

Dogs and tennis balls

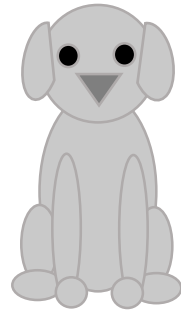
An ROC and AUC example

Emily Miller

Reference

Fawcett, T. (2006). An introduction to ROC analysis. *Pattern recognition letters*, 27(8), 861-874.

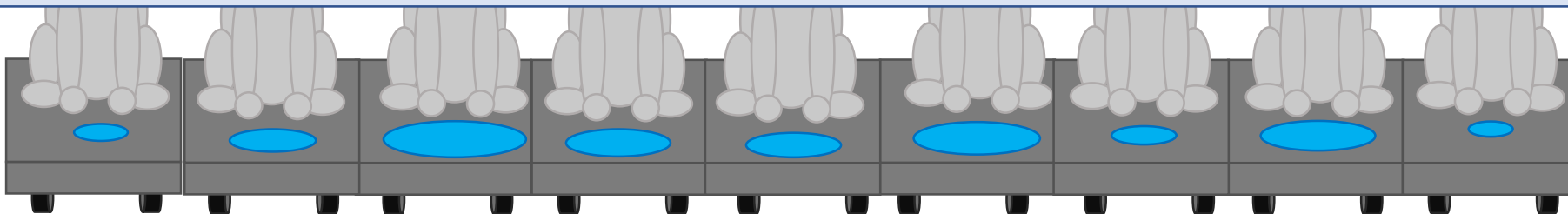
9 dogs, each on
a rolling platform
behind a screen



4 tennis balls



Each tennis ball is
held by a dog; each
dog can hold at most
one tennis ball.



9 dogs, each on
a rolling platform
behind a screen

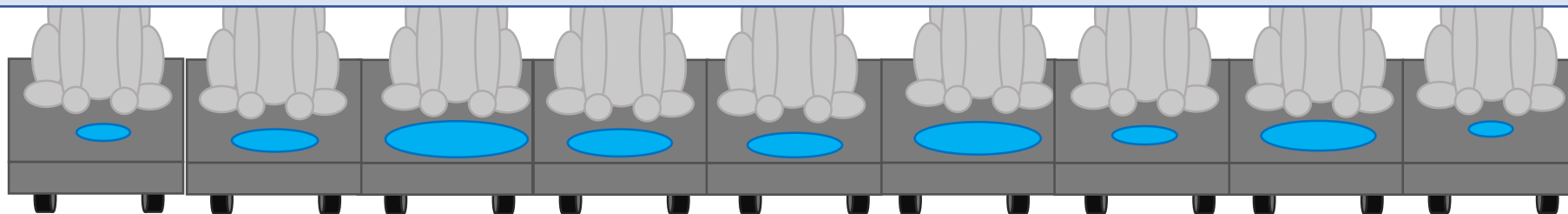


4 tennis balls



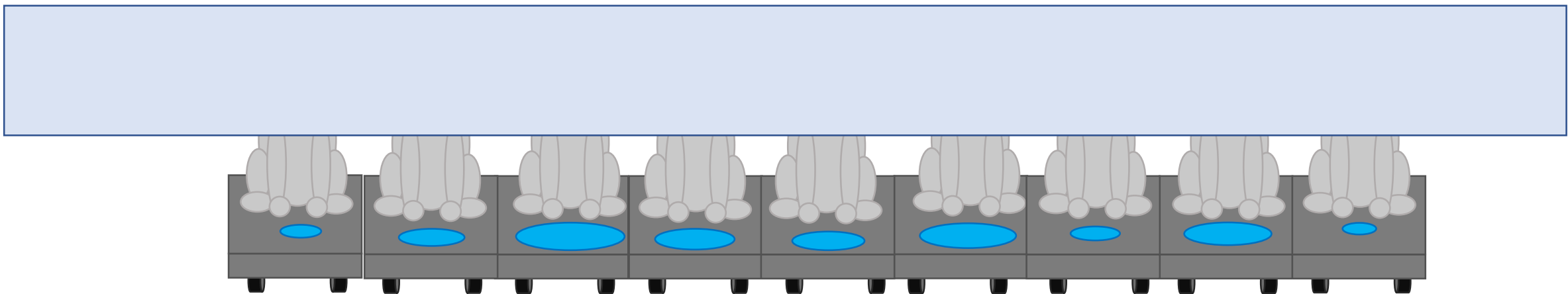
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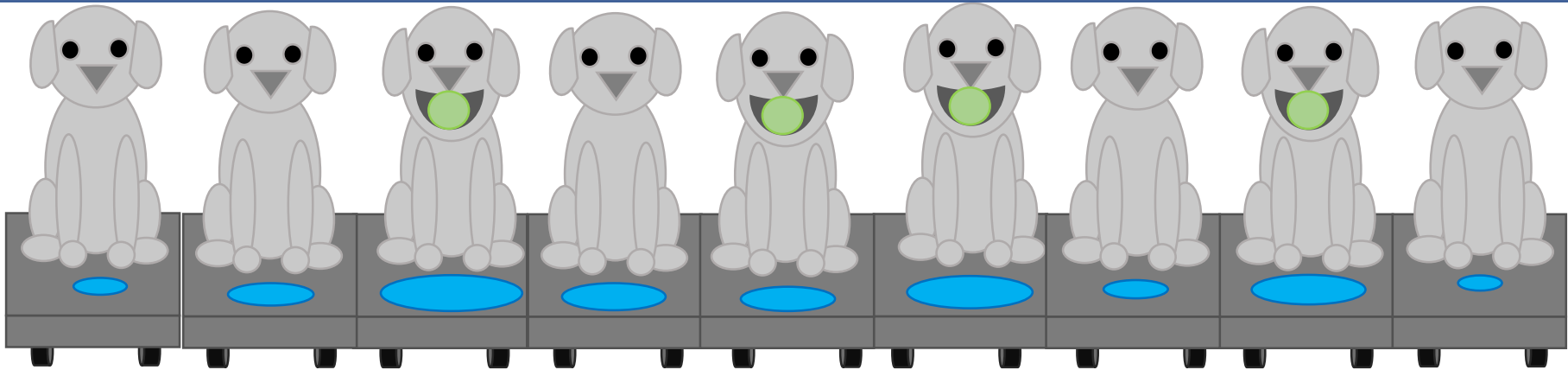
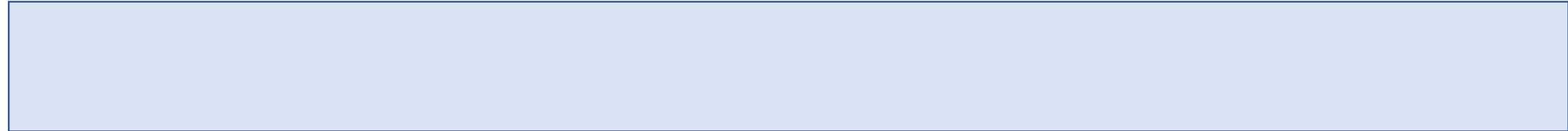
Let's consider two models that predict which dogs are holding tennis balls.



Model 1

We suppose that the farther a dog is now sitting to the left, the more likely that dog is to have a tennis ball.

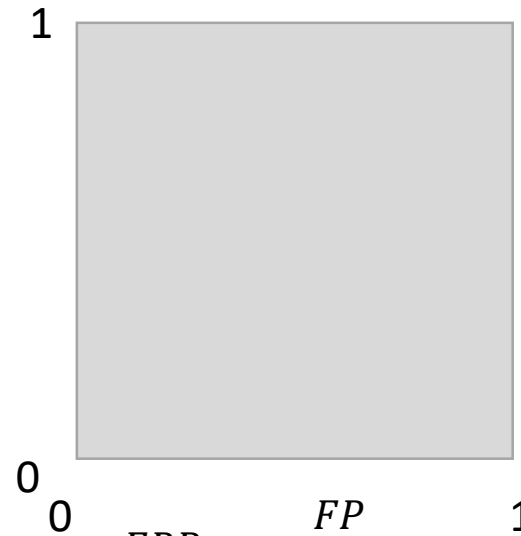




What fraction of the dogs with tennis balls we've included in the set of dogs we've predicted to have tennis balls.

