data used for the exercise consists of time-series it makes sense that a dynamic factor model is better suited for prediction here. In the case of the dynamic model the FPE criterion selects too many lags of y_t which results in a higher RMSE. The IC_{p2} criterion chooses $r = 4$ for the factor equation. For the forecasting equation, however, it seems to be better to increase the number of factors if the factors are also lagged in the forecasting equation. Also in this situation it does not necessarily make a difference in terms of RMSE if the number of factors differs in the factor equation and in the forecasting equation. This can be seen

[21] 6 is the optimal lag number in terms of RMSE.

[22] Essentially, table 1 uses the same models as are used in table 2 with the added restriction $q = 0$.

Table 1: Static factor model, model selection

	r_{factor}	$r_{forecast}$	s	RMSE
	In-Sample			
BIC	13	13	5	1.30
FPE	15	15	5	1.23
IC_{p2} & BIC	4	13	5	1.30
IC_{p2} & FPE	4	15	5	1.23
IC_{p2}	4	4	10	1.21
	Out-of-Sample			
RMSE	1	1	1	0.71
RMSE & IC_{p2}	4	1	1	0.71

Notes: The results are derrived by minimizing the criteria in the first column. If both an Bai and Ng (2002) criterion and a standard information criterion is given the former is applied to the factor equation and the latter to the forecasting equation. The maximum number of factors r is set to 15. The maximum number number of the lags of the variable of interest s is set to 10.

as in the optimal out-of-sample models they differ in both the static and the dynamic model while the RMSE stays the same. This no surprise as factor models relying on principal component estimation contain the most information in terms of variance in the first few factors .

In both the static and the dynamic factor model, choosing the number of static factors using the IC_{p2} criterion outperforms the other methods in terms of forecasting accuracy.

4.3 Structural breaks

Structural breaks are estimated using Quandt-Andrews type supremum tests as defined above. The critival values for the test statistics are given in table I of Andrews (2003). The results can be seen in table 3.

It is apparent that there are two periods with major changes in the loadings. Firstly, there are structural breaks reported in the loadings of 8 variables in the first quarter of 1999 which coincides approximately with the Russian default of 1998 and maybe the aftermath of the Asian financial crisis in 1997. In the following this period will be

Table 2: Dynamic factor model, model selection

(a) Maximum number of factor lags q set to 3

	r_{factor}	$r_{forecast}$	s	q	RMSE
	In-Sample				
BIC	2	2	3	1	0.55
FPE	10	10	10	3	2.91
IC_{p2} & BIC	4	2	3	1	0.55
IC_{p2} & FPE	4	10	10	3	2.91
IC_{p2}	4	4	10	2	0.70
	Out-of-Sample				
RMSE	7	7	1	3	0.47
RMSE & IC_{p2}	4	7	1	1	0.47

(b) Maximum number of factor lags q set to 4

	r_{factor}	$r_{forecast}$	s	q	RMSE
	In-Sample				
BIC	10	10	10	4	19.16
FPE	10	10	10	3	19.16
IC_{p2} & BIC	4	10	10	4	19.16
IC_{p2} & FPE	4	10	10	4	19.16
IC_{p2}	4	4	10	4	0.78
	Out-of-Sample				
RMSE	7	7	1	3	0.47
RMSE & IC_{p2}	4	7	1	3	0.47

Notes: The results are derrived by minimizing the criteria in the first column. If both an Bai and Ng (2002) criterion and a standard information criterion is given the former is applied to the factor equation and the latter to the forecasting equation. The maximum number of factors r is set to 10. The maximum number number of the lags of the variable of interest s is set to 10. q here refers to the number of lags of the factors in the forecasting equation, not to the number of dynamic factors as defined above. The maximum number of the factor lags q is set to 3 and 4 respectively. The results using the traditional information criteria hinge on the maximum number of factor lags.

referred to as the first cluster of structural breaks. Secondly, a total of 22 changes in the loadings are reported for the period of the second quater of 2008 until the second quarter of 2009. This period will be referred to as the second cluster of structural breaks. TODO: talk about the series that are most affected (for that I need to fix the variable indices)

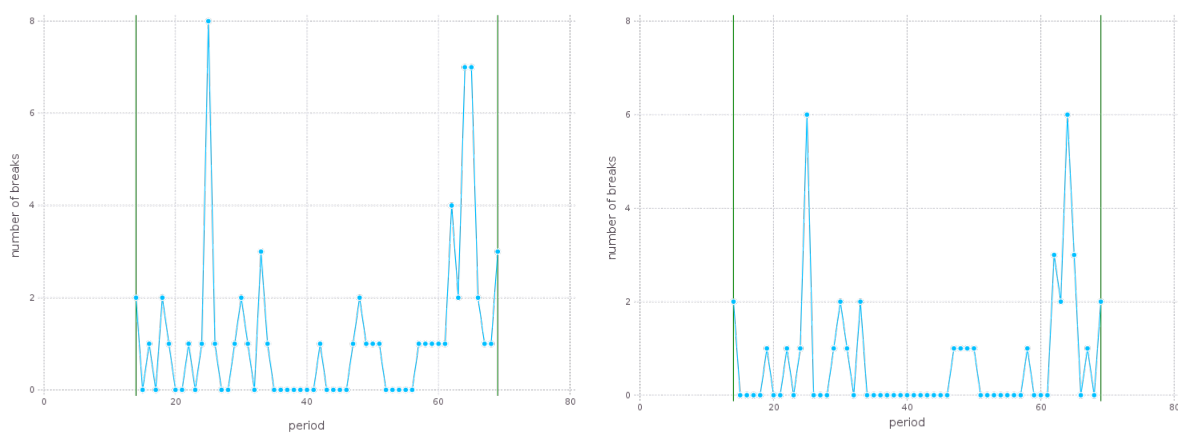
Table 3: Number of variable breaks per period

96:1	96:3	97:2	97:3	98:2	98:4	99:1	99:2	00:1	00:2	00:3	01:1	01:2	03:2	04:3
2	1	2	1	1	1	8	1	1	2	1	3	1	1	1
05:1	05:2	05:3	07:1	07:2	07:3	07:4	08:1	08:2	08:3	08:4	09:1	09:2	09:3	09:4
1	1	1	1	1	1	1	1	4	2	7	7	2	1	1

Notes: The entries report how many of the 169 variables are predicted to undergo a structural break by period. Reported are the numbers of significant values of $\mathscr{S}_{i,T}$ at the 5% significance level. The total number of structural breaks found in the 169 variables is 64.

Figure 1: Structural breaks per quarter

(a) 5% significance level (b) 1% significance level



4.3.1 Removing the effect of structural breaks

In the following the question of how to react to the presence of structural breaks is approached as in section 2 above. Structural breaks in the factor loadings influence the performance of the forecasting equation through at least two channels. Firstly the presence of structural breaks decreases the signal to noise ratio in the data and thus. Breitung and Eickmeier (2011) show that it is necessary to increase the number of factors r under the presence of structural breaks in order to estimate the space spanned by the factors. If the number of static factors in the forecasting equation is set equal to the number of static factors in the factor equation it becomes necessary to estimate a bigger number of parameters.

Either the length of the series is reduced by estimating the model only on data which is not subjected to structural breaks or the width of the data is reduced by removing the variables which are subjected to the structural break or a combination thereof.

For the purpose of this paper we are interested in forecasting performance of factor models which ignore structural breaks versus factor models which take the structural breaks into account. Thus the results of models which restrict the amount of data to reduce the effect of structural breaks are compared with those that do not. It is worth noting that the comparison is not perfect because as has been argued in section 2 it can be better to reduce the number of predictors to a set which is "targeted" towards the prediction of the variable of interest. It is possible that removing the variables with structural breaks "accidentally" targets the set of variables towards predicting the variable of interest independent of the structural break. In other words an improved result is not necessarily only due to the structural breaks being removed but might be due to a noisy predictor being removed. To be precise a series with a structural break could be considered a noisy predictor as well. However, it might be noisy independent of the structural breaks. Resultingly, a robustness check consists of targeting the set of variables for both approaches prior to the estimation to remove that effect.

A practical difficulty has to be addressed before the results are presented. Given the fact that there are two periods where a majority of structural breaks occur, each of which affects a different set of factor loadings, cutting off the values before the significant break periods would leave very few observations for estimating the new model.

