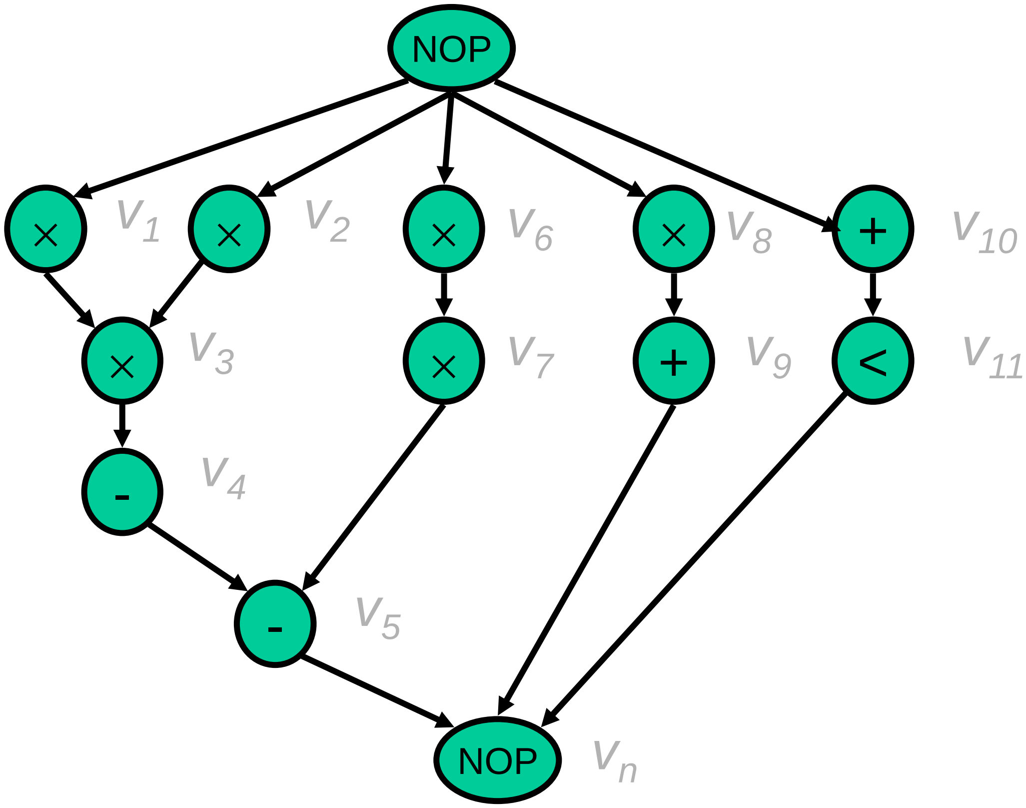
Mixed Integer Linear Programming

- A mathematical programming such that:
  - The objective is a linear function
  - All constraints are linear functions
  - Some variables are real numbers and some are integers, i.e., "mixed integer"
- It is almost like a linear programming, except that some variables are integers

NP-C problem

ILP Scheduling

- How to construct a mathematical model?



ILP Formulation of ML-RCS

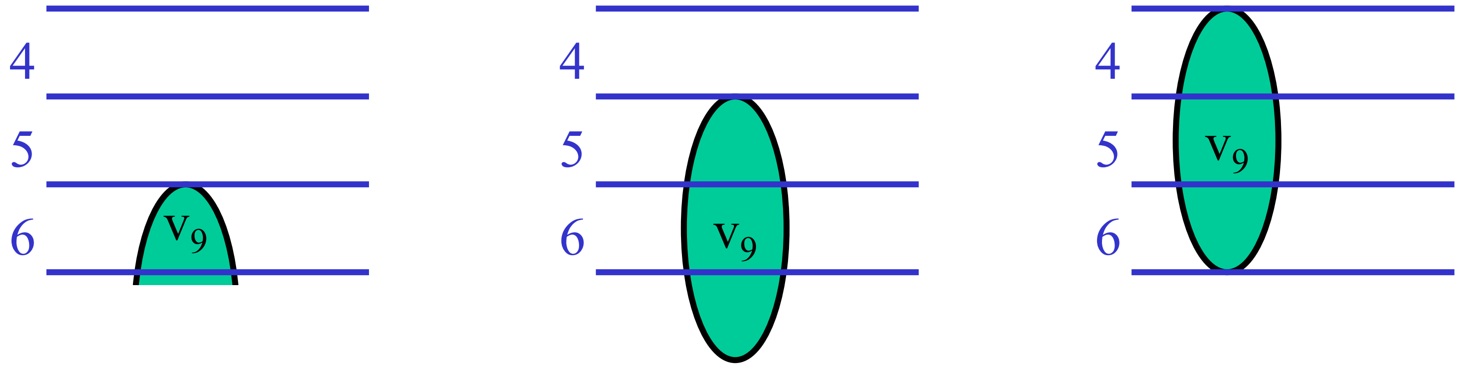
- Use binary decision variables
  - $i = 0, 1, ..., n$
  - $l = 1, 2, ..., \lambda' + 1$ λ': given upper-bound on latency
  - $x_{i,l} = 1$ if operation i starts at step l, 0 otherwise.
- Set of linear inequalities (constraints), and an objective function (min latency)
- Observations
  - $x_{i,l} = 0 \quad for \quad l < t_i^S \quad and \quad l > t_i^L$ start time feasibility
    $(t_i^S = ASAP(v_i),\ t_i^L = ALAP(v_i))$
  - $t_i = \sum_l l \cdot x_{i,l}$ t_i = start time of op i
  - $\sum_{m=l-d_i+1}^{l} x_{i,m} = 1$ If op v_i takes d_i steps, is op v_i (still) executing at step l?