$$TPR = \frac{TP}{TP + FN}$$

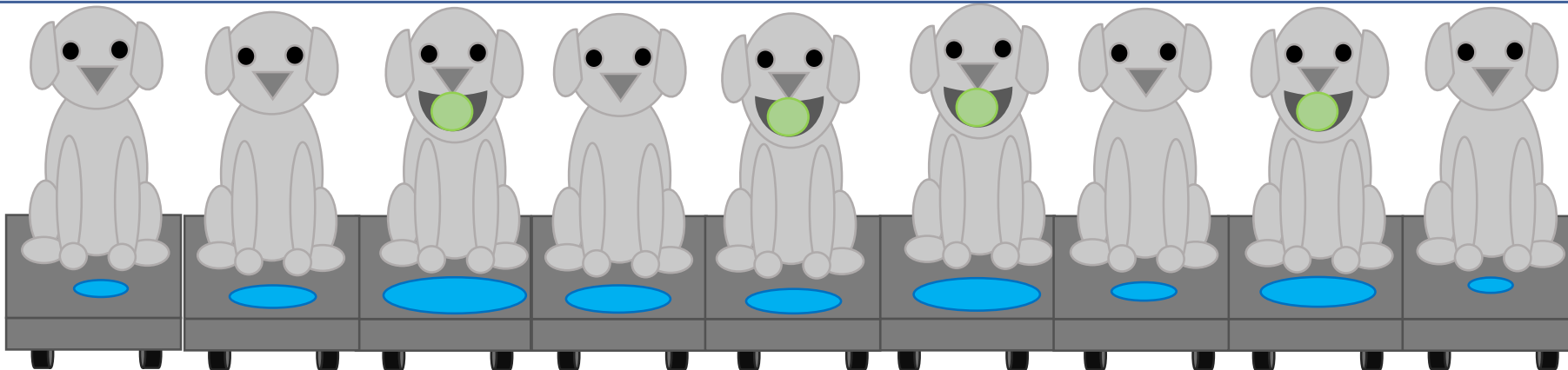
all actual positives



$$FPR = \frac{FP}{FP + TN}$$

all actual negatives

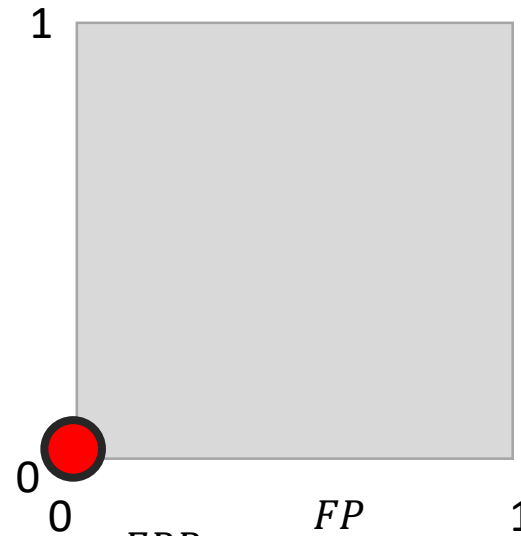
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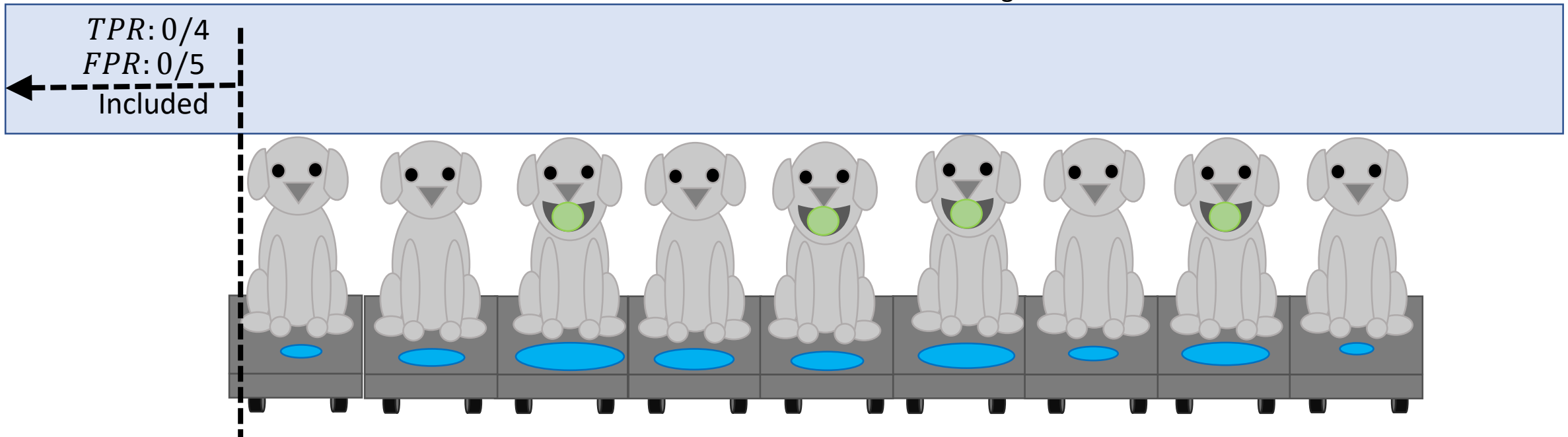
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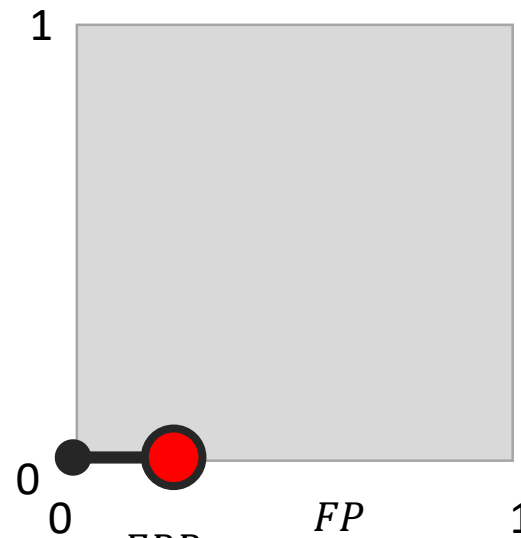
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all actual positives



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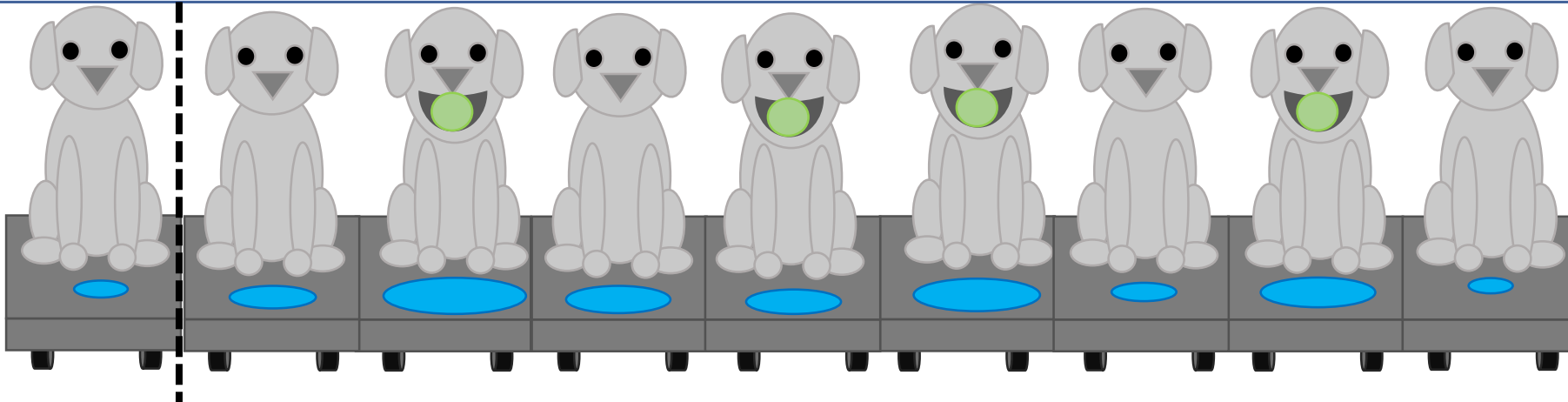
all actual negatives

What fraction of the dogs without tennis balls we've included in the set of dogs we've predicted to have tennis balls.

TPR: 0/4

FPR: 1/5

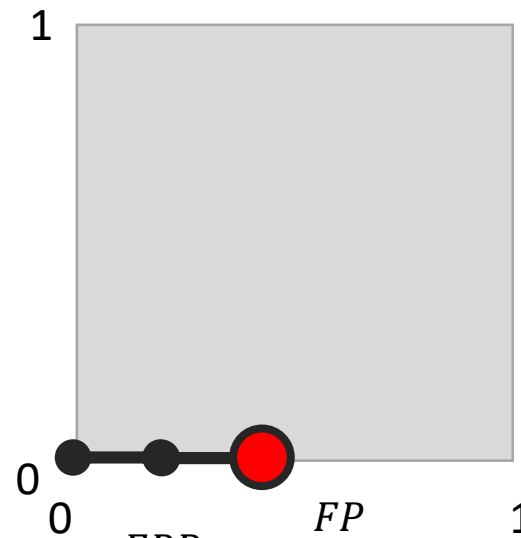
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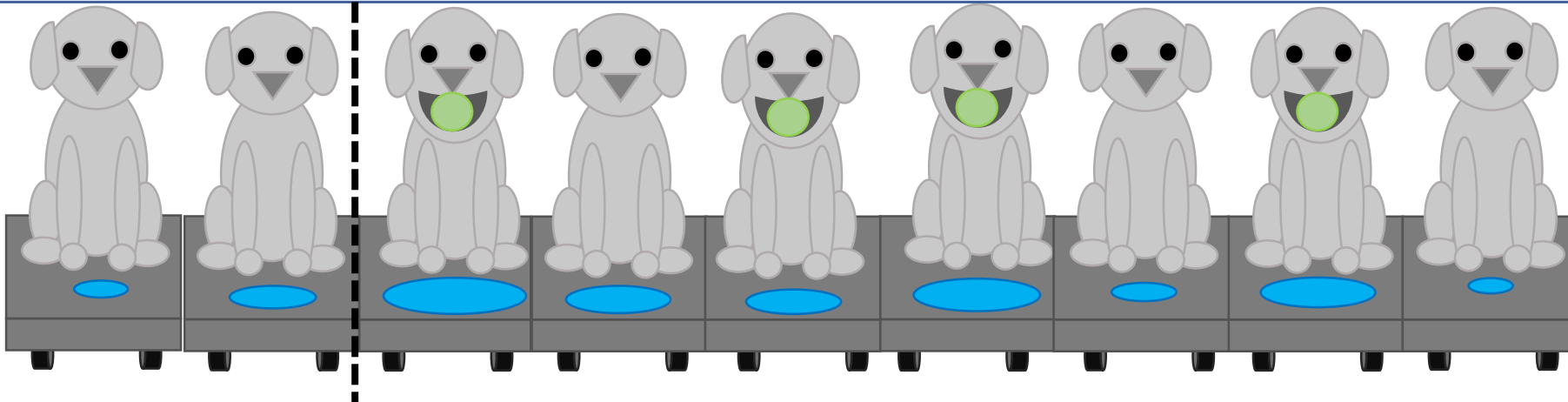
$$TPR = \frac{TP}{TP + FN}$$

all actual positives



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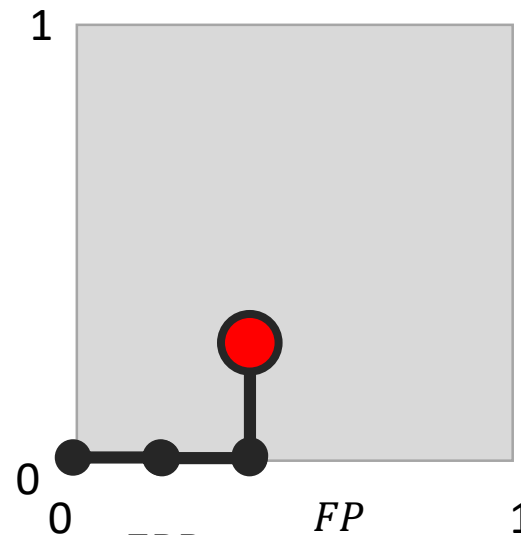
TPR: 0/4
FPR: 2/5
Included



What fraction of the dogs with tennis balls we've included in the set of dogs we've predicted to have tennis balls.

