Colour Perception: Mapping the Perceptual Limits of Surface Colour

Ujjayanta Bhaumik[1]

MSc Computer Graphics, Vision and Imaging

Supervised by

Dr. Lewis Griffin

Submission date: 4 September 2019

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"Why do two colors, put one next to the other, sing? Can one really explain this? No. Just as one can never learn how to paint".

-Pablo Picasso

Abstract

Colour is a sensation caused in the eye when light is reflected off or emitted by an object. Every image seems to have a tone that is set by the composition of colours present in it. However, if an image has a colour component that does not fit in according to the general setting of the image, the observer might find it unusual. The dissertation focuses on studying the perceptual limits of human colour vision and examines the sensitivity of human beings to shades of colours at the point of maximum saturation, given a particular amount of lightness.

A method was designed to study the similarity and differences between the ideal optimal colour solid and the psychological colour solids obtained from user studies. The experiment involved three stages: developing the ideal colour solid, development of an interface to collect user data, and comparing the psychological and ideal colour space.

The perceptual psychological stimuli responses to optimal colours were quite diverse among the participants, but for the lighter colours, the response was generally high. The findings show that the perceptual psychological solid is comparable to the optimal colour solid under ideal conditions. The two solids were found to be similar in structure, but the perceptual colour solid was shrunk radially inwards. The dissertation provides a path to study perceptual colour spaces more profoundly using a broader distribution of colours and also suggests some interesting developments of the perceptual colour solid worth looking into in detail.

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List of Abbreviations

RGB: **R**ed **G**reen **B**lue

HSV: **H**ue **S**aturation **V**alue

3D: **T**hree **D**imensional

sRGB: **S**tandard **R**GB

CIE: **I**nternational **C**ommission on **I**llumination

POI: **P**oint **O**f **I**ntersection

RAGB: **R**ed, **A**mber, **G**reen and **B**lue

CMYK: **C**yan, **M**agenta, **Y**ellow and **K**ey

STL: **S**tandard **T**riangle **L**anguage

LCD: **L**iquid **C**rystal **D**isplay

CRT: **C**athode **R**ay **T**ube

CMF: **C**olour **M**atching **F**unction

GUI: **G**raphical **U**ser **I**nterface

BRDF: **B**idirectional **R**eflectance **D**istribution **F**unction

Chapter 1

Introduction

Colour is an object's property to produce a sensation in the eye based on the light-emitting, reflecting or absorptive properties of the object. The perception of colour is very much controlled by the type of cells known as rods and cones which facilitate low light vision and colour vision respectively. Colour is something that makes the world a soothing and a beautiful place. Paul Gauguin, the famous French artist, once said, "Colour! What a deep and mysterious language, the language of dreams." This dissertation focuses on studying the perceptual colour limits of human vision using generated Mondrian style stimuli consisting of colours present vibrantly in natural scenes. The idea is to investigate when certain colour in a scene seems unrealistic to a person when compared to the overall chromatic tone of the image. To put things into perspective, fluorescence can be a good example. Fluorescence occurs when an object emits light after absorbing radiation. The emitted light often has a longer wavelength than the absorbed radiation. When a fluorescent element present in a normal image, it catches the observer's eye because it looks overwhelming as the fluorescent part looks brighter still. But the fact is that, not only fluorescent colours cause this effect. Even natural colours when fully saturated can look unnatural in an image.

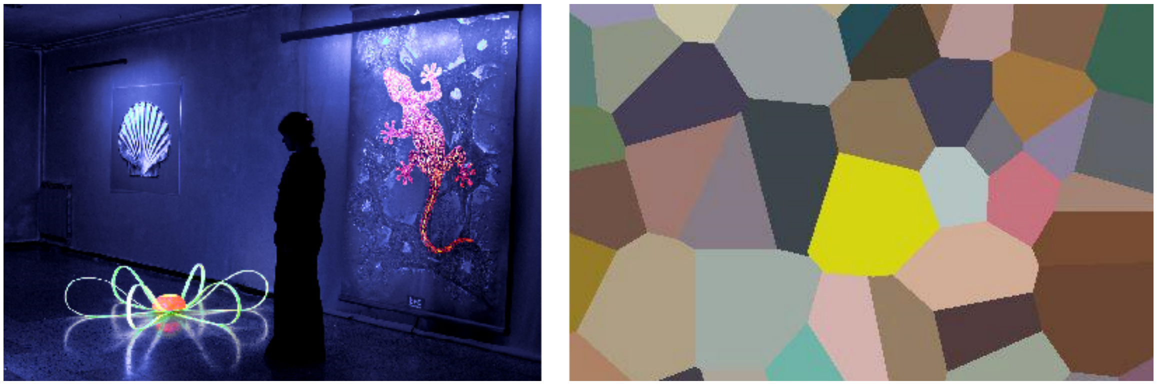


Figure 1.1: Fluorescence Paints Used in Painting versus Mondrian Style Stimuli (Left Image from [1])

In figure 1.1, the image on the left depicts a scene showing contemporary art where fluorescent paints are used. The fluorescent magical parts of the image would be immediately appealing to an observer as striking features. The image on the right hand side is a generated Mondrian style stimuli consisting of colours from natural scenes. One can see that the central polygon has a yellow colour which is really saturated and looks unnatural and vigorously striking out as compared to the other background colours. The dissertation examines different surface colours and how people react to them. By determining people's reaction to different colours on the RGB colour cube, the perceptual limits of colour vision are studied in detail.

1.1 Problem Statement

A colour map captures the vibrant range of colours that an individual can perceive. Across individuals, the colour perception spectrum varies dynamically according to the concentration of rods and cones in the eye. The focus would be to create a three dimensional colour solid that captures the perceptual limits of surface colour. By judgments of surface colours in Mondrian style stimuli, human colour perception limits would be studied. By comparing results across individuals, an average colour solid will be developed and its features looked into in detail.

1.2 Objectives

The main objective is to study the human colour perception limits and to find out how the colour solid looks like. The main targets are stated below:

- **Studying optimal colour solid**: Plotting the optimal colour solid using ideal reflectance functions with at most 2 transitions in wavelength and having value 0 or 1 at any point, and studying the solid structure

- **Understanding the perceptual colour limits**: Studying the responses to colour saturation limits using the Mondrian style stimuli to understand the perceptual colour limits and comparing the perceptual colour solid with the optimal colour solid to check any interesting results

1.3 Experimental Stages

- Stimuli Creation: Generation of stimuli using Python and MATLAB

- Setting up stimuli and presenting them using an application developed using MATLAB

- Doing user studies to gather data

- Fitting data and developing the 3D colour solid and studying the perceptual colour limits

1.4 Report Structure

Chapter 1. Introduction

This chapter introduces the theme of the dissertation, providing the problem statement and the stages and objectives of the project.

Chapter 2. Literature Review

A literature review is provided where a flashback into modern colour literature with main focus on the history of colourimetry, colour spaces and solids and psychophysical experiments regarding the human visual system is made.

Chapter 3. Technical Preliminaries

This chapter provides the technical prerequisites covering all the background concepts related to various aspects of the work for a smooth understanding.

Chapter 4. Experimental Methods

This chapter deals with the research design and has a broad discussion on the research methodology and the testing details.

Chapter 5. Implementation Details

This chapter deals with the coding aspects and optimization details.

Chapter 6. Results

All the major findings of the experiments are presented in this chapter.

Chapter 7. Discussion

A discussion of the results and corresponding insights are presented in this chapter.

Chapter 8. Conclusion

Chapter 8 concludes the dissertation and also provides a discussion on the limitations and possible future work.

Chapter 2

Literature Review

The human eye is a fascinating organ that has been the result of years of evolution and keeps on getting better. The different receptors present in the eye are hungry on capturing different shades of blue, green and red, and the combinations of the responses received are combined to get the colour perceived. For instance, if a high response of all the three- red, green and blue are received, the perceived colour is nearly white. A high response of only red and blue and no green would most likely create a magenta response. An absence of all response would correspond to being black. There is a closely related area called colour psychology, which relates hues to aspects of human behaviour. A perfect application would include colour choice affecting the effect of placebos[2]. In a nutshell, the colour of a drug can determine its fate-how well it would be judged by patients, how much they would like it and also how they could perform in the markets. Marketing makes close use of colour psychology. A sample cream for a burning treatment, if red, may not be taken quite well, whereas, if white, may just as well work. This exhibits how colour psychology can shape beliefs and how enormous, the area colour perception encompasses, is.

2.1 History of Colours and the Colour System

Colour system has captured the interests of scientists and philosophers starting from antiquity. Kuehni portrayed the history of colours from ancient Greece to the middle ages, and this gives a fascinating insight into colour science[3]. The Greek philosopher Plato defined brightness, white, red and black as the four primary colours from which all other colour sensations could be generated.

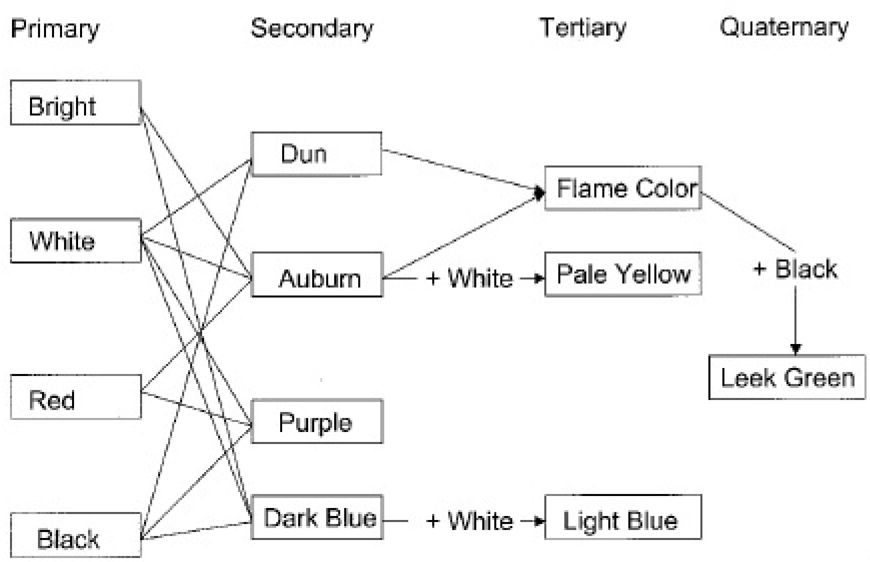


Figure 2.1: Colour system devised by Plato(Image from [3]) Plato defined brightness, white, red and black as the four primary colours from which all other colour sensations could be generated.

Another Greek philosopher Aristotle defined primary colours in a slightly different way. He defined blue and yellow to be the primaries and connected all colours to fire, air, earth and water[4].

Later Franciscus Aguilonius came up with a three colour system, as opposed to Aristotle and his system was based on red, blue and yellow.

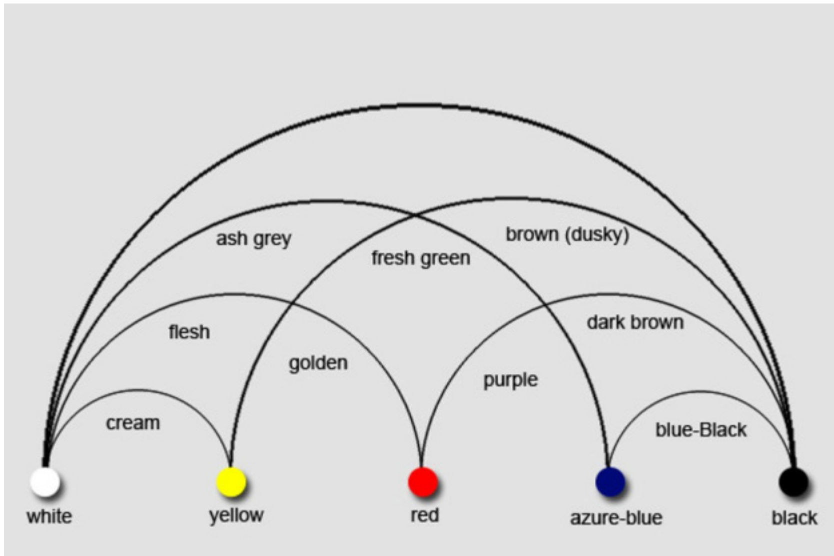


Figure 2.2: Tri colour system by Franciscus Aguilonius(Image from [5]) Franciscus Aguilonius' system of colour was based primarily on red, yellow, blue, and their combinations.

Isaac Newton, in his famous prism experiment, showed that white light, when passed through a prism, breaks into a spectrum of colours. If the spectra are passed through a second prism again, the first light is obtained[6].

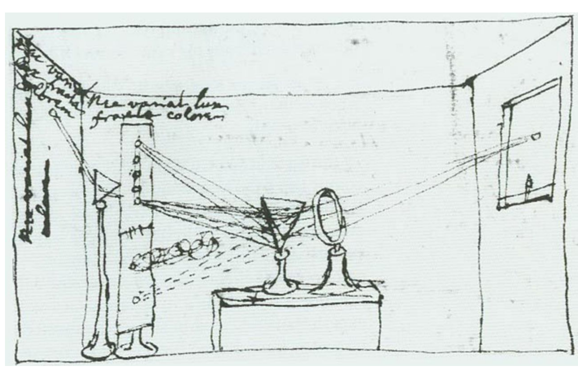


Figure 2.3: Newton's prism experiment(Image from [6]) Isaac Newton in his famous prism experiment showed that white light when passed through a prism breaks into a spectrum of colours.

Johannes Wolfgang von Goethe opposed the ideas of Newton that white light splits into different colours. He was more Aristotelian in his colour theory giving yellow and blue the primary significance[4]. His psychological approach laid the foundation for modern colour psychology.

Lars Sivik, Gunnar Tonnquist and Anders Hard created the Natural Colour System which is a universally accepted system today[7].

2.2 Colour Mixing

Colour plays an integral part in everyday life. Colour mixing, which is an essential part of this work, has been a historically addressed topic. Going back to Ptolemy's[8] disk of colours which when spun produced new colours, Forbes' addition and advancement of Ptolemy's technique by adding coloured sectors[9], Maxwell's replacement of the disk with a box that created beams of light and blended them too, a lot has happened over the past centuries.

Colour mixing can be additive, subtractive or partitive. Additive colour is based on the composition of different coloured lights while subtractive colour mixing is based on limited absorption of wavelength by an absorbing media like a pigment. Partitive colour mixing can be defined as the mixing of colours by putting them on a sectored disk and spinning it[10].

Additive Colour Mixing

Different coloured lights can be mixed to create an entirely new coloured light. Modern coloured displays like liquid crystal displays and cathode ray tubes utilize the RGB model, which is the additive model[11]. Isaac Newton's prism experiment to split white light gave rise to his famous colour wheel.

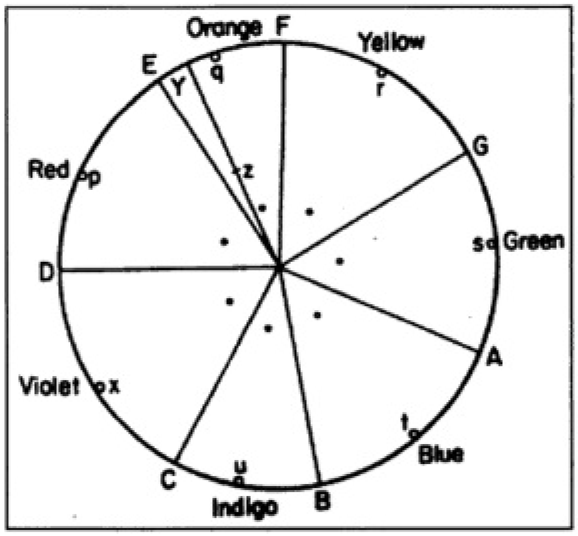


Figure 2.4: Newton's Colour Wheel(Image from [10]) Isaac Newton arranged the colours violet, indigo, blue, green, yellow, orange and red in a wheel. When the wheel was rotated, all the colours combined and produced the sensation of an almost white colour

James Maxwell designed a colour triangle, which is an intricate placement of colours with the vertices representing the three different primary colours. The other colours inside the triangle are obtained by adding different proportions these primaries.

Thomas Young and Hermann Helmholtz proposed the trichromatic colour theory, which states that human eye has three different types of cones which sense different ranges of wavelength[12]. Herman Helmholtz added further that the different classes of cones are: violet for sensing short wavelengths, green for mid-range wavelengths and red for the high-range wavelengths[13]. Helmholtz designed a colour circle where he used a different set of primary colours, namely violet, green and red as opposed to Maxwell, who used ultramarine, vermilion and emerald[10].

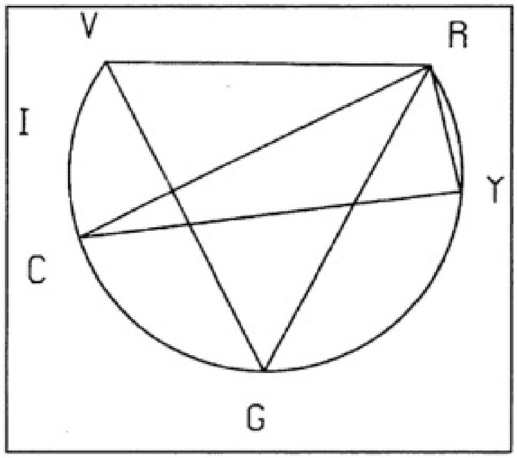


Figure 2.5: Helmholtz Colour Triangle(Image from [10]) Helmholtz colour circle had set of primary colours namely violet, green and red as opposed to Maxwell who used ultramarine, vermilion and emerald.

Mathematically, additive colour mixing can be represented as:

$L_o = mL_a + nL_b$, where L_o is the output light and L_a, L_b are the input lights with a radiant flux ratio of m:n[10].

Subtractive Color Mixing

When coloured light is passed through a filter, a part of the light is subtracted or filtered out, and only a certain part is transmitted. For instance, in the CMY or Cyan Magenta Yellow model, cyan acts as a complement to red, which indicates that a cyan filter does not allow red wavelength to pass. This is how subtractive colour mixing works.

The German philosopher Goethe's approach to colour mixing was similar to Aristotle, as opposed to Newtonian ways of additive mixing. Goethe had a subjective approach to colour mixing where he portrayed colours through a mixture of darkness and absence of it, while Newton's objective approach was more concerned with inspirations from the natural world around[10].

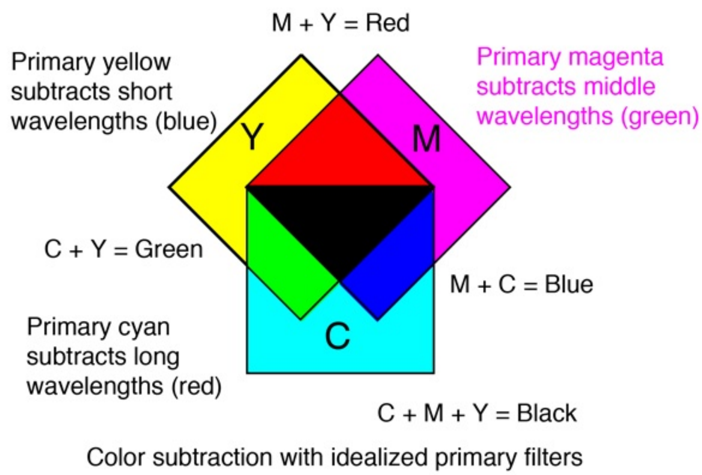


Figure 2.6: Subtractive Colour Mixing(Image from [14]) Subtractive colour mixing is based on the primaries cyan, yellow and magenta. Yellow takes away the wavelengths that are short, followed by magenta and then by cyan which takes away the high range wavelengths.