Start Time vs. Execution Time

- For each operation v_i, only one start time
- If $d_i=1$, then the following questions are the same:
  - Does operation v_i **start** at step l?
  - Is operation v_i **running** at step l?
- But if $d_i>1$, then the two questions should be formulated as:
  - Does operation v_i **start** at step l?
    - Does $x_{i,l} = 1$ hold?
  - Is operation v_i **running** at step l?
    - Does $\sum_{m=l-d_i+1}^{l} x_{i,m} = 1$ hold?

Operation v_i Still Running at Step l ?

- Assume that v_9 takes 3 steps, is v_9 running at step 6?
  - Is $x_{9,6} + x_{9,5} + x_{9,4} = 1$?

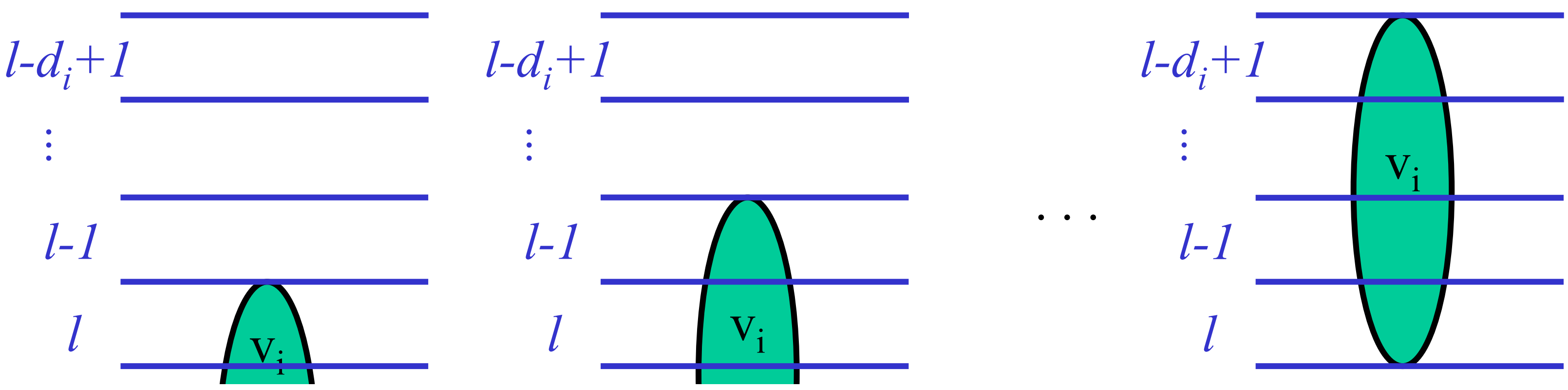


$x_{9,6}=1$ $x_{9,5}=1$ $x_{9,4}=1$

- Note:
  - Only one (if any) of the above three cases can happen
  - To meet resource constraints, we have to ask the same question for ALL steps, and ALL operations of that type

Operation v_i Still Running at Step l ?

- Is v_i running at step l ?
  - Is $x_{i,l} + x_{i,l-1} + \ldots + x_{i,l-d_i+1} = 1$?