If both periods with major structural breaks are treated in a way that only data after the structural breaks is used, this would leave at most 17 observations of which another 15 have to be removed for the first step in the pseudo out-of-sample forecasts.

In principle there is a simple way to overcome this issue. It can be remembered that some of the data columns are monthly series which have been merged into the quarterly data set by dropping two thirds of the observations. The data could be cast back into monthly form. Most of the series will have missing data which can be interpolated (e.g. GDP is only reported quarterly). While this method solves the problem in a technical sense[23], it has several drawbacks: firstly no additional information is added by the interpolation although there are more observations available to the model. At the same time the imputation adds additional uncertainty. Traditional standard error estimates

[23]For the first step of the pseudo out-of-sample period there are only 8 data points for estimation. For every quarterly step of the pseudo out-of-sample procedure there are an additional 3 observations. Thus, the ratio of observations to predictors improves quickly with each step of the pseudo out-of-sample prediction.

do not account for this and resultingly they may be downward biased (cf. Gelman and Hill (2006, chapter 25). It is thus likely that prediction based on this short data set performs badly in a forecasting sense. On the other hand, if predictions resulting from this shaved data set should perform better, it would constitute a strong indication that the structural breaks uncovered by the tests should not be ignored. In this case doing so clearly leads to a significant degradation of forecasting results. This has not been tested here.

Instead four approaches to removing the effects of structural breaks from the data are considered. First the variables in the second cluster of structural breaks between the second quarter of 2008 and the first quarter of 2009 are removed. Because this approaches ignores the first cluster of structural breaks in the first quarter of 1999, a second model is estimated to explore what happens if these series are excluded as well. A third model removes all series with structural breaks in the loadings. Finally, the fourth model leaves the *variables* with breaks in the first cluster in 1999 in the model but removes *observations* prior to the second quarter of 1999.

For all four approaches the best performing dynamic factor model in terms of RMSE is reported where the number of static factors in the factor equation is calculated using the IC_{p2} criterion and the number of factors in the forecasting equation as well as the number of factor lags is chosen according to the minimum of the RMSE of out-of-sample forecasts. Structural breaks are detected at the 5% significance level.

The figures in appendix D show the number of significant structural breaks left if the above adjustements to the data have been made. The reason to check for structural breaks again after some variables have been removed or the observations prior to the second quarter of 1999 have been removed, is that the estimation of the factor equation changes with a reduced set of variables which has an impact on the statistics of the structural break tests.

Table 4 shows that the exclusion of variables improves the RMSE considerably compared to ignoring the structural breaks. For this data set, the more variables with structural breaks at the 5% significance level are removed, the lower the RMSE becomes.

Figures for the number of structural breaks per period after variables have been dropped or the data length has been trimmed can be found in appendix D. Notably

	r_{factor}	$r_{forecast}$	s	q	RMSE
	Second break cluster removed $T = 82, N = 149$				
RMSE	7	7	1	3	0.45
RMSE & IC_{p2}	3	7	1	3	0.45
	First and second break cluster removed $T = 82, N = 141$				
RMSE	6	6	4	2	0.44
RMSE & IC_{p2}	3	6	4	2	0.44
	All variables with breaks removed $T = 82, N = 105$				
RMSE	6	6	3	2	0.40
RMSE & IC_{p2}	3	3	4	2	0.39
	Second break cluster removed, length trimmed $T = 57, N = 149$				
RMSE	1	1	5	1	0.51
RMSE & IC_{p2}	4	1	5	1	0.51

Table 4: Dynamic factor model, data adjusted for structural breaks

figure 5 shows a modest number of structural breaks at the 5% significance level *after* all structural breaks had been removed from the data set. Iterating the procedure by removing the variables with structural breaks again increases the RMSE to 0.72 which makes clear that blindly removing all variables which contain structural breaks need not be a good idea even though the previous results in table 4 seemed to indicate that removing as many variables with structural breaks as possible has a positive effect on RMSE.

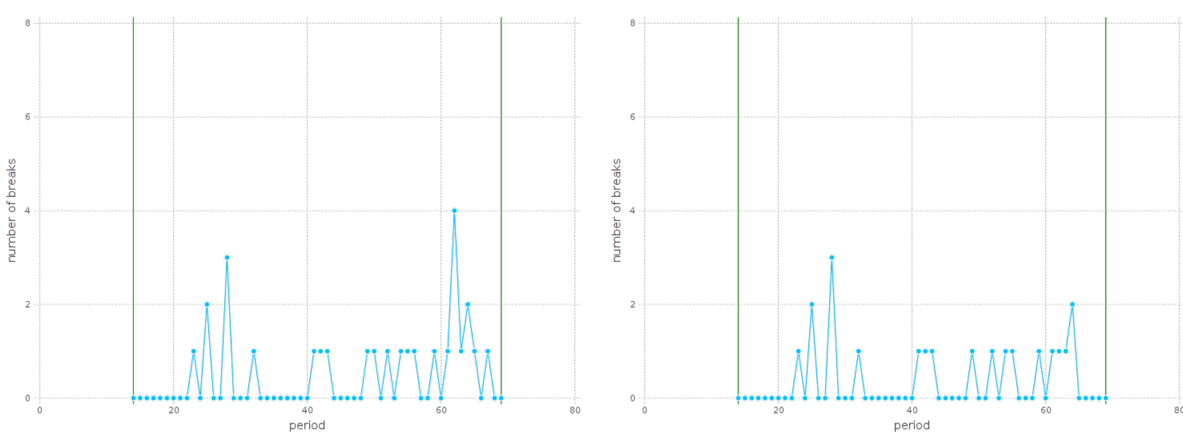
4.3.2 Robustness check Targeted predictors

It was found before that removing all variables with structural breaks can improve the forecasting performance. To test if targetting the predictors and removing the structural breaks has a similar effect, the variables with structural breaks are removed after the data set has been targeted. To do this the following sub section considers the residual effect of removing the structural breaks after the effect of targeting the predictors has been accounted for. Both soft and hard thresholding rules are applied.

Starting with hard thresholding, targeting the predictors at the 10%, 5% and 1% significance level (see Bai and Ng (2008) for details) leaves 85, 78 and 43 variables in the data set respectively. Figure 2 shows that targetting the data removes a part of the two clusters of structural breaks found in the original data. It can be inspected in table 5 that with hard thresholding the MSE is higher than it was for the original data set. For the complete data set the optimal model achieved an RMSE of 0.47. with hard thresholding the mse increases to about 0.50 for all three cut-off values. Regardless of the relatively bad performance of hard thresholding rules to target the predictors, removing the structural breaks from the targeted data set marginally improves the results.

	Hard thresholding				
	r_{factor}	$r_{forecast}$	s	q	RMSE
	$\alpha = 0.1$ $T = 82, N = 85$				
RMSE	1	1	5	2	0.50
RMSE & IC_{p2}	6	1	5	2	0.50
	Hard Thresholding $\alpha = 0.05$ $T = 82, N = 78$				
RMSE	1	1	5	2	0.50
RMSE & IC_{p2}	6	1	5	2	0.50
	$\alpha = 0.01$ $T = 82, N = 43$				
RMSE	1	1	5	2	0.49
RMSE & IC_{p2}	7	1	5	2	0.49

Table 5: Dynamic factor model, Targeted data, Hard Thresholding



(a) 5% significance level

(b) 1% significance level

Figure 2: Structural breaks per quarter, targeted predictors

	Hard thresholding, breaks removed					
	r_{factor}	$r_{forecast}$	s	q	RMSE	Δ
	$\alpha = 0.1$					
	$T = 82, N = 51$					
RMSE	11	11	2	1	0.44	0.06
RMSE & IC_{p2}	4	11	2	1	0.44	0.06
	$\alpha = 0.05$					
	$T = 82, N = 51$					
RMSE	1	1	5	2	0.50	0
RMSE & IC_{p2}	4	1	5	2	0.50	0
	$\alpha = 0.01$					
	$T = 82, N = 28$					
	Results omitted					

Notes: T and N denote the dimension of the targeted data sets where all variables with evidence of structural breaks found at the 5% level using the sup LM test have been removed. The column denoted with Δ calculates the difference between the RMSE of the best factor model calculated on the targeted data with structural breaks removed and the results from table 5 where the structural breaks have not been removed. Note that the set of variables for hard thresholding with $\alpha = 0.05$ differs although the number of variables coincides. Also note that there are no results for $\alpha = 0.01$ because the estiamted number of factors for this data set is 22 for wich there are no tabulated values to calculate the structural break tests with in Andrews (2003). The critical values have not been simulated here.

