A5

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Question 1

$$\theta = \int_{-1}^{1} \frac{1}{2}(x^4 + x^2) = [\frac{1}{10}x^5 + \frac{1}{6}x^3]_{-1}^{1}$$

where,$[\frac{1}{10}x^5 + \frac{1}{6}x^3]_{-1}^{1} = 2\int_0^1 \frac{1}{2}(x^4 + x^2)$ Therefore, $\theta = 2\int_0^1 \frac{1}{2}(x^4 + x^2)$

Question 2

Answer in py.file

Question 3

Answer in py.file

Question 4

$$Var[\theta^{MC}] = Var[\frac{2}{M}\sum_{m=1}^{M} X^{(m)}] = \frac{2}{M}Var[X]$$

$$Var[\theta^{HM}] = Var[\frac{2}{M}\sum_{m=1}^{M} Y^{(m)}] = \frac{2}{M}Var[Y]$$

where, X is crude MC method and Y is hit-or-miss.

$$Var[X] = \int_0^1 (\frac{1}{2}(x^4 + x^2))^2 - (\int_0^1 \frac{1}{2}(x^4 + x^2))^2 = 0.078095$$

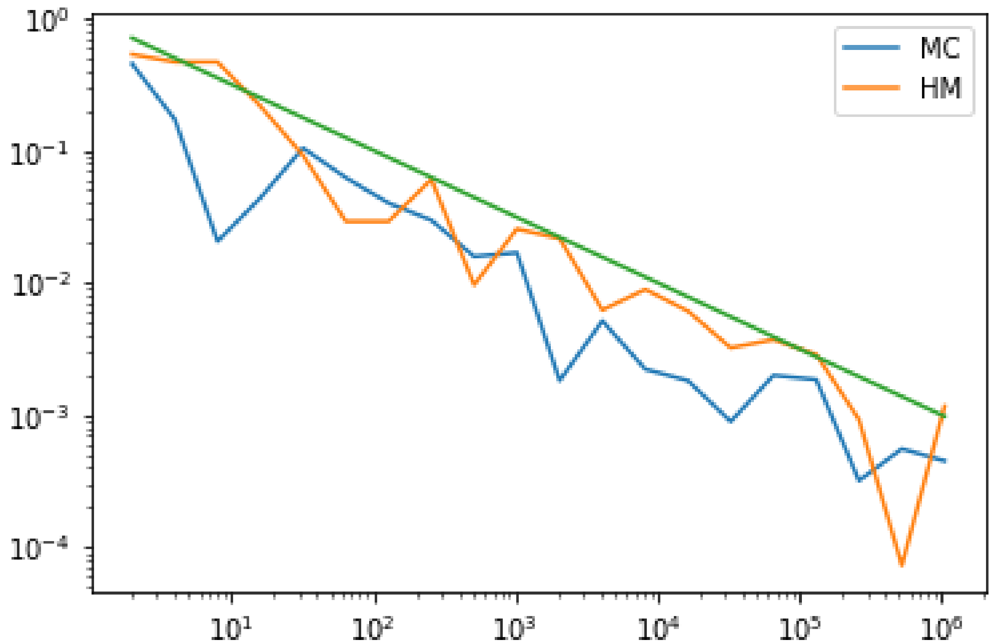
$$Var[Y] = \int_0^1 \frac{1}{2}(x^4 + x^2) - (\int_0^1 \frac{1}{2}(x^4 + x^2))^2 = 0.195555$$

Hence, Variance in MC and HM are $\frac{2}{M}(0.078095)$ and $\frac{2}{M}(0.195555)$, root mean squared are $(\frac{2}{M}0.078095)^{1/2}$ and $(\frac{2}{M}0.195555)^{1/2}$

Question 5

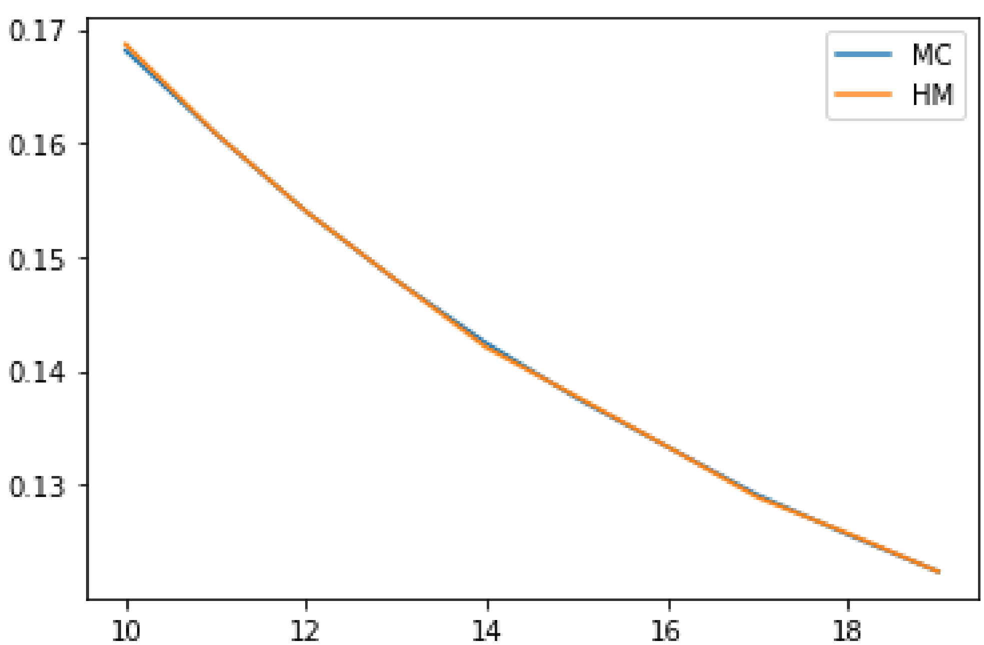
Calculate in python and it follows with our analytical results, if we have bigger sample, the variance and root getting smaller.

Question 6



From plot, error in MC and HM are both converge to zero when sample increasing. MC, generally, obtain smaller error than HM.

Question 7



From plot, Monte Carlo estimate with MC and HM provide us similar outcome. It's as expected since both method should provide same result. While increasing N, both estimate have converge at same speed and direction.