$x_{i,l}=1$ $x_{i,l-1}=1$ $x_{i,l-d_i+1}=1$

ILP Formulation of ML-RCS (Cont.)

- Constraints:
  - Unique start times: $\sum_{l} x_{i,l} = 1, \quad i = 0,1,\ldots,n$
  - Sequencing (dependency) relations must be satisfied

$$t_i \geq t_j + d_j \ \forall (v_j, v_i) \in E \ \Rightarrow \sum_{l} l \cdot x_{i,l} \geq \sum_{l} l \cdot x_{j,l} + d_j$$

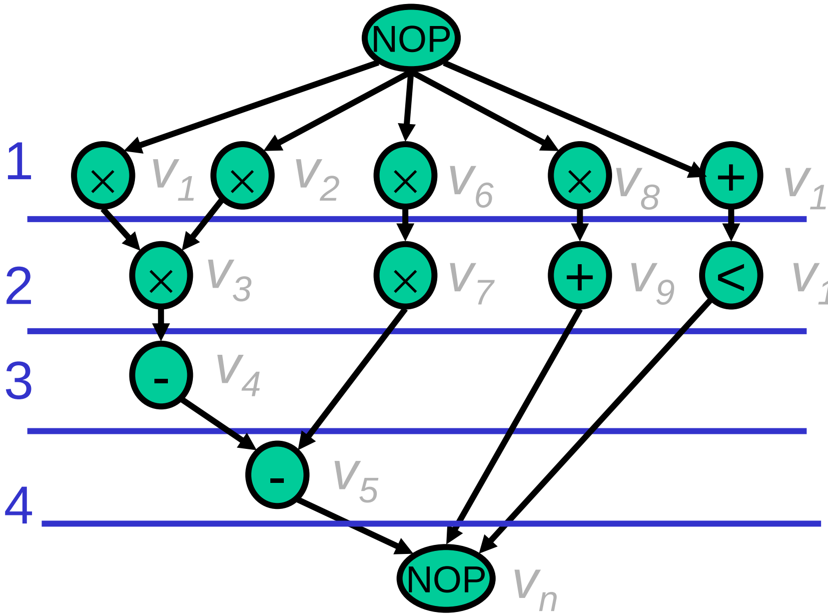
  - Resource constraints

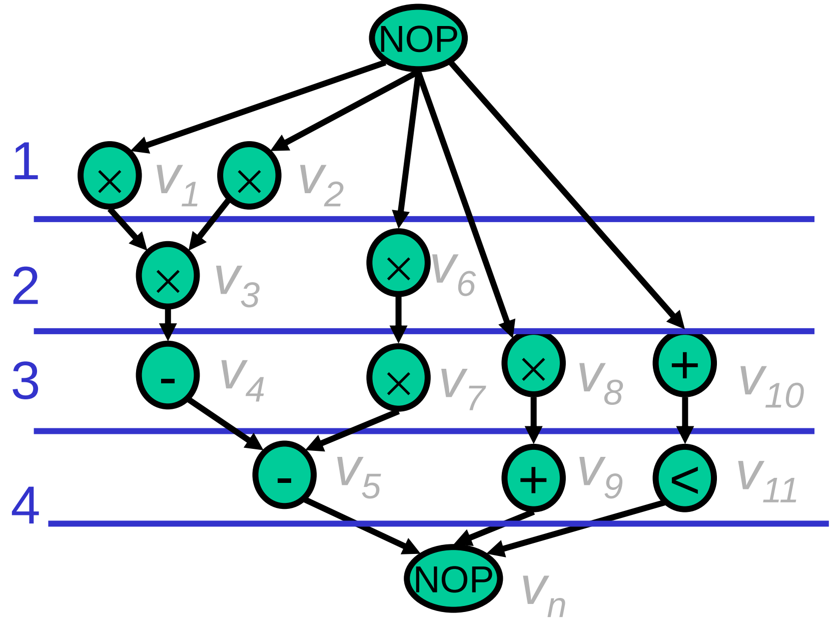
$$\sum_{i\,:\,T(v_i)=k} \ \sum_{m=l-d_i+1}^{l} x_{i,m} \leq a_k, \quad k = 1,\ldots,n_{res}, \quad l = 1,\ldots,\bar{\lambda}+1$$

- Objective: min $\boldsymbol{c}^T\boldsymbol{t}$.
  - $\boldsymbol{t}$ =start times vector, $\boldsymbol{c}$ =cost weight (ex: [0 0 ... 1])
  - When $\boldsymbol{c}$ =[0 0 ... 1], $\boldsymbol{c}^T\boldsymbol{t} = \sum_{l} l \cdot x_{n,l}$