Table 6: Dynamic factor model, Targeted data, Structural breaks removed

As noted in the introduction on targeted predictors above, the difficulty with soft thresholding lies in the choice of steps (denoted k from here on out) in the LARS-EN algorithm. This coincides with the number of variables chosen as each step adds one variable to the active set. It is conceivable that if the thresholding is too strict, i.e. the regularization parameter too large, the LASSO will have kicked out most of the variables which contain structural breaks. Some results of the removal of structural breaks after applying soft thresholding can be seen in table 7. Relative improvements of removing structural breaks for more values of k and Diebold-Mariano test statistics are shown in table 10. It can be seen that unless soft thresholding is very strict in the sense that only very few variables are selected, removing the variables with structural breaks can still improve the RMSE. However, significant improvements in forecasting accuracy can not be confirmed for customary levels of significance.

	Soft thresholding					
	r_{factor}	$r_{forecast}$	s	q	RMSE	Δ
	$k = 21$ $T = 82, N = 21$					
RMSE	2	2	6	1	0.31	-0.06
RMSE & IC_{p2}	1	2	6	1	0.31	-0.06
	$k = 25$ $T = 82, N = 25$					
RMSE	4	4	3	1	0.41	$+0.04$
RMSE & IC_{p2}	1	4	3	1	0.41	$+0.04$
	$k = 30$ $T = 82, N = 30$					
RMSE	7	7	3	1	0.44	$+0.06$
RMSE & IC_{p2}	3	7	3	1	0.44	$+0.06$

Notes: The entries report differnt optimal models for three choices of thresholding rules $k \in \{21, 25, 30\}$. $k = 21$ was chosen because it was the overall best performing model.

Table 7: Dynamic factor model, Targeted data, Soft thresholding

5 Conclusion

While factor models perform quite well in forecasting economic time series, structural breaks can, if not accounted for, undermine the forecasting performance significantly. Not only does one have to be careful with the actual prediction results but also interpretting the factors directly can become unfeasible as the number of static factors changes under structural breaks.

Estimating different model specifications under structural breaks on a large macroeconomic data set including German and US macroeconomic variables shows that removing variables with structural breaks tends to improve forecasting accuracy. This result holds even after the data set has been targeted on forecasting the variable of interest. The best performing models beat simple AR and moving average benchmark models by far.

Apendix

A Derrivation of Principal Components

The principal components of a data matrix X are defined as the transformation of X into the space spanned by the loadings [24] u_i where each u_i is a unit vector. The transformation is defined such that the transformed columns are uncorrelated (as are the loadings u_i) and such that the first transformed column (i.e. the first principal component) contains the largest amount of the variance of the original data, the second column the second largest amount (i.e. the second principal component), etc.

Let X have mean 0 (i.e. subtract the mean from each observation if it does not). For the first principal component, we want to find a loading vector vector u_1 such that the variance along the projection of x onto the first principal component direction u_1 is maximized where u_1 is a unit norm vector.

$$\max_{u_1:||u_1||=1} \frac{1}{T}\sum_{i=1}^{T}(x_i'u_1)^2 = \max_{u_1:u_1'u_1=1} u_1'(\frac{1}{T}\sum_{i=1}^{T}x_i'x_i)u_1 = \max_{u_1:u_1'u_1=1} u_1'(\Sigma)u_1$$

Setting up the Lagrangian yields

$$L(u_1,\lambda) = u_1'\Sigma u_1 - \lambda(u_1'u_1 - 1)$$

After taking the derivative with respect to u_1 and dividing by 2 we are left with

$$\frac{\partial L}{\partial u_1} = \Sigma u_1 - \lambda u_1 \overset{!}{=} 0$$

in other words u_1 is an eigenvector of Σ, the covariance matrix of x. The corresponding eigenvalue is λ and we must have $\Sigma = \lambda$ for the variance of the principal component to be maximal under the constraint.
The maximal variance of $x'u_1$ is thus $V(x'u_1) = u_1'\Sigma u_1 = u_1'\lambda u_1$ and it is achieved for the highest eigenvalue λ of the covariance matrix of x. The derrivation of the following principal components is accordingly subject to the additional constraint that the

[24]The u_i are also called scores or principal component directions.

principal components are uncorrelated. In the case of the second principal component this implies $\text{cov}(u_1'x, u_2'x) = u_1'\Sigma u_2 = 0$. The solution for the third and all following principal component is more complex but goes accordingly. The derrivations can be seen e.g. in Jolliffe (2005) or any other textbook on the subject. The solution for the i-th principal component is to set the loadings u_i to be eigenvector which belongs to the i-th highest eigenvalue. Also, for each principal component i we get that $\text{var}(u_i'x) = \lambda_i$.

B Estimation of the Factor Equation

Estimators of the factor equation solve (10).

$$\min_{F_1,\ldots,F_T,\Lambda} V_r(\Lambda, F) = \min_{F_1,\ldots,F_T,\Lambda} \frac{1}{NT} \sum_{t=1}^{T} (X_t - \Lambda F_t)'(X_t - \Lambda F_t)$$

$$\text{s.t. } N^{-1}\Lambda'\Lambda = I_r$$

The solution can be gotten by first optimizing over F_t. The first order condition is:

$$\frac{\partial V_r}{\partial F_t} = -2X_t'\Lambda + 2\hat{F}_t'\Lambda'\Lambda \stackrel{!}{=} 0$$

Resultingly $\hat{F}_t = (\Lambda'\Lambda)^{-1}\Lambda' X_t$. Inserting that back into the optimization problem gives $\min_\Lambda \frac{1}{NT} X_t' X_t - 2X_t'\Lambda(\Lambda'\Lambda)^{-1}\Lambda' X_t + X_t'\Lambda(\Lambda'\Lambda)^{-1}\Lambda'\Lambda(\Lambda'\Lambda)^{-1}\Lambda' X_t = \min_\Lambda \frac{1}{NT} X_t'[I - \Lambda(\Lambda'\Lambda)^{-1}\Lambda']X_t$.

This is equivalent to the problem $\max_\Lambda \operatorname{tr}\left\{(\Lambda'\Lambda)'^{-\frac{1}{2}}\lambda'(T^{-1}\Sigma_{t=1}^{T} X_t X_t')\Lambda(\Lambda'\Lambda)^{-\frac{1}{2}}\right\}$ which is the same as $\max_\Lambda \Lambda'\hat{\Sigma}_{XX}\Lambda$ s.t. $N^{-1}\Lambda'\Lambda = I_r$ where $\hat{\Sigma}_{XX}$ is the usual sample equivalent of the variance of X. This can be solved by setting $\hat{\Lambda}$ to eigenvectors of the r largest eigenvalues of $\hat{\Sigma}_{XX}$.

C Replication of Bai and Ng (2002) for $r = 7$, $r = 9$

Table 8: Replication of Table I of Bai and Ng (2002) for $r = 7$

DGP: $X_{it} = \sum_{j=1}^{r} \lambda_{ij} F_{tj} + \sqrt{\theta} e_{it}, r = 7$

N	T	PC_{p1}	PC_{p2}	PC_{p3}	IC_{p1}	IC_{p2}	IC_{p3}
100	40	6.4	5.9	6.97	4.93	3.46	6.73
100	60	6.72	6.27	7.02	6.34	4.74	6.99
200	60	6.93	6.87	7.0	6.78	6.46	6.96
500	60	6.99	6.96	7.0	6.94	6.92	6.99
1000	60	7.0	6.99	7.0	6.99	6.97	7.0
2000	60	7.0	6.99	7.0	6.98	6.99	7.0
100	100	7.0	6.77	7.35	6.89	6.32	50.0
200	100	7.0	7.0	7.0	7.0	6.99	7.0
500	100	7.0	7.0	7.0	7.0	7.0	7.0
1000	100	7.0	7.0	7.0	7.0	7.0	7.0
2000	100	7.0	7.0	7.0	7.0	7.0	7.0
40	100	6.39	6.12	7.02	4.86	3.49	6.78
60	100	6.79	6.37	7.01	6.01	5.0	7.0
60	200	6.93	6.85	7.0	6.82	6.58	6.98
60	500	6.96	6.97	7.0	6.97	6.9	6.99
60	1000	7.0	6.99	7.0	6.99	6.96	7.0
60	2000	7.0	7.0	7.0	6.97	6.98	6.99
4000	60	7.0	7.0	6.99	6.98	6.99	6.99
4000	100	7.0	7.0	7.0	7.0	7.0	7.0
8000	60	6.99	7.0	7.0	7.0	6.98	7.0
8000	100	7.0	7.0	7.0	7.0	7.0	7.0
60	4000	7.0	7.0	7.0	7.0	7.0	6.99
100	4000	7.0	7.0	7.0	7.0	7.0	7.0
60	8000	6.99	7.0	7.0	6.99	7.0	6.99
100	8000	7.0	7.0	7.0	7.0	7.0	7.0
10	50	5.0	5.0	5.0	4.76	3.95	5.0
10	100	5.0	5.0	5.0	4.58	4.18	4.96
20	100	7.04	6.5	7.98	3.14	2.07	6.94
100	10	5.0	5.0	5.0	5.0	4.96	5.0
100	20	7.1	6.64	8.05	3.3	2.08	8.2

Notes: Estimated number of factors averaged over 1000 simulations. The true number of factors is r and the maximum number of factors is ceil(min$\{T, N\}$)/2).