Partitive Colour Mixing

Partitive colour mixing can be defined as the weighted sum of colours to produce a new colour. If two input colours L_a and L_b are on two consecutive sectors of a disk with ratio of areas being m:n[15], the resultant mixture of them on the disk being spun rapidly is the output L_o:

$$L_o=\frac{m}{m+n}L_a + \frac{n}{m+n}L_b$$

The Partitive mixture can be extended to superpartitive mixture, which is also a linear combination of images[16]. The final image in a partitive mixture is a weighted sum of n images where the weights add up to one. If the weights are allowed to be negative, and they still add up to one, the resulting mixture is called a super-partitive mixture.

2.3 Colour Space and Colour Solid

Rolf G. Kuehni presented a well-defined history of colour spaces from ancient times to the present day[3]. He defines colour solid succinctly as:

"Colour space is a three-dimensional geometric space with axes appropriately defined so that symbols for all possible colour perceptions of humans or other animals fit into it in an order corresponding to the psychological order. In this space, each colour perception is represented as a point. The symbolic representations of colour perceptions in this space form the colour solid"

Kuehni argues that the colour solid varies from individual to individual, and this is quite expected as this represents the psychological colour space. Hermann Grassmann extended the idea of vector spaces to define the three dimensional colour space algebraically. Fearnley-Sander described his contribution[17] as:

"As noted first by Grassmann... the light set has the structure of a cone in the infinite-dimensional linear space. As a result, a quotient set (with respect to metamerism) of the light cone inherits the conical structure, which allows the colour to be represented as a convex cone in the 3- D linear space, which is referred to as the colour cone"

Grassmann laid down four laws, now known as Grassmann laws in Colour science[18]:

- A particular colour can be defined by its hue, its luminance and how much saturated it is.
- When a colour is mixed with its complement, gray colour is produced.
- For mixture of different lights, the total illuminance is equal to the sum of individual illuminance of the mixture components

- If X and Y are two colours both having hue and saturation, p and q respectively, and Z be the mixture of X and Y, then Z also has a hue and a saturation of p and q respectively

The CIE 1931 Colour Space was developed in 1931[19]. Based on an experiment performed on ten observers by William David Wright and another on seven observers by John Guild[19], the obtained results were used to develop the RGB Colour space from which the XYZ colour space was derived. Following this, the CIEUVW colour space was adopted in 1964, and CIELUV space was embraced in 1976[20].

Philipp Otto Runge was a German painter who developed a colour solid starting from three colours red, yellow and blue. The pure hues on the periphery of the sphere are the most saturated, and colours on antipodes are complementary to each other[21]. Johannes Itten developed a colour sphere that had twelve colours, with 3 primaries, 3 secondaries and the rest tertiary[22],[23].

The Munsell Colour system is a popular colour system developed by Albert H. Munsell. The system uses the axes of hue, chromaticity and value to define the perceived colour space. Value varies with luminance when it is reflective only[24].A cylindrical coordinate system is used with a ten point scale where 10 refers to a high degree of white representing a surface that is absorbing, while a 0 on the scale is an intuitive reference to blackness. The gray level can be controlled by changing C with appropriate H and V[22].

If the vertical cross section of the Runge, Itten and Munsell spheres are compared, it can be observed how differently they are arranged. For instance, chroma increases as one moves horizontally across the Munsell colour solid section while for Itten's section colourfulness increases in the same direction[25].

Optimal Colours

Optimal colour represents the purest form of a hue. David MacAdam discussed in detail how the development of better optimal colour standards are possible[26]. In fact, MacAdam developed the theorem that a colour can achieve its purest form with respect to a given wavelength if the associated spectrophotometric curve has at most two transitions and takes the value either 0 or 1 at each point. The end vertices of an optimal colour solid represent white and black.[25].

"A local colour is optimal if and only if the values of its reflectance function are either 0 or 1, with at most two transitions between those two values[27]."

This result was proven partially by Wilhelm Ostwald in 1916[28]. Erwin Schrodinger followed that four years later and proved that reflectance function corresponding to an optimal colour has a value 0 or 1 at any point and there can be at most two transitions[29]. There is a catch though, and the fact is that these reflectance functions produce optimal colours for humans only. West and Brill studied bees and found out that for different species, the nature of reflectance functions for producing optimal colours vary significantly[30].

Martinez-Verdu et al. devised a method to calculate the arrangement of optimal colours for a wavelength range using a lightness tolerance value. But the tolerance for lightness was selected by a trial and error method, and the process was very computationally intensive . Kenichiro Masaoka created a faster technique for the computation of optimal colours using trapezoidal integration[31]. Paul Centore showed that the colour solid corresponding to CIE XYZ colours is a zonohedron generated using vectors representing colours with reflectance value 1 for a wavelength interval and 0 elsewhere for their reflectance functions[27].

2.4 Psychophysical Colour Experiments

Colour perception is extremely sophisticated because it involves complex connections between how objects reflect light and how images are produced in the retina. Alongside the physical properties that objects possess, factors like geometry, position in space, illumination and spectral properties greatly define how an object looks. Several complex and exciting colour effects have been observed over the years. Ralph Pridmore and Manuel Melgosa studied the effects of psychophysical factors on the reception of colours[32]. Luminance, the observed light's purity, the most significant wavelength of the observed light are the psychophysical factors, while, the colour criteria included for the study comprise of the vibrancy, how bright and light the input is, how saturated input is, the hue and the chromaticity. Permuting and combining the different factors under normal conditions created nearly a hundred and a sixty combinations for testing. Colour science aims to capture these combinations and the effects they produce on individuals so that they can be applied in areas like commerce[32].

Colour effects

Psychophysical variables in different combinations can capture colour psychology in great detail. Perceived colour space is widely accepted to possess a three dimensional solid shape where the axes are defined by the factors hue, brightness level and the chromaticity. Colour science was formally defined by the Optical Society of America[20]. By altering these psychophysical variables, a lot of effects have been observed. Helmholtz–Kohlrausch effect named after Hermann Helmholtz and Rudolf Kohlrausch[33] shows that the human eye considers some colours to be brighter despite them being of the same intensity. The Hunt effect explains that with increasing luminance, the human eye notices a flurry of colours[34]. Stevens effect describes the perception of contrast with varying levels of luminance[35]. Crispening is a phenomenon where a similar background makes the perception of colour differences more easier[36]. Simultaneous Contrast is another interesting phenomenon where the same colour appears to be lighter or brighter depending on the contrasting background it is kept in, for instance, a level of gray can be more or less prominent depending on whether it is kept on a grayish or whitish background[37], [38].

An interesting fact about human vision is that binocular vision is not simply the summation of two individual monocular vision. It was accepted that three dimensional objects are treated with monocular vision and the presence of such objects in flat representation of pictures are considered imaginary. Berkeley in 1709 said that the aforementioned monocular representation is an impossibility and the claims were clarified and considered an assertion after the invention of binocular stereoscopy[39]. In binocular perception, both eyes obtain a different image of the same scene, and the depth is calculated from the different positioning of objects in both scenes. Koichi Shimono et al. presented a logical gate analogy of monocular vision and binocular vision to provide a psychophysical proof for the binocular system[40]. Stanikunas et al. worked on the psychophysical limitations in a quadrichromatic setup [41]. The tetrachromatic colour setup was created by a combination of the light emitting diodes of the colour red, amber, blue and green. The results of the RAGB setting is quite different from the CIE colour space, that is in standard use, and was developed by the International Commission on Illumination, back in 1931.

The verbal effects related to colour perception states that colour terms used to describe everyday colours have an effect on the responses that a person iterate[42]. Two views are related to this- one being the colour words solely used for communication purposes, the other being the cognitive system actually affected by choice of colour words. These alternatives reveal interesting options regarding the three dimensional colour solid in humans[42].

The Sapir- Whorf hypothesis says that if one takes speakers spread over a multitude of languages, they would perceive the colourful world quite differently[43].

Computer Graphics has become a really important tool in this area of study as it makes it possible to simulate conditions with varying intensity, geometry, lighting and other parameters. The simulations are really useful in duplicating natural conditions and placebo environments and are thus really helpful in psychophysical experiments, for instance, as in RenderToolbox[44]. Michael.F.Deering presented a simulated model of the human vision system, using a graphics renderer[45], to test the limits of perception of the human colour system. Full-fledged hardware comprising a stereo display with a frequency of 60 hertz and six degree depth association with the ability to render a collection of triangles, around ten billion, managed to represent the visual process in humans. Jan J. Koenderink and Andrea J. van Doorn defined a type of bidirectional reflectance function to address the scattering of light across an object's surface[46]. Applications in machine vision, graphical setups, image processing, require rendering of extremely intricate surfaces with varying geometry and thus reflectance functions are very important for accurate representations. The designed BRDF function can be used in Lambertian, Backscatter, Specular lobes. Mark D. Fairchild and Elizabeth Pirrotta[47] devised a method to describe the lightness and darkness of an object's chromaticity and this method can be used to tackle effects like the Helmholtz-Kohlrausch effect[33]. The devised parameter to predict the inherent lightness of an object can be given by:

$$L^* = 116(P/P_n)^{1/3} - 16$$

where P is the object lumination and P_n is the lumination of the white reference for the object. The ratio of a constant multiple of $(P/P_n)^{1/3}$ is thus related to the defined parameter L^*. L_1^* is an addition to L^*. L_1^* is given by:

$L_1^* = L^* + C^*$, where C^* is given by the CIELAB formulations. L_1^* can be further improved by adding to hue factor to get L_2^*[48].

$L_2^* = L^* + f_1(h^\circ)C^*$, where h is the hue factor.

$f_1(h^\circ)$ is defined as:

$f_1(h^\circ) = |a_1 sin(0.5 \times (h^\circ - 90))| + a_2$, a_1 and a_2 are constants.

The changes in root mean square over the experiments show that L_2^* is actually an improvement over L_1^*.

The report of the Optical Society of America regarding uniform colour scales[49] presents the fact that colours of a higher degree of saturation require a lesser amount of luminance compared to low saturated counterparts to appear similarly bright, based on experimental data. The report strongly emphasized that brightness and chromaticity are intricately related[49]. Advances in Computer Graphics have thus enabled experimental simulations that help to study these phenomenal colour effects in great detail.

Colour Constancy

Colour constancy refers to the fact that an object appears to be of the same colour in differently lit environments. Brainard et al., described in detail that the colour of an object perceived by humans and the material that define it are not simply related to the image that is created in the retina and thus the visualization of materials is quite a complex phenomenon[50]. Perception of materials is a primary stimulus for humans because the initial judgment comes from that. When a person eats something, the way the food looks can define whether the subject ultimately eats it or not. The way someone hesitates to touch something that looks spiky, may not be entertained by something that looks slimy, may not step on something that looks slippery, is all defined by the perception of colour and materials. The human visual system stabilizes itself to define a notion of colour constancy[51] so that an object's appearance is constant to changes in lighting and geometry. Colour constancy can be tested by a method called asymmetric matching, which includes two sets of images- one is a reference image, and the other is a secondary image under different illumination. Participants have to appear in a test where they try to match the secondary image by changing given parameters so that the secondary image looks like the reference image as much as possible. The correspondence established is used to study colour constancy[52],[53].

Factors like the structural organization of materials, shape and position can lead to changes in perception. Olkonnen and Brainard[54] studied the influence of geometry and shape of objects in the perception of materials. Different properties cause light interaction to vary across different surfaces and thus, the visual system responds differently. In the case of flat objects with simple geometry though, colour constancy is achieved fairly easily[55]. On the other hand, for non matte things with complex three dimensional geometry, the matter is not that simple. Brainard and Olkonnen concluded that lightness is not affected dramatically by factors called geometry and shape, while glossiness is quite intricately related to these factors.

Colour constancy can be affected by cues over time domain[56]. Colour constancy variants include displacement ones, where the colour appears constant despite changing positions of the object, atmospheric ones where the colour settles down despite changing mediums and relational constancy which assumes a comparison between surfaces with respect to changing factors. Standard colour constancy measuring laboratory techniques include using Mondrian style stimuli illuminated with different illuminants. The subjects generally have to match the given image to a target one by changing several parameters[56].

Reports show that in this type of asymmetric matching, perfect colour constancy is not quite obtained. The highest level of constancy ranges around seventy percent. Relational colour constancy changes are generally not picked by observers[56],[57]. Brian Funt et al. addressed colour constancy that comes out of mutual reflection. Mutual reflection is the leaking appearance of colours from one region to another under reflection[58]. Mutual reflection causes irregular scattering of light across surface points even though the reflectance across those points is constant. Brian Funt et al. devised an algorithm to negate the effect of mutual reflection and provide an approximate model of the complete spectral range. Cindy M. Goral et al. modelled a way to instantiate the reflection that occurs across two diffuse surfaces[59].

The ability to detect relational colour constancy, though, increases with the opportunity to harness more temporal cues. The experimental results strongly suggest that relational colour constancy is close to colour constancy of temporal Mondrian stimuli[60].

As prevalent in modern colour study literature, psychophysical experiments are extremely relevant and effective tools to find patterns in human colour vision and studying how the visual system works. The current work narrows down the focus strongly on optimal colours and builds on the optimal colour solid properties as defined by Erwin Schrodinger and Wilhelm Ostwald[28],[29]. A novel method is devised to compare the psychological colour solids obtained by user studies on 5 different individuals with the optimal colour solid developed as per the literature. Previous studies on optimal colours only deal with the optimal colour representation in various colour spaces and development of optimal colour solid in the ideal case. But, this piece of work focuses on optimal colour perception in human beings keeping in perspective the optimal colour solid developed in ideal conditions using optimal reflectance functions, and how individuals conceive optimal colours and the variations across various individuals.

Chapter 3

Technical Preliminaries

This chapter addresses all the central technical topics of the dissertation alongside a brief description for clear understanding.

3.1 Vectors

A vector is a mathematical entity that possesses a magnitude and direction. A line segment can be used to represent a vector.

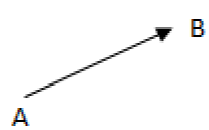


Figure 3.1: A vector Here the vector **AB** points in a definite direction from A to B and the length of the line segment AB represents its magnitude. The direction is given by **B-A** , where **B** is the vector from O to B, and **A** from O to A, with O as the origin.

Vectors can be added or subtracted from each other. One can follow the simple triangle rule to do so:

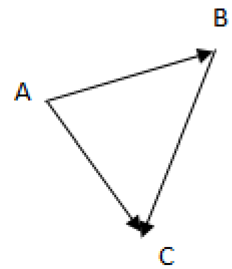


Figure 3.2: Triangle Law of vector addition Triangle Law of vector addition is an intuitive way of representing vector addition. The vectors **AB** and **BC** are added to get **AC** here

The simple, intuitive interpretation is: If a person starts from A and follows the direction AB, he ends at B. From B, the direction BC takes him to C. The person starts at A and ends at C. In vector addition terms,

$\mathbf{AB} + \mathbf{BC} = \mathbf{AC}$

One can change the sides to visualize subtraction as:

$\mathbf{AC} - \mathbf{BC} = \mathbf{AB}$

If one adds a vector to a point, one gets a vector. So, $\mathrm{A} + \mathbf{AB} = \mathbf{AB}$

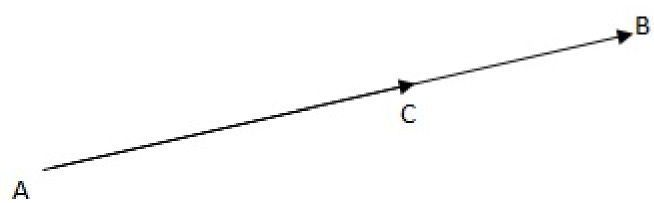


Figure 3.3: Straight line representation using vector
Given that, one knows the starting point of a line and the direction of the line, one can represent any point on the line.

In the above figure,

$\mathrm{A} + \mathbf{AC} = \mathbf{AC}$

$\mathrm{A} + \mathbf{AB} = \mathbf{AB}$

Say, $\mu = \mathrm{AC}/\mathrm{AB}$

One can write, A + μ**AB** = **AC**

A + μ(**B - A**)= **AC**

When μ=0, $A + \mu$(**B - A**) = **A**

When μ=1, $A + \mu$(**B - A**) =**B**

This is the line equation of a vector.

For instance, if A = (0.5, 0.5, 0.5), B = (1, 1, 1) and μ= 0.5,

A + μ(**B - A**) = (0.5, 0.5, 0.5) + 0.5 (1-0.5, 1-0.5, 1-0.5) = (0.75, 0.75, 0.75)

3.2 Voronoi diagram

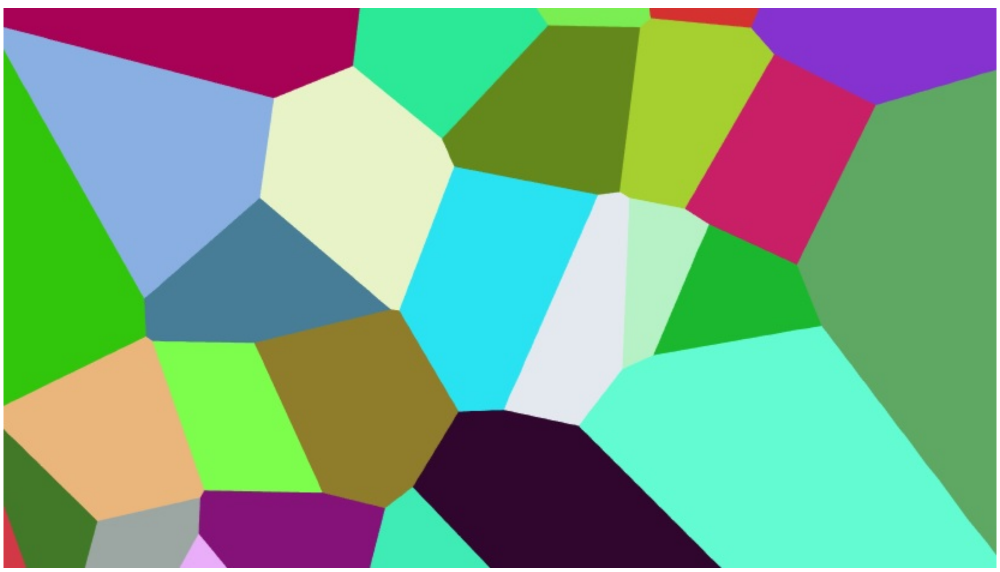


Figure 3.4: Voronoi diagram The plane above is partitioned into 28 regions and each of them is represented by different colour

A Voronoi diagram simply partitions a plane P into regions $P_1, P_2 \cdots P_N$ such that $P_1 \cup P_2 \cup P_3 \cdots P_N = P$ based on given n seed points. The partition is carried out in a way that if a seed point belongs to a particular region, all the points in that region will be closest to that seed point only[61].

In figure 3.4, the plane is divided into 28 parts. For calculating distance from a seed point, simple measures like Euclidean[1] or Manhattan distance[2] can be considered.