$$TPR = \frac{TP}{TP + FN}$$

all actual positives



$$FPR = \frac{FP}{FP + TN}$$

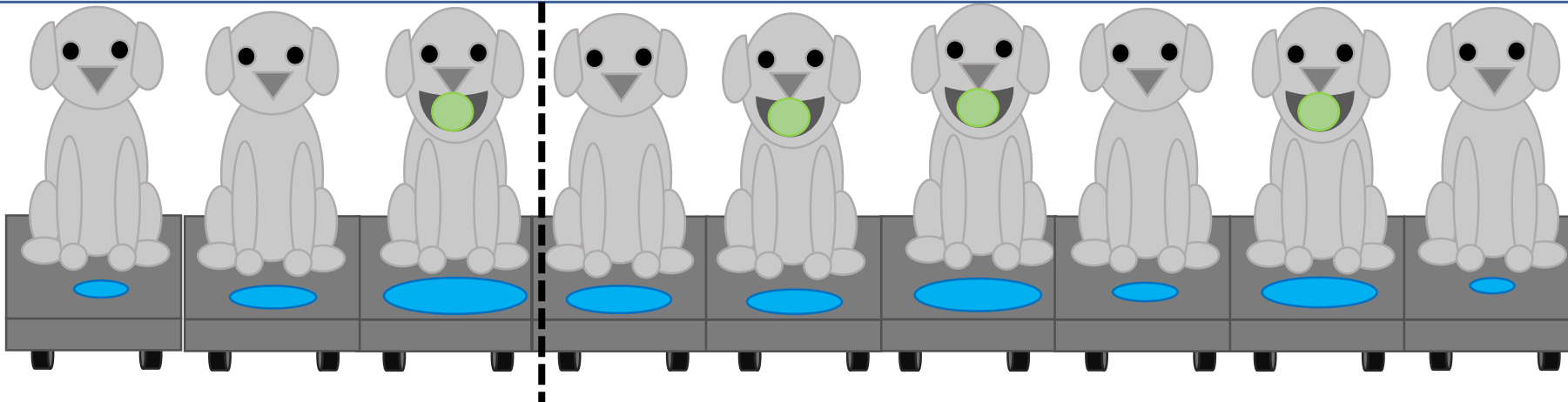
all actual negatives

What fraction of the dogs without tennis balls we've included in the set of dogs we've predicted to have tennis balls.

TPR: 1/4

FPR: 2/5

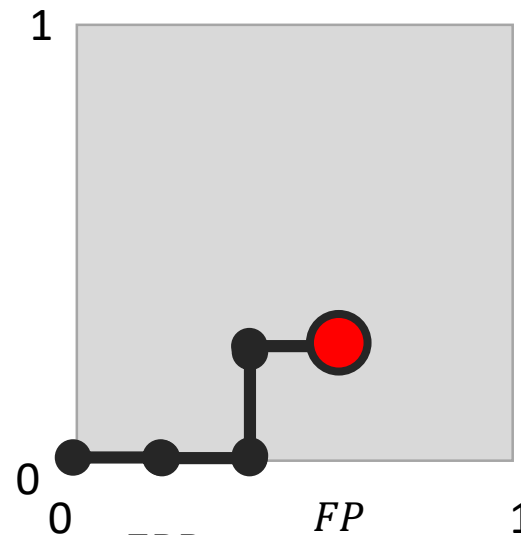
← Included



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$$TPR = \frac{TP}{TP + FN}$$

all actual positives

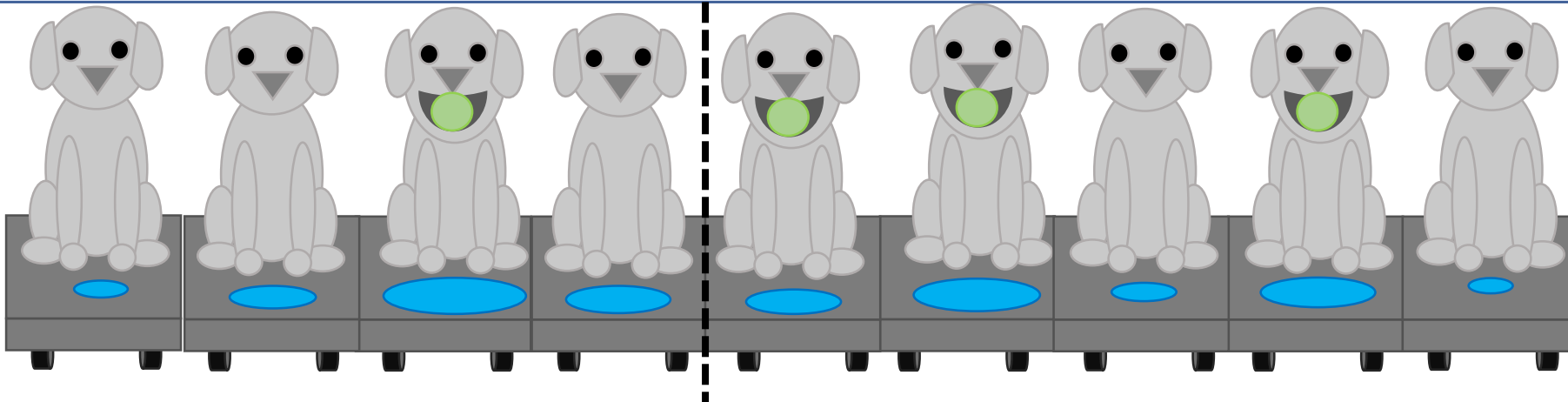


$$FPR = \frac{FP}{FP + TN}$$

all actual negatives

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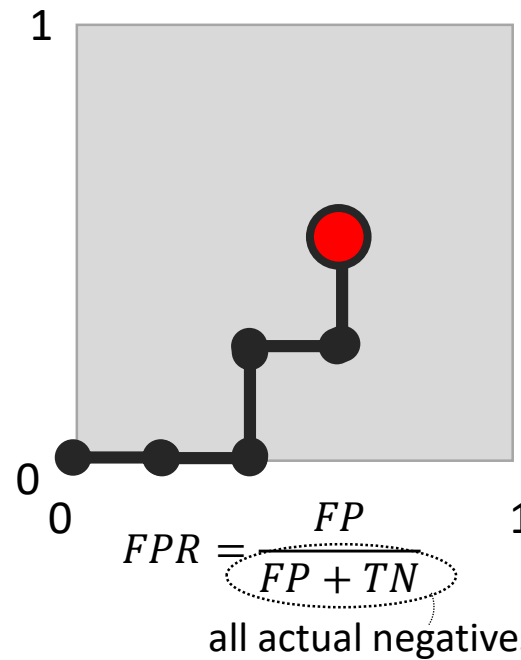
TPR: 1/4
FPR: 3/5
Included



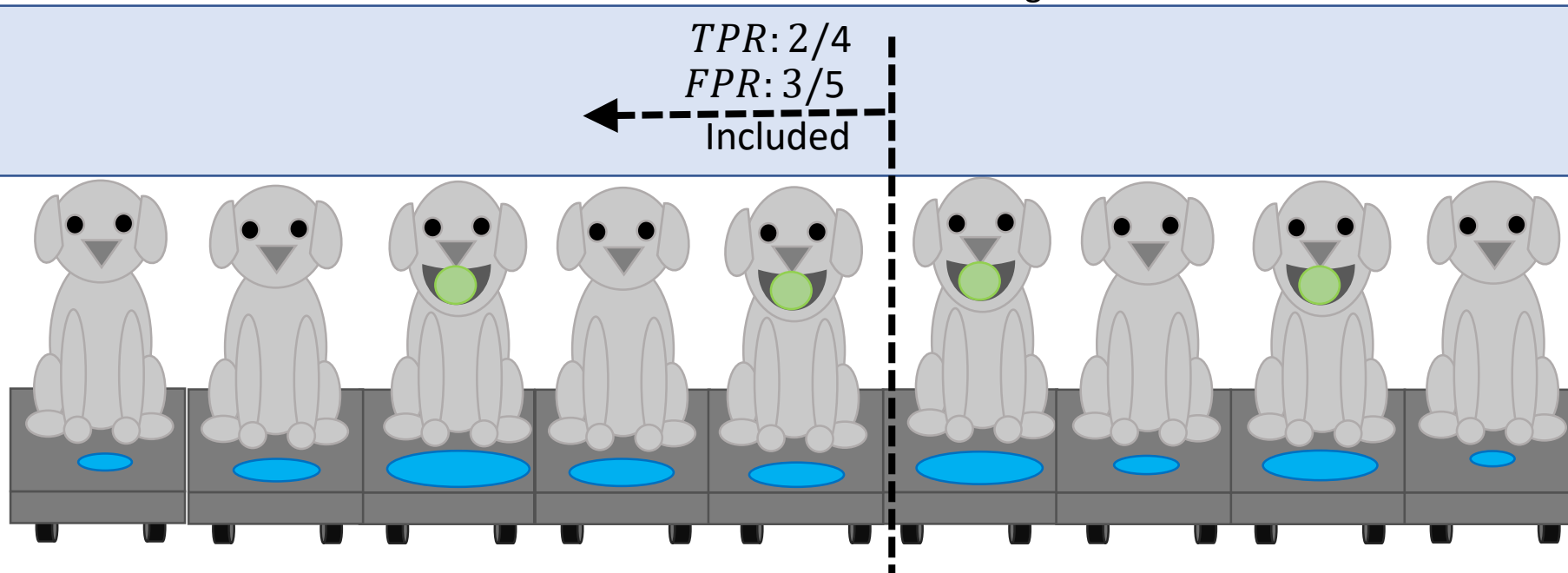
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all actual positives



