ASSIGNMENT

ON

GRAPH THEORY

Topic: Applications of Graph Theory

Submitted To

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INTRODUCTION

Graph theoretical concepts are widely used to study and model various applications, in different areas. They include, study of molecules, construction of bonds in chemistry and the study of atoms. Similarly, graph theory is used in sociology for example to measure actors prestige or to explore diffusion mechanisms. Graph theory is used in biology and conservation efforts where a vertex represents regions where certain species exist and the edges represent migration path or movement between the regions. This information is important when looking at breeding patterns or tracking the spread of disease, parasites and to study the impact of migration that affect other species. Graph theoretical concepts are widely used in Operations Research. For example, the traveling salesman problem, the shortest spanning tree in a weighted graph, obtaining an optimal match of jobs and men and locating the shortest path between two vertices in a graph. It is also used in modeling transport networks, activity networks and theory of games. The network activity is used to solve large number of combinatorial problems. The most popular and successful applications of networks in OR is the planning and scheduling of large complicated projects. The best well known problems are PERT (Project Evaluation Review Technique) and CPM (Critical Path Method). Next, Game theory is applied to the problems in engineering, economics and war science to find optimal way to perform certain tasks in competitive environments. To represent the method of finite game a digraph is used. Here, the vertices represent the positions and the edges represent the moves.

Graph theory is the study of points and lines. In particular, it involves the ways in which sets of points, called vertices, can be connected by lines or arcs, called edges. Graphs in this context differ from the more familiar coordinate plots that portray mathematical relations and functions. It has been enriched in the last decades by growing influences from studies of social and complex networks. A graph is a symbolic representation of a network and of its connectivity. It implies an abstraction of the reality so it can be simplified as a set of linked nodes. Graph theory is a branch of mathematics concerned about how networks can be encoded and their properties measured These are classified according to their complexity, the number of edges allowed between any two vertices, and whether or not directions are assigned to edges. Various sets of rules result in specific properties that can be stated as theorems.

Applications of Graph Theory

Graphs are among the most ubiquitous models of both natural and human-made structures. They can be used to model many types of relations and process dynamics in computer science, physical, biological and social systems. Many problems of practical interest can be represented by graphs. In general graphs theory has a wide range of applications in diverse fields. This paper explores different elements involved in graph theory including graph representations using computer systems and graph-theoretic data structures such as list structure and matrix structure. The emphasis of this paper is on graph applications in computer science. To demonstrate the importance of graph theory in computer science, this article addresses most common applications for graph theory in computer science. These applications are presented especially to project the idea of graph theory and to demonstrate its importance in computer science. In computer science, graphs are used to represent networks of communication, data organization, computational devices, the flow of computation, etc. For instance, the link structure of a website can be represented by a directed graph, in which the vertices represent web pages and directed edges represent links from one page to another. A similar approach can be taken to problems in social media, travel, biology, computer chip design, mapping the progression of neurodegenerative diseases, and many other fields. The development of algorithms to handle graphs is therefore of major interest in computer science. The transformation of graphs is often formalized and represented by graph rewrite systems. Complementary to graph transformation systems focusing on rule-based in-memory manipulation of graphs are graph databases geared towards transaction-safe, persistent storing and querying of graph-structured data.

Graphs in Chemistry

Graphs are used in the field of chemistry to model chemical compounds. In computational biochemistry some sequences of cell samples have to be excluded to resolve the conflicts between two sequences. This is modeled in the form of graph where the vertices represent the sequences in the sample. An edge will be drawn between two vertices if and only if there is a conflict between the corresponding sequences. The aim is to remove possible vertices, (sequences) to eliminate all conflicts. In brief, graph theory has its unique impact in various fields and is growing large now a days. The subsequent section analyses the applications of graph theory especially in computer science.

Algorithms and graph theory

The major role of graph theory in computer applications is the development of graph algorithms. Numerous algorithms are used to solve problems that are modeled in the form of graphs. These algorithms are used to solve the graph theoretical concepts which intern used to solve the corresponding computer science application problems.

Some algorithms are as follows:

1. Shortest path algorithm in a network
2. Finding a minimum spanning tree
3. Finding graph planarity
4. Algorithms to find adjacency matrices.
5. Algorithms to find the connectedness
6. Algorithms to find the cycles in a graph
7. Algorithms for searching an element in a data structure (DFS, BFS) and so on.

Various computer languages are used to support the graph theory concepts. The main goal of such languages is to enable the user to formulate operations on graphs in a compact and natural manner

Some graph theoretic languages are

1. SPANTREE – To find a spanning tree in the given graph.
2. GTPL – Graph Theoretic Language
3. GASP – Graph Algorithm Software Package
4. HINT – Extension of LISP
5. GRASPE – Extension of LISP
6. IGTS – Extension of FORTRAN
7. GEA – Graphic Extended ALGOL (Extension of ALGOL)
8. AMBIT – To manipulate digraphs
9. GIRL – Graph Information Retrieval Language
10. FGRAAL – FORTRAN Extended Graph Algorithmic Language

Use of graph enumeration techniques

Graph enumeration technique is used to identify the computerized chemical identification. The list of all distinct chemical structures will be generated based on the given chemical formula and the valence rules for any new substance. To identify the chemical compounds automatically, a computer language called DENDRAL has been developed.

