Econometrics Take Home Exam

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1 Problem

1a)

$$\ln L(\beta, X, Y) = \sum_i X_i'\beta - e^{X_i'\beta} Y_i$$

$$S(\beta) = \frac{\partial \ln L}{\partial \beta'} = \sum_i X_i' - X_i e^{X_i'\beta} Y_i$$

$$H(\beta) = \frac{\partial^2}{\partial \beta' \partial \beta} = -\sum_i X_i X_i' \exp^{X_i'\beta} Y_i$$

$I(\beta) = -E(H(\beta)) = \sum_i E(X_i X_i' e^{X_i'\beta} Y_i) = \sum_i E(E(X_i X_i' e^{X'\beta} E(Y_i|X_i)) = \sum_i E(X_i X_i' \frac{e^{X_i'\beta}}{\lambda_i})) = \sum_i E(X_i X_i')$

1b)

see gauss code

1c)

see gauss code

1d)

Parameters	Estimates	Std. err.	Est./s.e.	Prob.	Gradient
INTER	-2.4123	0.7916	-3.047	0.0023	0.0000
UI	-0.9945	0.2393	-4.156	0.0000	0.0000
RR	0.3305	0.5974	0.553	0.5802	0.0000
RRUI	0.5152	0.5690	0.906	0.3652	0.0000
DR	0.3645	0.8093	0.450	0.6524	0.0000
DRUI	-0.5876	0.9982	-0.589	0.5561	0.0000
LWAGE	0.1900	0.1038	1.830	0.0673	0.0000

The above table corresponds to the table printed by GAUSS after using the 'maxprt' command with the output from maxlik with the given dataset and a vector containing zeros as initial parameter vector. The p-values ('Prob.' column) indicate that only the intercept, UI and maybe LWAGE are significant at sensible significance levels (e.g. 1%, 5% or 10%). This could partly be explained by the interaction terms which capture some of the information of RR and DR.

Since $E(Y_i/X_i) = \frac{1}{\lambda_i} = e^{-X'\beta}$, in this model positive beta parameters result in lower UNDUR while negative values of the beta parameters result in higher estimations of UNDUR. Thus UI (reception of unemployment benefits) has a relatively strong positive effect on the duration of unemployment in our model albeit the negative sign of its parameter might suggest otherwise while RR (replacement rate) and DR (disregard rate) have a negative impact on UNDUR per unit of change.

The interaction terms capture the different effects that RR and DR have for people who received unemployment benefits. Interestingly the effect of DR on UNDUR is negative and the effect of DRUI is positive (although both parameters are not significantly different from 0) which hints at the disregard rate increasing unemployment durations for people which received UI. This could be interpreted as a sign that unemployment insurance negatively affects the effectiveness of the disregard rate in combating long unemployment durations.

Notably and intuitively LWAGE has a relatively high p-value (significant at 10% level). This can be explained as receiving higher wages might make it more attractive for people to find an open position or also (from a demand perspective) there could be more open positions with higher wages in the respective time frame. LWAGE is defined as the logarithm of the weekly wages before unemployment. Thus we would expect an increase of the weekly wage by 1% to decrease the unemployment duration by $e^{0.19*\ln(1.01)} \approx 1$.

1e)

$h(\beta) = exp(-X_i'\beta) - z$

From the lecture notes: $W = h(\hat{\theta}_U)'[\frac{\partial h(\hat{\theta}_U)}{\partial \theta'}\hat{V}(\hat{\theta}_U)\frac{\partial h(\hat{\theta}_U)'}{\partial \theta}]^{-1}h(\hat{\theta}_U)$

with $\theta = \beta$ and $\frac{\partial h(\hat{\beta}_U)}{\partial \beta'} = -X_i' \exp(-X_i'\hat{\beta}_U)$ we have:

$$W = h(\hat{\beta}_U)'[X_i' \exp(-X_i'\hat{\beta}_U)\hat{V}(\hat{\beta}_U) \exp(-\hat{\beta}_U' X_i)]^{-1}h(\hat{\beta}_U) =$$
$$h(\hat{\beta}_U)'[X_i' \exp(-2X_i'\hat{\beta}_U)\hat{V}(\hat{\beta}_U)X_i]^{-1}h(\hat{\beta}_U)$$

Since here $h(\beta)_{1x1} \Rightarrow W = h(\hat{\beta}_U)^2[X_i' \exp(-2X_i'\hat{\beta}_U)\hat{V}(\hat{\beta}_U)X_i]^{-1}$

For the rest of this subproblem, see GAUSS code.

1f)

$$PE(X_i) = \frac{\partial E(Y_i|X_i)}{\partial LWAGE_i} = \frac{\partial \frac{1}{\lambda_i}}{\partial LWAGE_i} = \frac{\partial \frac{1}{\lambda_i}}{\partial X_i^7} = -\beta_7 \exp(-X_i'\beta) = \frac{-\beta_7}{\lambda_i}$$

Where X_i^7 denotes the seventh value of the vector X_i.

The partial effect of $E(Y_i|X_i)$ with respect to LWAGE is the expected change of $E(Y_i|X_i)$ if LWAGE changes by 1 unit. The partial effect is negative (β_7 is positive) so if LWAGE increases, UNDUR will decrease in our model.

1g)

see GAUSS code.