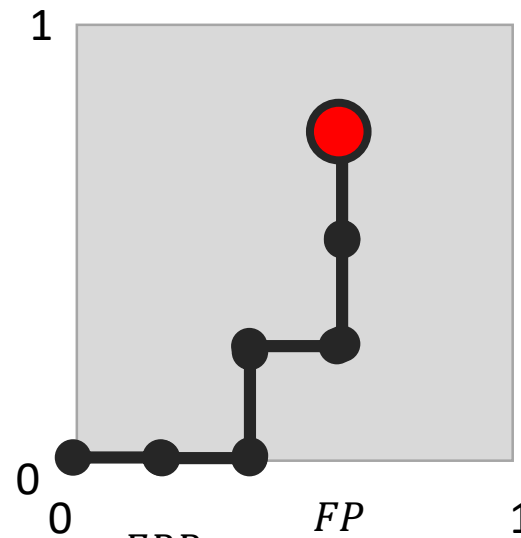
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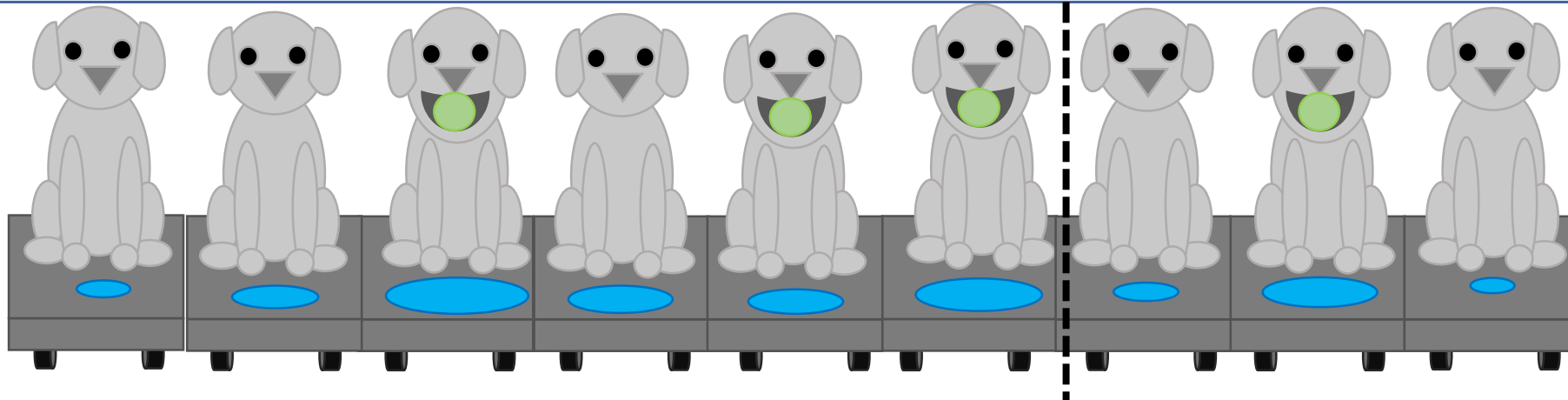
all actual negatives

What fraction of the dogs without tennis balls we've included in the set of dogs we've predicted to have tennis balls.

$TPR: 3/4$

$FPR: 3/5$

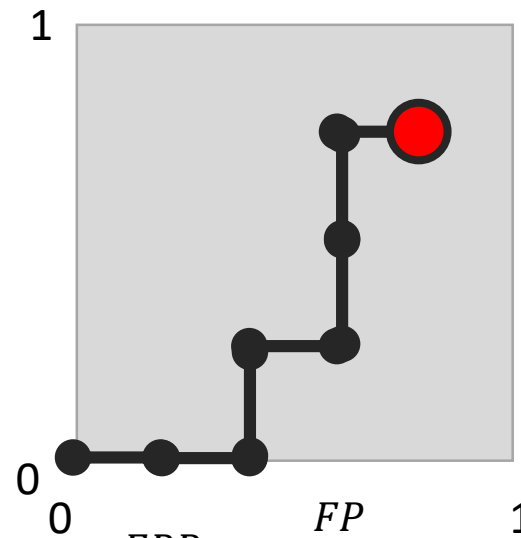
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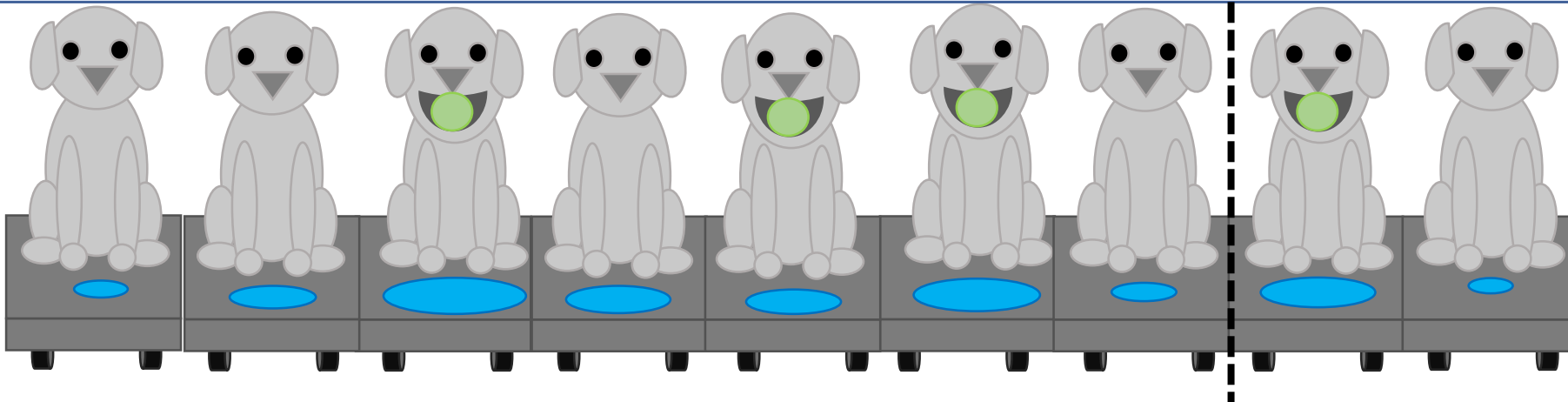


$$FPR = \frac{FP}{FP + TN}$$

all actual negatives

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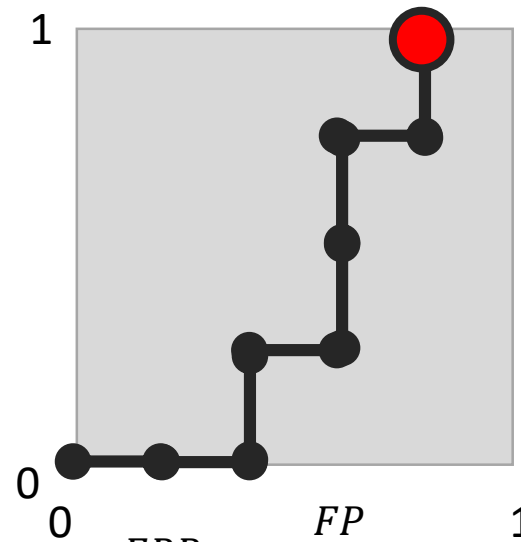
TPR: 3/4
FPR: 4/5
Included



What fraction of the dogs with tennis balls we've included in the set of dogs we've predicted to have tennis balls.

$$TPR = \frac{TP}{TP + FN}$$

all actual positives

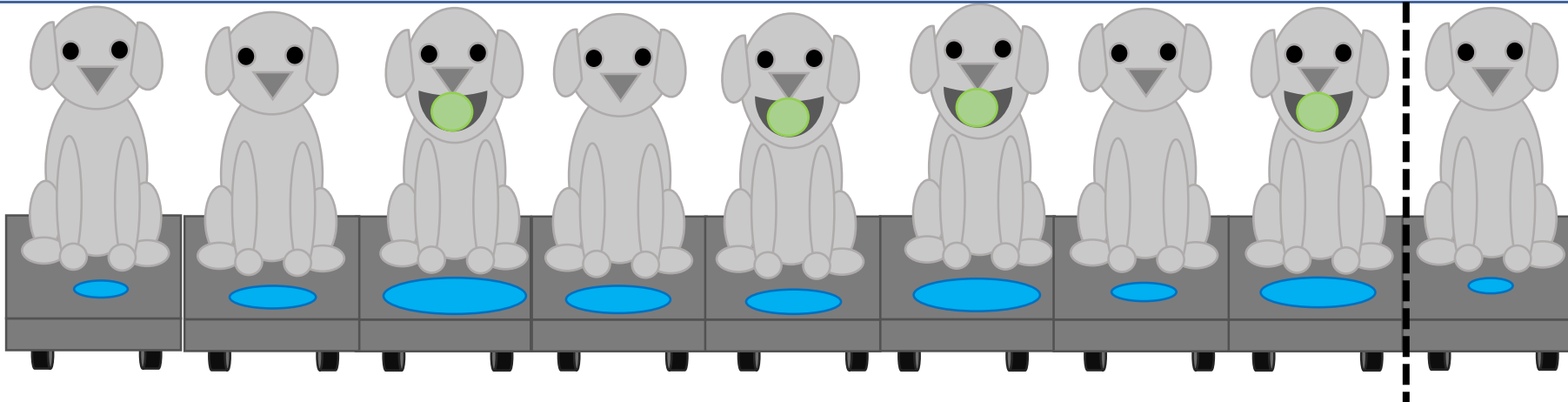


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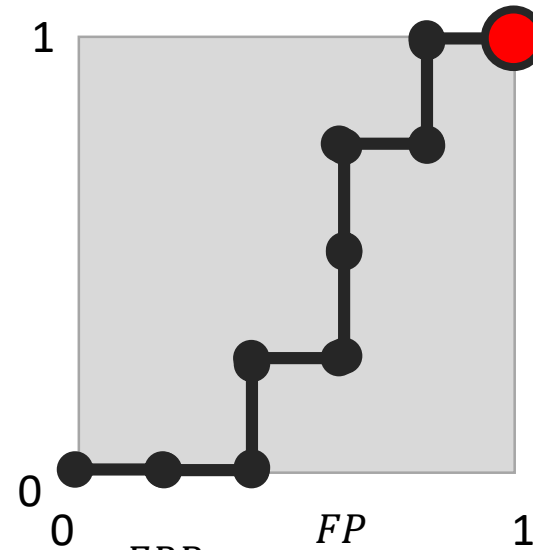
TPR: 4/4
FPR: 4/5
Included



What fraction of the dogs with tennis balls we've included in the set of dogs we've predicted to have tennis balls.