Graph Theory in OR

Graph theory is a very natural and powerful tool in combinatorial operations research. Some important OR problems that can be solved using graphs are given here. A network called transport network where a graph is used to model the transportation of commodity from one place to another. The objective is to maximize the flow or minimize the cost within the prescribed flow. The graph theoretic approach is found more efficient for these types of problems though they have more constraints

Graph Coloring

Graph coloring is one of the most important concepts in graph theory and is used in many real time applications in computer science. Various coloring methods are available and can be used on requirement basis. The proper coloring of a graph is the coloring of the vertices and edges with minimal number of colors such that no two vertices should have the same color. The minimum number of colors is called as the chromatic number and the graph is called properly colored graph

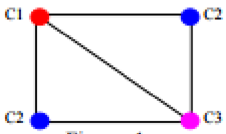

Figure -1
Proper vertex coloring with Chromatic number 3

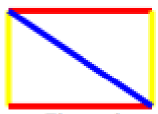
Figure -2
Proper edge coloring with Chromatic number 3

Graph coloring techniques in scheduling

Here some scheduling problems that uses variants of graph coloring methodologies such as precoloring, list coloring, multicoloring, minimum sum coloring are given in brief.

Job scheduling:

Here the jobs are assumed as the vertices of the graph and there is an edge between two jobs if they cannot be executed simultaneously. There is a 1-1 correspondence between the feasible schedulings of the jobs and the colorings of the graph.

Aircraft scheduling

Assuming that there are k aircrafts and they have to be assigned n flights. The ith flight should be during the time interval (ai, bi). If two flights overlap, then the same aircraft cannot be assigned to both the flights. This problem is modeled as a graph as follows. The vertices of the graph correspond to the flights. Two vertices will be connected, if the corresponding time intervals overlap. Therefore, the graph is an interval graph that can be colored optimally in polynomial time.

Bi-processor tasks

Assume that there is a set of processors and set of tasks. Each task has to be executed on two processors simultaneously and these two processors must be pre assigned to the task. A processor cannot work on two jobs simultaneously. This type of tasks will arise when scheduling of file transfers between processors or in case of mutual diagnostic besting of processors. This can be modeled by considering a graph whose vertices correspond to the processes and if there is any task that has to be executed on processors i and j, then and edge to be added between the two vertices. Now the scheduling problem is to assign colors to edges in such a way that every color appears at most once at a vertex. If there are no multiple edges in the graph (i.e) no two tasks require the same two processors then the edge coloring technique can be adopted. The authors have developed an algorithm for multiple edges which gives an 1-1 approximate solution.

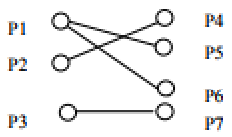

Figure – 3 Tasks allocated to processors

The diagram shows the tasks namely task1, task2, task3 and task4 are allocated to the processors (P1, P5); (P1, P6); (P2, P4) and (P3, P7) respectively

Precoloring extension

In certain scheduling problems, the assignments of jobs are already decided. In such cases precoloring technique can be adopted. Here some vertices of the graph will have preassigned color and the precoloring problem has to be solved by extending the coloring of the vertices for the whole graph using minimum number of colors.

List coloring

In list coloring problem, each vertex v has a list of available colors and we have to find a coloring where the color of each vertex is taken from the list of available colors. This list coloring can be used to model situations where a job can be processed only in certain time slots or can be processed only by certain machines.

Minimum sum coloring

In minimum sum coloring, the sum of the colors assigned to the vertices is minimal in the graph. The minimum sum coloring technique can be applied to the scheduling theory of minimizing the sum of completion times of the jobs. The multicolor version of the problem can be used to model jobs with arbitrary lengths. Here, the finish time of a vertex is the largest color assigned to it and the sum of coloring is the sum of the finish time of the vertices. That is the sum of the finish times in a multicoloring is equal to the sum of completion times in the corresponding schedule.

Electrical Engineering

The concepts of graph theory is used extensively in designing circuit connections. The types or organization of connections are named as topologies. Some examples for topologies are star, bridge, series, and parallel topologies.

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Different types of Graph

 Simple graph

A graph that includes only one type of link between its nodes. A road or rail network are simple graphs.

 Edge (Link)

An edge e is a link between two nodes. The link (i , j) is of initial extremity iand of terminal extremity j. A link is the abstraction of a transport infrastructure supporting movements between nodes. It has a direction that is commonly represented as an arrow. When an arrow is not used, it is assumed the link is bi-directional.

 Vertex (Node)

A node v is a terminal point or an intersection point of a graph. It is the abstraction of a location such as a city, an administrative division, a road intersection or a transport terminal.

 Multigraph

A graph that includes several types of links between its nodes. Some nodes can be connected to one link type while others can be connected to more than one that are running in parallel. A graph depicting a road and a rail network with different links between nodes serviced by either or both modes is a multigraph.

 Planar Graph

A graph where all the intersections of two edges are a vertex. Since this graph is located within a plane, its topology is two-dimensional. This is typically the case for power grids, road and railway networks, although great care must beinferred to the definition of nodes.

 Non-planar Graph.

A graph where there are no vertices at the intersection of at least two edges. This implies a third dimension in the topology of the graph 4 since there is the possibility of having a movement "passing over" anothermovement such as for air and maritime transport. A non-planar graph has potentially much more links than a planar graph.

Conclusion

Many problems of practical interest can be represented by graphs. In general graphs theory has a wide range of applications in diverse fields. This paper explores different elements involved in graph theory including graph representations using computer systems and graph-theoretic data structures such as list structure and matrix structure. The emphasis of this paper is on graph applications in computer science.. For instance, the link structure of a website can be represented by a directed graph, in which the vertices represent web pages and directed edges represent links from one page to another A graph is a diagram of points and lines connected to the points. It has at least one line joining a set of two vertices with no vertex connecting itself. The concept of graphs in graph theory stands up on some basic terms such as point, line, vertex, edge, degree of vertices, properties of graphs, etc.