Table 9: Replication of Table I of Bai and Ng (2002) for $r = 9$

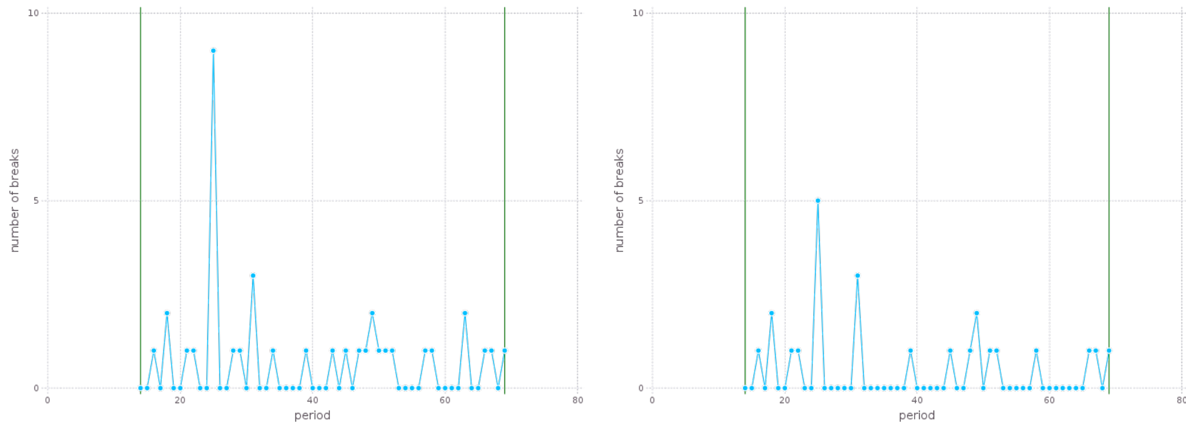
DGP: $X_{it} = \sum_{j=1}^{r} \lambda_{ij} F_{tj} + \sqrt{\theta} e_{it}, r = 9$

see Bai and Ng (2002) for specification of the DGP

N	T	PC_{p1}	PC_{p2}	PC_{p3}	IC_{p1}	IC_{p2}	IC_{p3}
100	40	6.51	5.93	8.06	3.55	1.49	7.89
100	60	7.02	6.25	8.77	5.4	2.74	8.9
200	60	7.74	7.31	8.65	7.33	6.26	8.74
500	60	8.09	8.02	8.52	8.18	7.97	8.58
1000	60	8.42	8.38	8.59	8.45	8.25	8.62
2000	60	8.47	8.32	8.59	8.67	8.56	8.64
100	100	8.03	6.93	9.01	7.68	4.85	50.0
200	100	8.85	8.48	8.99	8.92	8.58	9.0
500	100	9.0	8.99	9.0	9.0	8.99	9.0
1000	100	9.0	8.99	9.0	9.0	9.0	9.0
2000	100	9.0	9.0	9.0	9.0	9.0	9.0
40	100	6.63	6.06	8.08	3.38	1.66	7.79
60	100	7.03	6.33	8.85	5.51	2.68	8.94
60	200	7.79	7.43	8.6	7.1	6.05	8.73
60	500	8.16	7.98	8.61	8.28	7.96	8.66
60	1000	8.41	8.28	8.51	8.51	8.34	8.69
60	2000	8.4	8.4	8.58	8.57	8.56	8.74
4000	60	8.52	8.44	8.57	8.61	8.51	8.73
4000	100	9.0	9.0	9.0	9.0	9.0	9.0
8000	60	8.44	8.52	8.55	8.66	8.64	8.68
8000	100	9.0	9.0	9.0	9.0	9.0	9.0
60	4000	8.53	8.54	8.55	8.62	8.6	8.57
100	4000	9.0	9.0	9.0	9.0	9.0	9.0
60	8000	8.61	8.47	8.51	8.66	8.58	8.76
100	8000	9.0	9.0	9.0	9.0	9.0	9.0
10	50	5.0	5.0	5.0	4.66	3.29	5.0
10	100	5.0	5.0	5.0	4.32	3.59	4.77
20	100	7.25	6.84	8.1	1.76	1.28	7.21
100	10	5.0	5.0	5.0	4.96	4.74	5.0
100	20	7.37	7.02	8.28	1.92	1.25	8.89

Notes: Estimated number of factors averaged over 1000 simulations. The true number of factors is r and the maximum number of factors is ceil(min$\{T, N\}$)/2).

