Econometrics Take Home Exam

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1 Problem

1a)

$$\ln L(\beta, X, Y) = \sum_i X_i'\beta - e^{X_i'\beta} Y_i$$

$$S(\beta) = \frac{\partial \ln L}{\partial \beta'} = \sum_i X_i' - X_i e^{X_i'\beta} Y_i$$

$$H(\beta) = \frac{\partial^2}{\partial \beta' \partial \beta} = -\sum_i X_i X_i' \exp^{X_i'\beta} Y_i$$

$I(\beta) = -E(H(\beta)) = \sum_i E(X_i X_i' e^{X_i'\beta} Y_i) = \sum_i E(E(X_i X_i' e^{X'\beta} E(Y|X)) = \sum_i E(X_i X_i' \frac{e^{X'\beta}}{\lambda})) = \sum_i E(X_i X_i')$

1b)

see gauss code

1c)

see gauss code

1d)

Parameters	Estimates	Std. err.	Est./s.e.	Prob.	Gradient
INTER	-2.4123	0.7916	-3.047	0.0023	0.0000
UI	-0.9945	0.2393	-4.156	0.0000	0.0000
RR	0.3305	0.5974	0.553	0.5802	0.0000
RRUI	0.5152	0.5690	0.906	0.3652	0.0000
DR	0.3645	0.8093	0.450	0.6524	0.0000
DRUI	-0.5876	0.9982	-0.589	0.5561	0.0000
LWAGE	0.1900	0.1038	1.830	0.0673	0.0000

The above table corresponds to the table printed by GAUSS after using the 'maxprt' command with the output from maxlik with the given dataset and a vector containing zeros as initial parameter vector. The p-values ('Prob.' column) indicate that only the intercept, UI and maybe LWAGE are significant at sensible significance levels (e.g. 1%, 5% or 10%). This could partly be explained by the interaction terms which capture some of the information of RR and DR.

TODO: drop the following lines??: Dropping the interaction terms results in an increase of the p-value of RR and in an increase of the p-value of DR. However, RRUI seems to explain a substantial amount of RR and similarly DRUI has the opposite sign of DR which hints at a different behaviour among the persons receiving unemployment and those who do not.
Parameters Estimates Std. err. Est./s.e. Prob. Gradient

INTER -2.5134 0.7867 -3.195 0.0014 0.0000
UI -0.8257 0.0590 -13.996 0.0000 0.0000
RR 0.5598 0.4825 1.160 0.2459 0.0000
DR 0.1109 0.5144 0.216 0.8293 0.0000
LWAGE 0.1942 0.1051 1.848 0.0647 0.0000

Since $E(Y_i/X_i) = \frac{1}{\lambda_i} = e^{-X'\beta}$, in this model higher beta parameters result in lower UNDUR while lower (and negative) values of the beta parameter result in higher estimations of UNDUR. Thus UI (reception of unemployment benefits) has a relatively strong effect on the duration of unemployment in our model while RR (replacement rate) and DR (disregard rate) have a lower impact on UNDUR per unit change.

The interaction terms capture the different effects that RR and DR have for people who received unemployment benefits. Interestingly the effect of

DR on UNDUR is smaller than the effect of DRUI (although both parameters are not significantly different from 0) which hints at a stronger effect of the replacement rate on unemployment durations for people which received UI.

Notably and intuitively LWAGE has a relatively high p-value (significant at 10% level). This can be explained as e.g. receiving higher wages might make it more attractive for people to find an open position or also (from a demand perspective) there could be more open positions with higher wages in the respective time frame. LWAGE is defined as the logithm of the weekly wages before unemployment. Thus we would expect an increase of the log weekly wage by 1% to decrease the unemployment duration by $e^{0.19*\ln(1.01)} \approx 1$.

1e)

$$h(\beta) = exp(-X_i'\beta) - z$$

From the lecture notes: $W = h(\hat{\theta}_U)'[\frac{\partial h(\hat{\theta}_U)}{\partial \theta'}\hat{V}(\hat{\theta}_U)\frac{\partial h(\hat{\theta}_U)'}{\partial \theta}]^{-1}h(\hat{\theta}_U)$ with $\theta = \beta$ and $\frac{\partial h(\hat{\beta}_U)}{\partial \beta'} = -X_i' \exp(-X_i'\hat{\beta}_U)$ we have:

$$W = h(\hat{\beta}_U)'[X_i' \exp(-X_i'\hat{\beta}_U)\hat{V}(\hat{\beta}_U)\exp(-\hat{\beta}_U' X_i)]^{-1}h(\hat{\beta}_U) =$$
$$h(\hat{\beta}_U)'[X_i' \exp(-2X_i'\hat{\beta}_U)\hat{V}(\hat{\beta}_U)X_i]^{-1}h(\hat{\beta}_U)$$

Since here $h(\beta)_{1x1} \Rightarrow W = h(\hat{\beta}_U)^2[X_i' \exp(-2X_i'\hat{\beta}_U)\hat{V}(\hat{\beta}_U)X_i]^{-1}$

For the rest of this subproblem, see GAUSS code.

1f)

$$PE(X_i) = \frac{\partial E(Y_i/X_i)}{\partial LWAGE_i} = \frac{\partial \frac{1}{\lambda_i}}{\partial LWAGE_i} = \frac{\partial \frac{1}{\lambda_i}}{\partial X_i^7} = -\beta_7 \exp(-X_i'\beta) = \frac{-\beta_7}{\lambda_i}$$

Where X_i^7 denotes the seventh value of the vector X_i.

1g)

see GAUSS code.