3.3 Delaunay Triangulation

Voronoi diagrams can be created from Delaunay triangulation [62] of a set of seed points in a plane. Delaunay triangulation is a triangulation of the seed points such that for each triangle, the circumcircle has none of the seed points inside.

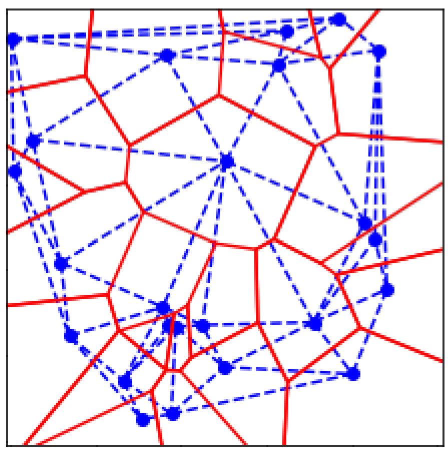


Figure 3.5: Delaunay Triangulation
The blue triangles represent the Delaunay triangles obtained using seed points in a plane. The red polygons represent the corresponding Voronoi diagram

3.4 Convex Hull

For a set of points in a space, a convex hull [63] is the smallest convex shape that covers the points completely. It can be visualized as minimally wrapping a gift cover over the

[1]Euclidean distance between points $P(x_1, y_1)$ and $Q(x_2, y_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
[2]Manhattan distance between points $P(x_1, y_1)$ and $Q(x_2, y_2) = |x_2 - x_1| + |y_2 - y_1|$

points.

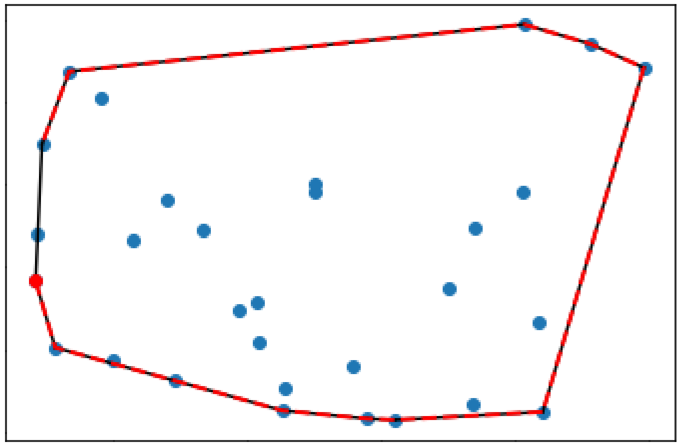


Figure 3.6: Convex Hull The red-black outlining polygon represents the convex hull of the blue points

3.5 Colour Space

A colour space is an arrangement of colours. A colour space can be combined with a colour model, a vector used to represent colours. For instance, in the RGB colour space, every colour can be represented by a vector, in this case, a triplet of numbers. Historical colour models like RGB, xyY and HSV used three element vectors to represent colours[64]. The CMYK colour space is an alternative colour space that uses a new variable black for better printing.

The RGB Colour space is defined by the strength of red, blue and green primaries and is based on the RGB colour model. Various colour spaces are based on the RGB colour model like Adobe RGB colour space, sRGB, Adobe Wide Gamut RGB, ProPhoto RGB colour space[64].

Different colour spaces can represent different ranges of colours. Some of them are bigger than the others and can represent a broader range of colours.

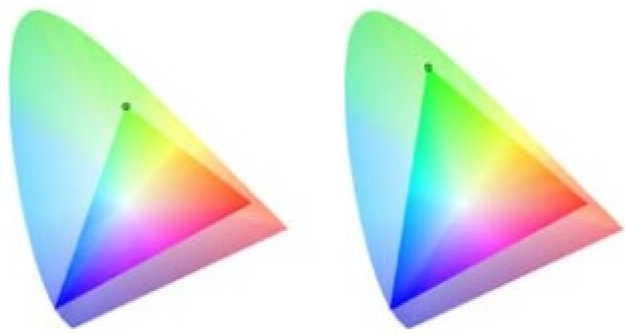


Figure 3.7: Colour Space Comparison(Image from [65]) The image on the left represents the sRGB colour space and the image on the right represents the Abobe RGB colour space from 1998 plotted over the CIE x,y Chromaticity diagram[65]. The triangles represent the total number of colours represented by the colour spaces. Since, the area of the triangles are different, the number of colours represented by sRGB and Adobe RGB colour space are different.

For instance, Adobe RGB 1998 represents a much broader range of colours as compared to sRGB colour space. The CMYK colour space can not represent some of the colours the sRGB colour space can and vice versa.

3.6 HSV colour model

HSV colour model stands for the 'hue-saturation-value' colour model[66]. Hue, saturation and value are three dimensions that can be used to represent any colour.

Hue names a colour like red, green, blue. Hue is represented by a circle and the colours are arranged by degrees[24],[67].

The colours red, yellow, green, blue and purple follow each other in the circle. Interestingly, the boundary of red and yellow consists of a red-yellow hue and similarly there are blue-green, purple-red, yellow-green, purple-blue hues [24]. The colours transition into one another.

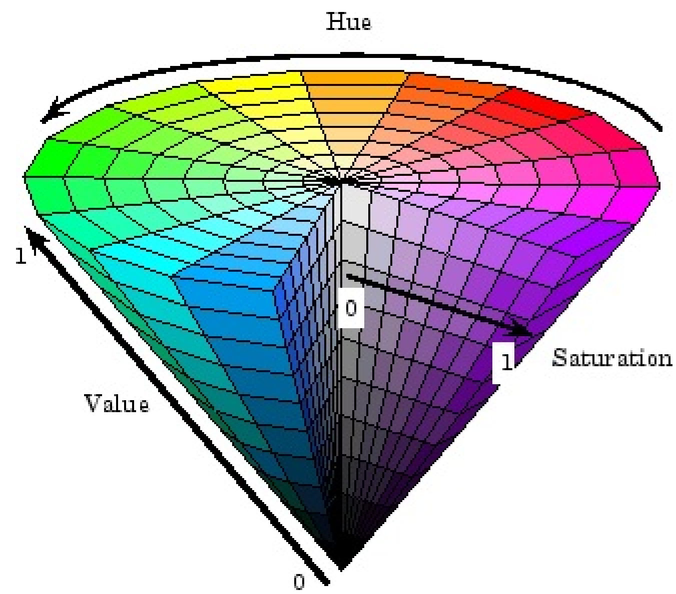


Figure 3.8: HSV Colour Space(Image from [68]) The HSV colour space has three dimensions hue, saturation and value. Hues are arranged in a circle. Saturation and value are both between 0 and 1.

Saturation defines the purity of a colour. A very pure red would appeal more than a dull red consisting lots of grays. The most saturated shades are along the periphery of the model.

Value indicates the amount of light in colour. The higher the colour's value, the brighter it looks.

The RGB colour model is the most common one and in fact, HSV model can be converted to RGB quite simply, as shown below[69],[70].

Hue H is between 0 and 360 degrees, both inclusive.

Saturation S, value V are between 0 and 1.

$H_{new}=\frac{H}{60°}$

$$R_i, G_i, B_i = \begin{cases} (VS, VS(1 - |H_{new}\%2 - 1|), 0), & 0 \leq H_{new} \leq 1 \\ (VS(1 - |H_{new}\%2 - 1|), VS, 0), & 1 \leq H_{new} \leq 2 \\ (0, VS, VS(1 - |H_{new}\%2 - 1|)), & 2 \leq H_{new} \leq 3 \\ (0, VS(1 - |H_{new}\%2 - 1|), VS), & 3 \leq H_{new} \leq 4 \\ (VS(1 - |H_{new}\%2 - 1|), 0, VS), & 4 \leq H_{new} \leq 5 \\ (VS, 0, VS(1 - |H_{new}\%2 - 1|)), & 5 \leq H_{new} \leq 6 \end{cases}$$

Here $a\%b$ gives the remainder when a is divided by b.

If H_{new} is not defined, $R_i, G_i, B_i = (0, 0, 0)$

The RGB values ultimately are: (R,G,B)=$(R_i + V - C, G_i + V - C, B_i + V - C)$

3.7 Conversion of XYZ Colour Space to CIE L*a*b* Colour Space

If , $\hat{X}, \hat{Y}, \hat{Z}$ be the tristimulus values that correspond to the given reference white chosen, for instance D65[20].

L* $= 116 \times (\frac{Y}{\hat{Y}})^{\frac{1}{3}} - 16$,if $(\frac{Y}{\hat{Y}}) > 0.008856$

L* $= 903.3 \times (\frac{Y}{\hat{Y}})$, ,if $(\frac{Y}{\hat{Y}}) < 0.008856$

a* $= 500 \times (F(\frac{X}{\hat{X}}) - F(\frac{Y}{\hat{Y}}))$

b* $= 200 \times (F(\frac{Y}{\hat{Y}}) - F(\frac{Z}{\hat{Z}}))$

Here, F(m)= $m^{1/3}$ if $m > 0.008856$,

and F(m) $= 7.787 \times m + 0.1379$,if $m < 0.008856$

3.8 Radiance Renderer

Radiance is a ray tracing software that was developed by Greg Ward. It first came out in 1985. An octree data structure is used to make ray tracing faster using this software. Radiance also implements global illumination.

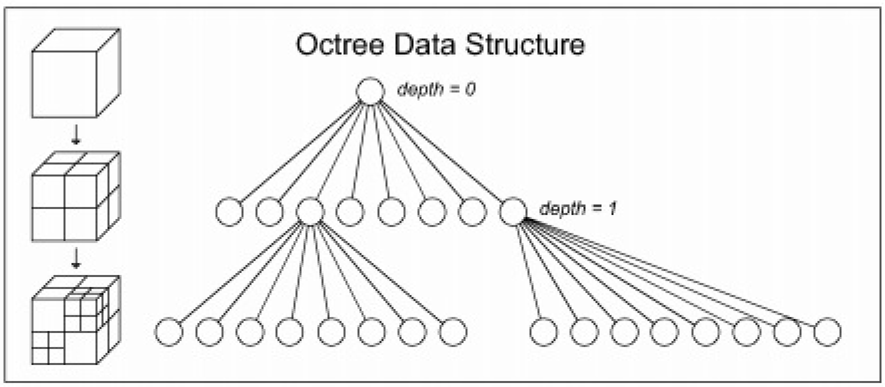


Figure 3.9: An Octree Data Structure(Image from [71]) In an octree, each node has 8 children. The root node here is represented as the big cube at top left and at every level, the divisions are refined.

3.9 Psychtoolbox

Psychtoolbox is a free toolbox that can be added as an extension to MATLAB. It is handy for doing psychological experiments in a controlled manner[72].

Psychtoolbox interacts with the hardware and gives capabilities like accessing display frame buffers, using vertical retrace for synchronising better, providing audio and video stimuli addition capabilities and easy processing of user inputs for psychophysical experiments.

3.10 D_{65} illuminant

D65 is a standard illuminant that simulates the average daylight environment in Northern and Western Europe. It was standardized by the CIE. D65 illuminant portrays typical daylight situation at a temperature of 6504 Kelvin. A standard illuminant is an instance of a light source in theory, and it has a defined spectral distribution profile. Standard illuminants make it possible to compare colours under various lighting conditions. The amount of sunlight that different places on earth receive is vastly different. Also, for artificial light sources, the emitted light depends on many factors like how old the source is and the voltage that is applied. Because of this variability, it is not possible to use these sources, precisely as they are, for colourimetric studies and specifications[73].

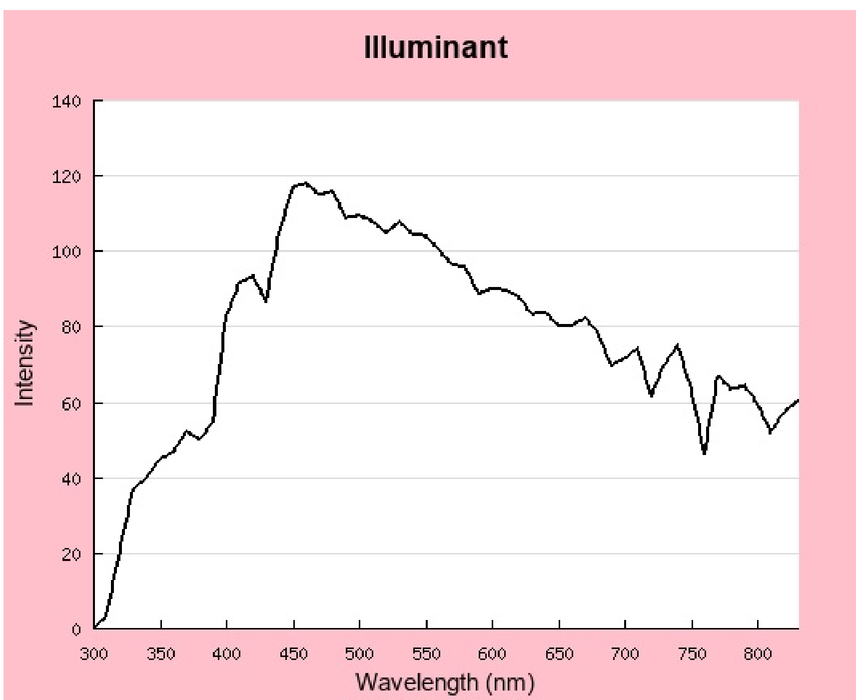


Figure 3.10: D65 Illuminant(Image from [74]) D65 illuminant portrays typical daylight situation at a temperature of 6504 Kelvin.

For this reason, standard illuminants with standard spectral signatures are defined, and D65 is one of the standard illuminants. There are several D series illuminants in the D series, as defined by CIE. D55 illuminant represents daylight for the noon sky at a temperature of 5500 Kelvin, and D75 illuminant represents daylight for the north sky at 7500 Kelvin.

3.11 Settings for Colour Study

This section defines various parameters that would be frequently used in describing the colour study in the later chapters. The details about how the whole colour study is conducted are given in Appendix C. The code for the experiment is listed in Appendix B.

Median Setting: For each vector originating from midGray point in different directions, the median setting defines the percentage of the length of the vector $\mathbf{V}$(from midGray point to the periphery of the RGB cube in that direction) at which the participant thinks the colour of the central Voronoi polygon is saturated and striking out as compared to the background Voronoi colours.

Inner Cube Intersection: For each vector originating from midGray point in different directions, the inner cube intersection setting defines the percentage of the length of the vector $\mathbf{V}$(from midGray point to the periphery of the RGB cube in that direction) at which $\mathbf{V}$ intersects the inner RGB cube from minimum Gray level (.112,.112,.112) to maximum Gray level (.819,.819,.819), where .112 and .819 are the low and high Gray levels for the experiment.

Solid Surface Intersection: For each vector originating from midGray point in different directions, the solid surface intersection setting defines the percentage of the length of the vector $\mathbf{V}$(from midGray point to the periphery of the RGB cube in that direction) at which **V** intersects the optimal colour solid.

Outer Cube Intersection: For each vector originating from midGray point in different directions, the inner cube intersection setting defines the percentage of the length of the vector **V**(from midGray point to the periphery of the RGB cube in that direction) at which **V** intersects the RGB colour cube. The following image defines the intersection points in detail.

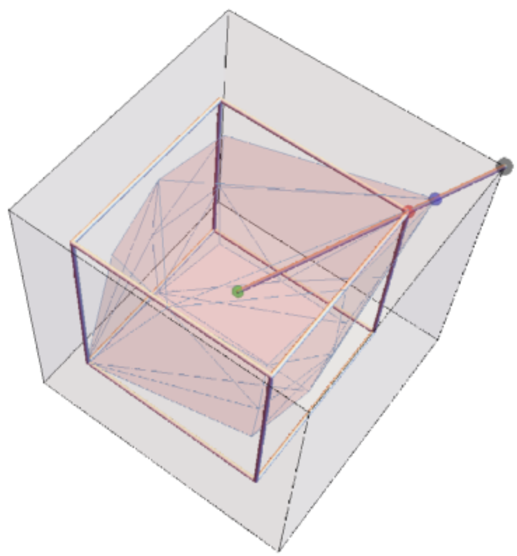


Figure 3.11: Intersection definitions The green sphere represents the midGray point (0.4655,0.4655,0.4655). The red tube from the green to the black sphere represents a vector or a particular direction. The red sphere here represents the inner cube intersection(the inner cube is the cube formed by the tubes and the outer cube representing full RGB colour cube is the gray bigger cube). The blue sphere represents the median setting and the black sphere represents the outer cube intersection. Say, the median setting is 0.59, and A="Distance between blue and green sphere" and B="Distance between green and black sphere". The blue sphere would be located at a point so that $\frac{A}{B} = 0.59$. The red polyhedron here represents the average colour solid and the intersection point(blue sphere) represents the median setting. Similarly, the Solid Surface Intersection represents the intersection of the tube or vector with the optimal colour solid and Background Intersection represents the intersection of the tube or vector with the background colour solid.

Chapter 4

Experimental Methods

This chapter explains in detail how the research was designed. The first part describes the stimuli generation, data collection and the user interface. The second part portrays the initial testing phase, which describes how the software prototype for the colour study was developed and the corresponding challenges.

4.1 Research Aim

The main aim was to study the human colour perception limits and to find out how the perceptual colour solid looks like. The work involved studying the perceptual psychological colour solid based on collected user data. The main focus was to compare the psychological and ideal colour solid to know more about the perceptual colour limits.

4.2 Preliminary Work

The initial step was deciding how to generate stimuli and present it to the participants using a Graphical User Interface. After a few initial trials, it was decided that the stimuli will be generated using Python and MATLAB, and the GUI would be solely generated using MATLAB.

MATLAB was the suitable choice for creating the GUI because it was possible to integrate Psychtoolbox easily with MATLAB. Psychtoolbox is an appropriate choice for psychophysical experiments because of its usefulness and accuracy to represent auditory and visual stimuli, and its ability to connect the hardware and MATLAB[72].

4.3 Methodology

4.3.1 Participants

Five students from University College London took part in the experiment voluntarily. All the participants had normal colour vision, and they had a mean age of 25 with a standard deviation of 4.28. The participants completed a session of the experiment per day over a period of 5 days.

4.3.2 Interaction with the Interface

When the user starts the Psychtoolbox application, there is a welcome screen that shows the instructions and the user is notified of the experiment configurations. The user can change the stimuli by using the slow keys: the **left** and the **right** key or the fast keys: **Y** and **U**. At the end of a trial, the user needs to press **S** key to store the preferred setting. After getting familiar with the trials, the user can proceed with the study. Before each trial, the trial number is notified and a Voronoi diagram is displayed. The Voronoi diagram has a polygon that is highlighted so that the user can understand that on pressing the fast or slow keys, the highlighted polygon will change its colour. After the buffer image is displayed, the user can proceed to the actual trial where he needs to use the fast or slow keys to adjust the highlighted polygon colour until it feels to them that the colour is saturated with respect to the surroundings. After 26 trials, a block is finished automatically and the data are stored.

4.3.3 Procedure

For the experiment, each stimulus is a Voronoi diagram that is made up of numerous polygons, one of which changes colour on being triggered by the user. For a trial within a block, all the Voronoi diagrams have the same background, and the central polygon has the changing shade of a particular colour. A block had 26 trails and each participant had to do a block a day over a period of 5 days. This section describes in detail how the stimuli were generated and presented to the participants.