ILP Example

- Assume $\bar{\lambda} = 4$
- First, perform ASAP and ALAP
  - (we can write the ILP without ASAP and ALAP, but using ASAP and ALAP will simplify the inequalities)





ILP Example: Unique Start Times Constraint

- Without using ASAP and ALAP values:

$$x_{1,1} + x_{1,2} + x_{1,3} + x_{1,4} = 1$$
$$x_{2,1} + x_{2,2} + x_{2,3} + x_{2,4} = 1$$
$$\cdots$$
$$\cdots$$
$$\cdots$$
$$x_{11,1} + x_{11,2} + x_{11,3} + x_{11,4} = 1$$

- Using ASAP and ALAP:

$$x_{1,1} = 1$$
$$x_{2,1} = 1$$
$$x_{3,2} = 1$$
$$x_{4,3} = 1$$
$$x_{5,4} = 1$$
$$x_{6,1} + x_{6,2} = 1$$
$$x_{7,2} + x_{7,3} = 1$$
$$x_{8,1} + x_{8,2} + x_{8,3} = 1$$
$$x_{9,2} + x_{9,3} + x_{9,4} = 1$$
$$\cdots$$

ILP Example: Dependency Constraints

- Using ASAP and ALAP, the non-trivial inequalities are: (assuming unit delay for + and *)

$$2 \cdot x_{7,2} + 3 \cdot x_{7,3} - 1 \cdot x_{6,1} - 2 \cdot x_{6,2} - 1 \geq 0$$

$$2 \cdot x_{9,2} + 3 \cdot x_{9,3} + 4 \cdot x_{9,4} - 1 \cdot x_{8,1} - 2 \cdot x_{8,2} - 3 \cdot x_{8,3} - 1 \geq 0$$

$$2 \cdot x_{11,2} + 3 \cdot x_{11,3} + 4 \cdot x_{11,4} - 1 \cdot x_{10,1} - 2 \cdot x_{10,2} - 3 \cdot x_{10,3} - 1 \geq 0$$

$$4 \cdot x_{5,4} - 2 \cdot x_{7,2} - 3 \cdot x_{7,3} - 1 \geq 0$$

$$5 \cdot x_{n,5} - 2 \cdot x_{9,2} - 3 \cdot x_{9,3} - 4 \cdot x_{9,4} - 1 \geq 0$$

$$5 \cdot x_{n,5} - 2 \cdot x_{11,2} - 3 \cdot x_{11,3} - 4 \cdot x_{11,4} - 1 \geq 0$$

ILP Example: Resource Constraints

- Resource constraints (assuming 2 adders and 2 multipliers)

$$x_{1,1} + x_{2,1} + x_{6,1} + x_{8,1} \le 2$$
$$x_{3,2} + x_{6,2} + x_{7,2} + x_{8,2} \le 2$$
$$x_{7,3} + x_{8,3} \le 2$$
$$x_{10,1} \le 2$$
$$x_{9,2} + x_{10,2} + x_{11,2} \le 2$$
$$x_{4,3} + x_{9,3} + x_{10,3} + x_{11,3} \le 2$$
$$x_{5,4} + x_{9,4} + x_{11,4} \le 2$$

- Objective:
  - Since λ=4 and sink has no mobility, any feasible solution is optimum, but we can use the following anyway: $Min \quad 1 \cdot x_{n,1} + 2 \cdot x_{n,2} + 3 \cdot x_{n,3} + 4 \cdot x_{n,4} + 5 \cdot x_{n,5}$

ILP Formulation of MR-LCS

- Dual problem to ML-RCS
- Objective:
  - Goal is to optimize total resource usage, $\mathbf{a}$.
  - Objective function is $\boldsymbol{c}^T\boldsymbol{a}$, where entries in $\boldsymbol{c}$ are respective area costs of resources
- Constraints:
  - Same as ML-RCS constraints, plus:
  - Latency constraint added:

$$\sum_{l} l \cdot x_{n,l} \leq \bar{\lambda} + 1$$

  - Note: unknown a_k appears in constraints.

Further Study

- Linear programming
  - http://www.cs.sunysb.edu/~algorith/files/linear-programming.shtml
- Linear programming tools
  - https://en.wikipedia.org/wiki/List_of_optimization_software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