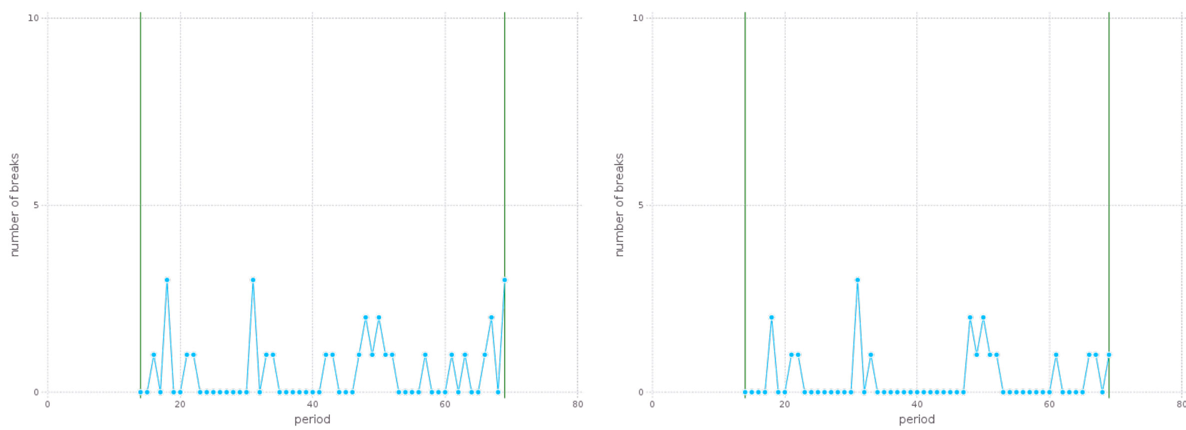
D Figures - Empirical Section



(a) 5% significance level

(b) 1% significance level

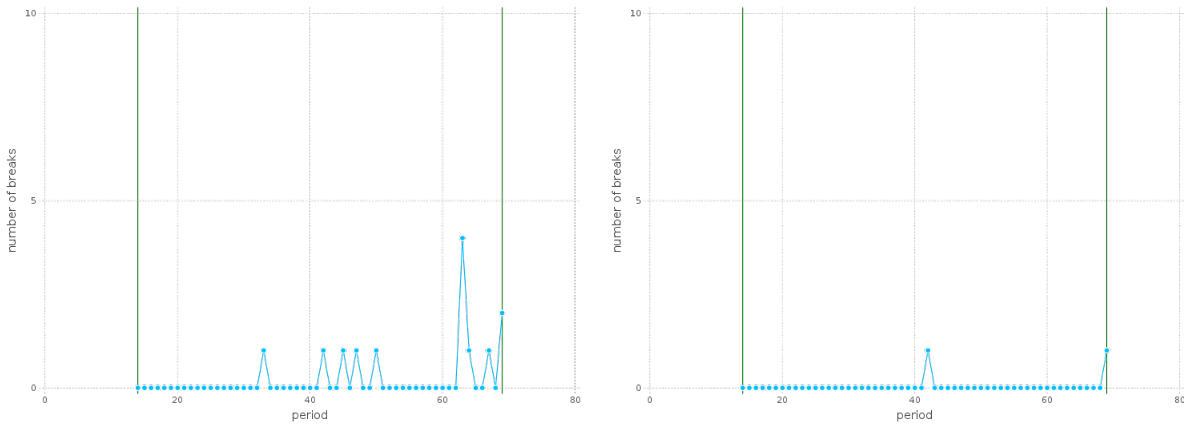
Figure 3: Second cluster removed



(a) 5% significance level

(b) 1% significance level

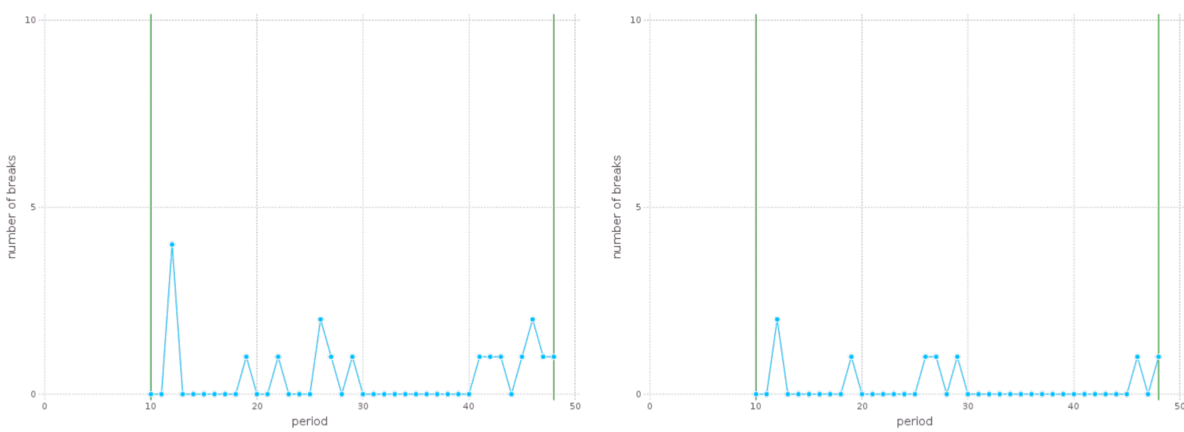
Figure 4: First and second cluster removed



(a) 5% significance level

(b) 1% significance level

Figure 5: All variables with breaks removed



(a) 5% significance level

(b) 1% significance level

Figure 6: Second cluster removed, data set trimmed before 1991 quarter 1

E Tables - Empirical Section

k	$N_{\text{breaks removed}}$	Δ	DM	p
20	13	-0.0565	-1.66	0.06
24	19	-0.0546	-1.21	0.14
26	19	0.0466	-1.52	0.08
28	18	0.0394	-0.23	0.41
30	21	0.0369	-0.36	0.36
31	22	0.0411	-0.22	0.41
32	21	0.0275	-0.28	0.39
33	22	0.0326	-0.19	0.42
34	21	0.0257	-0.14	0.45
35	24	-0.0311	0.90	0.81
36	22	0.0548	0.06	0.52
37	25	0.0603	-0.55	0.30

k	$N_{\text{breaks removed}}$	Δ	DM	p
20	13	-0.0565	-1.66	0.49
24	19	-0.0546	-1.21	0.53
26	19	0.0466	-1.52	0.52
28	18	0.0394	-0.23	0.54
30	21	0.0369	-0.36	0.55
31	22	0.0411	-0.22	0.56
32	21	0.0275	-0.28	0.57
33	22	0.0326	-0.19	0.48
34	21	0.0257	-0.14	0.60
35	24	-0.0311	0.90	0.48
36	22	0.0548	0.06	0.58

Critical values for the Diebold-Mariano test of improved accuracy:
$DM_{1\%} = 2.62$, $DM_{5\%} = 1.76$, $DM_{10\%} = 1.35$
Critical values for the Diebold-Mariano test of decereased accuracy:
$DM_{1\%} = -2.62$, $DM_{5\%} = -1.76$, $DM_{10\%} = -1.35$

Notes: The entries report the performance improvement achieved through removing the structural breaks from data sets which have been targeted using soft thresholding with k steps. Note that results with $k \in \{21, 22, 23, 25, 27, 29\}$ are not presented because the estimates for the number of factors was higher than 22 and there are no tabulated values for the sup LM statistic then. $N_{\text{breaks removed}}$ is the number of variables left in the model after structural breaks are removed. DM denotes the small sample corrected Diebold Mariano statistic from Harvey et al. (1997).

Table 10: Improvement in RMSE by removing structural breaks from targeted data

F Data series

Table 11: Bundesbank data series

	Series Id	Series Title
1	BBNA2.Q.DE.N.I. 021.CA.D.V.I05.A,	Actual earnings/ Overall economy/ Wages and salaries per employee (workplace concept)/ Index/ Germany
2	BBK01.EA4220,	Balance on current account
3	BBK01.DU7834,	Basic pay rates, overall economy, excluding ancillary benefits, excluding one-off payments, on a monthly basis, Germany
4	BBXS1.A.I7.N.EUBUCS. MANU.MAN013.TT.PCT.I00	Business and consumer surveys (European Commission) / Capacity utilization (man) / Euro area 18 (fixed composition)
5	BBXS1.A.DE.N.EUBUCS. MANU.MAN013.TT.PCT.I00	Business and consumer surveys (European Commission) / Capacity utilization (man) / Germany
6	BBK01.OU7744,	Capital / Total / All categories of banks
7	BBK01.BQ1351,	Central government debt - total debt
8	BBDE1.M.DE.Y.JAA9. P2XF50500.CB.V.ABA.A	Construction permits - estimated construction cost / At current prices, flows / Germany / Total (including construction work on existing buildings) - residential and non-residential buildings, total
9	BBXR1.M.HWWI.N.EURO. TOT- NXNGY.INDBEU.HE.M00	Construction price index / Germany / Unadjusted figure / Total
10	BBK01.OXA8R3	domestic Financial Vehicle Corporations - All categories of banks
11	BBDP1.M.DE.Y.VPI. C.A00000.I10.L	Consumer price index / Germany / Unadjusted figure / Total
12	BBK01.WX9141	DAX price index / End 1987 = 1000 / End of month
13	BBK01.WT4065	DE0001102358 / Prices / 1,500 Bund 14/24
14	BBK01.SUD106S	Effective interest rates of German banks / New business (estimated) / Households' deposits, overnight
15	BBK01.SUD210	Effective interest rates of German banks / New business / Non-financial corporations' deposits, overnight
16	BBDE1.M.DE.Y.GUA1. P2XF13020.BA.V.ABA.A	Employed persons (including proprietors and family workers) / Persons / Germany / Main construction industry
17	BBK01.US8CA0	Employees subject to social security contributions / Total / Eastern Germany
18	BBK01.US80CA	Employees subject to social security contributions / Total / Western Germany