$$TPR = \frac{TP}{TP + FN}$$

all actual positives



$$FPR = \frac{FP}{FP + TN}$$

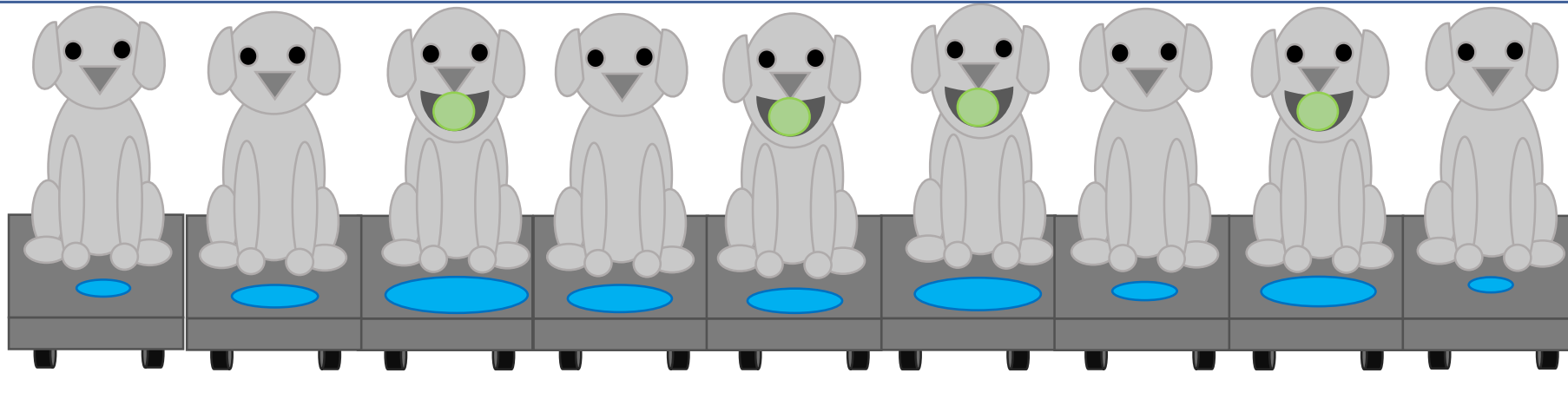
all actual negatives

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TPR: 4/4

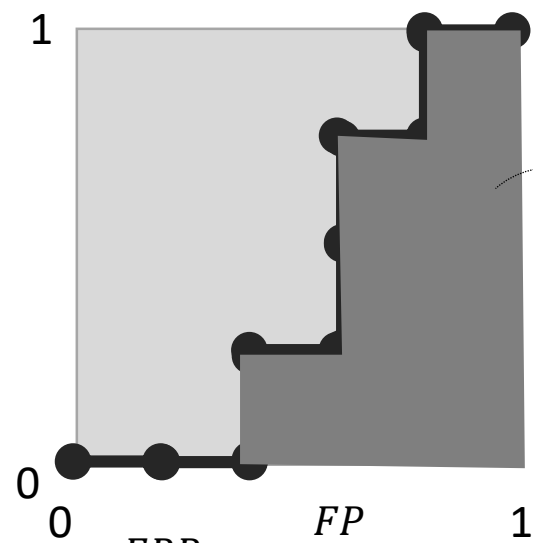
FPR: 5/5

Included



$$TPR = \frac{TP}{TP + FN}$$

all actual positives

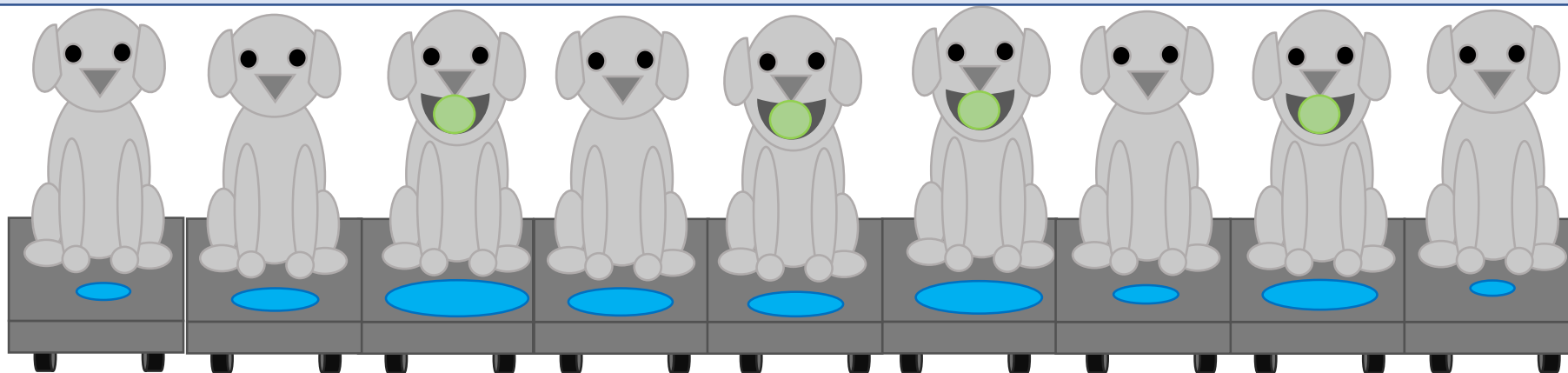


$$AUC = 8 * \left(\frac{1}{5} * \frac{1}{4} \right)$$

$$= 0.40$$

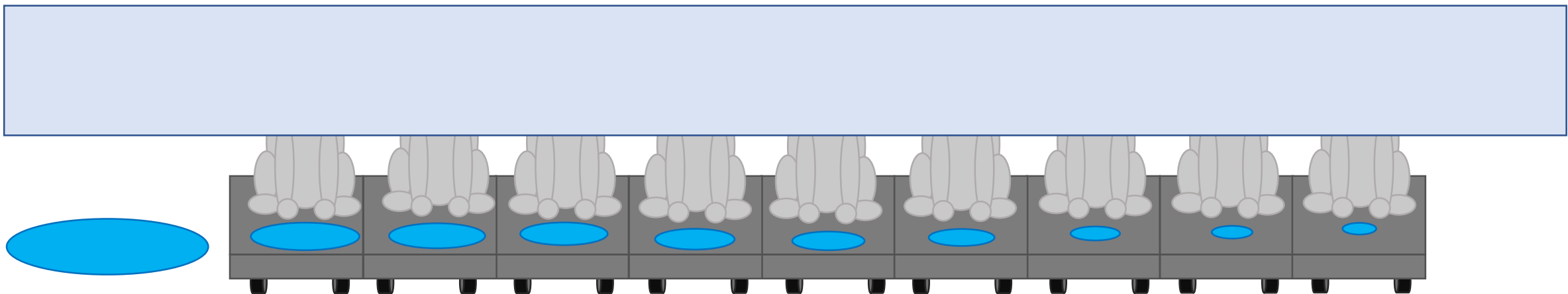
$$FPR = \frac{FP}{FP + TN}$$

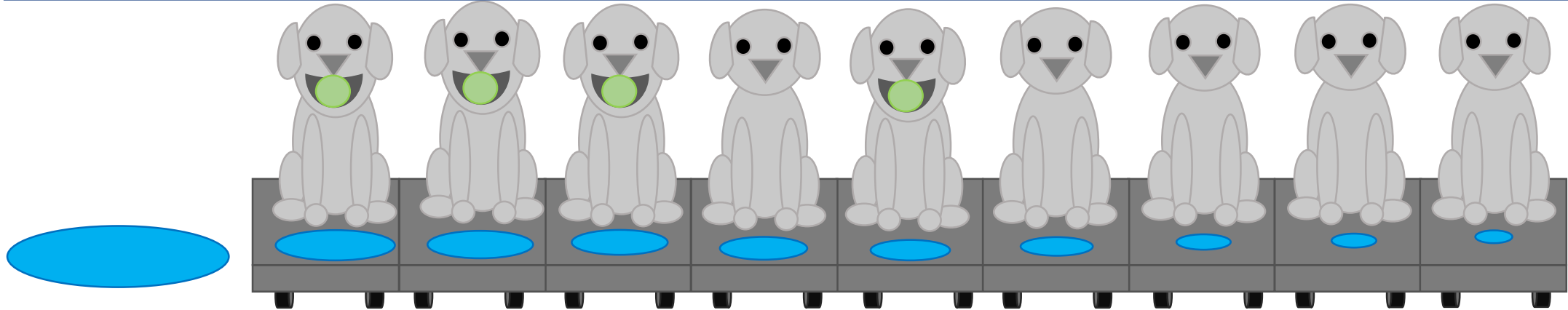
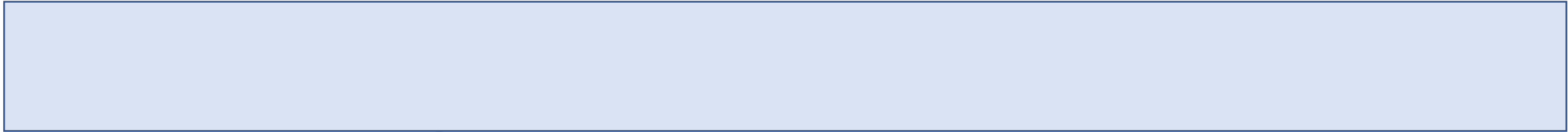
all actual negatives



Model 2

We suppose that the more a dog has drooled,
the more likely that dog is to have a tennis ball.

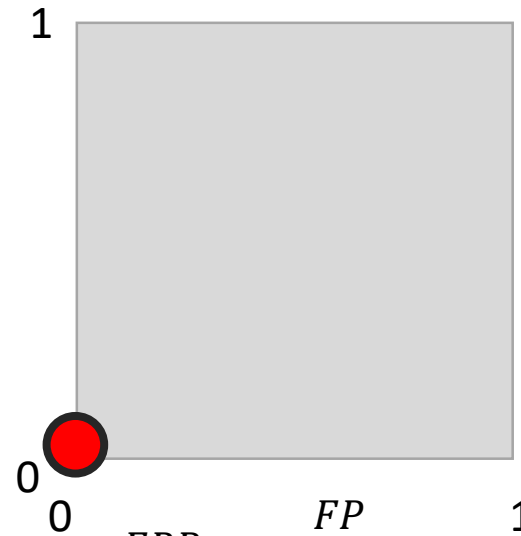




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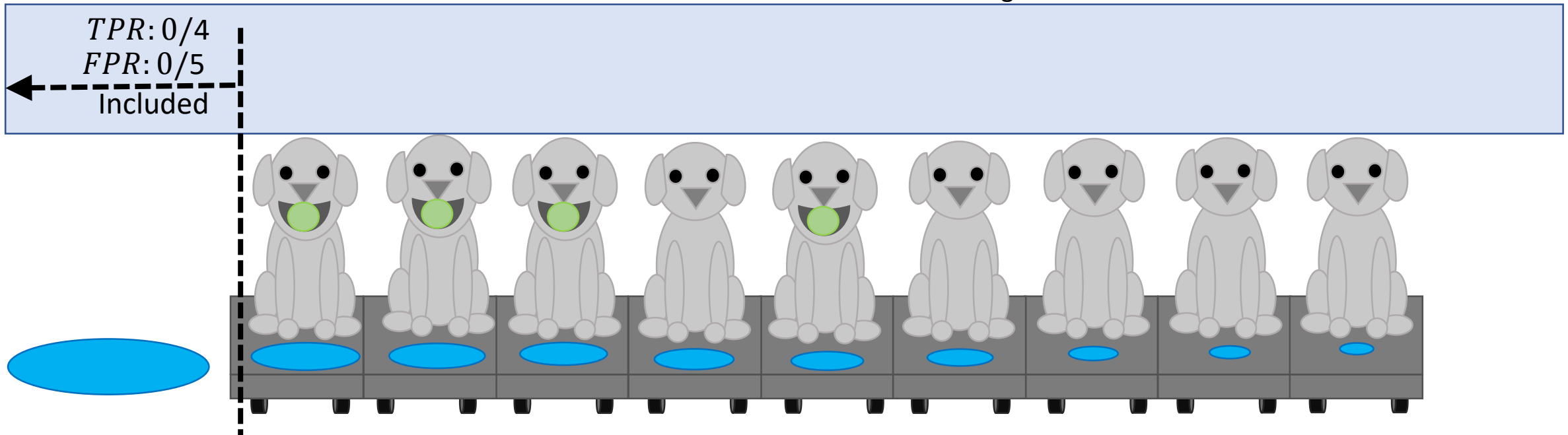
all actual positives