Selecting top and bottom gray levels

The top and bottom gray levels were selected by judging stimuli sets for different sets of $\{lowGrayLevel, HighGrayLevel\}$. At the start,

$\{lowGrayLevel, HighGrayLevel\}$=$\{0,0\}$

For a particular $\{lowGrayLevel, HighGrayLevel\}$ pair, it was checked for all 26 directions that when the maximum limit was reached, whether the colour looked saturated or not. At every step, the lowGrayLevel was decreased a bit, and highGrayLevel was increased a bit. The result section 6.1 shows all the calculations of the highGrayLevel and lowGrayLevel and how it was experimentally established that:

$\{lowGrayLevel, HighGrayLevel\}$=0.112, 0.819

MidGray= 0.4655= $\frac{(0.112+0.819)}{2}$

4.3.4 Stimuli Presentation

An RGB colour can be represented in the format (a,b,c) where a, b and c can be either 0 or 1. So, in a unit cube, every 3D point represents an RGB colour.

For instance (0,0,0) represents black, (1,1,1) represents white and (1,0,0) represents red.

The medium and high levels, m and M set experimentally, as described earlier are: m= 0.112, M=0.819

As a result, the medium gray level was set by (m+M)/2=0.4655

Therefore, the point in the unit cube that represents gray is (0.4655, 0.4655, 0.4655).

26 different directions were chosen in the unit cube, from the midGray point to the corners,midpoints of faces and the midpoints of edges.

For a particular vector (a,b,c), the line joining the middle gray point to an intermediate point in that direction is given by:

$$(0.4655, 0.4655, 0.4655) + \alpha(a, b, c) = (0.4655 + a\alpha, 0.4655 + b\alpha, 0.4655 + c\alpha)$$

When $\alpha = 1$, each of $0.4655 + a * \alpha, 0.4655 + b * \alpha, 0.4655 + c * \alpha$ has a chance to be 1. For instance, if the vector (0.5345,0,0) is considered:

$$(0.4655, 0.4655, 0.4655) + \alpha(0.5345, 0, 0) = (0.4655 + \alpha 0.5345, 0, 0)$$

In this case, when $\alpha = 1$, the right hand side becomes (1,0,0) that represents a pure saturated colour on the surface cube.

Following this method, in each of the different 26 directions, a different value of alpha gives colour on the surface, which represents the limit of perception. The stimulus is a

Voronoi diagram where the middle polygon represents the changing colour (0.4655, 0.4655, 0.4655)+α(a,b,c). The α is set by the person performing the experiment and is recorded. The person repeats the process for different blocks. In each block, the same set of 26 directions are taken but the Voronoi stimuli background and the central polygon shape changes. After the alpha values are set for the blocks, the median readings of the values are taken for each direction.

For different directions, the gradual increase in colour as alpha reaches 1 in the central polygon is shown below, in order for directions:

(0.5345,0,0), (0.5345,0,0.5345), (0.5345,0.5345,0)

Figure 4.1: Gradual Increase in Stimulus Colour In a particular block, the stimuli have their middle polygon changing colour. For a particular direction, the colour of the middle polygon starts from gray and intensifies accordingly. This figure shows how the colour gradually increases for 3 different directions. In total, there are 26 directions.

4.3.5 Optimal Colour Calculations

The concept of Optimal Colour was first given by Wilhelm Ostwald[28]. An optimal colour seems more bright than any other colour of the same chromaticity under the influence of a particular illuminant. Schrodinger[27],[29], showed that the reflectivity spectrum for generating optimal colour consists of the values 0 or 1 and can have at most two transitions. Four such types of spectra are possible:

Here, the reflectance functions can take either the values 0 or 1 and there can be at most two transitions.

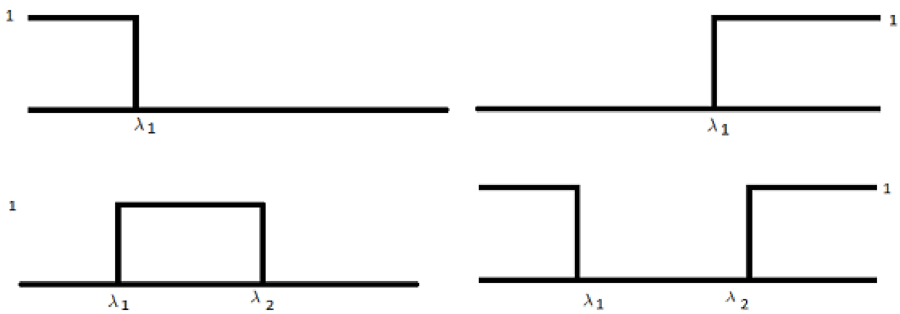


Figure 4.2: Reflectance Functions Four different types of reflectance functions are presented here. These functions always take the value 0 or 1 and they have at most two transitions.

Optimal colours can be represented with the help of X, Y and Z coordinates[27].

$$X = \frac{\int_{380}^{740} x(\lambda)r(\lambda)s(\lambda)d\lambda}{\int_{380}^{740} y(\lambda)s(\lambda)d\lambda}\text{.................(i)}$$

$$Y = \frac{\int_{380}^{740} y(\lambda)r(\lambda)s(\lambda)d\lambda}{\int_{380}^{740} y(\lambda)s(\lambda)d\lambda}\text{.................(ii)}$$

$$Z = \frac{\int_{380}^{740} z(\lambda)r(\lambda)s(\lambda)d\lambda}{\int_{380}^{740} y(\lambda)s(\lambda)d\lambda}\text{.................(iii)}$$

$\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$ represent the colour matching functions, r(λ) represents the reflectance function, s(λ) represents the d65 illuminant. The visible spectrum includes wavelengths between 380 and 740 nanometers.

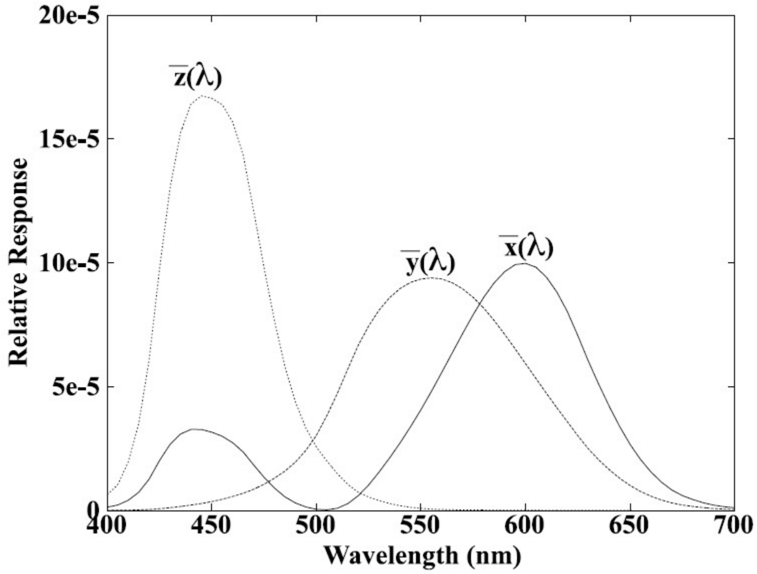


Figure 4.3: CIE Colour Matching Functions(Image from [27]) The standard observer function represents response of the human eye within two degree foveal arc on an average. The colour matching functions define the response of the standard observer for the three different primary colour functions that match a particular colour. The three primary colours had wavelengths 700 nm, 435.8 nm and 546.1 nm[20].

For calculating optimal colours, the four reflectivity spectra are used individually in equations (i), (ii) and (iii). From the XYZ coordinates, sRGB coordinates are calculated and plotted within a unit cube to obtain the optimal colour solid.

4.4 Testing Phase

Initially, for the stimulus presentation, the prototype was made using MATLAB, Python and Psychtoolbox. It presented the stimuli which were a set of Voronoi diagrams, and the participants could interact with it using a keyboard. The fully working final software was supposed to have a smooth interface that could present the stimuli without any time lag so that the experience was as comfortable as possible for the participants. The main aim of pilot testing was to detect the problems associated with the beta version of the final software.

4.4.1 Pilot Testing

The initial prototype was developed mainly using MATLAB and Python. For the purpose of checking the fact that the code was without bugs and the prototype was stable, the beta version was tested by two fellow researchers. The beta version did not have the full functionalities. It only presented a single experimental block as compared to the expected 5 blocks in the final software. The beta version worked smoothly but based on the feedback of the beta version testers, the following areas of improvement were identified.

- The beta version had only two options for stimuli transition- backward and forward. One could use the left and right keys for selecting a particular colour. It was decided that for the future version, an option would be included for a faster stimuli transition
- A buffer image describing which polygon in a stimulus, the corresponding Voronoi diagram would be added before each new trial in a particular block
- For the purpose of optimization, it was decided that the stimuli would be pre-computed and the images would be stored. This was to be done instead of real time computation of stimuli to avoid time lagging issues

All these necessary changes were addressed, and the final prototype was developed. The final prototype had a fully functioning block with 26 trials and was tested by 3 participants. The result was smooth, and the app was found to be fully functioning.

Chapter 5

Implementation Details

5.1 Setting up Voronoi diagrams

The Voronoi diagrams presented as the stimuli consisted of triangles, and they were randomly coloured using colours from a histogram consisting of colours that are generally present in natural scenes. An idea of how the histogram of natural colours looks is presented in Appendix A. A sample of stimulus is presented below:

Figure 5.1: Stimuli set The Voronoi diagrams presented as the stimuli consisted of triangles and they were randomly coloured using colours present in natural scenes.

The stimuli were presented to the users using a GUI. Each person had to press the left or the right key to select a particular image from a series of images. The colour of the central polygon in each stimuli changed slightly on pressing the left or the right key. For instance:

Figure 5.2: Gradual Increase in Colour of Central Polygon In a particular block, the stimuli have their middle polygon changing colour. For a particular direction, the colour of the middle polygon starts from gray and intensifies accordingly.

Each participant had to do six blocks where they had to press the left or the right key just to the point that the central polygon's colour seemed to be at the threshold of being unrealistic. They had to stop just before the central colour seemed too saturated or too striking as compared to the colours of the surrounding polygons.

This image shows the gradual progression of stimuli for a particular direction. As the user pressed the right or left key, they reached a different stimulus. Pressing the right key led to a stimulus where the intensity of the central polygon colour increased while pressing the left key led to a stimulus where the intensity of the central polygon colour decreased. Appendix C contains details of how the whole experiment was conducted.

5.2 The Colour Cube

Colour cube settings:

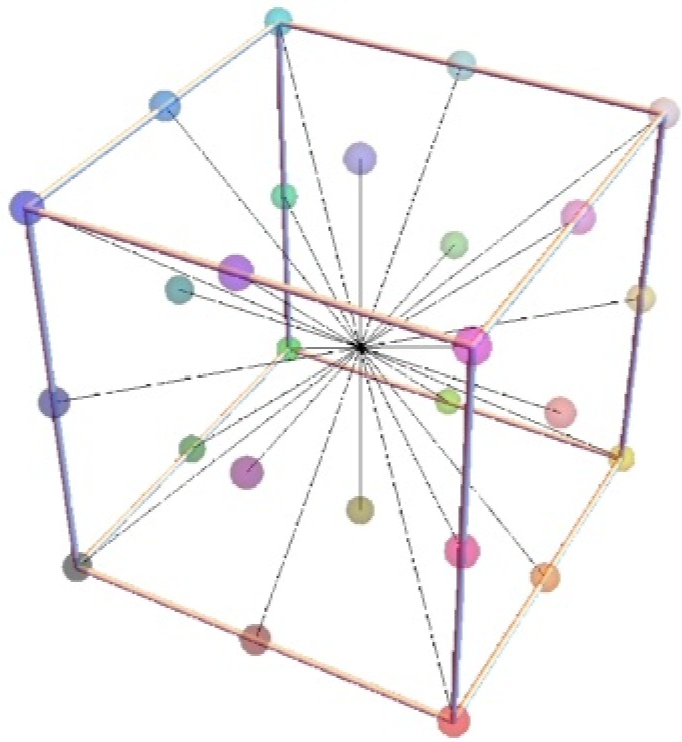


Figure 5.3: Colour Cube The colour cube depicts the 26 different directions represented by the black lines emanating from the central sphere. The spheres on the faces of the cube represent the 26 distinct surface colours.

Vectors were chosen along 26 directions in a unit cube, as shown in the figure. The directions were connecting the midGray point (0.4655, 0.4655, 0.4655) to points on the surface of the RGB colour cube.

5.3 Optimal Colours

The four different types of reflectance functions used for the development of the optimal colour solid are:

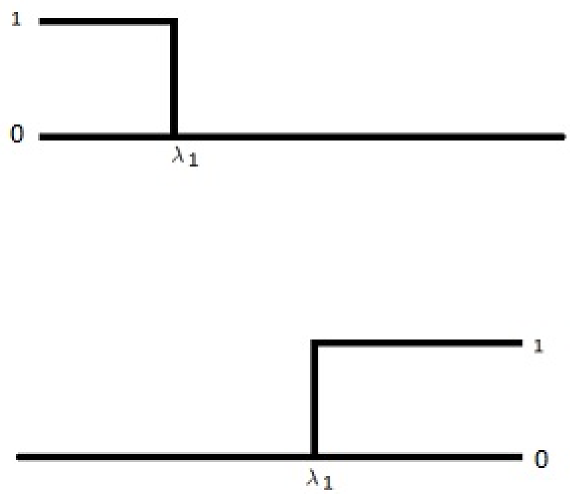


Figure 5.4: Reflectance Function Type A and B Reflectance function with only one transition at λ_1

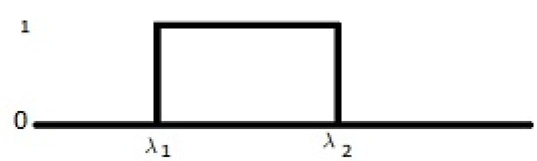


Figure 5.5: Reflectance Function Type C Reflectance function with two transitions at λ_1 and λ_2

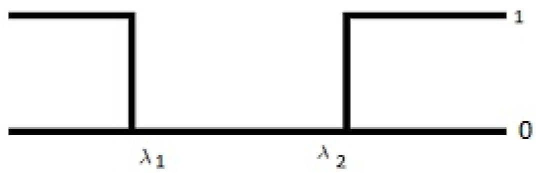


Figure 5.6: Reflectance Function Type D Second category of Reflectance function with two transitions at λ_1 and λ_2

For each different type of reflectance function, a different part of the optimal colour solid can be generated. So, using all four of them, the whole optimal colour solid is developed, as shown below:

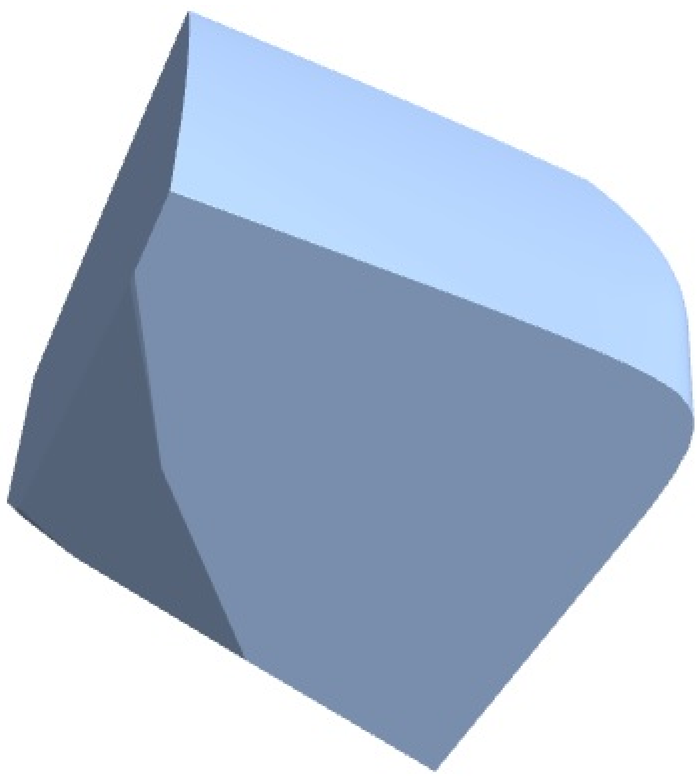


Figure 5.7: Optimal Colour Solid is a colour solid where all the optimal colours are on the surface of the solid.

The CIE standard D65 illuminant, intensity versus wavelength graph has been obtained from the Colour & Vision Research laboratory database from UCL[74]. The function is sampled at every wavelength interval of 1 nanometer starting from 390 nanometers up to 830 nanometers.

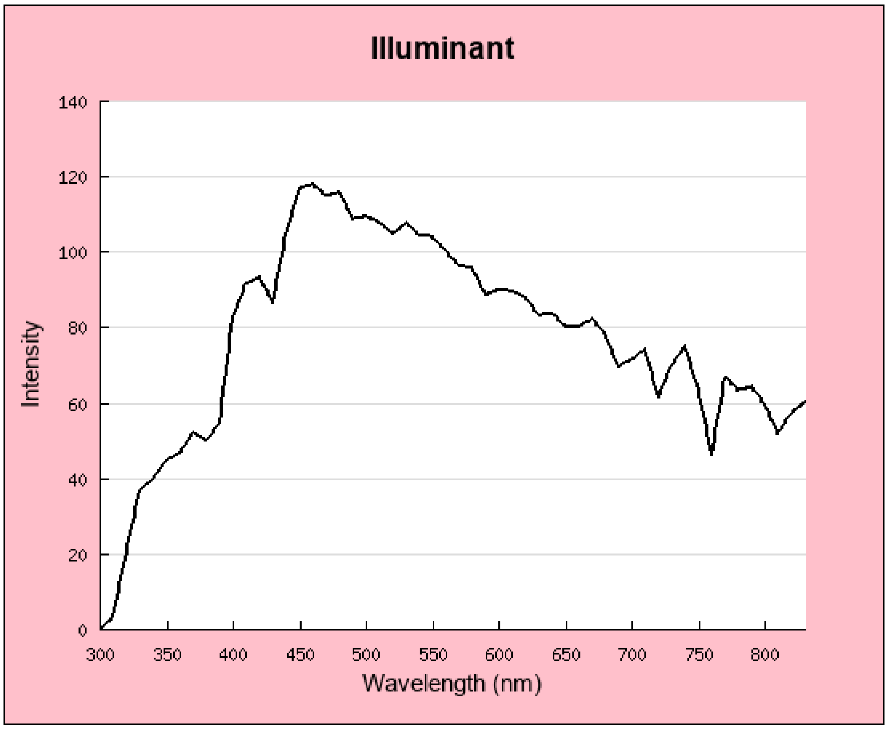


Figure 5.8: D65 Illuminant(Image from [74]) D65 illuminant portrays typical daylight situation at a temperature of 6504 Kelvin.

The colour matching functions are also obtained from the Colour & Vision Research laboratory database from UCL[74]. They are sampled every nanometer as well as each sample point is a triplet containing three colour matching values for red, green and blue, respectively.

$$X = \frac{\int_{380}^{740} x(\lambda)r(\lambda)s(\lambda)d\lambda}{\int_{380}^{740} y(\lambda)s(\lambda)d\lambda}$$

$$Y = \frac{\int_{380}^{740} y(\lambda)r(\lambda)s(\lambda)d\lambda}{\int_{380}^{740} y(\lambda)s(\lambda)d\lambda}$$

$$Z = \frac{\int_{380}^{740} z(\lambda)r(\lambda)s(\lambda)d\lambda}{\int_{380}^{740} y(\lambda)s(\lambda)d\lambda}$$

$x(\lambda), y(\lambda), z(\lambda)$ are the colour matching functions. r(λ) represents the reflectance function, s(λ) represents the d65 illuminant. The visible spectrum includes wavelengths between 380 and 740 nanometers.