	Series Id	Series Title
19	BBK01.USCA80	Employment in Germany / Employed persons in Germany / Germany
20	BBK01.JAB938	Enterprises
21	BBXN1.A.I7.N.NAG. B1QG00.0000.Z0N.IND.I00	Euro area 18 (fixed composition) / National accounts - Gross domestic product / Gross domestic product / Total / Indices
22	BBK01.XS5609,	Exports / European countries
23	BBK01.XX3382,	Exports of the Federal Republic of Germany / Special Trade / Total / 1 / 2 /
24	BBK01.EU4370,	Financial transactions / Other investment / Balance
25	BBK01.EU2518,	Foreign investment in Germany / Other investment / Total / Balance
26	BBK01.EU4510	Foreign trade / Exports (fob)
27	BBMF1.M.D0.EUR. GOVT.WYWG.Y10.BID	Geldmenge M1 / Veränderung saisonbereinigt / Jahresrate / EWU
28	BBK01.BQ2190	General government budgetary position (Germany) - total revenue
29	BBK01.BQM403	General government debt - of which currency and deposits
30	BBK01.BQM208	General government debt - of which loans
31	BBK01.BQ8068	General government debt as defined in the Maastricht Treaty - Germany - overall
32	BBK01.BJ8072	General government deficit / surplus as defined in the Maastricht Treaty - Germany - overall
33	BBK01.XS4204	German exports / special trade / values / 1 / 2 / 3 / 4
34	BBK01.XS4206	German foreign trade balance / special trade / values / 1 / 2 / 3 / 4
35	BBK01.XS4205	German imports / special trade / values / 1 / 2 / 3 / 4
36	BBK01.EC7750	German investment abroad / EU member states (28) / Total / Balance
37	BBK01.ED9065	German investment abroad / Emerging markets and developing countries / Total / Balance
38	BBK01.ED9004	German investment abroad / Euro-area member states / Total / Balance
39	BBK01.ED9001	German investment abroad / Europe / Total / Balance
40	BBK01.ED1096	German investment abroad / Other EU member states / Total / Balance
41	BBK01.ED9031	German investment abroad / Other European countries / Total / Balance
42	BBK01.JBA161	Gross domestic product / at current prices / Seasonally and working-day adjusted
43	BBK01.JAA026	Gross national income (GNP) / at current prices
44	BBK01.WU0042	Gross sales of domestic debt securities at nominal value / Total
45	BBK01.JAA327	Gross wages and salaries
46	BBXP1.M.U2.N.HICP. 000000.IND.I00	Hamonized Index of Consumer Prices / Euro area (changing composition) / Overall index / Unadjusted figure / Monthly
47	BBK01.TQ7162	Households - Household debt to GDP FSI-I033
48	BBK01.XS8953	Imports / Product Classification for Production / Statistics from 2009 Total excluding energy

	Series Id	Series Title
49	BBK01.XS8857	Imports / Value / Total, excluding energy
50	BBK01.EU4169	Income / Total / Receipts
51	BBDE1.Q.DE.Y.LCB1. A2N200000.A.L.I08.A1	Index of gross wages and salaries / Germany / Mining and quarrying, manufacturing and service activities (B-S)
52	BBDE1.Q.DE.Y.LCA1. A2N200000.A.L.I08.A1	Index of labour costs / Germany / Mining and quarrying, manufacturing and service activities (B-S)
53	BBDE1.Q.DE.Y.LCC1. A2N200000.A.L.I08.A1	Index of non-wage costs / Germany / Mining and quarrying, manufacturing and service activities (B-S)
54	BBDP1.M.DE.Y. VPI.C.SVXR.I10.A	Index of producer prices of industrial products sold on the domestic market / Germany
55	BBEE1.M.DE.AAA. XY0U12.R.AACPB.M00	Indicator of the German economy's price competitiveness against 24 selected industrial countries, based on the deflators of total sales
56	BBK01.XSC430	Indices of foreign trade prices - exports, total / Germany
57	BBXE1.M.I7.W. PROD.NS0020.IND.I00	Industrieproduktion / Gesamte Industrie (ohne Baugewerbe) / Index / nur kalenderbereinigt / Euro-Währungsgebiet-18
58	BBK01.EB0057	International Investment Position, all countries, net, total (from end 1997)
59	BBK01.OEAA56G	Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Baden-Württemberg
60	BBK01.OEAA21H	Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Bayern
61	BBK01.OEAA03I	Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Berlin
62	BBK01.OEAA52S	Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Brandenburg
63	BBK01.OEAA56J	Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Bremen
64	BBK01.OEAA01K	Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Hamburg
65	BBK01.OEAA56L	Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Hessen
66	BBK01.OEAA37R	Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Mecklenburg-Vorpommern
67	BBK01.OEAA56M	Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Niedersachsen
68	BBK01.OEAA56N	Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Nordrhein-Westfalen
69	BBK01.OEAA03O	Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Rheinland-Pfalz
70	BBK01.OEAA03P	Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Saarland
71	BBK01.OEAA56V	Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Sachsen
72	BBK01.OEAA02T	Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Sachsen-Anhalt

	Series Id	Series Title
73	BBK01.OEAA56Q	Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Schleswig-Holstein
74	BBK01.OEAA56U	Kredite an Nichtbanken (Nicht-MFIs) / Nichtbanken (Nicht-MFIs) insgesamt / Kredite insgesamt / Thüringen
75	BBK01.OEAB01G	Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Baden-Württemberg
76	BBK01.OEBB21H	Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Bayern
77	BBK01.OEAB01I	Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Berlin
78	BBK01.OEAB01S	Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Brandenburg
79	BBK01.OEAB01J	Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Bremen
80	BBK01.OEBB12K	Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Hamburg
81	BBK01.OELB02L	Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Hessen
82	BBK01.OEAB01R	Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Mecklenburg-Vorpommern
83	BBK01.OEAB01M	Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Niedersachsen
84	BBK01.OEAB01N	Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Nordrhein-Westfalen
85	BBK01.OELB03O	Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Rheinland-Pfalz
86	BBK01.OEAB01P	Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Saarland
87	BBK01.OEYB19V	Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Sachsen
88	BBK01.OEAB01T	Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Sachsen-Anhalt
89	BBK01.OEAB01Q	Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Schleswig-Holstein
90	BBK01.OEAB01U	Kredite an Nichtbanken (Nicht-MFIs) / insgesamt / zusammen / Alle Bankengruppen / Thüringen
91	BBK01.SU0009	Lending rates of banks / Current account credit less than EUR 100,000 / Average interest rate
92	BBK01.SU0511	Lending rates of banks / Long-term fixed-rate loans to enterprises and self-employed persons, EUR 100,000 and more but less than EUR 500,000, effective interest rate / Average interest rate
93	BBK01.SU0012	Lending rates of banks / Mortgage loans secured by residential real estate with interest rates fixed for 2 years, effective interest rate / Average interest rate
94	BBK01.OU7677	Lending to banks (MFIs) / Balances and loans / All categories of banks

	Series Id	Series Title
95	BBK01.OU7676	Lending to banks (MFIs) / Total / All categories of banks
96	BBK01.PC3767	Lending to computer and related activities, research and development / Total / All categories of banks
97	BBK01.PC3723	Lending to construction / Total/ All categories of banks
98	BBK01.OU7712	Lending to domestic banks (MFIs) / Credit balances and loans / All categories of banks
99	BBK01.TUD364	MONEY STOCK M1 (FROM 2002, EXCLUDING CURRENCY IN CIRCULATION) / GERMAN CONTRIBUTION
100	BBK01.TQ7036	Market liquidity - Average bid-ask spread in the securities market - government bills FSI-I035
101	BBK01.OU8045	Medium and long-term lending / to general government / Securities / All categories of banks
102	BBK01.OU8041	Medium and long-term lending to general government / Loans / Long-term / All categories of banks
103	BBK01.OU9829	Medium and long-term lending to general government / Loans / Medium-term / All categories of banks
104	BBK01.TS1303	Monetary aggregate M3 (from January 2002, excluding currency in circulation; from June 2010, excluding repos with central counterparties) / German contribution / Outstanding amounts at the end of the month (stocks)
105	BBK01.OUP001	Money market paper issued by banks (MFIs) / All categories of banks
106	BBK01.SU0295	Money market rates / EONIA / Monthly average
107	BBK01.SU0334G	Money market rates / EURIBOR three-month funds / Moving monthly average
108	BBK01.TVE309	Money stock M3 / EMU / Total / Changes
109	BBK01.JB5001	National accounts - domestic demand (price adjusted)
110	BBK01.JB5007	National accounts - employment / Germany / 1 / 2 /
111	BBK01.JB5005	National accounts - exports (price adjusted) /
112	BBK01.JQ5003	National accounts - government consumption (price adjusted)
113	BBK01.JB5004	National accounts - gross fixed capital formation (price adjusted)
114	BBK01.JB5008	National accounts - labour costs per employee /
115	BBK01.JQ5002	National accounts - private consumption (price adjusted)
116	BBK01.BJ9195	National accounts - total general government revenue (ESA 95) - Germany as a whole (western Germany up to and including 1990)
117	BBK01.BJ9049	National accounts - total general government revenue (GFS as defined by the ECB) - Germany as a whole
118	BBNA2.Q.DE.N.I.041.CA.D.V.I05.A	National accounts / Gross wages and salaries per hour worked by employees / Whole economy / Germany
119	BBK01.JJA327	National accounts/Households' income/Germany/ Gross wages and salaries (residence concept)
120	BBK01.JQA024	National accounts/Labour productivity per hour worked by persons in employment/ Whole economy
121	BBK01.JJB071	National accounts/Origin of GDP/Chain-linked index/ Production sector (excluding construction)