$$FPR = \frac{FP}{FP + TN}$$

all actual negatives

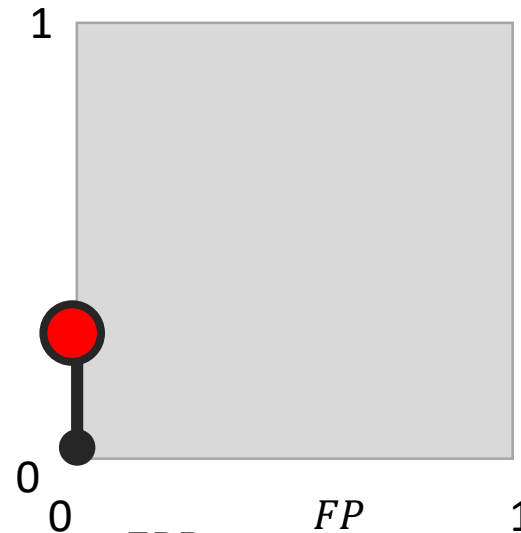
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$$TPR = \frac{TP}{TP + FN}$$

all actual positives

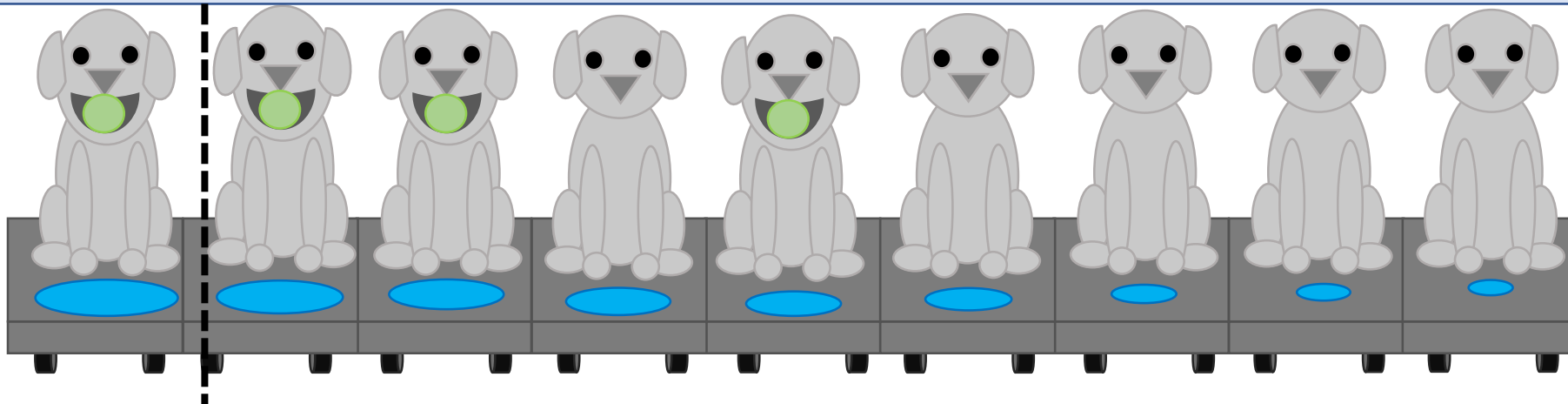


$$FPR = \frac{FP}{FP + TN}$$

all actual negatives

What fraction of the dogs without tennis balls we've included in the set of dogs we've predicted to have tennis balls.

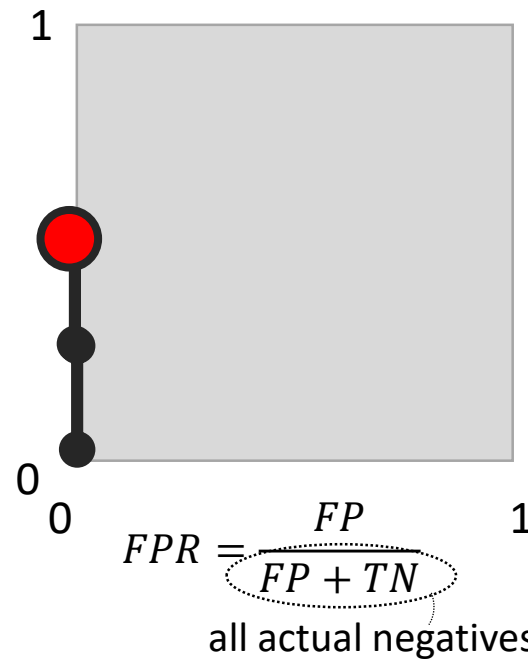
TPR: 1/4
FPR: 0/5
Included



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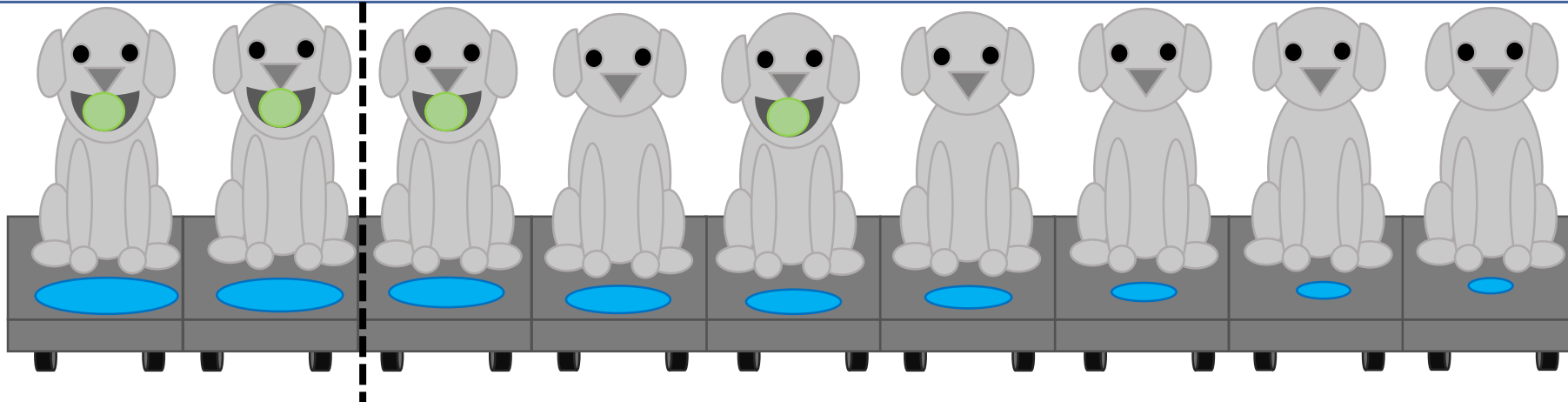
$$TPR = \frac{TP}{TP + FN}$$

all actual positives



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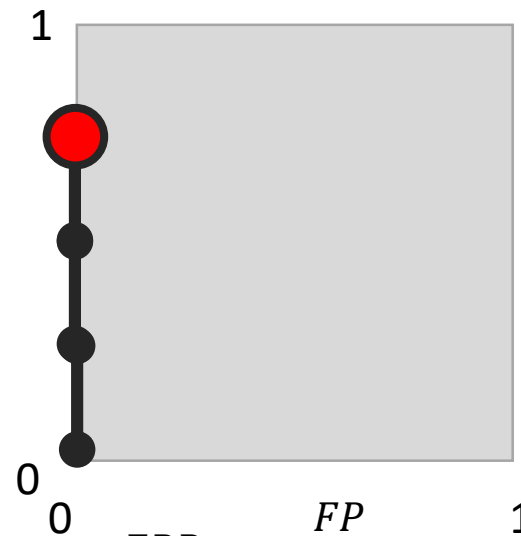
$TPR: 2/4$
 $FPR: 0/5$
← Included



What fraction of the dogs with tennis balls we've included in the set of dogs we've predicted to have tennis balls.

$$TPR = \frac{TP}{TP + FN}$$

all actual positives

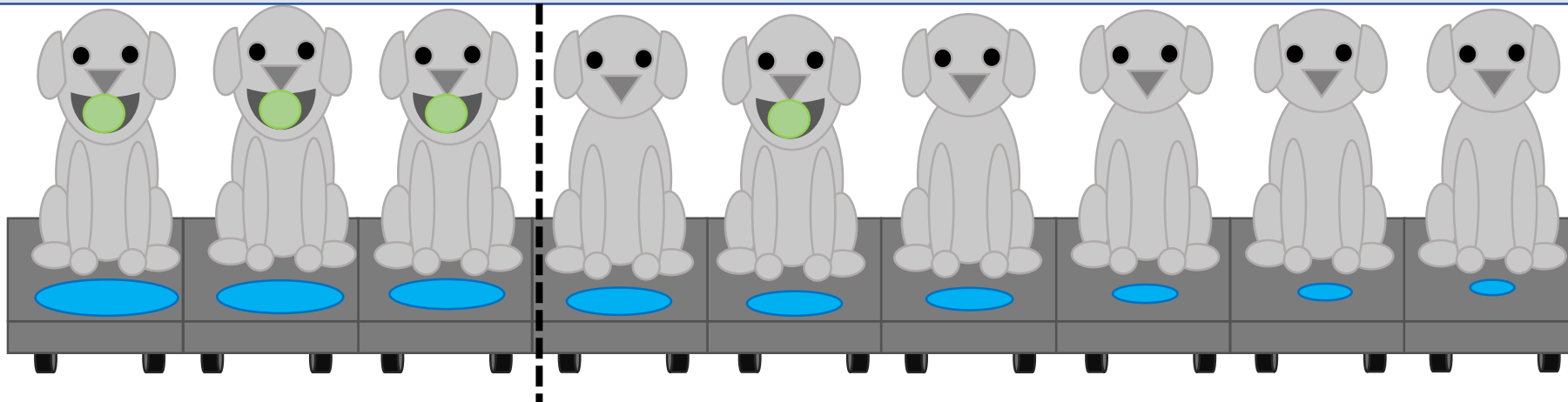


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$$FPR = \frac{FP}{FP + TN}$$