Here the reflectance functions, colour matching functions and the illuminant function, all are sampled at a wavelength of 1 nanometer. So, the integration problem can be discretized well.

$$X = \frac{\sum_{380}^{740} x(\lambda)r(\lambda)s(\lambda)}{\sum_{380}^{740} y(\lambda)s(\lambda)}$$

$$Y = \frac{\sum_{380}^{740} y(\lambda)r(\lambda)s(\lambda)}{\sum_{380}^{740} y(\lambda)s(\lambda)}$$

$$Z = \frac{\sum_{380}^{740} z(\lambda)r(\lambda)s(\lambda)}{\sum_{380}^{740} y(\lambda)s(\lambda)}$$

X_{white}, Y_{white} and Z_{white} are calculated using the above formulae using a reflectance function having value 1 at every point.

$$X_{reference} = \frac{\sum_{380}^{740} x(\lambda)s(\lambda)}{\sum_{380}^{740} y(\lambda)s(\lambda)}$$

$$Y_{reference} = \frac{\sum_{380}^{740} y(\lambda)s(\lambda)}{\sum_{380}^{740} y(\lambda)s(\lambda)}$$

$$Z_{reference} = \frac{\sum_{380}^{740} z(\lambda)s(\lambda)}{\sum_{380}^{740} y(\lambda)s(\lambda)}$$

X_{white}=10584.7789, Y_{white}=11170.2829, and Z_{white}=12010.7828

Normalizing by dividing X_{white}, Y_{white} and Z_{white} by Y_{white}, we get $X_{reference}$, $Y_{reference}$ and $Z_{reference}$.

$X_{reference}$=0.9505, $Y_{reference}$=1, and $Z_{reference}$=1.089

Now, for any one of the four different types of special reflectance functions with at most two transitions(Type A, Type B, Type C or Type D), stated above, and varying λ_1 and λ_2, we get a particular X,Y and Z.

Now, $\hat{X}, \hat{Y}$ and $\hat{Z}$ are defined as:

$\hat{X} = X\frac{X_{reference}}{X_{white}}$

$\hat{Y} = Y\frac{Y_{reference}}{Y_{white}}$

$\hat{Z} = Z\frac{Z_{reference}}{Z_{white}}$

$\hat{X}, \hat{Y}$ and $\hat{Z}$ are converted to X_{linear}, Y_{linear} and Z_{linear} using a matrix multiplication.

$$\begin{bmatrix} X_{linear} \\ Y_{linear} \\ Z_{linear} \end{bmatrix} = \begin{bmatrix} 3.2406 & -1.5372 & -0.4986 \\ -0.9689 & 1.8758 & 0.0415 \\ 0.0557 & -0.204 & 1.057 \end{bmatrix} \begin{bmatrix} \hat{X} \\ \hat{Y} \\ \hat{Z} \end{bmatrix}$$

The set lower and higher gray settings are respectively m=0.112 and M=0.819. Now, applying the inverse gamma function,

$\gamma^{-1}(g) = \frac{g}{12.92}$, if $g \leq 0.04045$
$\gamma^{-1}(g) = (\frac{g+0.055}{1.055})^{2.4}, ifg > 0.04045$

$\gamma^{-1}(m)$=0.0119, $\gamma^{-1}(M)$=0.636

Now, X_{linear}, Y_{linear} and Z_{linear} are rescaled to the range [$\gamma^{-1}(m)$,$\gamma^{-1}(M)$]= [0.112,0.819] to obtain $\bar{X}, \bar{Y}$ and $\bar{Z}$

$\bar{X} = 0.0119 + .6241 \times X_{linear}$

$\bar{Y} = 0.0119 + .6241 \times Y_{linear}$

$\bar{Z} = 0.0119 + .6241 \times Z_{linear}$

The final step is to pass $\bar{X}, \bar{Y}$ and $\bar{Z}$ through a non linear function for performing gamma correction and this gives the sRGB values.

$\gamma(w) = 12.92w$, if $w \leq 0.0031308$

$\gamma(w) = 1.055w^{\frac{1}{2.4}} - 0.055w$, if $w \leq 0.0031308$

$R = \gamma(\bar{X})$, $G = \gamma(\bar{Y})$ and $B = \gamma(\bar{Z})$

5.4 Finding Points where vectors from midGray point Intersect the Optimal Colour Solid

The midGray Point is defined as (0.4655, 0.4655, 0.4655). A particular direction can be represented by an ordered triplet (x,y,z). The optimal colour solid is converted into a triangulated mesh and for a particular point, it can assumed as being inside or outside the triangle by considering the surface normal direction. The MATLAB inpolyhedron function can test if a particular three dimensional point is inside a triangulated mesh[75].

Data: Polymesh, Input direction dir, Stepsize α

Result: Intersection point of ray starting from midGray point along a given direction dir

```
i=0;
while i ≤ 1 do
    currentPoint=midGray+dir×iα;
    if inpolyhedron(OptimalMesh.faces,OptimalMesh.vertices,currentPoint)==0 then
        return midGray+dir×iα;
        break;
    else
        continue;
    end
end
i=i+α;
```

Algorithm 1: Calculating intersection points

Polymesh is the triangulated mesh of the optimal colour solid obtained. The above algorithm is run for each of the different 26 directions. By changing the step size, the polygon intersection point can be determined more accurately.

5.5 Background Intersections

The background intersections represent the variability of colours in the Voronoi diagram backgrounds. The colours in the Voronoi background polygons come from the sampled natural histogram of colours.

For the background intersection, the following steps were undertaken:

- 100 different background images were generated for each direction.
- From each background image, the sampled colour points of background Voronoi polygons were obtained. A convex hull of all those colour points in RGB Space, which is a subspace of the whole RGB Cube, was generated.
- For each direction, the intersection of the vector from middle gray point in that direction with that polygon.

Chapter 6

Results

The results of the experiments are divided into various sections, as described. The first section displays how the choice of the minimum and maximum gray levels was made. The second section is about the optimal colour solid and its position in the whole RGB colour cube. The third section presents the results of the user studies and the main findings regarding the perceptual colour solid and optimal colour solid.

6.1 Finding the Minimum and Maximum Gray Levels

The minimum and maximum gray levels were determined experimentally to determine 26 different directions, which allowed a movement from mid- gray to the periphery of the colour solid without the colour getting saturated. For a particular minimum and maximum gray level pair, if the colour got saturated before reaching the periphery of the RGB Colour cube, the particular pair were discarded. For instance, for the particular direction (-0.5345, 0.5345, 0.5345), the colour got saturated before reaching the periphery of the RGB Colour cube along that direction from mid-gray, with minimum and maximum gray level pair being {0,1}. So, the particular {0,1} maximum and minimum gray level pair was discarded.

m	0	0	0.1	0.1	0.112	0.19	0.17	0.21	0.32
M	1	0.9	0.85	0.82	0.819	0.8	0.73	0.71	0.66
(0.5345,-0.5345,-0.5345)	N	N	Y	Y	Y	Y	Y	Y	Y
(0.5345,0.5345,-0.5345)	N	N	N	Y	Y	N	N	N	Y
(-0.5345,0.5345,-0.5345)	Y	N	--	N	Y	--	--	--	Y
(-0.5345,-0.5345,-0.5345)	N	N	--	--	Y	--	--	--	N
(0.5345,0,-0.5345)	N	N	--	--	Y	--	--	--	--
(0,-0.5345,-0.5345)	--	--	--	--	Y	--	--	--	--
(0,0.5345,-0.5345)	--	--	--	--	Y	--	--	--	--
(-0.5345,0,-0.5345)	--	--	--	--	Y	--	--	--	--
(0.5345,-0.5345,0.5345)	--	--	--	--	Y	--	--	--	--
(0.5345,0.5345,0.5345)	--	--	--	--	Y	--	--	--	--
(-0.5345,0.5345,0.5345)	--	--	--	--	Y	--	--	--	--
(-0.5345,-0.5345,0.5345)	--	--	--	--	Y	--	--	--	--
(-0.5345,0,0.5345)	--	--	--	--	Y	--	--	--	--
(0,-0.5345,0.5345)	--	--	--	--	Y	--	--	--	--
(0.5345,0,0.5345)	--	--	--	--	Y	--	--	--	--
(0,0.5345,0.5345)	--	--	--	--	Y	--	--	--	--
(0,0.5345,0)	--	--	--	--	Y	--	--	--	--
(0,-0.5345,0)	--	--	--	--	Y	--	--	--	--
(-0.5345,0,0)	--	--	--	--	Y	--	--	--	--
(0.5345,0,0)	--	--	--	--	Y	--	--	--	--
(-0.5345,-0.5345,0)	--	--	--	--	Y	--	--	--	--
(0.5345,0.5345,0)	--	--	--	--	Y	--	--	--	--
(-0.5345,-0.5345,0)	--	--	--	--	Y	--	--	--	--
(-0.5345,0.5345,0)	--	--	--	--	Y	--	--	--	--
(0,0,0.5345)	--	--	--	--	Y	--	--	--	--
(0,0,-0.5345)	--	--	--	--	Y	--	--	--	--

Table 6.1: Minimum and Maximum Gray Level Calculations

Table 6.1 shows the calculations for various gray level pairs and if the colour along a particular direction gets saturated before reaching the RGB Colour cube periphery.

Mid-Gray Calculations

In table 6.1, m and M in the first 2 rows represent the various minimum and maximum gray levels considered namely: {0,1},{0,0.9},{0.1,0.85}, {0.1,0.82},{0.112,0.819}, {0.19,0.8},{0.17,0.73}, {0.21,0.71} and {0.32,0.66}

The ordered triplets in the first column are the directions along which colour saturation is tested. For a particular direction (p,q,r), the expression that determines the way along which a person can reach the periphery of the RGB Colour cube from midGray point:

(midGray, midGray, midGray)+$\alpha \times (p, q, r)$

If α =1, then one is at the periphery of the RGB Colour Cube. In the above table, for a particular (m,M) pair, a 'Y' represents that for that direction, one can reach the periphery of the RGB Colour cube without the colour getting saturated. This is done for 26 different directions for an (m,M) pair. If along a particular column, an 'N' is reached meaning that colour gets saturated before reaching the RGB colour cube periphery, calculations are stopped for that column. A double dash '__' is used to indicate that no further calculations are done for that particular column, as 'N' is already an entry in that column, the idea being finding an (m,M) pair for which colour does not saturate along all the 26 selected directions. As evident from the table, for m=0.112 and M=0.819, for all 26 directions, an optimal colour can be reached at the periphery of the colour cube. So, midGray=(0.112+0.819)=0.4655 and the midGray point is (0.4655,0.4655,0.4655).

6.2 Optimal Colour Solid Intersections

The whole optimal colour solid developed using the colour matching functions, d65 illuminant and optimal colour reflectance functions, is shown below:

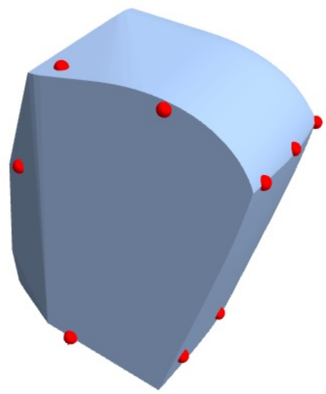


Figure 6.1: Intersection Points Some of the intersection points represented by red spheres. The vectors in 26 different directions all originate from the midGray point (0.4655, 0.4655, 0.4655). For each test direction, when the vector from midGray point intersects the surface of the optimal colour solid, the intersection point is a red sphere in this diagram.

Direction	POI	Direction	POI
(0.5345,-0.5345,-0.5345)	(0.85,0.076,0.0761)	(0,-0.5345,-0.5345)	(0.4655,0.0577,0.057)
(0.5345,0.5345,-0.5345)	(0.82,0.824,0.1063)	(0,0.5345,-0.5345)	(0.4655,0.8376,0.093)
(-0.5345,0.5345,-0.5345)	(0.10,0.830,0.1005)	(-0.5345,0,-0.5345)	(0,0.465,0)
(-0.5345,-0.5345,-0.5345)	(0.11,0.111,0.1114)	(0.5345,-0.5345,0.5345)	(0.8369,0.0941,0.836)
(0.5345,0,-0.5345)	(0.93,0.465,0)	(0.5345,0.5345,0.5345)	(0.8134,0.8134,0.813)

Direction	POI	Direction	POI
(-0.5345, 0.5345,0.5345)	(0.1110,0.8200,0.8200)	(0,0.5345,0.5345)	(0.4655,0.8229,0.8229)
(-0.5345, -0.5345,0.5345)	(0.0849,0.0849,0.8461)	(0,0.5345,0)	(0.4655,0.8450,0.4655)
(-0.5345, 0,0.5345)	(0.0740,0.4655,0.857)	(0,-0.5345,0)	(0.4655,0,0.4655)
(0,-0.5345,0.5345)	(0.4655,0.0804,0.8506)	(-0.5345,0,0)	(0,0.4655,0.4655)
(0.5345,0,0.5345)	(0.8462,0.4655,0.8462)	(0.5345,0,0)	(0.9664,0.4655,0.4655)

Direction	POI
(0.5345,-0.5345,0)	(0.8924,0.0386,0.4655)
(0.5345,0.5345,0)	(0.8225,0.8255,0.4655)
(-0.5345,-0.5345,0)	(0.0428,0.0428,0.4655)
(0,0,0.5345)	(0.4655,0.4655,0.8538)
(0.5345,0,0.5345)	(0.4655,0.4655,0)

Table 6.2: Intersection points, POI

For the direction (0.5345, -0.5345, -0.5345), where Algorithm 1 listed in section 5.4 gives Stepsize=0.7285, the intersection point is calculated as:

$(0.4655, 0.4655, 0.4655) + 0.7285 \times (0.5345, -0.5345, -0.5345)$=(0.8549,0.0761,0.0761)

The rest 25 different intersection points are tabulated in table 6.2. POI stands for the point of the intersection.

6.3 Voronoi Diagram Settings

As a part of the experiment, participants had to tune the settings of a Voronoi diagram so that they could increase or decrease the middle polygon's colour intensity in the Voronoi diagram. They had to determine the threshold at which the middle polygon's colour became saturated and looked striking out as compared to the background colours.

For a particular trial in a block, one had to increase or decrease the intensity of a polygonal area in a Voronoi diagram until the colour looked striking out as compared to surroundings. Here, the setting value for each stimulus number shows the percentage along a vector from the midGray point (0.4655,0.4655,0.4655) to the outer periphery of the full RGB Cube where the participant thinks that the colour gets saturated and seems more brilliant as compared to the colours in the background Voronoi cells. Here 2 different trial results for participant 1 are listed. The rest of the settings can be found in Appendix B.

Participant 1 settings

Trial 1

Stimulus no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Setting	51	86	79	90	90	73	75	75	59	94	78	59	88	92	96	97	72	98

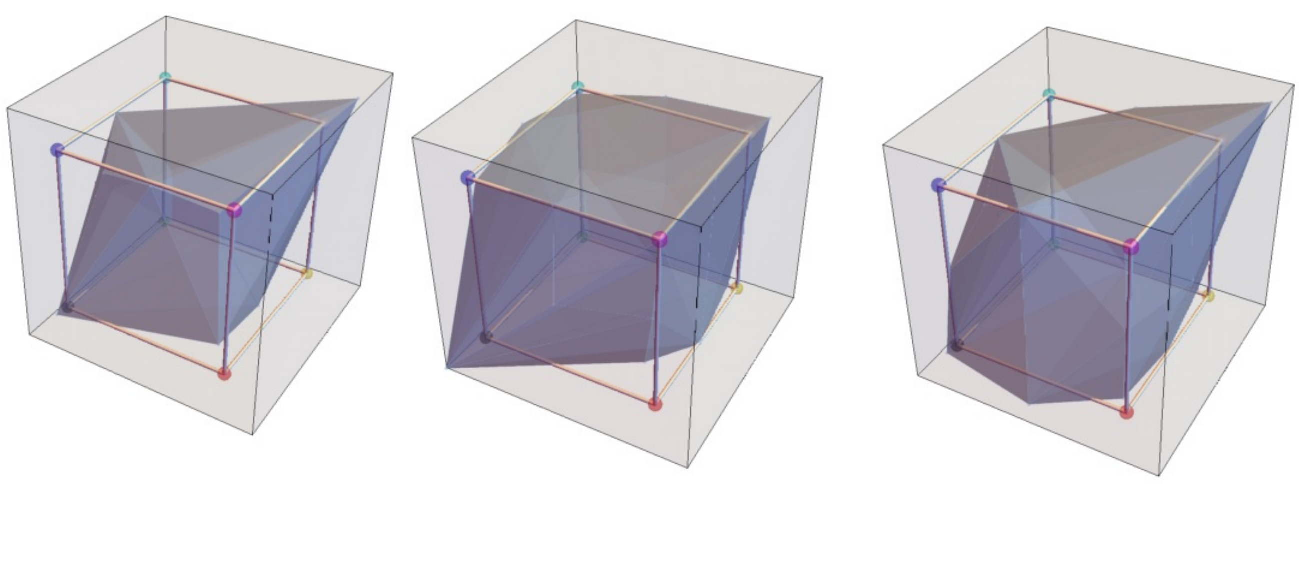
Stimulus no.	19	20	21	22	23	24	25	26
Setting	99	78	51	94	60	91	93	85

Trial 2

Stimulus no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Setting	41	41	25	52	48	72	79	86	75	80	58	77	89	81	76	70	71	84

Stimulus no.	19	20	21	22	23	24	25	26
Setting	94	84	69	85	33	64	90	80

6.3.1 Perceptual Colour Solids



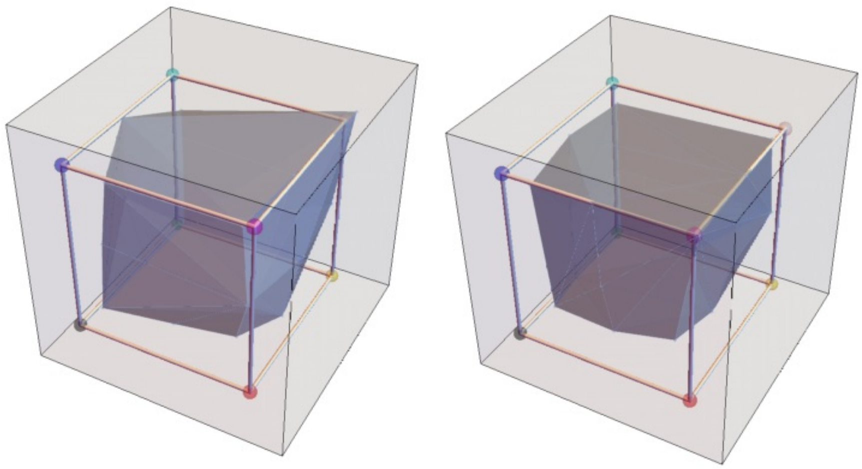


Figure 6.2: Perceptual Colour Solids These colour solids are developed from the median settings of the participants. The five different solids show the perceptual limits of colour vision for the various participants.