	Series Id	Series Title
122	BBK01.JQA001	National accounts/Overall economic view/ Compensation of employees - residents
123	BBK01.JQC000	National accounts/Overall economic view/At current prices/ GDP
124	BBK01.JJB949	National accounts/Unit labour costs per hour/ Whole economy (excluding agriculture, forestry and fishing, public services, education and health and other services)
125	BBK01.JQA111	National accounts/Use of GDP/At current prices/ Private consumption
126	BBK01.JQC111	National accounts/Use of GDP/Price index/ Private consumption
127	BBK01.OXA8T5	off-balance true sale of domestic banks (MFIs) - All categories of banks
128	BBEE1.A.I7.AAA. XZE021.R.AACPE.M00	Nominal effective exchange rate of the euro against the currencies of the EER-20 group
129	BBK01.TQ7032	Non-financial corporations sector - Total debt to equity FSI-I028
130	BBDE1.M.DE.Y.AEA1. A2P300000.F.C.I10.L	Orders received / At constant prices / Germany / Industry / Unadjusted figure
131	BBDE1.M.DE.Y.AEA1. P2XF04000.B2.C.I10.A	Orders received / At constant prices / Germany / Main construction industry
132	BBDE1.M.DE.W.AEA1. A2P340000.F.V.I10.A	Orders received / At current prices / Germany / Industry / Calendar adjusted only
133	BBDE1.M.DE.Y.AEA1. A2P300000.F.C.I10.A	Orders received / At current prices / Germany / Industry
134	BBDE1.M.DE.Y.AEA1. A2Q501000.F.V.I10.A	Orders received / At current prices, flows / Germany / 20+21 Manufacture of chemicals, chemical products; basic pharmaceutical products and pharmaceutical preparations
135	BBDE1.M.DE.W.AEA1. P2XF04000.B2.V.I10.A	Orders received / At current prices, flows / Germany / Main construction industry / Calendar adjusted only
136	BBDE1.M.DE.Y.AEA1. P2XF13010.B2.V.I10.A	Orders received / At current prices, flows / Germany / Main construction industry
137	BBDE1.M.DE.W.AEN1. A2P340000.F.V.I10.A	Orders received excluding large orders, domestic / At current prices, flows / Germany / Industry / Calendar adjusted only
138	BBDE1.M.DE.Y.AEN1. A2P350000.F.C.I10.A	Orders received excluding large orders, domestic / At current prices, flows / Germany / Industry
139	BBDE1.M.DE.Y.AEP1. A2P350000.F.C.I10.A	Orders received excluding large orders, euro-area countries / At current prices, flows / Germany / Industry
140	BBDE1.M.DE.W.AEN5. A2P340000.F.V.I10.A	Orders received excluding large orders, foreign / At current prices, flows / Germany / Industry / Calendar adjusted only
141	BBDE1.M.DE.Y.AEN5. A2P350000.F.C.I10.A	Orders received excluding large orders, foreign / At current prices, flows / Germany / Industry
142	BBDE1.M.DE.Y.AEP5. A2P350000.F.C.I10.A	Orders received excluding large orders, non-euro-area countries / At current prices, flows / Germany / Industry
143	BBDE1.M.DE.W.AEM1. A2P340000.F.V.I10.A	Orders received excluding large orders, total / At current prices, flows / Germany / Industry / Calendar adjusted only
144	BBDE1.M.DE.Y.AEM1. A2P350000.F.C.I10.A	Orders received excluding large orders, total / At current prices, flows / Germany / Industry

	Series Id	Series Title
145	BBDE1.M.DE.Y.AEB5. A2P350000.F.C.I10.A	Orders received from abroad / At current prices / Germany / Industry
146	BBDE1.M.DE.Y.AEB1. A2Q501000.F.C.I10.A	Orders received from the domestic market / At constant prices / Germany / 20+21 Manufacture of chemicals, chemical products; basic pharmaceutical products and pharmaceutical preparations
147	BBDE1.M.DE.Y.AEB1. A2P350000.F.C.I10.A	Orders received from the domestic market / At current prices / Germany / Industry
148	BBDE1.M.DE.Y.BAA1. P2XF00000.G.C.I10.L	Output in the production sector / At constant prices / Germany / Main construction industry / Unadjusted figure
149	BBDE1.M.DE.Y.BAA1. A2P300000.G.C.I10.A	Output in the production sector / Germany / Production sector including construction (B -F)
150	BBEX3.M.XAU. DEM.EA.AC.C02,	Price of gold in London / morning fixing * / 1 ounce of fine gold = USD ...
151	BBDE1.M.DE.Y.FG30. A2P200000.F.N.I10.A	Produktionsergebnis je Beschäftigten / Deutschland / Bergbau und Gewinnung von Steinen und Erden sowie Verarbeitendes Gewerbe (B + C) / Angaben für fachliche Betriebsteile / kalender- und saisonbereinigt
152	BBK01.TQ7040	Real estate markets - Residential real estate loans to total loans FSI-I039
153	BBK01.TXI362	TOTAL ASSETS OR LIABILITIES / GERMAN CONTRIBUTION
154	BBK01.TUB618	Total assets
155	BBK01.TUE365	Total assets or liabilities / Euro area
156	BBK01.TUB641	Total liabilities
157	BBK01.UJCY01	Unemployment rate (unemployment as a percentage of the civilian labour force) / Germany

Table 12: FRED data series

	Series Id	Series Title
158	A229RX0	Real Disposable Personal Income: Per capita
159	AAA	Moody's Seasoned Aaa Corporate Bond Yield©
160	AHECONS	Average Hourly Earnings Of Production And Nonsupervisory Employees: Construction
161	AHEMAN	Average Hourly Earnings Of Production And Nonsupervisory Employees: Manufacturing
162	AWHMAN	Average Weekly Hours of Production and Nonsupervisory Employees: Manufacturing
163	AWOTMAN	Average Weekly Overtime Hours of Production and Nonsupervisory Employees: Manufacturing
164	BAA	Moody's Seasoned Baa Corporate Bond Yield©
165	BOGMBASE	Monetary Base; Total
166	BUSLOANS	Commercial and Industrial Loans, All Commercial Banks
167	BUSLOANSNSA	Commercial and Industrial Loans, All Commercial Banks
168	CES0600000007	Average Weekly Hours of Production and Nonsupervisory Employees: Goods-Producing
169	CES0600000008	Average Hourly Earnings of Production and Nonsupervisory Employees: Goods-Producing
170	CES2000000008	Average Hourly Earnings of Production and Nonsupervisory Employees: Construction
171	CES3000000008	Average Hourly Earnings of Production and Nonsupervisory Employees: Manufacturing
172	CEU0600000007	Average Weekly Hours of Production and Nonsupervisory Employees: Goods-Producing
173	CEU0600000008	Average Hourly Earnings of Production and Nonsupervisory Employees: Goods-Producing
174	CEU1000000001	All Employees: Mining and Logging
175	CEU2000000001	All Employees: Construction
176	CEU3000000001	All Employees: Manufacturing
177	CEU3000000007	Average Weekly Hours of Production and Nonsupervisory Employees: Manufacturing
178	CEU3000000009	Average Weekly Overtime Hours of Production and Nonsupervisory Employees: Manufacturing
179	CEU3100000001	All Employees: Durable Goods
180	CEU3200000001	All Employees: Nondurable Goods
181	CEU9000000001	All Employees: Government
182	CIVPART	Civilian Labor Force Participation Rate
183	CLF16OV	Civilian Labor Force
184	CPIAPPNS	Consumer Price Index for All Urban Consumers: Apparel
185	CPIAPPSL	Consumer Price Index for All Urban Consumers: Apparel
186	CPIAUCNS	Consumer Price Index for All Urban Consumers: All Items
187	CPIAUCSL	Consumer Price Index for All Urban Consumers: All Items

