Advanced Econometrics

Take-Home Exam #2, WS 2012/13
January 17th – 24th, 2013 (16:00h)

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This exam contains 1 question which is worth ten (10) points.

Your solution should include answers (summary tables and interpretations), as well as your Gauss code. Please hand in the answers as a hard copy in Fabian Krüger's office (F309), and send your Gauss code to `Fabian.Krueger@uni-konstanz.de` in a file named after your student ID number.[1] Please document your answers and work steps in a way that makes it easy to understand what you did and why. Presentation of the results is part of grading.

The hard copy **and** the Gauss code have to be delivered until January 24th, 2013, 16:00h. Solutions delivered after the deadline will not be graded. Do not forget to state your student ID number.

Policy with regard to academic dishonesty:

The grade for your take home exam will be a part of your overall grade of your course. Students who wish to work together on assignment material may do so. However, each student must formulate and hand in their work independently.

Please note that plagiarism is a serious offence! It can be avoided by simply citing the original source of ideas or material that are not your own. Any attempt to plagiarize will be marked with zero (0) points for all take home exams in this course.

Good Luck!

[1] For example, "588497.gss".

Problem 1

This problem deals with the following heteroscedastic model for a sample $i = 1, \ldots, n$ of observations:

$$
\begin{aligned}
Y_i &= \beta X_i + \varepsilon_i, && (1)\\
\varepsilon_i &\overset{\text{i.i.d.}}{\sim} \mathcal{N}\left(0, \exp(X_i)\right), && (2)\\
X_i &\overset{\text{i.i.d.}}{\sim} \mathcal{U}\left(0, 5\right), && (3)
\end{aligned}
$$

where $\mathcal{U}\left(0, 5\right)$ in (3) denotes the uniform distribution with limits 0 and 5. Suppose you want to estimate β in (1), and consider the two following estimators:

1. The OLS estimator
$$\hat{\beta} = \frac{\frac{1}{n}\sum_{i=1}^{n} X_i Y_i}{\frac{1}{n}\sum_{i=1}^{n} X_i^2}, \qquad (4)$$
with estimated variance $\hat{\mathrm{V}}_W[\hat{\beta}]$ given by the White heteroscedasticity robust estimator discussed in the tutorial.
2. The estimator
$$\tilde{\beta} = \frac{\frac{1}{n}\sum_{i=1}^{n} \exp(-X_i) X_i Y_i}{\frac{1}{n}\sum_{i=1}^{n} \exp(-X_i) X_i^2}, \qquad (5)$$
with estimated variance given by
$$\hat{\mathrm{V}}[\tilde{\beta}] = \frac{1}{n} \frac{1}{\frac{1}{n}\sum_{i=1}^{n} \exp(-X_i) X_i^2}. \qquad (6)$$

a) Briefly define (in your own words) what heteroscedasticity is. Name an example relationship (i.e., economic variables Y_i and X_i) for which heteroscedasticity might be an issue. 1 P

b) Show that the estimator $\tilde{\beta}$ in (5) is a method of moments estimator of β. What is the corresponding moment function? 1 P

c) Based on the moment function derived in the last step, show that $\hat{\mathrm{V}}[\tilde{\beta}]$ in (6) is a valid estimator for the asymptotic variance of $\tilde{\beta}$. *Hint: Use Proposition 5.2.1 from the lecture notes.* 1 P

d) Do you expect the two estimators to be consistent for β? Which estimator would you expect to be more precise? Explain your answers based on results you know from the lectures or tutorials. 1 P

e) Write a Gauss procedure with the following structure.

- *Inputs:*
  - $n \times 1$ vector Y (dependent variable)
  - $n \times 1$ matrix X (regressor)
- *Outputs:*
  - $\tilde{\beta}$, scalar, the estimator in (5).
  - $\hat{\mathrm{V}}[\tilde{\beta}]$, scalar, the variance estimator in (6).

1 P

f) Perform the following simulation experiment. For $r = 1, \dots, R = 1000$:

- Simulate $n = 200$ observations from X_i, ε_i and Y_i, where $\beta = 1$ in (1). *Hint: In order to simulate a* $\mathcal{U}(0,5)$ *random variable, simply simulate a* $\mathcal{U}(0,1)$ *variable and multiply it by 5.*
- Apply the procedure from the computer tutorial to the $r-$th simulated data set to obtain $(\hat{\beta}_{(r)}, \hat{\mathrm{V}}_W[\hat{\beta}_{(r)}])$.
- Apply the procedure from part d) to the $r-$th simulated data set to obtain $(\tilde{\beta}_{(r)}, \hat{\mathrm{V}}[\tilde{\beta}_{(r)}])$.
- Compute two different t-statistics for the null hypothesis

$$H_0 : \beta = 0.8;$$

  one based on $\hat{\beta}_{(r)}$ and $\hat{\mathrm{V}}_W[\hat{\beta}_{(r)}]$, the other one based on $\tilde{\beta}_{(r)}$ and $\hat{\mathrm{V}}[\tilde{\beta}_{(r)}]$. For both of these t-statistics, record whether you do or do not reject the H_0 at a 5% significance level (two-sided test).

Compute and report the following quantities in order to summarize the results from $R = 1000$ simulated data sets:

- The average values of $\hat{\beta}_{(r)}$ and $\tilde{\beta}_{(r)}$
- The variances of $\hat{\beta}_{(r)}$ and $\tilde{\beta}_{(r)}$
- The average values of $\hat{\mathrm{V}}_W[\hat{\beta}_{(r)}]$ and $\hat{\mathrm{V}}[\tilde{\beta}_{(r)}]$
- The relative standard errors[2] for both estimators.
- The share of rejections for both t-statistics.

Interpret your results in detail, and compare them to your answers from part d).

Hint: Do not forget to set a random seed at the beginning of your program. State this random seed on your hardcopy solution, to make sure your results are replicable.

5 P

[2] These are often denoted by "RELSE"; see Section 2.5.2 of the course lecture notes.