6.3.2 Median Settings

In this section, the median settings of different participants are compared to each other. For each participant, a histogram that records the ratios of their median setting for a direction to the median of all participants' median settings in that direction, is plotted.

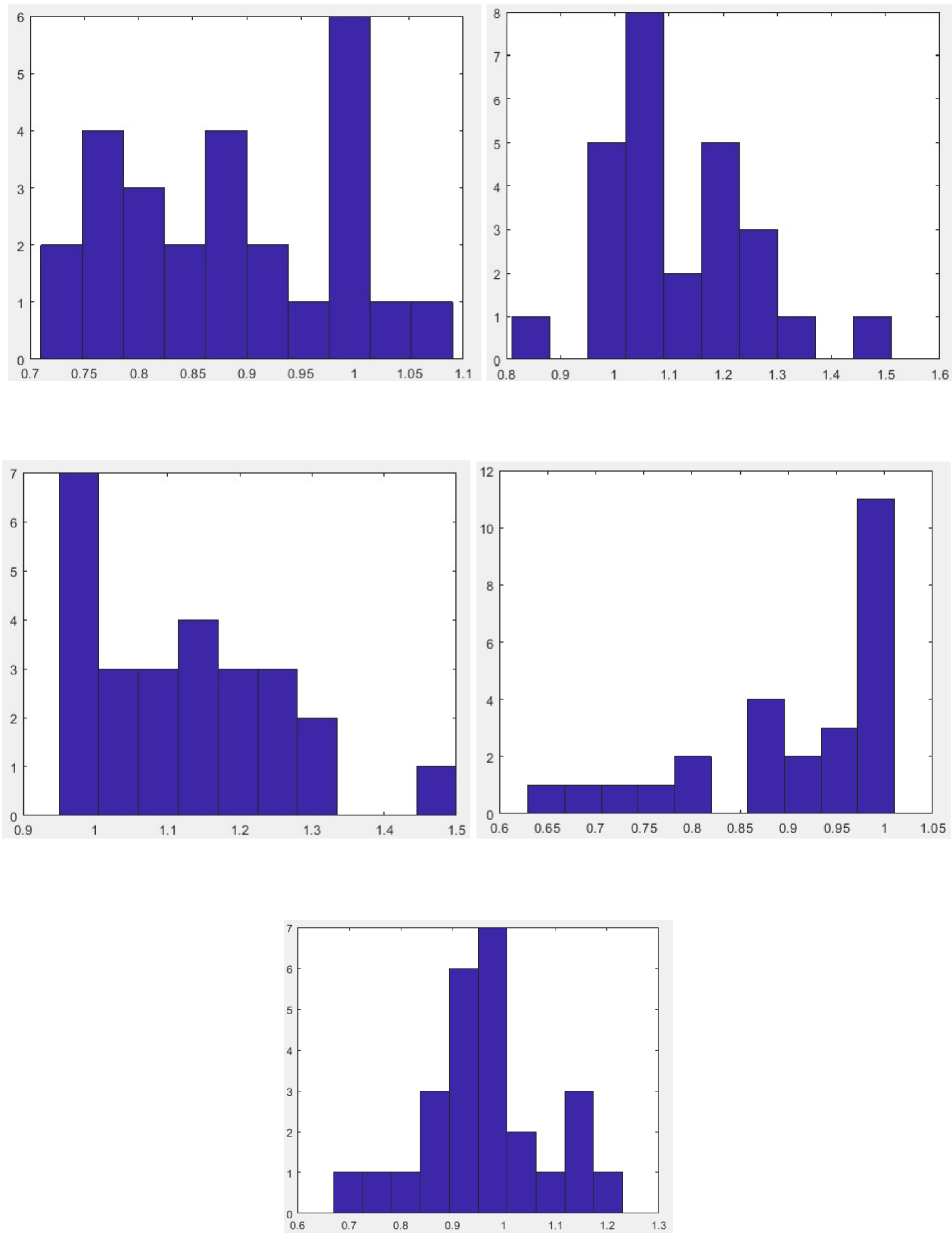


Figure 6.3: Median Settings Histogram These histograms depict the ratio of a participant's median setting to the median of all median settings for 26 different directions

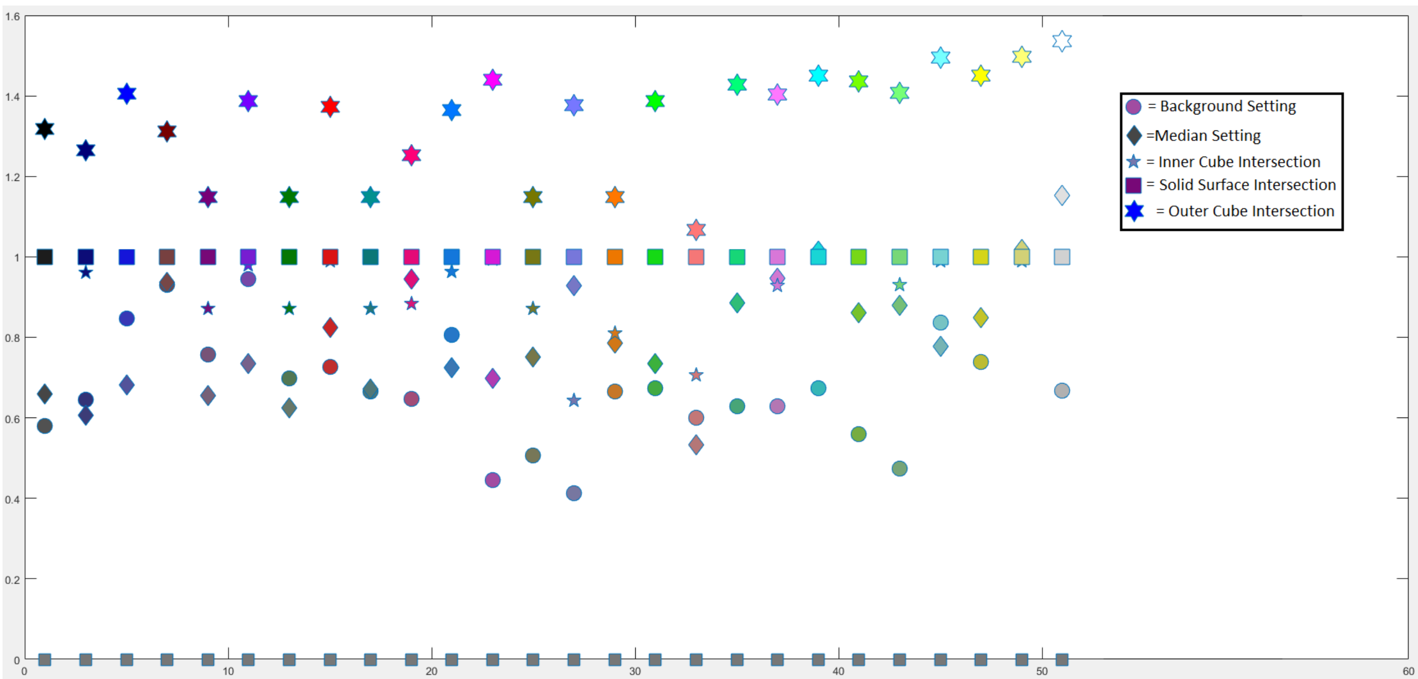


Figure 6.4: Median Settings The plot shows the median setting for a participant over the five experiments, alongside the intersection point of the vector from midGray point in 26 different directions with the background polygon created from background Voronoi colours, the whole outer RGB Cube, the inner RGB cube based on the low and high Gray settings(0.112 and 0.819) and the optimal solid surface.The colours are arranged in the increasing order of lightness following the formula: 0.3R+0.59G+0.11B, where R,G abd B are the red, green and blue components of a particular colour[76]. In general, for lighter colours towards the right end of the graph, participants chose comparatively higher settings.

6.3.3 Background Colour Polyhedrons, Median settings polyhedron and Optimal Colour Solid

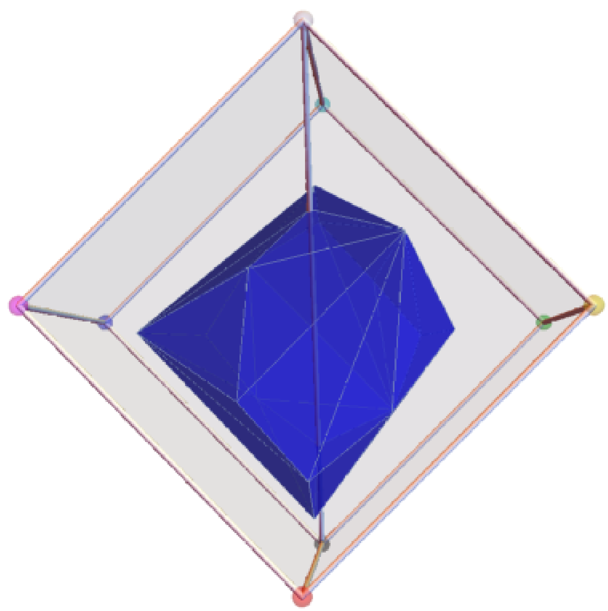


Figure 6.5: Polyhedron representing background colours The polyhedron represents the variability of colours in the Voronoi diagram background polygons. The colours in the Voronoi background polygons come from the sampled natural histogram of colours. From each background image, the sampled colour points of background Voronoi polygons were obtained. A convex hull of all those colour points in RGB Space, which is a subspace of the whole RGB Cube, is shown here in blue.

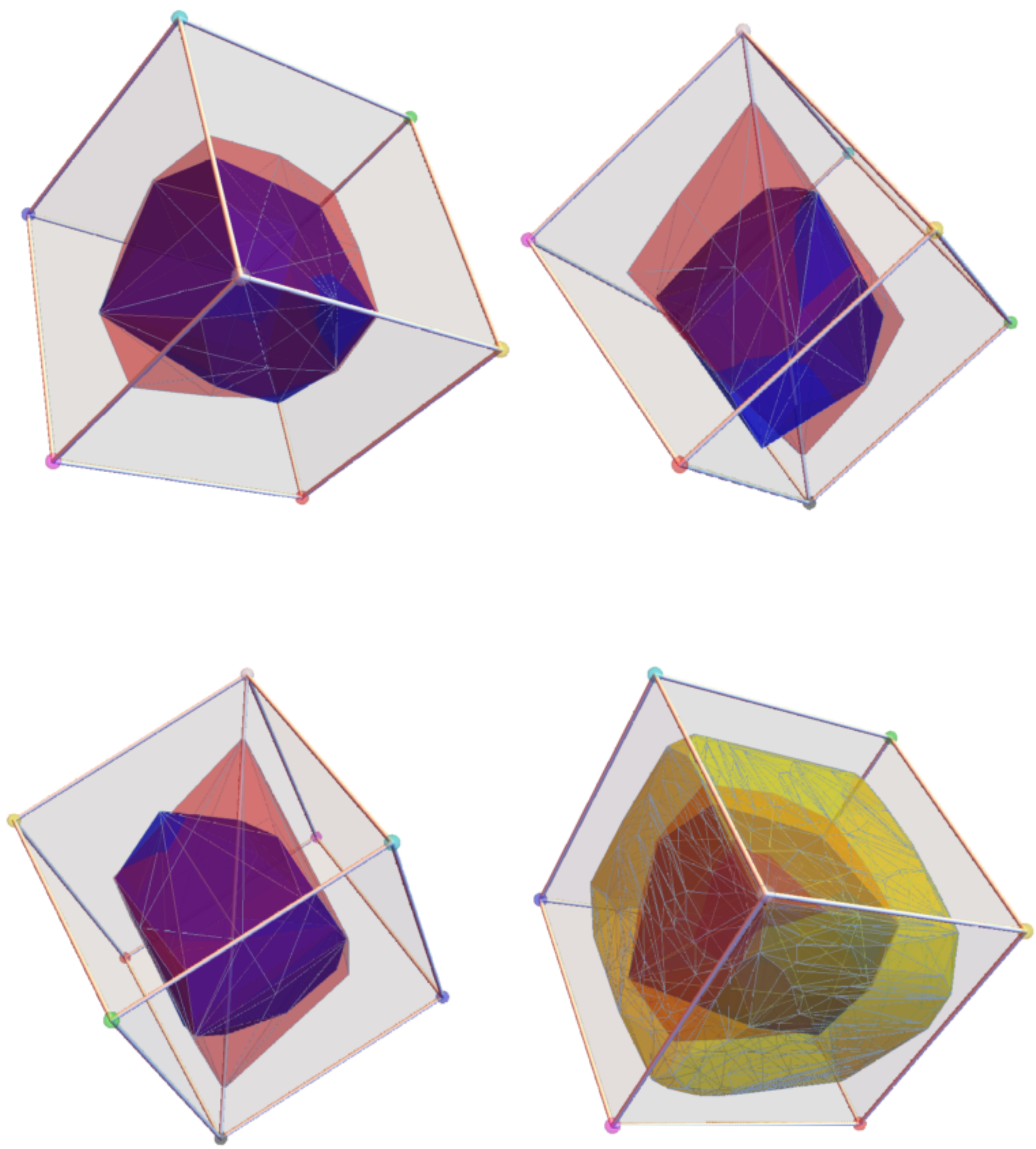


Figure 6.6: Median settings polyhedron, Background colours polyhedron and Optimal Colour solid The background polygon in blue obtained by plotting colours sampled from background colours is contained within the red polyhedron representing median settings. The fineness of the shape of the blue polygon can be bettered by taking more Voronoi diagrams per direction and sampling even more colours per direction. The figure at bottom right shows the Optimal colour solid in yellow, median settings polyhedron in red and the background colour polyhedron in light black.

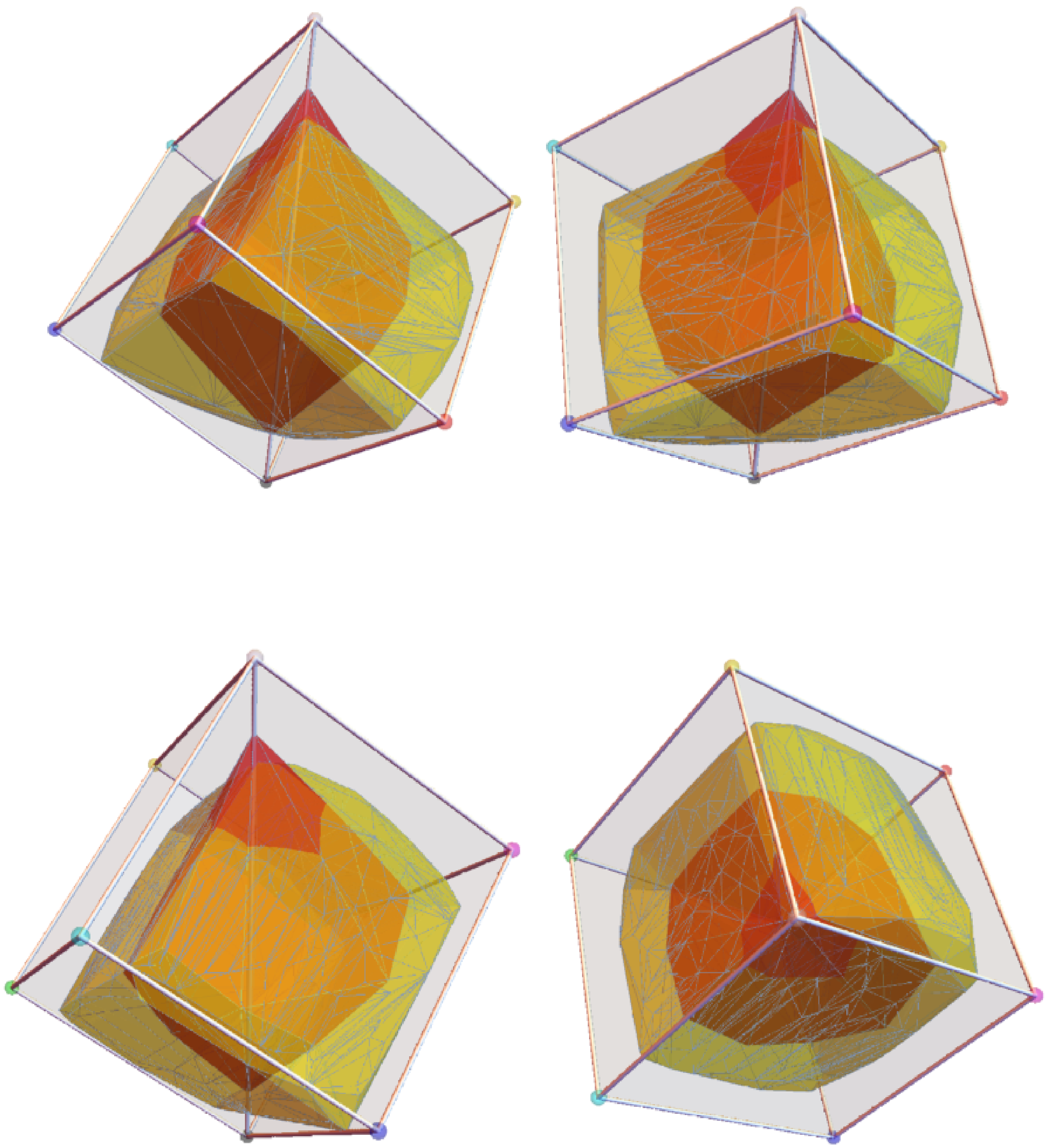


Figure 6.7: Optimal colour solid alongside polyhedron representing median colour settings of participants The yellow transparent solid represent the optimal colour cube and the red solid represents the convex hull of the median settings of the participants. The outer cube is the cube representing the whole RGB colour space with the spheres representing the colours at those points.

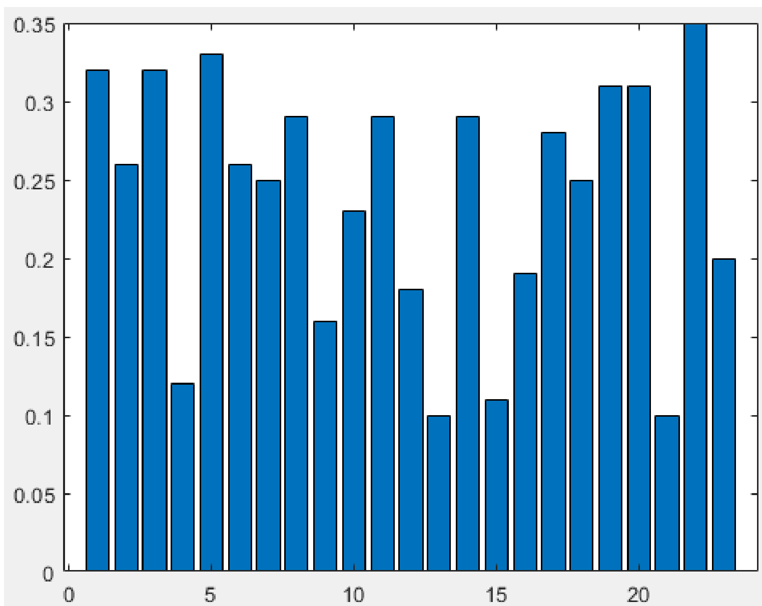


Figure 6.8: Radial Shrinkage Percentage The bars display the percentage of shrinkage in the direction of radial chromaticity. The optimal colour solid and perceptual colour solid look roughly the same but perceptual colour solid looks like a shrunk version.