	Series Id	Series Title
188	CPILFENS	Consumer Price Index for All Urban Consumers: All Items Less Food & Energy
189	CPILFESL	Consumer Price Index for All Urban Consumers: All Items Less Food & Energy
190	CPIMEDNS	Consumer Price Index for All Urban Consumers: Medical Care
191	CPIMEDSL	Consumer Price Index for All Urban Consumers: Medical Care
192	CPITRNNS	Consumer Price Index for All Urban Consumers: Transportation
193	CPITRNSL	Consumer Price Index for All Urban Consumers: Transportation
194	CURRNS	Currency Component of M1
195	CURRSL	Currency Component of M1
196	DMANEMP	All Employees: Durable goods
197	FEDFUNDS	Effective Federal Funds Rate
198	GS1	1-Year Treasury Constant Maturity Rate
199	GS5	5-Year Treasury Constant Maturity Rate
200	HOUST	Housing Starts: Total: New Privately Owned Housing Units Started
201	HOUSTMW	Housing Starts in Midwest Census Region
202	HOUSTMWNSA	Housing Starts in Midwest Census Region
203	HOUSTNE	Housing Starts in Northeast Census Region
204	HOUSTNSA	Housing Starts: Total: New Privately Owned Housing Units Started
205	HOUSTS	Housing Starts in South Census Region
206	HOUSTSNSA	Housing Starts in South Census Region
207	HOUSTW	Housing Starts in West Census Region
208	HOUSTWNSA	Housing Starts in West Census Region
209	INDPRO	Industrial Production Index
210	IPBUSEQ	Industrial Production: Business Equipment
211	IPCONGD	Industrial Production: Consumer Goods
212	IPDCONGD	Industrial Production: Durable Consumer Goods
213	IPFINAL	Industrial Production: Final Products (Market Group)
214	IPFUELN	Industrial Production: Fuels
215	IPFUELS	Industrial Production: Fuels
216	IPMAT	Industrial Production: Materials
217	IPNCONGD	Industrial Production: Nondurable Consumer Goods
218	LNS12032197	Employment Level - Part-Time for Economic Reasons, Nonagricultural Industries
219	LNU01000000	Civilian Labor Force
220	LNU01300000	Civilian Labor Force Participation Rate
221	M1NS	M1 Money Stock
222	M1SL	M1 Money Stock
223	M2NS	M2 Money Stock
224	M2SL	M2 Money Stock
225	MANEMP	All Employees: Manufacturing

	Series Id	Series Title
226	NAPM	ISM Manufacturing: PMI Composite Index©
227	NAPMEI	ISM Manufacturing: Employment Index©
228	NAPMII	ISM Manufacturing: Inventories Index©
229	NAPMNOI	ISM Manufacturing: New Orders Index©
230	NAPMPI	ISM Manufacturing: Production Index©
231	NAPMSDI	ISM Manufacturing: Supplier Deliveries Index©
232	NDMANEMP	All Employees: Nondurable goods
233	PAYEMS	All Employees: Total nonfarm
234	PAYNSA	All Employees: Total nonfarm
235	PERMITNSA	New Privately-Owned Housing Units Authorized by Building Permits: Total
236	PPIACO	Producer Price Index: All Commodities
237	PPICRM	Producer Price Index: Crude Materials for Further Processing
238	PPIFCF	Producer Price Index: Finished Consumer Foods
239	PPIFGS	Producer Price Index: Finished Goods
240	PPIITM	Producer Price Index: Intermediate Materials: Supplies & Components
241	SRVPRD	All Employees: Service-Providing Industries
242	TB3MS	3-Month Treasury Bill: Secondary Market Rate
243	TB6MS	6-Month Treasury Bill: Secondary Market Rate
244	UEMPMEAN	Average (Mean) Duration of Unemployment
245	UNEMPLOY	Unemployed
246	UNRATE	Civilian Unemployment Rate
247	UNRATENSA	Civilian Unemployment Rate
248	USCONS	All Employees: Construction
249	USGOOD	All Employees: Goods-Producing Industries
250	USGOVT	All Employees: Government
251	USMINE	All Employees: Mining and logging
252	W875RX1	Real personal income excluding current transfer receipts

References

Donald WK Andrews. Tests for parameter instability and structural change with unknown change point. *Econometrica: Journal of the Econometric Society*, pages 821–856, 1993.

Donald WK Andrews. Tests for parameter instability and structural change with unknown change point: A corrigendum. *Econometrica*, pages 395–397, 2003.

Jushan Bai. Inferential theory for factor models of large dimensions. *Econometrica*, 71 (1):135–171, 2003.

Jushan Bai and Serena Ng. Determining the number of factors in approximate factor models. *Econometrica*, 70(1):191–221, 2002.

Jushan Bai and Serena Ng. Forecasting economic time series using targeted predictors. *Journal of Econometrics*, 146(2):304–317, 2008.

Jushan Bai and Serena Ng. Principal components estimation and identification of static factors. *Journal of Econometrics*, 176(1):18–29, 2013.

Jean Boivin and Serena Ng. Are more data always better for factor analysis? *Journal of Econometrics*, 132(1):169–194, 2006.

Jörg Breitung and Sandra Eickmeier. Dynamic factor models. *Allgemeines Statistisches Archiv*, 90(1):27–42, 2006.

Jörg Breitung and Sandra Eickmeier. Testing for structural breaks in dynamic factor models. *Journal of Econometrics*, 163(1):71–84, 2011.

Jörg Breitung and Jörn Tenhofen. Gls estimation of dynamic factor models. *Journal of the American Statistical Association*, 106(495):1150–1166, 2011.

Jörg Breitung, Uta Kretschmer, and Deutsche Bundesbank. Identification and estimation of dynamic factors from large macroeconomic panels. Technical report, Discussion Paper, http://www. ect. unibonn. de/discussion. htm, 2004.

John G Cragg and Stephen G Donald. Inferring the rank of a matrix. *Journal of econometrics*, 76(1):223–250, 1997.

Russell Davidson and James G MacKinnon. *Econometric theory and methods*, volume 5. Oxford University Press New York, 2004.

Bradley Efron, Trevor Hastie, Iain Johnstone, and Robert Tibshirani. Least angle regression. *The Annals of statistics*, 32(2):407–499, 2004.

Mario Forni, Marc Hallin, Marco Lippi, and Lucrezia Reichlin. The generalized dynamic factor model. *Journal of the American Statistical Association*, 100(471):830–840, 2005. doi: 10.1198/016214504000002050. URL `http://dx.doi.org/10.1198/016214504000002050`.

Andrew Gelman and Jennifer Hill. Missing-data imputation. In *Data Analysis Using Regression and Multilevel/Hierarchical Models*, pages 529–544. Cambridge University Press, 2006. ISBN 9780511790942. URL `http://dx.doi.org/10.1017/CBO9780511790942.031`. Cambridge Books Online.

J Geweke. The dynamic factor analysis of economic time series', in (dj aigner and as golberger, eds) latent variables in socio-economic models, 1977.

Domenico Giannone, Lucrezia Reichlin, and Luca Sala. *Tracking Greenspan: systematic and unsystematic monetary policy revisited*. Centre for Economic Policy Research, 2002.

David Harvey, Stephen Leybourne, and Paul Newbold. Testing the equality of prediction mean squared errors. *International Journal of forecasting*, 13(2):281–291, 1997.

Ian Jolliffe. *Principal component analysis*. Wiley Online Library, 2005.

Denis Kwiatkowski, Peter CB Phillips, Peter Schmidt, and Yongcheol Shin. Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root? *Journal of econometrics*, 54(1):159–178, 1992.

Colin L Mallows. Some comments on c p. *Technometrics*, 15(4):661–675, 1973.

M Hashem Pesaran and Allan Timmermann. Selection of estimation window in the presence of breaks. *Journal of Econometrics*, 137(1):134–161, 2007.

Thomas J Sargent and Christopher A Sims. Business cycle modeling without pretending to have too much a priori economic theory. *New methods in business cycle research*, 1:145–168, 1977.

Christian Schumacher. Factor forecasting using international targeted predictors: the case of german gdp. *Economics Letters*, 107(2):95–98, 2010.

G William Schwert. Tests for unit roots: A monte carlo investigation. *Journal of Business & Economic Statistics*, 20(1):5–17, 2002.

Charles Spearman. " general intelligence," objectively determined and measured. *The American Journal of Psychology*, 15(2):201–292, 1904.

James H Stock and Mark W Watson. Implications of dynamic factor models for var analysis. Technical report, National Bureau of Economic Research, 2005.

James H Stock and Mark W Watson. Dynamic factor models. *Oxford Handbook of Economic Forecasting. Oxford University Press, USA*, pages 35–59, 2011.

Robert Tibshirani. Regression shrinkage and selection via the lasso. *Journal of the Royal Statistical Society. Series B (Methodological)*, 58(1):267–288, 1996.

Hui Zou and Trevor Hastie. Regularization and variable selection via the elastic net. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 67(2): 301–320, 2005.