Solution to QCQI

JOJO

2 Introduction to quantum mechanics

Exercise 2.1

Answer:
Since

$$(1,-1)+(1,2)-(2,1)=0$$

Thus $(1,-1),(1,2)$ and $(2,1)$ are linearly dependent.

Exercise 2.2

Answer:
$A=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$ w.r.t. the input basis $|0\rangle, |1\rangle$ and output basis $|0\rangle, |1\rangle$. If we take output basis as $\frac{|0\rangle+|1\rangle}{\sqrt{2}}, \frac{|0\rangle-|1\rangle}{\sqrt{2}}$, matrix representation of A is

$$A=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$$

Exercise 2.6

Answer:

$$\begin{aligned}(\sum_i \lambda_i |w_i\rangle, |v\rangle) &= (|v\rangle, \sum_i \lambda_i |w_i\rangle)^* \\ &= (\sum_i \lambda_i(|v\rangle, |w_i\rangle))^* \\ &= \sum_i \lambda_i^*(|v\rangle, |w_i\rangle)^* \\ &= \sum_i \lambda_i^*(|w_i\rangle, |v\rangle)\end{aligned}$$

Exercise 2.8

Answer:

Since $\| \left| v_k \right\rangle \| = 1$, we only need to prove that $\langle v_k | v_j \rangle = \delta_{kj}$.

$$\langle v_1 | v_2 \rangle = 0$$

Assume $\left| v_k \right\rangle$ is orthogonal to all $\left| v_i \right\rangle, 1 \leq i < k$. Thus $\forall 1 \leq i \leq k$

$$\begin{aligned}
\langle v_i | v_{k+1} \rangle &= \frac{\langle v_i | w_{k+1} \rangle - \sum_{i'=1}^{k} \langle v_i' | w_{k+1} \rangle \langle v_i | v_{i'}' \rangle}{\| \left| w_{k+1} \right\rangle - \sum_{i=1}^{k} \langle v_i | w_{k+1} \rangle \left| v_i \right\rangle \|} \\
&= \frac{\langle v_i | w_{k+1} \rangle - \sum_{i'=1}^{k} \langle v_i' | w_{k+1} \rangle \delta_{ii'}}{\| \left| w_{k+1} \right\rangle - \sum_{i=1}^{k} \langle v_i | w_{k+1} \rangle \left| v_i \right\rangle \|} = 0
\end{aligned}$$

By induction, $\left| v_{k+1} \right\rangle$ is orthogonal to all $\left| v_i \right\rangle, 1 \leq i \leq k$. Q.E.D.