Chapter 7

Discussion

This chapter explains the results presented in the previous chapter in detail and presents the corresponding analysis and insights. Also, the thoughts on how any improvements are possible and if any alternative ways of experimentation are possible, are presented.

7.1 Median Settings Across Participants

The perceptual colour solids developed from the median settings of the participants show the variability in sensitivity to optimal colours across the participants. On average, participants show a really high median setting for white colour and the black median setting is almost similar to the optimal colour solid mark for black colour. The direction connecting black and white is the one in which the average colour solid is most stretched out. A neat comparison of how the median settings of the participants fare against each other is presented in the median histograms in diagram 6.3.

For each participant, the histogram records ratios of their median setting for a direction to the median of all participants' median settings in that direction. If for a particular histogram, the total number of entries in the bins other than 1 is vast, that means the participant is mostly choosing a very high or shallow setting, as compared to others. The histograms show that two of the participants have consistently low ratios for the median settings, about 69.2% and 57% of the time. On the other hand, two participants show mostly high median settings.

The graph in figure 7.1 gives a far detailed colour profiling of the participants. In general, for lighter colours towards the right end of the graph, participants chose comparatively higher settings.

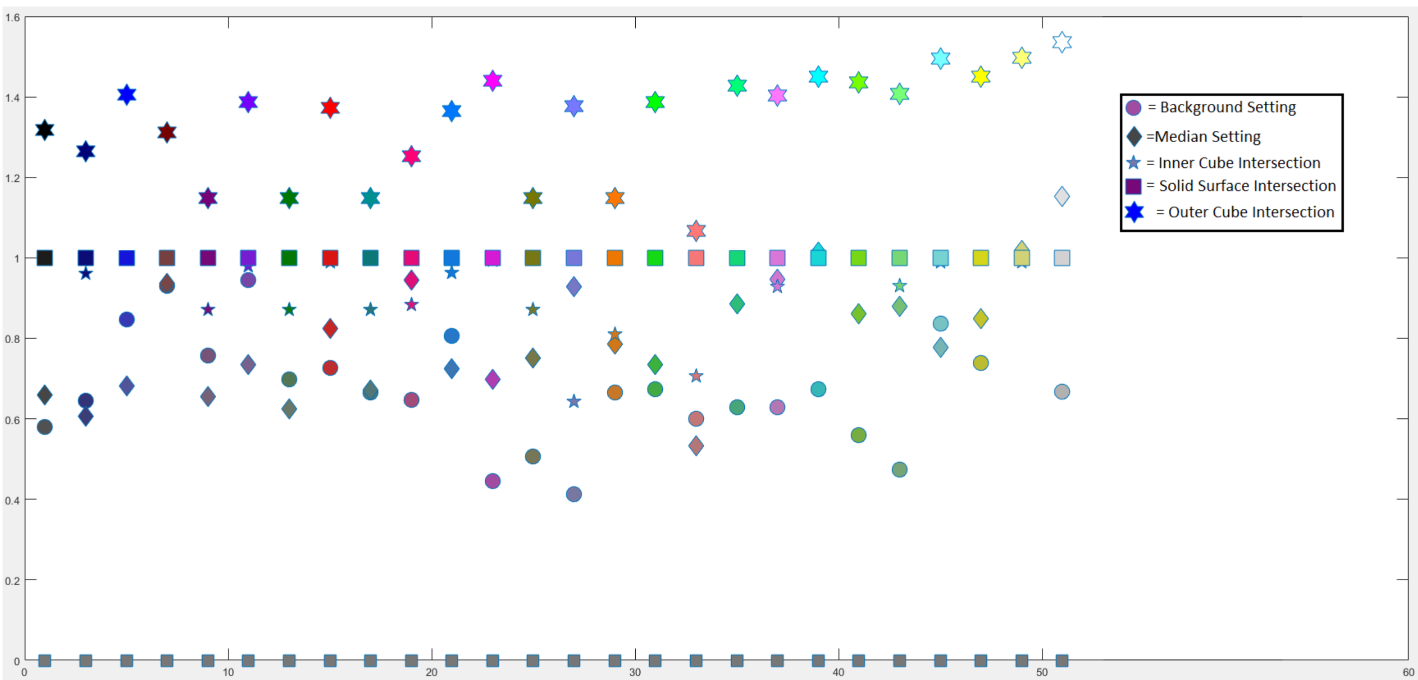


Figure 7.1: Median Settings The plot shows the median setting for a participant over the five experiments, alongside the intersection point of the vector from midGray point in 26 different directions with the background polygon created from background Voronoi colours, the whole outer RGB Cube, the inner RGB cube based on the low and high Gray settings(0.112 and 0.819) and the optimal solid surface.The colours are arranged in the increasing order of lightness following the formula: 0.3R+0.59G+0.11B, where R,G abd B are the red, green and blue components of a particular colour[76].

The colour change in a given direction can be perceived in detail in the following diagrams. These are 5 Voronoi diagrams that contain five special highlighted polygons in black outline.

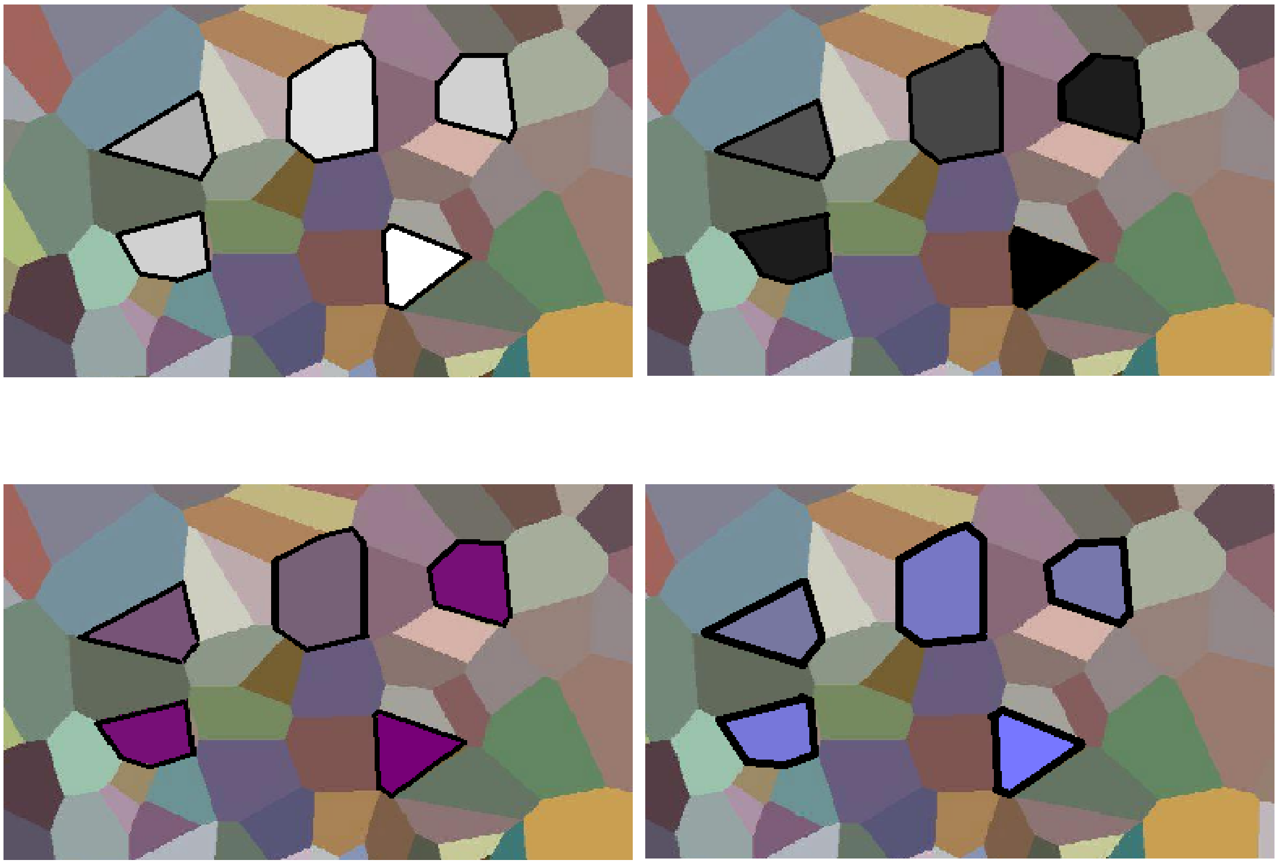


Figure 7.2: Colour Shades according to Participant Settings The purest colours are at the surface of the RGB Colour solid. The shades go from gray to the purest along a direction. Here four different directions are taken and the changing shades are shown.

Figure 7.2 contains 4 Voronoi diagrams, each with 5 highlighted polygons. In each of the diagrams, the middle polygon at the top row shows the shade of the particular white, black, purple or lighter shade of blue where the participant thinks that the colour is saturated and seems overwhelming and striking out compared to the unhighlighted polygons. The other polygons describe other settings: the leftmost polygon in the top row describe the colour shade at the background polygon intersection, the rightmost polygon at the top row represents the colour at the point of intersection of the vector from midGray point (0.4655,0.4655,0.4655) with the cube with diagonal connecting lowGray point (0.112,0.112,0.112) and highGray point (0.819,0.819,0.819), the inner cube. The left polygon at the bottom row represents the shade of colour at the point of intersection of the vector from midGray with the optimal colour solid and the rightmost polygon at the bottom represent the best shade of colour at the surface of the RGB colour solid.

7.2 A Juxtaposition of the Median Settings Polyhedron and Optimal Colour Solid

The polyhedron representing the median settings of the participants is mostly contained within the optimal colour solid of the participants, as evident from figure 6.7. The perceptual colour solid is shrunk radially, and median settings of the participants cross the optimal colour solid only in the direction of the white colour. The bar graph in figure 6.8 shows the radial shrinkage percentage in different directions.

The optimal colour solid is similar in shape to the perceptual psychological colour solid of the participants, but the optimal colour solid is expanding more. So, the perceptual visual limits of participants are in line with the optimal colour solid limits with a radially inward shrinkage. This suggests that the participant colour settings are agreeing to the optimal colour limits, although with a constant shrinkage in the radial chromaticity direction.

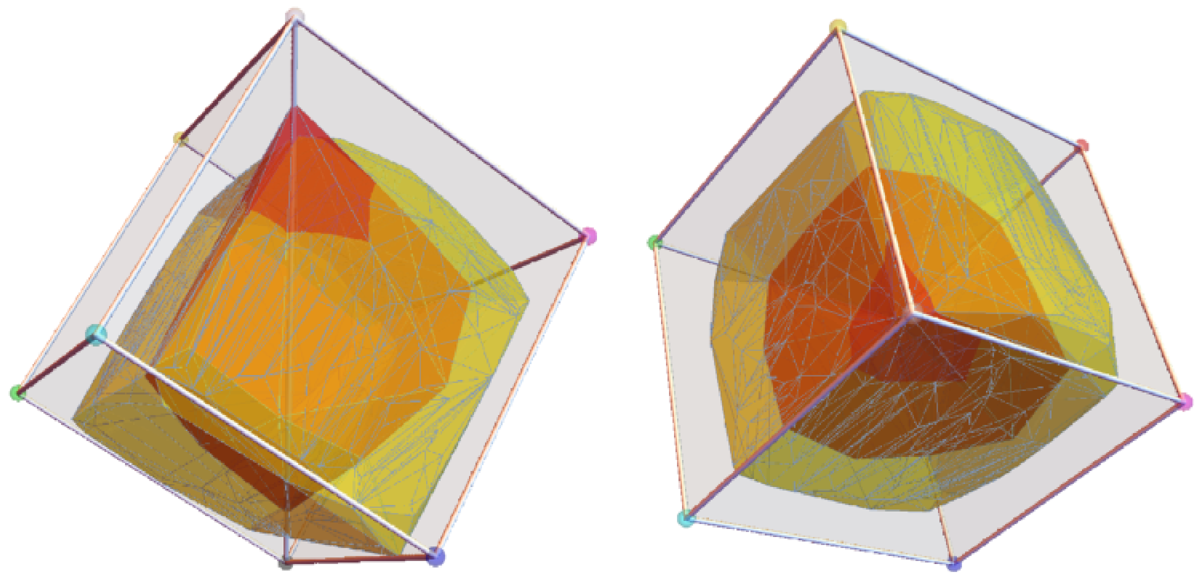


Figure 7.3: Optimal colour solid alongside polyhedron representing median colour settings of participants The yellow transparent solid represent the optimal colour cube and the red solid represents the convex hull of the median settings of the participants. The outer cube is the cube representing the whole RGB colour space with the spheres representing the colours at those points.

Figure 7.3 displays the perceptual colour solid inside the optimal colour solid. By merely observing the two solids, one can deduce the fact that it is not very difficult to transform the orange solid into the red solid. In the black and white direction, represented by the tube in figure 7.3, the red solid overshoots the yellow one, near the white direction. However, apart from that, the red solid is contained and a simple shearing and stretching in directions radially outwards from the centre, the red solid can be transformed into yellow.

The distribution of background colours clearly affects the stimulus reaction of the participants. It might be interesting to repeat the experiment with a larger sampling of background colours and study the corresponding response of the participants. If the experiment could have been done in a broader time frame, the same process can be repeated with a larger pool of participants, a more refined and generalized result can be obtained.

Also, as women generally see more shades of colour than men[77], it could be interesting to take that into effect too. If one closely sees the perceptual colour solid and the optimal colour solid together, the more exciting developments take place towards the extreme tapered ends of the solid, near the black and white colours. So, a more in-depth study around the direction of white light where the overshoot occurs and also the black region where the limits are almost same as the optimal colour solid could be seen as an extension to this experiment in the future. There might be something interesting about the ends near black and white, and thus it could be a good reference point for future work.

Chapter 8

Conclusion

The dissertation focuses on studying the perceptual limits of human colour vision. The research aim was to create a three dimensional colour solid that captures the perceptual limits of surface colour. On the basis of the experimental data obtained from five participants, a thorough comparison of the psychological colour space and optimal colour space under ideal conditions has been made. The research in the areas of optimal colours has been quite extensive but the connection of optimal colours with the perceptual psychological space is not that prevalent. The dissertation addresses this by presenting the connection between the optimal colour space and the perceptive psychological colour space and developing a three dimensional map of both for comparison.

The results show that the perceptual colour space varies quite a bit among the participants. Two of the participants chose comparatively high colour settings and two chose lower colour settings. Despite the fact that the perceptual colour space varies, there are a few trends that follow from the colour choice of participants. The perceptual colour space on an average is a subspace of the optimal colour space with shrinkage in the radial chromaticity direction. The percentage of shrinkage in the radial direction goes up to 35%.

The median responses of the participants are always not as high as the optimal conditions suggest, but for the lighter colours, participants tend to have generally higher settings. The perceptual colour limits are contained within the ideal limits. By eyeballing the optimal and the perceptual colour solid, it can be seen that the perceptual colour solid can be stretched radially outwards to be morphed into the optimal colour solid as the two solids are structurally quite similar. The interesting parts of the solid are near the polar areas of the black and white colours. The perceptual colour solid actually overshoots the optimal colour solid in the direction of white colour, the only region of the solid, in fact, where this phenomenon occurs. On the other hand, the perceptual colour solid almost matches the optimal limits in the direction of the black colour.

A more refined result can be obtained by finer sampling, taking even more directions across the colour space and repeating all the calculations. With the availability of more time, it would be possible to induct a larger number of participants, and it would be possible to generalize the results even more. From the experimental results, it can be concluded that the perceptual colour space varies significantly among individuals and the average perceptual limits of human vision are within ideal optimal colour limits. The dissertation scrutinizes the perceptual colour space by comparing it with the optimal colour space. For further experimentation with the perceptual colour space, studying the perceptual colour solid in the direction of the black and white colour directions could be a good starting point as these were the interesting findings in the solid structure. Also, taking a larger sampling of background colours can develop a refined background colour solid and how it affects the median responses of participants and the perceptual colour solid can be worth pondering upon.

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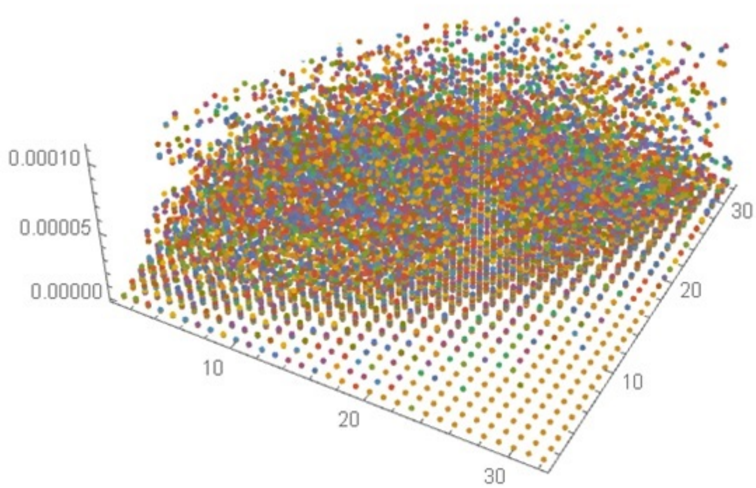
Appendix A

Natural Histogram of Colours

The ListPlot3D command in Mathematica gives a three dimensional view of values in an array. Scattering of colours in the natural colour histogram is shown below:



Appendix B

Voronoi Diagram Settings

Participant 1 settings

Trial 3

Stimulus no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Setting	90	60	41	42	51	48	48	79	85	71	58	67	51	50	66	76	43	54

Stimulus no.	19	20	21	22	23	24	25	26
Setting	64	83	43	45	47	52	85	54

Trial 4

Stimulus no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Setting	91	87	64	96	83	62	63	52	76	56	70	70	49	71	92	90	79	98

Stimulus no.	19	20	21	22	23	24	25	26
Setting	46	92	69	87	58	63	90	70

Trial 5

Stimulus no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Setting	49	62	56	85	99	43	63	55	51	72	69	59	96	74	51	61	86	72

Stimulus no.	19	20	21	22	23	24	25	26
Setting	46	82	94	48	94	82	78	92

Table B.1: Participant 1 Settings

Participant 2 settings

Trial 1

Stimulus no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Setting	62	26	42	72	77	78	55	44	30	36	51	42	67	70	98	74	65	95

Stimulus no.	19	20	21	22	23	24	25	26
Setting	76	96	71	86	83	95	92	83

Trial 2

Stimulus no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Setting	50	58	37	58	69	72	54	53	65	99	65	49	72	86	89	68	77	84

Stimulus no.	19	20	21	22	23	24	25	26
Setting	52	80	75	99	67	81	74	88

Trial 3

Stimulus no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Setting	73	76	52	70	61	86	50	67	52	88	68	48	73	82	99	91	80	70

Stimulus no.	19	20	21	22	23	24	25	26
Setting	77	96	62	55	79	72	99	85

Trial 4

Stimulus no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Setting	60	61	52	71	80	87	70	75	73	42	43	45	55	74	61	51	67	52

Stimulus no.	19	20	21	22	23	24	25	26
Setting	69	72	44	45	62	56	90	82

Trial 5

Stimulus no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Setting	64	61	50	79	61	81	57	74	65	91	55	52	66	81	84	87	65	55

