Stochastic data processing and simulation

A3 assignment

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Question 1:

Using the formula we obtain $S_{T_1}(t)\ to\ S_{T_3}(t)$ is $e^{-\sqrt{t}}$, $S_{T_4}(t)$is $e^{-\frac{1}{105}(t)^{\frac{21}{10}}}$, thus $S_T(t)$ for network is $\left(1-\left(1-e^{-\sqrt{t}}\right)^3\right)\cdot e^{-\frac{1}{105}(t)^{\frac{21}{10}}}$ and expected life length is 3.184948.

Question 2:

We obtain lambda function by network survival function, with command

*q2_a <- expression ((1-(1-exp(-sqrt(t)))^3) * (exp((-1/105) * t^(21/10))))*

*q2_b <- function (t) {(-1 * eval(D(q2_a,"t"))) / ((1 - (1 - exp(-sqrt(t)))^3) * (exp((-1/105) * t^(21/10))))}*

After this we can plot relationship between time(0 to 30) and lambda.

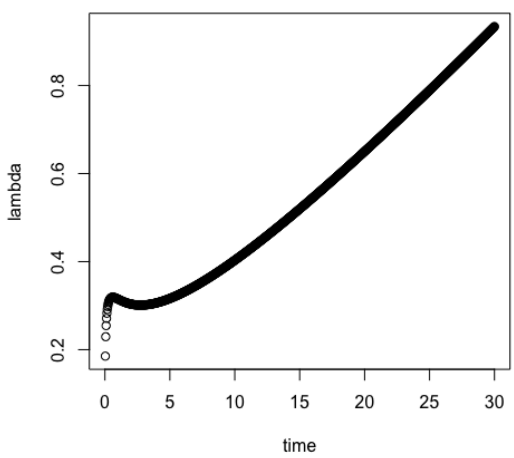

figure 1: relationship between time(0 to 30) and lambda

Question 3:

With *q3_a <- expression(exp((-1/105) * t^(21/10)))* and *q3_T4_fail <- function(t) {-eval(D(q3_a,"t")) * (1 - (1 - exp(-sqrt(t)))^3)}* , we can gen the T_4 fail rate function.

With command *integrate(q3_T4_fail, 0, Inf)* , we obtain 0.2187526 , which means that there's around 0.2187 probability failure of the network is caused by the failure of component T_4 .

Question 4:

Each component would have an exponentially distributed life length with parameter 1, therefore, we obtain $S_T(t)$ and $F_T(t)$ depending on k.

By trial and error, the minimum number of k is 7, so that the probability will be less than 0.2

Question 5:

Same as question 4, the minimum number of k is 6, so that the expected lifetime of the complete network is larger than 3.

Question 6:

We can obtain $S_T(t)$ and $F_T(t)$ for network via components' distributions. After this, I replace γ with $\left(\frac{10}{3} - 3\mu\right)$ via total cost should be 4, to reduce variable in $S_T(t)$.

It's hard to calculate $E[t]$ via $S_T(t)$ in R, therefore, I calculate μ which maximum $S_T(t)$ when t = Inf , with following command.

x.min <- optimize(q6_b, c(0.0001,1.1111), tol = 0.0001 , t = Inf, maximum = TRUE)
x.min

Result in, $S_T(t)$ has maximum value when μ close to $\frac{10}{3}$ and γ close to 0. Hence, $E[t]$ will also be maximum when μ close to $\frac{10}{3}$ and γ close to 0, given total cost should be 4.