all actual negatives

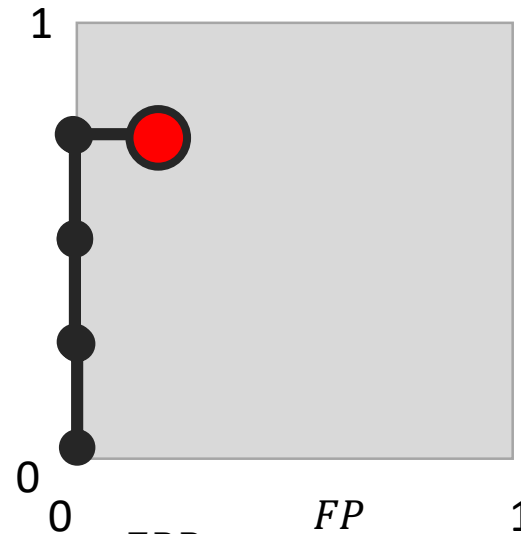
TPR: 3/4
FPR: 0/5
Included



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$$TPR = \frac{TP}{TP + FN}$$

all actual positives

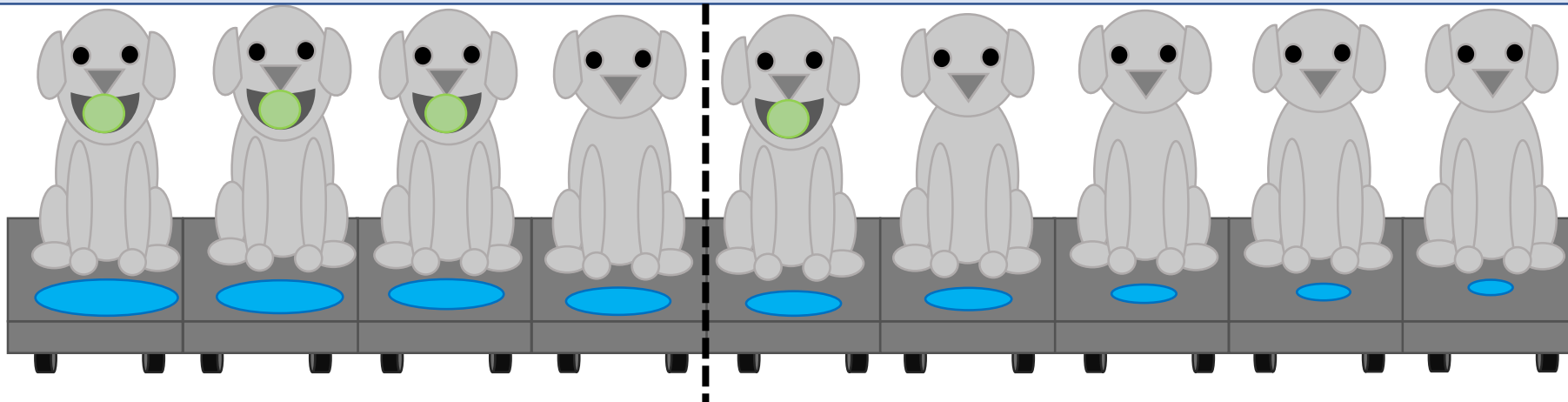


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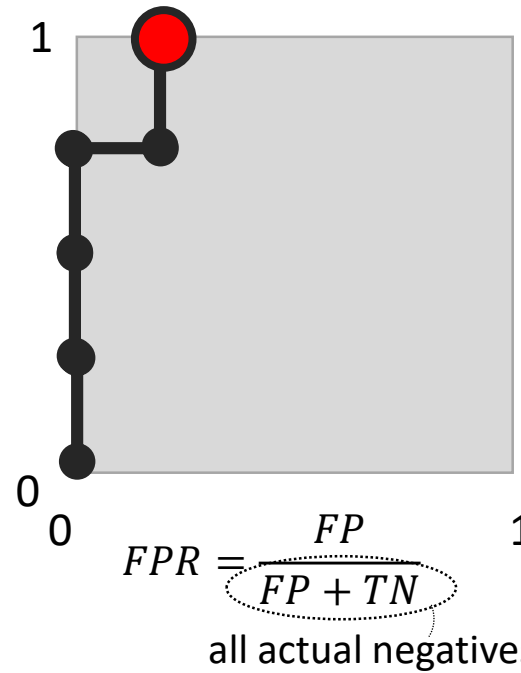
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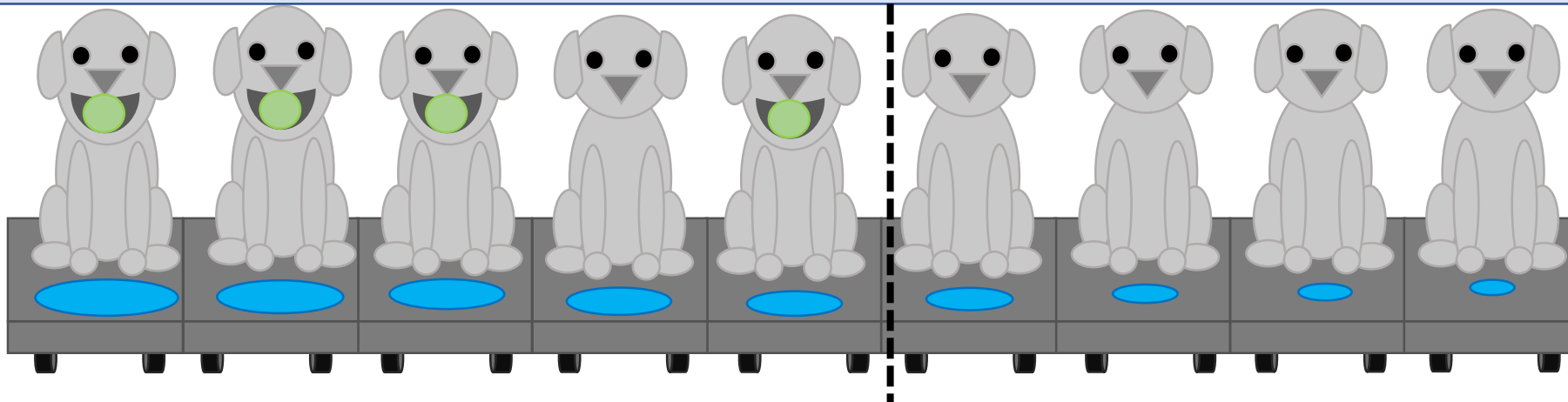
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all actual positives



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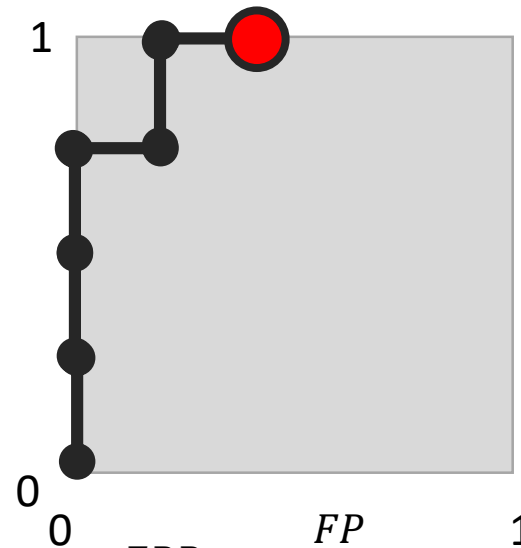
TPR: 4/4
FPR: 1/5
Included



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$$TPR = \frac{TP}{TP + FN}$$

all actual positives



$$FPR = \frac{FP}{FP + TN}$$

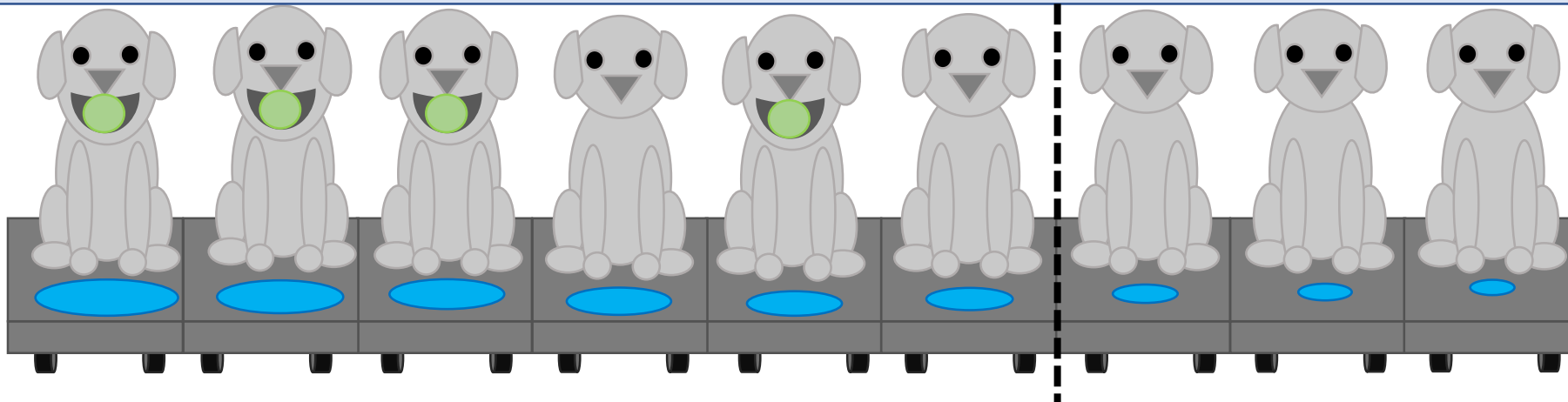
all actual negatives

What fraction of the dogs without tennis balls we've included in the set of dogs we've predicted to have tennis balls.