Stimulus no.	19	20	21	22	23	24	25	26
Setting	91	78	65	79	65	84	66	92

Table B.2: Participant 2 Settings

Participant 3 settings

Trial 1

Stimulus no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Setting	41	79	53	55	69	48	42	56	51	61	54	35	44	41	60	51	41	56

Stimulus no.	19	20	21	22	23	24	25	26
Setting	63	44	63	57	59	49	42	45

Trial 2

Stimulus no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Setting	58	62	32	54	71	46	48	44	41	64	53	50	78	55	60	74	56	53

Stimulus no.	19	20	21	22	23	24	25	26
Setting	72	89	85	96	52	47	98	90

Trial 3

Stimulus no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Setting	71	50	57	64	60	86	62	85	67	62	78	71	84	65	90	66	91	82

Stimulus no.	19	20	21	22	23	24	25	26
Setting	80	79	61	92	52	62	91	46

Trial 4

Stimulus no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Setting	49	31	28	67	54	64	38	60	51	79	47	47	77	43	73	43	44	55

Stimulus no.	19	20	21	22	23	24	25	26
Setting	53	61	45	54	50	40	42	58

Trial 5

Stimulus no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Setting	31	46	45	48	45	51	49	50	58	93	45	58	54	43	53	51	63	49

Stimulus no.	19	20	21	22	23	24	25	26
Setting	90	95	59	68	82	59	66	83

Table B.3: Participant 3 Settings

Participant 4 settings

Trial 1

Stimulus no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Setting	52	48	59	73	67	58	47	71	52	89	46	67	63	55	50	57	42	67

Stimulus no.	19	20	21	22	23	24	25	26
Setting	75	77	65	51	42	59	81	66

Trial 2

Stimulus no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Setting	36	50	36	67	50	48	52	54	45	66	46	44	75	56	49	45	48	48

Stimulus no.	19	20	21	22	23	24	25	26
Setting	70	52	47	36	39	41	51	53

Trial 3

Stimulus no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Setting	65	38	50	71	71	63	58	64	45	71	51	46	69	83	55	41	51	64

Stimulus no.	19	20	21	22	23	24	25	26
Setting	71	61	49	66	59	52	81	72

Trial 4

Stimulus no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Setting	64	46	50	69	62	66	46	47	46	77	48	38	68	46	58	61	48	44

Stimulus no.	19	20	21	22	23	24	25	26
Setting	68	49	42	55	44	42	69	54

Trial 5

Stimulus no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Setting	57	49	41	78	52	48	48	51	44	89	32	48	55	58	68	51	53	67

Stimulus no.	19	20	21	22	23	24	25	26
Setting	75	45	54	62	40	35	67	44

Table B.4: Participant 4 Settings

Participant 5 settings

Trial 1

Stimulus no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Setting	68	47	93	62	65	42	56	55	94	51	69	54	66	86	94	63	39	59

Stimulus no.	19	20	21	22	23	24	25	26
Setting	61	79	39	64	42	64	51	90

Trial 2

Stimulus no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Setting	41	47	56	53	65	63	74	74	90	69	92	57	69	70	92	57	49	61

Stimulus no.	19	20	21	22	23	24	25	26
Setting	85	60	78	87	46	51	66	92

Trial 3

Stimulus no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Setting	28	37	56	45	40	58	56	77	88	76	66	73	35	90	69	40	71	77

Stimulus no.	19	20	21	22	23	24	25	26
Setting	55	64	43	61	59	60	58	90

Trial 4

Stimulus no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Setting	45	45	59	51	51	56	42	66	65	89	46	55	66	58	63	79	91	65

Stimulus no.	19	20	21	22	23	24	25	26
Setting	73	76	56	87	52	57	81	78

Trial 5

Stimulus no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Setting	48	40	47	40	42	31	59	58	50	51	48	54	54	65	68	49	49	63

Stimulus no.	19	20	21	22	23	24	25	26
Setting	66	62	50	39	53	60	69	60

Table B.5: Participant 5 Settings

Appendix C

Code Listing

At first Voronoi diagrams were created using Python and then these images were processed in MATLAB for generating the stimuli for the experiment. For each individual trial, 2600 images were generated in total for 26 directions and there were 5 trials per participant.

Seed images used for stimuli generation:

MATLAB code for generating stimuli from Voronoi diagrams:

```

gmtest=0;
%seed images courtesy python
cd('\\images\1\');
for ij=1:100
    %aa=imread('C:\Users\hpw\Desktop\check\vornois\rr.jpg');
    aa=imread('6.jpeg');%there are 26 seed images for 26 directions
    aa=im2double(aa);
```

```
    %%
    gw=0.112;
    gb=0.819;
    w=gw*(1-gb);
    lmat=[];
    ind=1;
    v=[-0.5345, 0., -0.5345];%direction is represnted by this matrix v

    c=[663.9864 665.3802 709.9828 718.3457 737.8593 755.9791 729.4964];

    r=[383.1062 359.4111 317.5962 317.5962 324.5653 377.5309 419.3457 ];
    BW = roipoly(aa,c,r);


    imwrite(av11(aa,0.112,.819,gmtest,1,v,BW),sprintf('%d.jpg',ij-1));
    %%
    gmtest=gmtest+0.01;
end

function dost = av11(im,gb,gw,gm,lam,v,BW)
%%
im=double(im);
%%
w=gw*(1-gb);
for i=1:size(im,1)
    for j=1:size(im,2)
        im(i,j,:)=gb+w*im(i,j,:);
    end
end
for i=1:size(im,1)
```

```
    for j=1:size(im,2)
        if BW(i,j)==1
            tem=0.4655+v*gm;
            im(i,j,:)=gb+w*tem;
        end
    end
end
%imwrite(im,'C:\Users\hpw\Desktop\check\usefulImages\1.jpg');
dost=im;
%dost=double(im);
%

end
```

The code for generating optimal colour solid is listed below. The code and data files can be also be found in "https://github.com/jojo96/Perceptual-limits-of-human-color-vision". The colour matching functions and d65 illuminant sampled values are listed in an excel file. A snapshot of the file is given below. The column ind has the wavelength values, the points at which the functions are sampled. The columns x, y and z contain the values of the colour matching functions at the corresponding sampled points, and the d65 column contains the sampled values of the d65 illuminant.

ind			x	y	z		d65	
300			3.77E-03	4.15E-04	1.85E-02		54.6482	
301			4.53E-03	5.03E-04	2.22E-02		57.4589	
302			5.45E-03	6.08E-04	2.67E-02		60.2695	
303			6.54E-03	7.34E-04	3.21E-02		63.0802	
304			7.84E-03	8.84E-04	3.85E-02		65.8909	
305			9.38E-03	1.06E-03	4.61E-02		68.7015	
306			1.12E-02	1.27E-03	5.51E-02		71.5122	
307			1.33E-02	1.50E-03	6.58E-02		74.3229	
308			1.59E-02	1.78E-03	7.82E-02		77.1336	
309			1.88E-02	2.10E-03	9.28E-02		79.9442	
310			2.21E-02	2.45E-03	1.10E-01		82.7549	
311			2.60E-02	2.85E-03	1.29E-01		83.628	
312			3.04E-02	3.30E-03	1.51E-01		84.5011	
313			3.54E-02	3.80E-03	1.76E-01		85.3742	
314			4.11E-02	4.35E-03	2.05E-01		86.2473	
315			4.74E-02	4.97E-03	2.37E-01		87.1204	
....								
736			2.22E-06	8.89E-07	0.00E+00		59.1637	
737			2.10E-06	8.39E-07	0.00E+00		59.4509	
738			1.98E-06	7.91E-07	0.00E+00		59.7381	
739			1.87E-06	7.47E-07	0.00E+00		60.0253	
740			1.76E-06	7.05E-07	0.00E+00		60.3125	

```python
import pandas as pd
b=pd.read_csv(r'wrk.csv')
b=b[0:440]

rr=b['x']#the c
gg=b['y']
bb=b['z']
val=b['d65']
g=(b['ind'])
ind=0
xsum=0
```

```
ysum=0
zsum=0
intgr=[]
xlist=[]
ylist=[]
zlist=[]
colmat=[]#color matching functions
d65=[]

for i in range(1,440):
    h['val'])
    xlist.append(rr[ind])
    ylist.append(gg[ind])
    zlist.append(bb[ind])
    colmat.append(list([xlist[ind],ylist[ind],zlist[ind]]))
    ind=ind+1


darr=np.array(b['d65'])
np.array(d65)

amat = [[3.2406, -1.5372, -0.4986],
     [-0.9689, 1.8578, 0.0415],
    [0.0557, -0.204, 1.057]]
amat=np.array(amat)#
darr=darr[0:439]


prod=darr*ref
gk=(np.transpose(colmat)@prod)

```

```
xw=10584.77890613
yw=11170.28292098
zw=12010.78264773

xr=0.9505
yr=1
zr=1.0890


ref=0*np.ones(439)
for i in range(0,5):
    #ref[0:i]=1
    #ref=0.5*ref
    prod=darr*ref
    gk=(np.transpose(colmat)@prod)
    gk[0]=(xr*gk[0])/xw
    #print((xr*gk[0])/xw)
    gk[1]=(yr*gk[1])/yw
    gk[2]=(zr*gk[2])/zw
    z=amat@gk
    #print(z)
    z[0]=0.0119+0.6241*z[0]
    z[1]=0.0119+0.6241*z[1]
    z[2]=0.0119+0.6241*z[2]

    for k in range(0,3):
        if z[k]<0.0031308:
            z[k]=12.92*z[k]
        else:
            z[k]=1.055*pow(z[k],1/2.4)-0.055

```

```
    print(z)


ref=np.ones(439)
for i in range(0,439):
    ref=np.zeros(439)
    ref[0:i]=1
    #ref=0.5*ref
    prod=darr*ref
    gk=(np.transpose(colmat)@prod)
    gk[0]=(xr*gk[0])/xw
    #print((xr*gk[0])/xw)
    gk[1]=(yr*gk[1])/yw
    gk[2]=(zr*gk[2])/zw
    z=amat@gk
    #print(z)
    z[0]=0.0119+0.6241*z[0]
    z[1]=0.0119+0.6241*z[1]
    z[2]=0.0119+0.6241*z[2]

    for k in range(0,3):
        if z[k]<0.0031308:
            z[k]=12.92*z[k]
        else:
            z[k]=1.055*pow(z[k],1/2.4)-0.055

    print(list(np.clip(z,0,1)))


g=[]
for i in range(0,439):
```

```
    for j in range(0,439):
        ref=np.zeros(439)
        if i<j:
            ref[i:j]=1
            prod=darr*ref
            gk=(np.transpose(colmat)@prod)
            gk[0]=(xr*gk[0])/xw
            #print((xr*gk[0])/xw)
            gk[1]=(yr*gk[1])/yw
            gk[2]=(zr*gk[2])/zw
            z=amat@gk
            #print(z)
            z[0]=0.0119+0.6241*z[0]
            z[1]=0.0119+0.6241*z[1]
            z[2]=0.0119+0.6241*z[2]
            for k in range(0,3):
                if z[k]<0.0031308:
                    z[k]=12.92*z[k]
                else:
                    z[k]=1.055*pow(z[k],1/2.4)-0.055
            g.append(list(np.clip(z,0,1)))


with open(r'points.txt', 'w') as f:
    for item in g:
        f.write("\n%s" % item)



g1=[]
for i in range(0,439):
```

```
    for j in range(0,439):
        ref=np.ones(439)
        if i<j:
            ref[i:j]=0
            prod=darr*ref
            gk=(np.transpose(colmat)@prod)
            gk[0]=(xr*gk[0])/xw
            #print((xr*gk[0])/xw)
            gk[1]=(yr*gk[1])/yw
            gk[2]=(zr*gk[2])/zw
            z=amat@gk
            #print(z)
            z[0]=0.0119+0.6241*z[0]
            z[1]=0.0119+0.6241*z[1]
            z[2]=0.0119+0.6241*z[2]
            for k in range(0,3):
                if z[k]<0.0031308:
                    z[k]=12.92*z[k]
                else:
                    z[k]=1.055*pow(z[k],1/2.4)-0.055
            g1.append(list(np.clip(z,0,1)))

with open(r'points2.txt', 'w') as f:
    for item in g1:
        f.write("\n%s" % item)



ref=np.ones(439)
for i in range(0,439):
    ref=np.ones(439)
    ref[0:i]=0
```

```
    #ref=0.5*ref
    prod=darr*ref
    gk=(np.transpose(colmat)@prod)
    gk[0]=(xr*gk[0])/xw
    #print((xr*gk[0])/xw)
    gk[1]=(yr*gk[1])/yw
    gk[2]=(zr*gk[2])/zw
    z=amat@gk
    #print(z)
    z[0]=0.0119+0.6241*z[0]
    z[1]=0.0119+0.6241*z[1]
    z[2]=0.0119+0.6241*z[2]

    for k in range(0,3):
        if z[k]<0.0031308:
            z[k]=12.92*z[k]
        else:
            z[k]=1.055*pow(z[k],1/2.4)-0.055

    print(list(np.clip(z,0,1)))
```

The optimal solid obtained using the code given earlier was saved in a file named ace.stl and the STL file was processed in MATLAB to obtain intersection of a ray starting from midGray point along a given direction and the optimal solid.

```

fv = stlread('ace.stl');
grey=[.4655,.4655,.4655];
dir1=[-0.5345, -0.5345, -0.5345];

for i=1:100
```

```
    a1 = grey + (i/100)*dir1;
    if(inpolyhedron(fv.faces,fv.vertices,a1)==0)
        break;
    end
end
```

Radiance codes In Radiance , geometric objects can be created simply by following the syntax[78]:

Modifier type name

Examples:

To create a plastic, one can define:

void plastic plas

0 //string arguments (none here)

0 //integer parameters

5 7 .05 .05 0 .5 //5=number of parameters, (r,g,b)=(7 .05 .05), specularity =0, roughness=0.5

Then to create a red sphere,

plas sphere ball //creating a plastic sphere named ball of material plas created above

0

0

4 .1937 1.125 .565 // radius=4, coordinates of centre=(.1937 1.125 .565)

Optimal Colour Solid Intersections

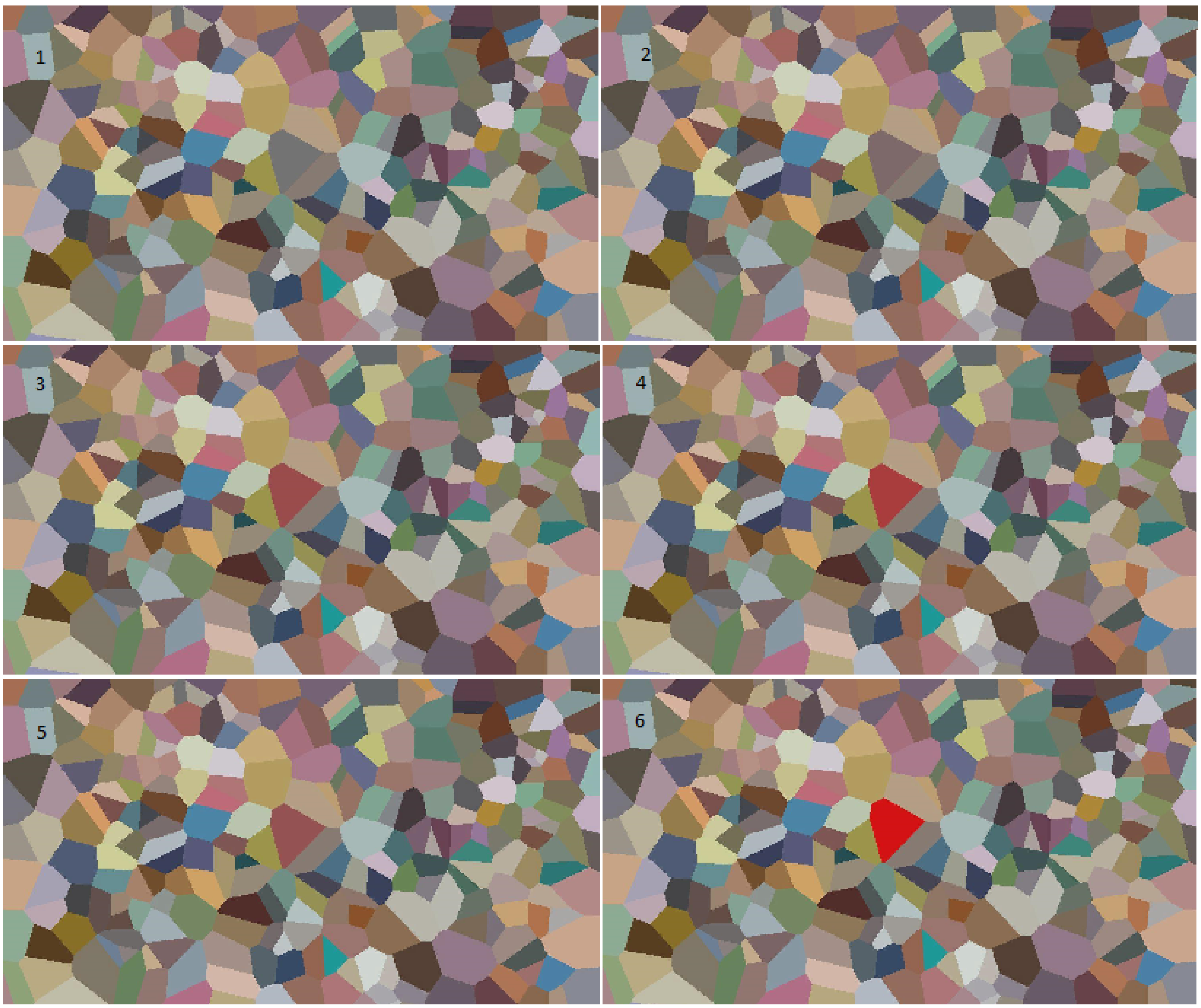
Algorithm for calculating intersection points

Data: Polymesh, Input direction dir, Stepsize α

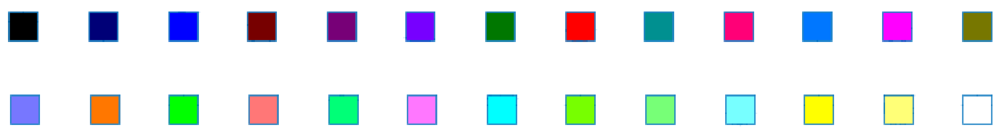
Result: Intersection point of ray starting from midGray point along a given direction dir

```
i=0;
while i ≤ 1 do
    currentPoint=midGray+dir×iα;
    if inpolyhedron(OptimalMesh.faces,OptimalMesh.vertices,currentPoint)==0
     then
        return midGray+dir×iα;
        break;
    else
        continue;
    end
end
i=i+α;
```

Experiment



The six images here show a progression where the colour of the central polygon changes from gray to deep red. A participant would see 100 of these images with the change in colour from gray to final colour(in this case red) being more gradual. When the participant thinks that the colour of the central polygon is saturating, they would stop. If they stop, say at the 73rd image, the median setting would be .73. A participant has to do 26 trials in an experimental block for 26 different directions or 26 different end colours.



These are the 26 different end colours. So, a participant had to do an experiment 6 times. the first data were discarded and the rest 5 recorded. For, each colour, the median of the five recodrdings is the median setting for a a participant. The median settings data are listed in Appendix B.