TPR: 4/4

FPR: 2/5

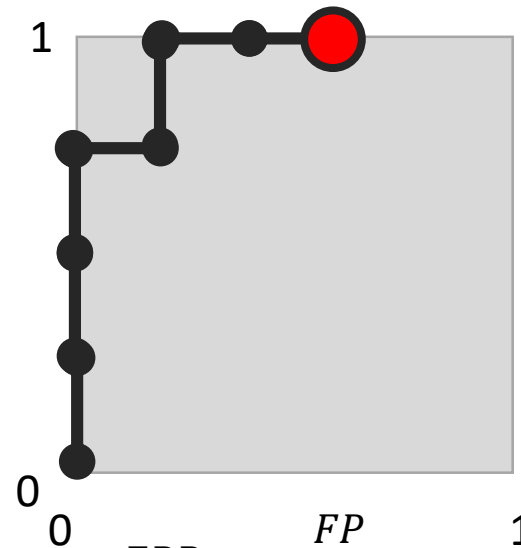
Included



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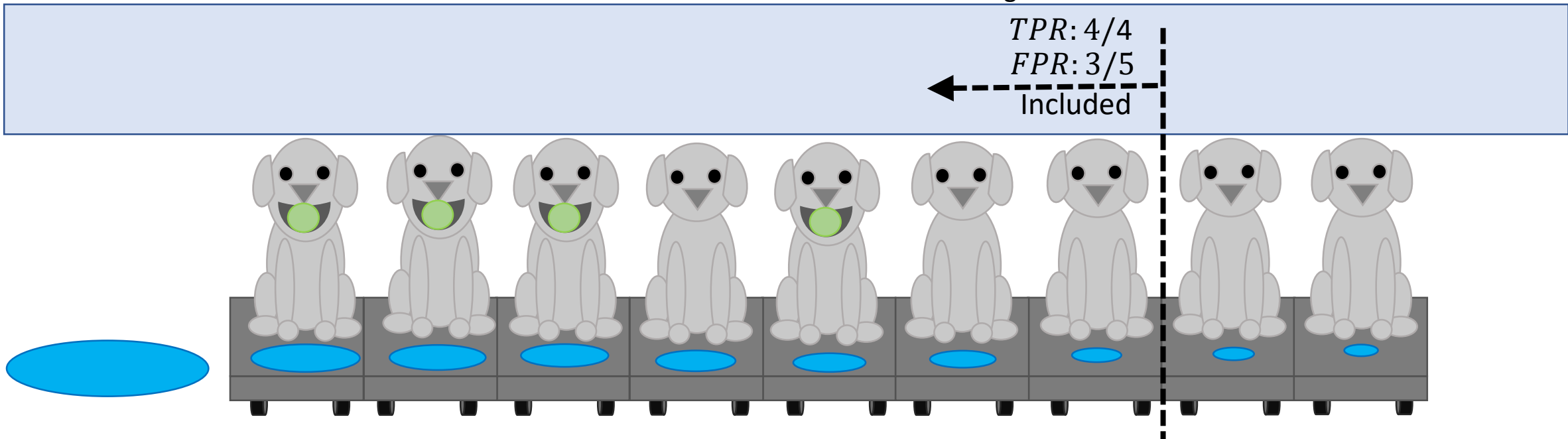
all actual positives



$$FPR = \frac{FP}{FP + TN}$$

all actual negatives

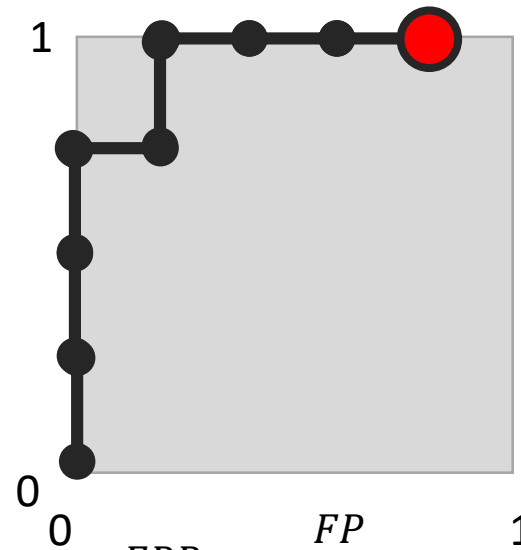
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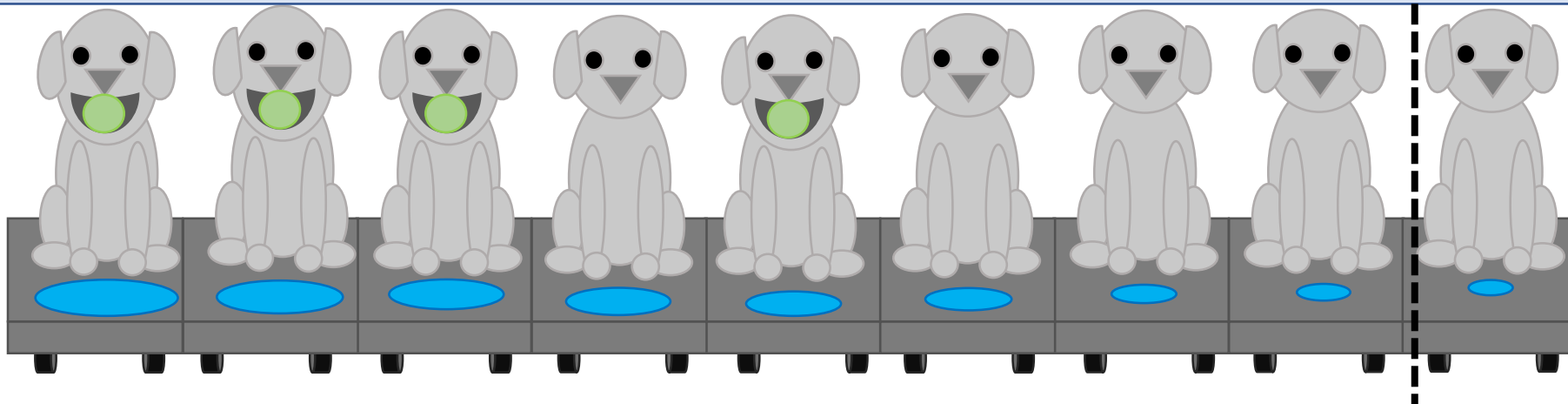
all actual negatives

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$TPR: 4/4$

$FPR: 4/5$

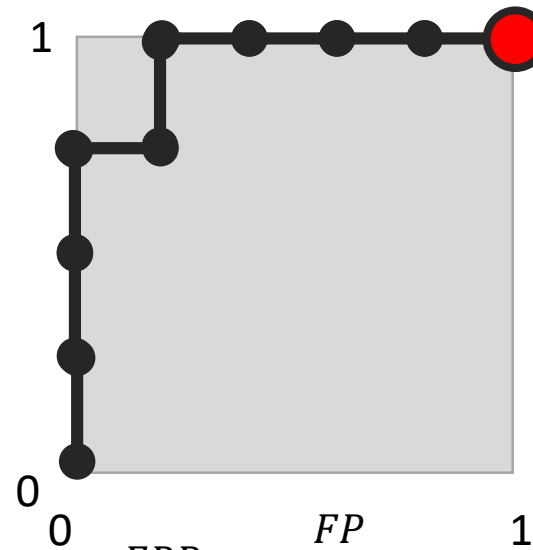
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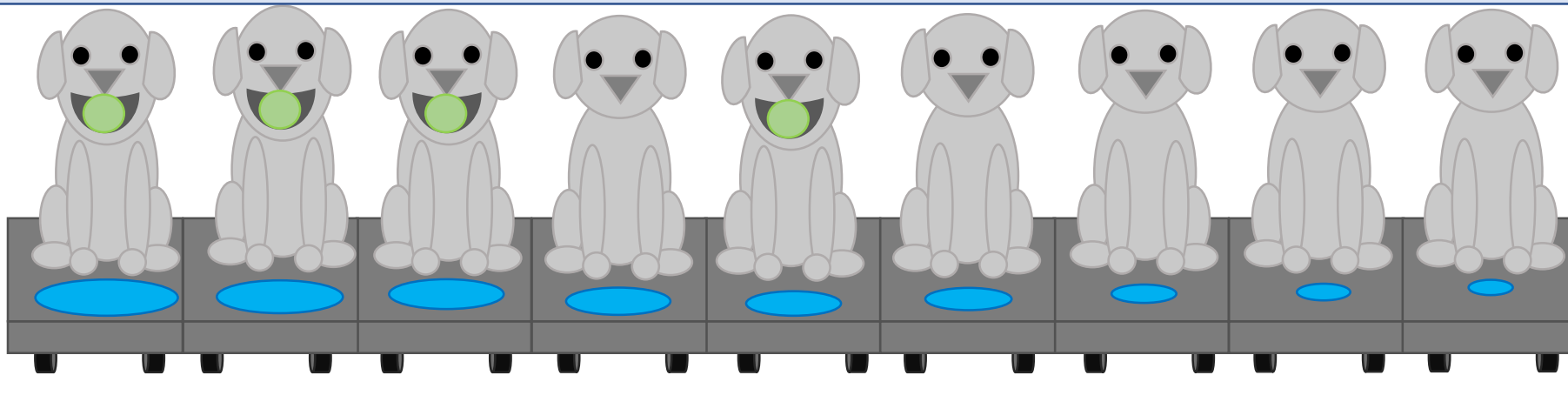
all actual negatives

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$TPR: 4/4$

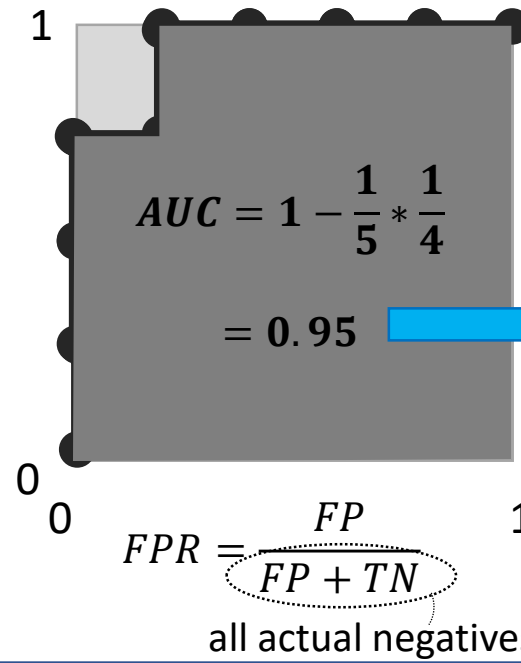
$FPR: 5/5$

Included



$$TPR = \frac{TP}{TP + FN}$$

all actual positives



Model 1's AUC: 0.40

Model 2's AUC: 0.95

Drool pool size is a much better classifier here.

